PHENOMENOLOGY OF SUPERSYMMETRY WITH LARGE TAN$\beta$.

A Dissertation

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by

Jason Edward Lennon, B.Sc.

Christopher Frank Kolda, Director

Graduate Program in Physics
Notre Dame, Indiana
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Abstract

by

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Heeding the theoretical evidence in favor of Supersymmetry (SUSY) as a candidate for physics beyond the Standard Model (SM), two important questions become immediately apparent. First, how can we search for SUSY? Second, what can we learn from these searches? It is these questions that guided the choice of investigation for this thesis. Here I present two practical phenomenological studies of SUSY parameter space, which are now in ongoing experimental development. The two processes I focus on are the anomalous magnetic moment of the muon, and the rare meson decay $B_s \to \mu^+\mu^-$. For both of these processes to be experimentally interesting, the ratio of the vacuum expectation values of the two Higgs fields, $\tan\beta = \langle H_U \rangle/\langle H_D \rangle$, must take on large values. Thus I spend some time motivating large values of $\tan\beta$ to facilitate interest in this region of parameter space. I shall show that the muon anomalous magnetic moment gives access to information about the slepton and gaugino sectors of the Minimal Supersymmetric Standard Model (MSSM), and the rare decay $B_s \to \mu^+\mu^-$ allows the extraction of information about the squarks, the Higgs sector, $\tan\beta$, and possibly the origins of SUSY breaking. Both of these have the possibility to provide circumstantial evidence for SUSY, or become useful constraints on SUSY models.
To my parents, Charles & Janie, and brother, Steve.

Without you none of this would have been possible.
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great career.
The Standard Model (SM) of particle physics is a remarkable scientific achievement. Since being placed on solid footing thirty years ago, it has provided a description of Nature which is theoretically beautiful, and has robustly stood up to a battery of experimental tests. Despite this success, one would be hard pressed to find a physicist that would make the claim that the SM is the final theory of Nature because the SM leaves unanswered some very important questions. With this goal of advancing toward the so-called Theory of Everything in mind, physicists have explored many ideas as to what new physics might lie beyond the SM. No matter what the form of this new physics might be, it is important that theorists explore new ideas and propose well-defined experimental tests in order to help drive the search for new physics. This will allow us to constrain possible models of new physics through a process of elimination, or hopefully, through the discovery of new particles. In the end, we drive onwards to a better understanding of how Nature functions on the most fundamental levels.

1.1 The Standard Model

In order to provide a foundation for the rest of this thesis, it is worthwhile to introduce the SM briefly. The construction of the SM is an intriguing story of the interplay between experimental discovery and theoretical insight. For a nice
introduction to this history, see Griffiths [1]. Presently we merely give a summary of the structure of SM. The SM particle content has two distinct divisions: matter and forces. The matter particles are the six quarks and six leptons, all of which are spin 1/2 fermions. Each of these has two chiralities\(^1\) which indicate how they participate in the weak interaction. The fermions occur in families, which are identical except for mass. Only the light first family fermions occur naturally. The heavier are visible to us in particle physics experiments. These together form the basis for all of particle interactions that we are most familiar with in nature, and in our laboratory experiments.

In addition to the matter particles, the SM contains three distinct force mediator particles which are Lorentz vectors (bosons). These mediators are responsible for governing interactions among the matter particles. The principle underlying the construction of each of these interactions is gauge symmetry. Each interaction is built upon a symmetry group, the difference between each interaction being which symmetry group the interaction obeys. The first of the SM forces is the strong interaction, which acts on quarks. The gauge boson for the strong force is the gluon and its gauge group is \(SU(3)\). The second interaction is the weak force, which acts on left handed quarks and leptons. It is built on the group \(SU(2)\) and its mediator is called the \(W\) boson. The third interaction of the SM is called hypercharge. It is based on a \(U(1)\) symmetry and its gauge boson is the \(B\) boson. It acts on all fermions. In this language we say that the gauge group of the SM is \(G = SU(3)_{C} \times SU(2)_{L} \times U(1)_{Y}\). The subscripts on the groups indicate our nomenclature for the charges the interaction acts upon.

The last ingredient of the SM is the Higgs boson, and as of this writing it is the only ingredient of the SM that is still unseen. This particle is a Lorentz scalar \(SU(2)\)

\(^{1}\)Neutrinos are a work in progress. There is evidence that some of the neutrinos must be massive [2].
doublet, and it plays a key role in the SM. The interactions mentioned above are not really what is seen in our experiments. What we see is a disguised version of them. In fact, Nature hid the symmetry group of the SM from us so well that it took a large amount of effort to decipher what it is. One of the key problems was that the weak interaction is chiral. Only massless particles, however, can have one chirality; massive particles must have both left and right handed components. This was the problem: how to construct a theory that had both chiral interactions and massive fermions. It is the Higgs that is responsible for generating fermion masses. The Higgs generates mass through spontaneous symmetry breaking. This means that while our SM Lagrangian is invariant under the symmetry group of the SM, the vacuum state is not. The Higgs has a preferred, intrinsic directionality in $SU(2)_L$ space, and its vacuum expectation value (VEV) is non-zero. Then the fermions that couple to the Higgs will, after the symmetry is broken, pick up masses that go like $y_f v$, where $y_f$ is the coupling strength of the fermion to the Higgs, and $v$ is Higgs VEV. The $SU(2)_L \times U(1)_Y$ part of the SM symmetry is then broken, i.e. hidden, to $U(1)_{QED}$. In addition to the fermions becoming massive, three of the four linear combinations of the gauge bosons of $SU(2)_L$ and $U(1)_Y$ become massive, which is why the weak force we deal with in experiments is short ranged. The resultant vector bosons we see are the massive $W^\pm_\mu$ and $Z^0_\mu$, and the massless photon $A_\mu$. All mathematical detail has been omitted from this discussion. The curious reader should see [3] for an excellent treatment. We list a summary of particles in the SM with their masses\(^2\), electric charges, and interactions in which they participate in Table 1.1.

\(^2\)Throughout this thesis, we use the natural system of units, in which $\hbar = c = 1$. In this system of units, the only unit of measure is energy (typically given in GeV); length is reciprocal to energy.
TABLE 1.1. PARTICLE CONTENT OF THE STANDARD MODEL. MASSES ARE TAKEN FROM REF. [46]. ELECTRIC CHARGES, $Q_{EM}$ ARE GIVEN IN UNITS OF THE PROTON CHARGE.

<table>
<thead>
<tr>
<th>Matter</th>
<th>Particle</th>
<th>$Q_{EM}$</th>
<th>Mass [MeV]</th>
<th>Interactions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>quarks</td>
<td>$u$</td>
<td>$+$2/3</td>
<td>$5$ 1400 176000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$c$</td>
<td>$+$1</td>
<td>$8$ 160</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$t$</td>
<td>$1$</td>
<td>$4800$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$d$</td>
<td>$-1$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$s$</td>
<td>$1/3$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$b$</td>
<td>$-2/3$</td>
<td></td>
</tr>
<tr>
<td>leptons</td>
<td>$e$</td>
<td>$-1$</td>
<td>0.511</td>
<td>electromagnetic</td>
</tr>
<tr>
<td></td>
<td>$\mu$</td>
<td>0</td>
<td>105</td>
<td>weak</td>
</tr>
<tr>
<td></td>
<td>$\tau$</td>
<td>0</td>
<td>1777</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\nu_e$</td>
<td>0</td>
<td>0</td>
<td>weak</td>
</tr>
<tr>
<td></td>
<td>$\nu_\mu$</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\nu_\tau$</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Force (Mediators) &amp; Higgs</th>
<th>Particle</th>
<th>$Q_{EM}$</th>
<th>Mass [GeV]</th>
<th>Interactions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>gluon</td>
<td>$g$</td>
<td>0</td>
<td>strong</td>
</tr>
<tr>
<td></td>
<td>$W$ boson</td>
<td>$W^\pm$</td>
<td>$\pm 1$</td>
<td>charged weak</td>
</tr>
<tr>
<td></td>
<td>$Z$ boson</td>
<td>$Z^0$</td>
<td>0</td>
<td>neutral weak</td>
</tr>
<tr>
<td></td>
<td>photon</td>
<td>$\gamma$</td>
<td>0</td>
<td>electromagnetic</td>
</tr>
<tr>
<td></td>
<td>Higgs</td>
<td>$H$</td>
<td>&gt; 115</td>
<td></td>
</tr>
</tbody>
</table>

1.2 Motivating Supersymmetry: The Hierarchy Problem

Given the remarkable success of the SM, why would we wish to go beyond it? Perhaps the most compelling theoretical argument for physics beyond the SM is the hierarchy problem. We know of two fundamental energy scales in nature. The first of these is associated with gravity. Newton’s constant, $G_N$, is the coupling constant for gravity. It is not only very weak, but also carries mass dimension. This sets the energy scale associated with gravitational interactions, which is called the Planck mass, $M_P = 1/\sqrt{G_N} \simeq 10^{19}$ GeV. The second scale Nature hands to us is the weak scale. Because of the masses of the electroweak vector bosons, we know that the electroweak $SU(2) \times U(1)$ gauge symmetry of the SM is spontaneously broken at an energy scale $M_W \simeq 10^2$ GeV. The huge difference in these energy scales already gives a clue as to the origin of the hierarchy problem. In a theory in which $M_P$ was set to one, $M_W$ would be alarmingly small, a dimensionless parameter of $\mathcal{O}(10^{-17})$!
Naively, we might expect that all energy scales in a theory would be relatively close to each other.

The true nature of the hierarchy problem is apparent only when we appeal to the full quantum theory. The idea put forth in the previous paragraph that all energy scales in a theory to be close to each other is often called Dirac naturalness, and it is really more of a bias than a precise requirement. The statement of the hierarchy problem in the full quantum theory is, however, much more precise. When one considers the radiative corrections to the Higgs mass squared, we find it is exquisitely sensitive to physics that lies at high energies. The reason for this is that quantum corrections to the Higgs mass squared are quadratically divergent. Because the Higgs is responsible for electroweak symmetry breaking (EWSB), we expect that the mass squared of the Higgs to be of order \((100 \text{ GeV})^2\). However, the radiative corrections to \(m_H^2\) can be of order \(M_P^2\), some thirty orders of magnitude greater than \(m_H^2\) itself. These problematic divergences come from gauge boson loops, Higgs loops, and fermion loops. For concreteness, we focus on fermion loops, but the argument for the others follows this vein. The coupling of fermions to the Higgs looks like \(\mathcal{L}_\text{SM} \ni -y_f H \bar{f} f\), a fermion loop diagram like the one on the left of Figure 1.1 will generate a correction to the Higgs mass squared of:

\[
\delta_f m_H^2 \simeq -y_f^2 \frac{\Lambda^2}{16\pi^2}
\]

where \(\Lambda\) is an ultra-violet (UV) cutoff. If we take \(\Lambda \sim \mathcal{O}(M_P)\), say, then \(\delta_f m_H^2\) will be some thirty orders of magnitude bigger than \(m_H^2\), thus if one considers instead the Higgs mass rather than the mass squared, we see that it must be fine tuned to roughly one part in one-hundred-million-billion. Thus the Higgs mass is not stable \((i.e.)\) natural against radiative corrections. What exactly do we mean by natural here? In this context, we mean technical naturalness. What this demands is that a number should be small only if the symmetry of the theory increases in the limit
that number goes to zero. A consequence of this is that radiative correction will be under control, because the corrections will be proportional to the small parameter itself. There is no symmetry that is gained by setting $m_H^2 = 0$, because scalar mass terms are totally invariant under the SM symmetries. This is why the radiative correction in Eq. (1.1) isn’t proportional to $m_H^2$ itself.

This exquisite sensitivity of the Higgs to its radiative corrections is quite alarming. Only scalar particles have this problem (fermions and vector bosons do not have this quadratic sensitivity to $\Lambda$), so it might be tempting to think this might only be an issue for the Higgs sector. From a minimalist point of view, it could be argued that this is acceptable; although it is unappealing, you only have to do this for one particle in the theory. However, the problem is more severe than that, because all of the fermions and vector bosons of the SM owe their masses to the Higgs VEV, so that the entire mass spectrum is potentially threatened by this troublesome divergence in the Higgs mass squared.

Is there a way around this problem? Imagine for a moment that our theory contained a scalar field $S$ that coupled to the Higgs so that the Lagrangian contained a term of the form: $-g_S|H|^2|S|^2$, then a diagram like that on the right of Figure 1.1 will generate a correction to the Higgs mass squared with a value of:

$$\delta_S m_H^2 \simeq +g_S \frac{\Lambda^2}{16\pi^2}.$$  

(1.2)
The salient feature of this contribution is that it is opposite in sign to the contribution in Eq. (1.1), which had a minus sign out front due to the fermion loop. So if one were somehow able to relate the fermion loop to the boson loop to have $g_S = y_f^2$, the problematic quadratic divergences would cancel. One could take a minimal approach, and attempt to set $g_S = y_f^2$ by hand. However, unless there is some symmetry protecting this relation, it will not stay intact at two loops. But we know there is no such symmetry from our discussion of technical naturalness. Thus we conclude that in order for this type of cancellation to work, there needs to be a symmetry of the theory that protects it.

It turns out that one proposed theory of new physics, Supersymmetry (SUSY), does relate fermions to bosons. Under the action of a SUSY generator, a boson goes to a fermion. Therefore the generators for SUSY must carry fermionic quantum numbers. In particular, they must carry the spin angular momentum of a fermion, that is, spin 1/2. Since spin is angular momentum, (remember that angular momentum is $L = r \times p$) SUSY is a space-time symmetry. Since SUSY generators are fermionic, they obey anti-commutation relations, and for the algebra we can heuristically write:

$$\{Q, Q^\dagger\} = \sigma^\mu P_\mu$$
$$\{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0$$
$$[P^\mu, Q] = [P^\mu, Q^\dagger] = 0$$

(1.3)

where we are being rather nonchalant about suppressing spinorial indices, appropriate factors of two, etc. In SUSY, every fermion has a partner boson with identical quantum numbers, but spin differing by a half. Actually, it is the degrees of freedom between fermions and bosons that match exactly, so that the partner to a Dirac fermion (four degrees of freedom) is two complex scalar fields. The partner to a massless vector boson (two degrees of freedom) is a Majorana fermion. This
relation between the fermionic and bosonic degrees of freedom is exactly what is needed to ensure the cancellation between the divergences in Eqs. (1.1-1.2).

So, SUSY solves for us the hierarchy problem at one loop. However, it is generic in quantum theories that renormalizations not generated at one loop can, and usually do, appear at higher loops, unless they are forbidden by some symmetry. It is considered one of SUSY’s most attractive features that the re-appearance of such dangerous quadratic renormalizations at higher loops is guaranteed not to occur. This is the so-called non-renormalization theorem [4]. The theorem states that quadratic divergences do not appear at any order of perturbation theory. The proof of this theorem employs supergraph techniques, in which the subtle cancellations between the fermion and boson contributions to a process are more transparent. We shall not give it here. Nonetheless the result is remarkable. It ensures that once the hierarchy between the weak and Planck scales is set, it remains intact, provided that the new SUSY states appear at an energy scale not too far above a TeV. The solution to the hierarchy problem taken in tandem with the non-renormalization theorem is usually taken to be the one of the most compelling theoretical arguments in favor of SUSY. Having given this primary motivation for SUSY, we now proceed to give the basics of building a SUSY model.

1.3 Building a SUSY Extension

We will now give the general ingredients to build a supersymmetric Lagrangian. The aim here is not to give a complete exposition of the introductory elements of SUSY model building. For two very excellent sources, I refer the curious reader to Balin & Love [5], and also to Martin [6]. We give here an overview of the important features, without the underlying arguments that enter into their construction. To form the matter sector, one needs the fermions $\psi_i$, and their partner scalars, $\phi_i$. For
the gauge sector with a gauge group with generators $T^a$ and structure constants $f^{abc}$, you must have the gauge vector bosons, $A^a_\mu$, and the partner gauge fermions, $\lambda^a$. Then we write the general supersymmetric Lagrangian:

$$\mathcal{L}_{\text{SUSY}} = -D^\mu \phi_i^* D_\mu \phi_i - i\psi_i^* \sigma^\mu D_\mu \psi_i - i\lambda^i \sigma^\mu D_\mu \lambda^i - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}$$

$$-\frac{1}{2} (W_{ij}\psi_i \psi_j + \text{c.c.}) - W_i^* W_i - \frac{1}{2} (g \phi_i^* T^a \phi_i)^2$$

$$-\sqrt{2} g \left[ (\phi_i^* T^a \psi_i) \lambda^a + \lambda^i (\psi_i^* T^a \phi_i) \right]$$  \hspace{1cm} (1.4)

Here we have introduced an object called the superpotential, $W$, which has the general form:

$$W = \frac{1}{2} M_{ij} \phi_i \phi_j + \frac{1}{6} Y_{ijk} \phi_i \phi_j \phi_k,$$  \hspace{1cm} (1.5)

and we take $W_i \equiv \partial W / \partial \phi_i$ and $W_i^* \equiv \partial W^* / \partial \phi_i^*$, and similarly for $W_{ij}$. This object is not really a scalar potential in the ordinary sense. It really just a holomorphic, gauge invariant function of the fields $\phi_i$. The exact form of the $M_{ij}$ and $Y_{ijk}$ in Eq. (1.5) are constrained from gauge invariance so that only a subset of them are non-zero.

The first four terms of Eq. (1.4) are the kinetic terms. The gauge covariant derivative $D_\mu = \partial_\mu - ig T^a A^a_\mu$ ensures the minimal coupling prescription of the matter fields to the gauge fields, and $F^{a\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$ is the usual gauge field strength tensor$^3$. The fifth term represents the Yukawa interactions and mass terms of the matter fields. The sixth and seventh terms yield the scalar potential of the model. The last two terms in square brackets are additional interactions that are allowed by gauge invariance and renormalizability which do not arise from the minimal coupling prescription. The coefficient of this term is obtained from the requirement of closure of SUSY transformations on the fields. In Eq. (1.4), the generators $T^a$ are in the representation of the field that it acts upon. The last two

$^3$Note that these formulas apply to non-abelian groups. For an abelian group, there is only one gauge field. Therefore the $f_{abc}$ are zero, and the $T^a$ are ordinary numbers.
terms, as well as the gauge kinetic and minimal coupling terms in the covariant
derivative must be replicated (with of course a different gauge coupling, $g$) for each
gauge group in the model.

We now focus our attention on building a SUSY extension of the SM. We have al-
ready introduced the SM particle content, along with their couplings, in Section 1.1.
Based on the previous discussion, we know that any SUSY extension must have spin
0 partners to the quarks and leptons, respectively called squarks and sleptons, spin
1/2 partners to the vector bosons, called gauginos (specifically the gluino, the wino
and the bino), and a spin 1/2 partner to the Higgs, called the higgsino. The exten-
sion of the Higgs sector is actually a little more subtle. In the SM, one Higgs doublet
can generate masses for both the up and down type quarks. This is accomplished by
coupling the up quarks to the Higgs, and the down quarks to the complex conjugate
of the Higgs. In SUSY, such a coupling is forbidden due to the requirement that
the superpotential be holomorphic. Actually, another way to see this is that an
operator like $Q_L d_R H$ would imply the existence of $Q_L d_R \tilde{H}$. However, if one tried
to couple this same Higgs to the up quarks, the SUSY equivalent operator would
be $Q_L \tilde{u}_R \tilde{H}$, which isn’t allowed because the $SU(2)$ indices have not been done
correctly. Therefore, we require a minimum of two Higgs fields, denoted $H_U$ and
$H_D$ to indicate which quark fields they couple to. Thus the extension of the Higgs
sector has these two Higgs doublets, along with their superpartner higgsinos. This
spectrum of particles is the Minimal Supersymmetric Standard Model (MSSM). A
summary table of the particles, along with their gauge quantum numbers, is given
in Table 1.2. The last ingredient we need is the superpotential:

$$W_{MSSM} = \bar{U} Y_U Q H_U + \bar{D} Y_D Q H_D + \bar{e} Y_e L H_D + \mu H_U H_D. \quad (1.6)$$

Now all that is left is to feed this superpotential through the machinery of this
section in order to construct the interaction Lagrangian. The particle inventory of
TABLE 1.2. THE PARTICLE CONTENT OF THE MINIMAL SUPERSYMMETRIC STANDARD MODEL, WITH THE GAUGE QUANTUM NUMBERS OF EACH SUPERMULTIPLIET.

<table>
<thead>
<tr>
<th>Chiral Supermultiplets</th>
<th>Names</th>
<th>spin $\frac{1}{2}$</th>
<th>spin 0</th>
<th>$(SU(3)_C, SU(2)_L, U(1)_Y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>quarks, squarks</td>
<td>$Q$</td>
<td>$(u_L d_L)$</td>
<td>$(\bar{u}_L \bar{d}_L)$</td>
</tr>
<tr>
<td></td>
<td>(3 families)</td>
<td>$\bar{U}$</td>
<td>$u_R^c$</td>
<td>$\tilde{u}_R^c$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\bar{D}$</td>
<td>$d_R^c$</td>
<td>$\tilde{d}_R^c$</td>
</tr>
<tr>
<td></td>
<td>leptons, sleptons</td>
<td>$L$</td>
<td>$(\nu_e e_L)$</td>
<td>$(\tilde{\nu}_e \tilde{e}_L)$</td>
</tr>
<tr>
<td></td>
<td>(3 families)</td>
<td>$\bar{e}$</td>
<td>$e_R^c$</td>
<td>$\tilde{e}_R^c$</td>
</tr>
<tr>
<td></td>
<td>Higgs, Higgsinos</td>
<td>$H_U$</td>
<td>$(H_U^+ H_U^0)$</td>
<td>$(\tilde{H}_U^+ \tilde{H}_U^0)$</td>
</tr>
<tr>
<td></td>
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<td>$H_D$</td>
<td>$(H_D^0 H_D^-)$</td>
<td>$(\tilde{H}_D^0 \tilde{H}_D^-)$</td>
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<tr>
<td>Vector Supermultiplets</td>
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<td>spin $\frac{1}{2}$</td>
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<td>$\tilde{g}$</td>
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<tr>
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<td>B boson, bino</td>
<td>$B^0$</td>
<td>$\tilde{B}^0$</td>
<td>$(1, 1, 0)$</td>
</tr>
</tbody>
</table>

the SM, and the gauge structure will supply the remaining terms. A comment about the form of this superpotential. As mentioned, two Higgs doublets are required in the MSSM by holomorphy. Therefore the MSSM is not merely the SM plus SUSY. It is really the two Higgs doublet SM, plus SUSY. In fact, the low energy limit of the MSSM is, in some cases, not the SM, because of effects due to the new structure of the Higgs sector. These effects can be very interesting. In fact, we will explore one of them in great detail in Chapter 4.

