ESSAYS
ON THE TERM STRUCTURE OF INTEREST RATES
AND EXCHANGE RATES

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This dissertation explores the determination of asset prices in the world through a low-dimensional set of risk factors. The economic intuition driving this work comes from the idea that investors form their future expectations on the economy based on observable risk patterns that determine the price of assets today. Given that those risk factors are summarized in current prices, studying the dynamics of interest rates at different horizons provides information on the market’s expectations of future macroeconomic fundamentals. I provide a summary of the literature that investigates asset pricing dynamics through term structure models. After analyzing the empirical applications of the yield curve and extracting the underlying factors that drive different assets and macroeconomic fundamentals, I pose the question: Can interest rate factors explain exchange rate fluctuations? In other words, is the information summarized in the yield curve enough to account for movements in the exchange rates? To answer this question, I expand a canonical no-arbitrage factor model of the term structure, in which the yields for the domestic and the foreign countries are determined by their own interest rate factors, and the ratio of their stochastic discount factors determines the (log) exchange rate changes, as predicted by the theory. My results suggest that yield curves do contain important information
to determine exchange rates, and potentially other asset prices, particularly at longer horizons. Moreover, extracting three or more factors from the yield curves (characterized as level, slope, and curvature) significantly improves the fit of the model. The information content in the yield curve can also account for the Uncovered Interest Parity puzzle, suggesting that financial markets contain information on the exchange rate risk premium. I also explore the ability of European countries to predict the path of the euro/U.S. dollar exchange rate with their own country specific factors. The results suggest that yield curve factors from many European countries have explanatory power over the euro dynamics. This work has important implications for understanding theoretical and empirical connections between interest rates and exchange rates through term structure models.
To my brothers, Diego and Nico, for making every day brighter.
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The motivation for this dissertation comes from the idea that there are a few factors that drive asset prices in the world. This idea has been explored in different contexts:

1. In the **Equity Returns** literature, work lead by the 2013 Economics Nobel Prize winners suggest that the price-dividend ratio is the best predictor of stock returns, particularly in the long run; see for example Fama and French (1988).

2. In the **Fixed Income** literature, work by Diebold, Li, and Yue (2008), among others, suggest that the factors driving interest rates at different horizons are similar across countries, suggesting that interest rates may be driven by a global component and an idiosyncratic component.

3. More recently, in the **Exchange Rate** literature, work by Greenaway-McGrevy et al. (2012) for example, suggests that fluctuations in the exchange rates of different currencies are driven by global components extracted from exchange rate data.

*Why do we think all assets might be driven by the same factors?*

Intuitively, an investor deciding how to allocate her wealth is going to consider portfolios with different risk profiles, assets at various horizons, and investment opportunities in different countries. In other words, we can think of a unique pricing mechanism that determines the prices of all these assets simultaneously, given a low-dimensional set of information available at each point in time.
In this dissertation, I primarily focus on understanding what drives interest rates and exchange rate fluctuations, and leave the incorporation of stock returns for future work. Specifically, I pose the question: Can interest rate factors explain exchange rate fluctuations?

**What are interest rate factors?**

The factors that drive interest rates have been identified in the literature as the main components of the yield curve. The yield curve shows the relationship between interest rates at different maturities, and its main components are the *level* or the average of the yield curve, the *slope*, and the *curvature*. These components are responsible for more than 99% of the cross-sectional variation of interest rates for any particular country.

I expand on the concept of how to extract interest rate factors and other empirical applications of the yield curve in Chapter 2. Most importantly, I am interested in analyzing whether the same factors that drive interest rates (i.e., the level, slope, and curvature of the yield curve) can also explain movements in other assets, specifically fluctuations in the exchange rate. I explore this concept in depth in Chapters 3 and 4, where I describe an international model that accounts for exchange rate movements between a domestic and a foreign country as a function of interest rate factors only. The results suggest that interest rate factors are able to explain exchange rate fluctuations, particularly at longer horizons.

I begin this dissertation by outlining the major breakthroughs in the term structure literature in Chapter 1, focusing on the most relevant research from the last decade and the challenges that lay ahead. Given the interesting applications of term structure models and the computational advances made in the past few years, the literature has been pushing the frontier further than even before.
1.1 Term Structure Models

Dynamic term structure models (DTSMs), originally developed in the finance literature, describe how yields of different maturities move over time, and more importantly, how they relate to each other. They have now become the model of choice when seeking the most comprehensive approach to understanding interest rate movements from a macro-finance perspective. Term structure models originated with Vasicek (1977), who presumed that the short term interest rate was a linear function of a vector that followed a Gaussian diffusion (joint normal distribution). This has been the building block of a large family of term structure models, formalized by Duffie and Kan (1996), characterized by Dai and Singleton (2000, 2003), and expanded by many others.\footnote{Comprehensive surveys on the literature can be found in Diebold, Piazzesi, and Rudebusch (2005), Piazzesi and Schneider (2007), Piazzesi (2010), and Gurkaynak and Wright (2012).}

1.1.1 Key Features

Term structure models consist of three main components. The first one is a model of the short rate. This can also be interpreted as a risk free or instantaneous rate, and in more sophisticated settings, as a monetary policy rule implemented by the Fed. The most common assumption, following Vasicek (1977), is that the short rate is an affine (linear) function of an underlying state vector that captures economic risks. This set of risks can be $N$-dimensional and move over time, typically as a linear autoregressive process, which becomes the second ingredient of term structure models. If the state vector follows an affine diffusion, then all of its conditional moments are also known under the actual distribution, which significantly facilitates the estimation of the model (Dai and Singleton, 2003). Moreover, the linear structure of these models, although not indispensable for estimation, allows for invariant transformations of the state vector and the model parameters that alleviate the computational burden.
associated with dynamic models. Dai and Singleton (2000) define as *invariant* any modification to the model that preserves the bonds and their distributions unchanged. Finally, these models specify how the short rate affects long-term bonds via a factor risk premium. The zero-coupon bonds and yields of various maturities are then determined as linear functions of the state vector, providing a comprehensive picture of the term structure of interest rates.

Although most of the literature was developed in continuous time, Duffie and Kan (1996) introduced the discrete-time versions of affine class models, where bond prices are exponential affine functions of the underlying state variables. Le, Singleton, and Dai (2010) show that it is possible, and in some cases convenient, to map a continuous-time DTSM to its discrete-time counterpart. Discrete time models gained popularity given that along some dimensions, they offer more flexibility and tractability in specifying the dynamics of the state process (Dai and Singleton, 2003; Le, Singleton, and Dai, 2010).

The key assumption of term structure models is that there are no arbitrage opportunities in the bond markets. This assumption implies that investors receive the same risk-adjusted compensation for bonds of different maturities. No-arbitrage is implemented in the form of cross-equation restrictions that make the yield curve consistent at every point in time. Hence the yields in DTSMs, are modeled as linear functions of the factors, in which the coefficients are restricted so that no-arbitrage holds, as in Duffie and Kan (1996). Although in reality this assumption may not hold at every point in time, given the liquidity of well-developed bond markets, violations to the arbitrage-free assumption are usually short lived, since bank holding companies and other large investors are able to immediately take advantage of those opportunities, acting as arbitrageurs in the economy.
1.1.2 Unobserved vs. Observed Factors

Models based on a reduced set of factors have proven to be a good representation of interest rate movements. Hence the assumption that a small number of factors drive the entire cross-section of interest rates is not only parsimonious but also effective. Besides deciding how to model the evolution of the factors over time, term structure models can take different stands on what the factors are, how they are estimated, and what restrictions they are subject to.

The most common representation of factors is through a latent (unobserved) state vector. This factor structure assumes that the economic risks that drive interest rate movements are unobserved and can be recovered by the bonds themselves through filtering techniques. Latent factors are usually unconstrained but the weights each factor places on the yields are restricted according to economic theory. Typically, three factors alone are enough to explain almost the entire cross-sectional variation of interest rates, with the first -and most persistent- factor having the largest explanatory power over interest rate movements. Although the source or meaning of these unobserved variables is unknown, these factors can be interpreted through the effect they have on the yield curve.

A more recent representation of the factors is obtained through principal component analysis in which factors are constructed as linear combinations of the yields themselves and are restricted to be orthogonal, although no constraints are imposed in the factor loadings. When dealing with observable factors constructed from the cross-section of yields, Dai and Singleton (2003) argue that internal consistency conditions need to be satisfied, so that the model correctly prices the state variables at each point in time.

Finally, Joslin, Priebsch, and Singleton (2014) show that allowing interest rate factors to be measured with or without error, does not yield significantly different results.
1.1.3 Stochastic Discount Factor

The stochastic discount factor (SDF) is the pricing mechanism that investors in each country develop in order to price assets in the local currency. Let $P^m_t$ be the price at time $t$ of an $m$–maturity zero coupon bond (no intermediate cashflows). Asset prices are risk-adjusted expected values of their future payoffs, so that

$$P^m_t = E_t \left[ M_{t+1} P^{m-1}_{t+1} \right], \quad (1.1)$$

where $M_{t+1}$ is the SDF that accounts for the adjustment of risk. The investors demand this risk premium to compensate for unaccounted fluctuations or uncertainty in the economy, such as macroeconomic risks like inflation, monetary policy risk, or maturity risk, among others. The SDF is also called pricing kernel in the finance literature and can be specifically linked to preferences through the intertemporal rate of substitution. SDFs are also related to the beta in conditional versions of the classical Capital Asset Pricing Model (CAPM) and its multifactor extensions. The basic intuition of the CAPM still holds true: the factors that determine asset prices also determine the risk premium, which compensates investors for weathering losses during bad times. However, the main goal of the CAPM is to express expected excess returns in terms of a security’s beta with a benchmark portfolio but does not take a stand on the characteristics of the pricing relationship. Term structure models parameterize the SDF directly as a function of the state vector implied by the market prices of risk, relating the risk premium to bonds across the yield curve without relying on assumptions about preferences and technology.

Let $\mathbb{P}$ represent the physical or actual process, such that

$$P^m_t = E^{\mathbb{P}}_t \left[ M_{t+1} P^{m-1}_{t+1} \right]. \quad (1.2)$$

Equations (1.1) and (1.2) only differ in notation, so that the new equation emphasizes the pricing of the asset under the physical process, which can also be interpreted as a historical process. The risk free rate equation, together with the equation of motion for the factors under $\mathbb{P}$, helps recover the historical moments of all bond returns.
Let $\mathbb{Q}$ represent the risk-neutral process, such that asset prices are expected values of their future payoffs discounted at the risk-free rate:

$$P_{tm} = e^{-rt} E_t^Q \left[ P_{t+1}^{m-1} \right]. \quad (1.3)$$

Equations (1.2) and (1.3) are equivalent representations of the asset pricing mechanism. However, under risk-neutrality, the prices have already been adjusted for risk and we do not need a pricing kernel to do so (Dai and Singleton, 2000). The existence of a risk-neutral distribution is guaranteed by the absence of arbitrage. Moreover, if $\mathbb{Q}$ is unique then bond markets are complete. The equation of motion for the factors under $\mathbb{Q}$, along with the equation for the short rate, prices all the assets in the economy. The reason it is necessary to specify the distribution of the state variables under both measures, is because what lays between the $\mathbb{P}$ and $\mathbb{Q}$ distributions of the state vector are adjustments for the market prices of risk: terms that capture agents’ attitudes toward risk. Term structure models evaluate the evolution of the factors under both distributions to determine more information on the pricing kernel as a function of a known state vector.

The specification of the SDF in the context of DTSMs implies that the risk premium is fully determined by the risk factors. This allows for a time-varying risk premium, which is consistent with empirical evidence on risk premium dynamics. Campbell and Shiller (1991) and Backus et al. (2001), among others, show that there is evidence from yield and forward rate regressions for a time-varying risk premium. Time-variance is also important to match the dynamics of yields in the data and deviations from the Expectations Hypothesis, as shown in Dai and Singleton (2002).

1.1.4 Estimation Techniques

Term structure models that are affine in the state vector are very popular because they lead to closed-form solutions to the prices of the bonds, which allows us to avoid
computationally costly techniques, such as solution methods for partial differential equations or Monte Carlo methods to estimate yields. If the factors are linear under the physical distribution, then all of its conditional moments are also known, even if the risk-neutral process exhibits non-linearities. Another popular assumption is that the factors follow a Gaussian diffusion. The assumption of Gaussian factors comes from the assumption that yields are Gaussian themselves, which implies a constant variance and allows the factors to take on negative values. For example, Dai, Singleton, and Yang (2007) assume that the risk factors follow a discrete-time Gaussian process. This allows for closed-form solutions for bond prices given their parameterization of the risk-neutral distribution and their flexible specifications of the market prices of risk. This Gaussian structure of the risk factors also allows for an analytic representation of the likelihood function for bond yields by Maximum Likelihood.

Similarly, Joslin, Singleton, and Zhu (2011) or JSZ build upon a similar framework and show that a latent vector can be replaced by observable factors. When the factors are latent, estimates governing the physical distribution necessarily depend on those of the risk-neutral distribution. JSZ show that under their framework, a *separation property* holds: the parameters that govern the physical and risk neutral distributions are different, except for the variance-covariance matrix of the innovations to the factors. However, the separation argument holds since the Maximum Likelihood estimates of the parameters are independent of the variance-covariance matrix, as shown in Zellner (1962). This line of work has simplified computational burden and provided state of the art estimation techniques that allow for more sophisticated specifications of term structure models.

One of the potential concerns within this framework is small-sample bias. Because the closer to a unit root, the larger the small-sample bias in Maximum Likelihood estimates, JSZ enforce a high degree of persistence of the factors under the physical
distribution to ensure a stationary process, by which they constrain all the eigenvalues that govern the speed of mean reversion to be strictly positive. However, Bauer, Rudebusch, and Wu (2012, 2014) argue that the JSZ method still leads to large biased estimates that may yield an inconsistent risk premium. Given the separation argument, only the parameters that characterize the physical density of yields are affected. They propose to replace the initial estimates of the factors’ vector autoregressive process under $P$ with bias-corrected estimates and then proceed to the Maximum Likelihood estimation. They are also able to show that a bias-corrected term premium exhibits countercyclical behavior, consistent with empirical and theoretical research as in Campbell and Cochrane (1999), Wachter (2006), and Cochrane and Piazzesi (2005).

Diez de Los Rios (2013, 2014) builds upon the JSZ framework as well to develop an Asymptotic Least Squares (ALS) estimator, which is consistent and asymptotically normally distributed. ALS is asymptotically equivalent to the Maximum Likelihood estimates from JSZ (under certain conditions) but takes advantage of the linearity in the pricing recursive relations of the bond coefficients to obtain parameter estimates with a reduced-form approach. Diez de Los Rios (2013, 2014) argues that this procedure helps avoid numerical optimization. Moreover, this work also suggests that imposing restrictions on the prices of risk helps to deal with persistence of the factors and eigenvalues close to one under both distributions.

1.2 Term Structure Models and Macroeconomic Variables

Theories on the term structure of interest rates attempt to shed light on the relationship between short-term and long-term interest rates. From a macroeconomic perspective, the short rate is one of the most important mechanisms through which monetary policy affects the real economy. However, most economic agents are concerned with interest rates at much longer horizons. How does monetary policy affect
the entire term structure of interest rates? Also, if monetary policy reacts to economic risks, how do macroeconomic shocks affect the yield curve? Does the yield curve in turn affect macroeconomic variables as well? Given that the shape of the yield curve has important implications for the economy, research on this area has shifted towards understanding the effect of monetary policy on the yield curve as well as the effect of the yield curve on the economy. The literature has mixed answers to these questions, but their relevance encourages further work through a macro-finance integrated approach.

1.2.1 Spanned Macroeconomic Risks

In order to incorporate macroeconomic variables into term structure models, the literature has mainly taken two directions. The first approach has been to provide economic intuition to the yield curve factors from term structure models by relating them to main macroeconomic variables, such as inflation and output growth. This line of work builds upon the idea that yield curve factors (the level, the slope, and the curvature) are a good statistical representation of the cross-sectional variation of interest rates. What are the economic forces that drive these interest rate factors? Evidence suggests that the level factor is highly correlated with aggregate supply shocks from the private sector (Wu, 2006) and shocks to preferences for current consumption and technology (Evans and Marshall, 2007), whereas the slope factor seems to be related to monetary policy shocks (Evans and Marshall, 1998, 2007; Wu 2006), since it disproportionally affects the short end of the yield curve. Diebold et al. (2005) provide a macroeconomic interpretation of the (latent) factors by relating them to the macroeconomy through unrestricted vector auto-regressive processes. They show that the level factor is highly correlated (40%) with inflation and the slope factor is highly correlated (40%) with real activity, measured by demeaned capacity utilization. However, the curvature factor, they find, is not correlated to any macro
variables. Their work allows them to study bidirectional feedback effects between yields and the macroeconomic variables through impulse response functions. Although they find strong effects of macro variables on future yields, evidence of yield curve effects on macro variables is weak, suggesting that the one-directional feedback approach (macro-to-yields) might be more important.

1.2.2 Unspanned Macroeconomic Risks

Another branch of the literature has instead proposed that, given that yield curve factors are not able to completely explain the time series properties of interest rates, there must be a set of new variables not captured by the yield curve factors, called unspanned factors, that need to be added to the state vector. Leading this approach is work by Ang and Piazzesi (2003), who took a first step towards understanding the joint dynamics of macro variables and yields in a Gaussian term structure model with time-varying risk premia. Although other papers had looked at VAR dynamics before, this study expands upon it by imposing no-arbitrage. Their work assumes there are five different factors: three latent factors (spanned by the yields) and two macro observable factors, independent from the yield curve. These two macro factors are constructed as the first principal components of a large set of macroeconomic series, which they label inflation and real activity. They also model short rate dynamics by drawing parallel arguments to the Taylor rule and its derivations. Similarly to Diebold et al. (2005), they find that shocks to the inflation factor affect the level of the entire yield curve, but shocks to real activity, affect the curvature - not the slope. More importantly, incorporating these additional macro factors seems to improve the forecasting performance of the model, given that it allows them to explain a significant amount of the time variation in bond yields, particularly for short-term bonds. Another important implication of their results is that the time variation in the pricing kernel seems to be driven by shocks to both observed macro factors as
well as unobserved factors.

An extension by Ang, Bekaert, and Wei (2008), specifies a no-arbitrage term structure model with nominal yields and inflation to explore the real and nominal sources of regime switches, by identifying the inflation risk premium component in the real term structure. Joslin, Le, and Singleton (2013) explore the relevance of measurement errors on the macro variables when fitting the yields. They argue that macro factors with no measurement error lead to poor bond pricing fit. More recently, work by Joslin, Priebsch, and Singleton (2014) studies how unspanned macroeconomic shocks affect the market prices of risk in a DTSM with no-arbitrage. Their findings suggest that unspanned macro risk can account for a substantial proportion of variance in the U.S. forward term premium, particularly at the short-end.

1.2.3 Monetary Policy and Forecasting

The analysis by Ang and Piazzesi (2003) has inspired a comprehensive line of research that incorporates macroeconomic variables to study nominal and real term structure dynamics, in and out of sample. Building upon previous work, Ang, Dong, and Piazzesi (2007) incorporate Taylor rules into a term structure model with time varying risk premia to study bidirectional macro-finance connections under no-arbitrage. They add output gap and inflation variables to a latent factor spanned by the yield curve. Since the model cannot be solved by Maximum Likelihood, they rely on Markov chain Monte Carlo for their estimation method. They find that allowing for bidirectional feedback can alter the amount of yield variation that can be attributed to macro factors, and that the bidirectional system attributes over half of the variance of long yields to macro factors alone. Rudebusch and Wu (2008) also develop a no-arbitrage term structure model, in which the short rate directly relates to macroeconomic fundamentals through a monetary policy reaction function (Taylor Rule). They find that the standard no-arbitrage term structure factors have clear
macroeconomic underpinnings, with one of the latent factors identified as a *perceived inflation target* and the other as a *cyclical monetary policy response to the economy*.

Several other papers have focused on the forecasting ability of asset prices for macro variables, particularly inflation and output. Ang, Piazzesi, and Wei (2006), for example, model the dynamics of yields jointly with GDP growth, where the state vector is comprised of yield factors and a GDP growth factor. In a comprehensive paper, Ang, Bekaert, and Wei (2007) explore different model specifications to study inflation forecasting, using four inflation measures and two different out-of-sample periods (post-1985 and post-1995). Their no-arbitrage model has two latent variables and quarterly inflation as risk factors. They conclude that term structure models are not helpful for forecasting inflation, which is in line with work by Stock and Watson (1999) and the results from Diebold et al. (2005) that suggest weak effects of term structure models on macroeconomic variables. Orphanides and Wei (2010) also study the relationship between macro variables (inflation, output growth) and bond yields, but they allow for evolving perceptions regarding the dynamic structure of the economy. In this framework, agents engage in real time re-estimation and updating of the vector auto-regressive process that models the economy, for a more realistic approach to investors' behavior. Their results suggest improved forecasting of yields and macro variables under an evolving dynamic environment.

Modeling the term structure of interest rates to account for spanned or unspanned macroeconomic dynamics is one of the most exciting new areas in the literature, with relevant implications for understanding how monetary policy affects the economy through yield curve dynamics and improved forecasting ability of macro factors.

### 1.3 Term Structure Models and Exchange Rates

Recently, term structure models have been extended to account for exchange rate movements. The motivation behind this modeling framework is that since long-
maturity yields are risk-adjusted expected values of average future short yields, the yield curve may contain information on the expected future path of economic fundamentals. Engel, Mark, and West (2007), for example, present evidence that short-run movements in exchange rates are primarily determined by changes in expectations. If investors' expectations are spanned by the yield curve, there is a natural framework to theoretically model exchange rates using information from the term structure of interest rates. Despite the large body of literature devoted to modeling exchange rates, there is a high degree of uncertainty about how exchange rates react to changes in expectations, particularly at shorter horizons. Understanding what the main drivers of exchange rate fluctuations are and their connections to movements in other assets, has encouraged prominent work relating interest rates and exchange rates through term structure models. I devote the remaining chapters of this dissertation to explore these connections empirically and theoretically.

