DESIGN OF CRASHWORTHY STRUCTURES WITH CONTROLLED BEHAVIOR IN HCA FRAMEWORK

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by

Punit Bandi

________________________________________
James P. Schmiedeler, Co-Director

________________________________________
Andrés Tovar, Co-Director

Graduate Program in Aerospace and Mechanical Engineering
Notre Dame, Indiana
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Abstract

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The field of crashworthiness design is gaining more interest and attention from automakers around the world due to increasing competition and tighter safety norms. In the last two decades, topology and topometry optimization methods from structural optimization have been widely explored to improve existing designs or conceive new designs with better crashworthiness. Although many gradient-based and heuristic methods for topology- and topometry-based crashworthiness design are available these days, most of them result in stiff structures that are suitable only for a set of vehicle components in which maximizing the energy absorption or minimizing the intrusion is the main concern. However, there are some other components in a vehicle structure that should have characteristics of both stiffness and flexibility. Moreover, the load paths within the structure and potential buckle modes also play an important role in efficient functioning of such components. For example, the front bumper, side frame rails, steering column, and occupant protection devices like the knee bolster should all exhibit controlled deformation and collapse behavior.

The primary objective of this research is to develop new methodologies to design crashworthy structures with controlled behavior. The well established Hybrid
Cellular Automaton (HCA) method is used as the basic framework for the new methodologies, and compliant mechanism-type (sub)structures are the highlight of this research. The ability of compliant mechanisms to efficiently transfer force and/or motion from points of application of input loads to desired points within the structure is used to design solid and tubular components that exhibit controlled deformation and collapse behavior under crash loads. In addition, a new methodology for controlling the behavior of a structure under multiple crash load scenarios by adaptively changing the contributions from individual load cases is developed.

Applied to practical design problems, the results demonstrate that the methodologies provide a practical tool to aid the design engineer in generating design concepts for crashworthy structures with controlled behavior. Although developed in the HCA framework, the basic ideas behind these methods are generic and can be easily implemented with other available topology- and topometry-based optimization methods.
Dedicated to my parents and Dr. John E. Renaud.
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SYMBOLS

\( E \)  Elastic modulus

\( F \)  External force

\( K \)  Stiffness matrix

\( K_p \)  Proportional control gain

\( \bar{K}_p \)  Adaptive proportional control gain

\( K_w \)  Proportional control gain for weight change

\( M^f \)  Mass fraction constraint target

\( \hat{N} \)  Number of neighboring cells

\( N \)  Total number of elements or design variables

\( S \)  Field variable state

\( \bar{S} \)  Effective field variable state

\( S^* \)  Setpoint or target setpoint

\( T^* \)  Average thickness target

\( c \)  Compliance of the structure

\( d, u \)  Displacement field

\( e \)  Error, typically between field variable and setpoint
\( i \) subscript typically used for element indexing

\( j \) HCA inner loop iteration counter

\( k \) HCA global iteration counter

\( p, q, r \) Penalization parameters

\( t \) Wall thickness

\( v, v_0 \) Velocity

\( w \) Weighting factor

\( x \) Design variable (relative density)

\( \Delta w_{\text{max}}^{\text{allow}} \) Maximum allowable change in weight

\( \Delta x_{\text{max}}^{\text{allow}} \) Maximum move limit on the design variable change

\( \mathcal{L} \) Lagrangian

\( \beta \) CA state

\( \epsilon \) Global convergence tolerance

\( \varepsilon \) Strain

\( \lambda \) Lagrange multiplier

\( \rho \) Density

\( \nu \) Poisson’s ratio

\( \sigma \) Stress

\( \sigma_y \) Yield stress

\( \sigma_{\text{eff}} \) Effective stress
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CHAPTER 1

INTRODUCTION

Automobile accidents kill an estimated 1.2 million people worldwide each year and injure about forty times this number \cite{79}. Over the past couple of decades, efforts have been made to reduce injury levels by adding many features in the car to avoid a crash, by improving traffic control and by enhancing highway construction. These measures have reduced injury rates, although the number of fatalities due to crashes are still increasing because of the ever increasing number of vehicles on the road. Figure 1.1 shows this trend for the United States based on data collected from the Fatality Analysis Reporting System (FARS). This trend has motivated automakers and the National Highway Traffic Safety Administration (NHTSA) to set higher safety standards for vehicles. This translates into better crashworthiness of the vehicle structure and more effective occupant restraint systems. With increasing fuel costs and competition among automakers, a lot of attention is paid to lowering vehicle weights, which makes achieving high standards of safety even more difficult.

1.1 Vehicle Crashworthiness: Goals & Requirements

First coined in the aerospace industry in the 1950s, the term “crashworthiness” provides a measure of the ability of a structural system or component to
Figure 1.1. Variation in the (a) fatality rate and (b) number of registered vehicles in the United States from 1994 to 2009 based on data collected from the FARS encyclopedia.

protect the occupants in survivable crashes. Similarly in the automotive industry, vehicle crashworthiness indicates a measure of a vehicle’s ability to plastically deform yet maintain sufficient space for occupants under reasonable crushing loads \cite{29}. Vehicle structures have evolved a lot over the last 50 years and with them, crashworthiness goals and requirements as well. Initially, most of the vehicle body was made of wood, and the goal of crashworthiness was to reduce vehicle deformations as much as possible. At present, vehicle bodies are manufactured primarily of stamped steel panels. Designers create vehicle structures to maintain the integrity of the passenger cabin while simultaneously controlling the crash deceleration pulse to fall below human survivable levels. Therefore, the goal of design for crashworthiness is to optimize a vehicle structure that can absorb significant crash energy by controlled deformations while maintaining adequate space so that residual energy can be managed by the vehicle restraint system to minimize crash
loads transferred to the occupants.

Each component in the vehicle body has a specific purpose in terms of crashworthiness. In order for a vehicle structure to yield a satisfactory deceleration pulse for a range of occupant sizes and crash speeds, the following component requirements must be met:

- The front structure should be stiff, yet deformable, with crush zones to absorb crash energy resulting from frontal impacts through plastic deformations and to prevent severe intrusions into the passenger cabin. For example, the front bumper beam and part of the frame rails prevent severe intrusion into the engine hood and passenger compartment, whereas the rear portion of the frame rails and shock tower help in reducing the crash force transferred to the occupant. Figure 1.2 shows the relative locations of these components in a typical vehicle structure.

- The side structure comprised mainly of the B-Pillar and side floor panel should be designed to minimize the intrusion in side impacts and prevent the doors from opening due to crash loads.

- The roof of the vehicle should be strong for rollover protection.

1.2 Modeling and Design for Crashworthiness

The increasing demand from customers, regulators and the media for safer and lighter vehicles is constantly driving the auto industry to design and build vehicles with improved crashworthiness and effective restraint systems. For example, the automakers need to build vehicles that comply with safety standards set by the Federal Motor Vehicle Safety Standard (FMVSS), New Car Assessment Program
Figure 1.2. Body-In-White (BIW) image of Chevrolet Cobalt 2010

(NCAP), and Insurance Institute for Highway Safety (IIHS), among others. These programs/agencies provide safety ratings for the vehicles based on their performance in some standard crash tests. In addition, competition among automakers to build high quality vehicles in shorter design cycles is making use of analytical and numerical modeling and design techniques preferable to actual prototype testing unless absolutely necessary.

Currently, most designers calculate vehicle and occupant response under crash loadings using either Lumped Mass-Spring (LMS) models or Finite Element (FE) models. LMS models, introduced in the early 1970s [51], approximate the vehicle system with a one-dimensional lumped mass-spring system. Nonlinear FE models, introduced in the mid-1980s, require a complete and detailed description of the component’s geometry and material properties. Because of their simplicity and fast turnaround time, LMS models are used in the early design stage for estimating the approximate stiffness of various components and conducting subsequent
parametric studies. However, there are some limitations of LMS models that make them less appealing during the design and optimization of modern vehicle structures. An LMS model is 1-D and does not accurately represent intricate modern sheet metal-based components. Moreover, the linear behavior of an LMS model cannot represent nonlinear behavior like buckling, plasticity, contact, etc., typically encountered during crash loadings. Due to these shortcomings, nonlinear FE models quickly gained popularity among designers. Continually increasing computational power and improved numerical structural mechanics techniques have further fueled this acceptance.

Developing in parallel, the field of structural optimization (see Section 2.1 for details) combined with (non)linear finite element modeling to open new doors for designers to systematically improve existing designs or develop new concept designs from scratch in shorter design cycles. In the beginning, use of structural optimization in vehicle design was limited to improving bending and torsional stiffnesses of the vehicle body. The earliest use of formal optimization techniques for crashworthiness design is found in the work of Bennett et al. [17], Song [98] and Lust [68]. More recent applications of optimization in vehicle crashworthiness design involving better representation and modeling of vehicle components and subsystems can be found in the work of Gu et al. [37], Kurtaran et al. [60], Dias and Pereira [26], Fang et al. [33, 34], Craig et al. [24], Liao et al. [65], Horstmeier et al. [43], and Hosseini-Tehrani and Bayat [44], among others. Although these efforts are significant in their attempt to consider full vehicle crash simulations and crashworthiness design objectives more systematically, most of them are limited to size and shape optimization with a small number of design variables. Since crashworthiness design problems are highly nonlinear and involve compu-
tationally expensive finite element analysis, shape and size optimization with a relatively small number of design variables using response surface methods, regression models or metamodels presents an attractive option. However, shape and size optimization can only help in improving an existing design. In order to conceive of new designs or improve existing designs significantly, topology and topometry optimization \[95\] are more commonly used.

The first practical use of topology optimization in crashworthiness design of continuum structures is credited to Mayer et al. \[71\], who considered an objective of maximizing the energy absorption subject to a volume constraint. Pedersen \[80, 81\] later devised a method for topology optimization using 2D frame structures. In his work, designs with a desired energy absorption history are developed using rigorously computed sensitivities. Numerical sensitivity calculations, a very large number of design variables, and computationally expensive finite element analysis make the use of topology and topometry optimization for crashworthiness design one of the most challenging problems in structural optimization.

In order to tackle these challenges, many heuristic methods have been proposed. In his heuristic scheme, Soto \[99\] implemented a prescribed plastic strain/stress (PPSS) criterion that varies the density within the design domain to achieve a prescribed distribution of plastic strains and stresses with a constraint on mass. Forsberg and Nilsson \[35\] proposed another non-gradient technique using thickness as the design variable and element internal energy density (IED) for design updates. However, this methodology can only handle a small set of problems by operating on the thickness of elements. Patel et al. \[76\] developed the HCA method, originally proposed by Tovar et al. \[107\], to address the crashworthiness design problem. The HCA method for crashworthiness design uses a concept sim-
ilar to a ‘fully stressed design’ [39] in which material is distributed within the
design domain to achieve a uniform internal energy distribution. Mozumder [72]
and Mozumder et al. [73] later extended the HCA method for topometry opti-
mization of sheet metal structures for crashworthiness design using thickness as
the design variable. Occupant injury-related criteria in the form of constraints
on maximum displacement and peak force are introduced in the HCA method by
controlling the mass of the structure. In this work, the basic HCA framework
with some modifications is used in developing new methodologies to tackle more
complex objectives in topology and topometry-based crashworthiness design.

1.3 Research Objectives

The overall objective of this research is to develop new methodologies to de-
sign crashworthy structures with controlled behavior. Compliant mechanism-type
(sub)structures are used in designing solid and tubular components that exhibit
controlled deformation and collapse behavior under crash loads. In addition, a new
method for controlling the behavior of a structure under multiple crash load cases
by adaptively changing the contributions from individual load cases is developed.
The main research objectives of this investigation are as follows:

1.3.1 Crashworthy Design with Controlled Energy Absorption

Various components and subsystems in a vehicle structure serve different pur-
poses to meet crashworthiness requirements and should work together for effect-
tive protection of occupants and important components in all forms of survivable
crashes. For example, the passenger compartment should be stiff with a peak load
capacity to support the energy absorbing members in front of it, without exhibit-
ing excessive deformation. The stiff compartment cage also helps in providing enough space for the restraint systems like airbags and seat belts to function effectively. Similarly, the B-pillar, side floor panels and roof should be stiff enough to protect occupants from injuries due to high intrusions during side impacts and rollovers. On the other hand, some other components, like the front bumper, side frame-rails, crash box beams and shock tower, which typically surround the passenger cabin and engine compartment, should show characteristics of both flexibility and stiffness in desired proportions. For example, the front portion of the vehicle, consisting mainly of the front bumper, should have a soft front zone to reduce the vehicle’s aggressivity in pedestrian-to-vehicle and vehicle-to-vehicle collisions. This soft zone should be followed by a stiffer zone(s) to maintain the overall integrity of the component and prevent excessive penetration into the engine compartment. Additionally, in-vehicle safety devices like the knee-bolster and energy-absorbing steering system should also have a combination of both flexibility and stiffness to provide a cushioning effect in the beginning of crash, yet maintain the overall integrity of the component.

Most of the work in the field of topology and topometry-based crashworthiness design is focused on maximizing energy absorption or minimizing intrusion. This results in stiff designs in most cases, and as discussed above, it is not always preferred. Patel et al. [76] suggested that reducing the overall mass fraction could achieve the desired level of flexibility expressed in terms of a peak force constraint. However, this approach leads to designs with thin structural members and increased risk for catastrophic failure in the case of low peak force requirements. Moreover, this approach lacks the ability to introduce flexibility at desired locations within the structure.
In this research, a new methodology based on compliant mechanism design is presented to introduce desired flexibility within a crashworthy structure. The ability of compliant mechanisms to transfer motion and force from the point of application of an input load to desired points in the structure is utilized to design crashworthy structures with controlled behavior. For this purpose, the design space is divided into two subdomains: a flexible subdomain (FSD) and a stiff subdomain (SSD). A defined FSD will create a compliant-mechanism-type substructure, while the SSD will generate a stiff structure solely based on the distribution of internal energy. This method, apart from introducing flexibility within the structure, also helps the designer in controlling the potential load path(s) and buckling behavior of the final design. This particular feature has advantages in designing structures with multiple load paths and progressively collapsing structures for specific applications.

1.3.2 Design of Progressively Collapsing Thin-Walled Structures

Energy-absorbing structures often take the form of thin-walled tubular metallic structures subjected to dynamic compressive loads. For example, the front frame rails in a vehicle play an important role during frontal collisions by absorbing crash energy through plastic deformation. For efficient energy absorption, these tubular structures should undergo progressive buckling, preferably starting from the loading end to ensure maximal usage of material for plastic deformation. However, the presence of imperfections or asymmetry in loading conditions can trigger buckling starting from the middle or even close to the rear end of these structures, leading to possible jamming or global (Euler-type) bending behavior, which adversely affects the energy absorption capability.
In order to ensure a progressive collapse behavior, preferably starting from the loading end, a new methodology based on compliant mechanism design is presented in this research. The underlying idea is to design the tube to trigger the symmetric mode of collapse near the impact end. Within topometry-based compliant mechanism design, output port definitions on the tube faces along its length create potential buckle zones in the final design. The method is currently implemented for thin-walled tubes with square cross sections; however, it can be easily extended to tubes with other cross-sections. A significant advantage of this method for designing thin-walled tube structures is the possible delay or complete avoidance of global (Euler-type) bending behavior during oblique loadings which otherwise adversely affects the load-carrying and energy absorption capacity of the structure. This method has potential application in designing S-rails typically used in automotive chassis. Figure 1.3 shows a longitudinally cut sectional view of a typical S-rail under axial loading from a rigid plate. Because of its shape, the loading is not purely axial for all sections, and the S-rail has a tendency to bend at sections II and IV. By designing the rail to trigger progressive collapse starting from section I, the potential bending in sections II and IV can be delayed, leading to more efficient use of the available material.

1.3.3 Adaptive Weighting Factors Method for Multiple Load Cases

Real world vehicle collisions are unique dynamic events in which the vehicle may collide with another vehicle of similar or different shape, stiffness and mass, or it may collide with a stationary object. To accommodate the uncertainty involved in the loading conditions, designers commonly use two approaches - (a) incorporating the notion of reliability into the design process and (b) approximating
several loading conditions with a few representative ones and treating the problem as a multi-load design problem. Reliability-based methods, although more systematic, are not practical for topology optimization in vehicle crashworthiness design due to the computationally expensive finite element analysis, large number of design variables, and inability to express sensitivities for reliability constraints in closed analytical form. Therefore, the second approach of approximating the uncertainty by using a few representative load cases in a multi-load case design problem is considered in this research.

The most common approach to handle multiple load cases in structural optimization is to superimpose the effect(s) from each load case. For example, in traditional topology optimization for minimum compliance design, a weighted sum of the compliance or strain energy from each load case for each element is taken to represent the final value [95]. The same approach is used for topology optimization of multiple load case crashworthiness design problems. In their work on multi-load case crashworthiness design problems, Forsberg and Nilsson [35] normalized the internal energy density (IED) of each element by the maximum IED among all elements for each load case and calculated a weighted sum over all load cases. Similarly, Patel [75] used a weighted sum of the internal energy (IE) for every element in each load case to obtain the associated field variable (see Section

Figure 1.3. A longitudinal cut-section view of a typical S-rail.
2.3.1 for details) in the HCA method for crashworthiness design. The value of the weight associated with an individual load case dictates the contribution of that load case in the final design.

In most of the work related to topology optimization of multi-load problems, the weights associated with the load cases are obtained by design experience, some external constraint or probability of occurrence. Also, the weights remain constant throughout the design process. For cases in which a few of the load cases dominate, the final design is dictated by those dominating load cases unless their weighting factors are deliberately chosen to be comparatively smaller. There should be a measurable performance criterion to be able to decide the relative dominance of one load case over the other, and subsequently, associated weights should be chosen. For crash applications, the amount of energy absorbed by the structure, peak force and maximum intrusion are widely accepted measures of design performance. Also, the performance under multiple load cases changes as the design itself changes, especially when the structure undergoes buckling or large deformations. Hence, the corresponding weighting factors should be updated accordingly.

In this research, a new strategy called the ‘adaptive weighting factors method’ is introduced to allocate weighting factors adaptively during the design process such that desirable performance is achieved for all load cases. This method helps in developing designs with more uniform performance under multiple load cases. The method is of particular interest when uncertainty in the loading condition is modeled using a few representative load cases with the objective of minimizing the difference in the performance of the final design under these load cases. In the present work, the amount of energy absorption and maximum intrusion are used as
the performance measures for a structure under crash loads. However, the method is general enough to be extended to handle other performance measures like peak force, plastic strain, maximum stress, or a combination of these measures as long as the variation in these quantities with respect to the weights is well understood.

1.4 Overview

This dissertation is organized in the following manner. Chapter 2 overviews the fundamentals of structural optimization, especially topology optimization. Some common problem formulations and existing algorithms for topology optimization are discussed, including the basics of the HCA method used in this research to develop new methodologies. The problem of topology optimization in crashworthiness design is then introduced along with its challenges and existing methods in the literature. Again, the HCA method for crashworthiness design \[76\] is briefly described with the help of two examples, which are referred to on many occasions in the subsequent chapters.

An ‘adaptive $K_p$ scheme’ is introduced in Chapter 3 for the existing HCA method to make it more easily applicable to a wider set of problems without convergence difficulties. The need for and advantages of using this scheme are presented through two example problems. Later in the Chapter 3, the issue of mesh-dependence or minimum member size control in the HCA method for topology optimization is investigated, and possible measures to achieve mesh-independent designs are described. Finally, the HCA method for compliant mechanism synthesis considering nonlinear effects is presented, as it is used in developing the new methodologies presented in Chapters 4 and 5.

The main contributions of this research are presented in Chapters 4, 5, and 6.
In Chapter 4, a new method based on the use of compliant mechanisms for designing crashworthy structures with controlled behavior is presented. This method is shown to develop designs with desired flexibility and potential load paths. Chapter 5 presents a method for designing thin-walled square tubes under axial and oblique impacts based on the use of compliant mechanism synthesis. The phenomenon of progressive collapse in thin-walled square tubes under axial and oblique loading is described along with potential challenges arising from unavoidable imperfections and asymmetries in the geometry and loading conditions. It is shown through examples that by using this method, square tubes can be designed to exhibit buckling starting from the loading (impact) end and systematically progressing toward the rear end even in the presence of reasonable geometric imperfections and asymmetries in the loading conditions.

Chapter 6 presents the development of an ‘adaptive weighting factors method’ for multi-load crashworthiness design problems. The method is of particular interest when the uncertainty in the loading condition is modeled using a few representative load cases with the objective of minimizing the difference in the performance of the final design under these load cases. In Chapter 7, the various methodologies developed as part of this research are summarized, and outstanding research issues and recommendations for the future work are also discussed.
CHAPTER 2

TOPOLOGY SYNTHESIS USING THE HCA METHOD

2.1 Structural Optimization

The objective of structural optimization is to maximize the performance of a structure or structural component. Optimal structural design is driven by limited material resources, environment impacts and technological competition, which demand lightweight, low-cost and high-performance structures. The design that minimizes or maximizes a performance objective within a set of constraints is considered the optimal design. A structural optimization problem can be expressed mathematically as

\[
\begin{align*}
\text{minimize} & \quad f(x) \\
\text{s.t.} & \quad g(x) \leq 0 \\
& \quad h(x) = 0 \\
& \quad x_{\text{min}} \leq x \leq x_{\text{max}},
\end{align*}
\]

where \( f(x) \) is the objective function to be minimized, \( x \) is the set of independent design variables, \( g \) is the set of inequality constraints, and \( h \) is the set of equality constraints. The design variable values are constrained by lower bound values \( (x_{\text{min}}) \) and upper bound values \( (x_{\text{max}}) \). Depending on the problem formulation
and requirements, various structural performance measures like weight, stiffness, critical load, stress, or displacement, among others, form the objective function and constraints. A commonly used optimization formulation is to maximize the stiffness of a structure subject to a mass constraint.

The three primary approaches to structural optimization are sizing, shape and topology optimization. Sizing optimization is to find the optimal cross-sectional properties of members in a truss or frame structure or the optimal thickness distribution of a plate structure. In sizing optimization, the shape of the design domain is fixed during the optimization process. On the other hand, in shape optimization, the surface of the structure is changed to achieve the best performance. The coordinates of the boundary or surface of the structure are the design variables. Shape and sizing optimizations are often used in the second phase of design after utilizing topology optimization. In topology optimization, the optimal number and locations of holes within a continuum design domain are determined. Theoretically, shape optimization is a subclass of topology optimization, but practical implementations are based on very different techniques, so the two types are treated separately [22]. Figure 2.1 shows the difference between the three approaches by considering an MBB-beam design problem.

2.2 Background

Various structural optimization methods have been developed in the last 50 years. Earlier work was based more on analytical methods searching for optimal configurations using the mathematical theory of calculus and variational methods [39, 56]. In the last two decades, the focus has shifted to numerical methods with advancement in computational power and the need to solve more complex
Figure 2.1. Categories of structural optimization. (a) Sizing optimization, (b) shape optimization, and (c) topology optimization. The initial designs are shown on the left side, and the optimal ones are on the right side. The figure is from Sigmund and Bendsøe [95].

practical problems with a larger number of design variables. Numerical methods generate optimal designs in an iterative fashion using finite element analysis and mathematical programming [90, 91].

Topology optimization is an iterative process that determines the best arrangement of a limited volume of structural material within a given spatial domain so as to obtain an optimal mechanical performance of the concept design. This computational technique for the optimal distribution of material in continuum structures was first introduced by Bendsøe and Kikuchi [10] using the homogenization approach. This approach is based on composite material models to describe the varying material properties in all dimensions with each element being a microstructure. Many researchers [8, 10, 11, 27, 74, 100] used and further extended the homogenization method for various applications in topology optimization. A parallel exploration based on the Solid Isotropic Material with Penalization (SIMP) approach, originally suggested by Bendsøe [15] and later formalized by Zhou and Rozvany [118], gained popularity because of its simplicity and ease of numerical
implementation.

Early work in topology optimization generally dealt with small-scale problems that used the assumptions of elastic material properties, small deformations, and static loading conditions. Traditionally, the objective of a topology optimization problem is to achieve maximum stiffness or minimum compliance with a constraint on the mass $M^*$. This can be formally expressed as

\[
\begin{align*}
\text{minimize} & \quad c(x) \\
\text{s.t.} & \quad m(x) \leq M^* \\
& \quad x_{\min} \leq x \leq x_{\max},
\end{align*}
\]

where $x$ is the design variable (relative density), $m$ is the mass, $M^*$ is the constraint on mass, and $x_{\min}$ and $x_{\max}$ being the bounds on the design variables. Other objectives and constraints that could be considered are global structural responses such as von Mises stresses, strains, eigen frequencies, and geometrical parameters such as volume or perimeter. Duysinx and Bendsøe [30] and Lipton [67] introduced a constraint based on local stress distribution. With appropriate choices of the objective and constraints, topology optimization has been extended to a wide variety of applications like compliant mechanism design, frequency response optimization, buckling problems, heat conduction, and stokes flow, among others. A comprehensive review of topology optimization can be found in Sigmund and Bendsøe [95], Rozvany [89], Eschenauer and Olhoff [32], and Bendsøe et al. [14].

Various methodologies have been developed for topology optimization over the past two decades. Most of the existing topology optimization algorithms fall into the categories of mathematical programming (MP), optimality criteria...
To solve the topology optimization problem, any nonlinear MP method may be used. Sequential Linear Programming (SLP) is an example that searches for the minimum in a nonlinear design space using a sequence of linear approximations for both the objective and constraints. Similarly, Sequential Quadratic Programming (SQP) methods approximate the objective and constraints with quadratic functions. The Method of Moving Asymptotes (MMA) developed by Svanberg is another widely used MP method for topology optimization. It is similar in nature to SLP and SQP in the sense that it works with a sequence of simpler approximate subproblems of a given type. For MMA, these subproblems are separable, convex and constructed based on sensitivity information. These methods are usually robust and can be applied to any sort of optimization problem. However, when applied to topology optimization problems, several calculations of the objective function, constraint functions and their derivatives are usually needed. Consequently, the use of mathematical programming methods for realistic topology optimization problems is somewhat impractical, as they usually require thousands of finite elements and therefore, thousands of design variables. To overcome this deficiency, OC methods have been considered.

OC methods derive or state conditions that characterize the optimum design, then find or change the design to satisfy those conditions while indirectly optimizing the structure. The OC equations (conditions) are derived either intuitively or rigorously. Fully stressed designs (FSD), constant internal force distribution designs (CIFDD), simultaneous failure mode designs (SFMD), and uniform strain energy density designs (USEDD) are examples of heuristic optimality criteria methods. Rigorously derived OC methods are often based on the Karush-Kuhn-Tucker
(KKT) optimality conditions. The use of OC methods for continuum design problems in topology optimization dates back to the pioneering work of Bendsøe and Kikuchi [16].

In contrast to the above mentioned methods, various topology optimization methods have been developed using evolutionary strategies that do not require gradient (sensitivity) information. An often used but less efficient approach is to use genetic algorithms (GAs). These methods are more likely to find the global optimum but require thousands of function evaluations. Another non-gradient based methodology developed by Xie and Stevens [112] is called Evolutionary Structural Optimization (ESO). In this method, inefficient material is progressively removed from the structure so that it evolves into an optimal design. Another approach developed by Tovar [105] and Tovar et al. [106, 107] that is inspired by the biological process of bone remodeling is known as the Hybrid Cellular Automaton (HCA) method. This method, with some extensions and modifications, is used as the basic framework for the present work and hence, is described in detail in the remainder of this chapter.

2.3 The HCA Method for Topology Synthesis

The hybrid cellular automaton (HCA) method is a computational technique that can be used to synthesize optimal topologies. This approach is inspired by the biological process of bone remodeling. Material is appropriated to uniformly distribute the effects of a specified mechanical stimulus, or field variable, throughout the design domain. The HCA method combines the cellular automaton (CA) paradigm with finite element analysis (FEA) for structural optimization.

A cellular automaton (CA) is a discrete model studied in computability theory
and mathematics [11]. It consists of a regular grid of cells, or lattice, where each is characterized by a finite number of states. The state of each cell at a given time, or generation, is a function of the states of a finite number of neighboring cells called its neighborhood. Every cell has the same set of rules, which are applied based on the information in its neighborhood. These rules are applied to the entire CA lattice for each generation. The best-known way in which cellular automata were introduced (and which eventually led to their name) was through work by John von Neumann in trying to develop an abstract model of self-reproduction in biology in late 1940s. According to Burks [20], the first CA proposed by von Neumann was a two-dimensional square lattice composed of several thousand cells. The CA rule made use of the state of each (central) cell plus the states of its four nearest neighbors, located directly to the north, south, east and west. Figures 2.2 and 2.3 depict some common two- and three-dimensional neighborhood layouts.

