TOWARDS A FICTIONALIST NOMINALISM

A Dissertation

Submitted to the Graduate School
of the University of Notre Dame
in Partial Fulfillment of the Requirements
for the Degree of

Doctor of Philosophy

by

Kenneth Boyce

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Michael Rea, Director

Graduate Program in Philosophy
Notre Dame, Indiana
July 2013
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Abstract

by

Kenneth Boyce

My dissertation has two aspects. On the one hand, it is a contribution to a first-order ontological dispute. I proceed by offering, over the course of four chapters, a sustained defense of nominalism (the view that there are no abstract objects). On the other hand, it is an exercise in metaontology. In particular, it is an attempt to undermine the framework for conducting ontological disputes handed down to us by Quine and his successors.

Neo-Quinean objectors to nominalism argue (to a first approximation) that we seem antecedently committed to endorsing various sentences that can be shown to imply that abstract objects exist. They further argue that there is no way that we can do without endorsing many of those sentences. We can’t, they argue, find workable, nominalistically friendly paraphrases for all of them; nor can we simply withdraw our assent (not without greatly impoverishing our ability to express various truths). It follows, they claim, that we are (on pain of logical inconsistency) “committed” to the existence of abstract objects.

I show how nominalists can successfully resist such arguments. I argue that we
can consistently talk and reason as if there were abstract objects without believing in them, and that we can do so in the absence of a successful paraphrase strategy. Accordingly, the approach that I take could aptly be described as a “fictionalist” one.

In Chapter 1, I show how to resist empirical arguments against nominalism that are advanced on the part of some neo-Quineans. In Chapter 2, I consider and reject an argument put forward by Peter van Inwagen (a prominent neo-Quinean) for the conclusion that we are committed to believing in properties. In Chapter 3, I develop a novel, formal, nominalist account of the inferential utility of reasoning as if there are monadic, first-order properties. Finally, in Chapter 4, I argue that the considerations raised in the previous chapters generalize so as to encompass other domains of our discourse that neo-Quineans claim commit us to the existence of abstract objects.
This dissertation is dedicated to my wife Sarah. Without her constant love and support, I surely would have never made it this far.
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ACKNOWLEDGMENTS

It is customary, I am told, to include in the acknowledgement section of a dissertation a list of those individuals to whom one has become indebted throughout one’s graduate student career. I am in no position to do that – not because I have no one to thank, but because I have far too many, and no means of choosing some over others.

I must, however, acknowledge the enormous debt that I owe to my wife, Sarah Boyce. She did not sign up for a husband who would spend the first eleven years of their marriage in graduate school. Nevertheless, things worked out that way. And she has been nothing but supportive and encouraging. I could not have done any of this without her. And without her, it would not have been worth doing.

I would also like to take this opportunity to thank my parents, David and Janet Boyce, for all of their love and support over the years.

Otherwise, I will limit the scope of my acknowledgements to those who in one way or another directly contributed to the writing of this dissertation. I must, of course, thank my advisor, Michael Rea, as well as my committee members Alvin Plantinga, Jeff Speaks, Meghan Sullivan, and Peter van Inwagen, for all of their input, mentorship, and support. I should also thank Robert Audi, Amelia Hicks, Graham Leach-Krouse, Bradley Rettler, Aaron Segal, and Patrick Todd for having read and offered comments on portions of the dissertation. I have also had enormously helpful conversations concerning the issues raised in the dissertation with Andrew Bailey, Josh Rasmussen, and Philip Swenson. I apologize to those whom I have failed to mention.
CHAPTER 1:
AGAINST EMPIRICAL ARGUMENTS FOR PLATONISM

1.1 Empirical Arguments for Platonism?

One of the most widely influential of the contemporary arguments for the existence of abstract entities is the so-called “Quine-Putnam indispensability argument.” Proponents of this argument maintain that we have strong empirical considerations, stemming from science, in favor of the existence of abstract entities. In particular, they maintain that the pervasiveness of mathematics in our best scientific theories, and its apparent indispensability for the purposes of adequately formulating those theories, afford us with strong empirical grounds for believing in the existence of mathematical entities.

There are various ways in which the Quine-Putnam indispensability argument for the existence of mathematical entities has been developed in the contemporary literature. Some of these have attempted to remain faithful reconstructions of the original versions of that argument.\(^1\) Others have simply been concerned with how to best develop it.\(^2\) I will not address, in this chapter, any interpretive issues pertaining to how Quine or Putnam developed the original versions of this argument.\(^3\) Nor will I attempt to address specifically each of the contemporary versions thereof. Instead, I will address what I take to be the most forceful anti-nominalist considerations put forward by the proponents of

\(^1\) (Quine 1969) and (Putnam 1979) are often taken to be classical sources for this line of argument.

\(^2\) See (Colyvan 2001) for one of many contemporary presentations of this line of argument.

\(^3\) See (Liggins 2008) for a challenge to the claim that Quine and Putnam actually put forward the sort of argument that has come to be labeled “the Quine-Putnam indispensability argument.”
that argument, and I will argue that these considerations fail to effectively undermine nominalism.

As Susan Vineberg has pointed out, nothing like the Quine-Putnam indispensability argument is likely to carry any freight with those who already deny scientific realism.\(^4\) Those who are already skeptical of whether considerations stemming from science afford us with good empirical grounds for believing in things like *atoms* are not likely to be moved by arguments for the conclusion that considerations stemming from science afford us with good empirical grounds for believing in things like *numbers*! For that reason, I take the best versions of the Quine-Putnam indispensability argument to be aimed at those who already have a strong penchant for scientific realism. And I think that the best way to advance the considerations put forward by the proponents of that argument is not to advance them in the form of an *argument* at all, but rather, in the form of a *challenge* to those who would accept scientific realism but deny the existence of mathematical entities. This challenge is described in the next section.

### 1.2 Scientific Realism but not Mathematical Realism?

Scientific realists believe that our best scientific theories are to be interpreted at face value (rather than instrumentally or in some other non-literal or non-factive way). They also believe that the empirical support we have for those theories gives us good reason to accept their implications regarding concrete unobservables.\(^5\)\(^6\) Similarly, mathematical

\(^4\) (Vineberg 1996, p. S259)

\(^5\) Scientific realism is thus opposed to views like that of van Fraassen’s (1980) constructive empiricism.
realists believe that our established mathematical theories (or at least a significant subset of them) are to be taken at face value and that we ought to accept their implications (including those involving existential quantification over mathematical entities).  

Scientific realists who do not wish to be mathematical realists face a challenge. It seems that our justification for believing in unobservable entities like quarks and neutrons primarily stems from the fact that their existence is implied by our best, most well-confirmed scientific theories. But our best scientific theories are shot through with mathematics and thereby also imply the existence of mathematical entities. Doesn’t epistemological consistency demand, then, that those who accept the existence of the concrete unobservables of science accept the existence of mathematical entities as well? As I indicated in the previous section, I take the strongest considerations raised by proponents of the Quine-Putnam indispensability argument to boil down to a version of this challenge.

The scientific realist who rejects mathematical realism might try to respond to the above challenge by attempting to carry out something like Hartry Field’s program. That is, she might attempt to show that our best scientific theories can be nominalized (i.e. reformulated in such a way that the reformulations dispense with the mathematical portions of the original theories while retaining all of their non-mathematical implications). Field himself tries to exhibit the feasibility of this program by offering a  

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6 Here I am following Leng’s (2005, p. 82) advice by characterizing scientific realism in such a way as not to render it incompatible with mathematical anti-realism (see also (Leng 2010, pp.11-12)). I also do not claim to have given a precise characterization of scientific realism. As I see it, ‘scientific realism’ denotes a family of views, rather than a single, precise thesis.

7 I hereby stipulate that part of what it is for a mathematical theory to be taken “at face value” is for its quantifiers to be given an objectual interpretation.
nominalistically acceptable reformulation of Newton’s theory of gravitation. It is controversial, however, whether the nominalization strategy that Field applies to Newton’s theory of gravitation can be successfully extended to contemporary scientific theories such as the general theory of relativity and quantum mechanics. And since the jury is still out when it comes to the success of Field’s program, the scientific realist who rejects mathematical realism would do well to cultivate additional strategies.

An alternative strategy would involve, not eliminating the mathematical portions of our best scientific theories, as Field attempts to do, but reinterpreting those portions so that they no longer imply the existence of mathematical entities. A scientific realist who takes this route might attempt, as philosophers such as Charles Chihara and Geoffrey Hellman have done, to show that our established mathematical theories (or at least the ones that are used in science) can be reformulated so as to eliminate quantification over mathematical entities. Such a strategy, as adopted by the scientific realist, however, threatens to be double-edged sword, one that might also be wielded against her by the scientific anti-realist. Once such “deviant” interpretations of the apparent quantification

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8 See (Field 1980) and (Field 1989). Another part of Field’s program (though one that is not as directly relevant here) is to attempt to show that the mathematical theories used in our best scientific theories are (what he calls) “conservative,” where (to a first approximation) a mathematical theory is conservative if and only if it is such that, when it is conjoined with any nominalistically acceptable theory, the resulting conjunction has all and only the same nominalistically acceptable consequences as does the original nominalistically acceptable theory. This part of Field’s program is aimed at helping to explain why the mathematics employed by our best scientific theories is instrumentally useful, even though ultimately dispensable.

9 See (Urquhart 1990) for reasons to doubt that Field’s strategy can be successfully applied to the theory of general relativity. See (Malament 1982) for reasons to doubt that it can be successfully applied to quantum mechanics, (Balaguer 1998b, chapter 6) for a response, and (Bueno 2002) for a counter response.

10 For an extremely helpful survey of many of the different kinds of strategies that are available here, one to which I am indebted for the present discussion, see (Burgess and Rosen 1997).

11 See (Hellman 1993), (Chihara 1990), and (Chihara 2007).
over mathematical entities found in our best scientific theories are permitted, why not similarly deviant interpretations of the apparent quantification over concrete unobservables found within those theories? On what principled grounds is the scientific realist able to insist that the apparent implications of our best scientific theories pertaining to the existence of concrete unobservables are to be accepted at face value, while denying that the same is true of the apparent implications of those theories that pertain to the existence of mathematical entities? In short, we are back to the challenge with which we began.

Elliott Sober, though not a scientific realist himself, suggests a line of response to this challenge of which scientific realists who reject mathematical realism might want to avail themselves. Sober has noted that there appears to be an asymmetry between the way in which claims about concrete unobservables are related to empirical observation and the way in which the claims of pure mathematics are so related. More specifically, he has noted that while we are often willing to take claims about concrete unobservables as capable of being disconfirmed by various empirical observations, we are often not willing to take pure mathematical claims as being disconfirmed by those same sorts of observations. This fact, Sober argues, indicates that pure mathematical claims are also not confirmed (at least not typically) by empirical observation. Sober’s point, if correct,

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12 Worries along these lines are suggested by some remarks of Burgess and Rosen (1997, pp. 61-63), as well as by the discussion found in (Colyvan 1999, pp. 4-5) and (Colyvan 2001, pp. 76-78).

13 A similar worry could be raised regarding Field’s program. If we are prepared to accept reformulations of our best scientific theories that eliminate the mathematical portions thereof, why shouldn’t we also be prepared to accept reformulations of those theories that eliminate quantification over concrete unobservables? See (Hawthorne 1996), however, for a powerful way in which a proponent of Field’s program can respond to this worry.

14 See (Sober 1993, especially pp. 49-50, 52-53). For similar lines of argument, see (Vineberg 1996), (Maddy 1997), (Bigaj 2003), (Peressini 2008), and (Leng 2002).
undercuts the sort of confirmational holism that many commentators have taken to serve as a key premise of Quine’s original version of the indispensability argument, one according to which the empirical observations that serve to confirm a scientific theory do so by confirming that theory as a whole, rather than by confirming some parts of that theory but not others.\footnote{See (Morrison 2010) and (Morrison 2012) for further discussion of the role that confirmational holism is often taken to play in Quine’s argument.}

Unfortunately, however, even if this line of response is entirely correct as far as it goes, from the perspective of the scientific realist, it doesn’t go far enough. It is still the case, the scientific and mathematical realist might argue, that our primary reason for accepting claims about concrete unobservables is that they are implied by our best scientific theories, these being well confirmed by empirical observation. The mathematical realist can believe, furthermore, that our empirical evidence gives us good reason to believe that those theories are \textit{true} (or, at least, approximately so), thereby making it rational for us to believe their known implications. The mathematical anti-realist, however, has no such luxury. Indeed, insofar as she believes that it is not likely that mathematical realism is true, she is committed to the claim that it is not likely that our best scientific theories are true (since a theory can be no more probable than its known implications).

Granted, it is true that if Sober’s response is correct, it may well be that some of the implications of our best scientific theories pertaining to concrete unobservables, unlike the mathematical implications thereof, are confirmed by the same empirical observations that confirm those theories. But to say this is the case is not to say which
implications are so confirmed; nor is it to say by how much. It may be, for all Sober’s response, taken by itself, can tell us, that none of those implications are confirmed to a degree sufficient to justify our belief in them. Perhaps this need not bother Sober, since he explicitly distances himself from scientific realism, but for the scientific realist, the challenge posed at the outset goes unmet.

Fortunately, as I argue in the remaining sections, there is a way to develop Sober’s line of response on behalf of the scientific realist who rejects mathematical realism so as to fill the above gap. I begin, in the next section, by developing a more detailed version of the Sober-inspired line of response sketched above. In the subsequent section, I show how to fill out that line of response even further so as to meet the objection just posed to it. The principles that will underlie that response, as it is developed in the following two sections, will be first approximations that are subject to counterexamples. In the final two sections, I will show how to qualify those principles so as to avoid these counterexamples, as well as provide additional arguments for their truth.

1.3 The Empirical Resilience and Isolation of Pure Mathematics

Above I noted Sober’s observation that we are typically unwilling to regard the claims of pure mathematics as capable of being disconfirmed by the sorts of empirical observations that we often bring to bear when testing scientific theories. Suppose, to take Sober’s own example, we are considering the “hypothesis” that 2+2=4. We note that this hypothesis, in conjunction with the auxiliary assumption that 2+2 is the number of apples on the

---

16 (Sober 1993, pp. 37, 41-42, 43-45, 48)
table, implies that there are four apples on the table. But suppose that we then observe that it is not the case that there are four apples on the table. Would that observation cast doubt on the hypothesis that 2+2=4? As Sober points out, it seems that the answer is “No”:

If there had failed to be 4 apples on the table, I do not think we would have concluded that 2+2 has a sum different from 4. Rather, we would have concluded that the auxiliary … assumption is mistaken. If this is how we comport ourselves, then the “experiment” just described need never have been run. If we hold our belief that 2+2=4 immune from revision in this experiment, then the outcome of the experiment does not offer genuine support of that proposition.  

We might summarize Sober’s observation by saying that the hypothesis that 2+2=4 is “resilient” in this empirical situation; it is not capable of being disconfirmed by any of the possible empirical observations that might be brought to bear upon it (in the context of the present experiment).

It is also tempting to draw a more general lesson from this example. It seems that similar considerations would apply to any claim of pure mathematics and to any empirical situation. That is, it is tempting to embrace something in the neighborhood of the following principle:

(Resilience) For any claim of pure mathematics, M, there are no possible empirical observations that count against M.

Sober himself stops short of endorsing (what I’m calling) Resilience (for reasons I will soon consider), contenting himself with the weaker assertion that something like that principle typically holds, with respect to the sort of pure mathematical claims implied by

17 (Sober 1993, p. 50)
our best scientific theories, and with respect to the kinds of empirical observations brought to bear upon those theories.\(^{18}\)

Before we consider objections to Resilience, however, it will be helpful to examine another conclusion that Sober draws from the above example. Note Sober’s assertion that “if we hold our belief that 2+2=4 immune from revision in this experiment, then the outcome of the experiment does not offer genuine support of that proposition.” Here Sober seems to be relying on something like the following principle of confirmation:

\[
\text{(Symmetry) } O \text{ favors } H \text{ if and only if } \neg O \text{ counts against } H.
\]

Sober’s reliance on this principle (or something like it, at any rate) flows out of his endorsement of another familiar principle of confirmation, the Likelihood Principle:

\[
\text{(LP) } O \text{ favors } H \text{ over } H' \text{ if and only if } P(O/H) > P(O/H').^{19}
\]

It can be shown (via the probability calculus) that LP entails

\[
\text{(LP') } O \text{ favors } H \text{ over } H' \text{ if and only if } \neg O \text{ favors } H' \text{ over } H.^{20,21}
\]

---

\(^{18}\) (Sober 1993, pp. 50-51, 56)

\(^{19}\) I am assuming that in order for O to favor H over H’, it must be the case that neither P(H) nor P(H’) is equal to an extreme probability (i.e. that neither is equal to 0 or to 1). This assumption ensures that in all cases in which the left hand side of LP is satisfied, the conditional probabilities that occur on the right hand side are well defined. I also stipulate that in those cases in which either of these conditional probabilities are not well defined, the right hand side of the relevant instantiation of LP is to be regarded as false.

\(^{20}\) For the “only if” direction, assume that O favors H over H’. By LP, P(O/H) > P(O/H’). By the axioms of the probability calculus, it follows that 1 – P(–O/H) > 1 – P(–O/H’) and therefore that P(–O/H’) > P(–O/H). By another application of LP, it follows that –O favors H’ over H. The “if” direction is proved by a parallel argument.
Let’s say that an observation *favors* a hypothesis if and only if it favors that hypothesis over its negation. Let’s also say that an observation *counts against* a hypothesis if and only if that observation favors its negation. By employing these definitions and substituting \( \sim H \) for \( H' \) in LP’, we may derive Symmetry.

Symmetry, in conjunction with Resilience, entails another strong principle regarding the bearing of empirical observation on mathematical theories:

(Isolation) For any claim of pure mathematics, \( M \), there are no possible empirical observations that favor \( M \).  

Isolation, furthermore, is just the sort of principle that would allow the mathematical anti-realist to resist empirical arguments for mathematical realism.

Unfortunately, however, both Resilience and Isolation can appear too strong. As they stand, both initially appear subject to counterexamples. Sober indicates, for

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21 Sober (1993, p. 44) explicitly endorses a version of LP’, noting that it follows from the Likelihood Principle.

22 It might be thought by readers familiar with Sober’s views that my employing the notion of a theory’s being favored *simpliciter* contradicts Sober’s (1993, p. 39) assertion that “the Likelihood Principle entails that the degree of support a theory enjoys should be understood relatively, not absolutely.” But this is not so. Sober makes it clear that what he means by this claim is that “the evidence we have for the theories we believe does not favor those theories *over all possible alternatives*” (p. 39, emphasis original). This does not entail that our evidence cannot favor a hypothesis over its own negation. In fact, Sober explicitly says otherwise: “The fact of the matter is that when scientists lack a developed substantive alternative to a theory, they contrast the theory with its own negation. This is a contrastive alternative that is always available” (pp. 52-53).

23 By “observations,” here and throughout this chapter, I really mean “observation reports” (which I am thinking of as propositions). A “possible observation” is an observation report that could (epistemically speaking) turn out to be true. Note that it follows from LP and the probability calculus that if \( O \) favors \( H \), \( 0 < P(O) < 1 \). It further follows from this that \( P(\sim O) > 0 \), which I take to be sufficient for \( \sim O \)’s being an epistemically possible observation (epistemically possible, at least, with respect to a given epistemic situation and body of background knowledge to which we might think of these probabilities as being implicitly relativized). Given all this, the following argument establishes that Isolation follows from the conjunction of Resilience and Symmetry. Assume (for *reductio*) that a possible observation report, \( O \), favors a claim of pure mathematics, \( M \). It follows from what was just said in this note that \( \sim O \) is a possible observation. It follows from Symmetry that \( \sim O \) is a possible observation that counts against \( M \). But this contradicts Resilience. So it follows (by *reductio ad absurdum*) that Isolation is true.
example, that he does not wish to deny that Plateau obtained empirical confirmation for propositions of pure mathematics when he dipped pieces of wire into soap suds in order to discover (for a number of cases) the surface of least area that is bounded by a given closed contour. A similar sort of counterexample might involve discovering the solutions to various mathematical problems by employing calculators or computer programs. Another kind of often cited example against principles like Resilience and Isolation is the alleged overturning of Euclidian geometry in the twentieth century on account of empirical considerations.

The first two of the above examples, I concede, are genuine counterexamples to Resilience and Isolation as stated. Call counterexamples of the same sort as these “calculator counterexamples,” on account of the fact that they involve the use of physical devices (such as calculators, computers, or wires dipped in soap suds) in order to discover the solutions to various mathematical problems. In Section 1.5, I will consider how to modify Resilience and Isolation so that they avoid counterexamples such as these. One thing that can be initially said about such counterexamples is that the alleged confirmation for the claims of pure mathematics that occurs in these cases is mediated by impure auxiliary assumptions, these being assertions concerning how a certain physical system models various pure mathematical theories. Furthermore, in those situations in which we are confident that our observations diverge from what we know to be implied by the relevant pure mathematical theory, in conjunction with the relevant auxiliary assumptions, our inclination is to lower our confidence in the auxiliary assumptions rather than in the pure mathematical theory. This suggests that the underlying pure

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24 (Sober 1993, p. 51)
mathematical theory informing our auxiliary assumptions is not itself being empirically tested (and therefore is not itself being empirically confirmed).  

Similar things may be said about the alleged overturning of Euclidian geometry on account of empirical considerations. Call such alleged counterexamples “Euclidian counterexamples.” Unlike calculator counterexamples, I do not regard Euclidian counterexamples as posing genuine counterexamples to Resilience and Isolation, not even as those principles are currently stated. Either we take Euclidian geometry as a pure mathematical theory or as a theory concerning the nature of physical space. If the former, then the thing to say in response to the relevant empirical observations is not that they have disconfirmed the pure mathematical theory, but that they have disconfirmed the hypothesis that the theory is appropriately modeled by physical space. If the latter, we have no genuine counterexample to Resilience (or to Isolation).  

As noted above, more will be said about calculator counterexamples and other alleged counterexamples to Resilience and Isolation in Section 1.5. I believe that enough has been said, for now, however, to support the plausibility of the assertion that principles in the neighborhood of Resilience and Isolation are true. My present concern, furthermore, is to investigate how much mileage the mathematical anti-realist might get out of a successful defense of these claims (or appropriately qualified versions thereof). In the next section, I will argue that if claims along these lines can be successfully

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25 For similar kinds of responses to cases of these sorts, see (Vineberg 1996), (Leng 2002) and (Bigaj 2003) and (Peressini 2008).

26 For a similar line of response to the claim that the alleged fall of Euclidean geometry affords an example of a mathematical theory being disconfirmed by empirical considerations, see (Leng 2002, pp. 402, 412) as well as (Leng 2010, pp. 80-81).
defended, mathematical anti-realists with inclinations toward scientific realism can develop a fully satisfying response to the challenge posed in Section 1.2.

1.4 Scientific Realism without Mathematical Realism

Given the likelihood principle (and the assumption that possible conjunctions of empirical observations are also possible empirical observations), the conjunction of Resilience and Isolation is equivalent to

\[(R&I) \text{ For any claim of pure mathematics, } M, \text{ and any possible conjunction of empirical observations, } E, \ P(E/M) = P(E/\neg M) \text{ (provided that } 0 < P(M) < 1).\]  

By means of the probability calculus, furthermore, it is easy to show that R&I entails

\[(\text{Independence}) \text{ For any claim of pure mathematics, } M, \text{ and any possible conjunction of empirical observations, } E, \ P(E/M) = P(E) \text{ (provided that } 0 < P(M) < 1).\]

But surely Independence, if true, is an instance of a more general principle that is also true. It is implausible, for example, that while no possible empirical observations could count for or against a claim of pure mathematics, our somehow learning (though non-empirical means) that there are neutrons could do so.

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27 Concerning the proviso that \(0 < P(M) < 1\), see note 19. If one is inclined to complain that the claims of pure mathematics are either necessarily true or necessarily false and therefore that, for any given pure mathematical claim, \(M\), \(P(M)\) is either 0 or 1 (thereby leaving, for any claim \(E\), either \(P(E/M)\) or \(P(E/\neg M)\) undefined), it should be borne in mind that the probabilities at issue here are to be regarded as epistemic probabilities rather than logical or objective probabilities. I will assume that claims that are necessarily true if true and necessarily false if false may nonetheless have non-extreme epistemic probabilities. I do not have the space here to address any skepticism that one might have concerning the applicability of the probability calculus to epistemic probabilities that allow necessarily true (or necessarily false) claims to have non-extreme probabilities. For one defense of the applicability of the probability calculus to epistemic probabilities of that sort, however, see (Garber 1983).

28 Proof: Suppose that \(P(E/M) = P(E/\neg M)\). By this and the total probability principle, it follows that \(P(E) = P(M)*P(E/M) + P(\neg M)*P(E/\neg M) = P(M)*P(E/M) + P(\neg M)*P(E/M) = P(E/M)*[P(M) + P(\neg M)]\) \(= P(E/M)*1 = P(E/M).\)
Let’s stipulate that a claim is to be regarded as “solely about non-mathematical entities” if and only if each of the quantifiers that occur in a sentence that expresses that claim are restricted to non-mathematical entities, and (given how all the non-mathematical entities in the world are intrinsically and in relation to one another) there being or not being mathematical entities makes no difference as to the truth value of that claim. A mixed mathematical claim such as *The ratio of an object’s gravitational mass to its inertial mass is equal to 1*, therefore, is not a claim that is solely about non-mathematical entities. By contrast, the claim *Neutrons are more massive than electrons* is a claim that is solely about non-mathematical entities. Let it also be stipulated that any claim that is solely about non-mathematical entities is not to be regarded as a claim of pure mathematics. Given these stipulations, we can now see that if Independence is true, then surely it is an instance of the following, more general, true principle:

\[
\text{(Strong Independence) For any claim of pure mathematics, } M, \text{ and any possible conjunction of claims that are solely about non-mathematical entities, } Q, \ P(Q/M) = P(Q) \text{ (provided that } 0 < P(M) < 1).\]

And Strong Independence, as I will now show, provides the mathematical anti-realist with a powerful resource for responding to the challenge that we have been considering.

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29 Let it also be artificially stipulated here that any abstract entities there are (whether the number 2 or the property of being feline) are to be regarded as “mathematical entities.”

30 It is a more general principle, at any rate, if (as I am assuming) any claim that genuinely reports an empirical observation is a claim that is solely about non-mathematical entities. I do grant that scientists often state their observations by using mixed mathematical claims. A scientist might report an observation by saying, for instance, “The mass of the sample was measured as being equal to 3.24 grams.” I assume, however, that, in a case such as this, the proposition that genuinely reports the empirical observation at issue would be something like *The digital readout screen displayed “3.24 g” when the sample was on the scale.*
Let ‘T’ stand for one of our best, most well-confirmed scientific theories, one that demonstrably implies some claim of pure mathematics, M.\(^{31}\) Let ‘E’ stand for the conjunction of a body of observation reports (reports solely about non-mathematical entities) that together constitute our empirical basis for accepting T. Suppose that P(T/M&E) is high enough to warrant belief in T given hypothetical certainty about M and E. Finally, let ‘U’ stand for some claim solely about non-mathematical entities that pertains to concrete unobservables (e.g. the claim that neutrons exist) and suppose that U is also demonstrably implied by T.

Strong Independence gives us the following claims:

1. P(E/M) = P(E)
2. P(U&E/M) = P(U&E)

By means of the probability calculus, it can be shown that these claims jointly entail\(^{32}\)

3. P(U/E) = P(U/M&E)

Since T demonstrably implies U, it is also the case that

4. P(U/M&E) ≥ P(T/M&E)

Finally, (3) and (4) together entail

\(^{31}\)I am, pace van Fraassen (1980), taking scientific theories to be propositional entities rather than model-theoretic ones, largely for the sake of convenience more than anything else.

\(^{32}\)Recall that, according to the standard definition of conditional probability, P(A/B) = P(A&B)/P(B). So, P(U/E) = P(U&E)/P(E). Thus, by 2, P(U/E) = P(U&E/M)/P(E) = P(U&E&M)/[P(M)*P(E)]. Also, by 1 and the standard definition of conditional probability, P(M&E) = P(M)*P(E/M) = P(M)*P(E). It follows from the above that P(U/E) = P(U&E&M)/P(M&E). Since it is also the case that, by the standard definition of conditional probability, P(U/M&E) = P(U&E&M)/P(M&E), it follows that P(U/E) = P(U/M&E).
(5) \( P(U/E) \geq P(T/M&E) \)

If we take these conditional probabilities to reflect what our rational credences ought to be, (5) tells us that our confidence in \( U \) given certainty about \( E \) ought to be at least as great as our confidence in \( T \) given hypothetical certainty about \( M&E \). But we stipulated that our confidence in \( T \) given hypothetical certainty about \( M&E \) ought to be high enough to warrant belief. It follows that our confidence in \( U \) given certainty about \( E \) ought to be high enough to warrant belief as well. This is all consistent, furthermore, with \( P(M/E) \) being quite low.\(^{33}\) The upshot is that the mathematical anti-realist who accepts Strong Independence may take herself to have principled grounds for accepting \( U \) (by seeing that \( T \) implies it) while rejecting \( M \) (even though she sees that \( T \) implies \( M \) as well). More generally, she may, in a principled way, accept those implications of our best, most well confirmed scientific theories that are solely about non-mathematical entities (including those that pertain to the existence of concrete unobservables) while rejecting the mathematical implications of those theories.

\(^{33}\) We may demonstrate the formal consistency of these claims by exhibiting a model in which they are all true. And we may do that by reassigning whatever propositions to ‘\( T \)’, ‘\( U \)’, ‘\( M \)’, and ‘\( E \)’ that we like (regardless of what these terms originally denoted), provided there is a possible state of affairs in which the resulting claims concerning the relevant conditional probabilities are all true. To that end, suppose we let ‘\( M \)’ denote the proposition that a fair, one thousand sided di (each side of which is assigned a unique whole number in the inclusive range of 1 through 1000) comes up 3 on a fair roll. Let ‘\( E \)’ denote the proposition that a certain coin (recently fairly tossed) came up heads. Let ‘\( U \)’ denote the same proposition as ‘\( E \)’. Finally, let ‘\( T \)’ denote the conjunction of \( M \) and \( E \). Clearly, given these assignments, \( P(T/M&E) \) is high enough to warrant belief in \( T \) given hypothetical certainty about \( M \) and \( E \) (its value equals 1, in fact) and \( T \) also demonstrably implies \( U \) and \( M \). Furthermore, the background knowledge could easily be such that claims (1) and (2) above are both true (in fact, it will be such that those claims are both true unless it is rather atypical). Finally, \( P(M/E) \) is quite low (1/1000 to be exact).
1.5 Qualifying Resilience and Isolation

The success of the above response turns, however, on a successful defense of Resilience and Isolation, or at least principles sufficiently in the neighborhood thereof. And, as conceded in Section 1.3, Resilience and Isolation (as there stated) are subject to counterexamples. The task of this section is to show how these principles can be qualified in such a way that they avoid those counterexamples. In the next section, furthermore, I will provide an argument, not merely for the plausibility of the resulting versions of these principles, but also for their truth (or at least for the truth of principles sufficiently in the neighborhood thereof).

Consider, then, what (in Section 1.3) I called “calculator counterexamples” to these principles, counterexamples that involve the use of physical devices in order to discover certain mathematical truths. As I also note in Section 1.3, one thing to be said about such cases is that they appear to be situations, not in which the underlying mathematical theory is itself being empirically tested, but in which individuals are gaining (through empirical means) information about various consequences of those theories. This suggests that we can avoid these sorts of counterexamples by reformulating Resilience and Isolation in such a way that they make reference, not to pure mathematical claims in general, but to some suitably general background mathematical theory. When it comes to contemporary mathematics (at least all of the contemporary mathematics that is needed to do science), furthermore, one cannot ask for a more general background mathematical theory than set theory.

Let ‘S’, then, denote some standard version of set theory that is well suited to supply us with all of the mathematics that we need to do science (Hartry Field suggests,
for example, that a version of Zermelo-Fraenkel set theory with choice, modified so as to allow for urelements, would do this job). Now consider the following modification of Resilience (which, in conjunction with Symmetry, entails the correspondingly modified version of Isolation):

(S-Resilience) There are no possible empirical observations that count against S.

S-Resilience is not subject to the calculator counterexamples offered against Resilience in Section 1.3. And if S is thought of as affording us with our most general background mathematical theory (at least for the purposes of doing science), S-Resilience is made plausible by the same kinds of examples that made Resilience plausible. While we may be inclined, in some experimental contexts, to blame a failed theoretical predication on a calculation error or a mistaken derivation, we will not be inclined to blame the failure on the falsity of our most general mathematical theory. Our most general mathematical theory seems to be immune from empirical disconfirmation.

Unfortunately, however, as Graham Leach-Krouse has pointed out to me, there are calculator-style counterexamples to S-Resilience as well. Below I put forward an adaptation of one of Leach-Krouse’s counterexamples. It will prove helpful to see this counterexample as being posed, not directly against S-Resilience, but against the following principle that (given the likelihood principle) is equivalent to it:

(S-Resilience*) For any possible empirical observation, E, $P(E/S) \geq P(E/\sim S)$.\(^{35}\)

\(^{34}\) (Field 1992, p. 112)

\(^{35}\) Here I drop the proviso that $0 < P(S) < 1$ on account of the fact that I’m tacitly assuming, from now on, that the background knowledge is such that the proviso is met (and also such that that the counterparts to this proviso in the various other principles discussed below are met).
Here is the counterexample: Suppose that we design a well-crafted machine (a computer running a well-designed proof-finder program, let’s say) to search for proofs of contradictions within S. Let ‘E’ stand for the claim that our machine is observed to report that it has found a contradiction within S. Given that our machine is well-crafted, E is less likely on the assumption that S is true than it is on the assumption that S is false, contrary to what S-Resilience* entails.

There is a principle in the neighborhood of S-Resilience*, however, that avoids this counterexample. Let ‘C’ stand for the claim that S is consistent. Now consider the following principle:

\[(SC\text{-Resilience}) \text{ For any possible empirical observation, } E, \ P(E/S&C) \geq P(E/\sim S&C).\]

SC-Resilience avoids the above counterexample by treating the claim that S is consistent as if it were a fixed item of background knowledge. Obviously, it is no more likely that our well-crafted machine will find a contradiction on the assumption that S is both true and consistent than it is on the assumption that S is both false and consistent. From SC-Resilience, furthermore, we may (via an argument that is exactly parallel to that given in Section 1.3, with C being treated as if it were an item of background knowledge) also conclude that the following principle holds:

\[(\text{Strong SC-Independence}) \text{ For any possible conjunction of claims that are solely about non-mathematical entities, } Q, \ P(Q/S&C) = P(Q/C).\]

Now let ‘T’ stand for one of our best, most well-confirmed scientific theories, formulated in such a way that the background mathematical theory that it uses is S, and also in such a way that it implies S (we can imagine, if we like, taking the scientific theory in question as it was originally formulated, reformulating it so that its background
mathematics conforms to S, and then conjoining it with S). As before, suppose that
P(T/S&E&C) is high enough to warrant belief in T given hypothetical certainty about
S&E&C. Likewise as before, let ‘U’ stand for some claim solely about non-
mathematical entities that pertains to concrete unobservables (one that is implied by T).
From Strong SC-Independence, we may also derive (via an argument that exactly
parallels the one given in Section 1.4) the following:

(5*) P(U/E&C) ≥ P(T/S&E&C)

If we take the above conditional probabilities to guide what our rational degrees of
confidence should be, it follows from all of the above that, in order to properly maintain a
high degree of confidence in U given our empirical evidence, we need not have a high
degree of confidence that the background mathematics in which we formulate T is true;
at most we need only have a high degree of confidence that it is consistent.36 And while
Gödel has shown us that it can’t be proven that our background mathematics is
consistent, our confidence that it is seems well justified.

SC-Resilience, therefore, both avoids the counterexamples discussed above, as
well as offers the scientific realist who denies mathematical realism a principled basis for
affirming what our best, most well-confirmed scientific theories imply about the

36 The claim that S is consistent is not, in this context, to be thought of as implying that S has a
model (in that case, the mathematical anti-realist couldn’t be highly confident that S is consistent without
being highly confident that there is at least one set-theoretical object). Nor is it to be thought of as
implying that S is metaphysically possible (the mathematical anti-realist might well think that it is a
necessary truth that there are no sets). It should be thought, rather, as the claim that it is not logically true
that not S, where the phrase ‘it is not logically true that not’ is to be read as functioning as a primitive
consistency operator (on this point, see Field (1992), who relies on such a primitive consistency operator
when he attempts to offer a nominalistic proof that mathematics is conservative in the sense described in
note 8). Or, if one is skeptical about the intelligibility of this way of understanding the consistency claim at
issue here, an alternative (one that involves adapting a suggestion made to me by Graham Leach-Krouse) is
to regard it as the claim that necessarily, no flawlessly designed, properly functioning, proof-finding
machine will report having found a contradiction within S.
existence of concrete unobservables, even while she also denies the implications of those theories that pertain to the existence of mathematical entities. SC-Resilience is also well supported by the fact that when we reflect on actual or hypothetical cases of scientific disconfirmation, we are not inclined to blame the failure on our background mathematical theory in those cases, but rather, on the other features of our scientific theory, or on some auxiliary assumption, or perhaps on some calculation error or mistaken derivation that caused us to be misled about what our background mathematical theory implies. There is also, I believe, an additional argument that can be made in favor of SC-Resilience (or at least a principle sufficiently in the neighborhood thereof), one that I provide in the next and final section.

1.6 Resilience and Causal Isolation

Let ‘there are S-sets’ abbreviate the claim that there are sets that collectively satisfy the axioms of S. Note that S implies that there are S-sets and that the claim that there are S-sets implies S (provided that S is a mathematical theory that is ontologically committed to the existence of sets, but to no entities that are not sets, and provided that S by itself doesn’t imply anything about how things are with the non-sets, beyond the fact that whichever non-sets there are enter into various set-theoretical relations). That is, the claim that there are S-sets is equivalent to S. Thus, if we let ‘S*’ stand for the claim that there are S-sets, SC-Resilience is equivalent to the following:

\[ (S^*\text{-C-Resilience}) \text{ For any possible empirical observation, } E, \quad P(E/S^*\&C) \geq P(E/\sim S^*\&C). \]
I submit, furthermore, that there is a reason to believe that S*C-Resilience is true that goes beyond the considerations raised in the previous sections.

Abstract entities (if they exist) are causally isolated from concrete things. And, for that reason, as other authors have noted, the bare claim that abstract entities exist does not (all by itself) afford us with any additional causal information that should alter our expectations concerning how concrete things are likely to behave. This fact about the causal isolation of abstract entities also (as other authors have pointed out) seems to point in favor of the truth of principles such as Resilience (or to principles in the neighborhood thereof). And it also seems to explain why there is the sort of asymmetry (that philosophers like Sober have pointed to) between how the existential claims of pure mathematics are related to our empirical evidence and how claims about concrete unobservables are so related. Concrete unobservables, like quarks and neutrons, if they exist, are not causally isolated in the way that abstract objects are; the claim that such entities exist does afford us with additional causal information concerning how other concrete entities can be expected to behave. The claim, furthermore, that the bare assumption that there are S-sets does not (by itself) afford us with any causal information about how concrete objects might be expected to behave, and that this affords us with a good reason to believe that something like S*C-Resilience is true, is just a more specific application of these more general considerations.

These claims about the causal isolation of abstract entities in general, and of S-sets in particular, however, should not be overplayed. The mere fact that a claim does not

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37 For authors who make arguments along the lines of those discussed in this paragraph, see (Balaguer 1998a), (Balaguer 1998b), (Balaguer 2009), (Leng 2005), (Leng 2010), and (Vineberg 1996).
(all by itself) afford us with additional *causal information* about how we might expect concrete entities to behave does not entail that it affords us with *no information at all* about how we should expect concrete entities to behave. Suppose, for example, that our background knowledge contained the conditional claim that if there are S-sets, then Higgs bosons exist. Then, by learning that S-sets exist, we would also learn that Higgs bosons exist, which would afford us with information about how we should expect particle accelerators to behave under certain conditions and the like.

But, of course, our actual background knowledge doesn’t contain such a conditional; nor does it contain anything else of the sort. The conditional probabilities referenced in principles like S*C-Resilience, furthermore, should be thought of as holding relative to a given body of background knowledge, and, more specifically, to a given body of background knowledge that is comparable to our own. Now we can concede that S*C-Resilience may not hold relative to certain bizarre bodies of background knowledge. (That is, we can concede that it may be that the sentence that expresses S*C-Resilience expresses a false claim under some interpretations in which the conditional probabilities to which it refers are understood as being relativized to a bizarre body of background knowledge). Even so, it is tempting to say that this is not how things are relative to our own body of background knowledge. There’s nothing in *our* body of background knowledge, we might want to assert, that links the bare existence of S-sets to rational expectations about how concrete objects are likely to behave.

If we do take up all of what we are tempted to assert here, we get a nice, clean argument for the claim that S*C-Resilience is true (and therefore for the claim that SC-Resilience, which is equivalent to it, is true). The argument is simply that since the bare
claim that there are S-sets does not (all by itself) afford us with any additional causal information about how we might rationally expect concrete objects to behave, and since there is nothing else in our background knowledge that links up that claim with rational expectations about how concrete objects might be expected to behave, we have no more reason on the assumption that S-sets exist (and that S is consistent) to expect certain empirical observations than we do on the assumption that S-sets do not exist (and that S is consistent). That is, we have good reason to believe that the universal generalization that S*C-Resilience makes holds true (provided, at least, that the conditional probabilities to which it refers are understood as being relativized to a body of background knowledge comparable to our own).

Unfortunately, the above argument, so put, is a little too clean. All I am attempting to argue in this chapter is that the mathematical anti-realist can sensibly deny that we have strong empirical grounds for the existence of mathematical entities. I am not attempting to argue here that the mathematical anti-realist can sensibly maintain that there are no strong a priori grounds for the existence of mathematical entities. So I want to concede, at least for the sake of argument, that there could be strong a priori grounds for the belief that S-sets exist. But once this is conceded, it looks like I also have to acknowledge that there are possible empirical observations (i.e. empirical observations that are not ruled out by our background knowledge) that link rational expectations concerning how various concrete objects will behave to the claim that S-sets exist.

Consider, for instance, the case of the Philosophical Guru. The Philosophical Guru is someone who we have good reason to believe is of enormous prowess when it comes to reasoning about a priori matters. When she opines on a given philosophical
issue that is to be settled on *a priori* grounds, one would do best to place a great deal of credence in what she says. Now, suppose we are about to ask the Philosophical Guru whether, in fact, S-sets exist. It is less likely, *on the assumption that S-sets exist*, that she will tell us that S-sets do not exist, than it is that she will tell us the same thing, *on the assumption that S-sets do not exist*. So, in this case, our background knowledge does contain information that links up rational expectations concerning how a certain concrete object will behave (namely, concerning how the Philosophical Guru will behave) to the claim that S-sets exist. In fact, not only does this case undercut the *argument* for S*C*-Resilience offered above, it also seems to afford us with a *counterexample* to that principle!^38

Something about this case seems amiss, however, if we take it not merely as a potential counterexample to S*C*-Resilience, but also as a reason to think that we might have empirical grounds for believing that S-sets exist. We can stipulate, in this case, that the Philosophical Guru’s grounds for her opinion concerning whether S-sets exist are entirely *a priori* in character. If she believes that S-sets exist, then she does not believe this for empirical reasons. If that’s so, however, then it also seems wrong to say that if she were to tell us that S-sets exist, that would afford us with *empirical grounds* for the claim that S-sets exist. Rather, it seems that what would have occurred in that case is that we would have learned through empirical means that the Philosophical Guru has strong, non-empirical grounds for believing that S-sets exist.

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^38 I thank Andrew Bailey for initially suggesting to me that there were counterexamples to principles like S*C*-Resilience that run along these lines.
This line of thought suggests that if we want to formulate S*C-Resilience in such a way as to make it more relevant to when we do or don’t have empirical grounds for believing that S-sets exist, we ought to think of it as applying, not directly to ourselves (at least not to those of us who fail to be philosophical gurus), but rather as applying to the Philosophical Guru herself. That is, we should think of the conditional probabilities to which it refers, not as those that hold relative to our background knowledge and epistemic situation, but as those that hold relative to the Philosophical Guru’s background knowledge and epistemic situation (provided that we stipulate that the empirical grounds that she has that bear on the issue are comparable to our own). More precisely, we should consider these conditional probabilities as holding relative to a *guru-like epistemic situation*, as well as to a body of empirical background knowledge that is comparable to our own – where an individual is in guru-like epistemic situation if and only if she has clear and salient access to all of the *a priori* grounds that bear on the question of whether S-sets exist (at least to all such *a priori* grounds that are available to beings like us).

I submit (with one *caveat* that I will discuss below) that the considerations that I’ve raised thus far are good reasons to believe that S*C-Resilience is true when the conditional probabilities to which it refers are taken to hold relative to a guru-like epistemic situation and to a body of empirical background knowledge comparable to our own, and therefore that SC-Resilience is true when its conditional probabilities are so understood, and therefore also that all of the claims that I argued follow from SC-Resilience are true when the conditional probabilities to which they refer are so understood. With this modification in place, the official conclusion of the arguments of this chapter is that it is consistent (at least with what all we non-gurus know to be the
case) that someone in a guru-like epistemic situation who possess a body of empirical background knowledge comparable to our own could consistently take herself to have principled grounds for believing in the concrete unobservables whose existence is implied by our best, most well-confirmed scientific theories, without believing in the mathematical entities whose existence is implied by those theories. The scientific realist who denies mathematical realism, furthermore, can take this fact as a reason to believe that she herself is not being epistemologically inconsistent in believing in things like neutrons and quarks while denying the existence of mathematical entities (even if she herself is not a philosophical guru).

The one caveat to all of this is that, in order to make the above modification work, I have to assume that the a priori grounds that are available to someone in a guru-like epistemic situation do not entitle her to be certain about whether or not S-sets exist. That is, they do not entitle her to assign an epistemic probability of 1 to the claim that S-sets exist; nor do they entitle her to assign an epistemic probability of 0 to that proposition. If matters were otherwise, if the Philosophical Guru were entitled to be certain about whether or not S-sets exist, then some of the conditional probabilities to which S*C-Resilience refers would go undefined. But the assumption that this is not the case is, I believe, a plausible one, at least if we are considering the sorts of philosophical gurus (or the approximations thereof) that might actually be found in our midst.39 Whatever a

39 If theism is true, then God may be thought of as a philosophical guru of sorts, one who is entitled to be certain about whether or not S-sets in fact exist. But God is also excluded from the present considerations by the stipulation that the philosophical guru in question is to be thought of as having empirical background knowledge comparable to ours, and God (by virtue of his being omniscient) fails to meet this condition (he knows vastly more about the relevant matters than we do). Theism does offer a potential counterexample to S*C-Resilience, however, insofar as God might choose to reveal to us (through a prophet whom we have good grounds to believe is a genuine prophet) whether or not S-sets exist; and it is less likely that God would reveal to us through a prophet that S-sets do not exist on the assumption that S*
priori considerations there are to be brought to bear on the issue of whether things like S-sets exist, I take it that they do not entitle anyone to be dead certain about the issue. At least they don’t if S is consistent, which we may assume to be the case, unless or until matters are shown to be otherwise (if S is inconsistent, then perhaps someone could be dead certain that there are no S-sets by seeing that the claim that there are S-sets formally implies a contradiction).

With all that said, I take myself to have established that nominalists can successfully resist empirical arguments for mathematical platonism even while remaining scientific realists. Thus I take Quine-Putnam indispensability arguments for the conclusion that we have strong empirical grounds for believing that there are abstract objects to fail.

But even if we don’t have strong empirical grounds for believing that abstract entities exist, we do talk and reason as if such entities exist, both when doing science and in the course of everyday life. And we don’t seem to be able to get by without talking and reasoning in this way. Perhaps, then, even if we don’t have good empirical grounds for believing in abstract entities, we can’t consistently deny that such entities exist. In the next chapter, I will consider an argument put forward by Peter van Inwagen, one that runs along these very lines, for the conclusion that we are committed to the existence of properties.

