EVALUATION OF PASSIVE BOUNDARY LAYER FLOW CONTROL
TECHNIQUES FOR AERO-OPTIC MITIGATION

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Abstract

by

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The effect of two passive boundary layer flow control techniques, Large-Eddy Break-Up (LEBU) devices and wall heating/cooling, on turbulent boundary layer induced aero-optical aberrations is experimentally investigated. A series of experiments is performed investigating the effect of LEBUs on levels of optical aberrations in the turbulent boundary layer. The results of these experiments are analyzed to determine the physical mechanisms responsible for the experimentally observed changes, and to characterize the sensitivity of optical distortions to different LEBU device configurations. The effect of moderate levels of boundary layer wall cooling, both for full and partial wall cooling, on aero-optic aberrations is also experimentally investigated, and the results are compared to a statistical model derived using the temperature-velocity relation from the Extended Strong Reynolds Analogy and a simple model for the development of thermal sub-layers in partially cooled boundary layers. A method is proposed to use wall heating to passively amplify aero-optic aberrations to measure wavefront distortions in boundary layers with normally weak aero-optical effects. The method is used to study
turbulent boundary layers with low Reynolds numbers. This work concludes with a brief summary of important findings, and a discussion of the implementation of these flow control techniques for real laser transmitting systems.
For Brette

“There is no more lovely, friendly and charming relationship, communion or company than a good marriage.”

~ Martin Luther
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CHAPTER 1:

INTRODUCTION

1.1 Motivation

Within the aerospace community, there is substantial interest in developing aircraft-based directed energy systems for a variety of applications, including air-to-air and air-to-ground, line-of-sight optical telecommunications systems. This type of system would use laser beams to relay data directly between optical transmitters and receivers on aircraft, ground stations, and satellites, and would allow for much greater communication bandwidth than is currently available on state-of-the-art aviation communication technologies.

One of the many challenges that must be overcome before line-of-sight optical communications systems can become a robust and reliable method of communication for airborne vehicles is the effect that the Earth’s atmosphere has on the propagation of light. This fluid-optic interaction, which will be discussed in more detail in Section 1.2, occurs when wavefronts propagating through the Earth’s atmosphere encounter regions of air that have non-uniform and/or unsteady distributions of density. Since density and index-of-refraction are proportional to one another, these regions of fluctuating density cause initially planar optical wavefronts passing through them to be distorted (Gladstone & Dale, 1863; Liepmann, 1952; Tatarski, 1961, 1971; Churnside & Shaik, 1989). These distortions can cause wavefronts to destructively interfere with themselves, leading to
significant reductions in the average on-target beam intensity that reduce the overall performance of directed energy systems (Fowles, 1973; Gilbert, 2013; Born & Wolf, 1999; Jumper & Fitzgerald, 2001). Unsteadiness in the density field has also been linked to instantaneous reductions in beam intensity, which can cause disastrous signal drop-outs for free-space communications applications (Gordev, Cress, and Jumper, 2013).

Figure 1.1. The Airborne Aero Optics Laboratory (AAOL); Top: Schematic of formation flight for data acquisition; Bottom: Image from flight test, with emphasis on laser beam from chase plane locked on to turret. [Image source, Jumper, et al. (2013)]

When wavefront distortions are the result of non-uniform density in the Earth’s atmosphere, they are referred to as the atmospheric-propagation problem (Tatarski, 1961,
Wavefront distortions that are the result of density fluctuations found in regions of turbulent flow surrounding aircraft are identified as the aero-optic problem (Gilbert & Otten, 1980; Jumper & Fitzgerald, 2001; Wang, Mani, & Gordeyev, 2012). Examples of flows that are aero-optically active include separated shear-layers (Jumper & Fitzgerald, 2004; Rennie, et al. 2008; Nightingale, et al., 2009), turbulent boundary layers (Cress, 2010; Wang & Wang, 2012; White & Visbal, 2011; White & Visbal, 2012; Gordeyev, et al., 2014), tip vortices (Porter, et al., 2013, Kelly, et al., 2013), and shocks (White & Visbal, 2014; Vorobiev, et al., 2014). The aberrations imposed on initially planar, collimated wavefronts by these different aero-optically active flows are likely to pose a significant problem for the performance of airborne optical systems, whether they are directed energy, imaging, or free-space communications applications, as small disturbances to optical wavefronts in the near-field can result in significant reductions in time-average and instantaneous on-target intensity at points very far away from the aircraft (Jumper & Fitzgerald, 2001).

In order to achieve a large field of regard, airborne directed energy systems and laboratories, including the Airborne Laser Laboratory (ALL), the Airborne Laser Testbed (ABL), and the University of Notre Dame’s Airborne Aero-Optics Laboratory (AAOL) (which is pictured in Figure 1.1), have gravitated towards the use of hemisphere or hemisphere-on-cylinder turrets in order to send and receive laser beams (Moler & Lamerson, 1998; Kyrazis, 2013; Jumper, et al., 2013). While these devices are mechanically simple, the complex three-dimensional flow that develops around turrets includes elements of turbulent boundary layers, flow separation, shear layers, wakes, and unsteady shocks (Cress, et al., 2007; Gordeyev & Jumper, 2010; Vorobiev, et al., 2014).
The density fluctuations which arise from the region of turbulent flow around turrets have been shown to impart problematic levels of aero-optic aberrations to beams passing to and from the turret, especially for backward-facing angles (De Lucca, Gordeyev & Jumper, 2013). Several different approaches have been considered for neutralizing the ill effects that turbulent flow around a turret has on the propagation of optical wavefronts. A number of passive and active flow control techniques, which aim to condition the flow around the turret to have less severe fluid-optic interactions, have been effective at reducing (although not eliminating) wavefront aberrations caused by the flow environment around turrets (Gordeyev, et al., 2010; Vukasinovic, et al., 2011; Wang, et al. 2012; Gordeyev, et al., 2013; Vukasinovic, et al., 2013; De Lucca, et al., 2013).

A second method for correcting wavefront aberrations caused by turbulent flow is to apply a pre-correction to the propagated laser beam that would eliminate the problem. This approach, known as adaptive-optics, is a well-known technique that has long been used to mitigate the atmospheric-propagation problem (Hecht, 2002). Adaptive-optic systems work by measuring the wavefront aberration caused by an external flow field and then applying a conjugate correction to the beam with a deformable mirror. As the pre-aberrated beam propagates through the aberrating flow field, the imposed aberration and conjugate correction cancel out, resulting in a planar corrected wavefront (Roggemann & Welsh, 1996; Hecht, 2002). The frequency response of traditional closed-loop adaptive optic systems, however, is not adequate to keep up with the temporal fluctuation of typical aero-optic wavefront aberrations, which are generally on the order of 50 kHz (Jumper & Fitzgerald, 2001). Recently, several novel methods have been proposed for improving the response of adaptive-optic systems for aero-optics applications.

Figure 1.2. Schematic of flow around the hemisphere on cylinder turret for subsonic freestream speeds. [Image source, Gordeyev & Jumper (2010)]

For applications in which a large field of regard is not an essential requirement, the flow-field over the optical aperture can be vastly simplified by eliminating the turret completely. Instead, the beam could be propagated through an aperture or phased array that is flush-mounted to the skin of the aircraft (Serati & Stockley, 2003; Cress, 2010; Whiteley & Gordeyev, 2013). In this configuration, the only aero-optically active flow field that must be traversed by the beam is the compressible, turbulent boundary layer (TBL). At present, aero-optic aberrations caused by the boundary layer have been studied analytically, experimentally and computationally for over six decades (Liepmann, 1952; Stine & Winovich, 1956; Rose, 1979; Wyckham & Smits, 2009; Cress, 2010; Wang &
Wang, 2012; White & Visbal, 2012; Gordeyev, et al. 2014), and much has been learned about the physics of TBL fluid-optic interactions. Important findings from research on the aero-optical characteristics of TBLs will be reviewed in detail in Chapter 2 of this dissertation, but can be summarized via a few main conclusions. First, aero-optic aberrations have been strongly correlated with the large-scale turbulent motions convecting in the outer part of the TBL (Gordeyev, et al. 2014, and the references therein). Secondly, analysis of time-resolved TBL wavefront measurements has revealed that even in cases where on-average target intensity is found to be high, large intermittent drop-outs in far-field signal intensity that lasted on the order of 1 ms were found (Gordeyev, Cress, & Jumper, 2013), which would be crippling to the performance of any potential airborne laser-based free-space communications systems.

These results clearly demonstrated that although the TBL is vastly simpler than the flow environment around a turret body, boundary layers are not guaranteed to be entirely benign with respect to aero-optics. As a result, there is substantial interest in developing and testing flow control schemes that would modify the boundary layer flow in such a way that would permit airborne directed energy and free-space communications systems to achieve high on-target intensity in both a time-averaged and instantaneous sense.

Jumper (1980) first suggested the use of parallel plate manipulators for aero-optic flow control. These devices, which are also commonly referred to as Large-Eddy Break-Up (LEBU) devices, were originally proposed as a method of flow control for achieving viscous drag reduction (Corke, et al. 1979). A detailed review of prior work on LEBUs is found in Chapter 3, but in summary, LEBUs were found to achieve this end by altering
the turbulence structure in the outer part of the TBL, and they significantly reduced integral length scales for long distances downstream of the devices (Corke, 1981; Savill & Mumford, 1988; Anders, 1990, and others). These effects, especially the suppression of large-scale turbulent structures in the outer part of the boundary layer, are very much aligned with the mechanisms identified as the dominant source of TBL aero-optic aberrations. Prior to this work however, no experimental or computational evaluation of LEBUs for aero-optic mitigation has been performed.

In fact, very few studies of flow control methods for mitigating TBL aero-optics have been performed to date (Cress, 2010; White & Visbal, 2011). One illuminating investigation was performed by Cress (2010) in which the effect of wall-temperature on TBL wavefront aberrations was experimentally investigated. Experiments were particularly focused on wall-heating, but both preliminary experimental measurements and a theoretical model for wall temperature effects indicated that wavefront aberrations could be reduced by as much as 80 % through the use of wall cooling (Cress, 2010). Additional data is still required, however, to validate Cress’ model over a wide range of temperatures and Mach numbers, but these suggest wall cooling is also a promising technique for aero-optic mitigation.

Finally, there is a pressing need to develop techniques to ‘close the gap’ in Reynolds number between experiments and computations of the aero-optic effects of TBLs. Due to both computational constrains and wavefront sensor limitations, experimental measurements of TBL aero-optic effects are performed at Reynolds numbers that are about an order of magnitude higher than results from high-fidelity simulations, making one-to-one validation of the computational methods used difficult to
date. Wall heating experiments showed that it was possible to passively amplify TBL wavefront aberrations (Cress, 2010), raising the possibility of using wall heating as a means of increasing the signal-to-noise ratio in experiments where aero-optic aberrations are otherwise very weak.

This dissertation will focus on experimentally evaluating the effect of boundary layer flow control techniques on the aero-optic characteristics of the turbulent boundary layer. Specifically, flow control methods and configurations that achieve sufficient levels of aero-optic mitigation will be presented. A novel method for obtaining low Reynolds number TBL wavefront measurements using wall heating flow control will also be presented. The relationship between modified boundary layers and the physics of fluid-optic interactions will also be discussed.

![Figure 1.3. Schematic of aero-optic aberrations from variable density field](image source Cress, 2010).
1.2 The Aero-Optic Problem

The index of refraction of a medium is defined as the ratio of the speed of light in a vacuum, \( c_0 \), and the local speed of light in the medium, \( c \);

\[ n = \frac{c_0}{c}. \]  

(1.1)

The index of refraction of a medium is also related to its density, \( \rho \), by the Gladstone-Dale relation,

\[ n(x, y, z, t) = 1 + K_{GD} \rho(x, y, z, t), \]  

(1.2)

where \( K_{GD} \) is the Gladstone-Dale constant (Gladstone & Dale, 1863). Strictly speaking, \( K_{GD} \) is actually a function the particular combination of light wavelength, \( \lambda \), and the gas mixture of the medium; however for the range of visible light (\( \lambda = 380 \text{–} 750 \text{ nm} \)) in air \( K_{GD} = 2.27 \times 10^{-4} \text{ m}^3/\text{kg} \) (Gardiner Jr., Hidaka, & Tanzawa, 1980). Here, \( n \) and \( \rho \) are defined as functions of a point in physical space, \( (x, y, z) \), at some instant in time, \( t \), which emphasizes the fact that density variations within an arbitrary medium will result in corresponding index-of-refraction variations. As different portions of initially planar (i.e. collimated) wavefronts propagate at different speeds depending on the local medium density, the wavefront is distorted from its initial collimated state. This effect is pictured schematically in Figure 1.3, with the wavefront distortions greatly exaggerated for demonstration purposes.

1.2.1 Optical Path Difference

As planar wavefronts propagate through these unsteady density distributions, the effect of turbulent density fluctuations on the propagation of light is typically quantified by defining the Optical Path Length (OPL). The approximate solution to the Maxwell’s
The equations for an electromagnetic wave propagating through some arbitrary medium is
\[ E(\mathbf{r}) = E(\mathbf{r}) e^{-jk_0 S(\mathbf{r})}, \]
where \( \mathbf{r} \) is the position vector, \( i \) is the imaginary unit, \( k_0 = 2\pi/\lambda_0 \) is the wavenumber in a vacuum, and \( E \) and \( S \) are the slowly varying wave amplitude and phase, respectively. Neglecting the wave amplitude (i.e. the ray approximation), the phase, or Optical Path Length of the wave can be expressed as the integral of the medium index-of-refraction, \( n(r) \), along the ray path,
\[ S(\mathbf{r}) = OPL(\mathbf{r}) = \int_{s_1}^{s_2} n(\mathbf{r}) ds, \quad (1.3) \]
where \( s \) is the path of propagation of the ray (Hecht, 2002). Assuming that rays propagate in the \( y \)-direction only, and substituting density for index-of-refraction via equation (1.2), \( OPL \) can be expressed as
\[ OPL(x, z, t) = \int_{y_1}^{y_2} \left[ K_{GD} \rho(x, y, z, t) + 1 \right] dy, \quad (1.4) \]
where \( y \) is the direction of beam propagation. The resulting deviation from the average \( OPL \) can then be expressed as the Optical Path Difference (\( OPD \)),
\[ OPD(x, z, t) = OPL(x, z, t) - \langle OPL(x, z, t) \rangle, \quad (1.5) \]
where the angled brackets denote spatial averaging. It may be shown that \( OPD \) is the mean-removed conjugate of the optical wavefront (Hugo R., 1995),
\[ OPD(x, z, t) = -W(x, z, t), \quad (1.6) \]
where the optical wavefront \( W \) is defined as the surfaces that join all points of equal phase at any given time (Hecht, 2002).
The extent to which wavefronts are distorted from their initially planar state is typically characterized by taking the root-mean-square (RMS) of the \( OPD \), or \( OPD_{rms} \). This quantity can be defined in two ways; one way is to take the RMS in the time domain,

\[
OPD'_{rms}(x,z) = \left[ \frac{1}{T_0} \int_0^T OPD^2(x,z,t) \, dt \right]^{\frac{1}{2}},
\]

where \( T \) is the length in time of \( OPD(x,z,t) \). Note that equation (1.7) gives \( OPD_{rms} \) as a function of location on the wavefront aperture. The second method of computing \( OPD_{rms} \) is to take the RMS of \( OPD \) over the aperture at each measurement in time:

\[
OPD_{rms}^{x,z}(t) = \left[ \frac{1}{A_{Ap}} \int_{A_{Ap}} \int OPD^2(x,z,t) \, dx \, dz \right]^{\frac{1}{2}},
\]

where \( A_{Ap} \) is the aperture area. The former definition from equation (1.7) is useful in applications where flow is not homogeneous over the optical aperture, as it gives an indication of the variation in wavefront aberrations in space. This definition can be used to identify regions over the aperture where there is a significant increase in wavefront distortion, such as flow separation or shock motion. The latter definition, equation (1.8), is useful for characterizing the instantaneous wavefront error and the effects of near-field wavefront distortions on far-field intensity, as will be shown in the next section. Cress (2010) showed that \( OPD_{rms}(t) \) follows a log-normal distribution for TBL wavefront aberrations, and so the time averaged value of \( OPD_{rms} \) is generally adequate for characterizing levels of aero-optic wavefront aberrations. Therefore for the remainder of this work, all references to \( OPD_{rms} \) will refer to the time averaged value of \( OPD_{rms}(t) \), as computed from the definition given in equation (1.8), unless otherwise noted.
1.2.2 Effect on Far-Field Intensity

In order to determine how to compute the intensity at some distance, \( L \), away from the source aperture, let us consider the Fresnel number, \( F = Ap^2 / (\lambda L) \), where \( Ap \) is the largest dimension (i.e. circumference) of the source aperture, and \( \lambda \) is the wavelength of the beam. This dimensionless quantity is a useful metric in optics for characterizing the influence of diffraction on beam propagation. For the case where \( F << 1 \) (i.e. \( > 10^{-2} \)), the beam is said to be in the ‘far-field.’ In the far-field, the effect of diffraction on the far-field intensity pattern can be calculated from the near-field (i.e. at the aperture) beam amplitude, \( E_0(x,z) \), and phase, \( \phi(x,z) \), functions by Fraunhofer diffraction, which is a simplified case of the more general Kirchhoff diffraction formula in the limit where \( L \) approaches infinity. The following section gives a mathematical description of Fraunhofer diffraction and discusses some of the implications of diffraction on far-field intensity of un-aberrated beams propagating from a circular aperture. Section 1.4.2 will provide a review of the effect of near-field aero-optic wavefront distortions to the far-field propagation problem.

1.2.2.1 The Airy Disk

Consider the simple case of a wavefront propagating from a circular aperture to a point some distance \( L \) away, where \( L \) is large enough that the Fresnel number \( F<< 1 \). At the aperture, the *aperture function* is defined as

\[
A(x,z) = A_0(x,z)e^{-i\phi(x,z)},
\]  

(1.9)

where \( x \) and \( z \) are the aperture coordinates, \( A_0(x,z) \) is the optical wave amplitude over the aperture, and \( \phi(x,z) \) is the phase of the optical wave over the aperture (Hecht, 2002).
Here the wave amplitude $A_0$ is related to the initial beam intensity profile across the aperture, and the phase function is related to the wavefront $W$ and its conjugate $OPD$:

$$\phi(x,z) = 2\pi W(x,z)/\lambda = -2\pi OPD(x,z)/\lambda.$$  

The complex amplitude in the far-field, $E(X,Z)$, where $X$ and $Z$ are the far-field spatial coordinates, is then calculated from the Fraunhofer diffraction equation (Hecht, 2002),

$$E(X,Z) = \iint_{Aperture} A(x,z)e^{-i2\pi\lambda(Xx+Zz)/\lambda} \, dx \, dz. \quad (1.10)$$

It is easily seen that the complex far-field amplitude is simply the Fourier transform of the complex wavefront aperture function. In the ideal case, illumination across the aperture is constant, $A = \text{const.}$, and the phase over the aperture is equal to zero; the solution to the Fraunhofer diffraction equation under these conditions is the Airy disk (Hecht, 2002).

The intensity function of the Airy disk in the far field is

$$I(X,Z) = I_0 \left[ 2J_1\left( \frac{2\pi}{\lambda} \left( \frac{X^2 + Z^2}{L} \right) \right) / \left( \frac{2\pi}{\lambda} \left( \frac{X^2 + Z^2}{L} \right) \right) \right]^2, \quad (1.11)$$

where intensity $I = EE^*$, $E^*$ is the complex conjugate of the complex wavefront amplitude, $J_1$ is the Bessel function of the first kind, and $I_0$ is the on-axis peak intensity (Fowles, 1975; Hecht, 2002). Peak intensity is also a function of the distance from the aperture $L$, aperture area, $A_{Ap}$, and the total power incident on the aperture, $P_0 = E_0^2\lambda^2A_{Ap}/2$ (Hecht, 2002):
Figure 1.4. Schematic of circular aperture and its corresponding far-field diffraction pattern, the Airy Disk.

\[ I_0 = \frac{P_0 A_{Ap}}{\lambda^2 L^2} \]  

It is emphasized that for a beam propagating from a circular aperture to the far field, the Airy disk is the solution for the ideal far-field performance. Even in this ideal case, with no aberrations imposed on the wavefront as it propagates to the far field, the $1/L^2$ functional dependence of far-field peak intensity $I_0$ is a considerable limitation on
the effective range of long-range optical systems. To increase the range in which on-target intensity will be sufficient for practical applications, system designers have sought to take advantage of the $1/\lambda^2$ dependence of $I_0$ by shifting to short wavelength laser (SWL) sources to power directed energy systems. Gilbert (2013) defined the SWL laser systems as having $\lambda < 2 \mu m$.

![Figure 1.5. Diffraction-limited far-field irradiance as a function of laser wavelength, $\lambda$.](image)

1.2.2.2 Effect of OPD on Far-Field Intensity

The performance of free space optical systems can be meaningfully quantified by defining the Strehl ratio $SR = I/I_0$, where $I$ is the on-target beam intensity in the far field, and $I_0$ is the diffraction-limited performance in the far field. In cases where aero-optic aberrations are present over optical apertures in the near-field, the phase argument of the aperture function is not equal to zero across the entire aperture. From equation (1.10, the
effect of near-field wavefront distortions on far-field beam intensity $I$ can be computed. If the beam initially has uniform intensity across the source aperture, then the Strehl ratio for the far-field on-axis intensity for the system with near-field aero-optic distortions $OPD(x, y, t)$ is (Born & Wolf, 1999)

$$SR(t) = \frac{I(t)}{I_0} = \frac{\left[ \int_{x,y} \exp \left[ i \frac{2\pi}{\lambda} OPD(x, y, t) \right] dx dy \right]}{\int_{x,y} dx dy}.$$

(1.13)

Figure 1.6 Effect of wavelength on Strehl Ratio for constant value of $OPD_{\text{rms}}$ (Jumper & Fitzgerald, 2001).

Assuming that $OPD(x, y, t)$ is normally distributed at each point in time, $t$, then equation (1.13) may be simplified as follows (Ross, 2009),

$$\frac{I}{I_0} = \exp \left[ i \frac{2\pi}{\lambda} \int_{x,y} OPD(x, y, t) dx dy \right].$$
\[ SR(t) = \exp \left[ - \left( \frac{2\pi OPD_{rms}(t)}{\lambda} \right)^2 \right]. \] (1.14)

This relation is known as the Maréchal approximation (Ross, 2009). It should be emphasized that as long as the assumption holds that the spatial statistical distribution of \( OPD \) is Gaussian, equation (1.13) is equivalent to (1.14) for any amplitude of \( OPD_{rms} \).

For cases in which the optical aperture is significantly larger than the characteristic turbulence scale in the aberrating flow it has been shown that equation (1.14) is approximately valid for all statistical processes, and that time averaged Strehl ratio may also be computed from the time-mean of \( OPD_{rms} \):

\[ \overline{SR} = \exp \left[ - \left( \frac{2\pi \overline{OPD_{rms}}}{\lambda} \right)^2 \right], \] (1.15)

where the overbar denotes temporal averaging (Steinmetz, 1982). This extension of the Maréchal approximation is known as the Large Aperture Approximation (LAA), and is generally only valid for \( OPD_{rms} = 0.1\lambda \) (Ross, 2009), although it has been shown to be valid up to \( 0.3\lambda \) under certain conditions (Porter, et al. 2013). In equations (1.13) through (1.15), the Strehl Ratio is a function of the \( OPD \) or \( OPD_{rms} \) relative to the wavelength of laser used in these systems. For a constant value of \( OPD_{rms} \), decreasing \( \lambda \) results in a significant decrease in the far-field Strehl ratio (see Figure 1.6).

Gilbert (2013) showed that in a real directed energy system, the real on-target intensity in the far field, \( I_{\text{Real}} \), can be represented as the product of the theoretical far-field intensity \( I_0 \) calculated from equation (1.12) and a two Strehl ratios expressing intensity losses resulting from different types of wavefront phase aberrations. Generally, this expression for real far-field intensity is given as
Figure 1.7. Comparison of far-field intensity versus wavelength for real and ideal directed energy systems.

\[ I_{\text{Real}} = I_0 \times \exp \left[ -\left( \frac{2\pi OPD_{\text{total}}}{\lambda} \right)^2 \right] \times \frac{1}{1 + \left( \frac{\theta}{\lambda} \right)^2} \]  

(Gilbert, 2013). The second term of this expression is the Strehl ratio for wavefront phase aberrations that have length scales \( \Lambda \) that are less than the Aperture diameter, \( Ap \), and the third term is the Strehl ratio expressing intensity losses due to wavefront phase aberrations with characteristic length scales \( \Lambda > Ap \) that result in some mean angular deflection for the entire beam (i.e. beam tip/tilt). Since \( Ap \) is typically greater than the turbulent length scales in the boundary layer aero-optic aberrations, and because wavefront errors associated with angular jitter are typically corrected using fast steering mirrors (FSMs), the effect of the latter term will be ignored for the remainder of this
work. The former term accounts for the effect of high-order phase aberrations from all manner of sources in a directed energy laser system. Gilbert (2013) presented a comprehensive list of sources of high-order wavefront aberrations, which includes, among other things, aberrations caused by aero-optic effects. Here, \( (\text{OPD}_{\text{rms}}^{\text{TOTAL}})^2 \) is equal to the sum of \( (\text{OPD}_{\text{rms}})^2 \) from each of the individual sources of wavefront error. To demonstrate the effect of these higher-order wavefront aberrations on far-field intensity, equations (1.12) and (1.16) are plotted as a function of wavelength in Figure 1.7.

It is immediately evident from the comparison in Figure 1.7 that high-order phase aberrations, such as those caused by TBL aero-optic effects, will substantially reduce the amount of laser energy that reaches the far field for SWL systems (that is, the ideal performance will not be achieved). It is also apparent that for a real SWL system there is some ideal wavelength \( \lambda^* \approx 2\pi\text{OPD}_{\text{rms}}^{\text{TOTAL}} \) at which the best system performance can be achieved (neglecting tip/tilt effects). One way to maximize the performance of a SWL directed energy system is to choose a laser source that closely matches the value of \( \lambda^* \) predicted for a system, so as to operate at the peak of the \( I_{\text{Real}}(\lambda) \) curve for a given system. However, if the level of high-order phase aberrations (i.e. total \( \text{OPD}_{\text{rms}} \)) used in the design of a SWL powered directed energy system is not precisely estimated, this ideal wavelength for an actual SWL system can deviate significantly from the value of \( \lambda^* \) used in the design process. This is illustrated in Figure 1.8, which shows the changes in \( I_{\text{Real}}(\lambda) \) when the total \( \text{OPD}_{\text{rms}} \) deviates from the value used in the design of a SWL system. For the case when the total \( \text{OPD}_{\text{rms}} \) from higher-order phase aberrations is larger than the \( \text{OPD}_{\text{rms}} \) used to design the SWL directed energy system, far field intensity can be
significantly reduced, especially at the design wavelength, $\lambda^*$. This reduced far-field intensity can result in a severe depreciation of system performance leading to marked reductions in range and effectiveness for SWL directed energy applications.

![Diagram showing the effect of OPD$_{rms}$ on the real far-field intensity, $I_{\text{Real}}(\lambda)$, as computed from equation (1.16).](image)

Figure 1.8. The effect of $OPD_{rms}$ on the real far-field intensity, $I_{\text{Real}}(\lambda)$, as computed from equation (1.16).

In the case where the total $OPD_{rms}$ of a SWL system is less than the value used in the design and choice of $\lambda^*$, the corresponding change in $I_{\text{Real}}(\lambda)$ will result in an increase in intensity in the far field. It should be noted though that in this scenario, the SWL system will not be operating at a wavelength corresponding to its actual peak efficiency (see Figure 1.8). Nonetheless, it is apparent from the analysis presented in Figure 1.8 and from equation (1.16) that reducing the total amount of high-order phase aberrations present in a SWL directed energy system will increase the achievable on-target intensity.
in the far field. This would yield both greater range and effectiveness for SWL systems; both of these goals are highly desirable for system designers. Therefore it is of interest for researchers to find means of reducing the total \( \text{OPD}_{\text{rms}} \) present along the optical path in order to improve the performance of directed energy systems. Some background on different approaches to this problem, including flow control methods for reducing aero-optic aberrations, has already been mentioned in Section 1.1. This dissertation will also focus on identifying and evaluating different passive flow control techniques that will achieve this end in the turbulent boundary layer.

1.2.2.3 Relating Changes in Strehl Ratio and \( \text{OPD}_{\text{rms}} \)

While Strehl ratio is a correct and practical metric for evaluating the far-field performance of airborne directed energy and communications systems, it is not the most suitable means of evaluating the effectiveness of aero-optic mitigation techniques. Equation (1.15) shows that the relationship between Strehl ratio and \( \text{OPD}_{\text{rms}} \) is non-linear, and is dependent on the particular laser wavelength, \( \lambda \), used in the system in question. Where either adaptive-optic or fluidic flow control aero-optic mitigation techniques are applied, the ‘corrected’ Strehl ratio, \( \text{SR}_{\text{corrected}} \), can be expressed as a function of the initial uncorrected Strehl ratio, \( \text{SR}_{\text{uncorrected}} \), and the ratio of \( \text{OPD}_{\text{rms, corrected}} \) to \( \text{OPD}_{\text{rms, uncorrected}} \) (Burns, Jumper, & Gordeyev, 2014):

\[
\text{SR}_{\text{corrected}} = \text{SR}_{\text{uncorrected}} \left( \frac{\text{OPD}_{\text{rms, corrected}}}{\text{OPD}_{\text{rms, uncorrected}}} \right)^2.
\]

(1.17)

It is immediately apparent from equation (1.17) the level of improvement in Strehl ratio resulting from some level of reduction in \( \text{OPD}_{\text{rms}} \) is highly dependent upon
the original uncorrected Strehl ratio, a quantity that is system dependent. Therefore, it is more practical to report the results of aero-optic mitigation techniques in terms of improvement in $OPD_{rms}$, rather than in Strehl ratio, since the relative improvement reported will be independent of the laser system used. These general results can be applied with ease to specific applications using equation (1.17), to translate a reduction in $OPD_{rms}$ achieved for a particular method to improvements in Strehl ratio for a specific airborne optical system.

1.3 Dissertation Organization

Chapter 2 of this dissertation provides a thorough overview of the current understanding of aero-optical aberrations caused by compressible, turbulent boundary layers. This chapter also provides some additional motivation and insight regarding investigations of passive flow control schemes for mitigating TBL induced aero-optic aberrations. A review of prior research on Large-Eddy Break-Up (LEBU) devices is presented in Chapter 3, and in Chapter 4 the effects of non-adiabatic wall temperatures on turbulent boundary layers is discussed. The wind tunnel facility and measurement techniques used to perform experimental measurements of passively modified TBLs are presented and discussed in detail in Chapter 5. The next two chapters present the results of experimental investigations of the aero-optics of LEBU devices. Detailed measurements of velocity and wavefront statistics are presented and compared in Chapter 6, while the effect of LEBU device configuration on wavefront measurements is discussed in Chapter 7. In Chapter 8, the experimental measurements of the effect of full and partial wall cooling on wavefront statistics are presented and compared to a modified statistical
model. Chapter 9 demonstrates a use for TBL flow control other than aero-optic mitigation; a scheme is presented that uses wall heating to passively amplify TBL wavefront distortions, and the effect of Reynolds numbers on TBL wavefront statistics is analyzed. In the final chapter, a summary of the major findings of this work is presented and discussed, and areas of research which merit additional research are also identified.
CHAPTER 2:

REVIEW OF THE AERO-OPTICS OF COMPRESSIBLE, TURBULENT
BOUNDARY LAYERS

Research on the effects of compressible, turbulent boundary layers (TBLs) on the propagation of light has been ongoing for more than sixty years, with analytical, experimental, and computational studies being performed by a significant number of authors during this time (Liepmann, 1952; Stine & Winovich, 1956; Rose, 1979; Wyckham & Smits, 2009; Cress, 2010; Wang & Wang, 2012; White & Visbal, 2012; Gordeyev, et al. 2014). This chapter provides a survey of the history of aero-optic investigations of boundary layers, as well as a review of important results from recent studies. This review will also highlight findings from these studies that will relate specifically to discussions in later chapters about potential aero-optic mitigation techniques for turbulent boundary layers.

2.1 Early Aero-Optics Research

The earliest known investigations of the effects of compressible, turbulent flow on the propagation of light can be traced to Liepmann (1952). In order to ascertain the effect that the side wall boundary layers of high-speed wind tunnels would have on the sharpness of Schlieren photographs, a theoretical formula was developed to predict the amount of angular jitter that would be induced on a ray of light as it passed through the
wind tunnel (Liepmann, 1952). Attempting to validate the results of Liepmann’s (1952) initial theoretical formulation, Stine and Winovich (1956) performed experimental photometric measurements on light traversing turbulent boundary layers. From these data, the authors found that the strength of TBL wavefront aberrations (i.e. the amount of radiant power scattered) was dependent on both the integral length scale and intensity of density fluctuations within the turbulent boundary layer. These results showed good agreement between the amount of optical scattering and turbulence characteristics, and caused Stine and Winovich (1956) to recognize that was possible to use measurements of optical aberrations to infer information about the turbulence scales present in an aberrating flow.

![Figure 2.1 Schematic of wavefront aberrations caused by density fluctuations in a turbulent boundary layer. [Flow visualization is from Corke, et al. (1979)].](image)
Building upon this and other early work, Sutton (1969) introduced the most widely cited theoretical formulation for calculating the effect of turbulent boundary layers on optical wavefronts from turbulence statistics. The equation derived by Sutton, referred to as the aero-optic ‘linking equation,’ was based heavily on Tatarski’s (1961) work describing the distortions of electromagnetic waves propagated through the Earth’s atmosphere. It related the time-averaged levels of $OPD_{\text{rms}}$ to turbulence density statistics. In its general form, the linking equation is given as

$$OPD_{\text{rms}}^2 = \frac{L L}{0 0} \text{cov}_\rho(y_1, y_2)dy_1dy_2,$$  

(2.1)

where $\text{cov}_\rho(y_1, y_2) = (\rho(y_1, t) - \rho(y_1, t))(\rho(y_2, t) - \rho(y_2, t))$ is the two-point density covariance function, with the overbar denoting temporal averaging. To simplify this expression, it is often assumed that the turbulent flow is homogeneous and isotropic, which permits the covariance function to be approximated using an analytic expression (Sutton, 1969; Hugo & Jumper, 2000). If an exponential form is assumed, the density covariance function becomes

$$\text{cov}_\rho(y_1, y_2) = \rho_{\text{rms}}^2 \exp\left\{-\left[\frac{(y_2 - y_1)^2}{\Lambda_\rho}\right]^{1/2}\right\},$$  

(2.2)

where $\rho_{\text{rms}}^2$ is the variance of the density fluctuations, and $\Lambda_\rho$ is the local density correlation length. Substituting equation (2.2) into (2.1), a simplified version of the linking equation can be obtained.

26
\[ \text{OPD}_{\text{rms}}^2 = 2K_{GD}^2 \int \rho_{\text{rms}}^2(y) \Lambda_{\rho}(y) dy. \] (2.3)

Jumper and Fitzgerald (2001) showed that Sutton’s linking equation is equivalent to Liepmann’s formulation if wavefront aberrations and their corresponding turbulent structures are assumed to convect with each other. This assumption was not proposed in the literature until the work of Malley, et al. (1992). Recent CFD studies of the aero-optics of TBL by Wang and Wang (2012), where wavefront aberrations were computed by integrating through the computed density field, have also shown good agreement predictions from the linking equation.

2.1.1 Strong Reynolds Analogy

One of the challenges of predicting aero-optic aberrations from turbulence density statistics is the lack of well developed techniques for making point measurements of mean and fluctuating density statistics. Typically, density based fluid measurement techniques are obtained via line-of-sight optical techniques such as interferometry, Shadowgraph, or Schlieren; by definition, these techniques are line-of-sight measurements which are affected by density fluctuations along the entire optical path, rather than at a single point or plane in the flow (Merzkirch, 2007). A technique called Focusing-Schlieren utilizes a novel approach that measures the effect of density fluctuations at a single focal plane in the flow, but the method is generally difficult to set up and use, and is only sensitive to the gradient of density in the flow (Weinstein, 1993; Merzkirch, 2007). Another recent and novel approach taken by Reid, et al (2013) at Auburn University used Planar Laser Induced Fluorescence (PLIF) to measure local density statistics in a turbulent wake formed downstream of a turret in transonic flow.
Since particle florescence is a function of temperature PLIF is typically used to measure temperature (Kowalewski, Ligrani, Dreizler, Schulz, Fey, & Egami, 2007), but using knowledge of local pressure, Reid, et al. (2013) modified this technique to compute density. While this approach is promising, its application is limited to flow regimes where PLIF is applicable, and requires additional knowledge of pressure in the turbulent flow.

One approach for getting around the considerable challenges of making density measurements in turbulent flow is to approximate density statistics from measured velocity statistics via the Strong Reynolds Analogy, or SRA (Morkovin, 1962). By assuming that pressure fluctuations, \( p' \), are negligible, the Reynolds-averaged form of the boundary layer momentum and energy equations for an adiabatic wall TBL can be simplified. This simplification yields a relationship between static temperature fluctuations, \( T' \), and fluctuations in the streamwise mean and fluctuating velocity components, \( U(y) \), and \( u'(y,t) \), respectively;

\[
T'(y,t) + \frac{U(y)u'(y,t)}{c_p} = 0,
\]

where \( c_p \) is the specific heat of the turbulent fluid. A more detailed discussion of this derivation can be found in Chapter 4. For isentropic flow, this relation can be re-written as

\[
\frac{T'}{T} = -(\gamma - 1)M^2 \frac{u'}{U},
\]

where \( M \) is the local Mach number. If \( p' \) is negligible, then it also follows from the ideal gas law that density fluctuations, \( \rho' \), and static temperature fluctuations are inversely proportional:
Combining (2.5) and (2.6) it is evident that the SRA implies that density fluctuations in the TBL are proportional to the product of the mean and fluctuating velocity: \( \frac{\rho'}{\bar{\rho}} = -\frac{T'}{T} \).

\[ \rho'(y) \sim U(y) u'(y). \]

Rose (1979) combined Sutton’s linking equation with the SRA to compute levels of \( OPD_{rms} \) indirectly from extensive hot-wire velocity profiles of TBLs. These indirect measurements of optical aberrations were supplemented by double-pulse interferometer measurements of wavefront aberrations obtained by Gilbert (1982). Although the data from Rose (1979) and Gilbert (1982) exhibited similar trends (i.e. \( OPD_{rms} \sim q \)), Masson, et al. (1994) found that there were substantial systematic differences between indirect and direct measurements of \( OPD_{rms} \). In spite of this apparent discrepancy between direct and indirect wavefront measurements of aero-optic effects, by the mid-1980s it was thought that the study of aero-optic phenomena was a mature discipline, and that the only necessary work in the field was to obtain turbulence statistics for a few additional aero-optically active flows (Sutton, 1985). It was also acknowledged at this time that as airborne optical systems moved to using shorter wavelength lasers (SWLs), aero-optic phenomena would become a more significant problem for airborne directed energy applications. As discussed in Chapter 1, this would require the reduction of \( OPD_{rms} \) for SWL systems in order to keep on-target intensity sufficiently high (Sutton, 1985; Gilbert, 2013).

The choice to move from mid/far-infrared wavelength lasers (\( \lambda \approx 10 \mu m \)) to shorter near-infrared wavelengths (\( \lambda \approx 1 \mu m \)) in airborne optical systems was made in
order to increase the systems’ diffraction limited range (Gilbert, 2013). As Sutton (1985) mentioned, the problems posed by aero-optically active turbulent flow surrounding an aircraft were then exacerbated for SWL systems because of the increased sensitivity to small wavefront phase errors at shorter wavelengths shown in section 1.2.2.2 (Gilbert, 2013). Consequently, research on the aero-optic effects of aberrating aerodynamic flows, including the turbulent boundary layer, saw a resurgence starting in the late 1990s. This research was focused on developing robust predictive models for levels of optical aberrations and on understanding the physical processes responsible for the bulk of optical aberrations. Then through an understanding of the fluid mechanics of the aberrating turbulent flows, methods of flow-control that would likely result in aero-optic mitigation could then be identified and tested (Jumper, 1980; Jumper & Fitzgerald, 2001; Gordeyev, et al., 2010; Vukasinovic, et al., 2013).

2.2 High-Bandwidth Aero-Optic Wavefront Sensors

The introduction of high-bandwidth wavefront sensing devices was of vital importance for advances in the aero-optic characterization of the turbulent boundary layer, and other aero-optically active aerodynamic flows. These two novel aero-optic wavefront sensors, the Malley probe 1-D wavefront sensor and the high-bandwidth Shack-Hartmann 2-D wavefront sensor, permitted the acquisition of wavefront measurements with high spatial and temporal resolution. The new sensors allowed for detailed study of the spatial and temporal characteristics of aero-optic wavefront measurements, which vastly improved on previous wavefront sensors that had been limited to averages of uncorrelated wavefront measurements.
2.2.1 Malley Probe 1-D Wavefront Sensor

Malley, et al. (1992) were the first to introduce a wavefront sensor capable of time-resolved measurements, which are now commonly referred to as the Malley probe 1-D wavefront sensor. The Malley probe measures the angle at which a small-aperture beam, which approximates a single ray of light, is deflected by turbulent flow as a function of time, $\theta(t)$. Since by definition, light rays propagate perpendicular to wavefronts, the deflection angle in a particular direction is equivalent to the slope of the wavefront in that direction. Therefore, in the streamwise direction, $\theta_z = \partial W/\partial x$, and in the spanwise direction, $\theta_y = \partial W/\partial y$. By assuming that wavefront aberrations and turbulent structures convect together at some convection velocity $U_C$, Malley, et al. established that a 1-D streamwise ‘slice’ of OPD could be reconstructed by first computing the OPL via the integral

$$OPL(x = -U_c t) = \int_0^t \frac{dOPL(\tau)}{dx} \frac{dx}{d\tau} d\tau = -U_c \int_0^t \theta_z(\tau) d\tau,$$

where $\tau$ is a placeholder for time; OPD is then computed using equation (1.5).
The first experiments using the Malley probe for aero-optic measurements of boundary layer induced aberrations were performed by Gordeyev, et al. (2003). This work was motivated by the apparent discrepancies between indirect and direct interferometer measurements of $OPD_{\text{rms}}$ from prior research, and by the use of shorter wavelength near-infrared lasers ($\lambda \sim 1 \ \mu m$) in airborne optical systems, as discussed in Chapter 1. These time resolved Malley probe wavefront measurements demonstrated the capability to generate consistent measurements that could be used to test existing and new scaling laws for a wide range of flow parameters in a relatively easy manner (Gordeyev S., Jumper, Ng, & Cain, 2003).

Fourier analysis of the deflection angle time series of each beam yields deflection angle amplitude spectra, $|\hat{\theta}(f)|$, which are related to the wavefront spectra via the Fourier version of equation (2.7):

$$\left|\hat{W}(k_x = 2\pi f / U_c)\right| = U_c \frac{|\hat{\theta}(f)|}{2\pi f}, \quad (2.8)$$

where $k_x = 2\pi f / U_c$ is a consequence of the frozen flow assumption, and $\hat{W}(k_x)$ and $\hat{W}(f)$ are the Fourier transforms of the 1-D wavefront aberrations $W(x)$ and $W(t)$, respectively. It was also shown that $U_c$ could be obtained directly from Malley probe deflection angle measurements by using two simultaneously sampled beams aligned in the flow direction. The spectral cross-correlation of the deflection angle signals from the two Malley probe beams is calculated,

$$S(f) = \left\langle \hat{\theta}_1(f)\hat{\theta}_2^*(f) \right\rangle. \quad (2.9)$$
Here $\hat{\theta}_1(f)$ and $\hat{\theta}_2(f)$ denote the Fourier transforms of the time series of deflection angle from the first and second beams (beam one is the upstream beam), and the star denotes the complex conjugate (Cress J., Gordeyev, Jumper, Ng, & Cain, 2007). From the slope of the phase argument of $S(f)$, the convective speed can be calculated as $U_C = \Delta/\tau$ for two beams separated by some distance $\Delta$, where the time delay $\tau$ is computed from the slope of the argument of the cross-correlation function $S(f)$:

$$2\pi\tau = \frac{d\{\text{Arg}[S(f)]\}}{df}.$$  \hfill (2.10)

Typical deflection angle spectra and cross-correlation phase curves obtained from the Malley probe are shown in Figure 2.3.

Early experimental measurements of aberrations from a single boundary layer (SBL) wavefront spectrum were shown to have a significant amount of low-frequency content due to mechanical vibration. These vibrations were largely due to the Malley probe return mirror being mounted on a strut inside the TBL test section, as shown in Figure 2.4a. To reduce the amount of low-frequency contamination in Malley probe measurements of a single boundary layer, later studies used an optical insert, shown in Figure 2.4b, to allow for the Malley probe return mirror to be mounted outside of the wind tunnel, isolated from mechanical vibrations in the test section (Cress, 2010). Double boundary layer (DBL) measurements, in which wavefronts are acquired for symmetric boundary layers on opposite walls of the test section, as shown in Figure 2.4c, were also used to reduce contamination from mechanical vibration, with the added benefit of an increased signal to noise ratio compared to SBL measurements (Wittich, et al. 2007, Cress 2010).
Figure 2.3: Typical results from Malley probe wavefront measurements; a) normalized TBL deflection angle spectra, and b) the phase of the cross-correlation, $\text{Arg}[S(f)]$, and convective velocity, $U_c$. 

$U_c = 0.82 U_\infty$
Figure 2.4. Boundary layer test section configured for Malley probe wavefront measurements: a) SBL configuration (Gordeyev, et al. 2003, Buckner, et al. 2005), b) SBL optical insert configuration (Cress 2010), and c) DBL Malley probe measurement configuration (Wittich, et al. 2007, Cress 2010).
Wittich, et al (2007) and Cress (2010) performed a number of experiments that directly compared results from SBL and DBL wavefront measurements. Results from these studies demonstrated that DBL and SBL wavefront measurements compared well after applying a statistical scaling, which assumed that the upper and lower wall boundary layers are statistically independent (Wittich, et al., 2007; Cress, 2010). Further details on this statistical scaling and its application to results are found in Appendix A.

Figure 2.5. Illustration of the Shack-Hartmann wavefront sensor operating principle.

2.2.2 Shack-Hartmann 2-D Wavefront Sensor

Shack-Hartmann wavefront sensors have been in use since the 1970s in the optics community for indirect wavefront measurements (Artzner, 1992; Platt & Shack, 2001). This wavefront sensor uses a microlens (or lenslet) array to measure the local gradient of a 2-D wavefront, averaged over the area of each microlens sub-aperture (see Figure 2.5); the actual OPD map can then be obtained by integrating the wavefront slopes across the
aperture (Platt & Shack, 2001). The low frame rates of commercially available systems like the Wavefront Sciences Shack-Hartmann type sensor (~ 30 frames per second) were found to be useful for obtaining ‘snapshots’ of aero-optic effects, but could not provide time-resolved wavefront measurements (Cress, 2010).

Figure 2.6. Illustrations of the first three Zernike modes: a) Piston, b) Tip (or X-Tilt), and c) Tilt (or Y-Tilt).

A novel approach to increase the bandwidth of Shack-Hartmann type wavefront sensors was introduced by Wyckham & Smits (2009), who coupled a microlens array with a high-speed CMOS camera in order to create a Shack-Hartmann wavefront sensor system capable of acquiring spatially resolved wavefront data at a much higher frame rate than had been seen before, up to 50,000 frames per second. The resulting wavefront
sensor from this novel development, now commonly referred to as the ‘high-bandwidth’ Shack-Hartmann wavefront sensor allowed for measurements of 2-D aero-optic aberrations with good spatial and temporal resolution. Throughout the remainder of this dissertation, all allusions to Shack-Hartmann wavefront sensors or 2-D wavefront sensors will be referring to the high-bandwidth Shack-Hartmann wavefront sensor.

