NUMERICAL SIMULATION OF TURBULENT FLOW OVER
HEMISPHERICAL TURRETS WITH APPLICATION TO AERO-OPTICS

A Thesis

Submitted to the Graduate School
of the University of Notre Dame
in Partial Fulfillment of the Requirements
for the Degree of

Master of Science
in
Aerospace Engineering

by

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July 2012
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Abstract

by

Bin Zhang

Hemispherical turrets are frequently employed on airborne aircraft for laser communication and targeting. However, the unsteady compressible turbulent flow around such a turret causes aberrations in the optical wavefront which degrades the performance of the system. To overcome this limitation it is necessary to have a deep understanding of the flow and optical environments around the turret, which are very challenging to study both experimentally and numerically. This thesis describes numerical study of this problem using advanced numerical techniques and high performance computers of the present days.

Two hemisphere-on-cylinder turrets, with base-height/diameter ratios $H/D = 0.375$ and $0.3125$, respectively, in a Mach 0.4 incoming flow are studied. The simulated flow fields confirm the findings from previous experimental studies, such as the existence of a necklace vortex around the base of the turret and flow separation in the central plane at an angle between $110^\circ$ and $115^\circ$. Furthermore, it is found that the necklace vortex is well below the optical aperture on the turret and thus has no direct impact on the aero-optical distortions, density fluctuations, which are directly related to optical aberrations, are strongest within the separated shear layer and wake region, and the recirculation bubble behind the turret experiences very little density fluctuations.
Optical computations also confirm the experimental results measured using 2-D wavefront sensor. Besides, the effects of elevation angle and azimuthal angle on optical aberrations are studied in a systematic way. It is found that the optical aberration grows almost linearly with elevation angle when the elevation angle is above $110^\circ$, but it has a more complicated relationship with the azimuthal angle. At small elevation angles it decreases with increasing azimuthal angle and the change is nonlinear; however, when the elevation angle is large, the aberration does not decrease much, and even increases with azimuthal angle at small azimuthal angles but keeps decreasing at large azimuthal angles.
To my parents

Without your love and encouragement this work would be impossible
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SYMBOLS

English

\( c_\infty \) Free-stream sound speed

\( C_p \) Pressure coefficient

\( D \) Turret diameter

\( H \) Turret height

\( K_{GD} \) Gladstone-Dale constant

\( K_{GD}^* \) Nondimensional Gladstone-Dale constant

\( Ma \) Mach number

\( Ma_r \) Reference Mach number

\( n \) Index-of-refraction

\( OPL \) Optical path length

\( OPD \) Optical path difference

\( p \) Pressure

\( P_{atm} \) Atmospheric pressure

\( Pr \) Prandtl number

\( Pr_t \) Turbulent Prandtl number

\( q_w \) Wall heat flux

\( rms \) Root mean square

\( Re \) Reynolds number
\( SR \) Strehl ratio
\( T \) Temperature
\( T_{atm} \) Atmospheric temperature
\( u, v, w \) Streamwise, wall-normal, and spanwise velocity components
\( U_\infty \) Free stream velocity
\( W \) Wavefront
\( x, y, z \) Coordinates for flow simulation
\( x', y', z' \) Coordinates for optical calculation
\( x^+, y^+, z^+ \) Distance in wall units

Greek
\( \alpha \) Elevation angle
\( \beta \) Azimuthal angle
\( \gamma \) Specific heat ratio
\( \mu \) Nondimensional molecular viscosity
\( \rho_{atm} \) Atmospheric density
\( \rho^* \) Nondimensional density
\( \rho_{ref} \) Reference density
\( \lambda \) Optical wavelength
\( \Omega \) Control volume
\( \partial \Omega \) Boundary of control volume
\( \tau_w \) Wall shear stress
ACKNOWLEDGMENTS

I would like to thank my advisor, Professor Meng Wang, for his consistent advice, guidance and support. His knowledge and experience benefited me immensely. Special thanks go to Dr. Kan Wang, with whom discussions were always stimulating and pleasant. I would also like to thank Professor Eric Jumper and my thesis committee members, Professor Scott Morris and Professor Stanislav Gordeyev for their advice.

This work was supported by the High Energy Laser Joint Technology Office (HEL-JTO) through AFOSR Grant FA 9550-07-1-0504.
CHAPTER 1

INTRODUCTION

1.1 Aero-optics Basics

1.1.1 Overview

As a planar optical wavefront propagates through a fluid medium with variable index-of-refraction, such as turbulent shear layers and atmospheric boundary layers, the wavefront becomes distorted (some parts have advanced farther than others). For airborne laser systems, the aberrating flow field is conventionally divided into two regimes [13, 17, 39].

The first regime is in the near field of the aperture where the optical aberration arises from unsteady or turbulent flow structures induced by flow interaction with the solid surface. The optical distortion in this regime is termed the “Aero-Optics Problem”. Examples include laser communication devices mounted on high speed airborne aircraft. Such a device introduces turbulent flows like separated shear layer around the aperture, which degrades the beam’s ability to focus in the far field. The second regime is the Earth’s atmosphere, which affects the long-range propagation of a laser beam. Optical distortions caused by atmospheric turbulence are referred to as “Atmospheric Propagation Problem”. They usually involve very large flow structures at very low frequencies, and thus can be corrected by static optical components or adaptive optical (AO) systems [13].
Aberrations induced by turbulence flow structures near the aperture are of high frequency and therefore difficult to correct by conventional AO systems [13, 17]. In the early days of laser technology, the aero-optical aberrations were of small magnitude (\(\sim 1\mu m\)) compared to the long optical wavelength (\(\sim 10\mu m\)) and therefore thought to be unimportant. However, since the 80’s, breakthroughs have been made in laser technology and lasers with much shorter wavelengths (\(\sim 1\mu m\)) have become popular. At the same time, the increased relative optical aberrations made aero-optics a serious problem. As a result, much research has been carried out in this area, as discussed in the paper by Sutton [40] and the review paper by Jumper and Fitzgerald [17]. The literature related to the present investigation is reviewed briefly in Section 1.2.1 and 1.2.2.

1.1.2 Aero-optical Aberrations

Optical aberrations originate from variations of the refractive index which is directly related to the air density through the Gladstone-Dale relation [8],

\[
n - 1 = K_{GD}\rho, \tag{1.1}
\]

where \(n\) is the refractive index, and \(K_{GD}\) is the Gladstone-Dale constant. For light with wavelengths between 1\(\mu m\) and 10\(\mu m\) traveling in air at standard condition (\(P_{atm} = 1.01325 \times 10^5\)Pa, \(T_{atm} = 288.15\)K, \(\rho_{atm} = 1.225\)kg/m\(^3\)), \(K_{GD}\) is approximately \(2.27 \times 10^{-4}m^3/kg\) [37]. In nondimensional form the above relation can be written as,

\[
n - 1 = K_{GD}^*\rho^*, \tag{1.2}
\]
where $K_{GD}^* = K_{GD} \cdot \rho_{ref}$ is the nondimensional Gladstone-Dale constant and
$
\rho^* = \rho / \rho_{ref}
$
is the nondimensional density, with $\rho_{ref}$ being the reference density. If the standard atmospheric density $\rho_{atm}$ is taken as the reference value, the dimensionless Gladstone-Dale constant is $K_{GD}^* = 2.8 \times 10^{-4} [24]$, and this value is used in the current study.

Wavefront is a surface with constant phase value [41] which is equivalent to that with constant optical path length (OPL) defined as below,

$$
OPL(t, x, y) = \int_{z_0}^{z_1} n(t, x, y, z) \, dz,
$$

(1.3)

where $z$ is the propagation direction of the beam, and $x$ and $y$ define the plane of the initial planar wavefront which is perpendicular to the propagation direction. Here an assumption has been made that the integration path is short such that there is no noticeable change in the direction of the beam [3], i.e., $\theta$ angle in Figure 1.1 (which is highly exaggerated) is very small. This assumption is valid in most aero-optics problems [17].