1.3.1 Common Sources of Exchange Rate Variation

Recent work by Graveline and Joslin (2011) and Greenaway-McGrevy, Mark, Sul, and Wu (2012), among others, suggests that exchange rates of different currencies may be driven by common sources of risk or global factors. That is, the comovements between different currencies may indicate the presence of underlying risk factors that drive exchange rate variation simultaneously. Graveline and Joslin (2011) analyze joint dynamics of exchange rates and swap rates for G-10 currencies with a no-arbitrage term structure model and conclude that there is one priced risk factor in G-10 swaps.

Lustig, Roussanov, and Verdelhan (2011) construct portfolios of currencies and extract two global exchange rate factors by principal component analysis: a level factor and a slope factor that are able to explain carry trade returns. Although the labels of these factors are the same as in the term structure literature, these
components capture common variation in exchange rates, not interest rates. Their interpretation of these exchange rate factors is as follows: the slope relates to the return in dollars on a zero cost strategy that goes long in the highest interest rate and short in the lowest interest rate currency; the level factor can be interpreted as the average portfolio return of a U.S. investor who buys all foreign currencies available in the forward markets. In a follow-up paper, Lustig, Roussanov, and Verdelhan (2013) construct a basket of currencies to predict excess currency returns by using forward exchange rates relative to future exchange rates. Although they develop a term structure model, they do not estimate the stochastic discount factor that determines exchange rates under no-arbitrage. Instead, they postulate a functional form and by Simulated Method of Moments, they choose parameters to match selected moments in the data.

Finally, Bauer and Diez de Los Rios (2012) develop a comprehensive approach to modeling interest rates and exchange rates in a multicountry framework, under an extended version of the JSZ model. They account for interest rate global risk, macroeconomic sources of risk, and exchange rate risk, which allows to capture higher volatility. They first estimate the risk-neutral parameters by minimizing the sum between predicted and actual yields and link those parameters to exchange rate dynamics. They then run OLS regressions to obtain the prices of risk (with restrictions), which allows them to obtain estimates of the physical distribution parameters. They find that economic restrictions have important implications for the model and that only the global interest rate factors command risk premia.

Term-structure models that incorporate different sources of risk and model multiple assets through a unique pricing mechanism, are still largely unexplored. This dissertation seeks to shed light on exchange rate and interest rate dynamics in a no-arbitrage framework.
1.3.2 Interest and Exchange Rates Connections

Clarida and Taylor (1997) estimate a vector-error-correction model for the term structure of the forward premium (interest rate differentials) and conclude that interest rates from two countries contain information that is useful to predict exchange rates. This line of work however, has produced mixed results. Supporting the idea that the same factors that determine the risk premium in the term structure of interest rates might also determine the risk premium in exchange rate returns is work by Hodrick and Vassalou (2002) and Dong (2006), among many others. Recent work by Ang and Chen (2010), finds that changes in the interest rates and slope of the yield curve are significant predictors of foreign exchange returns.

The literature has also mixed evidence on whether the information contained in both the domestic and foreign yield curves alone is useful in accounting for exchange rate movements. For example, Inci and Lu (2004) extend a quadratic term structure model in which global state factors have symmetric effects on pricing kernels in different countries, and provide evidence that interest rates factors alone may not be enough to determine exchange rate movements. Li and Yin (2010) reach similar conclusions in a no-arbitrage macro-finance framework, in which yield curve factors alone can only explain 13% of the euro/USD exchange rate. They find that incorporating macro fundamentals helps explain exchange rate movements, although 76% of the variation they find is driven by innovations to the factors.

Diez de Los Rios (2009) develops a two-country term structure model in continuous time, where the exchange rate is determined by each country’s risk free rate. This framework is able to provide statistically better forecasts than those produced by a random walk for the British pound and the Canadian dollar, although the results are negative for the German mark/Euro and the Swiss franc, which are explained by a rejection of the restrictions imposed by the term structure model.

One of the biggest challenges that the literature currently faces is whether interest
rate or other unspanned factors are able to produce better out-of-sample forecasting results of exchange rate movements at different horizons.

1.4 Term Structure Models and the Future

Incorporating macroeconomic variables, exchange rates, or both into term structure models of interest rates seems to be the most promising areas the literature has unraveled, with many unanswered questions and even more unexplored routes. But given the flexibility of term structure models, spanning the new advances in estimation techniques, there are still many applications to be considered within different areas. In this section, I summarize a few of the new directions term structure models have taken in very recent years.

1.4.1 Global Dynamics

Given the high degree of integration that financial markets have experienced in the past decades, a natural step in the literature is to identify the common sources of shocks across countries; i.e., global factors. Earlier work suggests that international bond return predictability can be explained by a global risk factor that relates to the world excess bond return (Ilmanen, 1995; Harvey, Solnik, and Zhou, 2002), and a second global factor related to foreign exchange risk (Harvey, Solnik, and Zhou, 2002), whereas Driessen, Melenberg, and Nijman (2003) find that a five-factor model, related to the level and slope of the yield curves, explains almost all of the variation in international bond returns. In addition, Perignon, Smith and Villa (2007) account for country-specific factors and determine that there is one source of global risk which is related to the level of country yield curves.

Foundational work in this area is the recent framework proposed by Diebold, Li, and Yue (2008). In their paper, they propose two global latent factors with simple auto-regressive dynamics as risk factors. This way, each country’s factors, the local
factors, load on the global variable and an idiosyncratic component that also exhibits auto-regressive dynamics. The local factors are estimated through simple OLS regressions. Diebold, Li, and Yue (2008) show that these local factors are related across countries, which motivates the idea of a global component embedded within each country’s risk vector. They then proceed to estimate the global model and find that the global level (or first unobserved factor) is related to global inflation (correlation of 75%) and the global slope is related to real economic activity (correlation to average GDP is 27%). However, they do not impose no-arbitrage, i.e., they do not derive the pricing kernel function within their framework.

Longstaff, Pan, Pedersen, and Singleton (2011) provide evidence on the existence of a global factor within sovereign credit spreads. They find that one factor alone can explain 64% of the variation in sovereign credit spreads during the 2000-2010 period, and that this global factor or principal component is negatively correlated with the U.S. stock market returns (-74%). They conclude that global factors may be more important than country-specific factors in accounting for variations in sovereign credit risk.

More recently, Dahlquist and Hasseltoft (2013) identify local and global components across international bond markets that are unrelated to the level, slope, and curvature factors. Their global factor is related to U.S. bond risk premia and international business cycles. They provide evidence of increased integration between markets and show that correlations between international bond risk premia have increased over time.

Given the intuitive interpretation of the international framework, the idea of global and idiosyncratic components driving asset prices deserves more attention. In particular, what do these factors mean and how much price variation are they able account for?
1.4.2 Stock Returns

From Campbell and Shiller (1988) to Lettau and Ludvigson (2001), there has been solid evidence of stock return predictability through a small number of factors, particularly at longer horizons. Are the factors driving all asset prices the same? Moreover, what do these factors represent? Motivated by the idea that under complete markets (assuming a representative agent) a unique stochastic discount factor should be able to account for prices of different assets, Campbell and Shiller (1991), Fama and French (1993), and Campbell and Ammer (1993), among others, model the dynamics of stock returns and bond yields, simultaneously.

Among the most influential work on this area, Cochrane and Piazzesi (2005, 2008) model joint dynamics by introducing the idea that a stock is the same as a bond although it is subject to cashflow risk, which makes its return uncertain. Under this view, the same variables or factors that predict bond returns should also help forecast stock returns. However, they reject the hypothesis that the main factors of the yield curve (level, slope, and curvature) alone can predict excess bond returns. They therefore introduce a *CP factor*: a tent-shaped linear combination of forward rates or bond yields that differs from the traditional principal component analysis. The CP factor is then unrelated to the level, slope, and curvature of the yield curve, but is able to predict future excess returns on 1 to 5 year bonds. Innovations to the CP factor represent good news about future economic performance. Indeed, the CP factor seems to be a strong predictor of the level of economic activity 12 to 24 months ahead. This CP factor suggests a connection between stock and bond returns that motivates modeling the time series of nominal bond returns and the cross-section of equity returns, within the same pricing context.

More recently, Lustig, Nieuwerburgh, and Verdelhan (2014) price both nominal bond yields and the aggregate stock market return in a no-arbitrage model in order to measure the wealth-consumption ratio in the data; they do not however, study
the cross-section of bond nor stock returns. Building upon this framework, Koijen, Lustig, and Nieuwerburgh (2012) propose a three-factor model that jointly prices the cross-section of returns on portfolios of stocks, the cross-section of government bonds, and time series variation in expected bond returns. Their three pricing factors are: the level of macroeconomic activity, the CP factor (from Cochrane and Piazzesi, 2005), and a DP factor, which is the dividend growth on value-minus-growth that is consistent with a risk-based resolution of the value premium puzzle. All of these factors are able to explain cross-sectional variation, but only the CP and DP factors seem to explain time variation. Finally, Baker and Wurgler (2012) show that government bonds comove most strongly with bond-like stocks, which are stocks of large, mature, low-volatility, profitable, dividend-paying firms that are neither high growth nor distressed. They propose a common sentiment indicator driving stock and bond returns.

1.4.3 Preferred Habitat

One interesting avenue that the literature has taken is to explore new applications of term structure models within different contexts, one of them being the preferred habitat framework. Preferred habitat models assume that investor-clientele have preference over specific maturities. These investors are only interested in investment opportunities over horizons that match their financial objectives. In this economy, there are also arbitrageurs: investors with no specific investment horizons, whose behavior helps to integrate maturity markets so that nearby bonds trade at similar prices and the term structure is arbitrage-free.

Ihrig et al. (2012) and Li and Wei (2013) incorporate the term structure framework into Vayanos and Vila (2009) preferred-habitat model in order to study the role of the Federal Reserve Bank in the bond markets. In these cases, besides the investors with preference for certain maturities and the arbitrageurs, there is a new risk-neutral
participant in the market: the Federal Reserve Bank. This new integrated approach allows one to study, for example, how changes in the supply of treasury securities affect the yield curve dynamics.

In joint work with Thomas Cosimano and Sebastian Roelands, I also explore the incorporation of the term structure of interest rates into a structural banking model with a preferred habitat world, as in Vayanos and Vila (2009). In this context, we examine optimal bank asset-and-liability allocation allowing for multiple maturities, given the incorporation of the term structure into the banking problem. This gives the bank the option to value assets and liabilities over a wide range of maturities and choose interest rates on loans and deposits relative to a reference rate (i.e., LIBOR) consistent with the corresponding horizon. The bank is also subject to capital requirements (Basel II) and liquidity constraints (Basel III). Finally, we test the predictions of the theory by using asset and liability data from the Call Reports (for the 500 largest commercial banks), which is divided into different maturity buckets. This allows for a more realistic approach to the portfolio allocation problem, since in this context the bank makes investment decisions relative to the state of the financial markets.

1.4.4 Regime Switching

The U.S. term structure of interest rates has experienced significant changes during the recent financial and economic crisis that has questioned whether standard term structure models are appropriate to study more recent interest rate dynamics. With short-term nominal interest rates essentially lowered to zero ever since December 2008, interest rate volatility has significantly dropped (particularly at the short-end), beginning a new regime of interest rate movements characterized by large asymmetries in the distribution of bond yields. What are the consequences for the economy? What are alternative methods the Federal Reserve Bank must now employ
in order to spur economic growth, reduce unemployment, or avoid deflation? Should the Fed transition back to conventional interest rate policy? More importantly, are term structure models still useful under this environment? Dai, Singleton, and Yang (2007), for example, argue that models that account for regime switches are able to better describe the historical distribution of interest rates than constant volatility models. They develop a discrete time regime-switching model with low and high volatility regimes and show that state-dependent regime-shifts risks are important in capturing variations in expected excess returns.

Christensen (2013) also develops a regime-switching model with a normal state and a special state that is referred to as the zero-bound state. The Treasury yield curve in the normal state is modeled with a standard three-factor term structure model, while the dynamics in the zero-bound state are augmented with a fourth factor that primarily affects the short end of the yield curve. His work reveals that the whole yield curve switches dynamics in the zero-bound state, particularly for short-term yields, and that regime-switching models outperform a standard three-factor term structure model, consistent with Dai, Singleton and Yang (2007).

Although the U.S. lower-bound phenomenon is very recent, studies looking at Japan yields during the liquidity trap have been able to shed light on the term structure dynamics when short interest rates are pegged to a constant rate for extended periods of time. For example, Kim and Singleton (2012) study the case of Japan, given that the country has experienced extended periods of near-zero interest rates. Modeling interest rates close to the zero bound is problematic, so they explore the goodness-of-fit of multi-factor DTSMs that enforce a lower bound on bond yields. Ignoring the zero bound when estimating DTSMs may lead to over-estimation of risk premium volatility as well as implausible Sharpe ratios. Standard Gaussian models, which allow for negative interest rates or non-negative square-root DTSMs have difficulties in replicating the distribution of bond yields when the short-rate is near its
lower bound. They find that shadow rate models and a special case of the quadratic-Gaussian model capture many features of the distribution of Japanese yields at the zero boundary quite well. Their results also suggest that at the zero bound, yield volatilities, bond risk premia, and the level of interest rates are positively related, for short and intermediate maturities.

Christensen and Rudebusch (2013a, 2013b) use an option-based approach to impose a zero lower bound and prevent the model from predicting negative interest rates with a positive probability. They argue that this approach is better than previous attempts to establish a zero bound, given the ability to obtain closed-form solutions and avoid over-identifying restrictions. They also claim to be better suited to fit yields in sample and out of sample. Their work compares a GDTSM to a zero lower bound model (a shadow-rate model) and finds that the fit and forecast performance of GDTSM tends to be worse. Shadow-rate models relate the short rate equation bounded below by zero to a shadow short rate. In such a model, the nominal short rate is a positive process within the shadow rate, which can take upon negative values. Importantly, Bauer and Rudebusch (2014) show that shadow-rate models can capture the substantial distributional asymmetry of future short rates that is due to the zero lower bound that nominal interest rates are subject to.

1.4.5 General Equilibrium Approach

Despite the progress term structure models have made in the past decade, Diebold, Piazzesi, and Rudebusch (2005) argue that the goal of a no-arbitrage macro-finance model specified in terms of underlying preference and technology parameters still remains a major challenge. Term structure models specify the stochastic discount factor as a function of a set of factors and the parameters that determine the price of risk, whereas preference-based models, derive the intertemporal rate of substitution given the consumption patterns of a representative agent. Relating these concepts
so that the model can correctly price assets in a general equilibrium context is an exciting direction towards the integration of macroeconomics and finance within the asset pricing literature. Work on this area has been expanded by Mehra and Prescott (1985), Campbell and Cochrane (1999), Bansal and Yaron (2004), Wachter (2006), and Piazzesi and Schneider (2007), among many others.

More recently, Le and Singleton (2010) derive the marginal rate of substitution equation from a model with Epstein and Zin preferences and a time-varying risk premium through standard log linear approximations. In this context, however, there is no clear intuition for what the risk factors that drive asset prices are. Le, Singleton, and Dai (2010) provide a descriptive summary on the state of the literature, and using historical data on consumption growth, inflation, and yields, estimate a habit-based model by Maximum Likelihood with nonlinear prices of risk. Given that the state vector is assumed to be linear under the risk-neutral measure, they obtain closed-form solutions for bond prices and avoid the need to determine prices numerically from the representative agent’s Euler equation. Along similar lines, Rudebusch and Swanson (2012) compare the implied pricing kernel from a dynamic stochastic general equilibrium model to the pricing kernel implied by an asset-pricing model. They also assume that households exhibit Epstein-Zin preferences, which allows for risk aversion to be modeled independently from the intertemporal elasticity of substitution. They are able to match both the basic macroeconomic and financial moments in the data by varying the parameter that governs the Epstein-Zin degree of risk-aversion.

Integrating term structure models into a preference-based framework is of interest not only to academics seeking to understand the theoretical connections of asset pricing dynamics, but also to monetary policy makers, bond investors, and other financial market participants concerned with fluctuations in asset prices from a general-equilibrium perspective.
2.1 Introduction

What are the factors that drive interest rates, exchange rates, inflation, and growth? Are these factors related to each other? This chapter begins by introducing in Section 2.2 basic mathematical and theoretical concepts that are necessary to understand the derivation of yield curves. Yield curves are constructed from interest rate data and provide a consistent representation of the evolution of yields throughout time. Section 2.3 presents a summary of the most popular method to construct a country’s yield curve, followed by an example in which this method is implemented to construct the Argentine yield curve in Section 2.4. Section 2.5 summarizes the evolution of interest rates throughout time for 20 different countries, as estimated by Bloomberg. A detailed analysis on the Bloomberg data is provided. Finally, Section 2.6 describes how the yield curve data can be used in order to extract interest rate factors and how these factors relate to the main drivers of exchange rates and macroeconomic variables.

2.2 Mathematical and Theoretical Concepts

Analyzing the term structure of interest rates requires the understanding of basic terms in the fixed income literature. Bonds are interest-bearing securities whose future payments are specified by an initial contract between borrowers and lenders. The largest issuer of debt in most countries is the government, issuing bills, notes, and
bonds to be repaid at different time horizons. **Zero coupon bonds**, or Treasury bills, are defined as government debt that repays the bondholders at the end of the bond’s life - or at maturity, a principal payment plus interest. This type of debt is usually up to one year. **Coupon bonds**, on the other hand, not only repay the principal at maturity, but also pay interest throughout their life in the form of coupon payments. In general, most long-term debt, from 1 to 30 years, is in the form of coupon-bearing instruments also known as Treasury notes and bonds. Looking at the yield on these instruments provides a general picture of the evolution of interest rates at various maturities, since the term structure theory allows for a uniform comparison of yields on these different instruments across time.

2.2.1 Zero Coupon Bonds

Zero coupon bonds will be the fundamental block for the construction of a country’s yield curve. For example, a zero coupon bond issued by the Argentine government in February 14, 2012 (at time \( t=0 \)) for a price of 87.86 Argentine pesos that expires a year later (at time \( t=1 \)), will pay the lender the price plus an annual yield, in this case 13.85% on February 13, 2013. The investor receives the yield as compensation for the opportunity cost of holding the bond. The yield on the investment will depend on different factors, the time to maturity being one of them, since the longer the waiting period until payment, the higher the opportunity cost of the investor, else being constant.

Let \( \delta_t(n) \) be the price of a zero coupon bond expressed in continuous compounding, also called the **discount function**, for any bond maturing \( n \) years from now and paying an annual interest, \( y_t(n) \), such that

\[
\delta_t(n) = e^{-y_t(n)n}.
\]  

(2.1)

\( \delta_t(n) \) is also the present value of the investment: the discounted value of the future amount to be obtained at time \( n \), which is normalized to 1 in this case. The yield
and the price of a bond are inversely related. This relationship is summarized by equation (2.1), which implicitly assumes the absence of arbitrage opportunities, as it is standard in the literature. Normally, if a bond were to sell at the same yield throughout its life, the price would rise steadily towards its face value as the bond approached maturity. A bond’s yield, however, is unlikely to be constant over time. The time-varying yield on a zero coupon bond paying \( n \) years from now is also called \textbf{spot rate} or zero coupon yield. Solving for \( y_t(n) \) in equation (2.1), we obtain

\[
y_t(n) = -\frac{\ln(\delta_t(n))}{n}.
\]  
(2.2)

2.2.2 Coupon Bonds

Treasury notes and bonds pay the instrument-holder interest equal to the face value times the interest (coupon) rate at which they are issued. A holder of a coupon bond receives a set of cashflow payments, \( c_t \), at the end of each period \( t \). Let \( r \) denote the given interest rate, so that the bond’s price is the present value of the future coupon payments plus the present value of the face value, in this case 1:

\[
p_t(n) = \sum_{t=1}^{n} \frac{c_t}{(1+r)^t}.
\]  
(2.3)

The actual prices paid in the new issue and secondary markets, \( p_t(n) \), are always the quoted price in dollars, \( p_t^{\text{clean}}(n) \), plus any accrued interest, \( a_t \):

\[
p_t(n) = p_t^{\text{clean}}(n) + a_t
\]  
(2.4)

The quoted price of a bond is also called the \textbf{clean price}, whereas the actual price of the transaction is called the \textbf{dirty price}. Although the clean price is most often quoted in the market, we are interested in the dirty price since it is the actual amount paid when buying the bond.

The structure of the cashflow payments allows us to think of coupon-bearing bonds as baskets of zero coupon bonds, which will prove extremely helpful in the construction of a zero coupon yield curve. The bond-pricing equation (2.3) can also
be expressed in terms of the discount function, $\delta_t(n)$, so that the internal rate of return of the cashflows is substituted by the corresponding discount factors:

$$p_t(n) = \sum_{i=1}^{n} c_t \cdot \delta_t(n).$$  \hspace{1cm} (2.5)

Substituting equation (2.1) into (2.5), allows us to express the price of a coupon bearing instrument in terms of spot rates, where

$$p_t(n) = \sum_{i=1}^{n} c_t \cdot e^{-y_t(n)n}. \hspace{1cm} (2.6)$$

The **yield to maturity** is the single discount rate that equates the present value of the bond’s remaining cashflows to its current price. If the yield to maturity is the same as the coupon rate, then the bond trades at *par*. When the yield to maturity is greater than the coupon rate, the bond sells at a *discount*. Zero coupon bonds, by definition, are discount bonds. Finally, if the yield to maturity is smaller than the coupon rate, the bond trades at a *premium*. The ability to express coupon bonds in terms of zero coupon rates allows for the integration of different instruments in the estimation of a yield curve at various horizons.