Processing of image data acquired with the high-bandwidth Shack-Hartmann sensor gives time series of 2-D wavefront measurements, $W(x,z,t)$ (Abado, Gordeyev, & Jumper, AAOL Wavefront Data Reduction Approaches, 2012). During this processing, the instantaneous piston, tip, and tilt, which are classically defined by the first three Zernike modes, $Z_j$ (Born & Wolf, 1999), are calculated for each wavefront realization. Piston is the mean value of the wavefront across the entire aperture, while tip and tilt are the average slope of the wavefront in the streamwise ($x$) and spanwise ($z$) directions, respectively, computed from a least-squares fit of a flat line (in 1-D wavefront measurements) or a plane (in 2-D wavefront measurements) to each wavefront realization (Siegenthaler, 2008). In the context of aero-optics measurements, wavefront tip/tilt modes can be related to both mechanical vibration and fluid-optic interaction. By dimensional arguments, however, any fluid-induced phenomenon related to beam tip/tilt must have a length scale that is several times the beam aperture size, making it very difficult to discern whether the tip/tilt is due to aero-optic phenomenon or mechanical vibration. Moreover in practical applications tip/tilt is often corrected using stand-alone fast-steering mirrors, or FSMs. FSMs are analog devices that have response rates on the order of 1 kHz (Tapos, et al., 2005). Since tip/tilt can be corrected for with these devices, and these modes cannot be uniquely linked to aero-optic phenomenon, it is useful to
remove them entirely from wavefront measurements. Piston is also removed, since it simply acts as a scalar offset of $OPL$. Steady lensing, which is defined as the time-average of $OPD(x, z, t)$, can also be removed from each instantaneous wavefront realization in cases where some imperfection in optical components or windows imparts a stationary wavefront pattern on the measurements.

After undergoing this processing, the time series of 2-D wavefront measurements are analyzed using a number of statistical measures. The root mean square wavefront error, $OPD_{rms}$, can be computed as a function of time by equation (1.8), or as a function of space via equation (1.7) (De Lucca, Gordeyev, & Jumper, 2013). Spatial and temporal wavefront spectra, correlation functions, and spectral cross correlations can also be computed.

![Diagram of a high-bandwidth Shack-Hartmann wavefront sensor](image)

Figure 2.7: Schematic of a typical high-bandwidth Shack-Hartmann wavefront sensor in a double-pass, DBL measurement configuration.

Not long after Wyckham & Smits’ work was published, the Airborne Aero-Optics Laboratory (AAOL) program at the University of Notre Dame began exploring a number of different high-bandwidth wavefront sensing options. A Phantom v711 high-speed
A CMOS camera from Vision Research was primarily used on the AAOL, due to its high sampling rates and onboard flash memory storage that allowed for rapid offloading from the camera’s onboard memory (Jumper E. J., Zenk, Gordeyev, Cavalieri, & Whiteley, 2013). Recently this camera has been upgraded with a v1610 high-speed CMOS camera for the next phase of the AAOL program. In addition to being used extensively for the AAOL program (De Lucca, Gordeyev, & Jumper, 2013), this particular Shack-Hartmann sensor was used to acquire TBL aero-optic wavefront measurements for several studies comparing results to Malley probe wavefronts (Smith, Gordeyev, & Jumper, 2013; Gordeyev, Smith, Cress, & Jumper, 2014).

Figure 2.8. Schematic of a typical high-bandwidth Shack-Hartmann wavefront sensor in a single-pass, SBL measurement configuration.

Similarly to the Malley probe wavefront sensor, high-bandwidth Shack-Hartmann wavefront measurements can also be made using a double-pass, DBL configuration, shown in Figure 2.7, or a single-pass, SBL configuration. This latter method was used by Wyckham & Smits (2009) in the initial use of the high-bandwidth Shack-Hartmann wavefront sensor using a slightly diverging source beam emanating from a fiber optic.
cable, as seen in Figure 2.8. In either the DBL or SBL configuration, simple adjustment of the beam expander optics allows for measurements to be obtained for different aperture diameters, $A_p$.

**TABLE 2.1**

**COMPARISON OF TYPICAL HIGH-BANDWIDTH SHACK-HARTMANN WAVEFRONT ACQUISITION PARAMETERS.**

<table>
<thead>
<tr>
<th>Authors</th>
<th>Frame Resolution (Pixels)</th>
<th>Lenslet Resolution</th>
<th>Frame Rate [fps]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wyckham &amp; Smits (2009)</td>
<td>$752 \times 752$</td>
<td>$17 \times 17$</td>
<td>1,500</td>
</tr>
<tr>
<td></td>
<td>$1128 \times 20$</td>
<td>$17 \times 1$</td>
<td>50,000</td>
</tr>
<tr>
<td>Smith, et al. (2013)</td>
<td>$1024 \times 512$</td>
<td>$60 \times 60$</td>
<td>9,500</td>
</tr>
<tr>
<td></td>
<td>$512 \times 512$</td>
<td>$30 \times 30$</td>
<td>25,000</td>
</tr>
<tr>
<td></td>
<td>$512 \times 256$</td>
<td>$30 \times 15$</td>
<td>49,000</td>
</tr>
</tbody>
</table>

Table 2.1 presents sampling rates and spatial resolution for the camera used by Wyckham and Smits, as well as for a number of studies that utilized the Phantom camera wavefront sensor from Smith, et al. (2013). For the Phantom camera, there is an increase in both spatial and temporal resolution by an order of magnitude over the sensor used by Wyckham and Smits in their experimental work. Also, 2-D wavefront measurements in Smith, et al. (2013) utilized the double-pass, double boundary layer (DBL) optical setup, which gave a substantial increase in signal-to-noise ratio. Comparisons of 1-D Malley probe, and 2-D Shack-Hartmann wavefront measurements have shown good agreement for turbulent boundary layers in the transonic ($M_\infty = 0.4 - 0.7$) regime when data were
compensated for the effects of finite aperture size (Smith, Gordeyev, & Jumper, 2013). This agreement validates the accuracy of the underlying assumptions of the Malley probe wavefront reconstruction for TBL measurements, although these comparisons have highlighted the capabilities and limitations of the 1-D and 2-D wavefront sensors.

2.2.3 High-Bandwidth Wavefront Spectra Processing

Temporal and spatial wavefront spectra obtained from either the Malley probe or Shack-Hartmann high-bandwidth wavefront sensors can be used to calculate a number of useful statistics, including average levels of \( OPD_{\text{rms}} \). Since the wavefront is the conjugate of \( OPD \), \( OPD_{\text{rms}} \) may be calculated from the 1-D wavefront amplitude spectrum, \( |\hat{W}| \):

\[
OPD_{\text{rms}}^2 = \frac{1}{\pi} \int_0^{\infty} |\hat{W}(k_x)|^2 dk_x = \frac{2}{U_c} \int_0^{\infty} |\hat{W}(f)|^2 df .
\]  

Therefore it follows that it is possible to calculate \( OPD_{\text{rms}} \) directly from deflection angle amplitude spectra, by substituting (2.8) into (2.11):

\[
OPD_{\text{rms}}^2 = 2U_c \int_0^{\infty} \left| \hat{\theta}(f) \right|^2 df .
\]  

A closer inspection of equation (2.11) and equation (2.12) reveals that these expressions are assumes that the optical aperture is of infinite length in the streamwise direction. This arises from using the frozen flow assumption to trade time for space in the derivation of these equations. However, it is well established that the boundary layer is developing in the streamwise direction, and thus the ‘infinite aperture’ assumption will break down as boundary layer statistics begin to vary significantly in the streamwise direction over an optical aperture. Therefore, finite aperture effects on wavefront spectra
must be considered. A method for estimating the amount of streamwise variation in TBL statistics for a given TBL Malley probe measurement location is presented in Appendix B, and can be used to help determine the upper limit on finite aperture size for particular measurement configurations.

Siegenthaler (2008) performed a thorough investigation of the effect of finite apertures on wavefront statistics. It was shown that the removal of tip/tilt and piston components from wavefronts for finite-aperture beams functions as a high-pass spatial filter. As a result, this process may reduce the overall level of aero-optical distortions computed from a tip/tilt removed wavefront measurement. To illustrate this effect, two simulated sinusoidal wavefronts are shown in Figure 2.9 with identical finite apertures and amplitudes, but different wavelengths. A wavefront containing structures significantly smaller than the aperture diameter is shown in Figure 2.9, top plot. A linear fit to this signal over the aperture shows only a small amount of tip/tilt, and the removal of tip/tilt only causes a minor change to the characteristics of the wavefront. In contrast, Figure 2.9, bottom plot, shows the case where tip/tilt is removed from a wavefront where \( Ap \) and structure size are close in magnitude. Here, a significant amount of the sinusoidal signal is removed along with tip/tilt is calculated from the linear-fit over the aperture, and the amplitude of the tip/tilt-removed waveform is significantly reduced. This emphasizes how, for structures of size \( \Lambda \), tip/tilt removal over some finite aperture \( Ap \) will reduce or remove the effect of wavefront aberrations that have length scales greater than \( Ap \).
Figure 2.9. Examples of the spatial filter effect of finite apertures and tip-tilt removal, with a) $Ap/\Lambda = 3.25$ and b) $Ap/\Lambda = 0.72$. 
Through this high-pass filtering effect (or finite aperture effect), the amount of $OPD_{rms}$ in the measured wavefronts is reduced. This is modeled in the 1-D wavefront energy spectrum through the addition of a one-dimensional wavefront aperture function term, $|\hat{W}(f)|^2 \rightarrow |\hat{W}(f)|^2 \cdot AF(Ap,f)$. The transfer function $AF(Ap,f)$, which is given in Gordeyev, et al. (2014), is a high-pass filter that captures the tip/tilt removal effects for 1-D wavefronts, $W(x = U_c t, z = \text{const.})$. Then, equation (2.12) becomes to be modified to include this aperture function:

$$OPD_{rms}^2(Ap) = 2U_c \int_0^\infty AF(Ap,f) \left( \frac{\hat{\theta}(f)}{2\pi f} \right)^2 df = 2U_c \int_0^\infty TF(Ap,f) \hat{\theta}(f)^2 df \quad (2.13)$$

where the transfer function $TF(Ap,f) = AF(Ap,f)/(2\pi f)^2$, which relates $OPD_{rms}(Ap)$ and deflection angle spectra, is a band-pass filter with the low-frequency filtering being a result of aperture effects, and the high-frequency filtering resulting from the $1/f$ relation between deflection angle and wavefront spectra (Siegenthaler, 2008; Gordeyev, et al. 2014). Analyzing the cumulative transfer function,

$$CTF(f) = \int_0^f TF(Ap,\xi) d\xi / \int_0^\infty TF(Ap,\xi) d\xi \quad ,$$

it becomes evident that the range $St_{Ap} = f \times Ap / U_\infty = 0.28$ to 20 contributes 95% of $OPD_{rms}^2(Ap)$ for any aperture size, which means for the ranges $St_{Ap} < 0.28$ and $St_{Ap} > 20$, the exact shape of the filter applied to deflection angle data is not important so long as all non-aero-optical components of the signal, like electronic interference or mechanical vibrations, are blocked by the filter.
2.3 Aero-Optical Properties of Turbulent Boundary Layers

The following sections will give an overview of recent experimental and computational results of investigations of the aero-optic effect of turbulent boundary layers. Particular emphasis is placed on results that shed light on the physical mechanisms responsible for wavefront distortions caused by TBLs. This understanding of the physics of TBL aero-optic phenomenon will later be utilized to identify passive flow control techniques that may be effective for aero-optic mitigation.

2.3.1 Convective Velocity

It was Malley, et al. (1992) that first identified that aero-optic wavefront aberrations should travel at the same convection velocity, $U_C$, as the corresponding aberrating flow. In early 2-beam Malley probe experiments at the University of Notre Dame, (Gordeyev, et al., 2003; Wittich, et al., 2007) found $U_C$ to be $0.82 - 0.85U_\infty$ for subsonic boundary layers. Measurements of convective velocity from surface pressure measurements in subsequent experiments were also in good agreement with optically measured convection velocities (Buckner, et al. 2005). Using refined Malley probe measurement techniques along with the high-bandwidth Shack-Hartmann wavefront sensor, the range of optically measured convective velocities was narrowed to $0.82 - 0.83U_\infty$ in subsonic compressible TBLs (Cress, 2010; Smith, et al. 2013), although several models suggest that $U_C$ is a function of Mach numbers(Gordeyev, Jumper, & Hayden, Aero-Optics of Supersonic Boundary Layers, 2012). Simulation of aero-optic effects of TBLs at $M = 0.5$ by Wang, et al. (2012) also showed that wavefronts convected at a constant speed of approximately $0.82U_\infty$. These velocities are consistent with non-optical measurements of convective velocity of large-scale structures in the turbulent
boundary layer using surface mounted unsteady pressure sensor arrays (Willmarth & Wooldridge, 1962; Bull, 1976) and multiple hot-wires (Udin, et al., 1997). These results together provide strong indirect evidence that the turbulent structures that are most aero-optically active (i.e. responsible for most of the aberrating effects) are large, and that they reside in the outer part of the TBL.

2.3.2 Wavefront Spectra

Analyses of wavefront and deflection angle spectra obtained from high-bandwidth wavefront measurements have also been critical for characterizing TBL aero-optic aberrations. Figure 2.10 a typical deflection angle spectrum from a Malley probe is presented along with two deflection angle spectra computed from tip/tilt removed Shack-Hartmann wavefront data obtained using two different aperture sizes, $Ap/\delta = 3.54$ and 2.21; these data were taken from Gordeyev, et al. (2014). The sharp peaks at $St_\delta = 4$ and 10 in the Malley probe spectrum are electronic-noise-related, while the small energy buildup on the high-frequency end of the SHWFS data is due to spectral aliasing, as the wavefront frequency resolution of 50 kHz is not high enough to resolve all features which are aero-optically active within the flow. In Figure 2.10a, it is evident that at low frequencies, there is also significant deviation between spectra which is dependent on aperture size, as the removal of global tilt from wavefront data acts as a high-pass filter on the local deflection angle with the transfer function

$$G_A(z) = \left[ 3 \sin(\pi z) - 3 \pi z \cos(\pi z) \right]/(\pi z)^3,$$  \hspace{1cm} (2.15)

where $z = Apf/U_C = (Ap/\delta) (U_{\infty}/U_C) St_\delta = 1.20 (Ap/\delta) St_\delta$ (De Lucca, Gordeyev, & Jumper, 2012).
Figure 2.10: Comparison between the (a) original streamwise deflection angle amplitude spectra computed from 2-D wavefront data for different apertures and (b) compensated for aperture effects and the Malley probe deflection angle spectrum [image source Gordeyev, et al. (2014)].
Using the transfer function equation (2.15), the finite-aperture deflection angle data can be ‘corrected’ for aperture effects, as shown in Figure 2.10b. Here the corrected deflection angle spectra from 2-D wavefront data collapse better at the low frequency range for different aperture sizes and the low-frequency end of the amplitude spectra was found to have a slope proportional to $f$ (Gordeyev, Smith, Cress, & Jumper, 2014). The Malley probe spectrum has more energy at the low end of the spectrum, compared to the spectra from 2-D wavefront data, because it only measures the local deflection angle and inevitable contamination from mechanical vibration cannot be properly removed from the Malley probe data. This very low end would affect wavefront statistics only for $Ap > 10\delta$, where the presented spectra-based analysis would fail anyway due to streamwise TBL evolution effects, which are discussed in Appendix B. On the other hand, mechanical vibrations add only a global tilt component to wavefronts and easily can be removed during data post-processing.

At the high-frequency end of the spectrum, the slope of the deflection angle spectra is approximately $\tilde{\phi} \sim f^{-2/3}$, which has already been seen in aero-optic characterizations of other turbulent flows (Abado, Gordeyev, & Jumper, 2012). This $f^{-2/3}$ behavior, which for wavefront spectra corresponds to $\tilde{W} \sim f^{-5/3}$ (since $\tilde{W} \sim \tilde{\phi} / f$) is a consequence of the dominance of Kolmogorov-type turbulence at small scales in turbulent flows. Tatarski(1961) showed that if the optical distortions are due to pressure or temperature fluctuations, which are proportional to a square of the velocity fluctuations, $(u')^2$ then the spectral density for two-dimensional wavefronts goes as $\Phi (k)$
\( \sim k^{-13/3} \) for large wavenumbers. Since the one-dimensional spectral density, \( \hat{W}(k) \), and two-dimensional spectral density, \( \Phi(k) \), are related by the expression,

\[
\Phi(k)dk \sim \Phi(k)dk = \hat{W}(k)^2 dk ,
\]

(2.16)

it follows that the one-dimensional amplitude wavefront spectrum should behave as

\[
\hat{W}(k) \sim (\Phi(k)k)^{1/2} = (k^{-13/3}k)^{1/2} = k^{-5/3}
\]

(2.17)

for large wavenumbers. Using the frozen flow assumption, this behavior can also be described as \( \hat{W}(k) \sim f^{-5/3} \) for high frequencies. Based on these observations, a new empirical fit is proposed for the streamwise deflection angle spectra:

\[
\hat{\theta}_{fit}(St_\delta) = \hat{\theta}_{peak} \frac{(St_\delta)}{1 + (St_\delta/0.75)^{5/3}} ,
\]

(2.18)

where \( \hat{\theta}_{peak} \) is the peak amplitude, which can be computed from \( OPD_{rms} \) via equation (2.12). As seen in Figure 2.10, the empirical fit does a good job of modeling the measured deflection-angle spectrum in the area of the peak location, and in both the low and high end of the spectrum.

From the relation given in equation (2.8), the deflection angle amplitude spectrum is analogous to the pre-multiplied wavefront amplitude spectrum, \( \hat{\theta} \sim f \hat{W} \), and therefore is indicative of the contribution of different frequencies to overall levels of \( OPD_{rms} \) (or the standard deviation of \( OPD \)). In Figure 2.10b, it is evident from both Malley probe and Shack-Hartmann deflection angle spectra that the maxima of the deflection angle spectra is near \( St_\delta = f\delta/U_\infty = 1 \). The square of the deflection angle spectrum is compared to the truly pre-multiplied wavefront energy spectrum, \( f \cdot [\hat{W}(f)]^2 \),
in Figure 2.11, which shows the ‘optical energy,’ or \((\text{OPD}_{\text{rms}})^2\) as a function of frequency. The pre-multiplied wavefront energy spectrum also shows that low frequencies \((St_\delta < 1)\) contain most of the optical energy. These results also support the notion that the turbulent structures in the TBL which play a dominant role in aero-optic aberrations are large (i.e. on the order of the boundary layer thickness), and are located in the outer part of the TBL. While the pre-multiplied wavefront spectra is useful for corroborating that large-scale structures contribute most to the total amount of \(\text{OPD}_{\text{rms}}\) in the TBL, inspection of the deflection angle spectra is more useful for investigating the turbulence structure (e.g. spectrum peak around \(St_\delta = 1, \sim f^{(-2/3)}\) Kolmogorov-like high-frequency decay). Therefore, aero-optic studies of TBLs typically present deflection angle spectra in order to clearly see the relationship between turbulence structure and wavefront aberrations.

![Figure 2.11. Comparison between deflection angle energy spectrum and pre-multiplied wavefront energy spectrum.](image)

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The dominance of large, $O(\delta)$, turbulent scales to overall TBL aero-optic distortions can also be inferred through examining the calculation of $OPD_{rms}$ from deflection angle spectra via equation (2.12). If a lower frequency bound, $f_{low}$, is defined for the integration of $\hat{\theta}(f)$, then the amount of optical energy contained above that frequency is defined as

$$OPD_{rms}^2 (f > f_{low}) = 2U_C^2 \int_{f_{low}}^{\infty} \frac{|\hat{\theta}(f)|^2}{(2\pi f)^2} df .$$  \hspace{1cm} (2.19)

Figure 2.12 shows the result of equation (2.19) for the spectra model equation (2.18) and Malley probe deflection angle spectra over a wide range of normalized lower-bound frequency values, $S_{\delta}^{low} = f_{low} \delta/U_x$. Assuming frozen flow to relate this lower frequency bound to wavenumber, $k_{\delta}^{low}/2\pi = f_{low}/U_C$, the estimated ‘wavelength’ or...
structure size, \( l \), of turbulent scales above flow is then \( l = U_C/f_{\text{low}} \), and by extension, \( l/\delta \approx 1/St_{\delta \text{low}} \). The results in Figure 2.12 show that frequencies \( St_{\text{low}} > 1 \) (which correspond to structures smaller than the boundary layer thickness) only contribute about 20\% to the total amount of optical energy. As the lower-bound of integration in frequency increases, the relative contribution of small scale structures continues to decrease, such that structures of sizes \( l < \delta/2 \) (i.e. \( St_{\delta \text{low}} > 2 \)) contribute only about 6\% to the overall optical energy measured in the TBL. This result gives strong statistical evidence both from the spectrum model equation (2.18) and experiments that small-scale turbulent structures are not significant sources of wavefront aberration in the TBL (Smith, Gordeyev, & Jumper, Aperture effects on Aero-Optical distortions caused by subsonic boundary layers, 2012). Results of LES simulations of the aero-optic effects of TBLs also demonstrated the relative unimportance of small-scale structures by using different optical computational grids to low-pass filter simulated wavefront results (Wang & Wang, 2012).

It is emphasized, however, that this finding does not suggest that small turbulent scales need not be modeled in boundary layer aero-optic simulations. The non-linear nature of the Navier-Stokes equations underscores the fact that small-scale turbulent fluctuations have considerable influence on the characteristics of large-scale structures in turbulent flow (Mathieu & Scott, 2000, and others). As a result, small-scale structures must be either included or properly modeled in any simulations that seek to accurately capture the structure and dynamics of the large-scale structures and their effect on TBL aero-optic aberrations.
2.3.3 Wavefront Correlations

From the deflection angle spectra, \( \hat{\theta}(f) \), several important statistical properties of the boundary layer can be computed besides \( \text{OPD}_{\text{rms}} \). The aperture function, \( G(Ap, \delta) \), is defined as

\[
G\left( \frac{Ap}{\delta} \right) \equiv \frac{\text{OPD}_{\text{rms}}(Ap)}{\text{OPD}_{\text{rms}}}, \tag{2.20}
\]

where \( \text{OPD}_{\text{rms}} \) and \( \text{OPD}_{\text{rms}}(Ap) \) are computed via equations (2.12) and (2.13), respectively. From the wavefront-deflection angle spectra relation in equation (2.8), the streamwise wavefront autocorrelation function can be computed from deflection angle spectra:
\[ R(\Delta x / Ap; Ap) = \int_{f_{low}}^{\infty} K(f Ap / U_x, \Delta x / Ap) \frac{\hat{\theta}(f)^2}{(2\pi f)^2} df, \]  

where the transfer function \( K \) is given in Gordeyev, et al. (2014). Comparisons of streamwise wavefront correlation functions computed from spectra via equation (2.21), and correlation functions computed directly from spatially resolved 2-D wavefront measurements have shown good agreement when finite aperture effects are accounted for (Gordeyev, et al. 2014).

2.3.4 Models for TBL Aero-Optic Aberrations

Rose (1979) and Gilbert (1982) performed some of the earliest attempts at developing a statistical model of aero-optic wavefront distortions from in the turbulent boundary layer from experimental measurements. Rose (1979) combined Sutton’s linking equation (2.3) with the SRA to compute levels of \( \text{OPD}_{rms} \) indirectly from extensive hot-wire velocity profiles of TBLs. These indirect measurements of optical aberrations were supplemented by double-pulse interferometer measurements of wavefront aberrations obtained by Gilbert (1982). Masson, et al. (1994) undertook detailed comparisons of indirect wavefront measurements of \( \text{OPD}_{rms} \) from Rose (1979) and direct measurements of Gilbert (1982) and showed that in both cases, levels of optical aberrations scaled with \( \text{OPD}_{rms} \sim (\rho M_x^2)^\eta \), where \( M_x \) is freestream Mach number. Velocity measurements from Rose indicated that \( \eta \approx 1 \), while Gilbert’s interferometer measurements showed \( \eta \approx 1.16 \) (Masson, Wissler, & McMackin, 1994). Although the comparison demonstrated consistent trends between direct and indirect measurements, the systematic differences between the magnitudes of hot-wire and interferometer wavefront error measurements...
were not sufficiently explained. As designers of airborne directed energy systems aspirations shifted towards using SWLs, this discrepancy between predictive models presented an obstacle for appropriately predicting the total \( OPD_{\text{rms}} \) from higher-order wavefront distortions.

Motivated by the need to resolve this issue, and bolstered by the development of Malley probe high-bandwidth wavefront sensor, Gordeyev, et al. (2003) found that for subsonic, compressible TBLs,

\[
OPD_{\text{rms}} \propto \frac{\rho_\infty \delta^*}{\rho_{SL}} M^2, \tag{2.22}
\]

where \( \delta^* \) is the boundary layer displacement thickness, \( \rho_\infty \) is the freestream density, and \( \rho_{SL} \) is the density at sea level. This model was initially developed from dimensional scaling arguments rooted in the assumption that optical aberrations were primarily caused by pressure variations in coherent Lamb-Oseen-like vortices in the outer part of the TBL (Gordeyev, et al., 2003). This analysis is similar to the model used for aero-optic distortions in compressible shear layers (Fitzgerald & Jumper, 2004). This early model was justified by the good agreement found between convective velocity measurements obtained independently from Malley probe wavefront and unsteady wall pressure measurements in the TBL (Buckner, Gordeyev, & Jumper, 2005).

Wyckham & Smits (2009) proposed an alternative approach to modeling TBL wavefront distortions. Coupling the Strong Reynolds Analogy with a bulk flow analysis, Wyckham & Smits derived the following model for \( OPD_{\text{rms}} \):

\[
OPD_{\text{rms}} = C_w K_{GD} \rho_\infty \delta M^2 \sqrt{C_f f_2^{-3/2}}, \tag{2.23}
\]
where \( C_w \) is an empirical constant, and \( C_f \) is the wall friction coefficient. For adiabatic walls, the temperature ratio was found to be

\[
 r_z = 1 + \frac{\gamma - 1}{2} M^2 \left[ 1 - \left( \frac{U_c}{U_\infty} \right)^2 \right],
\]

(2.24)

where \( r \) is the recovery factor. This model is valid for both subsonic and supersonic Mach numbers, and interestingly can be shown to be equivalent to the model presented in equation (2.22) (Gordeyev, Smith, Cress, & Jumper, 2014). It was also reported that for non-adiabatic walls, the temperature ratio would take the form

\[
 r_z = \frac{1}{2} \left( \frac{T_w}{T_\infty} + 1 \right),
\]

(2.25)

where \( T_w \) is the wall temperature. Equation (2.25) was found by assuming that the static temperature at the intermediate point \( y/\delta = 0.5 \) in the boundary layer is equal to the average of the freestream and wall temperatures: \( T(0.5\delta) = (T_w + T_\infty) / 2 \) (Wyckham & Smits, 2009). A quick examination of equations (2.23) and (2.25) will reveal that this model predicts a reduction in \( OPD_{rms} \) for increased wall temperature; however no experimental data was presented to validate this part of the model. Accompanying experimental measurements for adiabatic wall TBLs did indicate that the primary source of aero-optic aberrations are large-scale, order \( \delta \), turbulent motions in the outer layer, and that \( C_w \) was found to be in the range of 0.7 to 1.0 for Mach numbers between 0.8 and 7.8 (Wyckham & Smits, 2009). Unfortunately, the large signal-to-noise ratio in this groundbreaking wavefront sensor placed sizeable error bars on their experimental data, and made validation of their model difficult. However, subsequent experiments and statistical models would demonstrate that the return to the SRA was the correct approach for modeling TBL wavefront aberrations (Gordeyev, Smith, Cress, & Jumper, 2014).
Gordeyev, et al. (2014) used the SRA to derive a density-velocity relationship by assuming pressure fluctuations were negligible and using mixed scaling for velocity. The mixed velocity scaling used Morkovin scaling for the fluctuating velocity component (i.e. $g(y/\delta) \sim u_{rms}/U_{t}$) (Morkovin, 1962) and assumed a self-similar mean velocity profile, $f(y/\delta) = U(y)/U_{\infty}$ (Gordeyev, et al. 2014). The resulting expression for $\rho_{rms}(y)$ was then substituted into the simplified form of the linking equation (2.3), giving the following model for $OPD_{rms}$ in compressible TBLs:

$$\text{OPD}_{rms} = BK_{GD}\rho_{\infty}M^{2}\delta \sqrt{C_{f}} G(M),$$

(2.26)

where $G(M) \equiv 1 - 0.19M^{2} + 0.03M^{4}$, which accurate to within ±5% for $M < 1.5$ (Gordeyev, et al. 2014). Estimates of the model constant $B$ from experimental data found
Figure 2.15. a) Schematic of beam propagation at oblique elevation angle, and b) plots of $B(\gamma)$ for SBL data versus elevation angle, $\gamma$ [image source Gordeyev, et al., 2014].
\[ B = 0.19 \pm 0.01, \] which shows good agreement with the numerically computed value, \[ B_{\text{model}} = 0.19. \] This model was also shown to give very similar predictions compared to model equation (2.23), but with a different constant, \( C_w. \)

Experimental and computational investigations of large-scale structures in the TBL have firmly established that these vortical structures (i.e. hairpin vortex packets) have a preferred inclination angle with respect to the wall (Robinson, 1991; Adrian, et al., 2000; Hutchins, et al., 2005; Adrian, 2007; Wang, et al., 2012) ; see Figure 2.14. In light of this, it is expected that TBL aero-optic distortions should exhibit an anisotropic behavior for different oblique beam propagation angles. This anisotropy in wavefront aberrations has been observed both in TBL simulations by Truman & Lee (1990), Wang, et al. (2012), and White & Visbal (2012), and in experimental measurements (Gordeyev, Smith, Cress, & Jumper, 2014). Results for both experiments and numerical simulations show similar anisotropic dependence, as shown in Figure 2.15, where discrepancies between data sets can be attributed to Reynolds number effects. A simple isotropic modification to the model equation (2.26) was proposed, where becomes \( B(\gamma) = \frac{0.19}{\sin(\gamma)}, \) and it was shown that this shows satisfactory agreement with experimental data and some CFD in a limited range of elevation angles, \( 70^\circ < \gamma < 120^\circ. \) The anisotropic relationship between \( OPD_{rms} \) and elevation angle coupled with the knowledge that large-scale structures in the TBL display a preference for a particular inclination angle gives additional support to the theory that TBL aero-optic distortions are mainly caused by large-scale turbulent structures.

The results of experiments and simulations investigating the effect of non-adiabatic walls (e.g. wall heating and cooling) TBLs on aero-optic aberrations have been
performed as well (Cress, 2010; White & Visbal, 2012). Results of these studies showed that increasing wall temperature caused TBL wavefront aberrations to be *passively* amplified, with no noticeable effects on the turbulence structure in the heated wall TBL (Cress, 2010; White & Visbal, 2012). The increase in $OPD_{rms}$ with wall heating is contrary to the behavior predicted by Wyckham & Smits (2009) using the SRA to derive in equations (2.23) and (2.25). The findings of Cress (2010) and White & Visbal (2012) were also consistent with prior research on boundary layer heat transfer, which had already shown that the assumptions of the SRA were not valid for non-adiabatic wall TBLs, since total temperature was assumed to not fluctuate in the model (Spina, et al., 1994). An alternate form of the Reynolds analogy in which total temperature was allowed to fluctuate was developed by van Dreist (1951), which is now referred to as the Extended Strong Reynolds Analogy (ESRA) (Schlichting & Gersten, 2000; Smits & Dussauge, 2006).

Cress (2010) developed a modified aero-optic model that used the ESRA to model the effect of non-adiabatic walls on aero-optic distortions (van Dreist, 1951; Cress, 2010). This new model, which is discussed in more detail in Chapter 4, was correctly matched the dependence of $OPD_{rms}$ on wall temperature for the case of wall heating. The model also predicted that decreasing the wall temperature (i.e. wall cooling) would reduce levels of optical aberrations significantly (Cress, 2010). Cress (2010) showed that reductions in $OPD_{rms}$ could be achieved with a limited number of measurements in the wall cooling regime. However, the wavefront statistics obtained for wall cooling were limited, and so more measurements are required in order to validate his model for a wider range of cooling temperatures and Mach numbers. Chapter 4 will review these studies and the
details of the model modifications, since they are of interest for study as one potential method of passive flow control techniques for aero-optic mitigation in the TBL.

2.3.5 Instantaneous TBL Wavefront Characteristics

In addition to the time-averaged statistical analysis and modeling discussed in the previous sections, a limited number of studies have examined the instantaneous characteristics of time-resolved wavefront measurements. Wavefront data from (Gordeyev, Cress, & Jumper, 2013) obtained at $M = 0.4, 0.5$ illustrate that while average Strehl ratio can remain high for beams propagating through the TBL, time series of $OPD_{rms}(t)$, presented in Figure 2.16a show the presence of sudden, short lived increases in levels of $OPD_{rms}$. These brief spikes in instantaneous levels of $OPD_{rms}$ were shown to result in abrupt, significant drops in the corresponding Strehl ratio computed using equation (1.14), shown in Figure 2.16, right plot, over short ($\approx 1$ ms) time intervals (Gordeyev, Cress, & Jumper, 2013). It is thought that these spikes in $OPD_{rms}$ are related to large-scale coherent structures present in the outer layer of the turbulent boundary layer (Cress, 2010).

For airborne optical systems that require sustained on-target intensity, such as a theoretical free-space communication link, it was demonstrated that these instantaneous dropouts in Strehl ratio could be especially detrimental to far-field performance. Results also demonstrated that time-averaged levels of aero-optic aberrations caused by turbulent boundary layers might not be negligible at the current wavelengths of interest for airborne laser applications (Gordeyev, et al., 2013).
Figure 2.16. Time trace of $OPD_{rms}$ a) over an aperture of $10\delta$, $M = 0.5$, and b) the corresponding Strehl Ratio for $\delta = 4.8$ cm and 12 cm calculated for a laser wavelength of 1.0 $\mu$m [image source Gordeyev, et al., 2013].
2.4 Current Questions Regarding TBL Aero-Optic Effects

The review of research on TBL aero-optics research clearly shows that significant advances have been made in the field in recent years, both in experimental and computational studies. However, there are a number of critical research questions that remain unanswered. These open areas of study can be divided into two categories: 1) further research on the aero-optics of canonical TBLs, and 2) research on modified TBLs for aero-optic mitigation.

2.4.1 Aero-Optics of Canonical TBLs

Firstly, additional experimental measurements are required to validate the extension of statistical models for $OPD_{rms}$ into the low-Reynolds number and supersonic and hypersonic regimes. The latter area of research is currently being explored, and will not be dealt with in this dissertation. Experimental measurements of TBL wavefront aberrations in the low-Reynolds number regime present a non-trivial problem, as low-Reynolds number TBLs require either a thin boundary layer, small Mach number, or both. This makes for a very weak signal to be experimentally measured, which introduces a substantial amount of uncertainty due to decreased signal-to-noise ratios in high-bandwidth wavefront sensors. This is problematic, since good quality low Reynolds number experimental wavefront information is highly desired for several reasons. First, low Reynolds number TBL measurements are required in order to verify/correct aero-optic scaling models in this regime that were initially derived for high-Reynolds number TBLs. It is also important to obtain measurements in order to directly compare with results of computational simulations, as currently there is a large Reynolds number ‘gap’
between experimental measurements with typical $Re_\theta > 20,000$ and numerical simulations (Wang, et al., 2012; White & Visbal, 2013) with much lower $Re_\theta \sim 1,000$ to $3,000$.

A novel approach is discussed in this dissertation for obtaining good quality low Reynolds number TBL wavefront data. To overcome the weak aero-optic signature of low Reynolds numbers TBLs, wall heating could be used to passively amplify TBL aero-optic aberrations. Then, using the wall temperature model, $OPD_{rms}$ and other the adiabatic wall TBL wavefront statistics could be recovered from heated wall wavefront measurements. Chapter 4 will review the effect of wall temperatures on TBLs and aero-optic aberrations, and Chapter 8 will present the results of an experimental investigation of the validity of this approach for obtaining good quality low Reynolds number aero-optic measurements in TBLs.

The statistical models presented in the previous section have been shown to be in good agreement with experimentally measured wavefront distortions in the time-averaged sense (Gordeyev, et al. 2014). Interest in the instantaneous characteristics of TBL aero-optic distortions, which is driven by communications applications, motivates investigation as to whether or not the existing statistical models are valid in an instantaneous sense as well. Research in this area is ongoing, with experiments seeking to obtain simultaneous measurements of wavefronts and the TBL velocity field using particle image velocimetry, or PIV (Saxton-Fox, et al., 2014). These experiments are outside the scope of this work, but are a critical part of future investigations of TBL aero-optic effects.
2.4.2 Aero-Optics of Modified TBLs

Finally, while further investigations of canonical TBLs are important in their own right for understanding the physics of TBLs and their corresponding fluid-optic interaction effects, it is also necessary to identify and evaluate flow control techniques that will be effective at mitigating TBL wavefront aberrations in both the time-average and instantaneous sense. Two such techniques are to be studied in detail in this dissertation: 1) Large-Eddy Break-Up (LEBU) devices, and 2) Boundary layer wall cooling.

It has been firmly established through detailed analysis of fluid-optic interactions in the TBL that large-scale, outer-layer turbulent structures are a dominant source of wavefront aberrations in the TBL, and it has been shown from statistical models that $\text{OPD}_{\text{rms}}$ is proportional to the square root of TBL skin friction coefficient: $\text{OPD}_{\text{rms}} \sim (C_f)^{1/2}$. Extensive studies of Large-Eddy Break-Up (LEBU) devices as a means of reducing viscous drag of boundary layers showed that LEBUs reduced $C_f$ for distances $\sim O (100\delta)$ downstream (Corke, 1981; Anders, 1990) by altering the turbulence structure in the outer part of the boundary layer (Anders J. B., 1990). A more detailed review of the effect of LEBU devices on TBLs is found in Chapter 3. The use of this type of flow manipulator for TBL aero-optic mitigation was first suggested over three decades ago by Jumper(1980), and the similarities between recent findings on important features of TBL aero-optic aberrations and the effects of LEBU devices gives additional strong indications that LEBUs should be investigated for aero-optic flow control.

Cress (2010) identified wall cooling as another potential method of aero-optic mitigation in TBLs based on findings from investigations of the effect of wall
temperature on TBL wavefront aberrations, which preliminarily indicated that a nearly 80% reduction in $OPD_{rms}$ could be achieved for cooling over the entire wall in a $M_\infty = 0.3$ TBL (Cress, 2010). The implications for TBL aero-optic mitigation are obvious, but additional experiments are required in order to validate the wall-temperature effect model over a wide range of wall cooling temperature. It is also of interest to evaluate the efficacy of partial wall cooling on aero-optic mitigation in the TBL, since 1) full wall cooling may not always be feasible based on physical constraints in practical applications, and 2) because partial cooling could reduce the energy cost while still achieving reasonable levels of aero-optic mitigation. The wall temperature model for aero-optic aberrations is reviewed in Chapter 4, along with related work on non-adiabatic wall TBLs. The effect of step-changes in wall temperature on the temperature-velocity relationship in the TBL will also be reviewed in Chapter 4 because of its relevance to extending Cress’ (2010) aero-optic model of wall temperature effects to cases with partial wall cooling.
CHAPTER 3:
REVIEW OF LARGE-EDDY BREAK UP DEVICES

A great deal of effort in the field of fluid mechanics has been dedicated to identifying and evaluating different methods of passive flow control for turbulent boundary layers, including riblets (Walsh M. J., Riblets, 1990), vortex generators (Zaman, Hirt, & Bencic, 2012), honeycombs and screens (Yajnik & Ancharya, 1977), and large-eddy break-up (LEBU) devices (Corke, Guezennec, & Nagib, 1979). Studies of LEBU devices for reducing viscous drag in turbulent boundary layers has confirmed that $C_f$ could be reduced for distances $\sim O(100\delta)$ downstream (Corke, 1981; Anders, 1990) by substantially altering the turbulence structure in the outer part of the boundary layer (Anders J. B., 1990). The effectiveness of the LEBU device for modifying large-scale turbulent structures in the outer part of the TBL has caused it to be identified as a potential method of passive flow control that could be used to achieve aero-optic mitigation in the TBL (Jumper, 1980; Anders, 1985). This chapter will review the effects of Large Eddy Break-Up (LEBU) device on turbulent boundary layers, and discuss the relevant mechanisms discovered for viscous drag reduction and their potential effects on aero-optic aberrations.

The inspiration for the development of LEBU devices (also referred to in the literature as outer-layer manipulators (Corke, Guezennec, & Nagib, 1979), outer layer devices (Bandyopadhyay, 1986), or parallel plate manipulators (Savill & Mumford, 1986).
1988) evolved from of prior attempts at TBL drag reduction using honeycombs and screens (Yajnik & Ancharya, 1977). This work demonstrated that screen-type devices were highly effective at disrupting the natural balance of turbulent structures in the boundary layer, and large skin-friction reductions could be achieved over some distance downstream (Yajnik & Ancharya, 1977; Hefner, et al., 1979). Screens and honeycombs, however, have significant device drag which resulted in a net drag several times larger than the drag of the un-perturbed boundary layer, making them impractical for use as drag-reducing flow control devices (Hefner, Weinstein, & Bushnell, 1979).

Figure 3.1. An example of a pin-fence and screen-type passive flow control device used for aero-optic mitigation in separated shear layers. [image source Gordeyev, et al. (2013)]

In an effort to retain the effectiveness of outer-layer manipulators on viscous drag reduction in the TBL while reducing device drag, Corke, et al. (1979) and Hefner, et al. (1979), began investigating a new class of outer-layer manipulators, which are now widely referred to as LEBU devices. In general, LEBU devices consist of one or more thin flat plates or airfoils placed parallel to the wall within the turbulent boundary layer, as shown in Figure 3.2. These devices were designed to act as screens with sparse elements, significantly reducing blockage effects while retaining the ability to directly manipulate turbulent structures in the outer part of the boundary layer (Corke T., 1981).
The significant reduction in blockage compared to screen-type devices was expected to result in a net viscous drag reduction compared to the un-manipulated TBL (Corke, Guezennec, & Nagib, 1979). Some experiments have investigated LEBU devices with as many as four flat plate elements (Corke, 1981), however this work is limited to investigating LEBU devices with at most two flat plates.

Figure 3.2. Schematic of the effect of a single LEBU device on the TBL, and schematics of the a) Single LEBU, b) Multi-LEBU, and c) Tandem-LEBU configurations. [flow visualization image from Corke, et al. (1979)]

Single LEBU devices, shown in Figure 3.2a, consist of a single thin plates or airfoils of thickness $t$ and length $l$ arranged parallel to the wall (i.e. at zero angle of attack), offset by some height $h \leq \delta_0$. LEBU configurations with more than one parallel plate, sometimes referred to as multiple element LEBUs, are shown in Figure 3.2b and Figure 3.2c. Arrangements of vertically stacked parallel plates of identical length and thickness, referred to as Multi LEBU devices, are shown in Figure 3.2b, along with the
notation used to identify the height of each plate. For multi-LEBU devices, the heights of each plate, starting with the outermost and proceeding towards the wall, are denoted as $h_1$, and $h_2$. LEBU plates arranged in series in the streamwise direction are referred to as Tandem LEBU devices, as shown in Figure 3.2c. Each individual plate is identical in length $l$, and is mounted at the same height off the wall $h$, but plates are separated by some gap distance $s$ in the streamwise direction.

3.1 Effect of LEBU Devices on TBL Characteristics

The characteristics of LEBU modified boundary layers have been well documented using an array of diagnostic methods in order to quantify and maximize viscous drag reductions, and to investigate the mechanisms that were responsible for LEBU effects (Anders, 1990, and the references therein). Viscous drag has been experimentally determined using a variety of techniques, including momentum balance (Corke, 1981; Hefner, et al. 1983), oil film interferometry (Westphal, 1986), Preston tubes (Bertelrud, et al., 1982; Takagi, 1983), measurements of $dU/dy$ at the wall (Falco & Rashidnia, 1987), and others (Anders, 1990). Although there has been some debate in the literature about the validity of the use of some methods for computing $C_f$ (see Anders, 1990 for further discussion), direct measurement techniques have shown skin-friction reductions on the order of 30 % in LEBU manipulated boundary layers (Anders, 1990). These reductions in viscous drag have also been shown to persist for distances downstream on the order of 100 boundary layer thicknesses (Corke, 1981; Plesniak, 1984; Anders, 1990).
Figure 3.3. Smoke-wire flow visualization records for a) the unmodified TBL and b-d) manipulated boundary layers for different LEBU configurations. [image source Corke (1981)]
Smoke-wire flow visualization records (Corke, 1981; Savill & Mumford, 1988) showed a significant change in turbulence structure in the outer part of the LEBU modified TBL. In particular, the flow visualization images indicate that the size of turbulent structures in the outer layer are substantially reduced (inspiring the LEBU moniker), and showed an obvious reduction in intermittency in the TBL edge, which would reduce the entrainment of irrotational, high momentum freestream fluid (Corke, 1981).

These findings from flow visualization records were confirmed by later velocity measurements by Chang (1987) that found intermittency and velocity integral length scales were both reduced by up to nearly 40%, and persisted with decreasing effect as far as ~70δ downstream in the manipulated boundary layer. Single-wire and hot-wire rake velocity measurements in a LEBU modified TBL by Blackwelder & Chang (1986) also showed that between ‘bulges’ in the TBL edge, fluid sweeps towards the wall were reduced, an effect which was attributed to the no-slip boundary condition imposed by the LEBU device surface. Spanwise velocity length scales were also found to be significantly reduced over a similar range downstream of the LEBU, with recovery to un-perturbed levels persisting for 80 - 100δ (Wark, Naguib, & Nagib, 1989). The streamwise growth of the boundary layer was also slowed, and the integral thicknesses (i.e. displacement, momentum, etc.) were diminished compared to the un-modified TBL (Corke, 1981).

Figure 3.4 presents mean velocity measurements from Lemay, et al (1990) at several locations downstream of the LEBU that are typical of observations of the LEBU manipulated boundary layer. Immediately downstream of the device trailing edge, at $\xi = x/\delta_0 = 2$, a well-defined velocity deficit is clearly seen imprinted on the mean velocity.
This wake deficit weakens rapidly downstream, ‘smearing out’ almost entirely by $5 - 10\delta$ (Corke, 1981; Blackwelder & Chang, 1986; Lemay, et al. 1990). By $\xi = 20$, the wake deficit disappears entirely, and the modified TBL velocity profile is nearly identical to the un-modified boundary layer profile (Corke, 1981; Lemay, et al., 1990; Anders, 1990).

