ROBUSTNESS AND EFFICIENCY OF PLANAR BIPED WALKING ROBOTS

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David Christopher Post

James P. Schmiedeler, Director

Graduate Program in Aerospace and Mechanical Engineering
Notre Dame, Indiana
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Abstract
by
David Christopher Post

Legged robots are desirable for many applications, especially in man-made environments where having legs is a distinct advantage over having wheels. The legged robots in these applications must be both robust to disturbances and energetically efficient, and achieving these characteristics represents two of the most pressing challenges within the field. This work seeks to experimentally demonstrate that the use of curved feet under hybrid zero dynamics (HZD)-based control offers efficiency benefits and to make HZD-based controllers more robust to velocity disturbances. These aims were investigated using the biped robot ERNIE, which was transitioned from treadmill walking to continuous overground walking.

Efficiency improvements of curved feet over point feet were demonstrated by improving a previous model to appropriately account for curved foot impacts. Curved foot gaits, in general, had decreased specific resistance, a measure of energy consumption per distance traveled, and smaller joint errors compared to point-foot gaits at similar speeds. Robustness improvements were made by developing new controllers to reject velocity disturbances in experiment. An orbit-stabilizing control approach was successful at rejecting impulsive angular velocity disturbances applied to a simplified system in simulation, but control algorithm was too complex to implement in hardware. However, heuristic rules for disturbance rejection in hardware were developed from implementing the dominant control responses to disturbance from
the orbit-stabilizing controller. These control actions modify the trajectories of the torso and swing leg in response to a deviation from desired forward hip velocity. The heuristic-based control had increased efficiency (lower specific resistance) compared to the HZD-based control, smoothing the average step velocity around the room by reducing the accelerations and decelerations present with the original controller. Additionally, in response to both acceleration and deceleration disturbances, the heuristic-based controller returned to within 10% of the desired average step velocity in at most half the steps as with the HZD-based controller.
To Kristi and my family--I could not have done it without you.
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CHAPTER 1

INTRODUCTION

1.1 Motivation

Mobile robots, specifically legged robots, are desirable for many different applications. To consider a legged robot for an application it must have some advantage over a wheeled robot. Wheeled robots in general are more stable and efficient than legged robots. However, in man-made environments such as a stairwell or in uneven terrain, having legs is a distinct advantage over having wheels. The service industry is an exciting sector for legged robots, with possible applications including assisting the elderly in their homes or delivering the mail in an office building. Legged robots could also be used in disaster areas or dangerous environments. Additionally, humans are legged locomotors, and the study of robotic legged locomotion has the potential to provide insight into the dynamics of human locomotion. This insight can be used for the design of prostheses for amputees and for assisting stroke patients with walking rehabilitation. Although these applications are varied in nature, the legged robot in each case must be both robust and efficient, and achieving these characteristics represents two of the most pressing challenges within the field.

A robust control strategy that ensures stability is the first challenge in actuated legged robots, specifically bipeds. The stability metric used guides which control scheme is ultimately selected. The two main stability metrics are derived from the Zero Moment Point (ZMP) and limit cycle walking. The ZMP is used with fully actuated humanoid robots and is the point on the ground where the total moment
generated due to gravity and inertia equals zero \cite{63}. Postural stability is said to be maintained if the ZMP is constrained to lie within the convex hull of the foot-support area \cite{13}. Limit cycle walking is formally defined as “a nominally periodic sequence of steps that is stable as a whole but not locally stable at every instant in time \cite{25}.” A stable limit cycle corresponds to an attracting fixed point of the Poincaré map (a common tool used in analyzing non-linear dynamics \cite{61}) of the walker’s state at a certain point in the cycle, usually just after foot impact \cite{25}. Whether using the ZMP or limit cycle analysis, the current and past approaches have limited ability to reject and recover from disturbances, such as a change in ground height or velocity impedance. Robustness to large disturbances is an ongoing research problem in the field, no matter what strategy is employed.

The second challenge is producing a walking motion that is reasonably efficient energetically. Walking is an energy intensive motion compared to wheeled motion since a biped consumes energy to maintain its posture and balance, whereas wheeled robots generally stay in contact with the ground and are stably balanced. Walking requires cyclic acceleration and deceleration of the legs to achieve periodic forward motion, which consumes more energy than maintaining a constant speed with a wheeled robot. Walking must also overcome the energy losses from foot impacts, which are not present in wheeled robots. Because of these additional energy sinks, walking robots must have efficient gaits to minimize their total energy expenditure.

These challenges create a close coupling between mechanical design and control design to accomplish efficient, stable walking. This is particularly true for foot design, which varies greatly between ZMP-type and limit cycle walkers. ZMP-type walkers have large flat feet since they create a larger convex hull in which the ZMP can lie. They also usually have actuated 2-degree-of-freedom (DOF) ankle joints. This design allows the ZMP-type walker to execute a wide range of motions, notably standing still; however, this robustness comes at the expense of additional energy consumption.
and decreased efficiency. Limit cycle walkers usually have curved feet with no ankle joint, although some have flat feet with actuated 1-DOF ankles [52]. The curved feet make the limit cycle walker more efficient, but limit the walker’s robustness by preventing it from standing still.

The hardware used in this study, ERNIE [73], is rooted in the limit cycle approach. ERNIE is a planar, underactuated biped with 5 DOF, but only 4 are actuated—each hip and each knee. ERNIE has no ankle joint, and the uncontrolled rotation of the stance leg about the foot-ground contact point is the 5th DOF, which makes ERNIE underactuated by 1 DOF. ERNIE is controlled using the hybrid zero dynamics (HZD) approach [65]. The HZD-based approach is concisely described as extending the Poincaré analysis from limit cycle walking to underactuated robots. ERNIE has interchangeable feet, allowing the study of walking gaits using both point and curved bottom feet. The curved feet can be adjusted to move the center of curvature fore or aft of the leg shank. This adjustment is referred to as ankle offset, and the effects of it were studied in [43]. However, that study only modeled the effects of curved feet during the continuous swing dynamics and relied on a point foot assumption at impact. Limit cycle walkers also use curved feet, but thus far there is no explicitly defined design equation for their shape or known optimal value for their ankle offset. However, the ability to truly design HZD-based controllers for bipeds with curved feet could allow for more energetically efficient robotic gaits and more accurate investigation of human gait.

1.2 Objectives

This work sought to experimentally demonstrate that the use of curved feet under HZD-based control offers efficiency benefits and to make HZD-based controllers more robust to velocity disturbances. Accomplishing these aims required transitioning the ERNIE hardware from treadmill walking to continuous overground walking, because
the walking on the treadmill has undesirable stabilizing effects as noted in [66] and in Section 3.5.1 herein.

Efficiency improvements from curved feet were studied by improving the curved foot model from [43] to appropriately account for curved foot impacts. With a successful walking robot, the objective was to improve robustness by rejecting velocity disturbances in experiment. A mathematically rigorous approach was studied for a simplified system in simulation. The approach was too complex to implement in hardware, but it informed the development of heuristic rules that marry the simulation results with expert knowledge of the experimental setup. These rules were instead implemented in hardware, and disturbance rejection capabilities under these rules were evaluated.

1.3 Background

1.3.1 Metric of Stability and Influence on Robot Design

At one end of the control-influenced design spectrum are fully-actuated humanoid robots. WABOT-1 [31], built in 1973 at Waseda University, is the first anthropomorphic robot to successfully walk. Modern humanoid robots include ASIMO [54], WABIAN [23], HRP-2 [29, 30], KHR-1 [33], and many others. The walking control of WABOT-1 was based on a static stability metric and was aided by its large flat feet. Since WABOT-1, the dominant stability metric for humanoids employs the ZMP. Each specific humanoid robot has its individual strategies in design and control for assisting the ZMP metric, but the unifying feature most possess is large flat feet. This design and control combination is advantageous because it allows a very large design space for gaits. Speed, step length, foot height during the swing phase (to achieve ground clearance), and other parameters can be varied to reject disturbances and permit walking up and down stairs, turning, kicking a ball, recovering from a fall,
and some running. However, these benefits come at a considerable energy expense. For example, ASIMO has 34 DOF, each with a separate servomotor. Moving from distal to proximal, each motor in an arm or leg must get successively larger to carry the distal load, resulting in the motors comprising a significant portion of the robot’s mass and large energy-consuming torque requirements. Thus, for all ASIMO’s benefits in robustness, its relatively low efficiency means that ASIMO’s current operating time is only about 1 hour [5].

The other end of the design spectrum starts with an entirely passive approach. These bipeds use a sloped surface and the associated gravity contribution to walk with no other actuation! This approach was pioneered by McGeer [44, 45] who built straight-leg and kneed walkers. Like most passive walkers, these walkers are planar—they cannot turn. They are prevented from tipping over by means of some mechanism. Commonly, this is double set of feet, spaced in the coronal plane, which is the plane that divides the walker into front and back halves and is perpendicular to the plane of the walking motion. Additionally, the straight-legged walkers need some other mechanism to prevent “foot-scuffing” mid-stride. Several strategies have been employed, including slightly retracting the feet [44] and making the walker 3D by introducing hip sway [6]. Another approach is to instead modify the walking surface to a special checkerboard-like pattern.

Since McGeer, others have built successful passive walkers, including [6], which was made from Tinkertoys. Significantly, this was done by merely tinkering with the design until it worked, not running simulations or using other design tools. Collins [8] also employed the tinkering approach to build a more advanced version of a passive walker. This particular prototype has knees and specially shaped feet to achieve the side-to-side motion needed to prevent foot-scuffing. The prototype also has a compliant heel to restore some of the energy dissipated with each foot impact and counter-swinging arms to aid balance. The Cornell Efficient biped [7] is essentially
a powered version of Collins’ passive machine. With minimal actuation, the biped is able to walk on level ground, no longer needing the ground slope. The Cornell Ranger [53] takes the design to the extreme and managed to walk a record 65.24 km in 30 hr 49 min, without being touched, before its batteries died and it fell. This is significantly more efficient than ASIMO (by almost an order of magnitude), but definitely not as robust. Delft University has also built a series of minimally-powered curved-foot biped walkers including Mike and Denise [25]. A more recent design is Flame [24], a 3D walker, which uses ankle springs to mimic the curved foot motion, while still possessing a relatively flat foot overall and conforming to the idea of minimal actuation.

Moving up in complexity from minimally actuated walkers are underactuated bipeds. An underactuated biped has one or more DOF that is not actuated. An important class of walking bipeds underactuated by 1 DOF are ones that make point contact between the robot and ground. Modern examples include RABBIT [3], MABEL [58], and ERNIE [73], and like passive walkers, they are planar. These robots have stabilizing booms to prevent tipping in the coronal plane. RABBIT and MABEL were both able to walk and run, so they are underactuated by 1 DOF while walking and by 3 DOF while in flight [16, 47]. ERNIE is the experimental platform used for this work and has walked using both point and curved feet. With either set of feet, there is still underactuation by 1 DOF.

The dominant stability metric for the passive, minimally actuated, and underactuated robots is limit cycle walking. As mentioned previously, stability is determined from the Poincaré map of a single point of a gait cycle (usually instantaneously following foot impact). If the Poincaré map of this state is the state itself, $P(x^*) = x^*$, then a limit-cycle of the walker has been found. By slightly perturbing the limit cycle state and observing the mapped results, the Jacobian of the limit cycle is obtained. If the eigenvalues of this Jacobian are all of magnitude less than one, the limit cy-
cle is stable. If the largest eigenvalue has a magnitude of 1, the cycle is neutrally stable. Otherwise the cycle is unstable. Thus, the desired walking gait is achieved by finding a stable fixed point of the associated Poincaré map. A robot whose gaits are determined to be stable using Poincaré maps may not be statically stable at any instant, whereas the elementary ZMP approach requires significant time periods of static stability to perform many of its walking features.

Modeling limit cycle walking for biped walkers has been investigated by many [14, 26, 44], but Garcia et al. [11] developed the simplest walking model, which is a common starting point for many subsequent studies. The walker has a point mass at the hip and infinitesimal masses at the feet, employing point contact rather than the curved feet used in most of the passive walker prototypes. Kuo [35, 36] added an impulsive toe push-off and a hip spring to the model, which allow simulated walking on flat ground. Adamczyk’s simple model [1] adds curved feet to Kuo’s model. Wisse takes a different approach by adding a torsional spring at the pivot point, thought to mimic the flat-foot-with-ankle-springs motion of his most current prototypes [71].

For these passive and minimally actuated walkers, curved feet are important because they smooth the hip trajectory and expand the initial conditions that result in walking [70]. From human motion studies [12, 32, 55], it is expected that reducing the vertical displacement of a torso’s center of mass (CoM) during walking will reduce power consumption. Consistently, a curved foot creates less vertical displacement of the CoM compared to a point or flat foot. Hansen and Childress [20] showed that the complex motion of the human foot-ankle complex behaves much like a rigid foot with a circular radius, which suggests that curved feet could help a biped walker approximate human motion. Additionally, for kneed walkers, a forward ankle offset, the perpendicular distance between the leg’s shank and the foot’s center of curvature, keeps the knee locked for more of the gait cycle, allowing longer stride lengths and possible energy savings [43]. In [69], a curved-foot walker matches the step length
and approximate velocity of a point-foot walker using a shallower ground slope, showing that passive walkers with curved feet require less energy than point-foot walkers. Finally, in [1], a simple model shows that curved feet reduce the directional change in CoM velocity between the beginning and end of a step, which directly corresponds to reduced energy consumption. For all of these reasons, curved feet with an ankle offset are thought to create energy savings in biped walking.

1.3.2 Control of Underactuated Bipeds

1.3.2.1 Hybrid Zero Dynamics Control

The control strategy used for RABBIT, MABEL, and ERNIE is based on HZD [65]. HZD builds upon the non-linear analysis tool of zero dynamics [27] by examining systems with impact events. HZD analyzes the stability of a reduced-order system in a manner that guarantees stability of the full system. Westervelt’s [65] work and the preceding work by Grizzle [15] accomplish this by extending the use of Poincaré maps to underactuated robots. As originally developed, this approach required point-foot contact and rigid transmissions, which both RABBIT and ERNIE feature. HZD-based control has been successfully expanded to compliant drive-trains with MABEL [58], as well as to point-foot bipeds with parallel knee compliance with ERNIE [73]. HZD-based walking controllers have also been developed for point-foot biped running [59] and recent theoretical work has shown that HZD-based control can also be implemented on three dimensional, point-foot bipeds with two degrees of underactuation [4].

Currently, HZD-based control has only been formally developed for point-foot bipeds, and the point-foot controller has been used to achieve stable walking, with some difficulty, for curved foot bipeds [43]. Also, Grizzle et. al. have even demonstrated walking and running with passive feet in sneakers on the biped MABEL using a point-foot controller [18]. There is some evidence that human gait can be approx-
imated using an HZD-based or similar control strategy if feet are included in the model [37, 60]. The addition of feet can be accomplished by modeling the foot/ankle complex as a rigid circular arc fixed to the shank [22]. For humans, it is estimated that without feet, the energy required to walk would be approximately doubled [1]. For the five-link biped ERNIE, when its point feet were replaced with curved feet, the experimental energy savings in treadmill walking was approximately 60%, even though the process of designing the gaits did not fully consider the foot curvature [43]. Previous work applying HZD-based control to curved foot bipeds has either been robot-specific [34] or incomplete [60].

1.3.2.2 Limitations of HZD-based Control

In addition to not handling curved feet, HZD-based control assumes that a controller can stabilize the robot state to exactly the zero dynamics trajectory. This has been proven in theory, but in practice it is very difficult to do with real-world issues such as backlash, torque limits, sensor noise, unmodeled non-instantaneous dual-support phases, and noisy velocity observations. This can lead to errors in trajectory tracking and more importantly, to subsequent errors in velocity tracking. Since HZD-based control is only position-based, there is no mechanism to correct velocity errors from the zero dynamics orbit. For example, a gait under HZD-based control is designed to have a specific velocity. This gait could be pushed to twice that speed or slowed to half-speed, and the HZD-based control would increase or decrease the speed of the joint motions accordingly. However, the existence of the limit cycle is dependent on desired joint velocities at impact, and impacts with different velocities may not be successful. These disturbances to the gait are unaccounted for, and some additional control method is needed to correct for them. Because of these inherent issues with the control, improving the HZD-based control to reject velocity disturbances is a desirable step toward robustness.
1.3.2.3 Avenues for Improving HZD-based Control Robustness to Velocity Disturbances

An alternate set of coordinates for robot modeling may prove useful in identifying gait characteristics to exploit for disturbance rejection recovery. A potential coordinate set was developed for systems underactuated by 1 DOF, motivated by the fact that the set of reachable velocities from a nonzero velocity is not well understood, in the sense of geometric control theory [48]. The equations of motion for a system are formed using conventional position coordinates, but novel differential geometry-inspired quasi-velocity coordinates are used to express the velocity of a system in components in controllable directions and a single uncontrollable direction. An expression for the rate of change of the uncontrollable velocity is developed, and it is possible to analyze the influence of the controlled velocities on the uncontrolled velocity, with this influence being an instantaneous measure of the control authority of the system. In previous work [48, 49], the method was used to bring dynamical systems with arbitrary velocity to zero velocity, including a planar model of a rollerblader. However, the work detailed herein considers these coordinates to characterize the nonlinear coupling between links of a robot and interprets these coupling terms to produce rules for appropriate corrective control actions.

An alternative is rooted in model predictive (receding horizon) control. With this method, the current control action is obtained by solving a finite horizon open-loop optimal control problem, at each sampling instant, using the current state of the plant as the initial state. The optimization yields an optimal control sequence, the first of which is applied to the plant, and the process is repeated for the next instant [42]. This could be applied to walking robots by measuring error dynamics, which would show errors due to velocity disturbances, and computing the optimal control response for recovery. Model predictive control is successful in industrial control processes for which the dynamics are relatively slow, but has only been briefly touched
on with controlling walking robots. Most results for walking robot applications have only been demonstrated in simulation, with the major issue preventing experimental implementation being the ability to solve the optimal control problem online in real-time [2, 62, 67]. A promising avenue for overcoming this limitation first examines the simplified problem of stabilizing the pendulum on an inverted pendulum cart system to a desired orbit [56]. Using virtual constraints similar to those used in HZD-based control and a novel coordinate to measure deviation from a zero dynamic trajectory, a linearized model of the system’s transverse dynamics was stabilized with a time-varying LQR controller. With a small modification, this controller also stabilizes the original nonlinear system. This method was considered on further underactuated pendulum systems and steering of ships [57]. The method was expanded to hybrid systems with a devil-stick model [57], which paved the way for considering application to biped robots. A method for modeling the foot impacts of biped robots was accomplished in [10] using a three-link model with the same morphology as that discussed in [66], although plots of specific simulation results were never shown. Application to a two-link walker and demonstration in hardware with step disturbances was shown in [39]. The control algorithm from [39] is a receding horizon strategy, but limiting the planned footsteps to a periodic motion permits calculating the optimal control problem offline. A limited comparison to HZD was given, but did not focus on using the new control to reject velocity disturbances to maintain a periodic trajectory. Thus, this method was investigated further herein in an attempt to apply it to the five-link robot ERNIE.

1.4 Organization

Chapter 2 presents the design changes made to ERNIE to implement continuous overground walking. Chapter 3 covers the modeling and experimental results for walking with both curved and point feet under HZD-based control. Chapter 4
investigates theoretical approaches for improving control such that walking is robust to velocity disturbances. Chapter 5 details the control enhancements made in hardware to successfully reject disturbances in experiment. Finally, Chapter 6 outlines the overall conclusions and directions for future work.
CHAPTER 2

MECHANICAL DESIGN UPDATES FOR THE ERNIE HARDWARE

2.1 Motivation

The larger lab space at Notre Dame permitted an overground walking experiment setup that was not possible due to space constraints at The Ohio State University. Overground walking is desirable because walking on a treadmill has unique issues, including restorative forces from belt buckling \[72\] and coupling with control as discussed in Section \[3.5.1\]. In healthy human treadmill and overground walking, gait kinematics were similar, but joint powers in the sagittal plane for the hips and the knees were quite different \[38\]. These different power requirements are also anticipated for robots. Also, most other underactuated walking robots walk overground rather than on a treadmill. Although results among the robots are not directly compared, consistency with more typical experimental methodology is desirable.

Modifications were also made to the robot hardware. An extensive redesign of the torso was needed because the original design failed during preliminary experiments. The curved foot design was also modified because the original curved feet from \[43\] did not have an adequate arc length for many of the gaits designed with the updated curved foot model.
2.2 Center Structure

2.2.1 Overview

The ERNIE setup was adapted from a wall-mounted boom (Fig. 2.1) to a boom attached to a rotating center structure (Fig. 2.2). The new setup is similar to that used for MABEL [16]. A vertical column runs from floor to ceiling, with bearings at each end to allow rotation. The column is offset from the center of rotation so that the pivot point of the boom can be located on the spin center, preventing boom coupling dynamics from having greater influence on walking. To allow continuous walking around the room, a slip ring was used, which is an electromechanical device that allows the transmission of power and electrical signals from a stationary to a rotating structure. This was a critical component in the design and one of the first components selected; the final structure was designed around it. Further details of the design procedure are in the following sections.

2.2.2 Slip Ring

Since ERNIE was originally designed to walk on the treadmill, the electronic components (wall-mounted in Fig. 2.1) are quite large in terms of both size and weight. The robot RABBIT had electronic components mounted on the boom support structure, but the design required that RABBIT’s electrical cables be unwound after each trial. On the robot MABEL, a slip-ring was employed to prevent the cable wind-up issues present on RABBIT. Thus, a slip-ring is used on the overground walking setup of ERNIE. A slip-ring with over 100 connections would be needed to keep the ERNIE electronics stationary and was determined to be an undesirable solution because of the complexities of debugging potential connection problems. Instead, all of the electronics were moved onboard the rotating structure. A wooden platform was designed as discussed in Section 2.2.4 to hold the components and attached to the aluminum
support skeleton. Moving the electronics onboard required routing both power and signal circuits through the slip ring, and the company MOOG was selected as the slip ring vendor because of their range of available solutions. Power circuits were rated at 15 A and used to route standard 120 V AC power (3 circuits), the emergency stop signal (4 circuits), and the motor amplifier power (4 circuits). The motor amplifier power is provided via a step-down converter (kept stationary) from 120 V AC to 60 V AC. If all four motors draw large current, the load could exceed 15 A; therefore, the circuits required to provide the load were doubled. An ethernet cable provides the communication link between the PC and the dSPACE system, requiring 8 signal circuits, which were rated at 2 A each. Advised by an engineer at MOOG, the AC7036 slip ring was ultimately selected, which has 12 power circuits and 36 signal circuits. This design reduces the number of connections through the slip ring from over 100.
to 19, making identification of faulty connections easier. The mounting flange for the slip ring has a diameter of 4.5 in, and the body is a cylinder with diameter 3.07 in and length of 4.24 in. The rotating center structure was designed around these constraints.

2.2.3 Bearing Selection and Structure Design

Due to the large impact forces between ERNIE and the ground, bearings capable of high loads are needed to ensure that the structure will withstand walking experiments. The weight of ERNIE is approximately 40 lbs, and it was assumed that the impulsive forces due to its impact with the ground were three times this weight. The
weight of the electronic components and their platform was estimated to be 150 lbs, resulting in an estimated thrust load of approximately 300 lbs. Radial and moment loads are also a concern but were difficult to estimate because ERNIE and the boom are not rigidly attached to the center support structure. For example, the center of mass (CoM) of ERNIE is approximately 9.5 ft from the center line of the bearings. If rigidly attached and supported by the bearings alone, this is a moment load of 4,560 in-lbs, but the boom is pin-connected to the center support, allowing elevation changes and greatly reducing the potential moment load. To accommodate the slip ring mounting and the electronics platform, turntable bearings were considered. Available “Lazy Susan” bearings were not rated to the anticipated loads, so a custom tabletop was designed. To mount the slip ring and have diametrical clearance to attach the tabletop to a fixed support, the fixed platform needed to be at least 6 in. in diameter. To have a reasonable bearing cross section at this diameter, Kaydon slim-profile bearings were selected. Kaydon manufactures slim-profile bearings with a four-point contact design that transmits radial, axial, and moment loads without the need for the typical two bearing configuration. Table 2.1 contains the loading capacities for the KD-series bearing with a 6 in bore. The capacity for thrust load is at least an order of magnitude larger than the anticipated load. For moment loads, the capacity is larger than the maximum load from a rigidly attached robot, thus the bearing will be sufficient for the anticipated loads. With such a large factor of safety, it was assumed that this slim profile bearing could carry most of the loads of ERNIE, while the ceiling bearing’s primary purpose is reduced to keeping the column vertical.

An exploded view of the tabletop design is shown in Figure 2.3. The stationary platform of the tabletop is an aluminum extrusion with a center cut out which is press fit to the inside race of the bearing. The rotating platform is circular with a di-
ameter of 18 in and drilled with holes to attach brackets to mount the vertical column and holes to attach a wooden platform structure to carry the electronics. Two thin aluminum retaining rings were also machined to prevent the tabletop from disassembling under load. The table top is mounted on a 10 in diameter steel base platform, anchored to the ground with 1/4-20 concrete anchors. This two-piece (tabletop and base anchor) design was easier to machine and allows the entire tabletop assembly to be removed for inspection and replacement without removing the fixed anchor. The steel platform has a center bore of 3 in and a $1 \times 3$ in rectangular cutout for routing cables to the slip ring. The slip ring is mounted on the stationary platform, and a pin connects the rotating portion to the vertical column.

The vertical column is constructed of two 2040 profiles of 10 series 80/20 aluminum extrusion arranged in a “T” configuration. A close up of its attachment to the tabletop is shown in Figure 2.4. This profile was selected because the same cross section is used successfully on the MABEL experimental setup, with MABEL being at least twice the mass of ERNIE. Additionally, using 80/20 allows mounting the boom such that the height can be easily varied. In preliminary experiments, a single column of 2040 profile was used, but after a few steps around the room, the robot and

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column would noticeably vibrate at a high frequency. Stopping the robot, holding it in the air and tapping the column itself excited this vibration mode. Arranging another piece of extrusion to create the “T” profile reduced these vibrations considerably.

The ceiling anchor assembly is shown in Figure 2.5. A base plate is attached to 2 sets of Unistrut at 90 degree angles to each other. The top set of Unistrut is anchored to the ceiling. This configuration permits fine-tuning of the alignment of the anchor centers. A steel shaft of 2.0 in is attached to the plate and tapers to 0.75 in to fit a 0.75 in ID housed ball bearing. The ball bearing housing is attached to a rectangular plate, which is in turn attached to the 80/20 column. The 0.75 in end of
the shaft is drilled to mate with the shaft of a US Digital HD25 encoder, the body of which is attached to the 80/20 column. This encoder measures displacement of the boom around the room with 2500 lines of resolution. The preliminary design had a single bearing and plate on only one side of the 80/20 column. When determining solutions to the vibration issue, this was noted as a weak point in the design. The plate thickness of 0.50 in is slim compared to the rest of the structure, and the design is asymmetric, both of which probably contributed to the large vibrations. Thus, an identical plate and bearing assembly was also attached to the other side of the column to increase the structural rigidity and reduce vibrations. This design is not necessarily ideal because the bearings need to be aligned, but there is some float in
the bearing housings, so a very precise alignment was not required.

Contrary to the author’s specific request, the top anchor was originally attached to the plaster ceiling above the drop ceiling because the contracted carpenter insisted that it would be adequate to support the load. This attachment failed by shearing the plaster around the bolts attaching the Unistrut to ceiling after brief experimentation. A new carpenter then followed the author’s original request to anchor to the concrete girders, and this connection held for subsequent experiments. This also required ordering a new extrusion because the one originally cut to fit to the plaster height was now too short. The original extrusion was used to create the “T” profile mentioned above.

Figure 2.5. Ceiling anchor assembly for ERNIE.
2.2.4 Electronics Platform

A two-shelf wooden housing was designed to hold the electrical components of ERNIE and is as shown in Figure 2.6. It was designed to be easily removed from the rotating platform if needed for diagnostics. The bottom shelf carries the motor amplifiers, and the top carries the dSPACE system, E-stop circuitry, power supplies, and signal conditioning circuits. These components were placed by assuming each was a rectangular box, creating the components in CAD, and arranging them on a model of the wooden platform as shown in Figure 2.7. Each component model was given a uniform density based on dimensions and weights from specification sheets. In the CAD model, some of the components slightly overlap the structure because of this approximation. Using this model, the mass and inertia parameters for the additional load carried by ERNIE were calculated from the wooden assembly and all components on it. The vertical column and rotating portions of the aluminum skeleton were not included in this estimate because their inertia loads were considered small enough to neglect.