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is true than it is on the assumption that S* is false. But God has made no such revelations thus far (at least none that any of us are aware of). And it doesn’t seem that he is likely to do so any time soon (at least not on this side of the eschatological divide). And so we can deal with this sort of counterexample by treating the claim that God will make no such revelations (not on this side of the eschatological divide, at any rate) as if it were an item of background knowledge (in the same sort of way that we dealt with Leach-Krouse’s counterexample by treating C as if it were an item of background knowledge).
1.7 Works Cited


CHAPTER 2:
LIGHTWEIGHT PLATONISM UNDERMINED

2.1 Lightweight Platonism

In contrast to what we might call “heavy duty platonists,”40 “lightweight platonists” (as I will call them) deny that properties (and other abstracta) are to be assigned any significant explanatory role in accounting for the qualitative character of the concrete world.41 While, according to lightweight platonists, properties (perhaps along with other abstracta) exist, they do not perform such functions as accounting for similarity between objects; nor is it the case that the propositions expressed by simple predicative sentences like ‘This apple is red’ are identical to or to be analyzed in terms of propositions expressed by sentences like ‘This apple exemplifies redness’ (at least if propositions are taken to be sufficiently fine grained42). Rather, if anything, according to lightweight platonists, the explanatory order runs in the other direction; if anything, a given apple exemplifies redness because it is red, and not the other way around. According to them (as Theodore Sider has put it) “particulars, not properties, wear the pants.”43

40 I’m borrowing this label (as well as adapting it for my own purposes) from (Field 1985).
41 I hereby disassociate my use of this terminology from David Chalmer’s (2009) use of the term ‘lightweight realism’. Lightweight platonists are not to be considered, for example, as regarding platonism as a shallow, trivial or conceptual truth.
42 See note 46.
43 See (Sider 2006, p. 390). At least, this is what the lightweight platonist would say about the relationship between first order properties and the particulars that exemplify them. Of course, they’ll also say that first order properties are the pants wearers when it comes to their relationship to the second order properties they exemplify, and so on all the way up.
among the paradigmatic examples of lightweight platonists are philosophers such as David Lewis (at least when it comes to properties)\textsuperscript{44} and Peter van Inwagen.

Lewis maintains, for example, that properties are sets of possibilia, these being constructed out of concrete particulars existing throughout this and other possible worlds. The property of being a pig, according to Lewis, is, for example, nothing other than the set of all pigs, both possible and actual.\textsuperscript{45} Clearly, this is a view concerning the nature of properties that only a lightweight platonist could endorse. If Lewis’s picture is accurate, then surely the property of being a pig does not figure into any explanation as to why pigs are similar to one another. Nor does it figure into any explanation of the truths expressed by simple subject-predicate sentences such as ‘Wilber is a pig’. Rather, if anything, the explanatory order runs in the other direction. If anything, given Lewis’s view, the set that is the property of being a pig is qualified to be that property because it contains all and only pigs (and presumably this is, in part, because Wilber himself is a pig).\textsuperscript{46}

Van Inwagen’s view concerning the nature of properties is radically different from that of Lewis. Nevertheless, it is also clearly an instance of lightweight platonism.

\textsuperscript{44} Lewis (1983) flirts with invoking Armstrongian universals to explain certain facts concerning similarity, intrinsically and naturalness. But he explicitly distinguishes between Armstrongian universals, on the one hand, and properties on the other.

\textsuperscript{45} (Lewis 1986, pp. 50-69)

\textsuperscript{46} As I have characterized lightweight platonism, the lightweight platonist refuses to identify propositions expressed by sentences like ‘This apple is red’ with those expressed by sentences like ‘This apple exemplifies redness’ (at least if propositions are taken to be sufficiently fine grained). In identifying Lewis as a lightweight platonist, the parenthetical clause of this characterization becomes important. According to most platonists (Lewis included) the proposition expressed (in a given context) by ‘This apple is red’ is necessarily equivalent to the proposition expressed (in the same context) by ‘This apple exemplifies redness’. So distinguishing these propositions requires a view of proposition individuation that allows for the possibility of there being distinct but necessarily equivalent propositions. Lewis’s own standard conception of propositions (according to which propositions are to be identified with sets of possible worlds), however, does not allow for that possibility.
Van Inwagen identifies properties, not with sets of possibilia, but with what he calls “unsaturated assertibles” – “thing[s] that can be said of something.” As he explains,

> [My] theory identifies the property role with the role “thing that can be said of something”. This role is a special case of the role “thing that can be said”. Some things that can be said are things that can be said period, things that can be said full stop. For example: that Chicago has a population of over two million is something that can be said; another thing that can be said is that no orchid has ever filed an income-tax return. But these things – ‘propositions’ is the usual name for them – are not things that can be said of anything, not even of Chicago and orchids. One can, however, say of Chicago that it has a population of over two million … Let us call such things, propositions and things that can be said of things, assertibles. The assertibles that are not propositions, the things that can be said of things, we may call unsaturated assertibles.\(^47\)

For something to exemplify a property, on this view, is for an unsaturated assertible to be true of that thing (and therefore such that it may be truthfully said of that thing).\(^48\)

But if this is what property exemplification amounts to, then obviously facts concerning property exemplification are (if anything) to be explained in terms of the facts expressed by simple subject-predicate sentences and not the other way around.

Properties are, given Van Inwagen’s view, closely akin to propositions. And no one would say, for example, that electrons are charged because it is true that electrons are charged; if anything, the explanation proceeds in the other direction. True propositions merely represent things as they are; they do not figure into the explanation of why things are as they represent them to be. Likewise, no one with Van Inwagen’s view should say that electrons are charged because it is true of electrons that they are charged. Rather, if anything, they should maintain, it is the other way around; if anything, the property of

\(^47\) (Van Inwagen 2004, pp. 131-132)

\(^48\) At least in typical cases. One can imagine certain cases in which an assertible is true of something but may not be truthfully said of it – e.g. consider the assertible *that it never has anything said of it*.
being charged is true of electrons because the latter are accurately represented as being charged. For similar reasons, it is difficult to see how anyone with Van Inwagen’s view of properties could plausibly think of properties as figuring into a genuine explanation of the resemblance between objects or anything of the sort.

If properties are explanatorily idle in the way that either Lewis’s or Van Inwagen’s view suggests, why should we believe in them? There are various answers that the lightweight platonist might give to this question. She might claim, for example, that we have direct \textit{a priori} grounds for believing in such entities. This answer, however, won’t be persuasive to anyone who does not take herself to have such grounds. Is there any argument that might persuade those of us who remain unconvinced?

Lewis gestures toward such an argument when he states, in passing, that “we have frequent need, in one connection or other, to quantify over properties.” As a good neo-Quinean, Lewis demands that we believe in those entities that we appear to quantify over in our ordinary and technical endeavors except in those cases in which we are either prepared to paraphrase away such apparent quantification or to take back what we have said. Certainly it is true that many of the things that we say and believe seem to entail the existence of properties (or property-like entities). We believe that there are some

\footnote{Van Inwagen himself, in the question and answer portion of his talk at the “Relational vs. Constituent Ontologies” conference held at the University of Notre Dame (March 5-6, 2009), explicitly argued via this analogy between properties and propositions for the conclusion that facts expressed by simple subject-predicate sentences are not to be explained in terms of facts involving property exemplification.}

\footnote{Lewis 1986, p. 50}

\footnote{For an illustration of such Quinean reasoning in action, as Lewis believes it should be done, see (Lewis and Lewis 1970). For a particularly striking example of Lewis’s own use of such reasoning, see (Lewis 1973, p. 84).}
features that housecats and lions have in common, that human beings are capable of discerning at least five different shades of red, that there are some significant commonalities between elements found in the same column of the periodic table, etc. A Quinean such as Lewis would insist that, on pain of inconsistency, we either accept the existence of properties (or property-like entities), find some other way to express the above claims that does not even seem to involve quantification over such entities, or take back those claims. Assuming we are unwilling or unable to afford ourselves of one of the latter two options, Lewis would insist, we must afford ourselves of the first and accept the existence of properties (or at least entities capable of playing property-like roles).

Since Lewis’s own, independently motivated ontology already affords him with entities (namely, sets of possibillia) capable of playing (what he sees as) the property role, he does not feel the need to press a Quinean argument along the above lines for the conclusion that we ought to believe in the existence of properties (rather, he sees it as an advantage of his system that it delivers property-like entities with no additional ontological costs).\(^{52}\) Van Inwagen, however, in his “A Theory of Properties” does develop a particularly challenging version of such an argument, one that I will use as foil for the remainder of this chapter. I plan to show that Van Inwagen’s argument fails, and by so doing, to exhibit the fact that it is unlikely that any argument for platonism that runs along these lines succeeds. I will proceed in two steps. First, after summarizing Van Inwagen’s argument, I will, for the bulk of this chapter, present a reply that constitutes an *ad hominem* as directed towards Van Inwagen.\(^{53}\) In particular, I will argue that, given

\(^{52}\) (Lewis 1986, pp. 50-69)

\(^{53}\) William Lane Craig (unpublished) gestures toward an *ad hominem* reply to Van Inwagen along the lines that I will suggest below. But he does not develop such a reply in great detail.
Van Inwagen’s own views concerning the philosophy of language (as expressed in his book *Material Beings*), his argument fails. Second, in the final section, I will move beyond my *ad hominem* reply to Van Inwagen by arguing that the considerations raised by that reply can be developed, independently of his views concerning the philosophy of language, into an adequate response to the sort of Quinean argument that he puts forward.

### 2.2 Van Inwagen’s Argument for Properties

In order adjudicate the dispute between “platonists” (those who affirm the existence of properties) and “nominalists” (those who deny their existence), Van Inwagen recommends “the ontological method invented, or at least first made explicit, by Quine and Goodman.”[^54] He offers the following explanation of what he takes that method to consist in:

> According to Quine, the problem of deciding what to believe about what there is is a very straightforward special case of deciding what to believe… If we want to decide whether to believe that there are properties, Quine tells us, we should examine the beliefs we already have … and see whether they “commit us” (as Quine says) to the existence of properties.[^55]

[^54]: (Van Inwagen 2004, p. 113)

[^55]: (Van Inwagen 2004, p. 114)
are anatomical features that insects have and spiders also have’, “or ‘in the canonical language of quantification’,” to ‘It is true of at least one thing that it is such that it is an anatomical feature and insects have it and spiders also have it.’” And he also notes that it appears to be a “straightforward logical consequence” of the proposition expressed by this sentence that there are features. Thus, he concludes, it appears that the nominalist who affirms this proposition but denies that there are properties is simply being inconsistent.\(^{56}\)

Van Inwagen recognizes that the nominalist needn’t immediately bow to the force of the above argument. She has alternatives. As Van Inwagen sees it, her options are as follows:

1. She might become a platonist.
2. She might abandon her allegiance to the thesis that spiders share some of the anatomical features of insects.
3. She might attempt to show that, despite appearances, it does not follow from this thesis that there are anatomical features.
4. She might admit that her beliefs (her nominalism and her belief that spiders share some of the anatomical features of insects) are apparently inconsistent, affirm her nominalistic faith that this inconsistency is apparent, not real, and confess that, although she is confident that there is some fault in our alleged demonstration that her belief about spiders and insects commits her to the existence of anatomical features, she is at present unable to discover it.\(^{57}\)

As Van Inwagen sees these options,

Possibility (2) is not really very attractive … for at least two reasons. First, it seems to be a simple fact of biology that spiders share some of the anatomical features of insects. Secondly, there are many, many “simple facts” that could

\(^{56}\) (Van Inwagen 2004, pp. 114-115)

\(^{57}\) (Van Inwagen 2004, p. 115)
have been used as the premise of an essentially identical argument for the conclusion that there are properties… Possibility (4) is always an option, but no philosopher is likely to embrace it except as a last resort.58

“What [the nominalist] is likely to do,” according to Van Inwagen, “is to try to avail herself of possibility (3).” And if she does this, he claims, “she will attempt to find a paraphrase of ‘Spiders share some of the anatomical features of insects’,” one that “does not even seem to have ‘There are anatomical features’ as one of its logical consequences.”59 Unfortunately, Van Inwagen contends, the nominalist has no such suitable paraphrase available to her. More precisely, his contention is that the nominalist has no paraphrase of the spider-insect sentence that does not involve apparent quantification over properties “or something that a nominalist is … going to like any better than properties.”60 Van Inwagen completes his discussion by supporting his contention that there is indeed no such paraphrase. Rather than summarizing Van Inwagen’s arguments for that thesis, however, I hereby grant it for the sake of argument. Given this concession, how might the nominalist respond?

2.3 An Ad Hominem Reply to Van Inwagen

Suppose the nominalist were to avail herself of none of the options that Van Inwagen lists for her. Suppose instead that she offered up the following speech:

I do not deny that when English speakers, immersed in the ordinary business of life (or even in their less ordinary scientific endeavors), utter sentences like

58 (Van Inwagen 2004, pp. 115-116)
59 (Van Inwagen 2004, p. 116)
60 (Van Inwagen 2004, pp. 116-117)
‘Spiders share some of the anatomical features of insects’, they very often say true things. On the contrary, I affirm it. I acknowledge, furthermore, that the proposition such speakers express by uttering the sentence just alluded to entails the proposition that these same speakers, in the same sort of context, might also express when they utter the sentence ‘There are features’ (though it is more difficult to think of a context in which that sentence would be uttered in the ordinary business of life or even in technical scientific endeavors). But it must be kept in mind that these sentences are just that—sentences, not propositions. They themselves neither entail nor are entailed. Nor are they the objects of affirmation or denial. Furthermore, I contend, the propositions that an ordinary English speaker (or a scientist employing English to make scientific assertions) asserts by using these sentences are entirely consistent with the proposition that I, as a metaphysician, express by uttering the words ‘There are no features.’

One immediate objection that springs to mind to this speech, coming, as it does, from the lips of a nominalist, is that appears to involve reference to such abstract entities as propositions and sentence types, and these entities seem no more nominalistically acceptable than do properties. This, I concede, is an important objection to the nominalist’s employing the above speech, one concerning which I will have more to say later. For now, however, let us set this issue aside. Is there any other point at which a lightweight platonist such as Van Inwagen may sensibly object?

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61 It strikes me that the most plausible examples of such cases are those in which ordinary English speakers encounter philosophers who deny the existence of features but who (the ordinary speakers that is) fail to appreciate the subtleties of ontological debates. Imagine such a speaker responding to a such a philosopher as follows: “What do you mean there are no features!? Are you denying that anything has features in common with something else?! That’s absurd! You and I are both human, for example. There’s a feature we have in common. Of course there are features!” Plausibly, this speaker is talking past the philosopher at this point; what he means by uttering ‘There are features’ expresses a different proposition than the negation of the proposition asserted by the philosopher when she utters ‘There are no features’. At least that’s a plausible thing to say given the views concerning the philosophy of language sketched below.

62 Though I have construed the nominalist who makes this speech as availing herself of none of the options that Van Inwagen offers, one could construe her as availing herself of option 3. It depends on how the occurrence of ‘there are anatomical features’ in 3 is to be understood. If it is to be understood as expressing the proposition that, by this nominalist’s lights, ordinary English speakers would express by those words, the one that follows from the proposition that such speakers would express by the original spider-insect sentence, then this nominalist does not avail herself of option 3. If it is to be understood, rather, as expressing the proposition that a metaphysician, speaking as such, would express by uttering those words, then she does avail herself of that option. Nothing of interest hangs on this issue, however.
The first thing to note concerning the above is that it ought to ring familiar to anyone who has carefully read Van Inwagen’s *Material Beings*. There Van Inwagen argues with great sophistication that the only things that have proper parts are living organisms and therefore that there are no composite, non-living artifacts (such as tables and chairs). The above speech was deliberately patterned off of the following passage from *Material Beings* in which Van Inwagen defends his views against the charge that they contradict our ordinary beliefs:

I do not deny [that when ordinary English-speakers, immersed in the ordinary business of life, utter sentences like, ‘There are two very valuable chairs in the next room’ … they very often say true things]. In fact, I affirm it. “Now, look [comes the objection]. ‘There are two very valuable chairs in the next room’ entails ‘There are chairs’, which is what you deny.” The objection is misconceived. ‘There are two very valuable chairs in the next room’ and ‘There are chairs’ are sentences, not propositions. Therefore, they neither entail nor are entailed and they are not the objects of affirmation and denial. Moreover, any of the propositions that an English speaker might express by uttering ‘There are two very valuable chairs in the next room’ on a particular occasion … is, I would argue, consistent with the proposition that I, as a metaphysician, express by writing the words ‘There are no chairs’. 63

Aside from the obvious difference in ontological subject matter, there are some salient differences between the above passage and the speech that I have placed upon the lips of the nominalist. Some of these differences are discussed below. For now, however, I’d like to say a few words concerning what this passage (as well as the nominalist’s speech) presupposes about the relationship between ordinary discourse and that of ontology.

How is it, according to Van Inwagen, that the proposition that an ordinary English speaker expresses by uttering the words ‘There are two very valuable chairs in the next room’ fails to contradict the proposition that he, as a metaphysician expresses by writing

63 (Van Inwagen 1990, pp. 100-101)
the words ‘There are no chairs’? No doubt if another metaphysician (a misguided kind of Moorean perhaps) were to employ the sentence ‘There are two valuable chairs in the next room’ as a premise in an argument against Van Inwagen, the proposition that she would be expressing by means of that sentence would stand in contradiction to Van Inwagen’s views. How is it, then, that the proposition expressed by an ordinary English speaker, when she utters that sentence, fails to do so?\footnote{64}

Van Inwagen’s answer is that the ordinary English speaker and the metaphysician express different propositions when they utter that sentence (even abstracting away from differences in content that might arise owing to the indexical elements found therein). The proposition expressed by the metaphysician, presumably, is the one that the standard compositional semantics would assign to that sentence (a proposition that is true only if there is a value for the variable ‘x’ such that the open sentence ‘x is a chair’ is satisfied when x is assigned that value).\footnote{65,66} That proposition is true only if there are things

\footnote{64} I take it that what is primarily at issue here is what is sometimes called “speaker meaning” (i.e. content that a speaker asserts in using the sentence at issue) rather than “sentence meaning” (i.e. the semantic value had by the sentence itself). Van Inwagen, however, does not say much concerning the latter; nor does he say much concerning the relationship between these two sorts of meaning. I will follow his lead in that regard. (Thanks to Jeff Speaks for noting the relevance of this distinction here).

\footnote{65} While most formal developments of compositional semantics involve systematically assigning extensional truth conditions to sentences rather than propositions, I am assuming (as is not uncommon) that we are to think of these formal developments as attempts to model the compositionally determined meanings of the sentences in their domain, and that we are, in turn, to think of the compositionally determined meaning of a sentence as either being identical to a proposition or, perhaps (where indexicals are in play), to a function from contexts to propositions. Those who think of propositions as structured entities might think of the proposition assigned to a given sentence by the standard compositional semantics as the one whose structure mirrors the structure of that sentence and whose structural components correspond to the meanings of the terms of that sentence. Even those who do not think of propositions as structured may find this a useful picture. Thanks to Jeff Speaks and to Meghan Sullivan for some helpful correspondence and conversation concerning these issues.

\footnote{66} In speaking of “the proposition assigned by the standard compositional semantics” to a given sentence (as I do here and elsewhere), I am skirting past a variety of different issues. First, when discussing the semantics for sentences of natural language, it is an idealization to speak of “the standard
available to be quantified over that are, in fact, identical to chairs. Whereas, according to Van Inwagen, “this sentence [as it is ordinarily used] is sufficiently empty of metaphysical commitment that the proposition it ordinarily expresses is consistent” with the claim that, he, as a metaphysician, expresses by denying that there are chairs. What drives this point of view on Van Inwagen’s part, furthermore, is the Wittgensteinian thought that we should be inclined to regard the most commonly held beliefs of those found within our linguistic community as being true, provided at least that the world may be plausibly held to cooperate, in the right way, with the point of the speech and thought by which the people in our linguistic community are disposed to express those beliefs.

But in most of the ordinary discourse in which we make use of sentences involving

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composition semantics” for those sentences. For many sentences, it is controversial, even among adherents to a compositional semantics for natural languages, as to how their compositional structure is to be regarded and as to what content they are to be assigned. Nevertheless, there is relatively little such controversy (at least among compositional semanticists) when it comes to the class of sentences that are the focus of this particular discussion, namely, sentences with a relatively straightforward quantificational structure (the sort of sentences that may well show up as translation exercises in a standard first order logic textbook). (Thanks to Jeff Speaks for a helpful conversation concerning this issue.) Second, for some declarative sentences that we might make use of in everyday life, even among those sentences that we might successfully use non-metaphorically and non-idiomatically to make sober assertions, it is at least debatable (even among those who believe in the existence of propositions) as to whether there is any proposition available for a compositional semantics to assign to those sentences. Suppose, for example, that, strictly speaking, there are no such things as shadows. Suppose furthermore that, for these reasons, the English predicate ‘is a shadow’ is semantically defective. Given these suppositions, it is plausible that there just is no proposition available for a standard compositional semantics to assign to a sentence such as ‘There is quite a lengthy shadow being cast by the elm tree in the backyard this afternoon.’ Nevertheless, for ease of exposition, and since nothing in my argument ultimately turns on this issue, I will speak as if each of the sentences that I will discuss are such that there is a proposition assigned to them by the standard compositional semantics. (Thanks to Peter van Inwagen for pressing me on this issue.) Finally, on account of phenomena such as vagueness, ambiguity, and the like, one might think that, in many cases, there are several propositions assigned to a given sentence (as candidate meanings) by the standard compositional semantics, thereby rendering it inappropriate to speak of “the proposition” so assigned; I intend to ignore this complication as well.

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67 (Van Inwagen 1990, pp. 106-107)

68 This was communicated to me by Van Inwagen (in conversation).
apparent quantification over chairs, recondite metaphysical debates over the nature of composition are, far and away, beside the point.

This response, however, leaves us with two further questions. First, which proposition does Van Inwagen take the ordinary English speaker to be expressing when she utters ‘There are two very valuable chairs in the next room’? Second, just what does he mean when he asserts that this proposition is “consistent” with the one that he expresses by writing ‘There are no chairs’?

Regarding the first of these questions, Van Inwagen writes:

The only thing I have to say about what the ordinary man really means by ‘There are two valuable chairs in the next room’ is that he really means that there are two valuable chairs in the next room. And we all understand him perfectly, since we are native speakers of our common language.69

This is as explicit, in Material Beings, as Van Inwagen gets about which proposition is expressed by means of this sentence in ordinary speech. Van Inwagen is, however, willing to offer paraphrases of such sentences, and he is willing to say something about the relationship between the paraphrases that he offers and the original sentences (as used in ordinary speech).

Consider, for example, the sentence ‘There are some chairs that are not currently being sat upon’ as uttered in the course of everyday life (as it might be uttered, for example, by someone reporting to an event coordinator at a concert).70 Van Inwagen would propose the following as a paraphrase of this sentence: ‘There are some simples

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69 (Van Inwagen 1990, pp. 106-107)

70 I choose to use this sentence as an example here (rather than the one that we have been discussing) because it is an easier matter to apply Van Inwagen’s paraphrase strategy to it.
arranged chairwise that are not currently being sat upon’. Furthermore, according to Van Inwagen, while the original sentence and the paraphrase sentence fail to express the same proposition, there is an intimate relationship between them, insofar as they may be properly said to “describe the same fact” as one another. He also points out that the paraphrase sentence, unlike the original sentence, does not even appear to express a proposition that demonstrably entails that there are such items as chairs.  

What about the second of the questions mentioned above, the question of just what Van Inwagen means by saying that the proposition an ordinary speaker would express by uttering a sentence like ‘There are chairs that are not currently being sat upon’ is “consistent” with the proposition that he, as a metaphysician expresses by using the sentence ‘There are no chairs’? The most straightforward reading of this claim would be that it is metaphysically possible that the propositions expressed by these sentences are both true. And Van Inwagen has personally informed me (in conversation) that this is what he would now be inclined to say in response to the question “What do you mean by saying that these two propositions are consistent?”

This interpretation, however, does not fit well with some of the other things that Van Inwagen said in Material Beings. There, he said, for example, that the proposition ordinarily expressed by ‘There are two very valuable chairs in the next room’ (in a given context) is consistent with various claims to the effect that certain entailments hold and also consistent with the denial of the claims that those entailments hold. He says, for example, that the proposition ordinarily expressed by the above chair sentence is

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71 (Van Inwagen 1990, ch. 11)
consistent with the claim that there being materials arranged chairwise in a certain region of space entails that there is a chair-shaped composite object that exactly fills that region, and also that it is consistent with the denial of that claim. But since entailments either hold or fail to hold as a matter of metaphysical necessity, it must be the case that, for at least one of these claims, it is not metaphysically possible for both it and the proposition ordinarily expressed by the chair sentence to be true.

I don’t think that we need to really settle this interpretive issue, however, to see the point that Van Inwagen is driving at. The claim for which he is trying to argue is that one may consistently believe both the proposition that he expresses, as a metaphysician, by uttering ‘There are no chairs’ and the proposition that an ordinary speaker, speaking as such, expresses by uttering the words ‘There are chairs that are not currently being sat upon’. And presumably all that is at issue here is whether one can consistently believe both claims in the sense that there is no good argument by which it can be shown on purely logical grounds that one of these propositions entails the denial of the other. Furthermore, it is worth noting, one can consistently believe two propositions in this minimal sense even if it is metaphysically impossible for both propositions to be true.

So, to recap, we can see that Van Inwagen (or at least the Van Inwagen of Material Beings) appears to be committed to the following claims: (1) When a speaker, in the ordinary business of life, utters the sentence ‘There are two very valuable chairs in the next room’ (in a given context), the proposition that she expresses is not identical to the proposition that the standard compositional semantics would assign (in the given

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72 (Van Inwagen 1990, pp. 106-107)

73 I am grateful to Patrick Todd for some useful correspondence about how to put this point.
context) to this sentence (the latter of which is true only if, strictly speaking, there really are such things as chairs). (2) One may consistently believe both the proposition that an ordinary speaker (speaking as such) expresses by means of this sentence and the proposition that Van Inwagen (speaking as a metaphysician) expresses by uttering the words ‘There are no chairs’ at least in the sense that it is not possible to show, solely on logical grounds, that one of these propositions entails the denial of the other. As a bit of an aside, it is also worth noting here that, given Van Inwagen’s views, he could even say that the proposition an ordinary speaker (speaking as such) would express by means of the sentence ‘There are some chairs’ is consistent with the proposition that he, speaking as a metaphysician, expresses by means of the sentence ‘There are no chairs’. He could even concede in this vein that the former proposition is entailed by the proposition that an ordinary speaker would express by uttering ‘There are two very valuable chairs in the next room’ without thereby conceding that the denial of the latter is.

But, of course, it is open to the nominalist to say the exact same things, mutatis mutandis, concerning the proposition that an ordinary speaker might express by means of uttering the sentence ‘Spiders share some of the anatomical features of insects’ and the proposition that she (the nominalist) expresses, speaking as a metaphysician, by uttering the sentence ‘There are no features’. Our ordinary discourse involving apparent quantification over features is no more or less detached from recondite metaphysical debates concerning the status of universals than our ordinary discourse involving apparent quantification over chairs is detached from recondite metaphysical debates.

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74 Imagine, for example, that the curator of the antique museum is asked whether there are any chairs among the collection and that he responds by saying “There are some chairs. In fact, there are two very valuable chairs in the next room.”
concerning the nature of composition. So it would seem that if, on account of such detachment, the second kind of discourse is neutral with respect to the existence of non-living composite objects, then so is the first neutral with respect to the existence of abstract objects.

At least so it would seem initially. As Van Inwagen would no doubt point out, however, there is at least one salient difference between his own situation and that of the nominalist. Van Inwagen is prepared to offer paraphrases of the sentences of our ordinary discourse involving apparent quantification over chairs. The nominalist (at least as we are currently envisioning her), by contrast, is not prepared to offer any paraphrases of the sentences of our ordinary discourse that involve apparent quantification over features. In fact, the nominalist, as we are envisioning her, is even conceding for the sake of argument that it is not so much as possible in principle for her to offer adequate paraphrases of all of those sentences. No doubt this is a salient difference between the nominalist’s situation and that of Van Inwagen. But just what exactly is its significance? How does that difference in any way affect how the nominalist should evaluate the cogency of Van Inwagen’s argument?

Suppose that the nominalist adopts Van Inwagen’s views concerning the relationship between ordinary discourse and metaphysical discourse and she infers from that view that the proposition expressed by spider-insect sentence is in fact neutral with respect the existence of features, neutral in such a way that it cannot be shown, solely on logical grounds, that that proposition entails the denial of what she (speaking as a metaphysician) expresses by uttering the sentence ‘There are no features’. Then she already has an independent reason (stemming from her independently motivated views
concerning the philosophy of language) to think that there is no demonstrable inconsistency between the proposition expressed by the spider-insect sentence and her denial of the existence of features. She has this reason, furthermore, independently of any ability on her part to find a paraphrase of that sentence. But if that is so, then even prior to coming up with any suitable paraphrase of the spider-insect sentence, or even in the absence of an ability on her part to come up with such a paraphrase, the nominalist has reason to believe that Van Inwagen’s attempt to show that there is an inconsistency among the propositions that she affirms is a failure. Given her views concerning the philosophy of language (which she shares with Van Inwagen!), Van Inwagen’s argument doesn’t even so much as get off the ground.

But perhaps Van Inwagen might insist that, even given his views concerning the philosophy of language, the nominalist is not entitled to the claim that the proposition expressed by the spider-insect sentence is neutral with respect to the existence of features, unless she is able to give an adequate paraphrase of that sentence. Recall again that what drives Van Inwagen’s views concerning the relationship between ordinary and metaphysical discourse is the Wittgensteinian thought that we should be inclined to regard the most commonly held beliefs of those found within our linguistic community as being true, provided at least that the world may be plausibly held to cooperate, in the right way, with the point of the speech and thought by which the people in our linguistic community are disposed to express those beliefs. One thing that Van Inwagen’s paraphrase strategy does, furthermore, is exhibit just how it is that the world can be seen to cooperate with the point of our ordinary discourse involving apparent quantification over chairs without there actually being such entities as chairs. It makes no difference,
for example, to our ordinary practical purposes, whether there really are chairs that are
not currently being sat upon or whether there are merely some simples arranged
chairwise that are not currently being sat upon. Either way, the world is such that the
point of an ordinary utterance of the sentence ‘There are some chairs that are not
currently being sat upon’ is fulfilled. And, since it makes no practical difference which is
the case, the Wittgensteinian line thought that we are considering strongly lends itself
toward the view that the proposition ordinarily expressed by the aforementioned sentence
is one that can be true even if, strictly speaking, there are no such things as chairs, but
mere simples arranged chairwise. In the absence of a paraphrase on her part, however,
the nominalist is left without a similar argument for the claim that the proposition
expressed by the spider-insect sentence is neutral with respect to the existence of features.

We might put it this way: Let’s agree to say that the proposition that is
“perspicuously expressed” (in a given context) by a given sentence is the one assigned to
it by the compositional semantics (with respect to the given context). Let’s also stipulate
that we’ll call the proposition that would be expressed by means of a sentence were it
uttered in the ordinary business of life (in a given context) the proposition that is
“ordinarily expressed” by that sentence (in that context).\textsuperscript{75} Given this terminology, we
can say that, according to Van Inwagen’s views concerning the philosophy of language

\textsuperscript{75} Given certain widely held views in the philosophy of language, the distinction employed here
between the proposition that is “perspicuously expressed” by a given sentence and the proposition that is
“ordinarily expressed” by that sentence maps on to the commonly drawn distinction between what is
“semantically expressed” by a given sentence and what is “pragmatically conveyed” by that sentence (on a
given occasion of use). However, by employing the latter terminology, I would be taking on some fairly
substantive commitments concerning the philosophy of language (e.g. that what a sentence semantically
expresses corresponds to the proposition assigned to it by the standard compositional semantics)
concerning which I would prefer to remain neutral.
(in conjunction with his views concerning the metaphysics of composition), the proposition that is ordinarily expressed by the sentence ‘There are some chairs that are not currently being sat upon’ is true, even though the proposition that is perspicuously expressed by that sentence is false. What Van Inwagen’s paraphrase strategy allows him to do, furthermore, is find a sentence that perspicuously expresses a proposition that is equivalent to the proposition ordinarily expressed by the original sentence, one which does not so much as appear to express a proposition that is (on solely logical grounds) demonstrably inconsistent with his metaphysical views, and one which would serve just as well as far as the point of uttering the original sentence is concerned (at least when setting aside considerations of length and awkwardness). It is in this manner that Van Inwagen is able to exhibit the fact that the world can be seen to cooperate with the point of ordinary utterances of chair sentences even if in fact, strictly speaking, there are no such things as chairs.

Given all that we have granted for the sake of argument, however, the nominalist has no similarly adequate way of paraphrasing the sentence ‘Spiders share some of the anatomical features of insects’. In fact, given all that we have granted, the nominalist concedes that she is unable, even in principle, to supply a paraphrase of this sentence. Thus, given all that we have granted, the nominalist cannot exhibit, by means of a paraphrase, how the world might be seen to cooperate with the point of an ordinary utterance of that sentence in the absence of there really being such entities as features. And this, one might argue, gives the nominalist good reason to think that in order for the point of uttering the spider-insect sentence to come off, there must really be such entities as features, some of which both spiders and insects exemplify. And so, even given the
Wittgensteinian line of thought underlying Van Inwagen’s views, one might argue that the nominalist has good reason to think that the proposition ordinarily expressed by the spider-insect sentence is identical to the one that is perspicuously expressed by it, or, at the very least, that these propositions are logically equivalent to one another. And therefore, even given Van Inwagen’s views concerning the philosophy of language, one might argue that the nominalist has a good reason to think that the proposition expressed by the spider-insect sentence is inconsistent with her nominalist views.

The nominalist may rightfully object, however, that this line of thought moves by too quickly. Granted, she cannot exhibit, by means of supplying a paraphrase, how it is that the world cooperates with the point of an ordinary utterance of the spider-insect sentence. But that fact alone does not obviously entail that she has no other means of exhibiting how the world might be seen to cooperate with the point of such an utterance. Here’s an alternative strategy that the nominalist might adopt: Let’s say that a sentence is nominalistically friendly if and only if it meets each of the following conditions: (i) It does not even appear to involve quantification over nominalistically unacceptable entities. (ii) Each of the referring expressions that occurs in it refers to a nominalistically acceptable entity. (iii) It cannot be shown, on purely logical grounds (absent any question begging assumptions against the nominalist), to express a proposition that entails the existence of nominalistically unacceptable entities. (iv) When all of its quantifiers and quantificational phrases are restricted so as to range only over

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76 Some philosophers might think, for example, that it is a matter of logic that the following schema (at least when appropriately qualified to avoid Russell paradoxes) is valid: \("If \(a\) is \(F\), then \(a\) exemplifies the property of being \(F\)\) (where the propositions expressed by each instance of this schema is to be regarded as the proposition that is perspicuously expressed by that instance). The nominalist, however, could properly complain that it would be question begging against her for the platonist to simply take it for granted, in the context of their debate, that this schema is valid.
nominalistically acceptable entities, it perspicuously expresses a proposition with the same truth value as the one that it originally expressed.\textsuperscript{77} Now let's also say that a proposition is nominalistically acceptable if and only if it is perspicuously expressed by a nominalistically friendly sentence.\textsuperscript{78} One way for the nominalist to exhibit how it is that the world manages to cooperate with the point of an ordinary utterance of the spider-insect sentence, in lieu of finding a suitable paraphrase, would be for her to exhibit the fact that there are true, nominalistically acceptable propositions whose truth is jointly sufficient for the point of such utterances to be realized, even in the absence of there really being any such entities as features. If she can do that much, she can show how it is that the nominalistically acceptable facts are, by themselves, enough for the point of such an utterance to be fulfilled. And if the nominalistic facts are, by themselves, enough for the point of such an utterance to be fulfilled, then the Wittgensteinian line of thought that underlies Van Inwagen's own views concerning the relationship between ordinary and metaphysical discourse strongly lends itself to the view that the proposition that is ordinarily expressed by means of the spider-insect sentence is one that is itself neutral with respect to the truth of nominalism.

\textsuperscript{77} This last clause implies that whether a sentence is nominalistically friendly may not always be discernible absent certain items of non-linguistic knowledge. Consider, for example, the sentence ‘There are infinitely many things’ (where the quantifier that occurs therein is to be understood as being completely unrestricted). Suppose that this sentence does indeed express a true proposition. Whether it continues to express a true proposition when its quantifiers are restricted to concreta depends on whether or not there are, in fact, infinitely many concrete things, and whether or not there are that many concrete things may well be unknown.

\textsuperscript{78} Note that it does not follow from this characterization of what it is for a proposition to be nominalistically acceptable that every proposition that is not (on purely logical grounds) demonstrably inconsistent with nominalism is nominalistically acceptable. Suppose, for example, that the sort of nominalist whose views we are considering is correct in insisting that the proposition ordinarily expressed by the spider-insect sentence is not demonstrably inconsistent with the non-existence of features. Suppose also, however, that there is no sentence (not even of any suitably idealized language) that perspicuously expresses that proposition. Given these suppositions, the proposition expressed by the spider-insect sentence is not (by the current definition) nominalistically acceptable, in spite of the fact that it is also not demonstrably inconsistent with nominalism.
When it comes to the spider-insect sentence, furthermore, it is relatively easy for the nominalist who is sufficiently up to speed in terms of her entomological knowledge to find such true nominalistically acceptable propositions. The proposition perspicuously expressed by the sentence 'Spiders are segmented and insects are segmented' would do, for example. This proposition happens to be both true and nominalistically acceptable. And the nominalist (simply by virtue of being a competent English speaker) can see that if this nominalistically acceptable proposition is true, the point of an ordinary utterance of the original spider-insect sentence is fulfilled, regardless of whether or not there are any such entities as features (provided at least, as is plausibly the case, that the point of such an utterance is to convey accurate information about how spiders and insects are similar to one another and not to convey information about abstract entities).

Suppose, however, that the nominalist finds herself uncommonly ignorant of the entomological facts. Suppose indeed that her ignorance is so extensive that she finds herself unable to fill in the blanks of the following schema in such a way that she knows the resulting sentence to express a true proposition: "Spiders are _____ and insects are _____" (where the blanks are to be uniformly filled by an adjective that is itself an anatomical term). Even so, she knows that if the point of an ordinary utterance of the spider-insect sentence is to be fulfilled, there must be some English sentence that is both an instance of this schema and which expresses a true proposition, provided, at least, that English contains an adequate stock of anatomical predicates. Even if she is not certain, furthermore, that English contains an adequate stock of such predicates, she knows that if the point of an ordinary utterance of the spider-insect sentence is to be fulfilled, there must be some true proposition that could be expressed by a sentence that is an instance of
this schema in an expanded version of English, one that does contain an adequate stock of anatomical predicates. And so she can see that if the point of an ordinary utterance of the original spider-insect sentence is to be fulfilled at all, there must be some true, nominalistically acceptable proposition whose truth is sufficient (even in the absence of there really being such entities as features) for the point of such an utterance to be fulfilled. And so, even in the absence of her being able to cite such a proposition, the Wittgensteinian line of thought that she has taken up still pushes her toward the view that the proposition expressed by the original spider-insect sentence is one that is neutral with respect to the existence of features.

It is fortunate for the nominalist, furthermore, that her ability to take this Wittgensteinian line with respect to various sentences that involve apparent quantification over features does not crucially depend on her ability to actually cite nominalistically acceptable propositions whose truth is jointly sufficient for the point of ordinary utterances of those sentences to be fulfilled. That is because, in some cases, the cognitive and linguistic limitations that keep her from citing such propositions may well prove ineliminable. Consider, for example, the following sentence (suppose it is uttered by a famous entomologist while in the process of giving a rather dry lecture): ‘There are distinct female spiders of the same species, and any two female spiders of the same species share all and only the same anatomical features.’ It is not so easy, in this case, for the nominalist to come up with a list of sentences that express nominalistically acceptable propositions whose truth would be jointly sufficient for the world to cooperate with the point of the utterance of this sentence. Plausibly, any such list of sentences would have to be infinitely long, and (also plausibly) any language capable of expressing each of the
propositions that would have to be expressed on such a list would have to be a language that vastly outstrips any existing natural language in terms of its expressive resources.

Nevertheless, the nominalist can still see, simply in virtue of being a competent English speaker, that if the point of uttering the aforementioned sentence is to be fulfilled, there must be some such propositions (keep in mind here that we are still setting aside nominalistic scruples about quantifying over things like propositions and sentence types). She can see, for example, that if in fact there are distinct, conspecific female spiders) the point of an ordinary utterance of that sentence is fulfilled if and only if each of the propositions expressed by each of the instances of the following schema are true (where the instances of this schema are to be taken from an idealized language, one that is much like English, but one that lacks for no relevant expressive resources):

\[
\begin{align*}
\text{Any two conspecific, female spiders are such that if one is} & \quad \text{_____}, \quad \text{the other is} \quad \text{_____} \\
\end{align*}
\]

(where the blanks are to be uniformly filled by an adjective that is itself an anatomical term). It is clear (assuming, at any rate, that the relevant language is indeed adequately expressive) that these sentences, together with ‘There are distinct, conspecific, female spiders’ perspicuously express propositions that are jointly sufficient for the point of the entomologist’s utterance of the sentence we have been considering to be fulfilled, regardless of whether or not there are in fact such entities as features. Thus, the Wittgensteinian line that our nominalist has taken up also strongly lends itself to the claim that this sentence, as uttered by the entomologist, expresses a proposition that is neutral with respect to the existence of features.

\[79\text{I will assume throughout this discussion that, doubts about the existence of abstract objects aside, languages exist, as a matter of metaphysical necessity, “in Plato’s heaven” (as it were), regardless of whether they are ever instantiated by a linguistic community.}\]
Even given all of the above, however, the nominalist’s inability to provide a paraphrase of such sentences still threatens to supply another source of embarrassment for her. Suppose that the platonist and the nominalist both agree that when they are speaking “in the ontology room” (as Van Inwagen might put it), the semantics governing their sentences is to be regarded as strictly compositional; that is, the propositions expressed by each of their sentences are to be regarded as the propositions that are perspicuously expressed by them. Suppose, furthermore, that the platonist and the nominalist both agree (rightly or wrongly) that the sentence ‘There are distinct female spiders of the same species, and any two female spiders of the same species share all and only the same anatomical features’, as used in everyday life, expresses a truth. If the proposition ordinarily expressed by this sentence is not the one that the standard compositional semantics assigns to it, then perhaps neither the platonist nor the nominalist can express that proposition in the ontology room (where the semantics that governs their discourse is strictly compositional). Nevertheless, the platonist can use that very sentence in the ontology room to express a proposition that, by his lights, is true, equivalent to the proposition expressed by that sentence in ordinary life, and just as good for practical purposes. The nominalist, however, cannot. And given that she lacks an adequate paraphrase of that sentence, she cannot consistently assert, in the ontology room, any other true proposition that (by her lights) has these virtues. So it looks like the nominalist is committed to the view that there is a truth about the world that she can express outside the ontology room such that there is no truth equivalent to it that she can express in the ontology room. And this initially seems embarrassing. Isn’t one’s metaphysical theory supposed to give one (or at least to move toward giving one) a
comprehensive way of describing the world that perspicuously represents things as they are? And if so, isn’t it a mark against one’s metaphysical theory that one cannot perspicuously say all that one might wish to say about the world without expressing propositions that are demonstrably inconsistent with that theory?

“Not always,” the nominalist can reply. Consider again the case in which the nominalist finds herself ignorant of the entomological facts in such a way that she is unable to fill in the blanks of the following schema so that she knows the resulting sentence to express a true proposition: [Spiders are _____ and insects are _____] (where the blanks are to be uniformly filled by an adjective that is itself an anatomical term). The nominalist can still point out that if she could so fill in the blanks, she would be able to cite a true proposition that conveys at least as much information concerning how spiders and insects are similar to one another as does the proposition ordinarily expressed by the sentence ‘Spiders share some of the anatomical features of insects’, but one which she could express without even appearing to quantify over features. As it happens, she merely lacks the knowledge or the items of vocabulary required to fill in the blanks. But, she may rightfully point out, as Joseph Melia has done, that the fact that she is forced, merely on account of her cognitive and linguistic limitations, to appear to quantify over features (in order to say all that she might want to say) is not itself any good reason to think that there really are such things as features.80

And the same goes, mutatis mutandis, she might add (still taking her cues from Melia), for those cases in which the relevant cognitive and linguistic limitations are (even

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80 (Melia 1995)
in principle) ineliminable. There is no reason to think that human cognitive and linguistic limitations afford us with a reliable guide to what there is. So if she can plausibly maintain that it is merely on account of human cognitive and linguistic limitations that she cannot, without contradicting her nominalistic views, perspicuously say all that she might want to say about the world, she can also plausibly maintain that her inabilities in this regard do not afford her with any good reason to abandon those views.

The nominalist who adopts Van Inwagen’s views concerning the relationship between ordinary discourse and metaphysical discourse can, therefore (according to those views) properly regard the proposition expressed by the original spider-insect sentence as being neutral with respect to the existence of features, neutral in such a way that there is no logically demonstrable inconsistency between that proposition and the nominalist’s claim that strictly speaking there are no such entities as features. She can do so, furthermore, even in the absence of having an adequate paraphrase of that sentence. And, given that she can properly maintain all of this, it would seem that she is left with no burden to supply the sort of paraphrases that Van Inwagen would demand of her.

Van Inwagen suggests one other reason, however, as to why the nominalist should take herself to have the burden of supplying paraphrases for the original spider-insect sentence and those like it. Without such paraphrases, he argues, the nominalist has no adequate way of accounting for the logical relations that we take to hold among such sentences. In the next section, I flesh out this objection and consider how the nominalist might respond to it.
2.4 Inference without Paraphrase

The complaint that the nominalist cannot account for the logical relations among the relevant class of sentences is suggested by a criticism that Van Inwagen levels against a certain kind of nominalist reply to his argument, one that he places on the lips of his fictional, nominalist interlocutor, Norma. Norma proposes to paraphrase the sentence ‘Spiders share some of the anatomical features of insects’ with ‘Spiders are like insects in some anatomically relevant ways’. But she denies that, in using this sentence, she intends to quantify over such entities as ways. Rather, she maintains that she meant the open sentence ‘x is like y in some anatomically relevant ways’ to have no internal logical structure, or none beyond that implied by the statement that two variables are free in it.”

Van Inwagen objects to this response on the grounds that Norma’s failure to attribute the internal structure of quantification to this open sentence leaves a variety of its logical features unaccounted for. In particular, according to Van Inwagen, her denial fails to account for the logical relations between this open sentence and other sentences. Van Inwagen notes, for example, that there is obviously some structural or syntactical relation between the sentence ‘x is like y in some physiologically relevant ways’ and ‘x is like y in some anatomically relevant ways’ and challenges the nominalist to account for this relation. Here I agree with Van Inwagen that this is a legitimate challenge to Norma’s position, one which may well be insurmountable for her.

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81 (Van Inwagen 2004, pp. 119-120)
82 (Van Inwagen 2004, p. 120)
But the nominalist who avails himself of the reply to Van Inwagen’s argument that I have suggested above – call him “Norm” – does not face *this particular* challenge. Unlike Norma, he does not (or at least need not) deny that open sentences such as the above have the internal logical structure of quantification. Such structure, he would no doubt emphasize, is merely a matter of *syntax*, and he does not deny that these open sentences have the syntactical features they appear to have.\(^{83}\) Rather, he simply denies that the declarative sentences into which they figure are ordinarily used to express the propositions assigned to them by the standard compositional semantics.\(^{84}\) Accordingly, he can easily accept that the sentence ‘Spiders and insects are alike in some anatomically relevant ways’ expresses a truth, affirm that the open sentence ‘x is like y in some anatomically relevant ways’ has the internal logical structure of quantification, all the while consistently denying that any of these affirmations commit him to the existence of such entities as *ways*.\(^{85}\)

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\(^{83}\) Norm can refrain from denying this, furthermore, without committing himself to the claim that the syntactical features of a sentence that matter as far as its logical structure are concerned correspond to the syntactical features that can be read straightforwardly off of its surface grammar. Norm may allow for the possibility that various idiomatic devices and the like sometimes abbreviate or mask the genuine logical syntax of a given sentence. What matters, Norm can hold (at least as a heuristic device), is how the original English sentence (with its idiomatic components and the like taken into account) would be translated into the canonical language of quantification by a component English speaker who is excellent at logic but unscrupulous about matters of ontology.