### 2.2.3 Bond Price Sensitivity

In simple terms, the price elasticity of demand for bonds, $\varepsilon_p^D$, is given by

$$\varepsilon_p^D = -\frac{\%\Delta Q^D(n)}{\%\Delta p(n)},$$  \hspace{1cm} (2.7)

where the percentage change in the quantity demanded of bonds maturing $n$ years from now is given by $\%\Delta Q^D(n)$, and the percentage change in the price of the same bonds is given by $\%\Delta p(n)$. A bond’s **maturity** identifies how much time elapses until the bond’s final payment. However, this concept ignores all relevant information about the timing and magnitude of interim payments. **Duration** is similar to maturity; however, it uses a weighted average of the present value of the cashflows to measure the effective maturity of a security. By definition, the duration
of a zero coupon bond is equal to its maturity. Let duration, \( d \), measure how price-sensitive a security is to changes in interest rates. Simplifying notation and letting \( r \) be the interest rate,

\[
d \approx -\frac{\Delta p/p}{\Delta r/(1 + r)}, \tag{2.8}
\]

which can also be expressed in terms of price changes:

\[
\Delta p \approx -d \frac{\Delta r}{(1 + r)} p. \tag{2.9}
\]

Since duration, also known as Macaulay’s duration after its author, is a weighted average of the time until the expected cashflows from a security will be received, relative to the security’s price, we can express duration in terms of cashflow payments,

\[
d = \frac{\sum_{t=1}^{n} c_t (1 + r)^t}{\sum_{t=1}^{n} p_t (1 + r)^t}. \tag{2.10}
\]

Substituting equation (2.3) into equation (2.10),

\[
d = \frac{\sum_{t=1}^{n} c_t (t)(1 + r)^t}{p_t (n)}. \tag{2.11}
\]

From equation (2.11) it is clear that the duration of a bond depends on three variables: its current maturity, \( n \); its coupon payments, \( c_t \); and the interest rate, \( r \), at which future cashflows are discounted. Intuitively, all else constant, the duration of a security will be higher the longer its current maturity, the lower its coupon, and the lower the yield at which it currently trades.

Duration can also be interpreted as an approximate measure of the price elasticity of demand, so that the greater (shorter) the duration, the greater (lesser) the price sensitivity to changes in interest rates. Therefore, duration is a useful measure of price volatility. Let \( d’ \) denote modified duration

\[
d’ \approx \frac{d}{1 + r} \tag{2.12}
\]

and substitute equation (2.12) into equation (2.9) to obtain an estimate of price volatility in terms of modified duration,
\[ \% \Delta p = \frac{\Delta p}{p} \approx -d' \Delta r. \] (2.13)

Given the analysis on the elasticity of bonds, the following conclusions on price sensitivity are worth noting:

1. **Bond prices change asymmetrically to rising and falling rates.** For a specific absolute change in interest rates, the proportionate increase in price when rates fall exceeds the proportionate decrease in price when rates rise. This asymmetry is due to the convex shape of the curve when looking at the relationship between price and interest rates. When interest rates go down, there will be greater capital gains for the investor and when interest rates increase the investor will experience smaller capital losses.

2. **Maturity influences bond price sensitivity.** For bonds that pay the same coupon rate, long term bonds change proportionately more in price than short term bonds for a given change in interest rates. Investors will experience greater capital gains and losses in the long run when interest rates change. Recall that the price risk to which bonds expose the investor is larger the longer their current maturity. Thus, one would expect the yield curve to slope upwards over the full maturity spectrum.

3. **The size of the coupon influences bond price sensitivity.** For bonds that have the same maturity, low coupon bonds change proportionately more in price than high coupon bonds for a given change in interest rates. Therefore low coupon bonds will exhibit a higher price volatility relative to high coupon bonds.\(^1\)

\(^1\)For a more detailed discussion on bond price sensitivity refer to Koch and MacDonald (2010).
2.2.4 Floating Coupon Bonds

So far, the coupon payments on interest-bearing bonds have been certain to the investor. This is not the case for all bonds, since a security can pay interest tied to a current money market, making the future coupon payments unknown. This type of bonds with cashflow schedules that depend on another variable rate are also known as floating coupon bonds. The coupon payments earned on these investments, $\tilde{c}_t$, are the sum of a market index, $\text{Index}_t$, plus a spread, $\bar{c}$, which is usually fixed for the life of the investment, so that, $\tilde{p}_t(n)$, the price of a floating coupon bond maturing $n$ years from now is

$$
\tilde{p}_t(n) = E_0 \sum_{t=1}^{n} \frac{\tilde{c}_t}{(1 + r)_t},
$$

where

$$
\tilde{c}_t = \text{Index}_t + \bar{c}.
$$

Similarly to fixed coupon bonds, the price of a floating coupon bond will depend on the time to maturity and the coupon rate. In addition, it will also depend on the underlying benchmark index used to pay the investor. In the U.S., common market indexes include the Daily Federal Funds Effective Rate, and the one-month, three-month, and six-month London Interbank Offered Rate. In Argentina, for example, floating coupon bonds are tied to the Buenos Aires Deposits of Large Amount Rate, the average interest rate paid by private and/or public banks for deposits exceeding one million pesos or dollars.

Characteristics of floating coupon bonds:

- Floating coupon bonds tend to exhibit lower volatility relative to fixed-coupon bonds. Because the interest rate is not fixed and moves with changes in the level of market rates, the fair market value of floating bonds typically exhibits minimal price sensitivity to changes in interest rate levels.
• Another advantage of floating rate bonds, compared to traditional bonds, is lower interest rate risk. While an owner of a fixed-rate bond cannot take advantage of an increase in the prevailing interest rates, floating rate bonds will pay higher yields if rates go up. As a result, they will tend to perform better than traditional bonds when interest rates are rising. However, term-floaters adjust with the market, so if interest rates start to fall, the yield will decline.

• One of the main disadvantages of holding a floating coupon bond is uncertainty as to the future income streams from the investment. In contrast, an owner of a fixed-rate security knows exactly the amount to be received throughout the bond’s life. Holders of floating coupon bonds have to therefore speculate on the volatility of the benchmark index as well as the overall changes in the interest rate levels. Consequently, the price on this type of bonds might contain additional information on the investors’ perceptions about the future of the market.

2.2.5 Forward Rates

Forward rates are also useful when looking at the term structure of interest rates. A forward agreement establishes the price to pay for an asset at maturity. Hence a one-year forward rate, for example, is the interest contracted now to be paid for a year later. An $n \times n^*$-year forward rate is a contract for any asset where the buyer and seller agree on the asset’s price at time $n$, but defer the actual exchange until a specified future date, $n + n^*$. This implies that the investor pays $d_t(n + n^*)/d_t(n)$ at time $n$, to receive $1$ $n^*$ years later. Given a set of spot rates, we can derive $f_t(n, n^*)$, the forward rate of a future investment between time $n$ and $n + n^*$, such that

$$f_t(n, n^*) = -\frac{1}{n^*} \left\{ln \left[ \frac{d_t(n + n^*)}{d_t(n)} \right] \right\}. \tag{2.16}$$

From the definition of a forward rate agreement and equation (2.2), it follows that
\[ f_t(n, n^*) = -\frac{1}{n^*} \{ \ln e^{y_t(n+n^*)(n+n^*)} - \ln e^{y_t(n)n} \}, \] (2.17)

so that the forward rate at time \( n \) to be paid for at time \( n + n^* \) is

\[ f_t(n, n^*) = \frac{y_t(n + n^*)(n + n^*) - y_t(n)n}{n^*}. \] (2.18)

The **instantaneous forward rate** is the rate of interest that would apply on an agreement made at time zero to borrow at time \( t \) in the future for a very short amount of time, and can be derived by taking the limit of the forward rate from equation (2.18),

\[
f_t(n, 0) = \lim_{n^* \to 0} f_t(n, n^*) \\
= -\frac{\partial}{\partial n} \ln \left[ d_t(n) \right]. \] (2.19)

From equation (2.19), it follows that

\[
- \int_{x=0}^{n} f_t(x, 0)dx = - \int_{x=0}^{n} -\frac{\partial}{\partial x} \ln \left[ d_t(x) \right] \\
= \ln \left[ d_t(n) \right],
\] (2.20)

and therefore, the spot rate is hence identical to the average of the instantaneous forward rates:

\[
\frac{1}{n} \int_{x=0}^{n} f_t(x, 0)dx = -\frac{1}{n} \ln \left[ d_t(n) \right] \\
= y_t(n).
\] (2.21)

This relationship implies that the forward rate curve can be interpreted as indicating the expected future time path of these variables, whereas the instantaneous forward rate curve would represent the expected time path of future overnight rates.\(^2\)

The term structure defines the relationship between short-term and long-term interest rates and it is represented by the **yield curve**, which shows the spot rates

\(^2\)Refer to Svensson (1994) for more details.
for different maturities. Since forward rates, spot rates, and coupon bonds are all equivalent term structure descriptions, the spot rates and instantaneous forward rates will immediately follow from the set of estimated forward rates.

2.3 Estimation of Yield Curves

Ideally, we would like to have as many bonds as maturities we are interested in covering. However, the sample of homogeneous bonds to fit the yield curve is limited, leaving gaps over large periods of the maturity spectrum. Interpolation methods allow for a theoretical representation of the yield curve at all maturities implied from existing bonds. Yield curves, hence, are not uniquely defined by a group of market bonds. Once we have a consistent set of zero coupon yields, the methodology chosen to derive a continuous term structure depends on the specific objective of the analysis. Even though the overall goal of the estimation is to minimize pricing error, different techniques will yield different results in terms of smoothness and fit. For example, an investor studying the term structure of a country to identify any pricing anomaly will be mostly concerned with the fitness of the yield curve. In this case, McCulloch’s original cubic spline method might be more appropriate.\footnote{See McCulloch (1971) for more details on the cubic splines technique to fit the yield curve.} For monetary policy analysis, however, the need for precision is arguably less if one is concerned with understanding the macroeconomic fundamentals of the yield curve movements. An approach based on the Nelson and Siegel (1987) framework is therefore preferred when the primary goal of the estimation is to provide sufficiently smooth yield curves while ignoring variations from anomalous bond prices.

The Nelson-Siegel approach is an optimal strategy for the estimation of a yield curve for a number of reasons. First, the main objective in macroeconomics is to provide a general understanding of the evolution of interest rates. The Nelson-Siegel

\footnote{See McCulloch (1971) for more details on the cubic splines technique to fit the yield curve.}
estimation smoothes out pricing anomalies that are often present in the term structure. Second, the Nelson-Siegel framework seems appropriate when the number of observed bond prices is limited. The model is parsimonious in the number of parameters but allows for sufficiently rich shapes to match the movements of the yield curve. Finally, the estimation method is simple and the results are robust, making it very popular among central banks and other practitioners.

2.3.1 Nelson-Siegel Framework

Nelson and Siegel (1987) proposed a four-parameter parsimonious function for modeling the instantaneous forward rate, later extended by Svensson (1994), Diebold and Li (2006), and De Pooter (2007), among others. Despite the large number of options available, the Nelson-Siegel model and its extensions remain popular among practitioners due to their ability to produce a wide range of yield curve shapes.

The basic Nelson-Siegel function for modeling the instantaneous forward rate at a point in time \( n \) years ahead is

\[
\begin{align*}
    f_t(n,0) &= \beta_0 + \beta_1 \exp\left(-\frac{n}{\tau}\right) + \beta_2 \left[\left(\frac{n}{\tau}\right) \exp\left(-\frac{n}{\tau}\right)\right], \\
    \beta &= (\beta_0, \beta_1, \beta_2, \tau)
\end{align*}
\]

with the parameter vector \( \beta = (\beta_0, \beta_1, \beta_2, \tau) \) dependent on time.\(^4\) As expressed in equation (2.21), the spot rate can be expressed as the average of the instantaneous forward rates, hence, we can derive the spot rate function by integrating equation (2.22), such that

\[
\begin{align*}
    \frac{1}{n} \int_{x=0}^{n} f_t(x,0)dx &= y_t(n,\beta) \\
    y_t(n,\beta) &= \beta_0 + \beta_1 \left[1 - \exp\left(-\frac{n}{\tau}\right)\right] + \beta_2 \left[\frac{1}{n} \exp\left(-\frac{n}{\tau}\right) - \exp\left(-\frac{n}{\tau}\right)\right].
\end{align*}
\]

\(^4\)According to Bank for International Settlements (2005), Nelson-Siegel and its extensions are the most commonly adopted methodologies for fitting the yield curves.

\(^5\)The forward curve is assumed to be the solution to a second order differential equation with equal roots for spot rates (see Nelson and Siegel, 1987).
Hence, equation (2.24) gives us the corresponding zero coupon yields, which can be used to compute the discount function at any horizon.

Another attractive feature of this estimation strategy is the direct interpretation of the parameters. By taking the limit of equation (2.24) as $n$ approaches infinity,

$$\lim_{n \to \infty} y_t(n, \beta) = \beta_0, \quad (2.25)$$

the spot rate function converges to $\beta_0$, so that $\beta_0$ can be interpreted as the long-term interest rate. Under the assumption of positive interest rates, $\beta_0 > 0$. Looking at the spot rate function as $n$ goes to zero,

$$\lim_{n \to 0} y_t(n, \beta) = \beta_0 + \beta_1, \quad (2.26)$$

the function converges to $\beta_0 + \beta_1$, which can be seen as an approximation to the instantaneous short rate. Similarly, $\beta_0 + \beta_1 > 0$ under the assumption of positive interest rates throughout time. $\beta_2$ determines the shape of the interest rate curve, so that if $\beta_2 > 0$ interest rates at different maturities exhibit a concave shape and if $\beta_2 < 0$, interest rates are represented in a convex curve. Finally, the parameter $\tau$ specifies at which maturity $\beta_2$ achieves its maximum as well as governs the exponential decay rate. In particular, small values of $\tau$ produce slow decay and can better fit the curve at long maturities, while large values of $\tau$ produce fast decay and can better fit the curve at short maturities.\(^6\)

The discount function is estimated for each trade date by minimizing either the (weighted) sum of squared price errors or the sum of squared yield errors.\(^7\) Both approaches typically yield very similar parameter specifications. To obtain the estimated parameter vector of the spot rate function, let $y_t^M(n)$ be the actual market yields of the observed securities $n$ years from now, such that the objective function is to minimize the sum of the squared zero yield errors

---

\(^6\)Refer to Diebold and Li (2006) for more details on the economic interpretation of parameters in the Nelson-Siegel forward rate equation.

\(^7\)For a discussion on the advantages and disadvantages of each objective function refer to Svensson (1995).
\begin{equation}
\min_{\beta} \left\{ F(\beta) = \left[ y_t^M(n) - y_t(n, \beta) \right]^2 \right\}
\end{equation}

subject to the following parameter constraints:

\begin{align*}
\beta_0 &> 0, \\
\beta_0 + \beta_1 &> 0, \\
\tau &> 0.
\end{align*}

These restrictions are standard in the literature and consistent with the economic interpretation of the parameters for positive interest rates.

One of the main issues present in the Nelson-Siegel approach is that different combinations of parameter values can produce an equally good fit to the observed data. Therefore, if we care about the evolution of the parameter vector (given the importance of its economic interpretation), abrupt changes in their values become problematic. To avoid abrupt changes in the value of the parameters from one day to another, we can set upper \((u)\) and lower \((l)\) bounds on \(\tau\), such that \(0 < l \leq \tau \leq u\). These restrictions lead to smooth parameter series without catastrophic jumps in the parameters’ evolution throughout time.

2.4 Example: Argentina’s Yield Curve (2003 to 2012)

In this section, I construct weekly zero coupon and forward yield curves for Argentina from 2003 to 2012 using the Nelson-Siegel approach from Section 2.3. This case is of particular interest since the economy has been on the path to recovery from its 2001 financial default, and there are no high-frequency estimates of the Argentine yield curve available to researchers. I overcome issues of illiquidity and instrument heterogeneity by implementing the Nelson and Siegel (1987) methodology introduced

\footnote{Refer to Cairns and Pritchard (2001) for a discussion on catastrophic jumps.}
in the previous section, which is the preferred approach for most central banks, including the U.S. Federal Reserve (Gurkaynak, Sack, and Wright; 2007). Having a consistent set of interest rate data allows economists to document stylized facts on the Argentine yield curve and evaluate the health of its financial markets.

2.4.1 Data and Estimation

I construct weekly yield curves for the Argentine peso-denominated debt from 2003 to 2012 for a total of 521 weeks. The data are obtained from Bloomberg, L. P. 2012, the Ministry of Economy and Public Finance, and the Central Bank of Argentina. The reason I do not observe the term structure of Argentina previous to 2003 is that the country defaulted on $81 billion of debt in 2001. Since then, Argentina has restructured its debt in 2005 and 2010 and offered creditors new bonds in exchange for the non-performing securities. Therefore, through 2003, in the face of increased political uncertainty and weak fiscal data, the short term spreads are extremely high, causing the term structure to be steeply inverted and bond yields quite unstable. As expected, the estimates of the yield curve during early 2003 are more volatile.

Despite the increasing importance of emerging markets’ financial instruments, the number of Argentine securities is much smaller than U.S. Treasuries and even worse, they must be separated into different groups. One of the main conditions to meet when considering instruments for its estimation is homogeneity in the quality of bonds; that is, bonds should only differ in their maturities and coupon payments. However, Argentine securities differ in several aspects. First of all, not all debt is issued in the same currency, which implies different term structures, since each group

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10Not all creditors have accepted the restructured bonds offered by Argentina and are currently involved in law-suits against the government.
represents different levels of risk and cannot be considered into one single curve. Even bonds issued in the same currency may differ in their levels of default risk or the market of emission. Finally, unlike U.S. securities, most bonds in Argentina have a variable or floating coupon; they are tied to an index such as the inflation rate, GDP rate, or BADLAR, making the future coupon payments uncertain to the investor. These characteristics make the Argentine term structure fluctuate more than the U.S. yield curve from week to week. These concerns limit the number of bonds available for the estimation. Ideally, we would want to have the largest number of bonds possible each day. However, the Nelson-Siegel framework works well with the Argentine bond market, given that the number of observed bonds in a single day is greater than the number of parameters to be estimated. This is a significant advantage of such a parsimonious approach.

The dataset for this estimation includes the following zero-coupon bonds and long term coupon bonds issued by the government:

- The zero-coupon bond dataset includes Central Bank Bills (LEBACS) and Central Bank Notes (NOBACS), which are similar to U.S. Treasury bills and notes, respectively. LEBACS are zero coupon bonds usually issued with maturity up to 2 years. NOBACS are floating coupon bonds with longer maturities, tied to the BADLAR index (the average interest rate paid by private and/or public banks for deposits exceeding one million).

- Long-term debt is issued in terms of coupon-bearing securities. For the estimation, I exclude from the dataset defaulted bonds as well as securities not priced by Bloomberg (indicating lack of liquidity). All the coupon bonds considered are issued in Argentine pesos, in the domestic market, and under Argentine legislation. They are non-callable or bullet (callable bonds usually trade at substantially lower prices than similar non-callable bonds) and directly unre-
lated to the inflation rate\footnote{The inflation rate in Argentina is suspected of manipulation from the authorities. Refer to the Appendix for more information on inflation in Argentina.}

For the estimations I follow Ferstl and Hayden (2010a, 2010b), which allow for variable coupon payments and the possibility to specify the step size of the search grid as well as values for the $\tau$ constraints.\footnote{I constrain $\tau$ to be between 0.1 and 2.6, which allows for a smooth evolution of the beta parameters.} The variability of the estimated parameters is mitigated under this procedure, allowing for the economic interpretation of the parameters to hold, even in such unstable environment.

The yield curve is estimated by using a minimum of four different securities on a given day. For each week, I remove bonds that were extremely disengaged from similar yields. This unstable behavior of bond yields is not uncommon for the period considered; however, this is usually temporary and they tend to revert to the yield curve over time. There are 8 days within the dataset in which the yield curve cannot be inferred given the low number of observations. To avoid the interruption of the data series, I replace the missing observations with the values from the previous week\footnote{Days with low observed bond yields that need to be replaced with the parameters for the previous day are: May 25, 2004; June 1, 2004; November 28, 2006; July 31, 2007; August 21, 2007; August 28, 2007; May 27, 2008; and December 23, 2008.}

2.4.2 Results

The term structure under the Nelson-Siegel methodology is governed by the estimated parameters from equation \eqref{eq:2.24}. $\beta_0$ and $\beta_1$ share a similar evolution throughout time, with the highest values being in 2004-2005 and 2009. $\beta_2$ exhibits a more variable evolution, although centered around zero, whereas the value for $\tau$ fluctuates between 0.2 and 2.6, given the pre-imposed constraints on the objective function. $\beta_0$
shares a correlation of -0.42 with $\beta_1$ and -0.66 with $\beta_2$, whereas $\beta_1$ and $\beta_2$ exhibit a correlation of -0.29.

Given the estimated parameters and the relationship between zero coupon and forward rates, we can calculate spot rates, one-year forward rates, and instantaneous rates for each day in the dataset. An example of the three different yield curves for an arbitrary day, July 17, 2012, is shown in Figure 2.1.

In the context of the Nelson-Siegel model, the estimates of $\beta_0 + \beta_1$ can be interpreted as the short-term (3-month) spot rate, whereas the evolution of $\beta_0$ can be interpreted as the long-term (5-year) rate. In Figure 2.2 (a), we can observe that on average the parameters follow the evolution of the short term rate, although not during 2003, which is not surprising giving the instability of the domestic financial market after the default. The parametrization of the long-term rate from Figure 2.2 (b) exhibits a better fit during the more recent years as well (2006 to 2012).

The fit of the zero yield curve for October 20, 2009 is shown in Figure 2.3.
zero coupon yields are estimated from the set of coupon bonds available for that particular day, the Nelson-Siegel methodology allows for the interpolation of the yield curve for a continuous representation of the term structure, even for the maturities that were not available at that moment.

Figure 2.4 provides a three-dimensional representation of the yield curve. Estimates for the yield curve are reported at horizons that go as far out as possible without extrapolating far beyond the range of maturities that are actually outstanding on each date, as recommended by Gurkaynak et al. (2007). In general, the term structure slopes upwards, although during 2003 and 2009 the yield curve reverts and slopes downwards, followed by periods of particularly high spot rates, typical of times under greater uncertainty.

This is not surprising given the uncertainty in Argentina’s ability to meet its
Figure 2.3. Fit of the Argentine Yield Curve (October 20, 2009)

Figure 2.4. Three-Dimensional Argentine Yield Curve at Different Horizons (2003-2012)
payments after the 2001 default and the enhanced exchange rate risk after the de-
preciation of the Argentine peso. In 2003, the yield curve exhibits reversion at the
long end, indicating investors’ expectations of high near-term risk of default and fi-
nancial difficulties, reflected by very high short-term rates. In late 2003, for example,
the curve starts sloping upwards after exhibiting long-term reversion earlier in the
sample. The short-term spot rates are quite high, but the fit of the curve is rela-
tively good despite the low number of observations. In 2009, the spot rates reach
their maximum yield per maturity. This is not uncommon given the country’s sus-
ceptibility to international financial crises and changes in investors’ behavior towards
risk. According to Dabos and Bugallo (2000), the Argentine term structure is very
vulnerable to international crises, but tends to revert back to normal levels relatively
quickly. My results during this period, although with different methodologies, sup-
port their findings. We can also observe how the 2003 variability in the yield curve
affects mostly short-term debt, whereas the 2009 financial crisis mostly affects the
long-term rates. For example, for the 3-month, 6-month, and 1-year rates, the largest
deviation from the mean happens in early 2003, when the spot rates are more than
twice the long-run average. However, for higher-maturity rates, the largest deviation
from the mean happens in 2009. Finally, for the latest period in my sample, 2012,
the yield curve has been quite stable and very flat, around 15%. Table 2.1 shows the
correlation of yields with different maturities during the entire sample.

