Theoretical Investigation of Triaxial Strong Deformation and Tidal Waves in Nuclei

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Yingqiong Gu, B.S., M.S.

Stefan Frauendorf, Director

Graduate Program in Physics
Notre Dame, Indiana
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Abstract

by

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The appearance of triaxial nuclear shapes and the question how to prove their existence is a long-standing problem in nuclear physics. The recent discovery of wobbling rotational bands is considered as first clear cut evidence for this type of nuclear shape. So far wobbling bands have only been found in the Lu isotopes. In this thesis, the results of experimental search for wobbling bands in $^{163}\text{Tm}$ are theoretically analyzed. Calculations of the energy and electromagnetic transition probabilities are carried out in the framework of the microscopic Tilted Axis Cranking (TAC) model. The calculations account well for the data. It is found that the bands have a triaxial strong deformation (TSD). However, the two observed rotational sequences do not correspond to a zero and one wobbling phonon but to the two signature branches of the odd proton orbital, which is a very low lying particle-hole excitation. It is demonstrated that in general wobbling phonons compete with particle-hole excitations. With present-day types of experiments, the two modes can only be disentangled if the wobbling states are the lowest excitations, as in the Lu isotopes, whereas in many other cases, including $^{163}\text{Tm}$, the particle-hole excitations make up the spectrum of observed rotational TSD bands.

There exists a large group of transitional nuclei that show a behavior in-between harmonic vibration and rotation, a description of which has been a long-
standing challenge to nuclear theory. The new concept of nuclear tidal waves allows one to calculate the sequence of states with maximal angular momentum (yrast states). These states correspond to a surface wave running around the nucleus as tidal waves run over the earth’s oceans. Such a wave corresponds to a static deformation in the rotating frame of reference, which allows one to calculate its properties by means of the microscopic cranking model. For the first time such calculations are carried out in this thesis for nuclei with $Z = 44$, 46, 48 and $N = 56-66$. The calculations reproduce very well energies and E2-transitions probabilities of these nuclei. The low-spin parts show the expected gradual transition from vibration-like behavior of the nuclides near the closed shell ($Z=48, N=56$) to rotation-like behavior of the nuclide farthest in the open shell ($Z = 44, N = 66$). In addition, the strong irregularity (back-bending) caused by the alignment of the angular momentum of two $h_{11/2}$ neutrons with the rotational axis is well accounted for. At spins larger than $10 \hbar$, where the back-bending occurs, all nuclei behave more rotation-like, which is borne out by the calculations in accordance with experiment. Such a detailed theoretical description of the yrast states of transitional nuclei has been achieved for the first time in this thesis.
DEDICATION

To the past years!
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SYMBOLS

\( J \)  angular momentum
\( I \)  quantum number of angular momentum
\( \omega \)  rotational frequency
\( \mathcal{I}^{(1)} \)  kinematic moment of inertia
\( \mathcal{I}^{(2)} \)  dynamic moment of inertia
\( i_x \)  particle alignment
\( B(E2) \)  reduced probability of electric quadrupole moment
\( B(M1) \)  reduced probability of magnetic quadrupole moment
\( Q_\mu \)  quadrupole moment
\( Q_t \)  transitional quadrupole moment
\( \Delta \)  pairing gap
\( \epsilon_2 \)  quadrupole deformation
\( \gamma \)  triaxiality parameter
\( P \)  pairing operator
\( H_{sph} \)  the spherical part of Hamiltonian
CHAPTER 1

INTRODUCTION

The nucleus consisting of many nucleons is a quantum-mechanical many-body system. Quantum Mechanics is the fundamental theory to describe the nuclear structure, and the Schroedinger equation is the basic equation to describe the nuclear behavior. However it is almost impossible to exactly solve this problem for nuclei. For such a complicated system we have to introduce simple models which are tractable and provide insight. Let me review some models that are relevant to my dissertation.

1.1 Liquid Drop Model

The nuclear size is smaller than $2 \times 10^{-14}$ m. Some nuclei are spherical. However, many of them are not exactly spherical but slightly deformed or strongly deformed. The charge distribution of a spherical nucleus is well represented by the function

$$\rho(r) = \frac{\rho_0}{1 + e^{(r-R)/a}}$$

(1.1)

$r =$ distance from the center, which is called the Woods-Saxon potential [1]. The parameters $\rho_0$, $R$, and $a$ are adjusted to the experimental charge distribution measured by electron scattering. The parameter $R$ is the radius of the nucleus, and the parameter $a$ determines the surface thickness. The nuclear radius is given by:
\[ R = 1.2 \times A^{1/3} \text{ fm} \] (1.2)

where \( A = N + Z \), and \( N \) is the number of neutrons and \( Z \) the number of protons. The volume of a nucleus, is

\[ V = \frac{4}{3} \pi R^3 = \frac{4}{3} \pi \times 1.2^3 \times A \] (1.3)

and its density is

\[ \rho = \frac{A}{V} \approx 0.138 \text{ fm}^{-3} \] (1.4)

The nuclear density is almost the same for all nuclei. This is called the saturation of nuclear density. Let the mass of the nucleus be \( M \). If we assume that a nucleus was a simple collection of protons and neutrons, the mass of the nucleus would be the sum of these constituent nucleons’ masses, \( ZM_p + NM_n \), where \( M_p \) is the mass of the proton and \( M_n \) is that of the neutron. However, a nucleus is not a simple collection of protons and neutrons. They are bound to each other by a strong interaction, the nuclear force. As a consequence, the total mass of the system is less than the sum of the masses of the individual nucleons. This decrease of the mass of the system is called the mass defect:

\[ B(Z, N) = [(ZM_p + NM_n) - M(Z, N)]c^2 \] (1.5)

C. F.v. Weizsaecker[2] suggested that the nucleus is similar to a water drop, because the density of nuclear matter is constant inside the nucleus and it drops to zero in a thin surface layer (see equation (1.1)). For a heavy nucleus, the radius
$R \sim 6\text{fm}$ whereas the surface thickness $a \sim 0.6\text{fm}$. On the basis of this liquid drop model, Weizsaecker and H. A. Bethe introduced a simple formula for the nuclear binding energy.

$$B(Z, N) = C_v A - C_s A^{2/3} - C_c Z^2 A^{-1/3} - C_{sym} \frac{(N - Z)^2}{A} + \delta(A) \quad (1.6)$$

which is also called the semi-empirical mass formula. The first term is proportional to the nuclear volume and called the volume energy, which is the main part of the nuclear binding energy. The second term is proportional to the area of the nuclear surface, and it is considered as the surface energy. The third term is the Coulomb energy which reflects the repulsive electrostatic force between protons. The fourth term, which is called the symmetry energy, favors an equal number of protons and neutrons. This effect comes from the symmetry property in the nuclear force. The last term in the formula is called pairing energy. Equivalent nucleons tend to form Cooper pairs as electrons in a super conductor (proton-proton or neutron-neutron), which is caused by the attractive nuclear force.

As shown in figure 1.1, overall the experimental data can be reproduced very well by the Bethe-Weizsaecker mass formula. However, the detailed experimental data of the binding energies are not completely reproduced by the liquid drop model. They fluctuate around the values of the semi-empirical mass formula.

Figure 1.2 shows the deviations of the experimental binding energies from the values of the semi-empirical mass formula. The nuclei with neutron numbers’ 28, 50, 82 or 126 have especially large binding energies compared with the other neutron numbers. The same increased binding energy is observed for $Z = 2, 8, 20, 28, 50, 82, 126$. These numbers are called the magic numbers of nuclei.
Figure 1.1. Comparison of the binding energies between semi-empirical mass formula and experiment. The points indicate the experimental data of the binding energy per nucleon. The curve shows the values obtained by the Weizsaecker-Bethe mass formula.
Figure 1.2. The deviations of the experimental binding energies from the values of the semi-empirical mass formula.

1.2 Spherical Shell Model

The original idea of the nuclear shell model [3] is analogous with the atomic shell model. However, there are substantial differences between atoms and nuclei. Since the nucleus is like a liquid drop, the potential created by the nucleons must be almost constant inside of the nucleus and change to zero in a thin surface. However this potential does not yet reproduce the magic numbers.

M. G. Mayer and H. D. Jensen independently solved the problem by proposing an average potential, which included a strong spin-orbit term (See Fig. 1.3). The modern version is the spherical Woods-Saxon potential which reproduces the nuclear magic numbers very well. The shell structure of nuclei is one of the basic concepts of the nuclear-structure theory.

The nuclear shell model assumes that a nucleon moves in an effective attractive potential field created by all the other nucleons, which is roughly proportional to the nuclear density and can be expressed in the following form:

\[ V = -V_0(1 + \exp\left(\frac{r - R}{a}\right))^{-1} \]  

(1.7)

<table>
<thead>
<tr>
<th>Angular Momentum (n/2l/2m)</th>
<th>Spin-Orbit Coupling (1/2, 3/2, 5/2, 7/2)</th>
<th>Number of Nucleons</th>
<th>Magic Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>7  1j</td>
<td></td>
<td>1j 15/2</td>
<td>16</td>
</tr>
<tr>
<td>6  4s</td>
<td></td>
<td>4s 1/2</td>
<td>2</td>
</tr>
<tr>
<td>6  3d</td>
<td></td>
<td>3d 7/2</td>
<td>8</td>
</tr>
<tr>
<td>6  2g</td>
<td></td>
<td>2g 5/2</td>
<td>6</td>
</tr>
<tr>
<td>6  1h</td>
<td></td>
<td>1h 9/2</td>
<td>10</td>
</tr>
<tr>
<td>5  3p</td>
<td></td>
<td>3p 3/2</td>
<td>4</td>
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<tr>
<td>5  2f</td>
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<td>2f 5/2</td>
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<td>5  1h</td>
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<tr>
<td>4  3s</td>
<td></td>
<td>3s 11/2</td>
<td>12</td>
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<td>4  2d</td>
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<td>2d 3/2</td>
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<td>3  2p</td>
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<tr>
<td>0  1s</td>
<td></td>
<td>1s 1/2</td>
<td>2</td>
</tr>
</tbody>
</table>

Figure 1.3. Maria Goeppert Mayer’s Nuclear Shell Model. The sequence of energy levels is shown for a potential without spin-orbit coupling (left) and with spin-orbit coupling (right). The spectroscopic notation, \( n, l, j \) is explained in the left.
The parameters of this Woods Saxon potential model are:

\[ V_0 \approx 57 \text{ MeV}, \quad R \approx 1.25A^{1/3} \text{ fm}, \quad \text{and} \quad a \approx 0.65 \text{ fm}. \]

Fig. 1.4 illustrates the Woods-Saxon potential.

1.3 Deformed Shell Model

Nuclei are expected to be soft and flexible according to the liquid-drop model, i.e. the shape of a nucleus may strongly deviate from a sphere. Experimentalists found many nuclei with a remarkably deformed charge distribution in large regions of \( N \) and \( Z \) between the magic numbers.

The assumption of independent motion of nucleons in an average potential well is the theoretical basis of the shell model. For a particular nucleon, this potential well represents the nucleon’s interaction with all other nucleons in the nucleus. For spherical nuclei, the nuclear energy will be the same whatever is the orientation of the angular momentum of the nucleon. The nuclear states can be
Figure 1.5. Sketches of the nuclear spheroidal deformations

identified by their angular momentum $l$ and total angular momentum $j$. One uses
the spectroscopic notation $nl_j$, i.e. $ns_{1/2}, np_{1/2}, np_{3/2}, nd_{3/2}, nd_{5/2}$ etc. The orbital
angular momentum from $l = 0, 1, 2, 3, \ldots$ is indicated by the letters $s, p, d, \ldots$.
The levels for given $l_j$ are enumerated by $n$. For the deformed nuclei, one uses a
deformed Woods-Saxon potential which has a similar profile through the nuclear
surface as the spherical potential. The orbital and total angular momenta, $l, j$
will no longer be a “good” quantum numbers since they are not conserved. If
the potential well has an axially symmetric shape, the projection of the angular
momentum on the symmetry axis is still a good quantum number. We describe
the deformed potential in more detail in Chap. 3.1.

Figure 1.4 shows the sketches of the most prominent nuclear shapes, which
have spheroidal deformations. For spheroidal nuclei, the distance of the nuclear
surface from the origin is given by

$$R(\theta, \phi) = R_{\text{ave}} \left[ 1 + \beta_2 Y_{20}(\theta, \phi) \right]$$

as function of the two polar angle $\theta, \phi$. Here, $Y_{\lambda, \mu}(\theta, \phi)$ are the spherical harmonics.
The deformation parameter, $\beta_2 (= \alpha_{20})$, can be related to the axes of the spheroid by

$$\beta_2 = \frac{4}{3} \sqrt{\frac{\pi}{5}} \frac{\Delta R}{R_{av}}$$

(1.9)

in which the average radius, $R_{av} = 1.25 A^{1/3}$, and $\Delta R$ is the difference between the semi-major and semi-minor axes. Positive and negative $\beta_2$ values correspond to prolate and oblate shapes respectively. Higher-order axially-symmetric deformations need also to be taken into nuclear-structure theory, such as the hexadecapole deformations $\beta_4$.