2.3 Boom

A new stabilizing boom was also designed for the overground walking setup because the original boom was needed for the KURMET robot setup. Additionally, with a larger space, a longer boom was desired to reduce the effects of walking in a circle and more closely approximate straight line walking. The main section of the boom is a carbon fiber tube 3.5 in. in diameter and 8.0 ft long. At each end is an aluminum plug bonded to the carbon fiber tube using Loctite Hysol 9460 two-part epoxy. Like the original boom, clearance of 0.01 in was provided for the glue between the ID of the carbon fiber and the OD of the aluminum plug. This plug is hollow and tapped to attach face plates to mate the boom with both the robot and center
Figure 2.6. Wooden Platforms for carrying ERNIE electrical components

column. The shaft for mating with the robot was taken from the original design. The interface of the boom tube with the column was redesigned as shown in Figure 2.8. An aluminum extrusion extends from the boom face plate to mate with a horizontal shaft about which the boom pivots up and down. This extrusion was designed to mimic the design of the original boom, but the bolt pattern for connecting the part to the boom plate has a rather small diameter. Some experiments had issues with
these bolts loosening and the boom wobbling. The connection was tightened and Loctite was used, but future work should consider increasing the size of this part so a larger bolt pattern that better distributes the shear loads can be implemented. The aluminum extrusion is rigidly attached to the horizontal shaft on which a second HD25 encoder is mounted to measure boom elevation due to rotation about a
horizontal axis. (Currently this encoder is not used because the dSPACE board only has 6 digital encoder channels which are occupied by the displacement of the boom around the room, the 4 motor encoders, and the torso pitch encoder. The boom elevation is critical for running because there is a flight phase, but is not required for walking.) Two plates, one on each side of the 80/20 column, hold bearings for this shaft and are attached to the 80/20 with 8 bolts and T-nuts each. These plates are slid in unison up and down the 80/20 column to change the height of the boom. Six 1/4-20 bolts span the distance between the plates and are designed to keep the plates in alignment when adjusting boom height. Nuts on the inside faces of the plates on 2 of the bolts keep the plates from angling in towards center. These bolts also attach the bracket for the boom elevation encoder. Future work could consider redesigning this connection to be more robust to toe-in or toe-out of the plates as there is some play in the bolt connections. New plates could be added on the top and bottom of the space spanned by the 1/4-20 bolts and threaded to the existing plates, creating a box structure to keep better alignment. These plates, however, cannot interfere with the motion of the boom.

The boom height was adjusted to reduce foot scuff issues as in Figure 2.9. At foot impact and instantaneous dual support, the robot hips should be close to level as in Figure 2.9(a). If the boom is significantly below or above horizontal approaching this instant, the foot will touchdown earlier or later than expected (Figures 2.9(b) and 2.9(c)). Additionally, if the boom elevation is significantly above or below horizontal during the gait, large swing leg clearances are required for successful walking. Thus, the boom height was set such that it was level during dual support, approximately 0.73 m for curved feet and 0.71 m for point feet, which was successful in experiment. The height was determined once and used for all subsequent gaits of each foot type.
2.4 Floor

Experiments were attempted on the lab’s concrete tile-covered floor but were unsuccessful. The tile was too slick, and the robot slipped during most steps. The slipping was fixed by covering the floor with rubber gym floor matting (SBR bonded granular recycled rubber) with a thickness of 5/16 in. This surface provides good traction as well as a softening of the impact without being excessively spongy. Experiments on this covering, however, were still unsuccessful and resulted in motor gearhead damage, as discussed in Section 2.6. Thus, a raised floor was built, similar to that on which MABEL walks. The floor is 2×4 framing covered with 3/4 in
Figure 2.9. Ideal boom angle (a) with level hips at the instant of dual support. In (b), with the left-most leg the stance leg, the foot will hit the ground before the end of the gait cycle is reached. With the same stance leg in (c) the foot will still be above the ground at the end of the cycle, requiring the robot to fall farther forward to an unplanned gait configuration for the swing foot to impact the ground.

plywood, and the rubber mats were placed on top of the new floor. All experiments in subsequent chapters were executed on this raised floor without any of the issues associated with the concrete floor recurring.

2.5 Feet

Curved feet were originally constructed for ERNIE for the work in [43]. The arc length of these feet was rather short, and the gaits selected for use in these previous experiments did not have a constraint on arc rolling distance. Analysis of past gaits showed that in some cases the trajectory required rolling off the front edge of the arc. This may be a partial explanation for why there was difficulty in achieving stable gaits at high speeds in [43]. Based on preliminary curved foot gait generation (using the curved-foot impact map from Section 3.2.2), new feet were designed with an arc length that extended to approximately 48 degrees forward and 35 degrees backward from center. Additionally, a constraint was added to the optimization to prevent
motions that exceed these limits. The optimization and constraints are explained in detail in Chapter 3. The foot radius was shrunk from 8.0 in to 7.9 in because of a miscommunication to the machinist. However, 7.9 in is still approximately 30% of leg length, the desired percentage for mimicking human walking. An overlay comparison of the old and new curved foot is shown in Fig. 2.10. The bottom of the new curved feet are coated with grip tape for better traction, rather than the bicycle tire used on the old curved feet.

![Figure 2.10. Comparison of the curved foot from (blue) and the new curved foot design (gray).](image)

2.6 Torso

Repeated impacts with the concrete floor caused the gearheads of the original robot to fail. This failure was similar to that of the pins holding the planet gears in the final stage of the gearhead vibrating out of place and locking with the carrier arm of the next gearhead stage, as noted in [64]. Wensink fixed the original problem with a set screw and notched pins, which held up for treadmill walking. The problem
returned with overground walking, and two attempts were made to fix the gearhead problem. When the gearheads were disassembled, it was found that some set screws had loosened and several gear teeth had sheared. After the pins were returned to the desired positions and the set screws tightened, a second set screw was added behind the original set screw as a jam nut, and blue Loctite was again applied. Because gear teeth were broken, new gearheads were ordered and then cannibalized for parts because the motor gearhead assemblies on ERNIE had custom output shafts. This modification was not successful, and the gearhead problems quickly returned. As a last effort, the pins were welded into place. This solution also failed when a pin fractured after only a few steps around the room. These strategies were attempted first to avoid the cost and lead time required for purchasing all new motor assemblies.

To overcome the gearhead problem, new motor assemblies were selected for ERNIE. For compatibility with the existing control setup, the new motors are identical to the old motors (Maxon EC45 brushless 48 volt servomotors #136212), but new gearheads were chosen. To increase the torque capacity, a larger diameter gearhead (GP 52 C) with the same 91:1 reduction ratio as the original (GP 42 C) was selected. A comparison of the relevant characteristics of each gearhead is given in Table 2.2. The GP 52 C gearheads have twice the torque capacity of the GP 42 C gearheads for only a 67% gain in weight, which is a good trade-off.

The torso plates and motor pulleys were also redesigned to be compatible with the new motors. The original motors had custom machined output shafts that were much longer than the standard provided by Maxon. These longer output shafts had pulleys mounted to them with a keyway connection. These pulleys routed the cables that drive the joints. To keep the motor lead time to a minimum, it was decided to order standard output shafts and redo the pulley design. The new design has the pulley and shaft as an integrated piece that attaches to the motor output shaft via
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Note: GP 42 C were replaced by GP 52 C in the new design
a keyed shaft connection as shown in Figure 2.11. In the original design, holes were cut in the pulley faces to reduce weight. In the redesign, these holes were enlarged to ease the assembly and disassembly of the robot as discussed below. In the original design, the pulley cables had to be loosened, the set screw removed, and the pulley slid off the keyway to enable reaching the bolts face mounting the motor. To remove a motor for service or repair also required sliding the pulley off the output shaft. The opposite leg had to be partially disassembled as well because the longer output shaft makes it such that there is no clearance to remove the motor without moving the other leg. Originally, ERNIE was designed to have a much wider stance (as indicated by the extended cross braces), but the hips were made narrower and stiffer with the torsion tube. As a result, removing a motor required disassembling nearly the entire torso. With the new design, the motor pulley drivetrain can be left under tension and the motor removed from the torso without widening the hips. This is much more convenient for fixing future problems. Additionally, ERNIE’s joints could still function completely passively with the motors removed.

Since the torso was taller and the motors heavier, additional holes were drilled on the torso plates to save weight. The holes for face mounting the motors were drilled on each plate as well. These, along with the enlarged holes in the pulley faces, make it possible to tighten the motor mounting screws with a T-handle driver rather than an awkward standard hex key as shown in Figure 2.12.

Even with this upgrade to larger gearheads, the rotor inertias for ERNIE are relatively small compared to the dominant rotor inertias in [3], and the joints can be back-driven by hand. This upgraded design also added approximately 5 degrees of backlash in the drivetrain between the motor and joint positions due to the redesigned coupling between the motor output shaft and the drivetrain. This backlash is from the pulleys rocking on the keyways, even though oversized keys were cut to reduce the rocking. The keyway size was limited by the standard motor output shaft diameter.
The rocking was less prevalent in the old design with the same diameter output shaft. The keyway may be undersized for the larger inertia load of the new pulley design. Additionally, the motor output shaft is now cantilevered inside the new pulley design rather than bearing-supported at the distal end as in the original design. This changes the way the connection is loaded could contribute to the keyway rocking as well. These two facts (backlash and back-drivability) were of critical significance in implementing the control in experiment, as discussed in Chapter 3.

A 2-piece brace was also added in the middle of the torso as shown in Figure 2.11. The motors are face mounted, so the cantilever load of the motor body at impact could be significant and likely contributed to the deterioration of the original motor gearheads. To remedy this, an aluminum brace was fabricated with openings 0.12 in larger in diameter than the motor bodies. The openings were lined with rubber and the brace lightly clamped around the motor bodies. The motors were not designed for support in this way, but an engineer at Maxon advised that it should not be detrimental to motor performance.

The pitch angle of the torso was measured with a potentiometer as originally designed. The potentiometer reading was too noisy to use for feedback control in [72], so an alternative method was used therein for tracking gait progression. RABBIT and MABEL both have encoder readings for the hip joint; thus, an encoder was retrofit in the space previously occupied by the hip potentiometer on ERNIE. The encoder is a 15T thru-bore 10,000 line encoder from Encoder Products Company. A small shaft extension was machined to mate with the existing shaft on which the potentiometer was originally mounted so a thru-bore encoder could be mounted in its place. This shaft was attached with a screw that was accidentally sheared off after assembly. If further design modification is needed in the future, the original shaft and extension will have to be replaced.
2.7 Camera Mount

To provide a consistent view of the robot walking, a mount for a video camera was attached to the central column. This consisted of a tripod head and a machined plastic plate to attach the tripod head to an extension of 80/20. Putting the camera above the robot and on center gave a view that was particularly useful for comparing gaits during disturbance rejection trials.

2.8 Drawings

Assembly exploded views and drawings of the parts described in this chapter are collected in Appendix A.
Figure 2.11. Partial exploded view of the new torso design. The new motor pulleys are shown in red, and the gasket plate to reduce motor cantilever load is shown in green. Additional drawings and assembly views are gathered in Appendix A.
Figure 2.12. The method for installing and removing motors is shown for the knee motor. The larger holes on the faces of the pulleys and holes in the torso plates allow a T-handle hex wrench to reach the motor mounting screws. For the hip motor (not shown), this is the easiest way to tighten the screws as the distance between the pulley and screws is very narrow.
CHAPTER 3

COMPARISON OF EXPERIMENTAL POINT-FOOT AND CURVED FOOT
HZD-BASED CONTROL OF BIPED WALKING

3.1 Motivation

The work of [1] has shown that curved feet are more efficient than point feet. HZD-based control has been shown to work well for planar biped robots, but thus far only with point feet. It is desirable to have the efficiencies of curved feet combine with the theoretical soundness of the HZD-based control approach. The updates made to the ERNIE hardware and the new curved foot impact model from [40] make the comparison between point- and curved foot walking under HZD-based control possible.

3.2 Model

To validate the design of HZD-based controllers for curved foot bipedal robots, the five-link biped ERNIE was used (Fig. 2.2). ERNIE has two legs with knees and a torso, and the angle conventions for this robot are as shown in Fig. 3.2. The torso angle $q_5$ is measured counterclockwise (CCW) from the vertical. The stance-leg hip angle $q_1$ is measured CCW from the torso, as is the swing leg hip angle $q_2$. For both stance and swing legs, the respective knee angles $q_3$ and $q_4$ are measured CCW from the upper leg to the lower leg. ERNIE’s feet can be interchanged, allowing HZD-based control of both point- and curved foot walking to be compared using the same robot. Figure 2.2 shows ERNIE with curved feet and the current overground setup.
As currently designed, the curved feet have a radius of approximately 30% of the total leg length of ERNIE, which is consistent with the observed human foot ankle roll-over shape [21]. The point feet, being covered with racquetballs, actually have a radius of approximately 4% of the total leg length, but this is small enough in comparison to the curved feet to be neglected. As shown in Fig. 3.2, the perpendicular distance between the center of curvature of the foot and the line passing through the center of rotation of the knee and the lower leg’s COM is called the ankle offset $X_F$. The ankle offset was zero for all experiments here but can be adjusted for future work.

Each step is modeled as a finite time single support phase during which the swing leg moves from behind to in front of the stance foot and an instantaneous, inelastic collision during which the stance foot switches as shown for a simplified biped in Fig. 3.1. The single support equations of motion are continuous second-order ODE’s. The impact is modeled with an algebraic map relating the state of the robot immediately before impact to the state immediately after impact. For a gait to be one-step periodic, the state of the system just before impact must be the same for every step.

ERNIE has 4 actuators, one for each hip and knee, but these actuators are located in the torso so that the legs are lightweight. This design creates a coupled drivetrain, and the joint positions are related to motor positions by

\begin{align}
q_H &= m_H, \quad (3.1) \\
q_K &= m_K - (m_H - \pi), \quad (3.2)
\end{align}

where $m_i$ are the motor positions, $q_i$ are the joint positions, and the subscripts $H$ and $K$ refer to the hip and knee, respectively. The simulation and optimization of gaits is executed with actuators at the joints for simplicity. Because of this simplification, the coupled motor rotor inertia loads are mapped to the joints to correctly
Figure 3.1: Diagram of a biped walking from left to right. The swing leg (dashed) swings from behind the stance leg (solid) to in front of the stance leg. The swing leg then collides with the ground. For clarity during the impact event, the swing leg is called the impacting leg, and the stance leg is called the trailing leg. The hip displacement during a step is $x_H$, $L_i$ is the effective leg length at impact, and $\theta$ is a measure of gait progression. The superscripts ‘+’ and ‘−’ refer to just after and just before impact, respectively.
form the equations of motion. The resultant joint motions are later converted back to motor motions for execution on the hardware. This is explained in detail in Section 3.2.1. The mass and geometric properties for ERNIE are given in Table 3.1. These properties were determined from the solid models used for part design; the further identification of various friction parameters as in [50] was not performed. The reflected rotor inertia in Table 3.1 is the load seen by the joint actuator used in the simulation and derived in Section 3.2.1.

3.2.1 Continuous Phase

ERNIE was modeled as a planar five-link biped as in previous work [43, 72]. The model is derived by Lagrange’s method as in [66] with equations of motion

\[
D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = Bu,
\]

where \(q\) is a vector of joint angles, \(D\) is the \(N \times N\) inertia matrix, \(C\) is the \(N \times N\) matrix containing Coriolis and centripetal terms, \(G\) is the \(N \times 1\) vector containing gravity terms, \(u\) is the \(N - 1 \times 1\) vector of actuator torques and \(B\) is the \(N \times N - 1\) matrix mapping actuator torques to joint angles. For the models studied herein,

\[
B = \begin{bmatrix}
I_{N-1 \times N-1} \\
0_{1 \times N-1}
\end{bmatrix}.
\]

The motion of the biped during single support can be modeled by integrating Eq. 3.3 forward in time from the instant after impact until the instant before the next impact. The effect of the curved feet appears in \(D, C\) and \(G\) [43]. These effects were found in [43] by a variation on the Denavit-Hartenberg convention, but can also be found by integrating and embedding the rolling contact constraint.
Figure 3.2. A representative set of angle definitions for a five-link biped. The four actuated angles are given by $q_1, q_2, q_3,$ and $q_4$. The unactuated angle is $q_5$. This schematic also shows the geometric parameters for the biped ERNIE.
### TABLE 3.1

GEOMETRIC AND MASS PARAMETERS OF THE BIPED ERNIE

<table>
<thead>
<tr>
<th></th>
<th>Length (m)</th>
<th>COM (m)</th>
<th>Mass (kg)</th>
<th>Inertia (kg·m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Torso</td>
<td>0.279</td>
<td>0.132</td>
<td>14.5</td>
<td>0.100</td>
</tr>
<tr>
<td>Femur</td>
<td>0.360</td>
<td>0.125</td>
<td>1.47</td>
<td>0.024</td>
</tr>
<tr>
<td>Point-Foot Tibia</td>
<td>0.36</td>
<td>0.136</td>
<td>0.97</td>
<td>0.018</td>
</tr>
<tr>
<td>Point-Foot Radius</td>
<td>0.029</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Curved Foot Tibia</td>
<td>0.378</td>
<td>0.136</td>
<td>1.07</td>
<td>0.024</td>
</tr>
<tr>
<td>Curved Foot Radius</td>
<td>0.201</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Ankle Offset</td>
<td>0.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Boom</td>
<td>2.89</td>
<td>-</td>
<td>(3.14)</td>
<td>(7.25)</td>
</tr>
<tr>
<td>Center Support</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>4.48</td>
</tr>
<tr>
<td>Reflected Rotor Inertia</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.173</td>
</tr>
</tbody>
</table>

Note: The COM location is measured from the proximal joint.

The inertia is measured about the COM.

Values in parentheses were not used in the model.
As alluded to in Chapter 2, the boom and wooden platform structure were modeled as extra inertia loads carried by ERNIE. This is accounted for by adding an extra kinetic energy term in the simulation model as in [3] and [66]. The additional kinetic energy is

\[ K_a = \frac{1}{2} J_{\text{boom}} (\phi_h^2 + \phi_v^2) + \frac{1}{2} J_{\text{platform}} \dot{\phi}_h^2, \quad (3.5) \]

where \( \dot{\phi}_h \) and \( \dot{\phi}_v \) are the horizontal and vertical angular velocities of ERNIE about the center pivot point. These angular velocities are converted to be in terms of the generalized coordinates by recalling that \( v = r \omega \), and thus Eq. (3.5) is rewritten as

\[ K_a = \frac{1}{2} J_{\text{boom}} \frac{V_H^2(q)}{L_{\text{boom}}^2} + \frac{1}{2} J_{\text{platform}} \frac{V_{H,x}^2(q)}{L_{\text{boom}}^2}, \quad (3.6) \]

where \( L_{\text{boom}} \) is the length of the boom and \( V_H \) and \( V_{H,x} \) are the hip velocity and x-component of the hip velocity, respectively, expressed in terms of the joint coordinates. The values of the inertia terms were both determined from solid models of the associated components. The boom also adds a potential load to the system.

It was assumed that the boom mass could be neglected as in [43, 72] because it is lightweight in comparison to the center platform and electric components. However, as shown in Table 3.1, the inertia is a larger value than the platform inertia. This was discovered after gaits were generated and run successfully in experiment. The loads from the support electronics are at max 2.7% of the loads in the mass-inertia \( D \) matrix. Adding the kinetic and potential loads of the boom increases the values in \( D \) by up to 4.5%. A gait designed without the boom loads walked with a step velocity of 0.58 m/s. Adding the boom loads to this gait and recomputing a periodic gait trajectory resulted in a step velocity of 0.52 m/s. Note that when only adding the kinetic load of the boom, the change in speed was nearly imperceptible, and the potential load of the boom appears to be the most significant factor omitted from the model. Even neglecting the boom effect, successful experimental results were
obtained. In the future, the boom effects could be added to the model to improve speed correspondence between simulation and experiment.

In previous work [43, 72], it was assumed that the motor inertias were a diagonal term added to $D$ as in [66]; however, this is incorrect for the coupled drivetrain of ERNIE. Using Eqs. 3.1-3.2, the coordinate transformation between motor and joint coordinates is

$$ q = [R_{mq}] m + \begin{bmatrix} 0 \\ \pi \\ \pi \\ 0 \end{bmatrix} \tag{3.7} $$

and

$$ m = [R_{mq}]^{-1} q - \begin{bmatrix} 0 \\ 0 \\ \pi \\ \pi \\ 0 \end{bmatrix} \tag{3.8} $$

where

$$ [R_{mq}] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.9} $$

Similarly, the velocity relationships are found by differentiating the above equations. The additional kinetic energy from the motor inertia is equivalent whether it is expressed in joint or motor coordinates. The additional kinetic energy for each motor
is

\[ K_{i,mtr} = J_{i,mtr}m_i^2, \quad (3.10) \]

where the inertia term \( J_{i,mtr} \) is composed of

\[ J_{i,mtr} = J_{rotor}N^2 + J_{gearhead} + J_{pulleys} + J_{cable}, \quad (3.11) \]

where \( N \) is the gear ratio (\( N = 91 \) for ERNIE). For the motors on ERNIE, the first term in Eq. (3.11) is so dominant that it is the only relevant term in the calculation. From the motor data sheet, the EC 45 Maxon motor has a rotor inertia of 209 gcm\(^2\). Thus, the reflected inertia at the output shaft is

\[ J_{i,mtr} = (209e-7)(91)^2 = 0.1731 \text{ kg-m}^2, \quad (3.12) \]

the value given in Table 3.1 for reflected motor inertia. For comparison, the gearhead and motor pulley inertias are 16.7e-7 kg-m\(^2\) and 1.4e-4 kg-m\(^2\), respectively, neither of which are significant. The additional kinetic energy added to the system is

\[
K_{mtr} = m^T K_{mtr} m \\
= q^T [R_m q]^{-T} K_{mtr} [R_m q]^{-1} q \\
= q^T D_{mtr} q, \quad (3.15)
\]

where the resulting addition to the \( D(q) \) matrix in joint coordinates is

\[
D_{mtr} = \begin{bmatrix}
2J_{mtr} & 0 & J_{mtr} & 0 & 0 \\
0 & 2J_{mtr} & 0 & J_{mtr} & 0 \\
J_{mtr} & 0 & J_{mtr} & 0 & 0 \\
0 & J_{mtr} & 0 & J_{mtr} & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}. \quad (3.16)
\]
The principle of virtual work is used to find the conversion between joint torques and motor torques. The total virtual work of the non-conservative torques is

\[ \delta W = \sum F_i \delta_i \]  
\[ = \sum u_i \delta q_i \]  
\[ = (Bu)^T \delta q_i \]  
\[ = u^T B^T \delta q_i. \]

Rewriting the variation in motor coordinates,

\[ \delta W = u_1 \delta m_1 + u_2 \delta m_2 + u_3(\delta m_3 - \delta m_1) + u_4(\delta m_4 - \delta m_2), \]

which is also written

\[ \delta W = u^T B^T [R_m q] \delta m_q. \]

If \( \tau \) represents the motor torques, then Eq. 3.22 is equivalent to

\[ \delta W = (B\tau)^T \delta m_q. \]

Thus, the conversion from joint torques to motor torques is

\[ \tau = B^T R^T Bu, \]

and

\[ u = (B^T R^T B)^{-1} \tau. \]

As explained previously, motor inertias account for only a small percentage of the loads on ERNIE and do not dominate the dynamics. Although the simulations in [72] did not correctly model the motor inertias, the differences from using the
corrected model herein would be insignificant. This also points out that control laws in experiment based on the large motor inertia assumption can be revisited to improve performance. For a different robot with large motor inertia and a coupled drivetrain, it is critically important that the inertias are modeled correctly because the coupled motor dynamics will dominate the dynamics and an accurate control law accounting for the coupling may be needed. The conversion between joint and motor torques is particularly important when comparing experimental performance to simulation in Section 3.7.

3.2.2 Impact Details

The model of impacts for curved feet is modified from the one presented in [66] for point feet to account for the rolling contact between the foot and the ground that occurs with curved feet. The six key assumptions for a valid impact are [66]:

1. there is an impulsive force at the impacting foot,
2. the impact occurs over an infinitesimal period of time,
3. the position of the biped does not change over the duration of the impact,
4. the trailing foot lifts off the ground without interaction,
5. the impacting foot rolls without slip, and
6. the moments at the joints generated by the impulsive impact force are much larger than the moments generated by gravity and the actuators.

Based on these assumptions, the impact is a discrete event mapping end-of-step states to initial conditions of the subsequent step. This is what makes the model hybrid. Since the biped is left-right symmetric, only one impact map is needed, and a relabeling of the model coordinates is applied.

Using the nomenclature from Fig. 3.1, just before impact the trailing leg is in rolling contact with the ground. At impact, the biped is mathematically modeled as detached from the ground in order to retain Lagrange multipliers and find the
impulsive forces at impact. Just after impact, the impacting foot rolls without slip. Joint velocities are updated, reflecting the impulsive effects, and the coordinates are relabeled. The details of the full derivation of this impact map are found in [40], and the resulting impact map is summarized as

\[ q^+ = \Delta_q q^- \]  
\[ \dot{q}^+ = \Delta_q (q^-) \dot{q}^- , \]

where

\[ \Delta_q = \Delta_q (\Lambda_{11} - \Lambda_{12} E_1 S) , \]  
\[ \Lambda = I_{(N+2) \times (N+2)} - D_e^{-1} E^T (E D_e^{-1} E^T)^{-1} E \]  
\[ = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{bmatrix} \cdot \]  

The matrix \( I \) is the \((N + 2) \times (N + 2)\) identity matrix, \( \Lambda_{11} \) is an \(N \times N\) matrix, and \( \Lambda_{12} \) is an \(N \times 2\) matrix. The term \( D_e \) is found from the detached model with coordinates \( q_e = [q^T, p_x, p_y]^T \), where \( p_x \) and \( p_y \) are the x- and y-coordinates of the hip relative to an arbitrary point on the ground. The term \( E \) is obtained from the rolling contact constraints and for ERNIE is

\[ E = \begin{bmatrix} 0 & e_2 & 0 & e_1 & e_2 & 1 & 0 \\ 0 & e_4 & 0 & e_3 & e_4 & 0 & 1 \end{bmatrix} , \]  

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where

\[ e_1 = R - (L_L - R) \cos(q_2 + q_4 + q_5) + X_F \sin(q_2 + q_4 + q_5) \]
\[ e_2 = e_1 - L_U \cos(q_2 + q_5) \]
\[ e_3 = -(L_L - R) \sin(q_2 + q_4 + q_5) - X_F \cos(q_2 + q_4 + q_5) \]
\[ e_4 = e_3 - L_U \sin(q_2 + q_5). \]

The term \( E_1 \) transforms the pre-impact single support joint velocities into the pre-impact hip velocities and can be found by taking the first \( N \) columns of \( E(\Delta q q^-) \).

Thus, the complete hybrid system is

\[
\begin{align*}
M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G &= Bu \quad q^- \notin S \\
q^+ &= \Delta \dot{q} q^- \quad q^- \in S \\
\dot{q}^+ &= \Delta \dot{q} (q^-) \dot{q}^- \quad q^- \in S,
\end{align*}
\]

3.3 Zero Dynamics and Stability

The zero dynamics of the model are essential to optimize and simulate possible walking gaits because analysis of a reduced order model is much more efficient than simulating the full model. The zero dynamics \cite{27} are found following \cite{66} using feedback linearization and are summarized below. For convenience, the system dynamics in Equation 3.3 are modeled using coupled first order equations by defining \( x = [q^T, \dot{q}^T]^T \). The system can then be written as

\[
\dot{x} = f(x) + g(x)u,
\]

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where

\[
\begin{align*}
f(x) &= \begin{bmatrix} \dot{q} \\ -D^{-1}(C\dot{q} + G) \end{bmatrix}, \\
g(x) &= \begin{bmatrix} 0 \\ D^{-1}B \end{bmatrix}.
\end{align*}
\]

The desired configuration of the biped will be defined using the output function

\[
y = h(q),
\]

where \(h\) is a vector function of length \(N - 1\) and is the tracking error for the desired trajectory. The approach requires that the output is a smooth function of position only.