\(^{84}\) If one is tempted to say that the *mere* fact that the latter sentences are not ordinarily used to express the propositions assigned to them by the standard compositional semantics is a fact indicating that these open sentences do not have the internal logical structure of quantification, then Norm may reply that if *anything like* the distinction between what is semantically expressed by a sentence and what is pragmatically conveyed by it is correct, sentences are sometimes used to express or convey propositions that differ from those perspicuously expressed by them. Indeed, the very distinction at issue (however exactly it maps on to the semantically expressed/pragmatically conveyed distinction – see note 75), the distinction between what is ordinarily expressed by a sentence and what is perspicuously expressed by it, presupposes that the sentences in question express propositions other than those that are assigned to them by the standard compositional semantics in virtue of their logical syntax.

\(^{85}\) “If these open sentences have the internal logical structure of quantification,” one might ask, “then what are they quantifying over?” Assuming that there are no such entities as *ways*, I suppose that the
Similarly, Norm has a ready reply to Van Inwagen’s challenge that he account for the validity of inferences like the following:

\( (a) \) Either of two female spiders of the same species is like the other in all anatomically relevant ways.

\( (b) \) Hence, an insect that is like a given female spider in some anatomically relevant ways is like any female spider of the same species in some anatomically relevant ways.

As Van Inwagen notes, “if the premise and conclusion of this argument are read as having the logical structure that their syntax suggests, the validity of this argument is easily demonstrable in textbook quantifier logic.”\(^{86}\) Norma, denying as she does that these sentences have the internal logical structure of quantification, cannot avail herself of this account of their validity. Norm, however, has no such difficulty since he makes no such denial (or, at least, he needn’t make any such denial).

More precisely, what Norm has no difficulty accounting for is the \textit{formal} or \textit{syntactical} validity of such arguments. No doubt, however, that in denying as he does that sentences such as \( a \) and \( b \) express the propositions assigned to them by the standard compositional semantics, Norm leaves himself vulnerable to another kind of objection to his position. The whole point of developing a formal logic is to have a way of ensuring that various inferences one makes are \textit{truth preserving}. That is, one wants a guarantee

\(^{86}\) (Van Inwagen 2004, p. 120)
that if one reasons from true premises, in accordance with the rules of the formal logic one is employing, one will arrive at true conclusions. And, to secure that result, one will want it to be the case that various sentences follow from others according to the rules of one’s formal logic only if the propositions expressed by the latter sentences jointly entail the former.

The platonist has a ready account of how it is that we have a guarantee that the first order validity of the inference from $a$ to $b$ ensures that the proposition expressed by $a$ entails the proposition expressed by $b$. She may take $a$ and $b$ (respectively) to be synonymous with the following:

$(a')$ Either of two female spiders of the same species shares all and only the same anatomical features with the other.

$(b')$ Hence, an insect that shares some anatomical features with a given female spider shares some anatomical features with any female spider of the same species.

Furthermore, she may take the propositions ordinarily expressed by $a'$ and $b'$ to be (at the very least) necessarily equivalent to the propositions assigned to those sentences by the standard compositional semantics. She may then note that the inference from $a'$ to $b'$ is formally valid by the standard rules of first order logic and that the magnificent developments in metalogic from the twentieth century show us how this fact guarantees that the proposition assigned to $a'$ by the standard compositional semantics entails the proposition so assigned to $b'$. The nominalist, by contrast, has no such story to offer.

Now, one way that the nominalist could try to respond to this objection, the way that Van Inwagen suggests that she ought to try to respond, is by trying to offer
paraphrases of \(a\)' and \(b\)', paraphrases that appropriately mirror the logical relations between them, but which do not even appear to involve quantification over features or anything that the nominalist wouldn’t like any better than features. The nominalist can offer a satisfying response to the above challenge, Van Inwagen believes, only if she can offer such paraphrases. But (he also argues) she can’t. Therefore, Van Inwagen concludes, she has no satisfying response to the above objection.\(^{87}\)

Another way that Norm might respond to the above objection, however, is just to admit that, while he believes that formally valid arguments that employ sentences involving apparent quantification over features are truth preserving (perhaps he simply takes this as a pre-philosophical datum), he has no account to offer as to why this is so. This response may not be wholly intellectually satisfying. But it is hard to convict Norm of irrationality on that account (not the sort of irrationality, at any rate, that would be involved in his holding beliefs that he knows to be inconsistent with one another). In response to this objection, Norm might well be disposed to shrug his shoulders and offer the following reply: “I’m a metaphysician and am inured to mystery.”\(^{88}\) Norm might be so disposed because he takes himself to be in possession of powerful arguments against platonism, or he might simply find platonism so incredible that he is willing to live with mystery here rather than avail himself of the platonist’s way of resolving it. At worst, Norm is here forced to concede that here is one consideration that favors platonism over nominalism, one consideration to be balanced against others. That’s a far cry from the

\(^{87}\)(Van Inwagen 2004, pp. 120-121)

\(^{88}\)See (Van Inwagen 2007, p. 201)
original charge that Norm’s nominalism affords him with a demonstrably inconsistent set of beliefs.

As it stands, however, Norm is not forced to concede even that much. It’s not really all that mysterious, from his perspective, that arguments such as those discussed above are truth preserving. Before we consider how Norm might dispel whatever apparent mystery there is (from his nominalistic perspective) in accounting for the truth preserving character of the inference from \(a'\) to \(b'\), though, let us turn to a slightly easier example for him to deal with. Consider the inference encapsulated by the following sentences:

\[
(c) \text{ Shelob and Charlotte share at least two anatomical features in common.}
\]

\[
(d) \text{ Hence, there is an anatomical feature that Shelob has that Charlotte also has.}
\]

Now, though Norm concedes that he does not have a paraphrase to offer of either \(c\) or \(d\), Norm can say that in grasping the propositions ordinarily expressed by \(c\) and \(d\) (which he manages to do by virtue of his being a competent English speaker), he understands enough about the truth conditions of those propositions (even if he is unable to describe them in an informative way)\(^9\) to see that in each world in which the proposition

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\(^9\) I take it that Van Inwagen himself is committed to the claim that a competent English speaker may understand enough about the truth conditions of a given proposition to see what various of its entailments are without being able to describe those truth conditions in an informative way. If Van Inwagen is correct, for example, the truth conditions for the ordinary claim that there are some chairs that are not currently being sat upon are informatively described by the sentence ‘There are some simples arranged chairwise that are not currently being sat upon’. A competent English speaker, however, may well be unaware of that fact. Still, such a speaker might also understand enough about the truth conditions for this claim to understand that it entails the ordinary claim that there is some furniture that is not currently being sat upon. Such a speaker may well still grasp that under any possible conditions in which things are like \(that_1\) [the way things are when the ordinary claim that there are some chairs that are not currently being sat upon is true] they are also like \(that_2\) [the way things are when the ordinary claim that there is some furniture that is not currently being sat upon is true] without grasping that the possible conditions in which things are like \(that_1\) are those conditions in which there are some simples arranged chairwise.
ordinarily expressed by $c$ is true, the nominalistically acceptable facts are such that the proposition ordinarily expressed by $d$ is also true.\footnote{It is not clear that (for his current purposes) Norm needs to take the sort of apparent reference to and quantification over possible worlds that occurs here and in the following discussion all that seriously (as opposed to using it as a heuristic device). But, since we are currently prescinding from the question of whether, as a nominalist, Norm is entitled to help himself to apparent quantification over such entities as propositions and sentence types, we could, if we liked, allow Norm to identify possible worlds with propositions (in particular, with those propositions that are maximal with respect to entailment – see Van Inwagen, 1986).}

Norm can see, for example (doubts about the existence of propositions and sentence types aside!), that in every world in which the proposition ordinarily expressed by $c$ is true, its truth will be jointly entailed by two distinct, true propositions, each of which is perspicuously expressed by a sentence (of a suitably idealized, suitably expressive language) that conforms to the following schema: \(\text{\textquoteright}\text{Shelob is } ____ \text{ and Charlotte is } ____\text{\textquoteright}\) (where the blanks are to be filled in uniformly by an adjective that is an anatomical term). Norm (due either to his ignorance or to his limited vocabulary) may not be able to cite any such true propositions (at least not any that he knows to be true); nevertheless, simply by grasping the proposition ordinarily expressed by $c$, he can see that there must be some such true propositions in any world in which that proposition is true. It is also clear that any proposition that conforms to the above schema entails the proposition ordinarily expressed by $d$. Thus, without endorsing platonism or supplying a paraphrase, Norm can see that in every world in which the proposition ordinarily expressed by $c$ is true, there will be true, nominalistically acceptable propositions that entail the proposition ordinarily expressed by $d$. So it is no real mystery to him that the former proposition entails the latter.
Now consider the inference encapsulated by the following:

(a’) Either of two female spiders of the same species shares all and only the same anatomical features with the other.

(e) Shelob is a female spider and Charlotte is a female spider of the same species.

(f) Ladybug is an insect who shares an anatomical feature with Shelob.

(g) Hence, Ladybug shares an anatomical feature with Charlotte.

Again, simply in virtue of grasping the proposition expressed by a’, Norm can see that, in any world in which that proposition is true, each of the propositions perspicuously expressed by the infinitely many sentences (of a suitably idealized, suitably expressive language – I will, for the most part, suppress this qualification from now on) conforming to the following schema must also be true: "Any two conspecific, female spiders are such that one is ____ if and only if the other is ____" (where the blanks are to be uniformly filled in by an adjective that is itself an anatomical term). Furthermore, Norm can see that if the proposition ordinarily expressed by f is true, there must also be some true proposition expressed by a sentence of the form "Ladybug is ___ and Shelob is ___" (where the blanks are to be uniformly filled in by an adjective that is an anatomical term).

And it is obvious that any world in which the proposition ordinarily expressed by e is true, in which the infinitely many propositions perspicuously expressed by sentences conforming to the first schema are true, and in which a proposition perspicuously expressed by a sentence conforming to the second schema is true, is also a world in which there is a true proposition perspicuously expressed by a sentence that conforms to the following schema: "Ladybug is ___ and Shelob is ___ and Charlotte is ___" (where the blanks are to be uniformly filled in by an adjective that is an anatomical term). And,

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clearly enough, any proposition perspicuously expressed by a sentence conforming to this schema entails the proposition ordinarily expressed by \( g \).

So, though he cannot supply paraphrases of \( a', f \) and \( g \), Norm can nevertheless see (simply by grasping what sort of truth conditions the propositions ordinarily expressed by these sentences must have) that the propositions ordinarily expressed by these sentences, together with the proposition ordinarily expressed by \( e \), jointly entail the proposition ordinarily expressed by \( g \). And he can see this by virtue of seeing that in any world in which the first three propositions are true, the nominalistically acceptable facts must be such that the last of these propositions is also true. It is also clear (by “generalization from the arbitrary case,” as it were) that if the above inference is valid, then so is the inference with which we began our inquiry – namely, the inference from \( a' \) to \( b' \).

The above considerations render it plausible that the propositions ordinarily expressed by the class of sentences we are considering (i.e. sentences involving apparent quantification over features) are such that their entailment relations mirror the relations of formal, logical implication that hold among those sentences (at least those implication relations that are captured by first order logic), irrespective of whether or not there are features. In the next chapter, I will offer a formal proof to the effect that these considerations do in fact generalize, at least to a very wide class of sentences of the same sort as those that we’ve been discussing. If such mirroring does occur, however, it is no longer any mystery (even from a nominalistic perspective) that the formal arguments we have been considering are truth preserving. It is also no mystery, if such mirroring occurs, why it is that we would appear to quantify over features in ordinary (and even
technical) speech; the presence of such mirroring would render it useful, in many contexts, to talk and reason as if there were such entities, even if there were not.

The success of the above response, however, heavily depends on its being the case that, in giving it, Norm does not illicitly help himself to any resources that are not there to be had given his own ontology. In the next section, I consider an objection to the effect that, in giving the above response, Norm does just that.

2.5 Nominalistically Acceptable Truth Conditions

Until now I’ve been setting aside the issue of whether it is legitimate for the nominalist to engage in apparent quantification over entities such as propositions and sentence types. Thus, in the previous section, I permitted Norm to help himself to such quantification in answering the charge leveled against him in the previous section – namely, that his nominalism renders him unable to give a satisfactory account of the logical relations that hold among the various sentences involving apparent quantification over features. From now on, for the sake of convenience, I’ll refer to the challenge posed by the prospect of answering this charge as “the inference problem.”

Strictly speaking, Van Inwagen does not deny that those who disbelieve in the existence of features are able to resolve the inference problem. He readily grants that such individuals may offer paraphrases of the relevant class of sentences such that the logical relations among the paraphrases appropriately mirror the logical relations among the original sentences. He points out, for example, that one could replace apparent quantification over features in such sentences with quantification over concepts. But, as
he also notes, “it is certain that a nominalist will be no more receptive to an ontology that contains concepts (understood in a platonic or Fregean sense, and not in some psychological sense) than to an ontology that contains properties.”

The official conclusion of Van Inwagen’s argument, recall, is not that we cannot consistently get by without believing in properties; rather, his conclusion was that we cannot consistently get by without believing in properties or in other entities that are equally unacceptable from a nominalist point of view. And here one might think that Norm, in helping himself to apparent quantification of propositions and sentence types in the manner that he is portrayed as doing in the previous section, does not successfully avoid Van Inwagen’s conclusion. Rather, one might think, he merely trades one sort of nominalistically unacceptable commitment (i.e. to features) for another (i.e. to propositions and sentence types). In this section, I offer a response to this objection.

One thing to be said, right of the bat, on Norm’s behalf, is that there seems to be something dialectically infelicitous, in the current context, about any mere objection to his employing apparent quantification over entities such as propositions and sentence types. Norm, it should be remembered, is responding to an objection originally put forward by the platonist to the effect that various sentences that Norm endorses have logical consequences that Norm is unable to accept or for which Norm is unable to account. By raising that objection, it was the platonist who introduced the use of sentences involving apparent quantification over propositions and sentence types into the discourse. By engaging in such apparent quantification in kind, Norm is simply playing

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91 (Van Inwagen 2004, pp. 116-117)
along (it would be rather pedantic on his part if Norm replied by insisting that the platonist state the objection in a clearly nominalistically acceptable way before he is obliged to respond). Granted, if Norm ultimately finds such apparent quantification indispensable, then he may well be obliged, at some point, to defend himself against the charge that he cannot consistently engage in such apparent quantification while retaining his nominalism. Even so, that is not the charge directly at issue here and Norm cannot be legitimately expected to deal with everything at once.

Nevertheless, one might still suspect that there is something illegitimate about the way in which Norm engages in apparent quantification over propositions and sentence types in the process of answering the inference problem. In order to begin to dispel that suspicion, I would like to talk about a way in which Norm is not engaging in such quantification, a way that, I agree, would be dialectically illegitimate.

Consider again the paraphrase strategy described above, the one that involves replacing quantification over features with quantification over concepts. Note that a similar paraphrase strategy could be employed that makes use, not of concepts, but of sentence and word types, together with propositions. For instance, we could say that a predicate, \( F \), is “satisfied” by an object, \( O \), just in case there is a term (in the given language), \( N \), such that \( N \) refers to \( O \) and the sentence that results from filling in the blanks in the following schema with \( N \) and \( F \) (respectively) perspicuously expresses a true proposition: \( [\_ \text{ is } \_] \). And then, with this apparatus on hand, we could paraphrase the spider-insect sentence with the following: “There are some anatomical predicates (in a suitably idealized, suitably expressive language) that are satisfied by spiders and also satisfied by insects”.
What this paraphrase strategy essentially does is employ predicate types as surrogates for properties, just as the previous paraphrase strategy did with concepts. And, for that very reason, the sort of objection Van Inwagen offered against the previous paraphrase strategy applies; it is certain that a nominalist will be no more receptive to an ontology that includes predicate types (not to mention entities such as sentence types and propositions) than to one that contains properties. Indeed, predicate types (if such there be) are plausibly regarded as just being properties of a certain sort (properties had by their assorted tokens).

Though there is a certain superficial resemblance between the above paraphrase strategy and the way in which Norm is depicted in the previous section as employing apparent quantification over propositions and sentence types (as well as word types), Norm is most emphatically not engaging in that strategy (or in anything like it). He is not engaging in any paraphrase strategy that uses entities such as word and sentence types as surrogates for properties. In fact, Norm is not engaging in a paraphrase strategy at all.

To better describe what Norm is doing, it is helpful to introduce the following bits of terminology: Let’s say that a proposition has “nominalistically acceptable truth conditions” just in case it is not demonstrably inconsistent with nominalism and is also such that in every possible world in which it is true, there are true, nominalistically acceptable propositions (though perhaps not the same ones in each world) that entail it. With this terminology in hand, we can say that what Norm is doing is exhibiting the fact that propositions like the one ordinarily expressed by the spider-insect sentence not only fail to be demonstrably inconsistent with nominalism, but that these propositions are also such that they have nominalistically acceptable truth conditions, truth conditions that, in
turn, vindicate the entailment relations that we take (on formal grounds) to hold between those propositions and others. In so doing, Norm takes himself to be dispelling any apparent mystery there is in the claim that these entailments should hold given the truth of nominalism.

It is true that, in order to accomplish this, Norm is forced (owing to his own cognitive and linguistic limitations) to engage in apparent quantification over entities such as sentence types and propositions. But Norm (taking, once again, a cue from Melia) is also free to point out that what he has said allows us to recognize that a being who lacked such limitations could see that the relevant sentences have nominalistically acceptable truth conditions without engaging in any such apparent quantification. Such a being could directly entertain the relevant nominalistically acceptable, true propositions that, in our world, entail true propositions like those expressed by the spider-insect sentence. It could go on to do the same with respect to all the other possible worlds in which the relevant propositions are true. Such being could also directly see the relevant entailments between those propositions via its comprehension of their nominalistically acceptable truth conditions. It could do so, furthermore, without engaging in any apparent quantification over propositions or sentence types. Of course, if it were to do all this, it wouldn’t need to describe what it is doing as seeing the various entailments that hold between propositions (no more than Norm would need to describe what he’s doing, when he sees that, necessarily, if the folder is red all over, it is not green all over, as

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92 One difference between Norm and Melia (1995) (aside from the former’s being fictional), however, is that while Norm takes the relevant sentences to express truths, Melia takes them to be false but useful. What makes them useful, according to Melia, is that they allow us to get at truths about the concrete world that we would not (owing to our cognitive and epistemic limitations) be able to get at otherwise; though, a being that lacked our limitations would have no need to employ those sentences. Thus, Melia’s position is actually more akin to the error theory outlined in Section 2.6.
seeing the entailment that holds between the proposition that the folder is red all over and the proposition that the folder is not green all over\textsuperscript{93}. But we lowly, cognitively limited beings are forced to describe its activities in this manner.

As noted in the previous section, furthermore, Norm (still taking his cues from Melia) could then point out that if the only reason he is forced to engage in apparent quantification over propositions and sentence types (in order to say what he wants to say about these issues) is because of his own cognitive and linguistic limitations, that itself is no reason for him to think that he is committed to the existence of such entities as propositions and sentence types. He can also note that Van Inwagen himself has endorsed a similar point:

Even if it is humanly impossible to find a paraphrase for certain awkward sentences . . ., it does not follow that that the Wise Old Beings from the Galactic Core or an archangel or God cannot find a paraphrase for all of them. And suppose that the Wise Old Beings assure us that they know how to paraphrase any awkward sentence—although, unfortunately, human beings are incapable of understanding the paraphrases. If we believe them, then we shall no longer regard these sentences as creating a difficulty for nominalism. Once the Wise Old Beings have convinced human metaphysicists that all awkward sentences can be paraphrased, the human metaphysicists will no longer regard the awkward sentences as posing a difficulty for nominalism. If human beings cannot themselves understand the paraphrases, this fact will no doubt have a salutary effect on human intellectual self-satisfaction, but it will cast no doubt on nominalism.\textsuperscript{94}

\textsuperscript{93} It is also worth pointing out that the infinite being would not even have to explicitly endorse such strict conditionals in order to “see” (in the relevant way) the various entailments. The infinite being might, rather, directly infer a particular claim from various others by “using” those claims (as it were) without ever “mentioning” them, in the same way that I might infer a conclusion from various premises without ever referring to those premises or even explicitly, mentally endorsing any strict conditional that has the conjunction of those premises as its antecedent and the conclusion as its consequent. In this way, for example, the infinite being might be able to see that infinitely many claims jointly entail a certain conclusion, without quantifying over propositions or sentence types, even if it were unable (owing, say, to there being no infinitely long sentences in its language) to formulate a strict conditional that represents that entailment.

\textsuperscript{94} (Van Inwagen 2009, p. 303)
Granted, Norm does not hold that a being without our limitations could find *paraphrases* of the relevant sentences. But he does hold (with good reason) that such a being (even if it were itself a nominalist\(^95\)) would have no need of such paraphrases in order to consistently say (or, at least, consistently believe) all that one might want to say (or believe) concerning these matters. And surely the upshot as far as nominalism is concerned is the same.

Granted, I haven’t argued that *everything* that a nominalist such as Norm might want to say by means of sentences that appear to involve quantification over propositions and sentence types is legitimate. I, haven’t, for example, defended the use of such sentences in contexts such as the speech attributed to the nominalist at the beginning of Section 2.3. As I pointed out above, however, it would seem dialectically inappropriate, in the current context, for the platonist to object to the mere use of such sentences. Furthermore, I will have more to say about the issue of the nominalistic legitimacy of apparent quantification over propositions and sentence types in Chapter 4. For now, I believe that I have said enough to illustrate that there is nothing illegitimate about the way in which Norm has employed such quantification as a means of resolving the inference problem.

One lingering worry that one might have about how Norm resolves the inference problem, however, is that he does not merely appear to quantify over such entities as propositions and sentence and word types. He also, in various places, makes reference to

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\(^95\) Of course, if the relevant hypothetical being were, in virtue of lacking our cognitive limitations, omniscient, it would be question begging to ascribe to it the property of being a nominalist. I do not assume, however, that the relevant hypothetical being is omniscient. I assume only that it lacks those limitations that are germane to the current discussion.
schemas and various entities associated with them (such as the blanks that they contain). Isn’t the use of devices such as these, however, also nominalistically suspect? I do not believe so, at least not if we’re prepared to grant what has been said up to this point about the nominalistic legitimacy of Norm’s use of apparent quantification over propositions, sentence and word types. Norm’s apparent reference to schemas and the entities associated with them merely functions as a heuristic device for picking out various classes of sentence types and the accompanying classes of propositions perspicuously expressed by them. Since Norm is unable to directly consider all of the propositions in the relevant classes at once, he is forced to use some such device to pick them out. But, once again, a being that lacked Norm’s cognitive and linguistic limitations would have no such problem.

2.6 Beyond Ad Hominem

Given Van Inwagen’s views concerning the philosophy of language, the nominalist, as we have seen, may regard the propositions ordinarily expressed by sentences such as the spider-insect sentence as both true and neutral with respect to the existence of features. But what if we are not given Van Inwagen’s views concerning the philosophy of language? In particular, what if we were to reject the idea that sentences such as the spider-insect sentence ordinarily express propositions that are neutral with respect to the existence of features, insisting instead that such sentences are to be taken as expressing the propositions assigned to them by the standard compositional semantics? What should the nominalist say in that case?
In that case, the nominalist should adopt an error theory; she should say that the propositions expressed by such sentences are false. Nevertheless, she should also say that, in many contexts, these propositions are “correct,” where their correctness in a given context amounts to their having some alethic virtue other than truth, one that makes it appropriate to assert them – or better (since it is plausible that assertion amounts to endorsement of truth), to quasi-assert them – in ordinary contexts, contexts, that is, in which their ontologically controversial entailments are not at issue. Rather than distinguish between the propositions that these sentences are ordinarily used to express and those assigned to them by the standard compositional semantics, we can distinguish between the propositions expressed by these sentences being “ordinarily correct” and their being “strictly correct” (where the latter amounts to their simply be true and the

96 The fact that I am here portraying the nominalist error theorist as taking propositions, rather than sentences, to be the bearers of correctness introduces a complication that was already (to some extent) anticipated in note 66. If, in fact, there are no such entities as features, then one might think that, for that reason, the English predicate ‘is a feature’ is semantically defective and that, for that reason, there just is no proposition expressed by a sentence such as ‘Spiders share some of the anatomical features of insects’. I take it, however, that someone who held this view would still want to maintain that the aforementioned sentence is capable of being synonymous with other sentences. I take it, for instance, that such a person would want to maintain that it is possible to translate this sentence into other languages such that the resulting sentences are synonymous with it. Such a person would have to maintain, of course, that synonymy among declarative sentences does not require that those sentences express the same proposition (even in the same context). But I take it that if her view is to be at all plausible, she will have to maintain that such is the case. If so, the adherent of this position will be committed to the view that, doubts about the existence of sentence types aside, it is possible to type sentences by whether they are synonymous with one another, in such a way that two sentences may be said to belong to the same “synonymous sentence type” if and only if those sentences are synonymous with one another. And, as long as nominalist scruples about quantifying over sentence types are being set aside, this fact would allow a person who held the view that we are considering to regard synonymous sentence types (or better, perhaps, synonymous sentence types indexed to a given context), rather than propositions, as being the bearers of correctness. And this person could then modify the account given here accordingly.
former amounts to their having the relevant alethic virtue that makes it appropriate to quasi-assert them in ordinary contexts).\(^97\)

As Van Inwagen points out, anyone who holds to the sort of “austere philosophy of language” being considered here is going to need such a notion of correctness:

If someone maintains that ‘The sun moved behind the elms’ expresses a falsehood, he must still have some way to distinguish between this sentence and those sentences (like ‘The sun exploded’ and ‘The sun turned green’) that the vulgar would regard as the sentences that expressed falsehoods about the sun. He will require what we may call a “term of alethic commendation” which he can correctly apply to ‘The sun moved behind the elms’ and withhold from ‘The sun exploded’… If, I say, I accepted this austere philosophy of language … I should not be willing to say that people who uttered things like ‘There are two very valuable chairs in the next room’ very often said what is true. I should be willing to say only that they very often said what might be treated as a truth for all practical purposes.\(^98\)

The nominalist who adopts such an austere philosophy of language may adopt an error theory similar (except for its differing subject matter) to the one suggested by Van Inwagen.

Indeed, in some ways, it is easier for the nominalist to adopt such an error theory than it is to regard the propositions expressed by the relevant nominalistically offensive sentences as true. Such a nominalist escapes Van Inwagen’s charge of inconsistency by embracing one of the options that he attempts to force upon her. In particular, she embraces the option of abandoning her allegiance (supposing she ever had such an

\(^97\) Compare this with the strategy that Sider (1999) recommends on behalf of presentists. Rather than recommending that presentists seek to provide paraphrases of those sentences of ordinary discourse that appear to involve quantification over non-present objects, Sider recommends that presentists seek to render it plausible that such sentences are “quasi-true,” where their having that status involves their being underwritten by various “underlying truths.”

\(^98\) (Van Inwagen 1990, pp. 102-103).
allegiance in the first place) to the thesis expressed by the sentence ‘Spiders share some of the anatomical features of insects’. Rather than maintaining an attitude of belief toward the proposition expressed by this sentence, she maintains one of disbelief. However, this does not entail that she is not within her rights to maintain some other positive epistemic attitude toward that proposition. In particular, she may endorse that proposition as being ordinarily correct.

Furthermore, rather than claiming (along the lines sketched in Sections 2.4 and 2.5) that the proposition expressed by this sentence has nominalistically acceptable truth conditions, truth conditions that vindicate the sort of inferential uses to which she puts that sentence, she can maintain (after making suitable adjustments) that this proposition has “nominalistically acceptable ordinary correctness conditions” that vindicate those uses. The arguments that she employs using this sentence will not (by her own lights) be sound, but they will be “quasi-sound,” provided that the sentences she uses in those arguments express propositions that are ordinarily correct and their ordinary correctness conditions are such that whenever she manages to (correctly) infer a conclusion from her premises, that conclusion is also ordinarily correct.99 Furthermore, in those cases in which a proposition is ordinarily correct only if it is true (as will be the case for a great many nominalistically acceptable propositions100), an argument for that proposition will be quasi-sound only if that proposition is true.

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99 This terminology (i.e. that of referring to various arguments as “quasi-sound” or, as I do below, as “quasi-valid”) is borrowed from (Sider 1999, pp. 23-24). The manner in which I use these terms is similar, but not identical, to the way in which Sider uses them.

100 Though not all of them will be (by the lights of such an error theorist), not, at least, if she endorses any other “revisionary” views aside from nominalism. Sentences involving apparent quantification over shadows, for example, might, by the lights of such an error theorist, express
For example, the ordinary correctness conditions for the proposition expressed by the sentence ‘Any two female spiders of the same species share all and only the same anatomical features’ will be such that this proposition is ordinarily correct only if any two conspecific, female spiders are such that if one of them has fangs the other has fangs. So, by the nominalist error theorist’s lights, the following argument is quasi-valid (i.e. the propositions expressed by the premises of this argument are such that necessarily, if they are ordinarily correct, the conclusion is ordinarily correct).\(^1\)

(i) Any two female spiders of the same species share all and only the same anatomical features.

(ii) Shelob and Charlotte are female spiders of the same species.

(iii) Therefore, Shelob has fangs if and only if Charlotte has fangs.\(^2\)

The conclusion of this argument, furthermore, is ordinarily correct only if it is also true. Consequently, the fact that the above argument is quasi-sound (supposing this is so) guarantees not only that it has an ordinarily correct conclusion but also that it has a true conclusion.

\(^1\) Throughout this discussion, terms like ‘the ordinary correctness conditions’ and ‘ordinarily correct’ should be thought of as being \textit{rigid} (i.e. as being such that when we describe counterfactual situations, whether we are to say that a proposition is “ordinarily correct” and what we should say about what its ordinary correctness conditions are depends on our standards of evaluation for ordinary correctness, regardless of whether there are speakers in those situations who have different standards of evaluation).

\(^2\) Of course, this argument, as it stands, is \textit{not} \textit{formally} valid. But that could easily be fixed by adding additional sentences that also (by the nominalist, error theorist’s lights) express quasi-true propositions (though it is not, in the present context, worth taking the trouble to do so).
It might be objected that the error theorist is here assuming that whenever some propositions are ordinarily correct and entail a distinct proposition, the entailed proposition is also ordinarily correct, and that this assumption is questionable. However, the error theorist is making no such assumption. The fact that some ordinarily correct propositions jointly entail a proposition will guarantee that the entailed proposition is ordinarily correct, according to the error theorist, only if necessarily, the ordinary correctness conditions for the entailing propositions are satisfied only if the ordinary correctness conditions for the entailed proposition are satisfied. That will not, even by the error theorist’s own lights, always be so. Indeed, some of the propositions that the error theorist regards as ordinarily correct are (at least by her own lights) necessarily false and so entail any given proposition. But she will not infer from this fact that just any given proposition is ordinarily correct.

Likewise, the error theorist needn’t assume that every formally valid argument that contains premises each of which expresses an ordinarily correct proposition also has an ordinarily correct conclusion. Once again, this will be so, by her lights, only if the ordinary correctness conditions for the propositions expressed by the premises of the given argument are such that necessarily, they are jointly satisfied only if the ordinary correctness conditions for the proposition expressed by that argument’s conclusion are satisfied. Nevertheless, the error theorist can (by adapting, to her own purposes, the argument given in the previous section) plausibly maintain that this will be so in a large number of the cases that are of present concern.  

\[103\] Indeed, it is hard to think of clear cases in which this would not be so (at least if we set aside frozen idioms and the like, and in such cases it is not clear to begin with that we are dealing with sentences that have the same logical syntax as that suggested by their surface grammar). Michael Rea has suggested
Thus, I conclude, even the nominalist who does not adopt Van Inwagen’s views concerning the philosophy of language can adapt many of the considerations raised by the *ad hominem* response leveled against Van Inwagen in the previous sections for her own purposes. Either way, the nominalist has a good response to Quinean arguments for the claim that we are committed to the existence of properties, at least to Quinean arguments of the sort put forward by Van Inwagen. If such Quinean arguments fail and properties are to be thought of as explanatorily idle in the way that lightweight platonists take them to be, it becomes difficult for many of us to see why we ought to believe in them.

2.7 Works Cited


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to me that certain uses of metaphor might supply such cases and has offered the following example: ‘Juliet is the Sun’ may well express an ordinarily correct proposition (or at least a proposition that is correct in a given context). The conjunction of this sentence with ‘The Sun is a hot ball of gas’, furthermore, formally implies the sentence ‘Juliet is a hot ball of gas.’ But, obviously, the proposition expressed by the latter will not be correct in most of the ordinary contexts in which the proposition expressed by ‘Juliet is the Sun’ is correct. The first thing to be said about this example is that even if the error theorist were committed to there being no cases of formally valid arguments with ordinarily correct premises but incorrect conclusions (which she is not), she need not regard this case as falling within the purview of her theory. Her notion of correctness is a notion of truth-likeness, but she may adopt an account of metaphor according to which apt metaphors need neither be true nor truth-like. Even setting that aside, however, it is still far from clear that the proposition expressed by ‘The Sun is a hot ball of gas’ would be correct in contexts in which the proposition expressed by ‘Juliet is the Sun’ is correct. A person who uttered the former sentence, in such a context, would likely either be shifting the context, failing to “get” the metaphor, or making a joke (a joke whose status as such would turn on the fact that such a statement is incongruous in the given context). By contrast, the sentence ‘And the Sun is radiant’ would likely appear to express a correct proposition in such a context, as would the inferred conclusion ‘Juliet is radiant’.


CHAPTER 3:
A FORMAL ACCOUNT OF APPARENT QUANTIFICATION OVER FEATURES

3.1 Preliminaries

In Chapter 2 I argued that the nominalist can plausibly maintain that many sentences that appear to involve quantification over features (more specifically, over first order monadic properties) are ordinarily used to express propositions that can be seen to have (what I there called) “nominalistically acceptable truth conditions” or (less committedly) “nominalistically acceptable ordinary correctness conditions.” This is true, I argued, regardless of whether the propositions ordinarily expressed by those sentences are thought to differ from those assigned to them by the standard compositional semantics (as Van Inwagen’s Wittgensteinian views concerning the philosophy of language would have it) or not. In the former case, the nominalist might think that the propositions in question are not only ordinarily correct but true. In the latter case, the nominalist might adopt an error theory according to which it is (in most non-metaphysical contexts) the ordinary correctness of these propositions that is important, rather than their truth. In this chapter I intend to remain neutral about which (if either) of these views is correct.

Either way, I also argued, it is plausible to believe that formally valid arguments (at least of the sort that can be represented in first order logic) involving such sentences are correctness preserving. That is, it is plausible to believe that the arguments in question are such that if the propositions expressed by their premises are ordinarily correct, then so are the propositions expressed by their conclusions. I supported this
claim by providing examples of such arguments and by arguing that our grasp of the ordinary correctness conditions for the propositions expressed by their premises and conclusions allowed us to see that those particular arguments are indeed correctness preserving. Toward the end of Section 2.4, I also issued a promissory note to produce a formal proof to the effect that the considerations raised by these examples do in fact generalize over a large class of such arguments. In this chapter, I discharge that promissory note.

Most of this chapter will involve introducing a large amount of formal machinery and then using that machinery to establish a few key results. Since, without a clear view of the ends for which this machinery is being introduced, it would be all too easy for the reader to get bogged down in its details, I will use the remainder of this section to outline the overall structure of this chapter. My discussion will have five components.

First, I will describe a formal object language which can represent all of the arguments of the sort discussed in Chapter 2. It will be a language suited for first order logic, with its properly formed sentences defined up recursively in the usual sort of way. It will also differ from English in various other significant ways, ways that make it easier to obtain the sort of formal results that we will want. Nevertheless, it should be clear how the arguments that will be the focus of our attention could be represented in such a language.

Upon inspection, one may note that all of the arguments involving apparent quantification over features discussed in Chapter 2 met each of the following conditions: (i) They were all first order valid. (ii) None of them contained sentences that appeared to
involve quantification over any non-concrete entities other than monadic, first order properties. (iii) None of them contained sentences employing non-unary predicates (other than an identity predicate and a binary property having predicate) that could be satisfied (in whole or in part) by non-concrete items. I will, therefore, in this chapter, restrict my attention to arguments that meet these conditions, and limit the arguments that can be represented in the object language accordingly. Obviously, that restriction significantly limits the scope of my discussion. Nevertheless, treating the class of arguments that meet these conditions will suffice to discharge the promissory note issued in Chapter 2. And the exercise will prove instructive in other ways. In the next chapter, I will argue that the considerations put forward in this chapter plausibly generalize so as to apply to any discourse involving apparent quantification over abstract objects that would be desirable for the nominalist to accommodate.

The second thing I will do is take each “basic” sentence in the object language (where a basic sentence is either a closed, atomic sentence or the negation of such a sentence), and provide instructions for finding what I will call a “nominalistic surrogate” in the object language for that sentence. A nominalistic surrogate for a given sentence in the object language may (as matters were framed in Chapter 2) be thought of as a nominalistically friendly sentence that perspicuously expresses a proposition whose truth is both necessary and sufficient for the correctness of the proposition expressed by the original sentence. Once it has been specified how to find a nominalistic surrogate for each basic sentence, I will also specify how to find nominalistic surrogates for many (though not all) of the more complex sentences of the object language.
Third, I will define what it is for a sentence in the object language to be “nominalistically undergirded,” where a sentence of the object language being nominalistically undergirded may be thought of (in the terminology of Chapter 2) as that sentence expressing a proposition that is “ordinarily correct” by way of its “nominalistically acceptable ordinary correctness conditions” being satisfied. In the case of a basic sentence, its being nominalistically undergirded will simply amount to its nominalistic surrogate being true (i.e. to its nominalistic surrogate being such that it perspicuously expresses a true proposition). In the case of more complex sentences, what it is for each of them to be nominalistically undergirded will be recursively defined in terms of what it is for the basic sentences to be nominalistically undergirded. In Appendix A, I will also exhibit, by means of a detailed example, how the conditions under which a sentence in the object language is nominalistically undergirded correspond to the ordinary correctness conditions laid out for various of the propositions discussed in Chapter 2.

Fourth, I will set forward some axiom schemas and formal inference rules for the object language, of the sort found in a standard textbook on first order logic (appropriately tailored, in certain ways, for the peculiarities of the object language). I will then outline the steps of a proof of the following theorem concerning the object language (which I will refer to as the “Correctness Preservation Theorem”): 

*Correctness Preservation:* For any sentences of the object language, $\psi_1^*, \psi_2^*, \ldots, \psi_n^*, \varphi^*$, if each of the sentences $\psi_1^*, \psi_2^*, \ldots, \psi_n^*$ is nominalistically undergirded and, by one or more applications of the inference rules (provided for the object language), $\varphi^*$ may be inferred from $\psi_1^*, \psi_2^*, \ldots, \psi_n^*$, then $\varphi^*$ is nominalistically undergirded.
Since the object language and the rules that govern it will be tailored in such a way that formally valid arguments in English that contain English sentences that appear to involve quantification over features (and which meet the restrictions laid out above) may be translated into formally valid arguments in the object language (and since the notion of nominalistic undergirding will turn out, by design, to be the formal analog of the Chapter 2 notion of ordinary correctness), a proof of the Correctness Preservation Theorem will suffice to discharge the promissory note mentioned at the beginning of this section. The proof itself, it will turn out, is both rather straightforward – it mirrors a standard proof of the soundness of first order logic – and rather tedious. It will not be given in the main text but will be given in Appendix B.

Of course, the notion of ordinary correctness, of which the notion of nominalistic undergirding is meant to be a formal analog, is of interest only insofar as it is closely related (in the right way) to our notion of truth. In particular, it would not be of much philosophical interest to show that formally valid arguments in the object language are correctness preserving unless that also told us something informative about when their conclusions are true. Thus the fifth and final thing I will do is discuss the relationship between nominalistic undergirding and truth. Using a result proven in Appendix C, I will argue that whenever a conclusion of a formally valid argument in the object language with nominalistically undergirded premises has a nominalistically friendly conclusion (in the technical sense of what it is for a sentence to be “nominalistically friendly” laid out in Section 2.3), that conclusion perspicuously expresses a true proposition. The nominalist can appeal to this fact, I will further argue, to explain the utility of reasoning as if there were monadic properties that can be had by concrete things, even if in fact there are not.
3.2 The Object Language

In this section I describe the object language with which I will be working (and, where appropriate, I will also say a few things about the meta-language in which I will take myself to be working). I will not, for the most part, attempt to explain the rationale behind why I assign to the object language many of the various features and restrictions that I assign to it. The rationale behind many of these assignments will (hopefully) become clear when one comes to see the formal work that they do.

I will take myself to be working with an object language that includes singular quantifiers, the identity sign, the standard logical connectives, variables, constant symbols, and predicate symbols. I will take this language to come as already interpreted; that is, each of its predicate symbols and constants will be taken to have previously given, fixed meanings. I will also assume that the object language lacks for no expressive resources when it comes to its ability to characterize the concrete world, excepting those limitations that I artificially impose on it. A platonist could think of this assumption as entailing, among other things, that every relation (of any adicity) that can be expressed in a language that exhibits the restrictions that I impose on the object language, and which may be satisfied solely by concrete things, is in fact expressed by a predicate in the object language.

I will also assume that the object language contains a distinct atomic predicate symbol (keep in mind that all such symbols are to be thought of as already interpreted) for every case in which a predicate that applies solely to concrete objects may be defined in terms of other predicates found in the language (e.g. if there is an atomic predicate
symbol for being a cat and another for being a dog, then there is also one for being a cat or a dog, another for being both a cat and a dog, etc.). It should be apparent that the object language contains a vast number (indeed an infinite number) of predicate symbols.

Of course, given the above, it is doubtful whether any cognitively limited beings such as ourselves could fully learn such an object language, or even whether beings such as ourselves could single out and specify an interpretation for a class of sentences that (together with their assigned interpretations) would constitute such a language. But this will not be of any concern. My goal is to exhibit the fact that various propositions that are expressed by English sentences that appear to involve quantification over features have nominalistically acceptable correctness conditions, even if we cannot (owing to our own cognitive and linguistic limitations) completely and informatively describe those conditions. The object language need merely be an instrument suited to aid in that task.

Of course, if it isn’t possible for beings such as ourselves single out a class of interpreted sentences that has the features ascribed to the object language, it is a pretense on my part to speak of the object language (as if I have in fact singled out such a class of sentences, or at least as if I could do so in principle). Still, if we think of languages (complications having to do with indexicals and various other niceties aside) as something like parings between the members of a certain class of sentences with propositions (where any such paring counts as a language), the fact that we cannot single out a language that has the features of the object language does not (setting aside nominalist scruples about the existence of entities such as parings between sentences and propositions) entail that there is no such language. Thought of in this way, a language need not be learnable, or even specifiable, in order to be a language. And as long as there
is a language (nominalist scruples aside) with the features I ascribe to the object language, and as long as I can show that the relevant formal properties hold of any such language, that will be enough for my purposes.

Along the lines of what was suggested in Chapter 2, it is of heuristic value to think of the object language as one that is spoken (or at least thought in) by an infinite being who lacks our cognitive and linguistic limitations. The proof of the Correctness Preservation Theorem described below may be thought of as demonstrating that such a being, in virtue of its direct grasp of the conditions under which each sentence in the object language is nominalistically undergirded, could also directly see that each formally valid argument expressed in the object language is correctness preserving. As long as the object language is such that the formally valid arguments expressed in English that are of interest to us have translations within it, this result also establishes that such a being (supposing that it could also speak the relevant fragments of English) could see those arguments to be correctness preserving (via seeing this of their translations into the object language).

Let’s now return to the task of characterizing various features of the object language itself. Unary predicates (the atomic ones, that is) in the object language will be singled out and receive a more specialized treatment than non-unary predicates. These will be thought of as coming in two varieties – n-predicates and p-predicates. I will assume (though I will not specify how) that the object language marks this distinction syntactically. Intuitively speaking, the n-predicates may be regarded as predicates that (as the platonist might say it) “express properties” that can only be had by nominalistically acceptable objects (had by them intrinsically or in relation to one
another, but not in relation to any abstract objects) – properties such as *being massive*, *enjoying music*, *being concrete*, or, perhaps, a property such as *being concrete and being thought about* (but not, for example, *being thought about*, at least not if both features and concrete objects can be thought about, nor, for example, *exemplifying a property*, since that involves a relation to an abstract object). Intuitively speaking, the p-predicates express properties that may only be had by features (e.g. properties such as *being a feature*).

The object language will also contain a special purpose, binary predicate symbol, which I will represent with the single letter ‘H’, where the open sentence $xHy$ is to be read as ‘$x$ has $y$’, where the “having” at issue here is the sort of having involved in property exemplification. I will assume that (among the objects in the object language’s universe of discourse) only features may be “had” by other objects in this sense. Aside from this special purpose binary predicate and the identity predicate, however, none of the atomic, non-unary predicates in the object language “express relations” (as the platonist might put it) that can be entered into by objects that are not concrete.

The constant symbols in object language, like its unary predicates, will also come in two varieties; some will be what I will call “n-names” while others will be what I will call “p-names.” I will assume (though I will not specify how) that the object language marks the distinction between these two types of names syntactically (perhaps it does so by use of subscripts or by different sorts of font or by some other such means). At an intuitive level, n-names may be regarded as names that refer to nominalistically acceptable objects; these names, it will be assumed, are never empty; for each of them, there is an object that is its referent. By contrast, p-names may be regarded (intuitively
speaking) as names that refer to monadic properties (i.e. to features), and, even more specifically, to *first order* monadic properties that may only be had by concrete things. While I will often speak (for the purposes of exposition), however, of these names as “referring” to such properties, strictly speaking, I am *not* assuming that there really are entities to which these names refer. If nominalism is true (a point concerning which, for the purposes of characterizing the object language and giving the relevant formal proofs, I will remain neutral), then all p-names are empty names. It will be assumed, however, that the object language’s universe of discourse is exhausted by the entities referred to by its n-names and p-names (note that this assumption entails that every object in the universe of discourse has a name in the object language). It will also be assumed that no entity is suited to be both the referent of an n-name and the referent of a p-name. It will further be assumed that each name has at most one referent. It will also be assumed that each feature in the universe of discourse, but not necessarily each nominalistically acceptable object, has only one name in the object language. Finally, it will be assumed that there is at least one nominalistically acceptable object in the universe of discourse.

There is also, I hereby stipulate, an interesting semantic relationship between the unary n-predicates of the object language and the p-names thereof that goes as follows: For every p-name, there is a uniquely corresponding, atomic, unary n-predicate, the latter of which may be said to “project” the former. For example, if there is a p-name in the object language that refers to the property of being segmented, then this p-name may be said to be projected by (or to be the projection of) the atomic predicate in that language that means *is segmented*. If we are platonists and think of properties as Van Inwagen does, as assertibles, or at least as entities suited to be the contents of predicates, then we
may think of the p-name projected by a given predicate as naming the property expressed by that predicate. If we are nominalists, then we are likely to speak of p-names as “projections” of n-predicates in a more disparaging tone, as empty names that purport to denote entities that correspond to the contents of various predicates.

I further stipulate that the object language contains no synonymous atomic predicates, that each atomic n-predicate predicate projects exactly one p-name, and that no p-name is projected by more than one predicate. Note that these stipulations, in conjunction with the previous stipulation that each feature in the universe of discourse has exactly one p-name, entail that if there are features in the universe of discourse, they are individuated by the atomic n-predicates in the object language that express them. I will also take the p-names to be such that they (in some way) syntactically pick out the n-predicates that project them (perhaps they embed those predicates as subscripts).

I will also take there to be a further kind of semantic restriction on the meanings had by the p-predicates of the object language, which I will now proceed to explain. First, I will take the object language to be accompanied by a classification scheme that assigns each of its n-predicates to various categories. For example, the scheme might classify some n-predicates (say predicates that mean things like *is segmented* or *has eyes*) as “anatomical,” others as “psychological,” and so on. Some such categories (presumably) will apply to certain n-predicates in the object language but not others. Other categories in the classification scheme (such as that of being a predicate), however, may well be such that every n-predicate in the object language falls into them. Given the above, the semantic restriction I have in mind comes to this: every p-predicate in the object language “corresponds” (in an intuitive way) to one of the categories in the
classification scheme for n-predicates. For example, if one of the categories in the classification scheme characterizes certain n-predicates as “anatomical,” then a p-predicate that corresponds to it will be satisfied only by properties that are anatomical features.