The shape parameters introduced so far only describe axially-symmetric nuclear shapes, with quadrupole ($\lambda = 2$) deformations. Triaxial distortions are defined by the $\gamma$ degree of freedom, which describes a stretching along an axis perpendicular to the symmetry axis. The shape is given by

$$R(\theta, \phi) = R_{av}(1 + \beta \cos \gamma Y_{20}(\theta, \phi) + \frac{1}{\sqrt{2}} \sin \gamma (Y_{22}(\theta, \phi) + Y_{2-2}(\theta, \phi)))$$

(1.10)

where $\gamma = 0^\circ$ and $\gamma = 60^\circ$ correspond to prolate and oblate shapes, respectively. The most triaxial shapes have $\gamma = 30^\circ$.

In my thesis, I will mostly use the Deformed Shell Model[4]. In this model the Schroedinger equation is solved using a potential form that corresponds to the nucleus’ actual shape,

$$V = -V_0(1 + \exp(1 + \exp(\frac{r - R(\theta, \phi)}{a})))^{-1}$$

(1.11)

with $R(\theta, \phi)$ given by equation (1.10).
1.4 Nuclear Collective Model [5]

In addition to the motion of the individual nucleus inside the nuclear potential, the potential itself, rotates and vibrates, which is called collective motion. The collective model describes these excitation modes.

1.4.1 Rotation

A nucleus can generate angular momentum in two different ways, either collectively as rotations and vibrations of the droplet of the nuclear matter or by nucleon excitations in which a small number of unpaired nucleons rearrange to generate the angular momentum. In practice most nuclear states carrying large angular momentum are a mixture of these two modes.

The collective angular momentum $\vec{R}$ represents the motion of the nucleus as a whole, i.e. the rotation of the droplet. It is impossible to have collective rotation about an axis of symmetry, since the different orientations are indistinguishable in the quantum mechanics. In the important case of an axially-symmetric nucleus the collective rotation must be about an axis perpendicular to the symmetry axis.

Calling the angular momentum generated by the rearrangement of the valence nucleons, $\vec{i}$, the total angular momentum $\vec{J}$ of the nucleus will be

$$\vec{I} = \vec{R} + \vec{J} \quad (1.12)$$

This angular momentum coupling is shown schematically in Fig. 1.6.

The classical kinetic energy of a rotating body is

$$E = \frac{1}{2} \Theta \omega^2 \quad (1.13)$$
Figure 1.6. Schematic of the coupling of the collective angular momentum
where $\mathfrak{I}$ is the moment of inertia. In the quantum mechanics, it will turn into

$$E = \frac{\hbar^2}{2\mathfrak{I}} I(I + 1)$$  \hspace{1cm} (1.14)

For a band with a non-zero projection of $\vec{J}$ onto the symmetry axis, denoted by $K$, the energy becomes

$$E = \frac{\hbar^2}{2\mathfrak{I}(0)} [I(I + 1) - K^2]$$  \hspace{1cm} (1.15)

If the nucleus rotates very rapidly, the valence particles may contribute an amount of angular momentum $i$ (sometimes called also $i_x$) along the axis perpendicular to the symmetry axis, which is called aligned angular momentum or alignment (see Fig. 1.16). In this case the energy is

$$E = \frac{\hbar^2}{2\mathfrak{I}_0} (I + 1/2 - i)^2$$  \hspace{1cm} (1.16)

The aligned angular momentum $i$ is generated by the Coriolis force. The phenomenon called back bending appears when the Coriolis force breaks a pair of nucleons coupled to zero spin and aligns their spins. The rotational frequency of the ground band (before particle alignment) is $\omega_g = \frac{I+1/2}{3}$ and of the s band (after particle alignment) $\omega_s = \frac{I+1/2-i}{3}$. Since $\omega_s < \omega_g$, the experimental curve $J(\omega)$ bends back while the alignment occurs.

If the nuclear shape is triaxial, collective rotational motion around all three axes is possible. In addition to the uniform rotation about the axis with the largest moment of inertia, there is wobbling motion of the angular momentum, which is shown in Fig.1.7. Wobbling excitations are the quantized motion of this
type. The Hamiltonian of the triaxial rotor can be written as

\[ H_{\text{rot}} = \frac{I_x^2}{\mathfrak{3}_x} + \frac{I_y^2}{\mathfrak{3}_y} + \frac{I_z^2}{\mathfrak{3}_z} \]  

(1.17)

which has energy eigenvalues

\[ E(I, n) \approx \frac{I(I + 1)}{2\mathfrak{3}_x} + \hbar \omega_{\text{wob}} n + 1/2 \]  

(1.18)

where the wobbling energy \( \hbar \omega_{\text{wob}} \) is

\[ \hbar \omega_{\text{wob}} = \left( \frac{I}{3_x} \right) \sqrt{\frac{3_x^2}{\mathfrak{3}_x} - 1)(\frac{3_x}{\mathfrak{3}_y} - 1)(\mathfrak{3}_x > \mathfrak{3}_y, \mathfrak{3}_z)} \]  

(1.19)

The electromagnetic transition probabilities are an important experimental information in addition to the energies. The reduced probabilities \( B(M\lambda, I - > I') \) and \( B(E\lambda, I - > I') \) contain the information about the nuclear structure, where \( M\lambda \) refers to magnetic transitions of multipolarity \( \lambda \) and \( E\lambda \) to electric transitions of multipolarity \( \lambda \) [30]. The \( B(E2, I - > I-2) \) are of special importance for this thesis, because we measure the quadrupole deformation. It is common to introduce the transition quadrupole moment \( Q_t = \sqrt{\frac{16\pi}{5} B(E2)} \), which is approximately proportional to the deformation parameter.

1.4.2 Vibration

Nuclei can also generate collective angular momentum by vibrating. Vibrational quanta are classified as “phonons” of multipolarity \( \lambda \), which are characterized the multipolarity of the distortion of the nucleus surface. Phonons of \( \lambda=2 \) create the low-energy quadrupole vibration, which can have two forms in an axi-
Figure 1.7. Nuclear wobbling motion
ally deformed nucleus. The $\beta$ vibration was elongations along the symmetry axis and perpendicular to it, as shown in Fig. 1.8. The oscillation does not carry angular momentum. Therefore, such vibrational excitations are $0^+$ states. The second type of quadrupole vibration corresponds to an oscillation in the $\gamma$-shape parameter. It is illustrated in Fig. 1.8 as well. The $\gamma$ vibration is a traveling wave carrying angular momentum of $K=2$ along the symmetry axis. Octupole vibrations are associated with $\lambda=3$ phonons. They are not of interest in my thesis.

In this thesis we study quadrupole vibrations of nuclei with a spherical ground state. The collective motion of the nucleus will cause small surface deformations, which are described by the radius of the nucleus considered as a liquid drop:

$$R(\theta, \varphi) = R_0[1 + \sum_{\lambda \mu} a_{\lambda \mu} Y_{\lambda \mu}(\theta, \varphi)].$$  \hspace{1cm} (1.20)

The oscillator Hamiltonian is

$$H = \sum_{\lambda \mu} \frac{B_{\lambda}}{2} |a_{\lambda \mu}|^2 + \sum_{\lambda \mu} \frac{C_{\lambda}}{2} |a_{\lambda \mu}|^2$$ \hspace{1cm} (1.21)
Classically, \(a_{\lambda \mu}(t)\) oscillate with the frequency \(\Omega\).

The quantal description \([39]\) of a vibration is obtained by introducing the operator for the momentum of the multi-pole deformations \(\pi_{\lambda \nu}\), with \([\pi_{\lambda \nu}, a_{\lambda' \nu'}] = \delta_{\lambda \lambda'} \delta_{\nu \nu'}\). The oscillator Hamiltonian is then given by

\[
H = \sum_{\lambda \nu} \frac{|\pi_{\lambda \nu}|^2}{2B_{\lambda}} + \sum_{\lambda \nu} \frac{C_{2}^2}{2} |a_{\lambda \nu}|^2.
\] (1.22)

The quadrupole multi-polarity, \(\lambda = 2\), is the most important nucleus with a static deformation or nuclei that are in between well deformed and spherical types. The latter are described by introducing a more complicated potential \(V(\beta, \gamma)\) than the harmonic one \(C_{2}^2 \beta^2\), which may have its minimum at \(\beta_0 \neq 0\). Here the intrinsic deformation parameters \(\beta\) and \(\gamma\) are introduced as given by equation. (1.11). The form of the Hamiltonian is discussed in Chap. 4.2.

1.5 Goal of the dissertation

At present, the structure of triaxial strongly deformed nuclei is in the focus of nuclear physics. Nuclei with \(N \sim 94\) and \(Z \sim 71\) \([7]\) provide a possibility to study strongly deformed nuclear shape with triaxiality. The wobbling mode is uniquely related to the triaxiality. Evidence of the existence of the wobbling mode has been found and identified in \(^{163}\text{Lu}\)\([8]\) and its isotopes, \(^{165}\text{Lu}\) and \(^{167}\text{Lu}\)\([9]\), which encouraged people to consider wobbling as a general phenomenon in \(A \sim 163\) region. However, recently two interacting strongly-deformed bands in \(^{163}\text{Tm}\) have been identified by Prof. U. Garg’s group \([10]\) who did not find evidence for the presence of the wobbling mode. In order to resolve this discrepancy, we investigated the nuclei in this region, in order to explain the properties of new found bands and of other strongly deformed bands in Tm-Lu-Hf-Yb isotopes.
On the other hand, one of the major themes in nuclear structure research is shape evolution with nucleon number and angular momentum. Collective excitations of nuclei have two types of modes, one is vibration and another one is rotation. In $A \sim 110$ region, a transition from the vibrational motion at low spin to rotational motion at high spin has been suggested. The microscopic description of anharmonic vibrational motion has been a big challenge to theory. For the first time we applied the cranking method to interprete the behavior of surface vibrational nuclei, which is a crucial step forward in meeting this challenge.
2.1 Nilsson-Strutinsky approach-shell correction method [12]

As discussed in the introduction, the spherical shell model gives a good description of nuclei near closed shells. For deformed nuclei, the deformed Woods-Saxon potential is frequently used. An alternative form is the Nilsson potential [13]. Such a deformed potential is based on the modified harmonic-oscillator potential, which was first introduced by Nilsson to investigate the consequence of deformation for the single-particle energy levels. The Hamiltonian of the modified harmonic-oscillator potential is written as

\[
h_{Nil} = -\frac{\hbar^2}{2m} \nabla^2 + m \left( \omega_x x^2 + \omega_y y^2 + \omega_z z^2 \right) - 2\kappa \hbar \omega_0 [\vec{l} \cdot \vec{s} - \mu l^2 - \langle l^2 \rangle_N], 
\]

(2.1)

where the \((\vec{l} \cdot \vec{s})\) term represents the spin-orbit force and the term \(l^2 - \langle l^2 \rangle_N\) simulates the flattening of nuclear potential at the center. Respectively, the factors \(\kappa\) and \(\mu\) in the last term determine the strength of the spin-orbit and \(l^2\) terms.

The three oscillator frequencies \(\omega_x, \omega_y, \omega_z\) are expressed as functions of \(\omega_0\) and the deformation parameters \(\epsilon_2\) and \(\gamma\),

\[
\omega_x = \omega_0(\epsilon_2, \gamma)[1 - \frac{2}{3} \epsilon_2 \cos(\gamma + \frac{2\pi}{3})],
\]

(2.2)
\[ \omega_y = \omega_0(\epsilon_2, \gamma)[1 - \frac{2}{3} \epsilon_2 \cos(\gamma - \frac{2\pi}{3})], \quad (2.3) \]

\[ \omega_z = \omega_0(\epsilon_2, \gamma)[1 - \frac{2}{3} \epsilon_2 \cos \gamma)]. \quad (2.4) \]

The deformation parameter \( \epsilon_2 \) is approximately 0.95\( \beta_2 \), and \( \omega_0 \) is the oscillator frequency \( \hbar \omega_0 = 41A^{-1/3} \) MeV of the spherical nuclear potential. By using this deformation-dependent Hamiltonian, single-particle energies become functions of \( \epsilon_2 \). A neutron single-particle energy plot via deformation, which is known as a Nilsson diagram, is shown in figure 2.1.

The Nilsson energy levels are labeled by the quantum numbers \([Nn_z\Lambda]\Omega\), where \( N \) is the principal quantum number, \( \Omega \) is the projection of the single-particle angular momentum onto the symmetry axis, \( \Lambda \) is the projection of the orbital angular momentum onto the symmetry axis and \( n_z \) is the number of oscillator quanta along the symmetry axis. The parity of the state \( \pi \) is defined by \((−1)^N\).