Feedback linearization is applied to obtain a new system that is input/output linear. Differentiating Eq. 3.35 once gives

\[
\dot{y} = \frac{\partial h}{\partial x} \dot{x} = \begin{bmatrix} \frac{\partial h}{\partial q} \\ 0 \end{bmatrix} \begin{bmatrix} \dot{q} \\ -D^{-1}(C\dot{q} + G) \end{bmatrix} + \begin{bmatrix} \frac{\partial h}{\partial q} \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ D^{-1}B \end{bmatrix} \cdot u
\]

\[
= L_f h,
\]

where \(L_f h\) is the Lie derivative of \(h\) \([27]\). The derivative of \(h\) with respect to \(\dot{q}\) is
zero because $h$ is a function of configuration only. Since the input $u$ does not show up in $\dot{y}$, the equation is differentiated again,

$$\ddot{y} = \frac{\partial}{\partial x} (L_f h) \dot{x} \quad (3.37a)$$

$$= L^2_f h + L_g L_f h \cdot u. \quad (3.37b)$$

Because the input $u$ appears in Eq. 3.37, the system given by Eqs. 3.34 and 3.35 has a relative degree of 2, so the first $2(N - 1)$ new coordinates can now be defined as

$$\eta_1 = h(q), \quad (3.38)$$

$$\eta_2 = L_f h(q, \dot{q}). \quad (3.39)$$

The input is chosen as

$$u^* = (L_g L_f h)^{-1} (v - L^2_f h), \quad (3.40)$$

which creates the double integrator $\ddot{y} = v$. Choosing $v$ appropriately, for example as high gain PD feedback, collapses the dynamics to the zero dynamics manifold (i.e. the desired orbit is perfectly tracked). As a result, $\eta_1$ and $\eta_2$ are always zero. The final two coordinates (the zero dynamics) are chosen to be the same as in [66],

$$\xi_1 = \theta(q) \quad (3.41)$$

$$\xi_2 = L_f \theta, \quad (3.42)$$

where $\theta$ is required to be a smooth function of position only that monotonically increases over a step. Thus, the zero dynamics are

$$\dot{\eta}_1 = \eta_2$$

$$\dot{\eta}_2 = L^2_f \theta + L_g L_f \theta u^*. \quad (3.43)$$
A stable gait is determined from a Poincaré map of the zero dynamics just before impact \( \{\theta^-, \dot{\theta}^-\} \). The map is the result of applying the impact map to \( \{\theta^-, \dot{\theta}^-\} \) and integrating the zero dynamics back to a new \( \{\theta^-, \dot{\theta}^-\} \). The state \( \{\theta^-, \dot{\theta}^-\} \) is a fixed point of the map if
\[
\{\theta^*_-, \dot{\theta}^*_-\} = P(\{\theta^*_-, \dot{\theta}^*_-\}).
\] (3.44)
The fixed point is stable if the eigenvalue of \( P, |\lambda_P| < 1 \). Then the stable hybrid limit cycle \( \{\theta^*_-, \dot{\theta}^*_-\} \) can be expanded to a stable walking gait for the full dynamics. For analyzing gait stability, [66] chooses a different and clever choice for the second zero dynamics coordinate,
\[
\xi_2 = D_N(q)\dot{q},
\] (3.45)
where \( D_N \) is the \( N^{th} \) row of \( D \). This greatly simplifies the analysis because it avoids the need to invert the decoupling matrix in Eq. 3.43 and for point feet Eq. 3.44 becomes a differential equation of \( \xi_1^- \). For curved feet, Eq. 3.44 also simplifies to a differential equation of \( \xi_1^- \), but it is more complicated than the equation for point feet. The equation is still easily solvable, however, with the details of the differences presented in [40]. Also when the foot radius is zero, the stability check simplifies to that as originally developed for point feet. This is as expected because point feet are a degenerate case of curved feet.

To implement the control in simulation, the 4 output functions (Eq. 3.35) were chosen as
\[
h_i(q) = q_i - q_{i,\text{REF}}(\alpha_i; \theta),
\] (3.46)
where \( i = 1...4, \) \( q_{i,\text{REF}} \) is the reference trajectory for the joint—a sixth order Bézier polynomial defined by coefficients \( \alpha_i \), and \( \theta \) is a choice for the first zero dynamics variable \( \xi_1 \),
\[
\theta(q) = [-1, 0, -0.5, 0, -1] \cdot q.
\] (3.47)
Note that for point feet and equal thigh and shank lengths ($L_U = L_L$), Eq. 3.47 can be physically interpreted as the angle between vertical and the vector from the ground contact point to hip. As $\theta$ increases, the hips move forward, which is a critical characteristic for walking. For the curved foot configuration, Eq. 3.47 does not have the same physical interpretation as for point feet because of the rolling contact of curved feet and because $L_U \neq L_L$. However, the important characteristic of forward hip progression is still enforced, allowing Eq. 3.47 to also be used in simulation for curved foot bipeds. Thus, Eq. 3.47 encodes forward progression of the hips into the control for both point- and curved foot walking.

3.4 Gait Generation

Walking gaits for both point and curved foot bipeds were generated using a constrained numerical optimization to find the 28 coefficients that define the four 6th order Bézier polynomials specifying the joint trajectories (Eq. 3.46). Because the gaits were designed to be periodic, only 20 of these coefficients are independent, and the others are determined by constraints that enforce gait periodicity. Each individual gait has a specific trajectory, step length, and velocity. New point-foot gaits were needed to allow an accurate comparison because the results from [73] are skewed by treadmill effects and fixed step length. The MATLAB constrained nonlinear optimization routine \textit{fmincon} was used just as in [72], but the objective function for these gaits was

$$J = \frac{W_{\text{total}}}{x_H mg},$$

(3.48)

where

$$W_{\text{total}} = \sum_{i=1}^{4} \int_{0}^{t_f} |u_i \dot{q}_i| dt,$$

(3.49)

the sum of the work done by each actuator over a step, $x_H$ is the step length, $m$ is the total mass of the biped, and $g$ is the gravitational constant. The $J$ quantity is called
the specific resistance and is a non-dimensional measure of efficiency that allows a fair comparison between gaits with different speeds and step lengths, whereas previous research \[73\] instead used average mechanical absolute power over a step because step length was held constant. Because the optimization was sensitive to step length constraints, gaits were generated by sweeping a range of step lengths from 0.35 m to 0.55 m in 0.01 m increments using the result of one optimization as the initial seed for an optimization in the next step length range. This generated hundreds of gaits at particular speed, and in most cases, an optimal step length emerged. The gaits tested in experiment were selected to have approximately optimal step lengths as indicated by minimum specific resistance, unless the optimal step length exceeded 0.50 m, which is the experimentally determined limit of feasible step length. Beyond this step length, the hardware could not sustain stable walking, most likely due to motor torque limits.

Although gaits were designed for a specific speed, the optimization actually found gaits in a range of speeds around the desired speed due to how the optimization was implemented. The matrix \(\alpha\) of joint angle coefficients completely defines the gait step length and velocity. Initial conditions of the step \(x_0\) are a function of these matrix values. Finding a periodic orbit at an exact velocity and step length used the \(\alpha\) values as optimization variables, calculated \(x_0\), and simulated the gait to find its velocity. This method was very time consuming and had some trouble converging to a solution. An alternate approach picks \(x_0\) and some of the \(\alpha\) coefficients as the optimization variables, calculates the missing \(\alpha\) values, and simulates the gait. This method is much faster and contains a constraint that introduces tolerance on gait periodicity as used in previous work \[43, 72\]. Simulation steps using this approach will result in a 1st step that is exactly the desired speed. However, subsequent steps will approach the limit cycle determined directly from the \(\alpha\) matrix, as \(x_0\) is a function of \(\alpha\) but was determined independently via the optimization. Since most analysis is done on only
the first step, issues with bounds on the periodicity tolerance were not discovered until after gaits were generated and all experiments completed. For example, a set of 200 gaits optimized for 0.60 m/s and step lengths from 0.45 to 0.60 m actually had gait velocities ranging from 0.57 m/s to 0.63 m/s with an average of 0.58 m/s. This could contribute to the range of speeds observed in experiment. In the future, a better balance between optimization time and tolerance to desired speed can be found by changing the periodicity constraint tolerances. The exact periodicity method was attempted because the entire optimization code was updated by Anne Martin when the new curved foot model was created and the reasons for the periodicity constraint were not clear from the model used in [43, 72].

Besides the “standard” constraints for robot walking listed in [73], additional constraints were added to the optimization after preliminary experimental trials proved unsatisfactory. A constraint was added to prevent rolling off the ends of the curved feet. This constraint limited lower leg absolute angle to be between 25 degrees backward and 40 degrees forward from center. The swing foot height constraint was also modified. In previous work, the minimum height of the swing foot was constrained as a quadratic function of gait progression where the height at 50% of gait cycle was the parameter specified. On the ground, this constraint was too aggressive and resulted in gaits with significant foot scuffing in the later half of the gait cycle; increasing the height at the midpoint did not solve the problem. The possibility of joint errors due to backlash can allow the foot to actually dip below this constraint in the later half of the cycle. Thus, a new constraint was developed from preliminary experiments that walked successfully and is also shown in Figure 3.3. This constraint is a piecewise combination of a quadratic up to 50% of the gait cycle and a quartic from 50% to the end of the cycle. The increased height late in the gait cycle and steeper slope to zero foot height at the end of the gait cycle helped significantly with foot scuff issues.

Further constraints fall in two categories: end-of-step constraints and start-of-step
Joint errors and backlash in experiment can cause the swing foot to scuff the ground and transition to a new stance leg earlier in the gait cycle than planned, usually with the stance knee flexed significantly more than desired. This unexpected impact and subsequent incorrect start-of-step configuration create large trajectory tracking errors at the beginning of the step and can cause failure. The effects of such joint errors are mitigated by constraining the absolute angle of the lower swing leg to change by no more than 3 degrees during the last 20% of the step, which is similar to observed human swing leg motion [46]. Since upper swing leg motion is also small at the end of the gait cycle, even if the foot scuffs the ground early in the cycle, the error in step length and impact configuration is significantly reduced, leading to more successful experiments.

Due to the backlash in drivetrain, the joint angles of an impacting leg could have a near instantaneous change of up to 5 degrees. At the same time, impacts from overground walking will backdrive the motor angles as well because of the previously mentioned small rotor inertias. Without additional start-of-step constraints, in most

Figure 3.3. Example minimum foot height constraint
cases the motors did not have enough torque capacity to overcome these loads, and the experiment failed. Since the simulation does not account for the backlash or backdrive behavior and it was unclear whether a torque limit in simulation could prevent this behavior, start-of-step constraints were added to enforce at least 5 degrees of stance-leg knee flexion, hip flexion, and forward pitch of the torso during the first 20% of the gait cycle. These constraints are consistent with human hip and knee motion during the weight acceptance phase of gait (also comprising approximately 20% of the gait cycle) [51]. During weight acceptance, the joints absorb energy, and the muscles act eccentrically [68], which means a muscle elongates due to an opposing force greater than the contraction force generated by the muscle. The constraints thereby impose joint behavior in early stance that is consistent with the natural joint dynamics rather than allowing gaits with unnecessarily rigid joints shortly after impact. These constraints were needed on the torso, stance hip, and stance knee for point feet but only the torso and stance hip for curved feet. It is theorized that the stance knee constraint was not needed for curved feet due to the differing impacts between curved and point feet.

3.5 Hardware Control Implementation

3.5.1 Calculating the Phase Variable $\theta$

For mathematical simplicity, Eq. 3.47 can be normalized between 0 and 1 using

$$s = \frac{\theta - \theta^+}{\theta^- - \theta^+}. \quad (3.50)$$

Joint errors or drivetrain backlash can allow the hardware to have a configuration that corresponds to a $\theta$ value outside of the planned range from simulation. The behavior of the trajectory tracking polynomials (Eq. 3.35) outside of the planned range is not encountered in simulation and is likely not a desirable trajectory for
walking. Thus, the controller in hardware saturates at values of $s = 0$ and $s = 1$.

The monotonically increasing variable in Eq. [3.47] used to plan the desired joint trajectories in simulation is not equally appropriate for hardware implementation because the combination of non-zero tracking error and drivetrain backlash present in hardware is not accounted for in simulation. The most reliable sensors for feedback on the hardware are the motor and torso encoders because the joint potentiometer signals are noisy and filtering the potentiometer signals for reasonable performance would greatly reduce the control bandwidth. Substituting the relationships between motor and joint angles (Eqs. 3.1 and 3.2) into Eq. 3.47 gives the measurement of $\theta$ expressed in motor coordinates,

$$\theta = -\frac{m_H}{2} - \frac{m_K}{2} - q_5 - \frac{\pi}{2}$$

(3.51)

The coupled drivetrain causes the torso measurement $q_5$ to have twice the importance of the hip and knee motor position measurements, which is a particularly significant issue because of the drivetrain backlash. If it is assumed that $m_H$ and $m_K$ are perfectly tracking desired trajectories, the torso measurement $q_5$ could be pitched forward up to 10 degrees farther than desired due to the drivetrain backlash. In such a situation, $\theta$ would indicate that the hips have progressed much farther through the gait than they actually have, causing a significant error in the trajectory tracking (Eq. 3.35) and a subsequent large control action. This results in progression of $\theta$ inconsistent with the planned gait and causes either a walking speed that is much too fast or failure.

Since the forward progression of the hips is the key factor in walking, using a direct measurement of hip progression is a promising alternative method for calculating $s$. The horizontal displacement of the hips $x_H$ can be determined using an encoder on the boom that measures the angular position of the robot in the room. In previous
experiments on the treadmill, the horizontal displacement of the hips was determined by the boom encoder and the displacement of the treadmill belts. To relate $x_H$ to $s$, a cubic polynomial of $x_H$ was fit to the progression of $s$ given by simulating a step using Eq. 3.47. For a particular gait, the treadmill was set to the corresponding average step velocity. Since the step designed in simulation does not have a constant hip velocity, this control strategy provided some additional stabilization. The author found this strategy to be ineffective for overground walking in which the robot exclusively powers its own progression without the assistance of the treadmill belts to help drive it forward through the gait cycle. Additionally, a technical limitation of this strategy is that it requires the hip progression $x_H$ to be reset to zero at the start of each step. This reset assumes perfect joint trajectory tracking; therefore, an impact configuration with non-perfect tracking should correspond to a non-zero hip progression on the subsequent step. The reset to zero, however, creates a mismatch between the true configuration of the robot and the assumed configuration as used by the controller. Over several steps, the controller causes the mismatch error to increase, which eventually causes the gait to fail. Thus, one feasible solution to reduce the mismatch is using Eq. 3.51 to determine errors in the configuration at impact.

The approach taken in this work is to combine aspects of each approach, creating an averaged $s$ value that mitigates the disadvantages of each approach individually. This is accomplished by finding the relationship between hip displacement $x_H$ and gait progression $\theta$. If $\phi = \theta^- - \theta^+$, then as shown in Fig. 3.1, hip displacement is related to $\phi$ by observing that $L_i$, the effective leg length at impact, approximately forms 2 sides of a triangle with interior angle $\phi$ and third side $x_H^-$. (Note that it is exactly a triangle for a point-foot biped because the stance foot is fixed to the ground.) Thus,

$$x_H^- \approx 2L_i \sin \frac{\phi}{2},$$  \hspace{1cm} (3.52)
Assuming $\phi$ is a small angle,

$$\phi \approx \frac{x_H}{L_i},$$

(3.53)

which indicates that normalizing step length $x_H$ by $L_i$ is an approximately equivalent measure of $\phi$. Using this relationship, a new gait progression is defined in experiment,

$$\theta_{exp} = \theta + \frac{x_H}{L_i},$$

(3.54)

where $\theta$ is as in Eq. 3.51. The new progression $\theta_{exp}$ is still monotonically increasing over a step and

$$s_{exp} = \frac{\theta + x_H/L_i - \theta^+}{\theta^- + x_H/L_i - \theta^+}.$$  

(3.55)

This approach mitigates the effect of tracking errors increasing $s$ prematurely because of the $x_H$ term and also allows the errors in the impact configuration to be captured. This measure proved to be very successful in subsequent experiments.

This method of calculating $s$ differs from the methods implemented on the robots RABBIT [3] and MABEL [16]. RABBIT had dominant motor inertia terms and harmonic drives with almost no backlash; thus, joint errors were not an issue, and Eq. 3.47 was used. MABEL used the absolute angle of the virtual compliant leg relative to the ground as a measure of $\theta$, similar to Eq. 3.47. Even with a compliant drivetrain, the compliance was carefully modeled [50] and the drivetrain had little backlash, so the approach taken here was not required.

3.5.2 Implementing High-Gain PD Control

In previous work [43, 72], ERNIE was assumed to have rotor inertias that dominated the link inertia terms in the dynamics, as was the case for the robot RABBIT. On RABBIT, this permitted the use of high gain PD control on the motor positions directly and effectively ignored the link inertia loads [3]. As mentioned in Section
3.2.1 ERNIE does not have these dominant rotor inertias, but the assumption of controlling the motors directly was successful in previous walking on the treadmill. As mentioned in [66], any control that faithfully creates the desired joint trajectories should also be successful. The control from treadmill experiments was used for ground walking, but the gains were re-tuned. The re-tuned gains utilized a few additions to a generic PD control law, requiring a more detailed explanation of the actual law implemented than that presented in [72].

First, since the motor positions are directly controlled, motor error coordinates are needed for feedback,

\[ e_H = m_H - m_{H,ref} \]  
\[ e_K = m_K - m_{K,ref}, \]

where the reference trajectories \( m_{i,ref} \) are derived from the joint reference trajectories. Using Equations 3.1 and 3.2 to derive these references results in

\[ m_{H,ref} = q_{H,ref} \]  
\[ m_{K,ref} = (q_{K,ref} + q_{H,ref} - \pi) \].

Using Equation 3.59 as the knee motor reference trajectory has undesirable behavior as shown in Figure 3.4. Assuming a desired configuration of the swing leg is \( q_{H,ref} = 180^\circ \) and \( q_{K,ref} = -90^\circ \) results in motor angles of \( m_H = 180^\circ \) and \( m_K = -90^\circ \). The hip motor position is then changed to \( m_H = 200^\circ \), producing an error in hip motor tracking. Using Equation 3.59 the knee motor reference position is still \(-90^\circ\), resulting in joint angles of \( q_H = 200^\circ \) and \( q_K = -110^\circ \) as shown in scenario (b) of Figure 3.4. Thus, Equation 3.59 keeps the knee joint angle relative to the torso rather than the upper leg as desired and in this case would trigger an emergency-stop signal.
Figure 3.4. Comparison of calculating knee motor reference trajectories.

The desired joint position is as shown in (a) \(q_H = 180^\circ, q_K = -90^\circ, \ m_H = 180^\circ, \) and \(m_K = -90^\circ\). When a hip motor error of \(20^\circ\) is introduced by \(m_H = 200^\circ\), (b) is the undesirable result of using Equation 3.59 for knee motor reference \((q_H = 200^\circ, \ q_K = -110^\circ, \ m_H = 200^\circ, \) and \(m_K = -90^\circ)\), while (c) is the desired result \((q_H = 200^\circ, \ q_K = -90^\circ, \ m_H = 200^\circ, \) and \(m_K = -70^\circ)\) using Equation 3.60.

because \(q_K\) is less than \(-90\) degrees. Modifying the knee reference to use the actual hip motor position results in

\[
m_{K,ref} = (q_{K,ref} + m_H - \pi).
\] (3.60)

Using Equation 3.60 when \(m_H = 200^\circ\) results in a knee joint angle of \(q_K = -90^\circ\), as shown in scenario (c) of Figure 3.4. This keeps the knee joint angle relative to the upper leg as desired, is the way the knee reference trajectory was calculated in [43, 72], and is successful in overground walking experiments.
Motor and torso velocities were estimated using a 5-point numerical differentiator applied to the encoder outputs. Details of the estimation are in [9], and this is the same velocity estimation technique used on RABBIT [3]. An attempt was made to use the velocity observer from [17] to improve estimates immediately following impacts. However, the implementation proved to be too computationally expensive to compute the impact map online as needed.

Thus, the motor control law in experiment is

\[ u_i = -K_P(e_i) - K_d\dot{\hat{m}}_i, \]

(3.61)

where \( u_i \) is the voltage command sent to the motor amplifier, the proportional feedback is a function of the motor position error \( e_i \), and \( \hat{m}_i \) is the observed motor velocity. The ratio between motor command voltage and gearhead output torque is estimated as 10.2 Nm/volt, the same as in [72]. The proportional feedback function is

\[ K_P(e_i) = \begin{cases} 
K_{P1}e_i & \text{for } e_i \leq e_i,\text{threshold} \\
K_{P1}e_i,\text{threshold} + K_{P2}(e_i - e_i,\text{threshold}) & \text{for } e_i > e_i,\text{threshold} 
\end{cases} \]

(3.62)

which splits the proportional feedback at a threshold value. This function was available in [72], but for most results therein, \( K_{P1} = K_{P2} \), which reduces to simple proportional feedback. In [43], many trials set the gains such that \( K_{P2} \) is greater than \( K_{P1} \) and the threshold value is 4 degrees. For ground walking, the threshold value of 4 degrees was kept, but tuning resulted in \( K_{P1} \) greater than \( K_{P2} \). Tuning was done systematically by raising both proportional gains together until joint oscillations were visible, then slightly backing off the proportional gains. Next \( K_{P2} \), was increased to mimic the tuning from [43] with the thought that a large error should result in a larger control action. Resulting behavior, however, suggested that having a high gain far from the set point causes too much overshoot. Instead, with \( K_{P2} \)
greater than $K_{P_1}$, the overshoot behavior is largely eliminated while a higher gain holds to the set point more stiffly once inside the threshold. The derivative feedback term is not true derivative feedback, as in simulation, but is instead a damping term. This is typical of real-world derivative feedback in a system with poor signal-to-noise ratios and quick, large changes in set point. The derivative term in experiment is also low-pass filtered with a time constant of 0.02 s to remove some noise. It was attempted to implement true derivative feedback to be more faithful to the control in simulation and ideally obtain better performance, but this was unsuccessful because the observed velocity signal is not smooth enough. Increasing the number of points used in the observer or increasing the filtering time constant would increase the signal smoothness but introduce delay that is highly undesirable in a system with a quick-moving set point. Future work, however, could consider increasing the filter time constant and updating its contribution to the control action at a lower frequency. (A similar action was successful for the disturbance rejection controller in Chapter 5 but was unknown for these experiments and not implemented to preserve comparison between the control methods).

It was also attempted to implement the simulation feedforward term from Equation 3.40 $L_f L_g h^{-1}(L_j^2 h)$, by fitting this torque as a function of $\theta$. To properly implement feedforward control, however, a very accurate model is needed, including friction parameters and experimentally determined motor torque constants, as those from motor datasheets are for the ideal case and usually do not include gearhead inefficiencies. It was attempted to do a system identification procedure similar to that in [50], but the coupled drivetrain of ERNIE prevented reliably isolating parameters for identification. Assuming 100% efficiency and zero friction, the feedforward control was not reliable and thus not used.

Although ERNIE does not have dominant rotor inertia terms and feedforward control was not successful, high gain PD control on the motor positions effectively
TABLE 3.2

FEEDBACK CONTROL PARAMETERS FOR EXPERIMENTS WITH ERNIE.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{P1}$</td>
<td>50</td>
</tr>
<tr>
<td>$K_{P2}$</td>
<td>35</td>
</tr>
<tr>
<td>$e_{i, \text{threshold}}$</td>
<td>4.0 deg</td>
</tr>
<tr>
<td>$K_D$</td>
<td>1.5</td>
</tr>
<tr>
<td>$L_i \text{ point, curved}$</td>
<td>0.71, 0.73</td>
</tr>
</tbody>
</table>

enabled stable walking in experiment. The final feedback gain values used were as shown in Table 3.2.

3.5.3 Torso Offset

Also due to small differences between the simulation model and the experimental hardware, a torso offset was sometimes needed to achieve stable walking, as was the case in [72]. This was implemented as a constant addition to the desired angle of $q_5$ over a step, resulting in the torso pitching farther forward.

3.6 Performance Measures

The performance of the experiments was measured by energy consumption and trajectory tracking. First, efficiency was measured by specific resistance (Eq. 3.48).
but using

\[ W_{\text{total,exp}} = \sum_{i=1}^{4} \int_{t_{f}}^{t_{i}} |\tau_{i}\dot{m}_{i}| \, dt, \]  

(3.63)

where \( \dot{m}_{i} \) and \( \tau_{i} \) are the motor velocities and torques, respectively.

In addition, the errors in tracking the desired trajectories were also considered. Previous results reported only the maximum motor errors for hips and knees during an experimental trial [73]. Although max errors are important, it is also desirable to know how well the gait is tracked over the entire cycle. Thus, errors were measured in two ways, maximum error during the trial and an average RMS error,

\[ E_{\text{RMS}} = \frac{1}{M} \sum_{j=1}^{M} \sum_{k=1}^{N_{j}} \sqrt{\frac{\sum e_{k}^{2}}{N_{j}}}, \]  

(3.64)

where \( e_{k} \) is the motor or torso error at sample \( k \) of step \( j \), with \( M \) steps of \( N_{j} \) samples each. These measures give a succinct indication of the trajectory tracking ability of the control scheme.

3.7 Results and Discussion

The implementation from the previous section was used to conduct both point- and curved foot gait experiments. Data were collected over one complete lap of the room, which corresponds to a linear walking distance of approximately 18.2 m. Although every effort was made to keep the floor level and the center column vertical, there are relatively small, but consistent speed fluctuations as ERNIE walks around the room. Averaging values over one lap helps eliminate this source of noise and error. The number of steps required to walk once around the room were in the range of 34 to 45 steps for curved feet and 41 to 49 for point feet. The range of step number is an indication of the range of step lengths examined in experiment. Thus, the lower number of curved foot steps to complete one lap indicates that curved foot step
lengths were typically longer than the point-foot step lengths. Experimental results are drawn only from gaits that completed at least one lap of the room, requiring more robust gaits. The following experimental data is a subset of all data obtained for walking and represents the gaits with the lowest specific resistance values at particular speeds for both point and curved feet.

Fig. 3.5 shows the calculated specific resistance values versus average step velocity for both curved and point-foot experiments. Point-foot experimental trials were run with simulation gait profiles optimized for speeds of 0.40 to 0.60 m/s in 0.05 m/s increments, yet the actual speed range in experiment for point-foot gaits was 0.47 to 0.76 m/s. The speed at which a gait walked in experiment was not necessarily consistent with its designed speed, but most gaits walked faster in experiment than in simulation. The experimental speed was greatly influenced by the aforementioned torso offset, which was systematically increased from zero until a given gait could complete a lap of the room. Given the variability in resultant speed of point-foot gaits, curved foot gaits were designed for speeds of 0.40 to 0.70 m/s in 0.10 m/s increments, and the actual speed range in experiment for these curved foot gaits was 0.48 to 0.81 m/s. Curved foot gaits transferred more reliably from simulation to experiment than did point-foot gaits, as indicated by the more stringent constraints required to generate the point-foot gaits that were successful in experiment. The authors hypothesize that the larger impacts associated with point-foot gaits explain the greater difficulty in experimental implementation because at higher speeds, point-foot experiments failed due to post-impact stance-leg errors in excess of 30 degrees, which exceeded safety limits and required termination of the experiment.

Overall, curved feet are clearly more efficient than point feet for the range of speeds studied (Fig. 3.5). For a given speed, all of the experimental curved foot gaits had smaller specific resistance values than point-foot gaits. This is consistent with the predicted simulation efficiency results as shown in Fig. 3.6. Comparing Figs. 3.5 and 3.6, the specific resistance values for curved foot gaits are consistently lower than those for point-foot gaits across the range of speeds studied.
and [3.6], the trends from experiment are close to the trends from simulation, but the values are almost an order of magnitude larger, similar to the results in [72]. This discrepancy in magnitude occurs because of the ideal actuators, absence of backlash, and lack of error in the simulation.

![Graph showing specific resistance values versus gait speed for curved and point-foot gaits.](image)

Figure 3.5. Experimental specific resistance values versus gait speed for curved and point-foot gaits. The joint trajectories of gaits marked with an '*' are compared in detail in Fig. 3.9.

Fig. 3.7 shows the maximum error during a trial versus step velocity for both point and curved feet. For point feet, the stance hip generally exhibited the maximum error,
Figure 3.6. Simulated specific resistance values versus gait speed for curved and point-foot gaits. The joint trajectories of gaits marked with an ‘*’ are compared in detail in Fig. 3.9 but occasionally the torso angle exhibited the maximum error. For curved feet, the torso angle exhibited the maximum error except for a few cases in which the swing hip did. As expected, there is a trend of increasing error with increasing speed for both point and curved feet, because faster speeds require more motor power and have larger impacts. In all but one case, point-foot gaits have larger errors than curved foot gaits at similar speeds. This trend is also present when considering RMS errors as shown in Fig. 3.8. For both point and curved feet, the stance hip exhibited the maximum RMS error except for a few cases in which the torso angle exhibited the maximum error. Similar to the maximum error, there is an increase in RMS error
Figure 3.7. Maximum error for motor or torso tracking during experimental trial. The joint trajectories of gaits marked with an ‘*’ are compared in detail in Fig. 3.9

as step velocity increases. Figure 3.8 also shows that the minimum RMS errors for point and curved feet were roughly equal at the same speeds. The joint with minimum RMS error for point feet was the stance or swing knee, while the joint with minimum RMS error for curved feet was the swing knee. In nearly all trials, it appears that in addition to having improved energy efficiency, the curved foot gaits also have lower error states than the point-foot gaits, although at lower speeds the differences are less significant.

