INVESTIGATION OF A LASER-INDUCED BREAKDOWN SPARK AS A NEAR FIELD GUIDE STAR FOR AERO-OPTIC MEASUREMENTS

A Thesis

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by

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I would like to dedicate this to my wife, Rebecca, she put up with so much.
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## NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>Angle at the focal point between beam axis and the ray from the edge of the beam aperture.</td>
</tr>
<tr>
<td>AO</td>
<td>Adaptive Optic</td>
</tr>
<tr>
<td>CLSWT</td>
<td>Compressible Shear Layer Wind Tunnel</td>
</tr>
<tr>
<td>DE</td>
<td>Directed Energy</td>
</tr>
<tr>
<td>( D_C )</td>
<td>Aperture of a beacon light collecting lens</td>
</tr>
<tr>
<td>( D_L )</td>
<td>Aperture of a laser focusing lens</td>
</tr>
<tr>
<td>DM</td>
<td>Deformable Mirror</td>
</tr>
<tr>
<td>( \Delta t )</td>
<td>Laser pulse width</td>
</tr>
<tr>
<td>E</td>
<td>Laser pulse energy</td>
</tr>
<tr>
<td>( E_A )</td>
<td>Energy absorbed by spark</td>
</tr>
<tr>
<td>( E_{\text{MIN}} )</td>
<td>Minimum laser energy required to form a spark</td>
</tr>
<tr>
<td>( \varepsilon_h )</td>
<td>Phase error due to aero-optic disturbances in the DE beam path</td>
</tr>
<tr>
<td>( \varepsilon_b )</td>
<td>Phase error due to aero-optic disturbances in beacon beam path</td>
</tr>
<tr>
<td>( \varepsilon_{\text{hb}}^2 )</td>
<td>Anisoplanatic variance, the variance in phase between ( \varepsilon_h ) and ( \varepsilon_b )</td>
</tr>
<tr>
<td>( f )</td>
<td>Focal length of a lens</td>
</tr>
<tr>
<td>( f/# )</td>
<td>Ratio of lens focal length to lens aperture</td>
</tr>
<tr>
<td>( f_C )</td>
<td>Focal length of a beacon light collecting lens</td>
</tr>
<tr>
<td>( f_L )</td>
<td>Focal length of a laser focusing lens</td>
</tr>
<tr>
<td>( \phi_h )</td>
<td>Phase of DE beam</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Adiabatic index</td>
</tr>
<tr>
<td>( I_B )</td>
<td>Breakdown irradiance</td>
</tr>
<tr>
<td>IR</td>
<td>Infra-Red</td>
</tr>
<tr>
<td>( K_{\text{GD}} )</td>
<td>Gladstone Dale Constant</td>
</tr>
<tr>
<td>L</td>
<td>Distance from beam aperture to aim point of DE beam</td>
</tr>
<tr>
<td>( L_a )</td>
<td>Distance from beam aperture to aero-optic disturbance</td>
</tr>
<tr>
<td>( L_b )</td>
<td>Distance from beam aperture to near field beacon</td>
</tr>
<tr>
<td>LGS</td>
<td>Laser Guide Star</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Wavelength</td>
</tr>
<tr>
<td>( \Lambda )</td>
<td>Aberration repetition length</td>
</tr>
<tr>
<td>M</td>
<td>Mach number</td>
</tr>
<tr>
<td>MMSE</td>
<td>Minimal Mean Square Error</td>
</tr>
<tr>
<td>OPL</td>
<td>Optical Path Length</td>
</tr>
<tr>
<td>OPD</td>
<td>Optical Path Difference</td>
</tr>
</tbody>
</table>
\( r \) Radial co-ordinate in a cylindrical system
\( \rho \) Air density
\( S_t \) Strehl Ratio
\( \theta \) Azimuthal angle in a cylindrical co-ordinate system
\( U \) Streamwise velocity
\( \text{UV} \) Ultra-Violet
\( v \) Spark front velocity
\( w_0 \) Beam waist diameter
\( \text{WFS} \) Wavefront Sensor
\( x \) Position of spark wavefront

Subscripts

1 Low-speed flow of shear layer
2 High speed flow of shear layer
h High energy beam
b Beacon or spark beam
CHAPTER 1:
INTRODUCTION AND REVIEW

High field-of-regard, aircraft-mounted laser systems typically include parts of the operating envelope in which the laser must pass through highly-turbulent flow regions, such as a turbulent boundary layer or a shear layer associated with a separated flow region [1]. At subsonic and higher flight speeds, these turbulent flow regions become optically active such that the transiting laser beam will be distorted due to index-of-refraction variations within the flow [2, 3]. The study of the optical aberrations produced by these kinds of near field turbulent flows is called “aero-optics.”

The phase characteristics of the initial light beam can be restored using an adaptive-optic (AO) system [4] that places the conjugate waveform of the aberration onto the optical wavefront of the beam prior to its transmission through the aberrating flow field. In this case, computation of the conjugate waveform is predicated on an accurate determination of the original aero-optic aberration. Even for feedforward AO correction schemes, in which flow-control techniques are used to modify the frequency bandwidth of the aberrating flow, it is still anticipated that optical measurements will still be necessary to synchronize the AO scheme with the controlled flow aberrations [5, 6].

One method of measuring the aero-optic aberrations is to use the light from a guide star [7]. The guide star may be, for example, a naturally-occurring nearby star or even glint from the target; however there is considerable advantage to using artificial
guide stars since they can be placed at essentially any location around the flight vehicle. In practice, such a man-made guide star could be generated by focusing a high-energy pulsed laser at a point outside the aircraft, thereby creating a laser-induced air-breakdown spark with sufficient brightness for aero-optic measurements. For all types of guide stars, anisoplanatic effects occur when the light from the guide star does not pass through the same aero-optic flow as the outgoing laser beam. Furthermore, the optical character of laser-induced breakdown sparks is not well known. This thesis presents an experimental investigation into laser-induced breakdown sparks and their suitability as an illumination source for aero-optic measurements.

1.1 Aero-Optics

Aero-optics is the study of the effect that a fluid has on a traversing beam of light. The aberrations in an aero-optic flow originate from density variations in the fluid, and their associated variations in the index of refraction. According to [8] the index of refraction depends on the density of the fluid as follows:

\[ n = 1 + K_{GD} \cdot \rho \] (1.1)

where \( K_{GD} \) is the Gladstone-Dale parameter:

\[ K_{GD} = 0.223 \times 10^3 \times (1 + \frac{7.52 \times 10^{-15}}{\lambda^2}) \left( \frac{m^3}{kg} \right) \] (1.2)

For a beam of light that traverses the varying index-of-refraction field, the effective distance that each ray in the beam travels is given by the optical path length:

\[ \text{OPL}(x,y) = \int n(x,y,z) \, dz \] (1.3)

The optical path difference is then the difference in optical path length from the mean:
\[ \text{OPD}(x, y) = \text{OPL}(x, y) - \overline{\text{OPL}}(x, y) \] (1.4)

A simple evaluation of the effect of an aero-optic flow on a traversing beam of light can be obtained from the Strehl ratio, which is the ratio of the intensity of the beam of light on target when focused in the far field to the maximum intensity that can be obtained based on the diffraction-limited performance of the optical system:

\[ S_t = \frac{I}{I_0} \] (1.5)

Using the Marachel approximation for an infinite aperture, the Strehl ratio can be estimated as:

\[ S_t = e^{-\left[\frac{2\pi \text{OPD}_{\text{rms}}}{\lambda}\right]^2} \] (1.6)

As such, the far field performance of the optical system is seen to be directly linked to the root mean-square of the OPD variations of the outgoing beam.

1.2 Aberration Sources

Aero-optic aberrations can be categorized as “near field” or “far field,” depending on the proximity of the aberrating fluid to the aperture of the outgoing beam of light. Near field and far field aberrations have fundamentally different sources and can be observed having different characteristics, and therefore affect a traversing beam of light in fundamentally different ways.

As implied by the name, far field aberrations refer to aberrations that are located far from the optical aperture; these far field aberrations are most commonly associated with density and concomitant index-of-refraction changes in the atmosphere through
which the beam passes. In general, atmospheric aberrations originate from density variations associated with temperature differences between different layers of air. Atmospheric aberrations also tend to have a large scale compared to the beam aperture. As shown in [9], the large scale of atmospheric aberrations fundamentally influences the way in which the aberrations affect a traversing beam of light; in particular, due to the fact that the beam diameter is typically much smaller than the scale of the atmospheric aberration, the aberration appears primarily as a tip/tilt on the traversing beam. The large scale and slow convection speeds of atmospheric aberrations also means that atmospheric aberrations have a relatively-low frequency content; in particular, the frequency content of atmospheric aberrations is sufficiently low that they are now regularly compensated using currently-available feedback adaptive-optic approaches [10].

Aero-optic aberrations, on the other hand, are “near field” sources because they occur in the air that is in close proximity to the flight vehicle that is transporting the optical system under consideration. Furthermore, the physical cause of aero-optic aberrations is fundamentally different from atmospheric aberrations; as shown in [3], aero-optic aberrations arise primarily from the pressure wells and concomitant density depressions at the center of vortical structures in the turbulent flows surrounding flight vehicles traveling at compressible flow speeds. These turbulent flows are most commonly boundary-layer flows or shear layer flows; previous research into the aero-optic character of both of these types of flows can be found in [2] and [3, 5, 11, 12], respectively. Most importantly, because of the smaller scale turbulence from which aero-optic flows originate compared to atmospheric aberrations, aero-optic flows typically impose higher spatial frequency aberrations onto a traversing beam of light that cannot be removed by
simple tip/tilt correction; furthermore, the much higher flow speeds associated with aero-optic aberrations means that the temporal frequency content of aero-optic aberrations is also beyond the capability of conventional adaptive optic (AO) systems.