In general, the difference between a distorted wavefront and its spatial mean position is of more interest than the wavefront itself, and this difference can be expressed by the optical path difference (OPD) [41, 52],

$$
OPD(t, x, y) = OPL(t, x, y) - \langle OPL(t, x, y) \rangle,
$$

(1.4)

where the angle bracket means spatial average over the aperture. Conventionally, OPD is used interchangeably with the term wavefront since OPD is the conjugate of wavefront’s variation part [37] and has the characteristics of the wavefront. OPD is thus a crucial quantity in AO systems to correct optical aberrations.
The overall effect of aberrations over the whole aperture in the farfield can be characterized by the Strehl ratio [41]

\[ SR(t) = \frac{I(t)}{I_0}, \]  

(1.5)

where \( I(t) \) is the on-axis irradiance and \( I_0 \) is the diffraction-limited on-axis irradiance. Using the large aperture approximation (LAA) [22, 34], the Strehl ratio can be estimated as

\[ SR(t) = e^{-(\frac{2\pi OPD_{rms}(t)}{\lambda})^2}, \]  

(1.6)

where \( \lambda \) is the wavelength and \( OPD_{rms}(t) \) is the root mean square of \( OPD(t, x, y) \) over the aperture. A more widely used measure of the far-field optical quality is the time averaged Strehl ratio

\[ \overline{SR} = e^{-(\frac{2\pi OPD_{rms}}{\lambda})^2}, \]  

(1.7)
where overbar means time averaged quantities. The above relation makes $\overline{OPD}_{rms}$ one of the most important quantities in aero-optics.

As previously discussed, $OPD(t, x, y)$ is the conjugate of a wavefront, thus a wavefront can be written as $[5, 37]$

$$W(t, x, y) = -OPD(t, x, y),$$  \hspace{1cm} (1.8)

Based on a Taylor series expansion of the wavefront, one can obtain $[13, 41, 52]$

$$W(t, x, y) = A + B_1(t) \cdot x + B_2(t) \cdot y + W_{HighOrder}(t, x, y),$$  \hspace{1cm} (1.9)

where $A$ is a constant referred to as steady (or piston) component, $B_1(t)$ and $B_2(t)$ are the instantaneous tip/tilt components, and $W_{HighOrder}(t, x, y)$ is the high order term with small amplitude but high frequency. The steady and tip/tilt components can generally be corrected using AO systems, but the high order component cannot be corrected, making it the most detrimental to optical systems. It is therefore customary to focus on the wavefronts with steady and tip/tilt components removed in the study of aero-optics.

1.2 Previous Work on Hemispherical Turrets

Hemispherical turrets (see Figure 1.2) are one of the preferable platforms for airborne optical systems. They are generally made up of two parts: a cylindrical base and a hemispherical cap. There are several merits for choosing this configuration: it gives the maximum freedom of rotation, which means beam can be projected to a wide range of directions; if a conformal window aperture is used, the flow environment is independent of the projection direction. Although it has
a very large potential range of regard, the useful part of the hemispherical turret is currently limited only to the forward quadrant where the incoming flow is attached, while the aft quadrant is less useful due to severe optical aberrations caused by the very complicated flow environment [10] with separated shear layers, interaction between base necklace vortex and the wake bubble, etc. In the past few years, much effort, both experimentally and numerically, has been made to understand the flow and the aero-optics around hemispherical turrets. The following is a brief review of related work to date.

Figure 1.2. Schematic of a hemispherical turret (from [52])
1.2.1 Experimental Investigations

Gordeyev et al. [12] were among the first to measure the aero-optical effects around a hemispherical turret. Their experiment was carried out in a subsonic wind tunnel at the Air Force Academy. In the experiment, the incoming flow has a Mach number of 0.35 to 0.45, the turret has a diameter of 12" and a cylindrical base of height 4.5". Both steady and unsteady pressure were measured and flow visualization was carried out to investigate the flow topologies. They showed that flow in the central plane separates at an angle of 115°, and a necklace vortex was observed at the base of the turret which makes the flow quite unsteady (see schematic in Figure 1.3). Three types of sensors were used to measure optics: Malley probe, 2-D wavefront sensor and a $8 \times 8$ high bandwidth sensor. Qualitative agreement was seen in the $OPD_{rms}$ measured by different sensors, and it was found that $OPD_{rms}$ increases dramatically with elevation angle.

Later, Gordeyev and Jumper [9] further investigated the flow and optics around the same turret. They showed that when the Reynolds number is below 200,000 the boundary layer is laminar before separation, and when the Reynolds number is above 300,000, the boundary layer becomes turbulent before it separates at around 120°. They also found that once the Reynolds number is beyond this critical value, the optical aberrations exhibit only weak dependence on the Reynolds number. In addition, starting from potential flow theory they proposed scaling relations for both the steady lensing aberration in the attached region and unsteady aberrations in the separated region. The $OPD_{rms}$ is shown to be proportional to the free-stream density and the square of free-stream Mach number. They showed that when appropriate coefficients are chosen there is reasonable agreement between the scaling relations and their experimental results.
In 2011, a flight test was carried out by researchers at the University of Notre Dame. Large amount of data were obtained as reported by Porter et al. [33]. The optical aberrations from different viewing angles were studied in a systematic way, and the normalised $OPD_{\text{rms}}$ confirmed the scaling relation proposed by Gordeyev and Jumper [9]. A comparison between flight data and wind tunnel data was also performed which showed qualitative agreement, while the discrepancies were explained in terms of the different flow environments between the flight test and wind tunnel test. The effect of the near-field distortions on the far-field optical pattern was also investigated.

In addition to aero-optical measurements and mechanism studies, mitigation strategies using flow control have been actively pursued. For example, researchers at Georgia Tech and Notre Dame [11, 42–46] have explored the control of flow around hemispherical turrets. The turret has a diameter of 24" and a base height of 7.5". Control strategies such as splitting plates and synthetic jets were applied.
to reduce flow unsteadiness and hence optical aberrations, and significant reductions in the $\bar{OPD}_{rms}$ have been achieved. Since the current work is not about flow control, these investigations are not reviewed in detail here.

A thorough review of previous work on hemispherical turrets from the 1980’s to 2010 can be found in the review article of Gordeyev and Jumper [10].

1.2.2 Numerical Simulations

Morgan and Visbal [27] used a hybrid approach combining Reynolds-Averaged-Navier-Stokes (RANS) methods with Implicit Large-Eddy Simulation (ILES) to simulate flow over a small hemispherical turret ($D=1.5''$, $H=0.45''$) in a Mach 0.5 flow. Their simulation captured the main features of flow over turret, such as the separated shear layer and the necklace vortex. However, the mean velocity field and steady pressure coefficient showed obvious discrepancies from the experimental results, and optical computation was not performed in this work. Later on, Morgan and Visbal [28] used the same computational method to study the flow over a larger turret which has the same geometry as in the experiment of Gordeyev et al. [12]. Their computational domain was designed to match the wind tunnel configuration in the experiment. This time better agreement with the experiment in the mean flow field was obtained but no optical results were reported.

Nahrstedt et al. [29] performed a Partially Averaged Navier-Stokes (PANS) simulation of flow over the same optical turret. Although the mean flow field shows reasonable agreement with the experiment, the optical aberrations (say $\bar{OPD}_{rms}$) at almost all elevation angles are several times smaller than the experimental results. This is likely because the method they used is not able to capture a sufficiently wide range of flow structures that are important to optical distortions.
Ladd et al. [20] studied the same problem using unsteady RANS techniques and Detached Eddy Simulation (DES). Again the mean flow field is shown to be in reasonable agreement with the experimental measurements, but the optical predictions are highly dependent on the turbulence model and computational grids they used, which makes the solutions difficult to judge.

White et al. [53] used the same approach as in [27, 28] to study the optics around a turret that has the same geometry as in the experiment of Gordeyev et al. [12]. They found that the steady lensing and tip/tilt components removed $OPD_{rms}$ is much smaller than the experimental value. Since no flow field results were reported in their work, by simply looking at the numerical approach they used, the reason might be similar to that in Nahrstedt et al. [29], which implies that their hybrid RANS/ILES approach is unable to predict aero-optics accurately.

