β-DELAYED NEUTRON EMISSION STUDIES OF NEUTRON-RICH PALLADIUM AND SILVER ISOTOPES

A Dissertation

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The rapid-neutron capture process \((r\text{-process})\) is attributed as the source of nearly half the elements heavier than iron in the solar system. The astrophysical scenario responsible is still unknown, making the specific astrophysical conditions uncertain. To gain insight into the \(r\)-process nucleosynthesis, uncertainties in the nuclear properties of the involved isotopes must be minimized. The \(r\)-process path traverses nuclei far from stability through many isotopes which have not yet been produced in the laboratory. This forces astrophysical models to rely heavily upon theoretical models of the involved nuclear physics. To help constrain parameters used in both astrophysical and nuclear models, an experiment was performed to measure properties of neutron-rich nuclei just below the \(N = 82\) shell closure. These nuclei are believed to be responsible for production of the \(A = 130\) peak in the solar \(r\)-process abundance pattern. The first measurements of \(\beta\)-decay half-lives and neutron branching ratios, \(P_n\) values, were performed for neutron-rich isotopes of palladium and silver.
To Kristen. I’d be nowhere without you.
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1.1 Nuclear Astrophysics Overview

Nuclear astrophysics seeks to explain the formation of the elements observed in the universe. Specifically, the abundance distribution observed in our solar system. The study of the content of our sun has a long history starting with the quantum mechanical description of stellar fusion by Atkinson & Houtermans [1] in 1929 and the introduction of nucleosynthesis within the stellar evolution by Hoyle [2] in 1946. The knowledge of the period was then summarized in the seminal papers by Burbidge et al. [3] and Cameron [4] in 1957. These two papers outlined the possible stellar processes and provides the modern basis for current research in the field of nuclear astrophysics.

The general trends of the solar abundance distribution, as shown in Figure 1.1, can be explained by a number of cosmic processes. During the Big Bang, the large fraction of observed hydrogen and helium was produced. Beryllium and boron were produced through cosmic spallation, while the heavier metals were produced through stellar processes. The first and primary source of energy generation in stars is from hydrogen and helium burning generating heavier material. In massive stars this is followed by burning of heavier elements through neon, oxygen, and silicon burning.
This process of fusion eventually terminates in what is referred to as the iron-peak. Elements heavier than iron are primarily produced through neutron capture processes. There are two dominant processes a slow and a rapid neutron capture process, known respectively as the $s$-process and $r$-process. In addition, a small fraction of material is produced by a proton process, known as the $p$-process. [5] The following section expands on the various processes with a significant focus on the $r$-process.

![Figure 1.1. Solar abundances highlighting the processes responsible for their production [6].](image-url)
1.2 Big Bang Nucleosynthesis

The universe formed during a massive expansion phase called the Big Bang. During this phase the process of Big Bang Nucleosynthesis (BBN) [7, 8] was responsible for the formation of the early universe, which was composed of mainly hydrogen and helium, as well as a small amount of deuterium and lithium. Although there is some discrepancy in the $^7$Li abundance [9] the mass fraction of hydrogen, 74%, and helium, 25%, is considered to be well understood. This early material condensed to form the first stars.

1.3 Stellar Fusion

The formation of the first stars marked the end of the cosmic dark ages, in which no visible or infrared sources of light existed. These early stars, called population III, were composed almost entirely of the hydrogen and helium ejected from the Big Bang. These stars initially burned the hydrogen fuel until it was exhausted. This was followed by helium burning of material from the star’s formation and that which was produced during hydrogen burning. The helium burning phase enriched the early stars with metals. After the exhaustion of helium, these metals continued to produce energy through fusion until reaching the most bound nuclei at which point energy generation by fusion is no longer favorable. For a detailed review of stellar evolution see Carroll & Ostlie, 2006 [10] or Iliadis, 2007 [11].
1.3.1 Hydrogen Burning

The majorly of a star’s life is spent in the hydrogen burning phase. This is the principal energy generation process during a star’s lifetime. There are two main reaction sequences responsible for the destruction of hydrogen, the p-p chain and the CNO cycle.

1.3.1.1 p-p Chain

The p-p chain converts protons into helium and releases $26.22\text{MeV}$ of energy. The primary reaction branch, pp I, undergoes the following reactions:

\[
\begin{align*}
    p + p & \rightarrow ^2\text{H} + e^+ + \nu_e \\
    p + ^2\text{H} & \rightarrow ^3\text{He} + \gamma \\
    ^3\text{He} + ^3\text{He} & \rightarrow ^4\text{He} + 2p
\end{align*}
\]

This process is responsible for a majority of the energy production in the pp-chain. There are additional reaction chains that release various amounts of energy. These include the pp-II, pp-III and the pep chains [12, 13]. The pp-chains are the dominant form of energy generation in low mass stars.

1.3.1.2 CNO Cycle

The second process responsible for energy generation by hydrogen burning is the CNO cycle. This process requires that the stellar material has already been enriched with material heavier than hydrogen and helium. Due to the requirement of enrichment this process cannot occur in population III stars until the onset of
helium burning. The material used in the formation of population II stars has been enriched by the prior generation’s stellar nucleosynthesis and this process is accessible prior to the helium burning phase. Massive star energy generation is dominated by the CNO cycle over the previously described pp-chain.

During the CNO cycle four protons are consumed and a helium nucleus is produced generating a net energy release of $26.73\,\text{MeV}$. The process undergoes the following reactions:

\[
p + ^{12}\text{C} \rightarrow ^{13}\text{N} + \gamma \\
^{13}\text{N} \rightarrow ^{13}\text{C} + e^+ + \nu_e \\
p + ^{13}\text{C} \rightarrow ^{14}\text{N} + \gamma \\
p + ^{14}\text{N} \rightarrow ^{15}\text{O} + \gamma \\
^{15}\text{O} \rightarrow ^{15}\text{N} + e^+ + \nu_e \\
p + ^{15}\text{N} \rightarrow ^{12}\text{C} + ^4\text{He}
\]

The limiting reaction is the $^{14}\text{N}(p,\gamma)^{15}\text{O}$ reaction. This reaction sets the time-scale of the cycle and causes an increased abundance of $^{14}\text{N}$. These reactions describe the main CNO cycle although there exist smaller branches into other similar cycles. Under high temperature conditions, this cycle may be modified into the Hot CNO cycle. These temperatures are achieved in more extreme cases such as novae and x-ray bursts [14].

1.3.2 Helium Burning

After the hydrogen fuel has been depleted in the stellar core, the core begins to collapse until the increasing temperatures become high enough that helium burning
may proceed. Helium burning proceeds initially through the triple-\(\alpha\) process in which three helium nuclei, \(\alpha\) particles, are fused to form \(^{12}\text{C}\) and releases 7.273 MeV of energy. This process involves a two-step reaction:

\[
\alpha + \alpha \rightarrow ^{8}\text{Be} \\
\alpha + ^{8}\text{Be} \rightarrow ^{12}\text{C}
\]

This reaction chain is not significant until high stellar temperatures are reached. To successfully produce \(^{12}\text{C}\), the fusion of another \(\alpha\) particle with the beryllium nucleus must occur immediately following the first reaction, prior to the rapid decay of the unstable \(^{8}\text{Be}\) nucleus [15].

This process is followed by subsequent \(\alpha\) captures producing \(^{16}\text{O}\). Fusion beyond this point is inhibited as it becomes an endothermic process due to decreasing binding energies per nucleon. Figure 1.2 shows the measured nuclear binding energies as a function of nucleon number, \(A\). The trend rises from hydrogen until reaching its maximum at \(A = 56\).
Figure 1.2. Binding energy per nucleon as a function of nucleon number. Each point represents a single isotope. Fusion continues increasing nucleon number until reaching the maximum at $^{56}\text{Ni}$. Only measured binding energies are shown. Data taken from the Atomic Mass Evaluation 2012 [16].

1.3.3 Heavy Element Fusion

Once the helium material is exhausted, nucleosynthesis continues through fusion of the heavier metals in the star. As with the onset of helium burning, the stellar core will collapse until temperatures are sufficiently high for fusion to overcome the increasing Coulomb barrier. The heavy element fusion processes occur by fusion of nuclei in the following order: carbon, neon, oxygen, and silicon burning. In the silicon
burning phase the fusion of two $^{28}$Si nuclei is highly unlikely and the process instead proceeds by dissociation of lighter nuclei. The dissociated neutrons, protons and alpha particles are then captured forming more tightly bound nuclei. This process continues eventually terminating near the most bound nucleus, $^{56}$Ni, forming what is referred to as the iron-peak in the solar abundance pattern. At this point, stars with sufficient mass will undergo a supernova explosion where the surface of the star is ejected into space enriching the interstellar medium for future generations of stars.

1.4 Neutron Capture Processes

To achieve the production of heavier masses, a different mechanism is necessary as fusion becomes prohibitive. Charged particle reactions are inhibited due to increasing Coulomb barriers that can only be overcome at temperatures greater than that which can be achieved in most astrophysical scenarios. The neutron capture processes provide a mechanism to move beyond the iron-peak. Nearly all the elements heavier than iron were formed through neutron capture reactions processes. There are two major types of these processes: a slow-neutron capture process, $s$-process, and a rapid-neutron capture process, $r$-process. Each of these processes account for nearly half of the abundance distribution of the heavy elements. In addition to these processes, a small fraction of the material is generated by the $p$-process and $rp$-process.
1.4.1 Slow Neutron Capture Process (s-Process)

When heavy seed nuclei such as $^{56}$Fe are exposed to a flux of neutrons, neutron capture reactions occur. In this process, the flux of neutrons is small such that the time-scale for $\beta$-decay is much shorter than that of the neutron capture. In this regime the $\beta$-decays can be considered nearly instantaneous. This suggests the name slow neutron capture process (s-process). The process starts with a seed nuclei $(Z, N)$ that will capture a neutron changing it to $(Z, N + 1)$. If the resulting nuclei is stable then it will undergo another neutron capture. Unstable nuclei will be transformed under $\beta^-$-decay from $(Z, N)$ to $(Z + 1, N - 1)$. This process tends to follow the valley of $\beta$ stability. In general, the process cannot reach neutron-deficient stable nuclei if they are separated by one that is unstable to $\beta^-$-decay. These unstable nuclei may be overcome when the decay rate is of similar order to the neutron-capture rate. The nuclei located where the process splits following two paths are known as branching points. A diagram of the s-process path is shown in Figure 1.3.

As the neutron capture reactions are the limiting factor, the abundances as a function of mass number, $A$, involved in the process can be determined by the differential decay chain equation. Where production of the species $A$ is given by the neutron capture on species $A - 1$ reaction rate, $\langle \sigma \nu \rangle_{A-1}$, and destruction is given by the capture reaction rate of species $A$, $\langle \sigma \nu \rangle_A$.

$$\frac{dY_A}{dt} = -N_n \langle \sigma \nu \rangle_A Y_A + N_n \langle \sigma \nu \rangle_{A-1} Y_{A-1} \quad (1.1)$$

This steady-state flow approximation has shown to correctly reproduce the s-process
Figure 1.3. Diagram of the $s$-process path. Stable isotopes are shown in grey. The path originates with the seed nucleus $(Z = 0, N = 0)$. The nucleus $(Z = 0, N = 1)$ is a branching point with the dashed line representing the new path due to the branching.

The time-scale of the $s$-process is set by the neutron capture rates with typical lifetimes of $\tau_{n\gamma} \gtrsim 10$ yr. The isotopes with small capture cross-sections will tend to pile up with increasing abundance. Those with a large cross-section will quickly be destroyed and processed into heavier nuclei. This pile up and destruction leads to peaks and troughs, respectively, in the $s$-process abundance. Figure 1.1 shows the solar abundance distribution with two significant, narrow peaks attributed to the $s$-process centered at $A \approx 138$ and 208 corresponding to the closed shell numbers $N = 82$ and 126. The addition of a neutron to a nucleus with a closed shell requires substantial energy to overcome the large difference in neutron separation energy as shown in Figure 1.4. This reduces the available excitation energy thus forcing the reaction to occur where the level density is small, resulting in a reduction in the cross
The \( s \)-process continues along the valley of stability until reaching its termination point. The last stable nucleus along the path is \(^{209}\text{Bi}\), at which point further neutron captures will produce isotopes unstable to \( \alpha \) emission [18].

![Figure 1.4](image.png)

**Figure 1.4.** Upper panel show the two neutron separation energies for even \( N \) nuclei. Bottom panel shows the difference in two neutron separation energy between nuclei with neutron number \( N \) and \( N+2 \). Red dashed lines indicate the neutron magic numbers at \( N = 28, 50, 82, \) and 126.
At least two different components of the s-process are required to correctly reproduce the solar s-process abundance distribution [19]. The main component is responsible for production of isotopes from $A = 90$ to 205. The weak component which contributes far less to overall production is responsible for the majority of the abundance of isotopes with $A < 90$. Both components, the main and weak s-process, have different astrophysical sites.

The main component of the s-process is believed to originate from the late thermal pulsing of low mass ($1.5 - 3 \, M_\odot$) Asymptotic Giant Branch (AGB) stars [20–22]. In this scenario the AGB star alternates between hydrogen and helium burning. As the hydrogen burning produces more helium, the helium layer mass increases until it ignites. This new source of energy expands the start causing the hydrogen layer temperature to decrease extinguishing the hydrogen burning. This expansion phase is referred to as a thermal pulse. During the thermal pulse the helium intershell between the hydrogen and helium layers becomes completely convective mixing carbon products from helium burning with protons from the hydrogen layer. Between thermal pulses the carbon in the helium intershell undergoes the following reactions:

\[
p + ^{12}\text{C} \rightarrow ^{13}\text{N} + \gamma \\
^{13}\text{N} \rightarrow ^{13}\text{C} + e^+ + \nu_e \\
p + ^{13}\text{C} \rightarrow ^{14}\text{N} + \gamma
\]

These reactions form a pocket of $^{13}$C and $^{14}$N. Once the temperature is high enough, the lifetime of the neutron producing reaction $^{13}$C($\alpha$,n)$^{16}$O is shorter than the time between pulses, a sufficient neutron flux is generated. Produced neutrons are then
captured by seed nuclei fueling the s-process. The neutron density during this period is relatively low, \( N_n \approx 10^7 \text{ cm}^{-3} \), such that most branches are not activated.

As the hydrogen shell burning phase ends, the star contracts, increasing the temperature, until the helium shell is again ignited forming a thermal pulse. Higher temperatures are achieved and the \(^{14}\text{N}\) is converted to \(^{22}\text{Ne}\) through the reactions:

\[
\alpha + ^{14}\text{N} \rightarrow ^{18}\text{F} + \gamma \\
^{18}\text{F} \rightarrow ^{18}\text{O} + e^+ + \nu_e \\
\alpha + ^{18}\text{O} \rightarrow ^{22}\text{Ne} + \gamma
\]

The neutron source \(^{22}\text{Ne}(\alpha,n)^{25}\text{Mg}\) is activated leading to higher neutron densities. This phase is shorter and does not contribute substantially to production. The higher neutron density activates the branches thus changing the overall shape of the distribution.

As the helium layer becomes inactive and contracts the hydrogen shell will ignite again repeating the process. The \(^{13}\text{C}(\alpha,n)^{16}\text{O}\) provides a neutron source of density, \( N_n \approx 10^7 \text{ cm}^{-3} \), active for \( \sim 20 \times 10^3 \text{ yr} \) and the \(^{22}\text{Ne}(\alpha,n)^{25}\text{Mg}\) source provides densities of \( N_n \approx 10^{10} \text{ cm}^{-3} \) for a few years.

The weak component of the s-process occurs in core helium burning of stars with mass greater than 13 \( M_\odot \) [23, 24]. The more abundant \(^{14}\text{N}\), produced during the
CNO cycle, is converted to $^{22}\text{Ne}$ through the following reactions:

\[ \alpha + ^{14}\text{N} \rightarrow ^{18}\text{F} + \gamma \]
\[ ^{18}\text{F} \rightarrow ^{18}\text{O} + e^+ + \nu_e \]
\[ \alpha + ^{18}\text{O} \rightarrow ^{22}\text{Ne} + \gamma \]

Neutrons are then provided by the reaction $^{22}\text{Ne}(\alpha,n)^{25}\text{Mg}$ towards the end of the helium burning phase, once the temperature is sufficient to overcome the Coulomb barrier. In addition, $^{22}\text{Ne}(\alpha,n)$ competes with $^{22}\text{Ne}(\alpha,\gamma)$ and is dominated by the later at lower temperatures. More massive stars will produce more neutrons as stellar temperatures increase as a function of mass.