![Figure 3.4](image source, Lemay, et al. 1990)

Figure 3.4. Mean velocity profiles at several locations downstream of a single LEBU device; dashed line represents the un-perturbed TBL case, horizontal line denotes LEBU height. [image source, Lemay, et al. 1990]

Figure 3.5 also presents the results of hot-wire velocity measurements published by Corke (1981) that were obtained in an experiment performed by H. Nagib and R. Westphall at Stanford University (Corke, 1981). In this study, velocity measurements were obtained downstream of a tandem-LEBU device with plates of chord length $l = \delta$, placed $h = 0.5\delta$ from the wall, and separated $s = 8\delta$ in the streamwise direction. The mean velocity results, which are shown in Figure 3.5a, are consistent with the results presented in Figure 3.4; the LEBU device induces a wake deficit profile just downstream of the device that ‘smears out’ after a short distance downstream. By $45\delta$ downstream of the
device, the mean profile is nearly identical with the baseline state at that location. Values of the root-mean-square of the fluctuating velocity component from this experiment are also presented in Figure 3.5b. These data show that the LEBU has a significant effect on the magnitude and distribution of turbulent velocity fluctuations in the TBL. Just downstream of the LEBU trailing edge, there is a significant reduction in $u_{\text{rms}}$ for the whole region of the TBL below the LEBU device, $y < h$. Just above the LEBU device, however, there is a narrow region of the TBL in which there is a localized spike in the value of $u_{\text{rms}}$ above the baseline measured value. This region of increase disappears by about 10.5$\delta$ downstream of the LEBU trailing edge. The reduction in $u_{\text{rms}}$ below the LEBU device persists at $x = 10.5\delta$, but near the wall the value of $u_{\text{rms}}$ is slightly increased compared to $x = 1.2\delta$. By $x/\delta = 45$, $u_{\text{rms}}(y)$ was found to be nearly the same as the baseline profile obtained for the canonical turbulent boundary layer.

Similar developments were measured in the turbulent kinetic energy (TKE) balance measured using x-wire probes by Lemay, et al. (1990). Just downstream of the LEBU the TKE balance is dramatically altered, with the manipulated TBL appearing to be divided into two separate regions (Lemay, et al. 1990). Near the LEBU trailing edge ($\xi = 2$) production is locally increased in the region above the LEBU device, and in the region below the LEBU production was near zero for a substantial range of $y/\delta$ (Lemay, et al. 1990). By $\xi = 20$, Lemay (1990) found that the manipulated boundary layer was nearly relaxed to the canonical state, with the manipulated TBL balance bearing a good resemblance to baseline TKE measurements and to balances from other studies of canonical TBLs (Klebanoff, 1955; Murlis, et al., 1982).
Figure 3.5. Hot-wire velocity measurements of a) $U(y)$ and b) $u_{rms}(y)$ in a Tandem-LEBU modified TBL with $l = \delta$, $h = 0.5\delta$, $s = 8\delta$ (Corke, 1981).

Figure 3.6 from Lemay, et al. (1990) clearly demonstrates that the alteration of the production term in the modified TBL is largely due to the change in the mean velocity profile gradient caused by the LEBU wake, as the main part of the production term is proportional to $dU/dy$. The imposition of the wake deficit on an already shear flow profile appears to be what divides the manipulated TBL into two distinct regions, and also what determines the characteristics of the inner and outer regions (Lemay, et al., 1990). This mechanism for redistributing TKE in the boundary layer was first proposed by Corke.
(1981) based on evidence from mean and fluctuating velocity measurements in the near-LEBU wake region, and was confirmed by the work of Lemay, et al. (1990).

One-dimensional velocity spectra measured by Corke (1981) at $y = h$ in the LEBU-manipulated boundary layer are presented in Figure 3.7. These data showed that the low-wavenumber region (corresponding to large-scale structures) of the velocity spectrum just downstream of the LEBU device ($x/\delta = 1$) is substantially suppressed. In the high-wavenumber regime, there is an increase in turbulent velocity fluctuations that is consistent with the production of small scale turbulent structures in the LEBU wake. This result also supports the notion that large-scale structures are suppressed in the LEBU manipulated boundary layer, which was shown in the flow visualization images obtained.
by Corke (1981) (see Figure 3.3). At distances further downstream of the LEBU manipulator, the spectra eventually recover to the characteristic shape observed for the un-manipulated TBL.

Figure 3.7. One-dimensional streamwise velocity wavenumber spectra obtained by Corke (1981) for several locations downstream of a single LEBU manipulator: $V_\infty = 10.4$ m/s, $Re_\theta = 5,500$ [image source Corke (1981)].
3.2 Mechanisms for LEBU Drag Reduction

There have been a number of competing ideas proposed regarding the mechanisms that are responsible for drag reduction in LEBU modified turbulent boundary layers (Anders, 1990). The streamwise longevity of drag reductions, and the obvious reductions in scale of turbulent structures in manipulated TBLs indicated to early researchers that the \( V^\prime = 0 \) boundary condition imposed by the plates in the outer part of the boundary layer served to limit vertical velocity fluctuations in the outer part of the boundary layer, effectively ‘breaking up’ large turbulent structures (i.e. eddies) (Corke, 1981; Blackwelder & Chang, 1986; Anders, 1990).

Other researchers, however, have argued that these changes to the outer part of the TBL are not a result of the breakup of large eddies, but rather the introduction of small scale turbulence into the TBL from the LEBU wake which de-correlates the zones of the TBL on either side of the wake (Savill & Mumford, 1988; Falco & Rashidnia, 1987). In this explanation of the mechanism for LEBU turbulence modification, which is referred to as the \textit{wake effect}, the LEBU wake introduces a vortex street that acts as buffer between the outer and inner parts of the TBL, effectively de-correlating these regions of the boundary layer and preventing the transport of high-speed fluid across the LEBU wake boundary (Savill & Mumford, 1988). Instead of breaking up large eddies in the TBL, Savill & Mumford (1988) suggested that LEBU devices instead modified turbulent structure in the TBL through the interruption of the slow overturning of hairpin vortex packets (Head & Bandyopadhyay, 1981). In order to significantly interrupt this motion, it was recommended that device chord should be at least \( 2\delta \) (Savill & Mumford, 1988).
To determine whether the wake effect accounts for the drag reductions found in LEBU modified boundary layers, experiments were performed using wires or small cylinders in the place of plates, since wires will produce a wake with no significant limitation of vertical velocity fluctuations in the boundary layer (Takagi, 1983; Lynn, 1987; Savill & Mumford, 1988). Results of these studies concluded that although $C_f$ reductions that were similar to LEBU results were achieved, the reductions were not sustained nearly as far downstream as had been shown for LEBU devices in similar configurations (Savill & Mumford, 1988; Lynn, 1987); see Figure 3.8. Considering the rapidity of the recovery of the manipulated boundary layer from the wake deficit profile to its natural state (Hefner, et al. 1979; Corke, 1981; Blackwelder & Chang, 1986; Lemay, et al. 1990), and the results of wire manipulator experiments, it is evident that while the LEBU wake effect does play an important role in modifying the TBL for viscous drag reduction, it is not the only significant mechanism at work (Anders, 1990).

Instead, mechanisms responsible for skin friction reductions in the LEBU manipulated boundary layer are considered to be a combination of both plate and wake effects, where the $V = 0$ boundary condition imposed by the LEBU (i.e. the plate effect) upsets the large-scale turbulent motions, and the introduction of small-scale turbulence structures in the LEBU wake impedes their re-formation for long distances downstream (Anders, 1990). Also contributing to this delay is the de-correlation of the inner and outer zones of the manipulated boundary layer caused by the presence of the device wake, which acts as a shield that inhibits sweeps of high velocity potential fluid from penetrating below the LEBU wake (Savill & Mumford, 1988; Willmarth & Lu, 1972). LEBUs have also been shown to alter the dynamics of the boundary layer through
Figure 3.8. Schematics of manipulated TBLs based on flow visualization: a) Single LEBU: $l/\delta = 1.8$, $h/\delta = 0.75$. b) Single LEBU: $l/\delta = 1.6$, $h/\delta = 0.25$. c) Cylinder: $D/\delta = 0.05$, $h/\delta = 0.5$. [image source Savill & Mumford (1988)]
analysis of ‘burst’ events, or ejections of low-velocity fluid away from the wall. Corke (1981) showed that these burst events, which have been shown to be a significant contribution to Reynolds stress in the boundary layer (Corino & Brodkey, 1969), were reduced by about 17% in the manipulated boundary layer.

3.3 LEBU Optimization for Net-Drag Reduction

In addition to determining the mechanisms by which viscous drag reduction was achieved in LEBU modified turbulent boundary layers, a major focus of investigations sought to identify LEBU device configurations that would result in the largest net drag reductions. Results of these studies identified optimal ranges for different parameters, including device height, chord, thickness, angle of attack, and spacing. In addition studies also investigated different configurations of multiple element devices, including tandem and multi-LEBU devices.

Experiments by (Plesniak, 1984) and (Savill & Mumford, 1988) also showed that device chord (or length), \( l \), should be greater than \( \delta \) in order to be effective on breaking up large scale structures in the TBL. Both studies tested devices up to \( l/\delta \approx 5 \) and found continued reductions in \( C_f \) with increasing chord length, as shown in Figure 3.9 (Savill & Mumford, 1988). The increased surface area of these long-chord LEBUs, however, increased the surface area and thus, device drag. As a result, no net drag reductions were found for long devices (Plesniak, 1984; Savill & Mumford, 1988), and the bulk of experiments on LEBU devices have determined the optimal range of \( l/\delta \approx 0.8 – 1.6 \) (Anders, 1990).
Variations in device height, $h$, also have a significant impact on the viscous and net drag reductions achieved in LEBU manipulated boundary layers (Anders, 1990). Savill & Mumford (1988) showed that generally, devices placed closer to the wall achieved large maximum reductions of $C_f$ closer to the device trailing edge, but the recovery towards the un-perturbed boundary layer skin friction was rapid with increasing downstream distance. In contrast, LEBUs further out in the boundary layer produced shallow, long-lasting reductions in skin friction that resulted in the largest decrease in integrated viscous drag (Savill & Mumford, 1988). For typical laboratory Reynolds numbers, $Re_\theta \sim O (10^3)$, the optimal device height for achieving net drag reductions is found to be approximately $h/\delta = 0.75 - 0.8$ (Plesniak, 1984; Anders, et al., 1984; Poll &
Westland, 1987; Savill & Mumford, 1988), although Savill & Mumford (1988) indicated that for larger \( Re_\theta \), there is some indication that the optimal value of \( h/\delta \) may decrease.

The effect of device thickness was also investigated in several studies, and was found be minimally influential on \( C_f \) reductions in the modified TBL, however thinner LEBU devices were desired in order to keep device drag small and give net drag reductions (Hefner, et al., 1983; Anders, et al., 1984; Savill & Mumford, 1988). Tandem LEBU configurations were found to give net drag reductions when the streamwise spacing of the devices, \( s \), was between 5\( \delta \) – 15\( \delta \), with no strong indication of a definite optimum value within that range (Anders, 1990).

3.4 Potential for Aero-Optic Mitigation

As mentioned previously in Chapter 2, prior experimental work exploring the aero-optics of boundary layers has shown strong evidence for the dominance of large-scale (i.e. outer-region) turbulent structures as a source for boundary layer induced aberrations. It is also suspected that large instantaneous drop-outs in far-field beam intensity caused by TBLs are related to large-scale motions (LSMs) in the outer part of the TBL (Gordeyev, Cress, & Jumper, 2013) such as turbulent bulges and hairpin vortex (Cress, 2010). Conveniently, LEBU devices are effective at reducing streamwise and spanwise integral length scales, boundary layer intermittency, and the strength of bulges and sweeps without introducing a strong shear layer or other optically un-desirable flow features, making them an attractive candidate for use as an aero-optic mitigation application for TBLs.
While the potential for using LEBUs for aero-optic mitigation was recognized over three decades ago by Jumper (1980), and again several years later by Anders (1985), these suggestions have not been seriously investigated until only recently (Smith, et al., 2013; Smith, et al., 2014). In this effort, it is important to recall that prior studies of mechanisms and optimization of LEBU devices have been focused on reducing skin friction, which is related to the condition of the TBL locally near the wall. While the characteristics of the entire boundary layer have an effect on the local stress at the wall, this relationship is not straightforward in a perturbed boundary layer. Therefore a direct correlation between reductions in aero-optic aberrations and in $C_f$ should not be assumed in the LEBU modified boundary layer, as equation (2.26) might suggest. Also, the condition of having a small amount of LEBU device drag, which was an important factor for LEBUs drag reduction optimization, is not necessarily a restrictive factor for aero-optic mitigation. In real directed energy applications, it may be desirable to accept a small additional localized drag penalty in exchange for improved aero-optic performance over a finite aperture.

This is not to say, however, that prior research on LEBUs is of no value to the present investigation. The results of experiments reviewed in this chapter have provided valuable insight into the structure and physical mechanisms present in LEBU-modified TBLs, and serve as a robust starting point for examining the effect of LEBU devices on TBL induced aero-optic aberrations. Chapters 6 and 7 of this dissertation will present the results of experimental investigations of the effect of Large Eddy Breakup Devices on aero-optic wavefront measurements, in order to 1) verify whether or not LEBU devices do indeed reduce TBL wavefront aberrations, 2) ascertain the mechanism or mechanisms
responsible for changes to the aero-optic characteristics of the TBL, and 3) to provide
guidance for identifying LEBU devices that would be most effective for aero-optic
mitigation of the turbulent boundary layer.
CHAPTER 4:
REVIEW OF WALL TEMPERATURE EFFECTS ON TURBULENT BOUNDARY LAYERS

This chapter will review the effects of non-adiabatic wall temperature effects on aero-optic aberrations caused by turbulent compressible boundary layers. In the first section, some background will be given on the derivation of a model for relating velocity and temperature fluctuations in non-adiabatic wall compressible TBLs; the modified Crocco-Busemann relation (or the Extended SRA). Section 4.2 will review the application of this velocity-temperature relation to modeling the effect of wall temperature on levels of aero-optic aberrations from the TBL, and present the aero-optic wall temperature models to be used later on in this dissertation. Finally, section 4.3 will review research on the effect of partial wall-temperature modification (partial heating and/or cooling) on TBLs, since it is of interest for studying the effectiveness of partial wall cooling for aero-optic mitigation.

4.1 Temperature-Velocity Relationship

The coupling between scalar fluid properties, such as temperature and density, and fluid velocity is a consequence of the Navier-Stokes equations, which are the governing equations of fluid mechanics (Schlichting, 1979). One consequence of this coupling is that an area of significant temperature gradient, or a thermal boundary layer,
will be present in the boundary layer because of the local variation in velocity near in the wall bounded layer. A number of temperature-velocity relationships have been derived for different flow and boundary conditions, but this section will give some background on the derivation of the particular temperature-velocity relationships that are used to model the density fluctuations in a non-adiabatic wall, compressible, turbulent boundary layer.

4.1.1 Energy Equation

To derive the temperature-velocity relation in the boundary layer, it is convenient to consider the energy equation written in terms of enthalpy, or thermodynamic potential. For a steady \((d/dt = 0)\), two-dimensional \((d/dz = 0, \ w = 0)\), turbulent boundary layer in which static pressure fluctuations are assumed to be negligible \((p' = 0)\), the energy equation can be written as

\[
\bar{\rho}\bar{u}\frac{\partial H}{\partial x} + \bar{\rho}\bar{v}\frac{\partial H}{\partial y} = \frac{\partial}{\partial y}\left[\left(\frac{\bar{\mu}}{Pr} + \frac{\mu_t}{Pr_t}\right)\frac{\partial H}{\partial y}\right] + \frac{\partial}{\partial y}\left[\bar{\mu}\left(1 - \frac{1}{Pr}\right) + \mu_t\left(1 - \frac{1}{Pr_t}\right)\right]\frac{\partial}{\partial y}\left(\frac{\bar{u}^2}{2}\right),
\]

(4.1)

where \(H\) is the local total enthalpy, \(\rho\) is the local density, \(u\) and \(v\) are the velocity components in the streamwise \((x)\) and wall-normal \((y)\) directions, \(\mu\) is the viscosity, \(Pr\) is the Prandtl number, and \(\mu_t\) is the ‘eddy viscosity’ and \(Pr_t\) is the turbulent Prandtl number. The overbar indicates time-averaged quantities, and the tilde indicates Favre mass-averaged quantities (i.e. \(\bar{u} \equiv \rho\bar{u}/\bar{\rho}\)). The total enthalpy is \(H = h + u^2/2\), where \(h\) is the local static enthalpy. Assuming constant specific heat capacity at constant pressure, \(h = c_p T\), where \(T\) is static temperature. The Prandtl number is defined as the ratio of the
viscous diffusion rate or kinematic viscosity, \( \nu = \mu / \rho \), and thermal diffusion rate, \( \alpha = k / (\rho c_p) \);

\[
\text{Pr} = \frac{\mu c_p}{k},
\]

(4.2)

where \( k \) is the thermal conductivity. The eddy viscosity term arises from assuming that Reynolds stress in a fluid, \( \rho u_i u_j \), is can be modeled as an additional viscous stress term with some viscosity \( \mu \); this assumption is one of the oldest approaches for overcoming the closure problem in the Reynolds averaged Navier-Stokes equation (Mathieu & Scott, 2000). The turbulent Prandtl therefore the ratio of the eddy viscous diffusion rate, \( \nu_t = \mu_t / \rho \), and the eddy thermal diffusion rate, \( \alpha_t = k_t / (\rho c_p) \);

\[
\text{Pr}_t = \frac{\mu c_p}{k_t}.
\]

(4.3)

Total enthalpy \( H \) can be decomposed into mean and fluctuating components,

\[ H = \bar{H} + H'' , \]

\[
\bar{H} + H'' = \bar{h} + h'' + \frac{1}{2} (\bar{u} + u'')^2 = \text{constant} .
\]

(4.4)

It follows that

\[
\tilde{H} = \bar{h} + \frac{\tilde{u}^2}{2} = \text{constant} ,
\]

(4.5)

\[
H'' = h'' + \bar{u}u'' = 0,
\]

(4.6)

where the \((u'')^2\) term is disregarded since it small compared to the \( \bar{u}u'' \) term. It can be shown that the solution to the compressible TBL energy equation (4.1), assuming that \( \text{Pr} = \text{Pr}_t = 1 \) and \( dp/dx = 0 \), can be written as,
\[ H(y) = H_w + \left( H_\infty - H_w \right) \frac{\tilde{u}(y)}{U_\infty}, \]  

(4.7)

where \( H_w \) is the total enthalpy at the wall (and is equal to \( h_w \), since \( u_w = 0 \)), and \( H_\infty \) is the total enthalpy in the freestream. Re-writing the solution for total enthalpy from equation (4.7) in terms of its mean and fluctuating components,

\[ \tilde{H}(y) = H_w + \left( H_\infty - H_w \right) \frac{\tilde{u}(y)}{U_\infty}, \]

(4.8)

\[ H^*(y) = \left( H_\infty - H_w \right) \frac{u''(y)}{U_\infty}. \]

(4.9)

Recasting the expression for mean total enthalpy, equation (4.8), in terms of mean temperature yields the following solution,

\[ \frac{\tilde{T}(y)}{T_\infty} = \frac{T_w}{T_\infty} + \frac{T_{\infty,0} - T_w}{T_\infty} \left( \frac{\tilde{u}(y)}{U_\infty} \right) - \frac{(\gamma - 1)}{2} \frac{M_\infty^2}{\left( \frac{\tilde{u}(y)}{U_\infty} \right)^2}, \]

(4.10)

where \( T_{\infty,0} \) is the freestream total temperature. Total enthalpy fluctuations, equation (4.9), can be re-written in terms of static enthalpy fluctuations and velocity fluctuations using equation (4.6):

\[ h''(y) + \tilde{u}(y) u''(y) = \left( h_w + \frac{U_\infty^2}{2} - h_w \right) \frac{u''(y)}{U_\infty}. \]

(4.11)

Equation (4.11) can then be rewritten to solve for fluctuating temperature,

\[ \frac{T''(y)}{T_\infty} = \frac{T_{\infty,0} - T_w}{T_\infty} \left( \frac{u''(y)}{U_\infty} \right) - (\gamma - 1) \frac{M_\infty^2}{\tilde{u}(y)} \left( \frac{u''(y)}{U_\infty} \right), \]

(4.12)

which can also be expressed as

\[ \frac{T''(y)}{\bar{T}} = \frac{T_{\infty,0} - T_w}{\bar{T}} \left( \frac{u''(y)}{U_\infty} \right) - (\gamma - 1) \bar{M}^2 \left( \frac{u''(y)}{\tilde{u}(y)} \right). \]

(4.13)
These equations are referred to as the Crocco-Busemann relations for compressible turbulent boundary layers (Schlichting & Gersten, 2000; White, 2006, and others). If the total temperature is constant through the boundary layer (i.e. $T_{\infty} = T_w$) then equation (4.13) simplifies to the Strong Reynolds Analogy shown in equation (2.5). In review, the assumptions required to get to equations (4.10) and (4.13) from the steady, 2-D compressible TBL energy equation (which ignores static pressure fluctuations) were that $Pr = Pr_t = 1$, that the streamwise pressure gradient $dp/dx = 0$. To obtain the SRA, it is also assumed that total temperature is constant across the boundary layer.

4.1.2 “Extended” Strong Reynolds Analogy

There is strong evidence from experiments and computations that the Strong Reynolds Analogy (SRA), shown in equation (2.5), forms an important and useful temperature-velocity relationship in compressible, adiabatic wall TBLs (Morkovin, 1962; Cebeci & Smith, 1974; Smits & Dussauge, 2006). For boundary layers with non-adiabatic wall conditions though, the SRA does not adequately model the temperature-velocity relationship in the TBL; a consequence of assuming that pressure fluctuations are negligible is that through the equation of state, it follows from that total temperature fluctuations are also assumed to be negligible. Smits and Dussauge (1996) have shown that total pressure in isothermal wall TBLs, fluctuations of total temperature can be large, with $T_o''$ being on the order of $4 – 9\%$ of $T_o$, and as large as $60\%$ of the static temperature fluctuations, $T''$. Therefore, SRA is not sufficient to handle turbulent boundary layer heat transfer problems with non-adiabatic wall conditions (Spina, Smits, & Robinson, 1994).
To derive a temperature-velocity relationship in which total temperature was permitted to fluctuate, van Driest (1951) assumed temperature, and therefore static enthalpy, to be a function of the streamwise velocity component; \( \bar{h} = c_p \bar{T} = h(\bar{u}) \), and substituted this into the energy equation for steady, 2-D turbulent boundary layers. Solving the energy and momentum equations, van Driest (1951) found that the solution took the form

\[
h(\bar{u}) = a_0 + a_1 \bar{u} - \left( \bar{u}^2 \right) / 2,
\]

(4.14)

where \( a_0 \) and \( a_1 \) are constants of integration. Using equation (4.14) and assuming constant Prandtl number slightly less than one, van Driest (1951) derived a modified set of Crocco-Busemann relations that permitted total temperature fluctuations across a boundary layer with Prandtl number not equal to one and a non-adiabatic wall temperature. The resultant temperature-velocity relations are

\[
\frac{\bar{T}(y)}{T_\infty} = \frac{T_w}{T_\infty} + \frac{T_{aw} - T_w}{T_\infty} \left( \bar{u}(y) / U_\infty \right) - r \left( \gamma - 1 \right) M_\infty^2 \left( \bar{u}(y) / U_\infty \right)^2,
\]

(4.15)

\[
\frac{T''(y)}{T_\infty} = \frac{T_{aw} - T_w}{T_\infty} \left( u''(y) / U_\infty \right) - r \left( \gamma - 1 \right) M_\infty^2 \left( \bar{u}(y) u''(y) / U_\infty^2 \right).
\]

(4.16)

These equations are referred to by several different names: the Walz equations, the modified Crocco-Busemann relations, or the Extended Strong Reynolds Analogy (Schlichting & Gersten, 2000; Smits & Dussauge, 2006). The recovery factor, \( r \), which is defined as

\[
r = \frac{T_{aw} - T_w}{T_{aw,\infty} - T_\infty},
\]

(4.17)
indicates the increase of temperature seen at the wall from the conversion of kinetic energy to heat as velocity is slowed across the turbulent boundary layer, from $U_\infty$ in the freestream to zero at the wall. Although it is functionally dependent on Prandtl number, in turbulent boundary layers with air as the active medium, typically $r = 0.89$.

![Graph](image-source, Pirozzoli, et al. (2004))

Figure 4.1. Mean temperature of a spatially developing Mach 2.25 TBL from Pirozzoli, et al. (2004) as a function of $y^+$; (□) from DNS at $Re_\theta = 3530$ and (○) at $Re_\theta = 4263$; lines were computed from the ESRA. [image source, Pirozzoli, et al. (2004)].

DNS simulations of a compressible, supersonic turbulent boundary layer by Guarini, et al. (2000) found that although total and static temperature fluctuations were on the same order of magnitude, equation (4.16) was still found to be approximately valid. Mean temperature distributions obtained from direct numerical simulations (DNS) of a Mach 2.25 supersonic TBL by both Gatski and Erlebacher (2002) and Pirozzoli, et al. (2004), which are shown in Figure 4.1, were found to be in good agreement with the
temperature distributions predicted by the ESRA for $y < 0.9\delta$. In the outer layer, the assumptions behind the ESRA begin to break down, and a discrepancy is expected; however, the difference between the DNS and ESRA results is around just 10% of $T_\infty$ (Gatski and Erlebacher, 2002). Therefore, the temperature-velocity relation given in the ESRA was found to be the most promising for the derivation of an aero-optic model for wall temperature effects (Cress, 2010).

4.2 Aero-Optic Model for Wall Temperature Effects

The model developed using the SRA by Wyckham and Smits (2009) for predicting levels of aero-optic aberrations caused by non-adiabatic wall TBLs predicted opposite trends of what has been observed in experiments (Cress, 2010) and computations (White & Visbal, 2012), which is expected since the SRA is derived using the assumption that the TBL has an adiabatic wall and $Pr = 1$. To account for non-adiabatic wall effects, Cress (2010) used the ESRA temperature-velocity relation in order to derive a model for wall-temperature effects on levels of aero-optic aberrations caused by heated or cooled wall TBLs.

4.2.1 Review of Model Derivation

Since below $M = 3$, there is less than a 1.5% difference between Reynolds-averaged and Favre-averaged quantities (Smits & Dussauge, 1996), Reynolds averaged quantities can be substituted for Favre averaging. Replacing the fluctuating temperature and velocity terms in equation (4.16) with the root-mean-square of the temperature, $T_{rms}$, and velocity, $u_{rms}$, and squaring both sides, the following expression was obtained:
\[
\left( \frac{T_{rms}(y)}{T_\infty} \right)^2 = \left( \frac{u_{rms}(y)}{U_\infty} \right)^2 \times \left[ \left( \frac{\Delta T}{T_\infty} \right)^2 + 2r(\gamma - 1)M_\infty^2 \left( \frac{\Delta T}{T_\infty} \frac{u(y)}{U_\infty} \right) + r(\gamma - 1)M_\infty^2 \left( \frac{u(y)}{U_\infty} \right)^2 \right] \quad (4.18)
\]

where \( \Delta T = T_w - T_\infty \) (Cress, 2010). Recall from equation (2.6) that it follows from the ideal gas law, \( p = \rho RT \), that if \( p' \) is negligible, as the ESRA assumes it to be, density fluctuations are proportional to temperature fluctuations in the TBL. Substituting RMS quantities for fluctuating quantities, equation (2.6) becomes

\[
\left( \frac{\rho_{rms}}{\rho_\infty} \right)^2 = \left( \frac{T_{rms}}{T_\infty} \right)^2 \quad (4.19)
\]

By combining (4.18) and (4.19) to an expression for the root-mean-square of density fluctuations in the TBL was obtained:

\[
\left( \frac{\rho_{rms}(y)}{\rho_\infty} \right)^2 = \left( \frac{u_{rms}(y)}{U_\infty} \right)^2 \times \left[ \left( \frac{\Delta T}{T_\infty} \right)^2 + 2r(\gamma - 1)M_\infty^2 \left( \frac{\Delta T}{T_\infty} \frac{u(y)}{U_\infty} \right) + r(\gamma - 1)M_\infty^2 \left( \frac{u(y)}{U_\infty} \right)^2 \right] \quad (4.20)
\]

Substituting this expression into the simplified form of Sutton’s linking equation found in (2.3), Cress (2010) derived the following expression for levels of \( OPD_{rms} \) caused by TBL with heat transfer, as a function of the wall temperature,

\[
OPD_{rms} = B_0 K_{GDP} \rho_\infty \delta \sqrt{
\left[ B_1 M^4 + B_2 \frac{\Delta T}{T_\infty} M^2 + B_3 \left( \frac{\Delta T}{T_\infty} \right)^2 \right]^{1/2}
\]

where the constants \( B_0 \) thru \( B_3 \) are defined as
\[ B_0 = \sqrt{2} \frac{\delta^*}{\partial} \sqrt{\frac{C_f}{\delta}} (y - 1), \]

\[ B_1 = \int_0^y \left[ r^2 \left( \frac{U(y)u_{rms}(y)}{U_\infty^2} \right) \frac{\Lambda_y(y)}{\delta^*} \right] d\left( \frac{y}{\delta^*} \right), \]  

\[ B_2 = \int_0^\infty \left[ \frac{2r}{\gamma - 1} \frac{U(y)}{U_\infty} \left( \frac{u_{rms}(y)}{U_\infty^2} \right)^2 \frac{\Lambda_y(y)}{\delta^*} \right] d\left( \frac{y}{\delta^*} \right), \]

\[ B_3 = \int_0^\infty \left[ \left( \frac{u_{rms}(y)}{U_\infty (y - 1)} \right)^2 \frac{\Lambda_y(y)}{\delta^*} \right] d\left( \frac{y}{\delta^*} \right). \]

Rewriting equation (4.21) with \( B = B_0 B_1^{1/2}, C_1 = B_2/B_1, \) and \( C_2 = B_3/B_1, \) the model equation becomes

\[ OPD_{rms} = BK_G\rho_\infty \delta \sqrt{C_f M^2} \left[ 1 + C_1 \frac{\Delta T}{T_\infty M^2} + C_2 \left( \frac{\Delta T}{T_\infty M^2} \right)^2 \right]^{1/2}. \]  

(4.23)

Cress (2010) reported that using velocity measurements for a boundary layer with \( M = 0.5, C_1 \) and \( C_2 \) were theoretically predicted to be 6.38 and 10.28, respectively. A linearized form of equation (4.23) was also found to work well for modeling wavefront measurements for wall heating, since for \( \Delta T > 0, \) the third term of (4.23) is much smaller compared than the first two;

\[ OPD_{rms} = BK_G\rho_\infty \delta \sqrt{C_f M^2} \left[ 1 + D_1 \left( \frac{\Delta T}{T_\infty M^2} \right) \right], \]  

(4.24)

where the empirical constant \( D_1 = C_1/2 \) (Cress, 2010). Likewise, wavefront deflection angle spectra were shown to scale in a similar manner for wall heating:
\[
\hat{\theta}_{NORM}(S_{t\delta}) = \frac{\hat{\theta}(f)}{\rho_c \delta \sqrt{C_f M^2 \left(1 + D_1 \frac{\Delta T}{T_c M^2}\right)}}.
\] (4.25)

For equations (4.23) through (4.25), in the case of \(\Delta T = 0\) these relations reduce to previous \(OPD_{rms}\) and deflection angle scaling relations for the adiabatic wall boundary layer.

4.2.2 Experimental Validation

To validate the model for aero-optic effects of boundary layers with heat transfer, Cress (2010) performed a series of experiments investigating the effect of wall heating for a wide range of temperature differences \(\Delta T\) and freestream Mach numbers, \(M = 0.12 - 0.6\). Examinations of the deflection angle spectra of Malley probe wavefronts clearly showed that the linear temperature scaling from equation (4.25) collapses the spectra over a wide range of frequencies. This is shown in Figure 4.2 for the \(M = 0.4\) heated wall TBL from Cress (2010).

From a careful examination of equation (4.18), it is evident that a critical assumption in the derivation of this model is that the velocity structure in the TBL is not altered by the introduction of a non-adiabatic wall condition, but rather the difference in adiabatic and actual wall temperatures \(\Delta T\) acts as a scalar multiplier affecting the strength of the fluctuating temperature and density profiles in the TBL. The spectral collapse demonstrated by Cress (2010) shows the validity of these assumptions for moderate positive values of \(\Delta T\), at Reynolds numbers greater than 9,000.

\(OPD_{rms}\) computed from Malley probe measurements of heated wall TBLs also showed good agreement between experimental results and the model equations (4.23)
Figure 4.2. Mach 0.4 TBL deflection angle amplitude spectra for adiabatic wall and $\Delta T = 19.2^\circ C$ heated wall, normalized by equation (4.25) (top) without accounting for wall temperature and (bottom) considering wall temperature effects. [image source Cress (2010)]
and (4.24), as shown in Figure 4.3 and Figure 4.5 for a wide range of wall temperatures and Mach numbers. Figure 4.3 shows heated wall $OPD_{rms}$ data from Cress (2010) plotted against the linearized scaling from equation (4.24), using empirically obtained values of $D_1$, which differed from the values of constants predicted from integrating velocity profiles using equation (4.22). These experiments demonstrated that $OPD_{rms}$ could be successfully modeled using the linear scaling relationship over a wide range of wall temperature differences and subsonic Mach numbers. The results of these tests also highlighted the fact that even moderate values of wall heating could significantly increase wavefront aberrations above levels encountered for adiabatic wall temperature conditions only (Cress, 2010).

Figure 4.3. $OPD_{rms}$ versus the linearized heated wall temperature scaling, equation (4.24), using empirically determined $D_1$ constants. [image source Cress (2010)]
For the case where $\Delta T$ is negative (i.e. wall cooling) the linearized form of the model given in (4.24) is not valid, as the third term inside of the brackets in equation (4.23) is not negligible compared to the second term in this regime. The parabolic form of the temperature terms in equation (4.23) also indicates that there is some optimal value of wall cooling which results in the largest reduction in $OPD_{rms}$. Taking the derivative of equation (4.23) with respect to $\Delta T$, the optimal value of $\Delta T$ that gives the largest reduction in $OPD_{rms}$ can be found as

$$
\frac{\Delta T_{Optimal}}{T_o} = -\frac{1}{2}\left(\frac{C_1}{C_2}\right)M^2,
$$

(4.26)

which is purely a function of Mach number and the empirical constants $C_1$ and $C_2$. Using the constants computed from the $M = 0.5$ TBL by Cress (2010), Figure 4.4, left plot, shows predicted levels of $OPD_{rms}$ from (4.23) as a function of wall temperature for several Mach numbers.

Notice that the theoretical model predicts a reduction in $OPD_{rms}$ of about 80% at the optimal wall cooling temperatures. The optimal wall-cooling temperature, $\Delta T_{optimal}$, calculated from (4.26), is also shown in Figure 4.4, right plot as a function of freestream Mach number. It can also be shown that the amount of reduction in $OPD_{rms}$ at $\Delta T_{optimal}$ can also be expressed as a function of $C_1$ and $C_2$ by substituting equation (4.26) back into (4.23):

$$
\frac{OPD_{rms}(\Delta T)}{OPD_{rms}(\Delta T = 0)} = \sqrt{1 - \frac{1}{4} \left(\frac{C_1}{C_2}\right)^2}.
$$

(4.27)
Figure 4.4. a) Predicted $OPD_{rms}$ versus wall cooling temperature for different Mach numbers, and b) predicted optimal wall cooling temperatures using (4.26).
Cress (2010) also showed good agreement between experimentally measured $OPD_{rms}$ and the full wall-temperature effect model equation (4.23), again using empirically determined constants $C_1$ and $C_2$, for the heated wall data presented in Figure 4.3 as well as preliminary measurements of $OPD_{rms}$ for wall cooling for $M = 0.3, 0.4,$ and 0.5. The comparison between $OPD_{rms}$ for heated and cooled wall experiments and equation (4.23) from Cress (2010) is presented in Figure 4.5, with $OPD_{rms}$ normalized by the adiabatic wall value, $OPD_{rms} (\Delta T = 0)$. This allows the amplification and suppression

$$\frac{OPD_{rms}}{OPD_{rms,\Delta T=0}} = \left[ 1 + C_1 \left(\frac{\Delta T}{T_{\infty} M^2}\right) + C_2 \left(\frac{\Delta T}{T_{\infty} M^2}\right)^2 \right]^{0.5}$$

Figure 4.5. $OPD_{rms}$ normalized by adiabatic wall value plotted against RHS of equation (4.23) for a wide range of Mach numbers and temperature differences. [image source Cress (2010)]
of wavefront aberrations with respect to the baseline adiabatic wall more easily. In particular Cress (2010) found that measurements of $OPD_{rms}$ for wall cooling at $M = 0.3$ aberrations were reduced by almost 80% with respect to the adiabatic wall. He also found some indication that there may have been a minimum value of $OPD_{rms}$ in the $\Delta T < 0$ regime, although the limited number of wall cooling measurements did not allow for the clear identification of this predicted feature.

Large Eddy Simulations of compressible TBL aero-optic effects by White & Visbal (2011) showed good agreement with the trends predicted by Cress’ model, although there were significant quantitative differences between simulation data and model predictions. One possible reason for this discrepancy is the order-of-magnitude difference in Reynolds numbers between experiments and simulations (White & Visbal, 2011).

4.3 Effect of Step Changes in Wall Temperature

As discussed previously, the predictions from Cress’ model raises the possibility of using wall cooling as a method for aero-optic mitigation. In practical applications where this is of potential use, however, the energy expense of cooling of the full TBL development length in an airborne application might be impractical (Cress, 2010). One alternate means of applying wall cooling flow control might be to apply cooling over only partial lengths of the TBL development length upstream of the optical aperture. Therefore, this section will review research on the effects of step-changes in wall temperature boundary conditions on the turbulent wall bounded shear layer (TBL), and discuss how this knowledge might pertain to studies of partial wall temperature
modification on aero-optic aberrations in the TBL. In particular, literature focused on the following configurations of wall temperature step changes will be discussed:

- the transition across a step change with an increase in wall temperature,
- a step change resulting in a decrease in wall temperature downstream of the step, and
- multiple step-changes that are combinations of the above two cases.

Several schematics depicting these configurations are presented in Figure 4.6.

4.3.1 Effect of a Single Step Change in Temperature

Some of the earliest investigations of the effect of step-changes in wall temperature were performed by Reynolds, Kays, and Kline (1958a) as part of a series of reports on wall temperature effects on turbulent, incompressible boundary layers. To address the effect of step changes in wall temperature, they introduced an integral analysis using assumed 1/7th power-law profiles for both the mean velocity and temperature. Their analysis also assumed that a thermal sub-layer originated at the location of the step change, and grew internally independent of the velocity profile. This analysis was found to be in good agreement with temperature and velocity profiles measured in a boundary layer encountering a step-change in temperature, from an adiabatic to heated wall conditions (Reynolds, et al., 1958a).

Additional investigations of the step change from an adiabatic wall to a non-adiabatic wall boundary condition were also performed by a number of authors, including Gran, et al. (1974), Antonia, et al., (1977), and Charnay, et al. (1977). These studies built on earlier work by expanding these investigations to compressible flow (Gran, et al., 1974) and measuring the effect on fluctuating velocity and temperature profiles in
Figure 4.6. Schematics of three different step change configurations to be investigated in partial wall cooling experiments.
addition to the mean profiles (Antonia, et al., 1977; Charnay, et al., 1977). The results of these studies showed that the thermal disturbance introduced by the step significantly disrupts the self-similarity of the thermal boundary layer, requiring a distance on the order of $100\delta_0$, where $\delta_0$ is the TBL thickness at the step, to recover to a self-preserving state (Antonia, et al., 1977). The disturbance in the thermal profile takes the form of a thermal sub-layer that develops at the thermal step and propagates downstream; the sub-layer grows in thickness until it asymptotically approaches the boundary layer thickness.

For experiments in which the boundary layer encountered a ‘step-up’ in wall temperature (e.g. adiabatic wall to heated wall), the thickness, $\delta_T$, of this internal thermal layer was shown to grow approximately as $\sim x^{0.8}$ (Antonia, et al., 1977). Antonia, et al. (1977) also found that the streamwise growth of the thermal sub-layer is very similar to the way Townsend (1965) reported that a disturbance from an abrupt transition from smooth to rough walls propagates through the boundary layer (see Figure 4.7). Measurements of the thermal sub-layer thickness by Gran, et al. (1974) in a supersonic, compressible TBL with a step change in thermal boundary conditions from a cooled to an adiabatic wall reported a thermal sub-layer growth rate of $\delta_T \sim x$, which is similar to the rate found by Antonia, et al. (1977) for the adiabatic-to-heated wall step change.

Much like the external boundary of the TBL, the internal layer boundary between the canonical TBL and the developing thermal sub-layer was shown to be intermittent. The intermittency factor, $\gamma_{\delta_T}$, was found to be normally distributed about the mean sub-layer thickness, $\bar{\delta}_T$ (Antonia, et al. 1977).

Measurements also showed that sudden changes in wall heat flux conditions have little effect on the TBL velocity statistics for subsonic TBLs (Antonia, et al., 1977;
Charnay, et al., 1977). Similarly, Debiève, et al. (1997) showed that the transition from adiabatic to heated walls also did not appreciably alter the velocity statistics of a Mach 2.3 supersonic compressible TBL. Analysis of velocity and temperature measurements of supersonic TBL data did show deviation from the ESRA for the highest level of heating that persisted for distances downstream on the order of 50\(\delta\), although for moderate amounts of wall heating the TBL was found to be in good agreement with the ESRA at this streamwise location.

![Figure 4.7](image source Antonia, et al. (1977))

Figure 4.7. Streamwise development of internal thermal sub-layer thicknesses \(\delta_{\text{rms}}\) and \(\delta_T\) for the adiabatic to non-adiabatic wall step change. [image source Antonia, et al. (1977)]
Step changes where there is a reduction in wall temperature across the discontinuity (see Figure 4.6b) have also been studied in some detail by a number of authors (Gran, et al. 1974; Subramanian & Antonia, 1981; Hattori, et al., 2013). Similarly to the ‘step-up’ case discussed previously, it was found that an internal sub-layer also develops inside of the TBL. The growth rate of the sub-layer for the step-down case, however, is more rapid than for the step-up case. Subramanian & Antonia (1981) found from experiments that for a thermal step-down, the internal sub-layer growing approximately as \( \delta_T \sim x^{0.64} \) for a boundary layer flowing from a heated wall to an adiabatic wall. Gran, et al. (1974) reported similar results for supersonic TBLs undergoing a step change from an adiabatic to cooled wall, with the growth rate of the internal layer being approximately \( \delta_T \sim x^{0.5} \). Both of these experiments found results similar to the behavior recorded by Antonia & Luxton (1971, 1972) for a step change from a rough-wall TBL to a smooth wall, where the recovering internal layer grew as \( \delta_i \sim x^{0.43} \).

As shown in the previous section, the interface between the TBL and the ‘cold’ sub-layer induced by the step-down in temperature is also intermittent, the temperature fluctuations induced by changes in wall boundary conditions act passively, with no measurable effect on the velocity statistics (Subramanian & Antonia, 1981). Mean and fluctuating temperature profiles in the internal recovery layer were found to be self-similar across a majority of the internal layer when normalized by the sub-layer thickness and the maximum average temperature difference in the local profile (Subramanian & Antonia, 1981).
4.3.2 Effect of Multiple Step Changes

Relatively few investigations of the effect of a non-adiabatic strip on TBLs have been performed (Reynolds, Kays, and Kline, 1958b; Andreopoulos, 1983; Hattori, et al., 2013). However several authors (Reynolds, et al., 1958b; Andreopoulos, 1983) indicated that the effect of this arrangement of step changes could be considered as a superposition of the adiabatic to non-adiabatic step change (Antonia, et al. 1977) and the non-adiabatic to adiabatic step change (Subramanian & Antonia, 1980), both of which are discussed in the previous sections. Both step changes induce the development of an internal layer in the boundary layer, and the growth rate of the first transition is greater than that of the second (Andreopoulos, 1983), which is consistent with results from the previous sections.
4.4 Wall-Temperature Effects Review

In the first part of this chapter, the derivation of the Extended Strong Reynolds analogy was reviewed, and Cress’ (2010) application of the ESRA to modeling aero-optic aberrations of TBLs with heated and cooled wall was reviewed. It was found that both the ESRA model and experimental measurements showed that wall temperature acts as a passive scalar on TBL wavefront aberrations; some discrepancy was found in the model constants between experiments and theoretical predictions, but overall the theoretical model accurately captured the physics of the non-adiabatic wall aero-optic effects (Cress, 2010). In the Section 4.3, the effect of step changes in TBL heat transfer was reviewed. Step-changes in the thermal boundary conditions at the wall were shown to introduce a thermal sub-layer starting at the step location that grows up through the boundary layer as it propagates downstream. This growth rate of the thermal sub-layer was shown to depend on the ‘direction’ of the thermal step (i.e. adiabatic to non-adiabatic or vice versa). The thermal sub-layer boundary was found to be intermittent, and streamwise distances on the order of 100 boundary layer thicknesses were required in order for the boundary layer to recover to a self-preserving state.

The effect of wall temperature on aero-optic aberrations is certainly of interest for use in two ways in future research on the aero-optics of TBLs. The first area of interest is further investigating the use of wall cooling as a method of mitigating TBL induced wavefront aberrations. This model also predicted that wall cooling can result in a reduction of nearly 80% in $OPD_{rms}$. Preliminary measurements of $OPD_{rms}$ in the wall-cooling regime were presented by Cress, however there is a need to obtain a wide range of points to validate the statistical model predictions in this regime (2010). Additional
experimental measurements for this problem will be presented in Chapter 8 of this dissertation, along with results of comparison to the model from Cress (2010) reviewed in this chapter.

In addition to the necessary task of acquiring additional aero-optic wavefront measurements in order to validate Cress’ model for full wall cooling, investigations of the effect of partial wall cooling on TBL aero-optic mitigation are also desirable. This is because the energy expense of cooling of the full TBL development length in an airborne application might be impractical and/or cost-prohibitive, since the temperature difference at which optimal aero-optic mitigation is achieved increases as the square of the Mach number. The results of experimental measurements of the effect of partial wall cooling on TBL wavefront measurements will be presented in Chapter 8 in order to address this issue, and the effects of step changes in heat transfer at the wall will be investigated.

The results of experimental (Cress, 2010) and computational studies (White & Visbal, 2011) have also clearly demonstrated the capability to passively amplify TBL wavefront aberrations by heating the boundary layer wall. Experimentalists could use this effect to great advantage by using wall heating to increase the TBL aero-optic signal without altering the dynamics of the TBL. If the wall heating model is valid for low Reynolds number TBLs, then this technique can be used to obtain good quality aero-optic wavefront measurements for better comparison between computational and experimental results. The validity of the simplified wall-heating model on TBL wavefront statistics ($OPD_{rms}$ and spectra) as low as $Re_\theta = 9,000$ (Cress, 2010), however this is still several times the typical Reynolds numbers from computational studies (Wang & Wang, 2012; White & Visbal, 2012). Therefore, additional experiments are required in order to
determine the validity of the statistical model at Reynolds numbers that are comparable to those seen in recent simulations. The results of these experiments are presented in Chapter 9 of this dissertation, along with the methodology for recovering TBL wavefront statistics for adiabatic wall boundary layers from heated-wall wavefront measurements.
CHAPTER 5:
EXPERIMENTAL FACILITIES

The following chapter describes the experimental facilities and high-bandwidth wavefront sensors that were used to obtain the aero-optic and fluid dynamic measurements of compressible, subsonic, turbulent boundary layers. Section 4.1 contains a description of the Transonic Wind Tunnel (TWT) at Hessert Laboratories, University of Notre Dame, which is the wind tunnel where all experimental measurements presented in this dissertation were obtained. Section 4.2 presents detailed descriptions of the high-bandwidth Malley probe 1-D wavefront sensor used to obtain aero-optic wavefront measurements in all experiments.

5.1 Hessert Transonic Wind Tunnel (TWT) Facility

Experimental results presented in this dissertation are from experiments in the Transonic Wind Tunnel (TWT) facility located in Hessert Laboratory for Aerospace Research at the University of Notre Dame. The TWT, which is shown schematically in Figure 5.1, is composed of an inlet contraction, with an area ratio of 150:1 and screens and honeycombs to reduce freestream turbulence intensities (measured w/ hot-wires to be on the order of 2%). Following the inlet contraction is a modular smooth wall boundary layer development section, an optical measurement section, and a diffuser section that leads to the vacuum pump plenum.
Figure 5.1: Schematic of the Hessert Laboratory Transonic Wind Tunnel (TWT) configured for boundary layer aero-optics measurements.