For an in-depth comparison, a point-foot gait and a curved foot gait both designed for a speed of 0.60 m/s were compared. In experiment, the average velocity of the
Figure 3.8. Maximum and minimum RMS error for motor and torso tracking.
point-foot and curved foot gaits were 0.57 m/s and 0.61 m/s, respectively. The corresponding efficiency and error data for these gaits are shown with an additional asterisk symbol in Figs. 3.5-3.8. Table 3.3 shows the individual motor and torso errors for these gaits. Note that the errors are larger for the stance leg joints than for the swing leg joints due to the increased loads on the stance joints. For the point-foot gait, the maximum motor and torso errors were in the range of 4.1 to 14.1 degrees, and the average RMS errors range from 1.9 to 5.4 degrees. For the curved foot gait, the maximum errors range from 5.3 to 10.0 degrees and the average RMS errors from 2.2 to 4.3. The most significant improvement in error tracking for curved feet over point feet for these gaits is tracking the desired torso angle. This is consistent across the rest of the trials as well.

Plots comparing the actual joint trajectories of these gaits and their reference values over the step progression are shown in Fig. 3.9. The most significant difference in gait trajectory between the point- and curved foot gaits is the motion of the stance knee. For the curved foot gait, the optimized motion is almost static, in contrast to the mid-cycle extension favored by the point-foot gait. This is directly related to the rolling motion of the curved foot shifting the hip position forward, while with the point foot, knee extension is required to do so.

As one would expect, the virtual stance knee angle (the angle formed between the upper leg and the vector from the knee to the ground-contact point) of the curved foot gait more closely resembles the stance knee motion of the point-foot gait, particularly for the first half of the gait cycle as shown in Figure 3.10. The gait characteristics explained here are representative of most other experiments as well.

The estimated torque profiles for these gaits were as shown in Figure 3.11. To find torque in experiment, the amplifier current during the trial is recorded, and an ideal torque constant of 3.731 Nm/amp is used, where gearbox inefficiencies are ignored.
TABLE 3.3

COMPARISON OF ERRORS FOR POINT- AND CURVED FEET WALKING AT SPEEDS OF 0.57 AND 0.61 M/S

<table>
<thead>
<tr>
<th></th>
<th>Stance</th>
<th>Swing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hip</td>
<td>Knee</td>
</tr>
<tr>
<td>Point max</td>
<td>13.7</td>
<td>6.9</td>
</tr>
<tr>
<td>Curved max</td>
<td>8.3</td>
<td>6.6</td>
</tr>
<tr>
<td>Point RMS</td>
<td>5.4</td>
<td>2.6</td>
</tr>
<tr>
<td>Curved RMS</td>
<td>4.3</td>
<td>2.6</td>
</tr>
</tbody>
</table>

Limits were set on the motor commands such that ideal torque saturated at approximately ±40 Nm to prevent burning out the motors. These limits were set based on the original motors and not updated for the new larger capacity gearheads. Referring back to Table 2.2, this limit is well beyond the intermittently permissible torque for the old gearheads but within the range for the new ones. This could have contributed to the breakdown of the old gearheads or indicate that the limits on the motor commands can be increased with the new gearheads. Being conservative, the limits from the original design were used but could possibly be increased in the future. For both point- and curved foot walking, the most significant deviations from simulated torque values are just after impact up to approximately 20% of the gait cycle, where the torque magnitude is much larger as this is the weight acceptance phase mentioned in Section 3.5. For both point- and curved foot walking, trends in swing leg torque match simulation. For stance leg torques, point-foot trends are a worse match than curved foot trends. In both cases, torque limits were not implemented in simulation because the torques in simulation are mostly reasonable, and most gaits successful in
Figure 3.9: Comparison of joint trajectory tracking (degrees) for point-feet (top) and curved feet (bottom) for similar speeds (0.57 & 0.61 m/s). Data is color coded to indicate which leg was the stance leg for a step.

Simulation were successful in experiment. Additionally, with some error between the model and experiment and uncertainty of how much motor overloading is acceptable, an accurate limiting value is thus far undetermined. Future work could find an ideal value or investigate whether extremely aggressive torque limits would optimize to significantly different gait patterns and the resulting impact on gait robustness and
efficiency.

These results validate that HZD-based control is possible for curved foot bipeds. Not only is it possible, but overall for this hardware, curved foot gaits had better performance in terms of both efficiency and trajectory tracking than point-foot gaits over the range of speeds investigated. This significantly expands the design space for robots that can be controlled with HZD-based control while also having improved performance.

3.8 Extended Curved Foot Results

Additional curved foot experiments were conducted that are not directly comparable to the point-foot results. These experiments are collected below.
3.8.1 Self-Optimized Gaits

In the process of developing the optimization constraints, much time was spent hand-tuning the polynomial coefficients of Equation \ref{eq:3.35}. This was done because the optimization was converging to gaits with very bent legs that were not very successful in experiment and had problems with foot scuffing. In tuning these gaits, the only constraints that were definitely enforced were that the trajectories be invariant through impacts and the joint angles be reasonable. This was an iterative process between defining the trajectories and testing on the robot. Several successful gaits were found with this method; however, these gaits walked much faster than their designed speed. These gaits had much lower specific resistance (Equation \ref{eq:3.48}) than those from optimization, as shown in Figure \ref{fig:3.12}. The same was not necessarily true in simulation, as shown in Figure \ref{fig:3.13}. One possible explanation is that these gaits violate some constraint that is too aggressive in the optimization routine. Another possibility is these gaits are at a local minimum that the optimization has not found. The joint trajectories and torque profile for one of these gaits that walked at 0.60 m/s is shown in Figures \ref{fig:3.14} and \ref{fig:3.15}. The torques for this gait appear to be much closer to zero than those in Figure \ref{fig:3.11}, which partially explains the lower specific resistance.

3.8.2 Curved Foot Walking up to 1.0 m/s

There was much difficulty finding point-foot gaits at speeds faster than about 0.60 m/s. Gaits designed for curved feet at 0.70 m/s, however, were successful in experiment. For one of these gaits, stable walking was found with a torso offset of 2 degrees. The offset was increased in 2 degree increments up to 12 degrees, and data were recorded. As the torso offset was increased, the gait continued to walk around the room, but joint errors at these speeds were large, and the gait became evidently two-step periodic. The specific resistance of these gait trials is shown in Figure \ref{fig:3.16}.
A plot of the gait joint trajectories and torque profiles at the fastest speed of 1.0 m/s is as shown in Figures 3.17 and 3.18. Although the resultant gait was not ideal, several important facts about the hardware can be gleaned. First, offsetting the torso affects gait speed in a consistent way. A very large torso offset makes a gait walk faster than designed, but does not necessarily cause failure. This is important, and points toward using the torso offset to automatically regulate gait speed. It may even be possible to automate the torso angle to recover from a sudden velocity change, such as a disturbance. This has essentially already been done manually and on a slower time scale when the experiment procedure calls for torso offset to get a gait to walk.

3.9 Summary and Future Work

In nearly all cases, curved foot gaits had better performance in terms of decreased specific resistance and smaller joint errors compared to point-foot gaits at similar speeds. As the gait speed increased, the energy savings of curved feet over point feet was more apparent, likely due to the energy saving effect of curved foot impacts.

Having some backlash in the drivetrain between the motor and joint probably contributes to the noise in specific resistance results, and finding a way to decrease the backlash could make trends between point- and curved foot performance more clear. However, backlash is unavoidable in a real system. The robot designed in [16] went to great lengths to eliminate backlash and identify system friction parameters to match the simulation and hardware as closely as possible. This work shows that reasonable performance from this theoretical control theory can still be obtained with a reasonably simple model of the hardware, even when the hardware has real-world characteristics, such as backlash, that are beyond the scope of the theoretical control theory.

The optimization constraints used to generate experimentally feasible walking
gaits could be adjusted because better performance was obtained by tuning gaits by hand. Identifying the constraints violated by the hand-tuned gaits could be investigated as a first step. Additionally, a different optimization routine may be better suited for obtaining desired results.

The cost function Eq. 3.48 is only a function of the continuous dynamics of the system. Since it is believed that the impacts play a large role in the difference between point- and curved foot performance, future work could consider a cost function that tries to reduce impact losses. Some loss at impact helps stabilize the gait, but the work of [1] could be considered to add a component to the cost function that is a function of the direction change between pre- and post-impact velocity.
Figure 3.11. Comparison of point-foot (top two rows) and curved foot (bottom two row) estimated motor torques for similar speeds (0.57 & 0.61 m/s).
Figure 3.12. Specific resistance for hand-picked curved foot gaits compared to the computer optimized gaits.

Figure 3.13. Simulated specific resistance for hand-picked curved foot gaits compared to the computer optimized gaits.
Figure 3.14. Joint trajectory tracking (degrees) for a hand-picked curved foot gait at 0.60 m/s. Data is color coded to indicate which leg was the stance leg for a step.
Figure 3.15. Estimated motor torques for the hand-picked curved foot gait at 0.60 m/s (Figure 3.14)
Figure 3.16. Specific resistance for a curved foot gait varying torso offset from 2 to 12 degrees in 2 degree increments, compared to the gaits from Figure 3.5.

Figure 3.17. Joint trajectory tracking (degrees) for an evidently two-step periodic curved foot gait at 1.00 m/s. Data is color coded to indicate which leg was the stance leg for a step.
Figure 3.18. Estimated motor torques for an evidently two-step periodic curved foot gait at 1.00 m/s (Figure 3.17)
4.1 Motivation

A limitation of the HZD-based control approach is that there is no control mechanism to stabilize from velocity disturbances to the HZD orbit. For example, if an impulsive force pushes the state of the robot off the zero dynamics orbit, the current control formulation does not have a feedback component to guide the trajectory back to the zero dynamics orbit. If the impulsive force is large enough to cause the zero dynamics velocity \(\dot{\theta}\) to become negative, the robot would follow the cycle in reverse and fall back. Thus, there is a need to find a controller that can reject velocity disturbances and drive the walking motion back to the desired cycle. This can be thought of as a funnel of states around the \(\{\theta, \dot{\theta}\}\) trajectory that can be controlled back to the periodic trajectory.

Two controller designs in two different sets of coordinates were compared for accomplishing this task. The first set of coordinates are the conventional position and velocity coordinates used in Chapter 3. The second set of coordinates is an alternate formulation of the equations of motion with differential geometry-inspired quasi-velocity coordinates developed by [48]. These coordinates attempt to characterize the nonlinear coupling between links of this robot, and observing these coupling terms may produce rules for appropriate corrective control actions. The first controller design is partial feedback linearization as in Chapter 3. The second controller design
is an orbit-stabilizing control method first developed by [56] and applied to walking robots by [39]. This method uses a modified LQR controller with error dynamics to stabilize a funnel of trajectories around a desired trajectory. Manchester [39] briefly mentions how to possibly integrate this approach with HZD, and his work is modified and fully worked out here. Thus, there are three new conditions to compare to the existing approach of partial feedback linearization in conventional coordinates.

These approaches were first considered on a model problem, the inverted pendulum cart. This eliminates the hybrid aspect of the problem—foot impacts—reducing the complexity of the control problem while still retaining enough complexity to evaluate the methods.

4.2 Inverted Pendulum Cart

Stabilizing the inverted pendulum cart is a standard control problem and was a useful simple system for investigating new control schemes to be implemented on ERNIE. Instead of stabilizing the pendulum about the unstable upright equilibrium, the desired system behavior is a periodic orbit around that upright equilibrium. In this way, the inverted pendulum cart is a reasonable approximation for the biped ERNIE because both systems are underactuated by one DOF and the desired motion is a periodic orbit. The main difference is that the inverted pendulum cart does not have any impact events.

4.2.1 Dynamics

The system is shown in Figure 4.1. For simplicity, the cart mass $M$, pendulum mass $m$, and pendulum length are all assumed to be 1. Then the equations of motion
for the system can be written

\[
2\ddot{q}_1 + \cos(q_2)\ddot{q}_2 - \sin(q_2)\dot{q}_2^2 = u
\]  
(4.1)

\[
\cos(q_2)\ddot{q}_1 + \dot{q}_2 - g \sin(q_2) = 0,
\]  
(4.2)

where \(q_1\) is the horizontal displacement of the cart, \(q_2\) is the angle of the pendulum from vertical, \(g\) is the gravitation constant, and \(u\) is the input force applied to the cart in the horizontal direction. These equations are commonly written in matrix form

\[
M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G = Bu,
\]  
(4.3)

where

\[
M = \begin{bmatrix}
2 \cos(q_2) \\
\cos(q_2) & 1
\end{bmatrix} \quad (4.4)
\]

\[
C = \begin{bmatrix}
0 & -\dot{q}_2 \sin(q_2) \\
0 & 0
\end{bmatrix} \quad (4.5)
\]

\[
G = \begin{bmatrix}
0 \\
-g \sin(q_2)
\end{bmatrix} \quad (4.6)
\]

\[
B = \begin{bmatrix}
1 \\
0
\end{bmatrix}.
\]  
(4.7)

Note that the matrix \(C\) is not unique and Equation \([4.5]\) is one valid possibility. \(M\) is called the mass matrix and is of importance for the coordinate transformation. The system potential

\[
V = g \cos(q_2)
\]  
(4.8)

is also needed for the coordinate transformation. These 2nd order equations were
further manipulated into a system of coupled first order equations

\[
\dot{x} = F(x) + G(x)u, \quad \text{(4.9)}
\]

where \( x = [q_1, q_2, \dot{q}_1, \dot{q}_2]^T \) and

\[
F(x) = \begin{bmatrix}
\dot{q} \\
-M^{-1}(C\dot{q} + G)
\end{bmatrix}, \quad \text{(4.10)}
\]

\[
G(x) = \begin{bmatrix}
0 \\
M^{-3}B
\end{bmatrix}. \quad \text{(4.11)}
\]

4.2.2 Dynamics in Quasi-Velocity Coordinates

An alternate description of the system dynamics was constructed by following the work of Nightingale [48]. This formulation uses quasi-velocity coordinates derived from viewing the system through the lens of differential geometry. Instead of describing the system states by the positions \( q \) and the velocities \( \dot{q} \), velocities \( w \) and \( s \) are used instead, where \( w \) is the velocity in a direction parallel to the control input
and \( s \) is the velocity in a direction orthogonal to the input.

The resulting formulas for the system dynamics are presented here, while the theory behind them is left to [48]. The required elements from the usual representation are the mass matrix \( M \), potential function \( V \), and the input matrix \( B \).

Two operations need to be defined to construct these coordinates, and the first is the inner product in this representation. Given a mass matrix \( M \), the inner product of the coordinate representation of vector fields \( X \) and \( Y \) is

\[
\langle\langle X, Y \rangle\rangle = X^T M Y.
\]  

(4.12)

The symmetric product of vector fields \( X \) and \( Y \) is the vector field defined by

\[
\langle X : Y \rangle = \nabla_X Y + \nabla_Y X,
\]  

(4.13)

where \( \nabla_X Y \) is the covariant derivative of the vector field \( Y \) with respect to \( X \). Further details of these operations are found in [48, 49]. The vector field \( Y^* \) aligned with the system input is

\[
Y^* = M^{-1} B;
\]  

(4.14)

however, the formulation requires the vector field to be normalized, \( \langle Y : Y \rangle = 1 \). Thus, the vector field \( Y \) is found as

\[
Y = \frac{Y^*}{\sqrt{\langle\langle Y^*, Y^* \rangle\rangle}}.
\]  

(4.15)

The uncontrollable velocity is in the direction of the vector field orthogonal to \( Y \) with respect to the kinetic energy matrix \( M \) and is also normalized. Examining \( B \),
an orthogonal field is found by inspection, \( B_\perp = [0 \ 1]^T \), and

\[
Y_\perp = \frac{B_\perp}{\sqrt{\langle\langle B_\perp, B_\perp \rangle\rangle}}. \tag{4.16}
\]

The vector fields \( Y \) and \( Y_\perp \) fully span the velocity space and are related to \( \dot{q} \) by

\[
\dot{q} = wY + sY_\perp. \tag{4.17}
\]

The magnitude of the controllable velocity \( w \) is found from taking the inner product of \( Y \) with Equation 4.17 and rearranging,

\[
w = \frac{\langle\langle Y, \dot{q} \rangle\rangle}{\langle\langle Y, Y \rangle\rangle}. \tag{4.18}
\]

The magnitude of the uncontrollable velocity \( s \) is found similarly using the inner product of \( Y_\perp \) and Equation 4.17,

\[
s = \frac{\langle\langle Y_\perp, \dot{q} \rangle\rangle}{\langle\langle Y_\perp, Y_\perp \rangle\rangle}. \tag{4.19}
\]

The rate of change of \( s \) with respect to time is

\[
\frac{ds}{dt} = -\frac{1}{2} [w \ s] \mathbf{Q}^s \begin{bmatrix} w \\ s \end{bmatrix} - \langle\langle \text{grad} \ V, Y_\perp \rangle\rangle, \tag{4.20}
\]

where

\[
\mathbf{Q}^s = \begin{bmatrix} \langle\langle Y : Y \rangle, Y_\perp \rangle & \langle\langle Y : Y_\perp \rangle, Y_\perp \rangle \\ \langle\langle Y_\perp : Y \rangle, Y_\perp \rangle & \langle\langle Y_\perp : Y_\perp \rangle, Y_\perp \rangle \end{bmatrix}, \tag{4.21}
\]

and

\[
\text{grad} \ V = M^{-1} \frac{dV}{dt}. \tag{4.22}
\]

It is important to notice that the control input \( u \) does not appear in \( \frac{ds}{dt} \) since
\( \langle Y^*, Y_\perp \rangle = 0 \). Thus, the only way the control input can influence the \( s \) dynamics is through the coupling terms \( Q^s \). The rate of change of \( w \) is found similarly

\[
\frac{dw}{dt} = -\frac{1}{2} \begin{bmatrix} w \\ s \end{bmatrix} Q^w \begin{bmatrix} w \\ s \end{bmatrix} - \langle \langle \text{grad } V, Y \rangle \rangle + \langle \langle Y^*, Y \rangle \rangle u,
\]

\( (4.23) \)

where

\[
Q^w = \begin{bmatrix} \langle \langle \langle Y : Y \rangle, Y \rangle \rangle & \langle \langle Y : Y_\perp \rangle, Y \rangle \rangle \\ \langle \langle Y_\perp : Y \rangle, Y \rangle \rangle & \langle \langle Y_\perp : Y_\perp \rangle, Y \rangle \rangle \end{bmatrix}.
\]

\( (4.24) \)

Using these quasi velocities, converting to coupled first order equations, and manipulating to the same form as Equation \( 4.9 \), the dynamics are

\[
\dot{x} = \hat{F}(\hat{x}) + \hat{G}(\hat{x}) u,
\]

\( (4.25) \)

where \( \hat{x} = [q_1, q_2, w, s]^T \) and

\[
\hat{F}(\hat{x}) = \begin{bmatrix} wY + sY_\perp \\ \hat{F}_3(\hat{x}) \\ \hat{F}_4(\hat{x}) \end{bmatrix},
\]

\( (4.26) \)

\[
\hat{G}(\hat{x}) = \begin{bmatrix} 0_{2 \times 1} \\ \hat{G}_3(\hat{x}) \\ 0 \end{bmatrix}.
\]

\( (4.27) \)

To verify that these these alternate dynamic descriptions result in identical dynamic responses, the two descriptions were coded in MATLAB using \texttt{ode45}, and responses were found for the naive time-varying control input of \( u = 5 \cos (2\pi t) \).

Simulations were run for \( t = 5.0 \) s, and results were as shown in Figure \( 4.2 \). Equation \( 4.17 \) was used to convert the output of Equation \( 4.25 \) from quasi- to actual velocities. The systems are indeed identical to numerical tolerances, confirming that the
Figure 4.2. Comparison of the alternate coordinate descriptions. The output of the $s$ and $w$ simulation was converted to $\dot{q}_1$ and $\dot{q}_2$ for plotting.

coordinate transformation was valid.

4.2.3 Control

Although there has been much research in various methods for controlling the inverted pendulum cart system, only two are considered here. The first is PD control after performing a change of coordinates via partial-feedback linearization, which is the foundation of HZD-based control used on biped robots. The second approach is to add the orbital stabilization method from [56] to the first. The goal is to make it clear that the orbit stabilizing control has potential for application to biped robots.

4.2.3.1 Partial-Feedback Linearization and PD Control

To construct the controller for this section, the virtual holonomic constraint enforced by the resulting control system needs to be determined. This constraint was a function of the position variables only, consistent with HZD. For the inverted pen-
dulum cart system, the constraint enforced was

$$H(q) = q_1 + L \sin q_2 = 0. \quad (4.28)$$

In words, this constraint requires a point $L$ on the pendulum to stay on the vertical line $x = 0$ while the cart oscillates periodically. This virtual constraint results in different dynamics than an actual physical constraint. The following derivations are a combination of the work of [56] and [66]. First it was assumed that there is a static feedback control law that ensures that the constraint Equation 4.28 is invariant. Then the dynamics of this closed loop system are written

$$\ddot{q}_1 + L \cos q_2 \ddot{q}_2 - L \sin q_2 \dot{q}_2^2 = 0 \quad (4.29)$$
$$\cos q_2 \ddot{q}_1 + \ddot{q}_2 - g \sin q_2 = 0, \quad (4.30)$$

where Equation 4.29 is found from differentiating Equation 4.28 twice, and Equation 4.30 is a repeat of Equation 4.2. These two equations are combined by eliminating $\ddot{q}_1$,

$$(1 - L^2 \cos q_2)\ddot{q}_2 + L \cos q_2 \sin q_2 \dot{q}_2^2 - g \sin q_2 = 0. \quad (4.31)$$

This equation represents the zero dynamics of the system, which are the orbits of $q_2$ that the system follows for a given $L$. Figure 4.3 shows several of these orbits. Note that this equation is in the form

$$\alpha(\theta)\ddot{\theta} + \beta(\theta)\dot{\theta}^2 + \gamma(\theta) = 0, \quad (4.32)$$

with $\theta = q_2$. This form of the zero dynamics is useful in future sections.

It is desired to construct the controller to follow one of the level curves from Figure 4.3. The first step to accomplish this requires partial feedback linearization of
Figure 4.3. Several level curves ($\dot{\theta}$ versus $\theta$) of the virtual system for the inverted pendulum cart system.
the system. The constraint Equation 4.28 was considered as an output of the system to be zeroed,

\[ y = q_1 + L \sin q_2 = 0 \]
\[ y = q_1 - \phi(q_2) = 0 \] (4.33)
\[ H = x_1 - \phi(x_2) = 0. \] (4.34)

Differentiating this equation twice results in

\[ \dot{y} = L_f H = 0 \] (4.35)
\[ \ddot{y} = L_f^2 H + L_g L_f H u = 0, \] (4.36)

where

\[ L_f H = \frac{\partial H}{\partial x} F(x) \] (4.37)
\[ L_f^2 H = \frac{\partial L_f H}{\partial x} F(x) \] (4.38)
\[ L_g L_f H = \frac{\partial L_f H}{\partial x} G(x). \] (4.39)

The input \( u \) of the system is chosen as

\[ u^* = (L_g L_f H)^{-1} (v - L_f^2 H). \] (4.40)

This choice results in the system

\[ \ddot{\theta} = M(\theta, \dot{\theta}, y, \dot{y}, v) \] (4.41)
\[ \ddot{y} = v. \] (4.42)

An appropriate choice of \( v \) will enforce the constraint and stabilize the system as long
as the $\theta$ dynamics (zero dynamics) are also stable. One such choice is $v = -K_p y - K_d \dot{y}$, which is clearly globally stable for $y$.

The goal is to zero the output $y$, enforcing the desired virtual constraint. Since there are several orbits that would enforce the constraint, one is selected as the desired orbit. The orbit selected has initial conditions of $\theta = 0.1$, $\dot{\theta} = 0$, and is shown in red on Figure 4.3. An approximation of the orbit is

$$\theta_d(t) = 0.1 \sin \omega t$$

$$\dot{\theta}_d(t) = 0.1 \omega \sin \omega t,$$

where $\omega = \frac{2\pi}{1.41}$. This was found by a least squares fit to the numerical integration of the zero dynamics, Equation 4.32.

When starting the system on the desired orbit, the PD controller was successful in maintaining the desired orbit. However, if initial conditions or a disturbance cause the pendulum to leave the orbit, the specified controller does not correct for the disturbance and return to the orbit as shown in Figure 4.4. Initially, this may appear to be just non-zero steady-state error, which is what one would expect without integral control. However, once each trajectory in Figure 4.4 reaches a stable orbit, Equation 4.34 is zeroed because any level curve satisfies the constraint. Thus, this feedback control is not able to drive the system to the desired orbit because it has no indication that it is off the desired orbit. This is similar to the problem with regulating the speed of an step with a biped robot under HZD-based control. If a biped under HZD-based control is pushed off the desired hybrid orbit in such a way that the step velocity is increased, the biped will continue to walk at a faster speed until foot impacts gradually slow the robot back to the desired orbit. If the push forward is large enough, the biped will fail to complete the step and fall forward because the swing leg will not have enough time to move to the desired position. A
Figure 4.4. PD control fails to stabilize the inverted pendulum cart system to the desired orbit (red) when started at initial conditions off the desired orbit (blue circles).

similar situation occurs if the push off the orbit slows down the biped, but in this case, the push may be large enough that the biped fails to complete the step and falls backward.

4.2.3.2 Partial-Feedback Linearization and PD Control in Quasi-Velocity Coordinates

The feedback controller similar to that used in HZD-based control was also constructed for the \( \hat{x} \) coordinates. The difference is in finding \( \dot{y} \). The desired virtual constraint is differentiated as in Section 4.2.3.1 and found in the alternate coordinates
by substituting Equation 4.17 into Equation 4.35.

\[
\dot{y}_2 = L\hat{f}H \\
= \frac{\partial H}{\partial \hat{x}} F(\hat{x}) \\
= sL\cos(\theta) - \frac{w}{\sqrt{1 - \cos(2\theta)}} \frac{L\cos(2\theta) + L - 2}{\sqrt{2}}. 
\] (4.47)

Similarly,

\[
\ddot{y}_2 = L^2\hat{f}H + L\hat{g}\dot{L}\hat{f}Hu \\
= \frac{\partial L\hat{f}H}{\partial \hat{x}} F(\hat{x}) + \frac{\partial L\hat{f}H}{\partial \hat{x}} G(\hat{x})u. 
\] (4.49)

Note that a virtual system can be constructed in the new coordinates similar to Equation 4.32. In this case, the constraint only need be differentiated once since the time derivative of \(w\) does not show up in the \(s\) dynamics. The reduced system in these coordinates is found by solving Equation 4.47 for \(w\), assuming \(\dot{y}_2 = 0\),

\[
w = \frac{sL\cos(\theta)}{\sqrt{\frac{1}{2 - \cos^2(\theta)} (L\cos^2(\theta) - 1)}}. 
\] (4.50)

and substituting into Equation 4.20. This results in

\[
\dot{s} + \beta_2(\theta)s^2 + \gamma_2(\theta) = 0, 
\] (4.51)

where

\[
\beta_2(\theta) = -\frac{\beta(\theta)}{\alpha(\theta)^2} \\
\gamma_2(\theta) = \gamma(\theta). 
\] (4.52)
This is the same form as Equation 4.32 with $\alpha_2 = 1$. It is not known if the equations for $\beta_2$ and $\gamma_2$ hold for all systems. Several orbits were plotted for this system as shown in Figure 4.5. Additionally, a $u^*_sw$ similar to Equation 4.40 was found for this system, resulting in the alternate dynamic description of the system,

$$\dot{s} = M_2(\theta, s, y_2, \dot{y}_2, v_2)$$  \hspace{1cm} (4.54)

$$\ddot{y} = v_2.$$  \hspace{1cm} (4.55)

It is worth noting that the decoupling matrix $L_gL_fH$ is identical in either set of coordinates and the other terms are equal after transforming the coordinates.

This control had similar performance as the PD control in the original coordinates. However, choosing constant gains on the feedback in these coordinates would be equivalent to choosing varying gains in the original coordinates. It may be possible to exploit this for better control, but this control also did not drive the system to the
desired orbit, while the orbit-stabilizing controller was a more clear and promising option for doing so.

In either set of coordinates investigated, another measure is needed for feedback to drive the system back to the desired orbit. As discussed in Section 4.2.3.3, the work of [56] uses an additional coordinate that is essentially a measure of the distance of the system state from the desired orbit to provide this feedback.