We will also take ourselves (the speakers of the meta-language) to be in possession of an operator, ‘C’, which takes as arguments p-predicates from the object language and converts them into corresponding classifying predicates in the meta-language. For example, if F is a p-predicate that means the same thing as ‘is anatomical’ as it is used in the English sentence ‘Being segmented is an anatomical feature’, then C(F) would be a predicate in the meta-language that means the same thing as ‘is anatomical’ as it is used in the English sentence ‘is segmented’ is an anatomical predicate’.

In addition, I will assume that when a p-name is projected by a given n-predicate, F, then if there is an object to which that p-name refers, that object satisfies a given p-predicate, G, if and only if F is a C(G) predicate. Suppose, for example, that p is a p-name projected by an n-predicate, F, and that F means is segmented. Suppose also that G is a p-predicate in the object language that means the same thing as ‘is anatomical’ as ‘is anatomical’ is used in the English sentence ‘Being segmented is an anatomical feature’. Suppose furthermore that ‘C(G)’ is a predicate in the meta-language that means the same thing ‘is anatomical’ as ‘is anatomical’ is used in the English sentence ‘is segmented’ is an anatomical predicate’. Given all of these suppositions, any property that p denotes satisfies G if and only if F is an anatomical predicate. I.e. if there is such a property as being segmented for p to denote, then being segmented satisfies the p-predicate in the object language that means ‘is anatomical’ if and only if the predicate in the object language
language that projects p (namely, the predicate in the object language that means ‘is segmented’) is an anatomical predicate.

Now is also as good a time as any, I suppose, to mention a fact that is already no doubt apparent. I will, when speaking in the meta-language, unabashedly help myself to apparent quantification over various sorts of abstract entities (including expression types, operators, and the like). I would defend the dialectical appropriateness of this maneuver along the same lines that I defended Norm’s use of apparent quantification over propositions, sentence types, schemas, and the like in Chapter 2. My purpose is to exhibit the fact that various sentences in the object language that appear to involve quantification over features have suitable, nominalistically acceptable correctness conditions. Given my own cognitive and linguistic limitations, I am forced to use such devices as appearing to quantify over expression types in order to exhibit that fact. But, as I argued in Chapter 2, a being who lacked my limitations would not be so forced. Here I will here be content to allow the above remarks (and the discussion in Chapter 2) to suffice as a defense of my use, in the meta-language, of apparent quantification over abstract objects. I will, however, have more to say concerning this issue in the next chapter.

At last, with all of the above setup out of the way, we may define what it is for an expression to count as a sentence (whether open or closed) of the object language, in the usual sort of recursive way, as follows: First, we’ll say that an expression is a term if and only if it is either a variable of the object language or a constant symbol thereof. That said, for any expression, $\varphi^*$, $\varphi^*$ is a sentence (whether open or closed) of the object language if and only if
\( \phi^* \) is of the form \( t_1 = t_2 \) and \( t_1 \) and \( t_2 \) are terms,\(^{104}\) or

\( \phi^* \) is of the form \( Ft \) and \( F \) is a unary predicate and \( t \) is a term, or

\( \phi^* \) is of the form \( Rt_{t_2} \ldots t_n \) and \( t_1, t_2, \ldots, t_n \) are terms and \( R \) is an n-ary predicate (and \( n > 1 \)), or

\( \phi^* \) is of the form \( t_1Ht_2 \) and \( t_1 \) and \( t_2 \) are terms, or

\( \phi^* \) is of the form \( (\neg \alpha) \) and \( \alpha \) is a sentence of the object language, or

\( \phi^* \) is of the form \( (\alpha \lor \beta) \) and \( \alpha \) and \( \beta \) are each sentences of the object language, or

\( \phi^* \) is of the form \( (\alpha \land \beta) \) and \( \alpha \) and \( \beta \) are sentences of the object language, or

\( \phi^* \) is of the form \( (\alpha \rightarrow \beta) \) and \( \alpha \) and \( \beta \) are sentences of the object language, or

\( \phi^* \) is of the form \( (\exists v)(\alpha) \) and \( \alpha \) is a sentence of the object language and \( v \) is a variable of the object language.

3.3 Nominalistic Surrogates

I am now in a position to begin to lay the groundwork for what it is for a sentence in the object language to be nominalistically undergirded. I would also like to take this opportunity to note that, from here on, I will often speak (for the sake of convenience) as if it were sentences of the object language that were the bearers of truth/falsity (as well as

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\(^{104}\) I apologize for the sloppiness (or, as I prefer to call it, “notational shorthand”) that some of my readers may have already noticed here. Technically, the first occurrences of ’\( t_1 \)’ and ’\( t_2 \)’ are not variables in the meta-language for expressions, but expression placeholders in a schema. So, technically, this first clause should be something along the lines of the following: \( \phi^* \) is of the form \( t_1 = t_2 \) and the expressions found in the \( t_1 \) and \( t_2 \) positions are terms. Likewise, \textit{mutatis mutandis}, for each of the other clauses found below. This is not the first place that I will use such “notational shorthand”; it is, however, the one and only place in which I will apologize for it.
correctness/incorrectness) rather than (as I have been speaking until now) the propositions expressed by those sentences.

For my purposes, a “basic sentence” of the object language will be characterized as a closed sentence that is of one of the following forms: \( t_1 = t_2 \), \( \neg t_1 = t_2 \), \( Ft \), \( \neg Ft \), \( Rt_1t_2...t_n \) (where \( n > 1 \)), \( \neg Rt_1t_2...t_n \) (where \( n > 1 \)), \( t_1Ht_2 \), \( \neg t_1Ht_2 \). As noted in Section 3.1, a nominalistic surrogate for a sentence in the object language may be thought of as a nominalistically friendly sentence whose truth is both necessary and sufficient for the correctness of the original sentence. Accordingly, we will want set things up in such a way that the correctness conditions for each basic sentence may be regarded as being identical to the truth conditions for its nominalistic surrogate.

The most straightforward cases are those in which the basic sentence in question meets the following condition:

It is a basic sentence that contains no p-predicates and no p-names.

Any such sentence will fail to contain any terms that (putatively) refer to nominalistically unacceptable objects, and it will fail to contain any predicates that can be satisfied (in whole or in part) by such objects. Consequently, any such sentence is nominalistically friendly as it stands. And so, in every such case, the nominalistic surrogate for the basic sentence in question may simply be identified with that basic sentence itself.

Next on the list of the most straightforward cases, perhaps, are those that meet the following condition:

The basic sentence is of the form \( t_1Ht_2 \) or of the form \( \neg t_1Ht_2 \) and \( t_1 \) is an n-name and \( t_2 \) is a p-name.
Recall that it was stipulated that each p-name in the object language is to be thought of as referring to a first order monadic property that can only be had by concrete things. Recall also that each p-name is to be regarded as being uniquely “projected” by a single, atomic, unary n-predicate (where each p-name also, in some way, marks syntactically the predicate that projects it). Recall, furthermore, that (intuitively speaking) the p-name projected by a given n-predicate is to be thought of as referring to the property expressed by that predicate. So in the case in which a basic sentence is of the form $t_1 H t_2$ and $t_1$ is an n-name and $t_2$ is a p-name (recall that such a sentence reads “$t_1$ has $t_2$” where the having at issue here is that of property having), the most natural choice for its nominalistic surrogate is a sentence of the form $F t_1$, where $F$ is the n-predicate that projects $t_2$.

Likewise, in the case in which the basic sentence is of the form $(\sim t_1 H t_2)$ and $t_1$ is an n-name and $t_2$ is a p-name, the most natural choice for its nominalistic surrogate is a sentence of the form $(\sim F t_1)$, where $F$ is the n-predicate that projects $t_2$.

Another class of cases are those involving basic sentences that do not fall under any of the cases discussed above and whose correctness or incorrectness in no way depends on which nominalistically friendly sentences are true. These come in two varieties. The first variety includes sentences which may (given the stipulations governing the object language) be seen to be correct or incorrect because of their syntactical features alone.\textsuperscript{105} The second variety includes sentences which may be seen to be correct or incorrect, regardless of which nominalistically friendly sentences are true.

\textsuperscript{105} Note that a sentence’s being incorrect, in this context, does not amount to its being syntactically ill formed. Syntactically ill-formed “sentences” are, by definition, not sentences of the object language at all. The notion of correctness at issue here is a semantic one, not a syntactical one. Nevertheless, some sentences of the object language may be seen to have this semantic feature (or its absence) merely in virtue of their syntactical features.
on account of their semantic features, but not on account of their syntactical features alone.

Let’s begin by considering basic sentences that fall under the first variety. Some such cases include sentences that contain the identity sign (or the negation of such sentences). Given the stipulation that each property in the universe of discourse has exactly one p-name, each sentence that meets one of the following conditions is correct, regardless of which nominalistically friendly sentences are true:

The sentence is of the form $t_1 = t_2$ and $t_1$ and $t_2$ are each the same p-name.

The sentence is of the form ($\sim t_1 = t_2$) and one of $t_1$ and $t_2$ is a p-name and the other is a different name (whether a different p-name or an n-name).

Likewise, each sentence that meets one of the following conditions is not correct, regardless of which nominalistically friendly sentences are true:

The sentence is of the form $t_1 = t_2$ and one of $t_1$ and $t_2$ is a p-name and the other is a different name (whether a different p-name or an n-name).

The sentence is of the form ($\sim t_1 = t_2$) and $t_1$ and $t_2$ are each the same p-name.

Other basic sentences falling under the first variety are those in which n-predicates are combined with p-names, p-predicates are combined with n-names, or non-unary predicates (which have been stipulated to be such that they are satisfied only by concrete objects) are combined with p-names. For example, each sentence that meets one of the following conditions is correct regardless of which nominalistically friendly sentences are true:

The sentence is of the form ($\sim Ft$) and $F$ is an n-predicate and $t$ is a p-name.
The sentence is of the form \((\neg Ft)\) and \(F\) is a p-predicate and \(t\) is an n-name.

The sentence is of the form \((\neg Rt_{1}t_{2}...t_{n})\) and some of \(t_{1}, t_{2}, ..., t_{n}\) are p-names (and \(n > 1\)).

Likewise, each of the following sentences is not correct regardless of which nominalistically friendly sentences are true:

The sentence is of the form \(Ft\) and \(F\) is an n-predicate and \(t\) is a p-name.

The sentence is of the form \(Ft\) and \(F\) is a p-predicate and \(t\) is an n-name.

The sentence is of the form \(Rt_{1}t_{2}...t_{n}\) and some of \(t_{1}, t_{2}, ..., t_{n}\) are p-names (and \(n > 1\)).

Similar cases falling under the first variety involve the object language’s property having predicate. Given the stipulations that all of the p-names in the object language are to be thought of as referring to first order monadic properties that can only be had by concrete things, that all of the n-names refer to nominalistically acceptable objects, and the assumption that no nominalistically acceptable object can be had in the way in which properties are had, each sentence that meets the following condition is correct regardless of which nominalistically friendly sentences are true:

The sentence is of the form \((\neg t_{1}Ht_{2})\) and \(t_{1}\) is a p-name or \(t_{2}\) is an n-name.

Likewise, each sentence that meets the following condition is not correct regardless of which nominalistically friendly sentences are true:

The sentence is of the form \(t_{1}Ht_{2}\) and \(t_{1}\) is a p-name or \(t_{2}\) is an n-name.

Basic sentences of the second variety, those that are either correct or incorrect on account of their semantic features (but may not be seen to be so on account of their
syntactical features alone), involve cases in which p-predicates are combined with p-names. These are examples of “pure platonist sentences.” They pertain to the characteristics had by the non-concrete entities in the universe of discourse, regardless of how those entities are related to anything concrete. Examples of such sentences that are correct regardless of which nominalistically friendly sentences are true are those that meet one of the following conditions:

The sentence is of the form $Ft$, where $F$ is a p-predicate, $t$ is a p-name, and the predicate that projects $t$ is a $C(F)$ predicate.

The sentence is of the form $(\sim Ft)$, where $F$ is a p-predicate, $t$ is a p-name, and the predicate that projects $t$ is not a $C(F)$ predicate.

For an example of a case that meets the first of the above conditions, suppose $F$ is a p-predicate in the object language that means the same thing as ‘is anatomical’ as it is used in the sentence ‘Being segmented is an anatomical feature’, $t$ is a p-name that refers to the property of being segmented, and $G$ is the n-predicate that projects $p$, a predicate that means is segmented. Suppose also that $C(F)$ is a predicate in the meta-language that translates ‘is anatomical’ as it is used in the English sentence ‘‘is segmented’ is an anatomical predicate.’ Then the object language sentence $Ft$ means the same thing as the English sentence ‘Being segmented is anatomical’, and what the first condition says is that this object language sentence can be seen to be correct on account of the fact that $G$ (the n-predicate that projects $t$, where $t$ names the property of being segmented) is an anatomical predicate. I’ll leave it as an exercise for the reader to come up with an example of a case that meets the second condition.
Likewise, reversing the above, examples of such sentences that are not correct regardless of which nominalistically friendly sentences are true are those that meet one of the following conditions:

The sentence is of the form \( Ft \), where \( F \) is a p-predicate, \( t \) is a p-name, and the predicate that projects \( p \) is not a \( C(F) \) predicate.

The sentence is of the form \( \neg Ft \), where \( F \) is a p-predicate, \( t \) is a p-name, and the predicate that projects \( t \) is a \( C(F) \) predicate.

What nominalistic surrogates should we assign to sentences (of either the above varieties) that are correct/incorrect regardless of which nominalistically friendly sentences are true? Let’s allow the symbol ‘\( \bot \)’ to abbreviate some arbitrarily chosen, closed sentence from among those sentences of the object language that include no p-predicates or p-names, one that also happens to be a logical absurdity. The sentence \( \exists x(\neg x=x) \) would do nicely, for example. Then let ‘\( \neg \bot \)’ abbreviate the negation of that sentence. Now we can assign ‘\( \neg \bot \)’ as the nominalistic surrogate for each sentence of one of the above varieties that is correct regardless of which nominalistically friendly sentences are true, and we may assign \( \bot \) as the nominalistic surrogate for each sentence of one of those varieties that is not correct regardless of which nominalistically friendly sentences are true.

The cases discussed above exhaust every possibility for a basic sentence of the object language. Thus each basic sentence in the object language has been assigned a nominalistic surrogate. Given that these assignments have been made, we may also assign nominalistic surrogates for many (though not all) of the more complex sentences of the object language.
In general, we’ll say that for any sentence, \( \phi^* \) and any sentence, \( \psi^* \), \( \psi^* \) is a nominalistic surrogate for \( \phi^* \) if and only if

\( \phi^* \) is a closed sentence that contains no quantifiers and \( \psi^* \) is the result of taking each basic sentence in \( \phi^* \) and replacing it with its nominalistic surrogate.

Not every sentence in the object language is assigned a nominalistic surrogate by this definition. Nevertheless, the above will allow us to characterize what it is for any sentence in the object language, no matter how complex, to be correct, and to do so in terms of which nominalistically friendly sentences are true. That will be the task for the next section.

Before I move on to the next section, however, I would like to take this opportunity to address the role that the use, in the meta-language, of apparent quantification over entities such as expression types plays in the assignment of nominalistic surrogates to basic sentences. In some such cases it may be especially tempting to regard such sentences in the meta-language as themselves providing paraphrases of the relevant sentences in the object language. But if we are to avoid simply taking entities such as expression types as surrogates for entities such as features (thereby trading apparent quantification over one sort of nominalistically unacceptable entity for another), this is a temptation that must be resisted.

Consider, for example, a case in which we assign \( \neg \perp \) as the nominalistic surrogate for a sentence in the object language of the form \( Ft \) where \( F \) is a \( p \)-predicate and \( t \) is a \( p \)-name. If we have made this assignment correctly, then there is some \( n \)-predicate in the object language, \( G^* \), such that \( t \) is the projection of \( G^* \) and \( G^* \) is a \( C(F) \) predicate. To make the example even more specific, consider, as we did above, the case
in which \( F \) is a \( p \)-predicate in the object language that means the same thing as ‘is anatomical’ (as in the English sentence ‘\( \text{Being segmented} \) is an anatomical feature’).

Suppose (as we also did above) that \( t \) is the projection of a predicate in the object language that means the same thing as the English predicate ‘is segmented’ and that \( C(F) \) is a predicate in the meta-language that means the same thing as the English phrase ‘is anatomical’ (as used in the sentence ‘‘is segmented’ is an anatomical predicate’).

In that case we may translate into English (from the meta-language) the condition under which the sentence in the object language at issue is to be assigned (~\( \bot \)) as its nominalistic surrogate as follows:

The sentence that, in the object language, most directly translates ‘\( \text{Being segmented is anatomical} \)’ is to be assigned the nominalistic surrogate (~\( \bot \)) if and only if the atomic predicate that, in the object language, means the same thing as ‘is segmented’ is an anatomical predicate.

Given that the nominalistic surrogate assigned to the relevant sentence is also such that its truth conditions match the original sentence’s correctness conditions, the above is also equivalent to the following:

The sentence that, in the object language, most directly translates ‘\( \text{Being segmented is anatomical} \)’ is correct if and only if the atomic predicate that, in the object language, means the same thing as ‘is segmented’ is an anatomical predicate.

Now suppose, just for the moment, that our ontology includes such entities as expression types (both of English and of the object language) but no such entities as features. Then, given the truth of the above, the sentence ‘The atomic predicate that, in the object language, is synonymous with ‘is segmented’ is an anatomical predicate’ would serve as a nice paraphrase (from the perspective of our momentarily adopted ontology) of the
sentence that (in the object language) most directly translates ‘Being segmented is anatomical’. But (the previous supposition aside) this better not be how we are regarding the former sentence. We are interested in supplying nominalistically acceptable correctness conditions to the relevant sentences, not in engaging in a paraphrase strategy that employs such nominalistically unacceptable entities as expression types as surrogates for features.

If anything is to be regarded, from our perspective, as serving as a paraphrase of the sentence that (in the object language) translates ‘Being segmented is anatomical’, it is not the condition that we are able to state in the meta-language, but its nominalistic surrogate, the object language sentence (~ ⊥). Granted, as far as paraphrases go, this isn’t much of one. Using (~ ⊥) as a paraphrase for the original sentence is analogous to using an English sentence like ‘Everything is self-identical’ as a paraphrase for the English sentence ‘2 is a prime number’. Such a “paraphrase” (if one could bring oneself to call it that) is far from adequate, at least for most of the philosophical purposes for which paraphrases are employed. Nevertheless, it has this much going for it: its truth conditions are identical to the correctness conditions for the original sentence. And that, it will turn out, is enough for our purposes.

If that is so, however, then what role are the conditions that we are able to state in the meta-language actually playing? The answer is that they allow beings like us, even with our cognitive and linguistic limitations, to assign nominalistic surrogates to infinitely many sentences of the object language all at once. Our old friend the infinite being, however, who (we are now presuming) thinks in the object language, would have no need to make use of these conditions. It could run through each of the basic sentences
in the object language, directly see what its correctness conditions are (merely in virtue of grasping that sentence, in the same way that we can see that the sentence ‘Being segmented is an anatomical feature’ is correct under all conditions, merely in virtue of grasping that sentence), and assign an appropriate nominalistic surrogate to that sentence accordingly. And it could also do the equivalent of the latter, it is worth pointing out, without quantifying over entities such as expression types or conditions. If, for example, $Ft$ is a sentence in the object language whose nominalistic surrogate is $(\neg \bot)$, the infinite being could do the equivalent of assigning the latter sentence as the nominalistic surrogate of the former by endorsing (or “quasi-endorsing” in the manner described in Section 2.6) the sentence $((Ft \rightarrow (\neg \bot)) \& ((\neg \bot) \rightarrow Ft))$ (where endorsing or quasi-endorsing a sentence need only involve using that sentence and not mentioning it).

### 3.4 Nominalistic Undergirding

As noted in Section 3.1, the notion of a sentence’s being “nominalistically undergirded” (to be characterized in this section) is the formal analog of the Chapter 2 notion of what it is for a proposition to be such that its nominalistically acceptable ordinary correctness conditions are satisfied.

We may begin characterizing this notion by simply laying out a recursive definition of what it is for a closed sentence of the object language to be nominalistically undergirded: For any closed sentence, $\varphi^*$, $\varphi^*$ is nominalistically undergirded if and only if

$\varphi^*$ contains no quantifiers and the nominalistic surrogate for $\varphi^*$ is true, or
\( \varphi^* \) is of the form \((\sim \alpha)\) and \(\alpha\) is not nominalistically undergirded, or

\( \varphi^* \) is of the form \((\alpha \lor \beta)\) and either \(\alpha\) is nominalistically undergirded or \(\beta\) is nominalistically undergirded, or

\( \varphi^* \) is of the form \((\alpha \land \beta)\) and \(\alpha\) and \(\beta\) are each nominalistically undergirded, or

\( \varphi^* \) is of the form \((\alpha \rightarrow \beta)\) and either \(\alpha\) is not nominalistically undergirded or \(\beta\) is nominalistically undergirded, or

\( \varphi^* \) is of the form \(\exists v(\alpha)\) and there is some sentence, \(\alpha^*\), such that \(\alpha^*\) is a sentence that results from uniformly replacing every free occurrence of \(v\) in \(\alpha\) with a name and \(\alpha^*\) is nominalistically undergirded.

This definition allows one to take any closed sentence of the object language and (to turn a phrase from Quine) “chase correctness up the tree of grammar.” In the terminology of Chapter 2, the above definition ultimately tells us (if, for the sake of brevity, we continue speak of sentences rather than propositions as the bearers of truth and correctness) what sorts of nominalistically friendly sentences (of the object language) must be true in order for any given sentence (of the object language) to be ordinarily correct.

Consider, for example, the English sentence ‘There is a feature that Charlotte and Shelob have in common’. A direct translation of this sentence into the object language would consist of a sentence of the form \((\exists x)((Fx \land (cHx \land sHx)))\) where \(F\) is a p-predicate that means is a feature and \(c\) and \(s\) are names for Charlotte and Shelob respectively. The definition of nominalistic undergirding tells us, furthermore, that this sentence is nominalistically undergirded just in case at least one sentence that results from deleting the left most quantifier from the original sentence and replacing every free occurrence of \(x\) in the resulting open sentence with a name is also nominalistically undergirded. A sentence of the latter sort would be a sentence of the form \((Fp \land (cHp \land sHp))\) (where, if
that sentence is nominalistically undergirded, p must be a p-name). And, if a sentence of that sort is nominalistically undergirded, its nominalistic surrogate will be of the form \((\neg \bot) \&(Gc \& Gs)\) (where G is the n-predicate that projects p). This sentence, of course, will in turn be equivalent to a sentence of the form \((Gc \& Gs)\). So, in effect, our definition of nominalistic undergirding tells us that a translation into the object language of the English sentence ‘There is a feature that Charlotte and Shelob have in common’ is correct just in case there is at least one true sentence in the object language that translates an English sentence (or a sentence of an idealized language much like English except lacking for no relevant expressive resources) of the form \(\lnot \text{Shelob is } \square \text{ and Charlotte is } \square \) (where the blanks are to be filled in uniformly by an adjective). Note that here the definition of nominalistic undergirding coincides with what I said in Chapter 2 about the ordinary correctness conditions for the propositions expressed by sentences such as the one we have been considering. In Appendix A, I further exhibit how the above definition of nominalistic undergirding coincides with the notion of ordinary correctness found in Chapter 2 by showing this to be the case for a much more complex example.

The above definition tells us what it is for a closed sentence of the object language to be nominalistically undergirded. We will have occasion, however, to count various open sentences of the object language as nominalistically undergirded as well. In particular, we will want to count an open sentence as nominalistically undergirded whenever it is such that it is correct no matter how its free variables are uniformly “filled in” (so to speak). We will want, for example, to be able to say that any open sentence (of the object language) of the form \(x = x\) counts as nominalistically undergirded.
We may do this as follows: First, we’ll characterize what it is for one sentence to be a “corresponding closed sentence” for another:

For any sentence, \(\varphi^*\), and for any closed sentence, \(\psi^*\), \(\psi^*\) is a corresponding closed sentence for \(\varphi^*\) if and only if \(\psi^*\) is a sentence that results from taking each free variable in \(\varphi^*\) and uniformly replacing it with a name.

Given the above, we may characterize what it is for an open sentence of the object language to be nominalistically undergirded as follows:

For any open sentence, \(\varphi^*\), \(\varphi^*\) is nominalistically undergirded if and only if each of its corresponding closed sentences is nominalistically undergirded.

3.5 Axioms and Inference Rules

With these definitions in hand, we are now in a position to lay out some axiom schemas and inference rules that will together afford us with a formal logic that may be applied to the various sentences of the object language. The axiom schemas and rules that I discuss are, with one exception, of the sort that one would find in many a standard textbook on first order logic,\(^{106}\) axiom schemas and rules for which (when applied to object languages that are to be interpreted according to the standard semantics for first order languages) the standard soundness and completeness results may be proved.\(^{107}\) Thus, first order valid arguments that can be represented in English will have their counterparts in the object language (provided that they conform to the restrictions I’ve imposed on the sorts of arguments that can be represented in the latter). The rules that license the various

\(^{106}\) In particular, the axiom schemas and inference rules (with the exception noted below excluded) that I lay out here are adapted from the system found in (Leary 2000).

\(^{107}\) Note, however, that if in fact there are no referents for the \(p\)-names of the object language, the object language does not conform to such a semantics, since the standard semantics for first order languages excludes there being constants with no assigned referents.
inferences made in the former arguments will be of the same sort as those that license the corresponding inferences in their object language counterparts.

The one exception to the above is that I will include the following among my list of axiom schemas: $\varphi$ is an axiom if it is a sentence of the form $((Fx \rightarrow xHp) \& (xHp \rightarrow Fx))$, where $x$ is a variable, $F$ is an $n$-predicate, and $p$ is the $p$-name that $F$ projects. From a platonist point of view (at least from the point of view of a platonist who takes properties to be sufficiently abundant) this schema is valid. From a nominalist point of view, however, it is not. Nevertheless, both the nominalist and the platonist may agree that it is “correct” (and, in fact, it can be proven, and is proven in Appendix B, that each sentence in the object language that conforms to this schema is nominalistically undergirded). I include this axiom schema because it (or rather, its ordinary language counterpart) is often treated as valid in much of our ordinary discourse about properties.

For expository purposes (especially when it comes to laying out the proof of the Correctness Preservation Theorem in Appendix B), it will prove useful to divide the axioms of the object language into two groups. For reasons that are obvious upon inspection, the first group of axioms will be referred to as the quantifier free axioms and the second as the quantifier axioms.

The quantifier free axioms are given as follows: For any sentence of the object language, $\varphi^*$, $\varphi^*$ is a quantifier free axiom if and only if $^{108}$$\varphi^*$ is of the form $x = x$, where $x$ is a variable, or,

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$^{108}$ In various places below I insert (or fail to insert) parentheses for the sake of convenience and readability, regardless of whether my doing so is correct by the official grammar that I have assigned to the object language.
φ* is of the form \((x = y) \rightarrow (Fx \rightarrow Fy)\), where \(x\) and \(y\) are variables, or,

φ* is of the form \((x_1 = y_1) \& (x_2 = y_2) \& \ldots \& (x_n = y_n) \rightarrow (Rx_1x_2\ldots x_n \rightarrow Ry_1y_2\ldots y_n)\), where \(x_1, x_2, \ldots, x_n\) and \(y_1, y_2, \ldots, y_n\) are variables, or

φ* is of the form \((x_1 = y_1) \& (x_2 = y_2) \rightarrow (x_1Hx_2 \rightarrow y_1Hy_2)\), where \(x_1, x_2, y_1,\) and \(y_2\) are variables, or

φ* is of the form \((Fx \rightarrow xHp) \& (xHp \rightarrow Fx)\), where \(x\) is a variable, \(F\) is an \(n\)-predicate, and \(p\) is the \(p\)-name that \(F\) projects.

Before I provide the axiom schemas for the quantifier free axioms, another technical notion is needed – the relatively standard notion (adapted to the object language) of a term’s being *substitutable* for a variable. For any sentence, \(\varphi^*\), any term, \(u^*\), and any variable, \(v^*\), \(u^*\) is substitutable for \(v^*\) in \(\varphi^*\) if and only if

φ* is of one of the following forms: \(t_1 = t_2, Ft, Rt_1t_2\ldots t_n, t_1Ht_2\), or

φ* is of the form \((\neg \alpha)\) and \(u^*\) is substitutable for \(v^*\) in \(\alpha\), or

φ* is of the form \((\alpha \Box \beta)\) (where ‘\(\Box\)’ stands in for one of the binary truth functional connectives) and \(u^*\) is substitutable for \(v^*\) in \(\alpha\) and \(u^*\) is substitutable for \(v^*\) in \(\beta\), or

φ* is of the form \((\exists x)(\alpha)\) and either \(v^*\) is not free in \(\varphi^*\), or \(u^*\) is not \(x\) and \(u^*\) is substitutable for \(v^*\) in \(\alpha\).

With the above in place, here is the definition of what it is for a sentence in the object language to be a quantifier axiom: For any sentence, \(\varphi^*\), of the object language, \(\varphi^*\) is a *quantifier axiom* if and only if

φ* is of the form \(\neg \exists x(\neg \varphi) \rightarrow \varphi[x/t]\) (where \(x\) is a variable, \(t\) is a term, \(\varphi[x/t]\) is the result of replacing every free occurrence of \(x\) in \(\varphi\) with \(t\), and \(t\) is substitutable for \(x\) in \(\varphi\)), or

φ* is of the form \(\varphi[x/t] \rightarrow \exists x(\phi)\) (where \(x\) is a variable, \(t\) is a term, \(\varphi[x/t]\) is the result of replacing every free occurrence of \(x\) in \(\varphi\) with \(t\), and \(t\) is substitutable for \(x\) in \(\varphi\)).
Below I will also state the inference rules that are to apply to the object language. Before all of them can be stated, however, I will need to introduce a few more items of machinery. In addition to the object language, we will take ourselves to be working with another language, one suited for propositional logic. This language – call it “the propositional language” – comes equipped with the following countably infinite stock of propositional variables: \( P_1, P_2, \ldots \). We will think of these variables as coming in an order, with their ordinal numbers being the numbers that correspond to their subscripts. The propositional language also comes equipped with the standard truth functional connectives (the same ones as those found in the object language). The formation rules for sentences in the propositional language are the standard ones.

Any sentence, we’ll say, that results from replacing some of the subsentences of a sentence in the object language with propositional variables is to be referred to as a “mixed sentence.” For any given sentence of the object language, \( \varphi^* \), the propositional correlate of \( \varphi^* \) is found by executing the following procedure:

1. Find the first subsentence of \( \varphi^* \) (reading from left to right) of the form \( \exists v(\alpha) \) that is not in the scope of another quantifier (if there is such a subsentence) and uniformly replace each distinct occurrence of that same subsentence with \( P_1 \). Take the resulting mixed sentence and repeat this step (except instead of using the previously used propositional variable to make the replacements, use the next one) over and over again until there are no more subsentences of the form \( \exists v(\alpha) \) that are not in the scope of another quantifier. Then proceed to step 2.

2. Find the first (reading from left to right) atomic subsentence of the object language (i.e. the first subsentence of one of the following forms: \( t_1 = t_2, Ft, R(t_1t_2\ldots t_n, t_1Ht_2) \) of the (mixed or unmixed) sentence that remains after carrying out step 1 (if there is any such subsentence) and uniformly replace each distinct occurrence of that same subsentence with the next, unused propositional variable. Repeat this step until there are no remaining atomic subsentences of the object language.
We’ll also say that for any sentences, $\psi^*_1, \psi^*_2, \ldots, \psi^*_n$, $\phi^*$,

$\phi^*$ is a *propositional consequence* of $\psi^*_1, \psi^*_2, \ldots, \psi^*_n$ if and only if the propositional correlate of $\psi^*_1 \& \psi^*_2 \& \ldots \psi^*_n \rightarrow \phi^*$ is a truth functional tautology.

With the above in hand, we may now list the inference rules for our formal system as follows:

**Axiom Rule:** For any sentence, $\phi^*$, if $\phi^*$ is an axiom, one may infer $\phi^*$ from any (or no) sentences.

**Propositional Consequence:** For any sentences, $\psi^*_1, \psi^*_2, \ldots, \psi^*_n$, $\phi^*$, if $\phi^*$ is a propositional consequence of $\psi^*_1, \psi^*_2, \ldots, \psi^*_n$, one may infer $\phi^*$ from $\psi^*_1, \psi^*_2, \ldots, \psi^*_n$.

**Conditional Universal Generalization:** From any sentence of the form $\psi \rightarrow \phi$, one may infer the corresponding sentence of the form $\psi \rightarrow \exists x(\sim \phi)$, provided that the variable bound by the leftmost quantifier of the consequent of the latter sentence is not free in the antecedent of the former sentence.

**Conditional Existential Generalization:** From any sentence of the form $\phi \rightarrow \psi$, one may infer the corresponding sentence of the form $\exists x(\phi) \rightarrow \psi$, provided that the variable bound by the leftmost quantifier of the antecedent of the latter sentence is not free in the consequent of the former sentence.

### 3.6 An Outline of the Proof of the Correctness Preservation Theorem

As noted above, the proof of the Correctness Preservation Theorem is straightforward insofar as it mirrors a standard sort proof of the soundness of first order logic. The actual proof is given in Appendix B. An outline of that proof may be given as follows:

It is to be shown that, for any sentences of the object language, $\psi^*_1, \psi^*_2, \ldots, \psi^*_n$, $\phi^*$, if each of the sentences $\psi^*_1, \psi^*_2, \ldots, \psi^*_n$ is nominalistically undergirded and, by one or more applications of the inference rules (provided for the object language), $\phi^*$ may be inferred from $\psi^*_1, \psi^*_2, \ldots, \psi^*_n$, then $\phi^*$ is nominalistically undergirded.
First, it will be proved that each of the axioms is nominalistically undergirded.

Second, it will be shown that the inference rules preserve nominalistic undergirding. That is, it will be shown that, for any sentences $\psi^*_1, \psi'^*_2, \ldots, \psi^*_n, \phi^*$, if, according to one of the inference rules, $\phi^*$ may be inferred from $\psi^*_1, \psi'^*_2, \ldots, \psi^*_n$ via a single application of that rule, then if $\psi^*_1, \psi'^*_2, \ldots, \psi^*_n$ are nominalistically undergirded, $\phi^*$ is nominalistically undergirded.

Third, given the above proofs, one may easily prove the Correctness Preservation Theorem via mathematical induction on the number of applications of the inference rules.

### 3.7 On the Relationship Between Nominalistic Undergirding and Truth

In Appendix C, a proof is given of the following:

*Conservation Lemma:* For each sentence of the object language, $\phi^*$, such that $\phi^*$ is a closed, quantifier free sentence that contains no p-names, $\phi^*$ is nominalistically undergirded if and only if $\phi^*$ is true.

It is easy to see that, in conjunction with the Correctness Preservation Theorem, the Conservation Lemma entails the following:

*Conservation Theorem:* For any sentences of the object language, $\psi_1^*, \psi_2^*, \ldots, \psi_n^*, \phi^*$, if each of the sentences $\psi_1^*, \psi_2^*, \ldots, \psi_n^*$ is nominalistically undergirded and, by one or more applications of the inference rules (provided for the object language), $\phi^*$ may be inferred from $\psi_1^*, \psi_2^*, \ldots, \psi_n^*$, then if $\phi^*$ is a closed, quantifier free sentence that contains no p-names, $\phi^*$ is true.

This theorem, in turn, can be appealed to by the nominalist in order to explain why reasoning as if there were such items as monadic properties that can be had by concrete things is useful, even if in fact there are no such entities as properties.
Consider, for example, the following argument as given in *English*:

(P1) Charlotte and Shelob share all and only the same anatomical features.

(P2) Charlotte has the property of being segmented.

(P3) The property of being segmented is an anatomical feature.

(P4) Shelob has the property of being segmented if and only if Shelob is segmented.

(C) Therefore, Shelob is segmented.

This argument is clearly valid. Both the nominalist and the platonist, furthermore, might find themselves in the position of wanting to endorse this argument as a *good* one, one that gives us (in conjunction with whatever support we have for the truth or correctness of its premises) good reason to believe that its conclusion is true. But if the nominalist and the platonist do both find themselves in that position, the nominalist initially seems to be at a disadvantage. Whereas the platonist can think that the above is a good argument on account of the fact that it is both valid and such that all of its premises are strictly speaking true (i.e. such that its premises perspicuously express true propositions), the nominalist cannot think this. There is, as a matter of fact, not a single premise of the above argument that the nominalist can consistently regard as strictly speaking (non-vacuously) true.

Nevertheless, the nominalist might well be in a position to believe that each of the premises of the above argument are *correct*. Since, furthermore, the above argument will have a translation into the object language, the nominalist might well be in a position
to believe that each of the premises of that translation are nominalistically undergirded.
If so, the Correctness Preservation Theorem assures her that the translation of the
conclusion of the above argument into the object language is also nominalistically
undergirded. In addition, since the conclusion of the above argument translates into the
object language as a closed, quantifier free sentence that contains no p-names, the
Conservation Theorem assures the nominalist that the conclusion of the above argument
is also true. So the nominalist, like the platonist, can consistently regard the above
argument as a good one (one that gives us good reason to believe its conclusion), even
though (unlike the platonist) she is committed to denying that its premises are strictly
speaking true.

These considerations generalize even further, so as also to apply to several
arguments that don’t have quantifier free, closed sentences as their conclusions.
Consider, for example, a first order valid argument (one which has a translation into the
object language), with premises employing apparent quantification over monadic
features, each of the premises of which are correct (i.e. such that their translations into the
object language are nominalistically undergirded), that has the following as its
conclusion:

\[(C^*) \text{There is an } x \text{ and a } y, \text{ such that } x \text{ is not identical to } y, \text{ and such that } x \text{ is a spider, and } y \text{ is a spider, and } x \text{ is conspecific with } y, \text{ and if } x \text{ is segmented, then } y \text{ is segmented.}\]

\(C^*\) is obviously not a quantifier free sentence. Nevertheless, given the above stipulations,
the Correctness Preservation Theorem assures us that its translation into the object
language is nominalistically undergirded. Inspection of the conditions under which
sentences in the object language are nominalistically undergirded (in conjunction with the background knowledge that spiders are concrete things), furthermore, reveals that C*'s translation into the object language is nominalistically undergirded if and only if there is a nominalistically undergirded, closed, quantifier free sentence in the object language that contains no p-names, one that asserts of two nominalistically acceptable objects that they are distinct, that they are spiders, of a given one of them that it is conspecific with the other, and of that same one that if it is segmented then so is the other. The Conservation Lemma assures us, furthermore, that any such nominalistically undergirded sentence is true. And it is obvious that if there is such a true sentence in the object language, then C* is also true.

It is also obvious that the above considerations generalize so as to apply to any formally valid argument that can be translated into the object language, has correct premises, a conclusion whose only referring expressions refer to nominalistically acceptable objects, and a conclusion that can be seen to have the same truth value when each of its quantifiers are restricted so as to range only over nominalistically acceptable objects. It is obvious, that is, that the above considerations generalize so as to apply to any argument that can be translated into the object language, that has correct premises, and that has as its conclusion a nominalistically friendly sentence (in the technical sense laid out in Section 2.3 of what it is for a sentence to be “nominalistically friendly”).

It is of some interest to compare this result with an analogous result that Hartry Field has argued holds for mathematics. As noted in Chapter 1, Field argues that our mathematical theories are “conservative,” where (to a first approximation) a mathematical theory is conservative if and only if it is such that, when it is conjoined
with any nominalistically acceptable theory, the resulting theory has all and only the
same nominalistically acceptable consequences as does the original nominalistically
acceptable theory. As Field also points out, however, this initial characterization of what
it is for a mathematical theory to be conservative isn’t quite right. That’s because a
nominalistically acceptable theory might say things that are inconsistent with a
mathematical theory (e.g. the former might rule out the existence of mathematical
entities). Field handles this by insisting that mathematics is conservative with respect
to theories that are agnostic concerning the existence of mathematical entities. He further
notes that a nominalistically acceptable theory can be rendered agnostic concerning the
existence of mathematical entities by restricting the quantifiers of the sentences that
express it so that they only range over non-mathematical entities.

So, as Field points out, the claim that a mathematical theory is conservative
should really be stated as the claim that when it is conjoined with a nominalistically
acceptable theory expressed by sentences whose quantifiers are restricted so as to range
only over non-mathematical entities, any nominalistically acceptable consequence of the
resulting theory (that can be expressed by a sentence whose quantifiers are also so
restricted) is also a consequence of the original nominalistically acceptable theory. Similarly, the Conservation Theorem delivers the result that whenever we have a
formally valid argument (that has a translation into the object language) from correct
premises involving apparent quantification over monadic, first order properties, to a
conclusion whose only referring expressions refer to nominalistically acceptable objects,

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109 Field (1980, p. 11)

110 (Field 1980, pp. 11-12)
a conclusion which also can be seen to be true when all of its quantifiers are restricted so as to range only over such objects, that conclusion is true.

Nevertheless, in spite of the above analogies, the similarity between my approach and Field’s ends fairly quickly. I do not show, nor is it a concern of mine even to attempt to show, for example, that there is some version of property theory that is conservative in the same sense in which Field argues that mathematical theories are conservative. That is, it is no concern of mine to argue that there is a suitable version of property theory such that, when it is conjoined with any nominalistically acceptable theory (expressed by sentences whose quantifiers are restricted so as to range only over nominalistically acceptable entities), the resulting theory has no nominalistically acceptable consequences (expressed by sentences whose quantifiers are likewise so restricted) that are not had by the original theory. I don’t deny this, but it is not what I take to be at issue. My concern is simply to argue that we have good reason to believe that when we get such nominalistically acceptable consequences, via formally valid arguments (that have translations into the object language) that employ correct premises, those consequences are strictly speaking true.

It doesn’t follow from what I argue, however, that the resulting conclusions follow solely from whatever nominalistically acceptable theories we might have employed to furnish us with the premises of our argument. I do argue (in effect) that in such cases the resulting conclusions are entailed by the various nominalistically acceptable facts in virtue of which the premises of those arguments are correct (i.e. by the nominalistically acceptable facts in virtue of which the translations of those premises into the object language are nominalistically undergirded). But that is entirely consistent with
its being the case that none of our nominalistically acceptable theories state or entail those facts. It is entirely consistent with its being the case that we are ignorant of those facts, and even with its being the case that we couldn’t possibly state them or comprehend them.

Accordingly, my approach does not demand (as Field’s does) that we begin with nominalistically acceptable theories and use reasoning as if there were abstract entities merely as an instrumentally useful way of discovering the nominalistically acceptable consequences of those theories. It may be as Melia (1995) has argued (and as I conceded for the sake of argument in the previous chapter) that (on account of our cognitive and linguistic limitations) we don’t (and can’t) have access to a fully adequate theory of the world that the nominalist can consistently regard as strictly speaking true. Nevertheless, we might be able to get the next best thing – namely some reassurance that reasoning as if there are abstract objects is appropriately undergirded by the nominalistically acceptable facts, undergirded by those facts in such a way that when we properly infer nominalistically acceptable claims by way of such reasoning, we (quite often) have good reason to believe that those claims are true. My project in this chapter has been to provide such reassurance when it comes to a certain fragment of that reasoning (i.e. the fragment that can be represented by arguments that translate into the object language I have characterized). And in the next chapter, I argue that there is good reason to think that the considerations I have advanced in this chapter generalize beyond that fragment, indeed, that they generalize to any sort of reasoning involving apparent quantification over abstract objects that would be desirable for the nominalist to accommodate.
3.8 Works Cited


4.1 Overview

In this chapter I argue that there is good reason to think that the considerations raised in the previous two chapters generalize so as to apply to any sort of reasoning involving apparent quantification over abstract objects that would be desirable for the nominalist to accommodate. I do so in three steps. First, I make some general observations in support of the claim that the apparatus developed in the previous chapter can be expanded so as to apply to broader fragments of our ordinary and theoretical discourse involving apparent quantification over abstract objects (from now on “our platonist discourse”). I also illustrate how these suggestions might be applied in Appendix D and Appendix E.

Second, I argue that even in cases in which we (owing to our cognitive and linguistic limitations) are unable to generalize the apparatus developed in the previous chapter so as to apply it to a certain fragment of our platonist discourse, it is still plausible to think the considerations raised in the process of developing that apparatus apply to that fragment (at least if that particular fragment of our platonist discourse is worth keeping). Finally, I address the possible objection that the use of apparent quantification over propositions and sentence types that I’ve deployed in previous chapters is dialectically illegitimate, or at least that it would be self-defeating for the nominalist to make use of it. I complete this last step by arguing that the considerations that I raised in defense of the nominalistic legitimacy of apparent quantification over features in Chapter 2 generalize so as to
include apparent quantification over linguistic entities and propositions, or at least to
include the uses to which I’ve put such apparent quantification.

4.2 On the General Applicability of the Chapter 3 Apparatus

What’s needed to expand the apparatus of Chapter 3 to encompass some other given
fragment of our platonist discourse are the following ingredients: First, we have to have
some way of making stipulations that guarantee that the object language has names for all
of the abstract objects that (we speak as if) fall within the range of our quantifiers when
we are engaged in that fragment of our discourse. Second, there has to be some way of
appropriately assigning nominalistic surrogates to all of the basic sentences of the
suitably expanded version of object language that contains those names, or at least to
enough of those basic sentences that the conditions under which the other sentences of
the object language are nominalistically undergirded can be defined in terms of them.
Third, we have to have some reassurance that our assignments of the conditions under
which the sentences of the expanded version of the object language are nominalistically
undergirded are consistent (i.e. that they don’t count some of the same sentences as both
nominalistically undergirded and not nominalistically undergirded). Finally, we need to
be able to show that the inference rules associated with the expanded version of the
apparatus found in Chapter 3 are correctness preserving.

In Appendix D, I provide an example (one that is met to be illustrative) of how
these ingredients can be had with respect to our discourse involving apparent
quantification over sets. In Appendix E, I sketch some further strategies for generalizing
that apparatus so as that it applies to apparent quantification over non-monadic relations,
as well as over higher order properties. I submit for the time being, however, that, even prior to attempting to expand the Chapter 3 apparatus in these sorts of ways, we have good reason to believe that the above ingredients can be had for a considerable amount of our platonist discourse.

I’ll begin to support this claim by starting with the observation that much of that discourse is guided by some fundamental bridge principles that posit equivalencies between nominalistically friendly sentences and sentences that aren’t nominalistically friendly. Consider the following schemas (where what occurs on the left hand side is to be replaced by a nominalistically friendly sentence of the same form):

\[ n \text{ is } F \equiv n \text{ has the property of being } F \] (where \( n \) is a name and \( F \) is a predicate)

\[ \text{There are some } Fs \equiv \text{there is a set of the } Fs \] (where \( F \) is a predicate)

\[ \text{There are } n \text{ Fs } \equiv \text{The number of the } Fs \text{ is identical to } n \] (where \( n \) as it occurs on the left hand side is a numerical adjective and where \( n \) as it occurs on the right hand side is a name for a number)

\[ p \equiv \text{the proposition that } p \text{ is true} \] (where \( p \) is a truth-evaluable declarative sentence)

Schemas such as these can play multiple roles when it comes to expanding the apparatus of Chapter 3.

First, such schemas can assist us in introducing into the object language all of the names that we want. The first schema, for example, was the inspiration behind the stipulation in Chapter 3 that every \( n \)-predicate of the object language be regarded as “projecting” a corresponding \( p \)-name. Parallel stipulations could be made with respect to the others of the above schemas in order to introduce names for different kinds of abstract
entities into the object language. In Chapter 3, I also stipulated that the p-names projected by the n-predicates of the object language be regarded as syntactically marking the predicates that project them. This was no mere artificial device. The property names that play a role in our natural language discourse concerning properties, property names that figure into schemas like the first one above, behave in just that way. The same goes (mutatis mutandis) for the kinds of set names, numerical names, proposition names, etc. that play a role in schemas such as those above.