The $s$-process may be impeded by reactions that act as neutron sinks or poisons. In fact, the neutron source reaction $^{22}\text{Ne}(\alpha,n)^{25}\text{Mg}$ is self-poisoning since the product $^{25}\text{Mg}$ has a high neutron-capture cross-section. Other reactions may also act as poisons, such as $^{12}\text{C}(n,\gamma)^{13}\text{C}$ and $^{16}\text{O}(n,\gamma)^{17}\text{O}$. In the case of neutron capture on $^{12}\text{C}$, the neutrons are eventually recycled by $^{13}\text{C}(\alpha,n)^{16}\text{O}$ thus reducing its effectiveness as a neutron poison. In the $^{16}\text{O}$ case, the recycling reaction $^{17}\text{O}(\alpha,n)^{20}\text{Ne}$ competes with the $\alpha$-capture reaction $^{17}\text{O}(\alpha,\gamma)^{21}\text{Ne}$. It has been shown that the ratio of the $^{17}\text{O}(\alpha,n)$ to $^{17}\text{O}(\alpha,\gamma)$ reactions is such that the neutron capture of $^{16}\text{O}$ is an efficient neutron poison [25, 26].

Astrophysical modeling of these two components has been very successful and generally reproduce the solar $s$-process well. The $s$-process is responsible for nearly half the production of elements heavier than iron. Once the $s$-process abundances have been determined they can be subtracted from the solar abundance pattern and
the remainder will require another process to explain their production.

1.4.2 Rapid Neutron Capture Process (r-Process)

The well understood s-process does not provide a mechanism for the production of some neutron-rich nuclei that are shielded by isotopes with very short $\beta$-decay half-lives. To produce these, a mechanism with a much higher neutron flux is required. In this regime, the time scale for neutron-capture is much smaller than that of the $\beta$-decay. This leads to a successive and rapid capture of neutrons giving the mechanism the name rapid neutron capture, $r$-process [27]. The astrophysical site providing this high neutron flux is still unknown, two favored scenarios will be described in Section 1.4.2.2. The initial discussion of the $r$-process mechanism will assume we have conditions with a high neutron flux that last for some length of time. This process is responsible for production of the remaining half of the solar abundance heavier than iron, including the peaks in the solar abundance pattern, Figure 1.1, at $A = 80, 130$ and $195$.

The solar $r$-process pattern is determined by subtracting the s-process pattern from the total solar abundance pattern. The remainder of the pattern must be generated by the $r$-process. Recent models of the s-process have provided an increased predictive confidence providing a reliable $r$-process abundance pattern. Any uncertainties in the s-process abundances will directly impact the $r$-process abundances. The s-process pattern can be confirmed by comparison of stable nuclei that are shielded from the $r$-process by stable neutron-rich nuclei. These shielded nuclei can only be produced by the s-process and are called s-only nuclei. The shielding
isotope is typically separated from the s-process path by an unstable nucleus such that only the r-process is able to produce that isotopes. These shielding isotopes are referred to as r-only nuclei. The r-only nuclei provide an unambiguous confirmation of the solar r-process abundance pattern.

1.4.2.1 Classical r-Process

The path of the r-process is much further from the valley of stability than that of the s-process, passing through very neutron-rich isotopes. In the simple classical model of the r-process, the nuclei are exposed to a temperature of $T \geq 1\,\text{GK}$ and neutron density of $N_n \geq 10^{21}\,\text{cm}^{-3}$. In these conditions, both the neutron capture, $(n, \gamma)$, and the photo-disintegration, $(\gamma, n)$, reactions have time-scales much smaller than the $\beta$-decay rates. With sufficient temperature and neutron flux, an equilibrium along an isotopic chain is established such that the rate of abundance change for a fixed $Z$ will be constant. The abundance of an isotope along this chain, $Y(Z,N)$, can then be determined by a ratio to its neighbor using the Saha equation.

\[
\frac{Y(Z, N + 1)}{Y(Z, N)} = N_n \left( \frac{\hbar^2}{2\pi k T} \frac{M_{Z,N+1}}{M_{Z,A} M_n} \right)^{3/2} \frac{g_{Z,N+1} G^\text{norm}_{Z,N+1}}{g_{Z,N} g_n p_{Z,N}^\text{norm} e^{Q_{\gamma}/k T}}
\]  \hspace{1cm} (1.2)

where $g = (2j + 1)$ and $M$ describe the spin and mass of the involved particles, $G^\text{norm}_{Z,A}$ is the normalized partition function of nucleus $Z, A$ and $Q_{\gamma}$ is the Q-value, or neutron separation energy of the neutron capture reaction. Repeatedly applying this equation $m$ times one can determine the abundance of species $m$ relative to the
initial species $m = 0$.

\[ Y_m = Y_0 N_n^m \left( \frac{\hbar^2}{2\pi kT} \right)^{3m/2} \left( \frac{M_m}{M_0 M_n^m} \right)^{3/2} \frac{g_m}{g_0 g_n^m} \frac{G_m^{\text{norm}}}{G_0^{\text{norm}}} \exp \left[ \frac{1}{kT} \sum_{i=0}^{m-1} Q_i \right] \] (1.3)

One can see from this equation that the determination of the most abundant species is strongly dependent on neutron separation energies. The process can be thought of as successive neutron captures continuing until the neutron separation energy of the next nucleus is less than the a particular constant value established for a given astrophysical condition. As the nuclear properties are fixed, the only variation in the $r$-process is due to the neutron density and temperature. An increase in neutron density will shift the abundance maximum toward more neutron-rich, smaller neutron separation energies. An increase temperature will increase the rate of photo-disintegration pushing the maximum toward the less neutron-rich nuclei, larger neutron separation energies. Material will typically build up in one or two isotopes in the isotopic chain. Due to pairing effects, these isotopes are typically those with even number of neutrons, even–$N$. Even–$N$ nuclei have large separation energies while odd–$N$ nuclei have small separation energies. The isotopic chain will continue capturing neutrons until reaching the equilibrium separation energy, which typically occurs between an even–$N$ and odd–$N$ isotopes.

The isotopes with the largest abundance in an isotopic chain then represent waiting-points at which the process may not proceed until a $\beta$-decay occurs. The $\beta$-decay transfers material between isotopic chains and controls the ratio of abundances between chains, $Y_Z = \sum_N Y(Z, N)$. Two isotopic chains are related by the
differential decay chain:

\[ \frac{dY_Z}{dt} = -\lambda_Z Y_Z + \lambda_{Z-1} Y_{Z-1} \]  \hspace{1cm} (1.4)

where \( \lambda_Z \) is the decay constant for the isotopic chain given by

\[ \lambda_Z = \sum_N Y(Z,N) \frac{\lambda_\beta(Z,N)}{Y_Z} \]  \hspace{1cm} (1.5)

where \( \lambda_\beta(Z,N) \) is the \( \beta \)-decay constant for nucleus \((Z,N)\). The solution to this type of differential equation is described in Section 3.5. In the steady flow approximation the rate of abundance change between isotopic chains is constant giving the following relation between chains:

\[ \frac{dY_Z}{dt} = 0 \]

\[ \lambda_Z Y_Z = \lambda_{Z-1} Y_{Z-1} = const \]  \hspace{1cm} (1.6)

The solar \( r \)-process abundance pattern peaks at \( A = 80, 130 \) and \( 195 \) can be attributed to the neutron magic numbers \( N = 50, 82 \) and \( 126 \). These peaks are shifted to lower mass than the corresponding peaks in the \( s \)-process. As the \( r \)-process path reaches a nucleus with a magic number of neutrons, the separation energies drop significantly. This leads to a build-up of abundance in the nucleus with a magic number creating a waiting point. The path then continues after a \( \beta \)-decay, where the situation is again repeated in the next isotopic chain. The path will follow the magic numbers to increasing \( Z \). Meanwhile, as the process continues along the waiting points at the magic numbers the \( \beta \)-decay half-lives become increasingly long, as well as increase in neutron separation energy. Eventually, the process is able
to continue by neutron capture once the neutron separation energy is high enough. The waiting point with the largest $\beta$-decay half-life will have the largest abundance along the magic numbers. These peaks correspond to the classical $r$-process waiting point nuclei of $^{80}$Zn, $^{130}$Cd and $^{195}$Tm.

A schematic view of the $r$-process is shown in Figure 1.5. The figure demonstrates the successive neutron capture and subsequent $\beta$-decay proceeding along a path of constant neutron separation energy with typical values between 2-3 MeV [28]. The figure also shows the successive waiting points along a magic number.

When the neutron source is exhausted the unstable nuclei will undergo successive $\beta$-decays along constant isobars toward stability. The final abundances will be influenced by the duration, $\tau$, of the neutron flux determined by the astrophysical scenario. At the time of freeze-out, when the neutron flux is exhausted, the abundance pattern has a strong odd-even effect due to variation in neutron separation energy as explained above. This odd-even effect is much stronger than observed in the solar abundance pattern. It has been shown that this pattern is smoothed by the process of $\beta$-delayed neutron emission [28]. Many of the $\beta$-decays have Q-values larger than the neutron separation energy. This allows those nuclei to emit a neutron immediately following the $\beta$-decay shifting the abundance pattern from isobar $A$ to $A - 1$. The neutron emission branching ratio describes the probability that a $\beta$-decay leads to a neutron emission and is often labeled as a $P_n$ value.

In addition, heavier nuclei may also undergo $\beta$-delayed fission. This fission recycling may be an additional source of neutrons that could provide a short rejuvenation to the $r$-process. Finally, the heaviest isobars will reach $\beta$-stable nuclei that
Figure 1.5. The $r$-process path follows successive neutron captures (right arrows) until a waiting-point is reached. The waiting-points are determined by a specific neutron separation energy (red contour), dependent on the astrophysical properties. The path may then only proceed following a $\beta$-decay (diagonal arrows). The path tends to have numerous waiting-points along neutron shell closures, such as $N = 82$, (black rectangle) where the separation energy drops significantly. Separation energies (blue contours) are shown in 1 MeV steps, data from the Atomic Mass Evaluation 2012 [16] and the Finite Range Droplet Model 1995 [29]. The nearest stable nuclei $^{130}$Tc (gray box) is shown.
are unstable to $\alpha$ emission. These long lived $\alpha$ emitters may be used as nuclear chronometers [30].

The combination of the waiting point approximation and the steady flow approximation provides a simple model that demonstrates the importance of astrophysical and nuclear properties on the $r$-process. This describes the classical $r$-process, a first approximation, a more dynamic calculation would follow the subsequent neutron freeze-out, neutron production via $\beta$-delayed neutron emission, and subsequent neutron capture period. The process depends on the astrophysical scenario which sets the neutron flux of density, $N_n$, temperature, $T$, and duration, $\tau$, as well as the nuclear data providing the neutron separation energies, $S_n$, the $\beta$-decay rates, $\lambda_\beta$, and the neutron branching ratios, $P_n$ values. Properties of nuclei along the $r$-process path are difficult to determine experimentally as the nuclei are very neutron-rich and thus hard to produce. Many nuclei along the path have not been produced in a laboratory setting and current astrophysical models rely heavily on theoretical calculations. Precision determination of the nuclear data will reduce the ambiguity in the astrophysical model as well as assist in constraining nuclear models. This will allow more precise studies of the astrophysical scenario without ambiguity due to the properties of the involved nuclei.

1.4.2.2 Astrophysical Site of the $r$-Process

The astrophysical site of the $r$-process is still unknown [31, 32]. Many parametric studies of the process have been performed and have shown that a single constant neutron density and temperature is insufficient to reproduce the $r$-process pattern. A
superposition of at least three components is required [28] indicating multiple sites or a dynamic process in which the neutron flux and temperature vary. Although many sites have been proposed [33], there are currently two preferred sites: type II supernovae and neutron star-mergers [5]. Both sites offer a dynamic environment, sufficient neutron densities and temperatures that are not so large as to increase the dissociation rate.

The first of the preferred \( r \)-process sites is the type II supernova scenario which occurs at the end of a massive star’s life [34, 35]. Once the star has completed the silicon burning phase the star’s structure is composed of concentric spheres of various material, with heavier material toward the center. The outer core comes into nuclear statistical equilibrium, in which all the reaction are balanced by their counterpart. The central core is formed by the tightly bound \( ^{56}\text{Fe} \) nuclei which no longer has a nuclear source of energy generation. The stellar core becomes electron degenerate as its mass continues to increase. The electron degeneracy pressure maintains the growing core which continues to increase in temperature. Once the Chandrasekhar limit (1.4 \( M_\odot \)) has been reached the electron degeneracy is not sufficient to prevent its eventual collapse.

In addition, at these temperature the photo-dissociation of iron occurs. This produces helium and neutrons, the former of which is also dissociated into protons and neutrons. This proceeds through the following reactions:

\[
^{56}\text{Fe} + \gamma \rightarrow 13\,^4\text{He} + 4n \\
^4\text{He} + \gamma \rightarrow 2p + 2n
\]  

(1.7)
With a large abundances of protons, electron captures on protons, $p + e^- \rightarrow n + \nu_e$, occur removing the electron degeneracy pressure and accelerating the collapse of the core. This reaction produces a substantial number of neutrinos, which then stream out of the core toward the outer layers. As the core collapses, the density will continue to increase until reaching the nuclear density, at which point the core stiffens due to the repulsion caused by the strong force. This stiffening will cause the core to bounce and create an outgoing shock wave. The shock wave will propagate outward through the star, as it encounters more inward falling material the temperature is increased and leads to further dissociation robbing the shock wave of energy and forcing the shock to stall. This stalled shock wave is dense enough that the outgoing neutrinos interact with the material depositing energy and thus providing a heating source. The heating causes the shock to continue outwards leading to a delayed explosion. This phenomenon is called a neutrino-driven wind.

The outer core, which was in nuclear statistical equilibrium at high temperatures, will be composed of mostly protons and neutrons with a large neutron excess. As the material expands and cools, $\alpha$-particles are formed and eventually become the dominant particle. At this point, the equilibrium collapses and the large excess of $\alpha$-particles assisted by the neutrons produce material beyond $^{56}$Ni into the $A = 100$ region by $(\alpha, n)$ and $(n, \gamma)$ reactions. As the temperature continues to drop, the alpha-induced reactions slow and if the neutron excess is large enough an $r$-process begins using the $A = 100$ material as seeds. At this point, the neutron density will still be large and the temperature of the material will have dropped sufficiently to allow the process to run far from stability. At the termination of the $r$-process the
enhanced material will be ejected along the outgoing shock wave.

The second promising site is the merging of two neutron stars [36–39]. This environment provides a low temperature, low entropy, high neutron density environment that has been shown to reproduce the solar $r$-process abundances with $A > 140$. Neutron stars that become gravitationally bound will continue to spiral and in-fall until merging to form a single body. During this merging phase, the stellar material is shocked enough that temperatures increase such that nuclear statistical equilibrium (NSE) is formed. As with the core-collapse scenario the NSE material expands and cools and alpha particle fusion produces seed material. As the electron fraction in this scenario is lower there will be a larger fraction of available neutrons. As temperature drops more quickly the environment is considered relatively cold. The lower temperatures and increased neutron density will cause the process to run further from stability producing waiting-point nuclei with short half-lives. This allows the process to continue more quickly to the heavier mass regions. The $r$-process enhanced material is then ejected by tidal forces.

There are some drawbacks to these scenarios. The neutrino-driven wind models have not been successful in one-dimensional models. The shock stalls and does not become dense enough to allow for sufficient heating. Recent explorations of multidimensional models have shown that a shock revival at later times is possible [40]. The neutron star merger scenario suffers from an unknown occurrence rate that may be too low to produce sufficient enrichment of the interstellar environment [41].

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1.4.2.3 Nuclear Parameters Relevant to the $r$-Process

Three nuclear properties define the $r$-process path and eventual decay to stability, these are the neutron separation energies, $\beta$-decay half-lives, and $\beta$-delayed neutron emission branching ratios. The involved nuclei are neutron-rich and thus far from stability making experiments difficult due to low production rates. Typically, the half-lives and branching ratios are accessible by experimental measurements prior to measurements of nuclear masses and thus separation energies.