In Fig. 2.1, at zero deformation, the (2j+1)-fold degeneracy of a given energy state is seen. When the deformation is introduced, the j-states split into two-fold degenerate levels, because states with a different angular momentum projection on the symmetry axis of the potential have different energy.

In order to calculate the total binding energy, Strutinsky merged the concepts of the shell model and the liquid-drop model and was very successful in reproducing the observed ground-state energies and deformations. The basis of this method is that the total energy is split into two terms. The first term is the smoothly varying energy derived from the liquid-drop model, and the second term, a rapidly varying part, called the shell energy is calculated from the single-particle energies,
Figure 2.1. Nilsson diagram for neutrons 150 < A < 190. The figure shows the energies of states in the axial symmetric potential as function of deformation parameters $\epsilon$ which is called in $\epsilon_2$ in the text. The energy unit $\hbar \omega_0 = 41 MeV A^{-1/3}$ is used.
\[ E_{sc} = E_{ldm} + E_{shell(\text{protons})} + E_{shell(\text{neutrons})} \]  

(2.5)

The proton and neutron shell energies, \( E_{shell} \), are calculated independently and can be considered as the consequence of the difference between the actual discrete density of the single particle levels at \( e_i, g \), and a “smeared” level density, \( \tilde{g} \). The discrete and smeared level densities are defined as

\[
g(e) = \sum_i \delta(e - e_i), \quad \text{(2.6)}
\]

and

\[
\tilde{g}(e) = \frac{1}{\gamma \sqrt{\pi}} \sum_i f_{cor} \left( e - e_i \right) \exp \left( -\frac{e - e_i^2}{\gamma^2} \right). \quad \text{(2.7)}
\]

Here \( \gamma \) is an energy larger than the shell spacing \( \hbar \omega_0 \), and \( f_{cor} \) is a correction function that takes into account the long-range variations of \( g \) over energies much larger than \( \hbar \omega_0 \). The shell energy is calculated as

\[
E_{shell} = 2 \sum e_i - 2 \int e \tilde{g}(e) de, \quad \text{(2.8)}
\]

where the double degeneracy of the deformed energy levels gives the factor 2.

The total energy in the liquid-drop model \( E_{LD} \) is given by

\[
E(I = 0) = -a_v(1 - k_v \left( \frac{N - Z}{A} \right)^2)A + \frac{3 e^2 Z^2}{5} \left( \frac{B_c(\epsilon)}{R_0} - \frac{5 \pi^2}{6} \left( \frac{d}{R_0} \right)^2 \right) A^2 B_s(\epsilon)
\]

\[
+ a_s(1 - k_s \left( \frac{N - Z}{A} \right)^2) A^{2/3} B_s(\epsilon) \quad \text{(2.9)}
\]

where \( B_c(\epsilon_2) \) and \( B_s(\epsilon_2) \) describe the shape dependence of the Coulomb and surface energies. The second term in the Coulomb energy is a diffuseness correction.
In present approach, the parameters used for the liquid-drop model are,
\[ a_v = 15.4941\text{MeV}, \quad a_s = 17.9439\text{MeV}, \quad k_v = K_s = 1.7826, \quad d = 0.544\text{fm}, \]
\[ R_0 = 1.2249A^{1/3}\text{fm}. \]

This method is very successful in predicting the existence of stably deformed nuclear ground states.

2.2 Cranking model

The cranking model [14] was introduced by Inglis. This model gives a microscopic description of the influence of rotation on the single-particle motion. The collective angular momentum can be calculated from a summation over the single-particle angular momenta. Thus the theory can be used to describe single-particle motion as well as collective rotation.

The rotation is treated classically. The nucleons are considered as independent particles moving in the rotating potential well. The calculation is performed in the intrinsic coordinates of the rotating system. The rotation axis is assumed to coincide with one of the principal axes of the nucleus, which is chosen to be the \( x \)-axis.

The cranking Hamiltonian of a single-particle is
\[
h^w = h^0 - \hbar \omega j_x. \tag{2.10}
\]

In this formula, \( h^0 \) is the Hamiltonian containing the non-rotating deformed potential, for which we take \( h^0 = h_{Nil} \). The term \(-\hbar \omega j_x\) accounts for the uniform rotation. It generates the Coriolis and centrifugal forces, which modify the nucleon orbitals. The Coriolis force aligns the angular momenta of the nucleons with the rotation axis. The eigenvalues of this Hamiltonian \( e_v^\omega \) are called single-particle
Routhians and its eigenstates $\nu^\omega$.

Since the cranking axis $x$ coincides with one of the principal axes and we consider only the case that the deformed potential is reflection symmetric, the cranking Hamiltonian is invariant under a rotation by $\pi$ about the $x$ axis. This implies the signature quantum number $\alpha$ for the single particle states

$$e^{-ij_\pi} |\nu^\omega\alpha\rangle = e^{-i\alpha\pi} |\nu^\omega\alpha\rangle$$

The total signature $\alpha$ of a configurations, which is the sum of signatures of all occupied single particle states (or excited quasiparticles) determines the possible values of the total angular momentums: $I = \alpha + \text{even number}$. In Section 2.3 we describe the total energy in the rotating frame $E^\omega$ is calculated. The total angular momentum is the sum

$$J^\omega = \sum_\nu <\nu^\omega | j_x | \nu^\omega >,$$

The total energy in the laboratory frame is

$$E = E^\omega + \omega J(\omega).$$

The Cranking Model also takes the effects of the pairing interaction into account. For this, the Cranking Hamiltonian will be modified to include a pairing term,

$$H = h^0 - \hbar \omega j_x - \Delta (P^+ + P) - \lambda N$$

where $\Delta$ is the pair gap and $P^+$ generates pair of nucleons in time-reversed states. The term $-\lambda N$ is for keeping the expectation value of the total number of par-
particles constant. The eigenvalues of the Hamiltonian (2.14) are the quasiparticle Routhians $E^\nu_\omega$. For treatment of the pairing and calculation of the total energy see section 2.3.

The calculated single-particle Routhians $e^\nu_\omega$ or quasiparticle Routhians $E^\nu_\omega$ may be directly compared with experiment. In the experiment, we define the rotational frequency as

$$\hbar \omega(I) = \frac{1}{2}(E(I) - E(I - 2)) \quad \text{and} \quad J_x(I) = I - 1/2. \quad (2.15)$$

The aligned angular momentum is given as $J_x(\omega) - J_{\text{ref}}(\omega) = i_x$, where $J_{\text{ref}}$ is the angular momentum of the reference, which is considered as purely collective.

In order to transform the experimental energies to the rotating frame, we must define an expression for the total experimental Routhian which is

$$E^\omega_{\text{expt}}(I) = \frac{1}{2}[E(I + 1) + E(I - 1)] - \omega(I) J_x(I) \quad (2.16)$$

The experimental quasi-particle Routhian is

$$e^\omega_{\text{expt}}(I) = E^\omega_{\text{expt}}(I) - E^\omega_{\text{ref}}(I). \quad (2.17)$$

The reference energy is given by:

$$E^{\text{ref}}_x(\omega) = -\frac{1}{2} \omega^2 \mathbb{S}_0 - \frac{1}{4} \omega^4 \mathbb{S}_1 + \frac{1}{8} \hbar^2 \mathbb{S}_0. \quad (2.18)$$

and the reference angular momentum is $J_{\text{ref}} = \omega \mathbb{S}_0 + \omega^3 \mathbb{S}_1$.

Then we can compare the experimental Routhian $e^\omega_{\text{expt}}(I)$ with the theo-
retical Routhian as well as the experimental alignment with the calculated one
\( <\nu^\omega|j_x|\nu^\omega> \).

The Cranked Nilsson-Strutinsky (CNS) approach generalizes the Nilsson-Strutinsky shell correction method. From the single particle \( e_{\nu^\omega} + <\nu^\omega|j_x|\nu^\omega> \), a shell correction \( \delta E_{sh}(\omega) \) is calculated in the same way as equations 2.6-2.8. The total nuclear energy \( E_{tot} \) is calculated as a sum of the rotating liquid drop energy and shell energy,

\[
E_{tot}(\epsilon, I_0) = E_{LD}(\epsilon, I = 0) + \frac{1}{2\mathbb{I}_{rig}(\epsilon)(I_0^2 + bI_0^4)} + \delta E_{sh}(\omega(I)). \tag{2.19}
\]

\( I_0 \) refers to the specific spin we want to calculate, and \( b \) is determined by calculating the smoothed sum at several frequencies.

### 2.3 Tilted-Axis Cranking Method

Tilted-Axis Cranking (TAC) method is a generalization of the cranking model in which the rotational axis need not to coincide with the principle axes of the deformed potential.

Kerman and Oenishi [15] first pointed out the possibility of cranking about a non-principal axis. Frisk and Bengtsson [16] studied such solutions for realistic nuclei. Frauendorf [17] found the first fully self-consistent solutions and gave the interpretation of the titled cranking solutions in terms of \( \Delta I = 1 \) rotational bands. Since then the TAC approach has been quite successful in describing the energies and transition probabilities of rapidly rotating nuclei. If the TAC solution corresponds to rotation about a principle axis, the TAC reduces to be the conventional cranking model.

We will use two versions of the TAC method.
i) The pairing plus quadrupole-quadrupole model (PQTAC)

ii) Shell correction method (SCTAC)

Version (i) (PQTAC) will be applied to the small nuclear deformation and
version (ii) (SCTAC) for larger nuclear deformations.

We first discuss the PQTAC. We assume the rotational axis is the $z$ axis and
start with the two-body Routhian:

$$ H' = H - \omega j_z \quad (2.20) $$

Then we use Pairing+Quadrupole-Quadrupole (PQQ) model for the two-body
Hamiltonian:

$$ H = H_{sph} - \frac{\chi}{2} \sum_{\mu} - 2 Q^{+}_\mu Q_\mu - GP^+ P - \lambda N \quad (2.21) $$

The spherical part:

$$ H_{sph} = \sum_k \varepsilon_k c_k^+ c_k \quad (2.22) $$

coincides with the Nilsson Hamiltonian.

The pair correlations are taken into account by means of the monopole pair

$$ P^+ = \sum_{k > 0} c_k^+ c_k \quad (2.23) $$

Here $\bar{k}$ is the time reversed state of $k$. The quadrupole-quadrupole interaction is
defined as:

$$ Q_\mu = \sum_{k,k'} \sqrt{4\pi \over 5} \langle k | r^2 Y_{2\mu} | k' \rangle c_k^+ c_{k'} \quad (2.24) $$

Using the Hartree-Fock-Bogoljubov (HFB) mean-field approximation and neglect-
ing exchange terms, the HFB-Routhian becomes

\[ h' = h_{sph} - \sum_{\mu = 1}^{2} q_\mu Q^\dagger - \Delta(P^+ + P) - \lambda N - \omega j_z. \]  

(2.25)

In the equation, the deformed part of the potential is determined by the self-consistency condition

\[ q_\mu = \chi \langle Q_\mu \rangle. \]  

(2.26)

The pair potential is determined by

\[ \Delta = G\langle P \rangle. \]  

(2.27)

The chemical potential \( \lambda \) is fixed by the condition

\[ N = \langle \hat{N} \rangle. \]  

(2.28)

The equations of motion:

\[ [h_\omega, \alpha_i^+] = e_i' \alpha_i^+ \]  

(2.29)

determine the quasi-particle operators:

\[ \alpha_i^+ = \sum_k (U_{ki} c_k^+ + V_{ki} c_k) \]  

(2.30)

and the quasi-particle Routhians \( E_i^\omega \).

The above set of HFB equations gives the self-consistent solution. The total Routhian

\[ E' = \langle H' \rangle \]  

(2.31)
will have extrema at
\[ \frac{\partial E'}{\partial q_{\mu}}|_{\omega} = 0 \] (2.32)
\[ \frac{\partial E'}{\partial \Delta}|_{\omega} = 0 \] (2.33)
which are just the self consistent solutions. The total energy as function of the angular momentum is given by
\[ E(J) = E'(\omega) + \omega J(\omega), \quad J(\omega) = \langle j_z \rangle. \] (2.34)

For a general TAC solution, the \( z \) axis does not coincide with one of the principal axes,
\[ e^{-i\pi J_z}|\pi,\omega \rangle \neq e^{-i\pi \alpha}|\pi,\omega \rangle \] (2.35)
The signature is no longer a good quantum number. The single particle or quasi particle configuration \( |\pi,\omega \rangle \) describes a \( \Delta I=1 \) rotational band of the given parity.