Figure 2.2. Typical 2-D neighborhoods for CAs. \( \hat{N} \) is the number of neighboring CAs: (a) \( \hat{N} = 0 \) (b) \( \hat{N} = 4 \), (c) \( \hat{N} = 8 \) (d) \( \hat{N} = 24 \).
2.3.1 Formulation

As in traditional topology optimization methods, the structural design domain is discretized into material elements. To use a finite element method for structural analysis, the design domain is represented using a finite element model that is discretized using continuum finite elements (FE). The states of the material elements in the design domain are represented using a lattice of CAs, where a one-to-one correspondence between the CAs and FEs generally exists, although this is not a requirement. For example, mapping a non-uniform FE mesh or a uniform FE mesh with a smaller element size to a uniform CA lattice with a bigger size requires many-to-one FE to CA correspondence as shown in Fig. 2.4. At a discrete position $i$ and time/iteration $k$, a CA is defined by a set of states that are operated on by a set of rules belonging to a given neighborhood of the CA. The state of each CA, $\beta_i$, is defined by design variables $x_i$ (e.g., relative density, thickness) and field variables $S_i$ (e.g., stress, strain, strain energy density, mutual potential energy or functions of these quantities). The field variables are computed by finite element analysis; hence, this is a hybrid approach since each cellular automaton is provided global information. The complete state of each cell at time/iteration $k$ is

![Figure 2.3. The corresponding 3-D neighborhoods for CAs: $\hat{N}$ is the number of neighboring CAs: (a) $\hat{N} = 0$, (b) $\hat{N} = 6$, (c) $\hat{N} = 26$, (d) $\hat{N} = 124$.](image)
$k$ is expressed by

$$
\beta_i^{(k)} = \begin{cases} 
S_i^{(k)} \\
x_i^{(k)} 
\end{cases}.
$$

(2.1)

Figure 2.4. Mapping of FE mesh to CA lattice.

The set of local rules operate according to local information collected in the neighborhood $N$ of each cellular automaton. The final state of a CA is defined by the state of itself and the states of the CAs within the neighborhood. For example, the information collected from a neighborhood can be expressed as

$$
\bar{S}_i = \frac{\sum_{j \in N_i} S_j}{||N_i||},
$$

(2.2)

where $\bar{S}_i$ is the effective field variable, $N_i = \{j : d(i, j) \leq r\}$, $r$ is the size of the neighborhood, and $||N_i||$ is the cardinal of $N_i$. A parallel can be drawn between the neighborhood averaging of field variables in HCA and the filtering of sensitivities in
traditional topology optimization methods [95] to prevent numerical instabilities of checkerboarding and mesh dependency. In practice, the size of the neighborhood is often limited to the adjacent cells but can also be extended depending upon the minimum member size requirements.

To obtain an optimal design, material is distributed throughout the design domain in order to achieve a uniform distribution of field variables (actually quasi-uniform, as bounds on the design variable restrict the values to achieve full uniformity). Accordingly, a setpoint is uniformly applied to all CAs in the design domain. In the HCA method, the field variable state of each CA is driven to this value. Mathematically, it can be expressed as

$$\begin{align*}
\text{minimize} & \quad \sum_{i=1}^{N} |\bar{S}_i(x_i) - S^*| \\
\text{s.t.} & \quad \frac{\sum_{i=1}^{N} x_i}{N} = M_f^* \\
& \quad x_{\text{min}} \leq x \leq 1,
\end{align*}$$

(2.3)

where $M_f^*$ is the desired mass fraction and a non-zero lower bound of $x_{\text{min}}$ is used for the design variable to avoid numerical instabilities due to singular matrices. Adapting the principles of a fully stressed design [39] and uniform strain energy density [84], HCA is utilized to allocate material based on the strain energy density of each element. In other words, strain energy density is used as the field variable to drive the design to an optimal configuration.

2.3.2 Material Distribution Rules

Material distribution in the HCA method is governed by control-based rules [107]. These rules manipulate the design variable based on known behavior. An inversely proportional relationship exists between stiffness and strain energy for
an elastic body under a fixed loading. Therefore, in the design of a structure, the relative density of the element (controls the stiffness) must be increased to reduce the strain energy. Similarly, the relative density must be decreased to increase the strain energy. Using a proportionality-based control rule, the change in relative density (design variable) of element \(i\) at the \(k^{th}\) iteration can be expressed as

\[
\Delta x_i^{(k)} = K_p e_i^{(k)},
\]  

(2.4)

where \(K_p\) is the proportional control gain and \(e_i\) is the error between the effective field variable and setpoint for the \(i^{th}\) element. The normalized (scaled) error can be expressed as

\[
e_i^{(k)} = \frac{\bar{S}_i^{(k)} - S_i^{*}(k)}{\| \| \bar{S}(k) \| \|_{\infty}},
\]  

(2.5)

where \(\bar{S}_i^{(k)}\) is the effective field state of a CA, which reflects the average field state of itself and its neighborhood, and \(S_i^{*}(k)\) is the target setpoint. The equilibrium in a CA is determined by the condition \(e_i^{(k)} = 0\). When this condition is not satisfied, the local rule modifies the relative density \(x_i\) to restore equilibrium. Also, a maximum move limit on the density change \(\Delta x_{\text{max}}^{\text{allow}}\) is used to avoid numerical instabilities due to possible large changes in one update. Hence, the material update rule from Eq. (2.4) can be rewritten as

\[
\Delta x_i^{(k)} = \max\{ -\Delta x_{\text{max}}^{\text{allow}}, \min\{ K_p(e_i^{(k)}), \Delta x_{\text{max}}^{\text{allow}} \} \}.
\]  

(2.6)

In the algorithm developed by Tovar [105], the setpoint was fixed (user-prescribed), and mass was unconstrained. However, to accommodate a constraint on mass, the setpoint must be modified at each iteration as explained in the following section.
2.3.3 Mass Constraint

In practice, a constraint on the mass of the synthesized structure is often required as a surrogate constraint on the cost of manufacturing. Hence, a mass constraint is used based on the setpoint modification. In order to satisfy the mass constraint at each design iteration, a secondary inner mass loop is used to update the setpoint based on the difference between the current and desired mass. Tovar et al. [107] used numerical schemes like the Newton-Raphson bisection method to obtain an appropriate setpoint such that the mass constraint would be satisfied. Patel et al. [76] suggested that an existing monotonic relationship (inversely proportional) between the mass and setpoint can be used to update the setpoint in a secondary inner loop until the mass constraint is satisfied. Accordingly, the setpoint update for the \((j+1)^{th}\) inner loop can be written as

\[
S^{*(j+1)} = S^{*(j)} \left( \frac{M_f^{(j+1)}}{M_f^*} \right),
\]

where \(M_f^*\) is the mass fraction target. This overall control-based density update scheme with mass equality constraint check followed by setpoint update is represented in Fig. 2.5.

2.3.4 Convergence Criteria

In an iterative design method, a convergence criterion needs to be defined in order to obtain a final design. Convergence is achieved when the difference between two successive designs becomes less than a user-defined tolerance limit. To quantify the difference between two designs, the sum of the absolute difference between the design variables is used in this work. Also, in order to avoid premature
convergence, three consecutive designs are compared as

\[
\|\mathbf{x}^{(k)} - \mathbf{x}^{(k-1)}\|_\infty + \|\mathbf{x}^{(k-1)} - \mathbf{x}^{(k-2)}\|_\infty < \epsilon, \quad (2.8)
\]

where \(\epsilon\) is an arbitrary small quantity.

2.3.5 Methodology

A step-by-step procedure for the HCA method is as follows:

**Step 1.** Define the design domain, material properties, load conditions, boundary conditions and initial design \(\mathbf{x}^{(0)}\).

**Step 2.** Evaluate the field variable (strain energy density) at each discrete location \(i\) using finite element analysis.

**Step 3.** Update the density distribution according to the design rules described in Eq. (2.6) to obtain a new design \(\mathbf{x}^{(k+1)}\).

**Step 4.** Check the mass constraint. If satisfied, go to the next step; otherwise, update the setpoint and subsequently design to satisfy the mass equality con-
straint.

**Step 5.** Check for design convergence. If the convergence criterion is satisfied, the final structure is obtained; otherwise, the iterative process continues from Step 2.

A systematic flowchart of the algorithm is shown in Fig. 2.6.

![Flowchart](image)

Figure 2.6. Illustration of the HCA method for topology synthesis.
2.4 HCA Method for Crashworthiness Design

Crashworthiness is the ability of a structure to resist the effects of an impact with another object. For the automotive industry, the term crashworthiness provides a measure of the ability of a vehicle component or system to protect the passenger in a survivable crash event. With improvements in the efficiency of topology optimization methods and computational power to simulate crash events, structural topology optimization is changing the design process in the automotive industry by providing better structures. The optimization problem formulation in crashworthiness design must accommodate conflicting criteria such as a maximum acceleration (or peak force) constraint to avoid passenger injuries due to high g-forces and a maximum deformation (intrusion) constraint to avoid passenger injuries due to penetration of the passenger cabin [29, 95]. The structure is designed to deform plastically in a controlled manner, thereby absorbing impact energy and reducing the potential for passenger injury. In the next section, a brief background on the use of structural optimization (particularly topology optimization) for crashworthiness design is presented.

2.4.1 Background

The first practical use of structural optimization in crashworthiness design began with the work of Lust [68] in which the rear mid-rail modeled by an approximate nonlinear spring-mass system was designed for minimum weight. Crashworthiness constraints in the form of displacement and vehicle crash severity index (VCSI) were used. Mayer [70] and Mayer et al. [71] presented a topology optimization method based on a homogenization technique for crashworthiness design in which an objective of maximizing the energy absorption subject to a volume con-
straint was used. Optimality criteria were derived from an optimization statement using density as the design variable.

Pedersen [80, 81] later devised a method for topology optimization using 2D frame structures. In that work, the objective was to obtain a desired energy absorption history for a crushed structure. The formulation used numerically approximate computed sensitivities. Although this sensitivity-based method was shown to work well for ground structures made of beam elements, it has never been extended to more realistic continuum-element-based structures. Sensitivity-based methods, although more accurate, are not always preferred in topology optimization for crashworthiness design due to: a) the inability to express sensitivities in closed form analytical expressions and b) the large number of design variables. Some researchers [34, 43, 66] used metamodeling methods in which the objective and constraints are modeled using response surface methods (RSM) or radial basis functions (RBF). Although these efforts are significant in their attempt to solve the optimization problem, they are limited to shape or size optimization with a small number of design variables.

In order to tackle topology optimization for crashworthiness design of more realistic continuum-element-based components with a significant number of design variables, many heuristic methods have been proposed. Soto [99] presented a methodology that does not require sensitivity information. In this heuristic scheme, Soto implements a criterion called prescribed plastic strain/stress (PPSS). The PPSS criterion varies the density within the design domain to achieve a prescribed distribution of plastic strains and stresses with a constraint on mass. This methodology utilizes a density approach with two base materials, one that is stiff and one extremely soft, to represent a metallic foam-like material. This method
cannot generate solid metal structures. Forsberg and Nilsson [35] proposed another non-gradient technique using thickness as the design variable. However, by operating on the thickness of elements, this methodology can only handle a small set of problems. Patel et al. [76] developed the HCA method, originally proposed by Tovar et al. [107], to address crashworthiness design problems. The HCA method for crashworthiness design uses a concept similar to the fully stressed design [39] in which material is distributed within the design domain to achieve uniform internal energy distribution. Occupant injury-related constraints in the form of maximum displacement and peak force are also introduced by controlling the mass of the structure. In the present work, the HCA method for crashworthiness design in its original or a modified form has been used at many places. Hence, it is described briefly with 2 examples later in this chapter.

2.4.2 Performance Measures in Crashworthiness

The performance of a structure under static loads can be measured in terms of its compliance or displacement at the nodal load location(s). Since the input or applied force is constant, nodal displacement or strain energy can be used to measure the stiffness of the structure. For example, consider two linear structures, loaded with a static force $F_{\text{input}}$, that deform by $d_1$ and $d_2$ at the point of application of the force. Figure 2.7 shows the corresponding force-displacement behavior for the two structures. The areas under the force-displacement curves represent the corresponding strain energies ($SE_1$ and $SE_2$). The structure with less deformation ($d_1$) is stiffer and has a higher slope $k_1$ for its force-displacement curve. This also means that it has less area under the force-displacement curve and hence, less strain energy. Therefore, in order to achieve an efficient stiff design
under static loads, displacement at the nodal force location or the strain energy is minimized.

On the other hand, crash loads are more complex due to their dynamic nature. Typically, the amount of energy absorption, peak force and maximum intrusion are commonly used as measures of performance for a structure under crash loading. Since crash loads are time dependent, the value of time at which the structure is analyzed needs to be considered. A structure under an impact can be analyzed until the impactor loses all of its kinetic energy and comes to rest or loses contact with the structure. However, it is not always practical to perform crash tests or
simulations to such a full extent, especially when the intensity of impact is high. For example, during the front impact test of a vehicle with a rigid wall, the performance of the structure is considered only up to a certain deformation range. The objective is to analyze and design the structure for survivable crashes. For cases in which the structure is analyzed up to a certain deformation, peak force and the amount of energy absorption are considered for measuring the performance. An efficient structure should absorb the maximum energy for the lowest possible peak force; however, there is a trade-off between these two quantities. Hence, a structure with maximum energy absorption and peak force just below the injury levels is preferred. For example, consider two designs with the same mass under a dynamic load of high intensity. The designs are analyzed up to a prescribed displacement \(d^*\), and Fig. 2.8(a) shows the corresponding force-displacement behavior. In this case, the design with more energy absorption (green, 2), characterized by the area under the force-displacement curve, is preferred as long as the corresponding peak force is below the injury level.

For medium and low intensity impacts, the structures are analyzed for the complete crash duration. For example, the low-speed front and rear impact tests of bumpers for ensuring minimal damage during fender-bender-type incidents. Since, all of the vehicle’s kinetic energy is exhausted as it comes to rest, an efficient design should have minimal intrusion. Hence, the intrusion can be used as the performance measure in this case. For example, consider two designs with the same mass under a low intensity impact for which the force-displacement behaviors are shown in Fig. 2.8(b). The simulations in this case are carried out for time \(t^*\) long enough for the impactor to dissipate all of its kinetic energy. The design with less intrusion (green, 2) is preferred in this case.
Figure 2.8. The force-displacement behavior for two design with the same mass under (a) a high-intensity, and (b) a low-intensity impact.

In this work, the amount of energy absorption for high intensity crash loads and maximum intrusion for the medium and low intensity crash loads are used as the performance measures. In both cases, peak force can be controlled (if it exceeds survivable injury levels) by reducing the mass fraction as suggested by Patel [75].

2.4.3 Nonlinearities in crashworthiness

Finite element modeling of structures under crash or impact loads is among the most challenging nonlinear problems in structural mechanics. During a crash incident, the structure experiences high impact loads which produce localized plastic hinges and buckling. This generally leads to large deformations and rotations with contact and stacking among the various components. Moreover, crash loads are time-dependent, so the governing equations of motion need to be solved in the space-time domain. Therefore, a typical crash simulation involves modeling for ge-
ometric and material nonlinearities, contact mechanics and use of time-integration schemes.

A structure generally undergoes large (finite) displacements, rotations and deformations (strains) during crash loads. For large deformations, small strain theory of linear analysis, which assumes a constant strain-displacement matrix $B$ for each finite element, cannot be used. The measures of interest, like strains, stresses and energy absorption, are determined by defining a displacement-dependent strain-displacement matrix $B$ such as

$$d\varepsilon = B(u)du. \tag{2.9}$$

A commonly used finite strain measure to model the strain-displacement relationship is Green-Lagrange strain,

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i} + u_{k,i}u_{k,j}), \tag{2.10}$$

where $u$ is the point-wise displacement and subscript “,$j$” means differentiation with respect to coordinate $j$.

Since the stiffness and/or loads are displacement dependent in nonlinear analysis, an iterative scheme is required to solve for the displacements ($u$) to satisfy the equilibrium of the finite element equations. A residual vector $R$ is defined as the error in obtaining the equilibrium.

$$R(u) = F - \int_V B(u)^T \sigma(u) dV, \tag{2.11}$$

where $F$ is the external force vector and $\sigma$ is the 2nd Piola-Kirchoff stress written in vector form and the integration is performed over the undeformed volume $V$. 35
Numerical root-finding schemes like the Newton-Raphson method are commonly used for this type of analysis to solve for the displacements.

Material nonlinearities refer to the nonlinear relationship in the stress-strain curve for the given material where plasticity, or permanent deformation, is involved. Material yielding and strain hardening must be accounted for in an elastic-plastic model. Figure 2.9 shows a typical stress-strain behavior for continuous uniaxial loading of a metal like steel. This behavior can be divided into two main regions - elastic and plastic. In the elastic region, the stress-strain behavior is for most part linear, with the slope characterized by the elastic modulus. Yielding occurs when the uniaxial stress reaches the value of the yield stress ($\sigma_Y$) in tension. For multiaxial stress states, a more complex criterion like Tresca or von Mises is required to define yield. Once the yield criterion is reached, plastic straining or strain hardening occurs. Strain hardening is an increase in mechanical strength due to plastic deformation. There are two common methods to model strain hardening: isotropic hardening and kinematic hardening. In isotropic hardening, the yield surface increases in size but maintains its shape as a result of plastic straining. In kinematic hardening, the yield surface translates in the direction of increasing stress without changing its shape. The necking phase is indicated by a reduction in cross-sectional area. Necking begins after the ultimate strength is reached. During necking, the material can no longer withstand the maximum stress, and the strain rapidly increases. Plastic deformation ends with the fracture of the material. See Bathe [13] for further details of plasticity theory.

Another nonlinearity that needs to be accounted for during a crash simulation is the boundary nonlinearity. A model exhibits this nonlinearity when the loads, constraints or load paths change throughout the application. Since the region
Figure 2.9. The stress-strain curve: A linear elastic behavior for small displacements, followed by nonlinear plastic behavior.

of impact between two colliding objects is usually not known a priori, it must be determined as part of the solution. Therefore, problems involving contact are nonlinear. In defining a contact interface in a finite element model for two objects that come into contact, one object is designated as the ‘master’ surface, and the other is the ‘slave’ surface. The geometry of the master surface is interpreted as a smooth curve, usually by interpolating between nodes. The surface of the slave is constrained not to penetrate into the master surface. Three distinct methods for handling contacts in finite element analysis are the kinematic constraint method, distributed parameter method, and penalty method [40].

In the kinematic constraint method that uses the impact and release conditions of Hughes et al. [47], constraints are imposed on the global equations by a transformation of the nodal displacement components of the slave nodes along the contact interface. In the distributed parameter formulation, one-half the mass of
each slave element in contact is distributed to the covered master surface area. Also, the internal stress in each element determines a pressure distribution for the master surface area that receives the mass. After this distribution, acceleration of the master surface is updated. Constraints are then imposed on the slave node accelerations and velocities to ensure their movement along the master surface. In the penalty method, each slave node is checked for penetration through the master surface. If it penetrates, an interface force is applied between the slave node and its contact point. The magnitude of this force is proportional to the amount of penetration. This may be thought of as the addition of an interface spring. In this work, the penalty method is used for contacts unless specified otherwise because of its numerical efficiency and less chance of exciting mesh hourglassing (spurious energy modes) [40].

Crash simulations are problems that require analysis of dynamic events. The dynamic response of a time-dependent system can be expressed by the state equation

\[ M \ddot{d}(t) + C \dot{d}(t) + K d(t) = F(t), \]

(2.12)

where \( M \), \( C \) and \( K \) are the mass, damping, and stiffness matrices, respectively. The parameters \( d(t) \), \( \dot{d}(t) \), and \( \ddot{d}(t) \) are the displacement, velocity and acceleration vectors as functions of time. The dynamic analysis can use an implicit or explicit time integration scheme to solve the equations of motion in Eq. (2.12).

Implicit integration schemes (generally using the Newmark forward differencing method) assume a constant average acceleration over each time step between \( t_n \) and \( t_{n+1} \). The governing equation is evaluated, and the resulting accelerations and velocities at \( t_{n+1} \) are calculated. Then, the unknown displacements at \( t_{n+1} \) are determined. Explicit integration schemes (generally using a central differen-
ence method) assume a linear change in displacement over each time step. The
governing equation is evaluated, and the resulting accelerations and velocities at
t\_n are calculated. Then, the unknown displacements at t\_n+1 are determined. For

For crash simulations involving extensive use of contact, multiple material models and
a combination of non-traditional elements, explicit solvers are more robust and

computationally more efficient than implicit solvers. In this work, the explicit
time integration scheme of the nonlinear finite element solver LS-DYNA is used
for simulation purposes unless specified otherwise.

2.4.4 Material Parameterization for Plasticity

Ultimately, the goal of topology optimization is to determine a material dis-

tribution within the design domain to achieve a specified objective. To solve this

problem numerically, a discretized design space is considered with the density of
each element as the design variable. In many applications, the optimal topology

of a structure should consist solely of a macroscopic variation of one material
and void, meaning that the density of the structure is given by a ‘0-1’ integer

parametrization (often called a black-and-white design).

One commonly used approach to solve this problem is to replace the integer

variables with continuous variables and then introduce some form of penalty that
drives the solution to discrete 0-1 values. This is called the density approach or

Solid Isotropic Material with Penalization (SIMP) method. In this method, an

interpolation function that expresses various physical quantities as a function of
continuous variables is introduced. For topology optimization of 3D structures,
relative density x is often used as the continuous design variable. The density and
elastic modulus of an element with design variable $x_i$ can be expressed as

$$\rho_i(x_i) = x_i\rho_0,$$
$$E_i(x_i) = x_i^p E_0 \quad (0 \leq x_i \leq 1),$$

(2.13)

where $\rho_0$ and $E_0$ are the density and elastic modulus of the base material. In SIMP, one uses the penalization parameter $p$ greater than 1 so that intermediate densities are unfavorable. In other words, $p > 1$ makes it uneconomical to have intermediate densities in the optimal design.

Since material nonlinearities need to be considered for crash applications, a linear elastic, piecewise linear plastic material model from LS-DYNA [40] is used. As shown in Fig. 2.10, the plastic portion is represented by a curve of effective stress vs. effective plastic strain values. A similar technique using the SIMP approach for nonlinear material behavior was implemented by Patel et al. [76] to map various plastic properties like yield stress ($\sigma_Y$) and effective stresses ($\sigma_{\text{eff}}^j$) to the design variable $x$. Therefore, the interpolation of the stress-strain relationship used in this work can be expressed as

$$\sigma_{\text{eff}}^j(x_i) = x_i^r \sigma_{\text{eff}}^j_0,$$
$$\sigma_Y(x_i) = x_i^q \sigma_Y_0,$$

(2.14)

where $q$ and $r$ are the penalization parameters and the quantities with subscript 0 are the base material properties.

### 2.4.5 Methodology

Most traditional topology optimization problems consider elastic material behavior. For a purely elastic body, the energy absorbed during loading by the
structure is measured by the elastic strain energy \( (U^e) \) as

\[
U^e = \int_{\varepsilon_{ei}^e}^{\varepsilon_{ef}^e} \sigma : d\varepsilon^e, \tag{2.15}
\]

where \( \varepsilon_{ei}^e \) and \( \varepsilon_{ef}^e \) are the initial and final elastic strain due to loading. When designing for crash involving large deformations, considering plastic material behavior becomes important. The energy absorbed during the inelastic deformation is measured by the inelastic strain energy or plastic work \( (U^p) \) as

\[
U^p = \int_{\varepsilon_{ei}^p}^{\varepsilon_{ef}^p} \sigma : d\varepsilon^p, \tag{2.16}
\]

where \( \varepsilon_{ei}^p \) and \( \varepsilon_{ef}^p \) are the initial and final plastic strain. The total energy absorption, also known as internal energy \( (IE) \), should account for both elastic strain energy \( (U_e) \) and plastic work \( (U_p) \),

\[
IE = U^e + U^p. \tag{2.17}
\]
With the dependence on time, the total internal energy of the discretized body is written as

\[ IE_{\text{total}} = \sum_{i=1}^{N} \left( \int_{t_0}^{t_f} \int_{\Omega_i} \sigma_i : \varepsilon_i d\Omega_i dt \right) , \]

(2.18)

where \( N \) is the total number of elements and \( t_0 \) and \( t_f \) define the initial and final times of the interval considered.

During a crash, the kinetic energy of the impactor is dissipated in the form of internal energy (energy in the deformation), sliding energy (friction), damping energy and heat, among others. In this work, internal energy is considered as the main source of energy dissipation as the other forms contribute much less in comparison for crash loads in the survivable range. Patel et al. [76] extended the original HCA method for crashworthiness design by using internal energy as the field variable. A concept similar to a fully stressed design is adopted to synthesize structures that absorb maximum energy or result in minimum penetration for a specified mass (or volume). A fully stressed design formulation states that ‘For the optimal design, each member of the structure that is not at its minimum gauge is fully stressed under at least one of the design load conditions’ [39, 78]. Extending the idea of a fully stressed design for nonlinear transient events, all elements in the structure should contribute to the energy absorption through plastic deformation. For this purpose, the internal energy \( (IE_i) \) of each element is driven to a specified target or setpoint \( (IE^*) \) by varying its relative density \( x \) to iteratively modify its stiffness properties. Mathematically it can be expressed as

\[
\begin{align*}
\text{minimize} & \quad \sum_{i=1}^{N} |IE_i(x_i) - IE^*| \\
\text{s.t.} & \quad \frac{\sum_{i=1}^{N} x_i}{N} = M^*_f \\
& \quad x_{\text{min}} \leq x \leq 1,
\end{align*}
\]

(2.19)
where $N$ is the number of elements, $M_f^*$ is the mass fraction constraint, and $x_{min}$ is the lower bound value. The control-based material update rule, field variable averaging, and mass constraint inner loop are the same as described earlier in Section 2.3. A detailed description of the HCA method for crashworthiness design can be found in Patel [75] and Patel et al. [76] [77].

2.4.6 Examples

In the present section, two example problems with crash loading are presented with the objectives of maximizing energy absorption and minimizing intrusion, respectively. In both examples, the explicit nonlinear finite element code LS-DYNA is used to perform dynamic simulations. A linear elastic, piecewise linear plastic material (*MAT24) is used to model the design domain with properties shown in Table 2.1. In the first example, a slender structure under a moderately high intensity impact is designed to maximize the energy absorption for a prescribed intrusion value. In the second example, a beam structure under a low intensity impact is designed to minimize the penetration.

2.4.6.1 2D Slender Beam

In this example, a 2D slender beam constrained at its base is considered under an impact load from a rigid pole at its top surface. A schematic of the design domain with dimensions and the location of impact are shown in Fig. 2.11(a). The design domain is discretized into an $80 \times 120$ mesh of 4-node, plane stress quadrilateral elements with 1 mm thickness. A rigid pole that has a mass of 15.7 kg impacts the beam with an initial velocity of 15 m/s. The coefficient of friction between the rigid pole and beam is 0.3. The performance of the structure
### TABLE 2.1

**MATERIAL PROPERTIES FOR BOTH EXAMPLES**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Density</strong></td>
<td>2700 Kg/m³</td>
</tr>
<tr>
<td><strong>Elastic Modulus</strong></td>
<td>70 GPa</td>
</tr>
<tr>
<td><strong>Poisson’s Ratio</strong></td>
<td>0.33</td>
</tr>
<tr>
<td><strong>Yield Stress</strong></td>
<td>180 MPa</td>
</tr>
<tr>
<td><strong>Effective plastic strain</strong></td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>0.40</td>
</tr>
</tbody>
</table>

is considered for the first 60 mm of intrusion. It is to be noted that since the simulation is carried out using an explicit time integration scheme, it is difficult to stop at an exact time corresponding to the prescribed displacement. In this work, the desired quantities of interest (internal energy) are read at the closest time resulting in the prescribed displacement value. One can also take a weighted average of the internal energies at the two times (separated by the timestep size) bracketing the prescribed displacement value.

Based on the discussions in Section 2.4.2, an objective of maximizing the energy absorption (during the prescribed intrusion) is considered subject to a mass fraction constraint of 30%. Basic HCA-related parameters like neighborhood size, penalization parameters, and global convergence tolerance, among others, are summarized in Table 2.2. The final design is shown in Fig. 2.11(b). As the design evolves, it becomes stiffer and consequently absorbs more energy for a prescribed intrusion as shown in Fig. 2.12(a). This behavior is a result of (re)distributing the material to achieve uniform internal energy distribution. In this process, the
elements contributing more efficiently to energy absorption gain density (mass) at the cost of equivalent removal of material from less efficient elements. For this example, the less efficient outer elements eventually reach the lower bound of density, resulting in a column-like structure in the middle. It is to be noted that the increase in energy absorption comes at the cost of increased stiffness or peak force as shown in Fig. 2.12(b). For the cases in which the resulting peak force exceeds the survivable limit, an inner mass constraint update loop can be used to iteratively decrease the mass fraction in order to reduce the peak force [75].

Figure 2.11. (a) Schematic of a 2D slender beam impacted by a rigid pole and (b) the final design with 30% mass fraction.
2.4.6.2 2D Bumper Beam

In this example, a 2D beam constrained at both ends is considered under an impact from a rigid pole at the center as shown in Fig. 2.13(a). This example approximately represents the pole test for the rear bumper of cars. The design domain is discretized into a 160 × 40 mesh of 4-node, plane stress quadrilateral elements with 20 mm thickness. A rigid pole of mass 76.6 kg impacts the beam with an initial velocity of 8 m/s. The coefficient of friction between the rigid pole and beam is 0.3. The crash simulation is carried out for 0.01 seconds, long enough for the pole to dissipate all of its kinetic energy. The pole bounces back after dissipating all of its kinetic energy due to elastic recovery in the beam. Like the previous example, the internal energy is read at the closest time corresponding to the maximum intrusion.