The test section was constructed of Plexiglas so that the interior square duct cross section that was 10.0 cm × 9.9 cm (or about 4”×4”). The length of the TWT test section was fixed by inserting or removing modular lengths of the TBL development section. In the present study, the boundary layer development length to the optical measurement location varied between 100 cm and 170 cm. In the optical measurement section, the upper and lower Plexiglas walls were replaced with optical quality glass in order to ensure good optical access for aero-optic wavefront measurements. Wind tunnel velocity was controlled by changing the pressure in the plenum with vacuum pumps and a
pressure bleed valve installed on the plenum. In the experiments presented in later chapters, freestream Mach number spanned the range from approximately 0.2 to 0.5. In the present work, modifications to the TWT test section were made for Large-Eddy Breakup (LEBU) devices for aero-optic mitigation, and for investigating the effects of both full and partial wall heating and cooling. The specific details of the modifications to the test section will be discussed at the start of each subsequent chapter, prior to the presentation of experimental results for each method of boundary layer modification.

5.1.1 Baseline TBL Characterization

![Figure 5.2. Prior characterization of the TWT wind tunnel TBL; (top) Normalized velocity profiles $U(y)/U_\infty$ and $u_{rms}(y)/U_\infty$ [image source Cress (2010)].](image)

At present, the TWT has been used extensively in numerous experimental investigations of TBL aero-optic aberrations for over a decade (Gordeyev, et al., 2003),
and the TBL characteristics of the tunnel have been well characterized (Cress, 2010). Prior characterization of the TWT boundary layer by Cress (2010) used a hot-wire anemometer to obtain mean velocity, \( U(y) \), and velocity RMS, \( u_{rms}(y) \), profiles at a number of streamwise locations and a constant \( M_\infty = 0.52 \). From these velocity profiles, the boundary layer thickness, \( \delta \), can be estimated, and the integral displacement thickness, \( \delta^* \), and momentum thicknesses, \( \Theta \), can also be calculated:

\[
\delta^* = \int_0^\infty \left( 1 - \frac{U(y)}{U_\infty} \right) dy, \tag{5.1}
\]

\[
\Theta = \int_0^\infty \frac{U(y)}{U_\infty} \left( 1 - \frac{U(y)}{U_\infty} \right) dy. \tag{5.2}
\]

The boundary layer shape factor, \( H = \delta^*/\Theta \), can also be computed.

These measurements demonstrated that mean and RMS velocity measurements of the TBL exhibited self-similarity throughout the boundary layer development length. The growth of the displacement thickness, \( \delta^* \), was also found to follow the Prandtl \( 1/7 \)th power law (Schlichting, 1979), with a virtual origin 0.15 m upstream of beginning of the boundary layer development test section (Cress, 2010). Theoretical models for aero-optic aberrations have been developed using normalized TBL velocity profiles obtained from the TWT self-similar profile, and predictions from this model have compared favorably with TBL wavefront measurements obtained in other facilities with different cross sections, boundary layer thicknesses, Mach numbers, and Reynolds numbers (Cress, 2010; Gordeyev, et al., 2014).
Figure 5.3. a) Mean and b) fluctuating baseline velocity profiles for the Hessert TWT from the present study, compared to data from Cress (2010).
In the present study, baseline velocity profiles were obtained at $M_\infty = 0.4$ for streamwise locations $x = 120$ cm and 170 cm in order to verify that the TBL was consistent with previous measurements. These locations were chosen because they corresponded to the locations of the optical apertures used for investigations of the effect of passive flow control techniques on TBL aero-optic aberrations. The mean and RMS velocity profiles, normalized by the freestream velocity, are shown in Figure 5.3 along with the average profiles from Cress (2010) obtained at a similar Mach number. The results show good agreement between both the mean and fluctuating velocity profiles from the present study and from previous measurements in this facility.

Figure 5.4. Comparison of displacement thickness, $\delta^*$, computed from the present study and by Cress (2010) as a function of streamwise measurement location. The Prandtl $1/7^{\text{th}}$ power law from Cress (2010) is also shown.
From these data, the boundary layer thicknesses were also computed. For the velocity profile at 120 cm, the boundary layer thickness, $\delta$, was found to be 1.9 cm, and the integral displacement and momentum thicknesses were 2.6 mm and 1.9 mm, respectively. At 170 cm, $\delta$ was found to be approximately 2.4 cm, $\delta^*$ was 3.6 mm, and the momentum thickness $\Theta$ was 2.75 mm. The shape factors for both of these cases ranged from $H = 1.31 - 1.33$, which is consistent with other TBLs at these Reynolds numbers (Nagib, et al. 2007). The boundary layer displacement thickness was also computed from the velocity data obtained in the present study, and is plotted in Figure 5.4 along with previous data from Cress (2010).

5.2 Malley Probe 1-D Wavefront Sensor

As mentioned previously in Chapter 2, the Malley probe 1-D wavefront sensor was initially proposed by Malley, et al. (1992). It uses a small diameter beam to measure the local deflection angle of wavefronts that are convecting with an aero-optically active flow. Several variations that use two or more beams to experimentally measure the local convective velocity have been developed since its initial inception; these multiple beam Malley probes can also be used to obtain spatial correlations (Hugo & Jumper, 2000; Cress, 2010).

The two-beam Malley probe used in this dissertation used a 20 to 30 mW continuous wave (CW) HeNe ($\lambda = 633$ nm) laser source that was spatially filtered to generate a collimated small diameter beam with high, near uniform intensity across the beam profile. The spatial filter was also used to control the diameter of the source beam, which was approximately 1 mm. The source beam was then split into two parallel beams.
of equal intensity using a 50/50 beam splitter plate and a steering mirror. The spacing between these beams, $\Delta$, is typically on the order of $5 - 10$ mm. This pair of parallel beams then passes straight through a beam cube (losing 50% of its intensity) and through the measurement region (typically a wind tunnel test section) to the return mirror. The return mirror is aligned so that the Malley beams are reflected back to the beam cube along the same path. If the deflection angles are small (which they are for aero-optic measurements) then this is effectively an analog doubling of the signal intensity for the Malley probe, since the speed of light $c_0 \gg U_\infty$. This is referred to as the double-pass technique, since the Malley probe beams pass through the interrogation region twice in order to increase the signal-to-noise ratio.

![Figure 5.5: Schematic of a typical two-beam Malley probe sensor in a double-pass, double boundary layer (DBL) wavefront measurement configuration.](image)

The beam cube reflects the return beams onto a configuration of turning mirrors that isolate each of the two parallel Malley beams. Each beam is then passed through a final focusing lens, which in this case has a focal length of 1 m, onto an analog position
sensing device (PSD) placed in the focal plane of the final focusing lens. The PSD measures the displacement \( \Sigma \) of the beam spot location on its surface (relative to the mean value). Since the PSD is located one focal length away from the final focusing lens, a ray incident on the lens that is deflected at some angle \( \theta \) will can be calculated from the beam displacement on the PSD: \( \theta = \tan^{-1}(\Sigma / F) \approx \left( \Sigma / F \right) \) for small angles.

![Figure 5.6. A schematic of the final focusing lens and position sensing device (PSD), highlighting the special relationship between beam deflection angle \( \theta \) and lateral displacement \( \Sigma \) when the PSD is located one focal distance \( F \) away from the lens. [image source Cress (2010)]

The PSD is a photo-sensitive electronic device that can be used to determine the location of a focused spot on the face of the sensor. In the present study tetra-lateral PSDs are used due to their increased sensitivity for \( \lambda = 633 \text{ nm} \) light (since we are studying aero-optic mitigation). These PSDs have a linear response over more than 60\% of the sensor surface, but outside of this area the devices non-linear response has the potential to be problematic. Therefore, care was taken in all experiments to ensure that the Malley probe beam spots were centered on the PSDs before acquiring data. The PSDs used also
have a response time of $1 \mu s$, which allows them to achieve sampling rates on the order of hundreds of kilohertz. In the present study, time series of deflection angles for the two Malley probe beams were simultaneous recorded at a sampling frequency of 200 kHz. These data are then processed in the manner described in Chapter 2 to obtain deflection angle and wavefront spectra, convective velocity, and RMS wavefront error; a flowchart of the typical data processing procedures is shown in Figure 5.8.

Figure 5.7. (Right) Image of the SC-10D 2-D photodiode used in the Malley probe wavefront sensor as a PSD and (left) its corresponding typical position detectability. [image source Tetralateral page at www.osioptoelectronics.com]

Malley probe used in this dissertation was constructed on a six-foot optical bench that was placed on top of six inflated rubber tubes. The tubes were used in order to dampen mechanical vibrations caused by the wind tunnel motors, isolating the optical components of the Malley probe. The Malley probe return mirror was also mounted to the optical bench, outside of the test section as pictured in Figure 5.5. This also contributed to preventing the contamination of aero-optic wavefront data from
Figure 5.8. Flowchart depicting the different data processing procedures for Malley probe wavefront measurements.
mechanical vibrations. Recall from Chapter 2 that this configuration (mirror out of wind tunnel) is referred to as the double boundary layer (DBL) method, as it measures the aero-optic aberrations from both of the side-wall TBLs. For the present work, all wavefront data were acquired using the DBL measurement configuration, due to the advantage of a high signal-to-noise ratio and reduced contamination from mechanical vibration. Previously, Wittich, et al (2007) and Cress (2010) have shown that DBL measurements can be scaled to SBL levels by assuming that the upper and lower wall TBLs are statistically independent; the details of this SBL scaling are given in Appendix A. Prior work comparing DBL and SBL wavefront measurements of TBLs demonstrated that it was indeed valid for canonical TBL measurements.

This statistical scaling relation was also used by Cress to isolate wavefront statistics for cases where only one wall of the wind tunnel TBL development length was heated, modifying the total temperature in that boundary layer (Cress, 2010). This approach, described in detail in Appendix A, requires DBL wavefront measurements of the symmetric un-modified baseline flow (shown in Figure 5.9a) in addition to the DBL measurements of the one un-modified and one modified TBLs, see Figure 5.9b. From these data, the SBL-scaled statistics for the modified TBL (SBL, Modified) can be computed from the following relations:

\[
OPD_{\text{RMS}}^{\text{SBL, Modified}} = \sqrt{\left(\frac{OPD_{\text{RMS}}^{\text{DBL, Single Modified}}}{\text{RMS}}\right)^2 - \frac{1}{2} \left(\frac{OPD_{\text{RMS}}^{\text{DBL, Baseline}}}{\text{RMS}}\right)^2}, \tag{5.3}
\]

\[
\hat{\theta}_{\text{SBL, Modified}}(f) = \sqrt{\left(\frac{\hat{\theta}_{\text{DBL, Single Modified}}(f)}{\text{RMS}}\right)^2 - \frac{1}{2} \left(\frac{\hat{\theta}_{\text{DBL, Baseline}}(f)}{\text{RMS}}\right)^2}. \tag{5.4}
\]
Figure 5.9. a) Baseline, b) Single-TBL modified, and c) Double-TBL modified configurations for DBL aero-optic wavefront measurements.
In equations (5.3) and (5.4), ‘DBL, Baseline’ refers to wavefront statistics from the baseline TBL measurements obtained as shown in Figure 5.9a and ‘DBL, Single Modified’ refers to wavefront statistics from single-modified TBL wavefronts obtained as shown in Figure 5.9b. In order to validate the use of this scaling for non-symmetrically modified TBLs, several experiments were performed in which TBLs were symmetrically modified with LEBUs and with wall cooling, as shown in Figure 5.9c. Results of these comparisons will be presented in subsequent chapters and in Appendix A in order to validate the use of the single modified TBL scaling for particular types of modified TBLs.
CHAPTER 6:
EFFECT OF SINGLE LARGE-EDDY BREAKUP DEVICES ON TBLS

The next three chapters will present the results of experimental studies on the effect of Large Eddy Break-Up (LEBU) devices on aero-optic aberrations caused by compressible, turbulent boundary layers. Prior work investigating the aero-optics of boundary layers has shown strong evidence for the dominance of large-scale (i.e. outer-region) turbulent structures as a dominant source of boundary layer induced aberrations. This evidence, which was presented in Chapter 2, strongly suggests that one passive flow control device that could be very effective for mitigating aero-optic effects is the Large-Eddy Break-Up (LEBU) device (Jumper, 1980; Anders, 1985). LEBU devices, which were reviewed in Chapter 3, have shown great effectiveness at altering large-scale structures in the outer part of the TBL, decreasing turbulence correlation lengths, and reducing skin-friction in the modified turbulent boundary layer.

In this chapter, the experimental arrangement used for studying LEBUs for aero-optic mitigation will be presented along with a full explanation of the LEBU configurations tested in this dissertation. In the section 6.2 and section 6.3 detailed results of velocity and wavefront measurements downstream of selected single LEBU devices (see Figure 6.1a). One goal of this investigation is to verify that the LEBUs used in this work modify the velocity field in a manner that is consistent with the results of prior studies discussed earlier in Chapter 3. The present work also study seeks to study the
relationship between the characteristics of the LEBU-modified TBLs and their corresponding effect on wavefront statistics. The next chapter (Chapter 7) will present the results of parametric studies of single and multiple element LEBU devices performed using the Malley probe wavefront sensor in order to determine the optimal LEBU configuration for aero-optic mitigation.

Figure 6.1: Schematic of (a) Single LEBU, (b) Multi-LEBU, and (c) Tandem-LEBU device configurations.

6.1 Experimental Configuration

Experimental measurements of the turbulent boundary layer were conducted in the Hessert Transonic Wind Tunnel (TWT) described in Chapter 5, with a boundary layer development length of approximately 170 cm to the start of the optical measurement section. In order to obtain good quality wavefront measurements downstream of the LEBU devices, 30 cm sections of the upper and lower Plexiglas test section walls were replaced with flush mounted, optical quality glass windows. The test section was also modified so that LEBU devices could be mounted in the TBL at adjustable heights (see
Figure 6.2, right) using a pair of parallel aluminum bars embedded in the wind tunnel side walls. The LEBU mounts were installed so that the device trailing edge was located 5 mm upstream the start of the optical window ($x_{TE} = 169.5$ cm).

Figure 6.2. Schematic of the LEBU experimental setup (left) and mounting details as seen from upstream of the device (right).

Figure 6.3. A photograph of several single LEBU devices of different lengths ($l = 19$ mm, 24 mm, 38 mm, and 96 mm).

The LEBU devices, pictured in Figure 6.3, were constructed 20 gauge (~1 mm thickness) steel sheet metal. A shear press was used to cut each device to the desired LEBU length, $l$, and about 12 cm in span. A 90° bend was then formed about 1 cm in
from each end of the 12 cm length dimension, reducing the span of the LEBUs to approximately 10 cm – the internal width of the test section. Two parallel holes were drilled into each of the bent end sections of the LEBU devices, which pointed in the same direction and were parallel to the wind tunnel wall, so that two modified 8-32 countersunk hex socket screws could be used to secure the LEBU device to the wind tunnel wall mounts, as pictured in Figure 6.3. The leading edges of the LEBU devices were smoothed using a grinder/wire wheel brush, while the trailing edges were milled to a half-angle of less than 10° in order to prevent separation from occurring. A number of LEBU device configurations were constructed and experimentally evaluated in the Hessert TWT.

A full record of the LEBU configurations that were investigated in this work is presented in Table 7.1 of the next chapter. During all experimental measurements, free-stream velocity was measured using either a Pitot probe mounted just upstream of the LEBU device location or static wall pressure taps. Held constant for all tests at approximately $M = 0.4$. From the baseline TBL hot-wire velocity measurements obtained at Mach 0.4, which were presented in Chapter 5, the boundary layer thickness, $\delta$, was found to be 2.4 cm at the LEBU mounting location.

This chapter will present the results of hot-wire velocity measurements and Malley probe wavefront measurements in modified boundary layers for selected LEBU devices. The goal of examining these selected cases is to investigate the relationship between the effect of LEBU devices on velocity statistics and wavefront measurements. The two cases that are presented in this chapter are the $l = 1.6\delta$, $h = 0.6\delta$ and $l = 1.6\delta$, $h = 0.8\delta$ single LEBU devices.
6.2 Velocity Measurements

To investigate the effect of the devices on the turbulent boundary layer, hot-wire velocity measurements were obtained at three streamwise locations \((x/\delta = 1.5, 3.5, \text{ and } 5.4)\) downstream of the single LEBU devices using the hot-wire anemometer described in Chapter 5. From these data, mean and fluctuating velocity profiles were computed, along with power spectral densities of fluctuating velocity, and compared to results of the baseline TBL in order to evaluate the effect of LEBU devices on the boundary layer. These data were also compared to prior LEBU studies (which were discussed in detail in Chapter 3) in order to confirm that the LEBU devices used in this experiment behave similarly to what previous authors have reported.

Mean velocity profiles are plotted in Figure 6.4 for hot-wire velocity measurements obtained at three streamwise locations \((x/\delta = 1.5, 3.5, \text{ and } 5.4)\) downstream of the LEBU trailing edge for the \(l = 1.6\delta, h = 0.8\delta\) device. The wall-normal coordinate \(y\) is normalized by the incoming TBL thickness measured at the LEBU location, \(\delta_0 = 2.4\) cm. The velocity profile for the un-perturbed boundary layer is plotted for reference, and a horizontal dashed line marks the LEBU height. Downstream of the LEBU devices, a well-defined wake deficit is observed imprinted on the TBL mean velocity profiles; this wake region is centered about the LEBU height, and as \(x/\delta\) increases the maximum velocity deficit decreases and the wake spreads in the spanwise direction. Outside of the LEBU wake region, there is no significant change observed in the mean velocity profile.

Profiles of the root-mean-square of the fluctuating velocity component for the same LEBU device, which are plotted in Figure 6.5 along with the baseline case, showed a significant reduction in \(u_{rms}\) in the LEBU modified boundary layer, with the maximum
Figure 6.4. Mean velocity profiles of the LEBU-modified TBL at several locations downstream of a Single LEBU, \( l = 1.6\delta \), \( h = 0.8\delta \).

Figure 6.5. Root-mean-square velocity profiles of the LEBU-modified TBL at several locations downstream of a Single LEBU, \( l = 1.6\delta \), \( h = 0.8\delta \).
reduction being located at the LEBU height. Unlike the mean velocity profiles, the magnitude of the reduction did not change significantly with increasing \( x \), but rather remained constant around \( u_{rms} \approx 0.03U_\infty \). Velocity fluctuations in the region of the boundary layer below the LEBU height also showed a reduction in magnitude with respect to the baseline, with the reductions extending further towards the wall with increased distance downstream of the LEBU trailing edge. Above the LEBU device, small peaks were found in the \( u_{rms} \) profiles at all streamwise locations that exceeded the baseline \( u_{rms} \) locally. The magnitude of these spikes in RMS decreased further downstream from the LEBU and the peak location migrated away from the wall slightly.

Mean and RMS fluctuating velocity profiles obtained at the same \( x \)-locations downstream of the \( l = 1.6\delta \), \( h = 0.6\delta \) LEBU device are shown in Figure 6.6 and Figure 6.7, respectively. A cursory inspection of these data reveals that in general, the trends observed are very similar to those found for the \( h = 0.8\delta \) LEBU device. A well-defined wake deficit centered on the LEBU height was found in the mean velocity profiles that reduces in magnitude and spreads in the wall normal direction with increasing downstream distance. In the RMS profiles, the maximum reduction was observed at the LEBU height location; below the LEBU, a region of substantial reductions in \( u_{rms} \) was identified that expanded towards the wall as \( x \) increased. Local ‘spikes’ in \( u_{rms} \) were also discovered above the LEBU height location that behaved similarly to those observed for the \( h = 0.8\delta \) LEBU velocity profiles. The localized RMS spikes, however, were not as large with respect to the baseline \( u_{rms} \) velocity profile for the \( h = 0.6\delta \) LEBU.

Upon inspection, it was found that these velocity data are consistent with typical mean velocity profiles obtained in previous studies (Corke, et al., 1979; Hefner, et al.,
Figure 6.6. Mean velocity profiles of the LEBU-modified TBL at several locations downstream of a Single LEBU, $l = 1.6\delta$, $h = 0.6\delta$.

Figure 6.7. Root-mean-square velocity profiles of the LEBU-modified TBL at several locations downstream of a Single LEBU, $l = 1.6\delta$, $h = 0.6\delta$. 
1979; Lemay, et al., 1990), which showed a similar velocity deficit in the region just downstream of LEBU devices. Previous studies also showed that the wake mean velocity profile is nearly recovered to the baseline state by about $7.5\delta_0$, (Lemay, et al. 1990), and beyond $\sim20\delta_0$, the mean profile of the LEBU-modified boundary layer recovers to the natural boundary layer profile (Corke, 1982; Anders, 1990; Lemay, et al., 1990).

### 6.2.1 Wake-Deficit Velocity Profiles

In the mean velocity profiles shown, the maximum velocity deficit in the wake region is shown to diminish as downstream distance increases, indicating that the modified boundary layer is recovering from the disturbance caused by the LEBU device, at least in the mean velocity profiles. The spanwise extent of the wake imprint spreads in the wall-normal direction with increasing distance downstream. Both of these characteristics are consistent with the behavior of planar self-similar wakes in constant mean flow like those investigated by Moser, et al. (1998). They found that for planar wakes, the wake velocity deficit, $U_0$, and the wake half-width, $b$, evolved in the streamwise direction as $b \sim x^{1/2}$ and $U_0 \sim x^{-1/2}$ (Moser, et al., 1998). The non-dimensional growth rate of the wake, defined as $\alpha = \Theta^{-1} d(b^2)/dx$ where $\Theta$ is the wake momentum thickness, was found to be $0.29 - 0.41$ for canonical self-similar plane wakes (Moser, et al., 1998).

To determine if the LEBU wake evolves in a self-similar manner within the TBL, a modified form of the wake deficit velocity profile, $U_{\text{Deficit}}(y)$, is computed by subtracting the LEBU-modified boundary layer profiles from the baseline velocity profiles (Moser, et al., 1998):
Figure 6.8. Mean wake velocity deficit profiles downstream of the $l = 1.6\delta$, $h = 0.6\delta$ single LEBU device.

Figure 6.9. Streamwise development of the maximum wake velocity deficit and wake half-width for the $l = 1.6\delta$, $h = 0.6\delta$ single LEBU device.
\[ U_{\text{Deficit}}(y) = U_{\text{LEBU}}(y) - U_{\text{Baseline}}(y). \] (6.1)

The maximum wake deficit velocity, \( U_0 \), is the maximum of the absolute value of the wake deficit profile, \( U_0 = \max \left| U_{\text{Deficit}}(y) \right| \). Note that where the baseline velocity is greater than the mean velocity of the LEBU profile, \( U_{\text{Deficit}} \) will be negative. The wake half-width, \( b \), is defined as the distance between the two points where \( -U_{\text{Deficit}}(y) = 0.5U_0 \) (Moser, et al., 1998).

Mean velocity deficit profiles for the LEBU modified boundary layer data were computed, and the results are shown in Figure 6.8 for the \( l = 1.6\delta, h = 0.6\delta \) LEBU device, along with the mean deficit profile from Moser, et al. (1998) for canonical plane wakes. The LEBU wake deficit profiles presented exhibit self-similarity that is consistent with canonical plane wake data on the outer part of the boundary layer (i.e. \( (y - h) / b > 0 \)) for all streamwise locations. For the inner part of the wake (i.e. \( (y - h) / b < 0 \)), the wake profiles deviate slightly from the self-similarity scaling, with the deviation becoming larger for locations further downstream. From these data, the non-dimensional growth rate was found to be approximately 0.15 for the LEBU modified TBL wake profiles. This was much less than the range of growth rates, 0.29 – 0.41, found for canonical self-similar plane wakes (Moser, Rogers, & Ewing, 1998).

The deficit profile for the RMS velocity fluctuations can be computed in a similar fashion to the mean velocity in equation (6.1):

\[ u_{\text{rms, Deficit}}^2(y) = u_{\text{rms, LEBU}}^2(y) - u_{\text{rms, Baseline}}^2(y). \] (6.2)

RMS velocity deficit profiles computed via equation (6.2), however, were not found to scale well with self-similar planar wake data, due to the large reductions in \( u_{\text{rms}} \) below the
LEBU device. The location of the ‘spike’ in $u_{rms}$ that occurs on the upper side of the LEBU in Figure 6.7 is found to be constant at about $(y-h)/\delta \approx 0.6$. This is an indication that the local increase in $u_{rms}$ above the LEBU is closely related to the wake deficit profile in the modified TBL.

![Figure 6.10](image1.png)

Figure 6.10. Wake root-mean-square velocity profiles downstream of the $l = 1.6\delta$, $h = 0.6\delta$ single LEBU device. Legend is the same as Figure 6.8.

Corke (1981) recognized that local changes in the mean velocity profile could have a significant impact on velocity fluctuations in the LEBU-modified boundary layer, since the dominant turbulence production term $\overline{u'v'}(dU/dy)$ is directly proportional to the local mean velocity gradient. Since the wake deficit mean profile induced on the LEBU within a sheared mean flow, the local velocity gradient below the LEBU device is decreased and the velocity gradient above the LEBU is increased. This is illustrated in Figure 6.11, where $dU/dy$ and the fluctuating velocity component in the LEBU-modified
boundary layer are plotted for easy comparison. Here, \( dU/dy \) was computed from experimental data using a first order central difference method. It is evident from Figure 6.11 that there is some correlation between the velocity gradient and \( u_{rms} \), especially in the neighborhood of the LEBU height, however changes in \( dU/dy \) for the LEBU-modified boundary layer does not fully account for reductions in velocity fluctuations below the LEBU device. Rather it is likely that both the changes in velocity gradient and the limitation of vertical velocity fluctuations (i.e. the plate effect), that suppressed \( u_{rms} \)

Figure 6.11. a) normalized mean velocity gradient and b) baseline-normalized \( u_{rms} \) profiles for LEBU modified boundary layer (\( l/\delta = 1.6, h/\delta = 0.6 \)). Legend is the same as Figure 6.7.
over such a wide range of the area below the LEBU device. Additional support for this notion was shown in Lemay, et al. (1990), where measurements showed suppressed production for \( y < h \) in the LEBU-modified boundary layer up through the location where the mean profile (and therefore the velocity gradient) is *nearly* recovered to baseline state \( (x/\delta = 7.5; \text{see Figure 3.6}) \).

6.2.2 Velocity Spectra

Power spectral densities of fluctuating velocity at the LEBU heights are presented for all three downstream locations in Figure 6.12 and Figure 6.13, normalized by the local \( u_{rms}^2 \). Frequency is presented in normalized form as a Strouhal number based on TBL thickness \( \delta \) and freestream velocity \( U_\infty \): \( St_\delta = f\delta/U_\infty \). The sharp discontinuities in the high-frequency range are the result of electronic interference from other laboratory equipment, and are not related to physical boundary layer phenomenon. Compared to the baseline velocity spectrum, there is a significant reduction in the range \( St_\delta < 2 \) for both devices at all streamwise locations. For both LEBU heights, there is an increase above the baseline spectra in high frequencies at \( x/\delta = 1.5 \) that corresponds to a Strouhal number greater than 2. This peak dissipates quickly, as it is not present in the spectra for the next streamwise station (located only \( 2\delta \) downstream) for either LEBU height. This observation is consistent with trends from velocity spectra obtained by Corke (1981) just \( 1\delta \) downstream of LEBU devices in the wake, which also showed a reduction in low frequencies and an increase in high frequencies (see Figure 3.7). This increase in high frequencies corresponds to the introduction of small-scale turbulent structures by the LEBU wakes that dissipate quickly.
Figure 6.12. Power spectral density of velocity fluctuations in the LEBU wake ($y = h$) downstream of the $l/\delta = 1.6$, $h/\delta = 0.8$ LEBU device.

Figure 6.13. Power spectral density of velocity fluctuations in the LEBU wake ($y = h$) downstream of the $l/\delta = 1.6$, $h/\delta = 0.6$ LEBU device.
To investigate what is happening to the LEBU velocity power spectral density elsewhere in the LEBU wake, the ratio of LEBU-modified power spectra to baseline spectra was calculated as a function of $y$ and Strouhal number,

$$C_u'(y/\delta, St_\delta) = \frac{|\hat{u}'_{LEBU}(y/\delta, St_\delta)|^2}{|\hat{u}'_{Baseline}(y/\delta, St_\delta)|^2}. \quad (6.3)$$

Surface plots of these results are presented in Figure 6.14 for each of the streamwise locations that data was obtained at downstream of both the $h/\delta = 0.8$ and 0.6 devices. Note that values of $C_u'< 1$ indicate a reduction in velocity fluctuations with respect to the baseline, while $C_u'> 1$ indicates an increase. Lines of constant frequency in the surface plots of $C_u < 0.5$ near $St_\delta = 3-4$ in Figure 6.14a–f, and of $C_u > 4$ near $St_\delta = 9$ in Figure 6.14a, d, and f correspond to electronic interference from devices in the laboratory environment that was present in some of the hot-wire anemometer data at these frequencies. In all of the plots shown in Figure 6.14, the LEBU height is denoted with a horizontal white line, and contours of $C_u' = 0.5$ and $C_u' = 1.0$ are marked with solid and dashed black lines, respectively.

At the first measurements station ($x/\delta = 1.5$) downstream of the $h/\delta = 0.8$ LEBU device, the baseline normalized velocity spectra presented in Figure 6.14a show a narrow region of the boundary layer in the LEBU wake where $C_u < 0.5$ (greater than a 50% reduction in velocity fluctuations). This region extends from $y/\delta = 0.74 – 0.82$ in the wall-normal direction and covers a broad range of low frequencies up to approximately $St_\delta = 1$. Below this region in the boundary layer, velocity fluctuations for $St_\delta < 1$ were also found to be reduced with respect to the baseline velocity spectra. This effect is diminished closer to the wall, with $C_u' > 0.8$ for $y < 0.1\delta$. For high frequencies, i.e. $St_\delta > 1$,
there is a strong increase in velocity fluctuations at \( y \)-locations just above the LEBU device for \( St_\delta > 2 \); in this range the peak levels of velocity fluctuations are over four times that of the unperturbed boundary layer at this streamwise location. In Figure 6.14b, spectra for the next measurement location, \( x/\delta = 3.5 \), are presented. This data shows similar characteristics to the spectra surface presented in Figure 6.14a for \( x/\delta = 1.5 \), but there are some evident differences. The region of \( C_u' < 0.5 \) is slightly increased in the wall-normal direction (extending slightly farther towards the wall), and the high-frequency peak above the LEBU device is shown to be somewhat reduced. The peak of this region is still greater than four times the baseline spectra, however the area of this peak is smaller compared to the upstream location. A small area is observed near the wall (\( y/\delta = 0.1 - 0.2 \)) where \( C_u' \) is slightly greater than one, however the increase over the unmodified TBL is less than 5%. The spectra surface for the furthest downstream measurement location, \( x/\delta = 5.4 \), is presented in Figure 6.14c. This data showed that there is a significant increase in wall-normal extent of the region where \( C_u' \) is reduced below 0.5, and that the high-frequency peak at locations above the LEBU device is substantially reduced, further supporting the notion that this high-frequency peak corresponds to quickly dissipating, small-scale structures introduced into the flow by increased turbulence production on the upper side of the LEBU wake.

Spectra surfaces for the \( h/\delta = 0.6 \) LEBU device (see Figure 6.14c, d, and f) show very similar trends to the \( h/\delta = 0.8 \) device: a region of significant spectra reduction in the LEBU wake that expands towards the wall with increased downstream distance, and a peak in the high-frequency range at locations above the LEBU that decays further downstream of the device. Aside from these shared characteristics, there are some
Figure 6.14. Surface plots of $C_u'$ computed at several locations downstream of the (a, b, and c) $l/\delta = 1.6$, $h/\delta = 0.8$ and (d, e, and f) $l/\delta = 1.6$, $h/\delta = 0.6$ LEBU devices. Contour $C_u' = 1$ marked by dashed black line: $- -$; contour $C_u' = 0.5$ marked by solid black line: $-$; LEBU height marked by dashed white line.
differences for spectra downstream of the lower, $h = 0.6\delta$, LEBU device. At the $x/\delta = 1.5$ streamwise location, shown in Figure 6.14d, the initial region of $C_u < 0.5$ is slightly larger in the wall-normal direction, and high-frequency peak above the device is much weaker compared to the peak observed in Figure 6.14a. Figure 6.14d also shows reductions in velocity fluctuations throughout the TBL, both above and below the LEBU for $St_\delta < 1$. At the second streamwise location ($x/\delta = 3.5$) in Figure 6.14e, the wallward expansion of the region of $C_u < 0.5$ is more rapid compared to changes shown between Figure 6.14a and b, and the high-frequency peak above the LEBU device is substantially reduced compared to both the previous streamwise station and the same station for the $h = 0.8\delta$ device. Again, reductions in velocity fluctuations were found throughout boundary layer below $St_\delta = 1$ and up to $y = \delta$. Spectra for the final streamwise measurement location for the $h = 0.6\delta$ LEBU, presented in Figure 6.14f, show the largest observed area of $C_u < 0.5$, which extends from $h/\delta = 0.4$ to 0.7, for $St_\delta \leq 1$.

In general, spectra for both LEBU devices show that the largest suppression of velocity fluctuations in the LEBU modified boundary layer occurs over a broad range of low frequencies ($St_\delta < 1$), at wall-normal locations that are concentrated around the LEBU height, but moderate reductions in velocity fluctuations are found at low frequencies throughout the boundary layer. For both LEBU heights presented in Figure 6.14, the region of the boundary layer where $C_u < 0.5$ starts out concentrated around the LEBU height and extends further down towards the wall with increasing downstream distance. This region of significant reductions in velocity fluctuations is slightly larger for the LEBU device that is placed lower in the boundary layer. Above the LEBU device there is an increase in velocity fluctuations that is concentrated at high frequencies, $St_\delta > 2$. 
Physically, this corresponds to the addition of small-scale turbulent structures in the modified boundary layer that is likely a consequence of the increased velocity gradient on the high side of the LEBU wake. This high-frequency peak decreases in intensity at locations further downstream, which is consistent with the dissipation of the small-scale turbulent structures introduced by the wake. The amplitude of this high-frequency peak is also reduced for the LEBU device placed lower in the boundary layer.

6.3 Aero-Optic Wavefront Measurements

This section presents the results of aero-optic wavefront measurements of the LEBU modified TBL for the two single LEBU devices presented in the previous section: \( l = 1.6\delta, h = 0.6\delta \) and \( l = 1.6\delta, h = 0.8\delta \). Measurements were obtained with the Malley probe 1-D wavefront sensor, which is described in detail in Chapter 5 along with its data reduction and analysis procedures. As mentioned in section 6.1, the majority of the wavefront data obtained for LEBU-modified boundary layers was measured for a single modified boundary layer (SMBL) and one un-modified boundary layer, and then data were scaled to isolate the wavefront statistics for the SMBL using the procedure outlined in Appendix A. This procedure has been used effectively for adiabatic wall boundary layers, as well as TBLs modified through wall temperature changes (Cress, 2010); it has not been previously validated for boundary layers with flow control devices that alter the turbulence structure. In order to investigate the validity of this scaling for SMBL wavefront data, Malley probe measurements were obtained for both single and double modified boundary layers (see Figure A.2). SBL scaled spectra from both of these cases were then compared in Figure A.3, and shown to compare well with one another. In light
of this good agreement, the rest of the LEBU devices characterized in this study using high-bandwidth wavefront sensors were studied with only a single modified TBL, as shown in Figure 5.8b. This choice was made primarily in order to reduce the complexity of the experimental setup.

6.3.1 Deflection Angle Spectra

Amplitude spectra of Malley probe deflection angle measurements were computed for measurements downstream of the $l = 1.6\delta$, $h = 0.8\delta$ and $l = 1.6\delta$, $h = 0.6\delta$ single LEBU devices at a number of locations up to $10\delta$ downstream of the device trailing edge. The results are presented for several streamwise locations in Figure 6.15 and Figure 6.16 where frequency is non-dimensionalized as Strouhal number based on freestream velocity, $V_\infty$, and the canonical TBL thickness at the LEBU mounting location, $\delta$. Note that for $St_\delta < 0.2$, deflection angle spectra are contaminated by mechanical vibration from the wind tunnel motors. Also, in the high frequency range of the spectra the sharp peaks near $St_\delta = 6$, 7, and 10 were found to be the result of electronic interference from sources within the laboratory environment. This is typical of Malley probe deflection angle measurements obtained in previous experiments in a number of facilities (Smith & Gordeyev, 2013; Gordeyev, et al. 2014).

In Figure 6.15a, the spectra just downstream of the $l = 1.6\delta$, $h = 0.8\delta$ single LEBU device, at $x = 0.8\delta$, show that initially there is a large reduction in low frequencies $St_\delta \approx 0.2 – 4$. Assuming that the flow is frozen, this is the range of frequencies that corresponds to convecting structures that are $0.25 – 5\delta$ long in the streamwise direction. Note that due to the Malley probe’s sensitivity to mechanical vibration, the effect of LEBU devices on deflection angle spectra below $St_\delta = 0.2$ (i.e. structures larger than $5\delta$) cannot be
Figure 6.15. Deflection angle spectra from Malley probe measurements of the modified TBL for the $l = 1.6\delta$, $h = 0.8\delta$ single LEBU device at streamwise locations a) from $x = 0.8\delta$ to $x = 4.1\delta$ and b) from $x = 4.1\delta$ to $x = 9.5\delta$. 
ascertained from analysis of the wavefront spectra. A slight increase above the baseline spectra was also found downstream of the LEBU trailing edge for high frequencies around \(\text{St}_\delta = 5\). Downstream of the LEBU device in the region from 1 to 4\(\delta\), the peak of the LEBU modified deflection angle spectra from \(\text{St}_\delta = 1 - 5\) increased with downstream distance, and there was a slight increase above the baseline spectra in the neighborhood of \(\text{St}_\delta = 3 - 5\). The low frequency range, 0.2 < \(\text{St}_\delta\) < 1, deflection angle spectra were found to increase less rapidly with downstream distance compared to the spectra peak and high-frequency regions.

Figure 6.15b presents deflection angle spectra for the same LEBU device for streamwise locations between 4 and 10\(\delta\) downstream of the LEBU trailing edge. Spectra in this region also show significant reductions in the low-frequency (\(\text{St}_\delta\) < 2) regime, indicating that reductions in large-scale structures in turbulent flow persist for at least 10\(\delta\). This conclusion is consistent with observations from a number of previous studies discussed in Chapter 3, especially with flow-visualization records for LEBU-modified TBLs (Corke, 1981). The spectra in Figure 6.15b also show that at downstream locations > 4\(\delta\), the increase in deflection angle spectra above the baseline measured spectra found in high frequencies disappears. Beyond 4\(\delta\), the spectra presented show that the entire high-frequency range begins to decrease with increasing \(x\).

Figure 6.16 presents deflection angle spectra obtained from Malley probe measurements downstream of the \(l = 1.6\delta\), \(h = 0.6\delta\) single LEBU device at the same streamwise locations shown in Figure 6.15. Just downstream of the device (\(x \sim 1\delta\)), the spectra presented in Figure 6.16a also showed significant reductions over the range of low frequencies, 0.2 < \(\text{St}_\delta\) < 4. However, the reductions were found to be larger than
Figure 6.16. Deflection angle spectra from Malley probe measurements of the modified TBL for the $l = 1.6\delta$, $h = 0.6\delta$ single LEBU device at streamwise locations a) from $x = 0.8\delta$ to $x = 4.1\delta$ and b) from $x = 4.1\delta$ to $x = 9.5\delta$. 
those observed in Figure 6.15a for the \( h = 0.8\delta \) single LEBU device. Spectra for the \( h = 0.6\delta \) device at streamwise locations \( x < 4\delta \) did not show any significant increase over the baseline deflection angle spectrum, in contrast to what was observed in Figure 6.15a. By \( x = 2.4\delta \) the peak of the LEBU-modified TBL spectrum, which is around \( St_\delta = 2 \), increased slightly. The peak amplitude then remained relatively constant through approximately \( x = 4\delta \). Portions of the high-frequency range of the spectra (\( St_\delta > 4 \)) and the low-frequency end of the spectra (\( St_\delta < 0.5 \)) were found to decrease with increasing \( x \) between \( 3 - 4\delta \) downstream.

Deflection angle spectra collected between \( x/\delta = 4 - 10 \) for the \( h = 0.6\delta \) single LEBU are shown in Figure 6.16b. In general, spectra from these distances downstream of the LEBU trailing edge were found to result in broadband reductions that were notably larger than what was found for the \( h = 0.8\delta \) device. The peak amplitude of the deflection angle was shown to be increasing for measurements further downstream of the LEBU, and the peak location was found to be shifting slightly back towards \( St_\delta = 1 \), the peak location for the un-perturbed TBL wavefront spectra. As \( x/\delta \) approached 10, there were notable increases in the low-frequency range of the spectra that were caused by the shift in peak frequency and increase in peak amplitude of the LEBU-modified TBL.

In summary, inspection of deflection angle spectra for the two single LEBUs presented in Figure 6.15 and Figure 6.16 demonstrated that the devices yielded immediate and significant reductions in the low-frequency range of the spectra. Note that assuming frozen flow (which has been shown to be a good assumption for Malley probe wavefront measurements in the TBL), the range of frequencies over which reductions were observed corresponds to turbulent structures that are up to \( 5\delta \) in length. For Malley
probe spectra for the LEBU modified TBLs, the peak location was found to initially shift to higher Strouhal numbers (~ 2), indicating that the characteristic length scales in the modified TBL are reduced via the LEBU devices and their effect on large-scale motions.

The behavior of the high-frequency portion of the deflection angle spectra gave some evidence that the energy formerly contained in the large scale turbulent structures was redistributed by the LEBU wake into smaller scale turbulent structures. The turbulent kinetic energy (TKE) contained in these small scale structures was quickly dissipated through the energy cascade. After the excess TKE is shed through the mechanism of dissipation, the high-frequency range of spectra is also reduced. These observations are consistent with measurements of the TKE balance in a LEBU modified TBL performed by Lemay, et al. (1990), which showed both a significant increase in the production term and an increase in the dissipation term in the LEBU wake in the neighborhood of the \( y = h \) just downstream of the LEBU device (approx. \( x/\delta \approx 2.5 \)). Within 10\( \delta \) downstream of the LEBU trailing edge, however, Lemay, et al. (1990) found that the dissipation term is nearly recovered to the baseline canonical TBL levels. This is consistent with the downstream distances found in the present study at which wavefront spectra were found to show reductions in the high-frequency end of the spectra.

The examination of deflection angle spectra for both the \( h = 0.8\delta \) and \( h = 0.6\delta \) LEBU devices also highlights that the effect of LEBU devices on both the boundary layer and wavefront characteristics are highly dependent on LEBU geometry. This result compliments results from LEBU drag reduction investigations, which also found the effects to be highly dependent on device geometry (Corke, 1981; Savill & Mumford,
The effect of geometry on levels of aero-optic mitigation will be discussed later on in Chapter 7.

To better visualize the evolution of deflection angle spectrum in the streamwise position as compared to the baseline deflection angle spectrum, the ratio of deflection angle spectrum for LEBU-modified TBLs to baseline TBL spectrum can be defined as

\[
C_\theta \left( \frac{x}{\delta}, St_\delta \right) = \left| \frac{\hat{\theta}_{LEBU} \left( \frac{x}{\delta}, St_\delta \right)}{\hat{\theta}_{Baseline} \left( \frac{x}{\delta}, St_\delta \right)} \right| - 1, \tag{6.4}
\]

where \( C_\theta \left( \frac{x}{\delta}, St_\delta \right) \) will be equal to zero where the LEBU-modified spectrum is equal to the baseline spectrum, positive where the LEBU-modified spectrum is increased compared to the baseline, and negative where the LEBU-modified spectrum is less than the baseline spectrum. Contour plots of \( C_\theta \left( \frac{x}{\delta}, St_\delta \right) \) are presented in Figure 6.17 for both single LEBU devices. For both cases, Figure 6.17 shows a significant reduction in large-scale structures below \( St_\delta = 3 \) that diminishes as distance from the LEBU trailing edge increases. Additionally, a decrease in high-frequency content of the deflection angle spectrum is also shown as the downstream distance increases, which is consistent with the results previously shown in Figure 6.15 and Figure 6.16.

One feature that is readily apparent in the LEBU-modified deflection angle spectrum plotted in Figure 6.17 is the evolution of the peak of \( C_\theta \left( \frac{x}{\delta}, St_\delta \right) \) with the downstream distance from the LEBU device, which likely corresponds to the development of the LEBU device wake. The shift of the peak in \( C_\theta \) to the lower frequencies occurs as the downstream distance increases, eventually approaching a value of \( St_\delta = 1 \). It was discussed in chapter 3 that several authors (Corke, 1981; Lemay, et al. 1990, and others) found that just downstream of the LEBU there is a momentum-deficit
Figure 6.17. Baseline-normalized spectrum surfaces $C_\theta(x/\delta, St_\delta)$ for single LEBU devices of height a) $h/\delta = 0.8$ and b) $h/\delta = 0.6$. Note that * denotes the peak location of $C_\theta$ at each value of $x$. 
wake that begins at the LEBU trailing edge, and grows with the increasing downstream distance, $x$, to a point where it is on the order of the TBL thickness $\delta$ and interacts with the near-wall layer. If it is assumed that the peak in $C_\theta$ corresponds to a coherent structure of some size, $\gamma$, then the relationship between the structure size $\gamma$ and the frequency $f_{\text{Peak}}$ at which the maximum in $C_\theta$ occurs can be expressed via the frozen flow assumption,

$$\gamma \approx \frac{U_*}{f_{\text{Peak}}} \delta \frac{\delta}{St_\delta^{\text{Peak}}},$$

where $St_\delta^{\text{Peak}}$ is the Strouhal number that corresponds to $f_{\text{Peak}}$. If it is assumed that the shift of $St_\delta^{\text{Peak}}$ toward lower frequencies corresponds to the growth in the size of the LEBU wake, then the half-width, $b$, of the LEBU wake can be estimated as a function of $x$ from the peak of the normalized spectrum;

$$b(x) \approx \frac{\gamma}{2} \frac{\delta}{2 \cdot St_\delta^{\text{Peak}}}.$$

The result of this estimation for the single LEBU, $h = 0.6\delta$ device is presented in Figure 6.18 along with measurements of the wake half-width computed from hot-wire velocity profiles presented earlier in Figure 6.9. The comparison shows that the estimate of the wake half-width, $b$, from the normalized spectrum peaks is in good agreement with direct measurements obtained previously for the same case, and verifies that the evolution of the peak of the normalized spectrum $C_\theta$ is a result of the growth of the LEBU wake within the modified TBL.

Downstream of the location where the LEBU wake reaches the wall (which is denoted as $x_w$), Lemay, et al. (1990) showed that the distribution of the turbulence production and dissipation in the modified TBL recovers to the pre-LEBU boundary layer state. Based on these findings, it is expected that after the LEBU-modified spectrum peak
reverses to $St_δ \approx 1$ (indicating wake-wall interaction), LEBU modified deflection angle spectrum at locations far downstream of $x_w$ will recover to the canonical TBL spectrum. In the present study, it is apparent from the shape of deflection angle spectrum approaching $10\delta$ that the single LEBU-modified TBL is not yet recovered to a canonical state. However, the increases in Malley probe deflection angle spectra in the neighborhood of $St_δ \sim 1$ towards the baseline values indicates that the boundary layer is beginning to recover towards the ‘baseline’ form.