1.4 SUSY Breaking

In Section 1.3 we introduced the basics of building a SUSY Lagrangian, and discussed the particle content of the MSSM. The glaring issue with a SUSY extension of the SM is that none of the proposed new particles has been observed. Since SUSY
tells us that two superpartners must have the same mass, it must be that SUSY (if realized in nature) is a broken symmetry. Theoretically we expect that SUSY is broken spontaneously, in a manner similar to how the symmetry of the SM is broken, so that at energies above $\Lambda_{\text{SUSY}}$, SUSY is exact, and is only hidden at our everyday energy scales. Like $SU(2)$ breaking in the SM, SUSY breaking will induce new mass terms in the Lagrangian, except that SUSY breaking induces masses for more than just gauge fields, it introduces one (or more) mass parameters for each particle. These new terms, called soft terms because they have positive mass dimension, parameterize the SUSY breaking. In a theory with an underlying mechanism for SUSY breaking, there will be relations between them derived from the manner in which SUSY is broken, analogous to the relationships like $m_W = m_Z \cos \theta_W$ in the SM. In Chapter 4, we will examine three well motivated models of SUSY breaking. They each make some set of simplifying assumptions on the soft terms at the high scale, but they differ in the mechanism of SUSY breaking, and how it is communicated to the fields of the MSSM. However, for the moment we take a more minimalist approach and write down the most general soft breaking Lagrangian one could have:

$$\mathcal{L}_{\text{soft}} = -\frac{1}{2} (M_\lambda \lambda^a \lambda^a + \text{c.c.}) - [M^2_\phi]_{ij} \phi_i^* \phi_j$$

$$- \left( \frac{1}{2} B_{ij} \phi_i \phi_j + \frac{1}{6} A_{ijk} \phi_i \phi_j \phi_k + \text{c.c.} \right)$$

(1.7)

The soft breaking parameters consist of gaugino masses, $M_\lambda$, for each of the gauge groups of the model, scalar (mass)$^2$ terms $(M^2_\phi)_{ij}$, scalar bilinears $B_{ij}$, and trilinear scalar couplings, $A_{ijk}$. Note that the bilinear and trilinear soft breaking terms have forms similar to those of the superpotential of Eq. (1.5). This is to be expected. The form of the terms in the superpotential was restricted only by renormalizability, gauge invariance, and R-parity. Thus a breaking term is allowed
only if the corresponding term is allowed in the superpotential.

Including the the soft breaking terms of Eq. (1.7), the MSSM has some 120 plus parameters. This makes effective phenomenological studies rather difficult, even with today’s computers. However, we already know that SUSY must be an “atypical” SUSY, because the MSSM with mixing and phases set to “reasonable” values and the masses all set around a TeV would have copious amounts of flavor changing, in contradiction with experiment. Thus, there must be some sort of organizing structure between the soft terms. All of the SUSY breaking scenarios we will consider in Chapter 4, provide such a principle, and throughout this thesis we will assume that some sort of organizing principle exists.

1.5 Other Motivations for SUSY

To date, no direct evidence for SUSY exists. Nevertheless, SUSY has enjoyed a great deal of theoretical attention. Why is this? Aside from a beauty and elegance that has many physicists convinced that Nature is sure to have SUSY in Her machinery somewhere, there are a number of results that strengthen the theoretical case for SUSY. You can think of these as, in a manner of speaking, motivations in hindsight. That is, if you accept the introduction of SUSY as a candidate for new physics solely because it can solve the hierarchy problem, then these resultant features appear as remarkable consequences.

Perhaps the most fascinating result is that SUSY can tell us why the weak scale is where we observe it to be. In order to generate electroweak symmetry breaking, the square of the Higgs VEV must be negative, so that the minimum of the Higgs potential is shifted away from the origin. Aside from doing this by hand, there is no dynamical mechanism to facilitate this in the SM. It is a remarkable fact that in SUSY this occurs in a very clean way. From the MSSM Higgs potential, the
conditions for EWSB are:

$$|B\mu|^2 \geq (|\mu|^2 + m_{H_u}^2)(|\mu|^2 + m_{H_d}^2)$$

$$2|B\mu| \leq m_{H_u}^2 + m_{H_d}^2 + 2|\mu|^2. \quad (1.8)$$

The first condition is what shifts the minimum of the potential away from the origin, and the second ensure that it is bounded from below. It turns out that these relations among the Higgs soft mass squares are generically true, because in the MSSM $m_{H_u}^2$ is driven negative at the weak scale, even if it is positive at the SUSY breaking scale. What drives this transition in $m_{H_u}^2$ is renormalization group evolution. The renormalization group equation (RGE) for $m_{H_u}^2$ is:

$$\frac{d}{dt} m_{H_u}^2 \simeq \frac{1}{8\pi^2} \left( -3g_2^2 M_2^2 - \frac{3}{5} g_1^2 M_1^2 + 3 y_t^2 \left( m_{Q_3}^2 + m_{u_3}^2 + m_{H_u}^2 + A_t^2 \right) \right), \quad (1.9)$$

where I have omitted the negligible 1st and 2nd family Yukawa terms. The terms proportional to $y_t^2$ are what helps to drive $m_{H_u}^2$ down, making the EWSB conditions attainable, but only if $y_t$ is big enough so that the terms proportional to $y_t$ can dominate and push $m_{H_u}^2$ down [8]. In fact this is the case, and thus EWSB is driven purely by quantum corrections in the MSSM. This phenomenon is called radiative electroweak symmetry breaking (REWSB), and it is a very compelling theoretical fact that SUSY provides such a natural mechanism for EWSB. We show in Figure 1.2 the evolution of $m_{H_u}^2$ with renormalization scale. Notice that it passes through zero, and at the weak scale is roughly $-(180 \text{ GeV})^2$. It should be noted that $m_{H_u}^2$ becoming less than zero is neither necessary or sufficient for REWSB, but achieving it without this is much more difficult and contrived. The “prediction” of REWSB is already circumstantial evidence for SUSY, since one of the key ingredients for REWSB is that the top mass be large, so that $y_t$ can be large. It is a matter of historical significance for SUSY that before the top quark was found experimentally, physicists who worked in SUSY generally believed that $m_t$ should be heavier than
100 GeV, while the consensus of the time was that $m_t$ could not really be greater than about 40 GeV. After searches at LEP, and observation of $B$-$\bar{B}$ oscillations, it became clear that the top was indeed heavy, and this became a noteworthy “feather in the cap” for SUSY.

Another significant result of SUSY lies in the gauge sector. One of the primary goals of physics beyond the SM is the construction of a unified picture of interactions between matter in the universe. Two promising ideas along this vein have been developed: string theory and grand unified theories (GUTs). In string theories, the SM gauge groups unify into a single group, with a single gauge interaction, at some very high energy scale, like the Planck scale. In GUTs, there may or may not be a single group, but the gauge couplings all come from the same source nonetheless. Either way, gauge coupling unification at high energies is a generic feature. However, if one evolves the gauge couplings using the SM $\beta$-functions, they miss unification by many standard deviations. But in the MSSM, the particle content alters the $\beta$-functions from their SM values by just the right amount so that the RGEs drive the couplings to unify at a scale $Q_{\text{unify}} \approx 10^{16}$ GeV. We see this explicitly in Figure 1.3, where we show the gauge coupling evolution for both the SM and the MSSM. That two lines cross in a plane is by itself perhaps not very intriguing. After all, in a plane, it is usually exceptional if they don’t cross. But that a third line should cross at the same point as the first two is a very interesting result indeed. Physically, this is a red flag to us that there is something special about the scale $10^{16}$ GeV.

What is the significance of the energy scale $10^{16}$ GeV? Motivated by the principle of gauge symmetry, and the unification of the weak and electromagnetic forces in the SM, physicists began exploring the possibility of embedding the SM gauge group in a larger group, typically $SU(5)$ [12] or $SO(10)$ [13], which would then be broken at some high energy scale down to the SM, in a manner analogous to how the
electroweak force is broken down to QED at the weak scale. The models that arose from this motivation, called Grand Unified Theories (GUTs), require the unification\textsuperscript{4} of the SM gauge couplings at a scale $M_{\text{GUT}} \simeq 10^{15-18}$ GeV. The lower limit of this unification window is set by the requirement for GUTs to be consistent with the non-observation of proton decay, and the upper limit is because our field theoretic picture breaks down at the Planck scale. Return once again to Figure 1.3. The vertical lines in the plot indicate the allowed window for unification in GUTs. In the vernacular, SUSY “splits the goalposts.” The result of numerical unification of gauge couplings is already an attractive outcome from the MSSM. But that it occurs precisely where GUTs require it has to is even more compelling since the two frameworks are \textit{a priori} independent. A GUT after all, is “just” a gauge theory. SUSY, while it contains gauge interactions, is fundamentally different since it by definition involves spacetime. Since many physicists feel that a GUT underlies the SM, this adds yet another theoretically attractive result: SUSY is remarkably compatible with GUTs.

SUSY is also capable of naturally incorporating gravity. As it stands, the SM does not include gravity. Indeed, understanding gravity as a quantum theory remains somewhat of a grail-like quest for physicists. By definition, any step towards a fundamental theory had better take forward steps to include all the known interactions into one framework. All of our discussion so far has focused only on SUSY as a global symmetry of Nature. However, in keeping with the the principle of a gauge symmetry, we expect that SUSY must also be gauged, that is, made local.

\textsuperscript{4}A note about unification. This word is used to mean three very different things. Strictly speaking, unification means there is a single gauge coupling, and gauge group, for all interactions. When people say that forces unify in the MSSM, they mean that the couplings constants run together. However, it does not imply that they then become a single force with one coupling. When people say that QED and the weak force are unified in the SM, it is very misleading. There is really only a relation between $SU(2)\_L$ and $U(1)\_Y$; the gauge couplings of these two forces are different at the weak scale.
Figure 1.2. Evolution of Higgs soft mass parameter $m_{H_u}$ with energy scale. This plot is actually a plot of $\text{sgn}(m_{H_u}^2)\sqrt{|m_{H_u}^2|}$. We show this over $m_{H_u}^2$ for convenience. Note that the parameter passes through zero and is driven to a value of about $-(180\text{ GeV})^2$ at the weak scale, as desired.
Figure 1.3. Evolution of the inverse gauge couplings versus $\log_{10}(Q/M_W)$. The dotted lines are the evolution in the SM, and the solid lines are the evolution in the MSSM.
Since SUSY contains as a subset the Poincaré group, it must be that local SUSY is, among other things, a theory of general coordinate transformations, or in other words, a theory of gravity. Exactly how to quantize gravity is still unclear, but the natural inclusion of the remaining fundamental force of Nature is a step in the right direction along the path to a Theory of Everything.

Despite its problems, string theory is presently the preferred candidate for a fundamental theory of Nature. And it should come as no surprise that string theory requires SUSY, since local SUSY is a low energy limit of string theory. In fact, one of the ways that SUSY was first discovered was as a symmetry of the two dimensional world sheet [11] swept out by a propagating string. All string theories which incorporate fermions must have this world sheet SUSY. But in addition, string models generically require spacetime SUSY in order to stabilize the vacuum and be free from tachyons. Models without spacetime SUSY have to overcome these problems, which are usually taken to be indicators of a sick theory. That our present candidate for a fundamental theory requires SUSY is another noteworthy feature of SUSY.

There are many other reasons for believing in SUSY. First, it is the only extension of the Poincaré group that does not violate the Coleman-Mandula theorem. In 1967, Sydney Coleman and Jeffery Mandula [7] showed that the most general extension of the Poincaré group $\mathcal{P}$ must have the form $\mathcal{P} \otimes \mathcal{H}$, where $\mathcal{H}$ is an internal symmetry group. This restricted form for any extension of $\mathcal{P}$ means that there cannot be any non-trivial commutators with the Poincaré generators. The problem with the theorem was that it assumed that the generators of any group that would extend $\mathcal{P}$ obey commutation relations. However, we know that SUSY generators obey anti-commutation relations, so the theorem does not apply.

Next, SUSY also provides an attractive candidate for dark matter. We know
from data on galactic rotations that there is some form of matter that we can only see through its gravitational effects [10]. Models of SUSY with R-parity\textsuperscript{5} have the feature that the lightest superpartner (LSP) will be completely stable. In many models, the LSP is gaugino admixture which is electrically neutral, thus making it an excellent candidate for dark matter.

In addition, SUSY adds an element of egalitarianism to the SM. A re-examination of the roles of the particles in the SM reveals that the SM is, in a manner of speaking, an oligarchic theory. There is a division of labor between the gauge fields (bosons) and matter fields (fermions). Furthermore, the Higgs has an especially conspicuous place in the SM, being the only scalar field in the theory. SUSY removes all of this. Massive gauge fermions can now mediate interactions, and there are as many scalars as fermions in the theory. In particular, the Higgs is like any other particle, apart from its R-parity transformation. It is just the scalar partner in a supermultiplet, the same as any other in the theory.

We have seen that although it has been amazingly successful, there is compelling evidence that the SM is not a true candidate for a fundamental theory of physics. Guided by the spirit of construction of the SM, we have taken a possible hint given to us by the hierarchy problem to introduce SUSY. We then proceeded to present the basics required to build a SUSY extension, and in addition how to break SUSY in a way that would be compatible with the reason for introducing it in the first place. This led us to the introduction of the soft breaking terms. Inspired by the beauty of SUSY, we saw that a number of very attractive features result immediately from the theory, adding further motivation to search for SUSY. We now proceed to examine which models of SUSY would be interesting to explore.

\textsuperscript{5}R-parity is phenomenologically necessary in order to avoid extremely rapid proton decay. Thus it is usually taken as a part of the MSSM.
In Chapter 1 we introduced SUSY, an extension of the Poincaré group which relates fermions and bosons, as a solution of the hierarchy problem. Accepting this as our primary motivation for exploring SUSY as a possibility for new physics, we introduced the basic tools needed to construct SUSY Lagrangians, and gave an inventory of the particle content of the MSSM. Observing that SUSY must be a broken symmetry, we introduced the essential ingredients of a SUSY breaking sector. We also saw that SUSY provides naturally some very attractive theoretical results, further strengthening arguments for it, and making it clear why it has enjoyed so much favor. We now proceed to examine which regions of SUSY parameter space are worthwhile to explore. We are especially interested in obtaining information about one parameter in particular. This parameter is called tan $\beta$.

2.1 Tan $\beta$

In Section 1.3 we learned that the Higgs sector of the MSSM contains two Higgs doublets, which were listed in Table 1.2. Both of these fields participate in EWSB, picking up VEVs:

$$\langle H_U \rangle = v_u$$
$$\langle H_D \rangle = v_d.$$  \hspace{1cm} (2.1)

The notation $H_U$ and $H_D$ indicate which quark fields each Higgs couples to. The VEVs of the Higgs fields are subject only to the constraint $v^2 = v_u^2 + v_d^2 =$
(174 GeV)$^2$, where $v$ is the VEV of the SM Higgs. Since only the sum of their squares is constrained, the ratio of $v_u/v_d$ is a free parameter, and we can write:

\begin{align}
  v_u &= v \sin \beta \\
  v_d &= v \cos \beta \\
  \frac{v_u}{v_d} &= \tan \beta.
\end{align}

The parameter $\tan \beta$ is related to Yukawa couplings through the quark mass via relations like $m_t = y_t \sin \beta$, so we can theoretically constrain $\tan \beta$ from perturbativity considerations on the Yukawa couplings. In order for the top Yukawa coupling to remain perturbative, it must be that $\tan \beta \gtrsim 1$; and similarly for the bottom Yukawa coupling to stay perturbative, $\tan \beta \lesssim 65$. This allowed region of $\tan \beta$ can be roughly divided into two regions. Models with $\tan \beta$ greater than about 20 are said to have large $\tan \beta$, while models with $\tan \beta$ less than about 10 are said to have small $\tan \beta$. This classification is not rigorous, but the division is useful because it delineates two qualitatively different regions of parameter space. The central point is that there is a significant difference between going from $\tan \beta$ of 5 to 10, and going from $\tan \beta$ of 30 to 35. (This is manifest from just looking at the graph of the tangent function.)

The parameter $\tan \beta$ is of central importance in any SUSY model. Virtually all SUSY observables depend on $\tan \beta$, either explicitly, or implicitly through the computed spectrum of the model. Therefore it is extremely worthwhile to take some effort to examine whether or not there is anything we can say about this parameter, in order to make studies of SUSY parameter space more productive. In particular, we wish to motivate which regime of $\tan \beta$ is likely to be realized in a SUSY world. We will see that there are good reasons to argue that $\tan \beta$ may be large.
2.2 Grand Unification and Large tan$\beta$

Consider for a moment the quark masses of the SM. The top quark is known to have a mass of 178 GeV $[9]$, while the accepted bottom quark mass is $3.5 \text{ GeV}^1$. Thus the ratio of their masses $m_t/m_b \approx 50$. This ratio of quark masses is related to the ratio of the Yukawa couplings from EWSB:

$$\frac{m_t}{m_b} = \frac{y_t v \sin \beta}{y_b v \cos \beta} = \frac{y_t}{y_b} \tan \beta. \quad (2.3)$$

In the SM, of course, the ratio of the quark masses is the same as the ratio of the Yukawa couplings, which demands a hierarchy between the Yukawa couplings. But in the MSSM, with two Higgs doublets, the Yukawas need not be too different from each other, because now $\tan \beta$ can be large to accommodate the large ratio of $m_t$ and $m_b$. This argument is naive, since we have not argued that the Yukawas should be similar, but we will soon strengthen it.

We saw in Section 1.5 that the MSSM predicts the unification of the gauge couplings at a scale $10^{16} \text{ GeV}$, in a striking compatibility with GUTs. Historically, SUSY and GUTs arose separately, without any requirement that they have anything to do with each other. But when taken in tandem, we find that there is some valuable information that can be garnered about SUSY parameter space, especially $\tan \beta$, from GUT considerations. In addition to gauge coupling unification, most GUTs also require the unification of some of the Yukawa couplings (typically those of the third family) at the GUY scale. In particular, one of the simplest and most well motivated GUTs is based on the “prettiest” gauge group $SO(10)$. Calling this gauge group the prettiest is perhaps a bias. However, the group $SO(10)$ has some very remarkable features. First, all fermions within a single generation reside

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$^1$It is known from the structure of QCD that we cannot observe free quarks, so there is some degree of ambiguity in defining their masses. The mass of the $b$ quark given here is the running $\overline{\text{MS}}$ mass evaluated at $m_Z$. The top quark mass given is its pole mass. The top quark $\overline{\text{MS}}$ mass evaluated at $m_Z$ is 166 GeV.
in a 16, including the right handed neutrino. Secondly, SO(10) is automatically anomaly free, so the “magical” cancellations of the SM anomalies are not miracles, but implies by the larger group structure. Finally models place the Higgs in a 10, so that the form of the Yukawa terms from which the fermions will get their masses is a $16 \cdot 16 \cdot 10$. Since each fermion generation resides in a 16, each generation will have the same Yukawa coupling so that $y_t = y_b = y_\tau$ at the GUT scale. Then, from Eq. (2.3), we can see that $\tan \beta$ should be about 50. Renormalization of the Yukawas down from the GUT scale changes this ratio only slightly.

In GUT models based on $SU(5)$, the discussion is somewhat more subtle. In these models, the $\tau$ and bottom quark reside in 5, while the top resides in a 10, so all of the third family cannot have the same Yukawa coupling. The $\tau$ and the bottom Yukawas, being in the same representation however, do unify at the GUT scale. The emergence of large $\tan \beta$ in these models results from a numerical study of the RGEs [14]. The one loop RGEs for the bottom and tau Yukawas are:

$$\frac{dy_b}{dt} = \frac{y_b}{16\pi^2} \left( X_g + y_t^2 + 6y_b^2 + y_\tau^2 \right)$$

$$\frac{dy_\tau}{dt} = \frac{y_\tau}{16\pi^2} \left( Y_g + 3y_b^2 + 4y_\tau^2 \right)$$

(2.4)

where $t = \log(Q/\text{GeV})$ and we have defined:

$$X_g = -\frac{7}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2$$

$$Y_g = -\frac{9}{5}g_1^2 - 3g_2^2.$$  

(2.5)

Rather than present a precise numerical study here, we instead make a rough calculation which will show us what we wish to know. We start from the RGEs above and take the following steps. First, approximate the RGEs as a finite differences:

$$y_G - y_b = \frac{y_b}{16\pi^2} \left( X_g + y_t^2 + 6y_b^2 + y_\tau^2 \right) \log \left( \frac{m_z}{M_{\text{GUT}}} \right)$$

$$y_G - y_\tau = \frac{y_\tau}{16\pi^2} \left( Y_g + 3y_b^2 + 4y_\tau^2 \right) \log \left( \frac{m_z}{M_{\text{GUT}}} \right).$$

(2.6)
Here the logarithm \( \log(m_Z/M_{GUT}) \simeq -33 \). Next, we trade the Yukawa couplings for the quark masses, this will introduce some hidden factors of \( \tan\beta \) that reside in \( y_b \) and \( y_\tau \):

\[
y_t = \frac{m_t}{v \sin \beta} \simeq \frac{m_t}{v} \\
y_{b,\tau} = \frac{m_{b,\tau}}{v \cos \beta} \simeq \frac{m_{b,\tau}}{v} \tan \beta.
\]

This last approximation is pretty good, even down to \( \tan \beta \simeq 4 \). We then substitute the numerical values for all the parameters, and solve these equations for \( \tan \beta \). From this approach, we can obtain \( \tan \beta \) in the range \( 25 \lesssim \tan \beta \lesssim 55 \), depending on the additional set of assumptions made. This range of \( \tan \beta \) is somewhat spread out, but it yields a key feature: \( \tan \beta \) is large. This is important, because as we mentioned before, \( \tan \beta \) being large or small is truly the relevant piece of information, much more so than its specific numerical value.

So far we have exploited the quark sector, and its sensitivity to Yukawa couplings through EWSB to argue that \( \tan \beta \) should be large. We will now proceed to explore what information can be garnered about \( \tan \beta \) from a different set of considerations.

### 2.3 The \( B \) Parameter and Large \( \tan \beta \)

In Section 2.2 we examined what SUSY GUTs have to say about \( \tan \beta \). From the ratio of quark masses, we inferred that \( \tan \beta \) can easily be large in both \( SU(5) \) and \( SO(10) \), two of the most commonly considered SUSY GUTs. The analysis of the top-bottom quark mass ratio was not significantly altered through loop corrections, so the result of large \( \tan \beta \) from this consideration had an attractive degree of robustness. We now proceed to argue large \( \tan \beta \) from a different approach.