Since the impacting pole dissipates all of the kinetic energy, an objective of
minimizing the maximum intrusion is considered subject to a mass fraction constraint of 30%. All of the HCA-related parameters are the same as in Table 2.2 except a neighborhood of size 8 and a convergence tolerance of 3.2 (0.05% of the total number of elements in the design domain) are used for this example. The neighborhood size generally dictates the size of the thinnest members in the resulting structure (see Section 3.2 for details) and hence, should be chosen by the designer based on strength and manufacturability requirements. Figure 2.13(b) shows the final converged design. The material in the top middle of the final structure resists the local plastic deformation, and it is connected to the support ends at the lowest points to generate maximum upward reaction forces. Since the intensity of the load is not too high, the elements in the lower middle part do not deform much and hence, reach the lower bound. In Chapter 4, the same problem with a higher impact speed is considered, and the resulting structure in Fig. 4.6
has members in the lower middle part of the design domain as well.

Figure 2.13. (a) Schematic of a 2D bumper beam impacted by a rigid pole and (b) the final design with 30% mass fraction.

Figure 2.14(a) shows the variation in the maximum intrusion of the pole into the beam with design iteration number. As the design evolves, it becomes stiffer and hence, dissipates the kinetic energy of the impacting pole with less deformation and higher contact forces as shown in Fig. 2.14(b). The maximum intrusion in the final design reduces to almost half of its value for the initial design, whereas the peak force almost doubles its value. Since it is a low intensity impact, a considerable increase in the peak force for significant reduction in the intrusion is acceptable.

In the two examples presented above, it is shown that distributing material within the design domain to achieve uniform internal energy results in stiff designs. Unlike problems with only static loads, increased (effective) stiffness of a structure under crash loads can be interpreted in different ways depending on the selected
Figure 2.14. (a) Variation in maximum intrusion of the pole as the design evolves and (b) comparison of the force-displacement behavior of the final and initial designs.

performance measure. In the first example, increased stiffness meant increased energy absorption for a prescribed intrusion, whereas in the second example, increased stiffness meant less intrusion as the total energy absorption remained the same. In both examples, the desired objectives were achieved at the expense of increased peak force, which is acceptable as long as it is below the injury levels.

In the next section, an update on the original HCA method (presented so far) is presented by introducing a scheme to adaptively assign a proportional gain value $K_p$ to improve the convergence characteristics.

2.5 Summary

The HCA method for topology synthesis is presented in this chapter. Originally introduced by Tovar et al. [107], the HCA method drives the topology to a
configuration that has a uniform distribution of a mechanical stimulus. For topology optimization to achieve minimum compliance, element strain energy density (SED) is used as the mechanical stimulus, and material is distributed to achieve a uniform distribution of SED. Patel et al. [76] later extended this idea further to use the HCA method for crashworthiness design. Element internal energy, which is the sum of elastic strain energy and plastic work, is used as the mechanical stimulus for crashworthiness design. This drives the topology to generate stiff, energy absorbing designs. It presents an attractive option for applications involving crashes at low-to-medium speeds in which peak forces do not reach injury levels and the objective is to effectively dissipate crash energy with minimal damage (intrusion).

Patel et al. [76] also introduced a constraint on peak force to extend the applicability of the HCA method for a wider set of problems in crashworthiness design. The constraint on peak force uses an inverse relationship between the peak force and total mass of the structure. However, decreasing the mass fraction to increase the flexibility (less peak force) is not always a preferred method as it may adversely affect the integrity of the overall structure. There are other techniques to reduce the peak force without decreasing the mass fraction. One such technique is the use of compliant mechanism-type substructures, which is the main focus of Chapter 4.

Overall, the present chapter introduces the concept of topology optimization followed by the HCA method for crashworthiness design. The subsequent chapters use and refer to many elements of the present chapter while building new methodologies for comparison purposes. In the next chapter, the existing HCA method is modified and extended to make it more suitable to use in developing
the new methodologies in the subsequent chapters.
In this chapter, the existing HCA method is modified by introducing an ‘adaptive $K_p$ scheme’ to make it more easily applicable to a wider set of problems without convergence difficulties. Subsequently, the issue of mesh-dependence or minimum member size control, which is a common problem in topology optimization, is investigated, and possible measures to achieve mesh-independent designs in the HCA method are presented. Finally, the HCA method is extended to synthesize compliant mechanisms with nonlinearities arising due to large plastic deformations, keeping in mind their potential applications in crashworthiness design. The need for this modification and extension in the HCA method along with their implementation is described in the remainder of the chapter.

3.1 Adaptive $K_p$ Scheme

Material distribution in the HCA method is governed by a proportional law that uses the error signal between the field variables and setpoint. It has been observed that convergence and the final design are affected by the choice of the proportional gain $K_p$. Therefore, it is important to choose an appropriate value for $K_p$. So far, a fixed value of $K_p$ is assigned by the user based on experience. However, there is no set rule or guideline to decide an appropriate $K_p$ value. A
relatively larger $K_p$ value results in non-convergence (or oscillations near convergence), while a relatively smaller value of $K_p$ requires more iterations to converge. Moreover, different $K_p$ values may result in different converged final designs. To avoid this issue of choosing an appropriate $K_p$ value for every design problem beforehand, an ‘adaptive $K_p$ scheme’ is proposed and used for all of the examples in the remainder of this dissertation.

3.1.1 Formulation

As mentioned earlier, material is updated based on an error signal between the effective field variable and the setpoint using a proportionality-based control rule in the HCA method as expressed in Eq. (2.6). To eliminate the need for the user to choose an appropriate $K_p$ value, which may be different for different problems, an adaptive $K_p$ scheme is proposed. In this scheme, $K_p$ is evaluated before each design update such that the maximum change in the design variable over the whole design domain is less than or equal to the maximum allowable change ($\Delta x_{\text{max}}$). Hence, the adaptive proportionality gain $\bar{K}_p$ for iteration $k$ can be written as

$$\bar{K}_p^{(k)} = \frac{\Delta x_{\text{max}}}{\|(|e^{(k)}|)\|_\infty},$$

(3.1)

where $e^{(k)}$ is the normalized error between the effective field variables and setpoint. This definition of adaptive $K_p$ helps in avoiding the problem of a large proportion of the design variables undergoing changes equal to the maximum allowable value $\Delta x_{\text{max}}$ when a fixed large value of $K_p$ is chosen. It helps in achieving convergence within a reasonable number of iterations by avoiding design oscillations. Moreover, to ensure that only the elements that can afford a change of $\Delta x_{\text{max}}$ in their current relative density values contribute to the $\bar{K}_p^{(k)}$ evaluation, a binary matrix $B_{\text{unsat}}$
of the size of the design variable matrix $\mathbf{z}$ is defined. This binary matrix has value 1 at the locations corresponding to the elements with densities between $x_{\min} + \Delta x_{\max}^{allow}$ and $1 - \Delta x_{\max}^{allow}$. Hence, Eq. (3.1) can be rewritten as

$$\bar{K}^{(k)}_p = \frac{\Delta x_{\max}^{allow}}{\left(\|\mathbf{e}^{(k)}\|_{\infty}\right) \cdot B_{unsat}}. \quad (3.2)$$

The scalar matrix product in the denominator ensures that only the elements with densities between $x_{\min} + \Delta x_{\max}^{allow}$ and $1 - \Delta x_{\max}^{allow}$ contribute in the $\bar{K}^{(k)}_p$ evaluation. By its definition, when adaptive $K_p$ is used instead of a user-defined fixed $K_p$, there is no need to put a maximum limit on the change in the design variables. Therefore, Eq. (2.6) can be rewritten as

$$\Delta x_i^{(k)} = K_p^{(k)} \left(e_i^{(k)}\right). \quad (3.3)$$

As the design moves closer to convergence, required changes in the design variables become smaller. However, with the use of the adaptive $K_p$ definition in Eq. (3.2), there is potential for delayed convergence as $K_p$ may keep growing bigger to balance the maximum required change to the maximum allowable change. To avoid this issue of delayed convergence, $\bar{K}^{(k)}_p$ is held constant once a certain percentage (say 90%) of the elements are saturated (reach either the lower or upper bound).

Since a maximum error value over the whole design domain is used for updating $\bar{K}^{(k)}_p$, an irregular zigzag pattern in its behavior with iteration number is often observed, especially for nonlinear problems with plastic deformations. Due to local plastic deformations or buckling, some elements may show significant change in their behavior during successive design iterations and affect the $\bar{K}^{(k)}_p$ value drastically.
One can bound the changes in $\bar{K}_p^{(k)}$ during successive design iterations to filter or remove these irregularities. Two methods for bounding the $\bar{K}_p^{(k)}$ changes have been tested. In the first scheme, called ‘adaptive bounded $K_p$-I’ for subsequent discussions, a relative maximum allowable change $K_p B$ (say 10%) is imposed as an upper and lower bound on $\bar{K}_p^{(k)}$ changes. This can be expressed as

$$\bar{K}_p^{(k), bounded-I} = \max \left\{ \min \left[ \bar{K}_p^{(k)}, (1 + K_p B) \times \bar{K}_p^{(k-1), bounded-I} \right], (1 - K_p B) \times \bar{K}_p^{(k-1), bounded-I} \right\}. \quad (3.4)$$

In this scheme, $\bar{K}_p^{(k)}$ is still evaluated using Eq. (3.2) at each iteration, but this value is used only if it is within $K_p B \times 100\%$ of $\bar{K}_p^{(k), bounded-I}$ from the previous iteration. This scheme is ‘conservative’ compared to the original adaptive $K_p$ scheme.

In the second scheme, called ‘adaptive bounded $K_p$-II’, the relative maximum allowable change $K_p B$ is only imposed as an upper bound, and $\bar{K}_p^{(k)}$ is not allowed to decrease at any iteration to seek faster convergence. This can be expressed mathematically as

$$\bar{K}_p^{(k), bounded-II} = \max \left\{ \min \left[ \bar{K}_p^{(k)}, (1 + K_p B) \times \bar{K}_p^{(k-1), bounded-II}, \bar{K}_p^{(k-1), bounded-II} \right] \right\}. \quad (3.5)$$

This scheme is more ‘aggressive’ compared to the original scheme as the gain (if not constant) always increases. Examples in the next section provide comparisons of the two schemes for bounding $K_p$ with the original unbounded $K_p$ scheme. Note that unless specified otherwise, the unbounded definition of $\bar{K}_p$ in Eq. (3.2) is used whenever adaptive $K_p$ scheme is mentioned in the following discussion.
3.1.2 Examples

Two examples are presented to illustrate the importance of using the proposed adaptive $K_p$ scheme as compared to using fixed $K_p$ values based on experience. First, each problem is solved with different fixed $K_p$ values to show the effect of $K_p$ on the final design and convergence. Next, the same problem is solved using the adaptive $K_p$ scheme(s).

3.1.2.1 2D Slender Beam

In the present example, the 2D slender beam from the example in Section 2.4.6.1 is considered. The material properties and loading conditions are also kept the same, but the simulation is carried out for 0.013 seconds, long enough for the impacting pole to dissipate its kinetic energy completely. Hence, an objective of minimizing the maximum intrusion is considered. The HCA method is used with the same parameters as described in Table 2.2. First, fixed $K_p$ values of magnitude 0.05, 0.1 and 0.2 are used, and the final designs for these three cases are shown in Figs. 3.1(a), (b), and (c), respectively, along with the number of design iterations to converge. Fixed $K_p$ values of 0.05 and 0.1 both result in convergence within a reasonable number of iterations, but the final designs are significantly different. The reason for getting an Eiffel Tower-like design with $K_p = 0.1$ is the relatively faster accumulation of material in the bottom left and right corners of the design domain due to a relatively higher $K_p$ value. Once the two legs (struts) in Fig. 3.1(b) develop significantly in the first few iterations, they start to function as the main load paths and eliminate any chance of evolution of a new load path. On the other hand, $K_p = 0.5$ keeps the changes over one iteration small enough so that the two slanted legs do not become the main load.
paths in the early design iterations and the chances of a new load path evolution remain fair. Another different design is obtained with the use of $K_p$ equal to 0.2. Moreover, the design does not converge numerically due to oscillating behavior in the design variables near convergence.

Next, the same design problem is solved using the adaptive $K_p$ scheme, keeping all of the other parameters the same as before, and the final converged design is shown in Fig. 3.1(d). This design resembles closely the design obtained with a fixed $K_p$ value of 0.05. The variation of $K_p$ with design iteration number is shown in Fig. 3.2(a). The value of $K_p$ increases (with some local decrease) for the most part during the initial design phase. Once a significant number of elements (95% in this case) saturate, $K_p$ is held constant until convergence is achieved. The force-displacement behavior for the four designs in Fig. 3.1 is compared in Fig. 3.2(b). The performance of the designs with a fixed $K_p$ value of 0.05 and the adaptive $K_p$ scheme are very similar (almost overlapping), whereas the intrusion for designs with fixed $K_p$ values of 0.1 and 0.2 are comparatively higher than the other two. Hence, the performance (measured by the maximum intrusion value) of the design obtained with the adaptive $K_p$ scheme is the same if not better than the design obtained with the most appropriate choice of fixed $K_p$ value. Although a fixed value of $K_p$ (other than the three values tried here) may exist that could result in a converged design with better performance, it is very difficult to know it beforehand without going through some trial and error.

The same problem is also solved with the two adaptive $K_p$ schemes with bounds on relative $K_p$ variations between successive iterations. A 10% bound ($K_pB = 0.1$) is used for this example. The final designs with both of these schemes are almost identical to the final design (Fig. 3.1(d)) using the unbounded adaptive $K_p$ scheme.
Figure 3.1. (a) to (c) - Final designs with 30% mass fraction using fixed $K_p$ values, and (d) final design when adaptive $K_p$ scheme is used.

Figure 3.2. (a) Variation of $K_p$ with design iteration number and (b) comparison of the force-displacement behavior of the final designs in Fig. 3.1.
The use of bounds acts as a filter to make the variation of $K_p$ with iteration number smoother as shown in Fig. 3.3. It is difficult to anticipate whether the bounded schemes will converge in fewer or more iterations as compared to the unbounded scheme. On one hand, the upper bound on the $K_p$ changes in the bounded schemes slows down the convergence. On the other hand, the lower bound has the opposite effect on the convergence rate. For the present example, the number of design iterations for convergence in both bounded schemes is slightly more than the number of iteration for the unbounded scheme. Hence, apart from a smoother $K_p$ variation, there is no gain in terms of number of iterations to converge or the performance of the final design by using the bounded adaptive $K_p$ schemes. In the next example, the same exercise is repeated to reassert the findings of the present example.

![Figure 3.3](image.png)

Figure 3.3. Variation of $K_p$ with design iteration number for the two adaptive $K_p$ schemes with bounds.
3.1.2.2 2D Bumper Beam

In this example, the 2D bumper beam from the example in Section 2.4.6.2 is considered. The loading conditions, material properties, and HCA-related parameters are all kept the same. First, fixed $K_p$ values of magnitude 0.05, 0.1 and 0.2 are used, and the final designs are shown in Figs. 3.4(a), (b), and (c), respectively. Like the previous example, two converged but different final designs are obtained with fixed $K_p$ values of 0.05 and 0.1, whereas the design does not converge even after 150 iterations for a $K_p$ value of 0.2. Next, the adaptive $K_p$ scheme is used, and the final design is shown in Fig. 3.4(d). Figure 3.5(a) shows the variation of $K_p$ with design iteration number. After an initial increase, $K_p$ fluctuates in the middle and then increases to finally become constant once 95% of the elements in the design saturate. The force-displacement behavior for the converged designs in Fig. 3.4 are compared in Fig. 3.5(b). The overall force-displacement performance for the three converged designs (Figs. 3.4(a), (b), and (d)) is very similar except the maximum intrusion for the design with the adaptive $K_p$ scheme is slightly larger than the other two designs. Hence, for this example, a fixed $K_p$ value of 0.05 works slightly better than the adaptive $K_p$ scheme in terms of the number of iterations required for convergence and performance of the final design. This shows that there is always a possibility that a fixed $K_p$ value may be better than using the adaptive $K_p$ scheme. However, it is difficult to anticipate that value of $K_p$ for every problem.

Similar to the previous example, the problem is solved using the two schemes for bounding adaptive $K_p$ values. The final designs are nearly identical to the design with the unbounded adaptive $K_p$ scheme. The variation of $K_p$ with iteration number for both schemes is shown in Fig. 3.6. The adaptive bounded $K_p$-I scheme
Figure 3.4. (a) to (c) - Final designs with 30% mass fraction using fixed $K_p$ values, and (d) final design when adaptive $K_p$ scheme is used.

Figure 3.5. (a) Variation of $K_p$ with design iteration number and (b) comparison of the force-displacement behavior of the final designs in Fig. 3.4.
reduces the fluctuations (zigzag) in $K_p$ values compared to the original unbounded scheme but takes more iterations to converge because of its relatively conservative nature. On the other hand, the adaptive bounded $K_p$-II scheme removes all of the fluctuations in $K_p$. This scheme takes almost the same number of iterations to converge as the unbounded $K_p$ scheme. This example reasserts the findings of the previous example that are - a) the importance of using the adaptive (unbounded) $K_p$ scheme over the user-defined fixed $K_p$ values which may result in different converged designs or non-convergence and b) bounding the changes in $K_p$ value using the two bounded $K_p$ schemes does not show any significant improvements (except smoothing the $K_p$ behavior) in terms of number of iterations for convergence or the performance of the final design.

Figure 3.6. Variation of $K_p$ with design iteration number for the two adaptive $K_p$ schemes with bounds.
3.1.3 Final Remarks

The adaptive $K_p$ scheme is introduced and implemented within the HCA method to (a) eliminate the need from the user to choose an appropriate fixed value of $K_p$, which may be different for different sets of problems, and (b) improve the probability of convergence. The scheme calculates the $K_p$ value before each design update based on field variable and target setpoint information. With the help of two examples, it is shown that the adaptive $K_p$ scheme results in a unique design within a reasonable number of iterations that is almost equivalent (if not better) in performance than designs obtained with fixed $K_p$ values chosen from experience.

The two modifications of the original adaptive $K_p$ scheme to bound the relative change in its value over successive iterations were found to result in almost identical final designs with no significant reduction (if any) in the number of design iterations to convergence. Moreover, these schemes introduce an additional user-defined parameter $K_pB$. The unbounded $K_p$ scheme can be considered as a special case of the adaptive bounded $K_p$-I scheme with a very large $K_pB$ value. Since there are no significant advantages in terms of the final design and number of iterations for convergence identified in using bounded schemes over the unbounded scheme except the smoothness in $K_p$ behavior, the unbounded adaptive $K_p$ scheme is used in the remaining examples.

3.2 Minimum Member Size Control and Mesh-Dependence

In topology optimization, minimum member size control is closely related to the problem of mesh-dependency of solutions. It has been observed that increasing mesh density results in designs with more structural members of decreasing
size. Ideally, mesh-refinement should result in a better finite element model of the same optimal structure and a better description of the boundaries - not in a more detailed and qualitatively different structure \[96,119\]. Theoretically, this reflects the problem of non-existence of solutions in its general continuum setting. Introduction of more holes without changing the structural mass/volume will generally increase the efficiency of a given structure. In the limit, this process of refinement will result in structural variations in the form of microstructures that have an improved use of material \[95\]. From a design and manufacturing point of view, the problem of mesh-dependence translates into the control of the minimum member size in the final design. A structure with a lot of thin members, even though very efficient, may not be practical due to manufacturing and cost-related issues. The degree of simplicity of a design that suits a designer’s manufacturability criteria is an important concern. Moreover, very thin members may create regions of stress concentration leading to failure.

The approach to generate mesh-independent and macroscopic solutions is to reduce the size of the admissible design space by introducing a global or local restriction on the variation of density, hence ruling out the possibility of formation of fine scale structures. These techniques for enforcing such a restriction can be classified into 3 main categories - a) perimeter or area control \[38\], b) global or local gradient constraint on the density variation \[86\], and c) mesh independent filters in the optimization implementation \[95\]. The filter modifies the design sensitivity of a specific element based on a weighted average of the element sensitivity.

Stable designs under mesh refinement while maintaining a minimum length-scale can be achieved by controlling the neighborhood size. Increasing the neighborhood size in the refined mesh model to approximately match the actual size
of the neighborhood in the original mesh model can ensure qualitatively similar
designs. Two such cases are shown in Fig. 3.7. The radius of influence of a 4-
neighbor neighborhood is the same as the element length r. In a refined model
with each element divided into four equal elements of length r/2, a 12-neighbor
neighborhood can maintain the same radius of influence around each element as
shown in Fig. 3.7(a). Similarly, an 8-neighbor neighborhood in a coarser model
is approximately equivalent to a 36-neighbor neighborhood in a refined model as
shown in Fig. 3.7(b).

Figures 3.8 and 3.9 show two examples of mesh-independent designs obtained
by increasing the neighborhood size with mesh-refinement using the HCA method.
In the first example, a 2D cantilever beam is considered under a point load acting
at the lower right corner. The final design with a 160 × 60 mesh discretization
and a 4-neighbor neighborhood is qualitatively similar to the final design with
a 320 × 120 mesh discretization and a 12-neighbor neighborhood as shown in
Fig. 3.8. In the second example, a 2D bumper beam from the example in Section
2.4.6.2 is considered. Again, mesh-independent designs were obtained with a 160
× 40 mesh discretization and an 8-neighbor neighborhood and a 320 × 80 mesh
discretization and a 36-neighbor neighborhood. In both examples, the final designs
with mesh-refinement have the same structure as the designs with a coarser mesh
and a better description of the boundaries.

From the two examples presented above, it is clear that by appropriately con-
trolling the neighborhood size, mesh-independence can be achieved with reason-
able success. The size of the neighborhood also governs the minimum member
size in the final design. A bigger neighborhood results in designs with fewer and
stronger structural members suited for easy and cost-effective manufacturing. This
(a) A 4-neighbor neighborhood (left), and a 12-neighbor neighborhood (right) in a refined mesh model.

(b) An 8-neighbor neighborhood (left) and a 36-neighbor neighborhood (right) in a refined mesh model.

Figure 3.7. Neighborhood size equivalence: In mesh-refinement, neighborhood size can be appropriately increased to approximately match the actual radius of influence to achieve mesh-independent designs.

A qualitative understanding of the relationship between neighborhood size and minimum member size along with user experience is sufficient to obtain structures with desirable complexity. However, a more rigorous quantitative relationship would help in introducing a minimum member size constraint as a surrogate minimum neighborhood size requirement in the optimization problem formulation.

So far, the HCA method for minimum compliance and crashworthiness design has been presented. As discussed earlier, while stiff and energy absorbing designs
obtained using the HCA method are preferred in some crashworthiness applications, they are not always the best solutions. For some applications, the structure should have sufficient flexibility at desired locations, which can be achieved by introducing compliant mechanism-type substructures within the main structure. This is the focus of next chapter. Before that, the HCA method for compliant mechanism synthesis is presented in the next section.

3.3 Compliant Mechanism Synthesis using HCA Method

Compliant mechanisms are single-piece, hingeless structures that transfer or transform displacement, forces or energy from points of application of input forces
to the desired points of interest in an efficient manner. These mechanisms use elastic deformation throughout their body to transfer displacement from input points to output points (as opposed to traditional rigid-body mechanisms that use movable joints). A typical compliant mechanism, a force inverter, is shown in Fig. 3.10(a), and an equivalent rigid-body mechanism force inverter is shown in Fig. 3.10(b). The main advantages of compliant mechanisms are that they can be built with fewer parts, require fewer assemblies and need no lubrication. Compliant mechanisms are especially well suited for Microelectromechanical Systems (MEMS) [58], as the typical assembly processes required for rigid-body mechanisms are difficult at such small scales. Furthermore, linked mechanisms may not perform efficiently since friction dominates at small scales. Apart from MEMS, compliant mechanisms are suitable for many applications like robotics [59], wing flapping mechanisms [55], compliant micromechanisms [57, 63], actuators [83], and snap-through mechanisms [18], among others.

Figure 3.10. A force or displacement inverter mechanism
3.3.1 Background

Two systematic approaches referenced in the literature for design of compliant mechanisms are the kinematic-based approach used by Howell \cite{45} and the continuum-based approach used by Ananthasuresh et al. \cite{12}, Sigmund \cite{93}, and Pedersen et al. \cite{82}, among others. In the kinematic-based approach, hinges in rigid-body mechanisms are replaced with elastic hinges to produce pseudo-rigid body mechanisms that are then converted to fully compliant mechanisms. The continuum-based approach uses a topology optimization method introduced by Bendsøe and Kikuchi \cite{16}. In these methods, an optimum structure is found within a continuum design domain that is divided into finite elements. Element density or thickness is used as a continuous design variable, so the material distribution problem can be solved as a continuous variable problem.

In the beginning, most compliant mechanism design research used a linear elastic material model with small strain theory. However, many applications involve compliant mechanisms undergoing significant displacements. Hence, nonlinearity should be addressed while designing mechanisms for such applications. Although some researchers \cite{19, 82, 92} studied the effect of considering geometric nonlinearity on compliant mechanism design, nonlinearity arising from plastic material behavior has been studied by very few \cite{50}.

One of the reasons for not considering plastic material behavior is the range of applications of compliant mechanisms. Most practical applications involve these mechanisms operating in the elastic range so that they can recover their original shape for multiple uses. However, in this work, compliant mechanisms are utilized for designing crashworthy structures, so nonlinearities due to plastic material behavior, contacts, etc., should be considered. One of the major challenges
in considering nonlinearities during the design of compliant mechanisms is the sensitivity calculations. Unlike the linear elastic case, sensitivities for mechanism design with nonlinearities cannot be expressed in closed analytical form, which leaves the user to evaluate sensitivities numerically. Numerical sensitivity calculations drastically limit the problem size and complexity due to their expensive nature.

In this work, a control-based strategy within the HCA method for material distribution in the design space is employed to achieve an optimal compliant structure for the linear elastic case. The Karush-Kuhn-Tucker (KKT) optimality conditions for the linear elastic case are expressed in the form of an HCA-based objective of distributing the material to achieve a uniform state of an appropriate mechanical stimulus or field variable. The same idea is then extended to nonlinear compliant mechanism design. Although the final design for the nonlinear case cannot be claimed to be ‘optimal’, the method produces a design that performs reasonably better than a corresponding design obtained using linear elastic behavior. Since the method does not use numerical sensitivities, it can be applied to fairly complex problems involving nonlinearities due to plasticity, contact, and dynamic behavior, among others.

3.3.2 Problem Formulation and Dummy Load Method

The objective of a compliant mechanism is to efficiently transfer forces and motion from an input actuation location (referred to as input port I/P) to the desired output locations (referred to as output ports O/P) in a structure. Depending on the choice among force, motion or a combination of both to optimize, an appropriate objective function is defined. Alternatively, these objectives can be
expressed in terms of geometric advantage (GA) and mechanical advantage (MA), as these quantities are frequently referred to in relation to rigid-body mechanisms. Referring to Fig. 3.11, GA and MA can be expressed as

\[
GA = \frac{d_{out}}{d_{in}}, \quad MA = \frac{F_{out}}{F_{in}} = \frac{k_{out}d_{out}}{F_{in}},
\]

(3.6)

where \(d_{in}\) is the displacement at I/P due to the input load and \(F_{out}\) is the output force experienced by a workpiece against which the mechanism is working (acting). Linear springs of stiffnesses \(k_{in}\) and \(k_{out}\) are attached at I/P and O/P to represent the actuator and workpiece. Depending on the application, one can choose to maximize GA, MA or a combination of these two quantities. A mass constraint and sometimes a constraint on the input displacement also accompany the problem definition.