4.2.3.3 Orbit-Stabilizing Control

The following section presents recreation of the results of [56] for completeness. Recall that the zero-dynamics of the system (Equations 4.41 and 4.42) are written

\[ \alpha(\theta)\ddot{\theta} + \beta(\theta)\dot{\theta}^2 + \gamma(\theta) = 0. \]  

(4.56)

The key observation from [56] is that Equation 4.56 has a general integral of motion \( I(\theta, \dot{\theta}, \theta_0, \dot{\theta}_0) \) that is zero along the solutions of the zero dynamics \( \{\theta(t), \dot{\theta}(t)\} \) provided the initial conditions \( \{\theta_0, \dot{\theta}_0\} \) are chosen on that orbit. This coordinate is used to construct an almost linear system in coordinates \([I, y, \dot{y}]\) called the transverse dynamics [39]. If the transverse dynamics are controllable over a full period of the desired orbit, a modified LQR control design achieves a nonlinear time-varying feedback control law that locally exponentially orbitally stabilizes a chosen periodic motion.

A formula for the integral of motion \( I(\theta, \dot{\theta}, \theta_0, \dot{\theta}_0) \) was given by the following theorem.

**Theorem 1**[56]. Suppose the function \( \alpha(\theta) \) has only isolated zeros. If the solution \([\theta(t), \dot{\theta}(t)]\) of Equation 4.56 with initial conditions \( \theta(0) = \theta_0, \dot{\theta}(0) = \dot{\theta}_0 \) exists and is
continuously differentiable, then along this solution the function

\[ I(\theta, \dot{\theta}, \theta_0, \dot{\theta}_0) = \dot{\theta}^2 - \psi(\theta_0, \theta) \left[ \dot{\theta}_0^2 - \int_{\theta_0}^{\theta} \psi(s, \theta_0) \frac{2\gamma(s)}{\alpha(2)} ds \right] \]  \hspace{1cm} (4.57)

with

\[ \psi(\theta_0, \theta_1) = \exp \left\{ -2 \int_{\theta_0}^{\theta_1} \frac{\beta(\tau)}{\alpha(\tau)} d\tau \right\} \]  \hspace{1cm} (4.58)

preserves its zero value.

The proof starts by introducing the variable

\[ Y = \dot{\theta}^2(t). \]  \hspace{1cm} (4.59)

It is easy to see that

\[ \frac{dY}{dt} = \frac{d}{dt}(\dot{\theta}^2(t)) = 2\dot{\theta}\ddot{\theta} \]  \hspace{1cm} (4.60)

and

\[ \frac{dY}{dt} = \frac{dY}{d\theta} \frac{d\theta}{dt} = \frac{dY}{d\theta} \dot{\theta}. \]  \hspace{1cm} (4.61)

Therefore, along any solution of the dynamical system, Equation 4.56, the identity

\[ \ddot{\theta} = \frac{1}{2} \frac{dY}{d\theta} \]  \hspace{1cm} (4.62)

holds. Then, Equation 4.56 is equivalently rewritten

\[ \alpha(\theta) \frac{1}{2} \frac{d}{d\theta} Y + \beta(\theta) Y + \gamma(\theta) = 0. \]  \hspace{1cm} (4.63)

This differential equation for \( Y \) is linear, with \( \theta \) being an independent variable (instead of \( t \)).

Considering the case in which the function \( \alpha(\theta(t)) \) is different from zero along the
solution \( \{\theta(t), \dot{\theta}(t)\} \), Equation 4.63 is

\[
\frac{d}{d\theta} Y + \frac{2\beta(\theta)}{\alpha(\theta)} Y + \frac{2\gamma(\theta)}{\alpha(\theta)} = 0.
\]  

(4.64)

The general solution to Equation 4.64 is

\[
Y(\theta) = \psi(\theta_0, \theta) Y(\theta_0) - \psi(\theta_0, \theta) \int_{\theta_0}^{\theta} \psi(s, \theta_0) \frac{2\gamma(s)}{\alpha(s)} ds,
\]  

(4.65)

with \( \psi(.,.) \) defined as in Equation 4.58. It follows that along any solution of Equation 4.56 the function

\[
I(\theta, \dot{\theta}, \theta_0, \dot{\theta}_0) = \dot{\theta}(t)^2 - Y(\theta(t))
\]  

(4.66)

is identically equal to zero. The proof in [56] continues with the solution being continuous and bounded but is omitted here. For the inverted pendulum cart system, the integrals in Equation 4.57 were solved explicitly

\[
I = \dot{\theta}^2 - \dot{\theta}_0^2 \frac{1 - L \cos^2(\theta_0)}{1 - L \cos^2(\theta)} + 2g \frac{\cos(\theta) - \cos(\theta_0)}{1 - L \cos^2(\theta)}.
\]  

(4.67)

The time derivative of the function \( I \) is also needed and is defined in the following theorem.

**Theorem 2 [56].** With \( x \) and \( y \) being some constants, the time derivative of the function \( I(\theta, \dot{\theta}, x, y) \) defined by Equation 4.57 calculated along a solution \( \{\theta(t), \dot{\theta}(t)\} \) of the system

\[
\alpha(\theta) \ddot{\theta} + \beta(\theta) \dot{\theta}^2 + \gamma(\theta) = W
\]  

(4.68)

can be computed as

\[
\frac{d}{dt} I = \dot{\theta} \left\{ \frac{2}{\alpha(\theta)} W - \frac{2\beta(\theta)}{\alpha(\theta)} I \right\}.
\]  

(4.69)

The proof of this begins by finding the total derivative of the function \( I = \)
I(θ, ˙θ, x, y) along a solution of Equation 4.68,

\[
\frac{d}{dt} I =  \dot{θ} \frac{∂}{∂θ} I + \ddot{θ} \frac{∂}{∂θ} I,
\]

(4.70)

where

\[
\frac{∂}{∂θ} I = 2 \dot{θ},
\]

(4.71)

\[
\frac{∂}{∂θ} I = \frac{2γ(θ)}{α(θ)} - \frac{2β(θ)}{α(θ)} [I - \dot{θ}^2],
\]

(4.72)

and ˙θ is defined by Equation 4.68. Therefore,

\[
\frac{d}{dt} I = \dot{θ} \left\{ \frac{2γ(θ)}{α} - \frac{2β(θ)}{α} [I - \dot{θ}^2] \right\} + \frac{(W - β(θ)\dot{θ}^2 - γ(θ))}{α(θ)} 2\dot{θ} \\
= \dot{θ} \left\{ \frac{2}{α(θ)} W - \frac{2β(θ)}{α(θ)} I \right\}.
\]

(4.73)

The goal now is to determine a feedback controller that guarantees invariance of the virtual constraints, Equation 4.28, and an orbital asymptotic stability of the chosen periodic solution for the closed loop system. This is accomplished by rewriting Equations 4.41 and 4.42 in the variables [I, y, ˙y]. Equation 4.41 can be manipulated such that its left-hand side matches the form of the virtual limit system Equation 4.56 and the right-hand side is the result of the substitutions

\[
q_1 = y + \phi(θ)
\]

(4.74)

\[
q_2 = \theta
\]

(4.75)

\[
u = u^*.
\]

(4.76)
This results in equations for the system of the form

\[
\alpha(\theta)\ddot{\theta} + \beta(\theta)\dot{\theta}^2 + \gamma(\theta) = W(\theta, \dot{\theta}, \ddot{\theta}, y, \dot{y}, v) \quad (4.77)
\]

\[
\ddot{y} = v, \quad (4.78)
\]

where \( W \) is a nonlinear function of the variables. Based on Theorem 2, the almost linear system is

\[
\dot{I} = \frac{2\dot{\theta}}{\alpha(\theta)} \{ W(\theta, \dot{\theta}, \ddot{\theta}, y, \dot{y}, v) - \beta(\theta)I \} \quad (4.79)
\]

\[
\ddot{y} = v. \quad (4.80)
\]

A linear approximation of this system is found by approximating \( W(.) \) about the desired orbit \((y = \dot{y} = 0)\),

\[
W \approx g_y(.)y + g_\dot{y}(.)\dot{y} + g_v(.)v, \quad (4.81)
\]

where \( g_i \) is the partial derivative of \( W \) with respect to coordinate \( i \) about the point \((y = \dot{y} = 0)\). For the inverted pendulum cart,

\[
W = g_v v = -\cos(\theta)v. \quad (4.82)
\]

Using Equation (4.81), the transverse dynamics are written

\[
\dot{z} = A(t)z + b(t)v, \quad (4.83)
\]
where \( z = [I, y, \dot{y}]^T \), and

\[
A(t) = \begin{bmatrix} k_1(t) & k_2(t) & k_3(t) \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}
\]

(4.84)

\[
b(t) = \begin{bmatrix} \rho(t) \\ 0 \\ 1 \end{bmatrix},
\]

(4.85)

with

\[
k_1(t) = -\frac{2\beta(\theta_d(t))}{\alpha(\theta_d(t))}\dot{\theta}_d(t)
\]

(4.86)

\[
k_2(t) = 0
\]

(4.87)

\[
k_3(t) = 0
\]

(4.88)

\[
\rho(t) = -\frac{2\cos(\theta_d(t))}{\alpha(\theta_d(t))}\dot{\theta}_d(t).
\]

(4.89)

If Equation 4.83 is completely controllable over a period of the motion, exponential stabilization can be achieved. To control this system, a feedback controller \( v = -K(t)z \) is chosen. Note that the PD control from the previous section is equivalent to \( K(t) = [0 \ K_p \ K_d]^T \) for all \( t \). An example of such a controller was given in \([56]\) and is presented here. There exists a matrix function \( P(t) \), \( P(t) = P(t + T) \) and \( P(t) = P(t)^T \) for all \( t \in [0, T] \), that satisfies the Differential Matrix Riccati Equation

\[
P(t) + A(t)^T P(t) + P(t)A(t) + G = P(t)b(t)R^{-1}b(t)^T P(t)
\]

(4.90)

\( \forall t \in [0, T] \) with \( A(t) \) and \( b(t) \) defined in Equation 4.83. Given this, the feedback controller

\[
v = -R^{-1}b(t)^T P(t)z
\]

(4.91)
renders the linear periodic system Equation 4.83 exponentially stable. This controller, however, does not stabilize the original system (Equations 4.77 and 4.78). By modifying the controller to

\[ v = -R^{-1}b(\theta, \dot{\theta}, y, \dot{y})^T P(t)z \]  

where

\[ b(\theta, \dot{\theta}, y, \dot{y}) = \begin{bmatrix} -2\cos(\theta(t)) \dot{\theta}(t) \\ 0 \\ 1 \end{bmatrix}, \]  

the original system is stabilized as proven in [56, Proof Theorem 3]. The difference is that in Equation 4.91 \( b(t) \) is a function of the desired zero dynamics orbit at time \( t \) as shown in Equations 4.85 and 4.89. In Equation 4.93 \( b(t) \) is now a function of the actual system state at time \( t \).

Finding the solution \( P(t) \) is not straightforward depending on the complexity of the problem. A naive attempt at a solution is to solve Equation 4.90 for \( \dot{P}(t) \) and integrate forward in time until the periodic solution emerges. This method, however, has serious numerical difficulties, and several more reliable methods were given in [19] with increasing complexity and accuracy. It was found that the simplest method, a one-shot method, seemed suitable for this problem and was implemented using standard MATLAB toolboxes. The algorithm followed is laid out in [19, Sec 3.1-3.2] and in more detail in [28]. The result is a numerical solution of \( P(t) \) over the orbit period \( t \). This solution is linearly interpolated to find \( P(t) \) at any instant in time. Alternatively, fitting a series of sine functions to the numerical solution \( P(t) \) may also be successful.

After \( P(t) \) was found, the controller was tested for the ability to return to a desired periodic orbit after disturbances. This was simulated by starting the simulation at
Figure 4.6. $\dot{\theta}$ versus $\theta$ basin of attraction for the orbit-stabilizing control method in conventional coordinates. A grid of initial conditions was considered for $\theta = [-0.55, 0.55]$ and $\dot{\theta} = [-3.0, 3.0]$. Blue circles are the points that successfully stabilized to the orbit.

Several initial conditions off the orbit. Figure 4.6 shows a crude basin of attraction for the desired orbit. The state and time at which the pendulum crossed zero with positive velocity were recorded. If the difference between the sum of the states at the zero crossing was less than $1e-4$, the system was considered to have reached the desired orbit. Otherwise, the trial was considered unsuccessful. The input $u$ was also limited to the range $[-50, 50]$. Additionally, since the decoupling matrix from Equation 4.40 is singular at $\theta \approx \pm 0.61$, the simulation was stopped if $\theta = \pm 0.60$, and the trial considered unsuccessful.

This controller has a much larger basin of attraction for the orbit than the previous PD controller, and the control approach is a good candidate approach for application to biped walking.
4.2.3.4 Orbit-Stabilizing Control in Quasi-Velocity Coordinates

Two approaches were investigated for applying an orbit stabilizing controller in the quasi-velocity coordinates. The first was using coordinate transforms. The second method was rederiving the $I$ coordinate as a function of $s$.

The coordinate $I$ can be rewritten in terms of the new velocities by substituting Equation 4.17 into Equation 4.67:

$$I_{\text{conv}}(\theta, s, w, \theta_0, s_0, w_0) =$$

$$
\left( s - w \cos(\theta) \sqrt{\frac{1}{2 - \cos^2(\theta)}} \right)^2 \cdot 
\frac{(1 - L \cos^2(\theta_0)) \left( s_0 - w_0 \cos(\theta_0) \sqrt{\frac{1}{2 - \cos^2(\theta_0)}} \right)^2}{1 - L \cos^2(\theta)} + 
\frac{2g (\cos(\theta) - \cos(\theta_0))}{1 - L \cos^2(\theta)}.
$$

(4.94)

Note that Equation 4.50, the solution for $w$, is not used since it only holds when the system is on the desired orbit. However, to find the corresponding initial conditions $w_0$ and $s_0$, Equation 4.50 is used and

$$s_0 = \hat{\theta}_0 (1 - L \cos^2(\theta_0))$$

(4.95)

$$w_0 = \frac{s_0 L \cos(\theta_0)}{\sqrt{\frac{1}{2 - \cos^2(\theta_0)}} \left( L \cos^2(\theta_0) - 1 \right)}.$$

(4.96)

Equation 4.79 for $\dot{I}$ can be similarly converted. To be consistent, one must apply the substitutions when $y$ and $\dot{y}$ are not yet zero, as this is needed to find $b(\theta, \dot{\theta}, y, \dot{y})$, but then set $y = \dot{y} = 0$ to find $A(t)$ and $b(t)$. Starting again with Equation 4.41, it is manipulated such that its left hand side matches Equation 4.77 with the substitutions

$$q_1 = y + \phi(\theta)$$

(4.97)

$$q_2 = \theta$$

(4.98)

$$u = u^*_w.$$
Note that the $u_{sw}^*$ from the quasi-velocity formulation is used here rather than the original coordinates. This yields

$$\dot{I}_{\text{conv}} = \frac{2\dot{\theta}}{\alpha(\theta)} (W(\theta, w, s, y, \dot{y}, v) - \beta(\theta) I).$$

(4.100)

The function $W$ is the same as in Equation [4.79] except expressed in the quasi-velocity coordinates. For the inverted pendulum cart,

$$W = g_v v = -v \cos(\theta).$$

(4.101)

Now, Equation [4.17] and the coordinate conversion from solving Equation [4.47] for $w$ are substituted into Equation [4.100],

$$\dot{I}_{\text{conv}} = \frac{2}{\alpha(\theta)} \left( s - \frac{\dot{y} \cos(\theta)}{\alpha(\theta)} \right) (g_v v - \beta(\theta) I).$$

(4.102)

Setting $y = \dot{y} = 0$, the differential equation simplifies to

$$\dot{I} = \frac{2s}{\alpha(\theta)^2} (g_v v - \beta(\theta) I).$$

(4.103)

Equation [4.83] for this system has parameters

$$k_1(t) = -\frac{2\beta(\theta_d(t))}{\alpha^2(\theta_d(t))} s_d(t)$$

(4.104)

$$k_2(t) = 0$$

(4.105)

$$k_3(t) = 0$$

(4.106)

$$\rho(t) = -\frac{2\cos(\theta_d(t))}{\alpha^2(\theta_d(t))} s_d(t),$$

(4.107)
and

\[
b(\theta, w, s, y, \dot{y}) = \begin{bmatrix} \frac{2(s - \dot{y}\cos \theta)}{\alpha(\theta)^2} g \nu \\ 0 \\ 1 \end{bmatrix}.
\] (4.108)

As an alternative, it was sought to derive the integral of motion (Equation 4.57) in terms of the uncontrollable velocity \(s\) rather than the zero dynamic velocity because it was hypothesized that this would result in a controller that would better use the \(w\) dynamics to influence the \(s\) dynamics. This required stepping back to the definition for \(Y\) in Equation 4.59 and proceeding with a new definition,

\[
Y_{sw} = s^2.
\] (4.109)

Using this equation yields two new identities,

\[
dY_{sw} \frac{dt}{dt} = 2ss \dot{s}
\] (4.110)

and

\[
dY_{sw} \frac{dt}{dt} = dY_{sw} \frac{d\theta}{d\theta} = \dot{\theta} \frac{d}{d\theta} Y_{sw}.
\] (4.111)

Applying Equation 4.17 and Equation 4.47 when \(\dot{y} = 0\), the second identity simplifies to

\[
dY_{sw} \frac{dt}{dt} = \frac{s}{\alpha(\theta)} \frac{d}{d\theta} Y_{sw}.
\] (4.112)

Then,

\[
\dot{s} = \frac{1}{2\alpha(\theta)} \frac{d}{d\theta} Y_{sw}
\] (4.113)

can be substituted into Equation 4.51 and after rearranging, results in

\[
\frac{d}{d\theta} Y_{sw} - \frac{2\beta(\theta)}{\alpha(\theta)} Y_{sw} + 2\alpha(\theta)\gamma(\theta) = 0.
\] (4.114)
This equation is in the same form as Equation 4.57, so the solution was found similarly,

\[ I_{sw} = s(t)^2 - Y_{sw}(\theta(t)). \]  

(4.115)

Now an expression for the rate of change of \( I_{sw} \) is

\[
\frac{d}{dt} I_{sw} = \frac{\partial I_{sw}}{\partial \theta} \frac{d\theta}{dt} + \frac{\partial I_{sw}}{\partial s} \frac{ds}{dt}
\]

\[
= \dot{\theta} \frac{\partial I_{sw}}{\partial \theta} + \dot{s} \frac{\partial I_{sw}}{\partial s}
\]

\[
= \frac{s}{\alpha(\theta)} \frac{\partial I_{sw}}{\partial \theta} + \dot{s} \frac{\partial I_{sw}}{\partial s},
\]

(4.116)

where

\[
\frac{\partial I_{sw}}{\partial \theta} = 2\alpha \gamma + \frac{2\beta}{\alpha^2} \left( I_{sw} - s^2 \right)
\]

(4.117)

\[
\frac{\partial I_{sw}}{\partial s} = 2s.
\]

(4.118)

Using these two equations and

\[
\dot{s} = W_2 + \frac{\beta}{\alpha^2} s^2 - \gamma,
\]

(4.119)

Equation 4.116 simplifies to

\[
\frac{dI_{sw}}{dt} = s \left( 2W_2 + \frac{2\beta}{\alpha^2} I_{sw} \right).
\]

(4.120)

In this system, \( W_2 \) is not a function of \( v \) but of \( \dot{y} \). Thus, a similar procedure can be followed to manipulate the system dynamics such that Equation 4.120 can be used
to form a linear system, Equation 4.83, where

\[ k_1(t) = \frac{2\beta}{\alpha^2} s_d(t) \] (4.121)

\[ k_2(t) = 0 \] (4.122)

\[ k_3(t) = 2s_d(t)\dot{g}_y \] (4.123)

\[ g_y = -\frac{2s_d(t)\sin(\theta)(L\cos(2\theta) + L + 2)}{(L\cos(2\theta) + L - 2)^2} = -s_d(t)\frac{(4 - 2\alpha)\sin(\theta)}{2\alpha^2} \] (4.124)

\[ \rho(t) = 0, \] (4.125)

and

\[ b(\theta, \dot{\theta}, y, \dot{y}) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \] (4.126)

Simulations were run for both of these controllers in \( s \) and \( w \) coordinates and compared to the controller from Section 4.2.3.3. The converted system had identical performance to the conventional coordinates and thus the same basin of attraction.

Using the rederived system resulted in a basin of attraction as shown in Figure 4.7. This basin is not as large as that of the \( \dot{\theta} \) controller and is irregularly shaped. The rederived controller actually required linear approximations to form Equation 4.83, while the controller for the original coordinates did not. This is one possible explanation for the smaller basin of attraction.

Advantages of using the quasi-velocities \( \{s, w\} \) are unclear. Using the converted controller could prove advantageous if it is numerically more stable or easier to observe the required quantities in experiment. The rederived controller appears to have worse performance than the other options. The velocity coupling information from the quasi-velocity coordinates could prove useful in future work with further investigation into using the coupling information to inform control or determine stability.
4.2.4 Conclusions

Moving forward, the orbit-stabilizing controller provides a feedback mechanism to reject disturbances and stabilize to a desired orbit. The orbit-stabilizing controller also uses virtual constraints which are used in existing HZD-based control techniques, making integration with those techniques more straightforward. The simplest model that captures the desired behavior of ERNIE is a three-link model. A two-link model was investigated, but removing the torso oversimplifies the control problem and is not representative of ERNIE. Including knees greatly increases complexity of the model, so a three-link model was chosen to prove the concept. The biggest addition to the formulation is incorporating the foot impacts which make the desired orbit hybrid.

4.3 Planar Three-Link Model

Additional tools are needed to apply the orbit stabilization control technique to biped walkers. A numerical approximation of the $I$ coordinate is needed because
the integrals for the three-link biped cannot be solved explicitly, and numerical approximating them during numerical integration of the walker dynamics is also not efficient. Additionally, the desired closest point on the orbit is different for a hybrid cycle because direction of progression through the cycle is important. For a continuous cycle, the recovery action can be driven back to any point on the cycle. For a hybrid cycle, the location is more important. Foot impacts create a hybrid cycle that must be followed; therefore, the impact map must be incorporated into the synthesis of the controller design. This further complicates obtaining the time-varying feedback coefficients by requiring the solution of matrix Riccati equations with jumps. A point-foot three-link model is considered initially for simplicity. Areas in which applying the control to a curved foot three-link model would be difficult or are currently not done are noted.

4.3.1 Dynamics and HZD Formulation

Figure 4.3.1 shows a three-link biped and coordinate choices consistent with those outlined in [66]. The dynamics of a three-link biped are similar to those of the five-link ERNIE model from Chapter 3.

\[
\begin{align*}
\mathbf{M}(q)\ddot{q} + \mathbf{C}(q, \dot{q})\dot{q} + \mathbf{G} &= \mathbf{B}u & q^- \notin S \\
q^+ &= \Delta_q q^- & q^- \in S \\
\dot{q}^+ &= \Delta_\dot{q}(q^-)\dot{q}^- & q^- \in S,
\end{align*}
\]

(4.127)

(4.128)

(4.129)

where the specific terms of these matrices are straightforward to derive and are also found in [66]. These can also be written as coupled first order equations,

\[
\begin{align*}
\dot{x} &= f(x) + g(x)u & x^- \notin S \\
x^+ &= \Gamma(x^-) & x^- \in S.
\end{align*}
\]

(4.130)

(4.131)
For this system, the monotonically increasing variable is

\[ \theta = q_1 + q_3, \]

\[ (4.132) \]

and

\[ y = \begin{bmatrix} q_1 - \phi_1(\theta) \\ q_2 - \phi_2(\theta) \end{bmatrix}, \]

\[ (4.133) \]

where the functions \( \phi_i(\theta) \) were chosen as 7th-order Bezier polynomials. For the transverse dynamics, a coordinate transformation between \( q \) and \((y, \theta)\) is required;
thus, using Equations \ref{equation:4.132} and \ref{equation:4.133}

\[ q = \Phi(\theta) + \begin{bmatrix} y_1 \\ y_2 \\ -y_1 \end{bmatrix}, \] (4.134)

where

\[ \Phi(\theta) = \begin{bmatrix} \phi_1(\theta) \\ \phi_2(\theta) \\ \theta - \phi_1(\theta) \end{bmatrix}. \] (4.135)

4.3.2 Hybrid Transverse Dynamics

The transverse dynamics for a hybrid system such as the three-link biped are

\[ \dot{z}(t) = A(t)z(t) + b(t)v(t) \text{ for } t \neq t_j \] (4.136)

\[ z(t^+_j) = F_j z(t^-_j), \] (4.137)

where \( F_j \) is a linearization of the impact map for the impact at time \( t_j \). The continuous transverse dynamics are found similarly to those for the inverted pendulum cart. Finding the linearization of the impact map is more involved.
4.3.2.1 Continuous Phase

For the three-link biped of Equation 4.131 the zero dynamic coefficients are found

\[ \alpha(\theta) = B^\perp M(\Phi(\theta))\Phi'(\theta), \]
\[ \beta(\theta) = B^\perp[C(\Phi(\theta), \Phi'(\theta)) \Phi'(\theta) + M(\Phi(\theta))\Phi''(\theta)], \]
\[ \gamma(\theta) = B^\perp G(\Phi(\theta)), \]
\[ \Phi'(\theta) = \frac{d}{d\theta}\Phi(\theta), \]
\[ \Phi''(\theta) = \frac{d^2}{d\theta^2}\Phi(\theta), \]

where \( \Phi(\theta) \) is from Equation 4.134 and \( B^\perp \) is a full rank matrix such that \( B^\perp B = 0 \).

For this system \( B^\perp = [0, 0, 1] \).

The \( g_i \) functions are found through extensive calculation to eliminate their dependence on \( \ddot{\theta} \). First, Equation 4.127 is manipulated to isolate the \( M(q)\ddot{q} \) term on the left-hand side of the equation. Next the coordinate transformation Equation 4.134 is applied and the mass matrix inverted. Further manipulations isolate \( \ddot{y} \) and \( \ddot{\theta} \) on the left-hand side. The control \( u^* \) is applied, which requires inverting the decoupling matrix. Multiplying the \( \ddot{\theta} \) equation by \( \alpha \) and adding \( \beta \dot{\theta}^2 + \gamma \) results in the zero dynamics on the left-hand side and \( W \) on the right-hand side. Then, partial derivatives can be taken of \( W \), and the \( g_i \) functions can be found by setting \( y = \dot{y} = 0 \).

There is an option of finding the \( g_i \) functions more simply by retaining dependence on \( \ddot{\theta} \). First the equations of motion (4.127) are rewritten by substituting the coordinate transformation Equation 4.134 and its derivatives. Then the annihilator \( B^\perp \) is applied to the equations of motion. The right-hand side is immediately 0 because \( B^\perp Bu = 0 \). The left-hand side becomes

\[ \alpha(\theta)\ddot{\theta} + \beta(\theta)\dot{\theta}^2 + \gamma(\theta) - W = 0. \]
Thus, $W$ is easily found by multiplying by $-1$ and subtracting the zero dynamics determined from Equations 4.138 to 4.142. The $g_i$ functions will now be functions of $\ddot{\theta}$, but the $g_{vi}$ functions will not be functions of $\ddot{\theta}$. This is important because the $g_{vi}$ functions are needed in the actual control loop, so they cannot depend on $\ddot{\theta}$. Eliminating $\ddot{\theta}$ places less dependence on the numerical accuracy of the zero dynamics because the $g_i$ functions are calculated directly from the state of the system. Retaining the $\ddot{\theta}$ dependence may make it possible to find the transverse linearization for curved feet, which is too complex with the first approach. Since the $g_i$ functions that depend on $\ddot{\theta}$ are only needed offline to initially calculate the controller gains, leaving the dependence is acceptable as long as an accurate zero dynamics trajectory is generated.

Thus, the matrices from Equation 4.136 are:

$$A(t) = \begin{bmatrix} a_{11}(t) & a_{12}(t) & a_{13}(t) & a_{14}(t) & a_{15}(t) \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(4.144)

$$b(t) = \begin{bmatrix} b_1(t) & b_2(t) \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(4.145)
where

\[ a_{11}(t) = -m_*(t)\beta(\theta_*(t)), \quad (4.146) \]
\[ a_{12}(t) = m_*(t)g_{y1}(\theta_*(t), \dot{\theta}_*(t)), \quad (4.147) \]
\[ a_{13}(t) = m_*(t)g_{y2}(\theta_*(t), \dot{\theta}_*(t)), \quad (4.148) \]
\[ a_{14}(t) = m_*(t)g_{dy1}(\theta_*(t), \dot{\theta}_*(t)), \quad (4.149) \]
\[ a_{15}(t) = m_*(t)g_{dy2}(\theta_*(t), \dot{\theta}_*(t)), \quad (4.150) \]
\[ b_{11}(t) = m_*(t)g_{v1}(\theta_*(t)), \quad (4.151) \]
\[ b_{12}(t) = m_*(t)g_{v2}(\theta_*(t)), \quad (4.152) \]
\[ m_*(t) = 2\dot{\theta}_*(t)/\alpha(\theta_*(t)). \quad (4.153) \]

4.3.2.2 Linearization of the Impact Map

As first described in [39], the transverse dynamics can be visualized as a moving Poincaré section that is normal to and follows the nonlinear zero dynamics orbit throughout the state space. The impact event surface \( S \) is not necessarily flat or parallel to the transverse dynamics planes; thus, some rectification is required for proper tracking of the transverse dynamics through an impact event. The orbital stabilization method of [56] was extended to handle impacts by [39] and results in Equation 4.137.