If we consider the case that the \( z \) axis lies in one of the three principal planes, we may assume it to be the 1-3 plane. The other planes are made by \( \gamma \) vary within the range \(-120^0 < \gamma < 60^0\). Then the tilt angle \( \vartheta \) between the 3- and \( z \) axis determines the direction of the rotational axis. In practical applications, it is convenient to reformulate the TAC in the frame of principal axes. Then the quasi-particle Routhian is given by
\[ h \omega = h_{sph} - q_0'Q_0' - q_2'(Q_2' + Q_{-2'}) - \Delta(P^+ + P) - \lambda N - \omega(\vartheta j_1 + \cos\vartheta j_3). \] (2.36)
This is numerically diagonalized, which gives the solution of the equations of motion (2.29). The self-consistent equations become

\[ q_0' = \chi < Q_0' >, q_2' = \chi < Q_2' > = \chi < Q_{-2}' > \]  

(2.37)

and the condition for the tilt angle

\[ \tan(\vartheta) = \frac{< j_1 >}{< j_3 >}. \]  

(2.38)

which states that the angular momentum has the same directions as the angular velocity \( \vec{\omega} \). These equations are solved numerically.

In the SCTAC approach, one finds the quasiparticle solutions of the Routhian

\[ h^\omega(\epsilon_2, \gamma, \omega, \theta) = h_{Nil}(\epsilon_2, \gamma) - \Delta(P^+ + P) - \lambda N - \omega(\sin \theta j_1 + \cos \theta j_3). \]  

(2.39)

at a given deformation parameters \( \epsilon_2, \gamma \). The tilt angle is found by means of the condition Eq. (2.37). The energy in the rotating frame is calculated for a given quasi-particle configurations as the sum of the rotational energy and the ground-state energy as given by the shell correction method (\( E_{sc} \)).

\[ E_{SCTAC}^\omega(\epsilon_2, \gamma) = E_{ROT}(\omega, \epsilon_2, \gamma) + E_{SC}(\epsilon_2, \gamma) \]  

(2.40)
Where

\[ E_{\text{ROT}}(\omega, \epsilon_2, \gamma) = \langle \pi, \omega | h^{\omega}(\omega, \gamma, v) | \pi, \omega \rangle - \langle \pi \omega = 0 | h^{\omega=0}(\omega, \gamma) | \pi \omega = 0 \rangle. \]

(2.41)

The deformation parameters \( \epsilon_2, \gamma \) are found by minimizing \( E_{\text{SCTAC}}(\omega, \epsilon_2, \gamma) \) for fixed \( \omega \). The energy in the laboratory system is obtained by means of equation 2.33, where \( J = \sqrt{<j_1>^2 + <j_3>^2} \). For the case of the tidal waves we use a different method, which is discussed in section 4.3.
CHAPTER 3

TRIAXIAL STRONG DEFORMATION IN A \( \sim 164 \) REGION

3.1 Appearance of triaxial strongly deformed shapes

The calculation by means of the Ultimate Crank[18], which is a version of the cranking model based on a modified harmonic-oscillator potential if CNS including paring, predict the occurrence of a Triaxial Strongly Deformed (TSD) nuclear shapes for nuclei with \( 162 < A < 174 \) at high spin. Major gaps of single-particle energy are created at proton number 71 and neutron numbers 94 and 97 at low and intermediate frequencies (see Figs. 3.4, 3.5). As an example, a potential energy surface for \( ^{163}\text{Lu} \) is shown in Fig 3.2. TSD bands are believed to be built on the proton \( i_{13/2} \) intruder configuration[19]. The stable and large energy gap of single-particle energy levels indicate the possible existence of triaxial deformed nuclear shapes. Since the equilibrium shapes occur at large nuclear deformation, we speak of “Triaxial Strongly Deformed” (TSD) nuclei.

Nuclei with \( A \sim 164 \) provide the possibility of studying TSD nuclear shapes. Experimentalists have found several TSD bands in the Lu isotopes. In Fig 3.1 shows four TSD bands in \( ^{163}\text{Lu} \), which are connected by strong electromagnetic interactions. The three bands TSD1, TSD2, and TSD3 have very similar dynamic moments of inertia and alignments over almost the full frequency range. The decay of the excited TSD bands proceeds through the yrast TSD band. As will
be discussed in section 3.2, this band structure indicates the existence of wobbling excitations, which represent a very strong evidence for triaxiality.

Figure 3.3 gives a schematic overview of the experimental investigations and results on Lu-Hf isotopes. More than 20 bands in this region have been connected to the TSD shape. For the even-\(N\) Lu isotopes, the yrast TSD bands with positive parity and signature \(\alpha = +1/2\) are based on a configuration with the odd proton in the \(i_{13/2}\) state.

As we can see in the Fig 3.2, TSD states are expected to appear in two regions in deformation space, which are located around \(\epsilon_2 \approx 0.4\) and \(\gamma \approx \pm 20^\circ\). Figures 3.4 and 3.5 show the single-particle energy levels for neutrons and protons for shapes close to the TSD minima. The energy gap at neutron number 94 is large at low rotational frequency. It plays a very important role for the appearance of the TSD states. While the rotational frequency increases, the size of \(N=94\) neutron energy gap decreases and the size of the \(N=97\) neutron energy gap increases. Finally the \(N=97\) neutron energy gap becomes dominant at high rotational frequency. Comparing the proton energy levels, one notices that for \(Z=69\) (Tm) the proton system has a large level density, i.e. it does not particularly favor the TSD shape. It is only the \(N=94\) neutron gap that drives the nucleus to the TSD shape, which appears at smaller frequency \(\omega\) and deformation \(\epsilon_2\) than the \(N=97\) gap. For \(Z=72\) (Hf) there is a gap in the proton spectrum at TSD, which drives the nucleus to large deformation. This gap combines with the gap in the neutron spectrum at \(N=97\), which explains why TSD shapes occur at higher frequency \(\omega\) and have larger deformation in the Hf and Ta (\(Z=73\)) isotopes, for which the most favored TSD neutron number is 97 and not 94.
Figure 3.1. The partial level scheme of $^{163}$Lu shows four TSD bands together with the connecting transitions to the ND structures. [34]
Figure 3.2. Potential-energy surface for $^{163}$Lu [34] for parity and signature $(\pi, \alpha) = (+, +1/2)$ at $I = 53/2\hbar$. The energy minimum at normal deformation (ND) is marked by the dot. The two triaxial strongly deformed minima (TSD) are marked by crosses.
3.2 Wobbling motion in $A \sim 160$ region

The measurements of the E2-transition probabilities showed that the TSD bands have a large deformation (c.f. Section 3.4). However they do not say anything about deviations from axial shape. The observation of the wobbling mode in $^{163}$Lu is direct evidence for nuclear traxiality in the mass $A \sim 164$ region [20].

In the wobbling mode (see Section 1.4.1), the total angular momentum is tilted away from the axis of the largest moment of inertia by a wobbling angle, which is proportional to the number of wobbling quanta $n_w$. Furthermore, for the wobbling mode it is expected that reduced transition probability $B(E2, n_w = 2 \rightarrow n_w = 1) \sim 2 \cdot B(E2, n_w = 1 \rightarrow n_w = 0)$, and that the values of $B(E2, n_w = 2 \rightarrow n_w = 0)$ are very small. Figure 3.6 provides the evidence for the existence of the wobbling
Figure 3.4. Single-neutron levels as functions of the deformation and rotational frequency calculated along a path from spherical to the TSD shape at $\epsilon_2 = 0.424$ and $\gamma = +19.3^\circ$[36]. The path passes near the equilibrium shapes of the nuclei in the $A\sim164$ region.
Figure 3.5. Single-proton levels as functions of the deformation and rotational frequency calculated along a path from spherical to the TSD shape at $\epsilon_2 = 0.424$ and $\gamma = +19.3^\circ$[36]. The path passes near the equilibrium shapes of the nuclei in the A~164 region.
Figure 3.6. Experimental and theoretical values of $B(E2)_{out}/B(E2)_{in}$. TSD1, TSD2, and TSD3 correspond to the $n_w=0$, 1 and 2. The figure gives the ratio of the transitions between different values of $n_\omega$, $B(E2)_{out}$, and transitions between different rotational states within wobbling band $B(E2)_{in}$.

mode and thereby traxiality in $^{163}$Lu. The experimental values are compared with calculations in frame of the Particle-Rotor model [37]. Wobbling bands were also observed in $^{165}$Lu and $^{167}$Lu.

3.3 Experimental Study of $^{163}$Tm

Wobbling excitations have been found so far only for TSD band in Lu[21], which has $Z=71$. The experiment on $^{163}$Tm (earlier data in [22]) was carried
Figure 3.7. Partial level scheme of $^{163}$Tm. The thickness of the arrows is proportional to the transition intensities.\textsuperscript{[37]}

out by Prof. U. Garg’s group in collaboration with Prof. R. Janssen’s group at ANL in order to find wobbling excitations for the different proton number of $Z=69$ and investigate their properties \textsuperscript{[37]}. Within this thesis, I carried out the theoretical analysis of these measurements, which are published in \textsuperscript{[37]}. In a second experiment, the lifetime of the transitions were measured by the same groups \textsuperscript{[38]}. The theoretical analysis was also carried out within this thesis (section 3.5).

Figure 3.8 shows the level scheme of $^{163}$Tm. Bands 1,2,5 have normal deformation (ND). The two newly found bands TSD1 and TSD2 are suggested to have TSD shape.

The aligned angular momentum $i_x$, and the dynamic moment of inertia $\mathcal{I}^{(2)}$ are shown in Fig 3.9. These plots demonstrate that the two TSD bands have very
Figure 3.8. Alignments $i_x$ (upper panel) and the experimental dynamic moments of inertia $J^{(2)}$ (lower panel) for the two TSD bands in $^{163}$Tm as a function of rotational frequency.
Figure 3.9. Excitation energies relative to a rotational reference for the two TSD bands and ND bands in $^{163}$Tm. The top panel shows the experimental energies. The bottom one shows the labels of theoretical calculations. The numbers (1) and (2) in the bottom panel correspond to the energy minima in Fig 3.10.
similar properties as compared with TSD bands earlier observed in this mass
region. Comparing Fig. 3.8 with Fig. 3.1, one sees that the TSD bands of $^{163}$Tm
decay in a different way than the wobbling bands in $^{163}$Lu, where the connecting
transition between wobbling bands always proceed from the band with a higher
$n_w$ value to that with a lower $n_w$. However the linking transitions between TSD1
and TSD2 in $^{163}$Tm go both ways between the two bands with equal strength. As
seen in Fig 3.10, TSD1 and TSD2 join to a $\Delta I = 1$ sequence for most of the spins.
Only at the highest spins TSD2 has a slightly higher energy. A $n_w = 1$ wobbling
excitation is expected to be at an energy of $\hbar \omega_{\text{wobbling}}$ above the $n_w = 0$ band.
These observations strongly suggest that TSD1 and TSD2 in $^{163}$Tm represent the
two signature branches of the odd proton state and not the $n_w = 0$ and $n_w = 1$
of wobbling bands. This conjecture was substantiated by my detailed theoretical
analysis of the structure presented in the next section.

3.4 Theoretical analysis of the spectrum

The calculations were done within the framework of the configuration-dependent
Cranked Nilsson-Strutinsky (CNS) model and the Strutinsky Renormalization
Tilted-Axis Cranking model (SCTAC). Figure 3.11 gives single-proton routhians
at the TSD minima of $^{163}$Tm as calculated by means of the CNS. In the bot-
tom panel, the large filled circles mark the occupied levels, which indicates the
TSD configuration that we assign to the observed band TSD1. It is the lowest
configuration with negative parity and small signature splitting at TSD shape.

The CNS method is a special case of the SCTAC model, which assumes that
the axis of rotation is one of the principal axes of the deformed potential. If the
rotational axis is stable on one of the principal axes, CNS provides a solution of
Figure 3.10. Potential-Energy surface for $^{163}$Tm calculated by means of the CNS model at $I^\pi=63/2^-$ . Two TSD minima are marked by 1 and 2. The energy step between the contours is 0.25 MeV.

the TAC problem. If the rotational axis is not stable, we have to use TAC method to find the solution with the tilted rotational axis. The CNS approach was applied first because it is technically simpler.

The potential energy surface in Figure 3.10 presents the CNS energy of the configuration with (parity,signature)\(=(\pi,\alpha)\) = \((-,-1/2\) as a function of the deformation $\epsilon_2$ and triaxility parameter $\gamma$ for $I^\pi = 63/2^-$. One can see a prolate minimum at normal deformation ($\epsilon_2 \approx 0.21$), which represents the ND band 2, and
the two TSD minima, which have almost the same energy and almost the same values of $\epsilon_2$ and $|\gamma|$. This indicates that both TSD minima correspond to the same shape, only the axis of rotation is different. The axis of rotation is the short one in minimum 1 (with $\gamma > 0^\circ$), while it is the intermediate one for minimum 2 (with $\gamma < 0^\circ$). Figure 3.9 shows that the minimum 2 is energetically favored at low spin, and minimum 1 at high spin. That is, the CNS calculations suggest that the rotation axis of the nucleus flips from the intermediate axis (minimum 2, $\gamma<0^\circ$) to the short axis (minimum 1, $\gamma>0^\circ$) when the spin becomes larger than $24\hbar$. Such a sudden flip is unphysical. It indicates that rotation about a principal axes is unstable. We have to apply the TAC model to find the stable solution. In fact, a TAC solution with a tilted axis of rotation was found that has a lower energy than the CNS solution. It connects minimum 1 with 2 smoothly. As shown in Fig. 3.13, the experimental $B(M1)/B(E2)$ value is substantial. Rotation about a principal axis would give a much smaller value. The agreement between TAC calculations and data is a strong evidence for the existence of the tilted solution.