Since these nuclei have been difficult to access in the laboratory, typical astrophysical models rely almost entirely on theoretical mass models. Commonly used mass models include the Finite Range Droplet Model (FRDM) of Möller et al. [29], the Extended Thomas-Fermi plus Strutinsky Integral (ETFSI-1) of Aboussir et al. [42], and a Hartree-Fock approach with Skyrme forces and Bardeen-Cooper-Schrieffer parining (HFBCS-1) of Goriely et al. [43]. These models include strong shell effects, in other words a large change in separation energies across magic numbers, that continue to neutron-rich nuclei. There has been some question whether these shell closures are quenched and thus weakened as nuclei become more neutron rich. An extension of ETFSI-1 by Pearson et al. [44] has produced a mass model (ETFSI-Q) including a reduction in shell strength, called shell quenching. It has been shown that the addition of shell quenching can improve reproduction of the solar abundances at masses just prior to the abundance peaks at $A = 130$ and 195. Figure 1.6 shows a comparison between the two ETFSI models. The ETFSI-1 model with strong shell effects shows a considerable under production prior to the abundance peaks that is less prominent when including shell quenching with the ETFSI-Q model.
Figure 1.6. A comparison of two $r$-process calculations with differing mass models compared to the solar $r$-process abundances (black). The first mass model, ETFSI-1 [45], has a strong shell effect, no shell quenching (blue) and shows large deviations prior to the abundance peaks. The second mass model, ETFSI-Q [44], extends the previous model with a reduced shell effect, including shell quenching (red) shows a closer agreement to solar values.

The unmeasured $\beta$-decay rates are determined using either semiempirical global models or a Quasiparticle Random Phase Approximation (QRPA) calculations [46–48]. The QRPA calculations make use of the same mass models described above. Comparison of measured half-lives with calculated values can help constrain these models to more accurately predict $\beta$-decay rates as well as improve mass models. For example, the quadrupole deformation term, describing the nucleus shape, plays a significant role in the determination of the decay half-life from these QRPA models [49, 50]. A comparison of experimental and theoretical values as a function of quadrupole deformation, Figure 1.7, can provide an indication of the required nu-
ucleus shape in that region. In addition, if the shape is already known, the half-life can constrain the Q-value through this type of calculation.

Figure 1.7. An example of a constraint on mass models from measured $\beta$-decay half-lives. Measured half life of $^{39}$Si (dashed line) compared to QRPA calculation (solid line) performed at various quadrupole deformations. To reproduce the measured half-life a large prolate or oblate deformation is required. Shaded region corresponds to range of QRPA calculations given the uncertainties in $Q_\beta$. Figure adapted from [49].

Finally, $\beta$-delayed neutron emission probabilities play an important role in the final stage of the process. These branching ratios are responsible for the relative odd-even smoothness of the final abundance distribution. These parameters are also calculated using QRPA models. An under prediction of these values tends toward a more staggered odd-even effect in the final abundances. Uncertainties in these
parameters can make it difficult to compare calculated final abundances to \( r \)-process abundances determined from the solar system.

Providing measurement in this neutron-rich region of the chart reduce uncertainties in the specific measured nuclei as well providing information for nuclear models. Variations in \( r \)-process models due to nuclear uncertainties can be significant. If further astrophysical constraints on the scenario are to be extracted from these models the uncertainties in the input parameters must be reduced.

1.5 Other Processes

A small deviation exists between the solar abundances and the sum of the \( s \)- and \( r \)-process solar abundances. These deviations occur in nuclei on the proton rich side of stability, which are shielded from both of the neutron capture processes by stable nuclei. These nuclei are referred to as p-nuclei as they are proton-rich. The abundance of the p-nuclei is nearly two orders of magnitude smaller than that produced by the \( s \)- and \( r \)- process. Two possible processes are discussed below.

1.5.1 \( p \)-Process

The \( p \)-process takes place in stellar environments where high temperatures can be achieved. The initial seed material is taken from the products of the \( s \)- and \( r \)-process and is photo-disintegrated to the proton-rich side of the valley of stability via \((\gamma,n)\) reactions. The process follows successive \((\gamma,n)\) reactions moving the abundance toward neutron-deficient nuclei until the neutron separation energies become large. Meanwhile, the proton and alpha separation energies decrease and open the \((\gamma,p)\)
and \((\gamma, \alpha)\) reactions \([51]\). These reactions shift the abundance to different isotopic chains. This process produces the p-nuclei as well as contribute to the s- and sr-nuclei, nuclei produced by both the s- and r-process.

This process was initially thought to take place in hydrogen rich layers of a star undergoing a supernova \([3]\). As the shock passes through these layers, the temperature and density would become sufficient to initiate the process. In addition, proton-capture reactions may also occur at these temperatures sufficient to overcome the Coulomb barrier. Achieving these temperatures and pressures was shown to be unreasonable and the current accepted scenario occurs in hydrogen poor regions such as the O-Ne rich layers \([52, 53]\). In these regions there is insufficient hydrogen to support \((p, \gamma)\) reactions.

The process is limited at closed shell nuclei where the alpha and proton thresholds become high preventing further dissociation. Once the increased temperatures fall, the unstable material undergoes \(\beta^+\)-decay toward stability.

1.5.2 Rapid Proton Capture Process

Another process with a path along the proton-rich nuclei follows a series of rapid proton captures, giving it the name rapid proton capture process or \(rp\)-process. This process unlike the \(r\)-process occurs with some \(\beta^+\)-decay time scales on the order of the proton capture reactions. The proton capture then competes with the photo-disintegration and the \(\beta^+\)-decays and even proton emission as the path follows nuclei along the proton drip-line. The process is inhibited by waiting points which have small or negative proton separation energies and relatively long
$\beta^+$ half-lives. At the exhaustion of the hydrogen fuel, the material $\beta^+$-decays back to stability and may undergo $\beta$-delayed proton emission.

This process has been shown to occur on the surface of neutron stars in binary systems [54]. In the latter case, the neutron star accretes hydrogen rich material from its neighboring companion. As the material falls onto the surface of the neutron star its temperature increases such that a break-out of the CNO cycle occurs, resulting in a type I X-ray burst. The breakout initially leads to the $\alpha p$-process, a series of ($\alpha, p$) and ($p, \gamma$) reactions that bypass many of the slow $\beta^+$-decays. The process then continues through the $rp$-process as described above. These environments have large gravitational potential and it is unlikely that the produced nuclei can be ejected into the interstellar medium.
CHAPTER 2

INVESTIGATION OF $\beta$-DELAYED NEUTRON EMISSION PROBABILITIES AND HALF-LIVES OF PD AND AG ISOTOPES NEAR $N=82$

2.1 Introduction

Astrophysical models of the $r$-process are sensitive to the astrophysical site, as well as nuclear data inputs. To understand the constraints of the astrophysical scenario the other uncertainties must be minimized. In an effort to provide precise nuclear data many experimental campaigns measuring relevant nuclear properties have been performed. These campaigns are pushing the current boundaries of rare isotope production. For very neutron-rich nuclei, current production rates are very low not permitting many types of experiments. The measurement of half-lives and $\beta$-delayed neutron emission branching ratios of neutron-rich nuclei along the $r$-process path is possible even with very small numbers of produced nuclei. These values are important themselves as nuclear data inputs for the $r$-process, but the also may provide constraints on theoretical mass models which are currently the only available source for mass information in much of the $r$-process region. Production methods for nuclei in this region also produce a large number of contaminants which must be separated. These low production rates coupled with contamination require a
number of experimental detector systems to provide clear identification of the type of produced nucleus as well as measure its decay properties.

2.2 Experimental Setup

An experiment to investigate half-lives and $\beta$-delayed neutron emission branching ratios of neutron-rich nuclei was performed at the Helmholtzzentrum für Schwerionenforschung (GSI), located in Wixhausen, Germany. See Figure 2.1 for a schematic layout of the facility. A $\sim$900 MeV u$^{-1}$, $^{238}$U beam was delivered by two stages of acceleration. The first stage of acceleration was accomplished by the linear accelerator, UNILAC [55], delivering ions at energies up to 20 MeV u$^{-1}$. The beam is then injected into the second acceleration stage, the SIS-18 [56] synchrotron. After multiple revolutions and accelerations, the accelerated and bunched beam was then delivered to a 2.5 g cm$^{-2}$ thick lead target placed at the entrance to the FRagment Separator (FRS). The primary beam intensity was measured by the SEcondary Electron TRAnsmission Monitor (SEETRAM) [57] at up to $2 \times 10^9$ ions/spill. After in-flight separation and identification, the fission fragments were implanted into the Silicon IMplantation Beta Absorber (SIMBA). This detector system was capable of time and position correlation of ions and their decays providing a method to determine the $\beta$-decay half-lives. Neutrons released from nuclei undergoing $\beta$-delayed neutron emission were detected by the surrounding BEta-deLayEd Neutron (BELEN) Detector providing a neutron branching ratio, $P_n$ value, determination.
Figure 2.1. Layout of the Helmholtzzentrum für Schwerionenforschung (GSI). Beam is accelerated from the ion sources via the UNILAC and SIS18 accelerators and then delivered to the Fragment Separator (FRS).

2.2.1 FRagment Separator (FRS)

The FRagment Separator (FRS)[58] is a high resolution spectrometer consisting of a series of bending sections (magnetic dipoles) and focusing sections (quadrupole doublets and triplets). It allows in-flight physical separation and identification of reaction products produced by projectile fragmentation or fission. The resulting beam can be studied at the final focal plane or transported to another experimental station. A schematic drawing of the FRS depicting the separation and focusing
elements as well as the various detector systems is shown in Figure 2.2. The system has two focal planes. The first, called the intermediate focal plane comes after the first separation stage, consisting of the first two dipoles. At this point additional systems can allow for a second separation stage, the final two dipoles. The final separated beam is then delivered to the final focal plane where the experimental end station was placed.

In the FRS, the primary beam first impinges upon a fragmentation or fission target. At the high energies provided by the SIS18 often both channels are available. In this experiment, we use the FRS to select the fission fragments, producing a secondary cocktail beam. This secondary beam contains a large number of species many of which are not particular interesting to any given experiment. The remaining primary beam and secondary contaminates are separated using the $B\rho - \Delta E - B\rho$ method. Isotopes are first separated by making use of the Lorentz Force Law:

$$\vec{F} = \frac{d\vec{p}}{dt} = q(\vec{E} + \vec{v} \times \vec{B}) \quad (2.1)$$

The FRS makes use of magnetic dipoles providing a magnetic field tangential to the ion’s velocity. We can simplify the Lorentz force law for this specific application in which there is only a magnetic field and equate it with the centripetal force with radius $\rho$.

$$F = qvB = \frac{vp}{\rho} = \frac{\gamma mv^2}{\rho} \quad (2.2)$$
Figure 2.2. Schematic view of the FRS. The beam direction is from left to right, impinging first on the fission target, intermediate focal plane, and final focal plane. The intermediate focal plane consisted of a scintillator, a degrader to assist in separation, and two TPCs used in conjunction with the scintillator for particle identification. The final focal plane consisted of another TPC, two MUSICs, another TPC, a scintillator, an additional degrader, another MUSIC and the final implantation end station consisting of SIMBA, a silicon stack for implant and $\beta$-decay detection, surrounded by BELEN-30, a neutron detector system to detect $\beta$-delayed neutron emission. The TPCs, scintillator and MUSICs allowed for particle identification. The variable degrader was used to slow ions such that they were implanted into SIMBA.

Solving for the magnetic rigidity, $B\rho$, we find:

$$B\rho = \frac{p}{q} = \frac{\gamma\beta m_0 c}{q}$$

where we have introduced $\gamma$, the Lorentz factor, and $\beta$ as the ratio of the velocity of ion to the speed of light, $c$. Finally, $m_0$ is the rest mass and $q$ is the charge state of the ion. The values of $\rho$ are fixed by the acceptance of the FRS. The magnetic dipoles
have a central radius of $\rho_0 = 11.2641$ m. Using the first two dipole sections, which deliver beam to the intermediate focal plane, we can choose a value of the magnetic field and make a selection in the mass to charge ratio, $A/Q$. The chosen $A/Q$ is physically centered while contaminant ions will be horizontally offset and can then be stopped with slits. The second $B\rho$ stage will then refocus the beam at the final focal plane.

To further separate the selected $A/Q$, a degrader is placed at the intermediate focal plane. As the beam passes through the degrader, the momentum of the particles is changed as a function of atomic charge, $Z$. The beam then passes through the final two dipole section to the final focal plane. Making use of the degrader, the second $B\rho$ stage, with a fixed $B\rho$, now separates these ions providing a physical separation as a function $Z$. The ion of interest is again centered, now at the final focal plane, and offset contaminants are again stopped using slits. In addition, the degrader can be offset from a tangential alignment so that there is an angle between the beam and degrader. This allows for delivery of the ions of interest at the final focal plane to be either focused in position or mono-energetic.

The FRS is capable of transmitting a number of distinct isotopic species depending on the chosen acceptance. This requires an unambiguous particle identification (PID) of the fragments on an event by event basis. This is achieved at the FRS by measurement of the energy loss, time of flight and position of the ion as determined by ionization chambers, plastic scintillators and time projection chambers, respectively. The combination of these values provides an identification via atomic number, $Z$, and mass to charge ratio, $A/Q$, of each fragment. A description of each of these
methods and corresponding detector systems follow.

### 2.2.1.1 Atomic Charge Determination

To identify the atomic charge of the ions and thus their elemental type, multiple ion chambers were employed. Three MUltiple Sampling Ionization Chambers (MUSICs) [59] were placed at the final focal plane to determine the fragments atomic charge. The MUSIC chambers are operated at room pressure and temperature and filled with P10 gas (Ar + 10% CH\textsubscript{4}). The chamber is operated under an electric field between the cathode plate and eight anode strips. As the beam passes through the gas, charged particles are released and drift toward the anodes. The deposited charge signal is then readout using a series of amplifiers and a peak-sensing ADC for each anode.

These chambers made use of the fact that the energy deposition is proportional to the atomic charge squared, described by the Bethe formula:

$$
\frac{dE}{dx} = \frac{4\pi e^4 Z^2}{m_e c^2 \beta^2} N_{mat} Z_{mat} \left[ \ln \left( \frac{2m_e c^2 \beta^2}{I} \right) - \ln(1 - \beta^2) - \beta^2 \right] \quad (2.4)
$$

where $N_{mat}$ and $Z_{mat}$ are the number density and atomic charge of the stopping material, $m_e$ and $e$ are the rest mass of the electron and the electronic charge, and $I$ is the mean excitation energy. $Z$ is the atomic charge of the material and $\beta$ is the velocity of the ion relative to the speed of light, $c$.

The energy loss in the MUSICs is determined by the geometric mean of the eight
anode segments:
\[
\Delta E = \left( \prod_{i=1}^{8} \Delta E_i \right)^{1/8}
\] (2.5)

The ion chambers were calibrated by measuring the energy deposition as a function of fragment velocity, determined in Section 2.2.1.2, of a known reference fragment. The energy loss of another fragment was determined by scaling the calibration by \( Z^2 \). This approximation can be written as:
\[
\Delta E = \left( \frac{Z}{Z_0} \right)^2 f(\beta)
\]
(2.6)
\[
f(\beta) = \left( a + b\beta + c\beta^2 \right)
\]

where \( Z_0 \) is the atomic charge of our calibration ion and \( f(\beta) \) is determined from fitting \( \Delta E \) as a function of \( \beta \) where \( Z = Z_0 \) as shown in Figure 2.3. The resulting parameters are shown in Table 2.1. The atomic charge, \( Z \) can then be found by:
\[
Z = Z_0 \sqrt{\frac{\Delta E}{f(\beta)}}
\]
(2.7)

This method provides a clear separation between elements and provides a distinct identification of the elemental type of the fragments. The \( Z \) identification for MUSIC 1 is shown in Figure 2.4.

The second MUSIC chamber was not necessary for this experiment. Another experiment in our campaign implanted heavy ions, \( A > 200 \), that may carry a charge state through the fragment separator. This affects the resolution of the particle identification. The second chamber permitted a high enough sensitivity to separate...
Figure 2.3. Energy loss for Silver reference fragments shown as a function of fragment velocity with respect to the speed of light. Multiple silver isotopes were used from multiple settings to increase the range of $\beta$ values. Two ionization chambers are shown, in order of interaction with beam, MUSIC 1 (blue) and MUSIC 2 (red).

these cases. See Benlliure et al. [60] for further details.