The procedure for constructing \( F_j \) is outlined below and follows the theory from [39] and the detailed description of a three-link model in different coordinates from [10]. Since the transverse dynamics at time \( t^-_j \) do not necessarily align with \( S \), the dynamics are projected to \( S \), the impact map is applied, and the result is projected back to the transverse dynamics at \( t^+_j \). This requires two projection operators and a linearization of the impact map.

A linearization of the impact map is taken around the point of the desired impact
condition. A first-order approximation of the impact map around the desired impact condition is

\[ \Gamma(x^+ + \delta x^+ \approx \Gamma(x^-) + \frac{\partial \Gamma}{\partial x} \delta x^- . \] (4.154)

For point feet, the Jacobian \( \frac{\partial \Gamma}{\partial x} \) is found analytically, but it was too complex to find analytically for curved feet. This is one limitation of applying the control method to curved feet, but this could be overcome by finding this Jacobian numerically. One possible method is similar to that used to estimate stability Jacobians for passive walkers as in [25].

To find the projection operators, a relationship between the linearized increments of the generalized coordinates and the transverse coordinates is needed. This relationship is

\[ [\Delta I \Delta y^T \Delta \dot{y}^T]^T = L(t) [\Delta q^T \Delta \dot{q}^T]^T , \] (4.155)

where for the three-link biped,

\[ L(t) = \begin{bmatrix}
-2\ddot{\theta} & 0 & -2\ddot{\theta} & 2\dot{\theta} & 0 & 2\dot{\theta} \\
1 - \phi'_1 & 0 & -\phi'_1 & 0 & 0 & 0 \\
-\phi'_2 & 1 & -\phi'_2 & 0 & 0 & 0 \\
-\phi''_1\dot{\theta} & 0 & -\phi''_1\dot{\theta} & 1 - \phi'_1 & 0 & -\phi'_1 \\
-\phi''_2\dot{\theta} & 0 & -\phi''_2\dot{\theta} & -\phi'_2 & 1 & -\phi'_2
\end{bmatrix} , \] (4.156)

and the function of time notation has been dropped for simplicity. The vector normal to the transverse dynamics section is

\[ n(t) = [\dot{q}_s^T (t) \dot{\dot{q}}_s^T (t)]^T , \] (4.157)

where \( \dot{q}_s \) and \( \dot{\dot{q}}_s \) are the joint angular velocities and accelerations corresponding to the desired zero dynamics trajectory. The switching surface is defined as the swing
foot touching the ground in front of the stance leg (and ignoring the early impacts). Those conditions can be simplified to

\[ S = q_1 + q_2 + 2q_3 - 2\pi = 0, \]  

and the vector normal to this surface in the \( q, \dot{q} \) space is

\[ m = [1 \ 1 \ 2 \ 0 \ 0 \ 0]^T. \]  

These are combined as in [10] to obtain the projection from the transverse surface to the switching surface,

\[ P_{n(t_f)}^- = \left( I_6 - \frac{n(t_f)m^T}{n^T(t_f)m} \right) \left[ \begin{bmatrix} L(t_f) \\ n^T(t_f) \end{bmatrix}^{-1} \begin{bmatrix} I_5 \\ 0_{1\times5} \end{bmatrix} \right], \]  

and the projection from the switching surface back to the transverse surface,

\[ P_{n(0)}^+ = L(0) \left( I_6 - \frac{n(0)n(0)^T}{n^T(0)n(0)} \right). \]  

Thus, the impact mapping in the transverse coordinates is

\[ F_j = P_{n(0)}^+ \frac{\partial\Gamma}{\partial x} P_{n(t_f)}^- . \]  

4.3.2.3 Approximating the Orbit Error Coordinate for Use with HZD

For simple systems, such as the inverted pendulum cart, the integral representation of the \( I \) coordinate, Equation 4.57, can be solved explicitly and written as a function of the state variables. However, for more complicated systems, such as the three-link biped, this is not the case and presents several problems both in simulation and experiment. In preliminary simulations, the \( I \) coordinate was found by running
a numerical integration (quad) scheme for each call of the system dynamics function. This was very computationally expensive and had singularity issues when the walker was falling or very far from the desired orbit. It was also attempted to make the coordinate $I$ an additional state in the system dynamics. However, this also produced numerical difficulties in the simulation, partly due to the need to define a mapping of $I$ at impact.

These issues also present problems for practical implementation on the ERNIE hardware. The value of $I$ is needed at every time step to implement the orbit-stabilizing control, and this calculation is time consuming and difficult to implement in real-time. An approximation for $I^2$ was given in [57] as

$$I^2(x_1, x_2, \theta^*_0, \dot{\theta}^*_0) = 4 \left[ \dot{\theta}^*(\rho_0)^2 + \ddot{\theta}^*(\rho_0)^2 \right]$$

$$\times D^2(x_1, x_2) + O(|x_1 - \theta^*(\rho_0)|^2) + O(|x_2 - \dot{\theta}^*(\rho_0)|^2), \quad (4.163)$$

where

$$D(x_1, x_2) = \min_{0 \leq \rho < t_f} \left\{ \sqrt{|x_1 - \theta^*(\rho)|^2 + |x_2 - \dot{\theta}^*(\rho)|^2} \right\} \quad (4.164)$$

is the Euclidean distance from $\{x_1, x_2\}$ to the point on the orbit that is closest, defined by

$$\arg \min_{0 \leq \rho < t_f} \{|x_1 - \theta^*(\rho)|^2 + |x_2 - \dot{\theta}^*(\rho)|^2\}. \quad (4.165)$$

Thus, $I$ could be found from the square root of Equation 4.163. It is assumed that the positive root is used, although this is unclear. An alternate approximation for $I$ was developed herein and compared to Equation 4.163. The basis for the approximation of $I$ is a 2nd-order Taylor Series expansion in two variables. The expansion is taken at the point $\{x_1, x_2\}$ about the function $I(\theta^*, \dot{\theta}^*, \theta^*_0, \dot{\theta}^*_0)$, where $\{\theta^*, \dot{\theta}^*\}$ is the desired zero dynamics orbit. For simplicity of notation, the constants $\theta^*_0$ and $\dot{\theta}^*_0$ are temporarily
neglected. The expansion of $I$ is

$$I(\theta^*, \dot{\theta}^*) \approx I(x_1, x_2) + \frac{\partial I(x_1, x_2)}{\partial \theta}(\theta^* - x_1) + \frac{\partial I(x_1, x_2)}{\partial \dot{\theta}}(\dot{\theta}^* - x_2)$$

$$+ \frac{1}{2} \frac{\partial^2 I(x_1, x_2)}{\partial \theta^2}(\theta^* - x_1)^2 + \frac{\partial^2 I(x_1, x_2)}{\partial \theta \partial \dot{\theta}}(\theta^* - x_1)(\dot{\theta}^* - x_2)$$

$$+ \frac{1}{2} \frac{\partial^2 I(x_1, x_2)}{\partial \dot{\theta}^2}(\dot{\theta}^* - x_2)^2$$

$$+ O[(\theta^* - x_1)^3] + O[(\dot{\theta}^* - x_2)^3].$$

(4.166)

The first partials are given in Equations 4.71 and 4.72 and repeated below

$$\frac{\partial I}{\partial \theta} = 2 \dot{\theta}$$

(4.167)

$$\frac{\partial I}{\partial \dot{\theta}} = \frac{2\gamma(\theta)}{\alpha(\theta)} - \frac{2\beta(\theta)}{\alpha(\theta)} [I - \dot{\theta}^2].$$

(4.168)

It is straightforward to calculate the 2nd partials,

$$\frac{\partial^2 I}{\partial \theta^2} = 2\alpha' - \frac{2\alpha' - \beta}{\alpha^2}(I - \dot{\theta}^2) - \frac{2\beta}{\alpha} \left(\frac{\partial I}{\partial \theta}\right)$$

(4.169)

$$\frac{\partial^2 I}{\partial \theta \partial \dot{\theta}} = \frac{\partial^2 I}{\partial \dot{\theta} \partial \dot{\theta}} = 0$$

(4.170)

$$\frac{\partial^2 I}{\partial \dot{\theta}^2} = 2,$$

(4.171)

where the prime indicates a derivative with respect to $\theta$. Recalling that $I(\theta^*, \dot{\theta}^*) = 0$ and collecting terms of $I$,

$$0 \approx I(x_1, x_2)(1 + f_1(x_1, x_2, \theta^*)) + f_2(x_1, x_2, \theta^*, \dot{\theta}^*),$$

(4.172)

where

$$f_1 = \frac{2\beta(x_1 - \theta^*)}{\alpha} + \frac{(\alpha' + 2\beta^2 - \alpha\beta')(x_1 - \theta^*)}{\alpha^2}$$

(4.173)
and

\[
\begin{align*}
    f_2 &= (x_2 - \dot{x}^*)^2 + 2x_2(\dot{x}^* - x_2) - \frac{2x_2^2\beta(\theta^* - x_1)}{\alpha} \\
    &= -\frac{2\gamma(x_1 - \theta^*)}{\alpha} + \frac{x_2^2(-\alpha'\beta - 2\beta^2 + \alpha\beta')(x_1 - \theta^*)^2}{\alpha^2} \\
    &= \frac{x_2^2(\alpha'\gamma(x_1 - \theta^*)^2 - 2\beta\gamma(x_1 - \theta^*)^2 + \gamma'(x_1 - \theta^*)^2}{\alpha^2}.
\end{align*}
\]

(4.174)

Thus, the approximation of \( I \) at a point \((x_1, x_2)\) off the desired orbit is

\[
I(x_1, x_2) = -\frac{f_2(x_1, x_2, \theta^*, \dot{\theta}^*)}{(1 + f_1(x_1, x_2, \theta^*))}.
\]

(4.175)

To increase the accuracy of the approximation, the point of expansion \((\theta^*, \dot{\theta}^*)\) should be as close to \((x_1, x_2)\) as possible. Since the HZD-based controller is already parametrized by \( \theta \) rather than time, the chosen point is

\[
\rho_0 = \arg\min_{0 \leq \rho \leq T} (\theta^*(\rho) - x_1)^2,
\]

(4.176)

rather than minimizing the distance to \( \theta \) and \( \dot{\theta} \) as in [57]. Thus, \( \theta^*(\rho_0) = x_1 \) except when \( x_1 \) is outside the range of \( \theta^*(t) \) and it saturates to the corresponding end point \( \theta^*(0) \) or \( \theta^*(t_f) \).

A comparison of Equations 4.175, 4.163, and the integral value (\textit{quad}) is shown in Figure 4.9 for a gait of the three-link biped following the zero dynamics orbit. The value of \( I \) for all three measures is zero to some numerical accuracy. Figure 4.10 shows the effect of offsetting \( \dot{\theta} \) by 0.1 rad/s and recalculating \( I \). Here, \( I \) will be non-zero, and the integral measurement is the true value of \( I \). The 2nd-order approximation tracks the integral value almost exactly, while the approximation from Equation 4.163 has significant errors. The approximation from Equation 4.163 has problems at bends in the \((\theta^*, \dot{\theta}^*)\) orbit. Since the original approximation just looks for the closest point, at bends in the zero dynamics orbit, much of the orbit could possibly be skipped.
because the peak of a bend is farther away than the rest of the orbit. Equation 4.175 forces the distance calculation to track $\theta$ and prevents this behavior from happening. This is consistent with desired behavior on a hybrid cycle— it is undesirable to move toward a $\theta$ earlier in the cycle. A possible area of future work is aiming for a $\theta$ later in the cycle rather than at the same value as the current state, as it could be useful to predict control at a future state.

Figure 4.9. Comparison of approximations of the $I$ coordinate to the integral value for a three-link biped gait on the zero dynamics orbit. The measures are all zero within numerical tolerances, as expected.

Equation 4.175 requires calculating $\alpha'$, $\beta'$, and $\gamma'$ and subsequently $\phi^{(3)}_i$. However, with the current formulation using Bezier polynomials, these are easily found and possibly simpler to compute than $\ddot{\theta}^*(t)$ needed by Equation 4.163. More importantly for implementation on hardware, the approximation is now a function of only the position and velocity states. This makes the calculation more feasible with the
Figure 4.10. Comparison of approximations of the $I$ coordinate to the integral value for a three-link biped gait with the $\dot{\theta}$ trajectory offset by 0.1 rad/s. The 2nd-order approximation is a close match for the integral value, while the Shiriaev approximation has significant errors.

4.3.3 Solving the Matrix Riccati Equation with Jump Conditions

The procedure for finding the required time-varying gains used for the inverted pendulum cart simulation needs to be modified for modeling biped walking because the desired trajectories have jumps caused by the impacts. Manchester [39] gives an algorithm for finding $P(t)$ using a receding-horizon optimal control strategy. This algorithm was given assuming the desired trajectory was parametrized by time and the robot was walking on uneven terrain, requiring that the desired trajectory be updated. For this work, velocity disturbances are being considered, and the desired trajectory is a pre-determined stable walking orbit. Thus, computing the receding-horizon control one time will give the desired $P(t)$ for all step conditions. The hardware setup of ERNIE and the typical sensor packages of most other robots.
procedure is to find the feedback controller that minimizes the cost function \[ J(x, u) = \int_{t_i}^{t_f} \left[ z(t)^T Q(t) z(t) + v(t)^T R(t) v(t) \right] dt + \sum_{j=1}^{i+h} z(t_j)^T Q_j z(t_j), \] (4.177)

that is commonly encountered in LQR control. This specific formulation has weighting functions \( Q(t) \), \( R(t) \) and \( Q_j \) on the states, inputs, and step-end positions, respectively. In [39], these were chosen as identity matrices. In this work, changing the weight on the \( I \) coordinate in \( Q(t) \) seemed to facilitate desirable behavior and

\[ Q(t) = \text{diag}(100 \ 1 \ 1 \ 1 \ 1), \] (4.178)

making tracking the \( I \) coordinate 100 times as important as tracking the other coordinates. Since HZD should track \( y \) and \( \dot{y} \) to zero on its own, this suggests that the \( Q(t) \) should have an even larger value on the \( I \) coordinate or the other entries should be zero. Zeroing out the other weights created some numerical issues in implementation, so 100 was chosen as a compromise.

The solution \( P(t) \) is found via the algorithm from [39],

\[
\frac{d}{dt} Z(t) = Z(t) A(t)^T + A(t) Z(T) - B(t) R(t)^{-1} B(t)^T \\
+ Z(t) Q(t) Z(t) \text{ for } t \neq t_j \\
Z(t_j^-) = \left[ F_j^T Z(t_j^+) F_j + Q_j \right]^{-1} \text{ for } j = i, i+1, \ldots, i+h-1, \] (4.179)

(4.180)

where \( Z(t) = P^{-1}(t) \) and the equations are solved backward in time from \( t_{i+h} \) to \( t_i \) with a final condition \( Z(t_{i+h}) = 0_{(n-1)^2} \). Here, \( Z(t) \) is the inverse of the cost-to-go matrix usually associated with LQR design and allows the initial conditions of the simulation to be set to zero, corresponding to an infinite cost on the final condition. Thus, the equations can be numerically integrated in reverse time from a known initial condition to an unknown final condition rather than using a shooting method.
in normal time to converge to a known final condition. This was implemented on the three-link biped by integrating 6 steps into the future using an *ode45* implementation of Equations 4.179 and 4.180. The 6-step horizon was selected because the sum of the absolute difference between the states at the end of steps 5 and 6 was 3e-7, indicating that the solution was sufficiently periodic because the tolerances on the integration scheme accuracy were 1e-6. The solution of \( Z(t) \) on step 6 was used to find the periodic gains for the controller. These trajectories could be approximated by polynomials, but instead, numerical interpolation was used. These solutions in time were mapped to corresponding \( \theta \)-varying gains,

\[
K(\tau) = -R^{-1}B_j(\tau)^T Z^{-1}(\tau),
\]

(4.181)

where

\[
\tau = \begin{cases} 
0 & \text{for } \theta < \theta^*(0), \\
t & \text{for } \theta = \theta^*(t), \\
t_f & \text{for } \theta > \theta^*(t_f),
\end{cases}
\]

(4.182)

which finds the time stamp of the numerically generated \( Z(t) \) for the current \( \theta \). The resulting feedback for the orbit stabilizing controller is

\[
v = K(\tau)z(t).
\]

(4.183)

For the inverted pendulum cart, the closest point on the desired orbit from the current position was selected as the target. However, since the orbit for the robot is hybrid, it does not make sense to target \( \theta^* \) that is smaller than the current \( \theta \) because this would make the robot walk backward. Thus, the present implementation uses the target \( \theta^* \) as equivalent to the current \( \theta \) unless the current \( \theta \) is beyond the limits for that step, in which case the desired target is \( \theta^- \). Future work may investigate
whether targeting to $\theta^*(t + \delta t)$ would be beneficial.

4.3.4 Control Simulation Results

The disturbance rejection capability of this orbit-stabilizing approach was compared to that of the original HZD approach using the three-link biped given in [66]. The mass and inertia parameters for this walker are given in Table 4.1.

Velocity disturbances were implemented by stopping the simulation at the desired point in the simulation loop and changing the velocity states. This simulates an instantaneous change in velocity, similar to an impulsive disturbance force. Because the HZD-based controller is constructed with high-gain PD control, trying to disturb the states directly controlled by zeroing $y$ and $\dot{y}$ results in nearly instantaneously recovery to the desired trajectory. For example, changing the angular velocity $\dot{q}_2$ will not induce a response from the orbit stabilization controller because $y_2$ is the error tracking for $q_2$. Thus, the velocity disturbances investigated are those that disturb the robot from the zero dynamics trajectory. In absolute coordinates, this is obviously the stance leg velocity $\dot{\theta}$. In relative coordinates, $\dot{\theta} = \dot{q}_1 + \dot{q}_3$, and preliminary trials confirmed that changing $\dot{q}_3$ was essentially handled by the HZD-based control. Therefore, disturbances were created by changing the $\dot{q}_1$ state.

An initial gait for comparing the controllers for the three-link biped was picked with polynomial coefficients,

$$
a = \begin{bmatrix}
2.3489 & 2.2212 & 3.1 & 2.9 & 3.2 & 3.0 & 3.0 & 3.1343 \\
3.1343 & 2.8327 & 3.1 & 2.8 & 2.2 & 2.2 & 2.3 & 2.3489
\end{bmatrix},
$$

with initial conditions vector $x_0 = [2.3489 \ 3.1343 \ 0.4000 \ -0.9510 \ -2.2464 \ 1.7867]^T$. The approximate velocity of this gait is 0.73 m/s.

An algorithm was written to systematically increase the disturbance to the $\dot{q}_1$
TABLE 4.1

MASS AND INERTIA PROPERTIES FOR A THREE-LINK BIPED

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l )</td>
<td>0.5 m</td>
<td>Torso length</td>
<td></td>
</tr>
<tr>
<td>( r )</td>
<td>1.0 m</td>
<td>Leg length</td>
<td></td>
</tr>
<tr>
<td>( m_T )</td>
<td>10 kg</td>
<td>Torso mass</td>
<td></td>
</tr>
<tr>
<td>( m_H )</td>
<td>15 kg</td>
<td>Hip mass</td>
<td></td>
</tr>
<tr>
<td>( m )</td>
<td>5.0 kg</td>
<td>Leg mass</td>
<td></td>
</tr>
</tbody>
</table>

coordinate throughout the gait cycle from \( s = 0.01 \) to \( s = 0.99 \) in 0.01 increments. The \( s = 0 / s = 1 \) case was avoided to concentrate on disturbances during the swing phase and not at impact. The magnitude of disturbance that could be rejected was found to 2 decimal places. A trial was considered successful once \(|\dot{\theta} - \dot{\theta}^*| < .005\), suggesting that the gait had very nearly returned to the zero dynamics orbit. If the walker fell or did not return to the orbit, it was considered a failure. In addition to measuring the magnitude of disturbance rejected, the number of steps for the controller to return to the desired orbit was also measured. This measures how quickly the disturbed gait is attracted back to the desired orbit. Figure 4.11 shows the maximum disturbances rejected by each controller compared with the nominal zero dynamics trajectory. Additionally, the number of steps for recovery to the cycle is shown in Figure 4.12 for the corresponding disturbance shown in Figure 4.11.

Looking at Figure 4.11, both controllers reject roughly the same magnitude disturbance for most of the gait cycle, except from 62 to 84 percent of the cycle where the orbit stabilizing controller rejects a significantly larger disturbance than the HZD-based controller. In all cases, the orbit-stabilizing controller took 4 steps to return to the orbit, while the HZD-based controller took between 5 and 7 steps.
Figure 4.11. Comparison of deceleration disturbances on a three-link biped for the HZD-based controller and the orbit-stabilization controller. Data points indicate the largest possible disturbance from the orbit from which each controller returned the robot to the zero dynamic orbit ($\dot{\theta}$ versus $\theta$).

An example of the response to a disturbance of -0.32 rad/s at 70% of gait cycle is shown in Figure 4.13 for both the HZD-based and orbit-stabilizing controllers. A previous undisturbed step ($t \approx -2.0$ to $t \approx -1.2$) is identical under either controller, and the disturbance is applied at $t = 0$, shown by the instantaneous change in value of $\dot{\theta}$. When there is a large discontinuous jump in the trajectory of $q_2$, a new step has started. The disturbance is the maximum disturbance that can be rejected by the HZD-based controller at this point in the gait cycle, but a disturbance 5 times this amount can be rejected by the orbit-stabilizing controller. For both controllers the gait successfully continues walking, but the responses are different. With the HZD-
Figure 4.12. Comparison of the number of steps for recovery for the disturbances in Figure 4.11.

Based controller, the robot will continue to follow the specified HZD joint trajectories even if the speed of the gait is slowed. For the disturbance in Figure 4.13, two steps are taken with the orbit-stabilizing controller in the time it takes for one HZD step following the disturbance. On the other hand, the orbit stabilizing controller deviates from the desired trajectory to produce a corrective action for the gait velocity. From after the disturbance until the end of the disturbed step ($t \approx 0.4$), the orbit stabilizing controller pitches the torso forward. On the subsequent step, the orbit stabilizing controller both pitches the torso forward and exhibits more flexion in the swing leg. These behaviors are consistent for larger disturbances—the actions are just larger in magnitude. These corrective actions shrink to zero as the desired zero dynamics
orbit is approached. A plot of the zero dynamic velocity just before impact for steps following the disturbance is shown in Figure 4.14. This plot shows the zero dynamics velocity at impact having a quicker return to the desired value for the orbit-stabilizing controller than that of the HZD-based controller. Figure 4.15 is a phase plot of the zero dynamics orbit in response to the disturbance. This plot also shows a quicker return to the zero dynamics orbit for the orbit-stabilizing controller compared to the HZD-based controller.

A similar procedure was implemented for acceleration disturbances. Because of the way the HZD-based controller is designed, the gait should recover from any reasonable push forward disturbance. Pushing the robot forward speeds up the gait cycle, which only causes the gait progression to execute more quickly. Unless the gait is pushed forward so quickly that the stance leg cannot get into place, the impacting foot bounces up off the ground, or the stance foot slips, recovery from the disturbance will be slow but successful. The HZD-based controller will slowly bleed off speed at impact until the desired speed is reached. However, the orbit-stabilizing controller will take corrective action to slow down more quickly. Since the disturbance rejected by either controller could be arbitrarily large, a set disturbance of 0.50 rad/s was used. The steps for recovery for this disturbance for both controllers are shown in Figure 4.16. Again, the orbit-stabilizing controller has better performance, in most cases needing only 4 steps to return to the desired cycle, whereas the HZD-based controller in most cases needed 8 to do so.

Figures 4.17 and 4.18 are similar to those for deceleration disturbances, but for an acceleration disturbance of 0.50 rad/s at 25% of the gait cycle. In this case, the HZD-based controller returns to the desired cycle much more slowly than the orbit-stabilizing controller does. The only way the HZD-based controller slows the gait cycle is energy loss at impact. The orbit stabilizing controller takes corrective actions to deviate from the planned trajectories to facilitate a quicker return to the
zero dynamics cycle. The torso is pitched backward, and the swing hip is extended back relative to the desired trajectory. These actions again work to return the gait to the desired zero dynamics cycle and dissipate to zero.

For large disturbances beyond those rejected by the HZD-based controller, the orbit stabilizing controller can have dramatic deviations from the desired trajectory. For the disturbance of -1.6 rad/s at 70% percent of gait cycle, the trajectory of the robot is shown in Figure 4.19. This disturbance actually pushes the robot backward briefly as the trajectories of the torso and hip modulate to reject the disturbance.

4.4 Summary and Implications for ERNIE Control

Overall, the orbit-stabilization controller has superior disturbance rejection capabilities compared to the HZD-based controller. The orbit-stabilization controller can reject larger disturbances than the HZD-based controller and return to the desired orbit more quickly. For very large disturbances, deviations from the desired cycle can be quite large and possibly problematic in future hardware implementation. It also appears that the orbit stabilizing controller works to manipulate the system’s CoM relative to the ground contact point to reject the disturbances. When a deceleration disturbance slows the robot, the torso is pitched forward and the swing leg brought forward, moving the CoM forward, while the opposite happens when a disturbance accelerates the robot. Viewing the three-link robot as an inverted pendulum, these actions make sense for speeding or slowing the gait.

This work gives numerical simulation results for rejecting velocity disturbances with an orbit-stabilizing controller with $\theta$-varying gains. Additionally, a new approximation of $I$ that is more adaptable to hardware implementation and meshes better with HZD-based control was developed. Previous implementations of an orbit-stabilizing controller did not test the controller or use a model of this complexity. In [10], a three-link biped was considered, but the matrix gains $K(\tau)$ were linearized.
The work was focused on constructing the controller and its stability properties, and results of numerical simulations were omitted. The work of [39] planned trajectories for step down disturbances, but used a two-link walker. Therein, orbit stabilizing control was briefly compared to HZD control; however, eliminating the torso reduces the controller complexity considerably.

The increased performance of the orbit-stabilizing controller comes at the cost of controller complexity. Key elements of the control technique, such as obtaining and linearizing $W$ and matrix inversions of $M$ and $L_f L_g H$, are already quite complex analytically for a three-link model. In [66], some advice is given for coordinate transforms to simplify matrix inverses. However, these tricks rely on the point-foot assumption and no $q_N$ in the mass matrix, so they will not be helpful for application to a curved foot biped. The author was unable to obtain these needed analytic representations for the five-link model of ERNIE with point feet, let alone curved feet.

However, based on the morphology of ERNIE, the orbit-stabilizing controller for the three-link biped captures the dominant controller dynamics that could be most successfully implemented in hardware. A five-link robot adds two more joints that could be manipulated by the orbit stabilizing controller to reject disturbances. The majority of the mass of ERNIE, though, is in the torso, and the legs are comparatively lightweight. During the majority of the stance phase of ERNIE, the stance knee motor is already maxed out under HZD-based control, so it is unlikely that orbit-stabilizing actions using it would be successful in experiment. Similarly, it seems unlikely that manipulation of swing knee joints by an orbit stabilizing controller would add significant disturbance rejection performance because of the small effect on the overall CoM of ERNIE. Thus, disturbance rejection on ERNIE would be best accomplished by manipulating the torso and swing hip trajectories.

The three-link biped investigated does not have a mass distribution representative
of ERNIE. However, a three-link model with a more representative distribution was found to be very sensitive to impact configuration. Small changes in the end-of-step trajectories created very large torso pitch motions at the beginning of the cycle because the trajectories are required to stay invariant through the impact. A trajectory was found with reasonable torso pitching, but the swing foot motion was undesirable—the foot got very close to the ground, then shifted forward just before impact. This type of swing foot motion is difficult because numerical accuracy can cause the foot to miss the ground when it should not. Attempting to fix the swing foot motion creates the large torso pitching motions. These problems would be overcome if the model had knees and are too limiting to use an ERNIE-like mass distribution to understand the capabilities of the orbit stabilizing controller. Thus, potential control actions for ERNIE are based on a three-link model representative of gross gait kinematics rather than mass distribution.
Figure 4.13. Time history of a three-link biped’s response to a disturbance to $\dot{\theta}$ of -0.32 rad/s at 70% of gait cycle for both HZD-based and orbit-stabilizing controllers.
Figure 4.14. Zero Dynamics velocity (rad/s) just before impact for steps after the disturbance of -0.32 rad/s at 70% of gait cycle
Figure 4.15. Phase plot of the zero dynamics (rad/s vs. rad) and controller responses after the disturbance of -0.32 rad/s at 70% of gait cycle.
Figure 4.16. Comparison the number of steps for recovery for the push forward disturbance of 0.50 rad/s.
Figure 4.17. Time history of a three-link biped’s response to a disturbance to $\dot{\theta}$ of 0.50 rad/s at 25% of gait cycle for both HZD-based and orbit-stabilizing controllers.
Figure 4.18. Zero dynamic velocity (rad/s) just before impact for steps after the disturbance of 0.50 rad/s at 25% of gait cycle.
Figure 4.19. Orbit stabilizing control response for a large disturbance
CHAPTER 5

PRACTICAL EXPERIMENTAL IMPLEMENTATION OF ORBIT STABILIZING CONTROL

5.1 Motivation

As mentioned in the previous chapter, although the orbit stabilizing controller was successful in simulation, its complexities make it difficult to implement in experiment. Insight gained from those simulations, however, can be combined with knowledge derived from other experimental results to identify simplifications and heuristic rules that capture the spirit of the controller.