We notice that the $i_{13/2}$ proton level is empty, which indicates that this level does not play the decisive role in forming the TSD minima, as claimed in Ref.[23]. Rather, it is the N=94 energy gap in the neutron spectrum at $\epsilon_2 \approx 0.39, |\gamma| \approx 17^\circ$ that stabilizes the TSD shape.

The possibility to experimentally identify a wobbling band is restricted by the competition of this collective excitation with the particle-hole (p-h) excitations. If the energy of the wobbling band is low and the energy of the p-h excitations is high, the wobbling band may become the first excited band above the yrast band. This situation occurs in the Lu isotopes. The opposite case occurs in $^{163}\text{Tm}$. The energy of the p-h excitations between the signature partners of the $h_{11/2}$ orbital is
much smaller than the wobbling energy. In the experiment, only the band of the p-h excitation appears to have received sufficient intensity for observation.

3.5 Theoretical analysis of the lifetime measurements

The results of the lifetime measurements are compared with our calculations in Table 3.1. We can see the large value of the transitional quadrupole moments of TSD 1,2 bands, which indicates that these bands are strongly deformed. Before discussing the significance of the difference in the measured $Q_t$ values for the ND bands 1 and 2, on the one hand, and the TSD1 and TSD2 sequences on the
TABLE 3.1
EXPERIMENTAL AND CALCULATED TRANSITION QUADRUPOLE MOMENTS IN $^{163}$Tm. FOR DESCRIPTION OF THE EXPERIMENT SEE [38].

<table>
<thead>
<tr>
<th>Band</th>
<th>$Q_t$ [eb]</th>
<th>$Q_t$ <a href="TAC">eb</a></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$6.40^{+0.57}_{-0.33}$</td>
<td>5.8</td>
</tr>
<tr>
<td>2</td>
<td>$6.39^{+0.33}_{-0.31}$</td>
<td>5.8</td>
</tr>
<tr>
<td>TSD1</td>
<td>$7.42^{+0.44}_{-0.37}$</td>
<td>8.7 (I=24), 9.6 (34&lt;I&lt;50)</td>
</tr>
<tr>
<td>TSD2</td>
<td>$7.70^{+1.04}_{-0.57}$</td>
<td>8.7 (I=24), 9.6 (34&lt;I&lt;50)</td>
</tr>
</tbody>
</table>

other, it is worth examining the relevance of the experimental results through a comparison with other nuclei in the region. Since bands 1 and 2 are based on the $[523]_{7/2}^-$ configuration, a search of the literature was undertaken for transitional quadrupole-moment measurements of this configuration in neighboring nuclei. The results are given in Table 3.2. The $[523]_{7/2}^-$ configuration is yrast in $^{163,165}$Ho, and, perhaps more importantly, in $^{163}$Lu, one of the isotopes where TSD and wobbling bands are known as well.

The $Q_t$ moments in Table 3.2 have been obtained using a different of techniques. It can be concluded from Table 3.2 that the $Q_t$ moments measured [38] for bands 1 and 2 ($Q_t \sim 6.4$ eb) are in good agreement with those reported for the same configuration in the literature. This observation provides confidence in
TABLE 3.2
QUADRUPOLE MOMENTS OF ND BANDS BASED ON THE [523]7/2− CONFIGURATION IN Tm, Ho, AND Lu NUCLEI. THE ERROR BARS ARE STATISTICAL ONLY AND DO NOT INCLUDE THE SYSTEMATIC UNCERTAINTY IN THE STOPPING POWERS. NOTE THAT FOR SOME ENTRIES IN THE TABLE, A RANGE OF VALUES IS GIVEN. THE READER IS REFERRED TO REF. [37] FOR FURTHER DETAILS.

<table>
<thead>
<tr>
<th>Nuclide</th>
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<th>$Q_t$ (eb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{163}$Tm</td>
<td>1</td>
<td>$6.40^{+0.57}_{-0.33}$</td>
</tr>
<tr>
<td>$^{163}$Tm</td>
<td>2</td>
<td>$6.39^{+0.33}_{-0.31}$</td>
</tr>
<tr>
<td>$^{163}$Ho</td>
<td>ND</td>
<td>$6.78 \pm 1.13$</td>
</tr>
<tr>
<td>$^{165}$Ho</td>
<td>ND1</td>
<td>$6.42 \pm 0.15, 6.78 \pm 0.04$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$6.74 \pm 0.04$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$6.57 \pm 0.06$</td>
</tr>
<tr>
<td>$^{165}$Ho</td>
<td>ND2</td>
<td>$5.76 \pm 0.07$</td>
</tr>
<tr>
<td>$^{163}$Lu</td>
<td>ND1</td>
<td>$4.88^{+1.36}<em>{-0.68} - 6.78^{+2.66}</em>{-1.39}$</td>
</tr>
<tr>
<td>$^{163}$Lu</td>
<td>ND2</td>
<td>$2.13^{+0.62}<em>{-0.43} - 6.72^{+0.77}</em>{-0.40}$</td>
</tr>
</tbody>
</table>
the new data.

The $Q_t$ moments of ND bands 1 and 2 are compared with my calculations in Table 3.1. We predict the value to be $Q_t = 5.8$ eb at spin $I = 30$, with an associated axial quadrupole deformation of $\epsilon_2 = 0.21$. The new data for $^{163}$Tm clearly indicate that the deformation associated with bands TSD1 and TSD2 is larger than that of the yrast structure. As can be seen from Table 3.1, the $Q_t$ moments of bands TSD1 and TSD2 ($\sim 7.5$ eb) exceed those for bands 1 and 2 by $\sim 1$ eb. The larger deformation agrees with the interpretation proposed. However, the magnitude of the increase in the transitional quadrupole moments is overestimated by the calculations. The TAC calculations indicate a $Q_t \sim 5.8$ eb for the ND band and $Q_t \sim 8.7$ eb at spin $I = 24$ increasing slightly to 9.6 eb for $34 < I < 50$. At present, this discrepancy between data and calculations is not understood. It is, however, not unique to $^{163}$Tm. Table 3.3 compares $Q_t$ moments for TSD bands in all nuclei of the region where this information is available. Just as in the $^{163}$Tm case studied in this thesis, a discrepancy between the measured and calculated $Q_t$ moments was found for the Lu isotopes: Ultimate Cranker calculations predicted values of $Q_t \sim 9.2$ eb for the configuration associated with a rotation about the short axis ($\gamma > 0$), being lower in energy [18]. These values were computed to be essentially the same for the three Lu isotopes ($A = 163, 164, 165$). As stated above, the physical origin of this discrepancy between theory and experiment is at present unclear, although it was suggest out that the exact location in energy of the $i_{13/2}$ and $h_{9/2}$ proton- and $i_{11/2}$ neutron-intruder orbitals is crucial for the deformation [18]. These orbitals are deformation driving and, hence, might have a considerable impact on the $Q_t$ moments. It is possible that the use of the standard Nilsson potential parameters, questioned above for ND
configurations, needs also to be reconsidered for the precise description of TSD bands.

Considering the fact that the errors quoted for the $Q_t$ moments in Table 3.1 are statistical only and do not include a possible additional systematic error of $\sim 15\%$ (see [38]), the agreement between experiment and theory can be considered as satisfactory. Nevertheless, the fact remains that deformations calculated with the Cranked Nilsson-Strutinsky (CNS) model [13], the Tilted-Axis Cranking (TAC) model or the Ultimate Cranker (UC) code [14], all using the same Nilsson potential, tend to be systematically somewhat smaller than the values derived from experiment, an observation that warrants further theoretical investigation. Our calculations have been extended to the case of $^{163}\text{Lu}$. The computed $Q_t$ moments for the TSD bands are larger than the measured ones, in agreement with the general findings discussed above [18]. They decrease slightly from 10.3 $eb$ at $I = 20$ to 9.7 $eb$ at $I = 40$ just as in $^{163}\text{Tm}$. The observation that the $Q_t$ value of $^{163}\text{Lu}$ are somewhat larger than the corresponding ones in $^{163}\text{Tm}$ reflects the additional drive towards larger deformation brought about by the $i_{13/2}$ proton orbital which is occupied in this case. However, it should be pointed out that within the framework of our calculations, the occupation of the $i_{13/2}$ proton orbital is not a necessary condition to achieve a TSD minimum. Rather, the strong deformation is generated mainly by the $N = 94$ neutron gap. This point is illustrated in Fig. 3.12, where the single-neutron rothians are presented. The large $N = 94$ gap associated with the TSD shapes at positive and negative $\gamma$ values is clearly visible. The corresponding single-proton rothians can be found in Fig. 3.11. The occupation of the $i_{13/2}$ proton level in the Lu isotopes adds an additional degree of shape driving towards larger deformation. However, as stated above,
TABLE 3.3: QUADRUPOLE MOMENTS OF TSD BANDS IN Tm, Lu, AND Hf NUCLEI. THE ERROR BARS ARE STATISTICAL ONLY AND DO NOT INCLUDE THE SYSTEMATIC UNCERTAINTY IN THE STOPPING POWERS. NOTE THAT FOR SOME ENTRIES IN THE TABLE, A RANGE OF VALUES IS GIVEN. THE READER IS REFERRED TO REF. [38] FOR FURTHER DETAILS.

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<th>Band</th>
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<td>TSD1</td>
<td>$7.42^{+0.34}_{-0.37}$</td>
</tr>
<tr>
<td>$^{163}$Tm</td>
<td>TSD2</td>
<td>$7.70^{+1.04}_{-0.57}$</td>
</tr>
<tr>
<td>$^{163}$Lu</td>
<td>TSD1</td>
<td>$7.4^{+0.7}<em>{-0.4}$, $7.7^{+2.3}</em>{-1.3}$, $7.63^{+1.46}<em>{-0.88}$, $9.93^{+1.14}</em>{-0.99}$</td>
</tr>
<tr>
<td>$^{163}$Lu</td>
<td>TSD2</td>
<td>$6.68^{+1.70}<em>{-1.02}$, $-8.51^{+0.95}</em>{-0.73}$</td>
</tr>
<tr>
<td>$^{164}$Lu</td>
<td>TSD1</td>
<td>$7.4^{+2.5}_{-1.3}$</td>
</tr>
<tr>
<td>$^{165}$Lu</td>
<td>TSD1</td>
<td>$6.0^{+1.2}<em>{-0.2}$, $6.4^{+1.9}</em>{-0.7}$</td>
</tr>
<tr>
<td>$^{167}$Lu</td>
<td>TSD1</td>
<td>$6.9^{+0.3}_{-0.3}$</td>
</tr>
<tr>
<td>$^{168}$Hf</td>
<td>TSD1</td>
<td>$11.4^{+1.1}_{-1.2}$</td>
</tr>
<tr>
<td>$^{174}$Hf</td>
<td>TSD1</td>
<td>$13.8^{+0.3}_{-0.4}$</td>
</tr>
<tr>
<td>$^{174}$Hf</td>
<td>TSD2</td>
<td>$13.5^{+0.2}_{-0.3}$</td>
</tr>
<tr>
<td>$^{174}$Hf</td>
<td>TSD3</td>
<td>$13.0^{+0.8}_{-0.4}$</td>
</tr>
<tr>
<td>$^{174}$Hf</td>
<td>TSD4</td>
<td>$12.6^{+0.8}_{-0.8}$</td>
</tr>
</tbody>
</table>
Figure 3.12. Single-neutron routhians as function of rotational frequency in TSD minima 1 (top) and 2 (bottom). The line convention is: (π, α) = (+,1/2) full, (+,-1/2) dot, (-,1/2) dash, (-,-1/2) dash dot. The deformation parameters used in the calculations are: $\epsilon_2 = 0.39$, $\epsilon_4 = 0.05$, $|\gamma| = 17^\circ$. 
the data indicate that its impact is rather minor. This is borne out by our calculations which give average deformations of $\epsilon_2 = 0.39$, $|\gamma| = 17^\circ$ for $^{163}$Tm and of $\epsilon_2 = 0.41$, $\gamma = +19^\circ$ for $^{163}$Lu. The nearly equal deformations find their origin the following. $^{163}$Lu does not make full use of the $N = 94$ gap because it has two fewer neutrons, but this absence is compensated by the additional drive provided by the $i_{13/2}$ proton. The large $N = 94$ gap makes it unlikely that the $^{163}$Tm TSD bands involve a three-quasiparticle structure with a proton coupled to a neutron particle-hole excitation, which would be a possible alternative interpretation.