The correlations are not consistently positive, with magnitudes that vary substan-
tially across different horizons. In particular, bonds with shorter term to maturity
tend to be more correlated than bonds at the long end of the curve. Moreover, bonds
with nearby maturities exhibit a higher correlation. While the 9-month and 1-year
rate exhibit a 0.97 correlation, the 2-year and 5-year rates have a correlation of 0.24.

To assess the goodness of fit it is important to measure the pricing and the yield
ingrors. Figure 2.5 illustrates the day by day evolution of the root mean squared
errors (RMSE) and the average absolute mean deviation (AABSE). A value of zero indicates a perfect fit, and the value increases as the fit worsens.

**TABLE 2.1**

**CORRELATION OF ARGENTINE YIELDS AT DIFFERENT MATURITIES**

<table>
<thead>
<tr>
<th>3m</th>
<th>6m</th>
<th>9m</th>
<th>1y</th>
<th>2y</th>
<th>3y</th>
<th>5y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.74</td>
<td>0.47</td>
<td>0.29</td>
<td>-0.05</td>
<td>-0.15</td>
<td>-0.13</td>
</tr>
<tr>
<td>1</td>
<td>0.93</td>
<td>0.82</td>
<td>0.42</td>
<td>0.09</td>
<td>-0.29</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.97</td>
<td>0.67</td>
<td>0.30</td>
<td>-0.26</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.81</td>
<td>0.46</td>
<td>-0.18</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.87</td>
<td>0.24</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.67</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The poorest fit is following the financial global crisis in early 2009, and the errors on the fitting of yields is exacerbated in the beginning of the sample. Table 2.2 reports the average and the maximum error for the entire sample as well as period by period. For example, we can observe that the average RMSE of the prices is 1.17 during 2010, whereas the average RMSE of the prices for the entire sample is 0.67.

It is important to compare the estimated spot rates obtained with the Nelson-Siegel model with an interest rate benchmark, such as the Buenos Aires Interbank
Figure 2.5. Fitting Errors of the Argentine Yield Curve (2003-2012)

Note: RMSE (root mean square errors) and AABSE (average absolute mean errors) for prices and yields constructed for Argentina using the Nelson-Siegel methodology. A value of zero indicates a perfect fit.
Offer Rate (BAIBOR)\textsuperscript{14} According to a Central Bank report, the BAIBOR is a useful benchmark rate for the Argentine financial system.\textsuperscript{15} Figure 2.6 shows the evolution of the estimated spot rates and the BAIBOR of the same maturity. Although the worst fit is for the 3-month yield, in general interest rates estimated with the Nelson-Siegel methodology exhibit similar patterns throughout time.

![Figure 2.6. Estimated Argentine Spot Rates and BAIBOR at Different Maturities](image)

(a) 3-month  
(b) 6-month  
(c) 1-year  
(d) 1.5-year

Note: The dashed line represents the Buenos Aires Interbank Offered Rate (BAIBOR) and the solid line represents the estimated spot rates constructed for Argentina using the Nelson-Siegel methodology.

\textsuperscript{14}The BAIBOR provides aggregate daily information on the average of fixed interest rates (for the total duration of the transaction) offered for loans granted to financial institutions within the country. Based on the British Bankers Association’s procedure to determine the LIBOR, it displays the simple average for non-extreme observations (20\% at each extreme for maturities up to 180 days, and 10\% at each extreme for maturities over 180 days). For more information refer to http://www.bcra.gov.ar.

\textsuperscript{15}http://www.bcra.gov.ar/pdfs/estadistica/bmf97_4i.pdf.
TABLE 2.2

AVERAGE AND MAXIMUM RMSE AND AABSE FOR ARGENTINE YIELDS AND PRICES

<table>
<thead>
<tr>
<th>year</th>
<th>RMSE Prices</th>
<th>AABSE Prices</th>
<th>RMSE Yields</th>
<th>AABSE Yields</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>average</td>
<td>max</td>
<td>average</td>
<td>max</td>
</tr>
<tr>
<td>2003</td>
<td>0.56</td>
<td>3.75</td>
<td>0.45</td>
<td>2.85</td>
</tr>
<tr>
<td>2004</td>
<td>0.51</td>
<td>2.76</td>
<td>0.43</td>
<td>2.15</td>
</tr>
<tr>
<td>2005</td>
<td>0.27</td>
<td>1.08</td>
<td>0.20</td>
<td>0.81</td>
</tr>
<tr>
<td>2006</td>
<td>0.99</td>
<td>2.91</td>
<td>0.75</td>
<td>2.28</td>
</tr>
<tr>
<td>2007</td>
<td>0.24</td>
<td>1.60</td>
<td>0.18</td>
<td>1.13</td>
</tr>
<tr>
<td>2008</td>
<td>1.10</td>
<td>6.76</td>
<td>0.83</td>
<td>5.40</td>
</tr>
<tr>
<td>2009</td>
<td>0.70</td>
<td>5.39</td>
<td>0.48</td>
<td>3.68</td>
</tr>
<tr>
<td>2010</td>
<td>1.17</td>
<td>2.29</td>
<td>0.79</td>
<td>1.57</td>
</tr>
<tr>
<td>2011</td>
<td>0.47</td>
<td>1.34</td>
<td>0.34</td>
<td>0.87</td>
</tr>
<tr>
<td>2012</td>
<td>0.35</td>
<td>1.09</td>
<td>0.24</td>
<td>0.62</td>
</tr>
<tr>
<td>2003-2012</td>
<td>0.64</td>
<td>6.76</td>
<td>0.47</td>
<td>5.40</td>
</tr>
</tbody>
</table>

Note: RMSE (root mean square errors) and AABSE (average absolute mean errors) for prices and yields constructed for Argentina using the Nelson-Siegel methodology. A value of zero indicates a perfect fit.
2.5 Evolution of Yields for 20 Countries (1999-2013)

This section analyzes the evolution of yields for major countries in the world. Monthly interest rate data from January 1999 to January 2014 are obtained from Bloomberg, L.P. 2014, for a total of 181 observations. The dataset contains interest rates at 12 different horizons: 3 months, 6 months, and 1 to 10 years for 20 countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Hong Kong, Ireland, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, and United States. Yields are end-of-month data expressed in the local currency.

Figures 2.7 to 2.9 show the evolution of interest rates across different maturities for each country. Interest rates in the U.S. increased from April 2004 to around 5% at the beginning of 2007. Starting September 2008, they dropped below 1% and have remained close to the zero bound ever since. This pattern is observed in most other countries as well. The U.S. one-year interest rate is 2.4% on average, ranging from 0.1% to 6.3% during the sample period. The longest maturity in the sample is the 10-year rate, which is around 4.5% for most countries. For example, the median long-term interest rate is 4.4% for the U.S. and 4.6% for the U.K. Australia’s rates tend to be higher on average than most of the countries. Its 3-month rate is on average 5% and the 10-year rate is around 6%. Japan has the lowest interest rates across all maturities: its average 3-month rate is 0.17% and its average 10-year rate is 1.43%. It also exhibits very low variability; for example, the variation in the one-year rate ranges from 0.01% to 0.22% in the entire sample.

The European countries have similar-shaped yield curves - except for Portugal, Ireland, Italy, and Spain. Portugal and Ireland exhibit the highest volatility for one-year rates during the European crisis, ranging from 0.8% to 17% and from 0.2% to 12%, respectively. Their 5-year interest rate reaches very high values during that period as well; 23% and 13%, respectively. Similarly, Spain and Italy have higher-
than-average rates during the last years in the sample, reaching up to 7%. Belgium is the only other country that is close to exhibit this type of behavior. Norway’s yields seem consistently higher than the rest of the European countries’ throughout all years, and Norway is also the only country to experience yield curve inversion in 1999.

Summary statistics for one-year interest rates are in Table 2.3. In general, the well-established stylized facts on U.S. yield curves also apply for the rest of the countries in the sample.  

1. In general, the yield curve for every country is upward sloping. Low-maturity yields are typically lower than high-maturity yields, except for times of crisis in which the yield curve flattens and even tends to revert. The 2001 and 2009 crises are reflected in every country by higher-than-average short term rates; the magnitudes depending on each country’s vulnerability to financial contagion, among other factors. During 2008 for example, Austria’s, Finland’s, Switzerland’s, and Norway’s short-term rates not only increased significantly, but the yield curves even inverted, being short-term rates higher than long-term yields for subsequent months. The European crisis is reflected in 2011 and the beginning of 2012 by higher short-term rates than usual in all European countries. Australia is the only non-European country in the sample that seems to have been vulnerable to the European crisis. Portugal and Ireland experience very high interest rates during the European crisis, in which mid-term rates reach 20% and 13%, respectively, so that the yield curve exhibits upward-sloping short-term maturities and inverts in long-term maturities at the same time, creating a pronounced humped shape.

---

16 For analysis on stylized facts on U.S. yield curves refer to Piazzesi and Schneider (2007).
### TABLE 2.3

**SUMMARY STATISTICS OF ONE-YEAR YIELDS FOR 20 COUNTRIES**

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>median</th>
<th>min.</th>
<th>max.</th>
<th>std.dev.</th>
<th>skewness</th>
<th>kurtosis</th>
<th>$\hat{\rho} (12)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>4.82</td>
<td>4.90</td>
<td>2.28</td>
<td>7.07</td>
<td>1.21</td>
<td>-0.42</td>
<td>2.48</td>
<td>0.35</td>
</tr>
<tr>
<td>Austria</td>
<td>2.52</td>
<td>2.41</td>
<td>0.04</td>
<td>5.32</td>
<td>1.54</td>
<td>-0.05</td>
<td>1.83</td>
<td>0.63</td>
</tr>
<tr>
<td>Belgium</td>
<td>2.43</td>
<td>2.31</td>
<td>0.07</td>
<td>5.27</td>
<td>1.46</td>
<td>-0.04</td>
<td>1.89</td>
<td>0.59</td>
</tr>
<tr>
<td>Canada</td>
<td>2.85</td>
<td>2.82</td>
<td>0.49</td>
<td>6.34</td>
<td>1.64</td>
<td>0.30</td>
<td>1.99</td>
<td>0.66</td>
</tr>
<tr>
<td>Denmark</td>
<td>2.64</td>
<td>2.56</td>
<td>0.01</td>
<td>5.94</td>
<td>1.65</td>
<td>-0.06</td>
<td>1.92</td>
<td>0.68</td>
</tr>
<tr>
<td>Finland</td>
<td>2.44</td>
<td>2.42</td>
<td>0.01</td>
<td>5.38</td>
<td>1.53</td>
<td>-0.10</td>
<td>1.90</td>
<td>0.65</td>
</tr>
<tr>
<td>France</td>
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<td>2.28</td>
<td>0.02</td>
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<td>-0.02</td>
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<td>2.29</td>
<td>0.01</td>
<td>5.16</td>
<td>1.54</td>
<td>-0.08</td>
<td>1.76</td>
<td>0.66</td>
</tr>
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<td>Hong Kong</td>
<td>2.17</td>
<td>1.38</td>
<td>0.06</td>
<td>6.91</td>
<td>2.09</td>
<td>0.77</td>
<td>2.25</td>
<td>0.64</td>
</tr>
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<td>Ireland</td>
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<td>3.19</td>
<td>0.18</td>
<td>12.08</td>
<td>1.74</td>
<td>1.53</td>
<td>8.02</td>
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<td>2.75</td>
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<td>0.15</td>
<td>2.15</td>
<td>0.34</td>
</tr>
<tr>
<td>Japan</td>
<td>0.20</td>
<td>0.12</td>
<td>0.01</td>
<td>0.81</td>
<td>0.21</td>
<td>1.34</td>
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</tr>
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<td>Netherlands</td>
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<td>2.39</td>
<td>0.02</td>
<td>5.31</td>
<td>1.55</td>
<td>-0.06</td>
<td>1.78</td>
<td>0.67</td>
</tr>
<tr>
<td>Norway</td>
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<td>2.88</td>
<td>1.26</td>
<td>7.59</td>
<td>2.04</td>
<td>0.43</td>
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<td>2.76</td>
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<td>2.05</td>
<td>0.44</td>
<td>1.76</td>
<td>0.67</td>
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</tbody>
</table>

Note: Monthly interest rate data (in %) from January 1999 to January 2014 come from Bloomberg, L.P. 2014. $\hat{\rho} (12)$ refers to the autocorrelation function at the 12-month lag.
Figure 2.7. Yield Curves for Countries with Independent Currencies

Note: Monthly zero coupon yields from January 1999 to January 2014 come from Bloomberg, L.P. 2014. Yields are in percentage for maturities of 3 months, 6 months, and 1 to 10 years for each country.
Figure 2.8. Yield Curves for Countries under the Euro

Note: Monthly zero coupon yields from January 1999 to January 2014 come from Bloomberg, L.P. 2014. Yields are in percentage for maturities of 3 months, 6 months, and 1 to 10 years for each country.
Figure 2.9. Yield Curves for Countries with Higher Probability of Default

Note: Monthly zero coupon yields from January 1999 to January 2014 come from Bloomberg, L.P. 2014. Yields are in percentage for maturities of 3 months, 6 months, and 1 to 10 years for each country.

2. **Standard deviation tends to be lower for long term yields than short term yields.**

   This is particularly true during the earlier years in the sample, where short term yields exhibit high volatility. During the last few years, however, yields have stayed close to the zero bound, exhibiting lower variability at the short end of the yield curve.

3. **Yields are very persistent.** Table 2.3 shows the autocorrelation of one-year rates at one-month, one-year, and 2.5-year lags. The first order autocorrelation is higher than 90% for all countries. In general, the most persistent countries are Canada, Denmark, Finland, Germany, Netherlands, Norway, U.K., and U.S., with an autocorrelation higher than 65% after one year. The least persistent countries are
Australia, Ireland, Italy, Portugal, and Spain, with a one-year autocorrelation below 35%.

4. Yields are highly correlated across countries. Table 2.4 shows the correlation between one-year interest rates of different countries. As expected, interest rates of the same maturity tend to be very correlated across countries. The one-year yields for most European countries (Austria, Belgium, Denmark, Finland, France, Germany, Netherlands, Sweden, Switzerland, and U.K.) have a correlation typically higher than 90%, and this is true for other maturities as well. Italy’s, Norway’s, and Spain’s rates are also highly correlated with other European countries (more than 70%). Another group of highly correlated countries is the U.S., Hong Kong, and Canada (94%-96% across the sample).

There are yield curves however, that behave quite differently from the majority of the countries in the dataset. For example, although Portugal and Ireland are highly correlated with each other (82%), they seem disengaged from other country’s interest rates movements. The third exception is Japan. The Japanese yield curve is very different from other countries’ term structure, particularly during the earlier years.

5. Yields of near maturity are highly correlated. Table 2.5 shows the correlation of the 3-month interest rates of each country with respect to different maturities from 6 months to 10 years. For all countries, the closest the maturity range, the highest the correlation between interest rates of the same country. For example, 3-month and 6-month rates are highly correlated, moving almost one-to-one for every country, but 3-month and 10-year rates are not so closely related anymore, ranging from 14% (Spain) to 87% (Germany).
### TABLE 2.4

**CORRELATION OF ONE-YEAR YIELDS FOR 20 COUNTRIES**

|  | AUL | AUS | BEL | CAN | DEN | FIN | FRA | GER | HK  | IRE | ITA | JAP | NET | NOR | POR | SPA | SWE | SWI | UK  | USA |
|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| AUL | 1   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| AUS | 0.83| 1   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| BEL | 0.81| 0.99| 1   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| CAN | 0.71| 0.86| 0.86| 1   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| DEN | 0.75| 0.98| 0.97| 0.85| 1   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| FIN | 0.79| 0.99| 0.98| 0.85| 0.99| 1   |     |     |     |     |     |     |     |     |     |     |     |     |     |
| FRA | 0.81| 0.99| 0.99| 0.87| 0.98| 0.99| 1   |     |     |     |     |     |     |     |     |     |     |     |     |
| GER | 0.81| 0.99| 0.98| 0.87| 0.98| 0.99| 1.00| 1   |     |     |     |     |     |     |     |     |     |     |     |
| HK  | 0.58| 0.75| 0.75| 0.94| 0.76| 0.75| 0.76| 0.77| 1   |     |     |     |     |     |     |     |     |     |     |
| IRE | 0.37| 0.39| 0.43| 0.26| 0.34| 0.37| 0.38| 0.36| 0.22| 1   |     |     |     |     |     |     |     |     |     |
| ITA | 0.65| 0.84| 0.88| 0.73| 0.79| 0.80| 0.83| 0.81| 0.64| 0.62| 1   |     |     |     |     |     |     |     |     |
| JAP | 0.50| 0.51| 0.47| 0.29| 0.46| 0.48| 0.47| 0.46| 0.26| 0.27| 0.48| 1   |     |     |     |     |     |     |     |
| NET | 0.80| 0.99| 0.98| 0.88| 0.99| 1.00| 1.00| 1.00| 0.77| 0.35| 0.81| 0.45| 1   |     |     |     |     |     |     |
| NOR | 0.53| 0.85| 0.85| 0.76| 0.88| 0.86| 0.86| 0.86| 0.68| 0.35| 0.74| 0.26| 0.87| 1   |     |     |     |     |
| POR | 0.14| 0.11| 0.19| 0.06| 0.06| 0.08| 0.11| 0.08| 0.06| 0.82| 0.50| 0.13| 0.08| 0.12| 1   |     |     |     |
| SPA | 0.60| 0.80| 0.84| 0.72| 0.75| 0.77| 0.80| 0.78| 0.63| 0.57| 0.97| 0.47| 0.78| 0.72| 0.44| 1   |     |     |
| SWE | 0.70| 0.92| 0.93| 0.82| 0.91| 0.92| 0.93| 0.93| 0.67| 0.43| 0.80| 0.30| 0.93| 0.92| 0.18| 0.78| 1   |     |
| SWI | 0.71| 0.92| 0.91| 0.77| 0.93| 0.91| 0.92| 0.91| 0.72| 0.30| 0.82| 0.61| 0.91| 0.80| 0.14| 0.80| 0.81| 1   |
| UK  | 0.81| 0.90| 0.90| 0.92| 0.89| 0.90| 0.91| 0.92| 0.81| 0.19| 0.70| 0.30| 0.92| 0.72| -0.03| 0.68| 0.85| 0.77| 1   |
| USA | 0.70| 0.81| 0.82| 0.95| 0.81| 0.81| 0.83| 0.83| 0.96| 0.22| 0.68| 0.36| 0.83| 0.64| 0.03| 0.67| 0.69| 0.77| 0.89| 1   |

Note: Monthly interest rate data from January 1999 to January 2014 come from Bloomberg, L.P. 2014. One-year yields’ correlation for: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Hong Kong, Ireland, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, and United States.

This is true for all countries except for Ireland. Even though its correlation between 3-month and 6-month rates is 99%, the correlation between 3-month and one-year rates drops to 41%, and then gets close to 0 for further maturities, becoming negatively correlated with 5- to 10-year yields.

6. Most yields are close to a normal distribution over the sample period. This is consistent with contemporary work that suggests U.S. yields have become more Gaussian in recent years - see for example, Piazzesi (2010). Benchmark normal distributions are symmetric around the mean so that skewness is 0 and kurtosis...
<table>
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Note: Monthly interest rate data from January 1999 to January 2014 come from Bloomberg, L.P. 2014. 3-month yields for: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Hong Kong, Ireland, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, and United States.
is 3. The least normally distributed yields are Portugal’s, Ireland’s, and Japan’s; they exhibit excess kurtosis (3.5-12.9) and skewness (1.34-2.76). For the rest of the sample, skewness for one-year yields ranges from -0.42 (Australia) to 0.77 (Germany) and kurtosis ranges from 1.5 (U.K.) to 2.5 (Australia). This suggests that modeling yields under the assumption of Gaussian behavior is quite reasonable, especially during the time frame considered in the sample.

Bloomberg uses coupon bonds, bills, swaps, and/or a combination of these instruments to derive the discount factors for each country using standard bootstrapping. The zero-coupon yields are then calculated step-by-step using the discount factors. A minimum of four instruments at different tenors are required for each yield curve. A comparison between the U.S. Bloomberg data and the U.S. Federal Reserve data revealed that these datasets are almost identical, hence although Bloomberg’s method is proprietary, it must use similar bonds and/or similar methodology to extract zero coupon yields than the Federal Reserve, who uses an extension of the Nelson-Siegel method explained in Section 2.3 (see Gurkaynak et al., 2007).

2.6 Principal Component Analysis

Principal Component Analysis (PCA) is a variable reduction procedure that helps identify patterns in high dimensional data through the extraction of principal components or factors. The factors are an artificial construct that can be interpreted as unobserved variables to account for most of the variation in the dataset. The first principal component captures the largest variance in the data. The subsequent principal components are the vectors which maximize variance among all directions orthogonal to the previous factor. One of the first papers on PCA appeared in 1901 (Pearson, 1901). A few decades later, Harold Hotelling formulated PCA theory and coined the term principal component (Hotelling, 1933). PCA has been used in multiple disciplines since then. The main purpose of PCA is (1) to visualize high
dimensional data, (2) to identify the patterns or unobserved variables in our dataset, and (3) to reduce a set of observed variables into a smaller set of artificial variables called principal components (dimensionality reduction).

2.6.1 Step by Step Implementation

1. **Explore the raw data.** Before starting PCA, it is useful to examine the raw data. Usually, we want to calculate the mean and standard deviation for all variables, as well as the correlation matrix. If the variables in the dataset exhibit high correlation, then PCA can help us identify the underlying (unobservable) drivers of those variables.