1.3 Objectives of the Present Study

To the author’s knowledge, no numerical simulations to date have accurately predicted both the flow and aero-optics around hemispherical turrets. All the aforementioned studies employed hybrid RANS/LES methods of the DES type which do not provide sufficient details of the flow structures and inherit many weaknesses of the RANS methods. Although fully-resolved LES has been employed in recent years to conduct fundamental scientific investigations of aero-optics (e.g. [23, 24, 48–50, 52]), it is at present not practical for the complex turret flows at high Reynolds numbers. The objective of this study is to explore the use of LES combined with a wall-layer model [31, 51] to study the flow and aero-optical environments around a hemispherical turret. In contrast to DES, which treats attached boundary layers with RANS, only the near-wall region in a
turbulent boundary layer is modeled by RANS in a wall-modeled LES whereas the outer region is simulated with LES, providing a higher-level of fidelity and more flow details. In addition to the turbulence modeling approach, the underlying numerical algorithm is important. A low-dissipative numerical scheme is employed in the present study to ensure that optically important flow scales are adequately captured and the subgrid scale model plays a useful role.

Two turrets will be investigated, one corresponding to the 12" \((H/D = 0.375)\) turret used in the experiment of Gordeyev et al. \([9, 12]\), and the other corresponding to the 24" \((H/D = 0.3125)\) turret in the experiment of Vukasinovic et al. \([42–46]\). The resulting flow fields will be analyzed first, followed by an investigation of the aero-optical aberrations at different elevation angles as well as azimuthal angles. The relation between optical distortions and flow-field characteristics will be examined as well.
CHAPTER 2

NUMERICAL METHODS

2.1 Numerical Methods for the Flow Solver

Large-eddy simulation with a wall model is used to obtain time accurate flow-field. A low-dissipative LES code developed at Stanford University by Shoeybi et al. [36] is employed, and a wall model is implemented in the course of the investigation. This section describes the governing equations and the spatial and temporal discretization schemes for the flow solver. The wall model and its implementation are discussed in Section 2.2.

2.1.1 Governing Equations

The equations solved are the spatially filtered continuity, momentum and energy equations for compressible flow. If we use $L_r$, $\rho_r$, $u_r$, $c_r$, $T_r$ and $\mu_r$ as the reference length, density, velocity, sound speed, temperature and molecular viscosity, the governing equations can be nondimensionalized as the following form

$$
\begin{align*}
\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial \bar{\rho} \bar{u}_j}{\partial x_j} &= 0, \\
\frac{\partial \rho \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j} \left( \rho \bar{u}_i \bar{u}_j + \bar{p} \delta_{ij} \right) &= \frac{\partial \bar{\tau}_{ij}}{\partial x_j} - \frac{\partial \tau_{ij}^{SGS}}{\partial x_j}, \\
\frac{\partial \bar{\rho} \bar{E}}{\partial t} + \frac{\partial}{\partial x_j} \left[ (\rho \bar{E} + \bar{p}) \bar{u}_j \right] &= \frac{\partial \bar{q}_j}{\partial x_j} + \frac{\partial}{\partial x_k} (\bar{u}_j \bar{\tau}_{jk}) - \frac{\partial \bar{q}_j^{SGS}}{\partial x_j},
\end{align*}
$$

(2.1)
where $\tilde{\rho}$, $\tilde{u}_i$, $\tilde{\rho}\tilde{E}$, $\tilde{p}$, $\tilde{\tau}_{ij}$, $\tilde{q}_j$ are the filtered density, velocity, total energy, pressure, viscous stress tensor, heat flux vector respectively, $\tau_{ij}^{SGS}$ and $q_j^{SGS}$ are the subgrid-scale (SGS) stress tensor and heat flux vector. The filtered total energy has the form\(^1\)

$$\tilde{\rho}\tilde{E} = \frac{\tilde{\rho}}{\gamma - 1} + \frac{1}{2} \tilde{\rho} \tilde{u}_k \tilde{u}_k,$$  \hspace{1cm} (2.2)

the viscous stress tensor is

$$\tilde{\tau}_{ij} = \frac{\mu}{Re} \left( \frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} - \frac{2}{3} \frac{\partial \tilde{u}_k}{\partial x_k} \delta_{ij} \right),$$ \hspace{1cm} (2.3)

and the heat flux vector is

$$\tilde{q}_j = \frac{\mu}{(\gamma - 1)RePrM_r^2} \frac{\partial \tilde{T}}{\partial x_j},$$ \hspace{1cm} (2.4)

where $\gamma$ is the specific heat ratio, $\mu$ is the nondimensional molecular viscosity, $Re = \rho u_r L_r / \mu_r$ is the Reynolds number, $Pr = \mu_r C_p / k$ is the Prandtl number, $M_r = u_r / c_r$ is the reference Mach number, and $\tilde{T}$ is the filtered temperature. The viscosity is assumed to vary with temperature according to the power law $\mu = \tilde{T}^n$ with $n = 0.7$ in the current simulation.

The subgrid-scale stress tensor and heat flux vector resulting from the filtering process are

$$\tau_{ij}^{SGS} = \tilde{\rho} (\tilde{u}_i \tilde{u}_j - \tilde{u}_i \tilde{u}_j),$$  \hspace{1cm} (2.5)

$$q_j^{SGS} = \tilde{\rho} (\tilde{u}_j \tilde{T} - \tilde{u}_j \tilde{T}).$$  \hspace{1cm} (2.6)

These two terms are modeled using the SGS model proposed by Moin et al. [26]

\(^1\)an SGS term $\frac{1}{2} \tau_{kk}^{SGS}$ has been dropped here for simplicity
with modifications proposed by Lilly [21].

The governing equations in Eq. (2.1) are closed by the equation of state

\[ \bar{p} = \frac{\bar{p} \bar{T}}{\gamma M_r^2}. \]  

(2.7)

In all the above equations, variables with an overbar denote spatially filtered quantities defined as

\[ \bar{f}(x) = \int_{\Omega} G(x - x') f(x') dx', \]  

(2.8)

where \( G(x) \) is the kernel of a spatial filter which can take various forms ([7, 35]). Variables with a tilde indicate Favre filtering

\[ \tilde{f} = \frac{\rho \bar{f}}{\bar{\rho}}. \]  

(2.9)

The governing equations can be written in a standard vector form as

\[ \frac{\partial U}{\partial t} + \frac{\partial F_j}{\partial x_j} = G_j, \]  

(2.10)

where

\[ U = \begin{pmatrix} \bar{\rho} \\ \bar{\rho} \bar{u}_i \\ \bar{\rho} \bar{E} \end{pmatrix}, \quad F_j = \begin{pmatrix} \bar{\rho} \bar{u}_j \\ \bar{\rho} \bar{u}_i \bar{u}_j + \bar{p} \delta_{ij} \\ \bar{\rho} \bar{E} + \bar{p} \bar{u}_j \end{pmatrix}, \quad G_j = \begin{pmatrix} 0 \\ \tilde{\tau}_{ij} - \tau_{ij}^{SGS} \\ \tilde{u}_i \tilde{u}_j + \tilde{q}_j - q_j^{SGS} \end{pmatrix}. \]  

(2.11)

\( U \) is the vector of conservative variables, \( F_j \) is the vector of convective flux, and \( G_j \) is the vector of viscous flux.

Since a finite volume method is used, the actual equations to be solved are the
integral form of Eq. (2.10)

\[
\frac{\partial}{\partial t} \int_{\Omega} U \, dv + \oint_{\partial \Omega} F_j n_j \, ds = \oint_{\partial \Omega} G_j n_j \, ds, \tag{2.12}
\]

where \( \Omega \) denotes the control volume, \( \partial \Omega \) the boundary of the control volume, and \( \vec{n} \) the normal vector of the boundary. Divergence theorem has been used to convert the volume integrals to surface integrals.

2.1.2 Numerical Schemes

As described by Shoeybi et al. [36], the flow solver is based on an unstructured mesh topology. Around each node a control volume is formed. Such a control volume is referred to as dual cell as exemplified in Figure 2.1.