While the data confirm that the TSD bands are associated with a larger deformation, the measured $Q_t$ moments are smaller than the calculated values. It was pointed out that this difference between theory and experiment appears to a general feature of the region, which requires further investigation.

3.6 Appearance and absence of wobbling excitations in TSD nuclei

Figure. 3.11 provides a natural explanation for the absence of collective wobbling excitations in $^{163}$Tm with $Z=69$ and their presence in the Lu isotopes with $Z=71$. The collective wobbling excitations and the particle-hole/quasiparticle excitations are two types of motion which compete with each other in the $A\sim164$ region. In the Lu isotopes, the wobbling band is the lowest excited state because it takes about 1 MeV to excite the $i_{13/2}$ proton from the $(+,1/2)$ orbit (full line) to the $(+,-1/2)$ orbit (dotted). Moreover, the gap at $N=94$ in the neutron diagrams prevents the neutron p-h excitations to compete with the wobbling mode in the Lu isotopes. The particle-hole excitation becomes the lowest band in $^{163}$Tm because the energy difference between the signature partners of the $h_{11/2}$ orbital is tiny, much smaller than the wobbling energy. In the experiment [37] only the
Figure 3.13. $B(M1)/B(E2)$ values vs. rotational frequency for the TSD bands. Squares with error bars denote the experimental data; the solid line for the TAC calculation.
first excited band receives sufficient intensity for observation.

Figure 3.12 shows that the single-proton routhians have higher density for \(Z=72\) and 73 at frequency range of 300-500 keV. While the large neutron energy gap at \(N=94\) prevents the neutron p-h excitation to compete with the wobbling mode in the Lu isotopes, around \(N=102-104\), the higher level density gives the neutron p-h excitations the possibility of being the lowest excited states. This means that low-lying states will be the p-h excitations for these nuclides, and explains why TSD bands in Hf and Ta do not show the characteristics of the wobbling motion. This means that for these nuclides many low-lying p-h excitations exist which make an identification of the wobbling mode impossible. Figure 3. 13 also gives solid evidence for existence of particle-hole excitation.

Three observations can be made from Table 3.3: (1) the \(Q_t\) values for the TSD bands in \(^{163}\text{Lu}\) and \(^{163}\text{Tm}\) are essentially the same, (2) the \(Q_t\) moments of the TSD bands decrease from \(^{163}\text{Lu}\) and \(^{163}\text{Tm}\) to \(^{165}\text{Lu}\), an observation already made for Lu isotopes, and (3) all the TSD bands in Hf nuclei are characterized by \(Q_t\) moments that are larger than those in Lu and Tm by \(\sim 4 - 6\) eb, possibly pointing to larger deformation of these bands caused by the \(N=97\) neutron gap.

In summary, the TSD bands in Tm nuclei appear to be particle-hole excitations. Our CNS and TAC calculations reproduce the experimental results quite well, and give a reasonable explanation for the absense of wobbling mode in the Tm, Hf, and Ta isotopes and its presense in Lu isotopes.
CHAPTER 4

NUCLEAR TIDAL WAVES IN A ∼ 110 REGION

4.1 Introduction

Shape and phase evolution and the mechanisms of generating angular momentum are major topics in nuclear structure research. In the collective model, vibrational and rotational modes have been manifested by the different ways of producing their angular momentum (c.f. section 1.4). In this thesis, we will present a simple method for describing the evolution from vibrational to rotational nuclear structure with nucleon number. The method is applied to the $A \sim 110$ region, in which a transition from vibrational to rotational motion is found.

The gist of the method is illustrated by the classical vibrational modes of the surface of a droplet. As discussed in section 1.4.2, the vibration modes have a certain multipolarity. We restrict our study to the most important type, the quadrupole vibrations. There are two types of vibrations, one with angular moment $J=0$ and one with $J=2$ (The third type $Y_{21}$ is irrelevant here.):

\begin{align*}
J = 0 : & \quad R(\theta, \phi, t) = R_0 [1 + a_0 \cos(\Omega t) Y_{20}(\theta, \phi = 0)] \\
J = 2 : & \quad R(\theta, \phi, t) = R_0 [1 + 2a_2 \cos(2\phi - \Omega t) Y_{22}(\theta, \phi = 0)].
\end{align*}

As shown in left panel of Fig 4.1, the surface vibration with $J=0$ looks as expected for a vibration. It is a standing wave. The other case with $J=2$ on the right panel is a running wave. It looks like the rotational motion. We call this
mode “tidal waves”, because it runs over the surface of the nucleus like the tidal waves in the oceans run over the earth’s surface. The similarity of the tidal wave and rotation suggests that the methods devised for a microscopic description of rotating nuclei will describe the tidal wave vibrations as well. Moreover one can expect that these methods also describe the yrast states of nuclei in the transition region between spherical and well deformed shapes.

The tidal wave in a frame rotating with frequency $\omega$ corresponds to a statically deformed shape, and thus, it allows us to apply the Cranking Model. For the limit of small deformation ($\epsilon \leq 0.1$), the RPA equations for the quadrupole vibrations are approximate solutions of the self-consistent equations of the cranking model [32].

The low-lying states in the nuclei with $A \approx 110, Z < 50$ are known to be good examples of quadrupole vibrational structures in the nuclear chart. For this reason we have studied the Cd($Z=48$), Pd($Z=46$), Ru($Z=44$) isotopes with $56 \leq N \leq 66$.

4.2 Tidal waves in the collective model

Before starting the calculations in the frame of the microscopic cranking model, the concept of tidal waves was checked in the framework of the collective model (see section 1.4.2).

The Bohr Hamiltonian describing the dynamics of the nuclear surface is taken from [39]

\[
H = -\frac{\hbar^2}{2B} \left[ \frac{1}{\beta^4} \frac{\partial}{\partial \beta} \beta^4 \frac{\partial}{\partial \beta} + \frac{1}{\beta^2 \sin 3\gamma} \frac{\partial}{\partial \gamma} \sin 3\gamma \frac{\partial}{\partial \gamma} - \frac{1}{4\beta^2} k \frac{J_k^2}{\sin^2(\gamma - \frac{2}{3}\pi k)} \right] + V(\beta, \gamma).
\]

In the case $V(\beta, \gamma)$ does not depend on $\gamma$, the eigenfunctions are of the form
Figure 4.1. Time Frames of two type of vibrational motions, J=0, J\neq 0.
The solutions $\phi(r, \theta_i)$ for “angular part” are common to all $\gamma$-soft problems, while the dependence upon the potential $V(\beta)$ is described by the “radial” $f(\beta)$ wave function. The equation for $f(\beta)$ is

\[
\left[\left(-\frac{1}{\beta^4} \frac{\partial}{\partial \beta} \beta^4 \frac{\partial}{\partial \beta} + \frac{\tau(\tau + 3)}{\beta^2}\right) + \nu(\beta)\right] f(\beta) = \epsilon f(\beta), \tag{4.2}
\]

where we introduced the scaled eigenvalue $\epsilon \equiv (\frac{2B}{\hbar^2})E$ and the scaled potential $\nu(\beta) \equiv (\frac{2B}{\hbar^2})V(\beta)$. In ref.[22], eq. (4.2) is called the E(5) Hamiltonian. In this eigenvalue problem, the separation constant $\tau$ assumes the value $\tau = 0, 1, 2, \ldots$ We are interested in the yrast states, which correspond to $I = 2\tau$. Replacing the radial function $f(\beta)$ in eq.(2) by $\varphi(\beta)/\beta^2$, we can simplify eq.(4.2) to

\[
-\frac{1}{2} \varphi'' + \frac{1}{2} \left(\frac{\tau(\tau + 3) + 2}{\beta^2}\right) + \nu(\beta))\varphi = \epsilon \varphi. \tag{4.3}
\]

First we consider the harmonic oscillation potential in the E(5) Hamiltonian.

\[
\nu(\beta) = \frac{c}{2} \beta^2, \ c > 0. \tag{4.4}
\]

The radial wave function is determined by the effective potential

\[
V_{eff} = \frac{\tau(\tau + 3) + 2}{\beta^2} + \frac{c}{2} \beta^2. \tag{4.5}
\]

The first derivative of the effective potential is

\[
V_{eff}' = -2 \frac{\tau(\tau + 3) + 2}{\beta^3} + c\beta. \tag{4.6}
\]
The second derivative of the effective potential is positive,

\[ V_{\text{eff}}'' = 6\frac{\tau(\tau + 3) + 2}{\beta^4} + c. \]  

(4.7)

Therefore the potential has minimum at

\[ V_{\text{eff}}'(\beta) = 0, \]  

(4.8)

which is located at

\[
\begin{cases} 
\beta_{\text{min}}^4 = \frac{2\tau(\tau+3) + 2}{c}, \\
V_{\text{eff}}''(\beta_{\text{min}}) = 4c, \\
V_{\text{eff}}(\beta_{\text{min}}) = \sqrt{2c(\tau(\tau + 3) + 2)}. 
\end{cases}
\]  

(4.9)

The tidal wave corresponds to a static deformation, i.e, to the energy at the minimum.

\[ E_{\text{min}} = V_{\text{eff}}(\beta_{\text{min}}) = \sqrt{2c(\tau(\tau + 3) + 2)} \approx (\tau + \frac{3}{2}\sqrt{2c}), \]  

(4.10)

\[ E_{\text{min}} = \sqrt{\frac{\hbar^2}{2B}}e_{\text{min}} = \hbar\Omega(\tau + \frac{3}{2}\sqrt{2c}) \]  

(4.11)

\[ \Omega = \sqrt{\frac{C}{B}}. \]  

(4.12)

This means that calculating the energy by finding the static deformation minimum and approximating the square root by the expression valid for \( \tau \gg 1 \) gives the correct energy of the yrast states with \( \tau = I/2 \).
Next we took harmonic vibrations in the $\beta$ degree of freedom into account. The effective potential is expanded around $\beta_{\text{min}}$.

$$V_{\text{eff}} = V_{\text{eff}}(\beta_{\text{min}}) + 2c(\beta - \beta_{\text{min}}^2) \quad (4.13)$$

Then the equation 4.3 describes a harmonic oscillator with the frequency $\sqrt{2c}$. The eigenvalue of the energy is

$$\epsilon = (n + \frac{1}{2})2\sqrt{2c} + \sqrt{2c}(\tau + \frac{3}{2}) \quad (4.14)$$

The energy is then given by $E = \sqrt{\frac{\hbar}{2B}}\epsilon = \hbar\Omega(2n + \tau + \frac{5}{2})$, which is the exact energy of harmonic quadrupole vibrations. We conclude that the energy sequence of the yrast states can be calculated by finding the static deformation for given $\tau$.

We put an anharmonic potential into the E(5) Hamiltonian,

$$\nu(\beta) = \frac{c}{2}\beta^2 + D\beta^4C < 0, D > 0. \quad (4.15)$$

The effective potential is

$$V_{\text{eff}} = \frac{\tau(\tau + 3)}{\beta^2} + \frac{c}{2}\beta^2 + D\beta^4. \quad (4.16)$$

The first derivative of the effective potential is

$$V_{\text{eff}}' = -2\frac{\tau(\tau + 3)}{\beta^3} + c\beta + 4D\beta^3. \quad (4.17)$$
Second derivative of the effective potential is

\[ V_{\text{eff}}'' = 6 \frac{\tau(\tau + 3) + 2}{\beta^4} + c + 12D\beta^2. \] (4.18)

The minimum lies at

\[ V_{\text{eff}}' = 0, \] (4.19)

which gives

\[ \beta_{\text{min}}^6 + \frac{c}{4D}\beta_{\text{min}}^4 - \frac{\tau(\tau + 3) + 2}{2D} = 0. \] (4.20)

Solving the eq. (4.20), we get the analytic solution

\[ \beta_{\text{min}}^2 = \sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3 + \sqrt[3]{\left(\frac{q}{2}\right)^2 - \left(\frac{p}{3}\right)^3 + \frac{a}{3}}}}}, \] (4.21)

where \( a = \frac{c}{4D}, p = -\frac{a^2}{3}, q = \frac{2a^3}{27} - \frac{\tau(\tau + 3) + 2}{2D} \). The energy is given by

\[ V_{\text{eff}}(\beta_{\text{min}}) = \frac{\tau(\tau + 3) + 2}{\beta_{\text{min}}^2} + \frac{c}{2}\beta_{\text{min}}^2 + D\beta_{\text{min}}^4. \] (4.22)

In this case, the ground state energy will be calculated by the numerical computation.