Since aero-optic flows occur in the near field, it is not necessary to project an artificial guide star exceedingly far from an aircraft in order to image the aero-optic aberrations using the return light from the guide star. Figure 1.1 shows schematically the general arrangement of an artificial guide star system that uses a laser-induced breakdown spark.

![Image of artificial guide star system](image)

**Figure 1.1** Artificial guide star system that could be used in a real-world system.

As shown in the figure, the breakdown spark, or beacon, might be only a few times farther away from the projecting aircraft than the source of the aero-optic aberrations; this means that artificial guide stars need only be projected a distance on the order of meters away from the parent aircraft. Note also that there are several differences between the return light from the guide star and the outgoing laser beam; in particular:

- the beacon return light samples a different part of the aero-optic region than the region through which the outgoing beam passes,
- the beacon return light has spherical wavefronts as opposed to the planar wavefronts of the outgoing beam
These differences are known as “anisoplanatism” and are treated in more detail in Chapter 2 of this thesis.

1.3 Feedforward Adaptive-Optic Corrections

As mentioned above, the frequency content of aero-optic flows is very high, so that it is not possible to correct for aero-optic aberrations using conventional feedback AO systems. An alternative approach for the correction of shear layer aberrations is to first regularize the shear layer by applying a mechanical forcing at its origin [11]. With the shear layer regularized by forcing, the aberrations caused by the shear layer become repeatable so that it is possible to correct for the shear layer aberrations using a feedforward AO approach. In this case, a deformable mirror (DM) is pre-programmed with a shape that is the conjugate of the regularized shear layer aberrations, after which the phase of the DM motion must be matched to the phase of the shear layer. Reference [5] describes a feedforward AO correction of this type in which the phase matching of the DM with the shear layer aberrations was performed by a human operator who observed the corrected beam reflected from the DM; using this method, the Strehl ratio of a laser beam passing through the shear layer was increased from 0.1 to 0.66. In reality, however, it would not be possible to observe the outgoing, corrected laser beam as performed in [5]; rather, for a realistic system, the phase and amplitude of the regularized shear layer would be measured using, for example, the return light from an artificial guide star such as described in this thesis. As such, this work complements and extends the feedforward AO measurements previously demonstrated at the University of Notre Dame [5].
The advantage of the feedforward AO approach is that it is not necessary for the AO system to respond to random, high-frequency aberrations associated with the un-regularized shear layer. In particular, as shown in [13], a conventional, feedback AO correction of a typical shear layer flow would require measurements of the aero-optic aberrations due to the shear layer at a frequency on the order of 100 kHz. On the other hand, for the feedforward AO approach, measurements of the shear layer aberrations are only necessary at a rate that is sufficient for initial adjustment of the phase and amplitude of the DM to match the regularized aberration, and to periodically update these parameters to correct for any long-term flow fluctuations; this measurement rate would be on the order of a few Hertz at most. This slow measurement rate greatly alleviates the demands on an artificial guide-star system that employs a laser-breakdown spark. In particular, as will be shown in Chapter 3, the laser energy required to achieve breakdown in air is sufficiently high that laser-induced breakdown can only be achieved intermittently using a pulsed laser. As such, the much lower pulse rate that would be required if the laser-beacon system were to be used in a feedforward AO approach, means that much smaller and inexpensive commercially-available lasers can be used.

1.4 Prior use of Laser breakdown as Near Field Guide Star

A previous investigation into the use of a laser-induced air-breakdown spark as an artificial guide star is described in [14]. This study was motivated by a need to compensate for the aero-optic aberrations around the optical seeker of high-speed (hypersonic) missile-defense interceptors. The investigation used an Nd:YAG laser that emitted at the fundamental frequency of 1064 nm and had a pulse width of 10 ns.
The investigation resulted in several important findings. First, it was found that the laser pulse energy required to achieve air breakdown increased as the static pressure was decreased, and that a pulse energy of approximately 400 mJ was required to achieve air breakdown at a static pressure of 10 torr (~1300 Pa). For the Standard Atmosphere, a static pressure of 1300 Pa corresponds to flight at an altitude of 95,000 ft [15]; as such, the 400 mJ pulse energy gives an estimate of the approximate largest pulse energy required for a laser-spark artificial guide star.

The study also reported a spark lifetime on the order of 100 ns. The spark lifetime has important implications on the ability to make accurate optical measurements using the return light from the spark. In particular, when the spark is created in a high-speed flow, the location of the spark will be convected by the flow a distance roughly equal to the freestream speed times the spark lifetime:

\[ x = U_\infty t_{\text{lifetime}} \]  

(1.7)

If the distance \( x \) as shown in Equation (1.7) above is too large, then the spark will appear as an elongated light source, rather than a point source, making it difficult to use the light from the spark for optical measurements.

Finally, measurements of the breakdown spark showed that both the size and effective location of the spark varied from ignition to ignition. It was noted that these variations in spark size and location, if too large, would make it difficult to measure aero-optic aberrations using the return light from the spark.

In summary, the results of [14] demonstrated that it was possible in concept to measure aero-optic aberrations using the return light from a laser breakdown spark. The study revealed, however, several potential problems with the approach, including an
overly long spark lifetime as well as poor reproducibility of the size, position and energy radiated by the spark

1.5 Objectives of the Current Research

This thesis details an experimental investigation into the use of a laser-breakdown spark as a near field guide star for the measurement of aero-optic flows. The thesis presents data on the size and optical character of laser sparks and the implications of the data on potential operational systems, and addresses the major issues raised in [14]

Following this chapter, methods for the compensation of anisoplanatism effects associated with aero-optic measurements using a near field guide star are discussed in Chapter 2. In Chapter 3 the non-point source nature of the laser spark is discussed and its effect on wavefront measurements using the return light from the spark. Chapter 4 deals with the spark lifetime and the effect of fluid flow on the spark. Chapter 5 presents the conclusion of this thesis.
CHAPTER 2:
CORRECTION OF ANISOPLANATISM EFFECTS

Due to the close proximity of the guide star, or due to other design factors that restrict the geometry of the optical system used collect the light from the guide star, it is often not possible to measure exactly the same region of flow with the guide star as the region of flow through which the main beam passes. Further, the close proximity of the guide star may also result in noticeable wavefront curvature. The differences in sampled region and wavefront shape between the guide star system and the outgoing laser beam are called “anisoplanatism.” In this chapter, procedures for the determination and correction of anisoplanatism effects are presented.

2.1 Focal Anisoplanatism with Aero-Optics

The analysis geometry for a beam propagating through an aero-optic disturbance is shown in Figure 2.1. The figure shows an aero-optics beacon that is used to correct the aero-optic aberrations of the outgoing laser. The beacon is located at a short distance away from the aperture compared to the propagation range of the outgoing laser. The outgoing beam is imparted with a phase error $\phi_b$ due to aero-optic disturbances in the propagation path, while the return light from the beacon is imprinted with a phase error $\phi_b$; the aperture-averaged phase variances over a scaled unit circle for the outgoing beam and beacon are therefore:
\[ \varepsilon_h^2 \equiv \int_0^{2\pi} \int_0^1 \langle [\phi_h(r, \theta)]^2 \rangle dr d\theta \] (2.1)

\[ \varepsilon_b^2 \equiv \int_0^{2\pi} \int_0^1 \langle [\phi_b(r, \theta)]^2 \rangle dr d\theta \] (2.2)

As the primary error metric for AO phase compensation, the “anisoplanatic variance” can be defined as:

\[ \varepsilon_{hb}^2 \equiv \int_0^{2\pi} \int_0^1 \langle [\phi_h(r, \theta) - \phi_b(r, \theta)]^2 \rangle dr d\theta \] (2.3)

This error metric can be further broken down into distinct components:

\[ \varepsilon_{hb}^2 = \tilde{\varepsilon}_{hb}^2 + \bar{\varepsilon}_{hb}^2 \] (2.4)

where the term \( \tilde{\varepsilon}_{hb}^2 \) represents the variance of aero-optics phase not measured because of the finite range \( L_b \) of the beacon, and the term \( \bar{\varepsilon}_{hb}^2 \) represents the variance of phase measured in error by the beacon due to probing a different aero-optics disturbance volume than the outgoing beam focused on a target at range \( L \). As can be seen in Figure 2.1 relating each of the components of residual error to characteristics of the propagation path depends on the distribution of the aero-optics disturbances over the path characterized by the scale length \( L_a \).