![Graph showing comparison of wake half-width $b$ estimated from $C_θ(x/δ, St_δ)$ and from hot-wire velocity measurements downstream of the $l = 1.6δ$, $h = 0.6δ$ from section 6.2.1.]

Figure 6.18. Comparison of wake half-width $b$ estimated from $C_θ(x/δ, St_δ)$ and from hot-wire velocity measurements downstream of the $l = 1.6δ$, $h = 0.6δ$ from section 6.2.1.

6.3.2 $OPD_{rms}$ Results

A number of wavefront statistics, including $OPD_{rms}$, may be computed from Malley probe deflection angle measurements using the methods described in detail in
Chapter 5. Figure 6.19 presents $OPD_{rms}$ computed for the $l = 1.6\delta$, $h = 0.8\delta$ and $h = 0.6\delta$ single LEBU devices as a function of streamwise location. Note that $OPD_{rms}$ is normalized by the baseline $OPD_{rms}$ (i.e. no LEBU device). The error bars are calculated from estimates of the uncertainty in $OPD_{rms}$; this uncertainty is primarily due to the ~10% uncertainty in the measurement of the Malley probe beam spacing, $\Delta$, which propagates to $U_c = \Delta/\tau$, and then to $OPD_{rms}$ through equation (2.7). The uncertainty in both the SMBL LEBU wavefront measurements and the DBL baseline measurements are combined using the rules of uncertainty propagation (Dunn, 2014) to calculate the error bars shown in Figure 6.19.

For both devices, $OPD_{rms}$ is reduced by approximately 25% at the first measurement station around $x \approx 1\delta$ downstream of the LEBU trailing edge. For the $h = 0.8\delta$ device, $OPD_{rms}$ in the manipulated boundary layer is reduced over the entire streamwise extent of the study by at least 25%, and remain relatively constant over the length of the study. $OPD_{rms}$ downstream of the $h = 0.6\delta$ single LEBU decreased to approximately 65% of the baseline value (a 35% reduction) by $x = 2\delta$, and this level of reduction is sustained through about $5\delta$. Downstream of this location, there appears to be a slow but steady increase in the $OPD_{rms}$ of the LEBU modified TBL. By $x = 10\delta$ the $h = 0.6\delta$ device gives approximately a 30% reduction in $OPD_{rms}$ over the baseline value. Note that a clear transition to increasing values of $OPD_{rms}$ further downstream is not observed by $10\delta$ for the $h = 0.8\delta$ LEBU. However, the spectral evidence from Figure 6.15 and Figure 6.17a (namely the shift in peak location towards $St_\delta = 1$ and slight increase in low-frequency range) suggests that for the $h = 0.8\delta$ single LEBU, $OPD_{rms}$ might begin to
recover towards the baseline level at some point further downstream of the last measurement station, 10\(\delta\).

![Figure 6.19. Streamwise development of baseline normalized \(OPD_{rms}\) for the \(l = 1.6\delta, h = 0.8\delta\) and \(h = 0.6\delta\) single LEBU devices.](image)

Comparing the results of \(OPD_{rms}\) for the two single LEBU devices, it appears that the LEBU placed lower in the boundary layer \((h = 0.6\delta)\) gives larger initial reductions in \(OPD_{rms}\), just downstream of the device trailing edge, but the effect of the device appears to ‘wear off’ more rapidly since \(OPD_{rms}\) was found to slightly increase with \(x\) downstream of a local minimum near \(x \approx 3 – 4\delta\). The device placed closer to the TBL edge \((h = 0.8\delta)\) will bring about sustained reductions over longer distances downstream of the device trailing edge, but the magnitude of these reductions is not as large. These trends are similar to those found for \(C_f\) reductions in parametric studies of LEBU device
height by Savill & Mumford (1988). They found that shorter LEBU devices produced
greater reductions of skin friction near the LEBU trailing edge, but that the recovery
towards baseline levels was found to be more rapid that for devices further away from the
wall (Savill & Mumford, 1988).

6.3.3 Streamwise Wavefront Correlations

The streamwise wavefront correlation functions of Malley probe wavefront data
were computed by taking the inverse Fourier transform ($F^{-1}$) of the single-boundary layer
wavefront frequency spectra to obtain the autocorrelation in time at each streamwise
measurement location:

$$R_W(\tau) = F^{-1}\{ |\check{W}(f)|^2 \}.$$  \hspace{1cm} (6.7)

The frozen-flow approximation, $\Delta x = U_C \tau$, was then applied to estimate the streamwise
wavefront correlation function, $R_W(\Delta x)$. The correlation coefficient was then defined by
normalizing $R_W$ by the peak value:

$$\rho_W(\Delta x) = \frac{R_W(\Delta x)}{R_W(0)}.$$  \hspace{1cm} (6.8)

The results of this calculation are presented in Figure 6.20 as contour plots for the
baseline TBL wavefront measurements, and for the $h = 0.8\delta$ and $h = 0.6\delta$ single LEBU
devices for each streamwise measurement location. In these cases, low pass filter was
applied to remove frequencies below $St_\delta = 0.2$ from the computation of the correlation
Figure 6.20 (b). Wavefront correlation functions, $\rho_w$ computed from equation (6.7) for selected streamwise locations for a) the $h = 0.8\delta$, and b) $h = 0.6\delta$ single LEBU modified TBLs.
function in equation (6.7). High-frequency spikes in the spectra due to electronic interference were also band-pass filtered before the $R_W$ was calculated.

For the baseline TBL $\rho_W$ was not found to have any significant variation in the streamwise direction. The first zero crossing of the correlation coefficient curves was found at approximately $\Delta x/\delta = 0.57$, while the first minimum in the correlation curves was found around $\Delta x/\delta = 1.28$. The value of $\rho_W$ at the minimum location $\Delta x/\delta$ is approximately $-0.31$ to $-0.28$. This result for the baseline correlation functions is not unexpected, as the characteristic shape of the TBL wavefront spectra was consistent over the streamwise extent of the study.

Correlations for the $h = 0.8\delta$ and $h = 0.6\delta$ single LEBU devices are shown in Figure 6.20a and Figure 6.20b. For these data, it is immediately evident that correlation functions from both LEBU devices are clearly altered with respect to the baseline boundary layer values. This result was expected because of the notable changes to the spectra shape caused by both of the passive manipulators. The correlations for the $h = 0.8\delta$ device show that the value of $\Delta x$ where $\rho_W$ first crosses zero is reduced, from $0.057\delta$ for the baseline TBL to around $0.49\delta - 0.53\delta$. The location of the first minimum is also decreased to $1.1\delta - 1.2\delta$, and the value of $\rho_W$ at the minimum location ranges from $-0.28$ to $-0.22$. Note that this is an increase with respect to the baseline value for $\rho_W$ at the first minimum location. For the $h = 0.6\delta$ LEBU device, there is also a decrease in the value of $\Delta x$ where $\rho_W$ first crosses zero to $\Delta x = 0.47\delta - 0.50\delta$ for streamwise locations up to $x = 4\delta$. Beyond $4\delta$, the location of the first zero crossing reduces further to approximately $\Delta x = 0.42\delta$ up to the end of the optical measurement section. Note that these values are less than the value found for the zero crossing of the $h = 0.8\delta$ LEBU. The first minimum in $\rho_W$
for the $h = 0.6\delta$ LEBU was also not found to be constant with $x$; just downstream of the LEBU around $x \approx \delta$ the minimum value of $\rho_w$ was found to be approximately $-0.22$, and was located around $\Delta x = 1.1\delta$. Between $x = 1-5\delta$, the minimum value of $\rho_w$ increased to about $-0.17$, and the location of the minimum remained approximately constant. Over the remainder of the streamwise measurement locations (from $5\delta$ to $10\delta$), the minimum location slowly increases to $\Delta x = 0.85 - 1.0\delta$ by $x = 10\delta$. Over this distance, the minimum value of $\rho_w$ also decreases to about $-0.19$ at the furthest downstream measurement station.

Based on the changes in the wavefront correlation functions observed in Figure 6.20 for the LEBU modified TBLs, it is expected that the streamwise wavefront integral LEBU devices will also alter the streamwise wavefront integral correlation length, $\Lambda_w$. To calculate the integral correlation length, the correlation coefficient $\rho_w(x, \Delta x)$ was integrated along the $\Delta x$ coordinate to the location of the first zero crossing, $\Delta x_{\text{zero}}$:

$$\Lambda_w(x / \delta) = \int_0^{\Delta x_{\text{zero}}} \rho_w(x, \Delta x) d(\Delta x).$$

This choice was made because the negative correlation values past the first zero crossing are not large in magnitude; it has also been shown previously that this region of the correlation function is highly sensitive to finite Aperture effects (Smith, et al., 2012; Gordyev, et al. 2015).

The integral correlation length was computed for the baseline TBL wavefront data and for both single LEBU cases. The results for both single LEBU devices are shown in Figure 7.4, normalized by the baseline correlation lengths, in order to determine the relative reduction in streamwise correlation length for the modified TBLs. For both
devices these results show definite reductions of 15 – 34% in correlation length for the LEBU modified TBLs, depending on the LEBU configuration and streamwise location. Similarly to the results for $OPD_{rms}$ presented in Figure 6.19, the $h = 0.6\delta$ gives the largest reduction in streamwise correlation length over the whole of the optical measurement section, with a maximum reduction of approximately 34% at $x = 4 – 5\delta$.

![Figure 6.21. Integral correlation length computed from equation (6.9) as a function of streamwise location.](image)

6.4 Summary and Conclusions

This chapter presented experimental results of hot-wire velocity and Malley probe 1-D wavefront measurements of two different configurations of TBLs modified using single LEBU devices. Measurements of mean and fluctuating velocity profiles downstream of an $l/\delta = 1.6$ LEBU device placed at two heights, $h/\delta = 0.6$ and 0.8, were
presented in this section. For both devices, the LEBU was found to cause localized reductions in mean velocity downstream of the LEBU device, as well as reductions in $u_{rms}$ throughout the modified boundary layer. The mean velocity deficit was found to behave nearly as a self-similar planar wake deficit, but this was not true for the velocity fluctuations. Significant reductions in $u_{rms}$ were found in the LEBU modified boundary layer spectra – especially at low frequencies – over a significant portion of the boundary layer thickness. Some evidence was found that LEBUs introduced a significant amount of small-scale turbulent structures into the LEBU wake; however these are shown to dissipate quickly in the velocity spectra. These findings are consistent with the results of velocity measurements from previous studies (Corke, 1981; Lemay, et al. 1990), which have been discussed previously in Chapter 3. In Chapter 2, it was shown that the most significant contribution to $OPD_{rms}$ in the turbulent boundary layer came from large-scale turbulent structures, with frequencies above $St_\delta = 2$ (i.e. structures smaller than $\delta/2$) contributing only about 6% of the total level of optical energy. Therefore, it was expected based on these velocity measurements that LEBU devices will be an effective means of mitigating aero-optic aberrations caused by the TBL.

Experimental measurements of the levels of aero-optic aberrations caused by LEBU-modified boundary layers have shown that these devices are indeed able to reduce the overall level of optical aberrations caused by the TBL. For the two single LEBU configurations tested in this chapter, reductions on the order of 25% were achieved over streamwise distances of about $10\delta$, with localized reductions as large as 35% for the $h = 0.6\delta$ LEBU device. These results substantiate earlier claims that LEBU devices would be
effective means of flow control for the purpose of ‘quieting’ the aero-optic effects caused by TBLs (Jumper, 1980; Anders, 1985).

From careful analysis of wavefront deflection angle spectra, it was also shown that it is possible to gain some physical insight about the changes to the modified TBL. For example, inspection of wavefront deflection angle spectra showed that the LEBU devices suppressed large-scale turbulent structures in the manipulated boundary layer. This result is consistent with the findings obtained of velocity measurements earlier in this chapter. Also, changes in turbulence production and dissipation, as well as spectral features associated with the growth of the LEBU wake were identified through analysis of wavefront data. This supports the idea that it is possible to study changes to compressible turbulent flow structure using non-intrusive aero-optic wavefront sensors, which was initially put forth by Sutton (1969).

Both $OPD_{rms}$ and wavefront correlation lengths were reduced by both single LEBU devices tested in this chapter, and the trends observed in this particular study regarding the magnitude and streamwise extent of $OPD_{rms}$ and $\Lambda_W$ reduction are very similar to the patterns observed by Savill & Mumford (1988) for local $C_f$ reductions in LEBU modified TBLs. In the present study, the LEBU device produced larger reductions in $OPD_{rms}$ locally when it was placed closer to the wall, but the streamwise extent of the region of reductions is less sustained than for LEBU devices located farther away from the wall. In prior work, this same trend was found to be true for local reductions in $C_f$ that were the result of LEBU-modified TBLs.
CHAPTER 7:
EFFECT OF LEBU CONFIGURATION ON AERO-OPTIC ABERRATIONS

In the previous chapter, it was shown that LEBU devices are capable of reducing $OPD_{rms}$ of wavefront aberrations caused by turbulent boundary layers by 30 – 35%. It was found that the LEBUs caused this reduction by suppressing large-scale turbulent motions in the modified TBL. These results confirm the theory that careful manipulation of large-scale turbulent structures in aero-optically active flows will be effective for mitigating aero-optic aberrations, and also demonstrate that LEBUs are an effective flow control device for achieving this goal. Prior research on LEBUs intended for viscous drag reduction has indicated that device configuration has a substantial impact on the way in which the TBL is manipulated, and on the overall effect on drag at the wall. The results of these works, which were reviewed in Chapter 3, strongly indicate that the aero-optic properties of LEBU-manipulated TBLs will also be heavily dependent on the configuration of the device that is used. It cannot be assumed though that the results of optimizations for net drag reductions will translate directly to the best configuration for aero-optic mitigation. Recall from Sutton’s linking equation (2.3) that $OPD_{rms}$ is dependent on the integrated TBL turbulence statistics, while $C_f$ is reduced by decreasing the local wall shear stress at the wall, $\tau_w$. It is also important to note that prior researchers sought to find LEBU configurations that would provide the maximum net drag reduction; these generally turned out to be devices that produced longer-lasting, moderate reductions
in $C_f$ over long distances (~ $100\delta$). LEBUs identified as optimal for net drag reduction also had low device drag.

For aero-optic applications, however, the streamwise extent over which reductions in $OPD_{rms}$ should be achieved only needs to be as large as the Aperture diameter, which is on the order of about $10\delta$ for most practical applications. The drag of the LEBU device is also not a factor in identifying an optimal configuration for aero-optic mitigation, since it may be permissible to trade a small drag penalty for a more favorable optical environment around the aperture. Therefore, a systematic investigation of the effect of LEBU device parameters is required in order to gain some insight as to what configurations will give the optimal reduction of aero-optic aberrations.

To this end, a parametric study of the effect of LEBU length and height on TBL wavefront aberrations was performed, and the results of these experiments are presented
in section 7.1. A more limited investigation of the effect of multiple-element LEBU devices, including multi-LEBU (vertically stacked) and tandem LEBU (horizontally spaced) devices is presented in section 7.2. These results were included to assess the effect of having LEBUs with additional elements on levels of $OPD_{rms}$ in the manipulated TBL, as certain multiple element configurations have been shown to work better for viscous drag reduction.

**TABLE 7.1**

**SINGLE LEBU DEVICE CONFIGURATIONS CHARACTERIZED USING A MALLEY PROBE 1-D WAVEFRONT SENSOR**

<table>
<thead>
<tr>
<th>Height $h/\delta$</th>
<th>Length, $l/\delta$</th>
<th>0.8</th>
<th>1.0</th>
<th>1.6</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.3</td>
<td>♦</td>
<td>♦</td>
<td>♦</td>
<td>♦</td>
</tr>
<tr>
<td>0.5</td>
<td>0.3</td>
<td>♦</td>
<td>♦</td>
<td>♦</td>
<td>♦</td>
</tr>
<tr>
<td>0.6</td>
<td>0.3</td>
<td>♦</td>
<td>♦</td>
<td>♦</td>
<td>♦</td>
</tr>
<tr>
<td>0.8</td>
<td>0.3</td>
<td>♦</td>
<td>♦</td>
<td>♦</td>
<td>♦</td>
</tr>
</tbody>
</table>

**Legend**

- ♦ Malley probe wavefront measurements single modified TBL
- ♦ Single and double modified TBL cases for wavefront measurements

Additional hot-wire measurements of velocity were also obtained for a limited number of single, multi, and tandem LEBU configurations using the same methodology discussed in the previous chapter. A list of these configurations is found in section 7.3 of this chapter, along some discussion of the mechanisms which are responsible for reductions in aero-optic aberrations caused by LEBU devices.
7.1 Single LEBU Parametric Study

In order to identify the optimal single LEBU device parameters that yield the largest sustained reductions in $OPD_{rms}$ across the optical measurement section in the present study, a parametric investigation of LEBU chord length, $l$, and height, $h$, was performed. Using the Hessert TWT facility configured in the same manner as in Chapter 6, a series of Malley probe wavefront measurements were obtained between $1 - 10\delta$ downstream of the LEBU trailing edge. For this investigation, the LEBU length was varied between $l = 0.8\delta$ and $4\delta$, and the height was varied between $h = 0.3\delta$ and either $0.8\delta$ or $1.0\delta$. The effect of LEBU device thickness and angle of attack on aero-optic aberrations was not explored in the present study.

To get a sense of the general effect of LEBU length and height on the aero-optical environment in the LEBU-modified TBL, the following sections will present wavefront statistics examining the effect of LEBU length when height is fixed, and vice versa. The results of the full parametric investigation will be presented, with a focus on the maximum level of reduction in $OPD_{rms}$ as well as the streamwise distance over which particular reductions are sustained over. The latter criterion is especially important for applying these findings to the design of passive flow control schemes to implement on systems with apertures that are larger than several $\delta$ in diameter.

7.1.1 Effect of Device Length and Height

To investigate the effect of LEBU device length on $OPD_{rms}$ in the manipulated turbulent boundary layer, Figure 7.2 presents the results of wavefront measurements obtained at fixed height $h = 0.6\delta$ for several different chord lengths. This particular height was chosen because it gave the best results for the preliminary single LEBU investigation.
presented in the previous chapter. Note that these data are normalized by the baseline $OPD_{rms}$, which was measured for the un-modified TBL.

Figure 7.2. Baseline-normalized $OPD_{rms}$ measured downstream of single LEBU devices with fixed height ($h = 0.6\delta$) for different chord lengths, $l$.

It is evident from the data presented in Figure 7.2 that device chord length, $l$, had a significant impact on the level of aero-optic mitigation that was achieved. In general, longer devices yield larger reductions than shorter devices; the longest LEBU device tested was 4d in length, and gave reductions in $OPD_{rms}$ on the order of 40 - 45% over a significant portion of the measurement section. Based on these results, it is possible that longer LEBUs could be even more effective at suppressing aero-optic aberrations in the TBL. This result is consistent with experimental findings from Plesniak (1984) and Savill...
& Mumford (1988) for $C_f$ reductions in LEBU manipulated TBLs (see Figure 3.9), who tested LEBUs up to $5\delta$ in length and found no global optimum value for $l/\delta$.

Figure 7.3. Baseline-normalized $OPD_{rms}$ measured downstream of a long single LEBU ($l = 4\delta$) at different plate heights, $h$.

Baseline-normalized $OPD_{rms}$ obtained for the long, single LEBU, $l = 4\delta$ device at several different heights are presented in Figure 7.3 in order to demonstrate the effect of LEBU height on the aero-optic characteristics of the manipulated TBL. For the case where the LEBU was the closest to the wall, $h = 0.3\delta$, there was about a 15% reduction in $OPD_{rms}$ just downstream of the device trailing edge. Between $x = 3 - 4\delta$, $OPD_{rms}$ reached a minimum value between $0.7 - 0.75 OPD_{rms}^{BASELINE}$. Downstream of this location, $OPD_{rms}$ increased to about $0.85 OPD_{rms}^{BASELINE}$ by $x = 6\delta$, and to nearly $0.9 OPD_{rms}^{BASELINE}$ as $x$ approached $10\delta$. For the LEBU placed at the next-to-lowest height in this study, $h = 0.5\delta$,
$OPD_{rms}$ was initially reduced by greater than 20%, and by $2\delta$ downstream of the device trailing edge it rapidly dropped to below $0.6 OPD_{rms}^{\text{BASELINE}}$ (i.e. $>40\%$ reduction). $OPD_{rms}$ remained below $0.55 - 0.6 OPD_{rms}^{\text{BASELINE}}$ for the region $x \approx 2 - 5\delta$. Between $x = 5 - 7\delta$, however, it increased to $0.7 OPD_{rms}^{\text{BASELINE}}$ and remained between $0.7 - 0.75 OPD_{rms}^{\text{BASELINE}}$ until the end of the measurement section around $10\delta$.

For the case in which the LEBU was placed slightly higher up in the boundary layer at $h = 0.6\delta$, initially $OPD_{rms}$ was equal to approximately 75% of the baseline value. This reduced to approximately $0.56 OPD_{rms}^{\text{BASELINE}}$ by $3\delta$ downstream, and $OPD_{rms}$ remained between $0.55 - 0.6 OPD_{rms}^{\text{BASELINE}}$ (i.e. a $40 - 45\%$ reduction in aberrations) up to $x \approx 8.5\delta$. Beyond this point, $OPD_{rms}$ increased from 0.6 to almost 0.7 as $x$ approached $10\delta$, indicating that the aero-optic environment in the manipulated TBL might have been beginning to recover back towards the canonical state. The final tested LEBU height, $h = 0.8\delta$, saw an initial reduction of about 15% in $OPD_{rms}$ just downstream of the LEBU trailing edge. The value of $OPD_{rms}$ was found to drop swiftly, to about $0.7 OPD_{rms}^{\text{BASELINE}}$ around $x = 1.5\delta$. Reductions in $OPD_{rms}$ greater than 30% were sustained for the remainder of the measurement section, with a local minimum of $0.63 OPD_{rms}^{\text{BASELINE}}$ around $x = 7\delta$.

From these data, it appears that near the wall the manipulated flow produces only moderate, short lived reductions in $OPD_{rms}$, while devices placed near the center of the TBL ($h = 0.5 - 0.6\delta$) yield the largest reductions in TBL induced wavefront aberrations. The LEBU placed slightly above the middle of the TBL at $h = 0.6\delta$ yielded the largest sustained streamwise length of reduction in $OPD_{rms}$. In this case reductions in $OPD_{rms}$
were on the order of 45% over nearly $6\delta$, and were greater than 30% for about $9\delta$. For the device closest to the TBL edge ($h = 0.8\delta$), $OPD_{rms}$ was reduced by more than 30% for about $8\delta$ in the streamwise direction, but the maximum level of reductions was less than that observed for the $h = 0.6\delta$ case.

7.1.2 Optimal Single LEBU Configuration

The results of the parametric study of LEBU length and height and their effect on aero-optic aberrations in the TBL were summarized concisely by analyzing a few key criteria for evaluating the performance of each configuration:

- the minimum value of $\frac{OPD_{rms}^{LEBU}}{OPD_{rms}^{BASELINE}}$ obtained for each configuration, and
- the continuous length $L$ over which $OPD_{rms}$ is reduced below some threshold value.

The threshold values of 20% and 30% reductions in $OPD_{rms}$ were chosen based on the observations from experiments, thus the lengths for each are denoted $L_{(20\%)}$ and $L_{(30\%)}$, respectively.

The first measure of LEBU effectiveness is plotted in Figure 7.4 as a function of LEBU device height for devices of different length. The results clearly show that for all chord lengths, the maximum reduction in $OPD_{rms}$ occurs where the LEBUs are placed around the center of the TBL near $h = 0.5 - 0.6\delta$. It is also evident from this data that the magnitude of reductions in $OPD_{rms}$ monotonically increases with device chord length regardless of LEBU height, with the exception of $l/\delta = 0.8$ and 1.0 at $h = 0.8\delta$. This behavior is consistent the findings of parametric studies from previous authors (Plesniak, 1984; Savill & Mumford, 1988) investigating local $C_T$ reductions in LEBU manipulated
TBLs (see Figure 3.9), who tested LEBUs up to $5\delta$ in length and found no global optimum value for $l/\delta$ with regard to the maximum reduction in $C_f$. Recall that the best value often cited for LEBU drag-reduction devices, $l = 0.8 – 1.6\delta$ (Anders, 1990), was found from optimizations seeking the minimum net drag; in these works long chord lengths were found to add too much device drag to be effective.

Figure 7.4. Minimum values of $\frac{\text{OPD}_{\text{rms}}^{\text{LEBU}}}{\text{OPD}_{\text{rms}}^{\text{BASELINE}}}$ as a function of device height for single LEBU devices of different chord lengths.

Figure 7.5 presents the streamwise lengths $L_{(20\%)}$ and $L_{(30\%)}$, over which $\text{OPD}_{\text{rms}}$ is reduced by 20% and 30% in the modified TBL. It should be noted here that streamwise extent of the study is limited to $10\delta$, so the maximum value of any $L$ obtained from this...
Figure 7.5. The streamwise distance over which $OPD_{rms}$ was reduced by a) more than 20% and b) more than 30% with respect to the value measured for the un-manipulated TBL.
experiment is $10\delta$. It is possible that for some single LEBU configurations where $L$ was found to be $10\delta$ that in reality $L$ is greater than $10\delta$. The present study refrains from speculation as to which cases this may or may not occur in, but an extension of the optical measurement section beyond $10\delta$ in future work can be used to investigate this further if it is deemed necessary. In Figure 7.5a it was found that the continuous length over which $OPD_{rms}$ was reduced by 20% or more is maximized for $h = 0.6\delta$ for all LEBU devices. LEBU length was also shown have a notable effect, with the value of $L_{(20\%)}$ increasing with $l/\delta$.

The results for $L_{(30\%)}$ are plotted in Figure 7.5b as a function of LEBU height, for devices of different length. Note that increasing the threshold to reductions greater than or equal to 30% in $OPD_{rms}$ only preserves cases in which $l = 1.6 - 4\delta$ and $h = 0.5 - 0.8\delta$. Again for these data, $h = 0.6\delta$ is the optimal height for producing long regions of aero-optic mitigation. The $l = 4\delta$, $h = 0.6\delta$ Long LEBU device gives the largest region of reduction over 30% $L_{(30\%)} \approx 9\delta$. Note that this configuration also provides the largest local reduction in $OPD_{rms}$, as shown in Figure 7.4.

7.2 Multiple-Element LEBU Devices

While single LEBUs have been shown to give sustained reductions in $OPD_{rms}$ on the order of 30%, the effectiveness of multiple-element LEBU devices, including multi-LEBU (vertically stacked) and tandem LEBU (horizontally spaced) devices, should be assessed to investigate whether or not the additional elements result in any further reduction in levels of $OPD_{rms}$. Malley probe wavefront measurements were obtained in the same manner as described in section 7.1 for the multiple-element LEBU devices
described in Table 7.2. For these data, the streamwise evolution of $OPD_{rms}$ and baseline-normalized surfaces of deflection angle spectra, computed via equation (6.4) are presented.

**TABLE 7.2**

MULTIPLE ELEMENT LEBU DEVICE CONFIGURATIONS CHARACTERIZED USING A MALLEY PROBE 1-D WAVEFRONT SENSOR

<table>
<thead>
<tr>
<th>Multiple Element LEBU Devices ($l/\delta = 1.6$)</th>
<th>$h_1/\delta$</th>
<th>$h_2/\delta$</th>
<th>$s/\delta$</th>
<th>Measurement Type(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multi-LEBU</td>
<td>0.9</td>
<td>0.6</td>
<td>-</td>
<td>◆</td>
</tr>
<tr>
<td></td>
<td>0.8</td>
<td>0.5</td>
<td>-</td>
<td>◆</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td>0.3</td>
<td>-</td>
<td>◆</td>
</tr>
<tr>
<td>Tandem LEBU</td>
<td>0.6</td>
<td>-</td>
<td>4.0</td>
<td>◆</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td>-</td>
<td>8.0</td>
<td>◆</td>
</tr>
</tbody>
</table>

Legend

◆ Malley probe wavefront measurements single modified TBL

◆◆ Single and double modified TBL cases for wavefront measurements

7.2.1 Multi-LEBU Devices

Overall levels of $OPD_{rms}$ were computed and normalized by the baseline $OPD_{rms}$; these results are presented in Figure 7.6. For the $h_1/\delta = 0.9, h_2/\delta = 0.6$ and $h_1/\delta = 0.8, h_2/\delta = 0.5$ multi LEBU configurations, aero-optic distortions were reduced by about 20% immediately downstream of the devices, which compares well with the reductions observed for the single LEBU device with $l/\delta = h/\delta = 0.6$. Further downstream of the LEBU device, the levels of $OPD_{rms}$ for both of these multi LEBU devices continue to
decrease with respect to the baseline measurements, reaching a maximum reduction of approximately 30% around the streamwise position of $7\delta$. Compared to the single LEBU result, this location of the maximum reduction is approximately $2\delta$ further downstream, and is approximately the same strength as the reductions from the single LEBU device. Beyond the location of the maximum reduction for both of these multi LEBUs, the levels of $OPD_{rms}$ for $h_1/\delta=0.8, h_2/\delta=0.5$ appear to begin recovery towards the un-modified boundary layer state, while $OPD_{rms}$ for $h_1/\delta=0.9, h_2/\delta=0.6$ is shown to be leveling out around 70% of the baseline value. These results appear to indicate that the effect for multi LEBUs do not seem to be additive, as the results for the $h_1/\delta=0.9, h_2/\delta=0.6$ multi LEBU do not show a simple increase in the reductions detected for the $h/\delta=0.6$ single LEBU device.

Figure 7.6. Streamwise development of $OPD_{rms}$ for multi LEBU devices of different heights.
For the $h_1/\delta = 0.6, h_2/\delta = 0.3$ multi LEBU configuration $OPD_{rms}$ was reduced by about 30% immediately downstream of the device trailing edge, and $OPD_{rms}$ continued to decrease, resulted in a large, relatively flat range of reduction between $2\delta$ and $6\delta$ where levels are about 40% less than those of the baseline $OPD_{rms}$. Beyond $6\delta$ downstream of this multi LEBU device, levels of $OPD_{rms}$ begin to recover toward the un-modified TBL levels, reaching about 80% of the baseline value by $10\delta$.

Contour plots of $C_\theta (x/\delta, St_\delta)$, which was defined previously in equation (6.4), are presented in Figure 7.7 for all three multi LEBU device configurations. For the $h_1/\delta = 0.9, h_2/\delta = 0.6$ device, shown in Figure 7.7a, in the region immediately downstream of the LEBU ($x = 1-4\delta$) there is a significant reduction in the deflection angle spectrum over the baseline values below $St_\delta \approx 2$, and an increase above the baseline values in the high-frequency, $St_\delta > 3$, end of the spectrum. This increase over the baseline value is reduced with increasing downstream distance, until after $x \sim 4\delta$, where the high-frequency end of the spectrum ($St_\delta > 2$) is reduced below the baseline values. For the low-frequency portion of the deflection angle amplitude spectrum ($St_\delta < 2$), the strength of the reduction of the multi-LEBU modified spectrum increases, especially around a value of $St_\delta = 0.4$. In addition, as the measurement location increases from 4 to $10\delta$, the region of low-frequency suppression achieved by this particular LEBU device is reduced slightly in Strouhal number space as the peak of the LEBU-modified deflection angle spectrum approaches the baseline level and baseline location around $St_\delta \approx 1$. 
Figure 7.7. Baseline-normalized spectrum surfaces $C_\theta (x/\delta, St_\delta)$ for all three Multi-LEBU devices; a) $h_1 = 0.9\delta$, $h_2 = 0.6\delta$, b) $h_1 = 0.8\delta$, $h_2 = 0.5\delta$, and c) $h_1 = 0.6\delta$, $h_2 = 0.3\delta$. (Pages 181 – 182)
a) Multi-LEBU; $h_1 = 0.9\delta$, $h_2 = 0.6\delta$

b) Multi-LEBU; $h_1 = 0.8\delta$, $h_2 = 0.5\delta$
c) Multi-LEBU; $h_1 = 0.6\delta$, $h_2 = 0.3\delta$
For the $h_1/\delta = 0.8, h_2/\delta = 0.5$ device, shown in Figure 7.7b, the normalized spectrum surface appears qualitatively similar to the results shown in Figure 7.7a, although quantitatively there are some notable differences. Just downstream of the LEBU trailing edge, a significant decrease in the low frequency range ($St_\delta < 2$) and a significant increase in the high-frequency ($St_\delta > 2$) range of the deflection angle spectrum with respect to the baseline spectrum is found. However, the strength (or amount of increase) of the high-frequency portion of the spectrum appears to be reduced slightly compared to the $h_1/\delta = 0.9, h_2/\delta = 0.6$ device. Furthermore, the streamwise extent of this region of high-frequency increase is limited to about $3\delta$. The reduction in the low-frequency range of the spectrum appears to be rather similar in strength and size in Strouhal number space, with the only major difference being that a portion of the deflection angle spectrum for the $h_1/\delta = 0.8, h_2/\delta = 0.5$ device in the neighborhood of $St_\delta \approx 2$ remains at the level of the baseline amplitude spectrum, and shifts to a $St_\delta$ of nearly 1 as the measurement station approaches $10\delta$.

The normalized spectrum surface, shown in Figure 7.7c, for the $h_1/\delta = 0.6, h_2/\delta = 0.3$ multi LEBU device bears some qualitative similarities to the spectrum shown in Figure 7.7a and Figure 7.7b, respectively, but quantitatively the effect of this LEBU is shown to be much more significant than the previous two cases. Immediately downstream of the LEBU trailing edge, the LEBU modified deflection angle spectrum is reduced by approximately 60% compared to the baseline deflection angle spectrum below $St_\delta = 2$. This strong low-frequency reduction is sustained up to approximately $6\delta$, where the low-frequency ($St_\delta < 2$) end of the deflection angle spectrum begins to recover towards the baseline spectrum. Also just downstream of the LEBU device, there is a
slight 20% increase in a portion of the high-frequency \((2 - 7\delta)\) end of the deflection angle spectrum. This increase disappears entirely beyond \(3\delta\) downstream of the LEBU device. Beyond \(3 - 4\delta\), the entire high-frequency range of the deflection angle spectrum shows a reduction over the baseline of approximately 40% above \(St_\delta = 3\), indicating a decrease in dissipation relative to the baseline flow. This effect is likely the result of the suppression of low-frequency \((St_\delta < 2)\) structures beginning immediately downstream of the LEBU trailing edge which yields an overall reduction in the amount of energy cascading into small scales. At \(x = 10\delta\), the peak of the LEBU modified spectrum is approaching \(St_\delta = 1\), which is another indication (along with the recovery of overall levels of \(OPD_{rms}\) towards the baseline value) that the aero-optic reduction effects of this LEBU configuration are starting to wear off by \(10\delta\).

7.2.2 Tandem-LEBU Devices

Malley probe wavefront measurements were also performed for the tandem LEBU devices described in Table 7.2. Overall levels of \(OPD_{rms}\) for each tandem LEBU device were computed for streamwise locations \(x = 0.8\delta\) to \(10\delta\), and the results, normalized by the baseline \(OPD_{rms}\), are shown in Figure 7.8. For both tandem LEBU configurations, the level of aero-optic distortions just downstream of the LEBU is reduced by approximately 30%. Further downstream, levels of \(OPD_{rms}\) for the \(s = 4\delta\) tandem LEBU device remain relatively constant around 70% of the baseline value until \(x \approx 10\delta\). Downstream of it, the aero-optical distortions appear to be increasing slightly towards the baseline value. For the \(s = 8\delta\) tandem LEBU device, \(OPD_{rms}\) continues to decrease further downstream until reaching a region of maximum reduction between \(4 - 6\delta\), where the levels of LEBU-modified \(OPD_{rms}\) are reduced by about 42% compared
to the baseline. Downstream of the streamwise location of 6\(\delta\), the levels of \(OPD_{\text{rms}}\) begin to recover slightly towards the baseline levels, but the rate of recovery is more gradual than what was observed for the \(h_1 = 0.6\delta, h_2 = 0.3\delta\) multi LEBU device (which had the largest maximum reduction in \(OPD_{\text{rms}}\)), with the \(OPD_{\text{rms}}\) at 9–10\(\delta\) being approximately 35% lower than the baseline measurement. This result indicates that the tandem LEBU device results in the most significant reduction in \(OPD_{\text{rms}}\) over a longer streamwise extent than any of the other tested LEBU configurations.

![Figure 7.8. Streamwise development of \(OPD_{\text{rms}}\) for Tandem LEBU devices with separations \(s = 4\delta\) and 8\(\delta\), compared to the best-performing single and multi-LEBU results.](image)

The streamwise evolution of the baseline-normalized deflection angle spectra are presented in Figure 7.9 for the \(s = 4\delta\) and 8\(\delta\) tandem LEBU devices. Results in Figure 7.9a show that for the \(s = 4\delta\) LEBU device, the deflection angle spectrum is increased by
Figure 7.9. Baseline-normalized spectrum surfaces $C_\theta(x/\delta, S_{t\delta})$ for a) $s = 4\delta$ and b) $s = 8\delta$ tandem LEBU devices.
about 20% over the baseline spectrum immediately downstream of the LEBU wake above $St_\delta = 10$. This increase persists in the deflection angle spectrum until the streamwise location of approximately $8\delta$ downstream. Just downstream of this tandem LEBU device, there is a significant decrease of the LEBU modified spectrum compared to the baseline below $St_\delta = 3$, particularly in the low-frequency end of the spectrum corresponding to $St_\delta < 1$. By $x = 3\delta$, the region of reduction in the deflection angle spectrum expands such that the spectrum is reduced in the range $St_\delta < 8$. This effect remains mostly the same over the streamwise extent of the study, with the exception of the shift in the location of the local maximum in the normalized deflection angle spectrum from $St_\delta \approx 5$ just downstream of the LEBU to $St_\delta \approx 1.5$ around $x = 10\delta$. This feature of the spectrum likely corresponds to the spanwise growth of the LEBU wake within the TBL as it moves downstream. Additionally, approaching $10\delta$, there is an increased suppression of the LEBU-modified spectrum in the region, $St_\delta > 2$, which is likely the result of the suppression of large-scale structures further upstream, similar to single LEBU devices.

Normalized deflection angle results for the $s = 8\delta$ tandem LEBU, shown in Figure 7.9b, do not show, unlike the $4\delta$ tandem LEBU, any increase in the spectrum amplitude with respect to the baseline spectrum. For the $s = 8\delta$ tandem LEBU device, the region of suppression in the low-frequency end of the spectrum ($St_\delta < 1$) shows an overall stronger suppression of aero-optic spectrum compared to the $4\delta$ tandem LEBU at the same frequency range. Additionally, for streamwise locations $x > 4\delta$ there is an increasingly strong reduction in the high-frequency end of the deflection angle spectrum. This is likely is a result a reduction in the amount of energy cascading into smaller-scales at which turbulence dissipation occurs. For this tandem LEBU, the baseline-normalized spectrum
peak is also observed to shift to lower-frequencies as the streamwise location increases, moving from a value around $St_δ = 6$ to about $St_δ = 1$. At $10δ$, however, the suppression of large-scale structures ($St_δ < 1$) is still rather strong, so it is possible that levels of optical aberrations will remain low for several more TBL thicknesses before beginning to relax to the baseline values.

7.3 Velocity Measurements

In the Chapter 6, velocity measurements at several streamwise locations in the LEBU-modified TBL were presented along with aero-optic wavefront measurements for limited number of Single LEBU devices. The effects of LEBUs on velocity and wavefront statistics were then compared, and the results from both measurement techniques indicated that frequencies associated with large-scale turbulent structures in the TBL were suppressed by the devices. Both measurements also able to identify LEBU-induced changes to the TBL turbulence structure in the modified boundary layer. Additional comparisons of velocity and wavefront statistics in the LEBU modified TBL are presented in this section for selected LEBU configurations at a single streamwise location: $x = 6δ$. This streamwise location was chosen as it is roughly in the middle of the optical measurement section, and it has been shown from the data presented in previous sections that substantial reductions in $OPD_{rms}$ were achieved at this location for several different LEBU devices. The particular manipulators that will be examined in this section are as follows:

- Single LEBU: $l = 1.6δ, h = 0.6δ$
- Long single LEBU: $l = 4δ, h = 0.6δ$
• Multi LEBU: \( l = 1.6\delta, h_1 = 0.6\delta, h_2 = 0.3\delta, \) and
• Tandem LEBU: \( l = 1.6\delta, h = 0.6\delta, s = 8\delta. \)

Velocity data was acquired with the same single hot-wire anemometer used to obtain baseline velocity measurements presented in Chapter 5 and single LEBU velocity measurements in Chapter 6.

Recall from Chapter 2 that a number of models for calculating \( OPD_{rms} \) for compressible TBLs have been derived using Sutton’s linking equation (2.2) and the Strong Reynolds Analogy, equations (2.5) and (2.6). The terms in the SRA are easily arranged to show that at a particular \( y \)-value \( \rho_{rms} \) in the TBL is proportional to the product \( U(y)u_{rms}(y) \):

\[
\rho_{rms}(y) = \rho_\infty r(\gamma - 1)M_\infty^2 U(y)u_{rms}(y) / U_\infty^2. \tag{7.1}
\]

One possible modification of the form of the SRA given in equation (7.1) can be arrived at by substituting the energy spectral density for the root-mean-square terms on both sides of the equation. Since the energy spectral density of any arbitrary variable \( \xi(t) \) describes how its total variance, \( \xi^2_{rms} \), is distributed over the component frequencies that \( \xi \) may be decomposed into. In other words, \( \xi^2_{rms}(f) = S_\xi(f) \), where \( S_\xi(f) = |F[\xi^2(t)]|^2 \) is the one-sided energy spectral density of \( \xi \), and \( F \) denotes the Fourier transform. Therefore, equation (7.1) can be re-written in the form

\[
\rho^2_{rms}(y, f) = S_{SRA}(y, f) \times \left( \dfrac{r(\gamma - 1)\rho_\infty M_\infty^2}{U_\infty^2} \right)^2, \tag{7.2}
\]

where
\[ S_{SRA}(y, f) = F\{U(y)u(y, t)\} \quad (7.3) \]

This decomposes the first term of the integrand in the linking equation (2.3) into a function of frequency and wall-normal location. Assuming that the SRA is valid in the LEBU modified TBL, this approach can be used to identify where in both frequency and wall-normal space the LEBU devices are causing reductions in \( \rho_{rms} \). To easily visualize this, the ratio of the LEBU-modified to baseline energy spectra was computed:

\[
C_{SRA}\left(\frac{y}{\delta}, St_\delta\right) = \frac{S_{SRA}^{LEBU}(y/\delta, St_\delta)}{S_{SRA}^{BASELINE}(y/\delta, St_\delta)}. \quad (7.4)
\]

Here it is emphasized that \( C_{SRA} < 1 \) indicates a reduction in density fluctuations in the LEBU-modified boundary layer, while \( C_{SRA} > 1 \) implies an increase in density fluctuations. Filled contour plots of equation (7.4) are shown in Figure 7.10 for selected LEBU devices at \( x/\delta = 6 \).

The characteristics of the spectra surfaces presented in Figure 7.10 are consistent with other velocity and wavefront spectra presented in this and the previous chapters. The SRA-estimated fluctuating density spectra show that the LEBU devices all produce reductions in low frequencies, \( St_\delta < 1 \) (i.e. large scales) below the LEBH height. Note that for Multi-LEBU device in Figure 7.10c, there is a ‘pocket’ of low-frequency turbulence suppression below both LEBU elements. These spectra also show that LEBUs cause an increase in high-frequencies, \( St_\delta > 1 \) (corresponding to small scale structures) at wall-normal locations \( y > h \), which corresponds to turbulence production due to the increased mean velocity gradient at this location. The magnitude of these increases is especially large for long single LEBU and multi-LEBU devices.
Figure 7.10. Contour plot of $C_{SR4} (y/\delta, St\delta)$ at $x = 6\delta$ for a) $l = 1.6\delta, h = 0.6\delta$ Single LEBU, b) $l = 4\delta, h = 0.6\delta$ Long Single LEBU, c) $h_1 = 0.6\delta, h_2 = 0.3\delta$ Multi-LEBU and d) $s = 8\delta$ tandem LEBU devices. LEBU element height is marked by white lines, and the solid black contour indicates where $C_{SR4} (y/\delta, St\delta) = 1.0$. (Pages 192 – 193)
Figure 7.11. Wall-normal density correlation length functions for the canonical turbulent boundary layer; \( \Lambda_1 \) presented by Gilbert (1982), \( \Lambda_2 \) measured by Rose & Johnson (1982), and \( \Lambda_3 \) computed from DNS of a \( Re_\theta = 3550 \) TBL by Wang & Wang (2012).

The SRA-estimated \( \rho_{rms}(y) \) profile for the LEBU-modified TBL is easily obtained from the energy spectrum \( S_{SR,A} \) by integrating in the frequency domain:

\[
\rho_{rms}(y) = \frac{r(y-1)\rho_\infty M^2_{\infty}}{U^2_{\infty}} \left( \int_0^\infty S_{SR,A}(y,f) \, df \right)^{1/2}.
\]  
(7.5)

Note that it can easily be shown that equation (7.5) is equivalent to equation (7.1); however, computing \( \rho_{rms} \) from equation (7.5) allows for the possibility of spectral filtering if there is significant noise in the velocity measurements.

It follows from equation (7.5) that the simplified form of Sutton’s linking equation (2.3) can be re-written as:
\[ \text{OPD}_{\text{rms}} = \sqrt{2} K_{GD} (\gamma - 1) \rho_x \delta M^2 \left[ \int_0^\infty \sqrt{2} \left( \frac{U(y) \mu_{\text{rms}}(y)}{U_\infty^2} \right)^2 \Lambda_\rho(y) \left( \frac{y}{\delta} \right) \frac{d}{dy} \left( \frac{y}{\delta} \right) \right]^{1/2}. \] (7.6)

Remember here that \( \Lambda_\rho(y) \) is the wall-normal density correlation length, which can either be estimated experimentally (although with some difficulty) from hot-wire velocity measurements, or computed directly from CFD results (e.g. Wang & Wang, 2012). Several different density correlation lengths from the literature that have been used for modeling TBL aero-optic aberrations are presented in Figure 7.11. These functions have all been shown to give similar results for models of canonical subsonic and supersonic TBLs (Gordeyev, et al., 2012; Smith & Gordeyev, 2013; Gordeyev, et al. 2014).

**TABLE 7.3**

**COMPARISON OF DIRECTLY MEASURED AND INDIRECTLY ESTIMATED VALUES OF \( \text{OPD}_{\text{rms}} \) IN LEBU MODIFIED TBLS AT \( x = 6\delta \).**

<table>
<thead>
<tr>
<th>LEBU Device Configuration</th>
<th>( \text{OPD}_{\text{rms}} ) (Malley Probe)</th>
<th>( \text{OPD}_{\text{rms}}^{\text{LEBU}} )</th>
<th>( \text{OPD}_{\text{rms}}^{\text{BASELINE}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single LEBU</td>
<td>( l/\delta = 1.6 ) ( h/\delta = 0.6 )</td>
<td>0.68</td>
<td>0.82</td>
</tr>
<tr>
<td>Long Single LEBU</td>
<td>( l/\delta = 4 ) ( h/\delta = 0.6 )</td>
<td>0.55</td>
<td>0.85</td>
</tr>
<tr>
<td>Multi-LEBU</td>
<td>( l/\delta = 1.6 ) ( h_1/\delta = 0.6 ) ( h_2/\delta = 0.3 )</td>
<td>0.64</td>
<td>0.78</td>
</tr>
<tr>
<td>Tandem LEBU</td>
<td>( l/\delta = 1.6 ) ( h/\delta = 0.6 ) ( s/\delta = 8.0 )</td>
<td>0.55</td>
<td>0.80</td>
</tr>
</tbody>
</table>
For LEBU-manipulated TBLs, it was not expected that the functional forms of $\Lambda_\rho$ presented in Figure 7.11 is an appropriate estimate for the actual wall-normal density correlation length, since prior research presented in Chapter 3 showed that LEBU devices significantly modify streamwise and spanwise correlations in manipulated boundary layers. However, if $OPD_{rms}$ is estimated from equation (7.6) using the any one of the approximations for $\Lambda_\rho$ for the canonical TBL, the influence of LEBU-modified wall-normal correlation length can be quantified by comparing predicted levels of reductions to those measured using the Malley probe.