The final MUSIC chamber, MUSIC 3, was placed after the degrader material at the final focal plane. As fragments pass through the degrader there is a probability that another fragmentation reaction may occur. This is due to the high energies of the fragments and the substantial amount of material, $\sim 9$ g cm$^{-2}$, necessary to slow the ions for implantation. This MUSIC chamber allowed identification of fragments
TABLE 2.1

MUSIC VELOCITY CORRECTION PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>MUSIC 1 Value</th>
<th>MUSIC 2 Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>6289.19</td>
<td>7476.40</td>
</tr>
<tr>
<td>b</td>
<td>−10807.1</td>
<td>−13950.3</td>
</tr>
<tr>
<td>c</td>
<td>5364.21</td>
<td>7421.96</td>
</tr>
</tbody>
</table>

that underwent an additional fragmentation event in which the atomic charge was changed. A proper velocity calibration of this detector was not possible due to the spread in velocities of the primary beam after impinging on the detector. Without this calibration the resolution of the detector is significantly reduced and only a cut on whether a charge changing reaction occurred is available, shown in Figure 2.5. Events undergoing this additional fragmentation are removed from further analysis as there exact properties are unknown. This method is not sensitive to fragmentation reactions in which only neutrons are removed. These neutron fragmentation reactions will contaminate our PID as they change species after the PID was performed.
Figure 2.4. Histogram of measured atomic charges, $Z$. There is a clear separation between the various atomic charges allowing for an unambiguous determination of $Z$. 
Figure 2.5. Energy loss in MUSIC 3, following the final focal plane degrader. Ions undergoing an additional fragmentation reaction will have lower atomic charge and will deposit less energy in the MUSIC chamber (red). Those events which have not undergone an additional fragmentation will deposit the most energy (black). Events undergoing another fragmentation are cut from the final analysis as there exact isotopic information is unavailable.
2.2.1.2 Mass Determination

Mass Determination is done by identifying the magnetic rigidity and the time of flight of each ion during the second half of the FRS. The horizontal positions and angles of each fragment are determined at the intermediate and final focal planes by four Time Projection Chambers (TPCs) [61]. The time-of-flight was determined by the scintillators that measured the time difference of the ion between the two focal planes.

The TPC is composed of a vertical drift chamber between a plate cathode and a shielding grid. Four proportional counters with C-pads were placed just below the shielding grid. Use of the C-pads allowed use in higher ion intensities. The drift volume was filled either with Ar+10% CH₄ (P10) at room temperature and pressure. The gas mixture was held under a uniform electrical field with voltages of up to 400 V cm⁻¹. Fragment interaction with the drift volume lead to the ionization of the gas. Produced electrons and corresponding ions drift toward the anode and cathode respectively. The four 20µm anode wires were surrounded by the C-pad cathodes, which were each connected to two independent integrated delay line chips.

The vertical position was determined by the electron drift time to each anode wire, providing four measurements. The horizontal position was determined by the difference between the left and right side of each delay line, providing two measurements. Calibration of the TPC was performed by coincidence with a series of scintillator fibers 1mm thick. The fibers were arranged into a grid with a horizontal separation of 2cm and vertical separation of 1cm. The relative offset was then determined by closing the slits to ±1mm. An example of this calibration is shown in
Figure 2.6. Time Projection Chamber (TPC) position Calibration. A set of fiber scintillators are inserted and coincident events are shown. The x-y positions (black, dots) clearly show the separation between the fiber grid. Histograms of projections of the horizontal (red upper pane) and vertical position (blue right pane).
With the known positions of the fragments at the intermediate focal plane, \( x_{F2} \) and final focal plane \( x_{F4} \), the magnetic rigidity can be determined via the following equation:

\[
B\rho = B\rho_0 \left( 1 + \frac{x_{F4} - f_{mag}x_{F2}}{f_{disp}} \right)
\] (2.8)

where \( B\rho_0 \) is the central magnetic rigidity and the magnification, \( f_{mag} \), and dispersion, \( f_{disp} \) are properties of the chosen ion optics between the intermediate and final focal planes. The magnification describes the change in image size of the beam between the intermediate and final focal planes. The slope from linear fit of the horizontal positions at both focal planes provides the magnification as \( f_{mag} = 1.115225 \).

The dispersion can be found by varying the magnetic fields by a known scaling factor and measuring the resulting change in the final focal plane position. The slope from a linear fit of the position as a function of scaling factor dispersion of \( f_{disp} = 7.2202681 \text{ mm/\%} \).

The time-of-flight was determined by a pair of scintillators at the intermediate and final focal planes. Each of these scintillators had photo-multiplier tubes attached to the left and right ends of the scintillators. Due to the high rate of ions at the intermediate focal plane, it is beneficial to allow fragments reaching the final scintillator to provide a start condition for the Time-to-Analog Converter (TAC). Many of the contaminant ions at the intermediate focal plane do not reach the final focal plane. The stop signal is provided by a delayed signal from the scintillator at the intermediate focal plane. The velocity of the particle is then determined by the
following relationship with the time-of-flight, $t_{\text{flight}}$:

$$\beta = \frac{1}{c} \frac{l_{\text{eff}}}{t_{\text{flight}}}$$  \hspace{1cm} (2.9)

The path length, $l_{\text{eff}}$, is nearly constant for all fragments, with some deviations due to varying flight paths. The effective flight path can be determined as a function of a fragment’s horizontal positions at the intermediate, $x_{S2}$, and final, $x_{S4}$, focal planes as well as the angle at the final focal plane, $a_{S4}$:

$$l_{\text{eff}} = l_0 + c_{x_{S2}}x_{S2} + c_{x_{S4}}x_{S4} + c_{a_{S4}}a_{S4}$$  \hspace{1cm} (2.10)

where $c_{x_{S2}}$, $c_{x_{S4}}$, and $c_{a_{S4}}$ are constant for each setting of the FRS. The central path length, $l_0$, is the path of the particle for which the FRS system tune was chosen. The time-of-flight offset, $t_{\text{offset}}$, is due to the delay in transmitting the signal from one scintillator to the other. Both $l_0$ and $t_{\text{offset}}$ can be determined using well known velocities of the $^{238}\text{U}$ primary beam as calibration. The measured value of the time of flight deviates by some offset such that:

$$t_{\text{flight}} = t_{\text{meas}} - t_{\text{offset}}$$  \hspace{1cm} (2.11)

Using this relation and the fact that the primary beam should follow the central path, rearranging Equation 2.9 we find the following relationship:

$$t_{\text{meas}}\beta = t_{\text{offset}}\beta + \frac{l_0}{c}$$  \hspace{1cm} (2.12)
We have selected a $\beta$ value and tuned the FRS to transmit the primary beam with this velocity along the central path. Since the scintillators has two photo-multiplier tubes we can determine the time of flight from the time difference between the left or right sides. We chose to use an average between these times. A simple linear fit using the above equation can then extract the central path length and the offset time. Figure 2.7 shows the linear fit, we find $t_{\text{offset}} = 201.7632$ ns and $l_0/c = 122.2968$ ns.

Figure 2.7. Time of flight calibration points (black dots) fitted with a linear function, Equation 2.12, (red line) to determine the time of flight offset and central path length.
We can rewrite Equation 2.3 to solve for the atomic mass to charge ratio, where we replace the particle mass, \( m_0 \), as the atomic mass, \( A \), of our fragments:

\[
\frac{A}{Q} = \frac{1}{f} \frac{B \rho}{\gamma \beta c}
\]  

(2.13)

Where \( f \) is the conversion factor between amu and MeV \( c^{-2} \). This allows us to determine the mass to charge ratio using a particle’s magnetic rigidity determined from Equation 2.8 and its velocity determination from Equation 2.9.

2.2.1.3 Particle Identification

Using the atomic charge, \( Z \), determined in Section 2.2.1.1 and the atomic mass to charge ratio, \( A/Q \), determined in Section 2.2.1.2 we can make a particle identification. Plotting \( Z \) vs \( A/Q \) there is a clear separation between different fragments. Two dimensional graphical cuts can be placed around each isotope to separate each case for an independent analysis. During the experiment three settings were used: centered on \(^{126}\)Pd, \(^{127}\)Pd, and \(^{128}\)Ag. The tune parameters for each setting are shown in Table 2.2. The maximum thickness of the final degrader provided at the final focal plane did not provide enough material to sufficiently reduce the beam energy such that ions of interest would be implanted in SIMBA. An additional aluminum plate of thickness 15.4 mm corresponding to an aerial density 4.158 g cm\(^{-2} \) was added. A particle ID of ions reaching the final focal plane prior to the final variable degrader is shown for each setting in Figure 2.8, Figure 2.9, and Figure 2.10. The amount of beam time spent on each setting varied.
## TABLE 2.2

### TUNING PARAMETERS OF THE FRS

<table>
<thead>
<tr>
<th>Centered Ion</th>
<th>Magnetic Field $B$ [Tesla]</th>
<th>Intermediate Focal Plane Degrader</th>
<th>Final Focal Plane Degrader</th>
<th>Beam Time [hr]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dipole 1</td>
<td>Dipole 2</td>
<td>Dipole 3</td>
<td>Dipole 4</td>
</tr>
<tr>
<td>$^{126}$Pd</td>
<td>1.2275</td>
<td>1.2275</td>
<td>1.0131</td>
<td>1.0128</td>
</tr>
<tr>
<td>$^{127}$Pd</td>
<td>1.2407</td>
<td>1.2407</td>
<td>1.0259</td>
<td>1.0256</td>
</tr>
<tr>
<td>$^{128}$Ag</td>
<td>1.2407</td>
<td>1.2397</td>
<td>0.9890</td>
<td>0.9887</td>
</tr>
</tbody>
</table>
Figure 2.8. Particle Identification for the setting centered on $^{126}$Pd. A clear separation between distinct groupings is visible. $^{126}$Pd is indicated as a reference (red circle).
<table>
<thead>
<tr>
<th>Atomic Charge (Z)</th>
<th>Atomic Mass to Charge Ratio (A/Z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>43</td>
<td>2.55</td>
</tr>
<tr>
<td>44</td>
<td>2.6</td>
</tr>
<tr>
<td>45</td>
<td>2.65</td>
</tr>
<tr>
<td>46</td>
<td>2.7</td>
</tr>
<tr>
<td>47</td>
<td>2.75</td>
</tr>
<tr>
<td>48</td>
<td>2.8</td>
</tr>
</tbody>
</table>

Figure 2.9. Particle Identification for the setting centered on $^{127}$Pd. A clear separation between distinct groupings is visible. $^{126}$Pd is indicated as a reference (red circle).
<table>
<thead>
<tr>
<th>Atomic Charge (Z)</th>
<th>Atomic Mass to Charge Ratio (A/Z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>43</td>
<td>2.55</td>
</tr>
<tr>
<td>44</td>
<td>2.60</td>
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<tr>
<td>45</td>
<td>2.65</td>
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<td>47</td>
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<td>48</td>
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<td>50</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
</tr>
</tbody>
</table>

Figure 2.10. Particle Identification for the setting centered on $^{128}\text{Ag}$. A clear separation between distinct groupings is visible. $^{126}\text{Pd}$ is indicated as a reference (red circle).
2.2.1.4 Isomer Tagging

The measured Particle ID (PID) was confirmed by studying microsecond isomers produced in the fission target, by using the Isomer TAGging (ITAG) \cite{62} system used at the FRS. This technique was first demonstrated by Grzywacz et al. \cite{63}.

This system consists of two plastic scintillators, a passive stopper and two high purity germanium detectors as shown in Figure 2.11. This system is placed at the final focal plane of the FRS to allow for particle ID explained above. During the isomer data collection, one of the germanium detectors was replaced with a series of LaBr$_3$ detectors. Degrading material was added upstream of the focal plane scintillator such that the ions would be implanted in the stopper. A second scintillator was placed down stream of the stopper and was used to veto ions that were not implanted. This second scintillator is not part of the productions setup and is removed after confirmation of the PID. Adjacent to this scintillator, the $\gamma$ sensitive detectors were placed. Lead shielding was placed around these detectors to block beam induced radiation upstream of the stopper. The $\gamma$-rays emitted by the transition from the isomeric state to the ground state were measured and correlated with the particle identification, providing an unambiguous identification.

The ITAG was used to confirm our Particle ID by examining the 1497 keV, 7.9 $\mu$s isomeric state of $^{205}$Bi. By selecting $\gamma$-decays from this specific state, only $^{205}$Bi appears on the PID. Making use of the long fission fragment tails and overlaying multiple settings we can confirm the PID to the region of interest by simply counting the isotopes from identified species. This confirmation is shown in Figure 2.12.
Figure 2.11. Schematic of Isomer TAGging (ITAG) system used at the FRS. The system consists of two high purity germanium detectors, two plastic scintillators and a passive stopper. Beam is implanted into the stopper and characteristic isomeric decays were detected with the germanium detectors. Figure used with permission from Farinon [62].
Figure 2.12. Particle Identification confirmed with ITAG. $^{205}$Bi was unambiguously identified through a correlation with a detected $\gamma$-ray from the decay of 1497 keV isomeric state. Multiple FRS setting are overlayed and a confirmation of the region of interest was performed by identifying various species by counting from the known $^{205}$Bi.
2.2.2 Silicon IMplantation Beta Absorber (SIMBA)

In order to study the decay properties of neutron-rich nuclei, an efficient detector system capable of detecting charged particles and neutrons was required. The charged particles were implanted and resulting decays were measured within a series of silicon detectors placed at the final focal plane of the FRS. Following the particle identification stages explained above, the beam interacted with a degrader with chosen thickness, see Table 2.2, such that ions would be stopped in these silicon layers. The highly-segmented silicon detectors that were utilized were placed in a closely packed stack to maximize geometric efficiency. The stack consisted of two tracking detectors, two Single-Sided Silicon Detectors (SSSDs), three Double-Sided Silicon Detectors (DSSDs) and two more SSSDs as shown in Figure 2.13.

The two tracking detectors were SSSDs with an active area of \(60 \times 60\) mm\(^2\), 300\(\mu\)m thick with 60 strips each. The tracking detectors were oriented in the horizontal and vertical direction. Each half of the tracking detector was read out via resistor chain. By performing a center of mass calculation on the charge read on either side of the detector, one can determine the location on the detector that the ion traversed. The following equation is used to determine the raw position in arbitrary units:

\[
u = \frac{E_a - E_b}{E_a + E_b}\]

(2.14)

where \(u\) is the raw position, and \(E_a / E_b\) are either the left / right or up / down signals respectively. The positions have a higher order position dependence which can be corrected by fitting the ratio of the calculated position to its maximum value.
Figure 2.13. Schematic view of SIMBA silicon layers. Beam direction is from the left to right, encountering following layers from front to back: vertical tracking, horizontal tracking, front SSSDs, DSSDs, and rear SSSDs. The separation between detectors has been expanded for ease of viewing.

as a function of energy deposited, $E$. The following functional form was used for the calibration function, $f(E)$:

$$f(E) = \frac{u}{u_{max(E)}} = c_1 + c_2(E - c_3)^{c_4}$$

(2.15)

where the constant $c_4$ is less than 1, providing an inverse power law. $u_{max(E)}$ is the position calculated at large energy deposition. As each resistor chain of the detector is independent, a separate calibration function was used for each side. Where the constants, $c_{1-4}$ are dependent on the side, $a$ or $b$. The corrected positions at any
energy can then be computed by the following relationship:

\[ u_{\text{corr}} = \frac{1}{f_a(E_a)f_b(E_b)} \frac{E_a - E_b}{E_a + E_b} \]  \hspace{1cm} (2.16)

Figure 2.14 demonstrates the higher order energy dependence correction. The maximum value of the tracking position in the detector, \( u_{\text{max}} \), is determined at the largest energy deposition, at roughly 3500 in Figure 2.14. This position is chosen as the true position value for the entire strip and \( f(E) \) describes the correction fraction needed to remove the energy dependence. Prior to the correction, the resolution of the outer strips position was poor, providing an implant position with a large uncertainty. After applying the correction each strip was well resolved with a \( 1\sigma \) uncertainty in both the horizontal and vertical direction of 0.05 mm. Parameters used in the calibration function are given in Table 2.3.

The absolute scaling for the tracking position was then determined by performing a polynomial fit between the central value of the energy corrected positions for each strip and the correct strip positions. The central values were computed as the centroid of a fitted gaussian.