![Figure 2.1 Analysis geometry for correction of aero-optic aberrations using near-field beacon.](image-url)
2.2 Mitigation of Focal Anisoplanatism

Increasing the range of the beacon relative to the aero-optics disturbance location \( L_a \) may appear to be an easy solution to the problem of measurement anisoplanatism, but will have practical limits. An alternate approach to mitigating the anisoplanatic degradation associated with a near field beacon is the use of optimal estimation. Optimal modal compensation \([16, 17]\) is a method that offers considerable advantage and flexibility to the problem of aero-optics focal anisoplanatism. In summary, this technique uses a linear estimation matrix, \( A \), applied to a modal measurement set \( c_m \) consisting of \( N \) modes from one or more reference beacons to yield an estimate for the desired correction:

\[
\hat{c}_h = A \ c_m
\]  

(2.5)

By the nature of the optimality criterion for the estimator, the residual modal anisoplanatism is indeed minimized, given the properties of the measurements and their relation with the desired phase correction. The minimum error resulting from application of the estimator is given by:

\[
\text{diag} \left[ \langle (c_h - \hat{c}_h) (c_h - \hat{c}_h)^T \rangle \right] = \text{diag} \left[ \langle c_h \ c_h^T \rangle - B \ C^{-1} B^T \right]
\]  

(2.6)

where the "< >" denotes the average over the entire set of data. This minimization requirement corresponds exactly with the optimality criterion of a \textit{minimal mean-square error} (MMSE) estimator for the reference beam modes. Forming such an estimator requires calculation of the covariance of the beacon measurement modes as well as the cross-covariance of the measurements with the desired correction, as follows:

\[
A = B \ C^{-1}
\]  

(2.7)

\[
B = \langle c_h \ c_m^T \rangle
\]  

(2.8)
\[ C = \langle c_m c_m^T \rangle \]  

(2.9)

These statistical quantities depend primarily upon the location of the aero-optics beacon relative to the characteristic location of the aero-optics disturbance, \( L_b/L_a \), and secondarily to the location of the beacon relative to the overall path length, \( L_b/L \), again as seen in Figure 2.1.

2.3 Experiment

Beacon anisoplanatism measurements were performed in the Compressible Shear Layer Wind Tunnel (CSLWT) at the University of Notre Dame, which was constructed specifically to investigate the optical characteristics of high-subsonic compressible shear layers. The CSLWT mixes co-directional high- and low-speed flows at high subsonic flow speeds (up to Mach 1.0) to create a shear layer that is aero-optically active, and representative of the kinds of aero-optic flows likely to be encountered on flight vehicles. Details on the design and flow characteristics of the CSLWT can be found in [5, 11].

Anisoplanatism effects were evaluated by comparing wavefront aberrations from a point-source “beacon” to those from a collimated reference laser beam after passing through the compressible shear layer flow. Wavefronts were measured using a Shack-Hartmann-type wavefront sensor (WFS) manufactured by Wavefront Sciences. The flow was interrogated using a 532 nm wavelength frequency-doubled pulsed Nd:YAG laser. The wavefront sensor was a CCD camera with a 33 x 44 lenslet array and, although the frame rate of the CCD camera was only 30 Hz, the \( M_2 = 0.78 \) flow was effectively frozen by the 8 ns laser pulses.
In practice, a laser-breakdown beacon would be generated by projecting a focused laser beam into the atmosphere, typically within the field of view of the correcting AO system. The focused laser beam is of sufficient intensity to induce air breakdown, creating a nearly point source of light which is then used to measure the aero-optic environment in the vicinity of the outgoing laser beam aperture. However, as will be shown in Chapter 3, laser-breakdown sparks exhibit qualities that would interfere with the accurate measurement of anisoplanatism effects, including a relatively-large size and a tendency to move slightly from spark to spark. As such, for these tests, the artificial guide star was simulated using the light projected from an optical fiber that was aimed along the axis of the optical setup. The fiber used in the tests was a single mode fiber with a core diameter of 3.6 μm. The light emitted from a cleaved end of the fiber diverged with a full angle of approximately 5°.

An overview of the basic geometry of the experimental setup is shown in Figure 2.2. A 50 mm diameter collimated reference beam was passed through the shear layer coaxially with the diverging beacon beam emanating from the fiber. A \( f = 600 \) mm lens, aperture to 50 mm, was located a distance 600 mm away from the fiber in order to collimate the beacon beam after passing through the shear layer. The shear layer, which contributed most of the aero-optic aberrations to the reference and beacon beams, was situated halfway between the fiber and the \( f = 600 \) mm lens; as such, due to the geometry of the setup, the diameter of the beacon beam, as it passed through the aberrating shear layer, was approximately half that of the reference beam. This geometrical difference in the beacon and reference beams, in addition to the difference in curvature of the beacon
Figure 2.2 Basic geometry of beacon anisoplanatism measurement

(spherical wavefronts) and reference (planar wavefronts) beams, constituted the anisoplanatic effects that were evaluated by the measurements.

A photograph of the optical setup is shown in Figure 2.3. The setup consisted of transmission optics mounted on a raised platform that was level with the top of the CSLWT test section, and receiving optics mounted below the test section. The transmission optics included the laser, a beam splitter, a fiber-optic coupler and single-mode fiber to generate the simulated diverging beacon signal, and a series of beam expanders that generated the 50 mm collimated reference beam. The beacon and collimated beams were directed down through the test section by a mirror.

The receiving optics underneath the test section were designed to separate, reduce, and align the beacon and reference beams into the wavefront sensor. The first element of the receiving optics was the 600 mm focal length lens apertured to 50 mm. This lens both collimated the beacon beam and acted as the first element of a 5:1 beam reducer for the reference beam. Just after the 600 mm lens, a beam splitter was used to direct the collected light into two different lens systems that reduced the apertured beacon
Figure 2.3 Schematic (left) and photograph (right) of optical setup

and reference beams into (approximately) 0.5 in. diameter beams. At this stage, both the beacon and reference beams were passed into both beam-reducer lens systems, due to the different starting wavefront shape of each beam (i.e. the collimated planar reference wavefront versus the diverging spherical beacon wavefront) each beam was effectively filtered out by the other beam reducer, leaving no noticeable cross talk in the wavefront sensor images. The two 0.5 in. beams were then guided using mirrors so that the two beams were projected side by side into the wavefront sensor. To enable alignment of the two beams and thus minimize differences in no-flow tip/tilt between the two beams, one of the mirrors was mounted on a translation stage. The final optical element was a 4:1 beam reducer that projected the two aligned 0.5 in. beams into the WFS camera. Test images acquired with the WFS camera with the flow off showed no noticeable spherical aberrations, and verified that the beacon image covered approximately the inner 50% of the reference beam at the shear layer location.
2.4 Measurements

In order to generate strong aberrations in the shear layer flow and hence good signal-to-noise ratio, the wind tunnel was run at close to its maximum wind speed with \( M_2 = 0.78, \ M_1 = 0.15 \). This operating point gives a shear layer velocity ratio \( U_1/U_2 \approx 0.19 \), which is representative of the kinds of shear layers that are created by separated flow regions, and has been used in other investigations [5, 12].

As shown in [3], optical aberrations in the shear layer are caused primarily by the low-pressure wells in the center of vortical structures within the shear layer. The dominant vortical structure size (and concomitant aberration size) in the shear layer scales with the shear layer cross-stream width, which increases linearly in the streamwise direction. To obtain an indication of aberration size on the measured anisoplanatism, data were acquired at several downstream locations.

The dominant shear layer structures size in the CSLWT at the test speed \( M_2 = 0.78/M_1 = 0.15 \) is plotted in Figure 2.4 [11]; which also shows the \( \text{OPD}_{\text{rms}} \) of the related aberrations in order to give an indication of the strength of the optical signal. The figure shows that, at small streamwise distances, the structures sizes are very small and result in small-amplitude optical aberrations, while farther downstream the structures become larger and produce strong aberrations. At the most upstream locations, the optical aberrations were too weak to be accurately measured by the WFS, so that measurements were made downstream of \( x = 200 \) mm. On the other hand, at the farthest downstream locations, the 50 mm aperture of the optical setup was too small to capture the entire aberration size of the much larger, downstream aberrations. The result was that at the farthest downstream locations, the 50 mm aperture effectively “filtered” the large-scale
aberrations so that the aberrations appeared primarily as tip/tilt within the 50 mm aperture. Further, because the entire aberration did not appear within the aperture, the OPD measured by the WFS was also reduced. As such, to avoid excessive aperture effects, the maximum downstream location at which measurements were made was limited to $x = 400$ mm.

Figure 2.4 Dominant shear layer structure size (unforced) and $OPD_{rms}$ in CSLWT, $M_2=0.78/M_1=0.15$ [11]

Measurements were also performed with the shear layer forced at several different forcing frequencies. The shear layer was forced using voice-coil actuators that applied a moving-boundary perturbation to the flow perpendicular to the flow direction at the trailing edge of the splitter plate [5, 11]. The shear layer was forced over the most-effective frequency range of the forcing actuator, 750 Hz to 1500 Hz, producing regularized shear layer structure (and aberration) sizes from 100 to 200 mm. The shear layer forcing also produced much stronger OPD’s and improved signal-to-noise ratios for the wavefront measurements. The test matrix of anisoplanatism measurements that were performed is summarized in Table 2.1.
### TABLE 2.1

**TEST CONDITIONS**

FOR ANISPLANATISM MEASUREMENTS

<table>
<thead>
<tr>
<th>(X) (mm)</th>
<th>Forcing (Hz)</th>
<th>Dominant Aberration Size (\Lambda) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>None</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>750</td>
<td>200</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>160</td>
</tr>
<tr>
<td></td>
<td>1500</td>
<td>100</td>
</tr>
<tr>
<td>300</td>
<td>None</td>
<td>120</td>
</tr>
<tr>
<td></td>
<td>750</td>
<td>200</td>
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<tr>
<td></td>
<td>1000</td>
<td>160</td>
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<tr>
<td></td>
<td>1500</td>
<td>100</td>
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<td>400</td>
<td>None</td>
<td>180</td>
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<tr>
<td></td>
<td>750</td>
<td>200</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>160</td>
</tr>
<tr>
<td></td>
<td>1500</td>
<td>100</td>
</tr>
</tbody>
</table>

2.5 Results

An example of wavefronts from the beacon and reference beams is shown in Figure 2.5. The wavefronts were acquired at a downstream location \(x = 300\) mm, and with the shear layer forced at a frequency of 750 Hz. The wavefront for the beacon beam in the figure has been scaled to a diameter of 25 mm, which is the diameter of the beacon beam as it passes through the optically-active region of the shear layer flow. As can be seen in the figure, the 50 mm aperture of the optical system captures only a small portion of the 200 mm long aberration wavelength associated with the 750 Hz forcing, with the result that the aberration appears mostly as a streamwise tilt. The figure shows that the
aberration detected using the beacon closely matches the inner 50% of the wavefront measured by the reference beam.