Let us examine Eq. (1.7) again. The parameters represented by the \( B_{ij} \) arise in a slightly different manner in the MSSM. This scalar bilinear parameter are of particular importance in the Higgs potential, where they appear in the term:
\[ V \supset B \mu (H_U H_D + h.c.). \] It should not be surprising that \( B \) can tell us information about \( \tan \beta \) since both are tied directly to the Higgs sector. In any general model, minimizing the Higgs potential yields the following condition on \( \sin 2\beta \):

\[ \sin 2\beta = \frac{-2B\mu}{m^2_{H_u} + m^2_{H_d} + 2\mu^2}. \]  

(2.8)

Now our interest in the parameter \( B \) is manifest. If there were a way to generate a small (relative to the weak scale) value for \( B \), the relation in Eq. (2.8) demands \( \sin 2\beta \simeq \cot \beta \ll 1 \), which means that \( \tan \beta \) is large. We will now argue that achieving a small \( B \) parameter does not require any strange assumptions, but rather arises fairly naturally in a wide class of models. This in turn allows for a generic motivation for \( \tan \beta \) to be large. I will however proceed to argue in a reverse manner, arguing first from the most specific case to the most general.

Models with gauge mediated SUSY breaking (GMSB) [49] can yield naturally small values for the \( B \) parameter. We will address these models in some more detail later, but the essential feature of this class of models is that SUSY breaking is communicated through the coupling of new heavy (often much heavier than the SUSY breaking scale) messenger fields, typically in the fundamental representations of some gauge group which contains the SM gauge group, so that these new messenger fields share some interactions with the fields in the visible sector. When SUSY is broken, these messenger fields communicate the SUSY breaking to our visible sector via their gauge interactions. In general, gaugino masses generated at one loop while scalar mass-squared parameters are generated at two loops. However, the parameter \( B \) is generated at the three loop level, and so at the SUSY breaking scale it is negligibly small. Furthermore, once such a small value is generated, it does not run appreciably from its initial value. This is because in GMSB models, the scale at which SUSY gets broken is comparatively much smaller than it is in other models, therefore the logarithms \( \log(M_{SUSY}/M_W) \) that arise from the RGEs
need not be very big. Therefore at the weak scale where the minimization of the Higgs potential takes place, $\sin 2\beta \ll 1$, implying that $\tan \beta$ is large, as desired.

A small $B$ parameter is not something that is necessarily specific to GMSB models. In general, in order to generate the breaking terms of the MSSM from some high scale, you actually need to break two symmetries. The first, of course, is SUSY. The second is called an R-symmetry. An R-symmetry is a continuous global $U(1)$ under which different components of a supermultiplet carry different charge. The charge assignments for the various components of matter and vector supermultiplets are:

\begin{align}
R(\phi_i) &= \mathcal{R} & R(\psi_i) &= \mathcal{R} - 1 \\
R(A^a) &= 0 & R(\lambda^a) &= 1.
\end{align}

(2.9)

Note that for different matter supermultiplets, $\mathcal{R}$ can in principle be different. The charges for the vector supermultiplets are determined by the reality condition. A term in the Lagrangian is allowed by this symmetry if it has $R(\mathcal{L}) = 0$. Refer once again back to Eq. (1.7); we see immediately that the only terms that would be consistent with R-symmetry are the scalar mass squared terms\(^2\), whereas the gaugino masses $M_\lambda$, the trilinear $A$ terms, and the bilinear scalar $B$ terms are not in general consistent with an R-symmetry. Generically, it is natural for which violate symmetries to have smaller coefficients than terms that do not violate the symmetry because the in the limit that those symmetry violating terms go to zero, the symmetry would be restored. Looking at the breaking terms, we see that the only terms that do not violate R-symmetry are the scalar (mass)\(^2\) terms. Therefore, it would be natural for $B$ to take smaller values than such terms, making it natural for the denominator in Eq. (2.8) larger than the numerator, in turn making large values of $\tan \beta$ easier to reach.

\(^2\)Actually, only the \textit{diagonal} scalar squared terms are necessarily uncharged, but this is a minor point; usually only diagonal scalar mass squared terms are considered.
This argument, although somewhat more general, is admittedly weaker than the argument for small $B$ in GMSB models. Even though $B$ terms are in general lighter than scalar mass-squared terms, there is nothing saying they must be negligible. Furthermore, even if they are very light, renormalization effects in other models of SUSY breaking play a more important role than in GMSB because the SUSY breaking scale can be much heavier than it is in GMSB. However, the goal of examining this weaker argument was to show that the smallness of $B$ is a rather generic feature of SUSY models, and not something specific to only one specific SUSY breaking scenario.

This generic smallness of the $B$ parameter relative to the other masses in the theory can be seen in a different light. To understand it, we make use of the superfield formulation of SUSY. The superfield formulation is a very elegant way to construct SUSY Lagrangians out of objects called superfields, written, $\Phi(x, \theta, \bar{\theta})$, which are functions not only of the spacetime $x^\mu$, but also of two component Grassman numbers $\theta^\alpha$ and $\bar{\theta}^\dot{\alpha}$, which transform as Weyl spinors. The necessary information about Grassman numbers needed for this discussion can be found in Appendix C. We call the combination of spacetime and Grassman coordinates superspace. In this formulation, superpartner fields within a supermultiplet become component fields along the $\theta$ and $\bar{\theta}$ “directions” of the superfield. For example, a matter supermultiplet can be written as one superfield:

$$
\Phi_i(x, \theta, \bar{\theta}) = \phi_i + \sqrt{2} \theta \psi_i + \theta \theta F_i + i \partial_\mu \phi_i \theta \sigma^\mu \bar{\theta} - \frac{i}{\sqrt{2}} \theta \theta \partial_\mu \psi_i \sigma^\mu \bar{\theta} - \frac{1}{4} \partial_\mu \partial^\mu \phi_i \theta \theta \bar{\theta} \quad (2.10)
$$

where $\phi_i$ and $\psi_i$ are ordinary fields, and the $F_i$ are auxiliary fields $^3$, which are the same as the derivatives of the superpotential $W_i$ from below Eq. (1.5). The factors

$^3$An auxiliary field is a non-dynamical field which can be eliminated from the Lagrangian by its equations of motion.
of Grassman numbers in Eq. (2.10) can look deceiving at first. Note that $\theta^\alpha \theta^\alpha$ (no sum) is zero, but that $\theta \theta = \theta^\alpha \theta_\alpha = \epsilon^{\alpha\beta} \theta_\alpha \theta_\beta$ (contracted sum) is not, because although it may look like a quadratic power of $\theta$, it is really an inner product and so need not vanish.

The Lagrangian we are accustomed to dealing with is that which gives us the action, that is, it is only a function of spacetime. Therefore we must integrate out the superspace dependence. This integration in effect “projects out” the appropriate $\theta$ and $\bar{\theta}$ components in order to ensure that the resulting Lagrangian is invariant under SUSY. The terms we desire are those that transform as a total derivative under the action of SUSY transformations. The terms with this property are the $\theta \theta$ and $\theta \bar{\theta} \bar{\theta} \bar{\theta}$ terms. Therefore we define the superspace volume elements:

$$
d^2 \theta = -\frac{1}{4} d\theta^\alpha d\theta_\alpha
$$
$$
d^2 \bar{\theta} = -\frac{1}{4} d\bar{\theta}_\alpha d\bar{\theta}^\alpha
$$
$$
d^4 \theta \equiv d^2 \theta d^2 \bar{\theta},
$$

(2.11)

which will be used to integrate out the terms of interest. Then, using this superfield/superspace language, and the tool of Grassman integration, we can cast the Lagrangian of Eq. (1.4) into the following form:

$$
\mathcal{L} = \int d^4 \theta K(\Phi, \Phi) + \left( \int d^2 \theta W(\Phi) + \text{h.c.} \right).
$$

(2.12)

Here $W$ is of course the superpotential\(^4\), and the object $K$ is an arbitrary function, usually called the Kähler potential, which need not in general be renormalizable, but only gauge invariant and real. The Lagrangian of Eq. (1.4) for example had $K(\Phi^\dagger, \Phi) = \Phi^\dagger \Phi$. In a general model, it will contain not only this operator, but others as well.

\(^4\)A caveat here. The function given in Eq. (1.5) was a function of the scalar components of the supermultiplets. In this instance the functional form of $W$ is the same, but it is now a function of the superfields $\Phi_i$.  

29
General models of SUSY breaking couple some hidden sector SUSY breaking spurion field $X = \phi_X + \theta \psi_X + \theta^2 F_X$ to the fields of the MSSM. Then, components of $X$ pick up VEVs which generate the soft terms. The two operators which could generate $B$ are:

\[
\begin{align*}
\text{Source I} &= \int d^2 \theta \frac{XW(\Phi)}{M} \\
\text{Source II} &= \int d^4 \theta \frac{X^t X}{M^2} (H_U H_D + h.c.)
\end{align*}
\]

where $M$ is some very heavy mass scale, such as the Planck scale. Remember that $W(\Phi) \supset \mu H_U H_D$. Since these terms are not renormalizable\(^5\), we suppress them by powers of some heavy scale, $M$, which is possibly the Planck scale. There are other terms that could contribute to $B$, but they require higher dimension operators, and so will suffer further suppressions from the heavy scale $M$. Let us now discuss the implications of each of these sources.

The contribution in Source I is not permitted by gauge invariance, unless $X$ is a singlet. However, if the field $X$ is a singlet, this term has a different, yet equally detrimental problem. Fields that are gauge singlets wreak havoc with the protection against the re-appearance of quadratic divergences guaranteed by the non-renormalization theorem [16], which was one of SUSY’s strongest motivations. Breaking SUSY in this manner makes no sense; it would destroy the reason we claim to introduce SUSY in the first place. So since $X$ cannot be a singlet, this term is not allowed by gauge invariance. Thus we should expect that Source I cannot contribute significantly to the $B$ parameter.

We now turn our attention to Source II. This operator suffers from the problem that it violates a Pecci-Quinn symmetry. This is a symmetry in which the $H_U$ and

\(^5\)Note that Grassman numbers must carry mass dimension, since they transform as spinors. In particular $\text{Dim}[\theta] = 1/2$ and $\text{Dim}[d\theta] = -1/2$. Any operator that gets integrated over in $\int d^2 \theta$ must be dimension three, and any operator that is integrated over in $\int d^4 \theta$ must be dimension two.
$H_D$ carry different charge. The $\mu$ term in the superpotential breaks this Pecci-Quinn softly, so $\mu$ carries $Q_{\text{PQ}} = 2$. Thus, in order for Source II to contribute, it must be modified slightly to include explicit factors of $\mu$ because the combination $X^\dagger X$ carries no Pecci-Quinn charge. Therefore we consider the modification:

$$\text{Source III} = \int d^4 \theta \frac{X^\dagger X}{M^3} (\mu H_U H_D + h.c.),$$

which, after performing the Grassman integration becomes:

$$\text{Source III} \rightarrow \frac{|F_X|^2}{M^3} \mu H_U H_D.$$

In the preceding equations we mean $F_X = \langle F_X \rangle$ in a rather standard abuse of notation. Now we can make the identification $B = |F_X|^2/M^3$. In order to determine the size of $B$, we must understand the size of $|F_X|/M$. To do this, we examine the operator which is responsible for generating squark masses:

$$\mathcal{L}_{M^2} = \int d^4 \theta \frac{X^\dagger X}{M^2} (Q^\dagger Q + h.c.)$$

$$= \frac{|F_X|^2}{M^2} \tilde{Q}^\dagger \tilde{Q}$$

$$= M^2 \tilde{Q}^\dagger \tilde{Q}.$$

In order that SUSY solve the hierarchy problem, it must be that $M_{\tilde{Q}} \simeq M_W$. Therefore, $|F_X|/M \simeq M_W$. This in turn implies that $B \simeq M_W^2/M = \epsilon M_W \simeq 10^{-14} \text{GeV}$ so that $B$ is negligibly small in comparison to the weak scale.

This argument for small $B$ does have the weakness that it is the least specific of the arguments given. However, this is also its strength; it uses only general symmetries of the operators. The overall point is that a small $B$ parameter can be generically natural in SUSY, which in turn implies large $\tan \beta$.

### 2.4 Large $\tan \beta$: Closing Remarks

We saw in the preceding two sections how a large value for $\tan \beta$ can be motivated in a broad class of SUSY models through two different approaches. Each of these
arguments was well grounded, and provided solid motivation that $\tan\beta$ should take on large values. In fact, taken together they constitute a whole that is greater than the sum of its parts. Since each motivation has its origins in very different physical sectors, taken together they cover a broad spectrum of physics. A small $B$ parameter is intrinsically tied to SUSY breaking and the soft terms, while Yukawa unification is tied to RG evolution and the quark and Higgs sectors. Let us take a moment to heuristically tie them together.

Our discussion of Yukawa unification in Section 2.2 strongly hinted at the broader theme for physics at large $\tan\beta$, although it did not explicitly state it. The Higgs sector with large $\tan\beta$ is more interesting than it is at lower $\tan\beta$. This is because at large $\tan\beta$ there are potentially three large Yukawa couplings in the theory, rather than just the usual single large top Yukawa. This is a very important fundamental difference between the SM and SUSY: the SM has only one large Yukawa coupling, while SUSY can have three. This possibility of large couplings opens the door for a much richer physics than would otherwise be possible. Interactions proportional to these Yukawas would, without the enhancement afforded at large $\tan\beta$, not be interesting.

These large Yukawas are also linked to SUSY breaking, through RG effects. We will see in Chapter 4 that not only the structure of the SUSY breaking sector, but also the manner in which it runs down from the high scale, can have important effects on physical processes. These large Yukawa couplings can make pieces in the RGEs more relevant than there would be otherwise, resulting in very different running.

We have thus far motivated SUSY as a viable candidate for new physics, and also discussed which class of SUSY models provide good hunting ground for signals of SUSY through our examination of $\tan\beta$. We shall now proceed to examine two
physical processes that are quintessential large $\tan\beta$ processes, which provide clean tests and constraints for SUSY at low energies.
CHAPTER 3

THE MUON ANOMALOUS MAGNETIC MOMENT

In Chapters 1 & 2 we introduced and theoretically motivated the case in favor of SUSY, and examined what regions of parameter space would be interesting to study. It is crucial that predictions be made that are accessible to current experiments, or that will be accessible in the near future. This will provide the means to rule out models, or discern which model we are seeing if a signal for new physics is found.

We now proceed to the first of two processes which constitute the main subject of my studies. These two processes are inherently large tan$\beta$ signals. The first of the two is the muon anomalous magnetic moment. Before entering into a full discussion of the muon anomalous moment, we present a history of these quantities in order to motivate our study of them.

3.1 Lepton Magnetic Moments

Magnetic moments have a rich history that is closely intertwined with the development of quantum mechanics. This is because spin effect in quantum physics were among the first effects explored by physicists. Any particle with spin has an intrinsic magnetic moment. A lepton in an “orbit” with angular momentum $L$ will couple to an external magnetic field and posses magnetic moment:

$$\mu = \frac{e}{2m_\ell} L.$$  \hspace{1cm} (3.1)
Thus in a state with spin $\mathbf{S}$ we would naively expect that the magnetic moment would be the same as that in Eq. 3.1 with $\mathbf{L} \rightarrow \mathbf{S}$. Experiments, however, revealed that this is not so. It turns out that the magnetic moment was twice as large as expected. In other words:

$$\mu = g_s \frac{e}{2m_\ell} \mathbf{S}, \quad (3.2)$$

with $g_s = 2$. This $g_s$ is often called the Landé $g$-factor. It was one of the first successes of Dirac’s theory was that this factor of two arose naturally. This is done by coupling the lepton to an electromagnetic field by letting $p^\mu \rightarrow p^\mu - eA^\mu$ in the Dirac equation. Then in taking the non-relativistic limit ($E \simeq m$ and $\mathbf{p} \simeq m\mathbf{v}$) we obtain the Hamiltonian:

$$H = \frac{1}{2m_\ell} (\mathbf{p} - e\mathbf{A})^2 + e\phi - \frac{e}{m_\ell} \mathbf{S} \cdot \mathbf{B} \quad (3.3)$$

and you can see that the last term which is the spin interaction term does in fact have the correct factor of two.

With the advent of quantum electrodynamics (QED), theorists explored the implications that radiative corrections have on physical processes. One of the first great triumphs of QED was the successful prediction of the electron magnetic moment to one part in a million. In QED the expression for the lepton-photon vertex can in general be written:

$$\Gamma^\mu(p',p) = \gamma^\mu F_1(q^2) + \frac{e}{2m_\ell} \sigma^{\mu\nu} q_\nu F_2(q^2). \quad (3.4)$$

Here $q = p - p'$ and the functions $F_1$ and $F_2$ are the so called form factors. These two functions together contain all the information about the electric charge and magnetic moment of the lepton, that is, how the lepton will couple to external $\mathbf{E}$ and $\mathbf{B}$ fields. Computing the amplitude for electron scattering from an external field in the Born approximation yields:

$$Q_\ell = eF_1(0)$$
\[ \mu_\ell = 2[F_1(0) + F_2(0)] \left( \frac{e}{2m_\ell} \right) S. \] (3.5)

The factor of two and the factor inside the square brackets play the part of the Landé \( g \)-factor. At lowest order, \( F_1(0) = 1 \) and \( F_2(0) = 0 \), and we recover the Dirac result. Higher order corrections, however, shift this value away from 2, and this has been verified experimentally. In fact, the agreement of measurement with the theoretical precision is astounding: lepton magnetic moments are the most precisely known quantities in the history of science. For instance the experimental and theoretical values of the electron anomalous moment, \( a_e \), agree to better than eight digits.

It is this property that makes the endeavor of lepton magnetic moments worthwhile. This is a quantity that i) theorists say they can calculate to great degree of precision, and ii) experimentalist say they can measure to a great degree of precision. This makes them an important constraint for any model of physics beyond the SM.

3.2 Recent Chronology of \( a_\mu \)

In February of 2001, the Brookhaven National Laboratory (BNL) experiment E821 announced deviation of the muon magnetic moment, \( a_\mu = (g - 2)_\mu / 2 \), from its SM value by about 2.7\( \sigma \) [18]. This spurred a flurry of activity analysing the reported excess in various frameworks of new physics, including SUSY [19, 20, 21, 22]. My collaborators and I also performed an analysis in August of 2001 [23], with interest in what bounds on SUSY partner masses can be inferred from the discrepancy. In October of 2001, efforts by theorists brought errors to light in the theoretical calculation within the SM. In particular, the sign of the light-by-light (LBL) hadronic contribution was in error [35]. The correction of this contribution shifted the theoretical value towards the E821 value and reducing the discrepancy to about a 1.6 \( \sigma \) effect, leaving hardly any indication of new physics.
In the interim there was continued activity on \(a_\mu\). In the months following this activity, E821 continued an analysis based on a data set four times larger than their original set of Ref. [18]. At the same time, theorists made new calculations not only of the LBL pieces, but also of the other hadronic pieces contributing to \(a_\mu\) [24, 25, 26, 27]. In August of 2002, E821 released their updated measurement of \(a_\mu\) using the larger data set. The resultant world average for \(a_\mu\) was slightly higher than the previous value, but with smaller errors. These efforts placed the discrepancy in the range \([1.5\sigma, 3.2\sigma]\). We will discuss this range of values shortly.

In light of the new experimental and theoretical numbers, my collaborators and I made a new analysis which took into account these developments.

Our “active” involvement with \(a_\mu\) ended here. The status of the discrepancy since then has been ongoing. Most recently, in August 2003 the CMD-2 experiment announced [28] a re-analysis of the cross-sections for \(e^+ e^- \rightarrow\) hadrons, which theorists use to calculate hadronic contributions to \(a_\mu\), after overestimating their luminosity due to omission of leptonic t-channel contributions to Bhabba scattering.

In light of this new information from CMD-2, theorists re-calculated the hadronic pieces using the corrected cross sections [29, 30, 31]. At present, the discrepancy in \(a_\mu\) is a 2.7\(\sigma\) effect. I now proceed to give the analysis done in August of 2002. It should be stressed that any future change in the discrepancy can be accommodated by this work. One need only to read off the information from the appropriate figures in this thesis.

3.3 SUSY and \(a_\mu\)

The anomalous magnetic moment, which is the quantity measured by the Brookhaven E821 collaboration, is the coefficient of the the dimension 5 operator:

\[
\mathcal{L}_{a_\mu} = \frac{a_\mu}{2m_\mu} \bar{\psi}\sigma^{\lambda\nu} \psi F_{\lambda\nu}.
\]
In the SM, $a_\mu$ receives contributions from QED, electroweak, and hadronic processes, with the hadronic contributions usually separated into vacuum polarization and light-by-light scattering. The QED and electroweak contributions are very well understood, and have the values (in $10^{-10}$ units):

$$a^{\text{QED}}_\mu = 11 658 470.56 (0.29) \quad [32]$$
$$a^{\text{EW}}_\mu = 15.2 (0.1) \quad [33].$$

The calculation of the hadronic pieces requires experimental input. The next-to-leading (NLO) order contributions have been known for some time, while the LBL contributions have been updated:

$$a^{\text{Had,NLO}}_\mu = -10.0 (0.6) \quad [34]$$
$$a^{\text{Had,LBL}}_\mu = 8.0 (4.0) \quad [35].$$

Davier et al. [26] have calculated the LO hadronic pieces using $e^+e^-$ scattering data, and also using $\tau$-decay data in conjunction with the $e^+e^-$ data. They obtain:

$$a^{\text{Had,LO}}_\mu = \begin{cases} 
684.7 (7.0) & \text{(no } \tau \text{ data)} \\
701.9 (6.2) & \text{(} \tau \text{ data)} 
\end{cases}$$

Hagiwara et al. [27] have also calculated the LO hadronic contribution to $a_\mu$ without $\tau$ data:

$$a^{\text{Had,LO}}_\mu = 683.1 (6.2) \quad (\text{no } \tau \text{ data}),$$

which is consistent with Eq. (3.9). In the following analysis, I will use the average of the two results, but the larger error. Summing up all the contributions yields the SM prediction for $a_\mu$:

$$a^{\text{SM}}_\mu = \begin{cases} 
11 659 167.7 (8.1) & \text{(no } \tau \text{ data)} \\
11 659 185.7 (7.4) & \text{(} \tau \text{ data)} 
\end{cases}$$

These two values are not consistent with each other, therefore I will not combine them into one prediction of $a_\mu$. 38
The measurement made by E821 is $a_{\mu}^{E821} = 11 659 204 (7)(5) \times 10^{-10}$ [36] yielding a world average of:

$$a_{\mu}^{\exp} = 11 659 203 (8) \times 10^{-10}$$  \hspace{1cm} (3.12)

from which one deduces a discrepancy between the experiment and the Standard Model of:

$$\delta a_{\mu} = \begin{cases} 
35(11) \times 10^{-10} & \text{(no } \tau \text{ data)} \\
17(11) \times 10^{-10} & \text{(} \tau \text{ data)} 
\end{cases}$$  \hspace{1cm} (3.13)

To arrive at the numbers in Eq. (3.13), I have added the theoretical and experimental numbers in quadrature. From this, we see that the deviation is either 3.2$\sigma$ or 1.5$\sigma$, depending on whether or not the $\tau$ data is used. In the analysis that follows, we present results using both values of $\delta a_{\mu}$ from Eq. (3.13).