Figure 2.1. Schematic of constructed dual cells (colored in cyan) in 2-D (from [36])
Equations (2.12) are solved inside each dual cell using second order finite volume operators ([14, 36]) that satisfy the Summation-By-Parts (SBP) property [30, 38], which means those operators are energy-conservative. The Simultaneous-Approximation-Term (SAT) technique [25] is used to implement the boundary conditions, which improves the overall robustness of the solver.

The time marching is hybrid implicit/explicit based on the Jacobian matrix of the flux vector [36], which allows relatively large time steps. In the implicit part, Newton iteration is used to solve the non-linear systems, which results in second order accuracy, while in the explicit part a 3rd-order Runge-Kutta scheme is used.

2.2 Near Wall Treatment

Due to the high Reynolds number of the flow, it is unaffordable to perform adequately resolved LES which requires extremely fine grid resolution in the near wall region [4]. To account for the effect of the unresolved but dynamically important eddies in the near-wall region, an LES with wall modeling approach is adopted.

2.2.1 Wall Model Equations

Over the past few decades, many wall models have been proposed for LES of incompressible turbulent flows while relatively few wall models exist for compressible flows. In the present work the equilibrium wall model proposed in [18, 19] is adopted. A pressure gradient term is also included since, as suggested by Wang and Moin [51], it improves model performance for flows with large pressure gradients. The final wall model equations in non-dimensional form are (the overbar
\[
\frac{d}{dy} \left[ (\mu + \mu_t) \frac{du}{dy} \right] = \frac{1}{(\gamma - 1)M_{ref}^2} \left( \frac{\mu}{Pr} + \frac{\mu_t}{Pr_t} \right) \frac{dT}{dy} = 0, \quad (2.14)
\]

where \( y \) is the wall normal direction, \( x \) is the wall tangential direction, \( \mu \) is the dimensionless molecular viscosity, \( Pr = 0.7 \) is the Prandtl number, and \( Pr_t = 0.9 \) is the turbulent Prandtl number. The turbulent eddy viscosity \( \mu_t \) is taken from a mixing length model
\[
\mu_t = \kappa \mu y^+ \left[ 1 - \exp \left( - \frac{y^+}{A} \right) \right]^2, \quad (2.15)
\]
where \( \kappa = 0.41, A = 17, y^+ = (y/\mu)\sqrt{\rho \tau_w} \) is the distance to the wall in wall units, and \( \tau_w = \mu du/dy \) is the viscous wall shear stress.

2.2.2 Implementation and Validation of the Wall Model

The wall model equations (2.13) and (2.14) are two coupled nonlinear ODE’s. They are solved on an imbedded Cartesian grid in the near wall region of the LES domain (Figure 2.2). To make the equations well-posed, two boundary conditions are needed: at the top, the LES solutions for tangential velocity, temperature and density are interpolated onto the edge of the wall model grid as Dirichlet boundary conditions, while the bottom boundary conditions are either adiabatic or isothermal with a no-slip wall.

The nonlinearity of those two equations makes them expensive to solve and thus a simplification is made: the coefficients of the derivatives are computed based on the solutions at the previous time step and are thus not dependent on the solutions at the current step. This simplification renders the two ODE’s
linear and easy to solve. A second-order central difference scheme is used for the discretization, which results in two sets of tridiagonal systems that can be solved using the Thomas algorithm [1].

Once the wall model equations are solved, we can compute the wall shear stress $\tau_w$ and wall heat flux $q_w$ (for isothermal wall). And these two source terms are added to the near wall control volumes in the LES domain to compensate for the under-resolved quantities.

In the present implementation, the computed wall shear stress is converted to a body force ($f = \tau_w \Delta s / \Delta V$) because it is easier to be added to the flow solver in use. Furthermore, for stability reason, the traditionally used slip-wall boundary condition for wall modeled LES [31, 32, 51] is replaced by the no-slip condition [15].

To validate the wall model equations, (2.13) and (2.14) are solved outside the flow solver. The top boundary condition is taken from a mean flow field of a flat plate turbulent boundary layer at $Re_\theta = 2800$ simulated by Wang and Wang [49]. The wall model grid is much refined with a grid spacing of $\Delta y^+ = 0.5$, Figure 2.3.
shows a comparison between the velocity profile from the wall model equations and the well known log law as well as the linear relation. Excellent agreement is seen, which confirms the correctness of both the wall model equations and the numerical implementation.

Figure 2.3. Comparison of the mean streamwise velocity from the wall model equations for a $Re_{\theta} = 2800$ boundary layer with the Log Law and the linear relation. $U^+ = 2.44 \ln y^+ + 5.2$

To test the effectiveness of the wall model in LES, turbulent flat plate boundary layer simulations with and without the wall model have been performed. The Mach number in the free stream is $Ma = 0.5$, and the Reynolds number based on momentum thickness is $Re_{\theta} = 2800$. The LES grid spacing are $\Delta x^+ = 100$, $\Delta y_{min}^+ = 30$, $\Delta z^+ = 50$ in the streamwise, wall-normal and spanwise directions,
respectively, and underresolved turbulent inflow is provided at the inlet as well.

Figure 2.4 shows contour of the instantaneous streamwise velocity from wall modeled LES, which shows realistic turbulence structures. Figure 2.5 shows the mean streamwise velocity profile from an underresolved LES without a wall model (the correct wall shear stress is used for normalization), and Figure 2.6 shows the mean streamwise velocity from a wall-modeled LES on the same grid. It is obvious that the solution has been improved significantly when a wall model is used. Figure 2.7 and 2.8 show the root-mean-square of streamwise and wall-normal velocity fluctuations, $u_{rms}$ and $v_{rms}$, respectively. By comparing the LES results with/without wall model and the experimental data in [6], it is seen that profiles for both quantities are improved when a wall model is used.
Figure 2.5. Mean streamwise velocity profile in a $Re_\theta = 2800$ boundary layer from an underresolved LES without wall model. Red, LES without wall model; Green, $U^+ = 2.44 \ln y^+ + 5.2$

Figure 2.6. Mean streamwise velocity profile in a $Re_\theta = 2800$ boundary layer from LES with wall modeling. Red, LES with wall model; Green, $U^+ = 2.44 \ln y^+ + 5.2$
Figure 2.7. Root-mean-square of streamwise velocity fluctuations in a flat-plate boundary layer. ○, experiment of DeGraaff and Eaton [6] at $Re_\theta = 2900$; ▲, LES with wall modeling at $Re_\theta = 2800$; ▲, LES without wall modeling at $Re_\theta = 2800$.

Figure 2.8. Root-mean-square of wall-normal velocity fluctuations in a flat-plate boundary layer. ○, experiment of DeGraaff and Eaton [6] at $Re_\theta = 2900$; ▲, LES with wall modeling at $Re_\theta = 2800$; ▲, LES without wall modeling at $Re_\theta = 2800$. 
2.3 Numerical Approach for Aero-Optics

To study the aero-optics of turret flow, the first step is to compute the OPL as discussed in section 1.1.2, which is rewritten as below.

\[
OPL(t, x', y') = \int_{z_0}^{z_1} n(t, x', y', z') dz'.
\]  

Hereafter in this thesis, \(x', y'\) and \(z'\) will be used to denote the beam coordinates while \(x, y\) and \(z\) the flow coordinates.

Using Gladstone-Dale relation, the above expression for OPL can be written as,

\[
OPL(t, x', y') = K_{GD} \int_{z_0}^{z_1} \rho(t, x', y', z') dz' + (z_1 - z_0).
\]

Thus the core part of computing OPL is the integration of density along the beam direction.

In general the optical beam direction is not aligned with the grid for flow simulation (see Figure 2.9). It is therefore necessary to interpolate the density from the flow simulation grid to a beam grid. A trilinear interpolation capability for polyhedron elements with arbitrary surfaces was added to the flow solver by Wang [47] and this capability is further extended in the present work to handle an arbitrary number of beam grids with arbitrary elevation angles as well as aperture shapes for a 3-D turret.