Figure 2.5 Sample wavefronts for collimated reference beam (left) and beacon beam (right) acquired at $x = 300$ mm and with shear layer forced at 750 Hz.

2.6 Mitigation of Anisoplanatism by Optimal Estimator

2.6.1 Unforced Shear Layer

The results of the anisoplanatism measurements for the unforced shear layer are summarized in Figure 2.6. The figure shows, first, the aperture-averaged phase variances measured for the reference and beacon beams (blue and green lines). The phase variances were computed for light with a wavelength of 1 μm, with tip/tilt removed, and are shown as the average of 50 individual wavefront measurements. As can be seen in the figure, the variance for the reference and beacon beams increases in the streamwise direction, which is an expected outcome of the increasing shear layer structure size and concomitant aberration strength, as the shear layer evolves in the downstream direction (cf Figure 2.2). Also plotted in Figure 2.6 are the anisoplanatic variance $\varepsilon_{hb}^2$ and the residual
anisoplanatism after correction using the MMSE estimator, $\hat{\varepsilon}_{hb}^2$ (red and black lines). The anisoplanatic residual after MMSE estimation $\hat{\varepsilon}_{hb}^2$ was computed by estimating the reference wavefront mode vector $\hat{c}_h$ via Equation (2.5), constructing estimated phase error distributions for the reference beam, $\phi_h$, and substituting these estimated phase error distributions into Equation (2.10):

$$\hat{\varepsilon}_{hb}^2 \equiv \int_0^{2\pi} \int_0^1 \left[ \phi_h(r, \theta) - \hat{\phi}_h(r, \theta) \right]^2 \, dr \, d\theta$$

Figure 2.6 shows that, at the most upstream locations, the anisoplanatism of the uncompensated data is almost as large as the wavefront variance of the reference beam itself. This result shows that even though it is easily possible to discern the similarities between wavefronts for the beacon and reference beams for individual measurements as shown in Figure 2.5, the differences between the two wavefronts that arise from the various sources of anisoplanatism (e.g. $\hat{\varepsilon}_{hb}^2 + \bar{\varepsilon}_{hb}^2$ in Equation (2.4)) are still significant. As such, use of the unmodified beacon wavefronts to estimate the reference beam phase variance (e.g. for the purpose of AO correction) would result in additional errors from the estimation that could be of the same order as the original reference beam variance. On the other hand, Figure 2.6 shows that the MMSE estimator reduced the anisoplanatism by a factor of 2 or more, so that the residual anisoplanatism between the corrected beacon wavefronts and the reference wavefronts ranges from approximately 50% to less than 30% of the original reference beam wavefront variance.
Figure 2.6 Anisoplanatism results for unforced shear layer.

Figure 2.7 Anisoplanatism results for the forced shear layer, $x = 200$ mm.

Figure 2.8 Anisoplanatism results for forced shear layer, $x = 300$ mm.
2.6.2 Forced Shear Layer

Anisoplanatism results are shown in Figures 2.7 to 2.9 for the shear layer forced at 750 Hz, 1000 Hz and 1500 Hz, at the $x = 200$ mm, 300 mm and 400 mm measurement stations. The results also show a slight reduction in the residual anisoplanatism $\hat{\epsilon}_{hb}^2$ when the shear layer is forced; this trend can be more clearly observed by the slight upward slant of the data in Figure 2.10 which plots the reduction in anisoplanatism realized by the MMSE estimator, $\epsilon_{hb}^2 - \bar{\epsilon}_{hb}^2$. This reduction in residual anisoplanatism is likely due
to the higher amplitude and improved signal to noise of the forced aberrations. More significantly, the improved performance of the MMSE estimator might also be an outcome of the regularized nature of the forced shear layer, in which random disturbances are suppressed in favor of the large-amplitude vortical structures with passing frequency dictated by the forcing frequency; the optical aberrations of these large vortical structures would show much greater correlation in the beacon and reference wavefronts once anisoplanatic effects are removed.

2.7 Discussion

The data presented in this chapter have illustrated the success of the MMSE estimation technique in reducing anisoplanatism between the point-source beacon and collimated reference beams. In this regard, these data represent the first step towards developing near-field beacons for measurement of aero-optic flows.

The results of this investigation have shown that regularization of the shear layer by forcing changes the statistics of the anisoplanatism so that the estimator reduces the anisoplanatism more effectively; this synergy between shear layer forcing and the optimal estimation technique suggests that the two approaches might be used concurrently. In this case, it should be noted that the forced shear layer produces strong, coherent and sinusoidal-like aberrations [5], and it is possible that the MMSE technique works more robustly for these kinds of aberrations at any scale.

It should be noted that correction of the beacon measurements using the MMSE technique relies on the availability of a simultaneously-measured set of beacon and reference wavefronts in order to evaluate the anisoplanatism of the beacon beam and
construct the estimation matrix; this requirement is apparent from the definition of the $B$ matrix, Equation (2.8). For the results presented in this chapter, separate sets of beacon and reference wavefronts were acquired for each test condition (streamwise location and forcing frequency), and a distinct estimation matrix $A$ was computed and used to correct the beacon anisoplanatism. For a flight application, the optical configuration could be “calibrated” in a set of pre-flight experiments to generate a database of estimation matrices that cover a specified range of flight conditions.
CHAPTER 3:
THE OPTICAL CHARACTER OF LASER-INDUCED BREAKDOWN SPARKS

In Chapter 2, data were presented of the aero-optic aberrations of a compressible shear layer using an artificial guide star that was simulated using the diverging light from an optical fiber. Although the results of these initial tests were promising, it should be noted that the light output from the optical fiber used to simulate the beacon had better optical properties than can be expected from a spark. In particular, the optical fiber used to create the diverging, simulated beacon had a core diameter of only 3.6 μm, so that the light from the fiber behaved effectively as a point source; in comparison, laser-induced breakdown sparks have sizes on the order of a few millimeters or larger [18]. Furthermore, the size and effective location of laser-induced breakdown sparks can fluctuate between pulses of the driving laser, while the effective origin of the light from the optical fiber could be held stationary. The next step in the investigation of an aero-optic beacon was therefore to evaluate the effects of the non-point-source optical character of laser-induced breakdown sparks and to demonstrate the viability of aero-optic measurements using an actual laser-generated beacon; this chapter documents the approach and results of this investigation.
3.1 The Physics of Laser-Induced Air-Breakdown Sparks

A schematic of a generic laser-induced, air-breakdown beacon system is shown in Figure 3.1. A laser beam is focused using a lens with f/# = f_L/D_L to create a breakdown spark, and the return light from the beacon is imprinted with the aberrations imposed by an intervening optically-active flow. The beacon light is then collimated using a lens with f/# = f_C/D_C after which it is directed on to a wavefront-measurement instrument. For an operational system, working values for f_L/D_L, f_C/D_C will likely be at least 10, and have a practical upper limit of around 40. The system shown in Figure 3.1 is very simplified and ignores many other design parameters that can affect the beacon system performance; however, it will be shown that the effect of the non-point-source behavior of the spark is determined primarily by the values f_L/D_L, f_C/D_C.

![Figure 3.1 Generic aero-optic beacon system.](image)

A diagram of a laser beam focused using a lens with f/# = f_L/D_L is shown in Figure 3.2. The waist diameter of a focused laser beam is given by [19]:

\[
w_0 = \frac{4 \lambda f_L}{\pi D_L}
\]

(3.1)

The average irradiance at the spot location for a beam with pulse energy \(E\) and pulse duration \(\Delta t\) is therefore:
Breakdown and spark formation occurs when the irradiance at the waist location exceeds the breakdown threshold of the ambient gas.