At this point a comment about the two data sets is prudent. It is clear that the two data sets are inconsistent with each other. The problem is that the extraction of the hadron polarization amplitudes differs considerably when using the $e^+e^-$ versus the $\tau$ data. While the extraction is theoretically clean, the $\tau$ data presents a challenge due in part to isospin breaking effects in the $\rho$ meson mass difference [37]. However, there is no clear consensus on which result is more reliable. Hopefully this will be resolved in the future.

The SUSY contributions to $a_{\mu}$ were explored early on in the history of SUSY and have become more complete with time [38]. In my analysis I will follow the notation of [21] which has the advantage of using the standard conventions of Haber and Kane [39]. Any convention not defined here can be found in either of these two papers.

### 3.3.1 SUSY Diagrams Contributing to $a_{\mu}$

In the mass eigenbasis, there are only two one-loop diagrams contributing to $a_{\mu}$. These are shown in Figure 3.1. The diagram on the left has an internal loop of
smuons and neutralinos, the diagram on the right a loop of sneutrinos and charginos. Working in the mass eigenbasis is calculationally more efficient. However, the gauginos in Figure 3.1 are themselves admixtures of the interaction eigenstates, and the physics is more transparent if we examine this process using diagrams in the interaction eigenbasis even though there are many more of them. Working in this basis we can more easily separate leading and sub-leading contributions with a few observations.

First, we note that the magnetic moment operator is a helicity changing interaction. So any diagram must contain a helicity flip somewhere along the fermion current. There are two distinct ways to accomplish this: a helicity flip on an external leg, and a helicity flip on an internal propagator. Those with the flip on the external leg scale like $m_\mu$, while those with the flip on an internal line scale like $M_{SUSY}$. Since $M_{SUSY} \gg m_\mu$, diagrams with external helicity flips are negligible, and our discussion will be restricted to diagrams with internal helicity flips.

Next, notice that the interaction of the charginos and neutralinos with (s)muons and sneutrinos is through their Higgsino or gaugino pieces. Therefore each vertex contains a factor of either $y_\mu$ or $g$ (the weak and/or the hypercharge gauge coupling). Since each diagram has two vertices, each diagram will scale as one of $g^2$, $y_\mu g$, or $y_\mu^2$. In the SM, $y_\mu$ is smaller than $g$ by about $\mathcal{O}(10^{-3})$. In the MSSM at low $\tan\beta$ this
ratio is unchanged, but at large tan $\beta$ this can be reduced to $O(10^{-1})$ because the muon Yukawa coupling scales as $1/\cos \beta$. Thus we can drop terms that scale like $y_\mu^2$, but retain the terms that scale like $y_\mu g$ or $g^2$. Note that any pieces we decide are negligible here are in fact retained in the numerical calculations.

The diagrams representing the contributions we keep are shown in Figures 3.2. In Fig. 3.2(a)-(e) are the five neutralino pieces that scale like $g^2$ or $y_\mu g$; the only surviving chargino contribution, which scales like $y_\mu g$, is shown in Fig. 3.2(f). The contribution of the $i^{th}$ neutralino and $m^{th}$ smuon from each of these interaction diagrams is:

$$
\delta a_\mu = \frac{1}{48\pi^2} \frac{m_\mu m_{N_i}}{m_{\mu_m}^2} F_2^N(x_{im}) \times \begin{cases} 
 g_1^2 N_{i1}^2 X_{m1} X_{m2} & (\tilde{B}\tilde{B}) \\
 g_1 g_2 N_{i1} N_{i2} X_{m1} X_{m2} & (W\tilde{B}) \\
 -\sqrt{2} g_1 y_\mu N_{i1} N_{i3} X_{m2}^2 & (\tilde{H}\tilde{B}) \\
 \frac{1}{\sqrt{2}} g_1 y_\mu N_{i1} N_{i3} X_{m1}^2 & (B\tilde{H}) \\
 \frac{1}{\sqrt{2}} g_2 y_\mu N_{i2} N_{i3} X_{m1}^2 & (\tilde{W}\tilde{H}) 
\end{cases} \quad (3.14)
$$

and from the $k^{th}$ chargino and sneutrino we have:

$$
\delta a_\mu = \frac{1}{24\pi^2} \frac{m_\mu m_{C_k}}{m_{\mu_p}^2} F_2^C(x_{k}) g_2 y_\mu U_{k2} V_{k1} (\tilde{W}\tilde{H}). \quad (3.15)
$$

The gaugino diagonalizing matrices $N$, $U$, and $V$, and smuon mixing matrix $X$, are given in Appendix A, along with the kinematic functions $F_{n,N,C}(x)$. A comparison to Eq. (A.1) in Appendix A will reveal that in Eq. (3.14 - 3.15) we have ignored a number of complex conjugations from the full expression. It has been shown [20, 21] that the SUSY contributions to $a_\mu$ are maximized for real mass matrices and so we do not retain these phases in the rest of this discussion.

Many previous analyses of the MSSM parameter space found that at large tan $\beta$ the chargino-sneutrino diagram can most easily generate values of $\delta a_\mu$ large enough to explain the discrepancy. Using this observation, one can obtain upper mass
Figure 3.2. SUSY diagrams contributing to $a_\mu$ at one loop in the interaction basis.
bounds on the lightest chargino and muon sneutrino. However this behavior is not completely generic. Martin and Wells [21] emphasized that the $\tilde{B} - \tilde{B} - \tilde{N}$ contribution could alone generate the older observed excess in $a_\mu$. Since this contribution has no explicit tan$\beta$ dependence, they could explain the older discrepancy with a tan$\beta$ as low as 3. We can understand this result in a simple way because the $\tilde{B} \tilde{B}$ contribution has a calculable upper bound at which the smuons mix maximally. If $m_{\tilde{\mu}_1} << m_{\tilde{\mu}_2}$, and $m_{\tilde{N}_1} << m_{\tilde{N}_{2,3,4}}$, and the lightest neutralino is completely bino. In this case:

$$|\delta a_\mu|_{(\tilde{B}\tilde{B})} \leq \frac{g_1^2}{32\pi^2} \frac{m_\mu m_{\tilde{N}_1}}{m_{\tilde{\mu}_1}^2} \simeq 3800 \times \left( \frac{m_{\tilde{N}_1}}{100 \text{ GeV}} \right) \left( \frac{100 \text{ GeV}}{m_{\tilde{\mu}_1}} \right)^2$$

(3.16)

where we are expressing $\delta a_\mu$ in $10^{-10}$ units. To arrive at this, we have used the fact that $X_{11}X_{22} \leq \frac{1}{2}$ and $F_2^N(x) \leq 3$. We have also included a 7% two-loop suppression factor, as discussed in Appendix A. Though In any real model this contribution will be smaller, this is still $10^2$ larger than needed experimentally.

This pure $\tilde{B}\tilde{B}$ world is actually an experimental worst-case, especially for hadron colliders. In this scenario, the only super partners that are required to be light are a single neutralino (which is essentially $\tilde{B}$-like) and a single smuon. The neutralino is difficult to produce, and if it is stable, impossible to detect directly. The neutralino could be observed as missing energy in the decay of a smuon, but smuon production at a hadron machine is highly suppressed. Thus in the worst possible world, E821 could be explained with only these two light sparticles, with the rest of the SUSY spectrum hiding above a TeV. Making matters even worse, the super partners required to be light can in principle be too heavy to produce at a 500 GeV linear $e^+e^-$ collider. While this case is in no way generic, it is sufficient to demonstrate that E821 does not provide any sort of “no-lose” theorem at the Tevatron, LHC, or even a possible NLC.

This raises an important question: how many of the MSSM states must be “light”
in order to explain the E821 discrepancy? In the worst-case, only two. But even in the more optimistic scenario where the chargino-sneutrino diagram dominates the contribution to $a_\mu$, the answer appears to also be only two: a single chargino, and single sneutrino. In this picture, we have:

$$|\delta a_\mu|_{(C\tilde{\nu})} \leq \frac{g_2 y_\mu m_\mu m_\tilde{\chi}_1}{24\pi^2 m_\tilde{\nu}^2} |F_2^C|_{\text{max}}$$

$$\lesssim 2600 \times \left( \frac{m_\tilde{\chi}_1}{100 \text{ GeV}} \right)^2 \left( \frac{100 \text{ GeV}}{m_\tilde{\nu}} \right)^2 \left( \frac{\tan\beta}{30} \right)$$

(3.17)

where we have bounded $|F_2^C(x)|$ by 10 by assuming that $m_\tilde{\nu} \lesssim 1 \text{ TeV}$. This discussion is a bit simplistic, however, as we will see shortly.

3.3.2 Mass Correlations

There are a total of nine super partners that can enter into the loop diagrams of Figure 3.1: the muon sneutrino, two smuons, two charginos, and four neutralinos. The spectrum of these particles in the MSSM is determined entirely by seven parameters: two slepton soft masses ($m_L, m_R$), two gaugino masses ($M_1, M_2$), the $\mu$-term, a soft trilinear coupling ($A_\mu$), and finally $\tan\beta$. We discuss in Appendix A that $A_\mu$ has a negligible effect, and so can be eliminated from consideration. Additionally, in some well motivated SUSY models, $M_1$ and $M_2$ are related to each other. Therefore, there are either five or six parameters for setting nine sparticle masses. From these arise non-trivial correlations which can be useful in setting mass limits on the various sparticles.

First, there are correlations among the gauginos. A light $\tilde{W}$-like chargino ($\tilde{C}_i \sim \tilde{W}$) implies a light $\tilde{W}$-like neutralino ($\tilde{N}_i \sim \tilde{W}$), and vice-versa. There are also correlations in “mixed” (i.e., gauginos and smuons) systems between the masses of eigenstates and the size of their mixing. Consider the case of the smuons in particular. Their mass matrix is given in Appendix A. Upon diagonalization, the
The smuon mixing angle is given by:

$$\tan 2\theta_{\mu} = -\frac{2[\mathcal{M}_{\mu}^2]_{12}}{[\mathcal{M}_{\mu}^2]_{11} - [\mathcal{M}_{\mu}^2]_{22}} = \frac{2m_\mu \mu \tan \beta}{M_L^2 - M_R^2} \quad (3.18)$$

The chargino contribution is maximized for large smuon mixing, which occurs when either the numerator of Eq. (3.18) is very large relative the the denominator (i.e. very large off-diagonals) or when the denominator is nearly zero (i.e. degenerate diagonal elements). For the smuons, because the numerator is suppressed by $m_\mu$, the numerator can only become large when $\mu$ is very big. The denominator gets close to zero when $m_L^2 - m_R^2 \simeq m_Z^2$. Both of these conditions can have significant impacts on the spectrum.

There is another correlation that we have imposed on the spectrum, that of slepton mass universality. The most general version of the MSSM is plagued with dangerous amounts of lepton flavor violation (LFV) unless some degree of order is placed on the spectrum. The simplest such order one can place is that sparticles with the same gauge quantum numbers be degenerate. This is a stringent constraint on the squark sector, but this also affects the sleptons because of non-observation of $\mu \to e\gamma$ and $\tau \to \mu\gamma$. And typically mechanisms which generate degeneracy in the squark sector usually do so in the sleptons too. Thus we assume $m_{\tilde{\tau}_L} = m_{\tilde{\mu}_L} = m_L$ and $m_{\tilde{\tau}_R} = m_{\tilde{\mu}_R} = m_R$. This makes the (mass)$^2$ matrix of the staus identical to the smuons with $\mu$-index $\rightarrow$ $\tau$-index in the off diagonals. This gives an enhancement of mixing in the staus by a factor of $m_\mu / m_\tau \simeq 17$. This implies that $m_{\tilde{\tau}_1} < m_{\tilde{\mu}_1}$. In particular, if

$$M_L^2 M_R^2 < m_\tau^2 \mu^2 \tan^2 \beta \quad (3.19)$$

then $m_{\tilde{\tau}_1}^2 < 0$ and QED would be broken by a stau VEV. With slepton universality, this imposes the following condition on the smuon (mass)$^2$ matrix:

$$M_L^2 M_R^2 > \left(\frac{m_\mu}{m_\tau}\right)^2 m_\tau^2 \mu^2 \tan^2 \beta \quad (3.20)$$
or alternatively on the smuon mixing angle:

$$\tan 2\theta_\mu < \left( \frac{m_\mu}{m_\tau} \right) \frac{2M_L M_R}{M_L^2 - M_R^2}.$$  \hspace{1cm} (3.21)

While this does not eliminate the possibility of $\theta_\mu \simeq 45^\circ$, Eq. (3.21) generically implies that in order to obtain large mixing, you need a fine-tuning of better than one part in seventeen. I do not impose any such fine-tuning, yet we will find that imposing slepton mass universality reduces the upper bounds on slepton masses which are obtained in our study of MSSM parameter space. (As a side note, if one were to assume this slepton mass universality condition at some SUSY breaking scale above the weak scale, the Yukawa-induced corrections from RG evolution would break universality and drive the stau masses down. This in turn would further reduce the bounds on masses and mixing angles.)

The preceding assumption of slepton mass universality has an especially significant impact on the $\tilde{B}\tilde{B}$ worst-case scenario previously discussed. For a generic point in MSSM parameter space, one would expect that $\tan 2\theta_\mu \lesssim 1/17$ which reduces the size of the $\tilde{B}\tilde{B}$ contribution by a factor of 17. Because of this, the masses that would be needed to explain the E821 anomaly get pushed back towards the experimentally accessible range.

3.4 Numerical Results

Now that the basic approach has been given, I will carry out the analysis in detail. I focus my efforts on three cases. The first, gaugino unification, is the one most often considered in the literature. Here it is assumed that the gaugino mass parameters ($M_1$ and $M_2$) are equal at the same scale at which gauge couplings unify. This means that at the weak scale $M_1 = (5/3)(\alpha_1/\alpha_2)M_2$. The second case we consider is gaugino unification with the added requirement that the lightest SUSY partner (LSP) be a neutralino. As mentioned in Section 1.5 there is some interest in a
neutral LSP for a possible explanation for dark matter. The last case we consider is what we will call the “general MSSM” case, in which we assume no relation between the input parameters, and do not require that the LSP be electrically neutral.

Then the basic approach is straightforward. Using the formulas in Appendix A, I then put down a logarithmic hyper-grid on the space of parameters for several choices of $\tan\beta$. The grid runs from 10 GeV for $M_1$, $M_2$, and $\mu$, and from 50 GeV for $m_L$, and $m_R$ up to 2 TeV for all parameters. For the case with gaugino unification, $M_2$ is not a free parameter, and so is not looped over. I only consider $\mu > 0$ because the sign of $\mu$ essentially determines the sign of the $\delta a_\mu$, and the data favors a positive discrepancy. For the bounds on $\tan\beta$ I used an adaptive mesh routine over the parameter space. Due to the method used, I estimate that the errors on the mass bounds to be less than $\pm 5\%$.

3.4.1 Lightest Sparticle Bounds

Perhaps the most important information that can be obtained from the E821 data is an upper bound on the scale of sparticle masses. In fact, we can place bounds on these masses as a function of $\delta a_\mu$. This is useful given the state of flux of this discrepancy. From the analysis, if/when $\delta a_\mu$ changes values, one can simply read off new values for sparticle mass bounds. We will extract mass bounds on the lightest slepton and chargino, and, in addition, I will extract bounds on the lightest few particles, independent of their identity.

These additional bounds on sparticles independent of their identity provides a very important piece of information. We mentioned in Section 3.3.1 that it could be that E821 is explained by a lone pair of light sparticles with the rest of the spectrum lying out of reach experimentally. However, the bounds on the additional sparticles will not only tell us at which scale we can find SUSY, but also how
much information we might hope to uncover at that scale. The more sparticles we can detect, the better equipped we will be to disentangle the structure of the soft breaking sector, and perhaps get some hints as to the origins of SUSY breaking.

In Figure 3.3-3.6 I show the bounds of the four lightest sparticles for the gaugino unification scenario. These bounds are ignorant of the species of sparticle (those bounds will come in Section 3.4.2 and will always be greater than these bounds) but simply bounds on whatever sparticle is lightest. The important points to note on these plots are: i) the maximum values of mass are for the largest value of tan$\beta$; ii) the $1\sigma$ lower limit without (with) $\tau$ data requires at least two sparticles to be lighter than 490 (990) GeV' and iii) for low values of tan$\beta$ a maximum value of $\delta a_\mu$ is reached. This maximum value of $\delta a_\mu$ is due to the fact that, at low tan$\beta$, the only way to generate a sizable value for $\delta a_\mu$ is to have sparticles that would be lighter than experimental mass bounds. This is not the case at large tan$\beta$, where the linear scaling of $\delta a_\mu$ with tan$\beta$ makes it possible to generate sizable with light sparticles. (In poker terms, at large tan$\beta$ you have more “outs” to generate a sizable value for $\delta a_\mu$.)

I could produce the same plots for the case where the LSP is required to be a neutralino, but I will show for this scenario only a plot of the LSP mass, rather than all four lightest sparticles. I show this in Figure 3.7. In this plot, the solid lines require a neutralino LSP, while the dotted lines do not (i.e. they match those from tan$\beta = 50$ plot from Figure 3.6). Notice that for $\delta a_\mu \gtrsim 40 \times 10^{-10}$ there is little difference between the two cases. Also for the extremal values of tan$\beta$ there is little difference. Only for intermediate values of tan$\beta$ is there an appreciable shift in the mass bounds. For tan$\beta = 10$, the bound is pushed down by as much as 50 GeV if one imposes a neutralino LSP requirement.

Finally I consider the general MSSM case. Here the mass correlations are far
Figure 3.3. Bounds on the four lightest sparticles as a function of $\delta a_\mu$ for $\tan\beta = 3$, assuming gaugino mass unification. The vertical lines indicate the 1$\sigma$ bounds using the $\tau$ decay data ($\delta a_\mu = 6$) or not ($\delta a_\mu = 24$).
Figure 3.4. Bounds on the four lightest sparticles as a function of $\delta a_\mu$ for $\tan \beta = 10$, assuming gaugino mass unification. The vertical lines indicate the $1\sigma$ bounds using the $\tau$ decay data ($\delta a_\mu = 6$) or not ($\delta a_\mu = 24$).
Figure 3.5. Bounds on the four lightest sparticles as a function of $\delta a_\mu$ for $\tan\beta = 30$, assuming gaugino mass unification. The vertical lines indicate the $1\sigma$ bounds using the $\tau$ decay data ($\delta a_\mu = 6$) or not ($\delta a_\mu = 24$).
Figure 3.6. Bounds on the four lightest sparticles as a function of $\delta a_\mu$ for $\tan \beta = 50$, assuming gaugino mass unification. The vertical lines indicate the 1$\sigma$ bounds using the $\tau$ decay data ($\delta a_\mu = 6$) or not ($\delta a_\mu = 24$).
less pronounced, and as a result masses get pushed higher (as can be seen from the dashed line in Figure 3.7), but we can still find some interesting bounds. For instance, the central value of E821 demands that at least three sparticles lie below 525 GeV. In Figure 3.8 I show this explicitly by plotting the masses of the four lightest sparticles for $\tan \beta = 50$ versus $\delta a_\mu$ in the general MSSM. The noticeable upward shift in the masses in this scenario is because in the neutral LSP, gaugino unified case, the LSP is usually a $\tilde{B}$-like neutralino, which is not by itself responsible for generating $\delta a_\mu$. In the general MSSM, the lightest sparticle we can bound is one that contributes to $\delta a_\mu$. So the there may or may not be particles lighter than the bound we give in the general MSSM scenario.

We summarize this information in Table 3.1 where I have shown the mass bounds using the 1σ limit of the E821 data on the LSP for various $\tan \beta$ values within the various scenarios I consider. The numbers are the mass bounds on the lightest sparticle without (with) the inclusion of the $\tau$ decay data. The last line of the table represents the upper bound for any model with $\tan \beta \leq 50$: $m_{\text{LSP}} < 475 \text{ GeV}$ (1 TeV) for the E821 1σ lower bounds of $\delta a_\mu$. Perhaps of equal importance are the bounds on the next two lightest sparticles ("2LSP" and "3LSP"). For $a_\mu$ calculated without the $\tau$ decay data, $m_{2\text{LSP}} < 485 \text{ GeV}$ and $m_{3\text{LSP}} < 630 \text{ GeV}$; with the inclusion of the $\tau$ data, $m_{2\text{LSP}} < 1 \text{ TeV}$ and $m_{3\text{LSP}}$ is pushed somewhat above a TeV. Note that the bound on the 2LSP is roughly the same as that on the LSP. Furthermore, the E821 discrepancy implies a fourth (third) sparticle below 1 TeV even in the general MSSM scenario.

3.4.2 Bounds on Sparticle Species

In the previous section I placed bounds on the lightest sparticles, independent of their identity. It is possible, however, to place mass bounds on individual species
Figure 3.7. Bound on the LSP mass as a function of $\delta a_\mu$ for $\tan \beta = 3, 10, 30,$ and 50. The dotted lines assume gaugino unification, and the solid lines assume in addition that the lightest particle is a neutralino. The dashed line is the LSP bound for the general MSSM case, for $\tan \beta = 50$. 
Figure 3.8. Bound on the LSP mass as a function of $\delta a_\mu$ for $\tan \beta = 50$. The dotted lines assume gaugino unification while the solid lines are for the general MSSM.
TABLE 3.1. BOUNDS ON THE LSP FOR THE GENERAL MSSM, MSSM WITH GAUGINO UNIFICATION, AND MSSM WITH GAUGINO UNIFICATION PLUS A NEUTRALINO LSP. THE ENTRIES ARE THE 1σ BOUNDS WITHOUT (WITH) THE INCLUSION OF τ DECAY DATA.

<table>
<thead>
<tr>
<th>Mass Bound</th>
<th>General MSSM</th>
<th>Gaugino Unification</th>
<th>+ Dark Matter</th>
</tr>
</thead>
<tbody>
<tr>
<td>tanβ = 3</td>
<td>205 (331)</td>
<td>140 (230)</td>
<td>115 (215)</td>
</tr>
<tr>
<td>5</td>
<td>235 (395)</td>
<td>170 (280)</td>
<td>135 (280)</td>
</tr>
<tr>
<td>10</td>
<td>280 (475)</td>
<td>215 (350)</td>
<td>150 (325)</td>
</tr>
<tr>
<td>30</td>
<td>340 (750)</td>
<td>305 (580)</td>
<td>275 (565)</td>
</tr>
<tr>
<td>50</td>
<td>475 (1000)</td>
<td>370 (765)</td>
<td>365 (765)</td>
</tr>
</tbody>
</table>

of sparticles, for example, charginos or smuons. These bounds will necessarily be higher than those of the last section, but this information is still useful because it can provide us with hints about how and where to look for SUSY at Run II of the Tevatron or even at the LHC.