Figure 3.11. A schematic of the compliant mechanism design problem
In this work, an objective of maximizing the output displacement \(d_{\text{out}}\) subject to a mass equality constraint is considered. For a given input force and workpiece stiffness, the objective of maximizing the output displacement is equivalent to maximizing the mechanical advantage (see Eq. (3.6)) or the output work which is an appropriate choice for the future crashworthiness applications. The optimization problem formulation can be expressed as

\[
\text{maximize} \quad d_{\text{out}}(x) \\
\text{s.t.} \quad \sum_{i=1}^{N} x_i = M_f^* \\
x_{\text{min}} \leq x \leq 1, \tag{3.7}
\]

where \(x_i\) is the relative density of an element (design variable), \(N\) is the total number of elements, and \(M_f^*\) is the desired mass fraction. A lower bound of a small positive number \(x_{\text{min}}\) is used to avoid numerical instabilities due to singularities in the stiffness matrix. The displacement at the output port can be alternatively written as

\[
d_{\text{out}} \equiv \begin{bmatrix} 0 & 0 & \cdots & 1 & \cdots \end{bmatrix} \begin{bmatrix} \vdots \\ d_{\text{out}} \\ \vdots \end{bmatrix} = F_d^T U_1 = U_d^T K U_1 = MPE, \tag{3.8}
\]

where \(F_d\) can be thought of as a “dummy” or fictitious force of unit magnitude acting at O/P in the direction of the desired output displacement. \(U_1\) and \(U_d\) are the displacement fields due to the input and dummy loads, respectively. The product \(F_d^T U_1\) is termed the mutual potential energy (MPE) or the complimentary virtual work in the literature [23]. For a discretized FE model, mutual poten-
tial energy, like strain energy, can be expressed as the sum of elemental mutual potential energies.

\[
MPE = \sum_{i=1}^{N} MPE_i = \sum_{i=1}^{N} U^T_{di} K_i U_{1i} = \sum_{i=1}^{N} \sigma^T_{di} \varepsilon_{1i}, \quad (3.9)
\]

where \(\sigma_d\) is the stress field due to the dummy load and \(\varepsilon_1\) is the strain field due to the input load. Using mutual potential energy, the optimization problem in Eq. (3.7) can be rewritten as

\[
\begin{align*}
\text{minimize} & \quad -MPE(x), \\
\text{s.t.} & \quad \sum_{i=1}^{N} x_i = M^*_f, \\
& \quad 0 < x_{\min} \leq x_i \leq 1 \quad \text{for } i = 1, 2, \ldots, N. 
\end{align*} \quad (3.10)
\]

To calculate the mutual potential energy, two load cases are considered. In the first load case, an input force \(F_{in}\) at I/P is applied, and in the second load case, a dummy load \(F_d\) is applied at O/P in the direction of the desired displacement. Springs of stiffnesses \(k_{in}\) and \(k_{out}\) are added at I/P and O/P to model actuator and workpiece stiffnesses as shown in Fig. 3.12.

3.3.3 KKT Optimality Conditions & HCA Framework

The Karush-Kuhn-Tucker (KKT) optimality conditions are derived for the optimization problem formulation in Eq. (3.10) for the linear elastic case. The Lagrangian for the optimization problem can be written as

\[
\mathcal{L} = -MPE + \sum_{i=1}^{N} \lambda_{1i}(x_i - 1 + s_{1i})^2 + \sum_{i=1}^{N} \lambda_{0i}(x_{\min} - x_i + s_{0i})^2 + \lambda_v \left( \frac{\sum_{i=1}^{N} x_i}{N} - M^*_f \right), \quad (3.11)
\]
Figure 3.12. An illustration of the dummy load method: (a) input load model and (b) dummy load model with applied dummy load at the output port in the direction of the desired displacement.

where $\lambda_{1i}$, $\lambda_{0i}$ and $\lambda_v$ are the Lagrange multipliers, while $s_{1i}$ and $s_{0i}$ are the slack variables to represent inequality constraints with equality constraints. The necessary conditions can then be obtained as

$$\frac{\partial L}{\partial x_i} = 0, \quad \frac{\partial L}{\partial \lambda_{1i}} = 0, \quad \frac{\partial L}{\partial \lambda_{0i}} = 0, \quad \frac{\partial L}{\partial \lambda_v} = 0 \quad \text{for} \quad i = 1, 2, \ldots N. \quad (3.12)$$

A rigorous step-by-step evaluation and simplification of these KKT conditions is presented in Appendix A. Depending on whether the design variable is an interior point or at one of the bounds, three conditions are obtained.

$$\frac{- (MPE)_i}{S_i} = \frac{- \sum_{\forall i: x_{min} < x_i < 1} (MPE)_i}{\sum_{\forall i: x_{min} < x_i < 1} x_i} = \begin{cases} \forall i : x_{min} < x_i < 1 \\ S_i \leq S^* \quad \forall i : x_i = x_{min} \\ S_i \geq S^* \quad \forall i : x_i = 1, \end{cases} \quad (3.13)$$
where $S_i$ and $S^*$ are the field variable and target (setpoint), respectively, in HCA terminology. Using the field variable and setpoint definitions from Eq. (3.13), the material is distributed within the design space to reduce the error between them. The proportionality-based control method described in Section 2.3.2 is used as the material update scheme. All other aspects, like neighborhood averaging, penalization of intermediate densities using SIMP, and the adaptive $K_p$ scheme, among others, remain the same as described earlier in this chapter except the secondary setpoint update loop to satisfy the mass equality constraint.

Unlike the minimum compliance design problem, there is no monotonic relation between the target setpoint and the mass of the structure. Hence, the mass control in this case is done by scaling up or down the whole design variable matrix until the equality constraint is satisfied within acceptable tolerance limits.

\[
x^{(j+1)} = x^{(j)} \left( \frac{M^*_f}{M_f^{(j)}} \right),
\]

where $j$ is the iteration counter for the secondary mass update loop.

To account for large deformations, stresses and strains from finite strain theory are used in defining mutual potential energy as

\[
MP E_i = \sigma_{di} : \varepsilon_{1i},
\]

where $\sigma_{di}$ is the true stress tensor due to the dummy load and $\varepsilon_{1i}$ is the true (logarithmic) strain tensor due to the input load for $i^{th}$ element. Plastic material behavior is also considered by using the extended SIMP-like approach for intermediate density penalization as described in Section 2.4.4. The commercially available nonlinear finite element code LS-DYNA is used to perform the nonlinear
analyses in this work. In the next section, an example is presented to demonstrate the application of the HCA method for compliant mechanism design and the advantage of considering nonlinearities during the design phase.

3.3.4 Example

As an example of a compliant mechanism design problem, a 2D displacement (or force) inverter in Fig. 3.13(a) is considered. The goal is to design a structure that converts an input displacement (as a result of an input force) on the left edge to a displacement in the opposite direction on the right edge. Assuming the input actuator is a linear strain-based actuator, it can be modeled by a set of linear springs with stiffness $k_{in}$ and a force $F_{in}$. Similarly, the workpiece can be modeled by a set of linear springs with stiffness $k_{out}$ at the output port. A schematic of the design domain with the input force, desired output displacement, input and output springs, and constraint location is shown in Fig. 3.13(b). Only the upper half of the design domain is considered by defining a symmetry boundary condition at the horizontal symmetry axis. The design domain is discretized into a 100 × 50 mesh of 2D plane stress, 4-node quadrilateral elements with unit thickness. A total input force of magnitude 12000N is applied at the input port. 4 linear springs of stiffness 1000 N/mm each are attached at both the input and output ports.

First, the problem is solved assuming a linear elastic material model for the design domain. The density, elastic modulus and Poisson’s ratio for the elastic material are 2700 kg/m$^3$, 70 GPa and 0.33, respectively. The basic HCA-related parameters are summarized in Table 3.1. The adaptive $K_p$ scheme is used to evaluate the proportionality gain $K_p$ before each design update. A continuation
method based on a penalization parameter suggested by Rozvany [89] is used in this example. In this method, penalization $p$ equal to 1 is used in the first computational cycle, and then it is increased in steps in subsequent cycles. Using only $p = 1$ for the whole design process results in a structure with a significant number of elements having intermediate densities. Increasing the penalization $p$ gradually changes gray regions of intermediate densities locally into black-and-white areas of the same average density. In this example, $p = 1$ is used for the first 100 iterations, and then $p$ is increased in steps of 1 after every 50 iterations.

The final design with 30% mass fraction is shown in Fig. 3.14(a). The variation of displacement of the lower right node in the left direction is shown in Fig. 3.14(b). During the initial design phase, the output displacement is negative, meaning that the output port is moving rightward. The output port starts to move leftward once the two arms making an inverted V shape evolve. The jumps
TABLE 3.1

BASIC HCA-RELATED PARAMETERS FOR THE DISPLACEMENT INVERTER DESIGN PROBLEM

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Desired Mass fraction</td>
<td>$M^*_f$</td>
<td>30%</td>
</tr>
<tr>
<td>Initial density distribution</td>
<td>$x^{(0)}$</td>
<td>0.3</td>
</tr>
<tr>
<td>Neighborhood size</td>
<td>$\hat{N}$</td>
<td>8</td>
</tr>
<tr>
<td>Minimum allowable density</td>
<td>$x_{\text{min}}$</td>
<td>0.05</td>
</tr>
<tr>
<td>Global convergence tolerance</td>
<td>$\epsilon$</td>
<td>2.5</td>
</tr>
</tbody>
</table>

in the output displacement correspond to the increase in the penalty parameter $p$. On application of the input force, the left slanted arm (marked 1 in Fig. 3.14(a)) pushes the horizontal arm (marked 3) upward at the elastic hinge point which forces the slanted right arm (marked 2) to move the output port leftward.

To show the importance of considering nonlinearities during the design phase, the same problem is now solved with a plastic material model for the design domain and nonlinear FE solver. The same elastic material properties from the previous case are used in addition to the plastic material properties from Table 2.1. Also, the same HCA-related parameters from Table 3.1 are used. Since this particular example involves removal of material at the input and output ports during the initial design phase, nonlinear FEA with plasticity may fail to converge. To avoid this issue, a well developed iteration 100 design from the linear elastic case is used as the initial design for the nonlinear plastic case. Moreover, a well developed initial design from the linear elastic case is a better seed (initial design) than a design domain with uniform density as it already inverts the input displacement. The final design for the displacement inverter considering nonlinear plastic behavior is shown in Fig. 3.15(a), and the variation of the output displacement with
Figure 3.14. Displacement inverter considering linear elastic behavior: (a) final design with 30% mass fraction and (b) variation of output displacement as the design evolves.

design iteration number is shown in Fig. 3.15(b). The nonlinear plastic design has more material at the input and output ports compared to the linear elastic design because of local plastic deformations. Also, the hinge area (intersection of 1, 2 and 3 in Fig. 3.15(a)) has more material, which helps in transmitting the loads to the output port with reduced local plastic yielding. An additional member, marked 4 in Fig. 3.15(a), helps the hinge point to move upwards by strengthening the slanted left member (marked 1).

In order to compare the performance of the two designs in Figs. 3.14(a) and 3.15(a), the final linear elastic design is simulated considering nonlinear plastic behavior. Figure 3.16 shows the comparison of the output displacement for the two designs. The nonlinear plastic design results in more output displacement. The difference in the output values is not very large because the input displacement does not get transferred efficiently after the point at which local plastic yielding
Figure 3.15. Displacement inverter considering nonlinear plastic behavior: (a) final design with 30% mass fraction and (b) variation of output displacement as the design evolves.

becomes significant. Figure 3.17 compares the effective plastic strains for the two designs. The linear elastic design has high effective plastic strain close to the constraint location, which is undesirable. On the other hand, the nonlinear plastic design has significantly less effective plastic strain. Moreover, the location of maximum effective plastic strain is close to the input force, which is reasonable. One of the reasons for less effective plastic strain close to the constraint end in Fig. 3.17(b) is the presence of the additional member marked 4 in Fig. 3.15(a). This additional member forces the hinge point to move upwards by bending the horizontal member (marked 3) as opposed to stretching it, which creates a stress concentration near the constraint end.
Figure 3.16. Comparison of the output displacement of the final design obtained by considering linear elastic behavior with that of the final design obtained by considering nonlinear plastic behavior. Note that the linear elastic design is simulated considering nonlinear plastic behavior for comparison.

Figure 3.17. Effective plastic strain contours for the two designs.
3.3.5 Final Remarks

The HCA method for compliant mechanism synthesis is presented with the help of an example problem. The simplified KKT optimality conditions for the linear elastic case are represented in terms of field variable and target setpoint definitions. The field variable definition gives a measure of mechanical stimulus, which should be uniform throughout the design space to achieve an optimal configuration. This definition of mechanical stimulus is then considered for designing nonlinear compliant mechanisms. Although, the final designs in nonlinear cases cannot be said to be ‘optimal’, they are good candidate designs with better performance than the corresponding designs considering only linear elastic behavior. One of the major advantages of using the HCA method for nonlinear compliant mechanism design is that it does not require computationally expensive numerical sensitivity evaluations. Moreover, considering its potential use in designing crash-worthy structures, which involve simulating highly nonlinear dynamic problems with plasticity and contacts, numerical sensitivity calculations become almost impractical.

3.4 Summary

The present chapter introduces a modification to the existing HCA method in the form of an adaptive $K_p$ scheme to improve its convergence characteristics. Material distribution in the HCA method is governed by proportionality-based control rules. It was found that convergence and the final design are affected by the choice of the user-defined proportional gain. There are no set rules to determine an appropriate gain value that will ensure convergence to a unique final design in a reasonable number of iterations. To mitigate this problem, an
adaptive $K_p$ scheme is introduced that evaluates the $K_p$ value before each design update based on the maximum allowable change in density and error signal. The advantage of using this scheme is shown through two examples.

The issue of mesh-dependent designs related to topology optimization in general is also addressed in this chapter. It has been observed that increasing the mesh density results in designs with more structural members of decreasing size. In HCA, this problem can be overcome by appropriately increasing the neighborhood size with increasing mesh density to keep the same region of influence on an absolute scale. Two examples are presented to show that mesh independent designs can be achieved by increasing the neighborhood size.

Since compliant mechanisms are utilized for designing crashworthy structures in this research, nonlinearities arising from large deformations and plasticity should be considered during the design phase. Therefore, the HCA method is extended to design compliant mechanisms considering nonlinear effects. In the present chapter, a control-based strategy within the HCA framework for material distribution in the design domain is developed to design compliant structures considering nonlinear effects. Overall, the present chapter introduces some auxiliary contributions of this research that are relevant and useful for the new methods developed in the subsequent chapters.
CHAPTER 4

CRASHWORTHY DESIGN WITH CONTROLLED ENERGY ABSORPTION

Vehicle crashworthiness design is a challenging problem that involves the modeling of energy transfer through multi-body interactions, plastic deformation, and material nonlinearities, among others. Another factor that makes the problem even harder is the optimization formulation. Unlike, traditional minimum compliance or eigenfrequency maximization problems, the objective in crashworthiness design can vary depending upon the component to be designed and its role in the vehicle system. While some system-level handling and vibration reduction problems in vehicles are solved by improving bending and torsional stiffness, a different approach is needed for component and sub-system-level crashworthiness design. For example, the front structure of a vehicle should be stiff, yet deformable, with crush zones to absorb crash energy resulting from frontal impacts through plastic deformation and to prevent severe intrusion into the passenger cabin. In other words, the structure should exhibit characteristics of both stiffness and flexibility.

The optimization problem formulation must accommodate conflicting criteria such as a maximum acceleration or peak contact force constraint to avoid driver and passenger injuries due to too high g-forces and a maximum deformation constraint to avoid passenger and driver injuries due to penetration of the passenger cabin \[95\]. A structure with a constant high contact force (just below the injury criteria) throughout the crash event satisfies these requirements in the best man-
ner. Sometimes, a structure is required to follow a prescribed force-displacement history for ideal operation under crash. Fang et al. [34] proposed a multi-objective crashworthiness optimization for a full-scale vehicle frontal impact simulation using metamodels. A weighted average of maximizing energy absorption and minimizing peak acceleration was used as the objective function, and thicknesses of the 13 most energy efficient components were used as design variables. Later, the authors [43] also introduced a constraint in the form of maximum intrusion and compared the results using injury-based versus energy-based criteria for a vehicle level side-impact crash test. Liao et al. [66] also proposed a multi-objective optimization scheme based on surrogate models in which the objective was constructed using mass, acceleration and intrusion in a full vehicle frontal impact. Although these efforts are significant in their attempt to express the crashworthiness objective more systematically and completely, they are limited to a small number of design variables.

Pedersen [80, 81], for the first time, used rigorous sensitivity-based topology optimization techniques to design crashworthy structures with desired energy absorption history. In his work, rectangular 2D-beam elements with plastic hinges were used to construct a ground structure that would approximately represent a vehicle component or sub-assembly. Although this sensitivity-based method works well for ground structures made of beam elements, it has never been extended to more realistic continuum-element-based structures. Moreover, sensitivity-based methods, even though very accurate, are not preferred for large problems because of their computationally expensive nature.

Apart from controlling peak contact force and maximum intrusion, other factors like load carrying paths, relative stiffness and potential buckle modes within
the crashworthy structure are also important to consider while designing for specific purposes. For example, the front bumper of a vehicle should deform in a controlled fashion upon collision to transfer loads toward the side frame rails in order to protect the engine and passenger compartments as shown in Fig. 4.1(a).

While designing protective restraint systems like the knee bolster, which comes in contact with the passenger’s knee(s) upon crash, the relative stiffness within the structure plays an important role. As shown in Fig. 4.1(b), the relatively flexible front portion of the structure provides the desired cushioning (to reduce injury due to high force transfer to the passenger), while the stiff back portion maintains the overall structural integrity of the component.

![Diagram](image-url)

(a) The front bumper of a vehicle impacted by a rigid pole  
(b) A typical knee bolster impacted by a passenger knee

Figure 4.1. Examples of crash scenarios in which prescribed load paths (a) and relative flexibility (b) within a structure play an important role in protecting important components and passengers.
In order to tackle these rather complex objectives, this investigation proposes the use of internal plastic mechanisms in the design of crashworthy structures such that energy can be dissipated in a controlled manner. The plastic mechanisms are one-time-use devices that transfer forces and motion from the input load location to desired points within the structure. Also, the flexible nature of these mechanisms helps in introducing flexibility at desired locations within the crashworthy structures. For this purpose, two connected subdomains are defined: a flexible subdomain (FSD) and a stiff subdomain (SSD). The topology design in the FSD is driven by the compliant mechanism design approach, while the SSD makes use of the concept of a ‘fully stressed design’ for which material is distributed to achieve uniform energy distribution. Controlling the relative volumes of the subdomains and the mass fraction for each subdomain gives an additional indirect control over the force-displacement behavior of the final structure. In the next sections, the proposed ‘Controlled Energy Absorption Method’ is described in detail along with its applications for designing crashworthy structures.

4.1 Force-Displacement Response & Energy Absorption

As mentioned in the previous section, one of the challenges in design for crashworthiness is the selection of performance criteria for the objective function. Many combinations of energy absorbed by the structure, peak acceleration, peak force and maximum intrusion are used as the objective function in various multi-objective problem formulations. It is important to understand these performance measures in more detail. While peak acceleration and/or peak force experienced by the passenger during a crash give an indication of potential injury levels, maximum intrusion can have two meanings depending on how it is
measured. When it is measured on the passenger, such as chest deflection due to seatbelt tightening, it must be below human survivable levels. On the other hand, when it is measured on a vehicle component like the front bumper during a frontal impact or the B-pillar during a side impact, its value should be restricted by the distance of the passenger or important components (to be protected) from the impact surface. Since this work focuses on design of vehicle components, the latter definition of maximum intrusion is used for subsequent discussions.

Maximizing the energy absorbed by a structure is also a frequently used objective in vehicle crashworthiness; however, it should be paired with constraints on peak force and maximum intrusion for completeness. For example, a structure can absorb the same amount of energy under a high peak force with low intrusion or a low peak force with high intrusion as shown in Fig. 4.2 where the area under the force-displacement curve represents the energy absorption. The designs in the two cases are referred to as ‘stiff’ (red curve) and ‘flexible’ (green curve), respectively. Note that this characterization of designs as stiff or flexible is relative. Typically, compliance is used as a measure for comparing the stiffness of two structures. Here, peak force and maximum intrusion are used as measures for comparing the ‘effective stiffness’ of two structures. In most cases, the role of a component in the vehicle dictates how it should absorb energy. For example, a B-pillar should absorb energy with low intrusion (even at a cost of high peak force) since the distance between the B-pillar and passenger is very limited. In contrast, the front bumper and front frame rails can afford a reasonable deformation, so energy should be absorbed at moderately high peak forces with affordable intrusion levels. The proposed controlled energy absorption method gives an indirect control over the force-displacement behavior of a structure under crash by
controlling relative stiffness (or flexibility) within the structure.

![Force-displacement curves](image)

Figure 4.2. Force-displacement curves for two designs that absorb the same amount of energy. The stiff design (red) absorbs energy with a higher peak force and lower displacement than the flexible design (green).

4.2 Controlled Energy Absorption Method

A crashworthy component or system should absorb energy to mitigate the effects of a crash deceleration pulse while keeping the penetration levels within limits. Equivalently, a good crashworthy structure should have the characteristics of both flexibility and stiffness. Introducing a compliant-mechanism-type substructure in a crash structure helps in two ways. First, it adds flexibility in the structure; second, it helps in transferring input loads to the desired areas of the structure. To this end, this work proposes the definition of two subdomains: a flexible subdomain (FSD) and a stiff subdomain (SSD) as shown in Fig. 4.3. A
defined FSD will create a compliant-mechanism-type substructure, while the SSD will generate a stiff structure solely based on the distribution of internal energy (IE). Like traditional compliant mechanism synthesis, output ports are defined within the structure where input loads are desired to be transferred efficiently. These user-prescribed output ports should be defined at the boundary of the two subdomains as shown in Fig. 4.3. The relative volumes of two subdomains and their mass fractions, number of output ports and locations are predefined by the user. Although various combinations of these parameters could yield multiple candidate designs, intelligent decisions about the choice of these parameters can be made with some experience with the method and understanding of the nature of the problem. The effect of some of these parameters on the force-displacement behavior of final designs are studied with the help of examples in Section 4.4.

4.3 Formulation

The material distribution rules for both the FSD and SSD are defined in the HCA framework in which the objective is to achieve a uniform distribution of field variables. For an FSD designed with a compliant mechanism approach, the dummy load method (Section 3.3.2) is used to define two load cases. Load case 1 comprises the original loading condition, while load case 2 has dummy loads at the locations of the desired output motion or load transfer ports. On the other hand, the SSD is designed using the fully stressed design approach in which internal energy (IE) corresponding to the original loading (crash) is considered for uniform distribution. Hence, the state $\beta_i$ of each cellular automata (CA) representing an
Figure 4.3. Schematic of the controlled energy absorption method. The design space is divided into two subdomains with output ports (O/P) defined at the boundary of the two subdomains.

Individual material element has three components.

\[ \beta_i = \begin{cases} 
S_{F_i} \\
S_{S_i} \\
x_i 
\end{cases}, \quad (4.1) \]

where \( S_{F_i} \) and \( S_{S_i} \) are the field variables and \( x_i \) is the design variable (density, thickness etc.). \( S_{F_i} \) is a function of mutual potential energy (MPE) as obtained from the KKT optimality conditions in Section 3.3.3 and is evaluated using the dummy load method with two load cases. \( S_{S_i} \) is equal to the internal energy (IE) corresponding to original impact load case as discussed in Section 2.4.5. Although both MPE and IE are evaluated based on analysis of whole design domain, \( S_{F_i} \)
and $S_{Si}$ are defined only in their respective subdomains. Hence,

$$S_{Fi} = -\frac{(MPE)_{i}}{x_{i}}, \quad S_{Si} = 0 \quad \forall i \in \text{FSD}$$

$$S_{Fi} = 0, \quad S_{Si} = IE_{i} \quad \forall i \in \text{SSD}. $$

In order to achieve a uniform distribution of these field variables in their respective subdomains, setpoints or targets are applied to all CAs. Again, separate setpoints are defined for the two subdomains. For the FSD, a setpoint ($S_{F}^{*}$) obtained from the KKT optimality conditions discussed in Section 3.3.3 is used, while an average of the element internal energies (IEs) is used as the setpoint ($S_{S}^{*}$) for the SSD. Hence

$$S_{F}^{*} = -\frac{\sum_{i \in F}(MPE)_{i}}{\sum_{i \in F}x_{i}}, \quad (4.2)$$

$$S_{S}^{*} = \frac{\sum_{i \in SSD}(IE)_{i}}{\sum_{i \in SSD}i}, \quad (4.3)$$

where $F = \{l : l \in \text{FSD}, x_{\text{min}} < x_{l} < 1\}$. Separate mass fractions ($M_{f}^{*}_F$ and $M_{f}^{*}_S$) for the two subdomains are specified as opposed to an overall mass fraction in order to avoid situations in which one subdomain absorbs most of the available material. For satisfying the mass equality constraint in the FSD and SSD, secondary inner mass loops are used as shown earlier in Eqs. (3.14) and (2.7).

A step-by-step procedure for the proposed HCA-based controlled energy absorption method is described as follows and systematic flowchart is shown in Fig. 4.4.

Step 1. Define the two subdomains and output port location(s) and direction(s).
Step 2. Create a second load case with dummy load(s) applied at the output port location(s) in the direction(s) of desired motion.

Step 3. Start with an initial design $x^{(0)}$ for both load cases.

Step 4. Evaluate the internal energy (IE) at each discrete location $i$ within the SSD and mutual potential energy (MPE) within the FSD using finite element analysis on both load cases. Note that IE corresponding to only the original load case is considered.

Step 5. Update the density distribution according to field variables and setpoints to obtain a new design $x^{(k+1)}$.

Step 6. Check the mass equality constraint in both subdomains. If satisfied, go to the next step; otherwise, go into the respective inner mass loops.

Step 7. Check for design convergence. If the convergence criterion is satisfied, the final design is obtained; otherwise, the iterative process continues from Step 4.

4.4 Examples

In this section, two example design problems are solved using the proposed controlled energy absorption method. The final designs in both examples are compared with designs obtained using the traditional HCA-based method for crashworthiness (Section 2.4) which results in stiff designs. Effects of varying the relative subdomain size and mass fraction on performance of the final design are also discussed along with a heuristic to intelligently choose these parameters depending on the problem type and performance requirements.
4.4.1 2D Beam with 2 Horizontal Load Paths

In this example, a 2D beam constrained on both ends is considered under an impact load from a rigid pole at its center. A schematic of the design domain with dimensions and the location of impact is shown in Fig. 4.5. The design domain is discretized into a 160×40 mesh of 4-node, plane stress quadrilateral elements with

Figure 4.4. Illustration of HCA-based controlled energy absorption method for designing crashworthy structures.
20 mm thickness. The beam is modeled with linear elastic, piecewise linear plastic material, and the impacting pole is modeled as rigid. The material properties for the beam are shown in Table 4.1. The rigid pole with a mass of 76.6 kg impacts the beam with an initial velocity of 15 m/s. The coefficient of friction between the impacting pole and the beam is assumed to be 0.3. The impact in all cases is analyzed for 0.01 seconds for uniformity. The dynamic simulations are performed using the explicit solver of the nonlinear finite element code LS-DYNA.

![Design Domain 160X40 mesh](image)

Figure 4.5. Schematic of a 2D beam impacted at its center by a rigid pole with an initial velocity

First, the problem is solved using the traditional HCA-based method for crash-worthiness design in which material is distributed to achieve a uniform internal energy distribution. The goal is to obtain a stiff design for a given mass fraction of 30%. Basic HCA-related parameters like the neighborhood, penalization parameter, convergence tolerance, etc., are summarized in Table 4.2. The final
TABLE 4.1

MATERIAL PROPERTIES FOR BOTH EXAMPLES - LINEAR ELASTIC, PIECEWISE LINEAR PLASTIC MATERIAL MODEL FROM LS-DYNA

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Effective plastic strain</th>
<th>Effective stress (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>2700 kg/m³</td>
<td>0.00</td>
<td>180.0</td>
</tr>
<tr>
<td>Elastic Modulus</td>
<td>70 GPa</td>
<td>0.01</td>
<td>190.0</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
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<td>0.02</td>
<td>197.0</td>
</tr>
<tr>
<td>Yield Stress</td>
<td>180 MPa</td>
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<td></td>
<td></td>
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<td>0.15</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>0.20</td>
<td>238.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.40</td>
<td>248.5</td>
</tr>
</tbody>
</table>

design is shown in Fig. 4.6(a). Figure 4.6(b) shows the performance comparison of the initial design, two intermediate designs, and the final design in terms of force-displacement behavior. It can be seen from Fig. 4.6(b) that as the design evolves, it becomes stiffer. In other words, the intrusion becomes smaller, and the peak force becomes larger.

The above design is appropriate when reducing intrusion is the main objective; however, high peak force is another major concern for crashworthy structures. In many cases, reducing high peak forces at the expense of a reasonable increase in intrusion is preferred. For such cases, the proposed method gives the designer an option to introduce flexibility at desired locations within the structure and hence, reduce the peak forces. To demonstrate this advantage, the same problem is now solved using the proposed method by defining flexible and stiff subdomains. In this case, the two subdomains are defined laterally as shown in Fig. 4.7 with the
flexible subdomain (FSD) covering half of the width and two stiff subdomains (SSD) on the sides covering the other half of the width in total. Two output ports are defined at the boundaries between the subdomains at half of the total height, as shown in Fig. 4.7. The flexible subdomain and output ports serve purposes apart from reducing the peak force in that they offer the freedom to predefine load paths and a desired use of the design space.

As mentioned earlier, for the FSD designed with the compliant mechanism approach, the dummy load method is used to define two load cases as shown in Fig. 4.8. Load case 1 consists of the original loading, while load case 2 has dummy loads $F_d$ defined at the output ports in the direction of desired motion as shown in Fig. 4.8(b). Traditionally, while designing compliant mechanisms, springs of stiffness $k_{in}$ and $k_{out}$ are added at the input and output ports to represent the actuator and workpiece stiffnesses. In the present case, the stiff subdomain (SSD) is the workpiece, so there is no need to add extra stiffness at the output ports. However, the actuation in this case is an impact from a rigid pole, and it is difficult to estimate the stiffness corresponding to the actuation.