Figure 3.2 Diagram of laser beam focused by lens with $f/# = f_l/D_L$

Equation (3.2) shows that the ability of a particular laser to achieve breakdown depends strongly on the emission wavelength and the pulse duration of the laser. Measurements [20-22] also show that the breakdown threshold varies with wavelength by several orders of magnitude, but that the threshold is lower at infra-red ($\lambda \approx 1 \mu m$) and ultra-violet ($\lambda \approx 350$ nm) wavelengths. Experimental measurements of the breakdown threshold at infra-red and ultra-violet wavelengths are presented in Section 3.2

When the focused laser irradiance exceeds the gas breakdown threshold, atoms in the gas become ionized by the large electric field strength in the focal region, and the return of these atoms back to the ground state is accompanied by the emission of secondary light. At the most common test pressures and laser wavelengths, the dominant mechanism of gas ionization is cascade ionization (inverse bremsstrahlung), although multi-photon ionization may become important at high altitudes (e.g. ~30,000 ft) where the gas pressure is significantly lower and if shorter (UV) laser wavelengths are used [23,
After breakdown is achieved, the gas at the breakdown location becomes effectively opaque to the laser radiation and absorbs most of the incident energy in the latter part of the laser pulse [24]. Figure 3.3 shows schematically the portion of the laser pulse that is absorbed for different cases. In particular, Fig. 3.3(a) shows a “baseline” laser pulse that just has sufficient energy to achieve the breakdown threshold at the top of the pulse, so that the energy absorbed by the breakdown spark is proportional to the energy in the second half of the pulse (shaded). Fig. 3.3(b) shows how a shorter pulse would reduce the energy absorbed by the spark, while Fig. 3.3(c) shows how the energy absorbed by the spark would be increased by any factor that increases the pulse energy required to achieve breakdown, specifically, as shown by Equation (3.2), a larger breakdown threshold, longer wavelength or large $f_l/D_L$. If the laser pulse energy is greater than the level required to achieve breakdown, then the additional energy is also absorbed by the breakdown spark, as shown in Figure 3.3(d).

The breakdown spark grows rapidly to a size that is several orders of magnitude larger than the focal region of the initiating laser beam. In particular, the breakdown spark tends to grow back towards the focusing lens within the solid angle containing the
incident laser beam. The spark growth has been well modeled as a radiation-driven
detonation wave [18, 25 – 28], with a front velocity given by

\[
v = \left[ \frac{2 (\gamma^2 - 1) E_A}{\pi \Delta \rho \tan \alpha} \right]^{1/3}
\]

(3.3)

where \( E_A \) is the energy absorbed, as shown approximately in Figure 3.3. In Equation
(3.3), \( \rho_2 \) is the air density behind the shock front:

\[
\rho_2 = \rho_1 \left( \frac{\gamma + 1}{\gamma} \right)^{\gamma/2}
\]

(3.4)

where \( \rho_1 \) is the initial air density. Assuming a square pulse shape, Eq. (3.3) can be
integrated to give the growth of the luminous front, or visible spark length:

\[
x = (5/3)^{3/5} \left[ \frac{2 (\gamma^2 - 1) E_A}{\pi \Delta \rho \tan \alpha} \right]^{6/5} t^{3/5}
\]

(3.5)

Note that in Equations (3.3) and (3.5), \( \alpha \) is the half angle of the focused incident laser
beam; as such:

\[
\frac{1}{\tan \alpha} = \frac{f_l}{D_L}
\]

(3.6)

More realistic solutions for the spark length computed, for example, for a Gaussian
shaped laser pulse, can be found in [26]; however, the essential result is that the spark
length scales with the absorbed pulse energy and \( f_l/D_L \) of the focusing lens. The scaling
of spark length with pulse energy is illustrated in Figure 3.4, which shows high-speed
photographic images of laser-induced breakdown sparks created using a YAG laser
emitting at 1.06 μm and at 355 nm, and shows that the much larger input energies
required for spark formation in the IR result in substantially larger sparks than can be achieved at UV wavelengths.

![Spark comparison](image1)

Figure 3.4 High-speed camera images of sparks created using (left) $\lambda = 1.06 \ \mu m$, $E = 135 \ \text{mJ}$, $f_l/D_L = 12.5$ and (right) $\lambda = 355 \ \text{nm}$, $E = 6 \ \text{mJ}$, $f_l/D_L = 15$ (sea level pressure).

In practice, the size of the spark can also vary from pulse to pulse, presumably due to small variations in the conditions affecting the formation of the spark. The resulting spark length fluctuations are likely a result of variations in the amount of energy absorbed by the spark; for example, in [24] a variation in the effective breakdown threshold of $\pm 15\%$ is reported which, as shown in Figure 3.3, would result in significant variations in the absorbed energy. From the model for the spark length given in Equation (3.5), the sensitivity of the spark length to fluctuations in the absorbed energy is given by

$$
\frac{dx}{dE_a} = 0.272 \left[ \frac{2(\gamma^2 - 1)}{\pi \Delta \rho_1 E_A^3} \left( \frac{f_L}{D_L} \right)^{2/3} \right]^{1/3} \Delta t^{1/5}
$$

where Equation (3.6) has been used for $\tan \alpha$. Equation (3.7) shows that the amount of pulse-to-pulse length variation is also larger for more narrowly-converging laser beams focused using large $f_l/D_L$.

Large spark sizes, and particularly large variations in the spark size that result in a motion of the effective spark location, can interact with the optical system used to collimate the spark light for wavefront measurements, creating spurious noise on the
spark wavefronts which can interfere with the measurement of aero-optic aberrations. This effect highlights the importance of using the smallest sized spark that satisfies the brightness requirements of the wavefront-measuring optical system. As shown in this section, small spark sizes are achieved by employing a small $f_L/D_L$, as well as using small laser pulse energies which, as shown by Equation (3.2), can be achieved by using short pulse durations and short laser wavelengths as well as, again, a small $f_L/D_L$ ratio.

3.2 Measurement of Air Breakdown Threshold

As shown above, the size of the breakdown spark depends on the energy deposited by the laser beam into the spark. This deposited energy depends on the breakdown threshold of the ambient gas at the wavelength of the spark-forming laser.

The breakdown threshold was measured using the test setup shown in Figure 3.5. The measurements were made using two different lasers, an Nd:YAG laser emitting at 1064 nm, and a second Nd:YAG laser emitting at the frequency-tripled wavelength of 355 nm; these wavelengths were tested because, as shown in [18], the breakdown threshold of air is slightly lower at IR and UV wavelengths. The laser beam was focused using a lens to create a breakdown spark, and the minimum laser energy $E_{MIN}$ required to create a consistent spark for all laser pulses was measured. As shown by Equation (3.8), the breakdown energy depends on the ratio $f_L/D_L$ and not just on the focal length $f_L$ of the focusing lens; as such, $E_{MIN}$ was measured for several different values of $f_L/D_L$ over the range $f_L/D_L \approx 10$ to 40. If necessary, the laser beam diameter was expanded using a beam expander to ensure that the beam was sufficiently diffuse as to avoid damage to the focusing lens. The beam expander was constructed using a negative focal-length lens
followed by a positive focal-length lens, and is illustrated in Figure 3.5. All of the lenses were anti-reflection coated for the appropriate wavelength range; furthermore, since normal optical glasses absorb energy at UV wavelengths, special fused-silica lenses were used for the tests at 355 nm.

![Figure 3.5 Schematic of air breakdown measurements](image)

Solving Equation (3.2) for the pulse energy shows that:

\[
E_{\text{MIN}} - E_{\text{loss}} = 4\pi\Delta t \left( \frac{\lambda f_L}{\pi D_L} \right)^2 I_B
\]  

Equation (3.8) shows that a plot of \( E_{\text{MIN}} \) against the term in brackets will have a slope equal to the breakdown threshold \( I_B \). An additional term \( E_{\text{loss}} \) has been included in Equation (3.8) to account for energy losses in the measurements. Plots of \( E_{\text{MIN}} \) vs. \( 4\pi\Delta T(\lambda f_L/\pi D_L)^2 \) are summarized in Figure 3.6 for both the IR and UV lasers. Data are shown for different beam diameters \( D_L \) at each wavelength. The plot shows that the data are linear and that slope of the data are fairly similar for the different \( D_L \) at each wavelength. Furthermore, the y intercepts of the data, which corresponds to \( E_{\text{loss}} \) in Equation (3.8), are reasonable based on the experimental setup. In particular, for both the IR and UV results, the y intercept is larger for the large \( D_L \) values; this result can be attributed to the fact that in each case, the larger \( D_L \) beams were produced using a beam expander which imposed greater spherical aberrations onto the beam and hence greater energy losses. This overall consistency in the results shown in Figure 3.6 provides confidence that the slopes of the curves are accurate evaluations of the breakdown...
threshold, despite the fact that there was some subjectivity in the measurement of $E_{\text{MIN}}$. Approximate values for $I_B$, taken from the mean of the results for each wavelength in Figure 3.6, are $\sim 6.7 \times 10^{12}$ W/cm$^2$ at 1064 nm and $\sim 1.8 \times 10^{12}$ W/cm$^2$ at 355 nm.

The threshold pulse energy to achieve air breakdown was also measured as a function of ambient pressure by creating a spark inside a vacuum chamber. As shown in [5, 7], the breakdown irradiance (or pulse energy) should have an exponential dependence on pressure. Figure 3.7 shows a reasonably close fit of the data to the expected exponential dependence; variation of the data in Figure 3.7 was primarily a result of the difficulty in determining the exact energy at which breakdown was consistently achieved at every laser pulse.