Figures 4.2-4.5 demonstrate that calculating the energy of the tidal-wave mode by finding the static equilibrium deformation in the rotating frame (at fixed \( \tau = I/2 \)) qualitatively reproduces the experimental relation \( \omega(I) = \frac{1}{2}(E(I) - E(I - 2)) \) as seen in Figs.4.6, 4.7. This encouraged us to apply the cranking model, which calculates the static deformation in the rotating frame in a microscopic way.
Figure 4.2. The harmonic radial potential, with $c=1$, $d=0$.

Figure 4.3. The fourth order anharmonic radial potential with $c=1$, $d=1$.
Figure 4.4. The yrast energy of the harmonic potential, with $c=1$, $d=0$, calculated by means of eq. (4.12).

Figure 4.5. The yrast energy of the anharmonic radial potential, with $c=1$, $d=1$, calculated by eq. (4.22).
Figure 4.6. The energy difference $2\hbar \omega(I) = E(I) - E(I - 2)$ for the harmonic radial potential, with $c=1$, $d=0$, calculated from the energies in Fig. 4.4.

Figure 4.7. The energy difference $2\hbar \omega(I) = E(I) - E(I - 2)$ for the anharmonic radial potential, with $c=1$, $d=1$, calculated from the energies in Fig. 4.5.
4.3 Experimental situation in the A=110 region

Figs 4.10 and 4.11 now show the experimental relations between the angular momentum \( J = I \) and the angular frequency \( \hbar \omega(I) = \frac{1}{2}(E(I) - E(I - 2)) \). The low spin part, \( I \leq 8 \), is characterized by a gradual transition from spherical shape to static deformation. \( Z=48, N=56 \) has a nearly constant frequency of \( \omega \sim 0.4 \text{MeV} \).

A harmonic oscillator with frequency \( \Omega \) has a constant rotational frequency of \( \omega = \Omega/2 \). (see Fig.4.8). With increasing \( N \) and decreasing \( Z \), the function \( J(\omega) \) gradually changes to the form of a rigid rotor, \( J = 3\omega \) (see section 1.4.1), which is approached for \( Z=44, N=66 \). This is expected for increasing number of neutrons above the closed shell \( N=50 \) and increasing numbers of the proton holes in the closed shell \( Z=50 \). Figs. 4.12, 4.13 show that the \( B(E2,2^{-}\rightarrow0) \) values grow from 0.1 \((e\text{b})^2\) in \( Z=48, N=56 \) to 0.25 \((e\text{b})^2\) in \( Z=44, N=66 \). The low-lying states in nuclei with \( A \sim 110 \) and \( Z < 50 \) have been considered as the best examples of quadrupole vibrational structures in the nuclear chart. One can see that for all nuclei the function \( J(\omega) \) deviates substantially from the limit of a vertical line, which corresponds to a harmonic vibrator. They have the form shown in Fig. 4.7, which corresponds to an anharmonic potential. This is in accordance with the observation that for most of the so called “vibrational” nuclei. The typical ratio for excitation energies of the yrast \( 4^+ \) and \( 2^+ \) states is close to 2.2 rather than the harmonic limit of 2.0. These nuclei have a rather small quadrupole deformation \((\beta_2 \neq 0.15)\).

Most of the nuclides develop a pronounced backbending of the function \( J(\omega) \) around \( I=10 \). This phenomenon is caused by the sudden alignment of a pair of \( h_{11/2} \) quasineutrons with rotational axis. After the backbend, the function \( J(\omega) \) is shifted up by almost 8 units, which represent the angular momentum of the
aligned pair of quasineutrons. In Ref. [34], the backbend has been ascribed to a transition from vibrational-like to rotational-like behaviour, which was based on the observation that the moment of kinematic inertia \( \Im^{(1)} = J/\omega \) rapidly increases below the backbend, as for a vibrator, and become much more constant above, as for a rotor. Ref. [34] discusses the inverse ratio \( R = 1/\Im^{(1)} \) in so called “E-GOS” plots, which changes stronger below and less above the backbend, due to the \( 1/x \) scaling. We will discuss the transition on the basis of our microscopic calculations, which will lead to a more detailed picture.

4.4 Cranking calculations

In the calculation, we considered only the quadrupole deformations which are found by the minimizing the energy \( E(J, \epsilon_2, \gamma) \) for fixed angular momentum \( J \). Minimizing the energy in the rotating frame at fixed frequency \( \omega \) becomes too inaccurate in practice, because the energy surface in nearly flat around the appropriate frequency \( \omega \sim \Omega_{\text{vib}}/2 \). The TAC code [28] provides the energy in the rotating frame \( E'(\epsilon_2, \gamma, \omega) \) for given angular frequency \( \omega \). The following method was used to calculate a \( \epsilon_2 - \gamma \) grid of \( E(\epsilon_2, \gamma, J) \).

In our calculations, we assume no hexadecapole deformation, i.e. \( \epsilon_4 = 0 \). The pairing gaps are set constant \( \Delta_n = \Delta_p = 1.1 \text{ MeV} \) for all isotopes. The chemical potentials, \( \lambda_n \) and \( \lambda_p \), are adjusted to have the correct expectation values of \( Z \) and \( N \) at zero rotational frequency. The \( \lambda \) values at non-zero frequency will be taken from the values at zero frequency.

The inner loop variable of code will be taken as \( \epsilon_2, \gamma \), the outside loop variable will be \( \omega \). The standard step size was \( \delta \epsilon_2 = 0.005, \delta \gamma = 0.5^\circ \), and \( \delta \omega = 0.005 \text{ MeV} \). The tilt angles are set \( \theta = 90^\circ, \phi = 0^\circ \), i.e. \( x \) is the rotational
axis. Where \( J_x(\omega) \) changes rapidly, a smaller step \( \delta \omega \) was taken. The energy is calculated from \( E(\omega, \epsilon_2, \gamma) \) and \( J_x(\omega, \epsilon_2, \gamma) \) provided by the TAC code by

\[
E(\omega, \epsilon_2, \gamma) = E(\omega, \epsilon_2, \gamma) + J_x(\omega, \epsilon_2, \gamma) \omega.
\]

To find the energy at fixed spin value, \( J_x = I \) \((I=0,2,4,6,8,....)\), we first determine a \( \omega_o(\epsilon_2, \gamma) \) giving \( J_x(\epsilon_2, \gamma) \) closest to \( I \) for each grid point. Then we interpolate the angular momentum as

\[
J_x = \xi_1 \omega + \xi_2 \omega^3.
\]  

(4.23)

To find \( \xi_1, \xi_2 \), a each grid point, we calculate the kinematic moment of inertia,

\[
J_x/\omega = \xi_1 + \xi_2 \omega^2.
\]  

(4.24)

With \( t = \omega^2 \),

\[
J_x/\omega = \xi_1 + \xi_2 t
\]  

(4.25)

Then for each \( \epsilon_2 - \gamma \) grid point, we have a sequence of points \((t, J_x)\). We fit a straight line by using polyfit\((t, J_x, 1)\) in the “MATLAB” library, which gives the coefficients \( \xi_1, \xi_2 \). The interpolated energies obey the relations

\[
dE/dJ_x = \omega
\]

, \( dE^\omega/d\omega = -J_x(\omega) \), (4.26)which imply

\[
E^\omega = - \int_{\omega_0}^\omega J_x(\omega)d(\omega),
\]  

(4.27)

\[
E = E^\omega(\omega) + \omega J_x(\omega),
\]  

(4.28)
Figure 4.8. The deformation energy at given spin $I$ for $Cd^{110}$. The triaxiality parameter $\gamma = 10^\circ$.

resulting in

$$E^\omega = -(1/2)\xi_1(\omega^2 - \omega_0^2) - (1/4)\xi_2(\omega^4 - \omega_0^4) + E_0^\omega(\omega_0,\epsilon_2,\gamma),$$

\[ (4.29) \]

$$E(I) = (1/2)\xi_1(\omega^2 - \omega_0^2) + (3/4)\xi_2(\omega^4 - \omega_0^4) + E_0^\omega(\omega_0,\epsilon_2,\gamma).$$

\[ (4.30) \]

Knowing the $\xi_{1,2}$ for each grid point $(\epsilon_2,\gamma)$, we determine $\omega(I)$ by solving $\xi_1\omega + \xi_2\omega^3 = I$ and find $E(I) = E(\omega(I))$ be means of eq. (4.30). We run the
Figure 4.9. Quasineutron routhians of $^{110}$Cd at a typical deformation ($\epsilon_2 = 0.15$, $\gamma = 10^\circ$, $\epsilon_4 = 0$). Solid lines for positive parity, dash lines for negative parity.
Figure 4.10. The Experimental results (solid dots) and calculations (open symbols) for angular momentum as function of the angular frequency for yrast levels.
Figure 4.11. The Experimental results (solid dots) and calculations (open symbols) for angular momentum as function of the angular frequency for yrast levels.
Figure 4.12. Calculations (open symbols) and experimental results (solid squares) for $B(E2)$ as function of the spin for yrast levels. The diamonds show a modification of the calculations for well deformed nuclei, which is discussed in the text.
Figure 4.13. Calculations (open symbols) and experimental results (solid squares) for $B(E2)$ as function of the spin for yrast levels. The diamonds show a modification of the calculations for well deformed nuclei, which is discussed in the text.
Figure 4.14. The ground state energy of Cd isotopes calculated by means of SCTAC.

### TABLE 4.1

The calculated \((\epsilon_2, \gamma)\) values in the yrast states of Ru isotopes.

<table>
<thead>
<tr>
<th>Spin (^{100}<em>{44} \text{Ru}</em>{56})</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{102}<em>{44} \text{Ru}</em>{58})</td>
<td>0.15</td>
<td>0.1515</td>
<td>0.1615</td>
<td>0.1665</td>
<td>0.1725</td>
<td>0.185</td>
<td>0.165</td>
<td>0.1745</td>
</tr>
<tr>
<td>(^{104}<em>{44} \text{Ru}</em>{60})</td>
<td>0.175</td>
<td>0.1765</td>
<td>0.1825</td>
<td>0.1885</td>
<td>0.1925</td>
<td>0.2055</td>
<td>0.185</td>
<td>0.205</td>
</tr>
<tr>
<td>(^{106}<em>{44} \text{Ru}</em>{62})</td>
<td>0.20.25</td>
<td>0.215</td>
<td>0.225</td>
<td>0.235</td>
<td>0.2355</td>
<td>0.205</td>
<td>0.205</td>
<td>0.215</td>
</tr>
<tr>
<td>(^{108}<em>{44} \text{Ru}</em>{64})</td>
<td>0.2</td>
<td>0.215</td>
<td>0.2195</td>
<td>0.2255</td>
<td>0.2355</td>
<td>0.2475</td>
<td>0.25</td>
<td>0.2185</td>
</tr>
<tr>
<td>(^{110}<em>{44} \text{Ru}</em>{66})</td>
<td>0.21</td>
<td>0.2115</td>
<td>0.225</td>
<td>0.235</td>
<td>0.245</td>
<td>0.255</td>
<td>0.215</td>
<td>0.225</td>
</tr>
</tbody>
</table>
Figure 4.15. The ground state energy of Cd isotopes calculated by PQTAC.

Figure 4.16. The ground state energy of Pd isotopes calculated by SCTAC.
Figure 4.17. The ground state energy of Pd isotopes calculated by PQTAC.

Figure 4.18. The ground state energy of Ru isotopes calculated by SCTAC.
Figure 4.19. The ground state energy of Ru isotopes calculated by PQTAC.

TABLE 4.2
The calculated for \((\epsilon_2, \gamma)\) values in the yrast states of Pd isotopes.

<table>
<thead>
<tr>
<th>Spin</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{102}<em>{46}Pd</em>{56})</td>
<td>0.1</td>
<td>0.125</td>
<td>0.145</td>
<td>0.1445</td>
<td>0.1505</td>
<td>0.165</td>
<td>0.185</td>
<td>0.205</td>
</tr>
<tr>
<td>(^{104}<em>{46}Pd</em>{58})</td>
<td>0.125</td>
<td>0.1355</td>
<td>0.1445</td>
<td>0.165</td>
<td>0.1655</td>
<td>0.1525</td>
<td>0.1555</td>
<td>0.1625</td>
</tr>
<tr>
<td>(^{106}<em>{46}Pd</em>{60})</td>
<td>0.126</td>
<td>0.155</td>
<td>0.165</td>
<td>0.175</td>
<td>0.1745</td>
<td>0.165</td>
<td>0.1645</td>
<td>0.175</td>
</tr>
<tr>
<td>(^{108}<em>{46}Pd</em>{62})</td>
<td>0.15</td>
<td>0.1515</td>
<td>0.165</td>
<td>0.175</td>
<td>0.185</td>
<td>0.1625</td>
<td>0.1645</td>
<td>0.1665</td>
</tr>
<tr>
<td>(^{110}<em>{46}Pd</em>{64})</td>
<td>0.152</td>
<td>0.1545</td>
<td>0.165</td>
<td>0.1725</td>
<td>0.215</td>
<td>0.225</td>
<td>0.165</td>
<td>0.195</td>
</tr>
<tr>
<td>(^{112}<em>{46}Pd</em>{66})</td>
<td>0.154</td>
<td>0.1565</td>
<td>0.1625</td>
<td>0.1745</td>
<td>0.215</td>
<td>0.235</td>
<td>0.175</td>
<td>0.25</td>
</tr>
</tbody>
</table>
TABLE 4.3

The calculated for \((\epsilon_2, \gamma)\) values in the yrast states of Cd isotopes .