Deepak et al. [25] in their

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**TABLE 4.2**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Desired Mass fraction</td>
<td>$M_f^*$</td>
<td>30%</td>
</tr>
<tr>
<td>Initial density distribution</td>
<td>$x^{(0)}$</td>
<td>0.3</td>
</tr>
<tr>
<td>Neighborhood</td>
<td>$\tilde{N}$</td>
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<td>Penalization</td>
<td>$p$</td>
<td>1</td>
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<tr>
<td>Penalization</td>
<td>$q$</td>
<td>1</td>
</tr>
<tr>
<td>Minimum allowable density</td>
<td>$x_{min}$</td>
<td>0.05</td>
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<tr>
<td>Global convergence tolerance</td>
<td>$\epsilon$</td>
<td>6.4</td>
</tr>
</tbody>
</table>
Figure 4.6. Final design (a) obtained using the traditional HCA-based method. In (b) the force-displacement behavior of the final design is compared with the initial and two intermediate designs.

comparative study of various formulations and methods for compliant mechanism synthesis emphasized that the stiffnesses of the actuator and workpiece play an important role. Higher stiffnesses of the actuator and workpiece increase the material continuity to the point where de facto ‘hinge’ regions are eliminated [102]. For the present case of structures designed for crashworthiness, material continuity and distributed compliance are desired. For this reason, stiffness at the input port in the dummy load case is increased to the limit (spring of infinite stiffness) where it is approximated by a displacement constraint as shown in Fig. 4.8(b). The width of 80 mm for this constraint is determined based on the approximate contact width of the rigid pole with the beam in the original load case.

Mass fractions for both subdomains are set to 30%, making the mass fraction for the whole design domain 30% as well. All HCA-related parameters are the same as those in Table 4.2. Since in this case, the FSD is designed using the
Figure 4.7. Schematic of 2D beam problem solved using controlled energy absorption method. The FSD and SSDs are defined laterally and two output ports are defined at the boundary to predefine load paths.

Figure 4.8. Two load cases are defined to design the FSD using the dummy load method.

compliant mechanism approach that generally does not converge to a crisp 0-1 structure due to neighborhood averaging of field variables, a restriction method is used. An often used restriction method to obtain crisp boundaries in filter-based (equivalent to neighborhood averaging in HCA) topology optimization is to gradually decrease the size and influence of the filter during the optimization process [94]. In the present case, the neighborhood size is reduced to 0 once a converged design (at a relatively higher tolerance value), which may have some
gray regions, is obtained. In other words, the final design is obtained in two stages. In the first stage, a neighborhood of size 8 is used with a convergence tolerance of 24. In the second stage, the neighborhood size is reduced to 0, and the convergence tolerance is reduced to 16.

The final converged design is shown in Fig. 4.9(a). As intuitively expected, the FSD develops into an inverted ‘V’ shape that opens outward during the impact and transfers loads to the SSD. The introduction of flexibility in the structure due to the FSD can be further confirmed by the force-displacement comparison of the final and initial designs in Fig. 4.9(b). The final design has a lower peak force and more intrusion compared to the initial design. Figure 4.10 compares the force-displacement behavior of the design obtained using the controlled energy absorption method (Fig. 4.9) and the design obtained using the traditional HCA-based method (Fig. 4.6). Clearly, the design obtained with the present method has a significantly lower peak force at the expense of increased intrusion. The two designs are quantitatively compared in Table 4.3. The total energy absorption for the two designs is very close, which indicates that the proposed method helps in reducing peak force without affecting the total energy absorption by much (2.7%). Another interesting thing to note is the energy distribution within the structure for the design obtained using the present method. Due to its flexible nature, the FSD absorbs a significant proportion of the total energy absorbed by the whole design. Because of its flexibility and significant energy absorption, the FSD acts like a mechanical fuse and might help in protecting its neighbors by sacrificing itself first in most cases.
Figure 4.9. Final design (a) obtained using controlled energy absorption method. In (b) force-displacement behavior of final design is compared with initial design.

TABLE 4.3

QUANTITATIVE COMPARISON OF TRADITIONAL HCA-BASED DESIGN AND DESIGN WITH CONTROLLED ENERGY ABSORPTION METHOD

<table>
<thead>
<tr>
<th>Method</th>
<th>Max. Intrusion (mm)</th>
<th>Peak Force (N)</th>
<th>Total Energy Absorbed (N-m)</th>
<th>Energy Absorbed in FSD (N-m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HCA</td>
<td>50</td>
<td>197440</td>
<td>8550</td>
<td>4400</td>
</tr>
<tr>
<td>CEA</td>
<td>80</td>
<td>123780</td>
<td>8320</td>
<td>6715</td>
</tr>
</tbody>
</table>

To understand the effects of varying the relative subdomain sizes and mass fractions on performance of the final design, two additional scenarios are considered as shown in Fig. 4.11. In the first scenario, the FSD size is reduced to 300 mm, and equal mass fractions of 30% are considered for both subdomains. A shorter
Figure 4.10. Force-displacement behavior comparison of design obtained with controlled energy absorption and design obtained with traditional HCA-based method.

FSD places more material in the stiffer subdomain compared to the previous case when both subdomains were equal in size. This should result into a stiffer design compared to the previous case. Figure 4.12(a) shows the final design, and Fig. 4.13 compares the force-displacement behavior of this design and earlier designs. It is clear from the force-displacement comparison that the design with a shorter FSD (green curve) has a slightly higher peak force and less intrusion compared to the design with equal subdomain sizes and mass fractions (blue curve).

In the next scenario, the mass fraction of the FSD is increased to 35%, and the mass fraction of the SSD is decreased to 25% with equal sized subdomains as shown in Fig. 4.11(b). The overall mass of the whole design remains the same as in previous designs. The final design for this scenario is shown in Fig. 4.12(b). Since the SSD has less mass in this case, the overall stiffness of the whole structure
should be less compared to the case when equal mass fractions were set for both subdomains (equal size). This can be confirmed by the force-displacement behavior comparison in Fig. 4.13. The design in the present scenario (black curve) has a lower peak force and higher intrusion compared to all of the other cases considered so far.

![Diagram](image1)

**Figure 4.11.** Two variations of the original (Fig. 4.7) subdomain and mass fraction definitions are considered to study their effects on final design.

![Diagram](image2)

**Figure 4.12.** Final designs for the two scenarios presented in Fig. 4.11.
Based on the results of the two variations presented above, it can be seen that the effect of varying relative subdomain size and mass fraction on the behavior of final design can be qualitatively predicted. However, it is difficult to generalize the trend or predict the changes quantitatively as they depend on the problem definition. The next example helps in understanding these effects in more detail.

4.4.2 2D Slender Beam with 2 Angled Load Paths

In this example, a 2D slender beam constrained at its base is considered under an impact load from a rigid pole at its top surface. A schematic of the design domain with dimensions and the location of impact is shown in Fig. 4.14. The
design domain is discretized into an $80 \times 120$ mesh of 4-node, plane stress quadrilateral elements with 1 mm thickness. The beam is modeled as linear elastic, piecewise linear plastic material with properties shown in Table 4.1. The rigid pole with a mass of 15.7 kg impacts the beam with an initial velocity of 15 m/s. The coefficient of friction between the rigid pole and beam is 0.3. The impact in all cases is analyzed for 0.01 seconds for uniformity.

Figure 4.14. Schematic of 2D slender beam impacted by a rigid pole with an initial velocity at its center
First, the problem is solved using the traditional HCA-based method in which the objective is to obtain a stiff design for a given mass fraction of 30%. The basic HCA-related parameters are summarized in Table 4.4. The final converged design is shown in Fig. 4.15(a), and its force-displacement behavior is compared with the initial and two intermediate designs in Fig. 4.15(b). It can be seen from the force-displacement comparison that as the design evolves, the peak force increases and intrusion decreases until a final thick column-like structure is obtained.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Desired Mass fraction</td>
<td>$M_f^*$</td>
<td>30%</td>
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<tr>
<td>Initial density distribution</td>
<td>$x^{(0)}$</td>
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<td>Neighborhood</td>
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<td>Penalization</td>
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<tr>
<td>Minimum allowable density</td>
<td>$x_{min}$</td>
<td>0.05</td>
</tr>
<tr>
<td>Global convergence tolerance</td>
<td>$\epsilon$</td>
<td>4.8</td>
</tr>
</tbody>
</table>

Apart from being too stiff, one more issue with the column-like design could be that it has only one load path. In many applications, multiple load paths are preferred for robustness. For example, if one load path fails due to some imperfection, other load paths can save the structure from a catastrophic failure which is not possible in a single load path structure. Hence, in order to introduce
flexibility and multiple load paths, the same problem is now solved using the controlled energy absorption method. In this case, the two subdomains are defined vertically as shown in Fig. 4.16 with the FSD covering the top $1/3^{rd}$ of the height and the remaining being the SSD. Two output ports are defined at the boundary of the subdomains close to the outer ends, making an angle of $45^\circ$ with the horizontal boundary as shown in Fig. 4.16. Output ports at an angle help in diverting some of the impact load sideways, which makes the structure flexible. The current choice of angle ($45^\circ$) for the output ports is user-dependent, and other values may be tried depending upon the design objectives. Output ports making a smaller angle (say $30^\circ$) with the horizontal boundary will help in diverting more impact load to the sides, which will make the structure more flexible. On the other hand, output

Figure 4.15. Final design (a) obtained using the traditional HCA-based method. In (b) the force-displacement behavior of the final design is compared with the initial and two intermediate designs.
ports making a larger angle (say 60°) with the horizontal will transfer more loads downward to the SSD, making the overall structure more stiff.

As described in the previous example, two load cases are considered to design the FSD using the compliant mechanism design approach. The first load case consists of the original loading, while load case 2 has dummy loads $F_d$ defined at the output ports in the direction of desired motion as shown in Fig. 4.17. Again, in order to obtain designs with material continuity and distributed compliance, the
stiffness at the input port in the dummy load case is represented by displacement constraints (vertical direction).

Figure 4.17. Two load cases are defined to design FSD using dummy load method.

Mass fractions for both subdomains are set to 30%. All the necessary HCA-related parameters are the same as in Table 4.4. Like the last example, in order to get a crisp 0-1 design, neighborhood size is reduced to 0 after the first convergence for which the tolerance is set to 48. The final converged design is shown in Fig. 4.18(a). The FSD region evolved as an inverted ‘V’ making approximately an angle of 45° with the boundary between FSD and SSD. While this inverted ‘V’
shape for the FSD is intuitive, the merit of the present method lies in developing
a suitable structure in the SSD to support the loads transferred from the FSD.
The force-displacement performance of this design is compared with the design
obtained using the traditional HCA-based method in Fig. 4.18(b). It is to be
noted that these force-displacement responses are obtained for final designs after
removing the lowest density elements from the model. The introduction of the
FSD and multiple load paths reduces peak force at the expense of a slight increase
in intrusion. Only a small increase in intrusion for the present case is due to the
simulation stopping at 0.01 seconds. While the rigid pole had transferred all
of its kinetic energy to the beam in the previous design (red curve) before 0.01
seconds, the current design still has scope to absorb more energy at low force if
the simulation time is increased. The present comparison, however, is sufficient
to show the merits of proposed method.

Like the previous example, two variations of the present example in the form of
relative subdomain size and mass fraction are considered and shown in Fig. 4.19
For ease of understanding, these variations are referred as CEA-2 and CEA-3 in the
following discussion, whereas CEA-1 refers to the first attempt shown in Fig. 4.16
In CEA-2, the FSD height is reduced to 150 mm, and equal mass fractions of 30% are set for both subdomains. The final design is shown in Fig. 4.20(a). In CEA-3, the subdomain sizes are the same as in CEA-1, but the mass fraction is reduced to 24% in the FSD and increased to 33% in the SSD so that the overall mass fraction remains 30%. The final design for this case is shown in Fig. 4.20(b).

Figure 4.21 shows the comparison of the force-displacement behavior of the
CEA-1, CEA-2, and CEA-3 designs along with the design obtained using the
traditional HCA-based method. The CEA-2 design (green curve) has a force-
Figure 4.18. Final design (a) obtained using controlled energy absorption method. In (b) force-displacement behavior of final design is compared with design obtained using traditional HCA-based method.

displacement behavior very similar to the CEA-1 design (blue curve) because the inverted ‘V’ in both cases is sufficiently strong and does not get crushed significantly before the two lower legs start to deform. However, when the mass fraction of the FSD is reduced (as in CEA-3), it becomes more flexible and crushes significantly before the lower legs start deforming. This behavior of CEA-3 is further confirmed by its having the lowest peak force and maximum intrusion among all designs. One important feature of the CEA-3 design is its ability to hold almost a constant force throughout the impact duration due to successive deformation (collapse) from top to bottom.
(a) Scenario-1 (CEA-2): Shorter FSD  
(b) Scenario-2 (CEA-3): Reduced $M_f$ in FSD

Figure 4.19. Two variations of the original (Fig. 4.16) subdomain and mass fraction definitions are considered to study their effects on the final design.

4.5 Summary

A new method based on the use of compliant mechanisms for designing crash-worthy structures with controlled energy absorption is presented. This method helps in introducing flexibility at desired locations within the structure, which in turn reduces the peak force at the expense of a reasonable increase in intrusion. For this purpose, the given design domain is divided into two subdomains - flexible (FSD) and stiff (SSD) subdomains. The design in the FSD is governed by the compliant mechanism synthesis approach for which output ports are defined at the boundary between the two subdomains. These output ports help in defining potential load paths and help the user make better use of a given design space. The design in the SSD is governed by the principle of a ‘fully stressed design’ for
Figure 4.20. Final designs for the two scenarios presented in Fig. 4.19.

Figure 4.21. Comparison of force-displacement behavior of all designs obtained using controlled energy absorption method and design obtained using traditional HCA-based method.
which material is distributed to achieve uniform energy distribution within the design space. Together, the FSD and SSD provide a combination of flexibility and stiffness to the structure which is desirable for most crash applications.

Two example problems are used to demonstrate the capability and applicability of the present method. In the first example, a 2D horizontal beam constrained at its ends is considered under an impact from a rigid pole at its center. An FSD defined at the center of beam beneath the impact helps in reducing the peak forces transmitted by the pole while transferring loads to the sides where the SSDs are located. The design obtained using the present method is compared with the design obtained using the traditional HCA-based method. Traditional HCA-based designs are stiff, and the comparison helps in measuring the extent of flexibility introduced in the structures when designed using the present method. Effects of variations in relative subdomain size and mass fraction are also studied and show that qualitative predictions about changes in force-displacement behavior can be made.

In the second example, a 2D slender beam constrained at its base is considered under an impact from a rigid pole at its top surface. The subdomains in this case are defined vertically with the FSD close to the impact end. In this case, the FSD reduces the peak force, but also helps in generating a design with two load paths as opposed to the single column design obtained using the traditional HCA-based method. The idea behind defining an FSD near the top of a slender structure is to provide a cushioning effect at the beginning of impact. This effect indeed is seen in the CEA-3 design as the FSD first collapses before deformation in the SSD starts. This progressive collapsing behavior also helps in maintaining a constant force for a long duration of impact, which is desirable.
One of the limitations of the present method is the inability to predict the force-displacement response ‘quantitatively’ beforehand. For this reason, a number of iterations with varying relative subdomain sizes and mass fractions are required to achieve a desired force-displacement response. For future work, the manual iterations of varying subdomain size and mass fraction can be automated in a two-stage design method (see Section 7.2.2 for details).
Thin-walled tubular components are extensively used as structural members in the majority of transportation vehicles because of their low cost, good energy absorption capability and relatively low density. They have the ability to absorb the kinetic energy of the impacting body in the form of plastic deformations, hence protecting the structure and passengers involved. These structures can be used in various loading conditions such as axial crushing, bending, oblique impact, and transverse loading, among others. Tubular structures show significant energy absorption for long strokes in an axial crushing mode and hence, present an attractive option in crashworthiness designs.

A great deal of research has been done to study the axial crushing of thin-walled tubes since the pioneering work of Pugsley [87] and Alexander [7] during the 1960’s. In the 1980’s, work [2-5] on both the static and dynamic responses of tubes focused more on theoretical and experimental studies than computational approaches. The concept of a superfolding element was introduced by Wierzbicki and Abramowicz [110] to better understand the mechanics of the crushing of thin-walled structures. With the advancement in computing power and the numerical implementation of finite element methods in the last two decades, considerable work has been done in creating equivalent numerical models for tube crushing.
and their validation with experimental tests [36, 48, 53, 61, 62, 85, 103]. With the help of advanced nonlinear software like LS-DYNA and ABAQUS, results from numerical models of axial and oblique loadings of thin-walled structures sufficiently match experimental results.

Various experimental and numerical studies revealed three dominant modes of deformation during the axial crushing of thin-walled structures: progressive buckling, global or Euler-type buckling, and dynamic plastic buckling [49]. Of these three, the progressive buckling mode is desired for crashworthy designs because of its efficient energy absorption and better force-displacement behavior. In progressive buckling, crushing starting at one end (often the end close to the impact) and progressing systematically toward the other end of the structure is preferred as it utilizes the maximum possible material for plastic deformation without jamming. Thornton and Magee [104] showed that collapse initiators, also known as triggers, stress concentrators, or imperfections, can be used to 1) initiate a specific axial collapse mode, 2) stabilize the collapse process, and 3) reduce the peak force during the axial crush. Many researchers came up with ideas for introducing buckle initiators or surface patterns to enhance energy absorption and buckling behavior. Chase et al. [21] enforce progressive buckling by introducing various crush zones in the structure. El-Hage et al. [31] studied the quasi-static axial crush characteristics of square aluminum tubes with chamfering and other triggering mechanisms. Lee et al. [64] studied the effect of triggering dents on the energy absorption capacity of axially compressed aluminum tubes. An overview of geometric and material modification techniques to improve the buckling behavior and energy absorption characteristics of thin-walled tubes under axial crushing can be found in a review article by Yuen and Nurick [115].
During an actual crash event, the tubular structure will seldom be subjected to pure axial loads. For example, thin-walled structures are subjected to both axial forces and bending moments in an oblique crash. During the study of thin-walled tubes under oblique impacts by Han and Park [41] and Reyes et al. [88], a phenomenon of global bending or Euler-type buckling was observed if the load angle was higher than a critical value. The onset of global bending severely reduces the energy absorption capability of tube structures.

In the methodology presented here, the compliant mechanism design approach is used for thickness-based (topometry) design of thin-walled square tubes under axial compression. The ability of compliant mechanisms to transfer motion and forces from an input load location to the desired points in the structure is utilized to achieve the desired buckle zones in the axial member. By suitably defining the output port locations and desired displacement directions, progressive buckling can be initiated at the desired locations. Moreover, using this method, thin-walled structures can be designed to show progressive buckling even in cases of oblique impact at angles higher than the critical value at which bending collapse dominates the axial collapse and leads to poor energy absorption. To perform thickness-based topometry design of mechanisms, the HCA-based method for compliant mechanism synthesis as described in Section 3.3 is modified by using thickness as the design variable. In the following sections, a detailed description of the proposed method and two illustrative examples are presented.

5.1 Progressive Buckling in Square Tubes

During pure axial crushing of thin-walled square tubes, the maximum peak force \( F_{\text{max}} \) appears when the buckling starts in the structure for the first time,
and after that, the force oscillates between local peaks and minimum loads as shown in Fig. 5.1. Each pair of peaks is associated with the development of a wrinkle or buckle. Usually, these wrinkles or buckles develop sequentially from one end of a tube so that the phenomenon is known as progressive buckling. Designers often ignore the oscillations in the force-displacement behavior and use a mean value $F_{\text{mean}}$ as an indicator of the energy absorption capability.

![Figure 5.1. Typical force-displacement curve for an axially crushed thin-walled square tube.](image)

Wierzbicki and Abramowicz [110] have identified two basic forms of collapse elements, Type I and Type II, as shown in Fig. 5.2, which they used to study the progressive buckling of square tubes with mean width $w$ and mean wall thickness $t$. Based on the geometric compatibility requirements at the vertical interfaces of the basic elements, there are four different collapse modes. The symmetric
crushing mode consists of four Type I collapse elements in each layer of the folds. Therefore, in each fold during a symmetric crushing mode, two of the parallel faces of the tube move inward while the other two parallel faces move outward. This mode of crushing is dominant in thin square tubes with $w/t > 40.8$. When $w/t < 7.5$, the dominant mode of crushing is extensional, and each layer of fold can be idealized with four Type II basic collapse elements. The asymmetric mixed mode A is idealized as two adjacent layers of folds having 6 Type I and 2 Type II basic collapse elements, whereas asymmetric mixed mode B is idealized as two adjacent layers of folds having 7 Type I and 1 Type II basic collapse elements. This type (mixed modes A and B) of crushing is predicted to form within the range $7.5 < w/t < 40.8$. In the present work, tubes are designed to invoke a symmetric mode of collapse by defining output ports on the two parallel faces pointing inward.

![Diagram](image)

Figure 5.2. Basic collapse element shapes for a quarter of a square cross section of a tube with width $w$ - (a) Type I and (b) Type II.
5.2 Methodology

During axial crushing of thin-walled square tubes, it is often desired that the folding or collapse starts at the front end (end closer to the impact) and progresses systematically toward the other end of the structure to utilize the maximum possible material for plastic deformation without jamming. Also, the progressive buckling from the front end to the rear end helps in protecting important components close to or behind these energy absorbing structures. For example, the damage for low intensity (∼7-10 mph) frontal impacts in automotives can be restricted to the bumper-beam and crash-box (hence saving the front frame-rail behind them) if the crash-box collapses progressively without jamming. Thin-walled square tubes, however, do not buckle progressively from the front end to the rear end in all cases. The buckling behavior is dependent on many factors such as loading conditions, geometry, imperfections and asymmetries in the structure. For example, Fig. 5.3 shows the deformation under axial compression of a tube of uniform thickness (1 mm) except for a small imperfection in the form of a patch (blue stripe) of thickness 0.8 mm. It can be seen that buckling starts at the location of imperfection and then progresses in both directions.

Using the proposed method, thin-walled square tubes can be designed to undergo progressive buckling starting from the front end during axial crushing regardless of geometric and material imperfections, asymmetries in the tube geometry and loading conditions, etc. The basic idea is to design the tube using a compliant mechanism approach to trigger the symmetric mode of collapse near the impact end. Figure 5.4 shows three sets of output ports (in compliant mechanism terminology) pointing inward, defined on the two parallel faces with the first set defined close to the impact end. The two parallel faces of the designed tube
Figure 5.3. The undeformed (a) and deformed (b, c) states of a thin-walled square tube with an imperfection, axially crushed by a rigid plate.

have the tendency to move inward at the output port location and hence, form a fold or wrinkle with four type I (Fig. 5.2) elements (symmetric collapse). For ease of understanding, the location of the output ports along the length of the tube is referred as the ‘enforced buckle zone’ in subsequent discussions.

The compliant mechanism design approach described in Section 3.3 is modified to perform thickness-based (topometry) design changes. For plane stress elements, the variation in relative density $x$ and thickness $t$ have the same effect on the element stiffness tensor $[95]$. Since plane stress shell elements are used to model thin-walled structures, relative density $x$ can be replaced by the element thickness.
Figure 5.4. Three sets of output ports, pointing inward, defined on the two parallel faces to trigger buckling.

\[
\begin{align*}
\text{minimize:} & \quad -MPE, \\
\text{s.t.} & \quad \frac{\sum_{i=1}^{N} t_i}{N} = T^*, \\
& \quad 0 < t_{\min} \leq t_i \leq t_{\max} \quad \text{for} \quad i = 1, 2, ..., N.
\end{align*}
\]

(5.1)

The original mass equality constraint is reformulated as an equality constraint on the average thickness value. Similarly, the relative density variable \(x\) is replaced by the thickness \(t\) in the field variable and setpoint definitions from Section 3.3.

As described in Section 3.3, the dummy load method is used to evaluate mutual potential energy (MPE) with the help of two load cases. In the first load case, dynamic axial crushing by the rigid plate is approximated by static axial loads, and in the second load case, a dummy load \(F_d\) is applied at the output ports in the direction of the desired displacement. Once the tube is designed to transfer motion
and forces from the input loading points to the prescribed output points efficiently with the approximated static loading, it will behave in the similar fashion with the application of the actual dynamic loading.

The ‘enforced buckle zones’ act as triggering mechanisms and help in reducing the peak force by facilitating the formation of folds with lesser resistance. In addition, these enforced buckle zones delay or completely eliminate the onset of global bending (Euler-type buckling) during oblique impacts with load angles higher than the critical value. In the next section, two example problems are considered in which the tubes designed using the proposed method are shown to perform better compared with equivalent uniformly thick tubes.

5.3 Examples

In both examples, the explicit nonlinear finite element code LS-DYNA is used to perform dynamic simulations of axial crushing of square tubes. A linear elastic, piecewise linear plastic material (*MAT24) is used for modeling steel tubes with properties shown in Table 5.1 Generally, thin-walled structures under axial compression are modeled with plane stress shell elements. In several investigations [85, 103], reasonable agreement has been found between shell-based finite element simulations and experimental results. A very efficient plane stress shell element formulation from LS-DYNA (ELFORM=16) that is not subject to hourglassing (spurious strain energy modes) due to 4 in-plane integration points is used to model tubes in this work. Five integration points are used throughout the thickness in order to accurately capture the local element bending. This element is a fully integrated shell with assumed strain interpolants used to alleviate locking and enhance in-plane bending behavior [40].
TABLE 5.1

MATERIAL PROPERTIES OF STEEL USED FOR MODELING
SQUARE TUBES

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<th>Property</th>
<th>Value</th>
</tr>
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<tr>
<td>Elastic Modulus</td>
<td>207 GPa</td>
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<tr>
<td>Poisson’s Ratio</td>
<td>0.29</td>
</tr>
<tr>
<td>Yield Stress</td>
<td>253 MPa</td>
</tr>
<tr>
<td>Effective plastic strain (MPa)</td>
<td></td>
</tr>
<tr>
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<td>253</td>
</tr>
<tr>
<td>0.048</td>
<td>367</td>
</tr>
<tr>
<td>0.108</td>
<td>420</td>
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<tr>
<td>0.148</td>
<td>442</td>
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<tr>
<td>0.208</td>
<td>468</td>
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<tr>
<td>0.407</td>
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<tr>
<td>0.607</td>
<td>561</td>
</tr>
<tr>
<td>0.987</td>
<td>608</td>
</tr>
</tbody>
</table>

5.3.1 Enforced Progressive Buckling

The axial crushing with a rigid flat plate of a thin-walled square tube constrained at one end is considered. The tube is 200 mm long, and the square cross section has a width of 50 mm. The tube has a uniform thickness of 1 mm, except an imperfection is added in the form of a small patch (4 mm long) of thickness 0.8 mm in the middle of one face. The schematic of the tube and the location of the imperfection are shown in Fig. 5.5 (all dimensions are in mm.). The rigid plate, which has a mass of 147 kg, is given an initial velocity $v_0$ (3, 5 and 10 m/s). The contact between the rigid plate and tube is modeled using a constraint algorithm (*CONTACT_AUTOMATIC NODES_TO_SURFACE) with a friction coefficient of 0.3 to allow sliding movement. To account for the contact between the lobes (folds) during deformation, a single surface contact algorithm with a coefficient of friction 0.1 is used.

A mesh convergence study is performed with various mesh refinements and two
values of initial velocity $v_0$ for the impacting rigid plate. The force-displacement behavior, amount of energy absorption, and lobe (fold) formation pattern are used to evaluate the mesh size sufficiency. The results corresponding to the three representative mesh sizes are shown in Figs. 5.6 and 5.7. The mesh resolutions referred in the figures correspond to the total number of elements in the square perimeter by the number of elements along the length of the tube. For all three mesh resolutions and two cases of initial velocities, the first fold forms at the imperfection. The next fold for the $80 \times 50$ mesh model forms above the first fold, whereas it forms below the first fold for the other two finer mesh models. This difference in deformation pattern for the $80 \times 50$ mesh model is also reflected in the force-displacement behavior and the amount of energy absorption. Qualitatively,
the deformation pattern and force-displacement behavior for the 100×100 and 200×200 mesh models are very similar in terms of locations of lobe formation and corresponding peaks in the force-displacement curves.

Peak force and total energy absorption values can be used for quantitative comparison of the three models. The peak force with an initial velocity of 3 m/s for the three models starting with the coarsest mesh are 47780N, 44480N, and 43525N, respectively. Similarly, the peak forces for the three models with an initial velocity of 5 m/s are 51815N, 50390N, and 49370N, respectively. Figure 5.7 shows the variation in total energy absorption with mesh refinement for initial velocities of 3m/s and 5m/s. Clearly, the differences in values corresponding to the models with the 100×100 and 200×200 mesh resolutions are comparatively less than the differences between the models with the 80×50 and 100×100 mesh resolution. In order to be more confident about the quantitative convergence, one more model with even higher mesh resolution (like 300×300) could be used, but the simulation time increases drastically for higher resolutions as the timestep size in the explicit scheme is dependent on the smallest element size in the model. Hence, based on the reasonable similarities between the models with 100×100 and 200×200 mesh resolution and computational practicality, a mesh resolution of 100×100 (element size of 2 mm × 2 mm) is used for further design studies.
Figure 5.6. Force-displacement behavior comparison of the square tube under axial impact modeled with 3 different mesh resolutions.