After the tracking detectors there were two SSSDs of dimension \( 60 \times 40 \) mm\(^2\), 1000 \( \mu \)m thick with seven strips each in the horizontal direction. These detectors allowed identification of decay-like background events, see Section 3.1.

Following the first set of SSSDs was the implantation region comprised of three DSSDs, each being \( 60 \times 40 \) mm\(^2\), 700 \( \mu \)m thick with 1 mm pitch between each strip. The vertical strips were read out by a system with a linear high-gain amplification.
Figure 2.14. The energy deposition as a function of raw position determined by center of mass calculation (upper, black) for the horizontal position detector. A single inverse power law calibration function (upper, red) is overlayed. The calibration is scaled for each strip for comparison. Energy dependence is removed by applying the calibration (lower, black). (The large groupings of positions correspond to various $Z$ values.)

to clearly identify decay events, depositing energies less than 1 MeV. Heavy-ions stopping or traversing the silicon, depositing energies up to $\sim$1 GeV, caused the linear amplification to overflow a number of neighboring strips. For this reason, we used the tracking detector previously described to identify the location of the
TABLE 2.3

SIMBA TRACKING ENERGY DEPENDANCE CALIBRATION PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Horizontal Tracking</th>
<th>Vertical Tracking</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Left</td>
<td>Right</td>
</tr>
<tr>
<td>$c_1$</td>
<td>0.04602</td>
<td>0.04602</td>
</tr>
<tr>
<td>$c_2$</td>
<td>0.6645</td>
<td>0.6645</td>
</tr>
<tr>
<td>$c_3$</td>
<td>1024</td>
<td>1024</td>
</tr>
<tr>
<td>$c_4$</td>
<td>0.04646</td>
<td>0.04646</td>
</tr>
</tbody>
</table>

implanted ion. The DSSDs horizontal strips, as well as the SSSDs, made use of a linear-logarithmic preamplifier that allowed sufficient resolution for both heavy ions and the subsequent decays, as well as discrimination between the two types of events. The Mesytec MPR-16 LOG pre-amplifiers provided a linear gain up to $\sim 10$ MeV and beyond that a logarithmic gain was provided. Each horizontal strip was gain matched to provide a rough energy calibration. The calibration function made use of a linear and exponential term as shown in Figure 2.15.

To reduce the amount of amplification and digitization electronics, the first and last six horizontal strips were grouped into four groups of three strips each so that the 40 strips of one DSSD could be read by a module with 32 channels. This left 28 strips in the middle section which were connected to independent amplification and digitization channels. Since each DSSD has 1920 pixels, the effective pixelation of
Figure 2.15. Each strip of each DSSD was gain matched using a pulser input on the linear-logarithmic pre-amplifiers. The manufacturer provided the capacitance $0.78(8)\,\text{pF}$ giving a conversion factor of $17\,\text{MeV}\,\text{V}^{-1}$. The pulser voltage was varied and the corresponding ADC channel was determined. The calibration function (red) was composed of a linear (black) and an exponential (blue) term. The linear region has been magnified in the inset.

the full implantation region was 5760 pixels.

After all corrections and calibrations have been applied for the tracking detectors, a comparison to the redundant position measurement provided from the DSSDs was performed. Figure 2.16 shows a nearly one to one correlation between the vertical / horizontal DSSD and the vertical / horizontal tracking detectors. The linear fit parameters are given in Table 2.4.
Figure 2.16. Comparison of measured positions between the tracking layer and the three DSSD layers: DSSD-0 (left), DSSD-1 (middle), and DSSD-2 (right). The horizontal positions (upper panes) have a poor resolution in the DSSD due to overflows in linear pre-amplifiers attached to the vertical strips. The horizontal strips made use of logarithmic pre-amplifiers allowing single strip resolution for the strips vertical positions (lower panes). The first two and last two DSSD vertical strip positions correspond to three physical strips as they were connected. Linear fits (red) show a near one to one correlation between the DSSD and Tracking positions.

Two more layers of SSSDs of the same type as previous followed the DSSDs. These layers provided a rejection for non-implanted events. An event that did not
<table>
<thead>
<tr>
<th>Tracking Detector</th>
<th>Parameter</th>
<th>DSSD-1</th>
<th>DSSD-2</th>
<th>DSSD-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal</td>
<td>Slope</td>
<td>0.9810</td>
<td>0.9805</td>
<td>0.9800</td>
</tr>
<tr>
<td></td>
<td>Offset</td>
<td>0.5860</td>
<td>0.4938</td>
<td>1.963</td>
</tr>
<tr>
<td>Vertical</td>
<td>Slope</td>
<td>0.9870</td>
<td>1.003</td>
<td>0.9933</td>
</tr>
<tr>
<td></td>
<td>Offset</td>
<td>-4.630</td>
<td>-5.180</td>
<td>-5.120</td>
</tr>
</tbody>
</table>

A stop in the DSSD would subsequently deposit energy in these SSSD layers provide a veto condition. In addition, these detectors were used to reject $\beta$-like background events, see Section 3.1.

The energy thresholds of each DSSD and SSSD strip were set independently such that each threshold was just above the noise peak in that strip. As we were only interested in fragments that impinged on SIMBA, we added a requirement of a detection above threshold in the SIMBA tracking detectors as another gating condition to the Particle ID. In addition, fragments depositing energy in the rear SSSDs are rejected. Figure 2.17 shows a particle identification of implanted events over the entire experiment using these additional constraints.
Figure 2.17. Particle Identification of implanted ions.

2.2.3 BEta-deLayEd Neutron (BELEN) Detector

BELEN-30 is a specific implementation of the BELEN detector system[64]. In this configuration, 30 $^3$He proportional counters were arranged in a polyethylene matrix in two concentric rings surrounding a large central bore. The inner ring was composed of 10 counters at a pressure of 10 atm and the outer ring of 20 counters at a pressure of 20 atm. The large central bore had a diameter of 223 mm to accommodate the SIMBA detector. An additional 20 cm of material was added on each side of the central matrix to shield from external neutron sources. A schematic drawing of this
Figure 2.18. BELEN-30 is composed of two concentric rings of $^3$He proportional counters embedded in a polyethylene matrix. The inner ring (teal) contained 10 counters at a pressure of 10 atm and the outer ring (red) contained 20 counters at a pressure of 20 atm. A large central bore (black) allows SIMBA to be placed inside. Additional polyethylene shielding is placed around the central matrix.
The polyethylene matrix acted as a moderating material reducing the neutron energy, by successive scatterings. After randomly scattering through the matrix, an encounter with a proportional tube may lead to a neutron capture reaction inside the proportional counters. The reduction of neutron energy increased the detection efficiency in the proportional counter as the $^3\text{He}$ neutron capture cross-section increases as neutron energy is decreased. The typical moderation time distribution, Figure 2.19, follows an exponential decay with a time constant of $110(15) \mu$s.

The proportional counters were maintained with a 1500 V potential between the central anode and the concentric cathode forming the counter’s exterior. The capture reaction, $^3\text{He}(n,p)^3\text{H}$, releases two charged particles with a Q-value of 764 keV. Assuming the neutrons have been completely thermalized and thus do not contribute additional kinetic energy, the outgoing particles are released with characteristic energies, the proton energy, $E_p = 573$ keV, and the tritium energy, $E_{^3\text{H}} = 191$ keV. These charged particles then ionize the gas. Electrons are attracted toward the anode and create an electron avalanche thus multiplying the output signal.

The output signal is proportional to the energy deposited in the gas. The reaction products have significant ranges in the gas and they may escape the counter without depositing their full energy. This phenomenon is called the wall effect and produces a stair step pattern in the detected energy as shown in Figure 2.20. This leads to a large range of energy depositions for a neutron detection from the smallest energy of a single product, $E_{^3\text{H}} = 191$ keV, up to the full Q-value, $Q = 764$ keV. Due to the moderation, the incident neutron energy is not detected.

The arrangement of the two concentric rings of proportional counters was chosen
Figure 2.19. Emitted neutrons are moderated in BELEN’s polyethylene matrix until becoming thermalized and detected in a proportional tube. This moderation distribution (grey) has been fit (red) with an exponential decay (green) with a time constant of $110(15)\,\mu s$ and a constant term (blue) accounting for uncorrelated background neutrons.

so that the detection efficiency would be flat with respect to neutron energy. A series of Geant4 [65] simulations were performed to optimize this configuration. The final orientation gave a relatively flat distribution in the detection efficiency from 10 keV to 1 MeV. The simulation efficiencies are shown in Figure 2.21. These simulations have shown a good agreement with similar systems and past measurements of well known branching ratios. In addition, the simulation of a $^{252}$Cf source with this
Figure 2.20. Deposited energy spectrum for $^3\text{He}(\text{n},\text{p})^3\text{H}$ demonstrating the wall effect. A full energy gaussian at 764 keV (teal) is produced when both products deposit their full energy. Additionally, two separate continua are produced if the proton escapes (green) or if the $^3\text{H}$ escapes (yellow).

detector configuration agreed with an efficiency measurement preformed prior to the experiment.
Figure 2.21. BELEN-30 detection efficiency as a function of neutron energy. The total efficiency (black, triangles) is relatively flat up to $\gtrsim 1$ MeV. Inner (blue, circles) and outer rings (red, squares) also shown.

2.2.4 Triggering Scheme

During the experimental runs the data acquisition (DAQ) was activated by an accepted trigger. A schematic view of the electronic modules and triggers is shown in Figure 2.22. During the acquisition period, the system would not accept additional triggers, although those additional triggers are registered using scaler modules.

There were two types of accepted trigger, implant and decay, as well as a correlation trigger to detect neutrons. The implant trigger was provided by a discriminator
Figure 2.22. The data acquisition (DAQ) system was triggered on an accepted trigger, either a decay or implant trigger. For a detailed description see Section 2.2.4. Trigger types are highlighted by grey filled boxes. Modules highlighted in blue are read out when the DAQ is triggered.
signal from a coincidence between the two photo-multiplier tubes attached to the plastic scintillator located at the final focal plane. The discriminated value was chosen to maximize detection of the ions of interest. In addition to reading out the FRS parameters needed for particle identification, all of the channels of the SIMBA detector were also read out providing the ion position and energy deposition in the silicon layers.

The decay trigger was provided by any of the linear-logarithmic preamplifiers signals coming from SIMBA. This included the tracking layers, SSSDs, and the horizontal strips in the DSSDs and excluded the vertical strips in the DSSDs. The level of the discriminator was set as low as possible to maximize the $\beta$-decay detection efficiency, while staying above the noise level. If the decay trigger was accepted an acquisition window was opened to allow for a BELEN trigger.

The BELEN system was read by two DAQ systems, one analog and one digital. This campaign made use of the first coupling of the BELEN system with a digital DAQ. The analog system was used to confirm its proper operation and provide a backup system. As the systems are different their read out also differs.

The BELEN analog DAQ system was initiated to record neutron events that occurred in coincidence with decay events. Following a decay trigger, a 200$\mu$s window is opened which allowed time for emitted neutrons to be moderated in the polyethylene before being detected in one of the $^3\text{He}$ proportional tubes. A BELEN trigger was produced by the discrimination of the neutron energy in any of the 30 $^3\text{He}$ counters. This trigger produced a gate to store the energy signal from the triggered proportional tube. Only the first neutron event energy was stored, all subsequent neutrons
were ignored by the analog ADC. The analog TDC recorded discriminated neutron event times over the entire window produced from the decay trigger.

The BELEN digital DAQ was self-triggered on neutron events making use of a trapezoidal filter to save the neutron event time and energy. The digital DAQ has a small dead time relative to the analog system. Data from each event were stored in on board memory. The memory of the digital modules are read out and cleared when an accepted trigger was received. The clocks of the various modules were synchronized and the accepted trigger time is also recorded. This measurement of the accepted trigger time allows for reconstruction of when the neutrons are emitted relative to the decay events. This system permitted measurement of the detected neutrons at all times during the experiment. Its live time was not restricted to the 200 µs window that the analog system was. This allowed for a more complete neutron background study to be performed.
CHAPTER 3

ANALYSIS

After the data collection phase of the experiment, a series of multivariate analysis steps were necessary to extract the resulting half-lives and neutron branching ratios. First, a proper identification of event type, implant or decay, was necessary prior to performing a space-time correlation between these events. Once decay events have been associated with their corresponding implant event a sufficient background estimation is necessary to remove any additional time dependance of the decay curve. As the lifetimes are short, the resulting decay curve is fit with a multi-generational decay model that may include the parent nucleus and its children, grand-children and great grand-children. Finally, a correlation between detected neutrons and $\beta$-decays is performed to determine the probability of $\beta$-delayed neutron emission. Details of this analysis are discussed below.

3.1 Event Identification

Implant and decay events were triggered with relaxed conditions to ensure a high measurement efficiency. This was balanced against losses due to increased dead-time. Additional software gates were applied to clearly discriminate between the three types of events: implants, $\beta$-decays, and $\beta$-decay-like background events.
First, implants were separated by requiring a valid MUSIC signal providing a $Z$ measurement. Only ions with $Z > 44$ were considered for further analysis. In addition, valid signals in the scintillators and TPCs at the intermediate and final focal planes were required such that a $A/Q$ measurement was provided. Finally, only ions which were implanted were of interest. An ion was considered an implant under the following conditions. It must first reach the SIMBA detector requiring a valid signal in both of the tracking layers. An ion was then considered implanted if and only if there is a large energy deposition above 50 MeV in a DSSD layer. The energy deposition in SIMBA provides a clear discrimination between decay and implant events, as shown in Figure 3.1. Implants were rejected if they punched through the DSSD layers. These punch through events were identified by a large energy deposition in one of the rear SSSD layers.

The second type of event, decay events, must not resemble ions coming from the FRS. This requires that these events have MUSIC signals below the noise threshold. To provide a position correlation with the associated implant, a decay event with energy deposition in both a horizontal and vertical strip of a single DSSD was required.

To maximize the resolution of the time-of-flight and atomic charge measurement the scintillator and MUSIC amplification gain was set such that signals from ions of interest could use a large fraction of the accepted range of the ADCs. Fission fragments with a smaller atomic charge will produce smaller signals with some signals being comparable to electronic noise. These small signals will not trigger the data acquisition through a implantation trigger since they are below the discrimination
Figure 3.1. Deposited energy into each strip of the DSSD layers for DSSD-0 (bottom), DSSD-1 (middle), DSSD-2 (top). A clear discrimination between decay events with low energy deposition and implant events with high energy deposition is evident. The chosen energy above which an event is considered an implant is indicated (green line).

threshold. On the other hand, the SIMBA threshold was set very low and a decay trigger was produced for nearly any charged particle in the detector including these light fission fragments. These events are of the third type, $\beta$-decay-like background events.

Without a rejection condition for these events they would be considered real
\( \beta \)-decay events. A real event would originate from one of the DSSD layers and propagate outwards. It is highly unlikely that a real event would deposit energy in both the front and rear SSSD layers. A light fission fragment will traverse all layers of SIMBA and deposit energy in every layer. An event depositing more than 300 keV in one of the front and one rear SSSD layers was considered a background event and was rejected for implant decay correlation.

3.2 Implant-Decay Correlation

The implant and decay events were recorded independently with no explicit correlation between them initially. The events must be correlated in both time and space. For every implant, decay events were searched for correlated events that are within the chosen space-time hyper-volume.

The chosen physical correlation area was a square of five by five pixels centered on the implantation pixel of the implant. This area was then extended onto neighboring silicon layers to ensure that \( \beta \)-decay events corresponding to the ions that were implanted near the surfaces of the DSSD were not lost. This forms a correlation volume around the implant. The desired volume was a cuboid, but the DSSD layers had some small offsets between each layer in the vertical and horizontal directions. Figure 3.2 shows the probability of a decay event depositing its maximum energy as a function of radius from the implant. Any further increase in correlation volume did not significantly increase the \( \beta \)-decay detection efficiency.

A time window relative to the implant of -1 to +4 seconds was chosen. This long window ensures that the nuclei of interest would undergo a number of half-lives.
The window is extended further in both positive and negative directions to allow that the background time-dependence was well understood. Any correlated events with a negative relative time could not be from the decay of the implant and must be background. At long times following the implant the probability of background is greater than the likelihood of a $\beta$-decay. This time window allows us to clearly see background correlated with the synchrotron spill structure of $\sim1$ s spills occurring.
every \( \sim 3 \) s.