5.2 Heuristics from a Three-Link Biped and Simplifications for a Five-Link Biped

To simplify the orbit-stabilizing controller, the corrective action of the controller for the three-link biped was viewed from a high-level. As in Chapter 4, angular deceleration impulse disturbances were applied to the model by instantaneously changing the angular velocity of the stance leg \( \dot{q}_1 \). Figure 5.1 shows the initial response of the orbit-stabilizing controller for the angular deceleration impulse disturbance of -0.30 rad/s at points of 5 to 95 percent of the gait cycle in 5 percent increments. From this figure, it is easy to see that the orbit stabilizing control deviates from the desired HZD joint trajectories to drive the system back to the zero dynamics orbit. From the beginning to approximately 50% of the gait cycle, the torso angle \( q_3 \) pitches backward in response to the slowing disturbance. From about 50% to the end of the gait cycle, the torso response is to pitch forward. For nearly the entire gait cycle, the response of
the swing hip $q_2$ is to increase flexion and bring the swing foot farther forward. Figure 5.2 shows the initial response to an instantaneous change of the angular velocity of the stance leg by 0.30 rad/s, an angular acceleration impulse, at the same points in the gait cycle as the angular deceleration impulse disturbances. Here, the deviations are in the opposite direction from the deviations for the push backward. The torso pitches forward in response to disturbances in roughly the first half of the cycle and then pitches back and the swing hip extends to move the swing foot backward in response to disturbances in the second half of the cycle.

These motions were generated from the $\theta$-varying feedback gains of the transverse dynamics coordinates (Section 4.3.3), which is quite mathematically complex. From Figures 5.1 and 5.2 the resultant motions of the torso and swing hip in response to a disturbance are easily identified. Thus, a good approximation of the behavior produced by the orbit-stabilizing controller is obtained by simply offsetting the desired torso and swing hip trajectories in response to a zero dynamics velocity error.
5.2.1 Implementing the Orbit Error Coordinate

Even with the 2nd-order simplification developed in Section 4.3.2.3, the $I$ coordinate is a very complex function of $\theta$ and $\dot{\theta}$, that is potentially difficult to calculate in real-time. Additionally in experiment, a torso offset was almost always used to obtain stable walking, which shifts the desired zero dynamic orbit that the robot should follow. This creates a significant problem for using $I$ to feedback velocity error. For example, Figure 5.3 shows the simulated zero dynamic orbits of a curved foot gait for ERNIE and the same gait with a torso offset of 5 degrees, which increases the step velocity in simulation from 0.69 to 0.92 m/s. Until experiments are run, it is unknown what torso offset will be needed for success in experiment, and the $I$ coordinate will measure errors from the original gait’s orbit. Assuming the 5 degree offset gait represents stable walking in experiment, an orbit-stabilizing controller will constantly try to slow the gait as it measures the zero dynamics velocity as too fast, likely causing the gait to fail.
Two simplifying assumptions were made to implement a velocity feedback error similar to the $I$ coordinate. As discussed in Section 4.3.2.3 with a hybrid orbit, $I$ essentially simplifies to a zero dynamic velocity error because it is undesirable to track to a $\theta$ other than the current value. Thus, it was assumed that any velocity error term in experiment that includes a measure of the deviation from the zero dynamics velocity could be used. The error in hip velocity is a very intuitive measure that satisfies this requirement and is also a reliable measure because it can be computed separately from both the boom’s angular velocity and the joint velocities. It was also assumed that the reference hip velocity could be shifted up or down in response to torso offset changing the gait speed. Figure 5.4 shows the simulated hip velocity for the original gait, the gait with a 5 degree torso offset, and the original gait velocity profile shifted up by 0.25 m/s. The shifted profile is a reasonable match for the torso offset profile and can be adjusted during experiment to tune the heuristic controller to the desired gait speed.

5.2.2 Desired Joint Trajectory Offset Rules

Simple rules for joint trajectories were developed based on the observations of the three-link biped motion at the beginning of this chapter. The first rule is that the torso should pitch forward in response to a slower than desired hip velocity and pitch backward if the hip velocity is too fast. The simplest control law to implement this is to simply make the torso offset a linear function of the velocity error,

$$ T_{\text{offset}} = K_T e_{VH}, \tag{5.1} $$

where $K_T$ is a tunable feedback gain and

$$ e_{VH} = V_{H,\text{ref}} - V_H \tag{5.2} $$

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Figure 5.3. Simulated zero dynamic orbit for a five-link curved foot gait and the same gait with 5 degrees of torso offset.
Figure 5.4. Simulated hip velocities for a five-link curved foot gait, the gait with 5 degrees torso offset, and the original hip velocity shifted by 0.25 m/s
is the hip velocity tracking error.

This law ignores early gait cycle behavior on the three-link model that commands the opposite action, causing the torso to pitch backward. An acceleration of the torso backward at this point in the gait cycle could briefly accelerate the stance leg forward, because their motions are coupled together as are the two links of an underactuated pendulum. In the absence of gravitational forces, this would be the only way to accelerate the stance leg forward. From observing previous experiments, however, the overall forward motion of the center of mass has a larger overall importance in producing stable walking motion. Additionally, this brief backward motion could be an artifact of the lack of a motor model in simulation and unrealizable with the ERNIE hardware. Failing to execute the backward motion nearly instantaneously in response to a disturbance would have a more detrimental effect than ignoring it completely. From previous experiments and simulations, it is also apparent that the torso offset does not need to be driven to zero at impact.

$T_{\text{offset}}$ does not currently have a limiting saturation value in experiment. The largest offsets are encountered in experiment when starting the robot from rest, with the velocity error approximately equal to the desired step velocity. For the values of $K_T$ and speeds tested in experiment, the value of $T_{\text{offset}}$ was not unreasonable and did not cause the robot to emergency stop. In the future, a saturation value should be added for safety purposes. Instead of saturating $T_{\text{offset}}$ itself, the actual angle of the torso should be saturated to prevent it from being commanded to a position beyond the hard stops (approximate 55 degrees forward and 35 degrees backward).

The rule for swing hip offset is slightly more complex. Because matching desired step length is a critical factor in successful walking in experiment, the modulation of the swing hip in response to a disturbance must be driven to zero close to the end of
the gait cycle to prevent step length errors. Thus, the swing leg offset rules are

\[ L_{\text{offset}} = \begin{cases} 
0 & \text{if } K_{L}e_{VH} \leq 0 \\
K_{L}e_{VH} & \text{if } s \leq s_{\text{threshold}} \\
(1 - \frac{s - s_{\text{threshold}}}{0.9 - s_{\text{threshold}}})K_{L}e_{VH} & \text{if } s > s_{\text{threshold}},
\end{cases} \quad (5.3) \]

where \( K_{L} \) is the feedback gain and \( s_{\text{threshold}} \) is the percent of gait cycle after which swing leg offset is linearly reduced in preparation for foot touchdown. Thus, the swing leg will be brought forward in response to a slow velocity, but at some threshold value, the swing hip offset will be linearly decreased such that it is zero by 90 percent of gait cycle. The value of 90 percent builds in some robustness to foot touchdown occurring earlier than planned in the gait cycle due to motor position errors and also robustness to foot touchdown at a longer than desired step length.

From simulation, the swing hip should extend, moving the foot backward in response to an acceleration impulse, but this behavior will be ignored because it is risky to implement in hardware. An early foot touchdown could result in a step length that is much too short. In the acceleration impulse case, it was thought better to not use swing leg offset for recovery because it is very likely the recovery motion would result in an undesirable step length. Thus, \( L_{\text{offset}} \) is prevented from becoming negative. In turn, this will lead to a larger torso offset action for recovery from an acceleration impulse to compensate for the lack of swing leg recovery motion. The maximum value of the offset saturated at 20 degrees to prevent wild swing leg motion responses. Thus, writing the desired joint trajectories including these offsets,

\[ h_{\text{dist}}(q) = h(q) + \begin{bmatrix} T_{\text{offset}} \\ T_{\text{offset}} + L_{\text{offset}} \\ 0 \\ 0 \end{bmatrix}, \quad (5.4) \]
where $h(q)$ are the original planned joint trajectories from Eq. 3.35. If the velocity error $e_{VH} = 0$, the offsets go to zero, and desired trajectories are just as they were originally. These rules were implemented in the Simulink model, which in conjunction with the real-time target dSPACE hardware, controls ERNIE.

5.3 Experimental Implementation

5.3.1 Simulink Control Diagrams

The rules from the previous section were implemented on ERNIE, and the relevant portion of the Simulink block diagram that controls ERNIE is shown in Figure 5.5. The hip velocity was measured in a fashion similar to how $\theta_{\text{exp}}$ was measured in Chapter 3–combining two measures of its value for better reliability. One measure of hip velocity is found from the boom position encoder. Here, the velocity observer uses a 10 point stencil rather than the 5 point one used for motor velocities as described in Section 3.5.2. An encoder resolution of 2,500 lines, relatively low boom angular velocity (approximately 0.2 rad/s) and a relatively high sampling rate (1 ms) prevented a 5 point stencil from having sufficient accuracy. The observed velocity was then passed through a saturation block to ensure that this measurement of velocity does not exceed 2.0 m/s. This is needed because the 10 point stencil has significant error at the first few sample points, and 2.0 m/s is twice as fast as the robot has ever walked. Hip velocity is also measured by using the motor and torso positions and velocities,

$$V_H = \begin{bmatrix} v_2 & 0 & v_1 & 0 & v_2 \end{bmatrix} \dot{q}, \quad (5.5)$$

where

$$v_1 = -R + (L_L - R) \cos(q_1 + q_3 + q_5) - X_F \sin(q_1 + q_3 + q_5) \quad (5.6)$$

$$v_2 = v_1 + L_U \cos(q_1 + q_5). \quad (5.7)$$
The two measurements are averaged to obtain the hip velocity used for feedback. This value is passed through a first-order low-pass filter with a time constant of 0.02 s, the same filtering for the joint-level velocity damping feedback. The reference hip velocity is found from a 7th-order polynomial fit of the hip velocity as a function of gait cycle \( s \). This was done instead of using the joint reference trajectories to calculate velocity to reduce computation time. Considering that reference is not an exact match when the profile is shifted to match the resulting gait speed from torso offset, the errors from this fit were small enough to not matter.
Figure 5.5. Simulink block diagram of disturbance rejection implementation on ERNIE.
With both the reference and actual hip velocities, the velocity error is calculated and gains applied to generate the joint offsets. With a sampling rate of 1 ms, it was thought that updating the offsets at the same rate would create large jumps in the desired offsets from one time step to the next which could be problematic for the motor position controllers. To smooth out the jumps, the offset signals were again low-pass filtered with a time constant of 0.10 s. The value of this time constant was determined experimentally by increasing it until the robot did not exhibit jitter. While it may be possible to decrease this somewhat conservative value to reduce delay, the existing delay does not appear to significantly inhibit disturbance recovery motions. The hip offset is passed directly to the \textit{ref\_partition} block to calculate reference trajectories, while the leg offset signal first passes through a logic block to apply the rules from Eq. \ref{eq:5.4} and determine if the left or right leg is the swing leg.

\begin{table}[h]
\centering
\caption{Disturbance Rejection Control Parameters for Experiments with ERNIE.}
\begin{tabular}{ll}
\hline
Parameter & Value \\
\hline
$K_T$ & 0.5 \\
$K_L$ & 0.5 \\
$s_{\text{threshold}}$ & 0.75 \\
$L_{\text{offset\_limit}}$ & 20 degrees \\
\hline
\end{tabular}
\end{table}
Table 5.1 lists the gains and parameters used in experiment. The gains $K_T$ and $K_L$ were tuned by gradually increasing from 0 in 0.25 increments, trying values up to 1.0 for each. Setting the gains at 0.75 or 1.0 resulted in underdamped behavior because the corrective actions would cause overshooting in tracking the desired hip velocity. Having a fast response without a large overshoot is the desired behavior and accomplished with gains of 0.5 for each. If $s_{\text{threshold}}$ is too small, swing leg offsets will be ineffective because there will not be adequate gait cycle available to bring the swing leg forward before the offset is driven to zero. With too large of a threshold value, the system was unable to drive the offset to zero before impact. A value of 0.75 was successful in experiment. Setting the swing leg offset limit too high (30 degrees) resulted in recovery motions in which the offset could not be driven to zero before impact. In some cases, this actually caused the gait to fail. A limit that is too small will not allow adequate recovery motion. A limit of 20 degrees worked well for the disturbances investigated.

5.3.2 Disturbances in Experiment

5.3.2.1 Repeatable Deceleration Disturbances

To implement repeatable disturbances to the walking cycle, a barrier similar to a track hurdle was constructed as shown in Figure 5.6. The barrier consists of 2 $2 \times 4$'s forming an L shape with a height of 34 in and length of 22 in. This barrier makes contact with the distal end of the boom, slowing the velocity of the robot hips. Weights were added to the horizontal leg of the barrier to increase the disturbance applied to the robot. Two locations for adding additional weight to the barrier are shown at the left end of the barrier in Figure 5.6. The weights added were actually the motor pulleys from the original robot setup before the motors were replaced and weigh approximately 0.28 kg. (0.617 lb.) each.
Figure 5.6. Photo of the disturbance barrier with all 4 weights attached.
5.3.2.2 Acceleration Disturbances

Acceleration disturbances were also tested but in a less repeatable manner by the human experimenter pushing the boom forward at approximately the same rate and position in the room.

5.3.3 Gait Selection and Experiment Procedure

For the best comparison, walking gaits under both the original (HZD) and enhanced (HZD+heuristics) controllers should encounter the disturbance at the same point in the gait cycle, and the previous step for each should be nearly identical. To facilitate these goals, a consistent starting location in the room was identified and referred to as the zero position in the room. Curved foot gaits were selected from those already successful in experiment spanning the range of successful walking speeds. Three gaits were selected, one at each of roughly 0.50, 0.65 and 0.75 m/s. In the following sections, these gaits are referred to as gaits A, B, and C, respectively. Trials were run for a particular gait with the original controller. A suitable torso offset was found such that the gait would complete at least 1 lap of the room, starting with a push to initiate walking from the zero position with the right (inside) leg as the stance leg. The average speed was recorded for a lap around the room, starting at 10 seconds after the initial push to eliminate the effects of variability in the initial conditions. With the robot again started at the zero room position with the right leg in stance, the same gait was executed with the disturbance barrier placed at approximately 180 degrees around the room from the zero position. The enhanced control was tested with the identical procedure. The same torso offset from the corresponding original controller trial was used, and an average speed for a lap of the room was found. If this average speed was not within 0.05 m/s of the average speed of the original controller’s gait, the desired hip velocity profile for the enhanced controller was adjusted in 0.05 m/s increments until the difference in average speed
between the two gaits was within this threshold. Matching average speed around the room is important for a fair comparison in much the same way that matching footfall locations and step velocities just before encountering the disturbance are important. In combination, these constraints help to ensure that the disturbance is encountered at the same point in each gait cycle and when the robot is moving with the same velocity for both gaits.

The number of weights on the barrier was increased until a difference between the original and enhanced controller was noted. For Gait A, this was done in conjunction with tuning the parameters of the enhanced controller. Only two weights were used and resulted in a disturbance from which both controllers could recover. Instead of adding weights and running Gait A again, experiments moved on to Gaits B and C to make sure the tuning from Gait A would be successful for other gaits. For Gait B and C, the same tuning as Gait A was successful and the number of weights was increased to 4 (max). At least 3 trials were run for each gait, controller, and disturbance. This was done to determine the probable gait behavior, as just one trial is prone to error but 3 trials gives a sense of the repeatability of the results.

Acceleration disturbances were investigated by the experimenter pushing the boom forward by hand at roughly the same point in the room and with roughly the same force for each trial. While not formally repeatable, this experimental procedure does demonstrate the qualitatively different responses between the two controllers under acceleration disturbances of slightly differing magnitude at modestly different points in the gait cycle. Future work could design a trigger mechanism to release a device that pushes the boom forward more systematically.

5.3.4 Performance Indicators

The enhanced controller was evaluated on the ability to maintain the desired gait cycle without external disturbances besides those naturally occurring from walking
around the room. The indicators here are the same as in Chapter 3: specific resistance and error tracking. The controllers were also evaluated on the ability to reject the deceleration and acceleration disturbances from Section 5.3.2. This is measured by the number of steps and time required for the gait to return to within 10% of the nominal velocity around the room. If the controller does not overcome the disturbance and falls backward, the time and number of steps in infinite.

5.4 Disturbance Rejection Results

5.4.1 Comparison of Specific Resistance and Error Tracking

The enhanced controller was first evaluated with the performance metrics from Chapter 3. The enhanced controller should have at least equal or better performance compared to the original controller for disturbance-free walking around the room, otherwise a criterion for switching between the controllers should be developed as to not sacrifice significant efficiency performance for gains in robustness. For a more robust comparison, the specific resistance and error values were collected for 3 laps around the room for each gait speed and each controller. A plot of the specific resistance, maximum error, and maximum RMS error for each lap is shown in Figure 5.7. Gait A walked successfully at 0.50 m/s in the experiments from Chapter 3 with a torso offset of 2 degrees. This offset was obtained starting the walking at some random point in the room. For a fairer comparison, all trials in this section were started from the zero position in the room. To reach stable walking starting from the zero position in the room, a torso offset of 4 degrees was required. This resulted in the gait walking at a faster speed than intended. Gaits B and C originally had torso offsets of 4 and 6 degrees, respectively, but for both of these gaits, an offset of 4 degrees was found to produce successful walking. Therefore, all three gaits using the original controller had an identical torso offset of 4 degrees for these experiments.
Based on these observations, torso offset could be largely dependent on the starting location in the room and less dependent on gait speed or specific gait design. Also the calibration routine for the robot is prone to errors of at least 1 to 2 degrees, especially considering the amount of backlash in the drivetrain.

![Figure 5.7. Comparison of performance measures for the original and enhanced controllers in the absence of external disturbances.](image)

In all cases, the specific resistance for the enhanced controller was lower than that for the original controller. Speeding up and slowing down around the room requires more energy than maintaining a constant speed, so the enhanced controller’s smoothing of gait velocity around the room is actually more efficient than the original
controller. The maximum joint error for the enhanced controller is only up to a degree smaller than the original controller, and thus not very significant. The improvement in RMS tracking is minimal as well. The enhanced controller does not significantly improve joint tracking because the control is really an augmentation to the existing control technique—the PD gains are the same—it is just the set points to track that are different. One possible explanation for the modest improvement in tracking may be due to the swing leg retraction before impact. This may move the the drivetrain to one limit of the backlash, such that at the subsequent impact, the motor is immediately backdriven, which is stiffer than compressing through 5 degrees of drivetrain backlash before the motor position is backdriven. This improves tracking because the motor position errors are fed back sooner before the joint flexes due to impact.

5.4.2 Gait A deceleration disturbance

For these trials, the disturbance hurdle had 2 weights on it, and the enhanced controller only used torso offset feedback (i.e. $K_L = 0$). The undisturbed laps around the room for both controllers are shown in Figure 5.8. Each data point indicates the average step velocity of the step that began at the corresponding hip position in the room (0 to 360 degrees), allowing comparison of the response on a scale that eliminates much of the intra-step noise. For this gait, the HZD-based controller walked with an average velocity of 0.65 m/s and the enhanced controller at 0.57 m/s. Thus, the threshold speed (10% below the average) for disturbance recovery is 0.59 m/s for the original controller and 0.51 m/s for the enhanced controller. An offset of 0.05 m/s to the reference velocity profile for the enhanced controller was attempted to better match average step velocity, but resulted in step speeds just before the disturbance location that were faster than those step speeds for the original controller. Since velocity before disturbance is important, the comparison gives the original controller a slight edge in this regard because it has slightly greater step
velocities before the disturbance location. In all trials with both controllers, ERNIE successfully overcame the disturbance and continued walking around the room as shown in Figure 5.9, with the larger torso offset for these experiments resulting in average walking speed greater than 0.5 m/s.

The vertical dotted black line indicates the position of the disturbance barrier and horizontal blue and red dotted lines indicate −10% from the average velocity for undisturbed walking with the original and enhanced controller, respectively. Table 5.2 lists the steps and times for recovery from the disturbance. The enhanced controller recovers in only 4 steps, while the original controller needs 13 or 14, which is a significant improvement. Figure 5.10 is a zoomed view of the steps near the disturbance. The trials of the original controller encountered the disturbance barrier at \( s = 0.85, 0.90, 0.93 \), and the trials with the enhanced controller encountered the disturbance barrier at \( s = 0.96, 0.97, 0.96 \). This is a maximum of 12 and an
Figure 5.9. Step velocity versus room position for Gait A and a disturbance at 180 degrees. Three trials for each controller are shown. Both controllers continue walking, but the enhanced controller recovers more quickly.

average of 7 percent of gait cycle difference, which is a reasonable error with this setup. The enhanced controller rejected the disturbance with smaller overall change in desired speed. Even though the original controller produced walking that was faster than that of the enhanced controller at the time of the disturbance, its gait slowed to a speed below that of the enhanced controller following the disturbance. Therefore, the enhanced controller exhibits superior disturbance rejection capability immediately upon encountering the disturbance.

Figure 5.11 shows the relevant time histories of joint trajectories and hip velocities in response to the disturbance. For comparison, Figure 5.12 shows undisturbed steps of a gait using the original controller that started at the same room position as the gait in Figure 5.11. The third original controller trial is compared with the first enhanced controller trial because the \( s \) value of disturbance encounter is the closest
TABLE 5.2

GAIT A DECELERATION DISTURBANCE RECOVERY MEASURES

<table>
<thead>
<tr>
<th>Gait</th>
<th>Steps for Recovery</th>
<th>Time for Recovery (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HZD-1</td>
<td>14</td>
<td>13.2</td>
</tr>
<tr>
<td>HZD-2</td>
<td>13</td>
<td>11.7</td>
</tr>
<tr>
<td>HZD-3</td>
<td>14</td>
<td>12.4</td>
</tr>
<tr>
<td>Enhanced-1</td>
<td>4</td>
<td>4.0</td>
</tr>
<tr>
<td>Enhanced-2</td>
<td>4</td>
<td>4.2</td>
</tr>
<tr>
<td>Enhanced-3</td>
<td>4</td>
<td>4.2</td>
</tr>
</tbody>
</table>

Figure 5.10. A close-up of Figure 5.9 for the steps near the disturbance.
for these two. The step on which the disturbance is encountered is called step 0, and the robot first encounters the disturbance at time 0. Subsequent steps are sequentially numbered positively, while undisturbed steps before the disturbance are numbered negatively. From the disturbance until the end of step 0, there is almost no difference in the behavior resulting from the two controllers. Throughout step 1 (up to about 0.5s since the disturbance), the enhanced controller keeps the torso pitched forward in response to the slowed hip velocity, whereas the original controller pitches the torso back to follow the desired HZD trajectory. The enhanced controller continues this behavior in step 2, approximately 1.2 to 2.4 seconds after disturbance. By step 3 (2.4 to 3.3 seconds), the enhanced controller is close to following the HZD trajectory. With this action, the enhanced controller completes steps 1-3 slightly faster than the original controller, indicating a quicker recovery to the desired average walking speed.

These trials show that using only torso offset was effective at rejecting disturbance, and even with this small disturbance, the recovery is much quicker than with the original controller. Trials at faster speeds incorporate both offsets and use a larger disturbance to show more differences between the controllers.

5.4.3 Gait B deceleration disturbance

For this gait, weights were added to the barrier, bringing the total to 4. For Gait B, both the torso and swing hip offsets were used, and the average undisturbed speed around the room was roughly 0.63 m/s for both controllers, a much closer match than for Gait A. Figure 5.13 plots the disturbance response of each controller. This disturbance caused failure in the original controller, but the enhanced controller was able to reject this disturbance and keep walking.

The HZD gait and enhanced-controlled gaits encountered this disturbance at $s$ values of 0.53 and 0.50, respectively, indicating a nearly ideal comparison between
Figure 5.11. Trajectory tracking comparison for the original and enhanced controllers on Gait A.
Figure 5.12. Trajectory tracking for the original controller for Gait A. This is a reference of undisturbed walking for comparison with Figure 5.11.
the two because of the small difference. Additionally, it is easy to see that the gaits of both controllers had nearly identical speeds for step -1, the last step before the disturbance was encountered, so the subsequent step 0 began under similar conditions. Both controllers completed step 0, with the original controller actually producing a slightly faster step speed. On step 1, though, the differences become apparent. The enhanced controller produced continued walking that was slowed significantly by the disturbance, whereas the disturbance caused the gait of the original controller to fail by falling backward. This is why there is no step velocity information for the original controller’s step 1 after the disturbance—the step was never finished. The enhanced controller’s gait had a very slow step 1, but it returned to the nominal speed by step 4 as indicated in Table 5.3.

<table>
<thead>
<tr>
<th>Gait</th>
<th>Steps for Recovery</th>
<th>Time for Recovery (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HZD-1</td>
<td>∞</td>
<td>∞</td>
</tr>
<tr>
<td>Enhanced-1</td>
<td>4</td>
<td>4.82</td>
</tr>
</tbody>
</table>

Figure 5.14 shows the velocity tracking and relevant joint trajectories in response to this disturbance for both the original and enhanced controllers. The initial impulse of contacting the barrier is handled similarly by each controller, and the two step
velocities are about the same until the end of step 0. The torso angles are similar, and the swing leg trajectory for the enhanced controller exhibits only slightly more flexion. Again, the different responses to the disturbance are apparent in step 1. The original controller attempts to continue following the same trajectory as steps -1 and 0. At approximately 1.2 seconds after impact, the hip velocity for the original controller becomes negative. This indicates that the hips are moving backward, so the torso and swing leg begin to track their trajectories in reverse. This in turn increases the negative velocity of the hips, resulting in the robot falling backward.

The enhanced controller has a different response. As step 1 begins, the torso stays pitched forward in stark contrast to the torso angle with the original controller. Additionally, the swing hip flexes nearly 20 degrees more than the nominal trajectory as designed with the original controller. The hip velocity for the enhanced controller approaches zero but then begins to speed back up as the torso and swing leg placement aid recovery to the cycle by shifting the overall CoM of ERNIE forward. These actions help complete step 1 with approximately the same hip flexion as the nominal trajectory but with increased torso pitch. Near the completion of step 1, the barrier has been knocked over and no longer impedes the walking progress. In step 2, similar behavior is exhibited, but the magnitudes of the corrections are smaller because the hip velocity has less error from its desired value. Finally in step 3, the enhanced controller has almost returned to the nominal trajectory.

These experiments show that the enhanced controller can reject a larger disturbance than the original controller. These experiments also show that modulating both the torso and swing hip offsets is a successful strategy in experiment.

5.4.4 Gait B acceleration disturbance

An additional experiment was performed with Gait B. Instead of a barrier disturbance, the robot was pushed forward at approximately 180 degrees in the room using
both the original and enhanced controller. Manually pushing the boom forward is not as repeatable as a static disturbance, but attempts were made to push at the same location with the same force. Three trials with each controller were executed, and the disturbance did not cause failure on any trial with the recovery steps listed in Table 5.4. The original and enhanced trials with the closest match in push location and duration are shown in Figure 5.15. The boom was pushed forward near the end of the step starting at 180 degrees until just before the step starting at approximately 200 degrees. The enhanced controller takes a faster step than the original controller immediately following the disturbance, but then the step speed decays more quickly than it does with the original controller. Figure 5.16 shows the gait trajectories for the acceleration disturbance. Here, $t = 0$ corresponds to the step that started near 180 degrees, with the push initiated near the end of this step and continuing for most of the subsequent step. The acceleration disturbance can be clearly identified near $t$
Figure 5.16, where the hip velocities in both gaits increase sharply. The peak hip velocity of the enhanced-controlled gait is slightly larger because the acceleration disturbance was also slightly larger. As steps continue, the speed of the enhanced-controller gait drops more quickly than that of the original controller, although they are quite similar for the first 2 steps. Examining the torso pitch, the enhanced controller leans the torso back to slow the gait, while the original controller continues with the original planned trajectory. The swing hip appears to modulate as well, but the hip offset is zero when the hip velocity is greater than the reference velocity (Equation 5.3). Thus, the apparent modulation is because the swing hip position is measured relative to the torso.