Figure 5.7. Variation of total energy absorption by the uniformly thick tube under axial impact modeled with 3 different mesh resolutions.
Several cases with different initial impact velocity are considered, and it was found that the buckling starts at the imperfection and can progress in any direction. The deformation pattern for two representative cases are shown in Figs. 5.8 and 5.9. In both cases, the simulation is carried out for 0.03 seconds. For the impact with an initial velocity of 3 m/s, the subsequent folds form below the first fold, whereas the subsequent folds form above the first fold when the impact velocity is 5 m/s. The buckling behavior is often affected by the presence of imperfections (geometric, material, manufacturing etc.), and depending on their location, the buckling can be triggered in the middle or lower end of the tube structures. Moreover, the direction of progressive buckling (subsequent fold formation) depends on many factors, and it is difficult to predict it beforehand.

![Image](image_url)

Figure 5.8. Deformation of the uniformly thick tube with an imperfection at various times when impacted by a rigid plate with an initial velocity of 3 m/s.
In order to enforce that buckling start from the impact end and have more certainty about the subsequent fold formations, the present method provides an attractive option. With the help of output ports defined close to the impact end, the thickness distribution of the tube is changed to trigger a symmetric mode of collapse. In the present example, one set of output ports close to the loading end is defined as shown in Fig. 5.10(a). The output ports defined at a distance of 24 mm from the free end of the tube on the two parallel faces A and A’ (not visible in the figure) point inward. The choice of location for the output ports is user- and problem-dependent, but a reasonable location would be half the wavelength of deformation (during the axial crushing of a uniformly thick tube) away from
the free end. The wavelength of deformation is the distance between two successive folds [64]. It gives an indication of the average length of the tube consumed in one fold during progressive buckling. As mentioned earlier, the dummy load method is used to synthesize a structure having compliant-mechanism-like behavior such that the faces A and A’ move inward at the location of the output ports to facilitate a (symmetric mode) fold. The thickness variable is allowed to vary between an upper bound ($t_{\text{max}}$) of 1.2 mm and a lower bound ($t_{\text{min}}$) of 0.8 mm. The average thickness target ($T^*$) is set to 1 mm so that the final mass of the designed tube is the same as the mass of the original uniformly thick tube. The imposed imperfection is not considered during the design process as it is impractical and difficult to identify and model the imperfections beforehand for real world applications. Moreover, the idea of the present work is to come up with designs that behave in a desired predictable manner even in the presence of imperfections.

Figure 5.10(b) shows the thickness distribution of the designed tube. The ‘hour glass’ shaped patch of maximum allowable thickness $t_{\text{max}}$ on face A (and A’) creates a stiffness distribution that forces the center of it to move inward. The corresponding thickness distribution on face B (and B’) ensures that it moves outward at the same location along the length as the output ports on face A. The deformation behavior of this designed tube for the two initial velocities of impact (3 m/s and 5 m/s) is shown in Figs. 5.11 and 5.12. It can be seen that the buckling starts at the desired location in the desired mode (symmetric) and progresses systematically to the other end for both cases, even in the presence of the imperfection. A similar deformation behavior is observed for several variations in the impact velocity and the mass of the rigid plate.
Figure 5.10. (a) Output port definition, and (b) thickness distribution for the designed tube.

Figure 5.11. Deformation of the designed tube with an imperfection at various times when impacted by a rigid plate with an initial velocity of 3 m/s.
Figure 5.12. Deformation of the designed tube with an imperfection at various times when impacted by a rigid plate with an initial velocity of 5 m/s.

Figure 5.13 shows the comparison of the force-displacement behavior of the designed tube and the original uniformly thick tube for initial velocities of 3 m/s and 5 m/s. For the impact with an initial velocity of 3 m/s, the designed tube has a significant drop in the peak force at the cost of increased displacement. This can alternatively be stated as the designed tube being flexible compared to the uniformly thick tube. This is due to the enforced buckle zone in the designed tube that helps initiate (trigger) the buckling. The drop in the peak force for the designed tube is desired as it helps in reducing injury, whereas an increase in displacement, although not desired, can be afforded as thin-walled structures are used to withstand long strokes. A similar effect (drop in peak force and increase in displacement) is also seen when the impact velocity is 5 m/s, although the drop in the peak force is not very large because the intensity of the impact is high in this case. Table 5.2 shows the quantitative comparison of the designed tube and the original uniformly thick tube. All values are recorded for the simulation time.
of 0.03 seconds. While increased flexibility reduces the peak force, it also reduces the amount of energy absorption as the designed tube shows less resistance to deformation. Still, the longer stroke due to the increased flexibility of the designed tube helps in recovering some additional energy absorption.

![Graphs showing force-displacement behavior of uniform and designed tubes for different velocities](image)

(a) $v_0 = 3\text{m/s}$
(b) $v_0 = 5\text{m/s}$

Figure 5.13. Comparison of the force-displacement behavior of the designed tube and the original uniformly thick tube.

The proposed method has been shown to result in a design with the desired buckling behavior in the present example. The designed tube under axial impact shows progressive buckling starting from the loading end. In addition, the enforced buckle zone also helps in reducing the peak force. In longer tubes, more than one set of output ports can be defined to ensure that progressive buckling continues even after the first buckle zone. In the next example, a relatively longer tube is
### Table 5.2

**Quantitative Comparison of the Behavior of the Designed Tube and the Original Uniformly Thick Tube**

<table>
<thead>
<tr>
<th></th>
<th>Peak Force (N)</th>
<th>Displacement (mm)</th>
<th>Total Energy Absorbed (N-mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform ($v_0 = 3 \text{ m/s}$)</td>
<td>44480</td>
<td>43</td>
<td>651870</td>
</tr>
<tr>
<td>Designed ($v_0 = 3 \text{ m/s}$)</td>
<td>37870</td>
<td>56</td>
<td>634790</td>
</tr>
<tr>
<td>Uniform ($v_0 = 5 \text{ m/s}$)</td>
<td>50390</td>
<td>105</td>
<td>1617130</td>
</tr>
<tr>
<td>Designed ($v_0 = 5 \text{ m/s}$)</td>
<td>45085</td>
<td>110</td>
<td>1414770</td>
</tr>
</tbody>
</table>

Considered with two sets of output port definitions. In addition, the advantage of using the proposed method for designing tubes undergoing oblique impacts is also presented.

#### 5.3.2 Oblique Impact

In this example, a thin-walled square tube of length 480 mm, width 80 mm and uniform thickness 1.5 mm is considered under an impact by a rigid plate. The tube is constrained at one end and an imperfection is added in the form of a small patch (8 mm long) of thickness 1 mm close to the lower end of the tube on one of its faces. The schematic of the tube and location of the imperfection are shown in Fig. 5.14(a). The rigid plate, which has a mass of 400 kg, is given an initial velocity of 5 m/s in the axial direction. Two different loading scenarios are considered: a) a pure axial impact; and b) an oblique impact with a loading angle of 10° as shown in Fig. 5.14(b). The simulation time for both cases is 0.03 seconds.
Figure 5.14. Schematic of a thin-walled square tube with an imperfection impacted (a) axially and (b) obliquely by a rigid plate.

Like the previous example, a mesh convergence study is performed with various mesh refinements for both the pure axial and oblique impacts. The results corresponding to the four representative mesh sizes are shown in Figs. 5.15 and 5.16. The force-time behavior of the model with the 80×60 mesh resolution for both axial and oblique impacts is significantly different than the other 3 models with finer mesh resolutions. During pure axial crushing, the first fold appears at the imperfection for all 4 models. The second fold forms above the first fold in the model with the 80×60 mesh resolution, whereas the second fold forms below the first fold in the other 3 finer mesh models. The peak force corresponding to the third fold in the model with the 80×60 mesh resolution is comparatively less than the corresponding force for the other 3 models. This results in overall less
energy absorption for the 80×60 model during pure axial crushing, which can be seen in Fig. 5.16(a).

During the oblique impact, global bending in the 80×60 model starts a little later than in the other 3 models. This slight delay in the global bending results in comparatively higher energy absorption for the model with the 80×60 mesh resolution. There is very good agreement in the energy absorption values for the models with the 160×240 and 240×240 mesh resolutions in the cases of both pure axial and oblique impacts. Hence, based on the force-time behavior, the amount of energy absorption and the deformation pattern, a mesh resolution of 160×240 (element size of 2 mm × 2 mm) is found to be sufficient and used for further studies. Note that 160 is the number of elements along the square perimeter (40 elements on each face).

![Figure 5.15. Mesh convergence study: Force-time behavior comparison of the square tube under (a) pure axial impact, and (b) oblique impact modeled with 4 different mesh resolutions.](image_url)
Figure 5.16. Mesh convergence study: Variation of the total energy absorption by uniformly thick tube modeled with 4 different mesh resolutions.

Figure 5.17 shows the deformation behavior of the uniformly thick tube with an imperfection (Fig. 5.14(a)) under pure axial crushing by the rigid plate with an initial velocity of 5 m/s. The buckling starts at the imperfection and progresses in both directions afterwards. When the angle of impact is increased to 10° during the oblique loading, the tube undergoes Euler-type buckling (global bending) after a few folds form in the top as shown in Fig. 5.18. As found by some other researchers [41, 88] in similar studies, the onset of global bending adversely affects the load carrying capacity and hence, the energy absorption of thin-walled structures. This can be confirmed by Fig. 5.19 in which the force-time and energy-time histories for the pure axial impact are compared with those for the oblique impact. The force drops to 0 and energy saturates once the tube loses contact with the rigid plate after the onset of global bending during the oblique impact.
Figure 5.17. Deformation of the uniformly thick tube with an imperfection at various times when impacted axially by a rigid plate with an initial velocity of 5 m/s.

Figure 5.18. Deformation of the uniformly thick tube with an imperfection at various times when impacted obliquely (10°) by a rigid plate with an initial velocity of 5 m/s. Note the onset of global bending in (b) at t=0.009 seconds.
Figure 5.19. Comparison of the (a) force-time and (b) energy-time histories for the uniformly thick tube under pure axial and oblique impacts.

In order to enforce that progressive buckling starts from the loading end during pure axial crushing and to avoid or delay the onset of global bending during the oblique impact, two sets of output ports close to the loading end are defined in this example as shown in Fig. 5.20(a). Two sets of output ports are defined in this case to keep the tube buckling progressively downwards after the first buckle due to its relatively longer size. Also, the two sets of output ports help the tube to buckle progressively for a longer duration during the oblique impact and hence, completely avoid or delay the transition to global bending. The first set of output ports (pointing inward) are defined close to the loading end at a distance of 40 mm from the free end of the tube. The second set of output ports are defined 80 mm below the first set of output ports assuming 40 mm to be half the wavelength of deformation in this case. A good starting choice for deciding the relative distance between the output ports is to use the wavelength of deformation from the simulation of the original uniformly thick tube. However, a more robust
technique would be to use the relative distance between the output ports as an additional design variable in a 2-stage design approach.

During the topometry design, the thickness variable in this case is allowed to vary between an upper bound ($t_{max}$) of 1.8 mm and a lower bound ($t_{min}$) of 1.2 mm. The average thickness target ($T^*$) is set to 1.5 mm so that the final mass of the designed tube remains the same as the mass of the original uniformly thick tube. Like the previous example, the imposed imperfection is not considered during the design process. Figure 5.20(b) shows the thickness distribution of the designed tube. Like the previous example, the face A has an ‘hour glass’ shape with a higher thickness (1.8 mm) in the first 80 mm of the tube length. This thickness distribution on face A along with the thickness distribution on face B for the first 80 mm of length initiates a fold at a 40 mm distance (location of O/P-1) from the free end. In addition, a second smaller hour-glass-shaped thickness distribution on face A corresponds to the second set of output ports (O/P-2).

Figure 5.21 shows the deformation behavior of the designed tube under pure axial crushing. Note that the imperfection considered for the original uniformly thick tube is added back to the designed tube in order to make reasonable comparisons. As predicted, the buckling starts at the first set of output ports (Fig. 5.21(a)) in the prescribed symmetric mode, and it continues downwards. Moreover, the tube also buckles at the location corresponding to the second set of output ports (Fig. 5.21(c)) in a desired fashion. This shows that the proposed method can design tubes with multiple predefined buckle zones.

Next, the designed tube is tested under oblique impact ($10^\circ$), and the deformation behavior at various times is shown in Fig. 5.22. Unlike the uniformly thick tube, the designed tube continues to buckle progressively, completely avoiding the
transition to global bending. This behavior of the designed tube represents an increase in the load-carrying capacity compared to the uniformly thick tube, which can be seen in Fig. 5.23(a). Also, the increase in load-carrying capacity means more energy absorption as shown in Fig. 5.23(b). The designed tube absorbs \( \sim 80\% \) more energy than the uniformly thick tube during the simulation time of 0.03 seconds.

The present example has shown that the proposed method can be used to design long thin-walled square tubes with multiple predefined buckle zones. In addition, to enforce progressive buckling starting from the loading end, these enforced buckle zones also can delay or avoid the onset of global bending during oblique impacts with load angles higher than the critical value. In both examples, a common trend in terms of design (hour glass shape) has been found that
Figure 5.21. Deformation of the designed tube with an imperfection at various times when impacted axially by a rigid plate with an initial velocity of 5 m/s.

Figure 5.22. Deformation of the designed tube with an imperfection at various times when impacted obliquely $(10^\circ)$ by a rigid plate with an initial velocity of 5 m/s.
effectively triggers the buckling in a symmetric mode at desired locations along the length of the tube.

5.4 Summary

A new method for designing thin-walled square tubes under axial and oblique impacts based on the use of compliant mechanism synthesis is presented. Using the present method, square tubes can be designed to exhibit buckling starting from the loading (impact) end and systematically progressing toward the rear end even in the presence of reasonable geometric imperfections and asymmetries in the loading conditions. Note that large imperfections and/or oblique loadings at high angles of impact may still result in undesirable behavior, and such cases should be handled differently. For example, improving the manufacturing process could reduce imperfections and making changes in the basic geometry of the design could
accommodate oblique loads with high impact angles.

The enforced buckle zones (output ports) act as triggering mechanisms to a) initiate a specific axial collapse mode (symmetric in the present case), (b) stabilize the collapse process, and (c) reduce the peak force, which is desirable for crashworthiness. The presence of enforced buckle zones in the top portion of the tube also helps in avoiding or delaying the onset of global bending during oblique impacts with load angles higher than the critical value. Apart from tubes with square cross sections, the present method can be used for tubes with other regular cross sections like rectangular, circular, polygonal, etc., or even open sections like C and Hat shapes as long as the meshing is uniform (needed to store the element data in a uniform CA lattice).

The designed tubes with variable thickness distributions obtained with the present method are manufacturable with modern manufacturing techniques typically used for ‘Tailor-Welded Blanks’ (TWB). A tailor-welded blank is one part made up of different strengths or thicknesses of steel, joined usually by a laser weld. TWBs are increasingly gaining popularity among the automakers to reduce the weight of various sheet metal components without compromising the strength. Some common applications of TWBs in the automotive industry are the door inners, upper front rails and B-pillars.

The designs obtained with the present method can be simplified for ease of manufacturing by grouping the elements based on their thicknesses. The elements with thicknesses equal or close to the upper bound ($t_{max}$) are collected in one group and assigned a thickness equal to $t_{max}$. Similarly, elements with thicknesses close to the lower bound are collected in another group and assigned a thickness equal to $t_{min}$. All of the remaining elements whose thicknesses are in general close
to the mean thickness ($T^*$) are collected in the last group and a common thickness equal to $T^*$ is assigned to them. Figure 5.24 shows the result of using such a simplification by grouping the elements for the final design obtained in the first example (Sec. 5.3.1). Since the final design has most of the elements with thicknesses close to the upper bound, lower bound or mean value, the simplified design is very similar to the original final design in terms of both thickness distribution and performance as shown in Fig. 5.25. These simplified designs with 3-4 discrete levels of thicknesses can be manufactured using patch TWB. A patch TWB overlays one blank of material on top of another blank to add thickness (strength) where it is needed [1]. The two blanks are joined, usually by spot welds, before forming.

A common practice to enforce progressive buckling, preferably starting from the impact end in the thin-walled tube structures is to introduce triggering dents or grooves at desired locations along the length of the structure [28, 64, 115]. While the option of introducing dents or grooves seems very attractive for its simplicity and ease of manufacturability, care should be taken when choosing the dent sizes (width and depth). A small dent size may not always produce the desired progressive buckling behavior (possibly due to imperfections being dominant), whereas a big dent size may become a source of stress concentration leading to potential failure. The results of a preliminary study to compare the tube designs with dent(s) and designs obtained using the present method for the two examples from Sec. 5.3 are presented in Appendix B. It is to be noted that this comparative study is preliminary in nature with the intention to put the present method alongside the existing practice in the industry. A more exhaustive study can be done in future to highlight the advantages and limitations of the present
method as compared to the existing method of introducing dents or grooves in the tube structures.

One of the limitations of the present method is the absence of a well-defined rule to decide the location of the first set of output ports and the distances between successive sets of output ports. Although placing these output ports based on the wavelength of deformation (distance between two successive folds) during the axial crushing of a uniformly thick tube is fairly reasonable, a more rigorous approach can be developed by incorporating the performance measures like peak force and energy absorption in deciding the port locations. For future work, the number of output ports and their locations along the length of the tube can be parameterized, and a two-stage design method can be formulated with the objectives of minimizing peak force and maximizing energy absorption.
Figure 5.25. Comparison of the force-displacement behaviors of the original final design (Fig. 5.24(a)) and the corresponding simplified design (Fig. 5.24(b)) under axial loading for two initial plate velocities of 3 m/s and 5 m/s.
CHAPTER 6

ADAPTIVE WEIGHTING FACTORS METHOD FOR MULTIPLE LOAD CASES

Most of the work in the field of crashworthiness design considers a single loading condition, resulting in designs with superior performance for only that load case and potentially unsatisfactory performance for other load cases. However, real world vehicle collisions are unique dynamic events in which the vehicle may collide with another vehicle of similar or different shape, stiffness and mass, or it may collide with a stationary object. Also, the occupant behavior in a crash scenario depends upon size, age and crash speeds for both genders. To accommodate the uncertainty involved in the loading conditions in real world crash events, designers predominantly use two approaches: introducing the notion of reliability into the design process \[52, 54, 69, 97, 109\] or approximating several loading conditions with a few representative loading conditions to treat the problem as a multi-load design problem. One may argue the appropriateness of one approach over the other depending upon the problem in hand. Generally, reliability-based design methods are preferred for linear static problems in which a reliability evaluation inner loop can be merged into the material distribution outer loop without significantly increasing computation time; however, this approach becomes impractical for large crash problems unless it is restricted to size and shape optimization with
a small number of design variables \[6, 113, 114\]. Hence, this work takes the latter approach.

6.1 Multiple Load Cases in Topology Optimization for Crashworthiness Design

A number of researchers have extended various structural optimization methods to treat multiple load case problems. Diaz and Bendsøe \[27\] and Allaire et al. \[10\] extended the homogenization method for topology optimization to handle multiple load case problems by considering a weighted average of the mean compliances for all load cases as the objective function. Sigmund and Bendsøe \[95\] extended a solid isotropic material with penalization (SIMP) method for topology optimization of minimum compliance design to treat multiple load case problems by formulating the objective function to be minimized as a weighted average of compliances (strain energies) for each load case. Similarly, other researchers \[9, 117\] have also extended their work for multiple load cases by considering the sum or weighted sum of compliance for each load case. In their work on crashworthiness design, Forsberg and Nilsson \[35\] normalized the internal energy density (IED) of each element by the maximum IED among all elements for each load case and calculated a weighted sum over all load cases. Similarly, Patel \[75\] used a weighted sum of the IED for every element in each load case to obtain the associated field variable during the design of a crashworthy structure under multiple load cases in the HCA method. Thus, the average field variable for the \(i^{th}\) element can be written as

\[
S_{av}^i = \sum_{LC=1}^{N_{LC}} w_{LC} S_{LC}^i,
\]

where the subscript \(LC\) refers to a particular load case, \(w_{LC}\) denotes the associated weight for load case \(LC\) and \(N_{LC}\) is the total number of load cases.
In almost all of the work mentioned above, the weights associated with the load cases are obtained by design experience, some external constraint or probability of occurrence of the load cases. Also, the weights are constant throughout the design process. For situations in which a few of the load cases dominate, the final design is dictated by those dominating cases unless their weighting factors are deliberately chosen to be comparatively smaller. There should be a measurable performance criterion to be able to decide the relative dominance of one load case over the other, and subsequently, associated weights should be chosen. Victoria et al. [108] extended their isolines topology design (ITD) algorithm for multiple load cases by calculating the weight for controlling the volume contribution of each load case to the global volume fraction as the ratio of the average von Mises stress for that load case to the maximum average von Mises stress among all load cases. Their method helps alleviate the need to choose appropriate weights beforehand; however, a more realistic and intuitive measure of relative dominance of load cases is preferred.

For crash applications, the amount of energy absorbed by the structure, peak force and maximum intrusion are widely accepted measures of design performance. Also, the performance under multiple load cases may change significantly as the design itself changes, especially when the structure undergoes buckling or large deformations. Hence, the corresponding weighting factors should be updated accordingly. In this work, a new strategy is introduced to allocate weighting factors adaptively during the design process such that similar performance is achieved for all load cases.
6.2 Adaptive Weighting Factors Method

In many crash applications, a structure may be impacted at any location within a specific area. For example, an occupant protection device like the knee bolster can be impacted at any location on its top surface depending on the size of the occupant, as shown in Fig. 6.1(a). To account for this uncertainty in the loading condition, the structure is designed and tested for a few representative load cases as shown in Fig. 6.1(b). To be able to protect an occupant, a knee bolster design should perform satisfactorily well for all representative load cases. As discussed in the previous section, the total amount of energy absorption or maximum intrusion into the structure can be used to measure the effectiveness of the design under these load cases. When the structure does not have enough material at the right location to support a particular load case, it is reflected in low energy absorption or high intrusion for that case. The energy absorption for a load case can be increased by rearranging material within the design space that in general decreases the energy absorption for at least one of the remaining load cases as the total amount of material is fixed. Similarly, the intrusion for a load case can be decreased by rearranging material within the design space at the expense of an increase in intrusion for at least one of the remaining load cases. This rearrangement of material can be facilitated by appropriately changing the weighting factors associated with the load cases.

For multiple load case crash problems in which the load cases are very different in magnitude and their application locations on the structure, typically a failure criteria based on the maximum intrusion or peak force is used to check if the design is safe for all load cases. For example, a thin-walled cylindrical tube designed mainly for axial loads may be required to support some transverse load
Knee bolster

Possible impact scenarios
Passenger heights
Small
Medium
Large

(a) Location of a typical driver side knee bolster in a car
(b) Three representative impact scenarios depending upon the passenger size

Figure 6.1. A typical driver side knee bolster and possible knee impact cases based on passenger size.

occasionally. The tube in this case is designed to improve the performance under axial loads making sure that it is strong enough to withstand occasional transverse loads without failure. In such problems, weighting factors for the less dominant load cases should be high enough to make sure that there is enough material (strength) to support these load cases without failure. However, for the multiple load case problems in which load cases are very similar in magnitude and their application locations on the structure, matching the performance of the design for all such load cases results in robust structures. For example, matching the energy absorption or maximum intrusion for the above mentioned three representative load cases for a knee bolster results in a more robust design.

In this work, an adaptive weighting factors method is proposed in which the weights are allocated to match the performance of a design under multiple load cases that are reasonably similar in their magnitude and application locations.
This is done by using the average performance of the design for all load cases as the target and changing the weights to bring the performance of the design for individual load cases closer to the target value. In the present work, the amount of energy absorption and maximum intrusion are used as the performance measures for the structures under crash loads (see Section 2.4.2 for details); however, the method is general enough that it can be easily extended to consider other performance measures like peak force or some combination of peak force, energy absorption and maximum intrusion. Moreover, the method can be modified by replacing the average performance measure as the target with the minimum acceptable performance criterion to avoid failure so that it can be used for the problems with multiple load cases of very different nature.

6.3 Formulation

During the HCA-based method for crashworthiness design with multiple load cases, the field variable state of each element \((S_i)\) is represented by a weighted sum of the internal energy (IE) for each load case. Thus, the average field variable for the \(i^{th}\) element can be written as

\[
S_{i}^{av} = \sum_{LC=1}^{N_{LC}} w_{LC} I E_{i}^{LC},
\]

where the subscript \(LC\) refers to a particular load case, \(w_{LC}\) denotes the associated weight for load case \(LC\) and \(N_{LC}\) is the total number of load cases. Once the average field variables are evaluated for all elements in the design space, the problem becomes similar to that of a single load case in which the objective is to achieve a uniform distribution of these averaged field variables. Clearly, the associated weights play an important role in determining the contribution of individual load
cases in the final design.

In order to match the performance of a design for multiple load cases, it is important to understand the effect of a change in weight for a particular load case on the performance of the design for that load case. When the amount of energy absorption is used as the performance measure, increasing the weighting factor corresponding to a load case increases its contribution to the weighted sum and hence, more material is allocated to the elements within the structure that efficiently contribute to energy absorption for that load case. This in turn increases the energy absorption for that load case in the next cycle. On the other hand, when maximum intrusion is used as the performance measure, increasing the weight corresponding to a load case will result in more material at the locations that strengthen the structure for that load case. This in turn reduces the maximum intrusion for that load case in the next cycle. Since the total amount of available material to be distributed within the design space is limited, strengthening the structure for a particular load case generally results into weakening of it for at least one of the remaining load cases. Since the goal for changing the weights is to achieve uniform performance (energy absorption or maximum intrusion) for all load cases, the average of the performance measure for all load cases is used as the target. A proportional control law is used to evaluate the required change in weights at each iteration as

$$\Delta w_{LC}^{(k)} = \pm K_w \left( \frac{PM_{av}^{(k)} - PM_{LC}^{(k)}}{PM_{av}^{(k)}} \right) \quad \text{for} \quad LC = 1, 2, \ldots, N_{LC}, \quad (6.3)$$

where $PM$ is the performance measure (energy absorption or maximum intrusion), and $K_w$ is the proportional gain for the control law. The '+' sign is used when energy absorption is chosen as the performance measure, whereas the '-' sign is
used when maximum intrusion is chosen as the performance measure. $PM_{LC}^{(k)}$ and $PM_{av}^{(k)}$ are the performance measure for load case $LC$ and the average of performance measure for all load cases, respectively, at the $k^{th}$ iteration.

For example, consider a three-load case problem in which $EA_1^{(k)}$, $EA_2^{(k)}$ and $EA_3^{(k)}$ are the amounts of energy absorbed by the structure at the $k^{th}$ design iteration for the three load cases and $EA_{av}^{(k)}$ is the average of these three values. For a particular instance, assume that $EA_1^{(k)}$ and $EA_2^{(k)}$ are greater than $EA_{av}^{(k)}$, while $EA_3^{(k)}$ is less than $EA_{av}^{(k)}$ as shown in Fig. 6.2(a). In order to bring the energy absorption values for the three load cases close to their average value, the weights corresponding to load cases one and two should be decreased, while the weight for the third load case should be increased. Similarly, when maximum intrusion is used as the performance measure, the trend in weight change is reversed as shown in Fig. 6.3(a).

![Figure 6.2](image-url)  
(a) Weight change without a tolerance band  
(b) Weight change with a tolerance band

Figure 6.2. Change in weights to match the energy absorption for a three-load case problem.
A user-specified tolerance band (say 10%) on performance matching, $PM_{tol}^\text{av}$, can also be used to avoid weight changes if the performance for a particular load case is sufficiently close to the average value. Hence, Eq. (6.3) can be rewritten as

$$\Delta w^{(k)}_{\text{LC}} = \begin{cases} \pm K_w \left( \frac{PM_{av}^{(k)} - PM_{av}^{(k)}_{\text{LC}}}{PM_{av}^{(k)}} \right) & \text{if} \quad \left| \left( PM_{av}^{(k)} - PM_{av}^{(k)}_{\text{LC}} \right) \right| > PM_{tol}^\text{av} \times PM_{av}^{(k)} \\ 0 & \text{otherwise.} \end{cases}$$

(6.4)

For the example in Fig. 6.2(a), if a tolerance band $EA_{tol}^2$ is used for energy matching such that $EA_{av}^{(k)}_2$ lies in this band, as shown in Fig. 6.2(b), the change in $w_2$ should be zero. This tolerance band helps in achieving saturated weights once the energy values for all load cases are sufficiently close to each other. Figure 6.3(b) shows the effect of using a tolerance band $D_{tol}^k$ when maximum intrusion is the performance measure.