Second, the above schemas afford us with ways of assigning nominalistic surrogates to basic sentences that are also what we might call “impure sentences” (i.e. sentences that are, in some intuitive sense that is easy enough to recognize but hard to define, neither solely about abstract objects nor solely about concrete objects). The basic sentence of the object language that translates ‘Shelob has being segmented’ (where ‘being segmented’ is a property name), for example, is such a sentence. This sentence can be thought of as invoking a relation (or “non-relational tie,” as some platonists would have it) between a concrete entity (assuming, as I have in previous examples, that ‘Shelob’ names a spider) and an abstract property (namely, the property of being segmented). But, in spite of the fact that this sentence can be thought of as invoking a relation to an abstract entity, it can also (via the fact that our discourse regarding properties is guided by the first of the above schemas) be regarded as correctness equivalent to a nominalistically friendly sentence – namely, to ‘Shelob is segmented’. In Chapter 3, I took advantage of this fact as a means of assigning nominalistic surrogates to basic sentences involving the property having predicate. Similar advantage could be
taken of the others of the above schemas when it comes to assigning nominalistic surrogates for basic sentences that (putatively) refer to other sorts of abstract entities.

Another thing that schemas of the above sort can do for us (though I didn’t take much advantage of this fact in Chapter 3) is allow us to make stipulations that generate names for “higher order” abstract entities, that is, for abstract entities that relate to other abstract entities in the same sort of way that their first order cousins relate to concrete entities. In Chapter 3, I only concerned myself with first order monadic properties, and for that reason only allowed n-predicates to have projections. But something like the first schema also governs our reasoning about higher order properties (i.e. properties of properties). We are inclined to infer from the sentence ‘Red is a color’, for example, the sentence ‘Red has the property of being a color’. This suggests that we can get a lot of the names for the higher order properties that we might want by also allowing the p-predicates of the object language to have projections. I say more about how to take advantage of that suggestion in the Appendix E. In Appendix D, I also exploit a similar strategy to stipulate into the object language names for “higher order” sets (i.e. sets that have sets as members). It is true that a lesson well learned from the history of philosophical logic is that, in order to avoid paradox, one needs to be careful about how one applies schemas such as those mentioned at the beginning of this section when it comes to reasoning about higher order abstracta. Nevertheless, if due caution is taken, they can be used to expand the apparatus of Chapter 3 so that it is capable of handling apparent quantification over such entities.

One thing that schemas like those above aren’t much help with, however, is the task of assigning nominalistic surrogates to those basic sentences of the object language
that we might call (and did call in Chapter 3) “pure platonist sentences.” These are sentences that (in some intuitive sense) are entirely about (or at least putatively entirely about) abstract entities. The basic sentence that translates ‘Being segmented is anatomical’ is an example of such a sentence. So is the basic sentence that translates ‘Being segmented is identical to being segmented’. Schemas such as those listed at the beginning of this section don’t tell us much about the nominalistically acceptable conditions under which these sorts of sentences are to be regarded as correct. But the task of finding nominalistic surrogates for sentences such as these also isn’t all that worrisome, *provided that* we already have a way of deciding in advance which of these sentences are to be regarded as correct. That’s because pure platonist sentences are correct or incorrect *regardless* of how things are with the concrete world. So their nominalistic surrogates needn’t say anything informative about the nominalistically acceptable conditions under which they are to be regarded as correct. It is for that reason that we can (as we did in Chapter 3) assign a tautological nominalistically friendly sentence to serve as the nominalistic surrogate for those pure platonist sentences that are correct and the negation of that tautological sentence to those of them that are incorrect. What’s more worrisome, from the current vantage point, is the question of how such basic sentences are to be assigned the status of being correct (or incorrect) in the first place.

I don’t think there is much of a problem in this regard, however, when it comes to the basic sentences of the object language that translate sentences like ‘Being segmented is identical to being segmented’. The latter sentence is a formal consequence of ‘There is such a property as being segmented’. So its correctness is simply a formal consequence
of other sentences to which we are (by engaging the discourse at issue) committed to regarding as correct. This kind of consideration easily generalizes, of course, to expansions of the apparatus in Chapter 3 that treat other fragments of our platonist discourse than the one treated in that chapter.

There also isn’t much to worry about when it comes to basic sentences of the object language that translate sentences like ‘Being segmented is anatomical’. This sentence isn’t a formal consequence of other sentences that we are committed to regarding as correct simply by virtue of taking up the platonist discourse at issue. Nevertheless, the sentence ‘If there is such a thing as the property of being segmented, it is an anatomical property’ is as good a candidate for expressing a conceptual truth as anything is. And so the correctness of ‘Being segmented is anatomical’ winds up being a formal consequence of sentences that express conceptual truths, in conjunction with other sentences that we are already committed to regarding as correct. This sort of consideration also generalizes, of course. It applies, for example, to an expanded version of the Chapter 3 apparatus that contains translations for sentences like ‘The empty set is a pure set’.

In other cases, the correctness of such basic sentences might be a consequence of certain nominalistically acceptable necessary truths that bear an intimate relationship to them. Consider, for example, the sentence of the object language that translates ‘Crimson is a shade of red’.\footnote{It’s actually not entirely clear, based on what was said in Chapter 3, whether there is a translation of this sentence into the object language as it was characterized there. Whether there is depends on just how detailed the classification scheme that was taken to accompany the object language is taken to be. If, in addition to including categories like being an anatomical predicate, being a psychological predicate, and the like, it is also taken to include categories like being a red-shade predicate, then this} This sentence is correct. But it is not a formal consequence of other
sentences that we’ve stipulated to be correct. Nor is it a formal consequence of those sentences in conjunction with sentences that express conceptual truths. Nevertheless, the fact that this sentence is correct is intimately related to the nominalistically acceptable fact that necessarily, anything that is crimson is thereby red.\(^{112}\) We can see (on a priori grounds) that this sentence is correct because this fact obtains, and that other correct atomic sentences employing the predicate that translates ‘is a shade of red’ (e.g. the atomic sentence that translates ‘Burgundy is a shade of red’) are correct on account of the fact that the parallel facts obtain with respect to them (e.g. the fact that necessarily, anything that is burgundy is thereby red).

In other cases, there may not be any factors like the ones listed above available to help us decide whether to regard a certain class of basic, pure platonist sentences as being correct. Consider, for example, the basic sentence of the object language that translates ‘Being trilateral is identical to being triangular’. Should we regard this sentence as being correct? More generally, should we think of properties as being such that there can be sentence is part of the object language. If not, then we can take this sentence to be included in an expanded version of the object language. It doesn’t matter which.

\(^{112}\) The occurrence of ‘thereby’ in the sentence ‘Necessarily, anything that is crimson is thereby red’ should be taken as marking an explanatory connection of some sort. These other sentences might have done just as well:

‘Necessarily, if something is both crimson and red, it is red because it is crimson’

‘Necessarily, if something is both red and crimson, it is red in virtue of being crimson’

Just what sort of explanatory connection is at issue in these sentences is murky (though it seems to be the sort of connection that the literature on grounding and metaphysical explanation attempts to explicate). Since I am claiming that the above sentences are nominalistically friendly, I have to take a stand and maintain that they aren’t to be analyzed as positing relations to properties or other sorts of abstract entities. Otherwise, I don’t know what to say (but no one else appears to agree about what to say about these sorts of explanatory connections either). I can’t find a way of adequately capturing the claim at issue, however, without invoking such a connection (though if someone else can do so, all the better). I’m also presupposing that the direction of the relevant explanatory relation is as seems intuitively right to me. It strikes me that, if anything, a thing has a determinable characteristic because it has the relevant determinate characteristic. But some of my colleagues have reported having the opposite intuition.
distinct but necessarily coextensive properties, or not? From the perspective of a believer in properties, this is a substantive question. From a nominalist perspective, however, at best, all that might be said in favor of one side of the dispute regarding this question is that it better captures our conception of what properties are like, or is a better fit with our discourse that involves talk of properties. If this may be truthfully said of one side of the dispute, then that may be a reason to assign correctness conditions in accordance with what is said by that side (if one is trying, as best as one can, to capture our ordinary conception of properties or our ordinary discourse), or it may not (if, for example, one is attempting to regiment our concept of properties or our ordinary discourse involving talk of properties for some philosophical purpose that is better served by taking the side that doesn’t fit quite as well). In any case, though, from a nominalist point of view, choosing what to make of sentences of this sort will be a pragmatic matter, one that depends on what our purposes are. And, in some cases, it might simply require an arbitrary decision. In any case, though, there will be a way to go about it.

So far, then, we’ve seen that the correctness of a basic, pure platonist sentence can often be determined in at least one of the following ways: (1) by its being a formal logical consequence of other sentences that were stipulated to be correct, (2) by its being a formal logical consequence of conceptual truths in conjunction with other sentences that were stipulated to be correct, (3) by its bearing the same sort of intimate relations to necessary, nominalistically acceptable truths that sentences like ‘Crimson is a shade of red’ bear to the fact that necessarily, anything that is crimson is thereby red, (4) by its being such that regarding it as correct fits best with our concepts of the relevant entities, (5) by its being a better fit with our discourse involving talk of those entities, or (6) by its
being a pragmatic matter to be settled either by our interests or by some arbitrary stipulation on our part. Such resources could be exploited to assign the status of being correct or of being incorrect to all manner of basic, pure platonist sentences that might be found in various expansions of the apparatus of Chapter 3.

It is plausible, then, that by exploiting all of the sorts of resources mentioned above, we could go a long way towards vastly expanding the apparatus of Chapter 3 so as to make it apply to a much larger fragment of our platonist discourse. At least, what has been said above makes it plausible that we could (for much of that discourse) get as far as supplementing the object language with additional names for the various abstract objects that we appear to quantify over, increasing the stock of predicates that can be combined with those names in atomic sentences to form correct sentences, and assigning nominalistic surrogates to all of the basic sentences that have thereby been added to the object language. In other words, it’s plausible that we can have the first two ingredients (of those mentioned at the beginning of this section) that are needed to expand the apparatus of Chapter 3.

To complete the relevant expansions of the apparatus of Chapter 3, however, it is not enough to have the first two ingredients mentioned at the outset of this section. The final two ingredients must also be had as well. That is, we need to have some reassurance that our assignments of the conditions under which the sentences of the object language are nominalistically undergirded are consistent (i.e. that they don’t count some of the same sentences as both nominalistically undergirded and as not nominalistically undergirded). And, we also need to be able to show that the inference
rules associated with the suitably expanded version of the apparatus found in Chapter 3 are correctness preserving.

With respect to the first of these, there won’t always be a *proof* that our assignments of the conditions under which sentences of the object language are nominalistically undergirded are consistent, at least not if (as I argue in Appendix D) that apparatus can be expanded to encompass things like iterative set theory (of something like the Zermelo-Fraenkel variety). Suppose that, in fact, we have expanded the apparatus of Chapter 3 in such a way that it is capable of accurately representing the conditions under which the sentences of iterative set theory are correct. In that case, a proof of the consistency of the conditions under which the sentences of the expanded version of the object language count as nominalistically undergirded would amount to a proof of the consistency of iterative set theory. But the claim that iterative set theory is consistent is one of those things that Gödel showed us we can’t prove. Nevertheless, insofar as we can be confident that our expanded apparatus accurately represents when the sentences employed in iterative set theory are correct, we can be assured that it is consistent (or at least that the fragment of it that pertains solely to iterative set theory is consistent) to the extent that we can be assured that iterative set theory is consistent. And the consistency of iterative set theory is not really in doubt.

More generally, if we’ve managed to expand the apparatus of Chapter 3 in such a way as to capture a larger fragment of our platonist discourse, and we’ve assured ourselves that we’ve managed to do this so that the expansion accurately captures the conditions under which the sentences that are constitutive of that discourse are correct, then we can be assured that the conditions that we’ve assigned are consistent to the extent
that we can be assured that the relevant fragment of our discourse is consistent. As the paradoxes of naïve set theory and naïve property theory make clear, some of our discourse involving apparent quantification over abstract objects is inconsistent (at least prior to its having been appropriately subjected to philosophical regimentation). Even so, we’re generally convinced that large fragments of that discourse are consistent. And that assurance seems to have quite a bit of epistemological backing (from both a priori as well as empirical considerations).

So much, then, for the third ingredient. The final ingredient for expanding the apparatus of Chapter 3 is that we have some way of showing that the inference rules associated with the expanded version of the object language are correctness preserving. But as long as the relevant expansion of the apparatus of Chapter 3 defines nominalistic undergirding in the same sort of recursive way that it was defined in Chapter 3, and as long as nominalistic undergirding for the expanded version of that apparatus continues to be suited to be the formal analog of correctness for the relevant body of sentences, we will be able to show this in the same sort of way that we did in Chapter 3. We can see that this is the case by examining just what it was about how nominalistic undergirding was defined in Chapter 3 that allowed us to prove the Correctness Preservation Theorem.

Each non-basic sentence of the object language characterized in Chapter 3 was assigned its status of being nominalistically undergirded, or its status of failing to be so, via recursive definition in terms of the conditions under which the basic sentences of the object language count as nominalistically undergirded. But the recursive definition by which the more complex sentences of the object language were assigned the status of being nominalistically undergirded (or failing to be so) formally related nominalistic
undergirding for non-basic sentences to which basic sentences were nominalistically
undergirded in exactly the same way in which truth for non-basic sentences is formally
related to which basic sentences are true. Since this was the case, the inference rules
associated with the object language were guaranteed to preserve correctness provided that
y they also preserved truth.

Now, as matters stood in Chapter 3, the inference rules associated with the object
language did not preserve truth, at least not if nominalism is true. That’s because it was
there stipulated that the following is to be regarded as an axiom schema: $\phi$ is an axiom if
it is of the form $((Fx \rightarrow xHp) \& (xHp \rightarrow Fx))$, where $x$ is a variable, $F$ is an n-predicate,
and $p$ is the p-name that $F$ projects. But from a nominalist point of view, the
 correspondi
ng closed sentences for the instances of this schema are not true (because they
imply that there are properties). Suppose, however, that we had excluded this axiom
schema from the apparatus of Chapter 3. In that case, the inference rules specified for the
object language in Chapter 3 would have just been the inference rules provided by a
standard textbook on first order logic. And those rules (as we all know) are truth
preserving. So (for the reasons given in the previous paragraph), the inference rules
associated with the object language (sans the inclusion of the axiom schema just
mentioned) were also guaranteed to preserve nominalistic undergirding. The rule that
one may infer instances of the above axiom schema from any or no sentences,
furthermore, also preserved nominalistic undergirding (as proved in Appendix B). So the
addition of that rule did not alter the fact that the rules associated with the object
language in Chapter 3 preserved nominalistic undergirding. The fact that the apparatus of
Chapter 3 had the above features, furthermore, explains why it was possible to prove that
the inference rules associated with that apparatus were correctness preserving by means
of a proof that mirrors a standard sort of proof of the soundness of first order logic.

More generally, then, we can be confident that any expanded version of the
apparatus of Chapter 3 is such that the inference rules associated with it can be shown to
be correctness preserving provided that the following conditions are met: (i) Nominalistic
undergirding continues to serve as a suitable formal analog for correctness. (ii)
Nominalistic undergirding for complex sentences continues to be formally related to
nominalistic undergirding for basic sentences in the same manner as truth for complex
sentences is formally related to truth for basic sentences. (iii) A subset of the inference
rules associated with the object language by that apparatus are truth preserving. (iv) Any
rules that the apparatus associates with the object language that are not truth preserving
can nonetheless be shown to be correctness preserving.

I submit that we have good reason to believe that these conditions are likely easily
met for large fragments of our platonist discourse. Suppose we take a certain fragment of
that discourse other than the one that was treated in Chapter 3 – a fragment the includes
much of our discourse involving the use of iterative set theory, for example – and expand
the apparatus of Chapter 3 so that it applies to that fragment. Suppose that the resulting
expansion of the object language now includes all of the names for abstract objects that it
needs to have to capture that fragment of our discourse, and translations for each of the
sentences that pertain to that discourse (suppose furthermore that all such translations can
be represented in the canonical form of first order logic). Suppose also that whenever the
conventions that govern the relevant fragment our discourse count a sentence as correct,
and that sentence has a basic sentence as a translation into the object language, the
expanded apparatus assigns that basic sentence a true nominalistic surrogate. That is, suppose that the conditions under which basic sentences of the object language count as nominalistically undergirded match the conditions under which the sentences of our discourse that translate into the object language as basic sentences are correct. Suppose also that the conventions governing that fragment of our discourse entail that if a given sentence of that discourse is to be counted as correct, any formal (first order) logical consequence of that sentence is also to be counted as correct. Suppose, finally, that the relevant fragment of our discourse is consistent – i.e. that the conventions that govern it do not commit us to regarding some sentences as both correct and not correct.

In such a case, conditions (i) through (iv) mentioned above can all be easily satisfied. Condition (i) will be satisfied with respect to the basic sentences of the object language because the conditions under which basic sentences of the object language count as nominalistically undergirded match the conditions under which the sentences of our discourse that translate into the object language as basic sentences are correct. And if condition (ii) is met (something we can make true by stipulation), condition (i) will also hold with respect to the more complex sentences of the object language. This follows from the fact that the conventions governing the relevant fragment of our discourse ensure that the first order consequences of the correct sentences of our discourse that have translations into the object language are also correct. And condition (ii) (in conjunction with the fact that condition (i) holds with respect to the sentences of the fragment of our discourse that have translations as basic sentences) ensures that the translations of the latter sentences into the object language are also nominalistically undergirded. Condition (iii) can also be met by us via stipulation (e.g. by our making it
the case that a subset of the rules associated with the object language are those found in a standard textbook on first order logic). And condition (iv) will be met provided that we are sufficiently careful about whatever additional rules that we add (e.g. if our expansion of the apparatus is met to cover a fragment of our discourse pertaining to iterative set theory, and the conventions governing that fragment commit us to the view that it is correct to say that for any two sets, there is a union of those sets, and we’ve been careful to make our stipulations in such a way that for any two names for sets in the object language, there is a name for the union of those sets, then no harm is done in adding an axiom to the apparatus that says that for any sets, there is such a thing as the union of those sets).

I take all of the above to suffice to show that it’s plausible that the ingredients necessary to successfully expand the apparatus of Chapter 3 so as to apply to wider fragments of our platonist discourse are in place for a significant portion of that discourse. And, as I’ve already noted, in Appendix D and Appendix E, I add further support to this claim by providing examples of how such expansions can be carried out. I also take it to be obvious that the apparatus of Chapter 3 could be expanded in other ways (e.g. we might supplement its logical apparatus with modal operators, plural quantifiers, or other logical devices that we might want, and extend the rest of the apparatus to accommodate those additions in the natural ways).

Even so, I admit that we might not be able to extend the apparatus of Chapter 3 so as to encompass the entirety of our platonist discourse. In some cases, it may be that the apparatus of Chapter 3 can’t be expanded to cover a certain fragment of that discourse, but it turns out that that fragment isn’t worth keeping anyway (perhaps we have reason to
believe that there is something inconsistent or confused about it). In other cases, it may be that there’s no motivation to apply anything like the apparatus of Chapter 3 to the fragment of our platonist discourse at issue (perhaps we don’t reason formally with the sentences that are constitutive of that fragment in the way that we do with sentences like the ones treated in Chapter 3).\textsuperscript{113} For these sorts of fragments of our platonist discourse, it’s no big deal if we can’t extend the apparatus of Chapter 3 so as to encompass them. But there may be also be cases in which our limitations prevent us from extending the apparatus of Chapter 3 as far as we might like.

Consider, for example, the sentence ‘Gloria prefers red to orange.’ The predicate ‘… prefers … to …’ has no translation into the object language as described in Chapter 3. That’s because it’s a non-unary predicate that (intuitively speaking) can be partially satisfied by abstract entities, and no such atomic predicates were allowed into the object language. Still, ideally, it would be nice to have an expanded version of the apparatus of Chapter 3 that accompanies an expanded object language that does include this predicate among its stock of non-unary predicates.

A problem with attempting to expand the apparatus of Chapter 3 in this way arises, however, when we consider the question of how to assign nominalistic surrogates to the basic sentences of the relevant expansion of the object language that translate sentences like ‘Gloria prefers red to orange.’ Sentences such as these (as is well known) are not readily amenable to nominalist paraphrase. One could try to paraphrase this sentence, for example, with the sentence ‘Gloria prefers red things to orange things’. But

\textsuperscript{113} Perhaps some fragments of our platonist discourse that employ poetic sentences (e.g. ‘Love is a many-splendored thing’) are like this.
this paraphrase fails on account of the fact that the original sentence is not (correctness) equivalent to it. It may be, for example, that the sentence ‘Gloria prefers red to orange’ is correct even though it’s false that Gloria prefers red things to orange things (perhaps all of the orange things are, from the standpoint of Gloria’s preferences, all-things-considered, superior to the red things). Perhaps a more elaborate paraphrase would do here (a paraphrase that, even if complex, is still tractable to beings like us). But suppose not. In that case, there is also no tractable way for us to assign a nominalistic surrogate to the basic sentence of the expanded version of the object language that translates ‘Gloria prefers red to orange’. At least there is no way for us to assign one that spells out, in an informative way, the nominalistically acceptable conditions under which that sentence is correct.

Nevertheless, we can all agree that if the sentence ‘Gloria prefers red to orange’ is correct, its correctness must supervene on the nominalistically acceptable facts. It is correct, we can all agree, on account of the fact that some (perhaps quite complex) nominalistically acceptable proposition is true (one that pertains to how Gloria is disposed to react to visual stimuli in certain conditions, and the like). Perhaps we can’t state that proposition. But that’s on account of our ignorance and cognitive limitations. The infinite being would have no trouble in that regard. The infinite being could add an atomic predicate to the object language that translates ‘… prefers … to …’, and it could

114 Here I borrow from the discussion of the difficulties that these sorts of sentences pose for nominalist paraphrase strategies found in (Loux 2006, pp. 55-58).

115 I am here thinking of ‘Gloria prefers red to orange’ as an interpreted sentence type, where, as a matter of necessity, nothing counts as a token of that type unless it means that Gloria prefers red to orange. (We are here, of course, still setting aside nominalist scruples about apparent quantification over propositions and sentence types.)
say when the nominalistically acceptable facts are sufficient for the correctness of the basic sentences involving that predicate.

Perhaps a sentence such as ‘Gloria prefers red to orange’ is correctness equivalent to some complicated, nominalistically friendly sentence that is part of the object language (even if it is beyond us to state it, or at least beyond us to discover it). If so, the infinite being could assign that complicated sentence as the nominalistic surrogate for the object language sentence that translates ‘Gloria prefers red to orange’. But perhaps not. Perhaps the object language lacks, for one reason or another, the expressive resources required to state that condition (perhaps stating it requires the use of modal operators, for example, and the object language has not been expanded to include such operators, or perhaps no finitely long, nominalistically friendly sentence expresses a proposition whose truth is both necessary and sufficient for the correctness of ‘Gloria prefers red to orange’ and the object language does not contain infinitely long sentences …). In that case, the infinite being won’t be able to find (within the object language) an informative nominalistic surrogate for the sentence at issue. Nevertheless, it (lacking any pertinent cognitive limitation) could still entertain what its nominalistically acceptable correctness conditions are and assign it the status of being correct (or incorrect) accordingly.

Suppose then that we (not being as fortunate as the infinite being in these regards) propose to expand the apparatus of Chapter 3 as follows: We’ll add an atomic predicate that translates ‘… prefers … to …’ to the object language. We’ll also imagine this predicate as being syntactically distinguished, in some way, from the non-unary predicates of the object language that can only be satisfied by concrete things (that way we can more conveniently set the former predicate off from the rest and single it out for
special treatment). We’ll say that in cases in which a basic sentence involving it contains all and only n-names, it’s nominalistic surrogate is just to be identified with itself (because, in such cases, the basic sentence at issue will already be a nominalistically friendly sentence). We’ll also say that in cases in which the basic sentence at issue is an atomic sentence in which the first slot of this predicate is filled with a p-name, the nominalistic surrogate to be assigned to the resulting sentence is ⊥ (because we know enough to know that if there were such things as properties, necessarily, none of them would prefer anything to anything else). Likewise, when the basic sentence at issue is the negation of such a sentence, we’ll assign (¬⊥) as its nominalistic surrogate. When it comes to basic sentences in which the second or third slot of this predicate is filled in by a p-name (but not the first), however, we’ll just go ahead and stipulate that such sentences are to be counted as nominalistically undergirded if and only if their nominally acceptable correctness conditions are satisfied (whatever they happen to be). We’ll also say that when such sentences are nominalistically undergirded, they are to be assigned the “proxy nominalistic surrogate” (¬1), and that when they are not nominalistically undergirded, they are to be assigned the proxy nominalistic surrogate ⊥.

We’ll not pretend that these proxy nominalistic surrogates tell us anything informative about what the nominalistically acceptable correctness conditions for these sentences are; the purpose of assigning them is merely to enable us to carry on, with the rest of the apparatus, in pretty much the same way as we did in Chapter 3 (and having something to call a nominalistic surrogate for every basic sentence aids us in doing that).

Admittedly, this way of attempting to expand the apparatus of Chapter 3 feels like cheating. And it’s not all that hard to see why. To attempt to expand the apparatus in
that way is to give up on one of the primary reasons that apparatus was introduced in the first place – namely, as a means of exhibiting, in an informative manner, what the nominalistically acceptable correctness conditions for various applied sentences that make use of apparent quantification over features are. Insofar as the current proposal gives up on that ambition, it’s not clear that it counts as a legitimate way of developing the apparatus introduced in Chapter 3. Nevertheless, we do get something out of this proposal. Namely, we can see that the proofs of the Correctness Preservation Theorem and the Conservation Lemma will proceed for this expansion of the apparatus pretty much as they did before. For that reason, we can be assured that the formally valid arguments we make use of, by means of employing sentences that can be translated into the resulting, expanded version of the object language, are correctness preserving. And to the extent that we can be assured that those sentences have nominalistically acceptable correctness conditions, we can be assured that the purposes for which we put forward those arguments (in contexts in which the existence of abstract entities is not of practical or theoretical concern) are satisfied, regardless of whether or not abstract entities exist.

The more general lesson to be gleaned from all of this is that, even for many of those fragments of our platonist discourse that are such that we are unable to say (in an informative way) what the nominalistically acceptable correctness conditions for all of the (truth-evaluable, declarative) sentences that are constitutive of that discourse are, the considerations raised in the previous two chapters give us good reason to believe that the purposes that underlie that discourse are fulfilled regardless of whether there are abstract objects. Even more generally, the considerations raised in those chapters give us good reason to believe that we can be assured that those purposes are fulfilled provided that the
conventions that underlie the use of that discourse are consistent, that those conventions entail that correctness for the sentences that constitute that discourse are closed under formal consequence, and provided that those conventions are also such that the correctness of the sentences constitutive of that discourse supervenes on the nominalistically acceptable facts. But, I submit (with one caveat that I will discuss below) that any fragment of our platonist discourse that is worth keeping satisfies these conditions (at least if we set aside poetic uses of that discourse and the like).

There is one caveat here. It may be that some of the (truth-evaluable, declarative) sentences associated with the portion of our platonist discourse that is worth keeping are such that neither they nor their negations are correct according to the conventions governing that discourse (in just the same way that neither ‘Tom Sawyer had three daughters when he grew up’ nor its negation is correct according to the conventions that govern our discourse that appears to be about Tom Sawyer). But, arguably (though not uncontroversially!), if a (truth-evaluable, declarative) sentence is true, then its negation is not (and vice versa). This is one way, then, that truth and correctness might pull apart in terms of their formal properties. But the arguments given above depended on the claim that nominalistic undergirding (which I have been claiming serves as the formal analog of correctness) has the same formal properties as does truth. This gives us a reason, then, to think that the arguments of the previous chapters can only be applied to those fragments of our discourse involving apparent quantification over abstract objects that are such that if one of the sentences of that discourse is correct, its negation is not, and vice versa. \footnote{This sort of phenomenon can create difficulties for various fictionalist accounts of other aspects of our discourse as well. See, for example, Rosen’s (1990, pp. 341-344) discussion of the difficulties that it creates for his proposed version of modal fictionalism.}
There is a way to modify the sort of apparatus developed in the previous chapters, however, so that it can apply even to those fragments of our platonist discourse that don’t have this feature. We can employ a method akin to supervaluationist approaches to vagueness. Suppose we are dealing with such a fragment of our platonist discourse (one that is otherwise like the fragments of our discourse to which I argued that the considerations of the previous two chapters were applicable). Suppose also that we have gotten as far in expanding the Chapter 3 apparatus so as to apply to that discourse as supplementing the object language so that it can represent all of the (truth-evaluable, declarative) sentences of that discourse. And let’s further suppose that all the basic sentences of the expanded object language that are correct according to the conventions governing the discourse at issue have been assigned the status of being nominalistically undergirded. Suppose, also, however, that some of the basic sentences of the object language have been assigned the status of being “indeterminate,” where a closed sentence is to count as indeterminate just in case the conventions that guide the discourse at issue fail to assign it the status of being correct and also fail to assign that status to its negation (and let’s say that a closed sentence is “determinate” otherwise). Let’s further say that a “precisification” of the expanded apparatus is the result of taking each indeterminate sentence of the expanded version of the object language and either assigning it or its negation the status of being nominalistically undergirded (in a way that respects formal consistency as well as otherwise respects the conventions governing the original discourse). We’ll now think of each precisification of the apparatus, not as modeling the original discourse, but a corresponding precisification of that discourse (one for which nominalistic undergirding for the relevant precisification of the apparatus does serve as a
formal analog of correctness). We’ll also think of the considerations raised in the
previous two chapters and in this section as applying, not directly to the original
discourse, but directly to each precisification of it. We’ll further say that, when it comes
to the original discourse, a (truth-evaluable, declarative) sentence of that discourse counts
as “correct” just in case every precisification of the expanded apparatus counts its
translation into the object language as nominalistically undergirded, “incorrect” just in
case every precisification of the expanded apparatus counts its translation into the object
language as not nominalistically undergirded, and as “neither correct nor incorrect”
otherwise. And now things may go on just as they did before.

Given all of the above, I take it to be plausible that the considerations raised in the
previous two chapters generalize to all of the fragments of our platonist discourse that are
worth keeping. In the process of developing these considerations, however, I have
shamelessly engaged in apparent quantification over linguistic entities and propositions.
But such entities are no more acceptable from a nominalist point of view than are sets or
features. And so there is a lingering worry that I have put forward these considerations in
a way that is itself nominalistically illegitimate. It is this worry that I intend to address in
the remainder of this chapter.

4.3 On Apparent Quantification Over Abstract Entities in Philosophical Contexts

In the next two sections, I will address the worry raised above by arguing that the same
kinds of considerations that I advanced in Chapter 2, in favor of the claim that sentences
involving apparent quantification over features are nominalistically legitimate, generalize
so as to apply to sentences involving apparent quantification over propositions and
linguistic entities, *including* the sentences involving apparent quantification over propositions and linguistic entities that I’ve been employing in this and previous chapters. In this section, I want to clear the ground for that project, however, by addressing a potential objection to it that might be made right at the outset.

The objection is that the considerations raised in Chapter 2 turned on there being an important distinction between ordinary discourse and philosophical discourse, so that sentences that are ontologically committing (to a certain class of entities whose existence is philosophically controversial) when used in philosophical discourse fail to be so when they are used in ordinary discourse. But (so the current objection goes) the kind of discourse that we’re talking about now is philosophical discourse. Thus, no such distinction is applicable.

Consider, for example, the following sentence: ‘In spite of the fact that many of the sentences we use involve apparent quantification over features, strictly speaking, there are no features.’ This sentence, unlike the spider-insect sentence, or sentences involving apparent quantification over things like chairs, is not the sort of sentence that is likely to be uttered in everyday life (or even in technical but non-philosophical endeavors). It is, rather, only likely to be uttered in the context of a philosophical discussion, a philosophical discussion, no less, that pertains to matters of ontology! It is the sort of sentence that (as Van Inwagen might put it) one is likely to utter only while “in the ontology room.” But the ontology room (so the one pushing this objection might press) is supposed to be the place where we are to take seriously the quantificational structure of the sentences we use. So (the objection goes) the nominalist cannot utter this
sentence in the ontology room without committing herself to the existence of sentences (and where but the ontology room would she have occasion to utter this sentence?).

I believe this objection is mistaken. Even in the ontology room, not all apparent reference or quantification is to be taken seriously. Consider, for example, a platonist who says the following to her nominalist colleague: “Look, we both accept a number of propositions that obviously entail the existence of abstract entities. We both accept the proposition, for example, that spiders share some of the anatomical features in common with insects.” The nominalist who responded as follows would be giving an overly pedantic response, one that is entirely beside the point: “I disagree. I do not accept any such propositions because (if I am right) there are no such things as propositions.” The nominalist’s rejoinder is no more to the point, in this context, than it would be for the nihilist to protest an ordinary utterance of a sentence involving apparent reference to chairs on the grounds that, if he is right, there are no chairs.

Here we would be well advised to take on board the following observation by Matti Eklund (who endorses a similar view concerning the relationship between ordinary and metaphysical discourse as that of Van Inwagen) as applicable, not merely to everyday discourse, but even to much of philosophical discourse (even to much of the philosophical discourse that takes place in the ontology room):

Even genuinely literal assertions have what we may call non-serious features, features that are not important to the point of the assertions, and among these features are normally the ontologically committing ones. More specifically: with respect to much that we say or imply we do not commit ourselves either to its literal truth or to its truth in any fiction; we are, simply, non-committed.117

117 Eklund (2005, p. 558)
When we use sentences that appear to involve quantification over abstract entities to make certain philosophical points, it is plausible that we often fail to commit ourselves to the existence of those entities, at least insofar as the existence of such entities is “beside the point.”

And given Van Inwagen’s Wittgensteinian views concerning the relationship between ordinary and metaphysical discourse, it is also plausible that when we use such sentences in this non-committal way, we manage to express propositions other than the ones those sentences perspicuously express, propositions that are neutral with respect to the apparent ontological implications of the sentences used to express them (or, at least, neutral with respect to those implications that these sentences have in virtue of their quantificational structure alone and not in virtue of their assertive content). The same applies, *mutatis mutandis*, if we deny Van Inwagen’s views concerning the philosophy of language, and think about matters in terms of the error theory sketched in the final section of Chapter 2. In that case, we can often think of such sentences (even as used in philosophical contexts) as being strictly speaking false (if the entities in question do not exist) but “correct,” provided that the world can be plausibly held to cooperate with the point of those utterances regardless of whether the entities in question exist.

It is worth noting that there will be instances in which the existence of the entities in question is *in one respect* beside the point of an utterance but also, in another respect, to the point. Consider the following sentence as uttered by the platonist (for example): ‘Many of the sentences we utter express propositions that entail that there are abstract objects – propositions which we cannot help but regard as true’. The nominalist would be unjustly accusing the platonist of begging the question by uttering this sentence if she
made this accusation on account of the fact that this sentence appears to involve quantification over propositions and sentence types (and that such entities, if they existed, would be abstract). That’s beside the point. Nevertheless, in another respect, that abstract objects exist is very much to the point of this utterance. A more interesting example of this sort of thing is the following sentence (as uttered by the nominalist): ‘In spite of the fact that many of the sentences we utter appear to express propositions that entail the existence of sentences, strictly speaking, there are no sentences.’ In the next section, I argue that (in spite of initial appearances) the nominalist does not say something self-referentially incoherent by uttering sentences such as these. For now, I would simply like to point out that it is plausible that, in uttering this sentence, the nominalist does not commit herself to the existence of sentences – that’s (in the relevant respect) beside the point.

I don’t have much to say about which apparent implications of our utterances are “to the point” and which are not. That is ultimately a matter of the pragmatics of philosophical discussion, and matters of pragmatics are (in general) notoriously difficult to systematize. Nevertheless, we appear to be capable of agreeing about how to apply the distinction, at least in many cases. I will assume, in the next two sections, that we have enough of a grip on this distinction to put it to some philosophical use.

4.4 On Apparent Quantification over Linguistic Types and Propositions

In this section I will assume that Van Inwagen’s Wittgensteinian views concerning the philosophy of language are correct and (accordingly) that the propositions asserted in this and previous chapters via sentences involving apparent quantification over propositions
and linguistic types are “neutral” with respect to the apparent ontological implications of the sentences used to express them (at least insofar as those implications are “beside the point” of those sentences, in the sense laid out in the previous section). For the sake of convenience, I will refer to these propositions (the ones that are neutral regarding the ontological implications of the sentences used to express them) as the “neutral propositions” expressed by those sentences. I will also argue in this section that these neutral propositions have *nominalistically acceptable truth conditions*, in the same way that I argued in Chapter 2 that the propositions ordinarily expressed by sentences involving apparent quantification over features have nominalistically acceptable truth conditions. I intend to argue thus that the nominalist may consistently assert propositions expressed by sentences involving apparent quantification over propositions and linguistic types (even in philosophical contexts) in the same sort of way that I argued in Chapter 2 that the nominalist may consistently assert propositions ordinarily expressed by sentences involving apparent quantification over features. In the next section, furthermore, I will also argue that the considerations raised in this section are still applicable, *mutatis mutandis*, if we adopt an error theory regarding the relevant class of sentences, along the lines of the error theory sketched in Section 2.6.

I propose to carry out the task of this section by arguing that apparent quantification over propositions and linguistic types can be eliminated in favor of apparent quantification over linguistic acts (understood as act types rather than as act tokens) and that the latter can be shown to be nominalistically legitimate. I will begin my discussion of the nominalistic legitimacy of apparent quantification over linguistic types by focusing on the nominalistic legitimacy of apparent quantification over sentence types.
For now, however, I will set doubts about the existence of sentence types aside and I will begin by saying a few words concerning how I am thinking of such types. There are, obviously, many ways of typing sentence tokens and accordingly, many different varieties of sentence types. For example, we can group sentence tokens together by way of their sharing the same morphology and syntax (or, in more nominalistically friendly terms, by their being relevantly morphologically and syntactically similar). When grouping sentence tokens together in this way, a token of the English sentence ‘The snow is white’ does not belong to the same type as a token of the Spanish sentence ‘La nieve es blanca’ (their syntax is similar enough, perhaps, but their morphology is too different). Alternatively, we can group sentence tokens together by their sharing the same meaning (or, rather, as the nominalist would have it, by virtue of their being synonymous). On this way of going about things, a token of the English sentence ‘The snow is white’ does belong to the same type as a token of the Spanish sentence ‘La nieve es blanca’.  

Of course, neither of these ways of grouping sentence tokens together, or any other, for that matter, is the correct way of doing so. We can group sentence tokens together in any way that best serves our purposes. It will best serve my purposes in this section to combine both of the ways of typing sentences mentioned above. That is, I propose to group sentence tokens together into the same type if and only if they are both synonymous and relevantly morphologically and syntactically similar to one another. Consider, for example, the following two English sentence tokens: The snow is white.

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118 Well, maybe not, if we insist on grouping by perfect synonymy, since it is controversial whether there ever is such a thing as perfect synonymy between sentences belonging to two distinct natural languages. I intend to ignore complications such as this one, however, as nothing I want to say turns on them.
The snow is white. These two sentence tokens go together under the same type because they are synonymous and because they are relevantly similar in syntax and morphology. A token of a sentence in another language (a code language, let’s say) that is shaped like one of the aforementioned sentence tokens, but which means the same thing as the English sentence ‘The grey-haired man is a spy’ is not a token of that type (because it is not synonymous with the other tokens of that type). Consider also, for another example, the following Spanish sentence token: La nieve es blanca. This sentence token also does not belong to the same type as the first two sentence tokens mentioned above. It does not belong on account of the fact that it is not sufficiently morphologically similar to those tokens.

I will refer to sentence types of this sort as “interpreted sentence types,” or “interpreted sentences” for short, or even just as “sentences.” I am also assuming, furthermore, that (doubts about the existence of sentence types aside) the mere possibility of there being a sentence token of a given interpreted sentence type is sufficient for there actually being such a type. For instance, there are many blades of grass in my yard that have no names and which never will have any names. But I could go out tonight and baptize one such blade of grass with a unique proper name. And then I would be capable of writing down all sorts of interpreted sentences that involve the use of that name. That is sufficient, I am assuming, for there actually being the numerous interpreted sentence types that would have had tokens had I named that blade of grass and written down those sentences. Those types, I am assuming (nominalist scruples aside), exist, even though they don’t actually have (and never will have) any tokens. It doesn’t matter, furthermore, in this case, that I (as an actual language user) could have generated tokens of those
types. I will assume (again, nominalist scruples aside) that a given interpreted sentence type exists if it is possible at all for it to have tokens, even if there are no actual tokens of that type, and even if no actually existing language user could have produced such tokens. (I take it to be safe for me to make these assumptions provided that, at the end of the day, the nominalistically acceptable truth conditions that I put forward for the neutral claim that a given sentence type exists are themselves conditions that hold as a matter of metaphysical necessity).

On various occasions, I have had the need not only to speak of sentence types, but also of languages. Given the above, languages may be thought of as classes of interpreted sentence types. I propose, furthermore, that we regard any such class as a language (of course, thought of this way, most “languages” aren’t natural, or systematic, or learnable, or anything of the sort). I also propose that we understand talk of classes in this context, not in terms of quantification over actually existing entities of which interpreted sentences are members, but rather in terms of plural quantification over the interpreted sentences themselves. For example, I propose that we regard talk of the class of English sentences as simply a manner of speaking of the English sentences. (In virtue of what does an interpreted sentence get to be classified as an English sentence? I’m not sure. That’s a question for linguists and philosophers of language to investigate, however, and not an immediate concern of mine.) Talk of languages, on this way of thinking about things, does not really involve reference to entities such as languages, but is merely a locution for plural quantification over interpreted sentences.

I have also, on occasion, spoken of languages as containing various subsentential expressions, such as predicates, logical connectives, and the like. I take it that such talk
can be assimilated to the above proposal as follows: First, we are to think of what we are doing when we appear to quantify over such subsentential expressions as engaging in apparent quantification over expression types, and more specifically (as with the case of apparent quantification over sentences) as over interpreted expression types. Second, while it is dubious as to whether entities such as sentence types would literally have subsentential expression types as parts or constituents, we should note that there is a clear enough sense in which sentence types would “involve” subsentential expression types. The interpreted sentence type ‘Snow is white’, for example, may not literally be partially composed by the predicate type ‘white’, but it involves that predicate type in at least the following sense: Necessarily, any token of ‘Snow is white’ is partially composed of a token of ‘white’. So we can say the following: A language may be said to contain a certain subsentential expression type if and only if one of its interpreted sentences involves that subsentential expression type.

I will now proceed to argue that apparent quantification over sentence types (as well as over subsentential expression types) and propositions can be eliminated in favor of apparent quantification over linguistic act types, and finally that the latter is nominalistically legitimate. In the meantime, in the process of so doing, I will continue to help myself to apparent quantification over entities such as sentence types, subsentential types, and propositions. If my argument is successful, then not only will the apparent quantification over propositions and sentence types that I made use of in previous chapters have been vindicated as nominalistically legitimate, but the use of such quantification in the process of stating that very argument will have been vindicated as well.
It’s fairly easy to replace several instances of apparent quantification over sentence types with instances of apparent quantification over linguistic acts. I take it as obvious (doubts about the existence of actions aside) that when someone, say, utters the sentence ‘Snow is white’, she performs a certain linguistic act. Intuitively, this act has the sentence ‘Snow is white’ as its object. But we needn’t think that such acts have sentence types as their objects. We could, instead, read the phrase ‘utters ‘Snow is white’’ (as it occurs in the sentence ‘Vivian utters the sentence ‘Snow is white’’) adverbially – as equivalent to the phrase ‘utters ‘Snow is white’-ly’. Call such linguistic acts, acts of the same sort as uttering ‘Snow is white’, “sentential utterances.” And call predicates such as ‘utters ‘Snow is white’’ “sentential utterance predicates.” We can individuate these linguistic act types, furthermore, in a way that exactly parallels the manner in which we proposed above to individuate sentence types.

With this machinery on hand, we can paraphrase sentences like ‘Vivian eloquently spoke many sentences’ with sentences like ‘Vivian eloquently performed many sentential utterances’. The proposition expressed by the latter sentence, furthermore, can be seen to have nominalistically acceptable truth conditions in the same way that the neutral proposition expressed by ‘Spiders share some of the anatomical features in common with insects’ can be seen to have nominalistically acceptable truth conditions. In any world in which the former proposition is true, it will be jointly entailed by true propositions perspiciously expressed by several sentences (of a language much like English except lacking for no relevant expressive resources) of the form ‘Vivian eloquently uttered _________’ (where the blank is to be filled in in such a way that the occurrence of ‘uttered _________’ forms a sentential utterance predicate). Since
the propositions expressed by sentences that conform to this schema are clearly nominalistically acceptable (as long as sentential utterance predicates are understood adverbially in the manner suggested above), furthermore, we may (given Van Inwagen’s Wittgensteinian views concerning the philosophy of language) regard sentences such as ‘Vivian eloquently performed many sentential utterances’ as ordinarily expressing truths, regardless of whether there are such entities as sentential utterances. Granted, in order exhibit this fact, I have also engaged in further apparent quantification over propositions and sentence types. But the same kind of defense of the dialectical legitimacy of this move that was given in Section 2.5 could also be given here.

I have had occasion not merely to appear to quantify over sentences that have actually been uttered (or written down) but over sentences that have never been used. But sentences involving apparent quantification over unused sentence types can also be seen to express propositions that have nominalistically acceptable truth conditions. Consider, for example, the sentence ‘There are sentences that have never been uttered’. We may paraphrase this sentence with the following: ‘There are sentential utterances that have never been performed’. The neutral proposition expressed by the latter sentence may be thought to be true, furthermore, if and only if a sentence of the form ‘Possibly, someone utters ______, though, in fact, no one ever utters ______’ (where the blanks are to be uniformly filled in such a way that the occurrences of ‘uttered ______’ form sentential utterance predicates) perspicuously expresses a true proposition. Likewise, the proposition expressed by the sentence ‘There are infinitely many sentential utterances that have never been performed’ will, in any world in which it is true, be jointly entailed by infinitely many true propositions that may be perspicuously
expressed by sentences that conform to this schema. Propositions expressed by sentences of this form, furthermore, are obviously nominalistically acceptable. (Or, at least, they are obviously nominalistically acceptable to those who are willing to grant nominalists the use of primitive modal operators). Thus, apparent quantification over sentence types, even apparent quantification over infinitely many sentence types, even over infinitely many sentence types that have never been used, can be seen to have nominalistically acceptable truth conditions.

Apparent quantification over propositions can also be eliminated in favor of apparent quantification over linguistic acts (the latter of which can then be argued to be nominalistically legitimate) in a similar manner. Doubts about the existence of linguistic act types aside, I take it that in addition to sentential utterances, there are what we might call “propositional utterances.” For example, an English speaker might (as the proposition theorist would put it) “express the proposition that the snow is white” by performing the sentential utterance uttering ‘The snow is white’. A Spanish speaker might also express the proposition that the snow is white, but do so by performing the sentential utterance uttering ‘La nieve es blanca’. In that case, the English speaker and the Spanish speaker may be said to have performed the same propositional utterances (on account of the fact that they both expressed the proposition The snow is white), even though they also performed different sentential utterances. While both parties were described above as “expressing the proposition that the Snow is white,” furthermore, the propositional utterances that they performed needn’t be understood as actually having as their object an entity that is a proposition. Rather, each party may simply be said to have expressed that the snow is white, where the clause ‘that the snow is white’ is not to be
understood as naming a proposition, but as forming an adverb (i.e. we may take the phrase ‘expressing that the snow is white’ to be equivalent to ‘expressing that-the-snow-is-white-ly’).\footnote{For a classic exposition of a view of propositional attitude sentences that takes this sort of line, see (Prior 1971, especially pp. 16-20).}

With this apparatus on hand, we can handle truth ascriptions to propositions in the manner that deflationists about truth often suggest. We can understand the neutral proposition expressed by the sentence ‘The proposition that the snow is white is true’, for example, to be true just in case snow is white. We can handle sentences such as ‘Everything that Jones says is true’ by noting that the neutral proposition expressed by this sentence will be jointly entailed by those infinitely many propositions that may be expressed by sentences of the following form: \("\text{If Jones says that } p,\text{ then } p\)\) (where \(p\) is to be replaced by a truth-evaluable declarative sentence).