There is one complication in obtaining these bounds. At low tanβ the E821 data is most easily explained by the neutralino-smuon diagram and therefore there must be at least one light smuon and one light neutralino. At larger tanβ the contributions from the chargino-sneutrino diagram can dominate, implying a light chargino and sneutrino. However, the correlations discussed in Section 3.3.2 preserve bounds on various species over the whole range of tanβ values. For example, a bound on $m_{\tilde{\phi}}$ implies a bound on $m_{\tilde{\mu}_1}$, and a bound on $m_{\tilde{\chi}_1}$ implies a bound on at least one of $m_{\tilde{N}_i}$. Furthermore, in certain cases (such as gaugino unification), the converse may also be true.

We show in Figure 3.9-3.10 the mass bounds in $\tilde{\mu}_1$ and $\tilde{N}_1$ assuming gaugino unification; a plot for $\tilde{C}_1/\tilde{N}_2$ will appear later in Section 3.4.4 when I discuss Tevatron physics. Note that $\tilde{N}_1$ must lie below 500 (950) GeV, even for large tanβ, thanks to the gaugino unification condition, whereas $\tilde{\mu}_1$ can be heavier but must still be lighter than 915 (1800) GeV at 1σ.
Finally I consider the most general MSSM scenario, that is, without gaugino unification. We show these results for $\tan\beta = 50$ in Figure 3.11 alongside the results for gaugino unification for comparison. We see from the figure that the bound on $\bar{\mu}_1$ is essentially the same as the bound from assuming gaugino unification. However the gaugino masses have shifted, which is expected because the lightest neutralino is no longer a $\tilde{B}$-like spectator to $a_\mu$, but a $\tilde{W}$-like piece of a participating chargino.

The results of these plots are summarized as follows. For the MSSM with gaugino unification, the lightest neutralino must be lighter than 500 (900) GeV at 1$\sigma$. The lighter smuon must lie below 915 (1800) GeV. Without the $\tau$ decay data, the lighter chargino must be lighter than 890 GeV; if one uses the $\tau$ data, the bound is pushed to above 2 TeV.

3.4.3 Bounds on $\tan\beta$

The final bound I investigate with the E821 data is a bound on $\tan\beta$. (We will examine bounds on $\tan\beta$ with an entirely different approach in Chapter 4.) After E821 announced their discrepancy, there was some debate in the literature about which values of $\tan\beta$ were capable of explaining it. In Figure 3.12 I show the minimum value of $\tan\beta$ required to attain a given $\delta a_\mu$ in the general MSSM. We have checked that adding the assumption of gaugino unification changes does not change the figure significantly.

The behavior of this curve has two clear regions. For $\delta a_\mu \gtrsim 36 \times 10^{-10}$ the chargino contribution dominates and $\delta a_\mu \propto y_\mu \sim 1/\cos \beta \sim \tan \beta$, implying a linear scaling with $\tan \beta$, which is evident in the figure. At lower $\tan \beta$, the both the neutralino and chargino contributions can become important and so it is possible to generate $\delta a_\mu$ with a smaller value of $\tan \beta$ than possible from charginos alone. In this region the value of $\delta a_\mu$ is essentially independent of $\tan \beta$. The value of the
Figure 3.9. Bounds on the mass of $\tilde{N}_1$ as a function of $\delta a_\mu$ for various $\tan\beta$ under the assumption of gaugino unification. The vertical lines indicate $1\sigma$ bounds without (with) the inclusion of $\tau$ decay data.
Figure 3.10. Bounds on the masses of $\tilde{\mu}_1$ as function of $\delta a_\mu$ for various $\tan \beta$ under the assumption of gaugino unification. The vertical lines indicate 1\sigma bounds without (with) the inclusion of $\tau$ decay data.
Figure 3.11. Bound on the mass of $\tilde{\mu}_1$, $\tilde{C}_1$, and $\tilde{N}_1$ versus $\delta a_\mu$ for $\tan\beta = 50$. The solid lines assume gaugino unification, and the dotted lines are the general MSSM.
E821 data implies no bound on $\tan\beta$ at $1\sigma$. Further reductions in the error bars will not change this, only a change in the central value of discrepancy can alter this.

### 3.4.4 Implications for the Tevatron

Since the measurement of $a_\mu$ involves physics in the lepton sector, the measured discrepancy has little direct impact on a hadron machine such as the Tevatron. In particular, the light smuons associated with $\delta a_\mu$ cannot be directly produced at the Tevatron. They can only be produced if heavier non-leptonic states are produced which then decay into sleptons. The only such sparticles that are involved in $a_\mu$ are the gauginos. These states, in contrast to the sleptons, can be produced copiously and in fact they are the initial state of SUSY trilepton signature.

The gauginos that are of particular interest for possible trilepton signatures are the lighter chargino and the 2nd lightest neutralino ($\tilde{C}_1, \tilde{N}_2$). Studies of the mSUGRA parameter space indicate that detection ability of the trilepton signal strongly depends on the mass of the sleptons which appear in the gaugino decay chains. For heavy sleptons, the Tevatron is only sensitive to gaugino masses in the range $m_{\tilde{C}_1, \tilde{N}_2} \lesssim 130$ to $140$ GeV for $10$ fb$^{-1}$ of luminosity, and $145$ to $155$ GeV for $30$ fb$^{-1}$ [17], where the quoted ranges span from low to moderate $\tan\beta$. However, for light sleptons (below about $200$ GeV) the range is considerably extended, up to gaugino masses of around $190$ to $210$ GeV. For large $\tan\beta$, the trilepton signal disappears since the staus become significantly lighter, so that they dominate the final state.

It is impossible in the kind of analysis here to comment on the expected cross sections for the neutralino-chargino production (for instance, there is no information contained in $a_\mu$ about the t-channel squark masses) but we can examine the mass bounds on $\tilde{C}_1$ and $\tilde{N}_2$. In Figure 3.13 I show the upper bound of the heavier of
Figure 3.12. Bound on $\tan \beta$ as a function of $\delta a_\mu$ in the general MSSM. The dotted lines indicate 1σ limits with and without the inclusion of the $\tau$ data.
either $\tilde{C}_1$ and $\tilde{N}_2$. A few comments about the figure are in order. First this figure assumes gaugino unification. This is in keeping with the analysis of Ref. [17] since they consider mSUGRA, in which the gauginos unify by design. Second, I assume a neutralino LSP. This is because the topology of the trilepton event includes missing energy, which is caused by a the lightest neutralino escaping the detector; this is precisely why I consider the mass of the second lightest neutralino for this process. Finally, what I have actually plotted is $m_{\tilde{C}_1}$. In every case examined, the mass difference between $\tilde{C}_1$ and $\tilde{N}_2$ was only a few GeV, which is because they are both primarily $\tilde{W}$-like in the unified case and to a rough approximation have masses $\simeq M_2$.

From the figure it is clear that it is not possible to devise a no-lose theorem for the Tevatron from $\delta a_\mu$. However, if $\tan\beta$ is small and $\delta a_\mu \geq 35 \times 10^{-10}$ (the current central value not using the $\tau$ data) then the gauginos may be within the Tevatron’s reach. It must be emphasized that the values in this analysis are upper bounds on sparticle masses, not best fits or preferred values. Thus, for small $\tan\beta$ there is reason to hope that Tevatron will be able to probe the gaugino sector in Run II or Run III; for larger $\tan\beta$ there is little information for the Tevatron implied by $a_\mu$.

3.4.5 Implications for a Linear Collider

A consensus has emerged lately in favor of building a $\sqrt{s} = 500$ GeV linear collider, which would presumably be a factory for sparticles with masses below 250 GeV. Can the measured discrepancy in $a_\mu$ tell us anything about our chances for seeing SUSY at such a machine?

The preceding analysis can place a lower bound on the number of observable sparticles at a linear collider as a function of $\delta a_\mu$ and $\tan\beta$. We show this information in a histogram in Figure 3.14. This figure shows the minimum number of sparticles
Figure 3.13. Bounds on the masses of $\tilde{C}_1$ and $\tilde{N}_2$ as a function of $\delta a_\mu$ for $\tan\beta = 3, 5, 10, 30, \text{ and } 50$. The vertical lines indicate 1$\sigma$ bounds without (with) the inclusion of $\tau$ decay data. This plot assumes gaugino unification and a neutralino LSP.
that lie below 250 GeV for $\tan \beta = 3, 10, 30, \text{ and } 50$, assuming gaugino unification. The number in the segments of each bar represents the value of $\tan \beta$ for the model. For instance, a deviation of $\delta a_\mu = 35 \times 10^{-10}$ would imply five sparticles below 250 GeV for $\tan \beta = 3$, three sparticles for $\tan \beta = 10$, and one sparticle for $\tan \beta = 30$. If $\tan \beta = 50$, then all sparticles could lie above 250 GeV. As expected, the number of light states increases for large $\delta a_\mu$ and decreases for large $\tan \beta$. However, note that for $\delta a_\mu \geq 40 \times 10^{-10}$ there is no segment of the bar that has $\tan \beta = 3$ because there is no way to explain such a large value of the discrepancy for such low values of $\tan \beta$.

We see immediately from the figure that at low $\tan \beta$ there is a $1\sigma$ guarantee that at least 1 to 4 sparticles will be produced at a 500 GeV linear collider. This guarantee depends on whether the $\tau$ decay data is used or not. The counting here does not include extra sleptons due to slepton mass universality; for example a light muon sneutrino implies light tau and electron sneutrinos, and likewise for a light charged smuon. We also see that for $\tan \beta \gtrsim 30$ there is no guarantee at all that a 500 GeV machine would produce on-shell states; this should not be taken to mean that one should not expect their production, only that $a_\mu$ cannot guarantee it.

Note that particular classes of models, such as mSUGRA or gauge mediated SUSY breaking, may not saturate the bounds here. That is, other constraints may rule out all regions of parameter space in which $\delta a_\mu$ exceeds some value. However if a model does allow a particular value of $\delta a_\mu$, the model must contain light sparticles consistent with the bounds given above.

3.5 Conclusions

In summary, the muon anomalous magnetic moment has long been advertised as a key place to search for indirect evidence of SUSY. The current experimental excess,
Figure 3.14. Minimum number of sparticles that would be directly observable at a $\sqrt{s} = 500$ GeV linear collider as a function of $\delta a_{\mu}$. The bars represent $\tan\beta = 3, 10, 30,$ and $50$. This graph assumes gaugino unification, but does not include the additional sleptons implied by slepton mass universality.
however, is too small, given the large theoretical uncertainties to provide compelling evidence for SUSY.

The SM calculations indicate an excess of either $3.2\sigma$ or $1.5\sigma$. If one were to accept the E821 discrepancy as evidence for SUSY (at either level), then a number of statements can be inferred at the $1\sigma$ confidence level. First, the lightest sparticle must lie below $475\,\text{GeV}$ ($1\,\text{TeV}$). Second, for models with unified gaugino masses, there must be at least 2 sparticles with masses below $585\,\text{GeV}$ ($1\,\text{TeV}$). Finally, there is no lower bound on $\tan\beta$. Bounds on individual species of sparticles are weaker, usually falling at or above $1\,\text{TeV}$.

There pessimistic bounds, however, are for all $\tan\beta$, which means that they are bounds for the largest values of $\tan\beta$ considered (50). For lower values of $\tan\beta$, these bounds get pushed down to lower values. For instance, if $\tan\beta = 3$, and gauginos are unified with a neutralino LSP, there must be a sparticle below $115$ ($215$) GeV, within the range accessible in the very near future. So, while the mass bounds placed on SUSY masses by $a_\mu$ are relatively weak when no constraints are placed on the MSSM, constrained models force these bounds lower to an experimentally interesting region.

Finally, while a real deviation in $a_\mu$ would not prove that $\tan\beta$ is large, it is at large $\tan\beta$ where such a deviation could be most easily accommodated. Thus, studies of $a_\mu$ are good probes of SUSY model space with large $\tan\beta$. Unfortunately, at large $\tan\beta$, a deviation in $a_\mu$ does not imply a no-lose theorem for the Tevatron, or a $\sqrt{s} = 500\,\text{GeV}$ linear collider. The problem is that there is a region of values for which $\delta a_\mu$ is not sensitive to $\tan\beta$ at all. Therefore, more experimental probes of this region would be prudent. Especially interesting would be probes which were more sensitive to $\tan\beta$. Another such probe is the subject of the next chapter.
We have considered as our first study of SUSY the muon anomalous magnetic moment. The analysis in Chapter 3 gave us avenues to probe the gaugino and slepton sectors of the MSSM. Continued efforts by both theorists and experimentalists can hopefully solidify the status of the discrepancy. Either it is there, in which case it is possible that new physics effects are present, or it is not there, and $a_\mu$ should prove to be an excellent constraint on new physics models. The utility of $a_\mu$ as a constraint on SUSY models leads naturally to consider whether it says anything at all about other processes. In fact, some authors investigated precisely this topic [41, 42, 43, 44], when they searched for correlations between $a_\mu$ and another process: the decay of a $B$ meson into two leptons mediated by a Higgs boson. This rare flavor changing neutral current (FCNC) process could serve as a very fruitful hunting ground for SUSY. It has the potential to be not only a pre-emptive search to direct sparticle production, but also a powerful constraint. Observation of this decay could provide will allow us to gain useful information about $\tan\beta$, the Higgs sector, the squark sector, and even provide some information about the origins of SUSY breaking. Let us now begin our discussion of $B_s \rightarrow \mu^+\mu^-$.}

4.1 Higgs Mediated FCNCS

The issue of Higgs mediated FCNCs was first addressed by Glashow and Weinberg [45]. It had been obvious that the Higgs of the SM could not have flavor
violating couplings, since the couplings of the Higgs to fermions is what defines the fermion mass eigenstates and subsequently the flavor structure of the SM. However, the SM, technically speaking, is actually a minimal SM. In a general (not necessarily SUSY) extension of the SM with more than one Higgs $SU(2)$ doublet, the Higgs interaction basis and fermion mass basis can be different, leading to flavor changing Higgs couplings. Such models were explored by Glashow and Weinberg, where they examined models with two Higgs doublets. Their interest was to find models which avoided these potentially dangerous flavor violating couplings. One such model is the Type II Higgs two doublet model (2HDM); in which the two Higgs fields are segregated between the up and down type quarks: $\mathcal{L} = \bar{Q}_L Y_U U_R H_u + \bar{Q}_L Y_D D_R H_d$.

This structure is protected by a discrete symmetry under which the Higgs fields transform differently:

$H_U \rightarrow H_U$  
$H_D \rightarrow -H_D$  
$D \rightarrow -D$  
$X \rightarrow X$.  

(4.1)

This parity will ensure alignment of the mass and Higgs interaction bases.

The MSSM an example of a model that contains two Higgs doublets. Indeed, two Higgs doublets are required by holomorphy, and anomaly cancellation. Is the MSSM a type II model? From Eq. (1.6), it is obvious that the MSSM does have the structure of a type II model. Holomorphy provides the necessary segregation to forbid such flavor violating couplings. However, the parity that would protect against these flavor changing couplings is not present; it is destroyed by the $\mu$-term. Thus, the MSSM is not a type II model even at the classical level. Furthermore, we know that SUSY must be broken, so the protection provided by holomorphy is not guaranteed to remain intact. Therefore the protection from flavor changing Higgs
couplings is gone. It has been shown that such couplings arise at one loop [53, 55]. In fact, Hall, Rattazzi, and Sarid [55] found that sizable contributions to nonholomorphic operators of the form $QD^cH^*_u$ can generate significant shifts in the $b$ quark masses, despite suffering loop supressions. This is because the non-holomorphic mass operator gets enhanced by $\tan \beta$ since it is proportional to $\langle H_U \rangle$. That is, at large $\tan \beta$, the number $\tan \beta/16\pi^2$ need not be small. Their interest, however, was investigating contributions to the $b$ quark mass in $SO(10)$ GUT models, and the application to FCNCs was not apparent at that time. The utility of these nonholomorphic operators to FCNCs was first realized by Babu and Kolda [56] where they examined Higgs mediated $B_s \to \mu^+\mu^-$. Since their observation, there has been a flurry of activity on this topic [57, 58, 59, 65, 66, 71]. Let us now proceed to understand why this source of flavor changing occurs and why it can be so significantly enhanced in SUSY.

4.2 $B_s \to \mu^+\mu^-$ in the MSSM

In this section we wish to understand why $B_s \to \mu^+\mu^-$ gets enhanced in SUSY. In the SM, this process suffers from suppression due to the GIM mechanism, helicity factors and loop supressions. Thus, there is an unavoidable factor of

$$\frac{V_{ts}V_{td}}{16\pi^2} \times \frac{f_B}{m_B} \times \frac{m_\mu}{m_B} \approx \frac{1}{4.5 \text{ million}}$$

in the amplitude. This supression is always there, and this is why the brancing ratio for this process is so small in the SM. Is it possible to overcome these supressions, and if so, where do the enhancements come from? The most general MSSM has copious amounts of flavor violation. Too much, in fact. This is the SUSY flavor problem. In order to avoid the appearance of flavor changing processes that would be in gross contradiction with what is observed, the assumption of Minimal Flavor Violation (MFV) [47] is often made. In this picture, any source of flavor violation in
SUSY must come from the Yukawa structure of the SM, in other words, the CKM matrix. This imposition is already a very significant restriction on SUSY models. It implies that of the most general MSSM with all 120 plus parameters totally free, only an exceptional subset, i.e. the subset with this restricted flavor structure, can be realized. This has the desired effect of eliminating the rampant flavor violation of the most general MSSM, but it also means that for the most part, nothing new happens. In this picture, SUSY contributions change the SM by at most $\mathcal{O}(1)$. Nevertheless, most coherent models of SUYS breaking are designed so that MFV is built in, and throughout this thesis it is this framework in which I will carry out my analysis.

Any model which purports to enhance $B_s \rightarrow \mu^+\mu^-$ has to overcome the double suppression mentioned earlier. We will see that SUSY indeed not only compensates for these supressions, but overcomes them, allowing the signal for this to become several orders of magnitude greater than in the SM. What allows this remarkable enhancement the amplitude for this process? Let us turn our attention to what features the amplitude has. From our discussion about which symmetries get violated for this process to go, we know that the amplitude must go like $\mu M_{SUSY}$. What else can be said about the amplitude? Turn your attention to Figure 4.1. From this diagram, we immediately see a factor of $1/m_\phi^2$. Normally the lightest
Figure 4.2. Feynman diagrams contributing to $\epsilon_1$ and $\epsilon_2$

propagator would dominate, but in order for this amplitude to have the correct
decoupling behaviour in the limit of the SUSY states getting heavy, we must have
$m_\phi \simeq m_A$. Thus the amplitude contains a factor of $1/m_A^2$. Additionally, from each
vertex we pick up a Yukawa coupling. In SUSY, however, the down type quarks
and leptons have $\tan\beta$ enhancements to their Yukawa couplings, therefore there is
a factor of: $y_b y_\mu = m_b m_\mu / v^2 \cos^2 \beta \simeq m_b m_\mu \tan^2 \beta / v^2$ in the amplitude. At this
point I tabulate the relevant scaling of the amplitude. Thus far it contains:

$$\mathcal{A} \propto \mu M_{\text{SUSY}} \frac{m_b m_\mu}{v^2 m_A^2} \tan^2 \beta \times \ldots \quad (4.3)$$

There is in fact another factor of $\tan\beta$ in the amplitude, but it is more opaque
compared to the two already discussed. It arises essentially from a re-diagonalization
of the flavor basis. To see its origin, we must examine the loop bubble in Figure 4.1
a little more closely. We show the leading contributions to this bubble in Figure 4.2.
The first diagram in Figure 4.2 always gives a non-zero contribution to flavor chang-
ing [66] in the MFV picture. In a basis where the down quarks couple diagonally
to $\tilde{H}_d^\pm$, the up quarks will have off diagonal couplings to $\tilde{H}_u^\pm$ proportional to CKM
elements, (e.g. $y_t V_{ts} \tilde{s}_L \tilde{t}_R \tilde{H}_u^-$ + h.c.). Thus, the diagram on the left in Figure 4.2
will generate an interaction vertex of the form: $\tilde{s}_L b_R H_u^+$, with a coefficient defined
to be $y_b \kappa_{MFV}$. The parameter $\kappa_{MFV}$ includes $y_t V_{ts}$ factors, as well as kinematic
factors and $16\pi^2$ suppressions arising from the loop calculation. Then, when $H_u$
gets its VEV ($\langle H_u \rangle = v_u$), we will get an off-diagonal contribution to the fermion
mass matrix:
\[ L = \begin{pmatrix} s_R & b_R \end{pmatrix} \begin{pmatrix} m_s & 0 \\ \kappa_{MFV} y_b v_u & m_b \end{pmatrix} \begin{pmatrix} s_L \\ b_L \end{pmatrix}. \tag{4.4} \]

This induced off-diagonal entry in the mass matrix de-aligns the down quark interaction basis away from the mass basis. At tree level, the transition $b \bar{b} \rightarrow \mu \mu$ via a Higgs exchange is possible, but because of the de-alignment, the amplitude can be decomposed using a mixing angle, $\theta$. Diagonalizing the matrix in Eq. 4.4 using a bi-unitary transformation will give the value for the mixing angle $\theta$ between $b_L$ and $s_L$. It is given by:

\[ \sin \theta \simeq \kappa_{MFV} \frac{y_b v_u}{m_b} = \kappa_{MFV} \frac{y_b v_u}{y_b v_d} = \kappa_{MFV} \tan \beta. \tag{4.5} \]

This can yield large mixing since the factor $\kappa_{MFV} \tan \beta$ can be $O(1)$. This re-diagonalization induces a $b \bar{b} \rightarrow \mu \mu$ transition, which can be written as $A_{\bar{b}b \rightarrow \mu \mu} \simeq \sin \theta A_{bb \rightarrow \mu \mu} \simeq \kappa_{MFV} \tan \beta A_{bb \rightarrow \mu \mu}$. We show this decomposition pictorially in Figure 4.3. The appearance of this 3rd factor of $\tan \beta$ in the amplitude is surprising, and also what allows this to become so enhanced. Including these new factors to Eq. (4.3) gives us the relevant scaling of the amplitude:

\[ A \propto \mu M_{SUSY} V_{ts} V_{tb} \frac{m_b m_{\mu}}{m_A^2} \tan^3 \beta, \tag{4.6} \]

where we have omitted the loop kinematic and suppression factors. We will carry this calculation out in more detail in Section 4.3, but the salient feature of the amplitude is apparent here. The $\tan^3 \beta$ scaling is what allows $B_s \rightarrow \mu^+ \mu^-$ in SUSY to become so large. This implies that $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$ scales like $\tan^6 \beta$!! Thus, when $\tan \beta$ is large, significant enhancement over the SM value becomes possible.