Figure 3.6 Summary of air breakdown data
3.3 Determination of Optimum Laser for Beacon Formation

The above results show that the frequency-tripled YAG laser has considerable advantages with regard to aero-optic beacon formation compared to the YAG laser emitting in the IR. In particular, as shown in Figure 3.6, the breakdown threshold measured at the $\lambda = 355$ nm wavelength of the frequency-tripled Nd:YAG laser is lower than at its fundamental wavelength at $\lambda = 1064$ nm. Furthermore, as shown in Equation (3.1), it is possible to focus the UV beam to a smaller spot diameter $w_0$, so that the required pulse energy is also smaller. Finally, commercially-available versions of this laser typically have considerably shorter pulse durations than other types of lasers, around 5 ns. All of these factors mean that the pulse energy required for breakdown will be comparatively smaller than other potential lasers, with concomitant smaller breakdown spark dimensions.

Based on this, a “Brilliant” Nd:YAG laser with triple-harmonic generator (THG) package ($\Delta t = 3.5$ ns, $\lambda = 355$ nm) manufactured by Quantel Inc. was selected for beacon formation.
formation for this project. The maximum pulse energy of the laser is 100 mJ, which based on Figure 3.7, should be adequate to achieve breakdown at altitudes over 30,000 ft. The pulse repetition frequency of the laser is fixed at 10 Hz. As discussed in Chapter 1, this pulse rate is more than adequate for synchronization of an AO correction system with the “regularized” aero-optic aberrations of a forced shear layer as part of a feedforward AO correction strategy [5, 7]. The remainder of this thesis shows results obtained using this frequency-tripled Nd:YAG laser.

3.4 High-Speed Camera Measurements

High-speed images of the breakdown spark were made using a Photron Fastcam SA1-1.1 monochrome high-speed imaging camera. The images were acquired at a frame rate of $5 \times 10^5$ images per second (2 $\mu$s per image). A schematic of the experimental setup is shown in Figure 3.8.

![Schematic of high-speed camera measurements.](image)

Selected high-speed camera images of the UV spark are shown in Figure 3.9. In the images shown in Figure 3.9, the direction of the laser beam is from right to left. For comparison, a typical series of spark images acquired using an IR-laser is included in the
The $f_l/D_L$ values listed in the figure give the ratio of focal length to incoming beam diameter for the laser-focusing lens. Figure 3.9 shows that the UV sparks are significantly smaller than the IR spark, as expected. Furthermore, almost all of the image sets acquired for the UV spark spanned only a single frame at the $5 \times 10^5$ image-per-second frame rate of the camera, with only a very few images spanning 2 frames. For the cases that spanned two frames, the second image of the spark was typically very dim as shown in case 3 of Figure 3.9, indicating that the spark was nearly extinguished. It should be noted however, that the high-speed camera images are not perfectly synchronized with the initiation of the spark, so that the spark may actually initiate near the end of the first camera image; as such, the spark duration from the high-speed camera images is at most 2 $\mu$s. More accurate measurements of the spark lifetime using a fast photodiode are presented in Chapter 4.

Results of an analysis of the high-speed camera images are summarized in Figure 3.10. The figure shows the variation of the spark length and diameter as a function of laser energy and $f/D$ of the focusing lens, where “length” is defined as the extent of the spark along the optical axis of the beam and “diameter” is the extent of the spark perpendicular to the optical axis. Figure 3.10(a) shows that the spark is generally much longer than its diameter, that the spark length increases with the pulse energy and that generally greater energies are required to generate the sparks as $f_l/D_L$ increases; all of these trends agree with the predictions of Equations (3.2) to (3.6). Figure 3.10(b) shows that variations in the spark dimensions, particularly the rms of the spark length, increase with increasing $f_l/D_L$, in agreement with Equation (3.7). It should be noted that, since a spherical lens was used to focus the laser, some of the variation in laser energy required
to generate the sparks may have been due to spherical aberrations produced by the focusing lens; however, Figure 3.10 generally conforms to the expected spark behavior outlined in Section 3.2.

Figure 3.9 High-speed camera images of air breakdown spark. Images are accurately scaled to each other.

Figure 3.10 Spark dimensions from high-speed camera images
3.5 Spark Wavefront Measurements

Measurements of the spark wavefronts were made using the experimental setup shown in Figure 3.11. The wavefront sensor was a Shack-Hartmann configuration with a 33 x 44 lenslet array manufactured by AMO Wavefront Sciences. As shown in Figure 3.11, there were no intervening aberrations between the breakdown spark and the wavefront sensor, so that the measured wavefronts show the “baseline” aberrations produced by the spark and optical system. The wavefronts were acquired in a nearly backscatter arrangement, which is the probable arrangement that would be used in an operational system; the 20° angle between the optical axes of the wavefront and laser beams was the minimum possible to avoid interference between the two sets of optical components.

![Figure 3.11 Optical setup for baseline beacon wavefront measurements.](image)

The wavefront data were acquired in the form of optical path differences (OPD’s) from the mean over the measurement aperture. These wavefront data were further processed by removing tip, tilt and piston, and by removing the mean wavefront of the data set. A plot of the root-mean-square of the OPD variations (i.e. $\text{OPD}_{\text{rms}}$) over the measurement aperture as a function of the laser pulse energy is shown in Figure 3.12 for
different ratios of \( f_C/D_C \) of the lens used to collimate the light from the beacon. The figure shows that the \( \text{OPD}_{\text{rms}} \) of the baseline beacon wavefronts decreases significantly as \( f_C/D_C \) increases, and increases as the pulse energy increases for a given value of \( f_C/D_C \).

Insight into the trends shown in Figure 3.12 can be obtained by examining the rms of the OPD variations over the measurement aperture for a typical run, shown in Figure 3.13. The large, circularly-shaped OPD variations around the center and edge of the aperture shown in Figure 3.13 are characteristic of variations in the overall curvature, or “focus,” of the spark wavefront. As shown in Figure 3.14, these kinds of focus variations of the collimated wavefront can be caused by an effective motion of the origin of a spherical wavefront towards or away from the collimating lens.

![Figure 3.12 OPD_{rms} of unaberrated beacon wavefront as a function of f/D of the collimating lens. f/D of lens used to focus the laser was f_l/D_l = 15.](image)

Since the wavefront sensor viewed the spark nearly parallel to the optical axis of the driving UV laser beam, the focus variation in the spark wavefront was therefore likely caused by a variation in the effective location of the spark which, as discussed in Section 3.1 and shown in Figure 3.10, is largest along the optical axis of the driving laser beam.
Figure 3.13 OPD$_{\text{rms}}$ (µm) over aperture for typical baseline beacon wavefront measurement.

Figure 3.14 Illustration of how motion of a point source towards/away from a collimating lens creates focus-type distortions of the resulting collimated wavefront.

A model for the OPD$_{\text{rms}}$ due to a small displacement $\varepsilon$ away from the focal length of a lens can be derived using the “lensmaker’s equation” [29]:

$$\frac{1}{f_c} = \frac{1}{S_1} + \frac{1}{S_2}$$  \hspace{1cm} (3.9)

where the lengths S1 and S2 are defined in Fig. 3.15. For the case under consideration, S1 = $f + \varepsilon$, and S2 is the residual radius of curvature R of the wavefront of the beam after passing through the lens:
\[
\frac{1}{f_C} = \frac{1}{f_C + \varepsilon} + \frac{1}{R} \quad (3.10)
\]

Solving for \( R \):

\[
R = \frac{\varepsilon}{f_C^2 + \varepsilon} \approx \frac{\varepsilon}{f_C^2} \quad (3.11)
\]

The OPL corresponding to the wavefront is thus:

\[
OPL = \sqrt{R^2 - (y^2 + z^2)} \quad (3.12)
\]

and the \( \text{OPD}_{\text{rms}} \) over an aperture with diameter \( D_C \) is

\[
\text{OPD}_{\text{rms}} = \sqrt{\frac{\iint (OPL - \bar{OPL})^2 \, dx \, dy}{\iint dx \, dy}} \quad (3.13)
\]

Solution of Equation (3.13) gives:

\[
\text{OPD}_{\text{rms}} = \frac{\varepsilon}{28 \left( \frac{f}{D_C} \right)^2} \quad (3.14)
\]

Figure 3.15 Schematic demonstrating the relationship shown in Equation (3.9)
Using Equation (3.14), the effective spark motion amplitude $\epsilon$ was computed for the data shown in Figure 3.7. Figure 3.16 shows that the resulting $\epsilon$ data, except for a few outliers, correlate well with the laser pulse energy. The positive correlation of $\epsilon$ with the laser pulse energy, as shown in Figure 3.16, agrees with the same positive correlation shown in the high-speed camera measurements in Figure 3.10, and with Equation (3.7). A linear curve fit to the computed $\epsilon$ data is:

$$\epsilon = 0.0239 \cdot E + 0.122$$

(3.15)

where $\epsilon$ is in mm and $E$ is in mJ. Substitution of Equation (3.15) into Equation (3.14) gives:

$$\text{OPD}_{rms} = \frac{0.854 \cdot E + 4.34}{\left(\frac{f_c}{D_c}\right)^2} \quad \text{(Note: } f_c/D_c = 15)$$

(3.16)

where $\text{OPD}_{rms}$ is in $\mu$m.