We also need to have a way of handling sentences such as ‘The sentence ‘Snow is white’ perspicuously expresses the proposition that snow is white’. The strategy that I will propose for handling sentences such as these, however, requires a little by way of setup.

First, when thinking about how to handle sentences such as the one just mentioned, it will be useful to keep firmly in mind that the sentence types that we are appearing to quantify over are to be thought of as \(\text{interpreted}\) sentence types. So (given that way of thinking of things) it is to be thought of as being a necessary truth that the sentence ‘Snow is white’ perspicuously expresses the proposition that snow is white (because, by the current way of typing sentences, necessarily, a sentence token that
doesn’t perspicuously express that proposition isn’t a token of ‘Snow is white’). It should not be taken to follow from this fact, however, that necessarily, whenever a speaker uses the sentence ‘Snow is white’, she expresses the proposition that snow is white. This is especially important in this section, where we are taking on board Van Inwagen’s views concerning the relationship between ordinary and metaphysical discourse. We want to allow, for example, that when a speaker utters in everyday life ‘There are some chairs that aren’t being sat upon’, she expresses a proposition other than the proposition perspicuously expressed by that sentence.

Nevertheless, we can maintain that when a speaker utters such a sentence, there is some important connection between her sentential utterance and the proposition perspicuously expressed by the sentence at issue (even if it is hard to say clearly and precisely just what that connection is). If we, as competent English speakers, hear someone use the sentence ‘Snow is white’, for example, and we understand what we are hearing, we are capable of discerning what proposition is perspicuously expressed by the sentence that the speaker just used, even if the speaker herself failed to express that proposition by using that sentence. Indeed, this ability on our part often seems crucial to our ability to grasp just what proposition a speaker is expressing (or pragmatically endorsing) by using a sentence, even when that differs from the proposition perspicuously expressed by that sentence. Suppose, for instance, that a speaker utters ‘Yeah, and snow is white’ as a means of expressing (or pragmatically endorsing) the claim that something another participant in the conversation said was a banal triviality. We understand what claim the speaker is getting across in such a context via our
understanding which proposition is perspicuously expressed by ‘Yeah, and snow is white’ (together with our background knowledge and various other contextual clues).

We might want to say, then, that when a speaker utters a given sentence, she “strictly and literally conveys” the proposition perspicuously expressed by that sentence, even if she fails to express (or assert, or commit herself to the truth of) that proposition by uttering that sentence. So we can say that while it may not be a necessary truth that a speaker expresses the proposition perspicuously expressed by ‘Snow is white’ by using that interpreted sentence, it is a necessary truth that she strictly and literally conveys that proposition by using that interpreted sentence.

It is also important to recognize, furthermore, that we needn’t understand what it is for a speaker to strictly and literally convey that snow is white in terms that involve apparent quantification over propositions and sentence types. Sentential utterances can themselves be thought of as having a compositional structure, one that (doubts about the existence of sentence types aside) mirrors the compositional structure of the sentences that are their objects. In order to utter ‘Snow is white’, for example, one must first utter ‘Snow’, then one must utter ‘is’, and then one must utter ‘white’, all in close succession. Performing the act of uttering ‘Snow is white’, that is, requires performing other linguistic utterances in a certain order. There is, therefore, a compositional structure associated with the performance of that linguistic action, one that involves other linguistic actions as its proper components. So we can think of a speaker as strictly and literally conveying that snow is white when she utters ‘Snow is white’ in virtue of its being the case that her sentential utterance has the compositional structure that it does (i.e. in virtue of the fact that necessarily, when a speaker utters ‘Snow is white’, she first
utters ‘Snow’, then utters ‘is’, then utters ‘white’, all in close succession …) rather than in virtue of the fact that some sentence that is the object of that utterance has the compositional structure that it does.

With all of the above setup out of the way, we may now say that the neutral proposition expressed by the sentence ‘The sentence ‘Snow is white’ perspiciously expresses the proposition that snow is white’ is true just in case the proposition perspiciously expressed by the following nominalistically friendly sentence is true: ‘Possibly, some individual utters ‘Snow is white’; and furthermore, necessarily, any individual who utters ‘Snow is white’ also strictly and literally conveys that snow is white.’ Since the latter sentence, furthermore, is a nominalistically friendly one, the original sentence can be seen to have nominalistically acceptable truth conditions.

Up to this point, I’ve been ignoring complications that arise from thinking about sentences that describe which propositions are expressed by sentences that involve indexical elements. But these sorts of complications are dealt with easily enough. Consider, for example, the sentence ‘The sentence ‘I am hungry’, when uttered by a given person, perspiciously expresses the proposition that that person is hungry’. We can take this sentence to express a neutral proposition that is equivalent to the proposition perspiciously expressed by the following nominalistically friendly sentence: ‘Possibly, someone utters ‘I am hungry’; and furthermore, necessarily, for any given individual, x, if x utters ‘I am hungry’, x strictly and literally conveys that x is hungry.’

\[^{120}\]

\[^{120}\] If one has qualms about the manner in which this sentence binds (from the outside) a variable that occurs within the scope of a ‘that’ clause, there is an alternative strategy available that avoids this. Instead of attempting to find a nominalistically acceptable proposition that is equivalent to the neutral proposition expressed by the original sentence, we can simply point out that the truth of the latter
It is also worth briefly noting, at this point, that sentences involving apparent quantification over subsentential expressions, as well as the contents thereof, can be handled in similar ways to those in which sentences involving apparent quantification over sentence types and propositions have been handled up until now. We can note, for example, that a sentence such as ‘The word ‘red’ means red’ expresses a proposition that has nominalistically acceptable truth conditions, on account of the fact that the neutral proposition it expresses is equivalent to the proposition perspicuously expressed by ‘Possibly, someone utters ‘red’; and furthermore, necessarily, anyone who utters ‘red’ strictly and literally conveys red (where the phrases ‘utters ‘red’’ and ‘strictly and literally conveys red’ are to be given adverbial readings).

We’ll also need a way of handling sentences like ‘The neutral proposition expressed by ‘There are features that spiders and insects share in common’ is not identical to the proposition perspicuously expressed by that sentence’. We may handle this sentence as follows: Let the result of prefixing the phrase ‘strictly speaking’ in front of a sentence be to ensure that the proposition expressed by that sentence is the one perspicuously expressed by it. With that little device in hand, we can take the neutral proposition expressed by the above sentence to be equivalent to the proposition perspicuously expressed by the following sentence: ‘Typical English speakers are disposed to be such that when it is not of practical or theoretical interest to them whether features exist (but they are otherwise interested in being taken literally), and they

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proposition is jointly entailed by all of the true nominalistically acceptable propositions perspicuously expressed by sentences (of a language much like English except that it lacks for no relevant expressive resources – one that, for instance, has a name for each individual) of the form ‘Possibly, someone utters ‘I am hungry’; and furthermore, necessarily, if ___ utters ‘I am hungry’, then ___ strictly and literally conveys that ___ is hungry’ (where the blanks are to be uniformly filled in by a proper name).
assertively utter ‘There are features that spiders and insects share in common’, they do not express that strictly speaking, there are features that spiders and insects share in common.’

The device just introduced, it is worth noting, can also be put to good use in other ways. It can be used, for example, to dispel any apparent inconsistency there is in nominalist’s endorsing sentences like these: ‘In spite of the fact that many of the sentences we use involve apparent quantification over sentences, strictly speaking, there are no sentences.’ On the face of it, such sentences appear to be self-referentially incoherent. Nevertheless, the neutral propositions they express can be seen to have consistent, nominalistically acceptable truth conditions. The above sentence, for example, could be paraphrased as follows: ‘In spite of the fact that many of the sentential utterances that we perform are such that, by engaging in them, we appear to be quantifying over sentences, strictly speaking, there are no sentences’. And this paraphrase can be shown (along the lines already sketched above) to have nominalistically acceptable truth conditions.

One important thing that the above example does illustrate about the use of this device, however, is that the nominalist will have to be careful how she employs sentences involving the phrase ‘strictly speaking’ (as used in the stipulative, technical sense introduced above) in formal reasoning. Since, furthermore, this issue will come to be of some importance in the next section, it is worth taking a brief diversion from the main topic of this section to focus on it in some detail.
Any formal logic that the nominalist develops for sentences involving the phrase ‘strictly speaking’ (where it functions as the technical device introduced above) is going to have to be such that it does not allow certain kinds of inferences involving that phrase to go through (at least if she wants to ensure that the arguments that are formally valid according to that logic are truth preserving). In particular, she will have to insist on the following rule (which may be put schematically as follows): One may not infer ‘p’ from ‘Strictly speaking, p’ (where p is to be replaced by a truth-evaluable, declarative sentence), unless certain special conditions are met (conditions that she will have to specify when she lays down the inference rules governing her formal logic). She’ll want to insist, for instance, that one may not infer ‘There are no features’ from ‘Strictly speaking, there are no features’ (at least not in a context in which sentences involving apparent quantification over features are to be held as expressing propositions that are neutral with respect to the existence of features). She can allow, however, that one may infer a sentence such as ‘Shelob is segmented and Charlotte is segmented’ from ‘Strictly speaking, Shelob is segmented and Charlotte is segmented’ (at least in contexts in which the former sentence can be seen to be “to the point” if and only if the proposition that it perspicuously expresses is true).

She can also allow (and will want to allow) sentences that have ‘strictly speaking’ prefixed to them to enter into or be taken out of Boolean combinations with other

121 Note that the former sentence would translate into the object language characterized in the previous chapter as a quantifier free sentence that contains no p-names. If the correctness conditions (in the relevant context) for sentences like ‘Shelob is segmented and Charlotte is segmented’ are held to match the conditions under which sentences in the object language of Chapter 3 may be said to be nominalistically undergirded, it follows (via the Conservation Lemma proven in Appendix C) that this sentence is correct if and only if it perspicuously expresses a true proposition. This illustrates the fact that the apparatus in Chapter 3 could be expanded so that the object language is made to include a ‘strictly speaking’ operator, with formal rules governing its usage of the sort sketched here, and an additional rule that allows it to be either dropped from or added to quantifier free sentences that contain no p-names.
sentences, in accordance with the standard rules of propositional logic. For example, she can allow that it is legitimate to infer from the sentences ‘There are sentences’ and ‘Strictly speaking, there are no sentences’ the sentence ‘There are sentences and strictly speaking, there are no sentences.’ She will also want to allow sentences prefixed by ‘strictly speaking’ to logically interact with one another (via their subsentential, syntactical structure) in the ways permitted by standard first order logic. While, for example, she will not want to permit the inference of ‘There are no features that spiders and insects share in common’ from ‘Strictly speaking, there are no features’, she will want to allow ‘Strictly speaking, there are no features that spiders and insects share in common’ to be inferred from ‘Strictly speaking, there are no features’.

With the above aside out of the way, it’s time to get back to the main topic of this section. In addition to the kinds of sentences involving apparent quantification over sentences that I’ve discussed so far, I’ve also had occasion, in this and other chapters, to speak of sentences that conform to some particular schema or other. I need some means of arguing that the sentences by which I have expressed such claims have nominalistically acceptable truth conditions as well. Here’s a cheap but effective trick for exhibiting that they do have such truth conditions: Consider the following sentence: ‘There is a true proposition that is perspicuously expressed by an instance of ‘[Shelob is ____ and Charlotte is ____]’ (where the blanks are to be uniformly filled in by an adjective that is itself an anatomical term)’. We can exhibit that there are nominalistically acceptable truth conditions for this sentence as follows: We may note that the neutral proposition expressed by the above sentence is true if and only if there is a true proposition perspicuously expressed by a sentence that conforms to the following
schema: 「Shelob is ____ and Charlotte is ____，and possibly, someone utters ‘Shelob is ____ and Charlotte is ____’; and furthermore, necessarily, anyone who utters ‘Shelob is ____ and Charlotte is ____’ strictly and literally conveys that Shelob is ____ and Charlotte is ____」 (where the blanks are to be uniformly filled in by an adjective that is itself an anatomical term). Any instance of this schema, we may further note, expresses a nominalistically acceptable proposition. Granted, I haven’t dispensed with apparent quantification over schemas in this way. But I have exhibited the fact that sentences employing apparent quantification over schemas (at least the sorts of sentence that I’ve made use of that do so) express propositions that have nominalistically acceptable truth conditions. That’s good enough.

How should we understand talk of entailments between propositions? When such talk is of entailment relations between only finitely many nominalistically acceptable propositions (the identities of which are all known), it is not difficult to find nominalistically friendly paraphrases. For example, the sentence ‘The proposition The snow is white and the proposition The grass is green jointly entail the proposition There are white things and there are green things’ may be paraphrased as ‘Necessarily, if the snow is white and the grass is green, then there are white things and there are green things’. Such nominalistically friendly paraphrases will not be available in all cases, however.

Consider, for example, the sentence ‘The proposition that Shelob and Charlotte are both segmented entails the neutral proposition that there is an anatomical feature that Shelob and Charlotte share in common’. For sentences like these, we will have to forgo the prospect of finding nominalistically friendly paraphrases and simply be content with
exhibiting the fact that the neutral propositions that they express have nominalistically
acceptable truth conditions. This can be done easily enough. In fact, it looks like it can
be done too easily. Recall that a proposition may be said to have nominalistically
acceptable truth conditions if and only if necessarily, if it is true, then there are true
nominalistically acceptable propositions that (individually or jointly) entail it. But claims
about which entailments hold are necessarily true if true. So the neutral proposition
expressed by the sentence ‘The proposition that Shelob and Charlotte are both segmented
entails the neutral proposition that there is an anatomical feature that Shelob and
Charlotte share in common’ is one that is necessarily true. But if that proposition is
necessarily true, then, trivially, any nominalistically acceptable proposition entails it. So,
if our only goal is to exhibit that the above proposition has nominalistically acceptable
truth conditions, we’re done.

But it can’t be that easy to establish the nominalistic legitimacy of sentences like
the one we are currently considering. Perhaps exhibiting that various sentences that
express contingent claims are such that the neutral propositions expressed by them have
nominalistically acceptable truth conditions is enough to establish their nominalistic
legitimacy. Perhaps the same is true of those pure platonist sentences that are formal
consequences of sentences that assert that the abstract entities associated with the kind of
platonist discourse at issue exist, or formal consequences of those sentences in
conjunction with certain conceptual truths, and the like. But, in this case, something
more seems to be required.

Intuitively speaking, entailment claims like the one expressed by ‘The
proposition that Shelob and Charlotte are both segmented entails the neutral proposition
that there is an anatomical feature that Shelob and Charlotte share in common’ are not just about how things must be among the pure platonist facts, but also about how things must be among the nominalistically acceptable facts. And (arguably, at least) to establish the nominalistic legitimacy of sentences that express such claims, we should be required to find some way of reassuring ourselves that things must, in fact, really be that way among the nominalistically acceptable facts.

I suggest, however, that to establish the nominalistic legitimacy of sentences like the above, sentences according to which some propositions (the entailing propositions) entail others (the entailed propositions), it is enough to exhibit that the following fact obtains: Necessarily, if the entailing propositions are true, then whatever nominalistically acceptable true propositions (individually or jointly) entail the entailing propositions also (individually or jointly) entail the entailed propositions. I suggest, for example, that it is enough to establish the nominalistic legitimacy of ‘The proposition that Shelob and Charlotte are both segmented entails the neutral proposition that there is an anatomical feature that Shelob and Charlotte share in common’ to exhibit the fact that necessarily, whatever nominalistically acceptable propositions entail that Shelob and Charlotte are both segmented also entail the neutral proposition that there is an anatomical feature that Shelob and Charlotte share in common.

For this particular example, furthermore, doing that is in fact pretty easy. In this case, the entailed proposition (namely, the neutral proposition that there is an anatomical feature that Shelob and Charlotte share in common) is (in any possible world) true if and only if it is entailed by some true proposition perspicuously expressed by a sentence of the form ‘Shelob is ____ and Charlotte is ____’ (where the blanks are to be uniformly
filled in by an adjective that is itself an anatomical term). Since the proposition perspicuously expressed by the sentence ‘Shelob is segmented and Charlotte is segmented’ is just such a proposition, furthermore, and since that proposition is nominalistically acceptable, it follows that any propositions that entail it also entail the neutral proposition that there is an anatomical feature that Shelob and Charlotte share in common. So we can see that the truth of the neutral proposition expressed by ‘The proposition that Shelob and Charlotte are both segmented entails the neutral proposition that there is an anatomical feature that Shelob and Charlotte share in common’ is underwritten by what the relationships between the nominalistically acceptable facts are like in every possible world.

I take it that the above suffices to illustrate that those sentences involving apparent quantification over propositions and linguistic types occurring in this and the other chapters may (given Van Inwagen’s Wittgensteinian views concerning the philosophy of language) be seen to express propositions that have nominalistically acceptable truth conditions. Of course, in order to defend this thesis (indeed, in order even to express it), I have had to use sentences that involve apparent quantification over linguistic types and propositions. But if the arguments that I have provided by means of these sentences are sound, then my use of such sentences is itself benign, even from a nominalistic perspective. Matters are much the same if, instead of adopting Van Inwagen’s Wittgensteinian views, the nominalist adopts the error theory sketched in the final section of Chapter 2. Although, as I indicate in the next section, the nominalist’s taking on board such an error theory does introduce a few new wrinkles.
4.5 Error Theory and the Charge of Self Defeat

For the most part, the strategies of the previous section for exhibiting the nominalistic legitimacy of apparent quantification over propositions and linguistic types simply carry over, *mutatis mutandis*, if we adopt an error theory along the lines sketched in the final section of Chapter 2. Recall that according to that theory, we are no longer to regard the propositions expressed by the relevant sentences as differing from those perspiciously expressed by them (even when those sentences are used in contexts in which their ontologically controversial implications are beside the point). Rather than regarding the propositions expressed by those sentences as true but neutral with respect to the existence of the relevant nominalistically unacceptable entities, we are to regard those propositions as strictly speaking false but correct. And so on.

Before I speak in more detail about the prospects for expanding the error theory sketched in Chapter 2 to encompass apparent quantification over propositions and sentence types, however, it will prove helpful to make a slight adjustment to that theory. For the sake of not introducing too many complications at once, I developed the error theory sketched in the final section of Chapter 2 in such a way that propositions were taken to be the bearers of both truth and correctness. For present purposes, however, it will prove more useful to develop that theory in such a way that interpreted sentence types, rather than propositions, are taken to be the bearers of correctness.

A simple example can help to illustrate the rationale behind this modification. Consider the following sentence: ‘There are features that spiders and insects share in common, in spite of the fact that strictly speaking, there are no features’. Given
apparatus of the previous section and the Van-Inwagen-inspired, Wittgensteinian view of
the philosophy of language that went along with it, this sentence can be taken to express a
proposition that is the conjunction of the neutral proposition expressed by ‘There are
features that spiders and insects share in common’ with the proposition perspicuously
expressed by ‘There are no features’. And given the philosophy of language operating in
the previous section, we were entitled (without contradiction) to regard the proposition
expressed by the sentence at issue as being true. Given the error theory operating in this
section, however, we are not entitled to claim that the sentence at issue expresses a truth.
But we will still want to say that this sentence is correct (in some derivative or non-
derivative sense).

But if we take the proposition expressed by the above sentence to be the thing that
is the primary bearer of correctness, we run into a problem. The error theory currently
under consideration denies that there is any such distinction as that between the
proposition perspicuously expressed by a sentence and the neutral proposition expressed
by that sentence. And it looks like the only plausible candidate for the proposition
perspicuously expressed by the sentence ‘There are features that spiders and insects share
in common, in spite of the fact that strictly speaking, there are no features’ is just the
proposition perspicuously expressed by the sentence ‘There are features that spiders and
insects share in common in spite of the fact that there are no features’. But the latter
sentence is a formal contradiction (or, at least, it formally implies one). So if the
proposition expressed by the sentence we’ve been considering is correct, the proposition
expressed by a formal contradiction is also correct. So if the sentence we’ve been
considering counts as (derivatively) correct, then so does the sentence expressed by a
formal contradiction. But if that’s the case, then either the formally valid arguments we are interested in that make use of correct premises won’t always be correctness preserving, or every (truth-evaluable) declarative sentence (in the language in which we are giving our arguments) is correct (since any such sentence is the formal consequence of a contradiction).

Nevertheless, since we are taking the sentence types that we are appearing to quantify over to be interpreted sentence types, and since interpreted sentence types are individuated by their morphology and syntax, as well as by their meanings, the sentence ‘There are features that spiders and insects share in common, in spite of the fact that strictly speaking, there are no features’ is not identical to the sentence ‘There are features that spiders and insects share in common in spite of the fact that there are no features’. So if interpreted sentences rather than propositions are thought of as the primary bearers of correctness, the first sentence can count as correct without the second sentence counting as correct. And even if, according to our error theory, the proposition expressed by the first sentence is identical to the proposition expressed by the second, the rules that were laid down in the previous section for formally reasoning with sentences that employ the phrase ‘strictly speaking’ (rules that are to be regarded in force here as well) do not allow us to formally derive a contradiction from the first sentence. So even if the first sentence expresses the same proposition as a formal contradiction, the first sentence is not itself a formal contradiction (nor does it formally imply one).

If the nominalist error theorist still allows for apparent quantification over propositions, however, the above modification is not sufficient to dissolve the threat of
her having to endorse formal contradictions as correct. To see why, consider the following “disquotational principle” for propositions:

(PDP) The proposition that p is true if and only if p

It is obviously no part of the nominalist’s error theory that all of the instances of PDP are strictly speaking true. PDP is ontologically inflationary. Endorsing its validity commits one, for example, to the claim that snow is white if and only if the proposition that snow is white is true, which in turn (in conjunction with the known fact that snow is white) commits one to the claim that there is such a thing as the proposition that snow is white. But the nominalist error theorist denies that strictly speaking, there is such a thing as the proposition that snow is white. Even so, since the nominalist’s error theory implies that many of the sentences that we use that involve apparent quantification over propositions are correct, and since (it can be argued) something like PDP seems to play a kind of fundamental role in guiding our ordinary and philosophical talk of propositions, the nominalist will have good cause to think of PDP as being correct (i.e. such that all of its instances are correct). And this is enough, it might initially seem, to get her into trouble.

Consider the following argument:

(1) There are anatomical features that spiders and insects share in common, in spite of the fact that strictly speaking, there are no features. [premise]

(2) There are features. [1]

(3) Strictly speaking, there are no features. [1]

(4) The proposition that strictly speaking, there are no features is true. [3, PDP]

(5) The proposition that strictly speaking, there are no features is identical to the proposition that there are no features. [premise]
The proposition that there are no features is true. [4,5]

There are no features. [6, PDP]

There are features and there are no features. [2,7]

The premises of this argument are found on lines (1) and (5). (1) is a sentence that the nominalist error theorist is committed to regarding as correct. The correctness of (5) appears to be a commitment that the nominalist error theorist incurs by denying that sentences such as ‘There are anatomical features that spiders and insects share in common’ express any propositions other than those perspicuously expressed by them. If the nominalist error theorist commits herself to the correctness of PDP, furthermore, she is committed to the claim that (4) is correct if (3) is and that (7) is correct if (6) is. The correctness of the other lines follow provided that (as the nominalist herself maintains) formally valid arguments that make use of correct premises are correctness preserving. But since (8) is a formal contradiction, if it is correct, then either formally valid arguments employing correct premises are not correctness preserving, or every (truth-evaluable, declarative) sentence is correct. In any case, it would seem, there’s bad news for the nominalist error theorist.

Since the nominalist error theorist must find a way to maintain both that formally valid arguments employing correct premises are correctness preserving and that not all sentences are correct, she must find a way to deny that she is committed to the correctness of (8). Since the correctness of (1) is non-negotiable, that leaves her with the prospect of either denying the correctness of (5) or the correctness of PDP. A nominalist of the sort described in the previous section, one who holds to Van-Inwagen-inspired, Wittgensteinian views concerning the philosophy of language, would have no trouble
denying the correctness of (5). She can consistently believe (doubts about the existence of propositions aside) that the that-clause ‘that strictly speaking, there are no features’ names a different proposition than the that-clause ‘that there are no features’. This option is not available to the nominalist error theorist, however.

Nevertheless, there is another way for the nominalist error theorist to deny the correctness of (5). She doesn’t have to concede that the that-clause ‘that strictly speaking, there are no features’ is a legitimate proposition name. Since the technical phrase ‘strictly speaking’ is a formal device of her own making, she gets to insist upon what counts as its proper usage. She is thereby entitled to stipulate that sentences that employ the phrase ‘strictly speaking’ (as she uses it) do not form legitimate proposition names when ‘that’ is prefixed to them. With this stipulation in force, (5) no longer counts as a syntactically well-formed sentence, and so does not get to count as a correct sentence. In fact, with this stipulation in force, the above attempt to derive (8) already fails at (4). That’s because, with this stipulation in force, ‘The proposition that strictly speaking, there are no features is true if and only if strictly speaking, there are no

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122 I will not be too draconian, in this section, about refusing to embed sentences prefixed by ‘strictly speaking’ in ‘that’ clauses. When I do, however, such ‘that’ clauses should not be regarded as forming proposition names (on some occasions they may be understood adverbially; on others, they may be understood as names for sentences rather than for propositions).

123 The nominalist of the previous section will not want to make such a stipulation, however, since she sometimes uses the phrase ‘strictly speaking’ in proposition names in order to make it clear that the named proposition in question is the proposition perspicuously expressed by whatever sentence is at issue, rather than the neutral proposition expressed by that sentence. But the nominalist of this section has no use for that sort of distinction and so does not need to employ the phrase ‘strictly speaking’ for that purpose.
features’ no longer counts as a legitimate substitution instance of PDP (because, with this stipulation in force, that putative instance is syntactically ill-formed).  

The nominalist error theorist, insofar as she needs to quantify over propositions and sentence types (in order to express her views), does find herself in the position of having to assert sentences, in the philosophy room, that she regards as expressing propositions that are strictly speaking false. Whether this fact can be leveraged against her so as to create a significant objection to her view is a matter that I will discuss below. For now, however, I want to suggest some stipulative conventions that she might insist be in force in the philosophy room, conventions that will assist her in coherently stating her views in ways that are not liable to be misunderstood (provided that all parties are aware of the fact that those conventions are in force).

The nominalist error theorist can stipulate that when she is speaking in philosophical contexts, she’ll only assert sentences that are, by the lights of her error theory, correct even if the propositions they express are strictly speaking false. And she won’t assert (and may even assert the denials of) sentences that are incorrect by the lights of her error theory (even if those sentences express propositions that are strictly speaking true). The sentence ‘Many sentences that involve apparent quantification over sentences

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124 There are also other “disquotational principles” in the neighborhood of PDP that the nominalist error theorist had better just go ahead and deny are correct. Consider, for example, the following:

(PDP*) The proposition expressed by ‘p’ is true if and only if p.

This principle can be employed in an argument that parallels the above to derive a formal contradiction from other sentences that the nominalist error theorist is committed to affirming are correct. But, at least to my ears, PDP* doesn’t have the same trivial-sounding ring to it that PDP does. In any case, since the nominalist error theorist is ultimately an anti-realist about proposition talk anyway, she can get away, to some extent, with making some *ad hoc* stipulations about which sorts of sentences constituting proposition talk get to count as correct and which sorts do not, provided that those stipulations do not undercut other things that she wants to be able to say, or the philosophical purposes of engaging in proposition talk that she want to preserve.
are correct’, for example, is, according to her error theory, correct, even though the proposition expressed by that sentence is strictly speaking false (because it entails that there are sentences). So in philosophical contexts, she may assert that sentence, and she will refrain from asserting the denial of that sentence. Since she has the use of the phrase ‘strictly speaking’ at her disposal, however, adopting this practice does not undercut her ability to express the nominalist content of her error theory. For example, even though the practice at issue does not permit the nominalist error theorist to assert (in philosophical contexts) the sentence ‘There are no sentences’, it does allow her to assert ‘Strictly speaking, there are no sentences.’ The latter sentence is (by the lights of her error theory) correct, in spite of the fact that the former sentence is not.

Things get a bit more tricky for the nominalist error theorist who adopts this practice when it comes to various things she might want to say about the truth values of propositions. According to her error theory, for example, the proposition that there are no propositions is strictly speaking true. The question is how she may consistently express that claim. She can’t (given the currently adopted convention governing her manner of talking) do so by asserting the sentence ‘The proposition that there are no propositions is true’, not, at any rate, if she endorses the correctness of PDP. Endorsing that sentence as correct, in conjunction with endorsing the correctness of PDP (and the claim that formally valid arguments are correctness preserving) commits her to the correctness of ‘There are no propositions’; but she denies the correctness of that sentence. She also can’t do so by asserting the sentence ‘Strictly speaking, the proposition that there are no propositions is true’. This sentence formally implies ‘Strictly speaking, there
is such a proposition as the proposition that there are no propositions’ and, as a
nominalist, she denies that this sentence is correct.

What she can do, however, is adopt the following (schematically expressed) rule:
It is permissible to assert ‘The proposition that p is strictly speaking true’ (where ‘that p’
is a syntactically well-formed proposition name) if strictly speaking, p. Note that (given
what she takes to be correct) this rule permits the nominalist error theorist to assert the
sentence ‘The proposition that there are no propositions is strictly speaking true’. Of
course, given this way of talking, one is not permitted (for example) to infer ‘The
proposition that there are no propositions is true’ from the sentence ‘The proposition that
there are no propositions is strictly speaking true’, no more than one is permitted to infer
‘There are no propositions’ from ‘Strictly speaking, there are no propositions’. So, for
instance, the following would (given the manner of talking governed by these
conventions) be a formally consistent (even though paradoxical) thing for the nominalist
error theorist to say: ‘The proposition that there are no propositions is not true in spite of
the fact that the proposition that there are no propositions is strictly speaking true’. A
parallel convention can also be regarded as being in force when it comes to speaking of
various propositions as being true in spite of the fact that they are strictly speaking false.

The nominalist error theorist also needs a way of adequately expressing her
disagreement with the Van-Inwagen-inspired, Wittgensteinian nominalist theory of
platonist discourse that was being assumed in the previous section. The “Vansteinian
nominalist” (as I will now refer to the nominalist who holds the sort of view described in
the previous section) can (given her own manner of speaking) contrast her views with
that of the error theorist’s by saying that, according to her, sentences such as ‘There are
propositions’ express (in contexts in which their ontologically controversial implications are beside the point) propositions that are true whereas, according to the error theorist, such sentences express propositions that are false. The error theorist who adopts the currently proposed manner of speaking, however, can’t quite put things that way. The sentence ‘There are propositions’ is, according to her, correct, and so her current manner of speaking commits her to asserting that the proposition it expresses is true.

What the error theorist can do, however, is note that, according to the Vansteinian, the sentence ‘There are propositions’ expresses (in contexts in which its ontologically controversial implications are beside the point) some proposition other than the proposition that there are propositions (where ‘that there are propositions’ is here to be understood as naming the proposition perspicuously expressed by the sentence ‘There are propositions’). The error theorist can further note that, according to the Vansteinian, the proposition that is expressed by the sentence ‘There are propositions’ fails (in the context at issue) to entail any proposition that is strictly speaking false. And the error theorist can agree that if a proposition fails to entail any proposition that is strictly speaking false, that proposition is itself strictly speaking true. So given all this, the error theorist can express her disagreement with the Vansteinian by noting that, whereas according to the Vansteinian, the proposition expressed by ‘There are propositions’ both differs from the proposition that there are propositions and is strictly speaking true, this is not so according to her. According to her, rather, the proposition expressed by ‘There are propositions’ just is the proposition that there are propositions, and furthermore, that proposition is strictly speaking false.
With these modifications and stipulative conventions in place, it is a fairly straightforward matter to adapt the argument for the nominalistic legitimacy of apparent quantification over propositions and sentence types given in the previous section so as to make it fit with an error theory of the sort sketched in the final section of Chapter 2. A few tweaks are needed here and there. For example, instead of thinking of the effect of appending the phrase ‘strictly speaking’ to the front of a sentence as ensuring that the proposition expressed by that sentence is the one perspicuously expressed by it, we are to regard the effect of appending that phrase as ensuring that the resulting sentence is correct if and only if the embedded sentence expresses a proposition that is strictly speaking true. Instead of taking ourselves to be exhibiting the fact that the propositions expressed by the sentences at issue have nominalistically acceptable truth conditions, we are to take ourselves to be illustrating that the sentences at issue have nominalistically acceptable correctness conditions. And so on.

There is one obstacle, however, that faces the nominalist who wishes to adopt such an error theory, one that does not, for instance, face a nihilist or an organicist about composition who proposes to adopt the error theory proposed by Van Inwagen. Unlike an error theorist of the latter sort, the nominalist error theorist (at least one who wants to endorse something along the lines of the argument sketched in the previous sections, suitably modified, of course, to comport with her own views concerning the philosophy of language) is forced (absent an adequate paraphrase strategy) to make use of sentences that perspicuously express propositions that (by her own lights) are strictly speaking false, even in the context of philosophical discussion. The arguments made on behalf of the nominalist in the previous sections were expressed by means of sentences that
involved apparent quantification over abstract entities (such as sentence types and propositions). The Vansteinian nominalist can believe that the propositions expressed by those sentences are *strictly speaking true* and therefore that the arguments employing them are sound. But the nominalist who endorses the error theory sketched here cannot regard these arguments as having premises that are strictly speaking true (even when suitably modified so as to comport with her error theory) and therefore cannot regard them as strictly speaking sound. Indeed matters are worse. Not only is the nominalist who endorses the error theory described above forced to concede that the arguments on behalf of nominalism found in the previous sections are strictly speaking unsound (even when suitably modified to comport with her own views concerning the philosophy of language), she is forced to concede that the error theory that she endorses is itself strictly speaking false (i.e. that some of the sentences of which it consists express propositions

125 An argument may be said to be “strictly speaking sound” if and only if it is valid and has premises that express propositions that are strictly speaking true. Of course, given her current way of talking, the nominalist can say that many of the arguments she puts forward in defense of her theory are sound, because they are formally valid and it is correct, by her lights, to say that their premises express true propositions (although she cannot say this in every case, not, for example, in cases in which the arguments in question have premises or conclusions like ‘There are propositions in spite of the fact that strictly speaking, there are no propositions’). But it would be disingenuous on her part to try to make a philosophically significant point out of that fact. She still holds that it is strictly speaking false that those arguments are sound.
that are strictly speaking false)\textsuperscript{126} because some of the sentences of which it consists involve apparent quantification over things like sentence types and propositions.\textsuperscript{127,128}

How bad is this for the nominalist who endorses such an error theory? It’s not as bad as it might seem. Granted, such a nominalist cannot consistently believe that her error theory is \textit{strictly speaking true}, but she can consistently believe that it is \textit{philosophically correct}, correct that is, insofar as its implications that are \textit{to the philosophical point} (in the sense outlined in Section 4.3) express propositions that are strictly speaking true. Well, almost, but not quite. Such a nominalist cannot consistently \textit{believe} that her error theory is philosophically correct; the proposition expressed by the

\textsuperscript{126}This is a good occasion to mention that in this section I will be thinking of the nominalist’s error theory as a collection of formally consistent interpreted sentences. It is better to think of it that way than to think of it as a proposition because, if the nominalist’s error theory is correct, some of the sentences used to articulate it express the same propositions as do formal contradictions. The sentence ‘Many of the sentences we use are correct, in spite of the fact that, strictly speaking, there are no sentences’ expresses the same proposition (according to the nominalist’s error theory) as ‘Many of the sentences we use are correct in spite of the fact that there are no sentences’. The latter sentence is a formal contradiction (or at least it formally implies one). The former sentence, however, is not, since the occurrence of ‘strictly speaking’ in that sentence prevents the formal derivation of a contradiction (given the rules laid down in the previous section for drawing inferences from sentences involving that phrase).

\textsuperscript{127}Indeed, to pile things on a bit more, the nominalist error theorist finds herself (given the manner of talking that she has adopted) in a position of being licensed to assert that some of the sentences of which her error theory consists express propositions that are not strictly speaking false, but false. For example, the nominalist error theorist regards the sentence ‘Strictly speaking, there are no propositions’ as correct. So her currently adopted manner of talking permits her to assert it. But according to her error theory, that sentence expresses the same proposition as does ‘There are no propositions’. The latter sentence, however, according to her, is incorrect. So her commitment to the currently adopted way of talking commits her (via PDP) to the claim that the proposition that there are no propositions is not true. So she is committed to regarding sentences like the following as correct (and therefore, by the conventions currently governing her speech, as assertable): ‘Strictly speaking, there are no propositions, in spite of the fact that the proposition expressed by ‘Strictly speaking, there are no propositions’ is not true’.

\textsuperscript{128}Compare what has been said in this paragraph with an objection that Rosen (1994, p.169) raises to Hartry Field’s program:

When Field says that set theory is false but useful, and useful because conservative, what precisely does he mean by “set theory”? A scattered mass of ink and chalk? A theory shaped region of spacetime? The general worry is that on a wide range of restrictive ontological views, theories turn out to be among the entities the theorist professes not to believe in. And whenever this is the case, the fictionalist way out is simply not available.
sentence ‘The nominalist’s error theory is philosophically correct’ is itself strictly speaking false, at least, that is, according to her own error theory! Nevertheless, she can quasi-believe the sentence that expresses that proposition. That is, she can hold a positive epistemic attitude toward that sentence, one that falls short of belief, but is much like belief, one that disposes her to treat that sentence as “getting it right” as far as the philosophical point it is being used to make is concerned. I will have more to say about what quasi-belief amounts to momentarily, but, for now, take what has just been said above as an adequate characterization.

Belief is here being thought of as primarily involving a relation between a subject and a proposition. So when I speak of someone as believing a sentence (which I may do on occasion), one should think of that as elliptical for the claim that the person in question believes the proposition expressed by that sentence. Quasi-belief, however (for reasons parallel to the rationale behind taking interpreted sentences rather than propositions to be bearers of correctness) cannot be thought of as primarily involving a relation between a subject and a proposition. Accordingly, I will insist that it be understood as primarily involving a relation between a subject and an interpreted sentence. So when I do things like combine the phrase ‘quasi-believes’ with a ‘that’ clause, for instance, that should be understood (doubts about the existence of sentence types aside, of course) as expressing a relation between the quasi-believer and the embedded sentence, rather than between the quasi-believer and a proposition.

Compare this with what Rosen (1994, pp. 145-154) says while discussing Van Fraassen’s distinction between belief and acceptance (the latter of which, as far as I understand it, is much like what I’m calling “quasi-belief”). Rosen speculates that Van Fraassen merely accepts the central philosophical theory that he (Van Fraassen) puts forward (namely his constructive empiricist account of science), that he immerses himself in it, that he talks as if he believes it, that he puts it forward in philosophical conversation in order to make certain philosophical points, that he regards it as philosophically adequate in various respects, etc., but that he does not actually believe it. Regardless of the merits Rosen’s interpretation of Van Fraassen, this is the sort of position that I’m suggesting that the nominalist error theorist must (on pain of inconsistency) take herself to be in with respect to her error theory.

A similar point can be made regarding assertion, insofar as assertion is closely connected to belief. In various places above, I have spoken of the nominalist as asserting various sentences that are correct according to her error theory. Plausibly, however, to sincerely assert a sentence is to express the contents of one’s beliefs. If that is so, then the nominalist error theorist cannot be taking herself (at least insofar as she is trying to be sincere!) to be asserting the sentences that constitute her error theory. Nevertheless, she can take herself to be quasi-asserting them, as putting them forward in conversation as philosophically correct, and as conversationally indicating that she quasi-believes them. On this same point, as applied to Van Fraassen’s distinction between belief and acceptance, see Rosen (1994, pp. 149-150), who also employs the term ‘quasi-assertion’. In the main text, however, I will ignore the distinction between assertion and quasi-assertion.
The nominalist error theorist, insofar as she is unable to say many of the things she wants to say without, by her own lights, asserting sentences that express strictly speaking false propositions, is no doubt in something of an awkward position. Things would be nicer for her if she could find a way to adequately express herself without being forced to assert such sentences. But there does not appear to be anything incoherent when it comes to her actual beliefs concerning these matters. Because she quasi-believes the sentences that compose her error theory, for example, and quasi-believes that the formally valid arguments she employs using those sentences are correctness preserving, she also quasi-believes the conclusions of those arguments. But she needn’t believe those conclusions when they themselves (according to her own error theory) express propositions that are strictly speaking false.

Since the sentence ‘Some sentences are correct, in spite of the fact that strictly speaking, there are no sentences’, for example, is a formal consequence of her error theory, the nominalist error theorist will quasi-believe that sentence. But she will not believe it. Since the formal rules that govern the deployment of sentences that involve the use of the phrase ‘strictly speaking’, furthermore, do not allow her to formally derive from this consequence the sentence ‘Some sentences are correct in spite of the fact that there are no sentences’, and thus not the sentence ‘There are sentences and there are no sentences’, she is not committed to quasi-believing any formally contradictory sentences as a result of quasi-believing this sentence.

Nevertheless, the nominalist error theorist will believe the implications of her error theory that are to the philosophical point of that theory (at least those that she can see to be such). That is the commitment that comes along with quasi-believing that her
error theory is philosophically correct. Since, for example, the sentence ‘Strictly speaking, there are no sentences’ is a formal consequence of her error theory, and since that sentence is to the philosophical point of that theory, she will not merely quasi-believe, but actually believe that sentence. That is, she will actually end up believing the proposition expressed by ‘Strictly speaking, there are no sentences’; i.e. she will actually wind up believing that there are no sentences.

We can employ this fact about the nominalist error theorist’s dispositions, furthermore, to help us better understand what quasi-belief amounts to. We can take it to be constitutive of what it is for the nominalist to quasi-believe her error theory that she be disposed in this way and in the related manners. That is, we can think of quasi-belief as a cluster of dispositions that involve things like dispositions to believe certain propositions, make certain kinds of assertions, certain kinds of inferences, and the like. Many of the dispositions in this cluster will overlap with the dispositions that are typically associated with belief, which is what makes quasi-belief a lot like belief. Still, in various crucial respects, the dispositions associated with quasi-belief will diverge from those typically associated with belief (e.g. the nominalist will be disposed, when pressed, to engage in various sorts of retracting behaviors, such as admitting that many of the sentences she quasi-believes are strictly speaking false).

132 The manner in which quasi-belief and belief are related might be made even clearer if we were to adopt an account of belief according to which belief itself is to be thought of as a cluster of dispositions, with quasi-belief being a cluster of dispositions that significantly overlaps those that are constitutive of belief. See (Schwitzgebel 2002) for such an account of belief. As Schwitzgebel makes clear, a dispositional account of belief of this sort can be developed in ways that free it from the behaviorist overtones historically associated with it.
Even if there is nothing incoherent about the nominalist’s *attitudes* toward her error theory, however, isn’t her error theory *itself* in some way self-referentially incoherent, and isn’t that a problem for her? The nominalist’s error theory is, according to itself, for example, strictly speaking false (i.e. it is such that, according to it, the sentences that constitute it express propositions that are strictly speaking false). That does *sound* bad. But just what exactly is so bad about it? The nominalist isn’t claiming for her error theory that it is strictly speaking true. She is claiming for it merely that it is an instrumentally useful vehicle for getting her philosophical point across, and also for engaging in philosophical reasoning about the relevant issues. In order for it to be suited to that task, it doesn’t need to be strictly speaking true (not even according to itself!). It just needs to be philosophically correct (which, according to itself, it is).

Perhaps the problem isn’t that the nominalist’s error theory is self-referentially incoherent. Perhaps it is, rather, that it is, in some way, self-undermining. Here is one way to flesh out such a charge: The nominalist wants to argue that it is instrumentally useful to reason as if there are abstract objects, even if there aren’t any. I supported that conclusion in this and the previous chapters by arguing that formally valid arguments with premises that involve apparent quantification over abstract objects are correctness preserving, and that when one arrives at a nominalistically friendly conclusion by means of such reasoning, that conclusion is not only correct, but expresses a proposition that is strictly speaking true.\(^{133}\)

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\(^{133}\) This is a bit of a simplification, since (as I pointed out in Chapter 2 note 100), there might be nominalistically friendly sentences that the nominalist error theorist regards as expressing propositions that are strictly speaking false for reasons that are independent of her nominalism (e.g. she might not only be a nominalist, but also a compositional nihilist who regards nominalistically friendly sentences involving apparent quantification over chairs as expressing propositions that are strictly speaking false). I’ve been
But there are two difficulties faced by the nominalist error theorist regarding the status of this argument. First, she cannot believe its conclusion. That is, while she can (as a nominalist) believe that it is instrumentally useful to reason as if there are abstract objects even if there aren’t any (she needn’t engage in any apparent quantification over abstract objects in order to state that conclusion), she cannot believe that the reason this is so is because formally valid arguments with correct premises are correctness preserving and are such that, when they have nominalistically friendly conclusions, their conclusions express propositions that are strictly speaking true. For her to believe that would be for her to believe a proposition expressed by a sentence that involves apparent quantification over things like propositions and sentence types, a proposition that, according to her own error theory, is strictly speaking false. Second, she cannot believe that the arguments made in this and the previous chapters in favor of the conclusion that it is instrumentally useful to reason as if there are abstract objects are strictly speaking sound. Those arguments, according to her own error theory, have premises that are strictly speaking false.

Now all of this would be fine if the nominalist were already justified in believing that it is instrumentally useful to reason as if there are abstract objects even if there are none. In that case, even though she takes the arguments that she makes by means of employing such reasoning to be strictly speaking unsound, she can still take them to valid, to have correct premises, and to be correctness preserving. Even though, in that case, she cannot consistently believe, furthermore, that the arguments that lead her to ignoring this complication since the beginning of Chapter 3, however, and I intend to go on doing so. We can stipulate, if we like, that we are in a context in which no nominalistically friendly sentence is to be regarded as correct unless it expresses a proposition that is strictly speaking true.
conclusions such as this one are correctness preserving, she can justifiably quasi-believe
the sentence ‘Formally valid arguments with correct premises are correctness preserving
and are such that when they have nominalistically friendly conclusions their conclusions
express propositions that are strictly speaking true’. And in quasi-believing that sentence,
she can consistently regard herself as being licensed in believing the propositions
expressed by the nominalistically friendly conclusions that she arrives at by means of
employing it as a premise – including, for example, the claim that it is instrumentally
useful to reason as if there are abstract objects even if there are none.

But all of this is wholly problematic, it seems, if the nominalist error theorist is
not already justified in believing that it is instrumentally useful to reason as if there are
abstract objects. In that case, the nominalist’s reliance on such reasoning in order to
justify her reliance on such reasoning looks objectionably circular. Van Inwagen has
made a similar charge regarding attempts, like that of Hartry Field, to prove that
mathematics is conservative:

The proof will be a piece of pure, not applied mathematics, and many of its
premises will be, according to the nominalist, simply untrue. How then can the
nominalist suppose that it has been proved that the applied arithmetical fiction is
conservative? … No doubt all the false premises of the proof are true “in the
mathematical fiction” … but that’s only to say that (although false in reality)
they’re true according to a certain story … One cannot recommend the applied
arithmetical fiction to prospective applied mathematicians on the ground that it is
conservative if the thesis that it is conservative is (non-vacuously) true only in
some story. That would be like recommending the Koran to perspective Muslims
as God’s final revelation on the sole ground that, although the Koran is a fiction,
it is true in that fiction that the Koran is God’s final revelation.\(^{134}\)

\(^{134}\) (Van Inwagen, unpublished)
I believe, however, that this objection misconstrues the dialectical situation. Neither the platonist nor the nominalist is in a position of needing to justify her reliance on reasoning as if there are abstract objects. That we are justified in reasoning as if there are abstract objects is one of those pre-theoretical beliefs of ours that is legitimately taken for granted by both parties at the outset of the discussion. There are also various plausible stories that both parties (with a few tweaks here and there) could consistently tell about how it is that we are justified in engaging in such reasoning. Certainly, there are some empirical grounds for relying on it – it works! Perhaps, to some extent, we can see a priori that such reasoning is instrumentally useful. Perhaps properly functioning human beings naturally reason in that way in accordance with a good design plan aimed at truth. And so on. In any case, though, which of these stories are correct is not germane to the current topic (that’s a question for epistemology).