The reduction in $OPD_{rms}$ relative to the value predicted from baseline velocity measurements is given in equation (7.7):

$$\frac{OPD_{rms}^{LEBU}}{OPD_{rms}^{BASELINE}} = \sqrt{\frac{\int_0^\infty [\rho_{rms}^{LEBU}(y)]^2 \Lambda_\rho(y) dy / \delta}{\int_0^\infty [\rho_{rms}^{BASELINE}(y)]^2 \Lambda_\rho(y) dy / \delta}}, \quad (7.7)$$

where the superscripts ‘$LEBU$’ and ‘$BASELINE$’ denote quantities for the LEBU modified and baseline TBLs, respectively. Table 7.3 presents the results of that the amount of reduction in $OPD_{rms}$ estimated from the linking equation is significantly under-predicted compared to experimental results. Depending on the correlation function and LEBU case, using the canonical TBL correlation function over-predicts $OPD_{rms}$ in the modified TBL by up to 30% more than the actual value. This result serves as a clear, although indirect, indication that LEBU-induced aero-optic mitigation is not simply achieved by reducing the overall magnitude of density fluctuations in the TBL. Rather, the de-correlating ‘plate’ effects which were found to be a critical mechanism for LEBU
drag reduction (Chapter 3; Anders, 1990, and references therein) are also an important mechanism for affecting the aero-optic characteristics of LEBU-modified TBLs.

7.4 Summary and Conclusions

This chapter presented the results of a two-part, systematic investigation of the effect of LEBU device parameters on measured aero-optic aberrations caused by manipulated turbulent boundary layers. In the first part, a parametric study of single LEBU chord length and device height was performed. Experiments showed that for all single LEBU configurations tested, reductions in \( OPD_{\text{rms}} \) were achieved in the TBL downstream of the device. This result suggests that in practical applications, as long as the LEBU device is adequately thin and mounted at zero angle of attack, there is little risk that a non-optimized LEBU geometry will worsen the aero-optic environment over an aperture submerged beneath a fully developed, zero pressure gradient turbulent boundary layer.

After analyzing all cases from the parametric investigation, the optimal tested single LEBU configuration was identified as the \( l = 4\delta, h = 0.6\delta \), ‘long’ LEBU device. This configuration resulted in a maximum reduction in \( OPD_{\text{rms}} \) of approximately 45%, and also gave sustained reductions over 40% for 6\( \delta \) in the streamwise direction, and reductions in \( OPD_{\text{rms}} \) over 30% for 9\( \delta \). The word ‘tested’ is emphasized above, as the optimal length found in this parameter space resided on the edge, which leaves open the possibility that LEBUs of even longer chord length will give even larger reductions in \( OPD_{\text{rms}} \), although LEBUs with increasing chord length will result in additional device drag. This chord length is significantly longer than the range identified as optimal for net
drag reduction of $0.8 - 1.6\delta$ (Anders, 1990, and references therein). However, the response of $OPD_{rms}$ to chord length is consistent with the findings of parametric studies from previous studies of local $C_f$ reductions in LEBU modified TBLs (Plesniak, 1984; Savill & Mumford, 1988). As shown in Figure 3.9, these studies found that for LEBUs up to $5\delta$ in length, there was no global optimum value for $l/\delta$ found with regard to the maximum reduction in skin friction (i.e. maximum $C_f$ continued to decrease with increasing chord).

On the other hand, the optimum height found in the present study, $h = 0.6\delta$, was observed to fall inside of the parametric space investigated for all cases. The value of the optimal height for aero-optic mitigation was found to be lower than the value of $h = 0.75 - 0.8\delta$ identified as optimal for LEBU drag reduction (Anders, 1990, and references therein). It is unclear if this divergence is due to a difference in the effect of the LEBU modified TBL on $C_f$ and $OPD_{rms}$, or if the difference can be attributed to Reynolds number effects. Recall that Savill & Mumford (1988) indicated that the optimum LEBU height might decrease with increased Reynolds number.

The effects of Multi-LEBU and Tandem-LEBU devices on aero-optic aberrations in the TBL were also investigated using a limited number of configurations. The best results for each of these types of LEBU are presented in Figure 7.12 along with the optimal Long LEBU and the Single LEBU presented in Chapter 6. The results demonstrated that the best-performing Tandem-LEBU configuration for aero-optic mitigation is two-element tandem LEBU with $l = 1.6\delta$, $h = 0.6\delta$, and $s = 8\delta$. This configuration gave the maximum reduction in $OPD_{rms}$ of more than 40% downstream of the device to about $5\delta$, compared to the baseline measurements, and reductions greater
than about 35% over a large streamwise range between $1\delta$ and $10\delta$. Similar reductions in $OPD_{rms}$ of 35% were also shown for a multi-LEBU device with $l = 1.6\delta$, $h_1 = 0.6\delta$, $h_2 = 0.3\delta$, although the streamwise extent of the reduction was limited to about $4\delta$, after which levels of $OPD_{rms}$ appeared be rapidly recovering towards the baseline state. In comparison to multi-element LEBUs identified as optimal for drag reduction in turbulent boundary layers, the results of this study are in agreement with previous studies where tandem-LEBU configurations with elements’ spacing on the order of $5 – 10\delta$ were found to perform well (Savill & Mumford, 1988; Anders, 1990, and others).

![Figure 7.12](image)

*Figure 7.12. Baseline normalized $OPD_{rms}$ measured with the Malley probe 1-D wavefront sensor downstream of selected LEBU device configurations.*

Additional velocity measurements were also obtained at a single streamwise station for several selected single, tandem, and multi-LEBU device configurations. The
fluctuating density spectra were estimated for each of these devices using the Strong Reynolds Analogy, and the results were showed that significant reductions in the contributions from large scales below the LEBU devices, and increased small scale turbulent fluctuations above the LEBU device that are consistent with increased production on the upper side of the LEBU wake. Estimates of $OPD_{\text{rms}}$ were also computed from velocity data through Sutton’s linking equation, and the results gave indirect evidence that changes in wall-normal correlation length are also a key mechanism responsible for reducing aero-optic aberrations in the LEBU-modified TBL.

Levels of aero-optic mitigation for two tested multi-element LEBU configurations were shown to be better than levels of mitigation achieved by the $l = 1.6\delta$ single-element LEBU device. However, the long $l = 4\delta$ LEBU performed better than both of the tandem and multi LEBU devices, although only just slightly. These results demonstrate that aero-optic aberrations caused by the TBL can be easily and significantly suppressed by 30 – 40% over several $\delta$ in the streamwise direction using Large Eddy Break-Up devices. The mechanical simplicity of LEBU devices, along with the experimental demonstration of their aero-optic mitigation abilities in this work, makes them very good candidates for incorporation into the design of airborne directed energy systems that must contend with aberrations caused by compressible TBLs.
CHAPTER 8:

EFFECTS OF WALL COOLING ON THE AERO-OPTICS OF TURBULENT BOUNDARY LAYERS

Cress (2010) proposed a simple analytical model for the effect of wall temperature on aero-optic aberrations caused non-adiabatic wall turbulent boundary layers. The model was derived using some of the assumptions from the Extended Strong Reynolds analogy, namely the temperature-velocity relation given in equation (4.16), which is repeated here in Reynolds averaged form,

\[
\frac{T''(y)}{T_\infty} = \frac{T_{aw} - T_w}{T_\infty} \left( \frac{u''(y)}{U_\infty} \right) - r(y - 1)M_\infty^2 \left( \frac{\bar{u}(y)u''(y)}{U_\infty^2} \right). \tag{8.1}
\]

The effects of pressure and total temperature fluctuations on aero-optic aberrations were also neglected in the derivation of the aero-optic wall temperature model, which resulted in the following relationship being assumed for density and temperature fluctuations in the TBL:

\[
\frac{\rho_{rms}}{\rho_\infty} = -\frac{T_{rms}}{T_\infty}. \tag{8.2}
\]

A more detailed discussion of the model derivation can be found in Section 4.2, but the analytical relation obtained by Cress (2010) related \( OPD_{rms} \) to the boundary layer wall temperature, \( T_w \), by the following aero-optic model equation:
\[ \text{OPD}_{\text{rms}} = BK_{\text{GD}} \rho_{\infty} \delta \sqrt{C_{J} M_{\infty}^{2}} \left( 1 + C_{1} \left( \frac{\Delta T}{T_{w} M_{\infty}^{2}} \right) + C_{2} \left( \frac{\Delta T}{T_{w} M_{\infty}^{2}} \right)^{2} \right), \] (8.3)

where \( \Delta T = T_{w} - T_{r} \), the difference between the wall temperature and the adiabatic wall recovery temperature, \( T_{r} \) (Cress, 2010; Gordeyev, et al. 2015). The case where wall temperature was greater than the adiabatic wall recovery temperature (i.e. \( \Delta T > 0 \)), is defined as wall heating. The case where \( T_{w} \) is less than \( T_{r} \) (i.e. \( \Delta T < 0 \)) is referred to as wall cooling. Note that for \( \Delta T = 0 \), equation (8.3) reduces to the aero-optic model equation (2.26) for adiabatic wall, subsonic compressible TBLs. Cress (2010) reported that the values of the model constants \( C_{1} \) and \( C_{2} \) predicted from equation (4.22) were 6.38 and 10.28, respectively.

One interesting feature of the wall temperature model is that the temperature dependent term inside of the square root of equation (8.3) is a quadratic function of \( \Delta T \). For moderate values of wall heating (i.e. \( \Delta T < 0.1 T_{w} \)) Cress (2010) showed that equation (8.3) could be reduced by dimensional arguments to a linear form that showed \( \text{OPD}_{\text{rms}} \) increased linearly with wall temperature. Good qualitative agreement was found for wall heating between model predictions and the aero-optic effects that were observed in experiments (Cress, 2010; Smith, et al., 2014; Gordeyev, et al., 2015) and in simulations (White & Visbal, 2012).

For wall cooling, the linearized model equation does not hold for wall cooling due to the model’s quadratic functional dependence on wall temperature, which is shown in Figure 8.1. A consequence of this functional dependence is that the model predicts that \( \text{OPD}_{\text{rms}} \) will decrease with wall cooling, but that it will reach a minimum value at

\[ \Delta T_{\text{Optimal}} = \left( 0.5 C_{1} / C_{2} \right) T_{w} M_{\infty}^{2}. \]

For \( \Delta T < \Delta T_{\text{Optimal}} \), the model predicts that \( \text{OPD}_{\text{rms}} \)
Figure 8.1. Model predicted dependence of $OPD_{rms}$ on $\Delta T$, normalized by $OPD_{rms} (\Delta T = 0)$ for $M_\infty = 0.3, 0.6$ using the values of $C_1, C_2$ analytically predicted by Cress (2010).

increases as the wall temperature decreases further. Using the values from Cress (2010) for constants $C_1$ and $C_2$, the model predicts that the optimum cooling temperature is $\Delta T_{\text{opt}} \approx -0.3T_\infty M_\infty^2$, and that at this temperature $OPD_{rms}$ is reduced by over 80%. A few preliminary measurements for wall cooling were obtained by Cress (2010) that suggested that $OPD_{rms}$ was decreased by as much as 80% for small amounts of wall cooling, $\Delta T/T_\infty > -0.35$. These data, however, were obtained for a single cooled TBL and a single adiabatic wall TBL, and then re-scaled using the method outlined in Appendix A. This approach, however, adds a considerable amount of uncertainty to the SBL scaled wavefront measurement, and data was only obtained for very few wall cooling temperatures. Therefore, additional wavefront measurements using improved techniques
are required in order to validate the functional form of Cress’ (2010) model for the effect of wall cooling on aero-optic aberrations.

As a potential aero-optic mitigation technique, the application of wall cooling over the full boundary layer development length may not always be practical. The availability of space in aircraft on which a wall cooling system might be installed is likely to be limited, which would impose a constraint on how much of the TBL development length could be cooled. Energy requirements are also a possible limitation, since the amount of power required is proportional to the product of the length of cooling and the temperature difference: \( \sim L_{cool} \Delta T \). Therefore, it is also of interest to investigate the effect of cooling only partial lengths of the boundary layer development section upstream of the optical aperture on the levels of aero-optic mitigation that can be achieved.

This chapter presents the results of a systematic investigation of the effect of full and partial wall cooling on aero-optical distortions using the Malley probe 1-D wavefront sensor. For full wall cooling, this study sought to improve the experimental techniques used for conducting wall cooling wavefront measurements, and to obtain good quality data that could be used to validate the functional form of the model proposed by Cress (2010). Using the same improved experimental techniques, the effect of partial wall cooling on aero-optic aberrations was also experimentally measured for a variety of configurations. The experimental facility and test section used in full and partial wall-cooling experiments is described in Section 9.1, and the resulting wavefront statistics for both full and partial wall-cooling are presented in Section 9.2. Section 9.3 gives a review of some recent developments in the aero-optic model for full wall cooling, and compares the results of experiments to the updated model predictions. Additional modifications to
the model that account for the effects of partial wall cooling are introduced and examined in Section 9.3.

8.1 Experimental Setup

Experimental measurements of the effect of wall cooling on the aero-optics of turbulent boundary layers were conducted in the Hessert TWT described previously in Chapter 5, with the boundary layer development section modified to investigate full and partial wall cooling. The total length of the TBL development section is variable, and can be lengthened or shortened using 40 cm long modular test sections to change the boundary layer thickness at the measurement section. For the results presented in this chapter three modular sections were used, making the total length to the beginning of the optical section is 120 cm. The modular test sections are denoted as Sections 1, 2 and 3, starting at the end of the inlet contraction and moving in the direction of the mean flow.

Figure 8.2. A (top view) schematic of the Hessert Transonic Wind Tunnel (TWT) configured for aero-optic wavefront measurements of boundary layer wall cooling.
The sidewalls of the wall cooling test sections were constructed of 3/8 inch thick aluminum plates to improve heat transfer, and reservoirs were installed on the outside of the wind tunnel to contain the cooling material; in this study, dry ice was used to cool the boundary layer side walls. Wall temperature was directly measured using thermocouples embedded in the aluminum plate in order to provide accurate wall temperature measurements over the length of the test section. These thermocouples were embedded roughly at the midsection of each modular test section piece, thus their corresponding streamwise locations were approximately $x = 20, 60, \text{ and } 100 \text{ cm}$.

![Figure 8.3. A CAD model (left) and photograph (right) of the modular test sections showing the Aluminum sidewalls and cooling reservoirs.](image)

To investigate the effects of the wall cooling on levels of aero-optical aberrations, the side walls of the boundary layer development section were uniformly cooled while the wind tunnel was off. After reaching a significantly low temperature (typically around $-30^\circ\text{C}$), the tunnel was switched on. After the freestream velocity reached a steady state, simultaneous measurements of wall-temperature, free-stream velocity, and wavefront data were obtained as the wall temperature increased towards the recovery temperature.
In all cases, as will be discussed in the next paragraph, the sample time for each wavefront measurements was several orders of magnitude smaller than the total time (approx. 20 minutes) that it took for the wall temperature to go from the initial cooled temperature to $T_r$.

Figure 8.4. a) Single-modified TBL (SMBL) and b) Double-modified TBL (DMBL) configurations for cooling-wall wavefront measurements.

One-dimensional wavefronts were acquired using the Malley Probe wavefront sensor (see Chapter 5) downstream of the cooling wall boundary layer development section at approximately $x_{MP} = 125$ cm. Wavefront deflection angle and wall temperature data were simultaneously recorded repeatedly at 200 kHz for 5 seconds until the wall reached the adiabatic temperature. The 5 second long wavefront samples were divided into fifty 100 ms long windows (i.e. 10 flow-through times) over which wall temperature and wavefront statistics were computed. For all cases, the rate of the change in wall
temperature \(dT_w/dt\) was less than 2 °C/s, meaning the temperature drift over any 100 ms long window was less than +0.2 °C, which is on the same order of magnitude as the error in the thermocouple. Therefore it was assumed that the wall appeared to the boundary layer passing over it to be at an equilibrium temperature within each 100 ms long window over which wavefront statistics were computed. Note that in Figure 8.2 and Figure 8.3 it is shown that the cooling wall test sections were designed to allow symmetric cooling of both the sidewall TBLs. This was done to conduct wavefront measurements of both single and double modified TBLs, as illustrated in Figure 8.4. All wavefront statistics were then properly scaled to SBL values using the procedures described in Appendix A.

Table 8.1 provides a full record of the different wall cooling cases experimentally investigated in the present study, which includes experiments on both full and partial wall cooling. The effect of buoyancy on TBL heat transfer can easily be assessed for the range of ‘moderate’ cooling temperatures tested in this experiment (i.e. \(\Delta T = -30\) to 0°C) by computing the Grashof number, \(Gr_\delta\), which is the ratio of buoyant and viscous forces:

\[
Gr_\delta = \frac{g\beta\Delta T \delta^3}{\nu^2}.
\]

(8.4)

In equation (8.4), \(g\) is acceleration due to gravity, \(\beta\) is the coefficient of thermal expansion, and \(\nu\) is kinematic viscosity (White, 2006). In the present study, the maximum magnitude of \(Gr_\delta\) was found to be approximately \(7 \times 10^4\). Compared to the relative size of the square of the Reynolds number, \(Re_\delta^2 \approx 5 \times 10^{10}\), it is evident that \(Gr \ll Re^2\), which indicates that the effects of buoyancy can be neglected (White, 2006), as they have been in the statistical model.
TABLE 8.1
PARAMETERS FOR FULL AND PARTIAL WALL COOLING CASES EVALUATED
USING A MALLEY PROBE 1-D WAVEFRONT SENSOR

<table>
<thead>
<tr>
<th>Sections Cooled</th>
<th>Case Description</th>
<th>$M_{\infty}$</th>
<th>$L_{COOL}/L_{DEF}$</th>
<th>Modified TBL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 2, 3</td>
<td>Full Wall Cooling</td>
<td>0.4.. 0.5</td>
<td>96%</td>
<td>S/DMBL</td>
</tr>
<tr>
<td>1</td>
<td>Far-Upstream (Short)</td>
<td>0.35.. 0.4</td>
<td>32%</td>
<td>DMBL</td>
</tr>
<tr>
<td>2</td>
<td>Upstream Strip (Short)</td>
<td>0.35.. 0.4</td>
<td>32%</td>
<td>DMBL</td>
</tr>
<tr>
<td>3</td>
<td>Near-Aperture (Short)</td>
<td>0.35.. 0.4</td>
<td>32%</td>
<td>DMBL</td>
</tr>
<tr>
<td>1, 2</td>
<td>Far-Upstream (Long)</td>
<td>0.4</td>
<td>64%</td>
<td>S/DMBL</td>
</tr>
<tr>
<td>2, 3</td>
<td>Near-Aperture (Long)</td>
<td>0.35.. 0.4</td>
<td>64%</td>
<td>S/DMBL</td>
</tr>
<tr>
<td>1, 3</td>
<td>Near/Far Cooling</td>
<td>0.4</td>
<td>64%*</td>
<td>DMBL</td>
</tr>
</tbody>
</table>

*Not continuous length of cooling.

8.2 Experimental Results

8.2.1 Full Wall Cooling

From the Malley probe measurements of the cooled-wall TBL, deflection angle spectra were computed, and SBL scaled using the procedure outlined in Appendix A. Recalling that the wall-temperature effects model derived in Chapter 4 assumes the wall-temperature mismatch, $\Delta T$, only acts as a scalar multiplier, and does not significantly alter the structure of boundary layer turbulence. Cress (2010) showed that for wall heating, wall temperature merely acted as a scalar amplifier of wavefront deflection angle spectra; if spectra for different wall temperatures were scaled by peak amplitude, $\hat{\Theta}_{\text{Peak}}$ (which is proportional to $OPD_{\text{rms}}$), they were found to be self-similar (see Figure 4.2). It follows then from equation (4.25) that for cooled walls,
Figure 8.5. a) Deflection angle spectra and b) spectra normalized by their peak values, $\hat{\theta}^{\text{Peak}}$, for selected wall temperatures, computed for full wall cooling.
\[
\hat{\theta}_{NORM}(St_\delta) = \frac{\hat{\theta}(St_\delta)}{\left(\rho_x \delta \sqrt{C_f M^2} \left[1 + C_1 \left(\frac{\Delta T}{T_x M^2}\right) + C_2 \left(\frac{\Delta T}{T_x M^2}\right)^2\right]\right)^{1/2}}.
\]

(8.5)

A similar analysis was performed in the present study, and the results are shown in Figure 8.5 for the \(M = 0.4\), full wall-cooling case for several selected wall temperatures. Un-normalized, SBL scaled deflection angle spectra are shown in Figure 8.5a. These data show that there is a definite amplification of measured deflection angle spectra with wall temperature, and that this effect is not linear with \(\Delta T\). In other words, based on the results shown in Figure 8.5a, it is evident that there is some value of \(\Delta T\) locally between \(-30\) and \(0\) K where the spectra peak amplitude is at a minimum. Also, the effect of wall temperature appears to be similar for a broad range of frequencies, excluding portions of the spectra at low-frequencies, \(St_\delta < 0.2\), where data are contaminated by mechanical vibration, and narrow spikes in the high-frequency region that are the result of electronic interference. In Figure 8.5b these spectra are normalized by their peak amplitudes; from this normalization it was readily apparent that deflection angle spectra obtained from aero-optic wavefront measurements of full-wall cooling were self-similar for a broad range of moderate wall-cooling temperatures. This result gives good support for the notion that wall cooling, like wall heating, merely acts as a passive amplifier on density fluctuations in TBL turbulence, and does not substantially alter the structure of TBL turbulence in the present study.

Convective velocity was also computed from the spectral cross-correlation method described in Chapter 5 for the DMBL, \(M = 0.4\) cooled-wall turbulent boundary layer case. The results, presented in Figure 8.6 for different values of \(\Delta T\), show that \(U_C\) of
Figure 8.6. Convective velocity of aero-optic aberrations for full wall cooling as a function of wall temperature.

Figure 8.7. Model-predicted effect of wall temperature on TBL density fluctuation profiles computed from equation (4.20) for a $M_\infty = 0.4$ TBL.
aero-optically aberrating structures is a function of wall temperature. For $\Delta T/(T_\infty M^2) > -0.2$, $U_C$ is approximately constant, equal to the adiabatic wall value of $0.83 U_\infty$. As $\Delta T/(T_\infty M^2)$ decreases from $-0.2$ to $-0.4$, there is a slight increase in the value of convective velocity to around $0.88 U_\infty$, and then a decrease to around $0.75 U_\infty$ near $\Delta T/(T_\infty M^2) = -0.6$. For large negative values of $\Delta T/(T_\infty M^2) < -0.5$, convective velocity is reduced to around $0.8 U_\infty$ or below.

The variation of $U_C$ with wall temperature is consistent with the model-predicted behavior, as the temperature-velocity (or density-velocity) model equation (4.20) is a function of local velocity $U(y)$ and $\Delta T$. Figure 8.7 shows the effect of both temperature and local velocity $U(y)$ on the model-predicted profiles of $\rho_{rms}(y)$ for several different wall-cooling temperatures. These model-predicted profiles show that for a wide range of TBL wall cooling cases, the variation in the fluctuating density profile that is consistent with the experimentally measured variation of wavefront convective velocity with wall temperature.

$OPD_{rms}(\Delta T)$ was computed from the full wall cooling wavefront measurements, and the results are plotted versus $\Delta T/(T_\infty M^2)$ in Figure 8.8, normalized by the baseline adiabatic wall temperature for several cases. These data show that wall temperature has a strong effect on $OPD_{rms}$ in the cooled-wall turbulent boundary layer, with good agreement between both single and double modified TBL experiments. There is an obvious minimum in $OPD_{rms}$ around $\Delta T/(T_\infty M^2) = -0.4$, where $OPD_{rms}$ is reduced by approximately 60% with respect to the baseline, adiabatic wall value. This result demonstrates the viability of wall cooling as a potent means of mitigating TBL-induced wavefront aberrations, although it falls short of the 80% reduction in $OPD_{rms}$ predicted by
The results of full wall-cooling experiments also verify the functional dependence of $OPD_{rms}$ on wall temperature (shown in Figure 4.4), and the existence of an optimal cooling temperature that is proportional to $M^2$.

![Graph showing normalized measurements of $OPD_{rms}$ as a function of wall temperature $\Delta T/(T_\infty M^2)$ for several full wall-cooling cases.]

Figure 8.8. Normalized measurements of $OPD_{rms}$ as a function of wall temperature $\Delta T/(T_\infty M^2)$ for several full wall-cooling cases.

To determine the model constants $C_1$ and $C_2$ from experimentally obtained $OPD_{rms}$, equation (4.23) was further simplified by normalizing by the baseline adiabatic wall temperature, $OPD_{rms}(T_{aw}) = BK_GD_P_\infty \delta \sqrt{C_f M^2}$, so that

$$OPD_{rms}^{\text{NORM}}(\Delta T) = \frac{OPD_{rms}(\Delta T)}{OPD_{rms}(T_{aw})} = \left[1 + C_1 \left(\frac{\Delta T}{T_\infty} \frac{1}{M^2}\right) + C_2 \left(\frac{\Delta T}{T_\infty} \frac{1}{M^2}\right)^2\right]^{1/2}.$$  \hspace{1cm} (8.6)

Notice that the normalized model equation (8.6) is purely a function of the empirical constants $C_1$ and $C_2$, and the non-dimensional term $\Delta T/(T_\infty M^2)$. A least squares fit was
performed to all of the $OPD_{rms}$ data presented in Figure 8.8 for $\Delta T/ (T_{\infty} M^2) > -0.7$, and the resulting fit is shown to be in good agreement with experimental data. The value of $C_1 = 4.42$ obtained in this experiment is in good agreement, within experimental error, with the value of $C_1 = 4.3$ obtained for heated-wall experiments (Cress, 2010; Gordeyev, et al. 2015). While the empirical values obtained from these cooled-wall wavefront measurements are in good agreement with those obtained experimentally for the heated wall, it is of note that neither this experiment, nor prior work has yielded empirical constants that agree well with the theoretically predicted values of $C_1 = 6.38$ and $C_2 = 10.28$ calculated by Cress (2010).

For $\Delta T < -0.7 T_{\infty} M^2$, experimental measurements and model predictions for $OPD_{rms}$ consistently deviate from one another, with measured values of $OPD_{rms}$ being larger than the model predictions in this range. The breakdown of the model in this regime of wall temperatures is not well understood at this time, but a few explanations are offered here that could possibly explain this phenomenon.

- The temperature-velocity relation from the ESRA used to derive the aero-optic wall temperature model equation may break down for large negative values of $\Delta T$.

- The increase in $OPD_{rms}$ in this region might be caused by the intermittent formation of frost on the TBL development section walls during the initial cooling of the wall in the no-flow state.

Further investigation is required in order to determine whether either of these hypotheses adequately explains why the model predictions and experiments deviate from one another below $\Delta T = -0.7 T_{\infty} M^2$, but this is outside of the scope of the present study. This problem will be revisited later on in Chapter 10 during a discussion of future work on this subject.
8.2.2 Partial Wall Cooling

In addition to the full wall cooling case, several different types of partial wall cooling configurations were evaluated using the Malley probe wavefront sensor. A set of near-aperture partial wall-cooling wavefront measurements were obtained, in which the cooling end location $x_1$ was fixed just upstream of the optical measurement section at $x = 120$ cm, and the location where cooling began, $x_0$, was varied in order to change the length of cooling, $L_{\text{cool}} = x_1 - x_0$. The second category of partial wall-cooling tests was a set of wavefront measurements for far-upstream cooling, in which $x_0$ was fixed, and $x_1$ was varied in order to change the cooling length. Measurements of two additional cases were also conducted: 1) a mid-stream ‘strip’ of cooling (i.e. Section 2 cooling) and 2) a combination of short near-aperture and far-upstream cooling (i.e. Sections 1 and 3 cooled). The full list of the wall-cooling cases tested in this experiment is given in Table 8.1 along with the streamwise locations of the start, $x_0$, and end, $x_1$, of the cooled wall segments. The ratio of $L_{\text{cool}}$ to the TBL development length to the Malley probe location, $R_{\text{cool}} = L_{\text{cool}}/x_{\text{MP}}$, are also given in Table 8.1.

$OPD_{\text{rms}}$ was computed from partial wall cooling wavefront measurements in the same manner as for full wall cooling; the results are presented in Figure 8.9 along with the $M = 0.4$, DMBL full wall cooling $OPD_{\text{rms}}$ data for comparison. The results for the three longer partial wall cooling configurations, where $R_{\text{cool}} = 64\%$, are shown in Figure 8.9a. For the far-upstream 64% cooling configuration, the minimum value of $OPD_{\text{rms}}$ was about $0.53 OPD_{\text{rms}} (\Delta T = 0)$, and value of $\Delta T_{\text{Optimal}}$ was approximately $-0.47 T_{\infty} M^2$. For the same length of wall cooling near the aperture, the optimal wall temperature was found to be about $-0.40 T_{\infty} M^2$, and effectiveness for aero-optic mitigation is slightly better than for
Figure 8.9. Normalized measurements of $OPD_{rms}$ as a function of wall temperature $\Delta T / (T_{\infty} M^2)$ for partial wall cooling cases: a) long lengths of cooling (64%) and b) shorter lengths of cooling (32%).
the far-upstream cooling, at $0.45OPD_{rms}(\Delta T = 0)$. The optimal cooling temperature for the far-upstream cooling case was about 20% lower than the value found for full wall cooling; however for near aperture cooling $\Delta T_{Optimal}$ was not altered significantly. Recall from the description in Table 8.1 that the third partial wall cooling configuration was a hybrid of short near-aperture and far-upstream cooling, each with $R_{cool} = 32\%$. Combined, this results in a total non-contiguous cooling area, $R_{cool}$, equal to 64% of the total TBL development length. The near/far combined configuration yielded an optimal cooling temperature estimated to be in the neighborhood of $\Delta T_{Optimal} = -0.40T_{\infty}M^2$ (although wavefront data was not obtained for $\Delta T/(T_{\infty}M^2) = -0.5$ to $-0.3$). For the whole range of wall temperatures measured for this case, $OPD_{rms}$ is slightly higher compared to the full wall cooling case.

Data for shorter lengths of wall cooling are presented in Figure 8.9b for three additional partial wall configurations: near-aperture cooling, far-upstream cooling, and ‘mid-range’ cooling, in which the middle one-third of the wall in the middle of the TBL development test section was cooled. For the short length of far-upstream cooling, $\Delta T_{Optimal}$ was found to be $-0.53T_{\infty}M^2$, and $OPD_{rms}$ was only reduced by approximately 28% at this temperature. For the short length of near aperture partial cooling, $\Delta T_{Optimal}$ was about $-0.38T_{\infty}M^2$, where aero-optic aberrations were only reduced by about 23%. These data show that the aero-optic mitigation performance is slightly better for the near-aperture configuration than the far-upstream case of the same length, but the differences are not very large. Cooling a strip of the wall in the middle of the TBL development section (i.e. mid-range strip cooling), resulted in a shift in $\Delta T_{Optimal}$ to a lower temperature ($\approx -0.5T_{\infty}M^2$) than was found for full wall cooling. However, this configuration was
found to result in a slightly larger reduction in \( OPD_{rms} \) (about a 44% decrease) than either of the other short length cooling configurations.

While \( OPD_{rms} \) measured for a variety of different partial wall cooling configurations possessed a functional form similar to the one that has been both observed and predicted for full wall cooling, changes in optimal temperature value and \( OPD_{rms} \) reduction were found to be highly dependent on the partial wall cooling configuration. It was shown in Chapter 4 that both the optimal temperature value and the amount of aero-optic mitigation are a function of the model constants \( C_1 \) and \( C_2 \). Therefore, it is likely that an effective means of modifying the model for partial wall cooling is to change the model constant definitions so that they are functionally dependent on the length and location of wall cooling. The specific model modifications, and an assessment of their effectiveness, will be presented in Section 8.3.

SBL scaled deflection angle spectra obtained for the different partial wall cooling configurations were computed and normalized using the following expression:

\[
\Phi(S_{\delta},\Delta T) = \frac{\hat{\theta}_{NORM}(S_{\delta},\Delta T)}{\hat{\theta}_{NORM}(S_{\delta},\Delta T = 0)} - 1, \\
= \left[ \frac{\hat{\theta}(S_{\delta},\Delta T)/OPD_{rms}(\Delta T)}{\hat{\theta}(S_{\delta},\Delta T = 0)/OPD_{rms}(\Delta T = 0)} \right] - 1, \\
= \left[ \frac{\hat{\theta}(S_{\delta},\Delta T) \ OPD_{rms}(\Delta T = 0)}{\hat{\theta}(S_{\delta},\Delta T = 0) \ OPD_{rms}(\Delta T)} \right] - 1, \\
\tag{8.7}
\]

recalling that \( \hat{\theta}_{NORM}(S_{\delta}) \propto \hat{\theta}(S_{\delta})/OPD_{rms} \). It follows that at frequencies where values of \( \Phi(S_{\delta},\Delta T) \) are less than 0, the suppression of the corresponding turbulent scales contributes more to the overall reduction in \( OPD_{rms} \) at this are more effectively suppressed by the effects of wall cooling. Similarly, at frequencies where \( \Phi(S_{\delta},\Delta T) > 0 \)
the turbulent scales corresponding to these frequencies are not as strongly suppressed by partial wall cooling. Deflection angle spectra scaled using equation (8.7) for several different partial wall-cooling configurations were computed, and the results are plotted in Figure 8.10 for a few selected values of $\Delta T$. The particular temperatures that data was shown for in each case was chosen by selecting a few temperatures representing 1) small amounts of wall cooling, 2) temperatures in the neighborhood of $\Delta T_{\text{Optimal}}$, and 3) larger values of $\Delta T$ that are well below the optimal cooling temperature.

Figure 8.10 (a, b) presents spectra for the far-upstream wall cooling configuration as a function of $St_{\delta}$ and normalized wall temperature, $\Delta T/ (T_{\infty}M^2)$. It should be noted that deviations from one in all spectra below $St_{\delta} = 0.2$ and above $St_{\delta} = 6$ should be neglected, as these regions are contaminated with noise from mechanical vibrations and electronic interference, respectively. The data shown in Figure 8.10(a, b) for far-upstream cooling shows that the reductions in $OPD_{\text{rms}}$ observed for these cases are not the result of even suppression of density fluctuations across all frequencies. Rather, far-upstream wall cooling was found to result in larger reductions in low frequencies ($St_{\delta} < 1 - 2$), and the effect is nearly twice as strong for the longer length of upstream cooling. This finding is consistent with the notion that far-upstream cooling creates a thermal sub-layer that rises and has more of an effect on the large-scale structures in the outer portion of the cooled TBL.

Spectra are plotted in Figure 8.10c and Figure 8.10d in the same manner for wall cooling cases where the boundary layer was cooled near the optical aperture, for about 32% and 64% of the boundary layer development length, respectively. Note that the spectra presented in Figure 8.10d appears to be more ‘noisy’ than those presented in
Figure 8.10a–c; this is due to the fact that these data were obtained from SMBL wavefront measurements, rather than DMBL wall cooling tests, and as a result of the SMBL scaling there is an increase in uncertainty in the results. It was found that high frequencies ($St_δ > 1$) were more effectively suppressed by near-aperture cooling, especially in the neighborhood of the optimal cooling temperature value. Similarly to the results for far-upstream cooling, the effect is more pronounced for the longer length of near-aperture cooling, which is shown in Figure 8.10d. For this case, it also appears that the frequency range of effective suppression is larger, covering frequencies greater than $St_δ = 0.5$.

The trends observed in Figure 8.10 for both the near aperture and far-upstream cooling cases can be explained by the characteristics of the thermal sub-layers which are likely generated by each of these configurations, as has been observed in prior experimental studies (Gran, et al., 1974; and others). Far-upstream cooling has been shown to generate a cooled sub-layer which rises further away from the wall after the step change from cooling to adiabatic wall conditions. Such a sub-layer residing in the outer-layer of the TBL would have a larger suppression effect on density fluctuations from large-scale turbulent structures which also reside in the outer layer; therefore wavefront aberrations at lower frequencies are more suppressed. Near aperture wall cooling, on the other hand, should generate a cooled sub-layer near the wall that grows thicker as the length of cooling increases. Therefore turbulent density fluctuations, and their corresponding wavefront aberrations, associated with smaller scale structures in the TBL should be more suppressed than large-scale structures for the near aperture cooling case.
Figure 8.10. Deflection angle spectra, normalized via equation (8.7), for selected wall-cooling configurations; a) far-upstream cooling, $R_{cool} = 32\%$; b) far-upstream cooling, $R_{cool} = 64\%$; c) near-aperture cooling, $R_{cool} = 32\%$; d) near-aperture cooling, $R_{cool} = 64\%$. (Pages 223 – 224)
Convection velocity was also computed from DMBL wavefront measurements, and the results for the shortest strips ($R_{cool} = 32\%$) for both near aperture and far-upstream cooling are plotted in Figure 8.11. The results of DMBL measured $U_C$ from full-wall cooling measurements (from Figure 8.6) are also shown for comparison. For far-upstream wall cooling, the convection velocity is reduced below the value of $0.83U_\infty$ measured for adiabatic wall TBLs, with $U_C$ decreasing monotonically with wall temperature to about $0.73U_\infty$ at $\Delta T = -0.8T_\infty M^2$. Variation in $U_C$ with wall temperature was also found in full wall cooling wavefront measurements, however for far-upstream partial wall cooling the measured values of $U_C$ showed were consistently lower than those found for full-wall cooling. For near-aperture cooling, convective velocity is actually increased over the value of $0.83U_\infty$ measured for adiabatic wall TBLs. As wall temperature was decreased to around $-0.45T_\infty M^2$, $U_C$ increased to a maximum value of about $0.88U_\infty$, and as temperature was decreased further, $U_C$ was found to decrease, reaching a value of about $0.78U_\infty$ around $\Delta T = -0.75T_\infty M^2$.

In Figure 8.11, it is clear that the functional dependence of $U_C$ on $\Delta T$ for partial wall cooling temperature is distinctly different than the functional dependence measured for full wall cooling. Measured values of $U_C$ for near aperture partial wall cooling were found to be generally greater than for full wall cooling, while $U_C$ measured for far-upstream cooling was generally less than the full wall cooling results. One possible explanation for these differences was discussed in Chapter 4, namely that step changes in wall temperature create distinct thermal sub-layers within the turbulent boundary layer. Assuming that this is the case, far-upstream cooling could generate a thermal sub-layer that would affect density fluctuations in the outer part of the turbulent boundary layer.
Conversely, a thermal sub-layer generated by near aperture cooling would tend to affect density near to the wall, reducing the aero-optic signature of this region. The experimentally observed effect on $U_C$ is consistent with this hypothesis.

![Convective velocity computed from DMBL partial wall cooling wavefront measurements.](image)

**Figure 8.11.** Convective velocity computed from DMBL partial wall cooling wavefront measurements.

### 8.3 Aero-Optic Models of Wall Temperature Effects

The experimental results presented in the previous section provide much needed experimental validation that full and partial wall cooling as an effective aero-optic mitigation technique. These data can also be used as a benchmark to evaluate existing models for the effect of wall temperature on aero-optic aberrations, and as a guide for the further develop analytical models for full and partial wall cooling.
8.3.1 Full Wall Cooling Model

Recent work by Gordeyev, et al. (2015) has sought to refine the model initially proposed by Cress (2010), which was reviewed in Chapter 4. A detailed discussion of the modified model can be found in Appendix C, but a short summary of the updated model is given here. For the present model, equation (4.20) was replaced with

\[
\left( \frac{T_{rms}(y)}{T_\infty} \right)^2 = \left( \frac{u_{rms}(y)}{U_\infty} \right)^2 \left[ \frac{\Delta T}{T_\infty} + A(y)(\gamma - 1)M_\infty^2 \frac{U(y)}{U_\infty} \right]^2
\]

(8.8)

where the recovery factor \( r \) is replaced with \( A(y) \) : a term from the SRA that accounts for the stress integral distribution in the TBL (Smits & Dussauge, 1996). The relationship between density and temperature fluctuations given in equation (4.21), which is a consequence of the Strong Reynolds Analogy, was discarded in favor of an expression that accounted for the effect of pressure fluctuations, \( p_{rms} \), as well:

\[
\left( \frac{\rho_{rms}}{\rho_\infty} \right)^2 = \left( \frac{T_{rms}}{T_\infty} \right)^2 \left( \frac{T_\infty}{T(y)} \right)^4 \left( \frac{p_{rms}}{P_\infty} \right)^2 \left( \frac{T_\infty}{T(y)} \right)^2
\]

(8.9)

In equation (8.9), \( T(y)/T_\infty \) is the average static temperature profile. In the present model, this profile was calculated from the modified Crocco relation, equation (4.16):

\[
\frac{T(y)}{T_\infty} = 1 + r \frac{\gamma - 1}{2} M_\infty^2 \left( 1 - \left( \frac{U(y)}{U_\infty} \right)^2 \right) + \frac{\Delta T}{T_\infty} \left( 1 - \frac{U(y)}{U_\infty} \right).
\]

(8.10)

For subsonic TBLs at moderate wall cooling temperatures, the second and third terms of equation (8.10) are generally small (contributing less than 5%) over most of the TBL profile. Therefore, the dependence of this term on \( \Delta T_\infty \) was neglected. Equation (8.8) and (8.10) were combined with (8.9) to obtain an expression for \( \left( \frac{\rho_{rms}(y)}{\rho_\infty} \right)^2 \), which was
then substituted into Sutton’s linking equation (2.3). This process yielded an expression for \( OPD_{rms} (\Delta T) \) that is of the same form that was previously derived by Cress (2010),

\[
OPD_{rms} = A_0 K_G D^2 \rho_\infty \delta \sqrt{C_f M^2_{\infty}} \left[ 1 + C_1 \left( \frac{\Delta T}{T_{\infty} M^2_{\infty}} \right) + C_2 \left( \frac{\Delta T}{T_{\infty} M^2_{\infty}} \right)^2 \right]^{1/2}, \tag{8.11}
\]

where the constants \( A_0, C_1, \) and \( C_2 \) are defined using the following integrals:

\[
A_0^2 = \int_0^\infty [(y-1)A(y)f(y)g(y)]^2 \left( \frac{T_{\infty}}{T(y)} \right)^3 \Lambda(y) \, dy \\
+ \gamma^2 \frac{C_L}{2} \int_0^\infty g^2(y) h^2(y) \left( \frac{T_{\infty}}{T(y)} \right)^2 \Lambda(y) \, dy, \tag{8.12a}
\]

\[
C_1 = \frac{2(y-1)}{A_0^2} \int_0^\infty A(y)f(y)g^2(y) \left( \frac{T_{\infty}}{T(y)} \right)^3 \Lambda(y) \, dy, \tag{8.12b}
\]

\[
C_2 = \frac{1}{A_0^2} \int_0^\infty g^2(y) \left( \frac{T_{\infty}}{T(y)} \right)^3 \Lambda(y) \, dy. \tag{8.12c}
\]

In equation (8.12), the normalized mean velocity, fluctuating velocity, and fluctuating pressure profiles were expressed as non-dimensional functions:

\[
f(y) = U(y)/U_\infty, \quad g(y) = u_{rms}(y)/u_\tau, \quad \text{and} \quad h(y) = p_{rms}/u_\tau^2 \quad \text{(Gordeyev, et al., 2015)}.
\]

Three functions of \( A(y) \) were also tested by Gordeyev, et al. (2015) : \( A(y) = 1, A(y) = r, \) and \( A(y) \) from Smits and Dussauge (1996). The latter function of \( A(y) \) is shown in Figure 8.12a along with the functional forms of the non-dimensional profiles used by Gordeyev, et al. (2015) to estimate the model constants with equation (8.12). The mean velocity profile \( f(y) \) was obtained from experiments in the Hessert TWT, while \( g(y) \) and \( h(y) \) were obtained from DNS simulations by Guarini, et al. (200). Three different choices of density correlation length from experiments by Gilbert (1982) and Rose (1979), and from

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an LES simulation by Wang & Wang (2012) are presented in Figure 8.12b. These functions were also used to investigate the sensitivity of the analytically determined model constants to changes in the correlation length used to make predictions.

It was found by Gordyev, et al (2015) that for the nine combinations of choices for \(A(y)\) and \(\Lambda(y)\) the updated model predicted values of \(C_1\) from 5.46 to 6.84, and that \(C_2\) was between 7.92 and 11.73. Compared to the results of this updated wall-temperature model, experimentally determined values of \(C_1\) and \(C_2\) are still less than the values that are predicted analytically for all 9 possible combinations of \(A(y)\) and \(\Lambda(y)\). For all of the combinations, the predicted optimal cooling temperature, \(\Delta T_{Optimal} / (T_\infty M^2)\), was between \(-0.33\) and \(-0.27\). Also, at the optimal temperature \(OPD_{rms}\) was found to be between 0.23 to 0.27 times \(OPD_{rms} (\Delta T = 0)\) if the effect of pressure fluctuations was included in the model, and between 0.10 and 0.17 times \(OPD_{rms} (\Delta T = 0)\) if pressure fluctuations were ignored by setting \(h(y) = 0\). For both of these cases, the reduction in \(OPD_{rms}\) at \(\Delta T_{Optimal}\) was over predicted, but the inclusion of pressure fluctuations in the aero-optic model was also found to somewhat improve the comparison between of model predictions and experimental measurements of \(OPD_{rms}\) for full wall cooling. Inspecting Figure 8.8, it is also evident that there is good qualitative agreement between the functional form of the wall-temperature model and experimentally observed trends.

8.3.2 Partial Wall Cooling Model

Based on characteristics observed in partial wall heating/cooling literature (see Chapter 4 for review) and in convective velocity measurements from different partial wall cooling cases, it is reasonable to assume that the variation in the effect of the length and location of wall cooling on aero-optic aberrations is caused by the differences
Figure 8.12. Different choices of a) normalized mean and fluctuating velocity profiles, fluctuating pressure profile, and \( f(y/\delta) \), and b) density correlation length \( \Lambda(y) \) that were used to evaluate the model constants via equation (8.12).
between thermal sub-layers generated by each unique configuration. Since the model constants \( C_1 \) and \( C_2 \) account for the effect of wall-temperature in the TBL aero-optic model, equation (8.11, it was assumed that these empirical constants would retain the functional form found in equation (8.6) for partial wall cooling, but with \( C_1 \) and \( C_2 \) being dependent on the start location, \( x_0 \), and end location \( x_1 \), of wall cooling:

\[
OPD_{\text{rms}}^{\text{NORM}}(\Delta T) = \left[ 1 + C_1(x_0, x_1) \left( \frac{\Delta T}{T_x M_x^2} \right) + C_2(x_0, x_1) \left( \frac{\Delta T}{T_x M_x^2} \right)^2 \right]^{1/2}.
\] (8.13)

This notion is supported by the general characteristics of \( OPD_{\text{rms}} (\Delta T) \) observed in experiments for a variety of different partial wall cooling configurations, as shown in Figure 8.9.