The data of Fig. 3.11 are replotted with the fit of Equation (3.16) in Figure 3.17, which shows a good comparison between the fit and the data except for the $f_c/D_c = 44$ data (which appear as “outliers” in Figure 3.16); it is likely that the wavefront distortion due to spark motion for the $f_c/D_c = 44$ data is sufficiently small that other noise sources besides the spark motion become more relatively important, making Equation (3.16) less accurate for this case.
Figure 3.16 Apparent spark motion $\varepsilon$ computed from OPD$_{\text{rms}}$ of baseline beacon wavefronts

![Figure 3.16](image)

Figure 3.17 Comparison of focus model for baseline beacon aberrations, Equation (3.16), with measured wavefronts. Red line shows the OPD$_{\text{rms}} = 0.05 \, \mu m$ noise level

![Figure 3.17](image)

Based on the good fit shown in Figure 3.16, Equation (3.16) can be used as a reasonably accurate model for the aberrations on the baseline spark wavefronts. In effect, Equation (3.16) represents a “noise” level below which it would not be possible to measure aero-optic aberrations using the return light from the spark because these aberrations would be smaller than the aberrations that already exist on the spark wavefronts. For a nominal wavelength of 1 $\mu m$, the large-aperture approximation, Equation (1.6), shows that the Strehl ratio for an OPD$_{\text{rms}}$ of 0.05 $\mu m$ is still over 90%; as
such, if the $\text{OPD}_{\text{rms}}$ of the baseline spark wavefronts was 0.05 $\mu$m, then it would still be possible to detect aberrations that cause just a 10% reduction in Strehl ratio. This 0.05 $\mu$m $\text{OPD}_{\text{rms}}$ level is shown in Figure 3.17 (red line) as a nominal noise “limit,” and shows that this noise limit can be met by a large number of realistic optical designs. The ability to measure realistic aberrations using the breakdown spark is investigated in further detail in Section 3.7.

3.6 Spark Brightness

Beacon optical data has been presented thus far as a function of the input laser pulse energy $E$. This method of presentation facilitates immediate comparison of the experimental results with relationships describing the physics of the breakdown spark, Equations (3.2) to (3.7). For the design of an operational aero-optic beacon system, however, design specifications are more likely to be placed on the beacon brightness required to achieve a satisfactory signal-to-noise ratio for the optical instrumentation, with the laser pulse energy being adjusted to whatever level generates this brightness.

The spark brightness was measured using a Newport model 842PE optical power meter with a model 918D-SL-003 photodiode-type photodetector. The photodiode-type detector is well suited for these measurements because of the high sensitivity of the detector. A drawback of the detector is that its sensitivity is wavelength dependent (although other types of detectors also exhibit some wavelength sensitivity); this means that an accurate measurement of the spark brightness requires additional information on the spectral composition of the spark as well as the wavelength sensitivity dependence of the detector.
The measured spectrum for an electrically-induced air plasma is shown in Figure 3.18 [30]. The spectrum shows that most of the visible light emitted from the plasma occurs at short wavelengths (i.e. violet/blue), and is associated with N\textsubscript{2} molecular transitions. A photograph of the plasma that was investigated in [30] is shown in Figure 3.19 and, although there may be some color distortion in the photograph, the color of the plasma in Figure 3.19 appears at least qualitatively similar to the color of the air breakdown sparks that were observed in experiments. As such, the spark brightness measurements were performed with the detector sensitivity fixed to blue light in the range 410 to 420 nm.

Figure 3.18 Visible-band emission spectrum for an air plasma, [30].
The brightness measurements were performed by focusing the spark light onto the detector using a 200 mm focal length lens with 23 mm diameter aperture. The detector was oriented at approximately a 45° angle to the optical axis of the laser, in the backscatter direction. The total light energy from the spark was computed from the measured light power based on the geometry of the experimental setup and assuming isotropic radiation from the spark. Figure 3.20 shows that the beacon light energy $E_b$ scales linearly with the input laser beam pulse energy:

$$E_b = 0.00243 E - 0.0242 \text{ mJ}$$  \hspace{1cm} (3.17)

The constant in Equation (3.17) represents the fact that a minimum level of laser pulse energy $E$ is required before breakdown is achieved and the spark begins to emit optical energy. The specific value of -0.0242 mJ from the fit to the data in Figure 3.20 is probably an inaccurate value for this constant due to losses of energy in the optical system, for example, losses in the lenses used to focus the laser beam and to focus the spark light onto the photodetector. Equation (3.18) is an estimated relation for the beacon
light energy, in which the constant in Equation (3.17) has been adjusted to give a nominal beacon light energy at the estimated breakdown energy of 3.28 mJ for sea level conditions determined from the results of Section 3.2.

\[ E_b \approx 0.00243 E - 0.0031 \]  

(3.18)

![Figure 3.20 UV beacon light energy.](image)

3.7 Wavefront Measurements of Static Aberrations

The optical characterization of the laser breakdown spark presented thus far has shown that the behavior of the spark conforms to established theory. Furthermore, the experimental data indicate that the noise levels on the spark wavefronts are caused primarily by spark-to-spark variations in the effective location of the spark, and baseline noise levels for the spark wavefronts in the absence of any intervening aberrations were measured, Figure 3.12. The next step in the investigation was to test the accuracy with which aberrations with realistic amplitudes could be measured using the return light from the spark. It was decided to first investigate aberrations with OPD’s corresponding to realistic compressible shear layer flows, since these flows represent one of the most important types of aero-optic flows.
Peak-to-peak and rms OPD’s measured in CSLWT are summarized in Table 3.1. This wind tunnel mixes co-directional high- and low-speed flows at high subsonic flow speeds (up to Mach 1.0) to create a shear layer that is aero-optically active. As shown in [5, 11, 31], the CSLWT flow and associated aberrations are representative of the kinds of aero-optic flows likely to be encountered on full-scale flight vehicles. Furthermore, the CSLWT shear layer flow can be regularized using mechanical forcing that oscillates the trailing edge of the splitter between the high- and low-speed flows in a direction perpendicular to the flow direction, resulting in more regular, sinusoidal aero-optic aberrations. Table 3.1 shows that the peak-to-peak OPD of the aberrations in the CSLWT at a typical measurement distance of 400 mm downstream of the splitter trailing edge is approximately 1 μm and slightly greater when the shear layer is forced.

The suitability of the spark for wavefront measurements was tested by placing stationary (non-moving) aberrations into the return light path from the spark. Using stationary aberrations allows testing of the aero-optic beacon system in a more controlled environment without the additional measurement noise due to vibrations that typically occur during wind-tunnel measurements of actual aero-optic flows. The simulated, stationary aberrations used in the tests were selected to give OPD amplitudes similar to the levels associated with typical compressible shear layer flows as shown in Table 3.1. Through some trial and error, it was found that aberrations of comparable amplitude could be produced by passing the light through a 1/16\textsuperscript{th}-inch thick sample of Plexiglas.
TABLE 3.1
AERO-OPTIC ABERRATIONS MEASURED IN THE NOTRE DAME
COMPRESSIBLE SHEAR LAYER WIND TUNNEL, AT A DISTANCE X = 400 MM
FROM THE SPLITTER [31].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean (μm)</th>
<th>Minimum (μm)</th>
<th>Maximum (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unforced OPD_{rms}</td>
<td>0.233</td>
<td>0.097</td>
<td>0.697</td>
</tr>
<tr>
<td>Unforced OPD_{pp}</td>
<td>1.119</td>
<td>0.567</td>
<td>2.563</td>
</tr>
<tr>
<td>Forced OPD_{rms}</td>
<td>0.291</td>
<td>0.1101</td>
<td>0.568</td>
</tr>
<tr>
<td>Forced OPD_{pp}</td>
<td>1.300</td>
<td>0.605</td>
<td>2.283</td>
</tr>
</tbody>
</table>

Note: The high-speed flow Mach number was 0.78 and the low-speed flow Mach number was 0.12.

The optical setup used for the stationary aberration measurements is shown in Figure 3.21. The spark was viewed in a backscatter arrangement at an angle of approximately 20° to the optical axis of the UV laser beam. The Plexiglas plate was placed in the optical path of the spark light and an aperture was placed on the collimating lens for the spark beam such that the diameter of the spark beam at the Plexiglas aberration was approximately 17 mm. A 25 mm diameter reference beam was also passed through the Plexiglas at the same location as the spark beam; the anisoplanatism [32] between the reference and spark measurements of the aberration therefore consisted of the difference in beam diameter and angle at the aberration, as well as the difference in wavefront shape at the aberration (spherical for the spark beam and planar for the reference beam).
Figure 3.21 Optical setup for stationary aberration measurements.

Lens parameters for the optical setup were $f_l/D_L = 15$, $f_C/D_C = 24$, and the laser pulse energy was $E = 11$ mJ. The measured OPD$_{rms}$ of the unaberrated spark wavefront was 0.036 μm, which closely agrees with the curve fit of Equation (3.16). Peak-to-peak OPD variations on the unaberrated spark wavefront were approximately 0.2 μm, or around 1/5$^{th}$ of the typical peak-to-peak shear layer aberrations shown in Table 3.1. Using the large-aperture approximation shown in Equation (1.6) for a nominal 1 μm wavelength, the Strehl ratio corresponding to the OPD$_{rms}$ of the unaberrated spark wavefront is 95%. This indicated that the baseline spark wavefront noise should not significantly impact the ability to measure the 1 μm peak-to-peak simulated aberration.