2. **Center the data.** Mean subtraction (or mean centering) is necessary for performing PCA to ensure that the first principal component describes the direction of maximum variance. If mean subtraction is not performed, the first principal component might instead correspond more or less to the mean of the data. A mean of zero is needed for finding a basis that minimizes the mean square error of the approximation of the data.

3. **Scale the data** (optional). PCA is sensitive to the scaling of the variables. In addition to centering, when the variables are measured with different units, it is customary to standardize each variable to have unit variance. This is obtained by dividing each variable by its standard deviation, so that the data are normalized to have mean zero and standard deviation of one.

4. **Calculate the covariance matrix.** Traditionally, principal component analysis is performed on the covariance matrix or the correlation matrix (if the variables have been standardized). The covariance matrix contains scaled sums of squares and cross products.
5. **Calculate the eigenvectors and eigenvalues of the covariance matrix.** The eigenvector with the highest eigenvalue will correspond to the principal component of the dataset. It represents the most significant relationship between the data dimensions. The subsequent eigenvectors will capture the remaining variation in the dataset, orthogonal to the previous eigenvector. We can extract as many eigenvectors as variables we have in our dataset.

6. **Calculate the factor scores.** Multiply the eigenvectors and eigenvalues and the original (centered and scaled) data to obtain the new variables. The values of these new variables for the observations are called factor scores, and these factors scores can be interpreted geometrically as the projections of the observations onto the principal components.

Let $X$ be the dataset with all the variables of interest and $\mu_X$ be the mean of each variable, so that $\bar{X}$ is the mean-centered dataset. By subtracting the mean from each variable we are only looking at the data variation within the sample and not at the level variation. Let our dataset have $I$ observations (rows), $J$ variables (columns), and $F$ factors or principal components. Then the mean-centered data are represented by the corresponding factor scores $S$, factor loadings $L$, and an error term, $\epsilon$:

$$X - \mu_X = \bar{X} = S \cdot L' + \epsilon.$$  \hspace{1cm} (2.28)

- The Factor Score $S$ represents the amount of the artificial variable for each particular variable.
- The factor loadings $L$ capture the correlation between a principal component and a variable. Each loading is a direction in space with numbers associated to it that represent the highest variance. In other words, a loading is the shape
that is best able to describe the variation across the entire sample. Note that factor loadings are orthogonal by construction.

- Each principal component or factor (PC) has a score and a loading: $PC = S \cdot L'$. Loadings are common to all variables, but the score is different for every sample. Technically, a principal component can be defined as a linear combination of optimally-weighted observed variables.

- The residual $\epsilon$ is the unexplained portion. The highest the number of factors extracted from the data, the smaller the residuals will be.

### 2.6.2 How to choose the number factors, $F$?

In reality, the number of components extracted in a principal component analysis is equal to the number of observed variables being analyzed. However, in most analyses, only the first few components account for meaningful amounts of variance, so only these first few components are retained, interpreted, and used in subsequent analyses (such as in multiple regression analyses). Informally, an ad-hoc rule is that there should be at least 4 or 5 variables per PC. In principal component analysis, one of the most commonly used criteria for solving the number-of-components problem is the eigenvalue-one criterion, also known as the Kaiser criterion (Kaiser, 1960). With this approach, one retains and interprets any component with an eigenvalue greater than 1.00. Another method is the scree test (Cattell, 1966): one can plot the eigenvalues associated with each component and look for a “break” between the components with relatively large eigenvalues and those with small eigenvalues. The components that appear before the break are assumed to be meaningful and are retained for rotation; those appearing after the break are assumed to be unimportant.

and are not retained. A third criterion in solving the number-of-factors problem involves retaining a component if it accounts for a specified proportion of variance in the dataset. More recently, Bai and Ng (2002) developed a formal statistical procedure to identify the optimal number of factors for large panel datasets.

2.6.3 Interest Rate Factors

Given the high correlation of yields within each country, the literature has taken advantage of dimensionality-reduction techniques to find underlying drivers of interest rates at all maturities. I extract the main principal components (PCs) of each country by the eigenvalue decomposition on the correlation matrix after standardizing the data. Typically, the first 3 PCs are enough to explain most of the variability in all the yields throughout time. PC analysis is very successful at capturing the cross-sectional variation in yields. Table 2.6 presents summary statistics on the country-specific PCs, which I call local factors. The factors have a mean of zero and are orthogonal to each other by construction. In every country, the first factor extracted from the data is responsible for more than 75% of cross-sectional variation and the first three factors combined account for more than 99% of the total variability. Since the first factor captures most of the variation in all yields within a country, it fluctuates more and has a higher standard deviation (around 3.4) than the second (from 0.7 to 1.5) and third (from 0.2 to 0.5) factors. Local factors are also very persistent, with the first factor being more persistent than the second one (except for Ireland, Italy, and Spain). In term structure models, yields are usually modeled as linear functions of the factors. In this type of setting, that persistence of yields comes from persistent factors, as observed in the data.

Figure 2.10 shows the evolution of the first three factors over time. In general, the first factor is downward sloping, representing the drop in interest rates that most countries have experienced in the last few years. The exceptions are Ireland, Italy,
Portugal, and Spain, whose yields have increased during the European crisis, and Japan, whose yields have been close to the zero bound since the beginning of the sample.

Figure 2.11 shows the loadings or weights that each factor places on the different maturities for every country. The factor loadings exhibit the same pattern: horizontal for the first factor (around 0.03), downward sloping for the second factor, and hump-shaped for the third factor. Litterman and Scheinkman (1991) attributed the labels level, slope, and curvature to these three factors given the effect they have on the yield curve. For example, the first factor affects all yields the same, hence shocks to the first factor generate parallel shifts on the curve, changing the level or average of yields. The second factor loads positively for short term maturities and negatively for long term maturities, thus shocks to the second factor move long and short yields in opposite directions, changing the slope of the yield curve.

Finally, the third factor loads positively on short and long term maturities, but negatively on mid-range maturities, thus shocks to the third factor affect the curvature of the yield curve. This well-established interpretation of the factors is very common in the literature.

The cross-country correlation among yields, motivates the notion of underlying drivers of interest rates common to all (or most) countries, i.e., global factors. After the 10th component, the remaining variables contribute less than 0.03% to explain interest rate variability. I focus on the first ten global factors for the remaining of the section. Table 2.7 shows the summary statistics for the first 10 factors extracted by principal component analysis on yields of 20 countries, denoted as global interest rate factors. These factors are interpreted as the underlying variables driving interest rates across countries. By construction, the global factors have a mean of zero, and as factors extracted from the same dataset are orthogonal to each other, their correlations are zero as well.
### TABLE 2.6

**SUMMARY STATISTICS OF LOCAL INTEREST RATE FACTORS**

<table>
<thead>
<tr>
<th>Factors:</th>
<th>median</th>
<th>minimum</th>
<th>maximum</th>
<th>std. dev.</th>
<th>$\hat{\rho}(12)$</th>
<th>% explained</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 2 3</td>
<td>1 2 3</td>
<td>1 2 3</td>
<td>1 2 3</td>
<td>1 2 3</td>
<td>Total</td>
</tr>
<tr>
<td>Australia</td>
<td>0.6 0.1 -0.0</td>
<td>-8.3 -2.4 -0.7</td>
<td>5.7 1.7 0.6</td>
<td>3.4 0.9 0.2</td>
<td>0.4 0.1 -0.1</td>
<td>92.9 6.7 0.3 99.9</td>
</tr>
<tr>
<td>Austria</td>
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<td>-7.1 -1.7 -0.4</td>
<td>6.1 1.7 1.2</td>
<td>3.4 0.8 0.2</td>
<td>0.7 0.2 0.1</td>
<td>94.3 5.0 0.5 99.8</td>
</tr>
<tr>
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<td>0.0 0.0 0.0</td>
<td>-7.5 -1.6 -0.7</td>
<td>6.3 1.7 0.5</td>
<td>3.4 0.9 0.2</td>
<td>0.6 0.3 0.2</td>
<td>93.4 6.0 0.5 99.9</td>
</tr>
<tr>
<td>Canada</td>
<td>0.7 0.1 0.0</td>
<td>-5.4 -2.1 -0.5</td>
<td>6.9 1.4 0.4</td>
<td>3.4 0.9 0.2</td>
<td>0.8 0.4 -0.1</td>
<td>93.8 6.0 0.2 100.0</td>
</tr>
<tr>
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<td>-6.8 -1.4 -0.4</td>
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<td>3.4 0.7 0.2</td>
<td>0.7 0.2 -0.1</td>
<td>96.2 3.5 0.2 99.9</td>
</tr>
<tr>
<td>Finland</td>
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<td>-6.7 -1.4 -0.4</td>
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<td>3.4 0.7 0.2</td>
<td>0.7 0.2 0.1</td>
<td>95.4 4.2 0.4 99.9</td>
</tr>
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<td>3.4 0.7 0.2</td>
<td>0.7 0.2 0.1</td>
<td>95.4 4.2 0.3 99.9</td>
</tr>
<tr>
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<td>5.7 1.1 0.4</td>
<td>3.4 0.7 0.2</td>
<td>0.7 0.2 0.1</td>
<td>96.2 3.5 0.3 100.0</td>
</tr>
<tr>
<td>Hong Kong</td>
<td>-0.3 0.1 -0.0</td>
<td>-4.8 -1.7 -0.5</td>
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<td>3.4 0.7 0.2</td>
<td>0.7 0.4 0.0</td>
<td>95.2 4.5 0.3 100.0</td>
</tr>
<tr>
<td>Ireland</td>
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<td>-4.2 -2.8 -2.5</td>
<td>14.2 2.8 1.2</td>
<td>3.1 1.5 0.5</td>
<td>0.2 0.5 0.2</td>
<td>77.9 19.8 1.7 99.4</td>
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<td>3.2 1.3 0.3</td>
<td>0.2 0.6 0.3</td>
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<td>3.1 1.4 0.4</td>
<td>0.5 0.4 -0.1</td>
<td>81.7 16.9 1.1 99.7</td>
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<td>3.4 0.7 0.2</td>
<td>0.7 0.2 0.1</td>
<td>95.8 3.9 0.3 100.0</td>
</tr>
<tr>
<td>Norway</td>
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<td>-5.5 -1.8 -0.5</td>
<td>6.2 1.5 0.7</td>
<td>3.4 0.7 0.2</td>
<td>0.7 0.1 0.1</td>
<td>95.5 4.0 0.4 99.9</td>
</tr>
<tr>
<td>Portugal</td>
<td>-0.8 0.2 -0.0</td>
<td>-3.1 -3.6 -2.2</td>
<td>15.4 3.2 1.4</td>
<td>3.2 1.3 0.4</td>
<td>0.3 0.3 -0.0</td>
<td>83.3 14.4 1.3 99.0</td>
</tr>
<tr>
<td>Spain</td>
<td>-0.6 0.1 -0.0</td>
<td>-5.8 -4.7 -1.4</td>
<td>7.8 2.5 0.7</td>
<td>3.1 1.5 0.3</td>
<td>0.3 0.6 0.3</td>
<td>79.0 19.6 1.0 99.5</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.6 0.0 0.0</td>
<td>-5.9 -1.9 -0.5</td>
<td>5.8 1.6 0.5</td>
<td>3.4 0.8 0.2</td>
<td>0.7 0.1 0.1</td>
<td>94.3 5.3 0.4 99.9</td>
</tr>
<tr>
<td>Switzerland</td>
<td>-0.3 0.0 -0.0</td>
<td>-5.5 -1.8 -0.5</td>
<td>7.0 1.9 1.3</td>
<td>3.4 0.8 0.3</td>
<td>0.7 0.3 0.2</td>
<td>93.4 5.8 0.6 99.8</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>1.4 0.1 0.0</td>
<td>-6.8 -1.7 -0.6</td>
<td>5.4 1.4 0.4</td>
<td>3.4 0.7 0.2</td>
<td>0.7 0.3 -0.3</td>
<td>96.2 3.5 0.2 99.9</td>
</tr>
<tr>
<td>United States</td>
<td>-0.1 -0.0 -0.0</td>
<td>-5.4 -1.6 -0.4</td>
<td>7.1 1.3 0.4</td>
<td>3.4 0.7 0.2</td>
<td>0.7 0.3 -0.2</td>
<td>95.3 4.5 0.2 100.0</td>
</tr>
</tbody>
</table>

**Note:** Local factors are the principal components extracted from each individual country. Principal component analysis is computed on the correlation matrix of standardized monthly interest rate data from January 1999 to January 2014. There are 12 yields per country (maturities 3 months, 6 months, and 1 to 10 years) and 181 observations per yield. 12 components are calculated but only the first 3 are reported. Mean of each PC is 0 by construction. $\hat{\rho}(12)$ refers to the autocorrelation function at 12 months. % explained refers to the percentage of variance explained by each factor, whereas Total accounts for the cumulative variation of the first three factors combined.
Figure 2.10. Evolution of Local Interest Rate Factors (1999-2014)

Note: *Local Interest Rate Factors* are the principal components extracted from each individual country. The red solid line represents the first factor, the blue circles represent the second factor, and the green dashes represent the third factor. Principal component analysis is computed on the correlation matrix of standardized monthly interest rate data from January 1999 to January 2014.
Figure 2.11. Local Interest Rate Factor Loadings (3 months to 10 years)

Note: Local Interest Rate Factor Loadings are the weights that principal components extracted from each individual country have on the corresponding maturity. The red solid line represents the first factor loading, the blue circles represent the second factor loading, and the green dashes represent the third factor loading. Principal component analysis is computed on the correlation matrix of standardized monthly interest rate data from January 1999 to January 2014.
### TABLE 2.7

SUMMARY STATISTICS OF GLOBAL INTEREST RATE FACTORS

<table>
<thead>
<tr>
<th>Factor</th>
<th>median</th>
<th>minimum</th>
<th>maximum</th>
<th>std. dev.</th>
<th>skewness</th>
<th>kurtosis</th>
<th>$\hat{\rho}(12)$</th>
<th>% explained</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>23.5</td>
<td>13.1</td>
<td>-0.2</td>
<td>2.1</td>
<td>0.7</td>
<td>71.0</td>
</tr>
<tr>
<td>2</td>
<td>-1.1</td>
<td>-8.2</td>
<td>22.1</td>
<td>5.5</td>
<td>1.2</td>
<td>4.5</td>
<td>0.3</td>
<td>12.6</td>
</tr>
<tr>
<td>3</td>
<td>-0.6</td>
<td>-6.1</td>
<td>7.3</td>
<td>3.5</td>
<td>0.4</td>
<td>2.2</td>
<td>0.6</td>
<td>5.0</td>
</tr>
<tr>
<td>4</td>
<td>0.1</td>
<td>-6.2</td>
<td>7.1</td>
<td>2.8</td>
<td>0.1</td>
<td>2.4</td>
<td>0.4</td>
<td>3.3</td>
</tr>
<tr>
<td>5</td>
<td>0.4</td>
<td>-5.5</td>
<td>5.0</td>
<td>2.6</td>
<td>-0.1</td>
<td>2.1</td>
<td>0.2</td>
<td>2.8</td>
</tr>
<tr>
<td>6</td>
<td>-0.1</td>
<td>-5.2</td>
<td>3.7</td>
<td>1.9</td>
<td>-0.2</td>
<td>2.8</td>
<td>0.0</td>
<td>1.4</td>
</tr>
<tr>
<td>7</td>
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<td>-7.6</td>
<td>4.4</td>
<td>1.5</td>
<td>-1.0</td>
<td>9.4</td>
<td>-0.4</td>
<td>0.9</td>
</tr>
<tr>
<td>8</td>
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<td>4.2</td>
<td>1.4</td>
<td>0.1</td>
<td>3.3</td>
<td>-0.2</td>
<td>0.8</td>
</tr>
<tr>
<td>9</td>
<td>0.0</td>
<td>-2.9</td>
<td>3.8</td>
<td>1.0</td>
<td>0.2</td>
<td>3.4</td>
<td>-0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>10</td>
<td>0.1</td>
<td>-1.8</td>
<td>3.2</td>
<td>0.9</td>
<td>0.0</td>
<td>3.0</td>
<td>0.0</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Note: *Global Interest Rate Factors* are the principal components extracted from all 20 countries and 12 maturities, for a total of 240 variables. Principal component analysis is computed on the correlation matrix of standardized monthly interest rate data from January 1999 to January 2014. There are 12 yields per country (maturities 3 months, 6 months, and 1 to 10 years) and 181 observations per yield. 240 components are calculated but only the first 10 are reported. Mean of each PC is 0 by construction. $\hat{\rho}(12)$ refers to the autocorrelation function at 12 months. % explained refers to the percentage of variance explained by each factor. Total cumulative variation explained by the 10 factors combined is 98.6%.

The first global factor explains 71% of the total variation in all yields, whereas the first 10 global factors combined account for 98.6% of the total cross-section, cross-country variation. This is not surprising given the high correlation that yields exhibit across countries, suggesting that movements of yields over time can be driven by some underlying global trends.

Although thinking of these factors as global drivers of cross-sectional and cross-country variation of yields is very intuitive, Bauer and Diez de los Rios (2012) argue that in order to capture truly global variables, an inter-battery factor analysis, such
as in Perignon, Smith, and Villa (2007) is more appropriate. The interpretation of these factors, as well as providing robustness checks to verify whether principal component analysis on the full interest rate dataset captures global trends, requires a more thorough analysis, which I leave for further research.

The first global factor loads positively on most countries, ranging from 0.06 to 0.08. Hence a shock to the factor translates into a positive shock to all yields. For the most dissimilar countries, Japan, Ireland, Portugal, Italy, and Spain, the loadings are either smaller or negative. This might indicate that the first factor that drives interest rates is mostly driven by shocks to safe or stable countries, e.g., Germany.

On the contrary, the second global yield factor is small and negative for all countries, except for Ireland, Italy, Portugal, and Spain, whose loadings are positive and large (up to 0.16), meaning that the second factor contributes largely to explaining the unusual behavior of the countries most vulnerable to the European crisis or with higher than average credit risk. The third factor loads high and positively on short-term maturities and small and negatively on mid- and long-term maturities. This pattern is observed for all countries. The only distinctions are Japan’s loadings, which are higher than the rest of the countries (0.20), although they still exhibit the same pattern. This factor seems to primarily drive short-term or low yields across countries.

2.6.4 Exchange Rate Factors

The exchange rate data are obtained from Bloomberg L.P, 2014. There are a total of 181 monthly observations per currency from January 1999 to January 2014. All the (end of month) exchange rates are expressed in a base unit, the U.S. dollar (USD). The 20 countries/areas considered in the estimation and their currencies are as follows:
Table 2.8 summarizes the main characteristics of the data and Table 2.9 shows the correlation between currencies. Exchange rates, like interest rates, are very persistent. There is a distinct group of currencies that are highly related among themselves, sharing a correlation of 75% or higher: the euro, the Australian, New Zealand, Singapore, and Canadian dollars, the Swiss franc, the Norwegian krone, the Swedish krona, the Thai baht, and the Czech koruna.

Latin American countries: Brazil, Chile, and Colombia, share at least an 82% correlation among their currencies, and are least related to the British pound and the South African rand. The British pound is most related to the Korean won (81%).

The South African rand and the Russian ruble are the currencies with lowest correlation to others and the least persistent (less than 35% one-year autocorrelation), they are most related to the Korean won and the British pound. With principal component analysis, the main constructed factors can help identify the underlying drivers of currency movements common to the entire exchange rate sample.

Table 2.10 shows the summary statistics of the first 10 global exchange rate factors; that is, the factors extracted by PCA from the 20-exchange-rate dataset. The
first global exchange rate factor accounts for over 60% of the variation in currency movements over time, whereas the second and third exchange rate factors account for 15% and 13% of the variation, for a total of 88.5% combined. The evolution of the first three global exchange rate factors is depicted in Figure 2.12.

The first global exchange rate factor loads positively on all currencies with weights ranging from 0.02 to 0.28. The currencies with the lowest loadings are the South African rand (0.02), Russian ruble (0.03), British pound (0.09), Korean won (0.13), and the Philippine peso (0.16).