What is at issue in the current dialectical context is not whether it is justifiable to believe that reasoning as if there are abstract objects is instrumentally useful, but whether both parties can give an adequate account of why such reasoning is instrumentally useful. The platonist (as Van Inwagen himself is happy to point out),\(^\text{135}\) has a ready account of why such reasoning is instrumentally useful; i.e. many of the arguments that people construct in the process of engaging in such reasoning are strictly speaking sound. Since this explanation isn’t available to the nominalist, however, she faces some dialectical pressure to provide an alternative account. That’s where her claim that such reasoning is correctness preserving comes in, as well as her claim that the nominalistically friendly

\(^{135}\) See (Van Inwagen, unpublished).
conclusions arrived at by means of correctly engaging in such reasoning express propositions that are strictly speaking true.

The question that the nominalist error theorist does face, in the current dialectical context, then, is whether it is a problem for her that she can’t believe her own account of what’s good about such reasoning. I don’t think it is. If that account is itself correct, then accounts that employ sentences involving apparent quantification over abstract objects do not need to be strictly speaking true in order to be good. Such accounts, rather, can be good by virtue of their being correct. What makes such correct accounts good (at least insofar as their correctness contributes to their goodness, in addition to whatever other explanatory virtues they may have) is that they reliably convey accurate information about what the concrete world is like; or, to put the matter the other way around, what makes them good is that the concrete world is as it needs to be in order for them to be correct. The nominalist’s account of what makes such accounts good, if it is in fact correct, has precisely the same virtue. So if the nominalist’s account of why it is instrumentally useful to reason as if there are abstract objects is correct, she can consistently believe that she can adequately account for why it is instrumentally useful to reason as if there are abstract objects, even if in fact there are none, without offering an account that is strictly speaking true. So the nominalist error theorist can successfully argue, from her own point of view, that she is able to consistently and adequately account for why it is instrumentally useful to reason as if there are abstract objects.

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136 Leng (2010, chapter 10), who adopts a fictionalist approach to mathematics, argues this point at length as it pertains to the use of mathematical explanations in science. Balaguer (2009, p. 134) similarly states that fictionalists about mathematics can regard mathematically laden physical theories as good accounts of physical reality insofar as “the physical world holds up its end of the ‘empirical-science bargain’.”
4.6 Works Cited


Van Inwagen, Peter (unpublished) “Fictionalist Nominalism and Applied Mathematics”.
APPENDIX A:
HOW NOMINALISTIC UNDERGIRDING CORRESPONDS TO ORDINARY
CORRECTNESS

Consider the following sentence that we examined in Chapter 2: ‘Either of two female spiders of the same species shares all and only the same anatomical features with the other.’ In Chapter 2, I suggested that, merely in virtue of grasping the proposition expressed by that sentence, we could see that that proposition is correct if and only if each of the propositions perspicuously expressed by the infinitely many sentences (of an idealized language like English but lacking for no relevant expressive resources) conforming to the following schema are true: 「Any two conspecific, female spiders are such that one is ____ iff the other is ____」(where the blanks are to be uniformly filled in by an adjective that is itself an anatomical predicate). Here I will illustrate how my definition (given in Chapter 3) of what it is for a sentence in the object language to be nominalistically undergirded mechanically reproduces something equivalent to this suggestion. Having done so, I will take myself to have displayed the fact that my definition of what it is for a sentence to be nominalistically undergirded does in fact serve as a formal analog of what it is, according to Chapter 2, for a proposition to have nominalistically acceptable ordinary correctness conditions that are satisfied.

First, we need to consider what a translation of the above sentence into the object language would look like. Let’s begin to do this by considering how we would go about translating that sentence into the canonical language of first order quantification, without worrying about the various distinctive features of the object language. First, let ‘C’ stand
for the binary predicate ‘conspecific’, let ‘S’ stand for the unary predicate ‘is a female spider’, let ‘A’ stand for the predicate ‘is anatomical’ where this predicate can only be satisfied by features, let ‘F’ stand for the predicate ‘is a feature’, and let ‘H’ correspond to the special purpose property having predicate found in the object language. With these stipulations in hand, we may translate the sentence at issue as follows:

$$(\forall x)(\forall y)(Sx\&Sy\&Cxy \rightarrow (\forall z)(Fz\&Az \rightarrow ((xHz \rightarrow yHz)\&(yHz \rightarrow xHz))))$$

or, equivalently (since the object language that I’ve characterized lacks a distinct symbol for the universal quantifier), we may use the following translation:

$$(S0) \sim(\exists x)\sim(\exists y)\sim(Sx\&Sy\&Cxy \rightarrow \sim(\exists z)\sim(Fz\&Az \rightarrow ((xHz \rightarrow yHz)\&(yHz \rightarrow xHz))))$$

The above sentence has the same syntactical structure as an object language sentence except that it fails to syntactically mark the fact that S functions as an n-predicate and that F and A function as a p-predicates. From this point on, I’ll ignore this fact and I’ll adopt the pretense that the above sentence is a sentence of the object language. Let’s assume that the above sentence is nominalistically undergirded and see what follows.

Our definition of what it is for a sentence to be nominalistically undergirded tells us that since this is a closed sentence that has a negation sign out front, it is nominalistically undergirded if and only if the sentence that results from deleting the leftmost negation sign is not nominalistically undergirded:

$$(S1) (\exists x)\sim(\exists y)\sim(Sx\&Sy\&Cxy \rightarrow (\exists z)\sim(Fz\&Az \rightarrow ((xHz \rightarrow yHz)\&(yHz \rightarrow xHz))))$$
Since S1 begins with a quantifier that binds the variable x, our definition tells us that it is
nominalistically undergirded if and only if there is some nominalistically undergirded
sentence that is the result of deleting the leftmost quantifier from S1 and replacing every
free occurrence of x in the resulting open sentence with a name. Suppose that a is an
arbitrarily chosen name from the object language. Then one arbitrarily chosen sentence
that results from S1 by doing the above is the following:

\[ (S2) \neg(\neg(\exists y)(Sa\&Sy\&Cay \rightarrow \neg(\exists z)\neg(Fz\&Az \rightarrow ((aHz \rightarrow yHz)\&(yHz \rightarrow aHz)))) \]

Since it is being assumed that S0 is nominalistically undergirded, and it was established
that S0 is nominalistically undergirded if and only if S1 is not nominalistically
undergirded, it follows from the above that S2 is also not nominalistically undergirded.

Since S2 has a negation sign out front, furthermore, it is nominalistically
undergirded if and only if the following sentence (that results from deleting the leftmost
negation sign from S2) is not nominalistically undergirded:

\[ (S3) \neg(\exists y)(Sa\&Sy\&Cay \rightarrow \neg(\exists z)\neg(Fz\&Az \rightarrow ((aHz \rightarrow yHz)\&(yHz \rightarrow aHz)))) \]

So it follows from all of the above that S3 is nominalistically undergirded.

But S3, in turn, is nominalistically undergirded if and only if the following
sentence (that results from deleting the leftmost negation sign from S3) is not
nominalistically undergirded:

\[ (S4) (\exists y)(Sa\&Sy\&Cay \rightarrow \neg(\exists z)\neg(Fz\&Az \rightarrow ((aHz \rightarrow yHz)\&(yHz \rightarrow aHz)))) \]
S4 is nominalistically undergirded, furthermore, if and only if there is some nominalistically undergirded sentence that results from deleting the leftmost quantifier from S4 and replacing every free occurrence of y in the resulting open sentence with a name. If we suppose that b is an arbitrarily chosen name in the object language, then one sentence that results from so doing is the following:

\[(S5) \neg (S_{a} \& S_{b} \& C_{a} \rightarrow (\exists z) \neg (F_{z} \& A_{z} \rightarrow ((aHz \rightarrow bHz) \& (bHz \rightarrow aHz))))\]

It follows from all of the above that S5 is not nominalistically undergirded.

From all this, furthermore, it follows that the following sentence is nominalistically undergirded:

\[(S6) (S_{a} \& S_{b} \& C_{a} \rightarrow (\exists z) \neg ((F_{z} \& A_{z} \rightarrow ((aHz \rightarrow bHz) \& (bHz \rightarrow aHz))))\]

Note that S6 is composed of the following two sentences:

\[(S7) S_{a} \& S_{b} \& C_{a}\]

\[(S8) \neg (\exists z) \neg ((F_{z} \& A_{z} \rightarrow ((aHz \rightarrow bHz) \& (bHz \rightarrow aHz))))\]

Now assume that S7 is nominalistically undergirded. Note that S7 is a closed sentence that contains no quantifiers. Thus S7 is nominalistically undergirded, according to our definition, if and only if its nominalistic surrogate is true. But since all of the predicates that occur in S7 are unary n-predicates or non-unary predicates (other than the identity predicate or the property having predicate), the nominalistic surrogate for S7 is either equivalent to \( \perp \) (this would be so if either \( a \) or \( b \) is a p-name) or to S7 itself. Given
the assumption that S7 is nominalistically undergirded, however, it can’t be the case that 
the nominalistic surrogate for S7 is equivalent to $\perp$. So it must be the case that S7 is its 
own nominalistic surrogate and the also the case that S7 is true. Note that this is so, 
furthermore, if and only if the names $a$ and $b$ each refer to a female spider and the 
referent of $a$ is conspecific with the referent of $b$.

It also follows from the assumption that S7 is nominalistically undergirded (and 
the established result that S6 is nominalistically undergirded) that S8 is as well. To make 
a long story short, furthermore, our definition tells us that S8 is nominalistically 
undergirded if and only if any sentence in the object language that results in taking S8, 
deleting the first negation sign, the first quantifier and the negation sign after that, and 
then uniformly replacing each occurrence of the variable $z$ with a name is itself 
nominalistically undergirded. That is, it follows from the assumption that S7 is 
nominalistically undergirded (and therefore true) that any sentence in the object language 
that results in replacing the variable $z$ in the following open sentence with a name is itself 
nominalistically undergirded:

$$(S9) \ (Fz \& Az \rightarrow ((aHz \rightarrow bHz) \& (bHz \rightarrow aHz)))$$

Now let $c$ be some arbitrarily chosen p-name that is itself the projection of an anatomical 
predicate. It follows that the following sentence is nominalistically undergirded:

$$(S10) \ (Fc \& Ac \rightarrow ((aHc \rightarrow bHc) \& (bHc \rightarrow aHc)))$$

Suppose further that G is the anatomical predicate that projects $c$. Then the nominalistic 
surrogate for S10, according to our definition, is the following sentence:
(S11) \(((\neg \bot) \& (\neg \bot)) \rightarrow ((Ga \rightarrow Gb) \& (Gb \rightarrow Ga)))

But, of course, it is easy to see that S11 is true if and only if the following sentence is true:

(S12) \((Ga \rightarrow Gb) \& (Gb \rightarrow Ga)\)

Now, by applying conditionalization and generalization from the arbitrary case the appropriate number of times, we obtain the following result: If S0 is nominalistically undergirded, then in every case in which a sentence that results from uniformly replacing the variables in the open sentence \(Sx\&Sy\&Cxy\) with names is true, any sentence is true that is the result of taking the following schema \((G'x \rightarrow G'y) \& (G'y \rightarrow G'x)\), replacing the variables \(x\) and \(y\) with the same names with which \(x\) and \(y\) (respectively) were replaced in \(Sx\&Sy\&Cxy\), and uniformly replacing the occurrences of \(G'\) with an anatomical predicate. I leave it as an exercise to the reader to apply the above reasoning in reverse and thereby verify that the converse is also true.

The conjunction of the above and its converse, however, is equivalent (given the fact that every item in the universe of discourse has a name in the object language) to the claim that S0 is nominalistically undergirded if and only if each of the infinitely many sentences (of an idealized language like English but lacking for no relevant expressive resources) conforming to the following schema are true: 'Any two conspecific, female spiders are such that one is ____ iff the other is ____?' (where the blanks are to be uniformly filled in by an adjective that is itself an anatomical predicate). Thus our recursive definition of nominalistic undergirding mechanically reproduces something equivalent to what we said in Chapter 2 about the correctness conditions for the
proposition expressed by the sentence that we have been considering. This, I take it, illustrates the correspondence between the formal notion of nominalistic undergirding as characterized in Chapter 3 and the intuitive notion of ordinary correctness characterized in Chapter 2.
APPENDIX B:
A PROOF OF THE CORRECTNESS PRESERVATION THEOREM

B.1 The Quantifier Free Axioms are Nominalistically Undergirded

It is to be shown that

For any sentence of the object language, \( \psi^* \), if \( \psi^* \) is a quantifier free axiom, \( \psi^* \) is nominalistically undergirded.

Let \( \psi \) be an arbitrarily selected sentence of the object language.

Suppose \( \psi \) is of the form \( x = x \) where \( x \) is a variable. Let \( \sigma \) be an arbitrarily selected, corresponding closed sentence for \( \psi \). Note that \( \sigma \) is of the form \( t = t \), where \( t \) is a name. Either \( t \) is an n-name or \( t \) is a p-name. If \( t \) is an n-name, then the nominalistic surrogate for \( \sigma \) is \( \sigma \) itself. Obviously, in that case, the nominalistic surrogate for \( \sigma \) is true. If \( t \) is a p-name, then the nominalistic surrogate for \( \sigma \) is \( \neg \bot \), which is, of course, true. Either way, \( \sigma \) is nominalistically undergirded. It follows by generalization from the arbitrary case that every corresponding closed sentence for \( \psi \) is nominalistically undergirded and therefore that \( \psi \) is nominalistically undergirded.

Suppose \( \psi \) is of the form \( (x = y) \rightarrow (Fx \rightarrow Fy) \), where \( x \) and \( y \) are variables. Let \( \sigma \) be an arbitrarily selected, corresponding closed sentence for \( \psi \). Note that \( \sigma \) is of the form \( (t_1 = t_2) \rightarrow (Ft_1 \rightarrow Ft_2) \) where \( t_1 \) and \( t_2 \) are names. Now, either the sentence \( t_1 = t_2 \) is nominalistically undergirded or it isn’t. If it isn’t, then it is trivially the case that \( \sigma \) is nominalistically undergirded. So suppose instead that it is. Either (i) \( t_1 \) and \( t_2 \) are each n-names (whether the same or different) or (ii) \( t_1 \) and \( t_2 \) are each the same p-name.
i is the case. In that case, \( t_1 = t_2 \) is its own nominalistic surrogate and it follows from the assumption that the sentence \( t_1 = t_2 \) is nominalistically undergirded that that sentence is also true. It therefore also follows that the referent of \( t_1 \) is identical to the referent of \( t_2 \).

Now, either \( F \) is an n-predicate or it isn’t. If it is, then the nominalistic surrogate for \( \sigma \) is \( \sigma \) and it obviously follows from the fact that the referent of \( t_1 \) is identical to the referent of \( t_2 \) that \( \sigma \) is true. If it isn’t, then the nominalistic surrogate for \( Ft_1 \) is \( \perp \), which is, of course, false; and so it follows that the consequent of the nominalistic surrogate for \( \sigma \) is true and therefore that the nominalistic surrogate for \( \sigma \) is itself true. Either way, \( \sigma \) is nominalistically undergirded. Suppose instead that ii is the case. In that case, \( Ft_1 \) is the very same sentence as \( Ft_2 \), and so the nominalistic surrogate for the former is also the same as that for the latter. So, obviously, the nominalistic surrogate for the consequent of \( \sigma \) is true, and therefore the nominalistic surrogate for \( \sigma \) is also true, and thus \( \sigma \) is nominalistically undergirded. In any case, \( \sigma \) is nominalistically undergirded. It follows by generalization from the arbitrary case that every corresponding closed sentence for \( \psi \) is nominalistically undergirded and therefore that \( \psi \) is nominalistically undergirded.

Suppose \( \psi \) is of the form \((x_1 = y_1) \& (x_2 = y_2) \rightarrow (x_1 Hx_2 \rightarrow y_1 Hy_2)\), where \( x_1, x_2, y_1, \) and \( y_2 \) are variables. Let \( \sigma \) be an arbitrarily selected, corresponding closed sentence for \( \psi \). Note that \( \sigma \) is of the form \((t_1 = s_1) \& (t_2 = s_2) \rightarrow (t_1 Ht_2 \rightarrow s_1 Hs_2)\) where \( t_1, t_2, s_1, \) and \( s_2 \) are names. Either the antecedent of \( \sigma \) is nominalistically undergirded or it is not. If it is not, then it follows straightforwardly that \( \sigma \) is nominalistically undergirded. Suppose instead that it is. Then either \( t_1 Ht_2 \) is nominalistically undergirded or it is not. If it is not, then the consequent of \( \sigma \) is nominalistically undergirded and so \( \sigma \) is nominalistically undergirded. Suppose instead that \( t_1 Ht_2 \) is nominalistically undergirded. In that case \( t_1 \) is
an n-name and $t_2$ is a p-name and the nominalistic surrogate for $t_1Ht_2$ is true.

Furthermore, the nominalistic surrogate for $t_1Ht_2$ is a sentence of the form $Ft_1$ where $F$ is an n-predicate and $t_2$ is the projection of $F$. Note further that since the antecedent of $\sigma$ is nominalistically undergirded, it is also the case that the sentence $t_2=s_2$ is nominalistically undergirded, and since $t_2$ is a p-name it follows that $s_2$ is the same p-name as $t_2$. Thus it follows that $s_2$ is the projection of $F$ and that the nominalistic surrogate for $s_1Hs_2$ is $Fs_1$.

Furthermore, since the antecedent of $\sigma$ is nominalistically undergirded, it follows that $t_1 = s_1$ is nominalistically undergirded. And since $t_1$ is an n-name, it follows that the nominalistic surrogate for $t_1 = s_1$ is just $t_1 = s_1$ itself, and that $t_1 = s_1$ is also true. Since $t_1 = s_1$ is true, and $Ft_1$ is true, it follows that $Fs_1$ is true. And since $Fs_1$ is the nominalistic surrogate for $s_1Hs_2$, it follows that the latter is nominalistically undergirded. From this it also follows that the consequent of $\sigma$ is nominalistically undergirded and therefore that $\sigma$ itself is nominalistically undergirded. In any case, therefore, $\sigma$ is nominalistically undergirded. By generalization from the arbitrary case, it follows that every corresponding closed sentence for $\psi$ is nominalistically undergirded and therefore that $\psi$ is nominalistically undergirded.

Suppose $\psi$ is of the form $(x_1 = y_1) \& (x_2 = y_2) \& \cdots \& (x_n = y_n) \rightarrow (Rx_1x_2\ldots x_n \rightarrow Ry_1y_2\ldots y_n)$ (where $n > 1$). Let $\sigma$ be an arbitrarily selected, corresponding closed sentence for $\psi$. Note that $\sigma$ is of the form $(t_1 = s_1) \& (t_2 = s_2) \& \cdots \& (t_n = s_n) \rightarrow (Rt_1t_2\ldots t_n \rightarrow Rs_1s_2\ldots s_n)$ where $t_1$, $t_2$, ..., $t_n$ and $s_1$, $s_2$, ..., $s_n$ are names. Now either $Rt_1t_2\ldots t_n$ is nominalistically undergirded or it is not. If it is not, then consequent of $\sigma$ is nominalistically undergirded and so $\sigma$ is nominalistically undergirded. Suppose instead, then, that $Rt_1t_2\ldots t_n$ is nominalistically undergirded. In that case, each of $t_1$, $t_2$, ..., $t_n$ is an
n-name and $Rt_1 t_2 \ldots t_n$ (which is its own nominalistic surrogate) is true. Now either the antecedent of $\sigma$ is nominalistically undergirded or it is not. If it is not, then $\sigma$ is trivially nominalistically undergirded. Suppose instead, then, that the antecedent of $\sigma$ is nominalistically undergirded. It follows that each conjunct of the antecedent of $\sigma$ is nominalistically undergirded. It further follows that the nominalistic surrogate for each conjunct of the antecedent of $\sigma$ is true. But since each of $t_1, t_2, \ldots, t_n$ is an n-name, if any of $s_1, s_2, \ldots, s_n$ were a p-name, the nominalistic surrogate for one of the conjuncts of the antecedent of $\sigma$ would be $\bot$ (and would therefore be false). So it must be the case that each of $s_1, s_2, \ldots, s_n$ is an n-name. So it must also be the case that the nominalistic surrogate for each conjunct of the antecedent of $\sigma$ is just that conjunct itself, and so it must be the case that each conjunct of the antecedent of $\sigma$ is true. So, for each $t_i$ and each $s_i$, the referent of $t_i$ is identical to the referent of $s_i$. Since $Rt_1 t_2 \ldots t_n$ is true, it follows that $Rs_1 s_2 \cdots s_n$ (which, since each of $s_1, s_2, \ldots, s_n$ is an n-name, is also its own nominalistic surrogate) is true. It follows that the consequent of $\sigma$ is nominalistically undergirded and therefore that $\sigma$ is nominalistically undergirded. In any case, therefore, $\sigma$ is nominalistically undergirded. By generalization from the arbitrary case, it follows that every corresponding closed sentence for $\psi$ is nominalistically undergirded and therefore that $\psi$ is nominalistically undergirded.

Suppose $\psi$ is of the form $(Fx \rightarrow xHp) \& (xHp \rightarrow Fx)$, where $p$ is a p-name and $F$ is an n-predicate that projects $p$. Let $\sigma$ be an arbitrarily selected, corresponding closed sentence for $\psi$. Note that $\sigma$ is of the form $(Ft \rightarrow tHp) \& (tHp \rightarrow Ft)$ where $t$ is a name. Now, either $t$ is an n-name or a p-name. Suppose that $t$ is an n-name. In that case, the nominalistic surrogate for $Ft$ is $Ft$ itself and the nominalistic surrogate for $tHp$ is also $Ft$. 

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And so the nominalistic surrogate for $\sigma$ is $(Ft \rightarrow Ft) \& (Ft \rightarrow Ft)$, which is obviously true. Suppose instead that $t$ is a p-name. In that case, the nominalistic surrogate for $tHp$ is $\bot$ and, since $F$ is an n-predicate, the nominalistic surrogate for $Ft$ is also $\bot$. So the nominalistic surrogate for $\sigma$ is $(\bot \rightarrow \bot) \& (\bot \rightarrow \bot)$ which is obviously true. Either way, $\sigma$ is nominalistically undergirded. By generalization from the arbitrary case, it follows that every corresponding closed sentence for $\psi$ is nominalistically undergirded and therefore that $\psi$ is nominalistically undergirded.

**B.2 Interlude: The Substitution Lemmas**

The proof that the quantifier axioms are nominalistically undergirded will make use of the following three lemmas (the first of these is not explicitly mentioned in the proof that the quantifier axioms are nominalistically undergirded but is used to prove the second lemma):

*First Substitution Lemma:* For any sentences $\varphi^*$ and $\varphi^*$', and for any distinct variables, $u^*$ and $v^*$, and for any term $t^*$, if $\varphi^*$' is a sentence exactly like $\varphi^*$ except that every free occurrence of $u^*$ in $\varphi^*$ is uniformly replaced by a name in $\varphi^*$', and $t^*$ is not $u^*$, then if $t^*$ is substitutable for $v^*$ in $\varphi^*$', $t^*$ is substitutable for $v^*$ in $\varphi^*$.

*Second Substitution Lemma:* For any sentences $\varphi^*$ and $\varphi^*$', any term $u^*$, and any variable $v^*$, such that $\varphi^*$' is a sentence that results from uniformly replacing each free variable other than $v^*$ that occurs in $\varphi^*$ with a name, if $u^*$ is substitutable for $v^*$ in $\varphi^*$, $u^*$ is also substitutable for $v^*$ in $\varphi^*$'.
**Third Substitution Lemma:** For any variables, $u^*$ and $v^*$, and for any sentences $\varphi^*$ and $\varphi^*[v^*/u^*]$ such that $\varphi^*[v^*/u^*]$ is the result of uniformly replacing each free occurrence of $v^*$ in $\varphi^*$ with $u^*$, if $u^*$ is substitutable for $v^*$ in $\varphi^*$, then $\varphi^*[v^*/u^*]$ is a sentence exactly like $\varphi^*$ except that in every place that a free occurrence of $v^*$ occurs in the latter, a free occurrence of $u^*$ occurs in the former.

Each of these lemmas will be proven by means of mathematical induction on the complexity of sentences.

**(B.2.a) A Proof of the First Substitution Lemma**

Let $\varphi$ and $\varphi'$ be sentences, and $u$ and $v$ be distinct variables, such that $\varphi'$ is exactly like $\varphi$ except that every free occurrence of $u$ in $\varphi$ is uniformly replaced by a name in $\varphi'$. Let $t$ be a term other than $u$, and suppose that $t$ is substitutable for $v$ in $\varphi'$.

**Base Case**

Suppose $\varphi'$ is of one of the following forms: $t_1 = t_2$, $Ft$, $Rt_1t_2...t_n$, $t_1Ht_2$. Since $\varphi'$ is exactly like $\varphi$ except that every free occurrence of $u$ in $\varphi$ is uniformly replaced by a name in $\varphi'$, $\varphi$ is also of one of these forms. It follows that $t$ is substitutable for $v$ in $\varphi$.

**Induction Step**

Suppose $\varphi'$ is of the form $(\neg \alpha')$ and $t$ is substitutable for $v$ in $\alpha'$. Note that $\varphi$ is of the form $(\neg \alpha)$, where $\alpha'$ is a sentence that results from uniformly replacing every free occurrence of $u$ in $\alpha$ with a name. It follows from the inductive hypothesis that $t$ is substitutable for $v$ in $\alpha$. It follows that $t$ is substitutable for $v$ in $\varphi$.  

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Suppose $\varphi'$ is of the form $(\alpha' \Box \beta')$ (where ‘$\Box$’ stands in for one of the binary truth functional connectives), $t$ is substitutable for $v$ in $\alpha'$, and $t$ is substitutable for $v$ in $\beta'$.

Note that $\varphi$ is of the form $(\alpha \Box \beta)$. Note furthermore that $\alpha'$ is a sentence that results from uniformly replacing every free occurrence of $u$ in $\alpha$ with a name, and $\beta'$ is a sentence that results from uniformly replacing every free occurrence of $u$ in $\beta$ with a name. It follows from the inductive hypothesis that $t$ is substitutable for $v$ in $\alpha$ and that $t$ is substitutable for $v$ in $\beta$. It follows that $t$ is substitutable for $v$ in $\varphi$.

Suppose $\varphi'$ is of the form $(\exists x)(\alpha')$ and either $v$ is not free in $\varphi'$ or $t$ is not $x$ and $t$ is substitutable for $v$ in $\alpha'$. Suppose first that $v$ is not free in $\varphi'$. Since $v$ is distinct from $u$, and since $v$ is not free in $\varphi'$, it follows that $v$ is not free in $\varphi$. It follows that $t$ is substitutable for $v$ in $\varphi$. Suppose instead that $t$ is not $x$ and $t$ is substitutable for $v$ in $\alpha'$. Either $u$ is $x$ or it is not. Suppose first that $u$ is $x$. In that case, $u$ is not free in $\varphi$. So $\varphi$ is just $\varphi'$. It follows that $t$ is substitutable for $v$ in $\varphi$. Suppose instead that $u$ is not $x$. In that case, $\varphi$ is of the form $(\exists x)(\alpha)$, where $\alpha'$ is a sentence that results from uniformly replacing every free occurrence of $u$ in $\alpha$ with a name. It follows from the inductive hypothesis, furthermore, that $t$ is substitutable for $v$ in $\alpha$. Since $t$ is not $x$, it follows that $t$ is substitutable for $v$ in $\varphi$. In any case then, $t$ is substitutable for $v$ in $\varphi$.

(B.2.b) A Proof of the Second Substitution Lemma

Let $\varphi$ be a sentence, $u$ a term, and $v$ a variable. Suppose that $u$ is substitutable for $v$ in $\varphi$. Let $\varphi'$ be a sentence that results from uniformly replacing each free variable other than $v$ that occurs in $\varphi$ with a name.

Base Case
Suppose \( \varphi \) is of one of the following forms: \( t_1 = t_2, Ft, Rt_1t_2...t_n, t_1Ht_2 \). Since \( \varphi' \) is a sentence that results from uniformly replacing each free variable other than \( v \) that occurs in \( \varphi \) with a name, \( \varphi' \) is also of one of those forms. It follows that \( u \) is substitutable for \( v \) in \( \varphi' \).

**Induction Step**

Suppose \( \varphi \) is of the form \( \neg \alpha \) and \( u \) is substitutable for \( v \) in \( \alpha \). Note that \( \varphi' \) is of the form \( \neg \alpha' \), where \( \alpha' \) is a sentence that results from uniformly replacing each free variable other than \( v \) that occurs in \( \alpha \) with a name. By the inductive hypothesis, furthermore, \( u \) is substitutable for \( v \) in \( \alpha' \). So \( u \) is substitutable for \( v \) in \( \varphi' \).

Suppose \( \varphi \) is of the form \( \alpha \square \beta \) (where \( \square \) stands in for one of the binary truth functional connectives), \( u \) is substitutable for \( v \) in \( \alpha \), and \( u \) is substitutable for \( v \) in \( \beta \). Note that \( \varphi' \) is of the form \( \alpha' \square \beta' \), where \( \alpha' \) is a sentence that results from uniformly replacing each free variable other than \( v \) that occurs in \( \alpha \) with a name, and \( \beta' \) is a sentence that results from uniformly replacing each free variable other than \( v \) that occurs in \( \beta \) with a name. By the inductive hypothesis, furthermore, \( u \) is substitutable for \( v \) in \( \alpha' \) and \( u \) is substitutable for \( v \) in \( \beta' \). So \( u \) is substitutable for \( v \) in \( \varphi' \).

Suppose \( \varphi \) is of the form \( \exists x (\alpha) \) and either \( v \) is not free in \( \varphi \) or \( u \) is not \( x \) and \( u \) is substitutable for \( v \) in \( \alpha \). Assume first that \( v \) is not free in \( \varphi \). In that case, \( \varphi' \) is of the form \( \exists x (\alpha') \), where \( \alpha' \) is a sentence that results from uniformly replacing each free variable other than \( x \) that occurs in \( \alpha \) with a name. Since \( v \) is not free in \( \varphi \), furthermore, it is also not free in \( \varphi' \). So \( u \) is substitutable for \( v \) in \( \varphi' \). Assume instead that \( u \) is not \( x \) and \( u \) is substitutable for \( v \) in \( \alpha \). Now either \( x \) is not free in \( \alpha \) or it is. Suppose first that \( x \) is not
free in $\alpha$. In that case, $\varphi'$ is of the form $(\exists x)(\alpha''')$, where $\alpha'''$ is a sentence in which every free variable in $\alpha$ other than $v$ has been uniformly replaced by a name. It follows from the inductive hypothesis that $u$ is substitutable for $v$ in $\alpha''$. So it follows that $u$ is substitutable for $v$ in $\varphi'$. Suppose then that $x$ is free in $\alpha$. In that case, $\varphi'$ is of the form $(\exists x)(\alpha''')$, where $\alpha'''$ is a sentence in which every free variable in $\alpha$ other than $x$ and $v$ has been uniformly replaced by a name. Now either $v$ is $x$ or $v$ is some other variable. Suppose first that $v$ is $x$. In that case, $v$ is not free in $\varphi'$. So it follows that $u$ is substitutable for $v$ in $\varphi'$. Suppose instead that $v$ is not $x$. Now let $\alpha'''$ be a sentence in which every free variable other than $v$ in $\alpha'''$ is uniformly replaced by a name. Note that $\alpha'''$ is also a sentence in which every free variable in $\alpha$ other than $v$ has been uniformly replaced by a name. So it follows from the inductive hypothesis that $u$ is substitutable for $v$ in $\alpha'''$. Since, furthermore, $\alpha'''$ is a sentence exactly like $\alpha'''$ except that every free occurrence of $x$ in $\alpha'''$ is uniformly replaced by a name in $\alpha'''$, and since $u$ is not $x$, and since $v$ is also distinct from $x$, it follows from the First Substitution Lemma that $u$ is substitutable for $v$ in $\alpha'''$. So it follows that $u$ is substitutable for $v$ in $\varphi'$. In any case, then $u$ is substitutable for $v$ in $\varphi'$.

**(B.2.c) A Proof of the Third Substitution Lemma**

Let $v$ and $u$ be variables, and let $\varphi$ and $\varphi[v/u]$ be sentences, such that $\varphi[v/u]$ is the result of uniformly replacing each free occurrence of $v$ in $\varphi$ with $u$, and such that $u$ is substitutable for $v$ in $\varphi$. 

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Base Case

Suppose $\varphi$ is of one of the following forms: $t_1 = t_2$, $Ft$, $Rt_1t_2\ldots t_n$, $t_1Ht_2$. It is obvious that in that case, $\varphi[v/u]$ is a sentence exactly like $\varphi$ except that in every place that a free occurrence of $v$ occurs in the latter, a free occurrence of $u$ occurs in the former.

Induction Step

Suppose $\varphi$ is of the form $(\neg\alpha)$ and $u$ is substitutable for $v$ in $\alpha$. Let $\alpha[v/u]$ be the result of uniformly replacing each free occurrence of $v$ in $\alpha$ with $u$. It follows from the inductive hypothesis that $\alpha[v/u]$ is a sentence exactly like $\alpha$ except that in every place that a free occurrence of $v$ occurs in the latter, a free occurrence of $u$ occurs in the former. It further follows that $\varphi[v/u]$ is a sentence exactly like $\varphi$ except that in every place that a free occurrence of $v$ occurs in the latter, a free occurrence of $u$ occurs in the former.

Suppose $\varphi$ is of the form $(\alpha \mathbin{\square} \beta)$ (where ‘$\mathbin{\square}$’ stands in for one of the binary truth functional connectives), $u$ is substitutable for $v$ in $\alpha$, and $u$ is substitutable for $v$ in $\beta$. Let $\alpha[v/u]$ be the result of uniformly replacing each free occurrence of $v$ in $\alpha$ with $u$. Let $\beta[v/u]$ be the result of uniformly replacing each free occurrence of $v$ in $\beta$ with $u$. It follows from the inductive hypothesis that $\alpha[v/u]$ is a sentence exactly like $\alpha$ except that in every place that a free occurrence of $v$ occurs in the latter, a free occurrence of $u$ occurs in the former. It also follows from the inductive hypothesis that $\beta[v/u]$ is a sentence exactly like $\beta$ except that in every place that a free occurrence of $v$ occurs in the latter, a free occurrence of $u$ occurs in the former. It further follows that $\varphi[v/u]$ is a sentence exactly like $\varphi$ except that in every place that a free occurrence of $v$ occurs in the latter, a free occurrence of $u$ occurs in the former.
Suppose $\phi$ is of the form $(\exists x)(\alpha)$ and either $v$ is not free in $\phi$ or $u$ is not $x$ and $u$ is substitutable for $v$ in $\alpha$. Now suppose first that $v$ is not free $\phi$. In that case, $\phi[v/u]$ is just $\phi$. So it trivially follows that $\phi[v/u]$ is a sentence exactly like $\phi$ except that in every place where a free occurrence of $v$ occurs in the latter, a free occurrence of $u$ occurs in the former.

Suppose instead that $u$ is not $x$ and $u$ is substitutable for $v$ in $\alpha$. Either $v$ is $x$ or $v$ is some other variable. Suppose that $v$ is $x$. In that case, $v$ is not free in $\phi$ and the previous reasoning establishes that $\phi[v/u]$ is a sentence exactly like $\phi$ except that in every place where a free occurrence of $v$ occurs in the latter, a free occurrence of $u$ occurs in the former.

So suppose instead that $v$ is not $x$. Let $\alpha[v/u]$ be the result of uniformly replacing each free occurrence of $v$ in $\alpha$ with $u$. Note that since $v$ is not $x$, $\phi[v/u]$ is the sentence $(\exists x)(\alpha[v/u])$. Since $u$ is substitutable for $v$ in $\alpha$, it follows from the inductive hypothesis that $\alpha[v/u]$ is a sentence exactly like $\alpha$ except that in every place that a free occurrence of $v$ occurs in the latter, a free occurrence of $u$ occurs in the former. Since $u$ is not $x$, it further follows that $\phi[v/u]$ is a sentence exactly like $\phi$ except that in every place that a free occurrence of $v$ occurs in the latter, a free occurrence of $u$ occurs in the former. In any case then, $\phi[v/u]$ is a sentence exactly like $\phi$ except that in every place that a free occurrence of $v$ occurs in the latter, a free occurrence of $u$ occurs in the former.

B.3 The Quantifier Axioms are Nominalistically Undergirded

It is to be shown that

For any sentence of the object language, $\psi^*$, if $\psi^*$ is a quantifier axiom, $\psi^*$ is nominalistically undergirded.
Suppose \( \psi \) is of the form \( \sim \exists x (\sim \varphi) \rightarrow \varphi[x/t] \) (where \( x \) is a variable, \( t \) is a term, \( \varphi[x/t] \) is the result of replacing every free occurrence of \( x \) in \( \varphi \) with \( t \), and \( t \) is substitutable for \( x \) in \( \varphi \)). Let \( \sigma \) be a corresponding closed sentence for \( \psi \). Either \( t \) is a variable or \( t \) is a name. Suppose that \( t \) is a variable. In that case, \( \sigma \) is of the form \\
\( \sim \exists x (\sim \varphi') \rightarrow \varphi'[t/n] \), where \( \varphi' \) is a sentence that results from uniformly replacing every free variable in \( \varphi \) other than \( x \) with a name, \( \varphi'[x/t] \) is a sentence that is exactly like \( \varphi' \) except that every free occurrence of \( x \) in \( \varphi' \) is uniformly replaced by an occurrence of \( t \) in \( \varphi'[x/t] \), and \( \varphi'[t/n] \) is a sentence in which all free occurrences of \( t \) in \( \varphi'[x/t] \) are replaced by \( n \), where \( n \) is a name that replaces in \( \varphi' \) any free occurrence of \( t \) that occurred in \( \varphi \). It follows from the Second Substitution Lemma that \( t \) is substitutable for \( x \) in \( \varphi' \). It thereby follows from the Third Substitution Lemma that \( \varphi'[x/t] \) is a sentence exactly like \( \varphi' \) except that in every place that a free occurrence of \( x \) occurs in the latter, a free occurrence of \( t \) occurs in the former. It follows that \( \varphi'[t/n] \) is a corresponding closed sentence for \( \varphi' \). Now either \( \exists x (\sim \varphi') \) is nominalistically undergirded or it is not. Suppose first that it is. In that case \( \sim \exists x (\sim \varphi') \) is not nominalistically undergirded and so \( \sigma \) is nominalistically undergirded. Suppose instead that \( \exists x (\sim \varphi') \) is not nominalistically undergirded. In that case, there is no corresponding closed sentence for \( \sim \varphi' \) that is nominalistically undergirded. It follows that every corresponding closed sentence for \( \varphi' \) is nominalistically undergirded and therefore that \( \varphi'[t/n] \) is nominalistically undergirded. It follows that \( \sigma \) is nominalistically undergirded. So either way, \( \sigma \) is nominalistically undergirded. It follows by generalization from the arbitrary case that every corresponding closed sentence for \( \psi \) is nominalistically undergirded and therefore that \( \psi \)
is nominalistically undergirded. Now suppose instead that \( t \) is a name. In that case \( \sigma \) is of the form \( \sim \exists x (\lnot \varphi') \rightarrow \varphi'[x/t] \), where \( \varphi' \) is a sentence that results from uniformly replacing each free occurrence of every variable in \( \varphi \) other than \( x \) with a name, and \( \varphi'[x/t] \) is the result of uniformly replacing each free occurrence of \( x \) in \( \varphi' \) with \( t \). Note that since \( t \) is a name, \( \varphi'[x/t] \) is a corresponding closed sentence for \( \varphi' \). So it follows by the reasoning given under the previous suppositions that \( \sigma \) is nominalistically undergirded, by generalization from the arbitrary case that every corresponding closed sentence for \( \psi \) is nominalistically undergirded, and therefore that \( \psi \) is nominalistically undergirded. In any case, then, \( \psi \) is nominalistically undergirded.

Suppose \( \psi \) is of the form \( \varphi[x/t] \rightarrow \exists x (\varphi) \) (where \( x \) is a variable, \( t \) is a term, \( \varphi[x/t] \) is the result of replacing every free occurrence of \( x \) in \( \varphi \) with \( t \), and \( t \) is substitutable for \( x \) in \( \varphi \)). Either \( t \) is a variable or \( t \) is a name. Suppose that \( t \) is a variable. In that case, \( \sigma \) is of the form \( \varphi'[t/n] \rightarrow \exists x (\varphi') \), where \( \varphi' \) is a sentence that results from uniformly replacing every free variable in \( \varphi \) other than \( x \) with a name, \( \varphi'[x/t] \) is a sentence that is exactly like \( \varphi' \) except that every free occurrence of \( x \) in \( \varphi' \) is uniformly replaced by an occurrence of \( t \) in \( \varphi'[x/t] \), and \( \varphi'[t/n] \) is a sentence in which all free occurrences of \( t \) in \( \varphi'[x/t] \) are replaced by \( n \), where \( n \) is a name that replaces in \( \varphi' \) any free occurrence of \( t \) that occurred in \( \varphi \). It follows from the Second Substitution Lemma that \( t \) is substitutable for \( x \) in \( \varphi' \). It thereby follows from the Third Substitution Lemma that \( \varphi'[x/t] \) is a sentence exactly like \( \varphi' \) except that in every place that a free occurrence of \( x \) occurs in the latter, a free occurrence of \( t \) occurs in the former. It follows that \( \varphi'[t/n] \) is a corresponding closed sentence for \( \varphi' \). Now either \( \exists x (\varphi') \) is nominalistically undergirded or it is not. Suppose
first that it is. In that case, it follows straightaway that σ is nominalistically undergirded.

So suppose instead that ∃x(φ') is not nominalistically undergirded. It follows that there is no corresponding closed sentence for φ’ that is nominalistically undergirded. It follows, therefore, that φ’[t/n] is not nominalistically undergirded. It follows that σ is nominalistically undergirded. So either way, σ is nominalistically undergirded. It follows by generalization from the arbitrary case that every corresponding closed sentence for ψ is nominalistically undergirded and therefore that ψ is nominalistically undergirded. Now suppose instead that t is a name. In that case σ is of the form φ’[x/t] → ∃x(φ’), where φ’ is a sentence that results from uniformly replacing each free occurrence of every variable in φ other than x with a name, and φ’[x/t] is the result of uniformly replacing each free occurrence of x in φ’ with t. Note that since t is a name, φ’[x/t] is a corresponding closed sentence for φ’. So it follows by the reasoning given under the previous suppositions that σ is nominalistically undergirded, by generalization from the arbitrary case that every corresponding closed sentence for ψ is nominalistically undergirded, and therefore that ψ is nominalistically undergirded. In any case, then, ψ is nominalistically undergirded.

**B.4 A Proof that the Inference Rules Preserve Nominalistic Undergirding**

It is to be shown that

For any sentences ψ*1, ψ*2, ..., ψ*n, φ*, if, according to one of the inference rules, φ* may be inferred from ψ*1, ψ*2, ..., ψ*n via a single application of that rule, then if ψ*1, ψ*2, ..., ψ*n are each nominalistically undergirded, φ* is nominalistically undergirded.
Suppose \( \varphi \) is a sentence that may be inferred from any or no nominalistically undergirded sentences because \( \varphi \) is an axiom. It follows from the fact that the axioms are nominalistically undergirded that \( \varphi \) is nominalistically undergirded.

Now suppose instead that \( \psi_1, \psi_2, \ldots, \psi_n, \varphi \) are some sentences such that \( \varphi \) is a propositional consequence of \( \psi_1, \psi_2, \ldots, \psi_n \). Assume that each of \( \psi_1, \psi_2, \ldots, \psi_n \) is nominalistically undergirded. Let \( \sigma \) be the sentence \( \psi_1 \& \psi_2 \& \ldots \& \psi_n \rightarrow \varphi \). Let \( \sigma_p \) be the propositional correlate of \( \sigma \). It follows that \( \sigma_p \) is a truth functional tautology. Now let \( \psi'_1, \psi'_2, \ldots, \psi'_n, \varphi' \) be corresponding closed sentences for \( \psi_1, \psi_2, \ldots, \psi_n, \varphi \) respectively. Note that since each of \( \psi_1, \psi_2, \ldots, \psi_n \) is nominalistically undergirded, it must be the case that every sentence that is a corresponding closed sentence for one of \( \psi_1, \psi_2, \ldots, \psi_n \) is nominalistically undergirded as well. It also follows from this and the definition of nominalistic undergirding that \( \psi'_1 \& \psi'_2 \& \ldots \& \psi'_n \) is nominalistically undergirded. Now let \( \sigma' \) be the sentence \( \psi'_1 \& \psi'_2 \& \ldots \& \psi'_n \rightarrow \varphi' \). Note that given what it is for one sentence to be a propositional correlate of another, the propositional variables that occur in \( \sigma_p \) may be regarded as placeholders for the subsentences of \( \sigma \) that they were used to replace.

Note furthermore that \( \sigma' \) is exactly like \( \sigma \) except (perhaps) that some of the free variables in the latter are uniformly replaced by names in the former. It follows that each of the subsentences in \( \sigma \), for which the propositional variables that occur in \( \sigma_p \) may be regarded as placeholders, have counterpart subsentences in \( \sigma' \), and that the latter subsentences are corresponding closed sentences for the former. It also follows, therefore, that each occurrence of a propositional variable in \( \sigma_p \) may be regarded as a placeholder, not only for the occurrence of the original subsentence that it was used to replace in \( \sigma \), but for the occurrence of the counterpart corresponding closed sentence to that subsentence that
occurs in σ’. Note furthermore that, upon inspection of the recursive definition of what it is for a closed sentence in the object language to be nominalistically undergirded, it is evident that nominalistic undergirding for closed sentences of the object language is formally related to the truth functional connectives in exactly the same way that truth for propositional variables is so related. It follows, therefore, from the fact that σ_p is a truth functional tautology that σ’ is nominalistically undergirded. It also follows from the definition of nominalistic undergirding that if σ’ is nominalistically undergirded, then either ψ’_1 & ψ’_2 & … ψ’_n is not nominalistically undergirded or φ’ is nominalistically undergirded. And since it has already been established that ψ’_1 & ψ’_2 & … ψ’_n is nominalistically undergirded, it follows that φ’ is nominalistically undergirded. By generalization from the arbitrary case, it follows whenever each of the free variables in the sentences ψ_1, ψ_2, …, ψ_n, φ are replaced uniformly by a name throughout all of these sentences, each of the resulting corresponding closed sentences are nominalistically undergirded. But it is a straightforward consequence of this fact that every corresponding closed sentence for φ is nominalistically undergirded. So it follows that φ is nominalistically undergirded.

Suppose φ is a sentence of the form β → ~∃x(¬α) and that ψ is a sentence of the form β → α and that x is not free in β. Assume that ψ is nominalistically undergirded. It follows that every corresponding closed sentence for ψ is nominalistically undergirded. Let ψ’ be such a corresponding closed sentence for ψ. Note that ψ’ is of the form β’ → α’ where α’ and β’ are corresponding closed sentences for α and β respectively, and where each free variable that occurs in α and β has been uniformly replaced with a name throughout each of these sentences. Now let α” be a sentence that is exactly like α’
except that in every place in which x in α is replaced by a name in α’, x occurs in α’’.