All of the flavor changing discussed so far arises only in the MFV picture. There is another contribution to the off diagonal Higgs couplings, however. They come from the diagram on the right in Figure 4.2. Contributions from $\tilde{\gamma} \tilde{q} q$ couplings
can be different from $g q q$, which can induce flavor non-diagonal propagators, i.e., mass insertions. We will call models with this source of flavor changing “general”. In these models, the parameter of flavor changing, $\kappa_{general}$ is in principle different, from $\kappa_{MFV}$ because it is a priori not proportional to CKM elements and Yukawa couplings. Rather, it is parameterized by a mass insertion, through a dimensionless quantity

$$\delta_{ab}^{ij} \simeq (\Delta m_{ab}^2)_{ij} / \Delta m_{ab}$$

where $(a, b)$ are either $L$ or $R$ and $(i, j)$ label the generation. These contributions, although a higher order effect, can be significant. Until recently they were ignored in analyses because they were believed to be subleading. However, they can be an $O(50\%)$ effect.

There is a very subtle point that needs to be made about this source of flavor changing. These mass insertion parameters are proportional to off diagonal terms in the squark mass matrices. When we say that this source of flavor changing is “general,” we mean in a manner which is independent of any mechanism of SUSY breaking. That is to say, these contributions, in specific SUSY breaking scenarios, often take a form similar to that of the MFV contributions because typical scenarios of SUSY breaking have built in some alignment assumptions for the soft terms at the SUSY breaking scale. In the MSSM the RGEs for the soft parameters contain terms proportional to Yukawa matrices \cite{61}, so when the squark mass matrices are run down, the off diagonal terms, which are RGE induced, are proportional to Yukawa couplings. In these models, the same diagonalizing assumptions of the MFV picture also affect these parameters, so the mass insertions will contain implicit Yukawa and
CKM factors. It should be stressed that in general, these two sources need not have anything to do with one another. This is what is meant by the term general. Our goal here is to identify and parameterize the sources of flavor changes. In a general analysis, such as in Ref. [66] the MFV and general sources of flavor changing are independent.

4.3 $B_s \rightarrow \mu^+\mu^-$ Calculation

Let us now focus our effort to the SUSY calculation of this process. I begin by writing the effective Lagrangian generated by the diagrams in Figure 4.2, which is responsible for the interaction of two Higgs doublets with the down quark fields:

$$-\mathcal{L}_{\text{eff}} = \bar{D}_R Y_D Q_L H_d + \bar{D}_R Y_D [\epsilon_1 + \epsilon_2 Y_U^T Y_U] Q_L H_u^* + h.c. \quad (4.7)$$

Here $Y_U$ and $Y_D$ are $3 \times 3$ Yukawa matrices and the two $\epsilon$ parameters that appear in the Lagrangian are computed from the diagrams in Figure 4.2 [56]. At this point it is already clear where the flavor changing will come from: the $Y_U^T Y_U$ structure that appears in the second term of the Lagrangian cannot be simultaneously diagonalized with $Y_D$. The coefficient $\epsilon_1$ is non-flavor changing; the leading contribution to it comes from the gluino diagram of Figure 4.2 and takes the form (see Appendix B for loop functions):

$$\epsilon_1 = \frac{2\alpha_3}{3\pi} \mu^* M_3 f_3(M_3^2, m_{Q_L}^2, m_{d_R}^2), \quad (4.8)$$

where $M_3, m_{Q_L}^2,$ and $m_{d_R}^2$ are the gluino, the left handed squark, and the down type squark mass parameters, respectively. The coefficient $\epsilon_2$ does contribute to flavor changing. There are two contributions to it. The first piece is due to the Higgsino diagram of Figure 4.2 and it is given by:

$$\epsilon_\chi = \frac{1}{16\pi^2} \mu^* A_U f_3(\mu^2, m_{Q_L}^2, m_{u_R}^2). \quad (4.9)$$
There is also a contribution to $\epsilon_2$ arising from the gluino diagram in Figure 4.2. In the limit of squark mass degeneracy, the gluino diagram will contribute no flavor changing. However, squark mass degeneracy is broken by Yukawa induced radiative effects, and therefore this diagram contributes another term to $\epsilon_2$ which has the flavor changing $\mathbf{Y}^\dagger_\mathbf{U} \mathbf{Y}_\mathbf{U}$ structure. (It additionally contributes terms that contain $(\mathbf{Y}^\dagger_\mathbf{U} \mathbf{Y}_\mathbf{U})^2$, $(\mathbf{Y}^\dagger_\mathbf{D} \mathbf{Y}_\mathbf{D})^2$, $(\mathbf{Y}^\dagger_\mathbf{U} \mathbf{Y}_\mathbf{U})(\mathbf{Y}^\dagger_\mathbf{D} \mathbf{Y}_\mathbf{D})$, etc., but these either have no flavor changing, or else they have no new flavor changing structure and are suppressed by loop factors.) This contribution to $\epsilon_2$ is proportional to a flavor changing mass insertion, which vanishes in the limit of squark degeneracy. This contribution takes the form:

$$\epsilon_\Delta = \frac{2\alpha_3}{3\pi y_t^2} \mu^* M_{\delta} \delta_{DD} f_4(M_3^2, m^2_{Q_L}, m^2_{d_R}, m^2_{Q_L}).$$  \hspace{1cm} (4.10)

Here $\delta_{DD}$ is the source of flavor changing we called general in Section 4.2. It is given by:

$$\delta_{DD} = M_{b_L}^2 - M_{s_L}^2 - \frac{1}{2} \left( M_{b_R}^2 - M_{s_R}^2 \right),$$  \hspace{1cm} (4.11)

where $M_{q_{L,R}}$ correspond to the entries in the squark mass squared matrices. We add $\epsilon_\Delta$ to Eq. (4.9) so that $\epsilon_2 = \epsilon_x + \epsilon_\Delta$. The reason for dividing by $y_t^2$ in $\epsilon_\Delta$ will be clear shortly.

Now let us return to our Lagrangian from Eq. (4.7). This can be simplified by choosing a basis where $\mathbf{Y}_\mathbf{U} = \mathbf{U}$ and $\mathbf{Y}_\mathbf{D} = \mathbf{DV}^\dagger$, where $\mathbf{U}$ and $\mathbf{D}$ are diagonal, and $V$ is the CKM matrix. After this diagonalization, keeping only the pieces responsible for flavor changing, specializing to the quark fields of interest to us, and some algebra, this becomes:

$$\mathcal{L}_{FCNC} = \bar{y}_b V_{ub}^* \chi_{FC} \left[ V_{td} \bar{b}_R d_L + V_{ts} \bar{b}_R s_L \right] \left( H_u^{0*} \cos \beta - H_d^{0} \sin \beta \right) + h.c$$  \hspace{1cm} (4.12)

where $\bar{y}_b \simeq y_b[1 + (\epsilon_1 + \epsilon_2 y_t^2) \tan \beta]$ is the HRS enhanced bottom Yukawa coupling [55], and we have defined:

$$\chi_{FC} = \frac{-\epsilon_2 y_t^2 \tan \beta}{(1 + \epsilon_1 \tan \beta)[1 + (\epsilon_1 + \epsilon_2 y_t^2 \tan \beta)]}.$$  \hspace{1cm} (4.13)
The reason for dividing by $y_t^2$ in $\epsilon_\Delta$ is now clear. We need it to cancel the factor of
$y_t^2$ that multiplies $\epsilon_2$ in $\chi_{FC}$.

At this point we carry our calculation over into an effective Hamiltonian framework for ease and quickness. The Hamiltonian for this $B_s \to \mu^+ \mu^-$ is:

$$H = -\frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} [C_{10} O_{10} + C_{Q_1} Q_1 + C_{Q_2} Q_2] + h.c. \quad (4.14)$$

where $\{O_{10}, Q_1, Q_2\}$ and $\{C_{10}, C_{Q_1}, C_{Q_2}\}$ are the non-local operators and Wilson coefficients, respectively. The operators are given by:

$$O_{10} = \frac{e^2}{4\pi^2} \bar{s}_L \gamma^\mu b_L \bar{\ell}_L \gamma_\mu \gamma_5 \ell \quad (4.15)$$
$$Q_1 = -\frac{e^2}{4\pi^2} \bar{s}_L b_L \ell \ell \quad (4.16)$$
$$Q_2 = -\frac{e^2}{4\pi^2} \bar{s}_L b_L \ell \gamma_5 \ell \quad (4.17)$$

In the SM, the dominant contributions come from W box and Z penguin diagrams [54] and give non-negligible contributions only to $C_{10}$. The source of $C_{Q_1}$ and $C_{Q_2}$ has been addressed in our calculation of the effective Lagrangian above. These coefficients are evaluated at the weak scale, $M_W$, and so their running must be accounted for. Since $O_{10}$ is a $(V - A)$ quark current, it has zero anomalous dimension, and therefore $C_{10}$ has trivial running. The $Q_i$ operators have the same form as a quark mass operator, and must have the same anomalous dimension. Since $C_{Q_1}$ and $C_{Q_2}$ are proportional to $m_b$, their running can be properly taken into account by replacing $m_b(m_W)$ with $m_b(m_b)$ upon evaluation. Explicitly, the Wilson coefficients are given by:

$$C_{10} = -\frac{Y(x_t)}{\sin^2 \theta_W} \simeq -0.997 \left( \frac{\bar{m}_t(m_t)}{166 \text{ GeV}} \right)^{1.55} \quad (4.18)$$
$$C_{Q_1} = \frac{2\pi}{\alpha} \chi_{FC}^* \frac{m_b m_\ell}{\cos^2 \beta \sin \beta} \left( \frac{\cos(\beta - \alpha) \sin \alpha}{m^2_{h_0}} + \frac{\sin(\beta - \alpha) \cos \alpha}{m^2_{H^\pm}} \right) \quad (4.19)$$
$$C_{Q_2} = \frac{2\pi}{\alpha} \chi_{FC}^* \frac{m_b m_\ell}{m^2_{A_0} \cos^2 \beta} \quad (4.20)$$

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Finally, the relevant hadronic matrix elements are given by [52]:

\[
\langle 0 | \bar{b} \gamma^\mu \gamma_5 s(x) | B_s(P) \rangle = i f_{B_s} P^\mu e^{-iP \cdot x} \tag{4.21}
\]

\[
\langle 0 | \bar{b} \gamma_5 s(x) | B_s(P) \rangle = -if_{B_s} \frac{m_{B_s}^2}{m_b + m_s} e^{-iP \cdot x} \tag{4.22}
\]

Here \( f_{B_s} \) is the \( B_s \) decay constant, the numbers taken from [64]. These lead to the following expression for the branching ratio:

\[
B(B_s \to \ell^+ \ell^-) = \frac{G_F^2 \alpha^2 m_{B_s}^3 \tau_{B_s} f_{B_s}^2 |V_{tb}^* V_{ts}|^2}{64 \pi^3} \sqrt{1 - \frac{4m_{\ell}^2}{m_{B_s}^2}} \times \left\{ \left( 1 - \frac{4m_{\ell}^2}{m_{B_s}^2} \right) \left| \frac{m_{B_s}}{m_b + m_s} C_{Q_1} \right|^2 + \left| \frac{2m_{\ell}}{m_{B_s}} C_{10} - \frac{m_{B_s}}{m_b + m_s} C_{Q_2} \right|^2 \right\} \tag{4.23}
\]

Here \( \tau_{B_s} \) is the \( B_s \) lifetime, \( G_F \) is the Fermi constant, and \( \alpha \) is the fine structure constant evaluated at the weak scale: \( \alpha^{-1} (M_W) \approx 128 \). To obtain the expression for the decay of \( B_d \) into leptons, simply let \( s \to d \) in the quark masses, the meson masses, the meson lifetime, decay constant, and CKM elements.

### 4.4 Current Experimental Status and Potential

Let us now discuss the current experimental situation for \( B_s \to \mu^+ \mu^- \), as well as some projections that experiments could hope to reach in the near future. The current bounds on the various \( B_q \to \ell^+ \ell^- \) processes and their SM predictions are given in Table 4.1. The uncertainties in the theoretical values are due primarily to the uncertainties in the decay constants and the CKM elements that enter into the calculation. We see immediately from the table that all of the experimental bounds are orders of magnitude above the SM predictions.

The projections for searches at the hadron machines is quite good. There have been some analyses [71, 72, 73, 74] exploring the experimental capabilities of the Tevatron’s Run II. All of them seem to agree that the Tevatron can realistically probe down to between \( 10^{-7} \) to \( 10^{-8} \) for \( B(B_s \to \mu^+ \mu^-) \), depending on the luminosity that
TABLE 4.1. EXPERIMENTAL BOUNDS AND SM PREDICTION FOR THE VARIOUS $B_q \rightarrow \ell^+\ell^-$ PROCESSES. NOTE THAT THERE IS CURRENTLY NO EXPERIMENTAL ANALYSIS FOR $B$ MESONS INTO $\tau$'S. THE BOUNDS IN THE TABLE ARE DERIVED BOUNDS, EXTRACTED BY THE AUTHORS OF REF. [69], BASED ON LEP SEARCHES FOR THE MODE $B^- \rightarrow \tau^-\nu$.

<table>
<thead>
<tr>
<th>$(q, \ell)$</th>
<th>Exp.</th>
<th>Bound (90% CL)</th>
<th>SM prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(d, e)$</td>
<td>Belle [70]</td>
<td>$&lt; 1.9 \times 10^{-5}$</td>
<td>$(3.2 \pm 2.0) \times 10^{-15}$</td>
</tr>
<tr>
<td>$(d, \mu)$</td>
<td>CDF [68]</td>
<td>$&lt; 1.5 \times 10^{-7}$</td>
<td>$(1.4 \pm 0.9) \times 10^{-10}$</td>
</tr>
<tr>
<td>$(d, \tau)$</td>
<td>Ref. [69]</td>
<td>$&lt; 0.05$</td>
<td>$(2.8 \pm 1.8) \times 10^{-8}$</td>
</tr>
<tr>
<td>$(s, e)$</td>
<td>L3 [67]</td>
<td>$&lt; 5.4 \times 10^{-5}$</td>
<td>$(8.9 \pm 2.5) \times 10^{-14}$</td>
</tr>
<tr>
<td>$(s, \mu)$</td>
<td>CDF [68]</td>
<td>$&lt; 5.8 \times 10^{-7}$</td>
<td>$(3.7 \pm 1.1) \times 10^{-9}$</td>
</tr>
<tr>
<td>$(s, \tau)$</td>
<td>Ref. [69]</td>
<td>$&lt; 0.015$</td>
<td>$(8.0 \pm 2.3) \times 10^{-7}$</td>
</tr>
</tbody>
</table>

the Tevatron achieves. In particular, with $\mathcal{L} = 2$ fb$^{-1}$ the Tevatron can probe down to $\mathcal{B}(B_s \rightarrow \mu^+\mu^-) \approx 10^{-7}$. With a Run IIb integrated luminosity of 15 fb$^{-1}$ the Tevatron can see $\mathcal{B}(B_s \rightarrow \mu^+\mu^-) \approx 10^{-8}$. Thus, all our analyses in Section 4.6 will focus on numbers in this range. If it turns out that the signal for this process is smaller than this, then it will only be visible to the LHC. There has been some amount of speculation about the possibility in the future of a super B factory to reinforce and extend the existing precision flavor studies. If constructed, it would be possible to probe down to $\mathcal{B}(B_d \rightarrow \mu^+\mu^-) = 5 \times 10^{-9}$ and $\mathcal{B}(B_d \rightarrow \tau^+\tau^-) = 2 \times 10^{-9}$ with 10 ab$^{-1}$ of data [75] with such a machine.

4.5 Experimental Utility

What information can we hope to gather from an investigation of $B_s \rightarrow \mu^+\mu^-$? To date, the experimental bounds for this process are well above the SM predictions (we will see this shortly). Thus, this process provides an "open window" opportunity to indirectly probe new physics. It is this opening we wish to take advantage of. We will see in this chapter that there are SUSY contributions which enhance the branching ratio for this process by several orders of magnitude over the SM, and this
is why $B_s \rightarrow \mu^+\mu^-$ is such an excellent hunting ground for SUSY. A search for this signal, if successful, would provide a strong case for SUSY. If not seen, this process would provide another important constraint, which could become competitive with $b \rightarrow s\gamma$ constraints as this measurement gets better.

In the past, the focus of SUSY searches at the Tevatron has been in the “gold-plated” trilepton modes [17]. The searches for these events, which have the topology $ar{p}p \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_1^+ \rightarrow \ell^+\ell^-\ell^{\prime+} + \slashed{E}_T$, have been geared towards Supergravity (SUGRA) type models, which will be discussed shortly. These modes are a good probe of low $\tan\beta$ and low $m_{1/2}$. These modes are sensitive to low $\tan\beta$ because for $\tan\beta > 30$, the decay of the intermediate gauginos becomes dominated by decay into staus, whose masses get driven to lower values for larger $\tan\beta$ [58]. The reason trileptons probe only low $m_{1/2}$ is that the intermediate neutralino and chargino states must be produced on shell in order for the rate for the process to be appreciable. It is not as sensitive to $m_0$ because once the gauginos are produced, they decay into either squarks or sleptons, and higher values of $m_0$ just cause the gauginos to live longer, up to the point at which $Z$ boson decay modes will start to turn on.

While trileptons probe SUSY with low $\tan\beta$, the process $B_s \rightarrow \mu^+\mu^-$ is a definitive large $\tan\beta$ signal. As we have seen, the branching ratio for the process scales like $\tan^6 \beta$. As it happens, $B_s \rightarrow \mu^+\mu^-$ becomes relevant at roughly the value of $\tan\beta$ where the trilepton signal would turn off. Furthermore, it extends the Tevatron’s reach in $m_{1/2}$ because this source of flavor changing does not decouple as $M_{SUSY} \rightarrow \infty$, provided that $m_A$ does not also become too large [56]. In Ref [17] the claim was made that the Tevatron will only be able to probe $\tan\beta \lesssim 45$ (which is arguably “large” $\tan\beta$ already) in Run II. However, we will see that is not the case, and in fact the Tevatron has a better chance of seeing a signal for $\tan\beta$ above 45, and is capable of probing up to the largest values of $\tan\beta$ possible. Thus an observation
of $B_s \rightarrow \mu^+ \mu^-$ will have the added benefit of telling us in what regime of $\tan\beta$ space lies. This is a very important piece of information. Nearly all observables depend on $\tan\beta$, but it is very difficult to measure, so even “merely” identifying the allowed range of values for $\tan\beta$ will have a profound impact on studies of SUSY parameter space [66].

4.5.1 SUSY Breaking Models

In addition to providing strong circumstantial evidence for SUSY, a measurement of $B_s \rightarrow \mu^+ \mu^-$ will also be able to provide information about the origins of SUSY breaking. In this analysis of $B_s \rightarrow \mu^+ \mu^-$ I will focus on three models of SUSY breaking. They are not, as a matter of practicality, exhaustive. They are, however, the most commonly studied models, which cover a broad spectrum of physics, so the restriction is not too limiting. In order to facilitate our discussion, we now introduce the essential features of each of these SUSY breaking scenarios.

The first class of SUSY breaking models we consider are Supergravity (SUGRA) models [6]. The defining feature of these models is that the hidden sector field(s) $X$ responsible for SUSY breaking couple to the MSSM fields via gravitational interactions. This means that our low energy Lagrangian will have terms that are suppressed by factors of $M_P$. (Remember that $G_N = 1/M_P^2$ sets the strength of gravitational interactions.) The Lagrangian will have the form:

$$-\mathcal{L}_{NR} = \left( \frac{F_X}{M_P} \sum_a \frac{1}{2} f_a \lambda^a \lambda^a + h.c. \right) + \left( \frac{F_X^2}{M_P^2} K_{ij} \phi_i \phi_j^* \right)
+ \frac{F_X}{M_P} \left( \frac{1}{6} y_{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} \mathcal{M}_{ij} \phi_i \phi_j \right) + h.c. \right) \quad (4.24)$$

where $F_X$ is the VEV of the auxiliary field in the SUSY breaking multiplet. Notice that this Lagrangian has exactly the same form as that of Eq. 1.7. If the condition $F_X/M_P \sim M_W$ is imposed (as required for a solution to the hierarchy
problem), then this yields the soft breaking Lagrangian of Eq. 1.7. The parameters $f_a$, $K_{ij}$, $y_{ijk}$, and $\mathcal{M}_{ij}$ are determined by the underlying theory. However, if one assumes a minimal form for the kinetic terms and gauge interactions, then $f_a = f$ will be independent of the gauge group, $K_{ij} = \delta_{ij}$ for the scalars, and the other parameters will be proportional to parameters in the superpotential, so that $y_{ijk} = \alpha Y_{ijk}$ and $\mathcal{M}_{ij} = b M_{ij}$. In this picture, all the soft breaking terms can be written in terms of just four parameters:

$$m_{1/2} = f \frac{F_X}{M_P}; \quad m_0 = \frac{F_X^2}{M_P^2}; \quad A_0 = \alpha \frac{F_X}{M_P}; \quad B_0 = b \frac{F_X}{M_P}. \tag{4.25}$$

All the soft terms of the MSSM at the SUSY breaking scale can then be set from these. All gauginos will have a common mass $m_{1/2}$, all scalar mass squared parameters will be equal to $m_0^2$, all the trilinear terms will have the same size $A_0$, and the $B \mu$ term can be determined from $B_0 \mu$. In practice, we eliminate $\mu^2$ by knowing $m_Z^2$, and trade the parameters $B_0$ in favor of $\tan\beta$. Therefore SUGRA models are specified completely by the five parameters:

$$m_0, \quad m_{1/2}, \quad A_0, \quad \tan\beta, \quad \text{sgn} \, \mu. \tag{4.26}$$

It is suspect whether or not this set of alignment assumptions in this parameterization is theoretically well motivated, but the phenomenological utility is manifest. This framework has been well studied [48], and constitutes a large fraction of the phenomenological studies of SUSY parameter space.

The second class of models we consider are those with Gauge Mediated SUSY Breaking (GMSB) [49]. These models introduce of a new sector of chiral superfields, called messengers, which couple to both the hidden SUSY breaking sector, and the MSSM fields through gauge interactions. These messenger fields must appear only above a high enough scale, $M_{\text{mess}}$, in order to avoid having been already seen.
A typical GMSB model has a messenger sector consisting of $N_5$ messenger fields $\Phi_i$ (we will address the subscript 5 in the parameter $N_5$ momentarily). These messengers will acquire their heavy masses through a term in the superpotential that couples them to a gauge singlet chiral superfield:

$$W_{\text{mess}} = \sum_i y_i S \Phi_i \Phi_i.$$  

Here the notation $\Phi_i$ does not imply that holomorphy is being violated, but means that these chiral superfields transform with the opposite quantum numbers as the $\Phi_i$. Both the scalar and auxiliary components of $S$ will acquire VEVs to break SUSY. The messengers will then become heavy: $M_{\text{mess}} \approx y \langle S \rangle$. Here we have assumed that all the messengers appear at the same scale, which is a good approximation provided that the $y_i$ are not too different from one another. This is usually the case, although it is in principle possible that there be hierarchies among the messenger particles. Then there will be several messenger scales $M_{i,\text{mess}}$ so that the contribution of each messenger multiplet would need to be incorporated at the corresponding messenger scale for that multiplet.