<table>
<thead>
<tr>
<th>Spin</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{104}\text{Cd}_{56})</td>
<td>0.0</td>
<td>0.09</td>
<td>0.113</td>
<td>0.125</td>
<td>0.140</td>
<td>0.135</td>
<td>0.145</td>
<td>0.1445</td>
</tr>
<tr>
<td>(^{106}\text{Cd}_{58})</td>
<td>0.05</td>
<td>0.105</td>
<td>0.125</td>
<td>0.135</td>
<td>0.155</td>
<td>0.145</td>
<td>0.155</td>
<td>0.155</td>
</tr>
<tr>
<td>(^{108}\text{Cd}_{60})</td>
<td>0.075</td>
<td>0.1145</td>
<td>0.1365</td>
<td>0.1455</td>
<td>0.1525</td>
<td>0.135</td>
<td>0.1345</td>
<td>0.145</td>
</tr>
<tr>
<td>(^{110}\text{Cd}_{62})</td>
<td>0.05</td>
<td>0.115</td>
<td>0.135</td>
<td>0.14915</td>
<td>0.15515</td>
<td>0.145</td>
<td>0.1425</td>
<td>0.1445</td>
</tr>
<tr>
<td>(^{112}\text{Cd}_{64})</td>
<td>0.0</td>
<td>0.125</td>
<td>0.1465</td>
<td>0.17410</td>
<td>0.1415</td>
<td>0.1525</td>
<td>0.165</td>
<td>0.1725</td>
</tr>
<tr>
<td>(^{114}\text{Cd}_{66})</td>
<td>0.05</td>
<td>0.1175</td>
<td>0.1465</td>
<td>0.1855</td>
<td>0.2085</td>
<td>0.175</td>
<td>0.1715</td>
<td>0.1745</td>
</tr>
</tbody>
</table>

MATLAB program to find where the minimum is in the potential energy surface \(E(I, \epsilon_2, \gamma)\) , generated from the grid points. The location of \(E_{\text{min}}(I, \epsilon_2, \gamma)\) will give the deformation of the nuclei at specific spin. Fig. 4.8 shows as an example \(E(I, \epsilon_2, \gamma = 10^9)\).

In order to compare with experiment we show in Figs. 4.10 and 4.11, plots of the function \(J_x = I\) vs. \(\hbar \omega(I) = \frac{1}{2} (E(I) - E(I - 2))\), which is calculated from the experimental energies \(E(I)\) and from the calculated energies \(E_{\text{min}}(I)\). Such plots show better the details of the spin dependence than a plot of \(E(I)\).

The TAC code calculates the \(B(E2)\) values for each grid points. As these values do not change strongly with the frequency \(\omega\), we take the value for \(\omega_o(I, \epsilon_2, \gamma)\). For well deformed axial nuclei, like \(^{110,108,106}\text{Ru}\), we have to introduce one more term to correct the current model. We multiply the \(B(E2)\) value of TAC result with \(I(I - 1)/(I^2 - 1/4)\) (see discussion in section 4.5). After the correction, the theoretical calculations coincide with the experimental results (Corrected results marked by diamond symbol in Fig. 4.18 ).
As discussed in section 2.3, there are two versions of the TAC model. PQTAC is based on the QQ interaction and SCTAC on the shell-correction method. We started the calculations with PQTAC, which allows one to determine the deformation from the self-consistency eqs. (2.37) in sect. 2.3. This is technically simpler than looking for the energy minimum. Then we used the SCTAC, which turned out to give a better descriptions of the experimental results. The reason for the difference can be seen by comparing Figs. 4.14-4.19. PQTAC and SCTAC give very similiar ground-state deformation energies for $\epsilon_2 \leq 0.15$, where as for larger deformation the SCTAC energy increases more rapidly. It is gratifying that SCTAC gives better agreement with experiment because this approach is considered to be more reliable for large deformation than PQTAC. Therefore we accepted the SCTAC results, which will be discussed in the next section.

4.5 Discussion of the results

Fig 4.11 and 4.12 demonstrate that the energies of the yrast states are remarkably well reproduced. The SCTAC calculations reproduce in detail the variations with $N$ and $Z$. The $B(E2)$ values are experimentally much less well known, and a detailed comparison with the calculations is not possible. The general trend of increasing $B(E2)$ values when moving into the open shells is well accounted for. Only for $Z=44$ and $N=62, 64, 66$, the SCTAC calculations give too large values of $B(E2)$. As seen in Table 4.1, these are nuclei already well deformed in the ground state, and behave like rotors. For good rotor and low spin, the semiclassical expression [17] used in TAC for $B(E2, I \rightarrow I - 2)$ become inaccurate. In the case of axial shape, it must be corrected by the factor $I(I - 1)/(I^2 - 1/4)$, which takes into account zero-point fluctuations of the orientation. If corrected
in this way (diamond symbol in Fig. 4.12), the calculations agree well with the data. This indicates that theory needs some improvement in case of the low-spin states in well deformed nuclei. On the other hand, the theory is correct for the proper tidal-wave excitations. Hence, in order to describe the transition from the tidal-wave region to good rotation, one has to take into account the zero-point vibration, which is rather complicated and out of the scope of the thesis.

Most of the functions $J(\omega)$ show a backbend around $J\sim 10$, which is caused by the alignment of two $h_{11/2}$ quasineutrons. Above the backbend the configuration in the SCTAC calculation has two $h_{11/2}$ quasineutrons occupied (s-band), below it corresponds to the zero quasiparticle configuration (g-band). The g-configurations corresponds in Fig.4.9 to all negative energy level occupied at low $\omega$, the s-configuration corresponds at low $\omega$ to the lowest two negative parity levels on the positive side occupied and the highest two levels on the negative side empty. These occupations are kept through the crossing at $\hbar \omega \sim 0.28$ MeV, which marks the frequency of the backbending. For $Z=44$, $N=62,64$ and $Z=46$, $N=56$ there is a gradual change from the g- to the s-configuration, which reflects a strong interaction between the configurations. It is well known (c.f. [11]) that the g-s-interaction changes rapidly with $N$. The calculations reproduce the change very well.

The tidal-wave region lies below the backbend. As expected from the ground-state deformation energies $E_0 = E(I = 0, \epsilon_2)$ in Figs. 4.14-19, the functions become more rotation-like ($J \propto \omega$) when going into the open shell (compare $Z=48$ $N=56$ with $Z=44$, $N=66$). However, all functions $J(\omega)$ differ substantially from the limit of the harmonic vibrator ($\omega$=constant), i.e. they show pronounced anharmonicities. One obvious source for the anharmonicities is the deviation of $E_0$ from
the quadratic form (see section 4.2). A second source comes from quasicrossing of
the $h_{11/2}$ orbitals at $\hbar \omega = 0.42$ MeV in Fig. 4.9. There, the g-configuration changes
by aligning the angular momentum of a (less favored than in the s-configuration)
pair of $h_{11/2}$ quasineutrons. This alignment causes the steep increase of $J(\omega)$ be-
tween $\hbar \omega = 0.4$ MeV and 0.5 MeV, seen in a number of nuclides. This mechanism
is not taken into account in the purely collective approaches to the description of
transitional nuclei, as the one discussed in section 4.2. As expected, the calcu-
lated $B(E2)$ values increase nearly linear in $I$ for the more vibrational-like nuclei
and become much less $I$ dependent for the nuclides in the open shell. It is noted
that the linear dependence of $B(E2)$ on $I$, characteristic for a harmonic vibrator,
appear in nuclei, the energies of which deviate substantially from the linear $I de-
pendence of the harmonic limit. The microscopic SCTAC calculations reproduce
this surprising behavior.

Above the backbend, the deformation $\epsilon_2$ drops and becomes more stable (see
Table 4.1), which is reflected by the $B(E2)$ values in Figs 4.12 and 4.13. This
means that the alignment of the two $h_{11/2}$ quasineutrons stabilizes the nuclear
shape at a smaller deformation. The yrast states change their character into a
mode called “Antimagnetic Rotations”, which was discussed for the vibration-like
nuclei in Ref. [11]. For the rotation-like nuclei, the backbend represents the well
known rotational alignment of high-$j$ quasiparticles. Of course, there is a gradual
transition between these two scenarios.
CHAPTER 5

SUMMARY AND CONCLUSIONS

In this thesis two phenomena were studied: The rotation of nuclei with a strong deformed triaxial (TSD) shape and the tidal-wave mode of nuclei in the transition region between spherical and well deformed shape.

We demonstrated that a microscopic description of the rotation of TSD nuclei based on the Tilted Axis Cranking model well reproduces the experimental results and leads to a qualitative understanding why wobbling motion has only been observed in Lu isotopes so far. The new experiments on TSD bands in $^{163}\text{Tm}$ were analysed in detail. Both energies and electromagnetic transition probabilities are well reproduced by the calculations. The calculations confirm that the new bands have indeed a TSD shape, which is caused by a large gap in the neutron single particle spectrum at $N=94$. The crucial role of this “deformed shell” in generating the TSD shape was demonstrated for the first time. This revises the so far accepted scenario, according to which the odd $i_{13/2}$ proton-orbital drives the nucleus to the TSD shape. In rotating TSD nuclei there are particle-hole excitations and collective wobbling excitations, which compete energy wise. The details of the level distribution depend sensitively on the proton and neutron number.

In the Lu isotopes the particle-hole excitations happen to have a large energy, such that the wobbling excitations are the lowest and can be well studied in experiments. In most other nuclei of this region, including $^{163}\text{Tm}$, the reversed
situation is encountered. There are many low lying particle-hole excitations among which the wobbling modes are dispersed and possibly also fragmented. That is the reason why wobbling bands could not be found in most of the studied nuclei of the $A \sim 164$ region. Nevertheless, the calculations indicate that bands built on TSD shapes exist for $Z=69-72$ and $N=92-104$. The calculations in this thesis as well as calculations in the literature seem to overestimate the nuclear deformation. The calculated transition quadrupole moments are about 1 eb larger than the experimental ones. We could not find an explanation for this discrepancy.

The tidal-wave approach for describing the yrast states of transitional nuclei was applied to even-even nuclides with $Z=44, 46, 48$ and $N=56, 58, ..., 66$. For the first time, the low-lying states with spin $I=0, 2, 4, 6, 8$, of vibration-like nuclei were described in a microscopic way by means of the self-consistent cranking model. The energies as well as the E2-transition probabilities are very well reproduced. The tidal wave corresponds to $\epsilon$ deformation of nuclear surface which rotates with constant angular velocity. In this respect, it is like a rotational motion, allowing the application of the cranking model. However there is also a difference. In case of nuclear rotation, the angular momentum increases due to an increase of the angular velocity whereas the deformation stays approximately constant. In case of a vibration-like tidal wave, the angular momentum increases due to increase of the deformation whereas the angular velocity stays approximately constant. There is a gradual change from vibration-like tidal waves near closed shell, to rotational motion of the well deformed nuclei in the open shell. The development with protons and neutrons is well accounted for by our calculations. In accordance with experiment it is found that yrast energies of the nuclei considered as “the best vibrators” differ substantially from the harmonic limit of the constant level
spacing. However the data on transition probabilities seem to suggest that the B(E2) values increase roughly linearly with spin, as expected for a vibration-like nucleus. The microscopic calculations reproduce this surprisingly different $I$ dependence of energies and transition rates. Whereas the structure of the $2^+$ and $4^+$ states can be understood in terms of the collective deformations degree of freedom, only the rotational alignment of the high-$j$ neutrons must be taken into account in the case of the $6^+$ and $8^+$ states.

The possibilities to carry out microscopic calculation for the tidal-wave mode allowed us to describe the backbending irregularity observed in the yrast sequence of all studied nuclei, which is caused by the sudden alignment of two $h_{11/2}$ quasineutrons. The calculations reproduce well the change between the backbend and smooth upbend when $Z$ and $N$ change. Above the backbend the two aligned quasineutrons stabilize the shape and the motion takes the character of Antimagnetic Rotation. In this mode the nuclear deformation is substantially smaller than in the tidal-wave mode below the backbend and does not change strongly with spin.

The successful application of the cranking model to the transitional nuclei in the $A=110$ region suggest that the tidal-wave concept is a promising approach to other regions of the nuclear chart between closed shells and well deformed nuclei.
APPENDIX A

BIBLIOGRAPHY