In addition, a maximum allowable change, $\Delta PM_{av}^{\text{allow}}$ (say 15%), for the average performance value can also be used to avoid drastic changes in its value between successive design iterations due to a sudden change in performance for
one or more load cases (often due to buckling or loss of contact between the im-
pactor and structure). Hence, the average performance measure for all load cases
at the \( k \)-th iteration can be written as

\[
P_{M_{av}}^{(k)} = \begin{cases} 
\min \left( \sum_{LC=1}^{N_{LC}} \frac{PM_{M_{av}}^{(k)}}{N_{LC}}, (1 + \Delta PM_{M_{av}}^{allow}) \times PM_{M_{av}}^{(k-1)} \right) & \text{if } PM_{M_{av}}^{(k)} > PM_{M_{av}}^{(k-1)}, \\
\max \left( \sum_{LC=1}^{N_{LC}} \frac{PM_{M_{av}}^{(k)}}{N_{LC}}, (1 - \Delta PM_{M_{av}}^{allow}) \times PM_{M_{av}}^{(k-1)} \right) & \text{if } PM_{M_{av}}^{(k)} \leq PM_{M_{av}}^{(k-1)}. 
\end{cases}
\]  

(6.5)

The updated weights are evaluated by adding the required change in weights
from Eq. (6.4) to the weights from the previous iteration.

\[
\bar{w}_{LC}^{(k+1)} = w_{LC}^{(k)} + \max \left( -\Delta w_{max}^{allow}, \min \left( \Delta w_{LC}^{(k)}, \Delta w_{max}^{allow} \right) \right) \quad \text{for } LC = 1, 2, \ldots, N_{LC}.
\]  

(6.6)

Again, a maximum allowable change in the weights, \( \Delta w_{max}^{allow} \), is specified to avoid
large changes in one iteration. \( K_w \) and \( \Delta w_{max}^{allow} \) play an important role in con-
trolling the speed and stability of the proposed method and should be chosen
carefully to achieve saturated and converged designs in a reasonable number of
iterations. A high proportional gain \( K_w \) results in large changes in the weights. If
the proportional gain is too high, the system can become unstable with oscillating
weights. On the other hand, a very low \( K_w \) value results in very small changes
in the weights and consequently a less responsive system which may not be able
to catch up with the outer density update loop. Like traditional control systems,
tuning methods can be used to obtain a reasonable value of \( K_w \) or even a method
similar to the adaptive \( K_p \) scheme described earlier (Section 3.1) can be imple-
mented. However, in the present study, fixed gain values based on user experience
are used. $\Delta w_{\text{max}}^\text{allow}$ helps in controlling large weight changes during one design iteration which could originate from a large value of $K_w$ or an abrupt change in the performance of the design for one or more load cases.

In addition, a lower limit on weights, $w_{\text{min}} > 0$, can also be specified to avoid the contribution of one or more load cases going to zero. Hence, Eq. (6.6) can be written as

$$\bar{w}_{LC}^{(k+1)} = \max \left( w_{LC}^{(k)} + \max \left( -\Delta w_{\text{max}}^\text{allow}, \min \left( \Delta w_{LC}^{(k)}, \Delta w_{\text{max}}^\text{allow} \right) \right), w_{\text{min}} \right)$$

for $LC = 1, 2, \ldots, N_{LC}$. (6.7)

The updated weights are normalized so that the sum of all weights remains one for all iterations.

$$w_{LC}^{(k+1)} = \frac{\bar{w}_{LC}^{(k+1)}}{\sum_{i=1}^{N_{LC}} \bar{w}_{i}^{(k+1)}}.$$ (6.8)

Although there is no technical need to normalize the weights, it helps in understanding the variations in contributions of the various load cases as the design evolves.

Initial weights at the start of the design process can be chosen to be proportional (maximum intrusion) or inversely proportional (energy absorption) to the performance measure for the load cases. This helps to more quickly achieve the objective of equal performance for all load cases. It is to be noted that the weights are chosen to be proportional (or inversely proportional) to the performance values only when the corresponding load case performance values are out of the tolerance.
band \((PM^{\text{tot}})\). Hence,

\[
\begin{align*}
w_{i,LC}^{(0)} &= \begin{cases} 
\left( \frac{PM_{i,LC}^{(0)}}{\sum_{i=1}^{N_{LC}} PM_{i}^{(0)}} \right)^{\pm 1} & \text{if } \left| PM_{av}^{(0)} - PM_{i,LC}^{(0)} \right| > PM^{\text{tot}} \times PM_{av}^{(0)} \\
\frac{1}{N_{LC}} & \text{otherwise}, 
\end{cases}
\end{align*}
\]

(6.9)

where ‘+’ is used when the performance measure is maximum intrusion and ‘-’ is used when energy absorption is the performance measure. Again, the weights should be normalized to ensure that the sum of all weights is one to maintain consistency.

6.4 Examples

In this section, three example problems are considered to illustrate the applicability and advantages of using the proposed method for computing adaptive weighting factors. In the first example, a 2D cantilever beam with three static load cases is designed using the adaptive weighting factors method and the final design is compared with a design obtained when fixed equal weights are used for all load cases. A more complex problem of a 2D beam with 3 dynamic load cases is considered in the second and third examples with energy absorption and maximum intrusion as the performance measures, respectively.

6.4.1 2D Cantilever Beam with 3 Static Load Cases

A 2D cantilever beam is considered under three separate load cases. A schematic of the design domain with dimensions and the locations of the static point loads is shown in Fig. 6.4. The design domain is discretized into a 160\times60 mesh of 4-node, plane stress quadrilateral elements with 1 mm thickness. A linear elastic
material model with properties shown in Table 6.1 is used. The three load cases are static point loads of magnitude 1000N each acting at the lower right end of the cantilever beam separated by 50 mm as shown in Fig. 6.4. The three load cases approximately represent uncertainty in the load location close to the tip of the cantilever beam.

![Design Domain](image)

Figure 6.4. Schematic of a 2D cantilever beam with 3 static load cases.

To understand the effect of each load case separately, three design problems considering one load case at a time are solved first. The objective in each case is to obtain a stiff design for a given mass fraction of 30% which is achieved by distributing the material within the design space to obtain uniform strain energy distribution as discussed in Section 2.3. Basic HCA-related parameters such as neighborhood, penalization parameter, and convergence criteria, among
TABLE 6.1

MATERIAL PROPERTIES FOR CANTILEVER BEAM

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>$\rho_0$</td>
<td>2700 $kg/m^3$</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>$\nu$</td>
<td>0.33</td>
</tr>
<tr>
<td>Elastic modulus</td>
<td>$E_0$</td>
<td>70 GPa</td>
</tr>
</tbody>
</table>

others, are summarized in Table 6.2. Figure 6.5 shows the final designs for the three separate problems considering each of the right, center and left load cases individually. The designs for the center and left load cases are similar to the design for the right load case in terms of relative location and connection between various structural members except there is no material on the right side of the respective point load locations. This is expected as the material on the right side of the load locations in the center and left load cases does not deform and hence does not contribute to the overall stiffness of the structure.

TABLE 6.2

BASIC HCA-RELATED PARAMETERS FOR 2D CANTILEVER BEAM PROBLEM

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Desired mass fraction</td>
<td>$\tilde{M}_f$</td>
<td>30%</td>
</tr>
<tr>
<td>Initial density distribution</td>
<td>$\tilde{x}^{(0)}$</td>
<td>0.3</td>
</tr>
<tr>
<td>Neighborhood</td>
<td>$\hat{N}$</td>
<td>12</td>
</tr>
<tr>
<td>Penalization parameter</td>
<td>$p$</td>
<td>1</td>
</tr>
<tr>
<td>Minimum allowable density</td>
<td>$x_{\min}$</td>
<td>0.05</td>
</tr>
<tr>
<td>Global convergence tolerance</td>
<td>$\epsilon$</td>
<td>4.8</td>
</tr>
</tbody>
</table>
In order to highlight the advantage of using the adaptive weighting factors method, the three-load case cantilever beam problem (Fig. 6.4) is first solved using fixed equal weights of magnitude 1/3 each. Again, the objective is to obtain a design with the minimum weighted sum of the compliances for all load cases for 30% mass fraction. The final design is shown in Fig. 6.6(a). Figure 6.6(b) shows the variation in the resultant nodal displacements at the location of the point loads for the three load cases as the design evolves. In addition, the average of the nodal displacements for the three load cases along with the 15% upper and lower bounds are also plotted. For linear static problems, the displacement at the load location is indicative of the stiffness of the structure for that load. Hence,
the decrease in displacements for the three load cases in Fig. 6.6(b) indicates an increase in stiffness of the structure for these three load cases as the design evolves.

Figure 6.6. Final design and the variation of the resultant nodal displacements for the three load cases when fixed equal weights are used.

Although the three load cases have forces of equal magnitude and the points of application are reasonably close to each other, the final design with fixed equal weights in Fig. 6.6(a) is dominated by the left load case (Fig. 6.5(c)). The reason for this behavior is an overlap of the portion of the structure affected by the left load case with the portions affected by the center and right load cases. This can be further understood with the help of Fig. 6.7 in which the 2D cantilever design space is divided into 3 sections marked as L (blue), C (green) and R (red). The
elements in section L deform for all the three load cases, whereas the elements in section C deform only for the center and right load cases. Similarly, the elements in section R are deformed only for the right load case. Hence, when the strain energies (compliances) are added for the three load cases at each design iteration, the elements in section L get contributions from all three load cases and dominate the elements in sections C and R. This results into very thin and weak structural members in sections C and R. The effect of this overlap can be compensated by choosing higher weights for the center and right load cases to develop a more uniform and robust structure. For this purpose, the present method for adaptively changing the weighting factors to match the performance of a design for all load cases is useful.

Figure 6.7. 2D cantilever beam divided into 3 sections to show the regions affected by the three load cases.
The same three-load case 2D cantilever beam problem is now solved using the proposed adaptive weighting factors method, and the resultant nodal displacements at the load locations are chosen as the performance measures. The objective for changing the weights is to match the displacements for the three load cases within a tolerance limit. The overall objective is the same as in the last case to achieve a design with the minimum value for the weighted sum of compliances for the three load cases. Various parameters associated with the proposed method (discussed in Section 6.3) that are used for the present case are summarized in Table 6.3. The final design is shown in Fig. 6.8(a). The structure in this case has more material or stronger members in the right most part (sections C and R) to support the center and right load cases more effectively. Figure 6.8(b) shows the variation in the resultant displacements at the load locations for the three load cases along with their average as the design evolves. The proposed method helps in achieving a robust design that has similar displacements for the three load cases within a 15% tolerance, unlike the design with fixed equal weights (6.6(b)).

**TABLE 6.3**

PARAMETERS FOR USING ADAPTIVE WEIGHTING FACTORS METHOD FOR 2D CANTILEVER BEAM PROBLEM

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportional gain for weight change</td>
<td>$K_w$</td>
<td>0.1</td>
</tr>
<tr>
<td>Tolerance on displacement matching</td>
<td>$D_{tol}$</td>
<td>15%</td>
</tr>
<tr>
<td>Maximum allowable change in average disp.</td>
<td>$\Delta D_{av}^{allow}$</td>
<td>20%</td>
</tr>
<tr>
<td>Maximum allowable change in weight</td>
<td>$\Delta w_{max}^{allow}$</td>
<td>0.05</td>
</tr>
<tr>
<td>Minimum allowable weight</td>
<td>$w_{min}$</td>
<td>0.1</td>
</tr>
</tbody>
</table>
Figure 6.8. Final design and the variation of resultant nodal displacements for the three load cases when adaptive weighting factors method is used. 85% and 115% of average displacement values are also plotted to represent 15% tolerance band on displacement matching.

The variation in the weights for the three load cases with design iteration number is shown in Fig. 6.9(a). The initial weights are calculated using Eq. (6.9) as the displacements for the three load cases are out of the tolerance limit at the start of the design process. As the design evolves, the weight corresponding to the right load case increases, and the weight corresponding to the left load case decreases until the displacements for the two load cases come inside the tolerance band or the lower allowable limit (0.1) is reached. The displacement for the center load case is always within the tolerance band, and ideally the weight corresponding to it should remain fixed. It decreases (Fig. 6.9(a)) only because of the normalization of weights after each update. The decreasing weight for the
center load case implies the decrease in its contribution to the global objective compared to the right load case. Figure 6.9(b) shows the variation in the change in weights (before normalization) for the three load cases with design iteration number. This confirms the reason for the decrease in \( w_2 \) being the normalization as the desired change in its value is zero before normalization. The sudden jumps to zero for the change in weights for the left and right load cases correspond to the design cycles in which their displacements enter the tolerance band.

Figure 6.9. (a) Variation of weights, and (b) change in weights for the three load cases with design iteration number for adaptive weighting factors method. Note that the weights become constant once the displacements for the three load cases are sufficiently close to each other.
It is to be noted that for the present example, the performance measure of the resultant displacement at the load location can be easily replaced by strain energy (compliance) without much difference in the final design because the problem is linear and static in nature and displacement at the load location is a direct indicator of the compliance. In the next examples, the adaptive weighting factors method is used for more complex problems of dynamic nature.

6.4.2 2D Beam with 3 Dynamic Load Cases - I

In order to demonstrate the real advantage of the present method, a more crash-like problem is considered. Three separate load cases are applied to a 2D beam constrained on both ends. A schematic of the design domain with dimensions and locations of the impacts for the 3 load cases is shown in Fig. 6.10. The design domain is discretized into a 160×40 mesh of 4-node, plane stress quadrilateral elements with 20 mm thickness. The beam is modeled as a linear elastic, piecewise linear plastic material, and the impacting pole in all 3 load cases is rigid. The material properties for the beam are shown in Table 6.4. The rigid pole weighing 76.6 kg impacts the 2D beam with an initial velocity of 8 m/s for all the three load cases. To demonstrate the application of the present method for a large intensity impact, the dynamic simulation results for the three load cases are considered only up to a fixed intrusion value of 25 mm even though the kinetic energy of the rigid pole may not be completely expended by that time (2.4.2). LS-DYNA’s explicit solver is used for simulation purposes in which the nodal displacements and other quantities like stress, energy, etc., are evaluated at various time steps. It is very difficult to anticipate a time step size whose multiple will have the exact prescribed intrusion value. Hence, a time is chosen that cor-
responds to the intrusion value closest to the prescribed value with a sufficiently small time step size.

---

**Figure 6.10. Schematic of the 2D beam with 3 dynamic load cases.**

To highlight the advantage of using the adaptive weighting factors method, the present 3-load case problem is first solved using fixed equal weights of magnitude 1/3 each. As discussed in Section 2.4.2, energy absorption is used as the performance measure in this case. The objective is to obtain a design with the maximum weighted sum of energy absorptions for the three load cases for a given mass fraction of 30%. Basic HCA-related parameters are summarized in Table 6.5. Note that in addition to penalization parameter p, another parameter q is also used to interpolate plastic material properties like yield stress and effective stresses for intermediate density elements according to Eq. (2.14). The final design is shown in Fig. 6.11(a). Figure 6.11(b) shows the variation in total energy absorption (up to the prescribed displacement) for the three load cases as the design evolves. In addition, the average energy absorption for the load cases and 5% upper and lower bounds are also plotted.
### TABLE 6.4

MATERIAL PROPERTIES FOR THE 2D BEAM WITH 3-LOAD CASE PROBLEM

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>2700 $Kg/m^3$</td>
</tr>
<tr>
<td>Elastic Modulus</td>
<td>70 GPa</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>0.33</td>
</tr>
<tr>
<td>Yield Stress</td>
<td>180 MPa</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Effective plastic strain</th>
<th>Effective stress (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>180.0</td>
</tr>
<tr>
<td>0.01</td>
<td>190.0</td>
</tr>
<tr>
<td>0.02</td>
<td>197.0</td>
</tr>
<tr>
<td>0.05</td>
<td>211.5</td>
</tr>
<tr>
<td>0.10</td>
<td>225.8</td>
</tr>
<tr>
<td>0.15</td>
<td>233.6</td>
</tr>
<tr>
<td>0.20</td>
<td>238.5</td>
</tr>
<tr>
<td>0.40</td>
<td>248.5</td>
</tr>
</tbody>
</table>

### TABLE 6.5

BASIC HCA-RELATED PARAMETERS FOR THE 2D BEAM PROBLEM WITH 3 LOAD CASES

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Desired Mass fraction</td>
<td>$M_f^*$</td>
<td>30%</td>
</tr>
<tr>
<td>Initial density distribution</td>
<td>$x^{(b)}$</td>
<td><strong>0.3</strong></td>
</tr>
<tr>
<td>Neighborhood</td>
<td>$\tilde{N}$</td>
<td>8</td>
</tr>
<tr>
<td>Penalization parameters</td>
<td>$(p, q)$</td>
<td>(1, 1)</td>
</tr>
<tr>
<td>Minimum allowable density</td>
<td>$x_{min}$</td>
<td>0.05</td>
</tr>
<tr>
<td>Global convergence tolerance</td>
<td>$\epsilon$</td>
<td>6.4</td>
</tr>
</tbody>
</table>
Figure 6.11. Final design and the variation of total energy absorption (up to prescribed displacement) for the three load cases when fixed equal weights are used.

As the design evolves, energy absorption for all load cases increases (except a few local drops) compared to the initial design; however, the relative difference between the energies of the load cases remains significant. The final design has more mass and stronger structural members in the left half of the structure because of the dominant left load case. It can be seen in Fig. 6.11(b) that the energy absorption for the left load case becomes constant after a few iterations because the structure becomes strong enough to absorb all of the kinetic energy of the impacting rigid pole. The reason for the left load case dominance is its relative proximity to the constraint end compared to the center and right load cases. The effect of this dominance can be compensated by choosing a higher weight for the right load case to develop a more uniform and robust design. For this purpose, the present method for adaptively assigning the weighting factors based on the
performance measure (energy absorption for this example) is useful.

The same 3-load case beam problem is now solved using the proposed adaptive weighting factors method, and the total energy absorption up to the prescribed intrusion (25 mm) is chosen as the performance measure. The objective for changing the weights is to match the energy absorption for the three load cases within a tolerance limit. The overall objective is the same as in the last case (fixed equal weights) to achieve a design with the maximum weighted sum of energies for the three load cases. Various parameters associated with the present method for allocating adaptive weighting factors that are used in this case are summarized in Table 6.6. The final design is shown in Fig. 6.12(a). The structure in this case has more material and stronger members in the right half (compared to the design with fixed equal weights) to support the right load case more effectively. Figure 6.12(b) shows the variation of energy absorption for the three load cases along with their average as the design evolves. The present method helps in achieving a robust (balanced) design that has similar energy absorption (within a tolerance limit) for the three load cases by adaptively changing the corresponding weighting factors.

The variations in weights for the three load cases with design iteration number are shown in Fig. 6.13(a). The method starts with equal weights (1/3 each) for the three load cases as their energy absorption is within the tolerance limit. As the design evolves, the weight corresponding to the right load case increases, and the weight corresponding to the left load case decreases until the energy absorption for these two cases comes within the tolerance band. By this time (design cycle), however the weight corresponding to the right load case is too high, and its energy absorption value increases out of the tolerance band. Similarly,
### Table 6.6

#### Parameters for Using Adaptive Weighting Factors Method for 2D Beam Problem - I

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportional gain for weight change</td>
<td>$K_w$</td>
<td>0.2</td>
</tr>
<tr>
<td>Tolerance on energy matching</td>
<td>$E_{tol}$</td>
<td>5%</td>
</tr>
<tr>
<td>Maximum allowable change in average energy</td>
<td>$\Delta E_{av}^{allow}$</td>
<td>20%</td>
</tr>
<tr>
<td>Maximum allowable change in weight</td>
<td>$\Delta w_{max}^{allow}$</td>
<td>0.05</td>
</tr>
<tr>
<td>Minimum allowable weight</td>
<td>$w_{min}$</td>
<td>0.1</td>
</tr>
</tbody>
</table>

![Design Iteration # vs Energy Absorption (N-mm)](image)

(a) Design with adaptive weights
(b) Variation of energy absorption and its average for the 3 load cases

Figure 6.12. Final design and the variation of total energy absorption (up to the prescribed displacement) for the three load cases when adaptive weighting factors method is used.

The weight corresponding to the left load case is too small such that its energy decreases out of the tolerance band. Therefore, the trend in weight change reverses at this point, the weight corresponding to the right load case starts decreasing,
and the weight corresponding to the left load case starts increasing. This reversal in trend happens once more before the weights finally become constant. This oscillating behavior of the weights is caused by a number of factors. One major reason for these oscillations is the tight tolerance band of 5%. The nature of the problem itself is a contributor too, as there is less region of influence common to the left and right load cases. Hence, the two load cases always compete for more material until a balance is achieved. Figure 6.13(b) shows the variation of the change in weights for the three load cases before normalization. The change in weight corresponding to the center load case remains zero for the most part as its corresponding energy absorption is generally within the tolerance band. The region of influence for the center load case overlaps with the regions of influence for left and right load cases, so its energy absorption is not affected much when material balance shifts from the left to the right half of the structure.

The present example shows that using the adaptive weighting factors method, the relative contributions from various load cases can be controlled based on a performance measure and a more robust structure with similar behavior for multiple load cases can be designed. In the next example, the same problem is treated with a different performance measure to show the applicability of the present method for low-to-medium intensity impact problems with multiple load cases.

6.4.3 2D Beam with 3 Dynamic Load Cases - II

In this example, the same 2D beam with three load cases (Fig. 6.10) from the previous example is considered. To represent a low-to-medium intensity impact, the simulation time is kept large enough so that the impacting rigid pole transfers all of its kinetic energy to the beam and rebounds back. Since the rigid pole has
the same mass and initial velocity in all cases, the energy absorption for the three load cases is the same, so it cannot be used as a performance measure in this scenario. As discussed in Section 2.4.2, the maximum intrusion is used as the measure of performance in this case.

First, the present 3-load case design problem is solved using fixed equal weights of magnitude 1/3 each. Basic HCA-related parameters from the previous example (Table 6.5) are used. The final design is shown in Fig. 6.14(a). Figure 6.14(b) shows the variation of maximum intrusion for the three load cases as the design evolves. In addition, the average of maximum intrusion for the three load cases along with 10% upper and lower bounds are also plotted. It is clear from Fig. 6.14 that the left load case dominates and the final design has more material and stronger members in the left half of the structure. Even though the total energy absorption for the three load cases is the same at all design iterations, it’s the
distribution of that energy within the structure that makes the left load case
dominant. Since the left load case is relatively closer to the constraint end, the
energy distribution in this case is more concentrated and hence, contributes more
locally to the (equal) weighted sum of the energies. This significant imbalance
of material distribution in the final design is also reflected in the huge difference
in the performance (maximum intrusion) of the design for the three load cases.
In order to compensate for the dominant left load case and its effect on the final
design, a higher weighting factor for the right load case should be used.

Figure 6.14. Final design and the variation of maximum intrusion for the
three load cases when fixed equal weights are used.
The same 3-load case problem is now solved using the proposed adaptive weighting factors method, and maximum intrusion is chosen as the performance measure. The weights are changed to match the maximum intrusion values for the three load cases within a tolerance limit. The parameters associated with the present method used for this example are summarized in Table 6.7. The final design is shown in Fig. 6.15(a). Unlike the design obtained with fixed equal weights (Fig. 6.14(a)), the new design has a more balanced distribution of material and structural members to support the three load cases more uniformly. This is further confirmed by Fig. 6.15(b) in which the variation in maximum intrusion values for the three load cases with design iteration number is shown. As the design evolves, the intrusion for all load cases decreases in comparison to the first iteration design (uniform material distribution), and their relative differences also decrease. In other words, the design obtained using the present method has more uniform performance (maximum intrusion) for all load cases.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportional gain for weight change</td>
<td>$K_w$</td>
<td>0.2</td>
</tr>
<tr>
<td>Tolerance on displacement matching</td>
<td>$D_{tol}$</td>
<td>10%</td>
</tr>
<tr>
<td>Maximum allowable change in average intrusion</td>
<td>$\Delta D_{allow}^{av}$</td>
<td>20%</td>
</tr>
<tr>
<td>Maximum allowable change in weight</td>
<td>$\Delta w_{allow}^{max}$</td>
<td>0.05</td>
</tr>
<tr>
<td>Minimum allowable weight</td>
<td>$w_{min}$</td>
<td>0.1</td>
</tr>
</tbody>
</table>
Figure 6.15. Final design and the variation of maximum intrusions for the three load cases when the adaptive weighting factors method is used.

The variation in weights for the three load cases with design iteration number is shown in Fig. 6.16(a). For the first 10 iterations, all three load cases have equal weights (1/3) as their corresponding maximum intrusion values are close (within the tolerance limit) to each other. Once the intrusion value for the left load case crosses the lower bound of the tolerance band, its corresponding weight is decreased. Similarly, when the intrusion value for the right load case crosses the upper bound of the tolerance band, its corresponding weight is increased. Eventually, the intrusion values corresponding to the three load cases become close enough, and the weights become constant. Figure 6.16(b) shows the variation in change in weights for the three load cases. The change in weight for the center load case is always zero as its corresponding intrusion value lies within the tolerance band at all iterations. The increase in weight for the center load case in Fig. 6.16(a) is due to the normalization of weights.
Figure 6.16. Variation in (a) weights, and (b) change in weights for the three load cases with design iteration number for the adaptive weighting factors method.

Figures 6.17(a) and 6.17(b) show the force-displacement behavior of the three load cases for the final designs obtained using fixed equal weights and the adaptive weighting factors method, respectively. The left load case dominant design with fixed equal weights has significant differences among the three load cases in terms of peak force and maximum intrusion, whereas the design obtained using the proposed method has very similar force-displacement behavior for the three load cases. Since the total energy absorption for the three load cases is the same, peak force and maximum intrusion are closely related in this case. For example, any attempt to decrease the intrusion results in an increase in peak force. Hence, in this example, choosing maximum intrusion as the performance measure has the same effect as choosing peak force as the performance measure would have on the final design.
Figure 6.17. Force-displacement behavior of the three load cases for the final designs obtained using (a) fixed equal weights and (b) adaptive weighting factors method.

6.5 Summary

A new method for allocating weighting factors adaptively within the HCA-based method for crashworthiness design for multiple load cases is presented. The method helps in developing designs with more uniform performance under multiple load cases. The method is of particular interest when uncertainty in the loading condition is modeled using a few representative load cases with the objective of minimizing the difference in the performance of the final design under these load cases. In the present work, the amount of energy absorption and maximum intrusion are used as the performance measures for a structure under crash (impact) loads. However, the proposed method is general enough to be extended to handle other performance measures like peak force, plastic strains, maximum stress, or a combination of these measures as long as the variation in
these quantities with respect to the weights is well understood.

At present, the method allocates the weights with the objective of bringing the performance measure for various load cases close to each other. For this purpose, the average of the performance measure at each design iteration is used as the target for changing the weights. For multi-load case problems with load cases very different in nature for which obtaining a design with uniform performance is not of primary interest, the present method can be modified by replacing the average performance measure with a failure-based performance measure.

One of the limitations of the present method is the need for the user to specify the proportional gain $K_w$ based on experience. If a relatively higher value for the proportional gain is chosen, the method may become unstable with oscillating weights. On the other hand, a very small value of $K_w$ can make the method less responsive and risk the possibility of design convergence before an appropriate selection of weights is achieved. Since the weight change algorithm is control based, tuning methods from the literature or a method similar to the adaptive $K_p$ scheme described in Section 3.1 can be used to evaluate a suitable value for $K_w$. 
CHAPTER 7
SUMMARY AND RECOMMENDATIONS

In spite of continuous decline in the traffic fatality rate, the number of injuries and deaths in highway accidents remains staggering as more vehicles are added to the roads each year. With increasing competition and tighter safety norms, automakers are exploring the field of vehicle crashworthiness with more interest and focus. Structural optimization, in particular topology and topometry optimization, provides a means to systematically improve existing designs or conceive new designs with better crashworthiness in shorter design cycles. In the last two decades, many gradient-based and heuristic methods have been developed for topology- and topometry-based crashworthiness design. Most of these methods result in stiff structures that are suitable only for a set of vehicle component designs for which maximizing the energy absorption or minimizing the intrusion (penetration) are the main concerns. Typical examples of such components are the passenger compartment, B-pillar(s), side floor panels and roof. However, there are some other components in a vehicle structure that should exhibit characteristics of both stiffness and flexibility. Moreover, the load (or energy flow) paths within the structure and the potential buckle modes also play an important role in efficient functioning of such components. For example, the front bumper, side frame rails, steering column, and occupant protection devices like the knee bolster should all exhibit controlled deformation and collapse behavior. In this work, new
methodologies are developed to tackle such complex objectives in topology- and
topometry-based crashworthiness design.

7.1 Summary of Original Contributions

The primary objective of this research is to develop new methodologies to de-
sign crashworthy structures with controlled behavior. In this research, the HCA
method introduced by Tovar et al. [107] is used as the basic framework to develop
new methodologies. The use of compliant mechanism-type (sub)structures in de-
signing crashworthy components is the highlight of the research. The ability of
compliant mechanisms to efficiently transfer force and/or motion from the point
of application of input loads (referred to as input ports) to desired points (referred
to as output ports) within the structure is used to design solid and tubular compo-
nents that exhibit controlled deformation and collapse behavior under crash loads.
In addition, a new method for designing crashworthy structures with controlled
behavior under multiple load cases by adaptively changing the contributions from
individual load cases is developed. A summary of original contributions of this
research is as follows:

7.1.1 Design of Crashworthy Components using Compliant Substructures

A new methodology based on compliant mechanism design is developed to in-
troduce flexibility within a crashworthy structure. A desired balance of flexibility
and stiffness within a crashworthy structure is achieved by defining a flexible and a
stiff subdomain. The flexible subdomain (FSD) creates a compliant-mechanism-
type substructure, while the stiff subdomain (SSD) generates a stiff structure
solely based on the distribution of internal energy (strain energy + plastic work).
The objective of the FSD is to transfer the input kinetic energy of the impact to the desired locations within the structure, typically defined by the SSD. Alternatively, the FSD creates potential load paths within the structure. One obvious advantage of having an FSD is the reduction in peak contact force which is desirable for satisfying injury-related criteria. Physically, the FSD provides a much needed cushioning effect in the early part of crash loading, and the SSD helps in maintaining the overall integrity of the structure. This method also helps in designing structures with multiple load paths and better usage of the available design space for cases in which traditional methods yield a single load path design. Designs with a single load path are often less preferred as they are stiffer and more susceptible to catastrophic failure in the presence of a crack or imperfection.