**TABLE 8.2**

**EMPIRICAL CONSTANTS COMPUTED FOR FULL AND PARTIAL WALL COOLING EXPERIMENTS**

<table>
<thead>
<tr>
<th>Configuration</th>
<th>( R_{\text{cool}} )</th>
<th>( C_1 )</th>
<th>( C_2 )</th>
<th>( \frac{\Delta T_{\text{Optimal}}}{T_x M_x^2} )</th>
<th>( \frac{OPD_{\text{rms}}(\Delta T_{\text{min}})}{OPD_{\text{rms}}(\Delta T = 0)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Wall</td>
<td>96%</td>
<td>4.42</td>
<td>5.71</td>
<td>-0.39</td>
<td>0.38</td>
</tr>
<tr>
<td>Far-Upstream (Short)</td>
<td>32%</td>
<td>2.09</td>
<td>2.18</td>
<td>-0.48</td>
<td>0.71</td>
</tr>
<tr>
<td>Far-Upstream (Long)</td>
<td>64%</td>
<td>3.07</td>
<td>3.32</td>
<td>-0.46</td>
<td>0.54</td>
</tr>
<tr>
<td>Near Aperture (Short)</td>
<td>32%</td>
<td>2.43</td>
<td>3.67</td>
<td>-0.33</td>
<td>0.77</td>
</tr>
<tr>
<td>Near Aperture (Long)</td>
<td>64%</td>
<td>3.95</td>
<td>4.94</td>
<td>-0.40</td>
<td>0.46</td>
</tr>
<tr>
<td>Mid-Range (Short)</td>
<td>32%</td>
<td>2.74</td>
<td>2.74</td>
<td>-0.50</td>
<td>0.56</td>
</tr>
<tr>
<td>Near/Far Combined</td>
<td>64%*</td>
<td>3.97</td>
<td>5.29</td>
<td>-0.38</td>
<td>0.51</td>
</tr>
</tbody>
</table>

*Length of cooling is not continuous.
To empirically determine the values of $C_1$ and $C_2$ that corresponded to each partial wall cooling case from experiments, equation (8.13) was fit to the results of each experiment over the range $-0.7 < \Delta T / M^2 < 0$, and the results are presented in Table 8.2 along with the values of $\Delta T_{\text{Optimal}} / \left( T_\infty M^2 \right)$ and $OPD_{\text{rms}} \left( \Delta T_{\text{Optimal}} \right) / OPD_{\text{rms}} (\Delta T = 0)$ computed from the empirically determined $C_1$ and $C_2$. For increasing values of $R_{\text{cool}}$, the experimentally determined values of both constants also monotonically increased towards the values obtained for full wall cooling wavefront measurements. The values of $C_1$ and $C_2$ calculated for far-upstream cooling were lower than those found for all other configurations. Out of all the configurations where $R_{\text{cool}} = 32\%$, the mid-range case (where Section 2 only was cooled) resulted in the largest aero-optic mitigation. For cases where $R_{\text{cool}} = 64\%$, the near-aperture cooling case gave the biggest reduction in $OPD_{\text{rms}}$.

![Figure 8.13](image.png)

Figure 8.13. Schematic of a simple thermal sub-layer used for modeling the effects of partial wall cooling on wavefront aberrations in the TBL.
Previous characterizations of thermal sub-layers caused by step changes in wall temperature (see Chapter 4; Gran, et al., 1974; Antonia, et al., 1977) were used to inform the modification of the existing wall cooling model for the effect of partial wall cooling. To simulate the effect of thermal sub-layer on boundary layer aero-optic statistics, the scalar $\Delta T$ in equation (4.18) was replaced with an arbitrary temperature difference function $F_{\Delta T}$, which is a function of $y$ and the upper and lower bounds of the thermal sub-layer, $\delta_0$ and $\delta_1$:

$$
\left(\frac{T_{rms}(y)}{T_\infty}\right)^2 = \left(\frac{u_{rms}(y)}{U_\infty}\right)^2 \left[\frac{F_{\Delta T}(\delta_0, \delta_1, y)}{T_\infty} + r(y-1)M_\infty^2 \frac{U(y)}{U_\infty}\right]^2.
$$

(8.14)

Comparing equation (8.14) to (8.8), it becomes clear that $F_{\Delta T}(\delta_0, \delta_1, y)$ can be thought of as the effective value of $\Delta T$ at any point $y$ in the TBL profile for a given wall cooling configuration. It follows then that $F_{\Delta T}$ can be written as the product of the wall temperature difference $\Delta T$ and a configuration-specific sub-layer mask function, $\Pi_T(\delta_0, \delta_1, y)$: $F_{\Delta T}(\delta_0, \delta_1, y) = \Delta T \times \Pi_T(\delta_0, \delta_1, y)$. Note that $\Pi_T$ is bounded between 0 and 1 and has the same functional form as $F_{\Delta T}$ (see inset of Figure 8.13). Substituting this relation into (8.14), the temperature-velocity relationship for partial wall cooling becomes

$$
\left(\frac{T_{rms}(\delta_0, \delta_1, y)}{T_\infty}\right)^2 = \left(\frac{u_{rms}(y)}{U_\infty}\right)^2 \left[\frac{\Delta T \times \Pi_T(\delta_0, \delta_1, y)}{T_\infty} + r(y-1)M_\infty^2 \frac{U(y)}{U_\infty}\right]^2.
$$

(8.15)

In effect, the addition of a mask function to the model limits the application of the ESRA temperature-velocity relation to the region of the TBL within the thermal sub-layer; outside of the sub-layer, where $\Pi_T = 0$, equation (8.15) reduces to the SRA.

This temperature-velocity relation was combined with equation (8.9) to find that
\[
\left( \frac{\rho_{rms}(\delta_0, \delta_1, y)}{\rho_\infty} \right)^2 = \left( \frac{u_{rms}(y)}{U_\infty} \right)^2 \left( \frac{T_\infty}{T(y)} \right)^4 \left[ \frac{\Delta T}{T_\infty} \frac{\Pi_T(\delta_0, \delta_1, y)}{T_\infty} + r(y - 1)M_\infty^2 \frac{U(y)}{U_\infty} \right]^2 
+ \left( \frac{P_{rms}(y)}{P_\infty} \right)^2 \left( \frac{T_\infty}{T(y)} \right)^2,
\]

(8.16)

where \( T(y) / T_\infty \) is given in equation (8.10). Equation (8.16) was then integrated using Sutton’s linking equation (2.3) to calculated \( OPD_{rms} \). The resulting aero-optic model equation reduces to the same functional form found in equation (8.11, with the sub-layer mask function being grouped into the integrals for \( C_1 \) and \( C_2 \):

\[
C_1(x_0, x_1) = \frac{2(y - 1)}{A_0^2} \int_0^\infty \Pi_T(y, \delta_0, \delta_1) \delta(y) g^2(y) \left( \frac{T_\infty}{T(y)} \right)^3 \Lambda(y) dy \quad (8.17a)
\]

\[
C_2(x_0, x_1) = \frac{1}{A_0^2} \int_0^\infty \Pi_T^2(y, \delta_0, \delta_1) g^2(y) \left( \frac{T_\infty}{T(y)} \right)^3 \Lambda(y) dy, \quad (8.17b)
\]

where \( \delta_0 \) and \( \delta_1 \) are functions of \( x_0 \) and \( x_1 \), respectively. Note that in the limit where the length of partial wall cooling goes to zero, \( \Pi_T = 0 \) for all values of \( y \) and the model equation reduces to the adiabatic wall aero-optic model equation (2.26). When the wall is cooled over its full length, \( \delta_1 \) will go to zero, \( \delta_0 = \delta \), and \( \Pi_T \) will be constant, equal to one across the entire boundary layer. In this particular case, equation (8.15) is easily shown to be the same as equation (8.8), and therefore equation (8.17) is reduced to the definitions of \( C_1 \) and \( C_2 \) given in equation (8.12).

In the present study, a piecewise step function was used to define the sub-layer mask function for the partial wall cooling (PWC) model:
\[ \Pi_T(y, \delta_0, \delta_1) = \begin{cases} 0, & \text{if } y < \delta_1, \\ 1, & \text{if } \delta_1 \leq y \leq \delta_0, \\ 0, & \text{if } y > \delta_0. \end{cases} \]  

(8.18)

The thermal sub-layer limits \( \delta_0 \) and \( \delta_1 \) were calculated at the Malley probe measurement location, \( x_{MP} \), using the following approximations:

\[ \delta_0 = \delta(x_{MP}) \left( \frac{x_{MP} - x_0}{x_{MP}} \right)^{0.5}, \]
\[ \delta_1 = \delta(x_{MP}) \left( \frac{x_{MP} - x_1}{x_{MP}} \right)^{0.8}. \]  

(8.19)

These growth rates for these internal sub-layer boundaries were estimated from the results of prior investigations of thermal step changes in the wall boundary conditions presented by Gran, et al. (1974), Antonia, et al., (1977), and others (see Chapter 4).

It follows from equation (8.13) that the optimal temperature difference \( \Delta T_{Optimal} \) at which maximum aero-optic mitigation is achieved for partial wall cooling is computed

\[ \frac{\Delta T_{Optimal}(x_0, x_1)}{T_x M_\infty^2} = \frac{1}{2} \frac{C_1(x_0, x_1)}{C_2(x_0, x_1)}. \]  

(8.20)

Subsequently, the value of the baseline normalized \( OPD_{rms} \) at the optimal temperature is

\[ OPD_{rms}^{NORM}(\Delta T_{Optimal}) = \left[ 1 - \frac{1}{4} \frac{C_1^2(x_0, x_1)}{C_2^2(x_0, x_1)} \right]^{1/2}. \]  

(8.21)

Figure 8.14 presents the model predicted values of \( C_1 \) and \( C_2 \), \( \Delta T_{Optimal} \), and \( OPD_{rms} (\Delta T_{Optimal}) \) for near aperture and far-upstream cooling for different selections of \( g(y) = u_{rms}(y) / U_r \), with \( \Lambda_\rho (y) = \Lambda_1 (y) \). Since using the function of \( A(y) \) from Smits & Dussauge (1996) gave model predictions that were closest to experimental results for full wall cooling (see Gordeyev, et al. 2015), this term will be used in the PWC model
predictions for the remainder of this work. The values of $C_1$ and $C_2$ determine from experimental measurements, and their corresponding values of $\Delta T_{\text{Optimal}}$ and $OPD_{\text{rms}}$ ($\Delta T_{\text{Optimal}}$) from are also presented for comparison with the model results. The values of $C_1$ and $C_2$ computed from the model using $g(y)$ from Guarini, et al. (2000) are significantly over-predicted for $R_{\text{cool}} \approx 1$; this outcome is consistent with the results of other efforts to analytically calculate the model constants (Cress, 2010; Gordeyev, et al. 2015). For wall cooling over shorter lengths, the model predictions of both empirical constants are over-predicted for near aperture cooling, although the model shows the same general trend as the experimentally obtained constants. For far-upstream cooling the model comes close to the experimentally measured value of $C_1$ for $R_{\text{cool}} = 0.64$, but over-predicts $C_2$. For smaller values of $R_{\text{cool}}$, $C_1$ and $C_2$ are both significantly under-predicted for far-upstream wall cooling. The predictions made using velocity data from Guarini, et al. (2000) also show a larger difference between the near aperture and far-upstream values of $C_1$ and $C_2$ than was found in experiments.

For the constants computed using $g(y)$ obtained from velocity measurements obtained in the TWT (see Chapter 5), the values of $C_1$ and $C_2$ were still over-predicted for $R_{\text{cool}} = 1$, and the PWC model predictions computed from measured $g(y)$ showed similar trends to those calculated using $g(y)$ from Guarini, et al. (2000). The constants calculated using measured velocity data, however, were found to be closer to the experimentally obtained values of $C_1$ and $C_2$. The difference between the near aperture and far-upstream PWC model predictions for both constants was also smaller when $g(y)$ from TWT measurements was used. An examination of the PWC model in equation (8.17) finds that the predictions for both of the model constants, and particularly $C_2$, are highly sensitive to
the profile of $g(y)$ that is used. Comparing the experimentally observed trends and the PWC model predictions given in Figure 8.14, it appears that predictions made using TWT measured $g(y)$ yield estimates of $C_1$ and $C_2$ that compare more favorably with experimental observations.

The values of $\Delta T_{\text{Optimal}}\left(T_{\infty}, M_\infty^2\right)$ and $\text{OPD}_{\text{rms}}^{\text{NORM}}\left(\Delta T_{\text{Optimal}}\right)$ were also computed from the experimentally obtained values of $C_1$ and $C_2$ via equations (8.20) and (8.21), and from the constants from both choices of $g(y)$; the results are shown in Figure 8.14 (c, d). Similarly to the results for $C_1$ and $C_2$, both predictions from the analytical model give similar trends as functions of $R_{\text{cool}}$, and the model predictions calculated using $g(y)$ measured in the TWT (see Chapter 5) give results that are more consistent with experimental results than the PWC model predictions computed using $g(y)$ from Guarini, et al. (2000). Taken together, the results presented in Figure 8.14 demonstrate that the model constants, optimal temperature, and level of $\text{OPD}_{\text{rms}}$ calculated via the model equation are all highly dependent on the velocity profiles used as inputs for the PWC model. Since there is slightly better agreement between experiments and model predictions calculated with the locally measured velocity data, the measured velocity RMS profile will be used for $g(y)$ in model predictions for the remainder of this work.

The effect of $\Lambda_\rho(y)$ on model predictions of $C_1$, $C_2$, $\Delta T_{\text{Optimal}}(T_{\infty} M_\infty^2)$, and $\text{OPD}_{\text{rms}}^{\text{NORM}}\left(\Delta T_{\text{Optimal}}\right)$ is shown in Figure 8.15 along with, the values calculated from experimental results. The choice of wall-normal density correlation function, $\Lambda_\rho(y)$, was found to have a strong influence on the shape of the functional dependence of model
predictions on $R_{cool}$. All of the PWC model predictions tended to over-predict the near aperture cooling constants with respect to the experimentally obtained values. The results
Figure 8.14. Effect of $u_{rms}$ ($y$) on model predicted constants a) $C_1$, b) $C_2$, c) $\Delta T_{Optimal}/(T_{\infty}M_0^2)$, and d) $OPD_{rms}^{NORM}(\Delta T_{Optimal})$; calculated using $\Lambda_1 (y)$ from Figure 8.12.
Figure 8.15. Effect of $\Lambda_p (y)$ on model predicted constants a) $C_1$, b) $C_2$, c) $\Delta T_{optimal} / \langle (T_{\infty} M_{\infty}^2) \rangle$, and d) $OPD_rms^{NORM} (\Delta T_{optimal})$. 

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of PWC model predictions using $\Lambda_2$ over predicted experimental results by as much as 40%, while model predictions using $\Lambda_1$ and $\Lambda_3$ only gave a 23% over prediction. For the far-upstream case, all three choices for $\Lambda_\rho$ gave decent PWC model predictions of $C_1$ and $C_2$ around $R_{cool} = 0.64$, but significantly under predicts for $R_{cool} < 0.64$, and over predicts for $R_{cool} \approx 1$. For all three choices of density correlation length, the PWC model gave estimates of $\Delta T_{Optimal}$ or $OPD_{rms}$ that were not in great quantitative agreement with experimental results, but had functional forms that were consistent with trends observed in experiments.

From these observations, it is apparent that the simple analytical PWC model proposed in equation (8.17) does a good job of qualitatively modeling the effect of partial wall cooling on aero-optic characteristics. Quantitatively, however, the model has been shown in the present study, and in Gordeyev, et al. (2015) to be highly sensitive to different choices for input parameters, and to have generally poor agreement with experimental results. To correct this quantitative discrepancy, the model predicted constants $C_1 (x_0, x_1)$ and $C_2 (x_0, x_1)$ were scaled so that they are equal to the experimentally obtained values of $C_1$ and $C_2$ for the full wall cooling case ($R_{cool} \approx 1$):

$$C_1^*(x_0, x_1) = \left( \frac{C_1^{Exp}(R_{cool} \approx 1)}{C_1^{Model}(R_{cool} = 1)} \right) C_1^{Model}(x_0, x_1), \text{ and}$$

$$C_2^*(x_0, x_1) = \left( \frac{C_2^{Exp}(R_{cool} \approx 1)}{C_2^{Model}(R_{cool} = 1)} \right) C_2^{Model}(x_0, x_1), \text{ (8.22a)}$$

In this equation, the superscripts ‘Exp’ and ‘Model’ denote the values of $C_1$ and $C_2$ obtained from experimental measurements and model predictions from equation (8.19), respectively, and the star denotes the resulting scaled model predictions. This is similar to
the approach used by Gordeyev, et al. (2015) for extending model predictions for full wall cooling of supersonic TBLs.

The results of this scaling are shown in Figure 8.16, for different values of $\Lambda_p (y)$ using the same parameters for the calculations presented in Figure 8.15. The results show that the model predictions for $\Lambda_1$ and $\Lambda_3$, corrected by the scaling in equation (8.22), match experimental data well for near-aperture cooling. For the case of far-upstream cooling, $\Lambda_1$ performs the best, but still under-predicts $C_1$ and $C_2$ and over-predicts $OPD_{rms}$ ($\Delta T_{Optimal}$) with respect to the experimentally determined values. Despite this discrepancy, the PWC model still does a reasonably good job of predicting the $\Delta T_{Optimal}$ for far-upstream cooling.

Model predictions of $C_1$, $C_2$, $\Delta T_{Optimal}$, and $OPD_{rms}$ ($\Delta T_{Optimal}$) were calculated using $\Lambda_1$ and scaled to match empirical full-wall measurements using equation (8.22) for all four of the different categories of PWC configurations that were experimentally tested (as shown in Table 8.1). The results show that for the mid-stream cooling case, the PWC model gives predicted constants $C_1$ and $C_2$ that are similar values to the values obtained from the experiment, but the predicted optimal temperature and $OPD_{rms}$ are significantly over-predicted by the model. PWC predictions of the model constants for the near/far combined cooling case were found to consistently below the experimentally measured values. The value of $OPD_{rms}$ ($\Delta T_{Optimal}$) was also in poor agreement with experimental results, being over-predicted for the combined cooling case. The PWC model predicted optimal temperature value, however, was in good agreement with the experimental data.
Figure 8.16. PWC model predictions for different choices of $\Lambda_\rho (\gamma)$, scaled by equation (8.22) to match empirically determined full-wall constants.
These results show that the empirically scaled PWC model performs well for near-aperture cooling, and gives good agreement for far-upstream cooling with large values of $R_{cool}$ (~0.64). Agreement is quantitatively poor between experimental data and model predictions for the other two cases, mid-stream cooling and near/far combined cooling. In most cases, however, the PWC model appears to predict trends that are similar to those observed in experiments. This good qualitative agreement, and the prior literature on step changes in wall temperature reviewed in Chapter 4, indicates that the thermal sub-layer assumption for modeling partial wall cooling is the appropriate approach, but the quantitative disagreement demonstrates that particular methodology used in the present study does not fully capture the physics of all partial wall cooling configurations.

Recall that the PWC model incorporated assumption of thermal sub-layer by assuming that the functional form of the sub-layer temperature distribution is a step-change (see Figure 8.13). This assumption was chosen because of its simplicity, and does not account for the intermittency of the thermal sub-layer interface, or temperature transport within the turbulent boundary layer. Additional measurements of the mean temperature profiles at the aperture location for all PWC configurations would provide valuable input on how to incorporate these effects into temperature profile model equation, $F_{\Delta T}(y,x_0,x_1)$. It is speculated that by using these results to further refine the model equation for the temperature distribution in the partially cooled TBL, the PWC model predictions would be significantly improved.
Figure 8.17. PWC model predictions made using $\Lambda_1(y)$, $A(y)$, for all experimentally measured configurations, scaled by equation (8.22) to match empirically determined full-wall constants.
Earlier in this chapter, it was shown that equation (8.16) predicts profiles of $\rho_{rms}$ in the PWC modified TBL. If it is assumed that density fluctuations convect at the average local mean velocity within the TBL, then it follows that the aero-optic convective speed is related to integral of density-fluctuation weighted velocity integral. As a result of this assumption, the convective velocity, $U_C$, of wavefront aberrations in the cooled wall TBLs can be estimated using relation

$$U_C(\Delta T, x_0, x_1) = \frac{\int |\rho_{rms}(y, \Delta T, x_0, x_1) f(y/\delta) dy}{\int |\rho_{rms}(y, \Delta T, x_0, x_1)| dy}. \quad (8.23)$$

Although the relation given in equation (8.23) is based on conjecture, it is worth noting that this technique was able to correctly predict the increase in $U_C$ of aero-optical
structures that was observed experimentally in the supersonic TBL (Gordeyev, et al. 2012).

The model-predicted dependence of $U_C$ on $\Delta T$ was computed from equation (8.23) for the shorter (32%) lengths of wall cooling for which wavefront measured values of $U_C$ were reported in Figure 8.11, and the predictions are compared to experimental data in Figure 8.18. Similarly to other model predictions, the results shown in this figure demonstrate that while the model-predicted values of $U_C$ are in poor quantitative agreement with experimental results, the configuration-dependent trends predicted by the model are very similar to those found experimentally.

8.4 Summary and Conclusions

This chapter presented the results of 1-D Malley probe wavefront measurements of aero-optic aberrations caused by turbulent boundary layers subjected to full and partial wall cooling. The results of these experiments were compared to a simple analytical aero-optic model, which was originally proposed by Cress (2010) and recently updated by Gordeyev, et al. (2015). Good agreement was found between the functional form of the model and with wavefront measurements of aberrations for $\Delta T > -0.7T_\infty M^2$ in a TBL with cooling applied over the entire wall; prior to this work, the functional form of the model had only been experimentally validated for boundary layer wall heating. The analytical model tended to give empirical constants that were consistently higher than the values computed from experimental results, but the experimental results for the empirical constant $C_1$ obtained in the present study were found to be consistent with the result computed from heated wall experiments by Cress (2010). The functional dependence of
\( \text{OPD}_{\text{rms}} \) on \( \Delta T \) predicted for full wall cooling by the model was also found to hold for partial wall cooling when \( \Delta T > -0.7 T_\infty M^2 \). However, the model constants \( C_1 \) and \( C_2 \), empirically obtained from partial wall cooling wavefront data by fitting the model equation to experimental data were found to decrease monotonically with \( R_{\text{cool}} \), the percentage of the wall cooled upstream of the optical aperture.

The aero-optic model for full wall heating and cooling (Cress, 2010; Gordeyev, et al., 2015) is a simple analytical model derived using the ESRA, which assumes that pressure fluctuations are negligible in the TBL. As a result, the ESRA model assumes that static temperature fluctuations are merely a function of velocity and the wall temperature; i.e. \( T_{\text{rms}} = f(\bar{U}, \Delta T) \); then it follows from the ideal gas law that \( \rho_{\text{rms}} / \rho_\infty = -T_{\text{rms}} / T_\infty \). Cress (2010) combined these relations with Sutton’s linking equation, and predicted that \( C_1 \) and \( C_2 \) were equal to 6.32 and 10.28, respectively, which predict a >80% reduction in \( \text{OPD}_{\text{rms}} \) occurring at approximately \( \Delta T \approx 0.3 T_\infty M^2 \). The experimentally obtained values of \( C_1 \) and \( C_2 \) for full wall cooling were found to be 4.42 and 5.71, with only about a 60% reduction in \( \text{OPD}_{\text{rms}} \) at \( \Delta T \approx 0.39 T_\infty M^2 \). In an attempt to resolve this discrepancy, Gordeyev, et al. (2015) updated the model by including the effect of pressure in the calculation of \( \rho_{\text{rms}} \), which resulted in predictions of \( C_1 \) and \( C_2 \) that were closer to the experimentally obtained values.

In spite of the disagreement between the model-predicted values for \( C_1 \) and \( C_2 \) with experimental results, however, this work demonstrated that the aero-optic model for full wall cooling does correctly model the functional form and trends for both full and partial wall cooling when the experimentally-adjusted constants are used. As a result, the full-wall cooling model was extended for partial wall cooling by introducing a simple
step function to model the internal thermal sub-layers created by discontinuities in thermal boundary conditions at the wall. The results of the PWC model, when adjusted to match experimental results, were found to give decent predictions of the model constants $C_1$ and $C_2$, the optimal wall cooling temperature, $\Delta T_{\text{Optimal}}$, and the level of $OPD_{\text{rms}}$ reduction. The model performed particularly well for near-aperture cooling, but also gave passable predictions for far-upstream cooling when $R_{\text{cool}} > 0.64$.

These results show that PWC aero-optic model presented in this chapter may be used as a valuable predictive tool for the design of applied aero-optic systems.

The results of wavefront measurements presented in this chapter effectively demonstrated the value of both full and partial wall cooling as a novel method for mitigating TBL aero-optic aberrations. Measurements of $OPD_{\text{rms}}$ for different partial cooling wall lengths and locations presented in this chapter showed that in general, cooling over the entire length of the boundary layer gave the largest reductions in $OPD_{\text{rms}}$ of about 60% at a temperature of $\Delta T_{\text{Optimal}}$ $(T_\infty M^2) \approx -0.4$. Cooling over only partial lengths of the wall showed that it was possible to achieve reductions in $OPD_{\text{rms}}$ on the order of 50% for cooling over 64% of the TBL development section, and reductions between 25 – 45% for cooling over just 32% of the TBL development length. For both the model predictions and experimental results, $OPD_{\text{rms}}$ was shown to monotonically decrease with increasing $R_{\text{cool}}$, indicating that there is no ‘optimal’ partial wall cooling configuration. Rather, the largest amount of aero-optic mitigation occurs for cooling over the full length of the TBL wall. The functional dependence of $OPD_{\text{rms}}$ on $R_{\text{cool}}$, however, indicates that for the near wall cooling case, cooling over partial lengths of the wall is more effective than far-upstream cooling for aero-optic mitigation.
CHAPTER 9:
EFFECTS OF WALL HEATING ON THE AERO-OPTICS OF TURBULENT BOUNDARY LAYERS AT LOW REYNOLDS NUMBERS

In the review of the wall-temperature model of TBL aero-optic aberrations presented in Chapter 4, equations (4.24) and (4.25) showed that wall heating acts as a passive amplifier of wavefront distortions. Cress (2010) confirmed the model prediction that the passive amplification is valid across the range of frequencies of the wavefront spectra, with no resulting shift in peak frequency or spectra shape. This passive amplification effect can be used advantageously to study low Reynolds number TBLs that have a weak aero-optic signature, such as those with low subsonic freestream velocities and/or TBLs that are very thin. By heating the wall it is possible to a passive ‘marker’ to make low-Reynolds number TBLs aero-optically ‘visible.’ Thus wavefront sensors can be used as non-intrusive diagnostic tools to allow for investigation of the dynamics of turbulent boundary layers at relatively low Reynolds numbers. The heating technique can also provide valuable experimental wavefront information that can be used to verify/correct various scaling models at low Reynolds numbers and to provide a direct comparison with computational simulations, as currently there is a large Reynolds number ‘gap’ between experimental measurements with typical $\text{Re}_\theta > 20,000$ (Gordeyev, et al. . 2014) and numerical simulations (Wang & Wang, 2012; White & Visbal, 2012) with much lower $\text{Re}_\theta \sim 1,000$ to 3,000.
9.1 Experimental Setup and Instrumentation

A series of measurements were conducted in the Hessert Transonic Wind Tunnel (TWT) at the University of Notre Dame, described in Chapter 4, with some modifications for wall-heating experiments. In the current study, the total length to the beginning of the optical section was shortened to approximately 100 cm to reduce the boundary layer thickness, and therefore the Reynolds number. Non-adiabatic wall temperatures were introduced by replacing the Plexiglas wall with an aluminum wall with an 8 mm thick Aluminum plate for the first 100 cm of the test section, and heating it from the outside surface using strips of flexible electric resistive coil heaters, similar to the experimental configuration utilized by Cress (2010) for prior experiments investigating wall-heating on TBL aero-optic aberrations. A schematic of the modified wind tunnel and test section is shown in Figure 9.1.

Figure 9.1. Schematic of the Hessert Transonic Wind Tunnel (TWT) configured for Malley probe wavefront measurements of aero-optic aberrations from heated wall turbulent boundary layers.

Wavefront measurements were acquired downstream of the heated wall boundary layer development section in both facilities using the Malley Probe 1-D wavefront sensor,
which was discussed in Chapter 5. Malley probe data were acquired in the Notre Dame TWT at a streamwise location of 105 cm, at the conditions described in Table 9.1.

**TABLE 9.1**

BOUNDARY LAYER WALL HEATING CONDITIONS EVALUATED USING A MALLEY PROBE 1-D WAVEFRONT SENSOR

<table>
<thead>
<tr>
<th>$V_\infty$ [m/s]</th>
<th>$M_\infty$</th>
<th>$\delta$ [cm]</th>
<th>$Re_\theta$</th>
<th>$\Delta T$ [K]</th>
<th>$f_{\text{samp}}$ [kHz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>64.8</td>
<td>0.18</td>
<td>1.2</td>
<td>4,200</td>
<td>0, 15-28</td>
<td>200</td>
</tr>
<tr>
<td>98.8</td>
<td>0.28</td>
<td>1.2</td>
<td>5,700</td>
<td>0, 9-24</td>
<td>200</td>
</tr>
<tr>
<td>118</td>
<td>0.35</td>
<td>1.2</td>
<td>7,900</td>
<td>0, 7-28</td>
<td>200</td>
</tr>
<tr>
<td>140</td>
<td>0.41</td>
<td>1.2</td>
<td>9,000</td>
<td>0, 7-24</td>
<td>200</td>
</tr>
<tr>
<td>140</td>
<td>0.41</td>
<td>2.4</td>
<td>20,000</td>
<td>0</td>
<td>200</td>
</tr>
</tbody>
</table>

From the deflection angle time series measured with the Malley probe, deflection angle amplitude spectra, one-dimensional wavefronts, and $OPD_{rms}$ were computed using the techniques described in Chapter 5. To remove the aero-optic contribution of the unheated TBL along the lower wall in the TWT facility in the Malley probe wavefront measurements, the SBL scaling technique described in Appendix A was applied to the resulting wavefront statistics.

Values of $OPD_{rms}$ computed from adiabatic wall Malley probe wavefront measurements are presented in Figure 9.2 (left) compared to $OPD_{rms}$ ($Re_\theta$) computed from the model equation (2.24), assuming constant $\theta$ and $C_f$ ($Re_\theta$) from the modified Coles-Fernholz relation from Nagib, et al. (2007). Comparison of experimental measurements and the theoretical model show good agreement, as the differences between the theoretical model and experimental measurements are within the
Figure 9.2. Results of adiabatic wall Malley probe wavefront measurements at low Reynolds numbers: a) values of $OPD_{rms}$ compared to the model equation (2.26) as a function of Reynolds number for constant $\theta$, $C_f$ from modified Coles-Fernholz (Nagib, et al. 2007) ; b) signal-to-noise ratio as a function of $OPD_{rms}$. 253
measurement errors. The signal-to-noise ratio was computed for the deflection angle signal by dividing $\theta_{rms}^2$ for each of the measurement cases by the deflection angle variance computed for a ‘no-flow’ ($V = 0$) measurement case: $\text{SNR} = \left( \frac{\theta_{rms}^2}{\theta_{rms,\text{No-Flow}}^2} \right)$.

Here, $\theta_{rms}^2$ was computed by integrating the deflection angle spectra over the same range of frequencies used for computing $\text{OPD}_{rms}$ (i.e. aero-optically active frequencies) so as to remove the contribution of low-frequency vibration contamination from the deflection angle signal. The SNR for each Reynolds number is plotted in Figure 9.2 (right) as a function of $\text{OPD}_{rms}$, and the results clearly show that at the lowest Reynolds number cases, the signal to noise ratio is clearly approaching one. This finding illustrates the necessity of finding ways to obtain wavefront measurements at low-Reynolds numbers with high signal-to-noise ratios if experimental validation of computational results is ever to be reliably performed.

9.2 Heated-Wall Wavefront Statistics

For wavefronts obtained in this experiment, the convective velocity was found to be $0.82U_\infty$, which is also consistent with previous measurements in this facility at higher Reynolds numbers. Since changes in Reynolds number in the boundary layer causes changes in the inertial sub-range of the TBL (near the wall), the independence of $U_C$ with respect to $\text{Re}_\theta$ gives indirect evidence that the most aero-optically influential components of the TBL are large-scale structures located in the outer region of the boundary layer.

Deflection angle spectra were computed from Malley probe measurements of the adiabatic and heated wall cases, and scaled to remove the effect of the un-heated wall in
Figure 9.3. Deflection angle amplitude spectra obtained at different values of ΔT for a) Reθ = 4,200 (M∞ = 0.18) and b) Reθ = 9,000 (M∞ = 0.4). Spectra are normalized by equation (4.25) with D1 = 0.
the spectra using the method outlined in Appendix A. Several of these spectra are presented in Figure 9.3 for the Re_θ = 4,200 and 9,000 cases, normalized by in equation (4.25) with D_1 = 0, so that temperature effects are not accounted for. Inspection of these data clearly shows that heated-wall deflection angle spectra exhibit the characteristic shape observed for canonical subsonic TBL Malley probe measurements (see Figure 2.3a) are amplified with increasing wall temperature. From equation (2.12), the levels of OPD_{rms} were computed from SBL scaled deflection angle spectra at different wall heating temperatures for Re_θ = 4,200-9,000. Figure 9.4 presents these data as a function of ΔT, and it is apparent that OPD_{rms} increases proportionally with wall temperature, consistent with the trends suggested by both the wall-heating model given in equation (4.24) and previous experimental results (Cress, 2010).

![Figure 9.4. Levels of wavefront aberrations as a function of wall temperature change for a range of low Reynolds numbers.](image-url)
The following sections will demonstrate the application of the wall-heating scaling laws from Chapter 4 in order to determine if they are valid at low Reynolds numbers. It should be noted that prior to this work, this scaling has only been shown to be valid for Reynolds numbers as low as 9,000 (Cress, 2010), while the majority of the data in the present study was obtained at Reynolds numbers that are substantially lower. If the validity of these scaling laws can be established at low Reynolds numbers, then a technique will be presented for estimating wavefront statistics for adiabatic wall TBLs from heated wall wavefront measurements.

9.2.1 Heated Wall Normalization

Cress (2010) showed that deflection angle spectra for heated-wall TBLs could be normalized using the scaling relationship previously presented in equation (4.25). This scaling is shown here again for convenience:

\[
\hat{\theta}_{\text{NORM}}(St_\delta) = \frac{\hat{\theta}(f)}{\rho_c \delta \sqrt{C_f M^2 \left(1 + D_1 \frac{\Delta T}{T_x M^2}\right)}}. \tag{9.1}
\]

The validity of the scaling for Re < 9,000 was checked by investigating the collapse of deflection angle spectra obtained for different heating conditions when the data was scaled using equation (9.1).

In order to accomplish this task, \(D_1\) was computed from experimental measurements of wavefront statistics measured for several values of \(\Delta T\) for each Mach number. Cress (2010) presented several ways of doing this, either from a modified form of equation (9.1) wherein only the peak locations are...
\[ D_1 \frac{\Delta T}{T_\infty} = M^2 \left( \frac{\hat{\theta}^\text{peak}_\text{heated} - \hat{\theta}^\text{peak}_\text{unheated}}{\hat{\theta}^\text{peak}_\text{unheated}} \right). \]  

(9.2)

This expression can be re-written so that the spectral peak value for the heated TBL is a linear function of temperature difference,

\[ \hat{\theta}^\text{peak}_\text{heated} (\Delta T) = \hat{\theta}^\text{peak}_\text{unheated} \left[ 1 + D_1 \left( \frac{\Delta T}{T_\infty M^2} \right) \right]. \]  

(9.3)

Using this relation, it is possible to compute both \( D_1 \) and \( \hat{\theta}^\text{peak}_\text{unheated} \) via linear regression as long as at wavefront data are obtained for at least two values of \( \Delta T \). The value of \( D_1 \) can be found in a similar manner from \( \text{OPD}_{\text{rms}} \) computed from heated wall wavefront measurements. If \( M^2 \) is factored out of the bracketed term of equation (4.24), the linearized wall-heating model can be re-written as

\[ \text{OPD}_{\text{rms}} = 0.2 K_{GD} \rho C_f \delta M^2 \sqrt{C_f} \left[ 1 + D_1 \left( \frac{\Delta T}{T_\infty M^2} \right) \right]. \]  

(9.4)

For the case of \( \Delta T = 0 \) (no wall heating), the bracketed term is equal to unity, and \( \text{OPD}_{\text{rms}}^{(\Delta T=0)} = B K_{GD} \rho C_f \delta M^2 \sqrt{C_f} \) is recovered for the un-heated wall. Substituting this back in to equation (9.4) and expanding,

\[ \text{OPD}_{\text{rms}} (\Delta T) = \text{OPD}_{\text{rms}}^{(\Delta T=0)} + \frac{D_1 \text{OPD}_{\text{rms}}^{(\Delta T=0)}}{T_\infty M^2} (\Delta T). \]  

(9.5)

Using either equation (9.3) and equation (9.5), measurements of wavefront aberrations for heated walls at several wall temperatures may be used to recover estimates of \( D_1 \), \( \hat{\theta}^\text{peak}_\text{unheated} \), and \( \text{OPD}_{\text{rms}}^{(\Delta T=0)} \) by obtaining the \( y \)-intercept value from a linear regression fit on the heated wall wavefront statistics, which are a function of \( \Delta T \). Assuming a relative error in heated-BL \( \text{OPD}_{\text{rms}} \) measurements of approximately \( \pm 10 \% \).
(from the measurement precision of ±8% of Malley probe beam spacing, and therefore $U_C$), and an absolute error in temperature measurement of ±1 K, a linear regression was performed on the heated wall data presented in Figure 9.4 for each Reynolds number using the method from York, et al. (2004) which also returns error bars on the $y$-intercept. A similar analysis can be performed for spectra peak values, $\hat{\theta}_{\text{peak}}(\Delta T)$, assuming a relative error of ±5%.

![Figure 9.5. Values of $D_1$ computed from heated wall wavefront measurements by equations (9.3) and (9.5), compared to values from Gordeyev, et al. (2015).](image)

Results of $D_1$ computed from these linear regression analyses are presented in Figure 9.5 along with values of $D_1$ computed from experiments and theoretical analysis by Cress (2010). Values of $D_1$ from the present study computed from spectra peaks (9.3) and from $OPD_{rms}$ (9.5) show good agreement with one another. There is also good agreement between these data and $D_1$ computed for values reported by Gordeyev, et al.
Reynolds numbers > 10,000. Although experimentally obtained values of $D_1$ from experiments are about 30% lower than the analytically predicted value of $D_1$ from Cress (2010), the results indicate that there is no strong dependence on Reynolds number for the model constant. Rather, the data shows that $D_1$ is approximately constant for Reynolds numbers spanning nearly an order of magnitude, from $\text{Re}_\theta = 4,000 – 28,000$.

To verify whether wall heating at low Reynolds numbers (i.e. less than 9,000) also acts as a passive scalar on wavefront statistics, deflection angle spectra for different heated wall cases may be normalized by equation (9.1) and examined to see if spectra for a wide range of $\Delta T$ are self-similar. The results of this normalization are shown in Figure 9.6 for $\text{Re}_\theta = 4,200$ and 9,000, where the values of $D_1$ used in the normalization were obtained from the method described previously. In general, good self-similarity was observed for these normalized deflection angle spectra for a large range of wall temperatures. Where individual spectra deviated from a self-similar curve (e.g. $\text{Re}_\theta = 4,200$, $\Delta T = 0$ case) the deviations are due to low-frequency vibration contamination ($\text{St}_\delta < 0.3$) or high-frequency spikes due to electronic interference ($\text{St}_\delta > 30$). The capability of using scaled heated-wall data to obtain low Reynolds number wavefront statistics is also demonstrated by comparing values of $\text{OPD}_{\text{rms}}$ ($\Delta T = 0$) computed from the linear regression analysis on equation (9.5) to values of $\text{OPD}_{\text{rms}}$ measured directly for the adiabatic wall. These data, shown in Figure 9.7, show that the heated-wall estimates and baseline data are in good agreement with one another and with the theoretical model for $\text{OPD}_{\text{rms}}$ from equation (2.26). The error bars on the heated-wall estimates are also lower in comparison to the adiabatic wall measurements, in part due to the increased signal to noise ratio for the passively amplified heated wall wavefronts.
Figure 9.6. Deflection angle amplitude spectra for different wall heating temperatures $\Delta T$ for a) $Re_\theta = 4,200$ ($M = 0.18$) and b) $Re_\theta = 9,000$. Spectra are normalized by equation (4.25) with $D_1$ computed from experimental measurements.
Figure 9.7. Values of $OPD_{rms}$ ($\Delta T = 0$) obtained from adiabatic wall measurements and estimated from heated-wall measurements, compared to the model equation (2.26).

These results give confirmation that using wall heating to amplify TBL aero-optic wavefront statistics does not noticeably alter the wall-normal integrated characteristics of TBL density fluctuations. The success of the normalization for both spectra and $OPD_{rms}$ shown in the present study also demonstrates that it is feasible to use the normalization scaling to estimate adiabatic wall baseline statistics from measurements obtained for heated wall wavefronts at low Reynolds numbers (as low as 4,200) by completing the following procedure.

- First, obtain aero-optic wavefront data for a heated wall TBL at several values of $\Delta T$. 

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• Then extract $OPD_{rms}$ and $\hat{\theta}_{peak}$ for each value of $\Delta T$ and perform a linear regression fit of equation (9.3) and (9.5) to recover estimates of $D_1$, $\hat{\theta}_{peak}^{\Delta T=0}$, and $OPD_{rms}^{\Delta T=0}$ from the slopes and y-intercept.

• Finally, use equation (9.1) and the newly found value of $D_1$ to recover the unheated wall deflection angle spectra.

This technique is noteworthy because of its obvious usefulness for circumventing the problem of aero-optic ‘invisibility’ that occurs when aero-optic aberrations caused by the TBL are weak; a problem that occurs in TBLs where aberrations are not sufficiently large enough to measure with a high degree of confidence. The following section will demonstrate the usefulness of this method technique for investigating the effect of Reynolds number on wavefront statistics.

9.3 Wavefront Statistics at Low Reynolds Numbers

Figure 9.8 presents normalized deflection angle spectra obtained using wall heating as a passive amplifier for the different Reynolds numbers spanning an order of magnitude, from $Re_\theta = 1,700$ to 20,000. In this figure, all of the data comes from measurements of heated wall wavefronts described in Table 9.1, except for the $Re = 1,700$ case. This spectra, which is taken from Smith, et al. (2014), was obtained using a Malley probe wavefront sensor in the Merrill Wind Tunnel at California Institute of Technology (Caltech) at a freestream velocity of 9.4 m/s ($M = 0.03$), $\delta = 2.7$ cm for a wall heated to $\Delta T = 21$ K.

In the vicinity of the spectra peak, and for the low end of the spectral peak ($St_\delta < 1$), all cases show good collapse. The peaks for all measured spectra are at approximately $St_\delta = 1.0$, independent of Reynolds number. In Gordeyev, et al. (2014) it was shown that
using Kolmogorov-like arguments for the inertial range, the deflection angle amplitude spectrum at large frequencies should behave as $\sim f^{(-2/3)}$; this curve is plotted in Figure 9.8 as a dashed black line. The spectrum at the lowest Reynolds number quickly falls off of this theoretically predicted behavior, indicating a fairly small inertial range, while the fall-off is less drastic for larger Reynolds numbers, implying a larger inertial range.

![Normalized deflection angle spectra for different Reynolds numbers.](image)

Figure 9.8. Normalized deflection angle spectra for different Reynolds numbers.

To model the observed changes in the (high frequency slope) size of the inertial range of deflection angle spectra for different Reynolds numbers, the spectral model from equation (2.18) was modified in order to account for changes in the spectra roll-off as a function of $Re_\theta$. The fall-off of the spectra was assumed to take a form inspired by Tatarski’s modification of Kolmogorov’s atmospheric wavefront spectrum to account for the presence of inner scale dissipative structures (Tatarski, 1971). Seeking a function in
the form of the original model times the exponential term, the modified spectral model was found to be

\[
\hat{\phi}_{fit}(St_\delta) = \hat{\phi}_{peak} \frac{St_\delta}{1 + (St_\delta/0.83)^{5/3}} \exp\left[-\left(\frac{St_\delta}{f(Re_\theta)}\right)^2\right].
\]  

(9.6)

The \( f(Re_\theta) \) term was empirically determined to be \( f(Re_\theta) \approx 1.6 Re_\theta^{0.22} \) by fitting equation (9.6) to the experimental data presented in Figure 9.8.

![Figure 9.9. Empirically determined values of \( f(Re_\theta) \) from equation (9.6) and the corresponding power law fit.](image)
Figure 9.10. Comparison of experimental deflection angle spectra and the modified empirical model from equation (9.6) for Reynolds number, \( \text{Re}_{\theta} \), a) 1,700 (Caltech), b) 4,200, c) 5,700, and d) 20,000 (Notre Dame). (Pages 267 – 268)
a) \( \text{Re}_\theta = 1,700 \) (Caltech)

b) \( \text{Re}_\theta = 4,200 \) (ND)
c) $\text{Re}_\theta = 5,700$ (ND)

\[ \frac{\hat{\theta}(St_{\delta})}{\hat{\theta}_{\text{peak}}} \]

- Model eqn. (2.18)
- Model eqn. (9.6)

\[ 10^{-1} \quad 10^{0} \quad 10^{1} \]

\[ St_{\delta} \]

---

d) $\text{Re}_\theta = 20,000$ (ND)

\[ \frac{\hat{\theta}(St_{\delta})}{\hat{\theta}_{\text{peak}}} \]

\[ 10^{-1} \quad 10^{0} \quad 10^{1} \]

\[ St_{\delta} \]
Figure 8.9 shows comparisons between experimental data, the original TBL spectral model from equation (2.18), and the modified model for four Reynolds numbers ranging from 1,700 to 20,000. For \( St < 0.5 \), the experimentally measured spectra and model spectra deviate from one another, with the experimentally measured spectra containing more energy. This behavior is consistent with results from previous studies, where it was shown that for Malley probe measurements, this increase in energy is a result of contamination from mechanical vibrations in Malley probe data that cannot be properly removed (Gordeyev, Smith, Cress, & Jumper, 2014). For \( St > 0.5 \), Figure 9.10 shows that the modified spectral model, equation (9.6), does a good job of describing the high-frequency roll-off behavior for a large range of Reynolds numbers.

Heated wall TBL wavefront statistics can also be used to evaluate the validity of scaling laws for \( OPD_{rms} \) at low Reynolds numbers. One method of doing this is to rearrange the terms of the TBL aero-optic model presented in equation (2.26) to solve for the model constant, \( B \):

\[
B = \frac{OPD_{rms}(\Delta T = 0)}{K_{GB} \rho_{c} \delta M^{2} \sqrt{C_{f}}}.
\]

Recall that this model was originally derived for high Reynolds number TBLs using the Strong Reynolds Analogy and the mixed scaling for the mean and fluctuating velocity profiles (see Chapter 2). Here the only term that explicitly depends on Reynolds number is \( C_{f} \), and there are a number of relations for computing \( C_{f} \) as a function of \( Re_{\theta} \) that compare with experimental measurements (Nagib, et al., 2007). Therefore if the assumptions used to derive equation (2.26) are valid at low Reynolds numbers, the model...
constant should be consistent with what was found at high Re, where \( B = 0.19 \pm 0.01 \) (Gordeyev, et al. 2014).

![Graph showing model constant B from equation (2.26) estimated from low-Reynolds number wavefront measurements from \( OPD_{\text{rms}} \) shown in Figure 9.7 and equation (9.7).]