Once OPL is obtained, we can compute the OPD (see Eq. (1.4)) and then remove the tip/tilt component (see Eq. (1.9)), in practice, these two steps can be combined into a single one. We use the least square surface fitting method to find
a surface \( A + Bx' + Cy' \) that minimizes [47]

\[
G = \int \int [OPL(t, x', y') - (A + Bx' + Cy')]^2 dx'dy',
\]  
(2.18)

over the aperture. The steady lensing and tip/tilt removed OPD, also known as high-order wavefront, is then

\[
OPD(t, x', y') = OPL(t, x', y') - (A + Bx' + Cy').
\]  
(2.19)

This quantity will be used to further compute \( OPD_{rms}(t) \) and \( \overline{OPD}_{rms} \), etc.
CHAPTER 3
FLOW FIELD AROUND THE TURRETS

3.1 Simulation Setup

Two turrets are considered in the present investigation. One corresponds to that used in the experiment of Gordeyev et al. [9, 12] which has a height of 4.5" and a diameter of 12" (henceforth referred to as Case A), and the other corresponds to that in the experiment of Vukasinovic et al. [42–46] with a height of 7.2" and a diameter of 24" (henceforth Case B). For both cases in the simulations, the incoming flow has a Mach number of 0.4 and a Reynolds number of $4.6 \times 10^5$ based on the free-stream velocity and turret diameter. The Reynolds number has been reduced from the experimental values of $2.3 \times 10^6$ and $4.6 \times 10^6$, respectively, to ease the resolution requirements. As indicated by Gordeyev and Jumper [9], the flow and optical properties are not sensitive to the Reynolds number as long as it is supercritical. The detailed parameters in the simulations and experiments are listed in Table 3.1.

The computational domain has a size of $12D$ in the streamwise ($x$) direction, $5D$ in the vertical ($y$) direction, and $10D$ in the spanwise ($z$) direction. The turret center is located $4D$ downstream of the inlet (at $x = 4D$) in the central plane ($z = 0$). This configuration results in a blockage ratio less than 2%. Figure
TABLE 3.1

SIMULATION AND EXPERIMENTAL PARAMETERS

<table>
<thead>
<tr>
<th>Case</th>
<th>Ma</th>
<th>Re</th>
<th>H/D</th>
</tr>
</thead>
<tbody>
<tr>
<td>simulation A</td>
<td>0.4</td>
<td>$4.6 \times 10^5$</td>
<td>0.375</td>
</tr>
<tr>
<td>experiment A</td>
<td>0.3 ~ 0.45</td>
<td>$2.3 \times 10^6 \sim 3.5 \times 10^6$</td>
<td>0.375</td>
</tr>
<tr>
<td>simulation B</td>
<td>0.4</td>
<td>$4.6 \times 10^5$</td>
<td>0.3125</td>
</tr>
<tr>
<td>experiment B</td>
<td>0.3 ~ 0.5</td>
<td>$4.4 \times 10^6 \sim 7.3 \times 10^6$</td>
<td>0.3125</td>
</tr>
</tbody>
</table>

3.1(a) shows a 3-D view of the computational domain, while Figure 3.1(b) shows an $x$-$y$ cut through the center of the turret.

The domain is discretized using multi-block structured hexahedral meshes. A boundary layer type mesh is employed around the turret with small stretching ratio and high orthogonality. The outer region is divided into 17 small blocks to diminish mesh skewness. A global mesh smoothing is applied to improve the overall mesh smoothness and orthogonality.

Two meshes are employed for each case. The fine mesh is used to generate the results presented in this thesis whereas the coarse mesh is used to assess the grid sensitivity of the solutions. The smallest grid spacing is approximately $5 \times 10^{-3} D$ for the coarse mesh and $2.5 \times 10^{-3} D$ for the refined mesh in the near wall region, which is approximately 120 wall units for the coarse mesh and 60 wall units for the refined mesh. The total number of grid points is 3.6 million for the coarse mesh and 29 million for the refined mesh (number of grid points doubled in each direction). For the fine mesh, the smallest stretching ratio is 1.01 in the near wall region, and the largest one is 1.05 close to the outlet. Figure 3.2(a) shows the
Figure 3.1. Schematic of the computational domain
surface mesh for Case A, and Figure 3.2(b) shows the mesh in the central plane ($z = 0$).

The inlet boundary condition is taken from a steady 2-D RANS simulation of a flat plate boundary layer in order to match the boundary layer thickness in the experiment of Gordeyev et al. [12]. Another 3-D RANS simulation of flow over the turret was performed to obtain the outlet boundary conditions. Freestream values are used as the top boundary condition, while adiabatic and no-slip wall boundary conditions are employed at the bottom wall and turret surface. In order to avoid reflections of vortical and acoustic waves at the boundaries, sponge layers ([2, 16]) with a thickness of 1D are applied along the inlet, outlet and top boundaries.

The time marching is carried out using a constant CFL number which is fixed at 1.0 for the coarse mesh simulation and 2.0 for the refined mesh case. In both simulations the time step size is approximately $\Delta t = 3 \times 10^{-3} D/U_\infty$. Statistics are accumulated for 5 flow-through times ($150D/U_\infty$).

3.2 The Instantaneous Flow Field

This section presents the instantaneous flow field results from the simulations. Since the major flow features in Case A and Case B are similar, only the results for Case A are presented here unless otherwise pointed out. Figure 3.3 shows the contours of the instantaneous density, pressure, temperature and streamwise velocity in the central plane ($z = 0$). It is noted that an incoming boundary layer impinges onto the lower front portion of the turret base and forms a vortex at the corner, which has lower density, higher pressure, higher temperature, and lower velocity than the surrounding fluid. Above this vortex and below the lower front
Figure 3.2. Computational mesh for flow over the hemispherical turret in Case A
portion of the hemispherical cap, a region with low streamwise velocity (close to zero), but high values of all other quantities indicates that the flow goes into stagnation within this region. The flow is accelerated at the top of the turret and forms a region with low density, low pressure, and low temperature. Due to the adverse pressure gradient, the flow then separates from the turret surface forming a separated shear layer. In the near-wake region, we find that the density and pressure are lower but the temperature is higher. We also notice reversed flow close to the bottom surface, which indicates that a recirculation bubble is present. A close look at the instantaneous flow field values indicate that the variations of

Figure 3.3. Instantaneous flow field in the central plane ($z = 0$)
the magnitude of density, pressure and temperature are all within 15% of the free-stream values due to the low Mach number ($Ma = 0.4$) simulated here. However, the variation of the magnitude of streamwise velocity can be as large as 60% of the free-stream value.

To illustrate the flow structures, the vorticity magnitude at different cut planes are shown in Figures 3.4 - 3.6. Figure 3.4 shows the instantaneous vorticity magnitude in the $x$-$y$ plane through the turret center. The head of the necklace vortex (or say, horseshoe vortex) in front of the turret can be noticed. A crude estimate shows that the center of the vortex at this time instant is about $0.22D$ upstream of the turret while the height is about $0.18D$. Vortical structures with a wide range

![Image](https://via.placeholder.com/150)

Figure 3.4. Instantaneous vorticity magnitude $||\omega||D/U_\infty$ in the central plane ($z = 0$)

of scales can be seen in the wake region. Figure 3.5 shows a $y$-$z$ cut at $x = 4$. Two large round vortices on both sides of the turret are observed, which are cuts of the two braids of the necklace vortex. A crude estimate shows that the two vortices are $0.27D$ away from the turret, with heights of approximately $0.23D$. Compared with the head vortex in Figure 3.4, this indicates that the necklace vortex slowly
The flow structures are different in these two planes. The right plots show structures resembling those behind a 2-D cylinder. However, the left plots show structures in a wider region: there is a braid structure on each side of the cylinder which comes from the necklace vortex. The head of the necklace vortex is not seen because it is below the cut plane. It is also noticed that the two braids interact with the cylinder wake generating more flow structures. However, the right plots indicate that the cylinder base is sufficiently high such that the necklace vortex has little direct effect on the flow at this height. By comparing the plots at different time instants, it is observed that the wake structures in both planes alternate with time, however, the wakes on the left do not alternate as freely as those on the right, which is probably due to the retardation caused by the bottom wall and
Figure 3.6. Vorticity magnitude $\|\omega\| D/U_\infty$ at two time instants (top and bottom) in two horizontal planes (left, $y = 0.188D$; right, $y = 0.375D$)

also the interaction with the two braid vortices.