Moving on to the spectrum in GMSB, these new couplings give rise to soft breaking terms that arise from Feynman loop diagrams involving virtual messenger particles. Such a diagram is shown in Figure 4.4. This diagram generates masses for the gauginos. The diagrams responsible for generating masses for the scalars...
appear first at the two loop level. Several of these are shown in Figure 4.5. The
A-terms are also induced at the two loop level, but are suppressed by an extra factor
of $\alpha_a / 4\pi$ relative to the gauginos, so they can be taken to be zero to a very good
approximation. Thus the soft terms will have the form:

$$
M_a = N_5 \Lambda \frac{\alpha_a}{4\pi} \times g(z) \quad (4.28)
$$

$$
M_{\phi}^2 = N_5 \Lambda^2 \sum_a \left( \frac{\alpha_a}{4\pi} \right)^2 C_a \times f(z) \quad (4.29)
$$

$$
A_{ijk} = 0, \quad (4.30)
$$

where $\Lambda = \langle F_S \rangle / \langle S \rangle$ is the SUSY breaking scale, $z = \Lambda / M_{mess}$, $C_a$ is the Casimir of
the group, and $f$ and $g$ are threshold functions, which are both $1 + \mathcal{O}(z^2)$. When
feeding these inputs into a model, it is customary that the scale $\langle S \rangle$ is traded for $\Lambda$
as the free parameter, so that GMSB models are specified by the five parameters:

$$
M_{mess}, \quad N_5, \quad \Lambda, \quad \tan\beta, \quad \text{sgn} \mu. \quad (4.31)
$$

Strictly speaking, I have cheated in writing the soft terms. In the sums over gauge
groups in Eq. 4.30, the correct thing to do is to have three sums that go like $\sum_a n_a(i)$,
where $n_a(i)$ is the Dynkin index for each $\Phi_i + \bar{\Phi}_i$ under the gauge group $a$. It turns
out the the most popular class of models [50] are those in which the messengers
consist of $N_5$ copies of the $5 + \bar{5}$ of $SU(5)$. In this case:

$$
N_5 = \sum_i n_1(i) = \sum_i n_2(i) = \sum_i n_3(i) \quad (4.32)
$$

The reason that these models are the most attractive is not because they lead
to this simpler for for the soft terms, but because messengers with masses below
$M_{GUT}$ can ruin the unification of gauge couplings since they alter the $\beta$ functions.
However, for complete multiplets of $SU(5)$, the unification of the coupling constants
can still occur if the messengers are not very different in mass. This is also why it is
customary to take all the messenger multiplets to appear at the same scale $M_{mess}$. 

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The third class of models we investigate are Anomaly Mediated SUSY Breaking (AMSB) models. In these models, the hidden sector SUSY breaking is communicated to the visible sector through the auxiliary component of the gravity multiplet. This is accomplished if the Kähler potential has the form:

$$K = -3 \log \left[ h(\Phi, \Phi^\dagger) + k(X, X^\dagger) \right]$$

(4.33)

with \( \Phi \) and \( X \) being the visible and hidden sector fields, respectively. The field \( X \) will pick up a VEV \( \langle X \rangle = 1 + F_X \theta^2 \), which is or order \( m_{3/2} \), the gravitino mass. The soft terms are found by expanding the lagrangian with non-vanishing \( F_{\Phi} \) to obtain:

$$M_a = -\frac{b_a \alpha_a}{4\pi} m_{3/2}$$

(4.34)

$$m_{ij}^2 = \left( -\frac{\hat{\gamma}}{4} m_{3/2}^2 \right) \delta_{ij}$$

(4.35)

$$A_{ijk} = \frac{1}{2}(\gamma_i + \gamma_j + \gamma_k) m_{3/2}$$

(4.36)

Thus the spectrum is determined entirely by the anomalous dimensions and \( \beta \)-functions, and a single scale \( m_{3/2} \). Notice that in the second equation above, some of the scalars could have a problem with becoming tachyonic, depending on the sign of \( \hat{\gamma} \). It turns out that this is an issue for the sleptons, and so we add a new mass
parameter input to the theory, $m_0$ and add $m_0^2 \delta_{ij}$ to the equation for $m_{ij}^2$. In “pure” AMSB, this term is absent. However this renders these models phenomenologically unviable. So although the addition of the $m_0^2$ term is somewhat *ad hoc* and destroys two very attractive features of AMSB (scale invariance and the fact that all soft terms arise from just one free parameter), it must be present in order to have a meaningful spectrum. There have been models put forward [51] that offer possible origins for this term, but we simply take the minimal approach and allow it to be a free parameter since it must be there. Thus AMSB models can be specified with the following four parameters:

$$m_0, \ m_3/2, \ \tan \beta, \ \text{sgn} \ \mu.$$  \hspace{1cm} (4.37)

Once the high scale inputs have been set by one of these models, the low energy spectrum is computed through RG evolution, checked against electroweak data for consistency. In this way it is possible to conduct effective studies of parameter space for each of these models.

4.5.2 What Can We Expect To See?

We have seen above that each of the three scenarios of SUSY breaking predicts different mass spectra which derive from how, and at what scale, SUSY is broken in the model. Thus observables which depend on the spectrum can have qualitatively different signatures in different models of SUSY breaking. This is how we will be able to garner information about SUSY breaking from $B_s \rightarrow \mu^+ \mu^-$ searches. These have two origins, $LR$ mixing, which is caused by diagonalizing the $2 \times 2$ squark mass squared matrices, and $LL$ mixing, or splitting, which is RGE induced. The first is maximized for large values of the A-terms, and the second is caused by the mass insertion $\delta^{LL}_{DD}$. We can see both of these sources explicitly in our formulae: $\epsilon_\chi$ scales like $A_t$, and $\epsilon_\Delta$ scales like $\delta^{LL}_{DD}$.
For the models we are considering, let us examine what we expect to see from an analysis of $B_s \rightarrow \mu^+ \mu^-$. Let us focus first on SUGRA models. Even though SUGRA models have degenerate squark masses by design at the high scale, the SUSY breaking scale is quite large, so sizeable squark splitting can be induced from large logarithms. This high SUSY breaking scale also allows the A-terms to run significantly from their boundary values. The A-terms run towards negative values, and usually become sizable even if set to zero at the high scale, so that SUGRA models also have generically sizeable $LR$ squark mixing. Both of these factors allow for a sizeable contribution to $B_s \rightarrow \mu^+ \mu^-$.

The rare decay $B_s \rightarrow \mu^+ \mu^-$ suffers in models with GMSB. In these models, the two things necessary to generate a sizeable branching ratio for $B_s \rightarrow \mu^+ \mu^-$ are absent by design. Firstly, $A_0 \simeq 0$ at the SUSY breaking scale. This by itself is not a problem (SUGRA models can also have this) but since the SUSY breaking scale is close to the weak scale in GMSB, there is not much of a chance for $A_0$ to run significantly. It therefore it stays small, causing squark mixing to be suppressed. Secondly, because running effects are small, much less $LL$ mixing is generated than in SUGRA models, further supressing the signal you can get from GMSB type models. Therefore we expect that models with GMSB should have a smaller signal for $B_s \rightarrow \mu^+ \mu^-$. 

AMSB models are an interesting framework as $B_s \rightarrow \mu^+ \mu^-$ is concerned. These models are in some sense very much like SUGRA, but with a very important difference. The phase between $\mu$ and $M_3$ is opposite to that in SUGRA models. This means that while AMSB models can have a large signal for $B_s \rightarrow \mu^+ \mu^-$ for reasons similar to those of SUGRA, models run into perturbativity problems with $y_b$ at lower $\tan \beta$ than in SUGRA models. We will see why explicitly in Section 4.6. Thus, in AMSB signals for $B_s \rightarrow \mu^+ \mu^-$ can be comparable in size to SUGRA, aside from
this issue with $\tan\beta$. We will return to all of these SUSY breaking scenarios in more
detail in Section 4.6 when we present detailed numerical studies.

4.6 Analysis & Results

Having discussed the generalities of $B_s \rightarrow \mu^+\mu^-$, I now proceed with my detailed
numerical studies. The approach was fairly straightforward. First, I compute spectra in SUGRA, GMSB, and AMSB models using the spectrum calculation code
SOFTSUSY [76]. The inputs $\tan\beta$, along with $A_0$ for SUGRA, or $N_5$ for GMSB,
were chosen for each case we ran. I then looped over a logarithmic grid of the input
parameters $(m_0, m_{1/2})$ in SUGRA, $(\Lambda, M_{mess})$ in GMSB, and $(m_0, m_{3/2})$ in AMSB.
In addition I computed spectra for a random sampling of models, by allowing the
inputs in each of the SUSY breaking scenarios to be chosen within a specified range
of values. I considered models only with $\mu > 0$ because the anomalous muon magnetic
moment discrepancy favors positive $\mu$ at about the 90% confidence level. From
the computed spectrum I then calculated the various branching ratios and mixing
matrices needed for the analysis. Obviously, the analysis has some degree of model
dependence, but since one of the goals is to gather information the SUSY break-
ing sector, this is allowed. In any event, limiting ourselves in this way is not too
constricting when one considers the representative class of models studied.

I impose following mass constraints on the models: i) the lightest neutralino must
be heavier than 37 GeV, ii) the lightest chargino must be heavier than 67 GeV, iii)
the gluino must be heavier than 195 GeV, iv) the first two families of squarks must be
heavier than 200 GeV, v) the third family of squarks must be heavier than 83 GeV
for stops, and 91 GeV for sbottoms, and vi) the slepton must all be heavier than
70 GeV [46]. Any model that does not satisfy these mass bounds is rejected.

In addition to these mass bounds, I impose constraints from experimental data
on $a_\mu$, and $b \to s\gamma$. I use Chapter 3 to calculate $\delta a_\mu$, and require that $\delta a_\mu$ lie in the range $[-5, 57] \times 10^{-10}$. The reason for such a wide range of values here is due to the fact that there are in essence two central values, each with error bars, depending on if $\tau$ data is used in concurrence with $e^+e^-$ data or not. This issue is discussed in Chapter 3. We use the method of Ref. [62] to calculate $\mathcal{B}(b \to s\gamma)$, and require that it lie in the range $[2.1, 4.5] \times 10^{-4}$ [46]. The reason for this rather wide range of values is due to the uncertainties in the theoretical calculation.

The situation for $b \to s\gamma$ is slightly complicated. Although the experimental value for $\mathcal{B}(b \to s\gamma)$ is very well known [46], the theoretical calculation is wrought with scale uncertainties which can affect the next-to-leading-order (NLO) SUSY calculation at about the 30% level. Even the well known “leading” NLO contributions from HRS enhanced bottom Yukawa couplings cause roughly a 10% effect if omitted from the calculation (this effect is roughly a 30% effect for AMSB models). Since this is such an important constraint on any model of new physics, and it seems that the theoretical value at present is exquisitely sensitive to the NLO pieces, the calculation does indeed need to be settled. In order to make effective comparisons of the theoretical and experimental numbers the uncertainties in the theoretical values need to be reduced to a level comparable to the experimental errors. The constraints from $a_\mu$ and $b \to s\gamma$ are quite important, and there has been some investigation regarding correlations between these processes and $B_s \to \mu^+\mu^-$ [59]. This issue will be addressed later in this section. Let us now begin our study of the information that can be garnered from this approach.

4.6.1 $B_s \to \mu^+\mu^-$ and SUGRA Models

SUGRA models provide perhaps the best chance to see $B_s \to \mu^+\mu^-$. As mentioned earlier, $B_s \to \mu^+\mu^-$ benefits in SUGRA models from both large $A$-terms and sig-
significant squark non-degeneracy. This allows for sizeable enhancement over the SM value for $B_s \to \mu^+\mu^-$ over a wide class of SUGRA models. Let us first examine the gross features of SUGRA models. In Figure 4.6 we show a scatter plot of the value of $\mathcal{B}(B_s \to \mu^+\mu^-)$ in a large sampling of SUGRA models versus $\tan\beta$. Each point on the plot represents a SUGRA model, with its input parameters chosen at random. I took $m_0$ and $m_{1/2}$ in the range $[100, 1500]$ GeV, $\tan\beta$ in the range $[20, 55]$, and the relation $A_0 = \{0, \pm m_0, \pm 2m_0\}$ enforced randomly. The most immediately striking feature of this plot is the chasm in the values of $\mathcal{B}(B_s \to \mu^+\mu^-)$. This gap is a direct result of imposing the experimental constraint for $b \to s\gamma$. The points above this gap all have $A_0 < 0$; a large portion of them in fact have $A_0 = -2m_0$. This bifurcation caused by $b \to s\gamma$ originates from interference between the SUSY and SM contributions to $C_7$ [60]. The SM value for $C_7$ is fixed, but the SUSY contribution varies. It passes through zero, and attains a value of approximately twice the SM value, but with opposite sign. The gap in the plot is because of this. We will see this banding due to $b \to s\gamma$ in another guise in a moment. These points above the gap also illustrate explicitly that $\mathcal{B}(B_s \to \mu^+\mu^-)$ is maximized with larger values of the A-terms. In SUGRA models, $A_t$ runs towards negative values, even if $A_0$ is zero or positive. If you start out with $A_0$ negative, then $A_t$ can run down to even larger negative values than it would otherwise. Note also that the CDF bound is already a non-trivial constraint. In fact, as the bound is lowered, it will first start to place pressure on models with the largest negative values of $A_0$.

There has been some discussion about what implied bounds on $B_s \to \mu^+\mu^-$ can be inferred from $a_\mu$ [41, 42, 43, 44], so it is worthwhile to address this. Figures 4.7 and 4.8 are scatter plots of $\mathcal{B}(B_s \to \mu^+\mu^-)$ versus both $\mathcal{B}(b \to s\gamma)$ and $\delta a_\mu$ in SUGRA models. These scatter plots were generated using the same range of values for the input parameters as in Figure 4.6. The first noticeable feature of these plots
Figure 4.6. Scatter plot of $\mathcal{B}(B_s \rightarrow \mu^+\mu^-)$ versus $\tan \beta$ for a random sampling of SUGRA models. The blue dashed line represents the latest Run II bound from CDF.
is the tail in Figure 4.7 for higher values of $b \to s\gamma$. This is due to constructive interference between the SUSY and SM contributions to $C_7$. But the real point of these plots is that there are no clear correlations between $b \to s\gamma$ and $B_s \to \mu^+\mu^-$, or $\delta a_\mu$ and $B_s \to \mu^+\mu^-$. The regions that are filled by the models points do not have any noticeable patterns. There may perhaps be some relations between the processes, but those relations are far too model dependent to be visible on these plots, where all the input parameters were allowed to vary from model to model.

Let us now focus on a restricted subset of parameter space, and turn our attention to the features of $\mathcal{B}(B_s \to \mu^+\mu^-)$ in the $(m_0, m_{1/2})$-plane of SUGRA parameter space. In Figures 4.9 - 4.12, four such plots are shown. In these plots the dark shaded regions represent parameter space where models are either inconsistent, or do not satisfy the imposed experimental mass bounds. The heavy dark lines are contours for $B_s \to \mu^+\mu^-$, the this black lines are contours for $b \to s\gamma$, and the lighter grey lines are contours for $\delta a_\mu$. The contours for the processes $b \to s\gamma$ and $\delta a_\mu$ mark the boundaries of regions in the $(m_0, m_{1/2})$-plane that are excluded by these processes. The excluded regions are “underneath” these curves.

I would first like to use these plots to return to the issue of correlations among the processes $a_\mu$, $b \to s\gamma$, and $B_s \to \mu^+\mu^-$. At first glance, these plots seem to be at odds with the conclusion drawn from the previous paragraph. These plots seem to support the existence of that there are correlations among these processes, just by examining the shape of the contours. This is slightly misleading. In essence, this is an illustration of the model dependence of the correlations. You can see that the lines for $\delta a_\mu$ and $b \to s\gamma$ which demarcate excluded regions of parameter space do not maintain proximity to the same $B_s \to \mu^+\mu^-$ contour for all values of $\tan\beta$ and $A_0$. So although functional forms stay the same, the relative size of these contours changes significantly. For instance, you can see that for $\tan\beta = 30$, practically all
Figure 4.7. Scatter plot of $B(B_s \rightarrow \mu^+\mu^-)$ versus $B(b \rightarrow s\gamma)$ for a random sampling of SUGRA models. The blue dashed line represents the latest Run II bound on $B(B_s \rightarrow \mu^+\mu^-)$ from CDF.
Figure 4.8. Scatter plot of $\mathcal{B}(B_s \to \mu^+\mu^-)$ versus $\delta a_\mu$ for a random sampling of SUGRA models. The blue dashed line represents the latest Run II bound on $\mathcal{B}(B_s \to \mu^+\mu^-)$ from CDF.
of what is the experimentally interesting parameter space is ruled out by these constraints, while for tan\(\beta = 55\), the interesting part of parameter space is free from these constraints.

Furthermore, these plots also illustrate that the \(\delta a_\mu\) constraint is, in essence, redundant. There is nothing ruled out by \(a_\mu\) that isn’t already ruled out by \(b \rightarrow s\gamma\). In fact, it is also clear from these plots that as the bounds on \(B_s \rightarrow \mu^+\mu^-\) better, \(\delta a_\mu\) will in a sense be doubly redundant, because \(B_s \rightarrow \mu^+\mu^-\) will become competitive, especially at larger tan\(\beta\), with the bound from \(b \rightarrow s\gamma\). The relative behavior of these processes can be understood by observing that the contours shown for these processes plots scale with tan\(\beta\) in the ratio 1 : 2 : 6 for \(\delta a_\mu : \mathcal{B}(b \rightarrow s\gamma) : \mathcal{B}(B_s \rightarrow \mu^+\mu^-)\). This is why the constraints become relatively less important at larger tan\(\beta\), and conversely, why the bound on \(B_s \rightarrow \mu^+\mu^-\) will continue to become more important. This feature is not specific to SUGRA models. We will see it again in the plots studied for AMSB.

Notice also, in Figure 4.12, which has \(A_0 = -2m_0\), that the gap in the scatter plot caused by negative \(A_0\) is visible in a different light. The thin band between the lower two contours and everything above the highest-lying contour of \(b \rightarrow s\gamma\) is allowed, the wide band between the highest and next highest contour, and everything underneath the lowest-lying contour, is excluded. Lastly, notice in these plots that the combined region of SUGRA parameter space in the \((m_0, m_{1/2})\)-plane excluded by both \(b \rightarrow s\gamma\) and \(B_s \rightarrow \mu^+\mu^-\) implies that \(m_\tilde{\chi} \gtrsim 350\) GeV, over all the models considered here.

The last information that can be obtained in this analysis are bounds on tan\(\beta\) and \(m_A\). The branching ratio for the \(B_s \rightarrow \mu^+\mu^-\) is proportional to \(\tan^6\beta/m_A^4\), making this to an excellent place to extract information about these two parameters. In Figures 4.13 & 4.14 I plot the contours of \(\mathcal{B}(B_s \rightarrow \mu^+\mu^-)\) in the \((m_A, \tan\beta)\) - plane.
Figure 4.9. Contours of $\mathcal{B}(B_s \to \mu^+\mu^-)$ in the $(m_0, m_{1/2})$-plane for a SUGRA model in which $(A_0, \tan\beta) = (0, 30)$. 

SUGRA
\[ \tan\beta = 30 \]
\[ A_0 = 0 \]
Figure 4.10. Contours of $\mathcal{B}(B_s \rightarrow \mu^+\mu^-)$ in the ($m_0, m_{1/2}$)-plane for a SUGRA model in which $(A_0, \tan\beta) = (0, 40)$. 
Figure 4.11. Contours of $B(B_s \to \mu^+\mu^-)$ in the $(m_0, m_{1/2})$-plane for a SUGRA model in which $(A_0, \tan \beta) = (0, 55)$. 
Figure 4.12. Contours of $\mathcal{B}(B_s \to \mu^+\mu^-)$ in the $(m_0, m_{1/2})$-plane for a SUGRA model in which $(A_0, \tan\beta) = (-2m_0, 40)$. 

SUGRA

$\tan\beta = 30$

$A_0 = 0$
The contours shown are maximum values of $\mathcal{B}(B_s \rightarrow \mu^+\mu^-)$, so the bounds implied here are “true” bounds over all the range of $m_0$, $m_{1/2}$, and $A_0$ considered. In these plots the shaded regions contain models that were inconsistent or that did not satisfy experimental mass bounds. The real power of these plots is realized in the event of observation at the Tevatron. The reason observation is so useful is that bounding $\tan\beta$ is practically as good as measuring it directly. Remember that $\tan\beta$ space has a large/small dichotomy, so discerning which region you happen to lie in is a far more important piece of information than the exact value of $\tan\beta$. Additionally, $\tan\beta$ is very difficult to measure directly, but it is a fundamental input to any SUSY model.

A bound on $m_A$ will be equally as interesting and useful, though $m_A$ is usually a quantity derived from radiative breaking of the Higgs potential and therefore in most scenarios of SUSY breaking not a fundamental input. From our discussion in Section 4.4, we know it is very possible for the Tevatron to see $B_s \rightarrow \mu^+\mu^-$ in Run II if the branching ratio is at the $10^{-7}$ to $10^{-8}$ level [66]. If $B_s \rightarrow \mu^+\mu^-$ is seen at the $10^{-7}$ level, then for SUGRA models it must be that $m_A \lesssim 540$ GeV and $\tan\beta \gtrsim 28$. And if $B_s \rightarrow \mu^+\mu^-$ is seen at the $10^{-8}$ level, then $m_A \lesssim 1.1$ TeV and $\tan\beta \gtrsim 14$.

4.6.2 $B_s \rightarrow \mu^+\mu^-$ and GMSB Models

The situation in GMSB models is in rather sharp contrast to the other models of SUSY breaking considered. The latest CDF bound is now becoming interesting for SUGRA and AMSB models. However, there is no interesting information to be gained from the bound in GMSB models. I stated in Section 4.5 that GMSB models would not yield large signals for $B_s \rightarrow \mu^+\mu^-$ because the low SUSY breaking scale prevents sizeable RGE induced squark splitting, and because of the naturally small size of the A-terms in the theories, which will lead to a smaller amount of squark mixing. We explicitly demonstrate this point in Figure 4.15. This is a scatter plot,
Figure 4.13. Contours of $B(B_s \to \mu^+\mu^-)$ in the $(m_A, \tan\beta)$-plane for a SUGRA model in which $(A_0, \tan\beta) = (0, 55)$. 
Figure 4.14. Contours of $B(B_s \rightarrow \mu^+\mu^-)$ in the $(m_A, \tan\beta)$-plane for a SUGRA model in which $(A_0, \tan\beta) = (-2m_0, 40)$. 
analogous to the plot for SUGRA models, but in GMSB parameter space. Values of \( \Lambda \) in the range \([15, 300]\) TeV, \( M_{mess} \) in the range \([10^6, 10^{15}]\) GeV, and \( \tan\beta \) in the range \([30, 55] \) were chosen at random for \( N_5 = \{1, 3, 5\} \).