Equation (9.7) was used to estimate \( B \) from experimental data obtained at low Reynolds number, using values of \( OPD_{\text{rms}} (\Delta T = 0) \) measured directly for adiabatic wall conditions and values of \( OPD_{\text{rms}} (\Delta T = 0) \) estimated from a linear fit of heated wall data using equation (9.5). The results of these estimates are shown in Figure 9.11, compared with the previously obtained experimental value of \( B = 0.19 \pm 0.01 \) from Gordeyev, et al. (2012). Good agreement was found between values of \( B \) estimated from equation (9.7) for both adiabatic wall wavefront measurements and heated wall wavefront measurements at low Reynolds numbers, especially for \( Re > 5,700 \). At \( Re = 4,200 \), there
is a slight increase in the value of $B_{estimated}$ to about 0.22 – 0.23, however the error bars for these estimates are large.

9.4 Wall Heating Summary

This chapter presented the results of aero-optic wavefront measurements of heated-wall boundary layers for a range of subsonic Mach numbers from 0.03 to 0.4, and Reynolds numbers $Re_\theta = 1,700 – 20,000$. For all data except the lowest Mach/Reynolds number condition, wavefronts were obtained in the Hessert Transonic Wind Tunnel (TWT) at the University of Notre Dame, which was described in detail in Chapter 5. The $M = 0.03$ wavefront data presented from Smith, et al. (2014) and was originally obtained at the Merrill Wind Tunnel (MWT) at the California Institute of Technology. For all cases, wavefront data were obtained with the Malley probe at sampling rates large enough to resolve the range of aero-optically active frequencies. The effect of increased wall temperatures on deflection angle spectra was analyzed, and it was found that heating indeed acted as a passive amplifier on TBL turbulence (spectra shape was found to be constant with respect to $\Delta T$), confirming that the wall heating scaling laws developed by Cress (2010) for higher Reynolds number TBLs were valid for Reynolds numbers at least as low as 4,200. Using the scaling laws, it was shown that equations (9.1) and (9.5) could be used to re-scale heated-wall wavefront data to recover wavefront statistics for adiabatic wall temperatures at low Reynolds numbers.

Wavefront spectra from both heated and un-heated walls were extracted and compared to investigate the effect of Reynolds number on the shape of boundary layer wavefront spectra. It was found that the inertial sub-range of aero-optic wavefront spectra
is reduced with decreasing Reynolds number, which is consistent with previous observations of other turbulence spectra (Tatarski, 1971). The empirical spectral model from equation (2.18) was modified in order to model this Reynolds number dependence by the addition of an exponential decay term (which is a function of Reynolds number) that approximates the high frequency roll-off observed in the experimentally measured wavefront spectra. The modified model was shown to agree well with spectra obtained from experiments spanning an order of magnitude in Reynolds number, from $\text{Re}_\theta = 1,700 – 20,000$. The model equation (2.26) for $\text{OPD}_{\text{rms}}$, which was originally derived for high Reynolds number boundary layers, was also found to work well for Reynolds numbers $> 4,000$.

The method developed in this chapter for extracting wavefront statistics TBLs from heated wall wavefront data is valuable, because it provides a novel and promising method for studying the aero-optic characteristics of TBLs that would otherwise be difficult to measure due to the problem of aero-optic ‘invisibility’ that occurs when Mach and/or Reynolds numbers in the TBL are low. This approach for obtaining low Reynolds/Mach number TBL wavefront data is especially useful for studying low-Reynolds number TBLs using non-intrusive aero-optic wavefront sensors. This technique, along with the modification proposed to the spectral model for Reynolds number dependence, are important developments in obtaining experimentally measured low Reynolds number TBL wavefront statistics for comparison to computational results, which are limited to low Reynolds numbers by limitations of practical parameters for simulations.
CHAPTER 10:
CONCLUSIONS AND DISCUSSION OF BOUNDARY LAYER FLOW CONTROL
TECHNIQUES FOR AERO-OPTIC CONTROL

10.1 Summary and Conclusions

The experimental investigations of aero-optical aberrations has sought to identify passive flow control techniques that are effective at reducing the overall level of optical distortions induced by subsonic, compressible turbulent boundary layers. This research has also endeavored to explore the possible physical mechanisms responsible for observed changes in levels of aero-optic aberrations. To achieve these ends, a series of experiments were performed to investigate the aero-optical properties of modified boundary layers. The effect of Large-Eddy Break-Up (LEBU) devices on both the velocity and wavefront statistics was studied in Chapters 6 for a limited number of simple device configurations. Chapter 7 showed the effect of LEBU device configuration on experimentally measured aero-optic wavefront statistics. The effect of wall cooling over both full and partial wall lengths of the TBL development length were presented in Chapter 8, while Chapter 9 demonstrated the effect of TBL wall heating at low Reynolds numbers, and proposed a unique method of utilizing wall heating to perform aero-optic measurements in wall-bounded flows that are not typically aero-optically active. In their own right, the results and conclusions from each of these experiments provide guidance and insight for effectively modifying TBLs to achieve some desired modification to
corresponding wavefront characteristics. Taken together however, the results of each of these experiments demonstrate that through the proper use of flow control, experimentalists and system designers are capable of a significant amount of control over aero-optic aberrations caused by turbulent boundary layers.

10.1.1 Aero-Optic Mitigation

Experimental measurements of the effects of LEBU devices and wall cooling on aero-optic aberrations caused by compressible, subsonic turbulent boundary layers have shown (in Chapters 6, 7, and 8) that significant reductions in $OPD_{rms}$ can be achieved through the use of passive flow control. An investigation of the sensitivity of $OPD_{rms}$ to single LEBU device configurations showed that the optimal device height, $h$, for maximizing aero-optic mitigation was approximately $0.6\delta$. Results also showed that the device with the longest chord length, the $l = 4\delta$, $h = 0.6\delta$, ‘long’ single LEBU device, performed the best out of the entire set of single and multiple element LEBU devices tested in this study. This configuration resulted in a maximum reduction in $OPD_{rms}$ of approximately 45%, and also gave sustained reductions over 40% for $6\delta$ in the streamwise direction, and reductions in $OPD_{rms}$ over 30% for $9\delta$. It is also worth noting that this ‘optimal’ chord length was found on the edge of the parametric space tested in the present work for single LEBUs, and also that the investigation of multi-element LEBUs was by no means parametric in any sense. Therefore, it is possible that there exists a LEBU configuration (possibly consisting of a longer single LEBU or a multiple element LEBU device with longer chord lengths) that will result in greater levels of aero-optic mitigation than those achieved in the present study.
Although no definitive global optimum configuration was identified amongst the 16 single and 5 multi-element LEBU configurations reported in this dissertation, it is also reassuring that all of the configurations tested showed at least some reductions in $OPD_{rms}$ in the modified TBL downstream of the device. This result suggests that in practical applications, as long as the LEBU device is adequately thin and mounted at zero angle of attack, there is little risk that a non-optimized LEBU geometry will worsen the aero-optic environment over an aperture submerged beneath a fully developed, zero pressure gradient turbulent boundary layer.

Experimental investigations of the effect of full and partial wall cooling have also successfully demonstrated notable aero-optic mitigation capabilities, with $OPD_{rms}$ being reduced by as much as 60% using this flow control scheme. This maximum reduction was achieved for cooling applied over the full wall, and experiments showed that $OPD_{rms}$ monotonically decreases with increasing $R_{cool}$, meaning that there is no ‘optimal’ partial wall cooling configuration. Rather, the greatest reduction of aero-optic aberrations comes from cooling the wall over the full length of the TBL. Results showed, however, that partial cooling can still provide significant reductions in $OPD_{rms}$: cooling over 64% of the wall resulted in reductions between 40 – 55%, while cooling over just 32% of the wall reduced $OPD_{rms}$ by about 20 – 40%. Partial cooling results were also highly dependent on the location of the applied cooling, with the largest reductions being achieved for cooling just upstream of the optical aperture over longer lengths of the wall. This result is especially beneficial for the design of any potential wall cooling aero-optic mitigation system where cooling over the full TBL development length is not possible due to space or energy cost constraints.
10.1.2 Temperature-Velocity Modeling

Experimentally measured levels of $OPD_{rms}$ for both full and partial wall cooling in compressible, subsonic TBLs were compared to the aero-optic model equation for the effect of temperature on levels of optical aberrations,

$$OPD_{rms} = A_0K_{GD} \rho \delta \sqrt{C_f M_x^2} \left[ 1 + C_1 \left( \frac{\Delta T}{T_x M_x^2} \right) + C_2 \left( \frac{\Delta T}{T_x M_x^2} \right)^{3/2} \right]. \quad (10.1)$$

The functional form of this equation, which was initially proposed by Cress (2010), was found to be in good agreement with experimental results for both full and partial wall cooling when $\Delta T > -0.7T_x M^2$. In the present work, this model was modified in Chapter 8 to account for the effect of partial wall cooling. This was done by modifying the definitions of the model constants $C_1$ and $C_2$ with the temperature mask function $\Pi_T(y,x_0,x_1)$, which limits the application of the ESRA temperature-velocity relation to a finite thermal sub-layer within the partially cooled TBL:

$$C_1(x_0,x_1) = \frac{2(y-1)}{A_0^2} \int_0^\infty \Pi_T(y,x_0,x_1) A(y) f(y) \left( \frac{T_x}{T(y)} \right)^3 \Lambda(y) dy \quad (10.2a)$$

$$C_2(x_0,x_1) = \frac{1}{A_0^2} \int_0^\infty \Pi_T^2(y,x_0,x_1) g^2(y) \left( \frac{T_x}{T(y)} \right)^3 \Lambda(y) dy, \quad (10.2b)$$

In the present study, this a simple piecewise step function was used to model the influence of the thermal sub-layer on aero-optic model predictions,

$$\Pi_T(y,x_0,x_1) = \begin{cases} 
0, & \text{if } y < \delta_1(x_1), \\
1, & \text{if } \delta_1(x_1) \leq y \leq \delta_0(x_0), \\
0, & \text{if } y > \delta_0(x_0) 
\end{cases} \quad (10.3)$$
Note that the sub-layer boundaries, $\delta_0(x_0)$ and $\delta_1(x_1)$, are defined in equation (8.19), and shown in Figure 10.1 below.

![Diagram](image)

Figure 10.1. Illustration of thermal sub-layer approximation used to model the effects of partial wall cooling on $OPD_{rms}$.

A comparison of the model constants $C_1$ and $C_2$ that were computed from experimental data and values computed with the model equations (10.2) and (10.3) showed that the analytical predictions were in good agreement with experimentally observed trends. This good agreement is an indication that the present approach of using the temperature-velocity relations from the Extended Strong Reynolds Analogy, which are given in equations (4.15) and (4.16), is appropriate. Although the ESRA as has been shown to be only approximately valid, the temperature-velocity relation from this particular Reynolds analogy matches well with experimental and simulations due to the particular relationship between total and static temperature fluctuations (Guarini, et al. 2000).
The good agreement between partial wall cooling experiments and model predictions is also evidence that the thermal sub-layer mask approach used in the aero-optic model is also a good approach, despite of the fact that the particular choice of \( \Pi_T \) given in equation (10.3) does not account for intermittency effects at the sub-layer boundaries. Additional experimental and analytical investigation of the characteristics of the temperature field in the partially cooled boundary layer could be performed in the future if further refinements of the sub-layer mask function and aero-optic model are desired. At this time, however, the model does an adequate job of modeling the aero-optic characteristics of the partially cooled TBL, especially when the results are empirically scaled to match full wall cooling experimental results.

Below \( \Delta T = -0.7T_{\infty}M^2 \), it was found that the model predictions and experimental measurements of \( OPD_{\text{rms}} \) consistently deviated from one another. The breakdown of the model in this regime of wall temperatures is not well understood at this time, but there are a few likely explanations that warrant additional study at this time. First, Debieve, et al. (1997) found that the distance required for TBL temperature profiles to return to equilibrium grew appreciably for increasing values \( \Delta T \) for partial wall heating, and before equilibrium was reached \( T(y)/T_{\infty} \) was in poor agreement with the profiles predicted from the ESRA. It is therefore possible that a similar effect is present over the optical aperture for large negative values of \( \Delta T \); as a result the assumed ESRA temperature-velocity relation used in deriving the aero-optic model may not be entirely valid for these temperatures. This phenomenon may also account for some of the discrepancies between experimental data and model predictions for partial wall cooling from equations (10.2) and (10.3). Another possible explanation for the deviation of experimental data from the
model-predicted dependence at large negative values of $\Delta T$ is the effect of buoyancy. However, the Grashof number computed at $\Delta T = -1.0 T_{\infty} M^2$ is on the order of $10^5$, and the square of the Reynolds number is $5 \times 10^{10}$. Since $Gr \ll Re^2$, it should be the case that forced convection is dominant and buoyancy effects are negligible (White, 2006). Although it seems unlikely that buoyancy effects are significant in the experimental results, some simple geometric changes to the test configuration (such as moving wall cooling from the test section side walls to the top and bottom walls) could be made to further study any possible effects of buoyancy.

One additional problem that could potentially be encountered in the implementation of a wall cooling aero-optic mitigation system is the potential for frost formation on the TBL wall at low temperatures. The increased wall surface roughness that could be caused by the formation of frost on sections of cooled wall upstream of the optical aperture may lead to an increased in $OPD_{rms}$, which would be counter-productive for aero-optic mitigation. Thus, the potential for frost formation over the cold wall should be considered in the design and implementation of wall cooling aero-optic mitigation schemes, especially for systems that will be operating in air with a significant amount of moisture content.

Overall, the partial wall cooling aero-optic model has been shown to be in good agreement with experimental results, which validates the choice of using the ESRA temperature-velocity relation in the model derivation. This good agreement also suggests that the thermal sub-layer approximation introduced in this work is also suitable for predicting the effects of partial wall cooling on aero-optic aberrations. The analytical model presented in this work may be used as a valuable predictive tool for the design of
applied aero-optic systems. The framework used in the derivation of this work may also prove to be useful for application to other problems outside of aero-optics that also involves non-adiabatic turbulent boundary layers.

10.1.3 Aero-Optical Sensors for Non-intrusive Flow Diagnostics

From measurements of aero-optic aberrations in the LEBU-modified boundary layer, changes in turbulence production and dissipation that were consistent with the results obtained in velocity measurements were found. Spectral features associated with the growth of the LEBU wake were also identified from wavefront data, and estimates of the LEBU wake thickness made from optical measurements were shown to be in good agreement with the values obtained from velocity measurements. Changes in spectra obtained for different partial wall cooling configurations also were found to be consistent with the notion that thermal sub-layers were formed within the partially cooled TBL.

Aero-optic characteristics of low-Reynolds number TBLs were also experimentally investigated in order to evaluate the potential for using wall heating as a means of passively amplifying wavefront distortions in TBLs where aero-optic aberrations are typically quite weak. The effect of $Re_\theta$ on TBL wavefront (and therefore turbulence) spectra was also studied in this dissertation, and it was found that the inertial sub-range of aero-optic wavefront spectra decreased with Reynolds number, which was consistent with previous observations for other turbulence spectra (Tatarski, 1971). These results demonstrate that it is possible to study changes in turbulent flow structure using non-intrusive aero-optic wavefront sensors, which was an application initially suggested by Sutton (1969).
10.2 Future Work

10.2.1 Aero-Optic Mitigation Techniques

Independently, both LEBU devices and wall cooling have proven to be effective methods of flow control for aero-optic mitigation in subsonic, compressible TBLs. The results of these experiments have also shown that the mechanisms by which both of these flow control techniques work are unique; LEBUs were found to work by altering the turbulence structure in the region downstream of the device, while wall cooling reduced density fluctuations by modifying the total temperature field across the TBL. Therefore, additional benefits may be acquired if both methods were applied in a combination LEBU/wall cooling scheme, which is illustrated in Figure 10.2. In this configuration near-aperture wall cooling over some length would reduce density fluctuations near the wall, while the LEBU device would modify outer-layer structures that otherwise might not be significantly suppressed by near aperture cooling.

It is likely that the combination of these two techniques would also be beneficial for several reasons that would be appealing to system designers. First, it is probable that the inclusion of the LEBU device will reduce the amount of energy required for wall cooling to reach a desired level of $OPD_{rms}$. Well-designed LEBUs have been shown to typically result in small net drag reductions, so the energy that could be saved by adding LEBUs to wall cooling would not be spent elsewhere overcoming device drag. Furthermore, the combination of these two flow control techniques into one system adds some redundancy to the system, meaning that if either the LEBU or wall cooling flow control should fail during operations, some level of aero-optic mitigation may still be achieved.
For practical applications, the fully three-dimensional nature of TBL turbulence, which includes some amount of spanwise velocity and temperature fluctuations, requires additional consideration. To reduce the energy cost of any aero-optic mitigation scheme involving wall cooling, the spanwise width of the cooling section, $W_{cool}$, ought to be minimized. Given that the $U$-component of velocity is much larger than the magnitude of fluctuations in the spanwise direction, it is not expected that there will be an overwhelming amount of temperature dissipation in the $z$-direction. However, $W_{cool}$ will likely need to be several times greater than $Ap$ in order to ensure that the desired aero-optic mitigation capabilities are not reduced significantly.

10.2.2 Non-Intrusive Optical Diagnostics

The success of the wall heating and re-scaling method presented in Chapter 9 has the potential to provide a novel and promising means of obtaining experimental...
wavefront data for TBLs that are typically aero-optically ‘invisible,’ such as low-Reynolds number or incompressible TBLs. One instance in which this method would prove to be useful would be in obtaining experimental wavefront data for very low Reynolds number TBLs that could be compared to the results of high-fidelity large-eddy simulations of TBL aero-optic effects, such as those of Wang & Wang (2012). Another application for this technique is the use of wall heating to obtain TBL wavefront measurements of low-speed, incompressible TBLs along with simultaneous velocity measurements, as has been done recently in a collaboration with McKeon’s group at California Institute of Technology (Saxton-Fox, et al., 2014, 2015). Using this approach, the relationship between different turbulent structures in the boundary layer structure and \( \text{OPD} \) can be carefully studied along with different density-velocity models. In addition, wall cooling could be useful for investigating the relationship between pressure and density fluctuations in the TBL, since cooling reduces the relative contribution of temperature fluctuations to density. Finally, the capabilities of non-intrusive aero-optic wavefront sensors for characterizing compressible, turbulent flow is applicable to a whole range of supersonic and hypersonic applications where intrusive measurement techniques are often not practical or possible. Using the techniques developed in this work, it should be possible to non-intrusively characterize the turbulence structure in supersonic and hypersonic turbulent flows from high-bandwidth aero-optic wavefront measurements.

10.2.3 Extension of Flow Control to Supersonic TBLs

The experiments contained in this thesis have undoubtedly demonstrated that significant reductions in \( \text{OPD}_{\text{rms}} \) can be achieved by using passive flow control; namely LEBUs and wall cooling, to modify subsonic, compressible TBLs. These results are quite
useful in the regime of subsonic flow; however it is also of interest to consider the possible relevance of the flow control techniques evaluated in this dissertation to supersonic TBL aero-optic mitigation.

Although a significant number of experimental investigations of LEBUs have been completed over the last several decades at numerous subsonic flow conditions, the author is not aware of any studies at this time that investigate the effect of LEBU devices on supersonic TBLs. Given that large-scale motions have also been found to be prevalent in supersonic TBLs (Smith, et al., 1998; Smits, 1991, and others) it is hypothesized that LEBUs would be effective at altering turbulence structure in the outer layer in a similar manner as for the subsonic TBL. However, the presence of a flat plate in the outer region of the TBL would likely lead to shock formation when the plate is locally in flow where \( M \geq 1 \). The effects of a LEBU-induced shock on aero-optic performance are not known at this time, but if the shock that is formed is relatively planar and stationary it may not pose a significant problem for aero-optic applications. It may also be possible to use adaptive-optic controls, such as fast steering mirrors, to help mitigate the effects of any potential shock formation. Overall, LEBUs appear to be promising candidates for turbulence modification in supersonic boundary layers, and further investigations of their performance in this flow regime would be highly valuable.

Wall cooling is also a promising candidate for extension to the supersonic regime. The assumptions used to derive the wall cooling aero-optic model, namely the temperature-velocity relations of the ‘Extended’ Strong Reynolds Analogy, have already been shown to work well for both at subsonic and supersonic TBLs (Debiève, et al., 1997; Gatski and Erlebacher, 2002; Pirozzoli, et al., 2004). Gordeyev, et al. (2015)
demonstrated analytically that the full wall cooling model can be easily extended for supersonic freestream velocities. For partial cooling, Gran, et al. (1974) already previously demonstrated that thermal sub-layer effects are present and similar for partial cooling in supersonic TBLs. Therefore, the partial wall cooling aero-optic model is also expected to extend to the supersonic regime. One limitation of cooling for supersonic TBL aero-optic flow control, however, is found by calculating the value of the optimal cooling temperature.

![Figure 10.3. Optimal wall cooling temperatures as a function of Mach number, computed from equation (4.26) using experimentally-obtained model constants, for different altitudes (1976 U. S. Standard Atmosphere).](image)

Rearranging equation (4.26), and using the experimentally measured model constants $C_1$ and $C_2$ presented in Chapter 8, it can be shown that for full wall cooling,

$$T_{\text{Optimal}} \doteq T_{aw} - 0.4T_\infty M_\infty^2.$$ (10.4)
From equation (4.17), the adiabatic wall temperature can also be written as a function of the recovery factor $r$ and the ratio of the total and static freestream temperatures:

$$\frac{T_{w}}{T_{\infty}} = r\left(\frac{T_{o,\infty}}{T_{\infty}} - 1\right) + 1.$$  \hspace{1cm} (10.5)

Assuming isentropic flow, $T_{o,\infty}/T_{\infty} = 1 + (\gamma - 1)M_{\infty}^2/2$, equation (10.4) can then be re-written as

$$\frac{T_{\text{Optimal}}}{T_{\infty}} = 1 + \left(\frac{r(\gamma - 1)}{2} - 0.4\right)M_{\infty}^2.$$  \hspace{1cm} (10.6)

![Figure 10.4](image.png)

**Figure 10.4.** Estimates of the minimum value of $OPD_{rms}$ from equation (10.1) that can be achieved when $T_w$ is constrained so that $T_w \geq T_{\text{Optimal}}$; for full wall cooling with $T_{\infty} = 229 \text{ K}$ (conditions at approximately 30,000 ft.)

Figure 10.3 presents estimates of $T_{\text{Optimal}}$ computed from equation (10.6) for several different freestream air temperatures that were estimated from the 1976 U. S. Standard Atmosphere. Note that in all cases $T_{\text{Optimal}}$ becomes equal to absolute zero (0 K).
at about $M_\infty = 2.13$, which sets the theoretical maximum speed at which the optimal wall cooling temperature is reachable. Since cooling the wall to 0 K is not feasible in practical applications, however, it is more practical to consider the temperatures of common cryogenic materials, such as liquid Oxygen (LO$_2$), liquid Nitrogen (LN$_2$), and liquid Hydrogen (LH$_2$), which boil at approximately 90 K, 77 K, and 20K, respectively (Brown & Stein, 2015). These temperatures are also plotted in Figure 10.3. Comparing $T_{\text{Optimal}}$ to these more realistic estimates of the minimum achievable wall cooling temperature shows that for LH$_2$ optimal cooling temperatures could be achieved for $M_\infty < 2$, and for LO$_2$ and LN$_2$ this range of Mach numbers shrinks to approximately $M_\infty < 1.6$.

This does not mean, however, that wall cooling would not be an effective means of aero-optic mitigation for supersonic TBLs with $M_\infty > 2$. In fact, the aero-optic model (again, using $C_1$ and $C_2$ obtained from experiments) predicts that even cooling the wall to temperatures that are greater than $T_{\text{Optimal}}$ will result in considerable reductions of aero-optic aberrations. Estimates of the minimum value of $OPD_{\text{rms}}$ that can be achieved for wall cooling temperatures greater than a set minimum value (i.e. $T_w \geq T_{\text{Optimal}}$) are presented in Figure 10.4 for LO$_2$, LN$_2$, and LH$_2$. The results show that even up to Mach 5, optical aberrations can be reduced by between 40 – 45% for full wall cooling to temperatures on the order of 20 – 100 K. It is also expected that for partial wall cooling, the amount of aero-optic be further reduced as $R_{\text{cool}}$ is decreased, as was shown for subsonic TBLs. While this is somewhat short of the roughly 60% reduction that was measured for the subsonic TBL, it is still a significant improvement. Therefore wall cooling should still be considered as a good candidate for use as an aero-optic mitigation technique in supersonic TBLs. It should also be noted that if LEBUs are found to be a
good candidate for aero-optic mitigation in supersonic TBLs, it is expected that the combined LEBU/wall cooling flow control scheme proposed earlier would also work well in the supersonic regime.

10.3 Concluding Remarks

In summary, this dissertation has demonstrated that optical aberrations caused by subsonic, compressible turbulent boundary layer can be significantly reduced through the use of two passive flow control techniques:

- Large-eddy break-up devices (LEBUs), and
- Boundary layer wall cooling.

Experiments showed that aero-optic aberrations caused by the TBL can be easily and significantly suppressed by up to 35% over up to $10\delta$ in the streamwise direction using Large Eddy Break-Up devices. The mechanical simplicity of LEBU devices, along with the experimental demonstration of their aero-optic mitigation abilities in this work, makes them very good candidates for incorporation into the design of airborne directed energy systems that must contend with aberrations caused by subsonic, compressible TBLs. Wall cooling was found to reduce optical aberrations by as much as 60% when applied over the full TBL development section, and cooling over partial lengths was also found to be effective, although less so than for full cooling. Wall heating was also found to be useful for passively amplifying wavefront distortions in the TBL, which is highly useful for obtaining good quality experimental data for validating computational investigations of TBL aero-optic effects. In total, the results presented in this dissertation highlight the effectiveness of passive flow control for managing aero-optic aberrations caused by
compressible subsonic TBLs, whether the application is for directed energy, line-of-sight communications, or for control of aero-optic effects in wind tunnel testing.
Cress (2010) and Wittich, et al. (2007) have previously developed a scaling model for double boundary layer (DBL) wavefront measurements of symmetric, un-modified turbulent boundary layers. This scaling model, which is used to find wavefront statistics for only a single boundary layer (SBL), assumes that TBLs on opposite walls of the wind-tunnel are statistically independent. It follows from this assumption that the squares of the $OPD_{\text{rms}}$ and wavefront deflection angle spectra, $\hat{\theta}(f)$, obtained from DBL wavefront measurements are composed of the sum of the squares of these statistical quantities from the individual TBLs:

\[
\left( OPD_{\text{rms}}^{\text{DBL}} \right)^2 = \left( OPD_{\text{rms}}^{\text{SBL,1}} \right)^2 + \left( OPD_{\text{rms}}^{\text{SBL,2}} \right)^2, \tag{A.1}
\]

\[
\left( \hat{\theta}_{\text{DBL}}(f) \right)^2 = \left( \hat{\theta}_{\text{SBL,1}}(f) \right)^2 + \left( \hat{\theta}_{\text{SBL,2}}(f) \right)^2, \tag{A.2}
\]

where SBL, 1 and 2 denote the individual upper and lower wall TBLs. If these opposite wall TBLs are statistically identical, then equations (A.1) and (A.2) simplify further so that the SBL wavefront statistics for either TBL can be expressed as

\[
OPD_{\text{rms}}^{\text{SBL}} = \frac{OPD_{\text{rms}}^{\text{DBL}}}{\sqrt{2}}, \tag{A.3}
\]
\[
\hat{\theta}_{\text{SBL}}(f) = \frac{\hat{\theta}_{\text{DBL}}(f)}{\sqrt{2}}. \tag{A.4}
\]

Comparisons between SBL measured wavefront statistics, obtained using the configurations shown in Figure A.1a and Figure A.1b, and SBL wavefront statistics scaled from DBL wavefront measurements via equations (A.3) and (A.4) have been shown to be in good agreement (Cress, 2010; Wittich, et al., 2007). This result has validated the use of this scaling model to obtain SBL wavefront statistics from DBL wavefront measurements for use in other aero-optic studies of compressible TBLs (Gordeyev, Smith, Cress, & Jumper, 2014).

A.1 Extracting Modified TBL Statistics from SMBL Wavefront Measurements

If TBLs 1 and 2 are not statistically identical, then the scaling in equations (A.3) and (A.4) will not hold. Consider the case in which one TBL is modified by some arbitrary means, and the second TBL is left un-modified. Then, equations (A.1) and (A.2) become

\[
\left(\text{OPD}_{\text{rms}}^{\text{DBL}}\right)^2 = \left(\text{OPD}_{\text{rms}}^{\text{Modified TBL}}\right)^2 + \left(\text{OPD}_{\text{rms}}^{\text{Un-Modified TBL}}\right)^2, \tag{A.5}
\]

\[
\left(\hat{\theta}_{\text{DBL}}(f)\right)^2 = \left(\hat{\theta}_{\text{Modified TBL}}(f)\right)^2 + \left(\hat{\theta}_{\text{Un-Modified TBL}}(f)\right)^2. \tag{A.6}
\]

From (A.5) and (A.6), the SBL statistics for the single modified TBL (SMBL) are defined as

\[
\text{OPD}_{\text{rms}}^{\text{Modified TBL}} = \sqrt{\left(\text{OPD}_{\text{rms}}^{\text{DBL}}\right)^2 - \left(\text{OPD}_{\text{rms}}^{\text{Un-Modified TBL}}\right)^2}, \tag{A.7}
\]

\[
\hat{\theta}_{\text{Modified TBL}}(f) = \sqrt{\left(\hat{\theta}_{\text{DBL}}(f)\right)^2 - \left(\hat{\theta}_{\text{Un-Modified TBL}}(f)\right)^2}. \tag{A.8}
\]
Figure A.1. Boundary layer test section configured for Malley probe wavefront measurements: a) SBL configuration (Gordyev, et al. 2003, Buckner, et al. 2005), b) SBL optical insert configuration (Cress 2010), and c) DBL Malley probe measurement configuration (Wittich, et al. 2007, Cress 2010).
Note that this relation is only useful for obtaining SBL statistics for the modified TBL if the un-modified TBL statistics are known. If a ‘Baseline’ DBL wavefront measurement is obtained for the un-modified upper and lower TBLs as shown in Figure A.2, then by equations (A.3) and (A.4), their SBL scaled statistics can be substituted into (A.7) and (A.8):

\[ \text{OPD}_{\text{rms, Modified TBL}} = \sqrt{\left(\text{OPD}_{\text{rms, DBL}}\right)^2 - \frac{1}{2} \left(\text{OPD}_{\text{rms, Baseline DBL}}\right)^2}, \]  

(A.9)

\[ \hat{\theta}_{\text{Modified TBL}}(f) = \sqrt{\left(\hat{\theta}_{\text{DBL}}(f)\right)^2 - \frac{1}{2} \left(\hat{\theta}_{\text{Baseline DBL}}(f)\right)^2}. \]  

(A.10)

Using these relations, SBL wavefront statistics for modified TBLs can be extracted, so long as 1) the opposite wall TBLs remain statistically independent, and 2) Baseline DBL measurements of the symmetric, independent un-modified TBLs are obtained for use in (A.9) and (A.10). The validity of this equation can be checked using the SBL wavefront measurement techniques shown in Figure A.1, or by obtaining DBL wavefront measurements of *symmetrically modified* TBLs and scaling them using equations (A.3) and (A.4).

A.2 Examination of SBL Scaling for LEBU-Modified TBLs

This scaling has already been shown by Cress (2010) and Wittich, et al. (2007) to work well for scaling DBL wavefront measurements of adiabatic, canonical TBLs. Cress (2010) also showed that this technique was useful for recovering SBL wavefront statistics from DBL Malley probe measurements of a single heated boundary layer and a single adiabatic wall TBL for a range of Mach numbers and wall temperatures.
Figure A.2. a) Baseline, b) Single modified TBL, and c) Double-modified TBL configurations for DBL aero-optic wavefront measurements.
For attached TBLs in which the turbulence structure has been altered by a LEBU device, however, this scaling technique has not been previously validated. To do so, wavefront data were obtained for both a single LEBU-modified TBL and a double modified boundary layer, as pictured in Figure A.2b and Figure A.2c, respectively. Results of these measurements for the \( l = 1.6\delta, h = 0.6\delta \) single LEBU device are shown in Figure A.3 for four different streamwise locations distributed across the measurement length. The good agreement between deflection angle spectra obtained from the double modified boundary layer (DMBL) measurements and single modified boundary layer (SMBL) measurements indicates that the SBL scaling technique outlined in Appendix A is valid for studying LEBU modified TBLs. In light of this good agreement, the rest of the LEBU devices characterized in this study using high-bandwidth wavefront sensors were studied with only a single modified TBL, as in Figure A.2b. This choice was made primarily in order to reduce the complexity of the experimental setup.
Figure A.3. Deflection angle amplitude spectra at several locations downstream of the $l/\delta = 1.6$, $h/\delta = 0.6$ single LEBU device obtained from both single and double modified TBLs. (Pages 297 – 298)
(a) $x/\delta = 1.3$

(b) $x/\delta = 3.8$
APPENDIX B:
LARGE APERTURES AND VARIATIONS IN BOUNDARY LAYER STATISTICS

To address the effect of boundary layer growth on Aero-optic wavefront measurements, streamwise variation in boundary layer thickness $\delta$ can be estimated by using the Prandtl $1/7$ power law, $\delta/x = 0.37 \text{Re}^{-1/5}$, for zero-pressure gradient turbulent boundary layers on a flat plate (Schlichting, 1979). Defining also the Reynolds number based on the boundary layer thickness $\delta$, this equation can be re-cast as $\text{Re}_\delta = 0.37 \text{Re}^{4/5}$.

Figure B.1. An illustration demonstrating the growth of the turbulent boundary layer thickness over an optical aperture with its upstream edge located at $x = x_0$. 
Over an aperture of streamwise length $Ap$, the boundary layer thickness changes over the aperture by some amount $\Delta \delta$, schematically shown in Figure B.1, which is proportional to $\text{Re}_{x+Ap}^{4/5} - \text{Re}_x^{4/5}$. The ratio of change in the boundary-layer thickness over the aperture to the thickness at the beginning of the aperture may then be expressed as

$$\frac{\Delta \delta(\text{Re}_{Ap})}{\delta} = \frac{0.37(\text{Re}_{x+Ap}^{4/5} - \text{Re}_x^{4/5})}{\text{Re}_\delta} = \frac{\text{Re}_{x+Ap}^{4/5} - \text{Re}_x^{4/5}}{\text{Re}_x^{4/5}}. \quad (B.1)$$

And, since $\text{Re}_{x+Ap} = \text{Re}_x + \text{Re}_Ap$,

$$\frac{\Delta \delta(\text{Re}_{Ap})}{\delta} = (\text{Re}_x + \text{Re}_Ap)^{4/5} - 1 = \left(1 + \frac{\text{Re}_Ap}{\text{Re}_x}\right)^{4/5} - 1. \quad (B.2)$$

Substituting for $\text{Re}_x$ from the equation $\text{Re}_\delta = 0.37 \text{ Re}_x^{4/5}$ the series can be expanded to show

$$\frac{\Delta \delta(\text{Re}_{Ap})}{\delta} = \left(1 + \frac{\text{Re}_Ap}{3.47 \text{ Re}_\delta^{5/4}}\right)^{4/5} - 1 = 0.23 \frac{\text{Re}_Ap}{\text{Re}_\delta^{5/4}} + H.O.T. \quad (B.3)$$

If the first term in the expansion $0.23\text{Re}_Ap / (\text{Re}_\delta^{5/4}) \ll 1$, then the variation in the boundary layer thickness over the aperture is negligible. For reported experiments, the maximum aperture was $Ap = 10.2$ cm and minimum $\text{Re}_\delta$ was $\text{Re}_\delta = 2.2 \times 10^5$, and the corresponding change in $\delta$ over the aperture was found to be approximately 4%. Therefore in this work, the streamwise variation of the boundary layer thickness and related statistics over the aperture can be neglected, and it can be assumed that the streamwise direction is homogeneous over the length of the apertures in this study. For flows where there is more rapid streamwise variation of boundary layer statistics, the validity of the frozen-flow assumption will need to be revisited.
APPENDIX C:
AERO-OPTIC MODEL FOR A BOUNDARY LAYER WITH NON-ADIABATIC WALL BOUNDARY CONDITIONS

This appendix gives a detailed outline of the derivation of the aero-optic model for full wall cooling, as derived in Gordeyev, et al. (2015). This work is based off of the original derivation of an aero-optic model for wall temperature effects presented by Cress (2010), which used the temperature-velocity relationship from the Extended Strong Reynolds Analogy (ESRA) to approximate density fluctuations in the TBL. Since this initial pioneering work, a number of additional components, including the effect of pressure fluctuations and the TBL stress integral distribution, have been included in the development of this analytical model equation for $OPD_{rms}$ as a function of wall temperature.

C.1 ESRA Temperature-Velocity Relations

Recall the Extended Strong Reynolds Analogy temperature-velocity relations (also referred to as the Crocco-Busemann relations or the Walz equations) presented in Section 4.1.2:

$$\frac{T(y)}{T_{\infty}} = \frac{T_w}{T_{\infty}} + \frac{T_{aw} - T_w}{T_{\infty}} \left( \frac{U(y)}{U_{\infty}} \right) - r \frac{\gamma - 1}{2} M_{aw}^2 \left( \frac{U(y)}{U_{\infty}} \right)^2, \quad (C.1)$$
In these equations, $T$ is the static temperature, $T_w$ is the wall temperature, $T_{aw}$ is the adiabatic wall temperature, $U$ and $u_{rms}$ are the mean and RMS streamwise velocity component, $U_{\infty}$ and $M_\infty$ are the freestream velocity and Mach number, and $r$ is the recovery factor, which is defined as

$$r = \frac{T_{aw} - T_w}{T_{aw} - T_{\infty}},$$ (C.3)

where $T_o$ is the total temperature. This factor indicates the increase of temperature seen at the wall from the conversion of kinetic energy to heat as velocity is slowed across the turbulent boundary layer, from $U_{\infty}$ in the freestream to zero at the wall. Although it is functionally dependent on Prandtl number, in turbulent boundary layers with air as the active medium, typically $r = 0.89$ (Schlichting & Gersten, 2000; Smits & Dussauge, 2006). Note that these relations are typically written in Favre-averaged form, but since they are being applied to the derivation of an aero-optic model for wall temperature effects in subsonic, compressible TBLs, and there is less than a 1.5% difference between Reynolds and Favre averaging for $M_{\infty} < 3$ (Smits & Dussauge, 1996), equations (C.1) and (C.2) are written in Reynolds-averaged form.

While Duan, et al. (2010) found that the ESRA give comparable results for $y < 0.5\delta$ their numerical results suggested that in general, equation (C.2) can be written in the following form,

$$\frac{T_{rms}(y)}{T_{\infty}} = \frac{T_{aw} - T_{w}}{T_{aw} - T_{\infty}} \left( \frac{u_{rms}(y)}{U_{\infty}} \right) - r(y - 1)M_\infty^2 \left( \frac{U(y)u_{rms}(y)}{U_{\infty}^2} \right),$$ (C.4)
where $A(y)$ takes into account the stress integral distribution in the boundary layer (Smits & Dussauge, 1996).

C.2 Model for Non-Adiabatic TBL Aero-Optic Distortions

From Equation (2.3) it follows that if the density fluctuations and their correlation lengths across the boundary layer are known, the optical distortions can be calculated. From the ideal gas law, $p = \rho RT$, the density fluctuations, $\rho'$, are related to the pressure, $p'$, and the temperature, $T'$, fluctuations and, in the case of small fluctuations, can be written as,

$$\frac{\rho'}{\rho(y)} = \frac{p'}{p(y)} - \frac{T'}{T(y)}. \quad \text{(C.5)}$$

While the pressure fluctuations in adiabatic boundary layers have been shown to be several times smaller than the temperature fluctuations (Smits and Dussauge, 1996; Spina, et al, 1994) for cooled walls the temperature fluctuations are smaller with respect to the adiabatic case. Therefore the magnitudes of the pressure and temperature fluctuations can be comparable with one another.

Combining equations (C.1) and (C.4), and letting $\Delta T = T_w - T_{aw}$, the following expression for profiles of $T_{\text{rms}}$ can be derived,

$$\left( \frac{T_{\text{rms}}(y)}{T_\infty} \right)^2 = \left( \frac{u_{\text{rms}}(y)}{U_\infty} \right)^2 \times \left[ \left( \frac{\Delta T}{T_\infty} \right)^2 + 2A(y)(y-1)M_s^2 \frac{\Delta T}{T_\infty} \frac{U(y)}{U_\infty} + A(y)(y-1)M_s^2 \frac{U(y)}{U_\infty} \right]^2. \quad \text{(C.6)}$$
Equation (C.6) can then be combined with the equation of state and used to compute the density fluctuations. Assuming that there is no correlation between the pressure and temperature, equation (C.5) can be re-written in terms of root-mean-square quantities,

\[
\left( \frac{\rho_{\text{rms}}}{\rho(y)} \right)^2 = \left( \frac{T_{\text{rms}}}{T(y)} \right)^2 + \left( \frac{p_{\text{rms}}}{P_\infty} \right)^2.
\] (C.7)

In this model, it is assumed that locally, pressure is constant across the TBL. The local density is then computed easily found from the equation of state,

\[
\rho(y) = \frac{P_c}{RT(y)} = \rho_\infty \left( \frac{T_\infty}{T(y)} \right),
\] (C.8)

where the mean temperature profile can be computed from Equation (C.1):

\[
\frac{T(y)}{T_\infty} = 1 + r \frac{(\gamma - 1)}{2} M_\infty^2 \left( 1 - \left[ \frac{U(y)}{U_\infty} \right]^2 \right) + \frac{\Delta T}{T_\infty} \left( 1 - \frac{U(y)}{U_\infty} \right).
\] (C.9)

For non-adiabatic boundary layers, the fluctuating velocity component, compensated for density changes near the wall using the Van Driest transformation,

\[
\frac{\bar{u}_{\text{rms}}}{u_*} \sqrt{\frac{\rho(y)}{\rho_w}},
\]

was found to be mostly unchanged over a wide range of Mach numbers (Spina, et al., 1994; Guarini, et al., 2000; Duan, et al., 2010). Therefore, a fluctuating velocity profile from the adiabatic boundary layer Guarini, et al. (2000) was used:
\[
\frac{u_{\text{rms}}(y)}{u_c} \sqrt{\frac{\rho(y)}{\rho_w}} = \frac{u_{\text{rms}}(y)}{U_\infty \sqrt{C_f/2}} \sqrt{\frac{\rho(y)}{\rho_\infty}} = \frac{u_{\text{rms}}(y)}{U_\infty \sqrt{C_f/2}} \sqrt{\frac{\rho(y)}{\rho_\infty}} = \frac{u_{\text{rms}}(y)}{U_\infty \sqrt{C_f/2}} \sqrt{\frac{T_\infty}{T(y)}} = g\left(\frac{y}{\delta}\right)
\]

(C.10)

The mean velocity profile was assumed to be independent of the wall temperature,

\[
\frac{U(y)}{U_\infty} = f\left(\frac{y}{\delta}\right),
\]

(C.11)

and finally, the fluctuating pressure profile from Guarini, et al. (2000) was used:

\[
\frac{P_{\text{rms}}}{(\rho_w u_c^2)} = \frac{P_{\text{rms}}}{\rho_w U_\infty^2 (C_f/2)(\rho_\infty/\rho_w)} = \frac{P_{\text{rms}}}{\rho_\infty U_\infty^2 (C_f/2)} = h\left(\frac{y}{\delta}\right)
\]

(C.12)

The functional forms of \(f(y), g(y),\) and \(h(y)\) were shown previously in Figure 8.12.

Substituting Equation (C.6) into the simplified equation of state in (C.7), and using equations (C.9) through (C.12) to rearrange the relation yields the following expression for \(\rho_{\text{rms}}:\)

\[
\left(\frac{\rho_{\text{rms}}}{\rho_\infty}\right)^2 = \left(\frac{T_\infty}{T(y)}\right)^2 \left[\left(\frac{T_{\text{rms}}}{T(y)}\right)^2 + \left(\frac{P_{\text{rms}}}{P_\infty}\right)^2\right]
\]

\[
= \left(\frac{T_{\text{rms}}}{T_\infty}\right)^2 \left(\frac{T_\infty}{T(y)}\right)^4 + \left(\frac{T_\infty}{T_\infty}\right)^2 \left(\frac{P_{\text{rms}}}{P_\infty}\right)^2
\]

\[
= \left(\frac{T_\infty}{T(y)}\right)^3 (C_f/2)g^2(y) \left[\frac{\Delta T}{T_\infty} + A(y)(y-1)M_\infty^2 f(y)\right]^2
\]

\[
+ \left(\frac{T_\infty}{T(y)}\right)^2 yM_\infty^2 (C_f/2)h(y)^2.
\]

(C.13)
Substituting equation (C.13) into equation (2.3) (Sutton’s linking equation) and integrating results in the following expression for $OPD_{rms}$:

$$OPD_{rms} = A_0 K_{GD} \rho_c \delta \sqrt{C_f} \left[ M^4 + C_1 \frac{\Delta T}{T_\infty} M^2 + C_2 \left( \frac{\Delta T}{T_\infty} \right)^2 \right]^{1/2} \quad (C.14)$$

where,

$$A_0^2 = \int_0^{\infty} \left[ (\gamma - 1) A(y) f(y) g(y) \right] \left( \frac{T_\infty}{T(y)} \right)^3 \Lambda(y) dy \quad (C.15a)$$

$$+ \gamma^2 \left( \frac{C_f}{2} \right) \int_0^{\infty} g^2(y) h^2(y) \left( \frac{T_\infty}{T(y)} \right)^2 \Lambda(y) dy,$$

$$C_1 = \frac{2(\gamma - 1)}{A_0^2} \int_0^{\infty} A(y) f(y) g^2(y) \left( \frac{T_\infty}{T(y)} \right)^3 \Lambda(y) dy,$$  \text{and} 

$$C_2 = \frac{1}{A_0^2} \int_0^{\infty} g^2(y) \left( \frac{T_\infty}{T(y)} \right)^3 \Lambda(y) dy. \quad (C.15c)$$

Note that for the adiabatic wall boundary layer, equation (C.14) reduces to the experimentally proven scaling relation, $OPD_{rms} \sim \rho_c \delta \sqrt{C_f M^2}$ (Gordeyev, et al., 2014; Gordeyev, et al. 2012).

By squaring both sides of equation (C.14), it is evident that $(OPD_{rms})^2$ is a quadratic function of $\Delta T$ that reaches a minimum at

$$\frac{\Delta T_{min}}{T_\infty M^2} = - \frac{C_1}{2C_2}. \quad (C.16)$$

The value of $OPD_{rms}$ at this corresponding minimum location is then expressed as,
\[
\frac{OPD_{\text{rms}}(\Delta T_{\text{min}})}{OPD_{\text{rms}}(\Delta T = 0)} = \sqrt{1 - \frac{1}{4} \left( \frac{C_1}{C_2} \right)^2}.
\] (C.17)

Note that for positive \(\Delta T\) (i.e. wall heating), equation (C.14) may be rearranged so that

\[
OPD_{\text{rms}} = A_0 K_{GbD} \rho_{\infty} \delta \sqrt{C_f} \left( M_x^2 + D_1 \frac{\Delta T}{T_x} \right)
\times \left[ 1 + \frac{D_2}{2} \left( \frac{\Delta T}{T_x} \right)^2 \right] + H.O.T.,
\] (C.18)

where \(D_1 = C_1/2\) and \(D_2 = C_2 - (C_1/2)^2\). Cress (2010) and Gordeyev, et al. (2015) showed that the second term in the square brackets in equation (C.18) is less than 0.08 for all of the wall heating cases considered in these studies. Therefore, equation (C.14) simplifies to

\[
OPD_{\text{rms}} = A_0 K_{GbD} \rho_{\infty} \delta \sqrt{C_f} \left( M_x^2 + D_1 \frac{\Delta T}{T_x} \right)
\] (C.19)

for \(\Delta T > 0\).
REFERENCES