To ensure that there is no ambiguity when correlating implants and \( \beta \)-decays, implants with overlapping correlation hyper-volumes were rejected. In other words, if a decay could be correlated with more than one implant, all implants were rejected.

The time difference between the implant and decay event were recorded. A histogram was then constructed from the time differences of each correlated event. A single implant event was allowed to have multiple decay events to account for multiple generations of decays. This histogram can then be fitted with a GOED chain to determine the parent half-life.

3.2.1 \( \beta \)-Decay Neutron Emission Correlation

In addition to the implant-decay correlation, we must also correlate neutron emissions with \( \beta \)-decays. The BELEN detector system made use of a large polyethylene moderator to increase the detection efficiency of the emitted neutron. This moderator reduces the neutron energy by successive scattering throughout the material. As the neutron energy decreases the \( ^3 \)He capture cross-section increases. This moderation has some time dependance that can be described by an exponential decay with a time constant of \( 50 \) µs. Although the neutron emission is nearly instantaneous, the neutron detection may occur a significant amount of time after the decay event. The use of the digital acquisition system with BELEN allowed for an independent detection of neutrons relative to the implant and decays events. These neutron detection must then be correlated with the proper decay event. The digital modules were read out with every accepted trigger and included a time stamp for the accepted trigger.
To ensure that the emitted neutrons were accounted for a time gate of 200 µs after the accepted trigger was used to identify detected neutrons. Any detected neutron was then associated with the respective decay event.

3.3 Background Estimation

Understanding the $\beta$-decay background in this type of experiment is difficult due to a time varying beam structure. One would like to know the rate at which background events occur within the correlation volume during the correlation time. One frequently used method to estimate this value is to measure the number of decay-like events within the correlation volume prior to implant arrival. A constant term is then added to the decay chain fitting for background events. This type of background estimation works well at facilities where beam is provided at a relatively constant rate over multiple correlation time windows.

At the GSI facility, where this experiment was performed, the beam is delivered in spills with varying time structure within each spill. Spills are typically gaussian, with possible substantial deviations. The spill length and separation can be specified. For this experiment, spills were provided with $\sim$1 s length and $\sim$3 s separation. For this reason, the method of background estimation above is not sufficient as it ignores any time dependence. Instead, an alternate method was used in which the exact time structure of the spill was preserved while varying the location on the detector. In this method, we created a virtual implant concurrently with the real implant and at a random location on the detector such that the correlation volume between the real and virtual implants did not overlap. During the implant $\beta$-decay
correlation, an additional correlation between the virtual implant and $\beta$-decays was also performed. Since the decay events correlated to the virtual implant were outside the real correlation volume it is very unlikely the event is a real decay.

This background estimate can be further improved by randomly selecting the virtual implant according to the implant distribution. In this way, statistically the same regions of the detector were sampled. This is important since the background events were typically caused by light fission fragments, which have a spatial distribution across the detector created by traversing the FRS.

A KDE with an adaptive bandwidth, see Section 3.4, was then generated from the background event time distribution. The KDE has the advantage of being a continuous function and spreads the probability of an event over a time period longer than the bin width permitted in a histogram generation of the same data set. The wide bandwidth for regions with low count rate allow a correct treatment of the region between the spills.

With this method, we have generated a background estimation that has precisely the same time structure and statistically the same positions as the real implants. This estimate is time-dependent and reproduces the expected spill time structure. Any time independent components were also incorporated into our estimate.

To avoid biasing the background estimation due to virtual implant location, the position selection was repeated and a number of distributions were created. A density plot of the various distributions is shown in Figure 3.3. It shows the uncertainty in the background estimate due to choice of virtual implant location. The decay histogram fitting was performed with a Monte Carlo technique selecting a random background
location to incorporate this uncertainty.

Figure 3.3. Variation of background time distribution due to location selection for $^{127}\text{Ag}$. This represents the uncertainty in the background due to the choice of virtual implant location. The average probability density per time bin (blue) is overlayed.
3.4 Kernel Density Estimation

Kernel Density Estimation (KDE) is a part of a class of non-parametric density function estimators which includes the frequency histogram. The frequency histogram is a non-continuous step-function defined by bin widths and heights. The heights are determined by the number of observations in a bin divided by the bin width.

The KDE model was independently introduced by Rosenblatt [66] and Parzen [67] and is also referred to as the Parzen-Rosenblatt window method. A KDE is created from a set of \( n \) data points, \( x_i \), drawn from an unknown distribution \( f(x) \). The KDE is generated from a sum of kernels placed at each point \( x_i \). The estimated function \( \hat{f}(x) \) is written as

\[
\hat{f}(x) = \frac{1}{nh} \sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right)
\]

where \( K(x) \) is the chosen kernel and \( h \) is a smoothing parameter called the bandwidth. An example of a KDE is compared to a histogram in Figure 3.4. Both are generated from the same set of six observations. If the KDE is destined to be a probability density function (pdf), then the kernel must integrate to one. A number of kernels can be chosen including the typical normal, gaussian kernel. Other kernels include: uniform, triangular, cosine, quartic (bi-weight), tri-weight and Epanechnikov [68]. The KDE inherits the properties of the kernel and depending on kernel choice may be a continuous and differentiable function. This inheritance property will also affect the smoothness. Choosing a smooth kernel will tend to smooth the entire KDE. Although the KDE is directly affected by the choice of kernel it is not
as critical to smoothing as the choice of bandwidth.

Figure 3.4. Comparison of a frequency histogram (black, dotted line) and a Kernel Density Estimate (KDE), (blue solid line). The KDE is formed from the sum of individual normal kernels (red, dashed line) with adaptive bandwidths.

The term bandwidth is equivalent to the bin width when considering a histogram. A smaller bandwidth shows more detail, but can be subject to statistical noise, while
a large bandwidth will lead to over smoothing and a loss of detail. This trend is the
same for the bandwidth of a KDE. Choosing an optimum bandwidth is the subject of
a large body of research. For our purposes, we have started with the fixed bandwidth
determined by Silverman’s Rule of Thumb [69], which specifies a bandwidth based
on the assumption that the distribution is nearly normal. This gives the bandwidth
parameter as

\[ h = \left( \frac{4}{3} \right)^{1/5} \sigma n^{-1/5} \tag{3.2} \]

where \( \sigma \) is an estimated from the provided data. This choice of bandwidth selector
behaves poorly when the data is non-gaussian. For example, a bimodal distribution
would not be modeled well with this global bandwidth.

We can extend this definition and determine the bandwidth point-wise based on
the local distribution of points by adaptive kernel estimation. We must redefine the
estimated distribution \( \hat{f}(x) \) as an adaptive distribution \( \hat{f}_a(x) \).

\[
\hat{f}_a(x) = \frac{1}{n} \sum_{i=1}^{n} \frac{1}{h_i} K\left( \frac{x - x_i}{h_i} \right) \tag{3.3}
\]

Where the bandwidth, \( h_i \), is now a variable quantity dependent on the data and point
\( i \) and its neighbors. Abramson [45] proposed an adaptive bandwidth parameter given
by the following expression :

\[ h_i = h / \sqrt{f(x_i)} \tag{3.4} \]

This relationship provides a adaptive bandwidth such that regions with high-density
will have a small bandwidth and those with low density will require wide bandwidths
to smooth out statistical fluctuations. This requires knowledge of the optimum band-
width, \( h \), and the true density distribution, \( f(x) \).

We start with the fixed bandwidth choice as our best guess giving a point-wise bandwidth of

\[
h_i = \rho \left( \frac{4}{3} \right)^{1/5} \sqrt{\frac{\sigma}{f(x_i)}} n^{-1/5}
\]  

(3.5)

where the term \( \rho \) has been introduced to solve an issue in which the tails influence is overestimated may over smooth the kernel. This term can be ignored, set to unity, unless the standard deviation of the local structure, \( \sigma_{\text{local}} \), is smaller than two orders of magnitude than the global structure, \( \sigma \). This gives the following relationship for \( \rho \)

\[
\rho = \begin{cases} 
1 & \sigma/\sigma_{\text{local}} \leq 10^2 \\
\sqrt{\frac{\sigma_{\text{local}}}{\sigma}} & \sigma/\sigma_{\text{local}} > 10^2 
\end{cases}
\]  

(3.6)

This method removes almost any dependence on the original choice of bandwidth in the fixed kernel method as shown by Abramson [70]. With the adaptive kernel bandwidth multi-modal distribution can be correctly modeled.

The discussion of other methods for optimum bandwidth determination are beyond the scope of this text. For further discussion on bandwidth selection and of Kernel Density Estimators in general refer to Scott [71], Wand & Jones [72] and Härdle et al. [73].

We have shown that the KDE method makes some gains over frequency histograms such as smoothness. Both non-parametric models are positive definite. KDEs are more computational intensive, but unlike frequency histograms KDEs are bin-independent, continuous and defined everywhere. The method is insensitive to kernel choice. For our purposes we make use of a gaussian kernel and have provided
a simple method for estimating an optimum bandwidth.

3.5 Differential Decay Solution

To determine the half-life of a radioactive species, one must solve the system of differential equations that describe the decay chain

$$\frac{dN_i(t)}{dt} = -\lambda_i N_i(t) + \lambda_{i-1} N_{i-1}(t)$$

(3.7)

where $\lambda_i$ is the decay constant for species $i$, and $N_i(t)$ is the number of nuclei at time $t$. The first term corresponds to the destruction of species $i$ with $\lambda_i$ as the destruction decay constant. The second term corresponds to the production of species $i$ due to the decay of species $i - 1$. Traditionally, the Bateman equation[74] has been used as the solution of this system of equations. The simplest case to consider is one in which the initial composition, $N_i(t = 0)$, is purely the first species such that $N_{i\neq0}(0) = 0$.

Presented here is Bateman’s solution to that case:

$$N_i(t) = N_0(0) \sum_{j=1}^{i} c_j e^{-\lambda_j t}$$

(3.8)

$$c_j = \prod_{k=1}^{j} \frac{\lambda_k}{\lambda_k - \lambda_j}$$

This decay chain and solution consider the situation in which there is no external production of activity and species $i$ decays only to a single child, species $i + 1$, with no branching during decays. Although there is no external production of activity during the previously described experiment, there is a significant branching fraction.
This requires modification of Equation 3.7 to properly treat the branching ratios. We need to account for the fact that a child can be produced from multiple parents and adjust the production term accordingly. We add the branching ratio, \( \rho_{i,j} \), describing the fraction of decays from species \( i \to j \). This modification gives:

\[
\frac{dN_i}{dt} = -\lambda_i N_i + \sum_{j>i} \rho_{j,i} \lambda_j N_j
\]  

(3.9)

To correctly solve this, we have adopted the methodology of Genealogically Ordered Exit-Only Decay (GOED) chains introduced by Yuan & Kernan [75]. A GOED is sorted with the parents being the first isotopes, having the lowest index, followed by its children, grand-children and so on. External production is not allowed thus the chain is \textit{exit-only}.

The GOED chain can be solved with a recursive algorithm as follows:

\[
N_i(t) = \sum_{j=1}^{i} C_{i,j} e^{-\lambda_i t}
\]

\[
C_{i,j} = \begin{cases} 
N_i(0) - \sum_{k=1}^{i-1} C_{i,k} & \text{if } i = j \\
\frac{1}{\lambda_i - \lambda_j} \sum_{k=j}^{i-1} \rho_{k,i} \lambda_k C_{k,j} & \text{if } i \neq j
\end{cases}
\]  

(3.10)

This relationship properly treats the decay of a single species into multiple children and the production of a single child via multiple parents. It is required that each branching ratio is between zero and one

\[
0 \leq \rho_{i,j} \leq 1
\]  

(3.11)
and that the branching ratios for a single species sum to one.

\[ \sum_{j=i+1}^{n} \rho_{i,j} \equiv 1 \]  

(3.12)

The latter condition is often difficult to implement into a fitting routine as limits are often applied only to single parameters.

To more easily enforce Equation 3.12 we used the following descriptions for the branching ratios:

\[
\rho_{i,j} = \begin{cases} 
0 & j \leq i \\
1 - \sum_{k=i+1,k\neq p}^{n} \rho_{i,k} & j = p \\
f_{i,j} \left( 1 - \sum_{k=i+1,k\neq p}^{j-1} \rho_{i,k} \right) & j \neq p
\end{cases}
\]  

(3.13)

where \( p \) is the index of the primary branch, chosen for convenience, and \( f_{i,j} \) is a scaling factor where \( 0 \leq f_{i,j} \leq 1 \). This allows simple single parameter limits to be set on \( f_{i,j} \), that enforce a limit on the sum of branching ratios, Equation 3.12.

This definition of \( \rho_{i,j} \) can be shown to obey Equation 3.11 through mathematical induction. First, we chose the case \( j \neq p \) and \( j = i + 1 \)

\[
\rho_{i,i+1} = f_{i,i+1} \left( 1 - \sum_{k=i+1}^{i} \rho_{i,k} \right)
\]  

(3.14)

\[ = f_{i,i+1} \]

since we require \( 0 \leq f_{i,j} \leq 1 \), we must have \( 0 \leq \rho_{i,i+1} \leq 1 \). We now examine the case
of \( j \neq p \) and \( j = (i + 1) + 1 \)

\[
\rho_{i,i+2} = f_{i,i+2} \left( 1 - \sum_{k=i+1}^{i+1} \rho_{i,k} \right)
\]

\[
= f_{i,i+2} \left( 1 - \rho_{i,i+1} \right)
\]

Again, we required that \( 0 \leq f_{i,j} \leq 1 \) and we have previously shown that \( 0 \leq \rho_{i,i+1} \leq 1 \). Using these two statements, it can be shown that \( 0 \leq \rho_{i,(i+1)+1} \leq 1 \) therefore,

\[
0 \leq \rho_{i,j} \leq 1 \quad \forall j \neq p
\]

(3.16)

For the case where \( j = p \), we choose \( p = i + 1 \) and first examine the minimal case \( n = i + 2 \).

\[
\rho_{i,p} = 1 - \sum_{k=i+1,k\neq 1}^{n} \rho_{i,k}
\]

\[
= 1 - \sum_{k=i+2}^{i} +2 \rho_{i,k}
\]

\[
= 1 - \rho_{i,i+2}
\]

(3.17)

As shown above in Equation 3.16, we have \( 0 \leq \rho_{i,j \neq p} \leq 1 \) and therefore \( 0 \leq \rho_{i,p} \leq 1 \)
1 for \( n = i + 2 \). For \( n = (i + 2) + 1 \) we find the following:

\[
\rho_{i,p} = 1 - \sum_{k=i+2}^{n=i+3} \rho_{i,k}
\]

\[
= 1 - \rho_{i,i+2} - \rho_{i,i+3}
\]

\[
= 1 - \rho_{i,i+2} - f_{i,i+3} (1 - \rho_{i,i+2})
\]

\[
= 1 - \rho_{i,i+2} - f_{i,i+3} + f_{i,i+3} \rho_{i,i+2}
\]

\[
= (1 - \rho_{i,i+2}) (1 - f_{i,i+3})
\]

Equation 3.18

Since we require \( 0 \leq f_{i,j} \leq 1 \) and have shown \( 0 \leq \rho_{i,j \neq p} \leq 1 \), it must be true that \( 0 \leq \rho_{i,j=p} \leq 1 \) for \( n = (i + 1) + 1 \) therefore,

\[
0 \leq \rho_{i,p} \leq 1 \quad \forall \ n
\]

Equation 3.19

Equation 3.16 and Equation 3.19 shows that the our definition of \( \rho_{i,j} \), Equation 3.13, satisfies the requirement that the branching ratio is between 0 and 1, Equation 3.11. This definition must also satisfy Equation 3.12. This can be shown by simply separating the sum into the terms from primary and secondary branches:

\[
\sum_{j=i+1}^{n} \rho_{i,j} = \rho_{i,p} + \sum_{k+1,k \neq p}^{n} \rho_{i,k}
\]

\[
= \left(1 - \sum_{k+1,k \neq p}^{n} \rho_{i,k}\right) + \sum_{k+1,k \neq p}^{n} \rho_{i,k}
\]

\[
= 1
\]

Equation 3.20

We can now use this definition of the \( P_n \) values to enforce that the sum of decay
branching ratios of one species is equal to one. This allows us to extend the decay chain from two daughters per decay to an arbitrary number. This definition also permits setting limits on known branching ratios. Using this in conjunction with the recursive solution to the GOED chains provides a parametric model, which can be used to extract half-life values.