7.1.2 Design of Progressively Collapsing Thin-Walled Square Tubes

A topometry-based design method for thin-walled square tubes under axial and oblique impacts is developed to achieve progressive buckling starting from the impact end. During axial crushing of thin-walled square tubes, buckling starting at one end (often the end close to the impact) and progressing systematically toward the other end of the structure is preferred as it utilizes the maximum available material for plastic deformation without jamming. However, the presence of unavoidable imperfections and asymmetries in the geometry and loading conditions may trigger buckling to start from the middle or close to the rear end of the tube. The underlying idea of the present method is to design a square tube to trigger a symmetric mode (see Section 5.1 for details) of collapse near the impact end using the compliant mechanism design approach. The output port definitions on the tube faces (perpendicular to the longitudinal axis) along its length create
potential buckle (fold) zones in the final design. The enforced buckle zones act as triggering mechanisms to a) initiate a specific collapse mode, b) stabilize the collapse process, and c) reduce the peak force, which is desirable for crashworthiness. The presence of these buckle zones in the top portion of the tube also helps in avoiding or delaying the onset of global bending during an oblique impact with load angles higher than a critical value which otherwise adversely affects the load-carrying and energy absorption capacity of the structure. The method has potential application in designing frame rails typically used in automotive chassis.

7.1.3 Design of Crashworthy Structures with Controlled Behavior under Multiple Load Cases

An adaptive weighting factors method is developed to adaptively allocate weights to the various load cases in a multi-load case crashworthiness design problem. The objective is to develop a design with more uniform performance under multiple load cases. The method is particularly useful when the uncertainty in the loading condition is treated as an approximate multi-load case problem using a few representative load cases. In the present work, the amount of energy absorption and the maximum intrusion are used as the performance measures for a design under crash loads. The weight associated with a particular load case is changed based on the difference between the performance measure for that load case and the average of the performance measure over all load cases. The underlying idea is to control the contributions from individual load cases to the final design by changing the associated weights to bring the performance for all load cases to a common level within acceptable tolerance limits. The method is general enough to be extended to handle other measures of performance such as peak force, plas-
tic strains or a combination of various other performance measures as long as the variation of these quantities with respect to the weights is well understood.

7.1.4 Auxiliary Contributions

Some auxiliary contributions apart from the above mentioned main contributions of this research are as follows:

- **Adaptive $K_p$ scheme** - All of the new methods introduced in this research are developed within the HCA framework. Material distribution in the HCA method is governed by a proportional law that uses the error signal between the field variables and target setpoint (see Section 2.3.2 for details). For the material update scheme, it is important to choose an appropriate value of the proportional gain $K_p$. It was found that the final design and convergence rate are dependent on the choice of $K_p$. There were no set rules or guidelines to choose a $K_p$ value that results in a unique final design within a reasonable number of design iterations. Therefore, an ‘adaptive $K_p$ scheme’ is introduced and implemented in the existing HCA method to (a) eliminate the need for the user to choose an appropriate fixed value of $K_p$ which may be different for different sets of problems and (b) increase the probability of convergence. In this scheme, $K_p$ is evaluated before each design update based on the current field variable and target setpoint information. It is shown through examples that the adaptive $K_p$ scheme results in unique designs within a reasonable number of iterations that are almost equivalent (if not better) in performance than designs obtained with fixed $K_p$ values chosen based on user experience.
• **Mesh-dependence studies** - In topology optimization, it has been observed that increasing mesh density results in designs with more structural members of decreasing size. Ideally, mesh-refinement should result in a better finite element model of the same optimal structure and a better description of the boundaries - not in a more detailed and qualitatively different structure. Many methods to generate mesh-independent solutions have been proposed in the literature, and using mesh-independent filters is one of the most popular methods among them due to its computational efficiency. An effect similar to mesh-independent filters can be achieved in the HCA method by controlling the neighborhood size. A number of examples demonstrate how mesh-independent designs can be achieved through appropriate control of the neighborhood size. The neighborhood size also governs the minimum member size in the final design. Larger neighborhood sizes result in designs with fewer and stronger structural members more suited for cost-effective manufacturing.

• **Nonlinear compliant mechanism design** - Since compliant mechanisms are utilized for designing crashworthy structures in this research, nonlinearities arising from large deformations, plasticity and contact should be considered during the design phase. Therefore, the HCA method is developed for compliant mechanism synthesis considering nonlinear effects. In this work, a control-based strategy within the HCA framework for material distribution in the design domain is employed to achieve an optimal compliant structure for the linear elastic case. The Karush-Kuhn-Tucker (KKT) optimality conditions for the linear elastic case are expressed in the form of an HCA-based objective of distributing the material to achieve a uniform
state of an appropriate definition of mechanical stimulus or field variable. The same idea is then extended to nonlinear compliant mechanism design. Although the final design for the nonlinear case cannot be said to be ‘optimal’, the method produces designs that perform better than corresponding designs obtained using linear elastic behavior.

7.2 Recommendations for Future Research

This research introduces new methodologies for designing crashworthy structures with controlled behavior in the HCA framework. Although the presented methods are developed to a significant level, there are some improvements and extensions that can be introduced to make these methods more comprehensive and user-independent. Moreover, the idea of using compliant-mechanism-type structures for crashworthiness design has great potential in designing progressively collapsing solid and cellular structures. These improvements in the present methods and developing new methodologies along similar lines are described in the next few subsections.

7.2.1 HCA Method for Body-Fitted Non-Uniform Mesh

Currently, the HCA method for crashworthiness design [76] and all of the methods developed in the present research are implemented for uniform rectilinear meshing of the continuum structures. However, there are many problems in which structures with irregularly-shaped physical domains need to be optimized. Such problems require the use of body-fitted finite element meshes, which are typically non-uniform and non-rectilinear. A simple example is the finite element modeling of a square plate with a circular hole at its center as shown in Fig. 7.1(a). The inner
circular boundary is well captured by the 4-node elements arranged in a circular pattern, whereas the region near the outer linear boundaries is well represented by the 4-node quadrilateral elements arranged in a rectilinear fashion. The same plate with a hole can also be modeled with a uniform mesh of 4-node quadrilateral elements as shown in Figs. 7.1(b) and (c). However, the inner circular boundary is not well captured in Fig. 7.1(b) using the uniform mesh with an element size approximately the same as in the non-uniform mesh in Fig. 7.1(a). One approach to capture the circular boundary more precisely with the uniform mesh is to use finer mesh elements as shown in Fig. 7.1(c), but it drastically increases the computational cost.

Figure 7.1. Finite element meshing of a square plate with a circular hole at its center using (a) body-fitted non-uniform mesh, (b) and (c) uniform meshes.
One of the reasons for using a uniform rectilinear mesh in the HCA method is the computational efficiency. By using a uniform mesh, the cellular automata (CA) lattice (which contains the design variable and field variable information) can be overlapped with the finite element (FE) mesh, and the neighbors for each CA can be found by just searching the closest CAs in the perpendicular directions, often along the global axes. This search for neighboring CAs is not computationally expensive because it only requires increasing or decreasing the cell (element) index by a certain number to get the neighbors’ information. For example, in the case of a von-Neumann (N=4) neighborhood for a 2D problem, neighboring cells for each CA can be found by increasing or decreasing the CA index by one to identify the four neighbors.

In the case of a body-fitted non-uniform mesh, the uniform CA lattice cannot be perfectly overlapped with the non-uniform FE mesh. One approach to tackle this problem is to immerse the design domain (FE mesh) in a uniform CA lattice with a prescribed cell size and map the FEs to the CAs as shown in Fig. 7.2(a). This may result in one-to-one, one-to-many or many-to-one correspondences between the FEs and CAs depending on their relative sizes. Another approach is to use a non-uniform CA lattice exactly overlapping the non-uniform FE mesh and search for neighbors by identifying the cells/elements enclosed in a prescribed region around each CA. Figure 7.2(b) shows an example of using a circular region of radius R around each CA to identify neighboring cells.

There is a trade-off in terms of computational cost between using a finer uniform mesh to represent an irregular design space and using a body-fitted non-uniform mesh. While a uniform mesh saves computational efforts in searching and storing neighborhood information, it may result in higher computational cost.
in FEA due to the larger number of elements required to capture the design domain with reasonable accuracy. On the hand, a body-fitted non-uniform mesh can save significant computational effort by using a smaller number of finite elements, but computing and storing the FE-CA mapping (in approach 1) or searching for neighbors around each CA (in approach 2) can be a computationally expensive process. For crashworthiness applications in which FEA computational cost significantly exceeds that of the FE-CA mapping or finding neighboring cells, using a body-fitted non-uniform mesh can save a lot of computational cost.

7.2.2 2-Stage Design Method for Crashworthy Structures with Controlled Energy Absorption

For the method described in Chapter 4 to design crashworthy structures with controlled energy absorption, there are currently a few design decisions that need to be made by the user before the start of the design process. For example, the
relative sizes of the flexible and stiff subdomains, the number of output ports and their relative locations, and the relative mass fractions for the two subdomains must all be established. At present, these decisions are made based on the problem objectives and user experience. For instance, the output port numbers and their relative locations within the design space are governed by the desired load paths and/or buckle modes in the final design. However, a more systematic and formal approach to make these design decisions would make the present method more comprehensive and user-independent.

One possible approach is to introduce these design decisions as design parameters in a 2-stage design process. For instance, the size $d_1$ of the FSD and the distance $d_2$ of the output ports from the top surface of the design domain for the example in Section 4.4.1 can be used as design parameters as shown in Fig. 7.3. The first stage would be the same as that described in Chapter 4 to generate a structure with a given choice of subdomains and output ports. In the second stage, the above mentioned design parameters would be varied in order to achieve a desired performance. The peak force and maximum intrusion constraints can be used as the drivers for changing these parameter values. The variation in the peak force and maximum intrusion with changing relative subdomain sizes and mass fractions has been studied qualitatively through the examples in Chapter 4. It was found that increasing the relative size of the flexible subdomain increases the overall flexibility of the final design which in turn results in reduced peak force and increased intrusion values. Similarly, decreasing the mass fraction of the flexible subdomain also has the same effect on the peak force and intrusion values. Based on this qualitative understanding, relative subdomain size and mass fraction values can be changed in an iterative fashion to satisfy the peak force and
maximum intrusion constraints.

\[
\frac{(L - d_1)}{2}
\]

\[
d_1
\]

\[
\frac{(L - d_1)}{2}
\]

\[
H
\]

\[
O/P
\]

\[
SSD
\]

\[
FSD
\]

\[
SSD
\]

\[
O/P
\]

\[
\downarrow v_0
\]

Figure 7.3. The output ports and subdomain definitions for the example in Section 4.4.1.

The effect of variation in the number of output ports and their relative locations within the design domain on the peak force and maximum intrusion of the final design is problem dependent. However, some conclusions about the effect of the number of output ports on the final design can be drawn based on the following reasoning. In most cases, more output ports result in more load paths and hence, more thinner (weaker) structural members since the overall mass is constrained. Therefore, more output ports should in general increase the flexibility of the structure. On the other hand, the effect of changing the relative locations of these output ports on the final design is more closely related with the design domain configuration. The user's understanding of the problem and design requirements can help make an intelligent initial choice for the locations of the output ports. A design of experiment study can then be performed to determine
the effect of output port locations on the peak force and intrusion values.

7.2.3 2-Stage Design Method for Progressively Collapsing Thin-Walled Tubular Structures

For the method described in Chapter 5, the number and location of output ports in thin-walled square tube structures are user defined. As described in Section 7.2.2, a 2-stage method can be developed to automate the choice of output port numbers and their locations along the length of the tube based on the peak force and total energy absorption values. Generally, increasing the number of output ports decreases the peak force and energy absorption values because more output ports create more buckle zones and make it easier to form new folds. The initial choice for the location of output ports can be made based on the wavelength of deformation (distance between two successive folds) during the axial crushing of a uniformly thick tube. The location of the output ports can then be varied to minimize the peak force and maximize the energy absorption.

7.2.4 Design of Progressively Collapsible Solid Structures

The idea of using the compliant mechanism synthesis approach to design thin-walled tubular structures with user-defined buckle zones (Chapter 5) can be extended to design progressively collapsible solid structures. In many applications, solid slender structures under compressive loads are required to collapse progressively for controlled deformation and reduced peak forces. This can be achieved by defining multiple sets of output ports pointing outward. Figure 7.4 shows an example of a slender structure under compressive loads designed with four sets of output port definitions. The points corresponding to the output ports in the final
design have a tendency to move outward upon application of compressive loads, which ultimately evokes progressive collapse behavior. Similar to the method in Chapter 5, this method can also be further extended by introducing the number and locations of the buckle zones as design parameters in a 2-stage design method.

![Design Schematic](image)

(a) A schematic of the design domain with desired buckle zones

(b) Final design with 30% mass fraction

Figure 7.4. 3D collapsible design with 4 predefined buckle zones
Periodic cellular structures are new energy absorbing structures finding use in crashworthiness and blast mitigation applications. Metal honeycombs and metallic foams are already in wide use as sandwich panel structures. Although very effective, metal honeycomb structures are very generic in terms of their unit cell design and the cellular arrangement of those unit cells. Metallic foams, on the other hand, have very few design variables like relative density (proportion of solid material in the foam) and the material properties of the solid used to make the foam structure. This lack of design parameters to tweak in honeycomb and metallic foam structures leaves fewer opportunities to develop a design specific to a particular problem. Many researchers are working on developing new methods in which the unit cell of a customized periodic cellular structure is designed specifically for a problem of interest [46, 63, 116].

There is a great potential for tailored periodic cellular structures in crashworthiness design. The compliant mechanism design approach can be used to develop unit cells that have desired load paths and deformation characteristics. For example, consider a 2D slender beam under compressive loads from a rigid plate of mass M moving with an initial velocity of $v_0$ as shown in Fig. 7.5(a). One of the design objectives for slender members under compressive impact loads is to divert the impact loads sideways. This helps in avoiding thick column-like designs that are stiff and may not be preferred due to high peak forces. The design domain is divided into a 6×3 arrangement of unit cells. Each unit cell is then designed to have a compliant mechanism-type structure by approximating the impact load with a static point load acting at the top surface as shown in Fig. 7.5(b). In order to divert the input loads sideways, two output ports are defined on the left and...
right surfaces. The unit cell is constrained at the base to approximately represent its connection with the other unit cells and the elastic base. Figure 7.5(c) shows the final design for the unit cell with 30\% mass fraction. The diamond-shaped final design opens up sideways when pushed downward on the top surface.

Figure 7.5. Tailored cellular structure design process. (a) a 2D slender design domain divided into 18 unit cells, (b) the design problem description for a unit cell, and (c) final unit cell design with 30\% mass fraction.
The final design of the whole structure in its undeformed and deformed configurations is shown in Fig. 7.6(a). It can be observed from the deformed configuration that the unit cells collapse (deform) in layers. The cells in the middle of the structure show maximum deformation, whereas the cells in the top and bottom layers show more rotation than deformation due to boundary effects. Figure 7.6(b) shows the corresponding force-displacement response of the structure. Since the structure deforms by collapsing layers of unit cells of approximately the same stiffness, the reaction force remains almost constant throughout the deformation. A structure deforming at a constant force just below the injury level is best suited for crashworthiness applications as it has an optimal balance of peak force and energy absorption. The almost constant force in Fig. 7.6(b) can be increased or decreased based on the problem requirements by controlling the mass fraction of the unit cells. Moreover, collapse behavior starting from the top layer and systematically progressing toward the bottom layer can be evoked by using layers of unit cells of increasing mass from top to bottom. At present, the method of using compliant mechanisms for unit cell design in developing periodic cellular structures is in its infancy, and there is much room for improvement. However, based on the preliminary work done so far, there is a great potential in this method.

7.3 Closing Remarks

As the field of crashworthiness design is gaining more attention and importance in the auto industry all across the world, the methods introduced in this dissertation present new ways to design complex crashworthy structures with more controlled behavior. One of the major contributions of this research is the use
Figure 7.6. (a) Final design of the whole structure obtained by arranging unit cells in a $6 \times 3$ pattern and (b) its force-displacement response under compressive impact loads from a rigid plate.

of compliant mechanisms in designing crashworthy structures. The efficient use of compliant mechanisms adds much needed flexibility to crashworthy structures along with control over desired load paths and/or buckle modes. Although all of the methods in this research are developed in the HCA framework, the basic ideas behind these methods are generic and can be easily implemented with other available topology- and topometry-based optimization methods.
APPENDIX A

KKT OPTIMALITY CONDITIONS FOR COMPLIANT MECHANISM DESIGN

The optimization problem formulation for the compliant mechanism design problem (Section 3.3.2) can be expressed as

\[
\begin{align*}
\text{minimize} & \quad -MPE(\mathbf{x}), \\
\text{s.t.} & \quad \frac{\sum_{i=1}^{N} x_i}{N} = M_f, \\
& \quad 0 < x_{\text{min}} \leq x_i \leq 1 \quad \text{for} \quad i = 1, 2, \ldots, N.
\end{align*}
\]  

(A.1)

Considering linear elastic material behavior and small strain theory, the equilibrium equations for the two load cases in Fig. 3.12 in the discretized form required to evaluate mutual potential energy \( MPE \) can be written as

\[
\begin{align*}
\mathbf{KU} &= \mathbf{F}, \\
\mathbf{KU}_d &= \mathbf{F}_d.
\end{align*}
\]  

(A.2)

where \( \mathbf{U} \) is the displacement field due to the input force \( F_m \), \( \mathbf{K} \) is a matrix representing the combined stiffness of the design domain and springs, and \( \mathbf{F} \) is the force vector comprising the input force. Also, \( \mathbf{U}_d \) is the displacement field due to the dummy load, and \( \mathbf{F}_d \) is the force vector comprising the dummy load. By
definition, $MPE$ can be written as

$$MPE = U_d^T KU. \quad \text{(A.3)}$$

To obtain the Karush-Kuhn-Tucker (KKT) optimality conditions, the Lagrangian $L$ is defined as

$$L = -MPE + \sum_{i=1}^{N} \lambda_{1i}(x_i - 1 + s_{1i}^2) + \sum_{i=1}^{N} \lambda_{0i}(x_{\min} - x_i + s_{0i}^2) + \lambda_v \left( \frac{\sum_{i=1}^{N} x_i}{N} - M_f^* \right), \quad \text{(A.4)}$$

where $\lambda_{1i}$, $\lambda_{0i}$ and $\lambda_v$ are the Lagrange multipliers, while $s_{1i}$ and $s_{0i}$ are the slack variables to represent inequality constraints as equality constraints. The necessary conditions can then be obtained as

$$\frac{\partial L}{\partial x_i} = 0 \quad \text{for} \quad i = 1, 2, \ldots N$$

$$\frac{\partial L}{\partial \lambda_{1i}} = 0 \quad \text{for} \quad i = 1, 2, \ldots N$$

$$\frac{\partial L}{\partial \lambda_{0i}} = 0 \quad \text{for} \quad i = 1, 2, \ldots N$$

$$\frac{\partial L}{\partial \lambda_v} = 0. \quad \text{(A.5)}$$

Using Eq. (A.4) and Eq. (A.5),

$$\frac{\partial L}{\partial x_i} = -\frac{\partial MPE}{\partial x_i} + \lambda_{1i} - \lambda_{0i} + \lambda_v \frac{1}{N} = 0 \quad \text{for} \quad i = 1, 2, \ldots N. \quad \text{(A.6)}$$

From the definition of $MPE$ in Eq. (A.3),

$$\frac{\partial MPE}{\partial x_i} = \frac{\partial U_d^T}{\partial x_i} KU + U_d^T \frac{\partial (KU)}{\partial x_i}. \quad \text{(A.7)}$$

Since the input force $F$, which is also equal to $KU$ (Eq. (A.2)), is independent of
the design variables, Eq. (A.7) can be simplified as

$$\frac{\partial MPE}{\partial x_i} = \frac{\partial U_d^T}{\partial x_i} K U.$$  

(A.8)

Differentiation of Eq. (A.2) with respect to \( x_i \) yields

$$\frac{\partial U_d^T}{\partial x_i} K + U_d^T \frac{\partial K}{\partial x_i} = \frac{\partial F_d}{\partial x_i} = 0 \implies \frac{\partial U_d^T}{\partial x_i} K = -U_d^T \frac{\partial K}{\partial x_i}.$$  

(A.9)

Using Eq. (A.8) and Eq. (A.9),

$$\frac{\partial MPE}{\partial x_i} = -U_d^T \frac{\partial K}{\partial x_i} U.$$  

(A.10)

Since the \( i^{th} \) design variable \( x_i \) only affects its corresponding element stiffness matrix \( k_i \), Eq. (A.10) can be rewritten as

$$\frac{\partial MPE}{\partial x_i} = -u_d^T \frac{\partial k_i}{\partial x_i} u_i.$$  

(A.11)

In this work, the Simple Isotropic Material with Penalization (SIMP) approach is used to interpolate material properties like elastic modulus for the elements with intermediate densities. Hence, the elastic modulus for an element with density \( x \) can be expressed as

$$E(x) = x^p E_0 \quad (p \geq 1),$$  

(A.12)

where \( p \) is the penalization parameter and \( E_0 \) is the elastic modulus for the base material. The element stiffness matrix is directly proportional to the elastic modul-
ulas. Hence, Eq. (A.11) can be rewritten as

\[ \frac{\partial MPE}{\partial x_i} = -u_d^T(p_{ki}^T)u_i = -p \frac{(MPE)_i}{x_i}, \]  

(A.13)

where \((MPE)_i\) is the mutual potential energy for the \(i^{th}\) element. Using the result from Eq. (A.13) in Eq. (A.6),

\[ \frac{\partial \mathcal{L}}{\partial x_i} = p \frac{(MPE)_i}{x_i} + \lambda_{1i} - \lambda_{0i} + \lambda_v \frac{1}{N} = 0 \quad \text{for} \quad i = 1, 2, \ldots N. \]  

(A.14)

There are three possible cases depending on whether \(x_i\) is unsaturated \(x_{min} < x_i < 1\), at the lower bound or at the upper bound.

**Case I**  \(x_{min} < x_i < 1 \Rightarrow \lambda_{1i} = \lambda_{0i} = 0\)

Using Eq. (A.14),

\[ -p \frac{(MPE)_i}{x_i} = \frac{\lambda_v}{N}, \quad \forall i : x_{min} < x_i < 1. \]  

(A.15)

Rearranging Eq. (A.15) gives

\[ -(MPE)_i = \frac{\lambda_v x_i}{pN}, \quad \forall i : x_{min} < x_i < 1. \]  

(A.16)

Adding the LHS and RHS terms for all unsaturated elements in Eq. (A.16) gives

\[ -\sum_{\forall i : x_{min} < x_i < 1} (MPE)_i = \frac{\lambda_v}{pN} \sum_{\forall i : x_{min} < x_i < 1} x_i. \]  

(A.17)

Defining \(\sum_{\forall i : x_{min} < x_i < 1} (MPE)_i\) as the global mutual potential energy \((MPE)_g\),
the Lagrange multiplier $\lambda_v$ can be evaluated as

$$\lambda_v = (pN) \frac{-(MPE)_g}{\sum_{\forall i: x_{min} < x_i < 1} x_i}. \quad (A.18)$$

Using Eq. (A.16) and Eq. (A.18), the KKT optimality condition can be expressed as

$$-\frac{(MPE)_i}{x_i} = -\frac{\sum_{\forall i: x_{min} < x_i < 1} (MPE)_i}{\sum_{\forall i: x_{min} < x_i < 1} x_i} \quad \forall i : x_{min} < x_i < 1, \quad (A.19)$$

where $S_i$ and $S^*$ are referred to as the field variable and target setpoint, respectively, in HCA terminology.

**Case II** \( x_i = x_{min} \quad \Rightarrow \quad \lambda_{1i} = 0 \quad \& \quad \lambda_{0i} \geq 0 \)

Using Eq. (A.14),

$$p \frac{(MPE)_i}{x_i} + \frac{\lambda_v}{N} = \lambda_{0i} \geq 0. \quad (A.20)$$

Alternatively,

$$-\frac{(MPE)_i}{x_i} \leq \frac{\lambda_v}{pN}. \quad (A.21)$$

Referring to Eq. (A.19), Eq. (A.21) can be rewritten as

$$S_i \leq S^*, \quad \forall i : x_i = x_{min}. \quad (A.22)$$

**Case III** \( x_i = 1 \quad \Rightarrow \quad \lambda_{0i} = 0 \quad \& \quad \lambda_{1i} \geq 0 \)

Working along the same lines as above,

$$S_i \geq S^*, \quad \forall i : x_i = 1. \quad (A.23)$$
In Chapter 5, a new method to design thin-walled square tubes under axial and oblique impacts based on the use of compliant mechanisms is presented. The output ports definition during the design stage results in tubes with enforced buckle zones that act as triggering mechanisms to initiate progressive buckling starting from the impact end. One common practice to enforce progressive buckling in thin-walled tube structures is to introduce dents or grooves that act as buckle initiators. In this Appendix, the design examples from Chapter 5 are reconsidered by introducing dents as triggering mechanisms. The performance of the tube designs with dents is measured for the same axial and oblique loading conditions that are considered in the examples of Chapter 5.

B.1 One set of dents

A thin-walled square tube from the example in Sec. 5.3.1 is considered with one set of dents located close to the impact end on the two parallel faces as shown in Fig. B.1. The dents in this case are introduced by reducing the thickness of the tube at desired dent locations to 0.8 mm. The choice for dent location and size (thickness) is made based on the design parameters used for tube design in Sec. 5.3.1. The tube with dents is subjected to axial crushing by a rigid flat plate.
of mass 147 kg and initial velocities of 3 m/s and 5 m/s. For an appropriate comparison, the same imperfection as shown in Fig. 5.5 is considered for the tube with dents during axial crushing simulations. The deformation behavior of this dented tube for the two initial velocities of impact (3 m/s and 5 m/s) is shown in Figs. B.2 and B.3. It can be seen that the buckling starts at the imperfection rather than the dent location during axial crushing at 3 m/s, possibly due to insufficient dent size to overcome the influence of imperfection. On the other hand, the same dents are effective in triggering progressive buckling to start at desired location during axial crushing at 5 m/s. However, the mode of collapse at dent location is not symmetric as intended. Hence, unlike the designed tube from Sec. 5.3.1 (Fig. 5.10), the dented tube with current dent size does not always produce the desired behavior.

B.2 Two sets of dents

A thin-walled square tube from the example in Sec. 5.3.2 is considered with two sets of dents as shown in Fig. B.4. Again, for consistency, the location of dents and their size are chosen based on the design parameters used for tube design in Sec. 5.3.2. Similar to the example in Sec. 5.3.2, the dented tube is subjected to an axial and an oblique loading from a rigid plate with an initial velocity of 5 m/s. Also, the same imperfection as shown in Fig. 5.14 is considered for the tube with dents during axial and oblique loadings. The deformation behavior of this dented tube for the two loading conditions (axial and oblique) is shown in Figs. B.5 and B.6. It can be seen that the current dent configuration is not effective in producing desired buckling at the dent locations during the axial crushing and avoiding global bending during the oblique impact.
Figure B.1. A uniformly thick square tube from Sec. 5.3.1 with one set of dents on two parallel faces close to the impact end.

It is to be noted that the above studies are preliminary in nature and the results are specific to the current choice of dent location(s), shape and size. The aim of this study is to put the new designs developed in Chapter 5 alongside the conventional designs obtained by introducing dents or grooves. For future work, a more exhaustive study can be done to compare the effectiveness and limitations of the method developed in Chapter 5 with the existing conventional techniques for thin-walled tube design.
Figure B.2. Deformation of the tube with one set of dents (Fig. B.1) at various times when impacted by a rigid plate with an initial velocity of 3 m/s. Note that an imperfection similar to the example in Sec. 5.3.1 is also considered for appropriate comparison.
Figure B.3. Deformation of the tube with one set of dents (Fig. B.1) at various times when impacted by a rigid plate with an initial velocity of 5 m/s.
Figure B.4. A uniformly thick square tube from Sec. 5.3.2 with two sets of dents on two parallel faces close to the impact end. All dimensions are in mm.
Figure B.5. Deformation of the dented tube (Fig. B.4) with an imperfection at various times when impacted axially by a rigid plate with an initial velocity of 5 m/s.

Figure B.6. Deformation of the dented tube (Fig. B.4) with an imperfection at various times when impacted obliquely (10°) by a rigid plate with an initial velocity of 5 m/s.