The second and third global exchange rate factors load positively on about half the currencies and negatively on the rest. A positive shock to the second factor affects CAD, CLP, CZK, EUR, GBP, KRW, NOK, NZD, RUB, SEK, and ZAR, with the highest loadings on the British pound (0.52) and the Korean won (0.41).
<table>
<thead>
<tr>
<th>Currency</th>
<th>mean</th>
<th>median</th>
<th>minimum</th>
<th>maximum</th>
<th>std.dev.</th>
<th>skewness</th>
<th>kurtosis</th>
<th>$\hat{\rho}(12)$</th>
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</thead>
<tbody>
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<td>0.49</td>
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<td>0.17</td>
<td>0.11</td>
<td>1.92</td>
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</tr>
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<td>0.49</td>
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<td>-0.31</td>
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<td>0.02</td>
<td>0.03</td>
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<td>0.64</td>
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<td>SEK</td>
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<td>0.02</td>
<td>-0.39</td>
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<td>0.82</td>
<td>0.56</td>
<td>1.27</td>
<td>0.18</td>
<td>0.20</td>
<td>2.07</td>
<td>0.78</td>
</tr>
<tr>
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<td>0.03</td>
<td>0.03</td>
<td>0.02</td>
<td>0.03</td>
<td>0.00</td>
<td>0.10</td>
<td>1.57</td>
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<td>0.00</td>
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<td>0.65</td>
<td>2.37</td>
<td>0.57</td>
</tr>
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</table>

Note: Exchange rate data from January 1999 to January 2014 come from Bloomberg, L.P. 2014. All currencies are expressed in terms of the U.S. dollar.
## TABLE 2.9

**CORRELATION OF 20 EXCHANGE RATES EXPRESSED IN USD**

|       | AUD  | BRL  | CAD  | CHF  | CLP  | COP  | CZK  | EUR  | GBP  | JPY  | KRW  | NOK  | NZD  | SGP  | THB  | TWD  | ZAR  |
|-------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| AUD   | 1    |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| BRL   | 0.53 | 1    |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| CAD   | 0.95 | 0.55 | 1    |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| CHF   | 0.95 | 0.40 | 0.91 | 1    |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| CLP   | 0.75 | 0.82 | 0.74 | 0.62 | 1    |      |      |      |      |      |      |      |      |      |      |      |      |      |
| COP   | 0.59 | 0.82 | 0.53 | 0.49 | 0.84 | 1    |      |      |      |      |      |      |      |      |      |      |      |      |
| CZK   | 0.88 | 0.47 | 0.94 | 0.90 | 0.59 | 0.42 | 1    |      |      |      |      |      |      |      |      |      |      |      |
| EUR   | 0.84 | 0.39 | 0.88 | 0.84 | 0.57 | 0.35 | 0.96 | 1    |      |      |      |      |      |      |      |      |      |      |
| GBP   | 0.29 | 0.03 | 0.36 | 0.16 | 0.21 | -0.07 | 0.30 | 0.57 | 1    |      |      |      |      |      |      |      |      |      |
| JPY   | 0.79 | 0.51 | 0.70 | 0.81 | 0.58 | 0.54 | 0.65 | 0.53 | -0.19 | 1    |      |      |      |      |      |      |      |      |
| KRW   | 0.42 | 0.25 | 0.51 | 0.27 | 0.48 | 0.16 | 0.40 | 0.50 | 0.81 | -0.06 | 1    |      |      |      |      |      |      |      |
| NOK   | 0.91 | 0.40 | 0.93 | 0.88 | 0.62 | 0.40 | 0.96 | 0.95 | 0.53 | 0.58 | 0.55 | 1    |      |      |      |      |      |      |
| NZD   | 0.95 | 0.36 | 0.92 | 0.91 | 0.67 | 0.46 | 0.87 | 0.87 | 0.46 | 0.64 | 0.55 | 0.93 | 1    |      |      |      |      |      |
| PHP   | 0.45 | 0.74 | 0.38 | 0.37 | 0.75 | 0.89 | 0.28 | 0.22 | -0.14 | 0.46 | 0.08 | 0.27 | 0.30 | 1    |      |      |      |      |
| RUB   | -0.03 | 0.39 | 0.02 | -0.20 | 0.33 | 0.28 | 0.03 | 0.16 | 0.61 | -0.32 | 0.50 | 0.12 | 0.01 | 0.26 | 1    |      |      |      |
| SEK   | 0.91 | 0.42 | 0.90 | 0.87 | 0.69 | 0.46 | 0.89 | 0.98 | 0.57 | 0.57 | 0.60 | 0.95 | 0.95 | 0.35 | 0.19 | 1    |      |      |
| SGD   | 0.93 | 0.56 | 0.90 | 0.95 | 0.72 | 0.66 | 0.85 | 0.73 | 0.04 | 0.83 | 0.24 | 0.80 | 0.84 | 0.57 | -0.17 | 0.80 | 1    |      |
| THB   | 0.91 | 0.65 | 0.90 | 0.88 | 0.79 | 0.71 | 0.85 | 0.78 | 0.18 | 0.75 | 0.35 | 0.82 | 0.84 | 0.66 | 0.02 | 0.81 | 0.94 | 1    |
| TWD   | 0.78 | 0.64 | 0.69 | 0.73 | 0.80 | 0.73 | 0.56 | 0.48 | 0.00 | 0.75 | 0.31 | 0.57 | 0.68 | 0.66 | 0.00 | 0.67 | 0.82 | 0.75 | 1    |
| ZAR   | 0.04 | 0.21 | 0.01 | -0.14 | 0.28 | 0.06 | -0.08 | 0.11 | 0.48 | -0.05 | 0.41 | 0.05 | 0.09 | 0.00 | 0.56 | 0.17 | -0.21 | -0.08 | 0.06 | 1    |

Note: Exchange rate data from January 1999 to January 2014 come from Bloomberg L.P. All currencies are expressed in terms of the U.S. dollar.
### TABLE 2.10

SUMMARY STATISTICS OF GLOBAL EXCHANGE RATE FACTORS

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<th>Factor</th>
<th>median</th>
<th>minimum</th>
<th>maximum</th>
<th>std. dev.</th>
<th>skewness</th>
<th>kurtosis</th>
<th>$\hat{\rho}(12)$</th>
<th>% explained</th>
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</thead>
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<td>6.6</td>
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<td>0.5</td>
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<td>3.8</td>
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<td>-0.1</td>
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Note: *Global Exchange Rate Factors* are the principal components extracted from all 20 currencies relative to the U.S. dollar. Principal component analysis is computed on the correlation matrix of standardized monthly exchange rate data from January 1999 to January 2014. There are 181 observations per currency. 20 components are calculated but only the first 10 are reported. Mean of each PC is 0 by construction. $\hat{\rho}(12)$ refers to the autocorrelation function at 12 months. % explained refers to the percentage of variance explained by each factor. Total cumulative variation explained by the 10 factors combined is 98.8%.

The third exchange rate factor loads positively on BRL, CLP, COP, KRW, RUB, PHP, THB, TWD, and ZAR, with the highest loadings on the Philippine peso (0.41) and the Brazilian real (0.39). The loadings on the remaining currencies are negative.

#### 2.6.5 Macroeconomic Factors

Similarly, macroeconomic factors are extracted from data on consumer price index (CPI) and industrial production (IP) for 17 countries:

---

18Greenaway-McGrevy et al. (2012) extract global exchange rate factors with a different sample and interpret these factors as exchange rate themselves.
Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, the Netherlands, Norway, Portugal, Spain, Sweden, the United Kingdom, and the United States.

Monthly CPI for all items and IP data are obtained from the IMF International Financial Statistics database, from January 1999 to October 2013. Table 2.11 shows summary statistics of both series for all 17 countries. Macroeconomic variables are also very persistent, and as shown in Tables 2.12 and 2.13, they are very correlated. CPI is highly correlated across countries (typically over 95%), with Japan being the only exception: negatively correlated with all countries, from -76 (Netherlands) to -66 (U.K.). Industrial production is positively correlated among Austria, Belgium, Finland, Germany, Ireland, the Netherlands, Sweden, and the United States. It is also positively correlated among Canada, Denmark, Italy, Japan, Norway, Portugal, Spain, and the United Kingdom.

I extract global CPI factors and global IP factors by principal component analysis on standardized data, and report the summary statistics of the first 10 factors on Table 2.14. The first global CPI factor loads positively (around 0.25) on all countries except for Japan (-0.18).

The second CPI factor loads very high on Japan (0.97) and either small or negatively on the remaining countries. The third factor has the highest loading on Ireland (0.74) and the lowest on the U.K. (-0.48). Given the high correlation of consumer price indices across countries, the first three global CPI factors account for 99.6% of the total cross-country variation. The first global IP factor loads positively on all countries, with the highest loads being for France, Italy, Spain, the U.K., Denmark, Finland, Sweden, Canada, and Japan. The second factor loads positively on half of the countries (Austria, Belgium, Finland, Germany, Ireland, Japan, the Netherlands, and Sweden) and negatively on the remaining countries.
### Table 2.11

**Summary Statistics of Macro Data for 17 Countries**

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<th>macro variable:</th>
<th>mean</th>
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<th>minimum</th>
<th>maximum</th>
<th>std.dev.</th>
<th>skewness</th>
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Note: Monthly macroeconomic data from January 1999 to October 2013 for 17 countries from IMF IFS database. CPI: Consumer Price Index and IP: Industrial Production. $\hat{\rho}(12)$ refers to the autocorrelation function at the 12-month lag.
TABLE 2.12

CORRELATION OF CONSUMER PRICE INDEX FOR 17 COUNTRIES

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Note: Monthly consumer price index data from January 1999 to October 2013 for 17 countries from IMF IFS database.
TABLE 2.13

CORRELATION OF INDUSTRIAL PRODUCTION INDEX FOR 17 COUNTRIES

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Note: Monthly industrial production index data from January 1999 to October 2013 for 17 countries from IMF IFS database.
### Table 2.14

**SUMMARY STATISTICS OF GLOBAL MACROECONOMIC FACTORS**

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**Note:** *Global Macroeconomic Factors* are the principal components extracted from all 17 countries from consumer price index data (CPI) and industrial production data (IP) from the IMF IFS database. Principal component analysis is computed on the correlation matrix of standardized monthly macroeconomic data from January 1999 to October 2013. There are 178 observations per country. 17 components are calculated but only the first 10 are reported. Mean of each PC is 0 by construction. $\hat{\rho}(12)$ refers to the autocorrelation function at 12 months. % explained refers to the percentage of variance explained by each factor.
### TABLE 2.15

**CORRELATION OF ALL GLOBAL FACTORS**

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Note: **YIELD (Y)** refers to the global yield factors, **XRATE (X)** refers to the global exchange rate factors, **INFLATION (I)** refers to the global inflation factors, and **GROWTH (G)** refers to the global growth factors. All factors extracted from principal component analysis.

#### 2.6.6 Relationship Between Factors

I select a sample of 16 countries based on all the available information from January 1999 to October 2013: Austria, Belgium, Canada, Finland, France, Germany, Ireland, Italy, Japan, the Netherlands, Norway, Portugal, Spain, Sweden, the United Kingdom, and the United States. Each country has data on: yields (from 3 months to 10 years), exchange rate (with respect to the U.S. dollar), inflation (estimated as the annual change in logs of the CPI), and economic growth (estimated as the annual change in logs of the IP). I extract three global factors from each series (yields, exchange rates, inflation, growth) and calculate the correlation among factors in Table 2.15. The first global yield factor is negatively correlated with the first exchange rate.
factor (-0.65%) and the second inflation factor (-0.64%) and positively correlated with the second exchange rate factor (53%). The second global yield factor is negatively correlated with the second exchange rate factor (-50%) and positively correlated with the first inflation factor (49%). Finally, the third global yield factor is positively correlated with the first two exchange rate factors and the second inflation factor.

Although more work needs to be done in order to determine what these factors represent and whether they are truly global, this chapter motivates the idea that underlying principal components of the data on interest rates, exchange rates, and macroeconomic variables are related to each other.
CHAPTER 3

CAN INTEREST RATE FACTORS EXPLAIN EXCHANGE RATE FLUCTUATIONS?

3.1 Introduction

In the asset pricing literature, as seen in Chapters 1 and 2, it has been established that a low-dimensional set of risk factors can determine asset prices in the world. From this perspective, theoretical and empirical work has been developed in the areas of equity returns, fixed income and interest rates, and more recently, exchange rates. Can the same factors that drive interest rates also help explain movements in other assets? Specifically, can they help explain fluctuations in the exchange rate? In pursuing this avenue of research, I develop a two-country term structure model where yields are modeled as linear functions of interest rate factors, and these factors in turn, help determine the changes in the exchange rate between them through the stochastic discount factor. The results suggest that interest rate factors are indeed helpful in explaining changes in the exchange rate, particularly at longer horizons.

In this chapter, I model yields and exchange rates only with interest rate factors in a two-country dynamic term structure framework. This implies that the risk factors that drive exchange rate fluctuations are spanned by the countries’ yield curves. The Gaussian term structure model in discrete time is based on Joslin, Singleton, and Zhu (2011). I expand upon it by (1) adding a foreign country so that the yields of two countries are modeled simultaneously, and (2) given the countries’ stochastic discount factors, I model the exchange rate changes between them. The extension preserves
the desirable properties of their original model, which allows for a more tractable estimation of the parameters estimated by Maximum Likelihood. More importantly, I do not impose restrictions on the actual distribution of the observable factors, which are constructed as linear combination of yields within each country’s yield curve. The main three factors for the countries have been identified in the literature as the level, slope, and curvature of the term structure. I model interest rates as linear functions of these factors, as it is standard in the term structure literature, in order to yield closed-form solutions for the bond prices. I then derive each country’s stochastic discount factor, which is the pricing mechanism that investors in each country develop in order to price the assets in the local currency under no arbitrage. The stochastic discount factor is constructed with time-varying prices of risk that are also linear in the factors, as in Duffee (2002). Exchange rates are then determined by the ratio of the countries’ unique stochastic discount factors under the assumption that markets are complete. In this framework, factors from the yield curve of two countries determine the exchange rate changes between them through the interest rate differential, the difference in each country’s time-varying risk premium, and the relative shocks to the prices of risk.

My findings suggest that the yield curve does contain important information for modeling exchange rate dynamics, particularly at longer horizons. For example, my model with three factors from each country can explain between 30% and 60% of the one-year change in the log exchange rate for seven different currencies relative to the U.S. dollar: Australia, Canada, Japan, Sweden, Switzerland, Norway, and the U.K. Moreover, the yield curve’s third factor, the term structure’s curvature, contains low explanatory power for the cross-section of interest rates (in general, less than 1%) but significantly improves exchange rate predictions for most countries in the data set. For example, the one-year change in the British pound increases the (adjusted) $R^2$ from 22% to 61% with the addition of a curvature factor. A fourth
factor extracted from the yield curve may also contribute to explain fluctuations in
the exchange rate. However, more work needs to be done in order to understand the
macroeconomic sources of these risk factors priced in the yield curve. In particular,
what are these factors and how much variation in exchange rates do they contribute
to explain? Robustness checks suggest that the results are not driven by potential
correlation and heteroskedasticity in the error terms or small-sample bias. Moreover,
when modeled in terms of the British pound as opposed to the U.S. dollar, exchange
rate fit improves, suggesting that the U.S. term structure may not be the optimal
base to choose given its shift in volatility and level regimes during 2008.

Within this framework I explore two different applications of term structure mod-
els. First, I evaluate the model performance in the context of the Uncovered Interest
Parity puzzle and test whether the Fama (1984) conditions are satisfied. Results sug-
gest that given the risk premium implied by the model, satisfying the Fama (1984)
conditions, term structure models partially account for the Uncovered Interest Parity
puzzle. Furthermore, I study the impact of factors from countries under the euro and
conclude that the risk factors that drive the exchange rate are priced into the yield
curves of all countries to different extents.

My work relates to previous studies on joint dynamics of exchange rates and yield
curves, such as Hodrick and Vassalou (2002), Diez de Los Rios (2009), and Chen
and Tsang (2011), among many others. However, it is different in the following di-
mensions. First, I introduce observable factors, extracted from interest rate data
from each country separately, to model exchange rates in a bilateral term structure
framework. Hence, the yield curves from the domestic and foreign countries are mod-
eled as linear functions of their own interest rate factors, whereas the (log) exchange
rate changes are modeled as the (log) difference between the countries’ stochastic
discount factors, which depend on both countries’ set of factors. Second, I do not
impose restrictions of interdependence between the two countries, which allows for a
more tractable estimation of the unrestricted time-varying risk premium as a linear function of the factors. This helps preserve the separation argument from Joslin, Singleton, and Zhu (2011), which significantly reduces the dimensionality of the parameter set to be estimated by Maximum Likelihood. Third, I explore changes in the exchange rate at different horizons (from 1 month to 5 years) for a sample of seven currencies relative to the U.S. dollar and evaluate the model’s ability to solve the Uncovered Interest Parity puzzle and satisfy the Fama (1984) conditions for the risk premium.

The reminder of this Chapter is organized as follows. Section 3.2 describes the canonical term structure model that expands the baseline model to account for two countries and incorporates exchange rates in a bilateral framework. Section 3.3 introduces the data and the estimation strategy by Maximum Likelihood to estimate the model parameters. Section 3.4 presents all the results for interest rates and exchange rates, including tests for the uncovered interest parity puzzle and the case of the euro, and Section 3.5 concludes.

3.2 Term Structure Model

The main goal of term structure models is to explain the cross-sectional variation in yields. I extend the model to account for the variation in yields for two countries at the same time, as well as for changes in the exchange rate between them. The key assumption is that there are no arbitrage opportunities in the bond markets, which is reasonable in light of how liquid the markets are, particularly during the period considered. In Section 3.2.1 I describe a canonical dynamic term structure model in discrete time, under the assumption that the state vector that drives yield movements follows a Gaussian vector auto-regressive process, as in Joslin, Singleton, and Zhu (2011) - JSZ. Section 3.2.2 describes the transformed model with the observable state vector that preserves the same properties as before and derives the pricing kernel as in
Dai and Singleton (2002) and Ang and Piazzesi (2003), in which the market prices of risk are linear functions of the state variable. Section 3.2.3 outlines an extension of the model that (1) jointly estimates the term structure of two countries, and (2) models the exchange rate between them under the assumption that markets are complete. The model is developed to preserve the properties from the JSZ framework that provide a low-dimensional form and relatively unrestricted parameterization, and a separation of the estimated parameters that significantly reduces computational burden. In this context, factors from the yield curve of two countries determine the exchange rate changes between them through the interest rate differential, the time-varying exchange rate risk premium, and the difference in the countries shocks to the prices of risk.

3.2.1 Term Structure Model with Latent Factors

In standard term structure models, the one-period (annualized) risk free rate, \( r_t \), is expressed as a linear function of an \( N \)-dimensional latent state vector \( X_t \) (as in Vasicek, 1977):

\[
    r_t = \rho_0 X + \rho_1 X \cdot X_t, \tag{3.1}
\]

where \( \rho_0 X \) is a constant and \( \rho_1 X \) is an \( (N \times 1) \) vector. Given equation (3.1), the entire term structure of zero coupon bonds with maturity \( m \) (in years) can also be expressed in terms of \( X_t \). This context assumes that no arbitrage holds, which makes the time evolution of yields and the cross-sectional shape of the yield curve consistent at any point in time. By the fundamental theorem of asset pricing, the condition of no arbitrage is equivalent to the existence of a risk-neutral measure, \( Q \). Therefore, the price of an \( m \)-maturity zero coupon bond at time \( t \), \( P_t^m \), can be expressed as the expected value of the future payoffs (in this case = 1) discounted at the risk free rate,
under risk-neutrality \( (\mathbb{E}^Q_t) \):

\[
P^m_t = \mathbb{E}^Q_t \left[ \exp \left( - \sum_{i=0}^{m-1} r_{t+i} \right) \right]
= \exp \left[ a^m_X + b^m_X \cdot X_t \right],
\]

where \( a^m_X \) and \( b^m_X \) solve first-order (Riccati) difference equations with initial conditions \( a^0_X = 0 \) and \( b^0_X = 0 \) as in Duffie and Kan (1996). This allows one to model the yields at different horizons as a linear function of the state variable \( X_t \), where the yield loadings are given by \( A^m_X = -a^m_X/m \) and \( B^m_X = -b^m_X/m \),

\[
y^m_t = A^m_X + B^m_X \cdot X_t.
\]

The evolution of the unobserved factors is expressed under two equivalent probability measures: \( \mathbb{P} \) and \( \mathbb{Q} \). \( \mathbb{P} \) is the probability measure of the random process \( X \), referred to as the actual, historical, or physical distribution of the state vector. This process can be represented, for example, by an unrestricted vector auto-regressive process that describes the evolution of the yields. \( \mathbb{Q} \) is an equivalent martingale measure\(^1\) referred to as the risk-neutral distribution of \( X \). Under this distribution, the process has already been adjusted for risk, allowing for a risk-neutral representation of the movement of factors over time. The state vector then evolves as an auto-regressive process under the two distributions:

\[
\Delta X_t = K^p_{0X} + K^p_{1X} \cdot X_{t-1} + \Sigma_X \epsilon^p_t,
\]

\[
\Delta X_t = K^Q_{0X} + K^Q_{1X} \cdot X_{t-1} + \Sigma_X \epsilon^Q_t,
\]

where \((K^p_{0X}, K^Q_{0X})\) are \((N \times 1)\) constant vectors, \((K^p_{1X}, K^Q_{1X})\) are \((N \times N)\) matrices, and errors are i.i.d Gaussian, \( \epsilon^p_t, \epsilon^Q_t \sim N(0, I_N) \). \( \Sigma_X \) is a lower triangular matrix such that \( \Sigma_X \Sigma'_X \) is the \((N \times N)\) variance-covariance matrix of \( X_t \). Changing distributions

\(^1\)Defined as a finite stochastic process in which each realization is independent from the previous one.
from $\mathbb{P}$ to $\mathbb{Q}$ removes the drift from the stochastic process, which is essentially a shift in the mean and not the variance of the error term, for which $\Sigma^Q_X = \Sigma^P_X = \Sigma^2_X$. The change from physical to risk-neutral distribution captures the investors’ adjustment for risk, which will be an integral component in deriving the stochastic discount factor. I elaborate on this concept in Section 3.2.2.

JSZ simplify the estimations by imposing identification conditions that yield an observationally equivalent model. Let $\rho_{0X} = 0$ and $\rho_{1X} = [1 \ 1 \ \ldots \ 1]'$ so that the risk-free rate is the sum of the factors: $r_t = \iota \cdot X_t$, where $\iota$ is a vector of ones, and let $\Sigma_X$ be lower triangular with positive diagonal. $K^Q_{1X}$ is then parameterized as a diagonal matrix with ordered $\mathbb{Q}$ eigenvalues, $\lambda^Q$, which can be interpreted as the speed of mean reversion of the factors under risk-neutrality. Finally, $K^Q_{0X}$ can be normalized by $K^Q_{0X} = [k^Q_\infty \ 0 \ \ldots \ 0]'$, where $k^Q_\infty$ is proportional to the long-run mean of the short rate whenever the model is stationary under $\mathbb{Q}$. Given $\lambda^Q$ and $k^Q_\infty$, there exists a unique set of parameters consistent with no arbitrage. This JSZ invariant transformation of the model simplifies the estimation process and significantly improves computational performance, which will be a key advantage when deriving the higher-dimensional model in Section 3.2.3.

3.2.2 Term Structure Model with Observable Factors

By Duffie and Kan (1996) we can normalize a DTSM so that the state vector is observable. Let $P_t$ be a set of $N$ observable factors. These factors are constructed as

2This is an application of the diffusion invariance principle, which describes the dynamics of stochastic processes when the physical measure is changed to an equivalent martingale measure but only the drift changes (Girsanov, 1958).

3Note that this identification assumes that the factors are stationary under the risk neutral distribution, which may not be true. It is not uncommon for the estimated roots to be close to 0, given the possibility of a unit root in the pricing factors. JSZ show that an equivalent renormalization of $K^Q_{0X}$ and $\rho_0$ allows for the preservation of the canonical form in the presence of zero or negative $\mathbb{Q}$ roots as well. Refer to Joslin, Singleton, and Zhu (2011) and their online appendix for more details.
$N$-linear combinations of zero coupon yields by principal component analysis, such that

$$
\mathcal{P}_t = W \cdot y_t,
$$

(3.6)

where $W$ is an $(N \times J)$ matrix containing the weights that factors have on the $J$ maturity yields. The model assumes that $\mathcal{P}_t$ and $y_t$ are priced perfectly, but the observed yields, $y_t^{obs}$, are measured with error; such that $e_t^y = y_t^{obs} - y_t$. The errors are assumed to be conditionally independent of their lagged values and satisfy consistency conditions as specified in JSZ.