An 3-D view of the flow structures around the turret is plotted in Figure 3.7, which shows the iso-surface of the $Q$ criteria (the second invariant of the velocity gradient tensor). The necklace vortex and the wake vortical structures are visualized in greater detail than those captured by previous computations. It should be noted that these small flow structures are closely related to the high order abberations in the optical wavefront.

3.3 Flow Field Statistics

Figure 3.8 shows the time averaged density, pressure, temperature and stream-wise velocity in the central plane ($z = 0$). Compared with the instantaneous contours in Figure 3.3, we see that the two share many common features upstream
Figure 3.7. Iso-surface of the second invariant of velocity gradient tensor with $Q/(D U_\infty)^2 = 125$ from two viewing angles
(a) density $\bar{\rho}/\rho_{\infty}$

(b) pressure $\bar{p}/p_{\infty}$

(c) temperature $\bar{T}/T_{\infty}$

(d) velocity $\bar{U}/U_{\infty}$

Figure 3.8. Time averaged flow field in the central plane ($z = 0$)
of the turret which have been discussed in the previous section. It is noted that in the time averaged flow field, the largest values of $\bar{\rho}$, $\bar{p}$ and $\bar{T}$ appear in the front portion of the turret while in the wake region these variables are not very different from the free-stream values.

The RMS values of the fluctuations (Figure 3.9) are just the opposite. They have low levels upstream of the turret except in the necklace vortex but high levels in the downstream wake region. It is noted that the RMS contours of density and pressure are very similar, which indicates that the density fluctuations are mainly caused by pressure fluctuations in the wake region. The temperature fluctuations are mainly within the shear layer, and their magnitude decreases...
rapidly towards downstream. A common feature of $\rho_{rms}$, $p_{rms}$ and $T_{rms}$ is that inside the recirculation bubble, their levels are very low. From the RMS contours we can draw the conclusion that the strongest fluctuations occur in the separated shear layer and the wake region downstream of the recirculation bubble which are most detrimental to optical beams.

Figure 3.10 shows the streamlines in the central plane, in which (a) is the full view from which we can see the necklace-vortex head and the recirculation bubble, and (b)-(e) show the flow topologies at different locations. It is noted that there are three small vortices around the big necklace vortex (Figure 3.10(b)). The location of the necklace vortex can be estimated from this figure, which is about $0.75D$ upstream of the turret center and is consistent with the experimental value by Vukasinovic and Glezer [42]. There is a tiny bubble underneath the recirculation bubble (Figure 3.10(c)) whose center is only $0.03D$ behind the turret. From the streamlines around the separation point (Figure 3.10(d)) the separation angle is found to be $112^\circ$ and is in good agreement with the value reported by Gordeyev et al. [9, 12] which is between $110^\circ$ and $115^\circ$. The location of the reattachment point is estimated to be about $1.32D$ downstream of the turret center (Figure 3.10(e)) for Case A, and for Case B (not plotted here) it is approximately $1.20D$ which is lightly larger than the experimental value of $1.15D$ in Vukasinovic and Glezer [42]. Considering the difference in the height of the turret between the two cases the location of the reattachment point for Case A is also reasonable. The 3-D streamlines are shown in Figure 3.11, and we can obviously see the helical structures within the necklace vortex and the wake region.

Figure 3.12 shows the mean streamwise velocity in two $x$-$z$ planes at different $y$ locations. One cuts through the middle of the turret base and the other cuts
Figure 3.10. Streamlines in the central plane ($z = 0$). (a) The overall view; (b) the front corner; (c) the back corner; (d) around the separation point; (e) around the reattachment point.
Figure 3.11. 3-D streamlines around turret A (colored by density)
through the top of the turret base. Flows at the two locations are quite different. The cut at the higher location shows a flow topology similar to that of flow over a circular cylinder while in the lower cut there are two additional large structures beside the cylinder wake which are from the necklace vortex. These two structures gradually merge into the main wake as they extend further downstream.

![Figure 3.12](image)

**Figure 3.12.** Mean streamwise velocity in $x$-$z$ planes at two $y$ locations. (a) $y = 0.188D$; (b) $y = 0.375D$

Since the largest density fluctuations occur within the separated shear layer, it is important to examine how the flow separates from the turret surface. Figure 3.13 shows the mean surface pressure distribution on the turret. It is noted that there is a band of low pressure region (in blue color) whose width varies from $0.25D$ to $0.38D$. Upstream and downstream of this region are regions with higher pressure. Thus the flow is first accelerated in the favorable-pressure-gradient region and then decelerated in the adverse-pressure-gradient region, eventually leading to separation from the surface. The shape of the downstream edge of the low-pressure region is indicative of separation positions: the flow separates earlier and earlier as one moves from the hemisphere top to its side and then it separates later and later as one continues to move downward along the base cylinder.
Shown in Figure 3.14 is the computed pressure coefficient $C_p$ along the center line of the turret together with the experimental data of Gordeyev and Jumper [9] as well as the potential flow solution of flow over a sphere. Overall, the simulation result agrees well with the experimental data. It is interesting to see that both the numerical and experimental data before 80° are very close to the potential flow solution, but after 80° it deviates from the potential flow solution. From the $C_p$ curve it is noticed that the pressure first drops from the leading edge to about 90° with a minimum value of $-1.2$, and then increases until 112° to the value of $-0.3$. After 112° the flow is separated and $C_p$ stays nearly constant.

The mean velocity profiles and RMS of velocity fluctuations at three locations for Case A and Case B are plotted in Figure 3.15. It is seen that the coarse mesh results and fine mesh results are close to each other, which indicates that the current solutions are insensitive to the grid resolution. The first location is 1.5$D$ upstream of the turret center and the velocity profile is similar to a typical boundary layer profile. The second location is a short distance downstream of the
reattachment point and therefore the velocity gradient is very small near the wall. Finally, at the third location, the boundary layer is fully reattached but the effect of the turret wake is apparent in the velocity profiles. It is noticed that although there is a 17% difference in the height of the turret base between Case A and Case B, the velocity profiles are similar.

Figure 3.16 shows the frequency spectra of the pressure fluctuations at a location on the bottom wall, it is seen that there is reasonable agreement between the simulation and the experiment of Gordeyev et al. [12]. And Figure 3.17 shows the time correlation and frequency spectra of the pressure fluctuations at two locations 0.75\(D\) downstream of the turret center, with one near the half height of the turret base (location 1) and the other near the top of the base (location 2). At the first location there is a spectral peak at reduced frequency of \(fD/U_\infty \approx 0.2\)
Figure 3.15. Streamwise velocity profiles in the central plane at $x = 2.5D$ (left), $x = 5.5D$ (middle) and $x = 6.5D$ (right). Top, Case A; Bottom, Case B. --- , fine mesh; ---- , coarse mesh.
Figure 3.16. Frequency spectra of pressure fluctuations at $(x, y, z) = (5.0, 0, 0.5)D$. Red, simulation; Green, experiment of Gordeyev et al. [12]

Figure 3.17. Time correlation and frequency spectra of pressure fluctuations for Case A. Red, at $(x, y, z) = (4.75, 0.2, 0.3)D$; Green, at $(x, y, z) = (4.75, 0.4, 0.2)D$; Blue, with slope of $-7/3$
which corresponds to the alternating frequency of the wake structure behind the cylinder base. The correlation coefficients differ at large time separations which may be caused by the necklace vortex structures at the lower location. A line with a $-7/3$ slope is also plotted. The parallelism between this line and the spectral curves indicates that the flow is fully turbulent.

3.4 Conclusions

A few conclusions can be drawn based on the flow-field results discussed in the previous sections.

From the instantaneous flow field we confirmed the existence of a necklace vortex whose size grows as it extends downstream, and its position slowly moves away from the center plane. We also observed the alternation of the wake structures in time behind the cylindrical base, and by performing a spectral analysis we found that the reduced frequency is approximately 0.2.

We also found from the time averaged flow field that the density fluctuations in the wake region are mainly caused by pressure fluctuations, and are mainly concentrated within the shear layer and the wake region after reattachment while the recirculation bubble does not contain much density fluctuation. From the streamlines the separation angle and reattachment location as well as the location of the necklace vortex are identified which shows reasonable agreement with the experimental measurements.