A sample of reference-beam and spark wavefronts for six different simulated aberrations is shown in Figure 3.22. In each of the 6 images, the aberration measured by the collimated reference beam is shown in the left while the aberration measured by the spark beam is shown in the right. The figure shows that the Plexiglas plate successfully produced aberration amplitudes in the range of Table 3.1. Furthermore, the aberrations
measured by the spark closely matched the aberration measured by the collimated reference beam, even without any corrections applied to the spark wavefronts to adjust for anisoplanatism effects [32]. The average cross-correlation value between the spark and collimated wavefronts, also without any corrections applied for anisoplanatism effects, was approximately 75%. It should be noted, however, that the worst cross-correlations between the two beams corresponded to situations in which the spark beam happened to measure a region of the Plexiglas plate over which the OPD variation was much smaller than the nominal 1 μm peak-to-peak variations that typically occur in shear layer flows, so that the baseline spark wavefront noise had an unrealistically-large effect on the comparison between the two beams. For cases in which the spark beam measured peak-to-peak OPD’s closer to 1 μm, the cross-correlation between the two beams was around 90%.

Figure 3.22 Samples of wavefronts measured through 1/16th-inch thick Plexiglas plate. Spark wavefront is shown on the right of each image.
3.8 Discussion

The data presented in this chapter have shown that the finite dimensions of the laser-induced air breakdown spark, and more importantly, the spark-to-spark variations in the effective location of the spark, have significant impact on the ability to use the return light from the spark to measure aero-optic aberrations. The data have also shown that the behavior of the spark size and shape conform to the established physics for the laser-breakdown process, and experimentally-determined relationships have been defined describing the nature and magnitude of wavefront “noise” associated with the return light from the spark. Finally, wavefront measurements of stationary (non-moving) aberrations with amplitudes similar to the aberrations produced in the University of Notre Dame Compressible Shear Layer Wind Tunnel have shown that it is possible to accurately measure realistic aberrations using the spark, despite the “noise” on the baseline wavefronts from the spark.
CHAPTER 4:
SPARK LIFETIME

The spark lifetime has important implications on the usefulness of the spark for measuring aero-optic aberrations. In particular, a long spark lifetime would raise the possibility that the position or dimensions of the spark might be significantly distorted by convection effects if the spark were created in a high-speed flow. This kind of distortion of the spark would also interact with the optical system used to measure the spark wavefronts, creating another source of spurious noise on the spark wavefronts in addition to the noise resulting from the spark size and position fluctuations discussed in Chapter 3.

4.1 Lifetime Measurements

The spark lifetime was measured using a FDS02 fast photodiode manufactured by Thorlabs. This photodiode is designed for telecommunications applications and has a nominal rise time of 47 ps. The measurements were made by focusing the spark light onto the photodiode using a 50 mm focal length lens. Figure 4.1 shows an average oscilloscope trace of 16 spark emissions for a laser pulse energy of 45 mJ, and indicates that the majority of the light emitted from the spark is emitted within a 20 ns time period. This lifetime is significantly shorter than values reported in [14] (170 ns) and [33] (~ 90 ns); however, the breakdown sparks investigated in these studies were created using YAG lasers emitting at the fundamental (1064 nm) and first harmonic (532 nm), with
longer pulse times, and using significantly greater pulse energies. As such, it is possible that the longer spark lifetimes reported in [14, 33] are the result of the different parameters of the driving laser beam. In particular, as shown in Section 3.1, the longer wavelengths and pulse durations used in [27, 28] would result in more energy absorbed by the spark, and therefore a longer lifetime.

![Oscilloscope time trace of emission from spark measured using fast photodiode.](image)

The maximum distortion of the spark corresponds roughly to the flow convection distance over the spark lifetime as seen in Equation (1.7). It is anticipated that the aero-optic beacon system will be deployed on aircraft that operate at subsonic velocities. In this case, the 20 ns spark lifetime results in a convection distance, \( x \), on the order of a few micrometers even for flow speeds as high as Mach 1. This convection distance is orders of magnitude less than the normal fluctuations in spark dimensions shown in Figure 3.9, indicating that the spark lifetime should have a negligible effect on the optical quality of the aero-optic beacon system.

4.2 Wind Effects on UV Laser Breakdown Spark

The effect of flow on the spark characteristics was measured in one of Notre Dame’s high-speed indraft wind tunnels. The measurements were performed in a 4” x 4”
test section at a Mach number of 0.75. Because of the indraft configuration of the tunnel, the wind-on test-section pressure at M=0.75 was reduced by 32 kPa or 0.31 atmospheres below atmospheric pressure.

High-speed camera images of the spark were acquired both wind-on and wind off using a similar experimental setup to that shown in Fig 3.8, except that the spark was formed inside a wind-tunnel. The mean and rms of the spark length and diameter from the camera images are summarized in Figure 4.2 for wind-off and wind-on conditions. The figure shows good agreement with the data plotted in Figure 3.10, except that the laser energies are slightly higher for the Figure 4.2 data; this may be a result of transmission loss through the fused silica window used to pass the laser beam into the wind-tunnel test section. Figure 4.2 shows that the length and diameter of the spark does not appear to be “smeared” or distorted by the wind; in fact Figure 4.2 indicates that the spark is slightly smaller, and exhibits less variation in its dimensions, with the wind on than with the wind off. This apparent wind-on “shrinking” of the spark is most likely a result of the fact that the test-section pressure in the indraft wind tunnel was significantly reduced at the M=0.75 wind speed. As shown in Figure 3.6, the breakdown threshold increases as the air pressure decreases and measurements showed that the breakdown threshold at the M=0.75 operating pressure was approximately 6 mJ greater than at the wind-off ambient pressure. The spark length and diameter data are replotted in Figure 4.3 with the wind-on energies corrected for the 6 mJ increase of breakdown threshold; this plot shows a very close comparison in the spark dimensions both wind-off and wind-on, and indicates that the wind has essentially no effect on the spark.
The high-speed camera images were also analyzed to determine the centroid of intensity of the images. This analysis showed that the position of the spark changed by less than 0.05 mm between the wind-off and wind-on conditions; this value is significantly less than the effective spark motion \( \varepsilon \) shown in Fig. 3.16 for wind-off conditions, showing that the spark was neither distorted nor convected significantly by the wind.

Figure 4.2 Comparison of wind-off and wind-on spark dimensions from high-speed camera images, \( f_L/D_L = 25 \). Test-section speed for wind-on measurements was \( M=0.75 \).
4.3 Discussion

The data obtained with the fast photodiode shows that the spark lifetime is on the order of 20 ns. Furthermore when the spark was formed inside the wind tunnel the spark centroid position did not change more than 0.05 mm when the tunnel is turned on.

These data indicate that it should be possible to use a laser-breakdown beacon system in high-subsonic flows with negligible distortion of the spark position and dimensions, or the optical character of the spark, due to flow effects.
Aero-optic flows, such as compressible turbulent boundary layers and shear-layer flows, have been shown to produce severe aberrations on a traversing beam of light. For the most part, approaches to correct for aero-optic aberrations, including both feedback or feedforward adaptive-optic approaches, are predicated on a method of accurately measuring the aberrations in the first place. The objective of this thesis has been to investigate issues related to the use of laser-induced air-breakdown sparks as illumination sources for the measurement of aero-optic flows.

In Chapter 2 of this thesis, the effects of anisoplanatism between the beacon beam and the main beam were investigated. It was shown that wavefront measurements acquired using the return light from the spark could be compensated using a Minimal Mean-Square Estimation technique to produce an estimate of the aberrations that existed on a collimated reference beam of light that traversed the same aero-optic flow, despite the fact that the light from the spark had significant spherical curvature and did not sample the full region traversed by the reference beam.

In Chapter 3, the optical character of laser-breakdown sparks was investigated in detail. The work showed that the finite dimensions of the laser-induced air-breakdown spark, and more importantly, spark-to-spark variations in the spark location, have significant impact on the ability to use the return light from the spark to measure aero-
optic aberrations. In particular it was shown that spark-to-spark motion of the effective location of the beacon produces distortions in the spark wavefronts; these wavefront distortions act as a baseline “noise” that limits the ability of the beacon system to detect aero-optic aberrations that have an amplitude below this noise level. A simple model was developed for predicting the magnitude of this wavefront “noise” based on the optical systems used to generate and measure the spark. Further, it was shown that the magnitude of the baseline noise was lowered if the spark size was minimized, which was shown to be achieved if the laser used to generate the spark operated at short wavelengths and had a short pulse duration.

In Chapter 3, the optical character of air breakdown sparks was investigated in wind-off conditions; the objective of Chapter 4 was to determine if the optical character of the spark would be affected by flow. Measurements made using a fast photodiode showed that the spark lifetime was on the order of only 20 ns when generated using a YAG laser operating in the UV, indicating that any motion or distortion of the spark due to convection effects would be negligible at typical aircraft cruise speeds (M~0.8). This finding was confirmed by high-speed photographs of the spark in a M=0.75 flow, which showed essentially no distortion of the spark wind-on dimensions.

In summary, the results of this investigation have shown that it is possible to measure aero-optic aberrations with adequate accuracy using a laser-breakdown spark as long as the optical systems used to generate the spark and collect the return light from the spark are designed to minimize noise due to spark “jitter.” Based on this result, it is possible to make the following suggestions for future work:

- determination of the spark brightness levels necessary for optical measurements with a realistic full-scale system,
• investigation of how the spark system would be integrated into a feedforward AO control approach.

Both of the above topics would generate additional guidelines for the design of the laser-spark system that would need to be accommodated with the guidelines determined in this thesis for the mitigation of the spark baseline noise.