It should be noted at this point that what was input into our spectrum calculation was really \( \sqrt{N_5} \Lambda \). In other words, the true range of \( \Lambda \) was \([15, 300]\)/\( \sqrt{N_5} \) TeV. This was done because in GMSB models, although \( N_5 \) and \( \Lambda \) are independent inputs, from a low-energy effective Lagrangian point of view, the parameters that determine the masses at the weak scale are \( \sqrt{N_5}\Lambda \) for the scalars, and \( N_5\Lambda \) for the gauginos. What I found was that \( B_s \to \mu^+\mu^- \) in GMSB models is dominated by scaling from the scalar mass squares, so the rest of this analysis was conducted using \( \sqrt{N_5}\Lambda \in [15, 300] \) TeV for GMSB models.

Returning to the analysis, we can see from the plot in Figure 4.15 that even for the highest values of \( \tan\beta \) the signal does not get any larger than about \( 4.5 \times 10^{-8} \). Thus the current CDF bound rules out none of the GMSB models considered. We can conclude however, in agreement with Ref. [42] that an observation of \( B_s \to \mu^+\mu^- \) at the Tevatron implies that GMSB must be a sub-dominant source of SUSY breaking. GMSB models simply would not be able to account an observed signal. The kind of information and bounds we are interested in would not be as useful in GMSB because of this relative reduction in signal. Because of this, we will not provide the same detailed analysis as for other models of SUSY breaking.

4.6.3 \( B_s \to \mu^+\mu^- \) and AMSB Models

Let us now turn our attention to AMSB models. AMSB models can also generate significant contributions to \( B_s \to \mu^+\mu^- \). In AMSB, the A-terms can become sizeable, and also the pseudo-scalar Higgs mass gets pushed down, which means that the suppression from \( 1/m_A^4 \) is not as severe. This can make it seem as if the effective
Figure 4.15. Scatter plot of $B(B_s \rightarrow \mu^+\mu^-)$ versus $\tan\beta$ for a random sampling of GMSB models. The blue dashed line represents the latest Run II bound from CDF.
tan\(\beta\) scaling of \(\mathcal{B}(B_s \rightarrow \mu^+\mu^-)\) in AMSB models is greater than \(\tan^6 \beta\). Figure 4.16 shows a scatter plot in the same spirit as the previous two. Here, \(m_0\) was taken in the range \([100, 1500]\) GeV, \(m_{3/2}\) was taken in the range \([10, 150]\) TeV, and \(\tan \beta\) in the range \([20, 50]\).

The most glaring feature of this plot is the sharp drop off in signal for \(\tan \beta \gtrsim 45\). This happens because \(a_\mu\) is exerting a strong hidden influence on AMSB parameter space. As mentioned earlier, I only consider models with positive \(\mu\) in order to be consistent with \(\delta a_\mu\). However, in AMSB models, the phase difference between \(\mu\) and \(M_3\) is opposite to that in SUGRA and GMSB models. This difference causes problems with bottom Yukawa perturbativity at lower values of \(\tan \beta\) than it would otherwise. Here’s why: the bottom Yukawa coupling can be written as

\[
y_b = \frac{m_b/v}{1 + \eta \epsilon \tan \beta},
\]

which is the HRS corrected bottom Yukawa coupling. Here \(\epsilon\) (which I take to be positive) contains effects due to diagrams like those of Figure 4.2. The variable \(\eta\) contains the information about the sign of \(\mu\). In SUGRA models, if \(\mu > 0\), then \(\eta = +1\) also. But in AMSB models, if \(\mu > 0\) then \(\eta = -1\), driving the right hand side of Eq. (4.38) to larger values for the same value of \(\tan \beta\). This is in part why \(B_s \rightarrow \mu^+\mu^-\) grows faster with \(\tan \beta\) than in SUGRA. It is also why the models become troubled at lower values of \(\tan \beta\): The denominator of Eq (4.38) becomes less than one in AMSB models for the same value of \(\tan \beta\) that it would be greater than on in SUGRA, causing this enhanced \(y_b\) to be larger for a given value of \(\tan \beta\).

Aside from this issue, AMSB models provide just as legitimate of a chance to see \(B_s \rightarrow \mu^+\mu^-\) as SUGRA models do. Indeed, you can see this by comparing the largest values of \(\mathcal{B}(B_s \rightarrow \mu^+\mu^-)\) attained in AMSB with those of SUGRA models; they are comparable to one another.

As with SUGRA models, we can discuss correlations bewteen \(\delta a_\mu\), \(b \rightarrow s\gamma\), and
Figure 4.16. Scatter plot of $B(B_s \to \mu^+\mu^-)$ versus $\tan \beta$ for a random sampling of AMSB models. The blue dashed line represents the latest Run II bound from CDF.
$B_s \rightarrow \mu^+ \mu^-$ in AMSB models as well. In Figures 4.17-4.8 I show plots of $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$ versus both $\mathcal{B}(b \rightarrow s \gamma)$ and $\delta a_\mu$, similar to those of SUGRA models. We can see from the plots that, as in SUGRA models, there are no clear cut correlations that can be drawn from them. Of course, in Figure 4.17 it appears that as $\mathcal{B}(b \rightarrow s \gamma)$ increases, there is also an increase in $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$, but this is not surprising. This is just an artifact of how each branching ratio scales with $\tan \beta$. Thus once again we can conclude there are no clear correlations between these processes.

Now let us turn our attention to plots in the $(m_0, m_{3/2})$-plane. In Figures 4.19-4.20 I show contours of $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$ for AMSB models, for $\tan \beta = 30$ and 35. In Ref. [42], Baek et al. made the claim that AMSB models are effectively ruled out by $b \rightarrow s \gamma$ constraints. Examine the contour plots in the $(m_0, m_{3/2})$-plane in Figure 4.19 and 4.20. The solid dark grey lines represent the demarcation of allowed parameter space from $b \rightarrow s \gamma$, calculated using the leading NLO corrections due to HRS factors. The dash-dot dark grey lines represent the same demarcation as the solid line, but these were calculated without using the HRS insertions. From the alarming difference in these two lines, it is clear that the statement that AMSB is ruled out by $b \rightarrow s \gamma$ is not robust. It is far too sensitive to the next order corrections in $C_7$. Furthermore, AMSB is not ruled out, no matter which calculation you wish to use. There are still regions of parameter space that are accessible, which is especially clear from Figure 4.20. Thus we can conclude that AMSB models, as well as SUGRA models, can accommodate a large signal for $B_s \rightarrow \mu^+ \mu^-$. Lastly, I show a plot of contours of $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$ in AMSB models in the $(m_A, \tan \beta)$-plane, in order to obtain bounds on both of these parameters. This plot was generated in a manner analogous to those for SUGRA models, and it is shown in Figure 4.21. Notice here the bottom Yukawa cutoff at $\tan \beta \simeq 45$, which we saw in Figure 4.16. Also, note that the current CDF bound implies that $m_A \lesssim 330 \text{ GeV}$ for
Figure 4.17. Scatter plot of $\mathcal{B}(B_s \to \mu^+\mu^-)$ versus $\mathcal{B}(b \to s\gamma)$ for a random sampling of AMSB models. The blue dashed line represents the latest Run II bound on $\mathcal{B}(B_s \to \mu^+\mu^-)$ from CDF.
Figure 4.18. Scatter plot of $\mathcal{B}(B_s \to \mu^+ \mu^-)$ versus $\delta a_\mu$ for a random sampling of AMSB models. The blue dashed line represents the latest Run II bound on $\mathcal{B}(B_s \to \mu^+ \mu^-)$ from CDF.
Figure 4.19. Contours of $\mathcal{B}(B_s \to \mu^+\mu^-)$ in the $(m_0, m_{3/2})$-plane for an AMSB model in which $\tan\beta = 30$. 
Figure 4.20. Contours of $\mathcal{B}(B_s \to \mu^+\mu^-)$ in the $(m_0, m_{3/2})$-plane for an AMSB model in which $\tan\beta = 35$. 
TABLE 4.2. BOUNDS ON $m_A$ AND $\tan\beta$ OBTAINED FROM THE PLOTS IN FIGURES 4.13, 4.14, AND 4.21 FOR HYPOTHETICAL OBSERVATION OF $B_s \to \mu^+\mu^-$ AT THE $10^{-7}$ $(10^{-8})$ LEVEL.

<table>
<thead>
<tr>
<th>Model</th>
<th>$m_A \lesssim$ [GeV]</th>
<th>$\tan\beta \gtrsim$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMSB</td>
<td>470(740)</td>
<td>28(25)</td>
</tr>
<tr>
<td>SUGRA ($A_0 = 0$)</td>
<td>380(830)</td>
<td>28(14)</td>
</tr>
<tr>
<td>SUGRA ($A_0 = -2m_0$)</td>
<td>730(1020)</td>
<td>21(18)</td>
</tr>
</tbody>
</table>

If $B_s \to \mu^+\mu^-$ is seen at Run II at the $10^{-7}$ level, then $m_A \lesssim 470$ GeV and $\tan\beta \gtrsim 28$. If measured at the $10^{-8}$ level, then $m_A \lesssim 740$, and $\tan\beta \gtrsim 25$. These bounds are rather comparable to those of SUGRA models, emphasizing the point that before the bottom Yukawa turn-off, AMSB models qualitatively resemble SUGRA models.

4.7 Summary

In summary, the rare decay $B_s \to \mu^+\mu^-$ provides an excellent opportunity for the current hadron machines to conduct an indirect search for SUSY. If seen, it would provide strong circumstantial evidence for SUSY. A measurement of $B_s \to \mu^+\mu^-$ would allow the extraction of information about $\tan\beta$, $m_A$, and the SUSY breaking sector. Observation of $B_s \to \mu^+\mu^-$ would disfavor GMSB models, since they do not generate enough of an enhancement to be able to account for an observed signal. However, both SUGRA and AMSB could accommodate a signal. Furthermore, since there are no correlations between $\delta a_\mu$, $b \to s\gamma$, and $B_s \to \mu^+\mu^-$, no inferred bounds can be implied. Further analyses of $\delta a_\mu$ or $b \to s\gamma$ will only alter the parts of parameter space that are allowed from these processes (except of course in the AMSB scenario, where $b \to s\gamma$ is exquisitely sensitive the NLO corrections).

A measurement of $B_s \to \mu^+\mu^-$ at the $10^{-7}$ level would imply that $m_A \lesssim 730$ GeV, and $\tan\beta \gtrsim 21$. Observation at the $10^{-8}$ level would imply that $m_A \lesssim 1020$ GeV, and
Figure 4.21. Contours of $\mathcal{B}(B_s \to \mu^+\mu^-)$ in the $(m_A, \tan\beta)$-plane for an AMSB.
$\tan \beta \gtrsim 14$. I summarize the $m_A$ and $\tan \beta$ bounds that can be inferred from Figures 4.13, 4.14, and 4.21 in Table 4.2. If not detected, then the continuing improvement of the experimental bounds would make $B_s \rightarrow \mu^+ \mu^-$ a complimentary, or even competitive, bound to existing bounds on other processes such as $b \rightarrow s \gamma$ and $a_\mu$.

The current CDF bound is now becoming relevant for studies of the SUGRA and AMSB parameter spaces. This bound will continue to become more relevant as it improves, until the LHC turns on. Thus we eagerly await the continued improvement of these bounds from the hadron machines, and possibly from the construction of a super B factory.
CHAPTER 5
CLOSING THOUGHTS

My research interests during my graduate career were geared towards the study of SUSY with large $\tan \beta$. Exercising my belief that an important function of the theorist is to keep close touch with experiment, my personal bias was to conduct analyses on quantities which would have experimental relevance, either in the present time frame, on the near future.

Guided by this sentiment, I capitalized on the reported discrepancy in the muon anomalous magnetic moment to conduct a study of the slepton and gaugino sector of the MSSM as my first project. The analysis allowed bounds to be placed on a number of sparticles. While the present discrepancy is not statistically convincing evidence for new physics, the situation is not fully settled. The current discrepancy is as it began, a $2.7 \sigma$ effect. However, the numbers now are more well understood than when the discrepancy was first announced. Efforts are still ongoing to settle this issue, and my analysis still proves to be relevant today.

This work on the muon anomalous moment led me to study papers investigating correlations between this, and the processes $b \rightarrow s \gamma$ and $B_s \rightarrow \mu^+ \mu^-$. While conducting studies of SUSY breaking parameter space for $B_s \rightarrow \mu^+ \mu^-$, my interest developed into a pure interest in $B_s \rightarrow \mu^+ \mu^-$ itself. This process could contain a potentially large amount of information for SUSY models, so my study of it was very worthwhile. The physics behind the enhancements this process receives is truly
fascinating, and even has applications to other processes.

My hope is that these analyses can be used in conjunction with experiment to place useful bounds and constraints on parameter space, or uncover significant evidence for SUSY. This particular model of new physics has so much going for it that it is difficult to think that Nature chooses not to utilize SUSY somewhere. Naturally this is all subject to the sometimes unforgiving scrutiny of experiment. This is the necessary dynamic, the ebb and flow, between theory and experiment. This is how the great book of science book got written. And I am grateful to my advisor and the University of Notre Dame for the chance to be a small part of it.
APPENDIX A

CONVENTIONS FOR $\alpha_\mu$

A.1 Kinematic Functions for $\delta \alpha_\mu$

Here we give the calculational details of the supersymmetric contributions to $\alpha_\mu$, with our conventions for our gaugino mass eigenbases. The general expression for $\delta \alpha_\mu$ including phases is (here we follow Ref. [21]):

$$\delta \alpha_\mu^{(N)} = \frac{m_\mu}{16\pi^2} \sum_{i,m} \left\{ -\frac{m_\mu}{12m_{\tilde{\mu}m}^2} (|n_{im}^L|^2 + |n_{im}^R|^2) F^N_1(x_{im}) + \frac{m_{\tilde{N}_i}}{3m_{\tilde{\mu}m}^2} \text{Re}[n_{im}^L n_{im}^R] F^N_2(x_{im}) \right\}$$

$$\delta \alpha_\mu^{(C)} = \frac{m_\mu}{16\pi^2} \sum_k \left\{ \frac{m_\mu}{12m_{\tilde{\nu}k}^2} (|c_k^L|^2 + |c_k^R|^2) F^C_1(x_{k\tilde{\nu}}) + \frac{2m_{\tilde{N}_k}}{3m_{\tilde{\nu}k}^2} \text{Re}[c_k^L c_k^R] F^C_2(x_{k\tilde{\nu}}) \right\} \quad (A.1)$$

where $i = 1, 2, 3, 4$, and $i,m = 1,2$ label the neutralino, smuon, and chargino mass eigenstates, respectively, and:

$$n_{im}^L = \sqrt{2} g_{1i} N_{i1} X_{m2} + y_\mu N_{i3} X_{m1}$$

$$n_{im}^R = \frac{1}{\sqrt{2}} (g_{2i} N_{i2} + g_{1i} N_{i1}) X_{m1}^* - y_\mu N_{i3} X_{m2}^*$$

$$c_k^R = y_\mu U_{k2}$$

$$c_k^L = -g_2 V_{k1} \quad (A.2)$$

where $y_\mu = g_2 m_\mu/\sqrt{2} m_W \cos \beta$ is the muon Yukawa coupling and $g_{1,2}$ are the $U(1)_Y$ and $SU(2)_L$ gauge couplings, respectively. The loop functions in Eq. (A.1) depend on the ratio $x_{\alpha\beta} = m_\alpha^2/m_\beta^2$ of masses and are given by:

$$F^N_1(x) = \frac{2}{(1-x)^4} \left[ 1 - 6x + 3x^2 + 2x^3 - 6x^2 \log x \right]$$

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\begin{align*}
F_N^2(x) &= \frac{3}{(1-x)^3} \left[ 1 - x^2 + 2x \log x \right] \\
F_C^1(x) &= \frac{2}{(1-x)^4} \left[ 2 + 3x - 6x^2 + x^3 + 6x \log x \right] \\
F_C^2(x) &= -\frac{3}{2(1-x)^3} \left[ 3 - 4x + x^2 + 2 \log x \right]
\end{align*}

For degenerate masses \((x = 1)\), the functions are well defined and have the limit \(F_N^1(x) = F_N^2(x) = F_C^1(x) = F_C^2(x) = 1\). In addition, we can bound \(|F_N^2(x)| \leq 3\), while \(|F_C^2(x)|\) is unbounded as \(x \to 0\).

### A.2 Mass Matrices

The neutralino and chargino mass matrices are given by:

\[
\mathcal{M}_{\tilde{N}} = \begin{pmatrix}
M_1 & 0 & -m_Z s_W c_\beta & -m_Z s_W s_\beta \\
0 & M_2 & m_Z c_W c_\beta & -m_Z c_W s_\beta \\
-m_Z s_W c_\beta & m_Z c_W c_\beta & 0 & -\mu \\
m_Z s_W s_\beta & -m_Z c_W s_\beta & -\mu & 0
\end{pmatrix}
\]

and

\[
\mathcal{M}_{\tilde{C}} = \begin{pmatrix}
M_2 & \sqrt{2} m_W s_\beta \\
\sqrt{2} m_W c_\beta & \mu
\end{pmatrix}
\]

where we have introduced the notation \(s_\alpha \equiv \sin \alpha\) and \(c_\alpha \equiv \cos \alpha\) for an angle \(\alpha\).

We diagonalize these with mixing matrices \(N_{ij}, U_{kl}\) and \(V_{kl}\) satisfying:

\[
N^* \mathcal{M}_{\tilde{N}} N^\dagger = \text{diag}(m_{\tilde{N}_1}, m_{\tilde{N}_2}, m_{\tilde{N}_3}, m_{\tilde{N}_4})
\]

\[
U^* \mathcal{M}_{\tilde{C}} V^\dagger = \text{diag}(m_{\tilde{C}_1}, m_{\tilde{C}_2})
\]

The smuon mass matrix is given by:

\[
\mathcal{M}_\mu^2 = \begin{pmatrix}
M_L^2 & m_\mu (A_\mu^* - \mu \tan \beta) \\
m_\mu (A_\mu^* - \mu \tan \beta) & M_R^2
\end{pmatrix}
\]
where

\[ M^2_L = m^2_L + (s^2_W - \frac{1}{2}m^2_Z)c_{2\beta} \]

\[ M^2_R = m^2_R - m^2_Zs^2_Wc_{2\beta} \]  \hspace{1cm} (A.8)

for the soft masses \( m^2_L \) and \( m^2_R \). The smuon mixing matrix \( X_{mn} \) is defined:

\[ X \mathcal{M}^2_{\bar{\mu}} X^\dagger = \text{diag}(m^2_{\bar{\mu}_1}, m^2_{\bar{\mu}_2}). \]  \hspace{1cm} (A.9)

We will define a mixing angle \( \theta_{\bar{\mu}} \) such that \( X_{11} = \cos \theta_{\bar{\mu}} \) and \( X_{22} = \sin \theta_{\bar{\mu}} \). We set \( A_{\bar{\mu}} \) to zero. We have checked that at large \( \tan \beta \) this makes only a slight numerical difference, while at large \( \tan \beta \) there is no noticeable effect at all. The muon sneutrino mass is related to the left-handed slepton soft parameter by:

\[ m^2_{\tilde{\nu}} = m^2_L + \frac{1}{2}m^2_Z \cos 2\beta \]  \hspace{1cm} (A.10)

Finally, the leading two-loop contributions to \( \delta a_\mu \) have been calculated [40]. They were found to suppress the SUSY contribution by a factor of \((4\alpha/(\pi) \log (M_{\text{SUSY}}/m_\mu)) \approx 0.07\). We include this 7% suppression in all of our numerics.
APPENDIX B

CONVENTIONS FOR THE HIGGS FLAVOR CHANGING VERTEX

The loop functions that arise from evaluation of the chargino and gluino diagrams from FIGURE in SECTION carry mass dimension and can be found in Refs. [55, 56]. They are fully symmetric in their arguments, a useful fact to exploit for numerical evaluations.

It is often convenient to work with related functions which take as arguments mass ratios, and therefore carry no mass dimension. Such functions are given in Ref. [65]. Both sets of functions carry the same physics, so which you use is a matter of preference. Here we give the following useful conversions between the dimensionful functions and the ones that appear in the formulas in SECTION.

The dimensionful functions \( f_n(m_1^2, \ldots, m_n^2) \) can be related to dimensionless functions \( I_{n-1}(x_1, \ldots, x_{n-1}) \) through the following relations:

\[
f_3(m_1^2, m_2^2, m_3^2) = \frac{1}{m_3^2} I_2(x, y)
\]

where
\[
x = \frac{m_1^2}{m_3^2}, \quad y = \frac{m_2^2}{m_3^2}
\]

(B.1)

and:

\[
f_4(m_1^2, m_2^2, m_3^2, m_4^2) = -\frac{1}{m_4^2} I_3(x, y, z)
\]

where
\[
x = \frac{m_1^2}{m_4^2}, \quad y = \frac{m_2^2}{m_4^2}, \quad z = \frac{m_3^2}{m_4^2}
\]

(B.2)

The dimensionless functions \( I_n(x_1, x_2, \ldots, x_n) \) can be derived using a recursive method
as found in [65]. Starting from the basic single variable function:

\[
I_1(x) = \frac{x \log x}{x - 1},
\]

(B.3)

the others can be generated using the recursive relation:

\[
I_n(x, y, z_1, \ldots, z_{n-2}) = \frac{I_{n-1}(x, z_1, \ldots, z_{n-2}) - I_{n-1}(y, z_1, \ldots, z_{n-2})}{x - y}.
\]

(B.4)

The functions \( f_n \) do have well defined limits for mass degeneracy. For the dimensionless functions \( I_n \) this is equivalent to all the arguments being unity. Some useful values in this limit are:

\[
I_1(1) = 1 \quad I_2(1, 1) = \frac{1}{2} \quad I_3(1, 1, 1) = -\frac{1}{6}.
\]

(B.5)
In this appendix we introduce Grassman numbers, which get utilized in our discussion of the superfield formulation of SUSY. Grassman numbers are ordinary numbers that anticommute:

\[ \{\xi, \eta\} = 0. \] (C.1)

Here “ordinary” means that the anticommutation relations do not imply that these are operators. They are just numbers, with a perhaps odd defining property. This defining property of Grassman numbers makes algebra very easy, since quadratic and higher power terms vanish. Therefore any function expanded in a Taylor series in a Grassman variable must terminates after the first power, making the most general function possible \( f(\xi) = a + b\xi \).

We are particularly interested in integration over Grassman numbers. This operation is somewhat simple since we need only consider integration of two terms: a constant and a linear power. The defining property we demand on our operation of integration is that it be invariant under shifts in the integration variable:

\[ \int d\xi \left( a + b(\xi + \eta) \right) = \int d\xi \left( a + b\xi \right). \] (C.2)

This implies for the two terms of interest:

\[ \int d\xi = 0 \quad \text{and} \quad \int d\xi \xi = 1. \] (C.3)

This completes the discussion of Grassman variables for our purposes.
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