The pressure distribution on the turret surface clearly shows flow separation at different locations on the turret. The pressure coefficient in the center plane indicates that in the front portion of the turret the flow is close to potential flow, whereas in the aft portion, the flow differs significantly from potential flow.
CHAPTER 4

AERO-OPTICAL ABERRATIONS

4.1 Optical Computation Setup

The geometrical parameters for the turret and the optical beam are illustrated in Figure 4.1. The turret itself can be defined by the base height $H$ and the diameter $D$, while the beam can be described by the aperture size $Ap$, elevation

Figure 4.1. Definitions of geometric parameters for a hemispherical turret with a conformal window
angle $\alpha$ and azimuthal angle $\beta$. For example, if the beam is in the spanwise central plane ($z = 0$) and tilts downstream then $90^\circ < \alpha < 180^\circ$ and $\beta = 0^\circ$, however, if it tilts upstream then $0^\circ < \alpha < 90^\circ$ and $\beta = 180^\circ$. A Cartesian coordinate system $(x', y', z')$ is used for the beam grid, with $z'$ in the propagation direction and $x'$ and $y'$ as shown in Figure 4.1.

Figure 4.2 shows a schematic of the beam grid together with the turret surface mesh. The beam grid has a square cross section with a side length of $D/3$ which is chosen to match the size of the effective portion of the aperture in the experiment of Gordeyev et al. [12], and the length of the beam grid in the propagation direction
is $1.8D$, which is sufficiently long to allow it to traverse the entire turbulent region for the elevation angles considered. The beam grid spacing is designed to match that of the LES grid at the same location in order to reduce interpolation errors.

A beam grid with a square cross section rather than a circular cross section is employed because with the same resolution on the perimeter a circular beam grid will require more grid points. Also, the data from a square beam grid is easier to process than that from a circular one. To approximate the aperture cross-section, only data within a radius of $D/6$ are used. The optical quantities (spatial and temporal series of $OPL$) are computed and stored at every five time steps during the flow simulation, and the statistics are calculated over a time period of five flow-through times ($150D/U_\infty$).

4.2 Instantaneous Optical Aberrations

Figure 4.3 shows the instantaneous wavefront distortions after steady and tip/tilt components are removed at three elevation angles $\alpha = 110^\circ, 130^\circ, 150^\circ$ and azimuthal angle of $\beta = 0^\circ$. At the elevation angle of $\alpha = 110^\circ$ most of the beam passes through the attached flow region where density fluctuations are small, and therefore the upstream portion of the wavefront has little variation; however, there is a large growth of wavefront variation at the downstream portion due to flow separation. At $\alpha = 130^\circ$ most part of the beam traverses the separated shear layer and only a very small portion travels through the attached boundary layer and therefore the wavefront is distorted severely over most of the aperture. At elevation angle of $150^\circ$, the entire beam traverses the separated shear layer and part of the wake region, where the flow structures are larger than those at $\alpha = 110^\circ$ and $130^\circ$, resulting in large-scale and large-amplitude wavefront distor-
Figure 4.3. Instantaneous OPD with steady and tip/tilt components removed at $\beta = 0^\circ$. Left, $\alpha = 110^\circ$; Middle, $\alpha = 130^\circ$; Right, $\alpha = 150^\circ$

tions. By comparing the plots at different elevation angles, it is noted that the wavefront distortions reflect the underlying flow field characteristics, including the separation of the boundary layer and the size and location of flow structures.

Figure 4.4 shows the wavefront distortions at the elevation angle of $\alpha = 130^\circ$ and azimuthal angle of $\beta = 30^\circ, 60^\circ$ and $90^\circ$, respectively (the case with $\beta = 0^\circ$ can be found in Figure 4.3). By comparing the wavefronts at these four azimuthal angles, it is noted that as the beam turns away from the central plane, the optical distortion decreases. By examining the instantaneous flow structures around the turret as shown in Figure 3.7, it is observed that at the same elevation angle as the azimuthal angle increases the part of the beam which traverses the attached boundary layer becomes larger and at the same time the propagation distance within the turbulence region becomes shorter. These two factors reduce the optical distortions to a very low level when the azimuthal angle approaches $90^\circ$. 

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Figure 4.4. Instantaneous OPD with steady and tip/tilt components removed at $\alpha = 130^\circ$. Left, $\beta = 30^\circ$; Middle, $\beta = 60^\circ$; Right, $\beta = 90^\circ$

Similar results can be observed in Figure 4.5 which shows the wavefront distortions at $\alpha = 150^\circ$ and $\beta = 30^\circ, 60^\circ$ and $90^\circ$, from which we observe a decrease in the magnitude of OPD as the azimuthal angle increases. However, by comparing the wavefront distortions at $\beta = 30^\circ$ and $\beta = 0^\circ$ (in Figure 4.3) it is noted that the distortion magnitude and scales are even larger at $\beta = 30^\circ$. This is because of the large elevation angle at which the beam propagates through a long distance in the wake region even when the beam is turned away from the central plane by $30^\circ$. As the beam is turned away from the central plane by a small angle it sees shear layer structures from the side of the turret which have large fluctuations.

4.3 Statistics of Optical Aberrations

Figure 4.6 shows the time history of $OPD_{rms}$ at three elevation angles in the central plane ($\beta = 0^\circ$) during one flow-through time. It is noted that the
Figure 4.5. Instantaneous OPD with steady and tip/tilt components removed at $\alpha = 150^\circ$. Left, $\beta = 30^\circ$; Middle, $\beta = 60^\circ$; Right, $\beta = 90^\circ$

magnitude of $OPD_{rms}$ grows with increasing elevation angle. A close examination reveals that the variations of $OPD_{rms}$ with time at different angles share similar patterns but there is a time lag as elevation angle increases. For example we see a peak around $tc_\infty/D = 18$ for all three curves but obviously the peak for larger $\alpha$ appears later with its magnitude amplified.

The effects of near-field optical aberrations in the far field can be characterized by the $OPD_{rms}$ which is shown in Figure 4.7 for both Case A and Case B. Although there is a 17% difference in the base height between the turrets in Case A and Case B, the $OPD_{rms}$ are very close to each other. The $OPD_{rms}$ in Case B is slightly larger than that in Case A at nearly all elevation angles, which is likely because the turret in Case B is lower than in Case A, and thus the bottom wall and the necklace vortex have stronger effects on the separated shear layer and the wake region. The experimental data from Gordeyev et al.[9] are also plotted
in Figure 4.7, which shows a reasonable agreement with the numerical results, especially at large elevation angles. Based on potential flow theories Gordeyev and Jumper [9] proposed a scaling relation for the $\overline{OPD}_{rms}$ which can be written as $\overline{OPD}_{rms} = 1.4 / \sin \alpha$, which is plotted in the same figure (here $\overline{OPD}_{rms}$ is defined as $\overline{OPD}_{rms} = \overline{OPD}_{rms}/(\rho/\rho_{SL}M^2D) \times 10^6$, where $\rho_{SL}$ is the air density at the sea level). This scaling relation predicts $\overline{OPD}_{rms}$ well at large elevation angles ($\alpha > 135^\circ$). It is noticed that the computed $\overline{OPD}_{rms}$ grows almost linearly with $\alpha$ when it is larger than $110^\circ$. By a least square fit, this linear relation can be approximated by $\overline{OPD}_{rms} = 3.087\alpha - 5.312$ where $\alpha$ is in radian. This approximation is also plotted in Figure 4.7.

Figure 4.6. Time series of $OPD_{rms}$ for Case A at $\beta = 0^\circ$. Red, $\alpha = 110^\circ$; Green, $\alpha = 130^\circ$; Blue, $\alpha = 150^\circ$. , instantaneous value; ---- , mean value

To investigate the effects of azimuthal angle, Figures 4.8 - 4.11 plot the time history of $OPD_{rms}$ and the time averaged values. At the elevation angle of $130^\circ$ shown in Figure 4.8, it is noted that although at some time instants the $OPD_{rms}$ is
larger at large azimuthal angles, the time averaged $OPD_{rms}$ decreases consistently with increasing azimuthal angle. From Figure 4.9 we can have a better idea of how $OPD_{rms}$ changes with $\beta$ at $\alpha = 130^\circ$. It shows that at small azimuthal angles ($\beta < 30^\circ$) $OPD_{rms}$ decreases slowly while at large azimuthal angles it decreases quickly and almost linearly.