3.6 Half Life Analysis

To determine the half-lives of the radioactive species implanted into the SIMBA detector a fit to a histogram of decay times was performed. The time of each decay was extracted as the time difference of an event clock between the implant and decay event. These decays time were then used to generate a histogram in which multiple decay times per implant were permitted. This allows for a fit to be performed with components from each subsequent generation.

The fitting was performed with a Monte Carlo method to properly account for systematic uncertainties in both the daughter decay properties and the background determination. The adopted GOED decay model was discussed above in Section 3.5. The model permitted the parent, \((Z, N)\), to decay to a daughter, \((Z + 1, N - 1)\), and step-daughter, \((Z + 1, N - 2)\), the species created from a \(\beta\)-decay and neutron emission. Each daughter was permitted to repeat this and decay into its daughter and step-daughter. Each subsequent decay half-life was chosen from a gaussian distribution centered at the adopted literature value with a width corresponding to the \(1\sigma\) uncertainty. Another gaussian distribution was used for the neutron branching ratio of the offspring species.
An additional component for the background was included. In most cases, there were sufficient statics to determine the background component without the need for a scaling factor. The background determination random generator seed was varied during the Monte Carlo analysis. This resulted in choosing virtual implants from different regions if the detector producing slight variations in the background time dependence. In this way, the systematic uncertainty due to virtual implant choice is determined.

The best decay curve fits with a randomly chosen background seed are shown for the palladium cases in Figure 3.5 and Figure 3.6 and for the silver case in Figure 3.7 and Figure 3.8.

The $\beta$-detection efficiency, $\epsilon_\beta$, can be determined by the ratio of the number of detected parent $\beta$-decays, $N_\beta$, to the number of implant events, $N_{\text{implants}}$. The determination of $N_\beta$ is performed in the fit using the following relation:

$$N_\beta = \frac{N_0}{\lambda \Delta t} = \frac{N_0 t_{1/2}}{\ln(2) \Delta t}$$

(3.21)

where $N_0$ is the initial activity found from the fit minimization and $\Delta t$ is the bin width of the fitted histogram. We then find the efficiency as

$$\epsilon_\beta = \frac{N_\beta}{N_{\text{implants}}} = \frac{1}{N_{\text{implants}} \ln(2) \Delta t} \frac{N_0 t_{1/2}}{\ln(2) \Delta t}$$

(3.22)

The $\beta$-detection efficiency for each isotopic case is shown in Figure 3.9. The average efficiency of SIMBA is roughly 60%. The figure shows a drop in efficiency for $^{123}$Pd and $^{124}$Pd. The chosen FRS settings physically centered more neutron-
Figure 3.5. Decay curve histograms (grey) are fit for $^{123-125}$Pd cases shown in logarithmic (left) and linear (right) scales. The fit function (black) is comprised of a genealogically ordered exit-only decay chain (green) accounting for components (red) from the parent generation (red, solid line), daughters (dashed line), grand-daughters (dotted line) and great grand-daughters (dashed-dotted line) as well as a background component (blue).
Figure 3.6. Decay curve fits for $^{126-128}$Pd shown in logarithmic (left) and linear (right) scales. Fit function (black) is comprised of decay components (red) from the parent generation (red, solid line), daughters (dashed line), grand-daughters (dotted line) and great grand-daughters (dashed-dotted line) as well as a background component (blue). The background subtracted fit (green) is plotted for comparison.
Figure 3.7. Decay curve fits for $^{124-126}$Ag shown in logarithmic (left) and linear (right) scales. Fit function (black) is comprised of decay components (red) from the parent generation (red, solid line), daughters (dashed line), grand-daughters (dotted line) and great grand-daughters (dashed-dotted line) as well as a background component (blue). The background subtracted fit (green) is plotted for comparison.
Figure 3.8. Decay curve fits for $^{127-129}$Ag shown in logarithmic (left) and linear (right) scales. Fit function (black) is comprised of decay components (red) from the parent generation (red, solid line), daughters (dashed line), grand-daughters (dotted line) and great grand-daughters (dashed-dotted line) as well as a background component (blue). The background subtracted fit (green) is plotted for comparison.
rich species onto the DSSD. These two cases have implant positions on the edge of
the DSSD detectors $\beta$ particles may be emitted such that do not interact with the
silicon layers. Thus the efficiency should decrease drastically at as the mass number
decreases.

![Figure 3.9. $\beta$-detection efficiency as a function of mass number for palladium (red) and silver (blue) isotopes.](image)

The final determination of the half-life was drawn from the mean value of the
Monte Carlo results. The upper and lower statistical uncertainties varied for each Monte Carlo run and final values were chosen in the same manner. The systematic uncertainty was then determined by the width of the resulting half-life distribution. The resulting values are tabulated and compared against literature values in Table 3.1. In addition, a comparison between the half-lives from this work and theoretical values from Fang et al. [76], Niu et al. [77], Borzov [78], and Möller et al. [48] is shown in Table 3.2.
TABLE 3.1

COMPARISON OF MEASURED HALF-LIVES TO PREVIOUS EXPERIMENTAL VALUES

<table>
<thead>
<tr>
<th>Isotope</th>
<th>This Work</th>
<th>Montes et al. [50]</th>
<th>Kratz et al. [79-81]</th>
<th>Wohr et al. [82]</th>
<th>Fedoseyev et al. [83]</th>
<th>Hill et al. [84]</th>
<th>Reeder et al. [85]</th>
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<tr>
<td>123Pd</td>
<td>170±38</td>
<td>178 (18)</td>
<td>170 (18)</td>
<td>170 (18)</td>
<td>176 (18)</td>
<td>174 (18)</td>
<td>172 (18)</td>
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<tr>
<td>124Pd</td>
<td>144±17</td>
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<td>147 (18)</td>
<td>147 (18)</td>
<td>150 (18)</td>
<td>149 (18)</td>
<td>148 (18)</td>
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<tr>
<td>125Pd</td>
<td>61±8</td>
<td>64 (3)</td>
<td>64 (3)</td>
<td>64 (3)</td>
<td>64 (3)</td>
<td>60 (3)</td>
<td>58 (3)</td>
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<tr>
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<td>56±11</td>
<td>59 (3)</td>
<td>59 (3)</td>
<td>59 (3)</td>
<td>59 (3)</td>
<td>58 (3)</td>
<td>56 (3)</td>
</tr>
<tr>
<td>127Pd</td>
<td>73±21</td>
<td>77 (3)</td>
<td>77 (3)</td>
<td>77 (3)</td>
<td>77 (3)</td>
<td>73 (3)</td>
<td>71 (3)</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
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<td>233 (6)</td>
<td>233 (6)</td>
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<tr>
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<td>117 (12)</td>
<td>117 (12)</td>
<td>114 (12)</td>
<td>112 (12)</td>
</tr>
<tr>
<td>128Ag</td>
<td>58±10</td>
<td>60 (5)</td>
<td>60 (5)</td>
<td>60 (5)</td>
<td>60 (5)</td>
<td>58 (5)</td>
<td>55 (5)</td>
</tr>
</tbody>
</table>

* All half-lives reported in milliseconds.
** Uncertainties listed: +stat−stat (sys).
*** Possible mixture of half-lives from the ground state and isomer identified by Kameda et al. [86].
**** Possible mixture of half-lives from the ground state and isomer identified by Kautzsch et al. [87].
† Possible mixture of half-lives from the ground state and isomer identified by Kratz et al. [79-81].
We report the first half-life measurements for $^{125}\text{Pd}$, $^{126}\text{Pd}$ and $^{127}\text{Pd}$. Other reported values agree with recent literature measurements, except for two deviating cases: $^{124}\text{Pd}$ and $^{129}\text{Ag}$. We also find that the theoretical calculations by Möller
et al. [48] overestimate the palladium half-lives. For the silver half-lives, the theoretical calculation seems to initially under estimate the half-lives, for the case of $^{124}\text{Ag}$ and $^{125}\text{Ag}$. We then observe a trend similar to that seen with the experimental values, except for $^{129}\text{Ag}$. This discrepancy may be due to a possible isomeric state, which is explained below.

The new values for palladium isotopes will have the effect of increasing the speed of the $r$-process as the flow can surpass the waiting-point more quickly. This will tend to increase the abundances of heavier nuclei and decrease that of lighter nuclei. This effect may help resolve the large under-abundances observed just prior to the $A = 130$ mass peak.
### TABLE 3.2

**COMPARISON OF MEASURED HALF-LIVES TO THEORETICAL CALCULATIONS**

<table>
<thead>
<tr>
<th>Isotope</th>
<th>This Work$^*$</th>
<th>Fang et al. [76]</th>
<th>Niu et al. [77]</th>
<th>Borzov [78]</th>
<th>Möller et al. [48]</th>
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<tr>
<td>$^{123}$Pd</td>
<td>170$^{+45}_{-30}$(18)</td>
<td>187.3</td>
<td>143</td>
<td>396.64</td>
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<tr>
<td>$^{124}$Pd</td>
<td>144$^{+17}_{-15}$(18)$^{****}$</td>
<td>138.5</td>
<td>114.6</td>
<td>105</td>
<td>288.69</td>
</tr>
<tr>
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<td>61$^{+8}_{-7}$(1)</td>
<td>104.5</td>
<td>76</td>
<td>265.44</td>
<td></td>
</tr>
<tr>
<td>$^{126}$Pd</td>
<td>56$^{+11}_{-9}$(2)</td>
<td>83.62</td>
<td>70.9</td>
<td>76</td>
<td>221.69</td>
</tr>
<tr>
<td>$^{127}$Pd</td>
<td>73$^{+23}_{-15}$(12)</td>
<td>62.38</td>
<td>51</td>
<td>210.12</td>
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</tr>
<tr>
<td>$^{128}$Pd</td>
<td>$&lt;262$</td>
<td>33.08</td>
<td>48.55</td>
<td>43</td>
<td>74.23</td>
</tr>
<tr>
<td>$^{124}$Ag</td>
<td>230$^{+27}_{-24}$(6)$^{****}$</td>
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<td>171</td>
<td>111.36</td>
<td></td>
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<tr>
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<td>157</td>
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<td></td>
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</tr>
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<td>79.97</td>
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</tr>
<tr>
<td>$^{128}$Ag</td>
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<td>60</td>
<td>79.64</td>
<td></td>
</tr>
<tr>
<td>$^{129}$Ag</td>
<td>95$^{+34}_{-27}$(9)$^\dagger$</td>
<td>29.46</td>
<td>56</td>
<td>31.66</td>
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</tbody>
</table>

$^*$ All half-lives reported in milliseconds.

$^**$ Uncertainties listed: $^{+\text{stat}}_{-\text{stat}}(\text{sys})$.

$^{****}$ Possible mixture of half-lives from the ground state and isomer identified by Kameda et al. [86].

$^{****}$ Possible mixture of half-lives from ground state and isomer identified by Kautzsch et al. [87].

$^\dagger$ Possible mixture of half-lives from the ground state and of isomer identified by Kratz et al. [79–81].
3.6.1 $^{124}$Pd Half-Life

The discrepancy observed for $^{124}$Pd between our measurement and the work done by Montes et al. [50] may be explained by the existence of a long lived isomer. Our production method at higher energies may be able to access this isomeric level. This may result in the measurement of decays from a mixture of two states, the ground-state and the long-lived isomer. A possible isomeric state was identified by Kameda et al. [86]. They were only sensitive to half-lives up to 20 $\mu$s and were only able to provide this as a lower limit.

A possible explanation for this isomer can be found in nearby even-even isotopes. A trend is observed in which the 7- level excitation energy drops with increasing neutron number while that of both the 6+ and 5- increase, see Figure 3.11. When the 7- level drops below the 6+ and 5- level the possibility of a M1 or E2 transition is lost. The next most likely transition is an E3 transition from 7- to 4+. Examining the specific cases in the tin isotopes we find $^{126}$Sn has a 7- isomer with half-life, 6.6 $\mu$s, that decays via an E2 transition to the available 5- level [88]. Moving to more neutron-rich nucleus, $^{128}$Sn, we find the 7- level has dropped below the 5- level [88, 89]. The next accessible transition is an E3 transition to the 4+ level. This causes the 7- isomer half-life to increase significantly to 6.5 s.

We see a similar trend in the palladium isotopes although data is unavailable for the most neutron-rich nuclei. Assuming that the measured half-life is purely the isomeric state we can estimate the excitation energy, $E_x$, of the 7- state. The
Figure 3.11. Excitation energies for levels with spin 7- (solid line), 6+ (dashed line), 5- (dotted line) and 4+ (dash-dotted line) as a function of neutron number are shown for even-N isotopes of palladium (blue), cadmium (red) and silver (yellow). Calculated Weisskopf estimates of the required energy for a 7- isomer in $^{124}$Pd with half-life from this work (circle) and Montes et al. [50] (square). This proposed isomer will not become significant unless the 7- level lies below both the 6+- and 5- level as in $^{128}$Sn ($N = 78$).

Weisskopf estimate for an E3 transition is

$$\lambda_W(E3)\hbar = 2.3 \times 10^{-14} A^2 E_\gamma^7$$  \hspace{1cm} (3.23)
Figure 3.12. Level schemes of $^{126}$Sn and $^{128}$Sn. The 7- state drops below the 5- state in $^{128}$Sn increasing the isomeric half-life.

Replacing the decay constant with $ln(2)/t_{1/2}$ and solving for $E_\gamma$ we find:

\[
E_\gamma = \left( \frac{ln(2)\hbar}{2.3 \times 10^{-4} A^2 t_{1/2}} \right)^{1/7}
\]  

(3.24)

This gamma energy plus the recently measured excitation energy of the 4+ state, $E_x(4+) = 1300(22)$ keV [90], allows us to estimate the required 7- excitation energy. Assuming the half-life from this work is completely due to the isomeric state we find $E_x(7-) = 1332$ keV. Assuming Montes et al. [50] measured only the isomeric
state we find $E_x(7^-) = 1326$ keV. Both of these values are reasonably close to the measured line at $E_\gamma = 62.2(16)$ keV from Kameda et al. [86].

3.6.2 $^{129}$Ag Half-Life

For the $^{129}$Ag case, Kratz et al.[79–81] indicated that a long-lived isomer may exist. They suggest a $\pi p_{1/2}$ isomer and using a QRPA model, which accounted for first forbidden decays, calculate a half-life value of 125 ms. After performing a reexamination of their experimental measurement, they proposed a rough value for the isomeric half-life of 160 ms.

In this work, a fit considering the isomer was performed. The ground-state half-life was fixed to the reported value of $46^{+5}_{-9}$ ms from Kratz et al.. The initial population was taken as a mixture of both the ground-state and isomeric-state. The isomeric half-life was extracted as $130^{+90}_{-50}(20)$ ms. This value is in agreement with the 160 ms value suggested by Kratz et al..

3.7 Detection Limits

In measurements with low counting statistics it is important to define the detection limits. The most widely used criteria to specify these limits was described by Currie [91]. First, one must decide if the measurement of a sample indicated a detection, this level is known as the critical level. At the critical level the number of false-positives should be kept small. A typical measurement of this type will provide the number of measured events, $N_m$, and the number of background events, $N_b$. The number of signal events would then be, $N_s = N_m - N_b$. 

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For multiple measurements of zero-activity, $\bar{N}_m = \bar{N}_b$, and $N_s$ will follow a gaussian distribution centered at 0 with a width of

$$\sigma_{s=0} = \sqrt{2\sigma_b^2}$$  \hspace{1cm} (3.25)

We would like to define a critical level, $L_c$, such that 95% of the measurements are below $L_c$ indicating a zero activity measurement. Since the distribution is gaussian we find the critical limit

$$L_c = 1.645\sigma_{s=0} = 2.326\sigma_B$$  \hspace{1cm} (3.26)

will provide a false-positive probability of less than 5%.