From equation (3.3), $\mathcal{P}_t$ can be expressed as a function of the latent vector $X_t$

$$
\mathcal{P}_t = W \cdot [A^m_X + B^m_X \cdot X_t] = W \cdot A^m_X + W \cdot B^m_X \cdot X_t.
$$

(3.7)

Equivalently,

$$
X_t = (W \cdot B^m_X)^{-1} \cdot (\mathcal{P}_t - W \cdot A^m_X).
$$

(3.8)

Now, the yields can be expressed in terms of observable factors by substituting (3.8) into (3.3):

$$
y_t = A^m_X + B^m_X \cdot [(W \cdot B^m_X)^{-1} \cdot (\mathcal{P}_t - W \cdot A^m_X)],
$$

(3.9)

where $A^m_P = A^m_X \cdot [I - B^m_X \cdot (W \cdot B^m_X)^{-1} \cdot W]$ and $B^m_P = B^m_X \cdot (W \cdot B^m_X)^{-1}$, such that $y_t = A^m_P + B^m_P \cdot \mathcal{P}_t$.

The risk free rate can also be expressed in terms of observable factors:

$$
r_t = \iota \cdot X_t
$$

(3.10)

$$
r_t = \iota \cdot [(W \cdot B^m_X)^{-1} \cdot (\mathcal{P}_t - W \cdot A^m_X)],
$$

(3.11)

where $\rho_{0P} = -\iota \cdot [(W \cdot B^m_X)^{-1} \cdot W \cdot A^m_X]$ and $\rho_{1P} = \iota \cdot (W \cdot B^m_X)^{-1}$, such that $r_t = \rho_{0P} + \rho_{1P} \cdot \mathcal{P}_t$.

Finally, equation (3.4) can be expressed as $\Delta \mathcal{P}_t = K_{0P}^\mathcal{P} + K_{1P}^\mathcal{P} \cdot \mathcal{P}_{t-1} + \Sigma_p e_t^P$, where
Similarly equation (3.5) can be derived in order to represent the evolution of factors under the risk-neutral distribution.

Given the introduction of observable factors, the model can be now summarized by the following equations:

\[ r_t = \rho_0 P + \rho_1 P \cdot \mathcal{P}_t, \]  
\[ y_t^m = A_m P + B_m P \cdot \mathcal{P}_t, \]  
\[ \Delta \mathcal{P}_t = K_0^P P + K_1^P P \cdot \mathcal{P}_{t-1} + \Sigma_P \epsilon_t^P, \]  
\[ \Delta \mathcal{P}_t = K_0^Q P + K_1^Q P \cdot \mathcal{P}_{t-1} + \Sigma_P \epsilon_t^Q, \]

where internal consistency conditions are satisfied, i.e., \( W \cdot A_m P = 0 \) and \( W \cdot B_m P = I_N \).

The yield coefficients are rotated so that

\[ A_m P = A_X \cdot \left[ I - B_X^m \cdot (W \cdot B_X^m)^{-1} \cdot W \right], \]
\[ B_m P = B_X^m \cdot (W \cdot B_X^m)^{-1}. \]

The parameters under the physical distribution, \( \Theta^P = \{ K_0^P, K_1^P, \Sigma_P \} \), can be estimated from an unrestricted vector auto-regression on \( \mathcal{P}_t \), such that \( K_1^P = I + K_1^{P, VAR} \) and \( \Sigma_P \) is the lower triangular Cholesky decomposition of the variance-covariance matrix. \( K_0^Q \) can be interpreted as the long-run average of the factors and \( K_1^Q \) as the speed of mean reversion in the process towards its long-run unconditional mean or the half-life of the factors. Finally, \( \epsilon_t^Q \) and \( \Sigma_P \) are the shocks that disturb the state vector from moving back to its mean and the variance-covariance matrix of the innovations with free parameters, respectively. The parameters under the risk neutral
distribution, \( \Theta^Q = \{ K^Q_{0P}, K^Q_{1P}, \Sigma_P, \rho_{0P}, \rho_{1P} \} \), can be expressed in terms of \( \lambda^Q \) and \( k^Q_{\infty} \) given the mapping between \( X_t \) and \( P_t \), and the yield coefficients determined by recursive rules \( \{ A^m_X, B^m_X \} \) along with the weighting matrix \( W \):

\[
K^Q_{1P} = (W \cdot B^m_X) \cdot K^p_{1X} \cdot (W \cdot B^m_X)^{-1}, \quad (3.21)
\]

\[
K^Q_{0P} = (W \cdot B^m_X) \cdot \begin{bmatrix} K^Q_{\infty} & 0 & \cdots & 0 \end{bmatrix}' - K^Q_{1P} \cdot (W \cdot A^m_X), \quad (3.22)
\]

\[
\rho_{1P} = \iota \cdot (W \cdot B^m_X)^{-1}, \quad (3.23)
\]

\[
\rho_{0P} = -\rho_{1P} \cdot (W \cdot A^m_X). \quad (3.24)
\]

The JSZ transformation allows for a significant reduction in the number of parameters to be estimated by Maximum Likelihood, since the risk-neutral distribution can then be fully specified by \( \Theta^Q = \{ \lambda^Q, k^Q_{\infty}, \Sigma_P \} \). This separation in the parameters estimated under the different probability measures is a key property of the model, which I preserve when integrating exchange rates.

**Pricing Kernel**

Asset prices are determined by the Euler equation: \( E_t [M_{t+1} R^m_{t+1}] = 1 \), in which \( R \) denotes the real return on a traded asset with no intermediate payoffs, such that \( R^m_{t+1} = \frac{P^m_{t+1}}{P^m_t} \). Given the absence of arbitrage, \( M_{t+1} \) is the pricing mechanism that determines current prices on the basis of future cash flows (Harrison and Kreps, 1979). Hence bond prices can be determined recursively by \( E_t [M_{t+1} P^m_{t+1}] = P^m_t \).

The pricing kernel, also called stochastic discount factor, is exogenously specified by the risk-free rate, \( r_t \), and the price of risk, \( \Lambda_t \), as an exponential quadratic function,

\[
M_{t+1} = \exp \left( -r_t - \frac{1}{2} (\Lambda'_t \Lambda_t - \Lambda'_t \epsilon_{t+1}^p) \right), \quad (3.25)
\]

where \( \epsilon_{t+1}^p \) is the vector of shocks to the factors. The price of risk measures how much the investor has to be compensated for fluctuations in the state vector. As in Duffee
(2002) and Ang and Piazzesi (2003), $\Lambda_t$ takes a linear functional form with respect to the factors,

$$\Lambda_t = \Sigma_P^{-1} (\Lambda_0 + \Lambda_1 P_t),$$  \hspace{1cm} (3.26)

where

$$\Lambda_0 = K_0^P - K_0^Q,$$  \hspace{1cm} (3.27)

$$\Lambda_1 = K_1^P - K_1^Q.$$  \hspace{1cm} (3.28)

The price of risk is the time-variant vector underlying the adjustment to the drift in the change of measure from $\mathbb{P}$ to $\mathbb{Q}$, i.e., it is determined by the difference between the risk neutral distribution and the actual distribution of the risk factors. This accounts for the fact that the movement between distributions captures the investor’s willingness to substitute payments across time. $\Lambda_t$ captures the prices of shocks to the risk factors per unit of volatility. Since it is independent of cashflow patterns of the security being priced, it is common to all securities with payoffs that are functions of the factors, and is implied by the fact that the factors are Gaussian under both distributions. Duffee (2002) and Dai and Singleton (2002) show that this equation is able to match many features of the historical distribution of yields really well. As in Dai and Singleton (2002), the market prices of risk are directly affected by the risk factors, and not only through the factor volatility matrix. $\Lambda_0$ represents the average prices of the factors assigned by investors, whereas $\Lambda_1$ measures how the market prices vary with respect to the risk levels summarized in $P_t$. Finally, $\Sigma_P^{-1}$ captures the interaction of the shocks to the the factors.

3.2.3 International Term Structure Model with Exchange Rates

I extend the JSZ framework into a two-country model, for which most of the JSZ conditions still hold, allowing for a still tractable estimation of the parameters by Maximum Likelihood. Each domestic ($d$) and foreign ($f$) country is affected by their
own set of observable interest rate factors, $\mathcal{P}_t^d$ and $\mathcal{P}_t^f$. Factors are extracted by principal component analysis, as it is standard in the literature, for which they are orthogonal linear combinations of the country yields. The underlying assumption is that factors are observed without error. For $i = \{d, f\}$ the model is now expanded for two countries:

\[
 r_t^i = \rho_0^i + \rho_1^i \cdot \mathcal{P}_t^i, \tag{3.29}
\]

\[
 y_t^{m,i} = A_m^i + B_m^i \cdot \mathcal{P}_t^i, \tag{3.30}
\]

\[
 \Delta \mathcal{P}_t^i = K_0^i \mathcal{P}_t^i + K_1^i \cdot \mathcal{P}_{t-1}^i + \Sigma_i^i \cdot \mathcal{P}_t^i, \tag{3.31}
\]

\[
 \Delta \mathcal{P}_t^f = K_0^f \mathcal{P}_t^f + K_1^f \cdot \mathcal{P}_{t-1}^f + \Sigma_i^f \cdot \mathcal{P}_t^f. \tag{3.32}
\]

Internal consistency conditions still hold, so that $A_p^{m,d} \cdot W^d = A_p^{m,f} \cdot W^f = 0$ and $B_p^{m,d} \cdot W^d = B_p^{m,f} \cdot W^f = I$. Each country has its own risk free rate, and the factors for each country are characterized by independent auto-regressive processes, i.e., there are two independent equations for $\Delta \mathcal{P}_t^d$ and $\Delta \mathcal{P}_t^f$ under both distributions $\mathbb{P}$ and $\mathbb{Q}$. What allows for the preservation of the JSZ framework is the fact that the parameters governing the physical distribution can be estimated from time series alone by OLS. Most importantly, no constraints are being imposed on the physical distribution of $\mathcal{P}_t^i$.

Extending the model to two countries allows for the parameterization of a pricing kernel for each country given equations (3.25) and (3.26)

\[
 M_{t+1}^i = \exp \left( -r_t^i - \frac{1}{2} \left( \Lambda_t^i \Lambda_t^i - \Lambda_t^i \mathcal{P}_t^i \epsilon_{t+1}^i \right) \right), \tag{3.33}
\]

with their respective prices of risk for the domestic and the foreign countries: $\Lambda_t^d = \left( \Sigma_d^i \right)^{-1} \left( \Lambda_0^d + \Lambda_1^d \mathcal{P}_t^d \right)$ and $\Lambda_t^f = \left( \Sigma_f^i \right)^{-1} \left( \Lambda_0^f + \Lambda_1^f \mathcal{P}_t^f \right)$. Given that in an international setting with no arbitrage, the exchange rate between two countries is governed by the ratio of their pricing kernels (Bekaert, 1996), this model is a natural framework.
for exploring exchange rate dynamics. Let the nominal spot exchange rate $S_t$ at time $t$ be the domestic currency price of one unit of the foreign currency. We thus have:

$$\frac{S_{t+1}}{S_t} = \frac{M_{f,t+1}}{M_{d,t+1}}. \tag{3.34}$$

By taking logarithms, so that $\Delta s_{t,t+1} = \log (S_{t+1}) - \log (S_t)$ and $\log \left( M_{i,t+1}^d \right) = -r_i^d - \frac{1}{2} \left( \Lambda_i^d \Lambda_i^d - \Lambda_i^d \epsilon_{t+1}^{\Delta r} \right)$, we can express the log exchange rate change as

$$\Delta s_{t,t+1} = \left( r_i^d - r_i^f \right) + \frac{1}{2} \left[ (\Lambda_i^d \Lambda_i^d) - (\Lambda_i^f \Lambda_i^f) \right] + \left( \Lambda_i^d \epsilon_{t+1}^{\Delta r} - \Lambda_i^f \epsilon_{t+1}^{\Delta r} \right). \tag{3.35}$$

This relationship formally describes the log exchange rate change as the sum of (1) the interest rate differential between the countries, (2) the exchange rate risk premium, and (3) the difference in the shocks to the prices of risk assigned by investors from each country. Recall that interest rates and the prices of risk are linear functions of the risk factors $\{ \mathcal{P}_t^d, \mathcal{P}_t^f \}$. In this context, the same factors that drive variation in the cross-section of interest rates in each country, drive the exchange rate changes between them. Also, taking advantage of the term structure framework of this model, I explore exchange rate changes at different horizons. Hence, $\Delta s_{t,t+k} = \log \left( M_{i,t+k}^f \right) - \log \left( M_{i,t+k}^d \right)$ for $k = 1, 2, ..., K$, where

$$\log \left( M_{i,t+k}^i \right) = \sum_{l=1}^{k} \log \left( M_{i,t+l}^i \right). \tag{3.36}$$

Finally, the model assumes that $\mathcal{P}_t$ and $\{ y_t^d, y_t^f, \Delta s \}$ are priced perfectly, but the observed variables, $\{ y_t^{d,obs}, y_t^{f,obs}, \Delta s_t^{obs} \}$, are measured with error; such that

$$e_t = \begin{bmatrix} e_t^{d,y} \\ e_t^{f,y} \\ e_t^{\Delta s} \end{bmatrix} = \begin{bmatrix} y_t^{d,obs} - y_t^d \\ y_t^{f,obs} - y_t^f \\ \Delta s_t^{obs} - \Delta s_t \end{bmatrix}. \tag{3.37}$$
3.3 Data and Estimation Strategy

Monthly interest rate and exchange rate data from January 1999 to January 2014 are obtained from Bloomberg, L.P. 2014, for a total of 181 observations. The dataset contains interest rates at 12 different maturities: 3 months, 6 months, and 1 to 10 years for

- countries with independent currencies: Australia, Canada, Japan, Norway, Sweden, Switzerland, the United Kingdom, and the United States; and
- countries under the euro: Austria, Belgium, Finland, France, Germany, Ireland, Italy, the Netherlands, Portugal, and Spain.

All interest rates are end-of-month data, annualized, and expressed in the local currency. All the (end of month) exchange rates are expressed in terms of U.S. dollars (USD). I calculate log exchange rate changes at different horizons: 1, 3, 6, 12, 18, 24, 30, 36, 42, 48, 54, and 60 months. The 7 currencies are: Australian dollar (AUD), Canadian dollar (CAD), Japanese yen (JPY), Norwegian krone (NOK), Swedish krona (SEK), Swiss franc (CHF), and British pound (GBP).

I extract the three main principal components by the eigenvalue-eigenvector decomposition on the variance-covariance matrix of each country’s set of yields. Typically, the first two or three main components are enough to explain more than 99% of the variability in the cross-section of yields within a country, and are interpreted to be the level, slope, and curvature of a country’s yield curve. In this model, the cross-sectional variation captured by the interest rate components also drives exchange rate fluctuations at different horizons.

The first step in the estimation strategy is to optimize over \( \{ \Sigma_p, \lambda^Q \} \), where \( \Sigma_p = \{ \Sigma^d_p, \Sigma^f_p \} \) and \( \lambda^Q = [ \lambda^{d,Q} \lambda^{f,Q} ]' \). \( \Sigma_p \) is parameterized by the (lower triangular) Cholesky factorization of the variance-covariance matrices from the vector auto-regressions under \( \mathbb{P} \) for both countries. I do not impose any restrictions between
factors of different countries since their dynamics are modeled separately; Joslin, Le,
and Singleton (2013) argue that these initial estimations of $\Sigma_P$ will be very close to
the optimal values. $\lambda^Q$ is assumed to be real and it is parameterized by the difference
in ordered eigenvalues. Therefore, the model is $Q$ stationary because the difference in
eigenvalues is negative by construction. The optimal estimates of \{\Sigma_P, \lambda^Q\} are ob-
tained by constrained nonlinear optimization, in which the eigenvalues are bounded
so that $\lambda^Q < 0$ and are constrained to be greater than -0.95. $\Sigma_P$ is unbounded ex-
cept for the diagonal elements in each country, which are bounded from below by
0 to preserve singularity. Forcing a positive diagonal for the state transition matrix
implies the stability criteria for arbitrage free models and does not conflict with the
trends of dynamic models.

To summarize, the parameters to be estimated are $\Theta = \{K^{P,i}_{0P}, K^{P,i}_{1P}, \Sigma_P, \lambda^Q, k^Q_i\}$
and the observable (interest rate) factors are $P_t = \left[ P^d_t, P^f_t \right]'$.

The conditional log likelihood function can be factorized into the risk neutral and
the physical conditional densities of the observed data:

$$\log \left( f \left( z^{obs}_t | z^{obs}_{t-1}; \Theta \right) = f^Q \left( z^{obs}_t | P_t \right) + f^P \left( P_t | P_{t-1} \right) , \right.$$ 

where $z^{obs}_t$ is the vector of observed yields for the domestic and foreign countries and
the observed log exchange rate changes at different horizons between the countries:

$$z^{obs}_t = \left[ y^{d,obs}_t, y^{f,obs}_t, \Delta s^{obs}_t \right]'$$. The $P$ conditional density of the observed vector
captures the time series properties of the factors, whereas the $Q$ conditional density
captures the cross-sectional properties of yields.

Using the assumption that $P_t$ is conditionally Gaussian, the log likelihood under
$P$ can be expressed as

$$f \left( P_t | P_{t-1} \right) = \frac{N}{2} \cdot \log (2\pi) + \frac{1}{2} \cdot \log \left| \Sigma_P \right|$$

$$+ \frac{1}{2} \sum_{t=1}^{T} \left( (P_t - E_{t-1}[P_t]) \cdot (\Sigma_P)^{-1} \cdot (P_t - E_{t-1}[P_t]) \right) , \quad (3.38)$$
where $N = N^d + N^f$ is the total number of observable factors in the model,

$$E_{t-1} [\mathcal{P}^i_t] = K_0^{P,i} + (I + K_1^{P,i}) \mathcal{P}^i_{t-1},$$  \hspace{1cm} (3.39)

$$\Sigma'_{\mathcal{P}} \Sigma_{\mathcal{P}} = \begin{bmatrix} 
\Sigma^d_{\mathcal{P}} \Sigma^d_{\mathcal{P}} & \Sigma^f_{\mathcal{P}} \Sigma^d_{\mathcal{P}} - \Sigma^d_{\mathcal{P}} \Sigma^f_{\mathcal{P}} \\
\Sigma^d_{\mathcal{P}} \Sigma^f_{\mathcal{P}} & \Sigma^f_{\mathcal{P}} \Sigma^f_{\mathcal{P}} 
\end{bmatrix}. \hspace{1cm} (3.40)$$

Imposing zeros on the off-diagonal elements of the joint variance-covariance matrix of the factors instead, does not seem to matter for estimation purposes; these numbers are actually very small. The parameters $(K_0^{P,i}, K_1^{P,i})$ that maximize the log likelihood function $f$ (conditional on $t = 0$ information) are given by the OLS estimates of the conditional mean of $\mathcal{P}_t$.

The log likelihood under $Q$ can be expressed as

$$f \left( z_{it}^{obs} \mid \mathcal{P}_t \right) = \frac{1}{2} \sum_{t=1}^{T} c_t^2 / \Sigma_e^2 + \frac{1}{2} (J - N) \cdot \log (2\pi) + \frac{1}{2} (J - N) \cdot \log (\Sigma_e^2), \hspace{1cm} (3.41)$$

where $c_t^2$ is the vector with square errors between the actual and the model-implied yields and exchange rates as in $(3.37)$ and $\Sigma_e = \sqrt{\frac{\sum_{t=1}^{T} c_t^2}{J-(J-N)}}$, where $J$ is the number of observed variables.

3.4 Results

In order to obtain a general measure of goodness of fit, I perform a linear projection of the data on the model-implied equations. In other words, given the model equations describing yields and exchange rates:

$$\hat{y}^m_t = A^m_{\mathcal{P}} + B^m_{\mathcal{P}} \cdot \mathcal{P}_t,$$  \hspace{1cm} (3.42)

$$\Delta s_{t+h} = \sum_{i=1}^{k} \left( r^d_{t+i-1} - r^f_{t+i-1} \right) + \frac{1}{2} \sum_{i=1}^{k} \left[ \left( A^d_{t+i-1} A^d_{t+i-1} \right) - \left( A^f_{t+i-1} A^f_{t+i-1} \right) \right]$$

$$+ \sum_{i=1}^{k} \left( A^d_{t+i-1} e^d_{t+i} - A^f_{t+i-1} e^f_{t+i} \right), \hspace{1cm} (3.43)$$

I perform an OLS regression of the data on the model-implied equation to obtain adjusted $R^2$ as a measure of overall fit for country/currency $i$: 96
\[ y_{t}^{obs,i} = \alpha_{i} + \beta_{i}y_{t}^{i} + \mu_{t}^{i} \]  \hspace{1cm} (3.44)
\[ \Delta s_{t}^{obs,i} = \alpha_{i} + \beta_{i}\Delta s_{t}^{i} + \mu_{t}^{i}. \]  \hspace{1cm} (3.45)

Table 3.1 reports the estimates along with the standard errors in parentheses and the adjusted \( R^2 \) for equation (3.44) at selected maturities. The model yields are a function of three country-specific factors, denoted level, slope, and curvature. The adjusted \( R^2 \) close to one indicates the model implied yields are able to reproduce the actual yields observed in the data very well. Moreover, the coefficients are significant at the 99% confidence level and are very close to unity. The weakest fit is for Japan’s yields; the 3-month and 7- and 10-year yields have an \( R^2 \) below 90%. Given Japan’s term structure, three factors may not be enough to fully account for the cross-sectional variation of interest rates during the period considered. Accounting for more factors may be important in the future, particularly for better explaining Japan’s yield curve. More importantly, incorporating exchange rates into the international JSZ framework does not compromise the fit of the yields.