Now either ∃x(¬α’’) is not nominalistically undergirded or it is. Suppose first that it is not. In that case, it follows that ¬∃x(¬α’’) is nominalistically undergirded and therefore that β’ → ¬∃x(¬α’’) is nominalistically undergirded. So suppose instead that ∃x(¬α’’) is nominalistically undergirded. In that case there must be some sentence, α*’’’, such that α*’’’ is a sentence that results from uniformly replacing every free occurrence of x in α’’ with a name and such that ¬α*’’’ is nominalistically undergirded. Let α’’’ be such a sentence. It follows that α’’’ is not nominalistically undergirded. Now consider the sentence β’ → α’’’. Note that since x is not free in β, this sentence is also a corresponding closed sentence for ψ. Thus, this sentence must also be nominalistically undergirded. And since α’’’ is not nominalistically undergirded, it must be the case that β’ is also not nominalistically undergirded. It follows that β’ → ¬∃x(¬α’’) is nominalistically undergirded. In any case, therefore, β’ → ¬∃x(¬α’’) is nominalistically undergirded. It follows by generalization from the arbitrary case that for any corresponding closed sentence for ψ of the form β*’ → α*’ (where α*’ and β*’ are corresponding closed sentences for α and β respectively), the corresponding sentence of the form β*’’ → ¬∃x(¬α*’’) (where α*’’ is a sentence exactly like α*’ except that in every place in which x in α is replaced by a name in α*, x occurs in α*’’) is also nominalistically undergirded. But it follows from this that every corresponding closed sentence for φ is nominalistically undergirded. It follows therefore that φ is nominalistically undergirded.
Suppose $\varphi$ is a sentence of the form $\exists x (\alpha) \rightarrow \beta$ and $\psi$ is a sentence of the form $\alpha \rightarrow \beta$ and suppose that $x$ is not free in $\beta$. Assume that $\psi$ is nominalistically undergirded. It follows that every corresponding closed sentence for $\psi$ is nominalistically undergirded. Let $\psi'$ be such a corresponding closed sentence for $\psi$. Note that $\psi'$ is of the form $\alpha' \rightarrow \beta'$, where $\alpha'$ and $\beta'$ are corresponding closed sentences for $\alpha$ and $\beta$ respectively and where each free variable that occurs in both $\alpha$ and $\beta$ has been uniformly replaced with a name throughout each of these sentences. Let $\alpha''$ be a sentence that is exactly like $\alpha'$ except that in every place in which $x$ in $\alpha$ is replaced by a name in $\alpha'$, $x$ occurs in $\alpha''$. Now either $\exists x (\alpha'')$ is not nominalistically undergirded or it is. Suppose first that it is not. In that case it follows straightforwardly that $\exists x (\alpha'') \rightarrow \beta'$ is nominalistically undergirded.

So suppose instead that $\exists x (\alpha'')$ is nominalistically undergirded. In that case there must be some sentence, $\alpha'''$, such that $\alpha'''$ is a sentence that results from uniformly replacing every free occurrence of $x$ in $\alpha''$ with a name and such that $\alpha'''$ is nominalistically undergirded. Let $\alpha'''$ be such a sentence. Now consider the sentence $\alpha''' \rightarrow \beta'$. Note that since $x$ is not free in $\beta$, this sentence is also a corresponding closed sentence for $\psi$. Thus, this sentence must also be nominalistically undergirded. And since $\alpha'''$ is nominalistically undergirded, it must be the case that $\beta'$ is nominalistically undergirded. It follows therefore that the sentence $\exists x (\alpha''') \rightarrow \beta'$ is nominalistically undergirded. In any case, therefore, $\exists x (\alpha''') \rightarrow \beta'$ is nominalistically undergirded. It follows by generalization from the arbitrary case that for any corresponding closed sentence for $\psi$ of the form $\alpha^* \rightarrow \beta^*$ (where $\alpha^*$ and $\beta^*$ are corresponding closed sentences for $\alpha$ and $\beta$ respectively), the corresponding sentence of the form $\exists x (\alpha'''') \rightarrow \beta'''$ (where $\alpha'''$ is a sentence exactly
like $\alpha^*$ except that in every place in which $x$ in $\alpha$ is replaced by a name in $\alpha^*$, $x$ occurs in $\alpha^{*'}$) is also nominalistically undergirded. But it follows from this that every corresponding closed sentence for $\varphi$ is nominalistically undergirded. It follows therefore that $\varphi$ is nominalistically undergirded.

**B.5 A Proof of the Correctness Preservation Theorem by Mathematical Induction**

It is to be shown that

For any sentences of the object language, $\psi_1^*, \psi_2^*, \ldots, \psi_n^*, \varphi^*$, if each of the sentences $\psi_1^*, \psi_2^*, \ldots, \psi_n^*$ is nominalistically undergirded and, by one or more applications of the inference rules, $\varphi^*$ may be inferred from $\psi_1^*, \psi_2^*, \ldots, \psi_n^*$, then $\varphi^*$ is nominalistically undergirded.

This will be established via proof by mathematical induction on the number of applications of inference rules.

Let $\psi_1, \psi_2, \ldots, \psi_n, \varphi$ be some sentences such that, by one or more applications of the inference rules, $\varphi$ may be inferred from $\psi_1, \psi_2, \ldots, \psi_n$ and such that each of $\psi_1, \psi_2, \ldots, \psi_n$ is nominalistically undergirded. (For convenience, let’s ignore the case in which $\varphi$ is inferred from no sentences via one or more applications of the inference rules; the following proof applies, *mutatis mutandis*, to that case as well).

**Base Case:** Suppose $\varphi$ may be inferred from $\psi_1, \psi_2, \ldots, \psi_n$ by a single application of an inference rule. From the fact that the inference rules preserve nominalistic undergirding, it follows that $\varphi$ is nominalistically undergirded.
**Induction Step:** Suppose $\varphi$ may be inferred from $\psi_1, \psi_2, \ldots, \psi_n$ by $m$ applications of the inference rules (where ‘$m$’ is some arbitrarily chosen numeral that corresponds to a positive integer). Consider the last step of a $m$-step proof in which $\varphi$ is inferred from $\psi_1, \psi_2, \ldots, \psi_n$. That step will consist in the inferring of $\varphi$ from some sentences $\sigma_1^*, \sigma_2^*, \ldots, \sigma_k^*$ by a single application of an inference rule, where each of $\sigma_1^*, \sigma_2^*, \ldots, \sigma_k^*$ was inferred by less than $m$ applications of the inference rules from $\psi_1, \psi_2, \ldots, \psi_n$. Let $\sigma_1, \sigma_2, \ldots, \sigma_k$ be the sentences from which $\varphi$ is inferred at this last step. By the inductive hypothesis, it follows that each of $\sigma_1, \sigma_2, \ldots, \sigma_k$ is nominalistically undergirded. By the fact that the inference rules preserve nominalistic undergirding, it follows that $\varphi$ is nominalistically undergirded.
APPENDIX C:
A PROOF OF THE CONSERVATION LEMMA

It is to be shown that

For each sentence of the object language, $\varphi^*$, such that $\varphi^*$ is a closed, quantifier free sentence that contains no p-names, $\varphi^*$ is nominalistically undergirded if and only if $\varphi^*$ is true.

Since any closed, quantifier free sentence of the object language is composed of Boolean combinations of basic sentences (recall that a basic sentence is a closed, atomic sentence, or the negation thereof), and since nominalistic undergirding for closed sentences is formally related to the truth functional connectives in the same manner as is truth, it suffices to prove the above to show that it is true of each basic sentence of the object language that contains no p-names.

Assume that $\varphi$ is a basic sentence of the object language that contains no p-names. $\varphi$ is thus of one of the following forms: $t_1 = t_2$, $(\neg t_1 = t_2)$, $Ft$, $(\neg Ft)$, $Rt_1t_2...t_n$ (where $n > 1$), $(\neg Rt_1t_2...t_n)$ (where $n > 1$), $t_1Ht_2$, $(\neg t_1Ht_2)$. And since $\varphi$ is closed and contains no p-names, each term that occurs in $\varphi$ is an n-name.

Suppose $\varphi$ is of the form $t_1 = t_2$ or of the form $(\neg t_1 = t_2)$. Since $t_1$ and $t_2$ are both n-names, the nominalistic surrogate for $\varphi$ is just $\varphi$ itself. So $\varphi$ is nominalistically undergirded if and only if $\varphi$ is true.

Suppose that $\varphi$ is of the form $Ft$. Either F is a p-predicate or F is an n-predicate. Suppose that F is a p-predicate. In that case, since t is an n-name, the nominalistic
surrogate for \( \varphi \) is \( \bot \), and so \( \varphi \) is not nominalistically undergirded. Furthermore, since no nominally acceptable object can satisfy a p-predicate, it is not the case that \( \varphi \) is true. So \( \varphi \) is neither true nor nominally undergirded. So, trivially, \( \varphi \) is nominalistically undergirded if and only if \( \varphi \) is true. Suppose, on the other hand, that \( F \) is an n-predicate. In that case, the nominalistic surrogate for \( \varphi \) is \( \varphi \) itself. So \( \varphi \) is nominalistically undergirded if and only if \( \varphi \) is true.

Suppose that \( \varphi \) is of the form \( \neg Ft \). Either \( F \) is a p-predicate or \( F \) is an n-predicate. Suppose that \( F \) is a p-predicate. In that case, since \( t \) is an n-name, the nominalistic surrogate for \( \varphi \) is \( \neg \), and so \( \varphi \) is nominalistically undergirded. Furthermore, since no nominally acceptable object can satisfy a p-predicate, \( \varphi \) is true. Thus, \( \varphi \) is both true and nominalistically undergirded. So, trivially, \( \varphi \) is nominalistically undergirded if and only if \( \varphi \) is true. Suppose, on the other hand, that \( F \) is an n-predicate. In that case, the nominalistic surrogate for \( \varphi \) is \( \varphi \) itself. So \( \varphi \) is nominalistically undergirded if and only if \( \varphi \) is true.

Suppose that \( \varphi \) is of the form \( Rt_1 t_2 \ldots t_n \) (where \( n > 1 \)) or of the form \( \neg Rt_1 t_2 \ldots t_n \) (where \( n > 1 \)). Since \( t_1 \ldots t_n \) are n-names, the nominalistic surrogate for \( \varphi \) is \( \varphi \) itself. So \( \varphi \) is nominalistically undergirded if and only if \( \varphi \) is true.

Suppose \( \varphi \) is of the form \( t_1 H t_2 \). Since \( t_2 \) (along with \( t_1 \)) is an n-name, the nominalistic surrogate of for \( \varphi \) is \( \bot \), and so \( \varphi \) is not nominalistically undergirded. Furthermore, since no nominally acceptable object can be had (in the sense in which properties are had), it is not the case that \( \varphi \) is true. So \( \varphi \) is neither true nor
nominalistically undergirded. So, trivially, \( \varphi \) is nominalistically undergirded if and only if \( \varphi \) is true.

Suppose \( \varphi \) is of the form \((\neg t_1 H t_2)\). Since \( t_2 \) (along with \( t_1 \)) is an n-name, the nominalistic surrogate for \( \varphi \) is \((\neg \bot)\), and so \( \varphi \) is nominalistically undergirded. Furthermore, since no nominalistically acceptable object can be had (in the sense in which properties are had), \( \varphi \) is true. Thus, \( \varphi \) is both true and nominalistically undergirded. So, trivially, \( \varphi \) is nominalistically undergirded if and only if \( \varphi \) is true.
APPENDIX D:  
THE CHAPTER 3 APPARATUS AND ITERATIVE SET THEORY

In this appendix, I illustrate some of the points made in Section 4.1 by showing how the apparatus of Chapter 3 can be extended so as to apply to iterative set theory.\footnote{For the inspiration behind the general strategy that I employ in this appendix, as well as for some of the specific moves that I make in executing it, I am indebted to Bigelow’s (1988, chapters 15-16) attempt to show that iterative set theory can be reduced to a theory of universals.}

Let’s begin our expansion of the apparatus of Chapter 3 to encompass iterative set theory by saying that a name is “in the nominal extension” of a unary, atomic predicate if and only if the atomic sentence that results from combining that name with that predicate is nominalistically undergirded. Let’s also say that for any given names of the object language, a predicate may be said to “nominally collect” those names if and only if all and only those names are in its nominal extension. With these definitions in hand, we may say, for example, that there is a predicate in the object language that nominally collects the names of human beings, provided that there is a predicate in the object language that has all and only the names of individual human beings in its nominal extension (remember that the object language is to be thought of as having a name for each concrete thing) – provided that is, that every atomic sentence that results from combining that predicate with one of those names is nominalistically undergirded. And in this case, given the expressive resources that the object language was stipulated to have, there clearly will be such a predicate (the atomic predicate that translates ‘is
human’ would be one example; perhaps the atomic predicate that translates ‘is a featherless biped’ is another).

Suppose then that F is one of the predicates of the object language that nominally collects the names in the object language that refer to individual human beings. We could also, if we like, stipulatively introduce another name into the object language (one not found there previously), a name that we might refer to as “the s-projection” of F (and we could also say that F “s-projects” that name). The s-projection of F is not to be identified with the projection of F as characterized in Chapter 3 (from now on, in this appendix, just to keep things straight, the projection of F as characterized in Chapter 3 will now be referred to “the p-projection of F”). Intuitively, the s-projection of F should be thought of as naming the set of humans, whereas the p-projection of F is to be thought of as naming a property (perhaps the property of being human, perhaps the property of being a featherless biped – it depends on just which predicate of the object language F happens to be). We may now stipulate that not only F, but every n-predicate of the object language is to be thought of as having an s-projection. We may also stipulate that s-projections are to be syntactically distinguished both from p-names and from n-names – call them “s-names”. We may further stipulate that the s-names are to be thought of as syntactically marking the predicates that s-project them. Finally, we should also stipulate that distinct s-names are to be thought of as being such that they co-refer if and only if they have the same nominal extension (thought of in this way regardless of whether they actually have referents or not).

We can also, if we like, introduce a special class of unary predicates – call them “the s-predicates,” which are to be regarded as syntactically distinguished from the n-
predicates and p-predicates of the object language. We may, if we like, introduce these predicates into the object language on an ad hoc basis, stipulating the conditions under which the basic sentences involving them are to be regarded as correct as we go along. We might introduce, for example, an s-predicate into the object language that translates ‘is a set’, and stipulate that an atomic sentence of the form $S t$, where $S$ is the predicate in question and $t$ is a name, is to be regarded as correct if and only if $t$ is an s-name.

We could also add to the object language some special purpose non-unary predicates that can be correct when combined with s-names. One that would prove especially helpful, for example, for the purposes of doing set theory (or rather, for the purposes of having the object language be capable of representing many of the arguments that we use when doing set theory) would be a binary predicate that translates ‘is a member of’. Suppose we stipulate that ‘$\in$’ is to be the symbol in the object language that stands for that predicate, and that any sentence of the form $t_1 \in t_2$, where $t_1$ and $t_2$ are names, is to count as a well formed sentence of the object language. We may also stipulate that a basic sentence of that form is to count as correct just in case $t_2$ is an s-name and the object language sentence of the form $F t_1$, where $F$ is the predicate that s-projects $t_2$, is nominalistically undergirded.

If we assume (just for the moment) that every s-name that has now (up to this point) been stipulated to be part of the object language has a referent, then we do not get the entire set-theoretic hierarchy (we do not yet have sets with sets as members), but we get all the sets of concrete objects that we could ever want. We get all those sets, at least, if we can be assured that for any concrete objects whatsoever, there is an n-predicate in the object language that nominally collects the names of those objects. And we can be
assured of this, given the stipulation that the object language lacks for no expressive resources other than those that are inconsistent with the limitations imposed upon it in Chapter 3 (in addition to the stipulation made in Chapter 3 that the atomic n-predicates of the object language are closed under definability in terms of other atomic n-predicates). I take it that for any xs, where the xs are concrete objects, it is meaningful to say of any object that it is “one of those_{xs}” (where ‘those_{xs}’ is a placeholder for a demonstrative term that plurally refers to the xs). If so, then the object language (given its vast expressive resources) will be such that for any concrete objects whatsoever, there is a predicate in the object language that nominally collects the names of those objects.

We also get one pure set under the assumption that all of the s-names of the object language have referents – namely, the empty set. Given the object language’s expressive resources (and especially given that the atomic n-predicates of the object language are closed under definability in terms of other n-predicates), some of the n-predicates of the object language will have no names in their nominal extension – the atomic predicate that translates ‘is a married bachelor’, for example. The s-projections of predicates such as these, if they refer to anything at all, refer to the empty set. We can get more of the set-theoretic hierarchy out of the assumption that all of the names we introduce into the object language have referents as follows:

First, we introduce yet another class of predicates into the object language – call them “gathering predicates.” We’ll stipulate that these predicates are to be thought of as being syntactically distinguished from the other kinds of predicates in the object language. We’ll further stipulate that each gathering predicate has at least one s-name in its nominal extension. We’ll also stipulate that these predicates are abundant in the
following way: They are such that for any xs in the object language, such that each of the xs is one of the n-names or s-names that has been introduced into the object language thus far, and such that at least one s-name is among the xs, there is a gathering predicate that nominally collects the xs.

For cases in which a gathering predicate has a finite number of names in its nominal extension, that predicate can be defined in terms of open sentences within the object language. Pretend, for example, that e is the name in the object language that is s-projected by the atomic predicate that translates ‘is a married bachelor’ and that t is the name in the object language that names the Eiffel Tower. In that case, the gathering predicate that has both e and t in its nominal extension is definable in terms of the following open sentence of the object language: \((x=t) \lor (x=e)\). That is, the result of combing that predicate with a name is to be regarded as correct if and only if replacing x in this open sentence with that same name results in a correct sentence.

Gathering predicates that have an infinite number of names in their nominal extension, however, are not definable in this way. Still, we can state, easily enough, the conditions under which an atomic sentence that results from combining such a predicate with a name is to be regarded as correct. We can say that such a sentence, a sentence of the form \(Ft\), where F is a gathering predicate and t is a name, is to be regarded as correct just in case t is in the nominal extension of F. One might worry, however, that this way of stipulating when atomic sentences involving gathering predicates are correct is objectionably circular in a manner in which the former method (that of defining such predicates in terms of other open sentences in the object language) is not. We can already state the conditions under which a corresponding closed sentence for an open
sentence like \((x=t) \lor (x=e)\) is correct, and we can do so in ways that do not already presuppose a knowledge of what the nominal extensions of any gathering predicates are. Because of that fact, such open sentences afford us with a means of supplying non-circular definitions of gathering predicates in the cases in which the nominal extensions of those predicates are finite. The current way of stating the conditions under which sentences involving those predicates are correct, however, affords us with no such advantage.

Even so, there is nothing objectionably circular about the current way of stating those conditions, not (at any rate) relative to the purposes at hand. In order to satisfy those purposes, we do not need to supply non-circular definitions for all of the gathering predicates that we’ve introduced into the object language. All that we need to do is make a stipulation that guarantees that the object language contains all of the gathering predicates that we want it to contain, and then we need a way of saying (as speakers of the meta-language) when sentences that involve those predicates are correct. We’ve already been able to do the former by stipulating that for any plurality of the names that have been stipulated to be part of the object language thus far that include at least one s-name among them, there is gathering predicate that has all and only those names in its nominal extension; that won’t give us all the gathering predicates that we will ultimately want, but it gives us enough of them for now.\(^\text{138}\) And we have a way of saying when atomic sentences that make use of these predicates are correct – namely, when they are combined with names that are in their nominal extensions.

\(^{138}\) Talk of “pluralities” of names here and elsewhere should be understood merely as a convenient device for plurally quantifying over names, not as a way of referring to additional entities (other than the names themselves) that are the pluralities of those names.
One might still worry, however. In particular, one might worry that unless our stipulations provide something like non-circular definitions for all of the gathering predicates in the object language, then at least some of those predicates are meaningless, and that this constitutes a problem of some sort. This issue didn’t arise when it came to the predicates that gave us all of the s-names for sets of concrete objects that we wanted. For any given plurality of concrete objects, we could be assured that the object language contained a meaningful predicate that had all and only the names of those objects in its nominal extension (even if all it meant was something like *one of those*). But (according to the nominalist, at any rate), the s-names in the object language are empty names. So some of the names in the nominal extensions of the gathering predicates are empty names. The infinite being could not, for that reason, introduce such a predicate into its language by gesturing to the referents of the names in its nominal extension and say that the predicate in question means *one of those*; such an attempt to define one of these predicates by means of ostentation would fail.

Even so, I’m not convinced that it’s true that the gathering predicates introduced into the object language by the above stipulation that can’t be defined in terms of open sentences in the object language are meaningless. One of the limitations of the object language is that it does not allow for infinitely long sentences. Perhaps, if it’s possible to have a language that contains infinitely long sentences, the gathering predicates of the object language could be defined in another language that lacks that limitation. Even if they cannot be so defined, furthermore, perhaps the gathering predicates of the object
language can be considered meaningful on account of the fact that they can be regarded as similar in meaning (in certain respects) to their definable cousins.\(^{139}\)

But even if those predicates are not meaningful, they could still be consistently used by someone who is capable of understanding the object language. One could imagine, if it helps, the infinite being taking up such a predicate into its vocabulary simply by starting to use it in sentences (it need not ever mention it), and by deciding, for a certain plurality of names, to regard atomic sentences involving that predicate as correct if and only if they are the result of combining that predicate with one of those names. It could do the latter, furthermore, without ever mentioning sentences or names. It could do so by simply endorsing (or quasi-endorsing) infinitely many sentences of the form \(Ft\), where \(t\) is a name and \(F\) is the predicate in question, while rejecting infinitely many other sentences of that form (i.e. by endorsing or quasi-endorsing all and only those atomic sentences that combine that predicate with what we, as the finite speakers of the meta-language, would say are part of its nominal extension).

The infinite being’s activities, in this case, would be similar to those of a child playing a game, one who makes up nonsense predicates and decides (in an entirely ad hoc manner, just making things up as she goes along) to apply those predicates to certain objects but not others (and perhaps even to endorse certain atomic sentences that combine those predicates with empty names but not endorse other such atomic sentences), with a stipulation in force that once such a predicate has been applied to a certain object, it is to

\(^{139}\) Some might consider even their definable cousins to be meaningless on account of the fact that the s-names that figure into their definitions are empty names. But I don’t think that’s right. The s-names introduced into the object language so far can all be regarded as meaning the same things as definite descriptions such as ‘the set of human beings’, and I take it that these descriptions are meaningful.
continue to be applied to that object. The result of the child’s activities in that case would be to generate a growing list of sentences that are correct according to the rules of her game – correct even if nonsensical. One could, in such a case, play along with that child and use her nonsense predicates by applying them to the same objects that she does. One could even use sentences involving those predicates to communicate truths – “That jiberjabber you’re holding right now belongs to your sister; you’d better put it away before she finds out that you have it.” In such cases, it would be the correctness of one’s usage of the predicates in question that counts (both for the purposes of rightly participating in the game as well as communicating the relevant truths), not whether the sentences that one employs by using them are meaningful. Similarly, the usefulness of introducing gathering predicates into the object language depends only on the possibility of sentences involving them having nominalistically acceptable correctness conditions, not on whether or not they are meaningful.

Now that we’ve stipulatively added these gathering predicates into the object language, we may also stipulate that each of them is to be regarded as having an s-projection. This introduces even more names into the object language. So let’s further stipulate that the object language is now to be thought of as having gathering predicates that nominally collect these additional names. I.e. Let’s further stipulate that for any of the xs, such that the xs are n-names or s-names of the object language, and such that the at least one of the s-projections of the gathering predicates that was introduced into the object language by means of the previous stipulation is among the xs, there is an additional gathering predicate that nominally collects the xs. Let it now be stipulated that these predicates are to be thought of as having s-projections. And so on, infinitely many
times. Given the assumption that every s-name that we manage to introduce into the
object language by means of our stipulations has a referent, this infinite sequence of
stipulations affords us with all the sets we could ever need (at least all of the ones that we
could ever need that can be built out of pure sets and out of concrete objects; we could
have also, if we liked, expanded this apparatus even further to include sets of properties,
but we’ll stick with what we’ve got for now).\textsuperscript{140}

Of course, since the whole point of this exercise is to exhibit that sentences
involving apparent quantification over sets have nominalistically acceptable correctness
conditions, we are not actually entitled to the assumption that the s-names that we’ve
introduced into the object language have referents. We are now, however, in a position to
specify the conditions under which sentences in this expanded version of the object
language are nominalistically undergirded.

First, we may consider cases in which we are able to assign nominalistic
surrogates to basic sentences involving s-names. If, for example, the basic sentence in
question is of the form $Ft$, where $F$ is an s-predicate and $t$ is an s-name, and the
stipulations governing the conditions under which atomic sentences involving the
predicate $F$ are to count as correct are met in this case, the sentence in question is to be
assigned ($\neg \perp$) as its nominalistic surrogate; otherwise it is to be assigned $\perp$. Likewise,$\textit{mutatis mutandis}$, for sentences of the form ($\neg Ft$), where $F$ is an s-predicate and $t$ is an s-

\textsuperscript{140} More precisely, to ensure that we do in fact get all of the sets we might need, what we should
really think of as happening here is that an initial, countably infinite sequence of stipulations is made along
the lines specified above. That sequence of stipulations constitutes “Round 1” (as it were). We should also
think of Round 1 as being followed by another infinite sequence of stipulations along the same lines. This
sequence of stipulations constitutes “Round 2.” Round 2, of course, is to be followed by Round 3, and
Round 3 by Round 4, and so on, infinitely many times.
name. Similarly, if the basic sentence in question is of the form $Ft$, where $F$ is a gathering predicate and $t$ is a name (perhaps an s-name, perhaps not), the sentence in question is to be assigned ($\neg \bot$) as its nominalistic surrogate if $t$ is in $F$’s nominal extension; otherwise it is to be assigned $\bot$. Likewise, *mutatis mutandis*, for sentences of the form $\neg Ft$, where $F$ is a gathering predicate and $t$ is a name.

If the basic sentence is of the form $t_1 \in t_2$, where $t_1$ is a name and $t_2$ is an s-name, then we may say that the “intermediate paraphrase” of this sentence is the sentence of the form $Ft_1$, where $F$ is the predicate that s-projects $t_2$. And we may say that the nominalistic surrogate for this sentence is identical to the nominalistic surrogate of its intermediate paraphrase. For example, if $F$ is an n-predicate and $t_1$ is an n-name, then the nominalistic surrogate for the sentence at issue will just be $Ft_1$ (since the nominalistic surrogate for an atomic sentence that combines an n-predicate with an n-name is just that sentence itself). If, on the other hand, $F$ is a gathering predicate and $t_1$ is a name that is in $F$’s nominal extension, then the nominalistic surrogate for the sentence at issue is $\neg \bot$.

And so on. Likewise, *mutatis mutandis*, for a basic sentence of the form $\neg t_1 \in t_2$.

If the basic sentence in question is of the form $t_1 = t_2$, where one of $t_1$ and $t_2$ is an s-name and the other is not an s-name, the nominalistic surrogate for that sentence is $\bot$. For a sentence of the form $\neg t_1 = t_2$, where one of $t_1$ and $t_2$ is an s-name and the other is not an s-name, the nominalistic surrogate for that sentences is $\neg \bot$. Basic sentences of the form $t_1 = t_2$ or of the form $\neg t_1 = t_2$, where $t_1$ and $t_2$ are both s-names, require special treatment. Call these sentences “set identity sentences.” Unlike the other basic sentences of this expanded version of the object language, set identity sentences will not be
assigned nominalistic surrogates (I will have more to say about what we will do with them shortly).

Otherwise, nominalistic surrogates are to be assigned to basic sentences in pretty much the same way that they were in Chapter 3 (extrapolating in the obvious ways from what was said in Chapter 3, as well as making the obvious adjustments when necessary). I do not believe that it is necessary to explicitly spell out the remaining details. As before, we’ll say that any basic sentence of the object language that has a true nominalistic surrogate is nominalistically undergirded. We’ll also say that any closed, quantifier free sentence of the object language that has no set identity sentences as subsentences is nominalistically undergirded just in case the sentence that results from replacing each of the basic sentences that occur within it with their nominalistic surrogates is true.

As I said above, set identity sentences – basic sentences of the form \( t_1 = t_2 \) or of the form \( \neg t_1 = t_2 \), where \( t_1 \) and \( t_2 \) are both s-names – require special treatment. We will not attempt to find nominalistic surrogates for them. Instead, we will attempt to find “replacement sentences” for them – sentences that are nominalistically undergirded under the same conditions in which the original sentences are to be regarded as correct.

We will say, for instance, that the replacement sentence for a sentence of the form \( \neg t_1 = t_2 \), where \( t_1 \) and \( t_2 \) are both s-names, is the sentence of the form \((\exists x)(((Fx&(\neg Gx))\lor (\neg Fx)&Gx))\), where \( F \) is the predicate that s-projects \( t_1 \) and \( G \) is the predicate that s-projects \( t_2 \). We’ll also say that this replacement sentence is nominalistically undergirded just in case there is a corresponding nominalistically
undergirded sentence of the form \(((F_n \& \neg G_n)) \lor ((\neg F_n \& G_n))\), where \(n\) is a name (we’ve already specified, furthermore, the conditions under which a sentence of this form is nominalistically undergirded). We’ll also want to say that a sentence of the form \(\neg t_1 = t_2\) (where \(t_1\) and \(t_2\) are both s-names) is nominalistically undergirded just in case its replacement sentence is nominalistically undergirded. Note that this condition is equivalent to the claim that a sentence of this form is correct just in case there is a name in the nominal extension of the predicate that s-projects \(t_1\) that is not in the nominal extension of the predicate that s-projects \(t_2\) or vice versa.

Similarly, we’ll want to say that the replacement sentence for a sentence of the form \(t_1 = t_2\), where \(t_1\) and \(t_2\) are both s-names, is the sentence of the form \(\neg (\exists x)(((Fx \& \neg Gx)) \lor (\neg Fx \& Gx)))\), where \(F\) is the predicate that s-projects \(t_1\) and \(G\) is the predicate that s-projects \(t_2\). And we’ll say that this replacement sentence is nominalistically undergirded just in case there is no corresponding nominalistically undergirded sentence of the form \(((F_n \& \neg G_n)) \lor ((\neg F_n \& G_n))\), where \(n\) is a name. And we’ll want to say that a sentence of the form \(t_1 = t_2\) (where \(t_1\) and \(t_2\) are both s-names) is nominalistically undergirded just in case its replacement sentence is nominalistically undergirded. That turns out, furthermore, to be equivalent to the condition that a sentence of this form is correct just in case there is no name in the nominal extension of the predicate that s-projects \(t_1\) that is not in the nominal extension of the predicate that s-projects \(t_2\) or vice versa.

The conditions under which set identity sentences and basic sentences other than set identity sentences are to count as nominalistically undergirded now afford us with the base cases for the recursive definition of nominalistic undergirding that goes along with
this expansion of the apparatus developed in Chapter 3. The inductive clauses of that
definition are just as they were before. I take it as obvious how adapt the proofs of the
Correctness Preservation Theorem given in Appendix B so as to fit with this expanded
apparatus, and that it is obvious that the adapted proof will go through in much the same
way as it did there.

A slight modification must be made, however, to the Conservation Lemma in
order for the corresponding adaptation of its proof (as given in Appendix C) to go
through. The Conservation Lemma of Chapter 3 was stated as follows:

(CL) For each sentence of the object language, $\varphi^*$, such that $\varphi^*$ is a closed,
quantifier free sentence that contains no p-names, $\varphi^*$ is nominalistically
undergirded if and only if $\varphi^*$ is true.

Of course, to adapt CL to fit the current expansion of the apparatus, we need to change
the clause that reads “contains no p-names” to “contains no p-names and no s-names”.
I’ll refer to the statement that results from making this modification as “CL*”. Even with
this modification in place, however, CL* is problematic.

The problem is that if the object language is expanded in the ways suggested
above, and if, in fact (for some of the reasons suggested above), some of the gathering
predicates of the object language (including some of the ones that form nominalistically
undergirded, atomic sentences when combined with n-names) are meaningless, there will
be counterexamples to CL*. Any atomic sentence that combines such a gathering
predicate with an n-name to form a nominalistically undergirded sentence (i.e. any
nominalistically undergirded sentence of the form $Ft$, where $F$ is such a gathering
predicate and $t$ is an n-name), would, for instance, be a counterexample to CT*. The
resulting sentence would be a nominalistically undergirded, quantifier free sentence that contains no p-names and no s-names, but it would not be true; rather, it would just be meaningless.

But this is fixed easily enough. We simply need to modify the clause in CT* that reads “contains no p-names and no s-names” to read “contains no p-names, no s-names, and no gathering predicates.” Or, if we wanted to be a little more fine grained than that, we could imagine the gathering predicates that are non-circularly definable in terms of open sentences found within the object language as being syntactically distinguished from those that are not so definable. We could call the former predicates “definable gathering predicates” and the latter predicates “undefinable gathering predicates.” And then we could modify the clause at issue to read “contains no p-names, no s-names, and no undefinable gathering predicates.” I take it as apparent that, with either of these modifications to CL* in place, an adapted version of the proof of the Conservation Lemma provided in Appendix C will go through in much the same way that it did there.

I take the above to suffice, then, to show that the apparatus of Chapter 3 can be expanded so as to encompass iterative set theory.

D.1 Works Cited

APPENDIX E:
THE CHAPTER 3 APPARATUS, RELATIONS, AND HIGHER ORDER PROPERTIES

My goal in this appendix is to provide a sketch of how the apparatus of Chapter 3 can be generalized so as to encompass much of our discourse involving apparent quantification over relations, as well as over higher order properties. My goal here is merely to provide a sketch of how this might be done. Much of what I will have to say, I admit, begs to be filled out in more detail. The topics of this appendix will also be treated separately and independently from those of the previous appendix.

First, let’s consider how the Chapter 3 apparatus might be expanded so as to handle apparent quantification over non-monadic properties (i.e. over relations). Let’s keep in place (for now) the restriction on non-unary predicates that we imposed in Chapter 3, namely that such predicates (other than the identity predicate and the property having predicate) be thought of as expressing relations that can only be entered into by concrete things. But instead of imagining the object language (as characterized in Chapter 3) as including only n-names and p-names that are projected by unary n-predicates, imagine it as including names that are projected by its non-unary predicates as well. And instead of imagining the classification scheme that accompanies the object language as only categorizing its unary n-predicates, imagine it as categorizing the object language’s non-unary predicates as well. And so on. Now call all of the predicates that apply solely to concrete objects (the original n-predicates as well as the non-unary
predicates) “n-predicates.” Continue, as before, to call the projections of the n-predicates of the object language (including the new additions) “p-names.” Let’s also add an additional stipulation, however, to the effect that the p-names now in some way syntactically mark the arity of the predicates that project them.

Now suppose that we modify the property having predicate of the object language, H, so that it functions as follows: Where σ is a sentence of the object language of the form $t_1t_2...t_nH_{t_m}$, such that $t_1$, $t_2$, ...$t_n$ are n-names and $t_m$ is the projection of an n-ary n-predicate, σ is to be read as indicating that $t_1$, $t_2$, ... $t_n$ stand in the $t_m$ relation to one another (and in that order). We are also to take the nominalistic surrogate for σ, in this case, to be $R_{t_1t_2...t_n}$, where R is the n-ary predicate that projects $t_m$. We then go about assigning all other nominalistic surrogates to basic sentences in the same sort of way (mutatis mutandis) as we did before. Modify the rest of the apparatus in Chapter 3, the axioms, proofs, and the like, in accordance with these developments. (Though I will not bother to demonstrate it, I take it as fairly evident that all of the modified proofs will continue to go through pretty much as they did before).

So far I’ve insisted that only n-predicates are to be thought of as having projections. But we can relax that constraint. We can stipulate that each p-predicate of the object language has its own projection as well, and we can thereby add the projections of p-predicates to our stock of p-names. The projections of p-predicates may be thought

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141 I don’t claim that it is entirely unproblematic to give natural English translations of these sorts of sentences. For some of the issues involved in doing so (or rather, for some issues involved in offering a formal treatment of relation names that naturally carry over as issues involved in providing natural English translations of these sorts of sentences), see (Van Inwagen 2006).

142 In cases in which a closed sentence is of this form except that $t_m$ is not the projection of an n-ary predicate (because $t_m$ is an n-name or because it is the projection of a predicate of an arity other than n), the nominalistic surrogate for that sentence is to be regarded as ⊥.
of, of course, as naming higher order monadic properties (i.e. as naming properties that may be had by properties). If we add such names to the object language, then we will also need to modify the conditions under which the sentences of the object language employing the property having predicate are nominalistically undergirded. As matters currently stand, any basic sentence of the form $pHt$, where $p$ is a $p$-name and $t$ is also a $p$-name, gets assigned $\bot$ as its nominalistic surrogate. But if we are going to allow $p$-predicates to have projections, that won’t do.

We can say this of such basic sentences instead: For any basic sentence, $\sigma^*$, of the form $pHt$, where $p$ is a $p$-name and $t$ is also a $p$-name, the “intermediate paraphrase” of $\sigma^*$ is a sentence of the form $Fp$, where $F$ is the predicate that projects $t$; and the nominalist surrogate for $\sigma^*$ is to be the nominalistic surrogate for $\sigma^*$’s intermediate paraphrase. Suppose, for example, that $G$ is the atomic, $n$-predicate of the object language that translates ‘is segmented’, that $p$ is the $p$-name projected by $G$, that $F$ is the $p$-predicate that translates ‘is anatomical’, and that $t$ is the $p$-name projected by $F$. In that case, the intermediate paraphrase of the sentence $pHt$ is the sentence $Fp$ (the latter being the object language sentence that translates ‘Being segmented is anatomical’). Since the nominalistic surrogate for $Fp$ in that case is ($\neg\bot$), the nominalistic surrogate for $pHt$ is also ($\neg\bot$). Likewise, the intermediate paraphrase for the sentence $tHp$ is $Gt$ (the latter being the object language sentence that translates ‘Being anatomical is segmented’). Since $Gt$ is a basic sentence that results from combining a $p$-name with an $n$-predicate, its nominalistic surrogate is $\bot$, and so the nominalistic surrogate for $tHp$ is likewise $\bot$.

For a slightly more tricky example, consider (keeping the above suppositions intact) the sentence $tHt$. The intermediate paraphrase for this sentence is the sentence $Ft$
(the latter being the object language sentence that translates ‘Being anatomical is anatomical’). Unfortunately, our current apparatus does not supply us with a nominalistic surrogate for $F_t$. Recall that, according to the apparatus of Chapter 3, an atomic sentence that combines a p-predicate, $F^*$, with a p-name, $t^*$, is to be assigned the nominalistic surrogate ($\bot$) if and only if the predicate that projects $t^*$ is a $C(F^*)$ predicate (where ‘$C$’ is an operator in the meta-language that takes p-predicates in the object language and converts them to predicate-classifying predicates in the meta-language). The problem here is that the classification scheme that we were taking to accompany the object language in Chapter 3 was held only to categorize n-predicates and not p-predicates. So to handle cases like this one, we’ll have to regard the classification scheme that accompanies the object language as having been expanded so as also to categorize p-predicates. In the current case, the classification scheme might now be taken, for example, to assign $F$ to the category of being a property describing predicate (as well as to other categories). But it won’t assign $F$ to the category of being an anatomical predicate (it isn’t, to put matters the other way around, the sort of predicate that can be intuitively thought of as predicating anatomical features of things). So $F$, the predicate that projects $t$, isn’t a $C(F)$ predicate. So the sentence $F_t$ is to be assigned the nominalistic surrogate $\bot$.

That takes care of the modifications that we need to be able to assign nominalistic surrogates to basic sentences of the form $pH_t$, where $p$ is a p-name and $t$ is also a p-name. Consider the other parts of the apparatus of Chapter 3 (including the modifications made to it up to this point) as adjusted to fit. Again, while I will not take the trouble to spell it all out, I take it as evident enough that the proofs given in Chapter 3, with these
modifications taken into account, will still go through in much the same way as they did before.

We can also relax the restriction imposed on the object language in Chapter 3 that its non-unary predicates be such that they can only be satisfied (in whole or in part) by concrete things. We’ve already seen in Chapter 3 how to do that for a couple of cases—namely, for the identity predicate and the property having predicate. For those predicates, we were able (given the other restrictions imposed on the object language) to “write in by hand,” as it were, the nominalistically acceptable conditions under which sentences that combined those predicates with p-names were satisfied. We can allow other such non-unary predicates into the object language provided that we can also find some way of stating the conditions under which they are nominalistically undergirded. For example, we can say that two monadic properties are “coextensive” if and only if they are exemplified by all and only the same things. We could then, if we liked, have introduced a predicate into the object language that means the same thing as ‘coextensive’ (as applied to monadic properties). Pretend that we introduced the symbol ‘E’ into the object language to serve that role and that we stipulated that any object language sentence of the form $Et_1t_2$ (where $t_1$ and $t_2$ are monadic p-names) is to be regarded as nominalistically undergirded if and only if the corresponding sentence of the form $(\exists x)(\neg (xHt_1 \rightarrow xHt_2) \& (xHt_2 \rightarrow xHt_1))$ is nominalistically undergirded.

The above example involves a non-unary predicate that is satisfied (if at all) solely by non-concrete items. But we can also allow for non-unary predicates that can be satisfied partially by concrete items and partially by non-concrete items. We can imagine
the object language as including, for example, an atomic, non-unary predicate that corresponds to the following open sentence: ‘x exemplifies y and z exemplifies y’. Let’s pretend that ‘A’ is the symbol that we introduced into the object language in order to express that predicate. Then we can say that an object language sentence of the form $At_1t_2t_3$ (where $t_1$, $t_2$, and $t_3$ are names) is nominalistically undergirded if and only if the corresponding sentence of the form $(t_1Ht_2&t_3Ht_2)$ is nominalistically undergirded.

We can generalize from the above examples as follows: Let’s say that a “specification” is any open sentence of the object language that is such that our apparatus already tells us the conditions under which its corresponding closed sentences are nominalistically undergirded. Let’s say that an “abbreviation” for a given specification is a predicate symbol of the object language that may be explicitly defined in terms of that specification. For example, E and A above may be regarded as abbreviations for the specifications $\sim(\exists x)((xHy \rightarrow xHz)\&(xHz \rightarrow xHy))$ and $(xHy\&zHy)$ respectively. Now we may stipulate that every specification of the object language has an abbreviation.

Some of the specifications of the object language will be open sentences whose abbreviations “express relations” (as the platonist would have it) that can only be entered into by concrete things. The abbreviations for these specifications will have the same meanings as the n-predicates of the object language. In fact, given the stipulation made in Chapter 3 that there are no synonymous atomic predicates in the object language (a stipulation that is still in force), it follows that the abbreviations for these specifications just are the n-predicates of the object language. Other specifications will be open sentences whose abbreviations “express relations” (as the platonist would have it) that
can only be entered into by abstract things. Some of their abbreviations will mean the
same thing as the original p-predicates of the object language and therefore (given the
stipulation that the object language contains no distinct synonymous atomic predicates)
just be the original p-predicates of the object language. The apparatus developed in
Chapter 3 (modified so as to accommodate apparent quantification over non-monadic
properties in the manner suggested above) already tells us (in a systematic way),
furthermore, the conditions under which the sentences of the object language that involve
only n-predicates and p-predicates are nominalistically undergirded.

In addition to the specifications associated with the previously existing n-
predicates and p-predicates, however, there will be others (such as those associated with
E and A above, for example) that are such that adding their abbreviations to the object
language adds something genuinely new (in comparison with what atomic predicates we
had in Chapter 3). Call these predicates “the specialized predicates” and imagine them as
being syntactically distinguished both from the n-predicates and the p-predicates. While
there is, furthermore, no systematic way of specifying the conditions under which
sentences involving the added specialized predicates are nominalistically undergirded
(without knowing what their specifications are), nevertheless, each of these predicates
will have specifications associated with them concerning which our apparatus does tell us
(in a systematic fashion) the conditions under which their corresponding closed sentences
are nominalistically undergirded. So the Correctness Preservation Theorem (at least the
version of it that we could prove for the version of the Chapter 3 apparatus that
incorporates the modifications suggested above sans the addition of these new predicates)
will still assure us that formally valid arguments that employ as premises sentences that make use of these predicates are also correctness preserving.

Adding specialized predicates to the object language does, however, falsify the Conversation Lemma as it was stated in Chapter 3. Recall that the Conservation Lemma of Chapter 3 was stated as follows:

(CL) For each sentence of the object language, $\phi^*$, such that $\phi^*$ is a closed, quantifier free sentence that contains no p-names, $\phi^*$ is nominalistically undergirded if and only if $\phi^*$ is true.

But consider the following specification: $\exists y (xHy)$. This sentence translates the open sentence ‘$x$ exemplifies something’. Since every concrete object satisfies at least some n-predicate of the object language or other, furthermore, any sentence of the object language that results from replacing $x$ in this specification with an n-name is nominalistically undergirded. Now pretend that $G$ is abbreviation of this specification.

For the reason just given, any atomic sentence of the object language that results from combining $G$ with an n-name will be nominalistically undergirded. Any such sentence, furthermore, will be a closed sentence of the object language that contains no p-names. But (at least if nominalism is true) any such sentence will also be false (on account of the fact that if nominalism is true, nothing exemplifies anything), contrary to CL. This difficulty can be dealt with, however, by modifying the clause of CL that reads “contains no p-names” to read “contains no p-names and no specialized predicates.” Otherwise, the proof of the Conservation Lemma can go on pretty much as it did before.

I’ve been assuming, up to this point, that only the n-predicates and p-predicates have projections and that none of the specialized predicates do. If we’re careful about it,
though, we can allow *some* (though not all!) of the specialized predicates to have projections. We can allow, for example, the specialized predicate that corresponds to the specification \( (\exists x)(xHy) \) (i.e. the specialized predicate that translates ‘is exemplified by something’) to have a projection. Suppose that we do. Pretend that \( S \) is the specialized predicate of the object language mentioned above and that \( s \) is its projection. Pretend also that \( n \) is some name from the object language and consider the following sentence: \( nHs \). We may now stipulate that in order to discover the conditions under which this sentence is nominalistically undergirded, we are, first, to regard \( nHs \) as being nominalistically undergirded if and only if the object language sentence \( Sn \) is nominalistically undergirded. And we are to regard this latter sentence as nominalistically undergirded if and only if the object language sentence \( (\exists x)(xHn) \) is nominalistically undergirded. Our apparatus already tells us, furthermore, the conditions under which this last sentence is nominalistically undergirded.

Suppose then that at least some of the specialized predicates of the object language do have projections (I’ll have more to say concerning which ones we should suppose this to be true of in a moment). Consider all of those predicates as also being categorized by the classification schema that accompanies the object language. And generalize (in the obvious ways) from the procedure outlined above (as well as from the procedures already given) for finding the conditions under which sentences containing their projections are nominalistically undergirded.

In order to fill out the apparatus of Chapter 3 as far as possible, we will want to allow as many specialized predicates as we can to have projections. We can’t allow *just*
any specialized predicate to have a projection, however. Consider, for example, the abbreviation associated with (~xHx). Pretend that N is abbreviation for this specification and suppose that we (inadvisably) added a projection of N to the object language; pretend, furthermore, that b is that projection. Now consider the object language sentence bHb. The procedure outlined above tells us to regard this sentence as being nominalistically undergirded if and only if Nb is nominalistically undergirded. It further tells us to regard Nb as being nominalistically undergirded if and only if (~bHb) is nominalistically undergirded. It follows that bHb is nominalistically undergirded if and only if (~bHb) is nominalistically undergirded. It further follows from the definition of nominalistic undergirding found in Chapter 3 (as it would be naturally adapted to the above modifications) that (~bHb) is nominalistically undergirded if and only if bHb is not nominalistically undergirded. So it follows that bHb is nominalistically undergirded if and only if bHb is not nominalistically undergirded. Contradiction! This, of course, is just the property version of Russell’s paradox, as it can be developed within the Chapter 3 apparatus (if we are not sufficiently cautious about how we go about expanding it).

So we do need to be careful about which of the specialized predicates of the object language we allow to have projections. But we can allow many of them to have projections. We can allow any one of them to have a projection provided that allowing it to do so does not render our apparatus inconsistent. And we’ll want to allow as many of them as we consistently can to do so. Which ones are we permitted to say have projections? That’s equivalent to the question of how to work out property theory in such a way as to allow for there being (according to the theory) higher order properties such as
being exemplified by something and the like without falling into Russell-type paradoxes.

And I’m happy to hand that question over to the logicians.

I believe that the arguments that I’ve made in the main sections of Chapter 4, as well as the illustrations of how the Chapter 3 apparatus can be expanded in this and the previous appendix, give us good reason to believe that the considerations raised in the previous chapters generalize so as to support the claim that the nominalist can consistently engage in any of the fragments of our platonist discourse that are worth keeping. I believe that I’ve supported this conclusion well enough, at any rate, to shift away any dialectical burden that the nominalist might have to give an adequate account of how it is that our platonist discourse manages to be useful in spite of the fact that there are no such things as abstract objects.

E.1 Works Cited