The situation is rather different for $\alpha = 150^\circ$ shown in Figure 4.10 and 4.11: The $OPD_{rms}$ does not have an obvious drop (Case B) and even increases (Case A) at small azimuthal angles. The reason for this has been explained in the previous section in terms of the instantaneous wavefront distortions and flow structures. At the same time, it is noticed that at large azimuthal angles $OPD_{rms}$ decreases...
Figure 4.8. Time series of $OPD_{rms}$ for Case A at $\alpha = 130^\circ$. Red, $\beta = 0^\circ$; Green, $\beta = 30^\circ$; Blue, $\beta = 60^\circ$; Cyan, $\beta = 90^\circ$. , instantaneous value; ..., mean value

Figure 4.9. Time averaged $OPD_{rms}$ at $\alpha = 130^\circ$ and different azimuthal angles. , Case A; ----, Case B
Figure 4.10. Time series of $OPD_{rms}$ for Case A at $\alpha = 150^\circ$. Red, $\beta = 0^\circ$; Green, $\beta = 30^\circ$; Blue, $\beta = 60^\circ$; Cyan, $\beta = 90^\circ$. , instantaneous value; ---- , mean value

Figure 4.11. Time averaged $OPD_{rms}$ at $\alpha = 150^\circ$ and different azimuthal angles. ---- , Case A; ----- , Case B
faster with $\beta$ for $\alpha = 150^\circ$ than for $\alpha = 130^\circ$. It is also noticed that the $\overline{OPD_{rms}}$ for Case A and B are very close to each other at azimuthal angles larger than $30^\circ$.

While $\overline{OPD_{rms}}$ describes the overall optical distortion over the aperture, it is of interest to examine how the distortions are distributed over the aperture. Figure 4.12 shows contours of the nondimensional quantity $OPD^*_{rms}(x, y)$ which is defined as $OPD^*_{rms}(x, y) = OPD_{rms}(x, y)/(\rho/\rho_{SL}M^2D) \times 10^6$, where $OPD_{rms}(x, y)$ is the temporal RMS of $OPD(x, y, t)$ at each point on the aperture. It is noticed that at $\alpha = 110^\circ$ the optical aberrations are mainly within the downstream portion of the aperture where flow is separated. As the elevation angle increases large aberrations also appear on the side of the aperture where flow separation also occurs.

Figure 4.13 shows the auto-correlations and frequency spectra of $OPD_{rms}$ at $\alpha = 110^\circ, 130^\circ, 150^\circ$, and $\beta = 0^\circ$. There are no obvious peaks in any of the three spectral curves. Note that the spectral level increases with increasing elevation angle, which is consistent with the variation of $\overline{OPD_{rms}}$. The differences are mainly at low frequencies.

4.4 Conclusions

In this chapter the aero-optical aberrations caused by the flow over the tur-rets are calculated and analyzed. The relation between the distortions and flow structures and the effects of the elevation and azimuthal angles are investigated.

It is observed that as the elevation angle increases in the central plane, the optical beam sees more flow structures and propagates a longer distance in the turbulence region resulting in larger optical aberrations.

The numerical results of time averaged $OPD_{rms}$ are in overall agreement with
Figure 4.12. Distribution of $OPD_{rms}^*(x,y)$ over the aperture at $\beta = 0^\circ$ and three elevation angles.
the experimental measurements using 2-D wavefront sensor by Gordeyev et al. [9] at large elevation angles. And the scaling relation proposed by Gordeyev et al. [9] is also confirmed, especially for large elevation angles. It is found that when the elevation angle is sufficiently large (α > 110°) the magnitude of optical aberrations increases almost linearly with the elevation angle.

The effect of azimuthal angle is more complicated and depends on the elevation angle. When the elevation angle is small, the level of aberrations decreases with increasing azimuthal angle, slowly at first and then more rapidly; When the elevation angle is large, the aberration magnitude does not drop and it may even increase with azimuthal angle at small azimuthal angles; as the azimuthal angle increases the aberrations begin to drop, and it drops faster than at small elevation angles.

The frequency spectra of the $OPD_{rms}$ show that the elevation angle dependence of the optical distortions occur mainly at low frequencies.
CHAPTER 5
SUMMARY AND OUTLOOK

5.1 Summary

This thesis describes a study of the flow and optical environments around two hemispherical turrets with conformal windows using large-eddy simulation. Due to the high Reynolds number involved, it is unaffordable to perform a fully resolved LES. To reduce the computational cost, a wall model approach is adopted to model the effect of near-wall turbulence structures instead of fully resolving them. Based on an unstructured-mesh LES code for compressible flow from Stanford University, a wall model for compressible flow is implemented and tested in a flat-plate turbulent boundary layer. The results show that the solution with a wall model is improved compared to that without a wall model, which confirms the effectiveness of the wall modeling approach.

Simulations of flow over two hemispherical turrets are carried out, and the results show agreement with experimental measurements in terms of the pressure coefficient distribution, recirculation bubble size and unsteady pressure frequency spectra in the separated shear layer. From the instantaneous flow field we identified a low density, low pressure and high temperature wake region, and confirmed the existence of a necklace vortex which grows toward the downstream direction. It is noticed that the wake structures behind the turret base are in an alternating motion, and they interact with the necklace vortex close to the bottom wall.
From the time averaged flow fields, the flow region with large turbulent fluctuations, the flow separation angle, the location and size of the necklace vortex, and the location of the reattachment point are identified. The major findings related to optical distortions are: 1) the necklace vortex is below the optical aperture and thus does not have a direct effect on the optical aberrations; 2) in the recirculation bubble, density fluctuations are small; 3) large density fluctuations are mainly within the separated shear layers and the wake region after reattachment, and are predominantly caused by pressure fluctuations.

Based on the fluctuating density field obtained from LES, the optical distortions are calculated and analyzed. The resulting $\overline{\text{OPD}}_{\text{rms}}$ shows reasonable agreement with the experimental results measured using a 2-D wavefront sensor. Additionally, the effects of elevation angle and azimuthal angle on optical aberrations are studied in a systematic way. It is found that in the central plane the optical aberration grows almost linearly with the elevation angle when the latter is above 110°, but it has a more complicated relationship with the azimuthal angle. At small elevation angles it decreases with increasing azimuthal angle and the change is nonlinear. However, when the elevation angle is large, the aberration does not decrease much, and even increases, with azimuthal angle at small azimuthal angles; at large azimuthal angles it decreases monotonously with the azimuthal angle.

5.2 Outlook

The aero-optics of flow over a hemispherical turret is a very challenging problem both experimentally and numerically. The current study is the first attempt to predict the flow and optical properties using LES coupled with a wall model, and
among the first to study the optical distortion mechanisms. Because of the pre-
liminary nature of the present study, there are many questions that remain to be
answered in the future. The predicted flow-field statistics, particularly the veloci-
ty statistics, need to be thoroughly validated against experimental measurements.
Further grid-sensitivity studies are warranted, and the solution dependence on the
computational domain size and inlet conditions should be examined. The simula-
tions need to take into account the wind tunnel blockage effect and the position
of the turret in the wind tunnel for a fair comparison. Accurate experimental
data are needed, particularly in and around the recirculation bubble. In terms of
the physical sources of optical aberrations, the following are a few issues that are
worth exploring: 1) the effect of the necklace vortex on the flow around the opti-
cal window and associated optical distortions; 2) the critical height of the turret
base with which the necklace vortex has a direct effect on the optical aberrations;
3) the evolution and breakdown of the separated turbulent shear layer from the
turret surface; 4) the effects of Reynolds number and Mach number.

Studying the flow and aero-optics around a turret is only the first step to-
ward the final goal of reducing the optical aberrations. Further investigations
can also include the optimization of the turret geometry and exploration of flow
control strategies to suppress the flow separation and thereby mitigate the optical
distortions.
BIBLIOGRAPHY