Table 3.2 displays results for equation (3.45), which accounts for the goodness of fit of the annualized model-implied exchange rate changes at different \( k \) horizons (from 1 month to 5 years). I compare the results obtained under two different factor specifications: the first column indicates the results for a two-factor model, constructed by using the first two factors (level and slope) of each country’s yield curve, for a total of four factors; and the second column, reports the results for a three-factor model, which incorporates the level and slope of the yield curve plus a third factor (curvature) from each country. This model determines the exchange rate changes as a function of six total factors. The domestic country is chosen to be the U.S. so that the exchange rates are in terms of USD prices. Therefore, an increase in \( \Delta s_{t}^{i} \) represents a depreciation of the U.S. dollar with respect to the foreign currency, \( i \). The results for \( \beta_{i} \), along with the corresponding standard errors and the adjusted \( R^2 \), are reported for selected horizons (1 month, 6 months, and 1, 3, and 5 years).
<table>
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<td>(0.02)</td>
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<td>0.97</td>
<td>0.97</td>
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<td>0.97</td>
</tr>
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<td>1.00***</td>
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<td>0.99***</td>
<td>0.99***</td>
<td>0.99***</td>
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<td>(0.01)</td>
<td>(0.01)</td>
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<td>(0.01)</td>
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<td>0.98</td>
<td>0.97</td>
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</tr>
<tr>
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<td>β</td>
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<td>1.00***</td>
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<td>0.98</td>
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### TABLE 3.2

#### MODEL FIT FOR LOG EXCHANGE RATE CHANGES

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<th>5-year</th>
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<td>2-factor</td>
<td>3-factor</td>
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<td><strong>β</strong></td>
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<td>0.85</td>
</tr>
<tr>
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<td>(0.23)**</td>
<td>(0.20)**</td>
<td>(0.10)***</td>
<td>(0.12)***</td>
<td>(0.09)***</td>
</tr>
<tr>
<td>N.W.</td>
<td>(0.22)**</td>
<td>(0.23)**</td>
<td>(0.17)***</td>
<td>(0.21)***</td>
<td>(0.15)***</td>
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<tr>
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<tr>
<td><strong>β</strong></td>
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<td>0.45</td>
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</tr>
<tr>
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<td>(0.12)***</td>
<td>(0.10)***</td>
<td>(0.13)***</td>
</tr>
<tr>
<td>N.W.</td>
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<td>(0.21)**</td>
<td>(0.19)***</td>
<td>(0.18)***</td>
<td>(0.24)***</td>
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<tr>
<td><strong>R²</strong></td>
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<td>0.01</td>
<td>0.05</td>
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<td>(0.13)***</td>
<td>(0.11)***</td>
<td>(0.08)***</td>
</tr>
<tr>
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<td>(0.27)***</td>
<td>(0.22)**</td>
<td>(0.23)***</td>
<td>(0.20)***</td>
<td>(0.19)***</td>
</tr>
<tr>
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<td>0.05</td>
<td>0.20</td>
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<tr>
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<td>-0.00</td>
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<tr>
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<td>0.58</td>
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<td>(0.12)***</td>
<td>(0.10)***</td>
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<td>(0.21)***</td>
<td>(0.23)***</td>
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<td>0.05</td>
<td>0.12</td>
<td>0.30</td>
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</table>
At the one-month horizon, the fit of the model ranges from 0% (Swiss franc) to 5% (Japanese yen, Norwegian krone, and British pound). Very short exchange rate fluctuations are quite difficult to predict. Meese and Rogoff (1983) claimed that the best forecasting model is obtained under the assumption that the exchange rate follows a random walk. However, at longer horizons the predictability of exchange rates improves significantly. This type of behavior is observed in other assets as well. Although short term movements are not well understood, it seems that at longer horizons the factors are better able to capture the signal that drives movements in asset returns. Under the three-factor specification, for example, the model is able to explain about half the variation in one-year changes, and more than 3/4 of the variation in the 5-year changes. The Canadian dollar and the Swiss franc exhibit the worst fit in the sample. The three-factor Canadian dollar has an $R^2$ around 30% at the one-year horizon and the Swiss franc, 57% for the 5-year horizon. The best fit is for the British pound (61% at the 1-year horizon and over 90% at the 5-year horizon). Figure 3.1 shows the model fit at the one-year horizon for both Switzerland and the United Kingdom.

One potential concern when modeling exchange rates is small-sample bias and the possibility of correlation and conditional heteroskedasticity in the error terms. Small-sample bias arises due to the high persistence of exchange rates, and becomes particularly problematic when the sample period is short and the dynamic process is very severe. To this end, I estimate robust (or White) standard errors to account for small-sample bias and Newey-West standard errors to account for potential error correlation and heteroskedasticity in the innovation vector. Similar to Lustig, Roussanov, and Verdelhan (2011), I follow Andrews (1991) to choose the number of lags for the Newey-West estimation to be the horizon $k$ (in months) plus one. After the 6-month horizon, all of the estimates are statistically significant at the 99% confidence level regardless of the method to calculate standard errors.
Figure 3.1. Exchange Rate Fit (Switzerland and the U.K.)
In general, the *three-factor model* outperforms the *two-factor model*; that is, adding a third factor to the model, the curvature factor, significantly contributes to explaining exchange rates, despite having low-content information to explain cross-sectional interest rate variation. For example, at the one-year horizon, the level and slope factor account for 19-37% of exchange rate movements, whereas the level, slope, and curvature together, explain 31-61%. I study the model performance under different factor structures: a *one-factor model* and a *four-factor model* in Table 3.3. The level-only model accounts for less than 5% of short-term movements for Norway, Canada, and the U.K, but it seems to capture between 4% and 67% of longer term movements, depending on the currency. The *four-factor model*, outperforms the *three-factor model* for every currency except for the British pound. An important direction in accounting for model-specification is the interpretation of the factors. Although the first three factors are related to movements in the yield curve (level, slope, and curvature), it is not clear what the macroeconomic sources of these factors are and how they relate to the exchange rates. A fourth factor, moreover, seems to contribute to explaining exchange rate fluctuations but has no interpretation at all. In addition, extracting higher dimensions of the yield curve can become problematic as the eigenvalues get closer to zero. Finally, it may be necessary to perform Chi-Square or other model-selection tests to determine the optimal factor-specification to model exchange rates under a parsimonious but still flexible approach.

Another potential concern might be the idea of some inherent property of the U.S. dollar driving the results, since it is chosen as the domestic country and the base for exchange rates in all previous estimations. The U.S. may not be an ideal base country since its term structure exhibits a regime change in 2008, in which short-term yields drop to the zero bound, exhibiting lower volatility and changes in the distribution. I therefore re-estimate the model by considering a different base country, the U.K., so that exchange rates are in terms of the British pound, instead of the U.S. dollar.
### TABLE 3.3

**MODEL FIT FOR EXCHANGE RATE CHANGES (OTHER MODELS)**

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<th>6-month</th>
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<th>5-year</th>
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<td>(0.14)***</td>
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<td>(0.12)***</td>
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TABLE 3.4
MODEL FIT FOR LOG EXCHANGE RATE CHANGES (GBP BASE)

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<td>(0.08)**</td>
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<td>(0.17)**</td>
<td>(0.13)**</td>
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<tr>
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<tr>
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</tr>
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<td>(0.17)**</td>
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<td>(0.21)**</td>
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<td>(0.15)**</td>
</tr>
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<td>0.01</td>
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</tr>
<tr>
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<td>0.58</td>
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<td>0.83</td>
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<td>(0.09)**</td>
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<td>(0.07)**</td>
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<td>(0.16)**</td>
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<td>(0.15)**</td>
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<td>0.05</td>
<td>0.21</td>
<td>0.30</td>
<td>0.52</td>
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</table>
Table 3.4 reports the coefficients, robust and Newey-West standard errors, and the adjusted $R^2$ for all countries in which the log exchange rate changes are modeled as the log difference of their pricing kernels with respect to the U.K. The exchange rate results under the new base are significantly better, indicating that the U.S. term structure may not be correctly capturing exchange rate dynamics under a constant volatility model.

3.4.1 Uncovered Interest Parity (UIP)

In this section, I evaluate my model in the context of the Uncovered Interest Parity (UIP) puzzle. UIP claims that any excess return on foreign-denominated deposits is offset by the expected depreciation against the domestic currency. That is, if we run the following regression

$$\Delta s_{t,t+k} = \alpha + \beta_{UIP} \sum_{i=1}^{k} (r_{t+i-1}^d - r_{t+i-1}^f) + \mu_{t+k}$$

we would expect that $\alpha = 0$, $\beta_{UIP} = 1$. However, empirical evidence suggests this proposition does not hold, since the coefficient on the interest rate differential is not only different from one, but also negative, and the explanatory power of the interest rate differential is very low. Several papers have focused on studying this phenomenon.\(^4\) In my model, the UIP condition does not hold, since the expected log exchange rate change is not only a function of the interest rate differential but also a function of a risk premium term implied by my framework:

$$E_t\Delta s_{t+k} = \alpha + \beta_{ID} \sum_{i=1}^{k} (r_{t+i-1}^d - r_{t+i-1}^f) + \beta_{RP} \left( \frac{1}{2} \sum_{i=1}^{k} \left[ (\Lambda_{t+i-1}^d \Lambda_{t+i-1}^d) - \left( \Lambda_{t+i-1}^f \Lambda_{t+i-1}^f \right) \right] \right) + \mu_{t+k}.$$

This is consistent with the idea proposed by Fama (1984) and Hansen and Hodrick (1983), that attribute the anomalous findings to a time-varying risk premium. I eval-

\(^4\)Extensive surveys on the UIP literature and anomalies can be found in Hodrick (1987), Froot and Thaler (1990), and Engel (1996).
uate the ability of my model to reproduce the UIP puzzle when equation (3.46) is estimated and I examine to what extent, the augmented UIP regression implied by my model - equation (3.47), can help solve the puzzle. Table 3.5 shows the results for the UIP and augmented-UIP regressions with the three-factor model, respectively. The coefficient for the UIP regression, $\beta_{\text{UIP}}$, is typically negative and statistically insignificant, although positive for short-term regressions in Canada, Sweden, and the U.K. Moreover, the adjusted $R^2$ is close to zero for every country. This is consistent with the UIP puzzle observed in the data. When incorporating the model-implied risk premium, the adjusted $R^2$ always increases, and the coefficients on the interest rate differential and the risk-premium, $\beta_{ID}$ and $\beta_{RP}$, are always positive, usually statistically significant, and much closer to one. These results suggest that the risk premium term is clearly important in modeling exchange rates, even when only interest rate factors are considered, and can (partially) explain the UIP puzzle.

I further examine each one of the components of the model exchange rate. Table 3.6 presents the correlation between the observed log exchange rate changes and the three components of the exchange rate implied by the model. Whereas the correlation between the data and the interest rate differential or the relative shocks to the price of risk is typically negative or small, the risk premium is always large and positively correlated with the exchange rate at all horizons. This confirms the idea that the risk premium is a key driver of exchange rates, which is consistent with the Fama (1984) and Hansen and Hodrick (1983) explanation of the UIP puzzle, and does not rely solely on large shocks to account for movements in the exchange rate.
### TABLE 3.5

**UNCOVERED INTEREST PARITY REGRESSIONS**

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<td>$\beta_{UIP}$</td>
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<td>(2.61)***</td>
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<td>0.03</td>
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<td>(2.31)***</td>
<td>(1.18)***</td>
<td>(1.11)***</td>
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<td>(2.37)***</td>
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<td>(1.31)***</td>
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<td>(1.19)***</td>
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<td>(0.86)***</td>
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<td>(1.71)***</td>
<td>(1.86)***</td>
<td>(1.46)***</td>
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Besides attributing the failure of UIP to the presence of a time-varying risk premium, Fama (1984) identified two conditions necessary for the risk premium to solve the UIP puzzle. First, the risk premium has to be more volatile than the interest rate differential, and second, the model-implied risk premium and the interest rate differential need to be negatively correlated. Table 3.7 shows that the Fama conditions hold for the three factor model, for all currencies, and for all horizons considered. The variance of the risk premium is higher than the variance of the interest rate differential, mostly twice as much. Also, the correlation between the interest rate differential and the risk premium is always negative and large, as expected.

3.4.2 Analysis of Countries Under the Euro

In the baseline model, the factors extracted from two countries help determine their exchange rate fluctuations through each country’s stochastic discount factor. What are the factors that determine movements in the euro/U.S. dollar exchange rate? I explore the determination of the path of the euro by constructing artificial currencies for ten countries under the euro regime: Austria, Belgium, Finland, France, Germany, Ireland, Italy, the Netherlands, Portugal, and Spain. That is, I estimate pricing kernels for those countries individually to explore whether their interest rate factors alone contribute to explaining changes in the euro determination. As expected, the yield curve factors for a country like Germany are important in determining changes in the euro. However, risk factors from countries with more volatile interest rates, such as Italy and Spain, also have high explanatory power on the exchange rate dynamics. The results suggest that the risk factors that govern fluctuations in the euro are priced in the yield curves of different European countries to different extents.

Table 3.8 reports the results for the OLS regression from equation (3.45) for ten European countries.
### Table 3.8

**Model Fit for Log Exchange Rate Changes (Euro)**

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All the countries seem to have some degree of explanatory power on the euro. In general, the same conclusion holds for this set of countries: exchange rate predictions improve at longer horizons and the addition of a curvature factor generally improves exchange rate fit. At the one-year exchange rate change, the model with three factors is able to explain between 28% and 58% of variation. Ireland and Portugal have the worst fit (35% and 28%), whereas Belgium, Spain, and Italy have the best fit (50%, 52%, and 58%). However, in the long-run, all countries explain between 82% and 89% of total variation in the euro. Interestingly, the model with two factors only is very good at predicting five-year changes as well (79% to 86%). This suggests that the curvature may not be able to contribute to long-run information on the determination of the euro. These results suggest that interest rate risk in European countries may contribute to the exchange rate determination of the euro. Understanding what these factors are and how they affect the exchange rate dynamics has important policy implications and requires further study.

3.5 Conclusion

In this Chapter, I examine whether interest rate factors alone can help determine fluctuations in the exchange rate between two countries. To this end, I expand a Gaussian dynamic term structure model to incorporate exchange rates, modeled as the ratio of two countries’ stochastic discount factors. My results suggest that a term structure model is in fact able to capture bilateral exchange rate dynamics using only interest rate factors, particularly in the long run -consistent with other assets. Moreover, a third and a fourth yield curve factor, despite having a low information content with regards to the cross-section of interest rates, seems to significantly contribute to modeling exchange rate fluctuations. These results are robust to different calculations of standard errors to account for sample bias and potential heteroskedasticity and correlation in the error vector, as well as a different specification of the base
currency. This has relevant implications for the international economics literature, since the estimated risk premium is the main component in explaining exchange rate movements, and it seems to (partially) account for the UIP puzzle, when modeled by observable factors from the yield curve. Moreover, the model-implied risk premium is consistent with the Fama (1984) conditions needed to account for violations in UIP. Understanding the connections within asset prices through the pricing kernel will help bring the macroeconomics and finance literature even closer.

This chapter provides evidence that the same stochastic discount factor (or equivalently, a representative agent) can price interest rates and exchange rates at different horizons relatively well, by only using observable interest rate factors. This suggests that investors in each country price different assets through the same pricing kernel, or the same stochastic discount factor, given a low-dimensional vector of information. Another interesting extension is to derive the necessary conditions for the exchange rates to be modeled under no arbitrage without the assumption of complete markets. The intuition behind this notion comes from the idea that the exchange rate today is the discounted value of the exchange rate in the future. In other words, the exchange rate is an asset, as in Engle and West (2005). For example, a two-year foreign bond bought by a domestic investor, if held to maturity, involves the exchange rate two years from today, since the investor would have to convert the proceeds from the foreign bond into the domestic currency.

Term structure models impose cross-equation restrictions to ensure no arbitrage, but they do so in the bond markets only, by restricting the coefficients in the exponentially affine bond pricing equation. However, in an extended model that seeks to price other assets as well, the literature is silent on whether no-arbitrage conditions should be implemented in the new pricing equations. The argument is as follows. If exchange rates are tradable assets themselves in well-developed markets, the equations that determine exchange rate fluctuations at different horizons should also be
subject to no-arbitrage restrictions. Therefore the arbitrage-free value of the current exchange rate can be expressed as an exponentially linear function of the domestic and foreign factors. I leave these questions for further research.


APPENDIX A

MANIPULATION OF INFLATION IN ARGENTINA (2007-2012)

The role of transparent institutions has been proposed as central to the development of a society in the economic growth literature. The Argentine case of data manipulation has not only affected the Latin American community but is also at the heart of world-wide debates around the globe. This topic is conceptually innovative as well as controversial, since the government is in denial of this phenomenon. Although the reasons for government’s intervention on national statistics have been explored, the consequences of this ongoing corruption are still uncertain. Unfortunately, corrupted statistics are not an isolated event but a phenomenon that has persisted throughout history in different political regimes that look for self-preservation of power. Next, I provide a summary of the current state of knowledge on the issue as well as an up-to-date comprehensive summary of Argentina’s data manipulation.

Argentina’s history has been dampened with political turmoil, devaluations, military coups, capital flights, multiple currencies, and increasing external debt since the beginning of the twentieth century. However, after the 2001 crisis following the devaluation of the peso, the economy was stable under a strict regime of price controls to prevent prices from rising. In anticipation of an inflationary outburst by the end of 2006, the government decided to take direct action by changing the measurement techniques used to estimate the Consumer Price Index (CPI) produced by the National Statistics and Census Institute (INDEC). Since then, the official CPI has been

doubted by academics and international organizations, given that the new methodology implemented was never fully disclosed and there have been serious accusations of data manipulation.

Before the intervention, the INDEC started to receive serious pressure from the government upon expectations of higher inflation and suspicion of noncompliance with the imposed price controls. During the last quarter of 2006, the Secretary of Commerce, Guillermo Moreno, inquired about the location of stores surveyed to construct the CPI in Argentina along with other confidential data. The INDEC technicians refused to provide such information, invoking Law 17,622 promulgated in 1968 upon the institute’s creation, under which surveyed data for statistical purposes are protected from political interference. Therefore, in February 6, 2007, the CPI Director was replaced, and Beatriz Paglieri was instated as the new director by the recently elected President Cristina F. de Kirchner. The arrival of Paglieri and a new CPI series, marked the beginning of what is known as the intervention of the national statistics. From then on, technicians of all ranks who argued or inquired about the methodological changes of statistics were replaced by new workers appointed by the government.

Given the discrepancies with other provinces’ inflation numbers, in May 2008 the INDEC stopped the publication of the National CPI, leaving the manipulated CPI constructed in Buenos Aires as the sole official source of inflation. However, rumors of manipulation were intensifying within the international community. This prompted the INDEC to create an Advisory Committee (Consejo Asesor), comprised of intel-

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3This protection is given in articles 13, 14, 15 and 17 of law 17,662.
lectuals from Argentine universities, to provide academic support to the intervention. This move backfired, however, when in December 2009 the results presented by the Advisory Committee concluded that the official CPI was no longer trustworthy and required immediate reform for the provision of transparent information.

Although the methodological changes were never fully disclosed, the government attempted to justify the intervention by claiming that previous CPI series had been purposefully inflated to benefit holders of inflation-linked securities. But the government’s attempt to lower the external debt by understating inflation is not enough justification to manipulate statistics, given that there are also securities tied to real GDP, which is now overstated and yields higher coupon payments for the holders of those securities. The government also claimed that high inflation rates were product of boycotts from the opposition or typing mistakes from the former technicians. Although the political motive behind the intervention is not clear, the most likely explanation points to a government’s reluctant attempt to control a potential inflationary outburst after years of stability that could compromise the government’s standing.

Regardless of the motive, the sudden change of methodology to measure the CPI did not comply with any of the requirements established by international regulations. The methodological changes reduced by half the basket of goods, which originally contained around 800 different products, and removed goods with high price variation. There were also price caps implemented, so that certain products could not register large price increases and those that were subject to price controls were not allowed to differ from the government’s agreed price. Under an environment of strict price controls and unreliable statistics, Argentina’s credibility has been shattered. The government intervention evidenced the fragility of national institutions, which bears

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a high political cost. Even during the 2009 swine flu outbreak, international organizations doubted the government’s numbers of reported swine flu cases, claiming they were grossly understated. This is only one example of the lack of credibility generated by dubious statistical reports.

Since the CPI series is incorporated into estimations of other crucial macroeconomic variables, such as the calculation of poverty rates and GDP, as well as the indexation of bonds, there is a general disbelief in all of the country’s official statistics. The latest press release by the IMF in February 2012 urges the Argentine authorities to correct the situation.

Given the obligations of all member countries to provide accurate data to the International Monetary Fund (IMF), on February 1, 2012, the Executive Board met to consider the Managing Director’s report on proposals for remedial measures that Argentina has to implement to address the quality of the official data reported to the Fund for the Consumer Price Index for Greater Buenos Aires (CPI-GBA) and Gross Domestic Product (GDP). The Executive Board regretted the absence of progress in aligning the CPI-GBA with international statistical guidelines and took note of the authorities’ intentions to adopt some remedial measures to address the quality of its reported GDP data. The Executive Board approved a decision that calls on Argentina to implement specific measures, within a period of 180 days, to address the quality of reported CPI-GBA and GDP data, with a view to bringing the quality of the data into compliance with the obligation under the Articles of Agreement. The Managing Director shall, by September 6, 2012, report to the Executive Board on the status

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of implementation by Argentina of the above-mentioned measures.

The lack of credible figures pushed the search for alternative ways to estimate inflation in Argentina. The IMF resolved to multiple indices to assess the situation in the country. Also, in the article Don’t Lie to me Argentina, The Economist explains why they stopped reporting official data and details the international sources they now choose to use. Cavallo (2011), for example, uses daily prices collected from online supermarkets to construct alternative CPI series for Argentina. He shows how his estimation with “scraped price indexes” yields no discrepancies with official statistics in Brazil, Chile, Colombia, and Venezuela, providing further evidence that Argentina’s government indices do not reflect the true level of inflation in the country. Private companies and consultancy agencies reverted to their own calculations of inflation as well. In response, the government started to dissuade unofficial statistics by intimidating their authors with threats of monetary fines and jail time.

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8 Official statistics - Don’t lie to me, Argentina, Why we are removing a figure from our indicators page, http://www.economist.com/node/21548242.