The second limit of interest ensures that the probability of false-negative measurements is minimized. This limit suggests a minimum detectable activity that could be achieved for a measurement. This limit of detection, $L_d$, would be chosen such that a distribution of $N_s$ would be centered at $L_d$ with a width $\sigma_{s>0}$ providing the probability of a measurement below $L_c$ is less than 5%. This requires that

$$L_d = L_c + 1.645\sigma_{s>0}$$  \hspace{1cm} (3.27)

If we first take the approximation $\sigma s > 0 = \sigma s = 0 = \sqrt{2}\sigma_b$ we find

$$L_d = L_c + 1.645\sqrt{2}\sigma_b$$

$$= L_c + 2.326\sigma_b$$  \hspace{1cm} (3.28)

$$= 4.653\sigma_b$$
This first approximation can be improved by using the correct form of $\sigma_{s>0}$, defined by the width of the distribution of the measured signal and the background signal as

$$\sigma_{s>0} = \sqrt{N_m + N_b}$$

$$= \sqrt{N_s + 2N_b}$$

Expanding for $N_s \ll N_b$ and using the first approximation for $L_d$ from Equation 3.28 we have

$$\sigma_{s>0} \approx \sqrt{2N_b \left( 1 + \frac{N_s}{4N_b} \right)}$$

$$= \sqrt{2N_b \left( 1 + \frac{4.653\sigma_b}{4N_b} \right)}$$

$$= \sqrt{2}\sigma_b + 1.645$$

Plugging this and Equation 3.26 into Equation 3.27 we find the Currie Equation:

$$L_d = 4.653\sigma_b + 2.706$$

This limit can be used as the minimum number of counts needed to ensure a signal is not a false-negative when the critical limit of activity is $L_c$.

3.8 Determination of Neutron Branching Probabilities

The probability of neutron branching, $P_n$, is determined by a simple ratio between the number of events detected with only a $\beta$-decay, $N_\beta$, and those with a
correlated $\beta$-decay and neutron emission, $N_{\beta,n}$. Correcting for the number of background correlations, $B_{\beta,n}$, and the neutron detection efficiency, $\epsilon_n$, gives the following relation:

$$P_n = \frac{(N_{\beta,n} - B_{\beta,n}) f}{N_{\beta}} \frac{f}{\epsilon_n}$$  \hspace{1cm} (3.32)

Where $f$ is a small correction due to a counting bias, explained below. Making use of the Monte Carlo simulations performed for the half-life analysis (Section 3.6) we have already determined a distribution of the number of detected $\beta$-decays from each performed fit. The $\beta$-detection efficiency, $\epsilon_\beta$, drops out as both $N_\beta$ and $N_{\beta,n}$ are corrected by this value. This leaves only the determination of values related to the neutron detection.

After performing the correlation of detected neutrons and $\beta$-decays (Section 3.2.1) a simple accounting of the number of events with a $\beta$-neutron correlation was performed. The $\beta$-neutron correlated events were counted within a time frame of ten half-lives after the implant event. This count was repeated for each Monte Carlo run, as the half-life varied, providing a distribution of $N_{\beta,n}$. In addition, we find the number of background events, $B_{\beta,n}$, with correlated neutron emission within the same $10\ t_{1/2}$ time window. Knowing the width of the background distribution, $\sigma_B$, we then apply the Currie equation, Equation 3.31, to determine that the distribution of $N_{\beta,n} - B_{\beta,n}$ must be above the critical limit, $L_d$, to allow for a quantitative analysis.

This leaves only the neutron detection efficiency unknown. An efficiency measurement was performed with a $^{252}$Cf neutron emission source. $^{252}$Cf decays by spontaneous fission emitting on average 3.735 neutrons per fission [92]. The source
provided a known activity, but unfortunately emitted neutrons at a large range of energies up to 15 MeV [93]. This exceeds the linear region of the detector efficiency. By using previously measured $P_n$ values and Geant4 simulations [65] an efficiency could be determined.

A similar detection system was used to previously perform measurements of well known branching ratios. A Geant4 simulation of these measurements was performed showing good agreement to experimental results. Another Geant4 simulation comparing an efficiency run with a $^{252}$Cf source and its large neutron energy distribution in the same system was performed and also showed good agreement. Using the new BELEN-30 configuration, another set of simulations were performed. The first of a $^{252}$Cf source matched our experimental measurement of the neutron detection efficiency, $\epsilon_n^{(252 \text{Cf})} = 40.1(4)\%$. Knowing that we find good agreement with the simulations we can adopt the simulation’s value of neutron detection efficiency in the linear region, $\epsilon_n = 40(2)\%$.

Finally, a small correction was necessary due to a bias toward detecting events with smaller Q-values. This bias is an artifact of the inability to set a non-zero threshold in the DSSD detectors of SIMBA. A $\beta$-decay to the a level of the daughter below the neutron separation energy will emit an electron with larger energy than a decay to a level above the neutron separation energy. Energy deposition into the strips of the DSSD is inversely proportional to the total electron energy, see Equation 2.4. Thus events emitting neutrons are more likely to deposit an energy above the DSSD threshold than an event not emitting a neutron.

This bias was corrected by performing simulations that were not restricted to a
non-zero bias. Using the Geant4 simulation tool kit [65], the $\beta$-decays were simulated across the DSSD surface reproducing the implant position profile. The maximum energy deposited within the correlation volume was then determined and plotted in a histogram, see Figure 3.13. The simulations were repeated for multiple $Q$-values covering a range from 2-13 MeV. The ratio of the number of events above the threshold between the $\beta$-decay at the $Q_\beta$ and $Q_{\beta,n}$ then provides a correction value, $f$, as a function of threshold energy. In addition, a decay to an excited state above the neutron separation energy is unlikely to occur exactly at the separation energy. An additional 500 keV was added to the $Q_{\beta,n}$ to account for the additional excitation energy of this level above the separation energy.

Plots of the resulting correction factors for each isotopic case are shown in Figure 3.14 and Figure 3.15. The correction factor has some uncertainty due to the unknown $Q$-value. We have adopted the uncertainties from the Atomic Mass Evaluation 2012 [16] extrapolations. Finally we can determine the correction for our DSSD thresholds set near 150 keV.

After establishing the efficiency and the bias correction factor, we can apply Equation 3.32 and find a distribution of $P_n$ values. This distribution indicates the most likely $P_n$ value, which was used as the adopted value. The statistical uncertainty was calculated in quadrature from $\sqrt{N}$ uncertainties for both the number of detected correlated $\beta$-neutrons, $N_{\beta,n}$, and the background, $B_{\beta,n}$. The uncertainty from the efficiency and the bias correction was also considered. The width of the $P_n$ distribution is a consequence of the Monte Carlo simulations of variations of known daughter parameters and thus are related to the systematic uncertainty. Including
Figure 3.13. Histogram of Deposited Energy within Correlation Volume for various Q-values. In our region of interest the \( \beta \)-decay values are larger than the neutron separation energies giving a positive \( \beta \)-delayed neutron emission Q-value. The change in Q-value between a decay to ground-state or an excited change above the separation energy shifts the electron energy spectrum. More electrons are detected at low energy, less than 150 keV, at a lower Q-value, decay to higher lying energy state (blue) than at a higher Q-value, decaying to ground-state (red).

The width of the distribution in the uncertainty accounts for systematic uncertainties generated from the half-life determination. The parameters relevant to determination of the \( P_n \) values, \( N_{\text{impalnts}} \), \( N_{\beta} \), \( N_{\beta\text{a},n} \), and \( B_{\beta\text{a},n} \) are tabulated in Table 3.3. The resulting \( P_n \) values are presented in Table 3.4.
We report the first measurements of the $P_n$ values for $^{123-126}$Pd and $^{125-128}$Ag. Due to low statistics we were only able to assign upper limits to the $P_n$ values for $^{127}$Pd, $^{124}$Ag and $^{129}$Ag. The only previously measured $P_n$ value in this region was $^{127}$Ag from Montes et al. [50]. Our upper limit for this isotope agrees with their reported value.
Figure 3.15. Silver neutron emission probability correction factors as a function of threshold energy. The DSSD detectors are biased toward detection of $\beta$ correlated with neutron emission due to their lower energy. Geant4 [65] simulations were performed to determine the necessary correction factors. Each panel shows a correction for a specific isotope of silver. The gray band shows the variation in correction factor due to uncertainty in Q-value.

We find that the theoretical estimates of Möller et al. [48] underestimate the neutron branching ratios for all cases in both palladium and silver. The use of our new values over those of Möller et al. [48] will tend to smooth the odd-even effect observed in the final abundances determined by $r$-process models.
<table>
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<tr>
<th>Isotope</th>
<th>(N_{\text{implants}})</th>
<th>(N_\beta)</th>
<th>(N_{\beta,n})</th>
<th>(B_{\beta,n})</th>
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<td>78</td>
<td>7</td>
<td>3</td>
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<td>(^{124}\text{Pd})</td>
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<td>255</td>
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<td>8</td>
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<td>Fang et al. [76]</td>
<td>Borzov [78]</td>
</tr>
<tr>
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<td>2.91</td>
</tr>
<tr>
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</tr>
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<td>6.8</td>
<td>7.1</td>
<td>9.84</td>
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</table>

* All branching ratios reported as percentages.
Figure 3.16. Comparison of half-lives measured in this work (blue) versus previous literature values (black) as well as theoretical calculations, Table 3.4
CHAPTER 4

R-PROCESS IMPLICATIONS

4.1 High Entropy Wind Model

To explore the impact of these half-life and branching ratio measurements an astrophysical model of the high entropy wind conditions thought to be found near the surface of the proto-neutron star in a core collapse supernova, as described in Section 1.4.2.2, was used. As a first look at the impact we use a simplified parametric model described by [36, 94, 95]. This model follows a number of adiabatic expansions described by an entropy per baryon, $S$, an initial electron fraction, $Y_e$, and an expansion velocity $v$. A full reaction network is coupled to this model which includes $\beta$-decay, neutron capture and photo-dissociation reactions. The network neglects the neutrino captures and fission. The astrophysical parameters are tuned to reproduce an $r$-process. Each expansion used fixed realistic values of $Y_e = 0.45$ and $v = 7500 \text{ km s}^{-1}$. A set of equidistant entropy values were used with values ranging from $S/k = 10 - 260$ in steps of 10. Although these values are larger than may be expected in a supernova they have been shown to reproduce a reasonable $r$-process [36, 95]. $\beta$-decay rate and $P_n$ values were chosen from experimental values where available and QRPA calculations otherwise [48]. Neutron capture reaction
rates were chosen from the NON-SMOKER statistical model [96] using the FRDM mass model [29].

The astrophysical parameters of the model were chosen to reproduce the $r$-process peak found at $A = 130$. A comparison of the final abundances from to model compared to solar abundances is shown in Figure 4.1. This simple model reproduces the expected peaks at $A = 130$ and 195, but under produces the rare-earth element peak.

To understand the impact of the new experimental measurements of $^{125−127}$Pd half-lives a series of calculations were performed, shown in Figure 4.2. First, the nominal calculation using all previous theory values was performed. The half-lives corresponding to the isotopes with new measurements were then varied up and down by a factor of five. This factor was adopted as a reasonable uncertainty in the theoretical half-life. This provided a band in which the final abundances may lay with the uncertainty from the nuclear theory. The calculations were then repeated with the new half-life values and their upper and lower limits. We observe an impact in the $A = 126$ to 129 region. The uncertainty in the final abundances decrease from 40% to 10%. We also observe that the nominal value shifted by 10% reducing the under and over production at $A = 126$ and 128 respectively.

To understand the impact of the $^{128}$Pd upper limit another set of calculations similar to the previously described set were performed, shown in Figure 4.3. Again the theory values were increased and decreased by a factor of five. With only a measured upper limit available we performed a single calculation at that limit. This only slightly reduced the uncertainty in the half-life and the impact in the $r$-process
Figure 4.1. A simple parametrized model of the high entropy wind conditions thought to be found during core collapse supernovae (black line) is compared to the solar $r$-process abundances (grey points) [6, 22].

abundances was also small.

Finally, the impact of newly measured neutron branching ratios, $P_n$ values, on the $r$-process was evaluated. A set of calculations, Figure 4.4, where the theoretical $P_n$ values of isotopes with first time $P_n$ measurements, $^{123-126}$Pd and $^{125-128}$Ag, were increased to 100% and decreased to 0% were performed to provide an initial band of possible $r$-process abundances. Using the new measurement values and their uncertainties a new band was determined. We observed an impact in the mass region of the new $P_n$ values as well as for lighter abundances from $A = 111$ to 128. We
found a significant constraint on the possible final abundance pattern with the new experimental values reducing the largest abundance uncertainty from a factor of 6 to 0.4. The nominal abundance value was found to increase for $A = 122 - 125$ and decrease for $A = 125 - 129$. 
Figure 4.2. Abundances (upper pane) determined from a high entropy wind model (lines) compared to solar $r$-process abundances [6, 22] (grey points). The effect on $r$-process abundances from uncertainties in theoretical half-lives is shown (light grey band). In addition, ratios are shown (bottom pane) relative to the baseline calculation (solid black line) which was calculated using the nominal theoretical values. The theoretical half-lives for isotopes with first time half-life measurements from this work, $^{125-127}$Pd, were increased (solid red line) and decreased (solid blue line) by a factor of five. Abundances determined with new experimental measurements from this work (dashed black line) and the corresponding upper (dashed red line) and lower (dashed blue line) limits show a reduction in the uncertainty (dark grey band) compared to those calculated with theoretical values.
Figure 4.3. Abundances (upper pane) determined from a high entropy wind model (lines) compared to solar \( r \)-process abundances [6, 22] (grey points). The effect on \( r \)-process abundances from uncertainties in theoretical half-lives is shown (light grey band). In addition, ratios are shown (bottom pane) relative to the baseline calculation (solid black line) which was calculated using the nominal theoretical values. The theoretical half-life for \(^{128}\)Pd was increased (solid red line) and decreased (solid blue line) by a factor of five. Abundances determined with new experimental upper limit from this work (dashed black line) show only a slight reduction in the uncertainty (dark grey band) compared to those calculated with theoretical values.
Figure 4.4. Abundances (upper pane) determined from a high entropy wind model (lines) compared to solar $r$-process abundances [6, 22] (grey points). The effect on $r$-process abundances from uncertainties in theoretical neutron branching ratios, $P_n$ values, is shown (light grey band). In addition, ratios are shown (bottom pane) relative to the baseline calculation (solid black line) which was calculated using the nominal theoretical values. The theoretical $P_n$ values for isotopes with first $P_n$ measurements from this work, $^{123-126}$Pd and $^{125-128}$Ag, were increased (solid red line) and decreased (solid blue line) to 100% and 0% respectively. Abundances determined with new experimental measurements from this work (dashed black line) and the corresponding upper (dashed red line) and lower (dashed blue line) limits show a substantial reduction in the uncertainty (dark grey band) compared to those calculated with theoretical values and a shift from the theoretical nominal value.
4.2 Conclusion

We have reported on the first measurements of the half-lives of $^{125-127}$Pd as well as previously measured values of $^{123,124}$Pd and $^{124-129}$Ag, some with significant improvements in their uncertainties. The only deviations observed may be explained by the possible existence of long-lived isomers. In addition, we have reported first time measurement or upper limits for neutron branching ratios of $^{123-127}$Pd and $^{125-129}$Ag as well as a repeated measurement of $^{124}$Ag.

These bulk properties of the nuclei can provide the first constraints on shape, masses and $\beta$-decay strength. Through systematic trends of the half-lives and branching ratios a constraint on both mass models, as well as QRPA models, can be obtained. These constraints lead to better nuclear modeling of even more neutron-rich nuclei, which in turn improves $r$-process modeling.

Comparison between the experimental values and QRPA calculations has shown some deviations for palladium isotopes and a surprising relative agreement for the silver isotopes.

These measured values are also used as direct inputs into $r$-process models helping to constrain the impact from uncertainties in nuclear properties. Half-lives affect the waiting-point nuclei and thus how much material is able to reach heavy masses. Branching ratios play a role after freeze-out smoothing the final abundance pattern of astrophysical calculations that is compared to the solar $r$-process pattern. Theoretical models also play an important role in the $r$-process, as the path runs far from stability through very neutron-rich nuclei that have been difficult to access experimentally. We have shown the impact on a high entropy wind model using our
newly measured half-lives and neutron branching ratios. We observe a reduction in the uncertainty in the final abundance pattern from $A = 124$ to 130 as well as a shift in the abundance pattern.

Continued investigation of nuclear parameters important to the $r$-process will help reduce the uncertainty due to nuclear data. The remaining sensitivity of models will be to uncertainties in astrophysical parameters. This will allow for better understanding of the astrophysical scenarios responsible for the $r$-process.


70. I. S. Abramson. ‘Arbitrariness of the Pilot Estimator in Adaptive Kernel Meth-