### TABLE 5.4

**GAIT B ACCELERATION DISTURBANCE RECOVERY MEASURES**

<table>
<thead>
<tr>
<th>Gait</th>
<th>Steps for Recovery</th>
<th>Time for Recovery (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HZD-1</td>
<td>6</td>
<td>3.46</td>
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<tr>
<td>HZD-2</td>
<td>5</td>
<td>2.80</td>
</tr>
<tr>
<td>HZD-3</td>
<td>5</td>
<td>2.84</td>
</tr>
<tr>
<td>Enhanced-1</td>
<td>4</td>
<td>2.40</td>
</tr>
<tr>
<td>Enhanced-2</td>
<td>5</td>
<td>3.07</td>
</tr>
<tr>
<td>Enhanced-3</td>
<td>3</td>
<td>1.77</td>
</tr>
</tbody>
</table>
Overall, the enhanced controller slows to the trajectory more quickly than the original controller. This is beneficial because a larger acceleration disturbance could cause the original controller to take several steps at a greatly increased speed. These steps could be fast enough that the swing leg might not respond quickly enough to ensure the appropriate step length, ultimately leading to failure by falling forward. The actions of the enhanced controller work to prevent this by leaning the torso back to slow down the gait more rapidly.

5.4.5 Gait C barrier disturbance

These trials were run with the same 4 weights on the barrier and the average undisturbed step velocity was 0.76 m/s for both controllers. With the increased speed of this gait, these trials had trouble hitting the disturbance at a consistent point in the gait cycle as shown in Figure 5.17. The original controller encountered the disturbance at $s = 0.06, 0.14, 0.08$ and the enhanced controller at $s = 0.78, 0.74, 0.66$. To match the speed of the enhanced controller to the original controller near 180 degrees in room position, a hip velocity offset of 0.20 m/s was needed. This seems to get a close speed match at 180 degrees in the room, which is one desirable criteria for a fair comparison. However, the footfall locations are more varied than for previous trials, so the gaits encounter the disturbance at different points in the gait cycle. A trial was run with an offset of 0.10, and the match between footfalls was better, but the speeds differed by more than 0.05 m/s.

These results are still important in that they demonstrate that the enhanced controller is effective at a range of speeds, but the comparison to the original controller is not as clear-cut as in the slower speeds. Table 5.5 lists the number of steps for recovery for each trial. Even considering that the disturbances are at different points in the gait cycle, the enhanced controller recovers to the cycle in much fewer steps.
than the original controller.

5.4.6 Results with Offset Starting Location

Due to differences between the two controllers, the same gait had slightly different step lengths around the room. Starting experiments under either controller at the same position in the room had issues with aligning foot falls near the disturbance barrier, most apparent in the results from the previous section. The experiment procedure was modified to start experiment trials with the enhanced controller at a different location in the room to obtain better correspondence with the pre-barrier gait conditions of the experiments under HZD-based control. Results obtained with this approach show the advantages of heuristic-based control more clearly.

For Gait B good correspondence between pre-barrier conditions was found for a disturbance that caused walking failure under the HZD-based control. Further experiments were run with a lighter barrier to compare the response when walking under
either controller successfully overcomes the disturbance. To obtain correspondence between pre-barrier gait conditions the enhanced controller was started 5 in behind the starting position of the HZD-based controller. Although good matching between pre-barrier gait conditions was obtained in Section 5.4.3 without a starting position offset, differences in the robot calibration required a starting position offset for these experiments. This value was obtained by first estimating an starting location offset from previous experimental results and then systematically increasing or decreasing the offset until the \( s \) values at impact from each controller were within 0.05 of each other. For this gait, 19 steps were taken before the disturbance barrier, thus the start position offset corresponds to a step length difference of about 7 mm (0.26 in).

Three trials were run with each controller and Figure 5.18 shows the control response to the disturbance barrier with 2 weights. Table 5.6 lists the the pre-barrier gait conditions and the steps and time to recovery. The range in gait cycle for all trials was 5% and the range in pre-impact step velocity was 0.04 m/s, indicating a very close match in pre-impact conditions. Both controllers were successful at rejecting this disturbance and returning to the desired step velocity. The enhanced controller was able to return to the desired velocity much quicker than the HZD-based controller, in only 3 steps compared to 14 or 15.
Figure 5.14. Comparison of relevant trajectory tracking information following a disturbance to Gait B
Figure 5.15. Comparison of step velocity versus room position following an acceleration disturbance to Gait B
Figure 5.16. Comparison gait trajectories following an acceleration disturbance to Gait B
Figure 5.17. Step velocity versus room position for Gait C and a deceleration disturbance at 180 degrees. The controllers encounter the disturbance at vastly different points in their trajectories, but all continue walking.
Figure 5.18. Step velocity versus room position for Gait B and a deceleration disturbance at 180 degrees. The controllers encounter the disturbance at nearly the same point in their trajectories and both continue walking.
<table>
<thead>
<tr>
<th>Gait</th>
<th>$s$ at Impact</th>
<th>Step Velocity before Impact</th>
<th>Steps for Recovery</th>
<th>Time for Recovery (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HZD-B1</td>
<td>0.39</td>
<td>0.61</td>
<td>14</td>
<td>13.45</td>
</tr>
<tr>
<td>HZD-B2</td>
<td>0.37</td>
<td>0.59</td>
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<tr>
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<td>0.37</td>
<td>0.62</td>
<td>14</td>
<td>13.38</td>
</tr>
<tr>
<td>Enhanced-B1</td>
<td>0.36</td>
<td>0.62</td>
<td>3</td>
<td>2.83</td>
</tr>
<tr>
<td>Enhanced-B2</td>
<td>0.34</td>
<td>0.62</td>
<td>3</td>
<td>2.83</td>
</tr>
<tr>
<td>Enhanced-B3</td>
<td>0.35</td>
<td>0.63</td>
<td>3</td>
<td>2.85</td>
</tr>
</tbody>
</table>
Experiments with different start location were also run with Gait C. A starting location offset of 3 in was needed for matching pre-barrier conditions. Figure 5.19 shows the control response to the disturbance barrier with 4 weights, while Table 5.7 lists the pre-barrier gait conditions and the steps and time to recovery. The range in gait cycle for all trials was 4% and the range in pre-impact step velocity was 0.03 m/s, again indicating a very close match in pre-impact conditions. Both controllers were successful at rejecting this disturbance and returning to the desired step velocity, in 4 steps for the enhanced controller and 8 steps for the HZD-based controller. Again, the enhanced controller greatly reduces the number of steps for recovery. Since experiments with both controllers were successful with the maximum number of weights on the barrier, it was not determined whether the enhanced controller can reject a larger disturbance than the HZD-based controller for this gait. However, based on the similarities with the results for Gait B, it was assumed that the enhanced controller will reject a larger disturbance than the HZD-based controller. To demonstrate this the barrier would need to be modified so more weight could be added.
Figure 5.19. Step velocity versus room position for Gait C and a deceleration disturbance at 180 degrees. The controllers encounter the disturbance at nearly the same point in their trajectories and both continue walking.
TABLE 5.7

GAIT C DECELERATION DISTURBANCE RECOVERY MEASURES
WITH BOTH CONTROLLERS SUCCESSFULLY REJECTING THE DISTURBANCE

<table>
<thead>
<tr>
<th>Gait</th>
<th>$s$ at Impact</th>
<th>Step Velocity before Impact</th>
<th>Steps for Recovery</th>
<th>Time for Recovery (s)</th>
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<tr>
<td>HZD-C1</td>
<td>0.68</td>
<td>0.78</td>
<td>8</td>
<td>6.05</td>
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<tr>
<td>HZD-C2</td>
<td>0.68</td>
<td>0.78</td>
<td>8</td>
<td>6.04</td>
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<td>8</td>
<td>6.07</td>
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<td>0.75</td>
<td>4</td>
<td>3.14</td>
</tr>
<tr>
<td>Enhanced-C2</td>
<td>0.66</td>
<td>0.75</td>
<td>4</td>
<td>3.08</td>
</tr>
<tr>
<td>Enhanced-C3</td>
<td>0.67</td>
<td>0.75</td>
<td>4</td>
<td>3.08</td>
</tr>
</tbody>
</table>
Acceleration disturbance experiments were also conducted using Gait C. The starting location offset of 3 in, from the deceleration trials, was also used for the acceleration disturbances. The boom was pushed forward just after the first foot touch down beyond 180 degrees in room position and the push was completed before the next foot touch down. Figure 5.20 compares the control response of 5 trials for each controller, while Table 5.8 lists the recovery metrics, with the range in step velocity before push being 0.03 m/s. When manually pushing the boom, determining the force of the push and the percentage of gait cycle when the push was applied is difficult. For these acceleration trials, the step velocity during the push was also recorded and as shown in Table 5.8. The range of step speeds during the push among these trials was 0.18 m/s. Similar step velocities during the push would indicate that the similar force was applied to the boom, and while a more repeatable experiment procedure is desirable, the range is step velocity during push is acceptable for comparing response of each controller. For this acceleration disturbance, the enhanced controller recovered to the desired trajectory in fewer steps than the HZD-based controller. Additionally, just after the push, the enhanced controller step velocities decrease on the very next step, while step velocities with HZD-based controller increase before decreasing. The increased velocity for the HZD-based controller is likely because the gait does not lose enough energy at impact to slow down immediately and additionally does not modify the joint trajectories to assist in slowing down the step velocity as does the enhanced controller.
Figure 5.20. Step velocity versus room position for Gait C and an acceleration disturbance at 180 degrees. The controllers encounter the disturbance at nearly the same point in their trajectories and both continue walking.
<table>
<thead>
<tr>
<th>Gait</th>
<th>Step Velocity before Push</th>
<th>Step Velocity during Push</th>
<th>Steps for Recovery</th>
<th>Time for Recovery (s)</th>
</tr>
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<td>1.13</td>
<td>6</td>
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<tr>
<td>HZD-C2</td>
<td>0.79</td>
<td>1.05</td>
<td>6</td>
<td>3.15</td>
</tr>
<tr>
<td>HZD-C3</td>
<td>0.79</td>
<td>1.06</td>
<td>7</td>
<td>3.45</td>
</tr>
<tr>
<td>HZD-C4</td>
<td>0.79</td>
<td>0.95</td>
<td>6</td>
<td>3.25</td>
</tr>
<tr>
<td>HZD-C5</td>
<td>0.80</td>
<td>0.98</td>
<td>6</td>
<td>3.21</td>
</tr>
<tr>
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<td>0.81</td>
<td>1.02</td>
<td>5</td>
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<td>1.08</td>
<td>3</td>
<td>1.51</td>
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<tr>
<td>Enhanced-C3</td>
<td>0.82</td>
<td>1.04</td>
<td>5</td>
<td>2.74</td>
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<tr>
<td>Enhanced-C4</td>
<td>0.82</td>
<td>1.01</td>
<td>5</td>
<td>2.76</td>
</tr>
<tr>
<td>Enhanced-C5</td>
<td>0.81</td>
<td>1.07</td>
<td>5</td>
<td>2.76</td>
</tr>
</tbody>
</table>
5.4.7 Overall Conclusions

In undisturbed walking the enhanced controller was more efficient than the HZD-based controller and also had a more consistent step velocity around the room. The enhanced controller rejected small deceleration disturbances in much fewer steps than did the original HZD-based controller. The enhanced controller was also able to reject larger deceleration disturbances than the HZD-based controller. Finally, the enhanced controller rejected acceleration disturbances in fewer steps than did the HZD-based controller. These behaviors were demonstrated for at least two gaits and it is probable that further experiments would be successful with other gaits. Overall the enhanced controller was superior to the HZD-based controller for all scenarios investigated. These performance gains were obtained with a simple approximation of the orbit-stabilizing controller from Chapter 4. Thus, even simple heuristic rules implemented much of the desired performance of a complex theoretical controller. With a more advanced implementation of that controller, even better performance is likely possible.

5.5 Three-Link Simulations of Heuristic-Based Controller

After demonstrating the success of the heuristic rules in experiment, three-link simulations using similar rules to those used in experiment were run to fully compare simulation controllers. Gains and parameters for the simulation were initially set as in experiment and tuned to close to these values. After tuning, the performance of the heuristic-based simulation controller was compared to the HZD-based and orbit-stabilizing controllers as shown in Figures 5.21 and 5.22. In simulation, this controller actually has worse overall performance than the original HZD-based controller. The magnitude of the disturbance rejected is not as large, and the steps until recovery are greater than those needed for recovery with the original controller. This is partially
explained by tuning starting with the parameters from experiment. If gains were
determined for the simulation model without regard to the experimentally values,
the performance may be better. Additionally, the early cycle behavior ignored by the
simple rules may be more important in simulation than it was in experiment. Thus,
although the three-link model provides insight for disturbance recovery behavior, it
is not necessarily a good model for determining the magnitude of disturbances that
the enhanced controller could reject in experiment. Additionally, it points to the
importance of having expert knowledge of the hardware and good insight into what
might be successful in experiment.

5.6 Summary

A simplified three-link model was useful for determining simple heuristic rules
to reject velocity disturbances on the more complex system in hardware. The en-
hanced controller developed from these rules was superior to the original HZD-based
control for all scenarios investigated: undisturbed walking, small decelerations, large
decelerations, and accelerations. Walking with the enhanced controller had a more
consistent speed around the room and also was more efficient with lower specific re-
sistance than the same gait under HZD-based control. The efficiency gain with the
enhanced controller is likely because it smooths the average step velocity around the
room by reducing the accelerations and decelerations present with the original con-
troller. Gaits using the enhanced controller recovered from deceleration disturbances
in much fewer steps than the same gaits under HZD-based control. Additionally, in
at least one case, the enhanced controller rejected a larger disturbance and continued
walking, while that same disturbance caused failure with the original controller. Re-
cover from acceleration disturbances was also quicker with the enhanced controller.
Thus, simple heuristic rules can implement much of the desirable behavior of a more
complicated theoretical controller with less overhead. Better performance could po-
potentially be gained by further refining the heuristics to capture more of the behavior of the full theoretical controller. Mismatches between the simulation and experiment prevented the three-link model from being useful for estimating the magnitude of disturbances that could be rejected in experiment. However, this is a minor disadvantage compared to the great benefits otherwise obtained from using the simplified model. This also highlights the importance of blending knowledge gained from simulation and experimental work, realizing the limitations of the simplified model and using expert knowledge of previous experiments to guide development of the heuristics.
Figure 5.22. Comparison of heuristic (black), orbit-stabilizing (blue), and HZD-based (red) control steps to recovery in simulation for a three-link walker.
6.1 Conclusions

This dissertation addressed aspects of two major issues with biped walking robots: efficiency and robustness. In Chapter 3, overground walking with curved feet under HZD-based control was more energetically efficient than that of point feet. Curved foot walking also had reduced joint trajectory errors compared to point-foot walking. Constraints added to the optimization to enforce “weight-acceptance” behavior just after swing foot touchdown were very important for generating gaits that were successful in experiment. These results were made possible by the designed transition from treadmill to overground walking and robot design improvements from Chapter 2. Thus curved foot walking was superior to point-foot walking and contributes to improving efficiency of biped robots.

In Chapter 4, different options were investigated for improving the velocity disturbance rejection capabilities of biped robots. A novel set of quasi-velocity coordinates provides insight into the coupling between robot link velocities, but it was unclear how to take advantage of this for disturbance rejection. On the other hand, simulations of a three-link biped showed superior rejection of impulsive velocity disturbances when using an orbit-stabilizing controller instead of an HZD-based controller. The orbit-stabilizing controller rejected larger velocity disturbances than the HZD-based controller and returned to the desired step velocity in fewer steps.

The controller algorithm from Chapter 4 was too complex to implement in hardware; however, in Chapter 5, heuristic rules were developed based on that controller
and knowledge of past experiments. This heuristic-based controller was superior to
the original HZD-based controller in all areas investigated. It was more efficient
in undisturbed walking, rejected deceleration disturbances in fewer steps, rejected
larger deceleration disturbances, and rejected acceleration disturbances in fewer steps.
These behaviors were demonstrated with at least 2 gaits at different speeds and based
on the results, could be easily demonstrated for other gaits. These great improve-
ments in robustness were obtained with simple rules that capture much of the impor-
tant behavior of a complex theoretical controller. Thus this heuristic control method
improves the robustness of planar biped walking.

Although the improvements to efficiency and robustness of planar biped walking
were demonstrated with the ERNIE hardware, the techniques are easily adaptable to
other robots. This work as a whole shows the importance of having a strong theoretical
base for understanding the dynamics and control of biped robots, but balanced
with utilizing heuristics, simplifications, and approximations to make practical im-
plementation of desired behaviors possible in experiment.

6.2 Future Work

In addition to the suggestions throughout this work, major areas for future work
are outlined below.

6.2.1 Optimization

The objective function in the gait generation optimization routine (specific resis-
tance) is only a function of the continuous phase of the gait. This objective function
does not consider the sensitivity of the impact map to joint position and velocity
errors. In experiment, the gait trajectory near impact was critical for successful
walking. For example, comparing a curved foot gait at approximately the same
speed before and after the weight acceptance constraints were implemented showed
a 33% increase in step length and a corresponding 15% drop in specific resistance post implementation. Also, the standard deviation in the specific resistance drops by 70% with weight acceptance constraints compared to without, even though the step lengths have about the same variability and more velocity variability with weight acceptance. This seems to suggest that weight acceptance avoids the inefficient steps and decreases the sensitivity to errors in impact conditions. Thus, optimizing to gaits with less sensitive impact configurations is highly desirable. The weight acceptance constraints push the optimization in the right direction, and it may be possible to incorporate the simple models of impact losses from [1] into the objective function to optimize to gaits with less sensitive impact configurations.

Another possible optimization improvement is changing the algorithm used to search the feasible gait space. As described in Chapter 3, the current approach sweeps a range of step lengths at a desired speed to find an optimal step length. Even so, the optimization could get stuck in a local minimum as the parameter variations from one step range to the next are not very large. Thus, step length may not be the best characteristic for searching the feasible gait space. Instead the gaits could be categorized by the basic shapes of the joint trajectories. For example, in nearly all gaits successful in experiment, the swing hip flexes forward to a peak value before retracting just before impact. This motion could be normalized such that gaits with the same characteristic joint trajectory shape, but different speeds and step lengths could be compared. The most efficient gaits may have a common characteristic, such as the location of the peak in the gait cycle. Identifying characteristics such as this could narrow the search space in a different way than the step length and velocity constraints do. Then searching within the restricted space for a desired speed or step length may yield better results.
6.2.2 Foot Shape and Ankle Offset

In this work, HZD framework was extended to curved foot bipeds and was successful in experiment. It may be possible to further expand the HZD framework to include feet with variable radius. Although a foot with circular radius captures the overall motion of the human foot-ankle complex, it does not capture the impulsive toe-off forces. Designing different foot radii at touchdown and lift-off could help mimic the toe-off forces or other desirable gait characteristics not available with a fixed-radius foot.

The effects of ankle offset in overground walking could also be investigated because the results from [43] are influenced by the treadmill and an inaccurate model of curved foot impacts used to determine gaits for experimental testing. Ankle offsets used in [43] were too small to detect any trends in experiment. Using the correct impact model may yield gaits successful in experiment with much larger ankle offsets, making the trends of ankle offset in experiment more clear.

6.2.3 Refining the Phase Variable Calculation

In this work, $\theta$ was calculated by combining two measures with equal weight on each. A different weighting may yield better performance but was not considered herein. It may also be possible to change the weights as the gait cycle progresses rather than have a static value, placing more weight on motor measurements near impact and the boom measurement mid-cycle. Other more advanced techniques from the field of sensor fusion, such as Kalman Filtering, may also be beneficial for calculating a reliable $\theta$.

6.2.4 Improving Disturbance Rejection

For some trials, comparing the performance between the HZD-based and heuristic controller was difficult because different footfalls lead to differences in gait cycle
percentage when first contacting the barrier and were not consistent between the two controllers. Systematically altering the starting location of the heuristic controller relative to that of the HZD-based controller to reduce the discrepancies near the disturbance would be a good way to alleviate this issue. If this strategy proves successful, it could also be used to record the disturbance responses at several points in the gait cycle by moving the room starting location for both controllers.

The heuristics and feedback on hip velocity error constituted a first pass at successful disturbance rejection in experiment. These rules could be further refined for better performance. Introducing heuristic $\theta$-varying gains should improve the performance, capturing more of the orbit-stabilizing behavior from simulation. This could be implemented by transitioning from linear heuristics to a higher-order polynomial rules.

The heuristics could also be replaced if scaling the orbit-stabilizing controller up to the five-link model is possible. Leaving the $g_i$ functions dependent on $\ddot{\theta}$ is key for developing the controller. It was originally thought that this dependency had to be resolved, which prevented constructing the controller because resolving the dependency was infeasible.

Improving the velocity observer to that of [17] would also be beneficial as that observer is more accurate just after impact and overall compared to the current observer. However, computational complexity is again the limiting factor, and some work must be done to simplify the calculations to implement in real-time.

6.2.5 Transition to 3D

Combining HZD-based controllers with other approaches may be an avenue to success with 3D bipeds. With a humanoid biped it may be possible to used a HZD-based controller for stability in the sagial plane while using a ZMP-based approach in the coronal plane. This interesting combination would allow the energetic benefits
of HZD-based control when walking, while still retaining the robustness of the ZMP approach for standing still. This would require careful consideration of actuation of the ankle motors. It may be possible to design walking gaits assuming a passive ankle in the sagial plane (making the robot underactuated in that plane), and allow the robot walking motion to backdrive the motor, similar to the weight acceptance constraints on ERNIE. The coronal plane ankle motor would still be actuated to control the location of the ZMP in that plane. When transitioning to a stand still, the torque command to the sagial plane ankle motor could be increased to gradually slow and stop the walking.
APPENDIX A

DETAILED DRAWINGS
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University of Notre Dame
Title: TABLE BEARING
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#7 DRILL ( 0.201 )  \( \Phi 0.750 \)  ( 4 ) HOLE

10-24 UN-C 2B TAP (0.150)  \( \Phi 0.500 \)  ( 8 ) HOLE

8X \( \Phi 0.257 \) THRU 1/4-20 UN-C 2B CLEAR EQ SP

\( \Phi 5.500 \)

4X EQ SP CUT OUT FOR BEARING DISASSEMBLY

DIMS NOT CRITICAL

AA-AASECTION

University of Notre Dame
Title: T_INNER
Material: Aluminum

Scale: 0.500 Date: 09-Jun-10 Drawn By: David C Post
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EQ SP

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EQ SP

1/4-20 UNC 2B CLEAR F DRILL (0.257) THRU-(4) HOLE
EQ SP

10-24 UNC 2B CLEAR #7 DRILL (0.201) THRU-(8) HOLE
EQ SP

6.37
3.900
22.5
2.00
0.10
5.500
Aluminum

David C Post

Drawn By: David C Post
Date: 09-Jun-10
Scale: 0.750

Material: Aluminum
MIRROR ACROSS AA PATTERN OF 8 HOLES MIRROR ACROSS AA
10-24 UNC- 2B CLEAR #9 DRILL ( 0.196 ) THRU -( 16 ) HOLE

6.63 PRESS FIT FOR KAYDON KD060XP0
7.000 -.001 - .002
7.500

1/4-20 UNC - 2B CLEAR
F DRILL ( 0.257 ) THRU -( 4 ) HOLE

EQ SP

16X φ 257 THRU 1/4-20 UNC - 2B CLEAR
PATTERN OF 8 HOLES MIRROR ACROSS AA

1/4-20 UNC - 2B CLEAR
F DRILL ( 0.257 ) THRU -( 4 ) HOLE

EQ SP

4X EQ SP CUT OUT FOR BEARING DISASSEMBLY
DIMS NOT CRITICAL

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Date: 09-Jun-10
Drawn By: David C Post
University of Notre Dame

Title: BOOM_PLUG

Material: Aluminum

Scale: 1.000
Date: 14-Jun-10
Drawn By: David C Post
University of Notre Dame

Title: BOOM_END_PLATE_CENTER

Material: Aluminum

Scale: 1.000  Date: 14-Jun-10  Drawn By: David C Post
THRU HOLE TOP SECTION
TAP THRU BOTTOM SECTION

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#25 DRILL ( 0.150 ) THRU-( 1 ) HOLE

10-24 UNC- 2B CLEAR
#9 DRILL ( 0.196 ) THRU-( 1 ) HOLE

2X Ø 0.1875 THRU
HOLE FOR 0.1875 DOWEL PIN

THRU HOLE TOP SECTION
TAP THRU BOTTOM SECTION

10-24 UNC- 2B TAP THRU
#25 DRILL ( 0.150 ) THRU-( 1 ) HOLE

10-24 UNC- 2B CLEAR
#9 DRILL ( 0.196 ) THRU-( 1 ) HOLE

2X Ø 0.1875 THRU
HOLE FOR 0.1875 DOWEL PIN

University of Notre Dame

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<td>1.1625</td>
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<tr>
<td>0.500</td>
<td>2X THRU HOLE FOR 0.1875 DOWEL PIN</td>
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<td>2.825</td>
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<td>0.500</td>
<td>10-24 UNC - 2B TAP 0.160</td>
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<td>0.150</td>
<td>#25 DRILL (0.150) THRU (2) HOLE</td>
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<td>0.30</td>
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<td>0.375</td>
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<td>1.00</td>
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<td>14-Jun-10</td>
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<td>Drawn By:</td>
<td>David C Post</td>
</tr>
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<td>MOTOR_CLAMP</td>
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SCALE 0.200

UNLESS OTHERWISE SPECIFIED
DIMENSIONS ARE IN INCHES
ANGLES = 0.5
2 PL = 0.01 3 PL = 0.005

DRAWN BY Post, D
DATE Apr-07-13

FILE: TORSO.ASM
SCALE: 0.200 SHEET 2 OF 2
M5 CLEARANCE
(Ø .2283 )
(4) HOLES

Ø 2.13
Ø 1.8110
Ø 1.260

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DRAWN BY Post, D
DATE Jul-14-11

UNLESS OTHERWISE SPECIFIED
DIMENSIONS ARE IN INCHES
ANGLES = 0.5
2 PL = 0.01 3 PL = 0.005

MATERIAL ALUMINUM
QUANTITY (4)

FILE: NEW_SPACER
SCALE: 1.000 SHEET 1 OF 1
DEPTH OF DRILL POINT
(2) HOLES
1.90  BC
.31
25.00°
THRU
(6) HOLES
40.0°
THRU
(4) HOLES
2.00 BC
SEE DETAIL A
SEE DETAIL B
#6-40 UNF TAP
#6-32 UNC TAP THRU
(4) HOLES
\(\phi\) 2.00 BC
\(\phi\).50 THRU
(6) HOLES
40.0°
PATTERN
\(\phi\) 1.65 BC

\(\phi\).31
\(\phi\) .336
DEPTH OF DRILL POINT
(2) HOLES
\(\phi\) 1.90  BC
\(\phi\).472
THRU
(12mm SHAFT)
\(\phi\).1875
THRU
\(\phi\) .400
25.00°
SLOT
\(\phi\).1875

\(\phi\).941
.276
.051
.10
.50
R.05

\(\phi\) 2.70
\(\phi\) 2.32
\(\phi\) .984

\(\phi\) .157
(4mm KEY)

\(\phi\) .307 REF

\(\phi\) .336
\(\phi\) .25.00

\(\phi\) .1875

\[22.5°\]

\[25.00°\]

\[100.0°\]

\[80.0°\]

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\text{University of Notre Dame}\]

\[\text{DRAWN BY Post, D}\]

\[\text{DATE Jun-16-11}\]

\[\text{SCALE: 0.750 SHEET 1 OF 1}\]

\[\text{UNLESS OTHERWISE SPECIFIED DIMENSIONS ARE IN INCHES ANGLES = 0.5} 2 \text{ PL = 0.01 3 PL = 0.005}\]

\[\text{CHECKED}\]

\[\text{MATERIAL STEEL FILE: MOTOR_PULLEY_BASE_HIP}\]

\[\text{QUANTITY (2) SCALE: 0.750}\]

\[\text{REV}\]
OTHER HOLES SAME PATTERN AS MOTOR_PULLEY_BASE

- \( \phi 25 \text{MM} \) (\( \phi 0.9843 \))
- \( \phi 2.70 \)
- \#6-32 THRU (4) HOLES
- \#4-40 TAP THRU (2) HOLES
- 2.70
- 25MM
- ( ) 0.9843
- R.05
- 22.5°
- .10

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DRAWN BY Post, D
DATE Jun-16-11
CHECKED

MATERIAL STEEL
QUANTITY (4)

FILE: MOTOR_PULLEY_COVER2
SCALE: 1.000
SHEET 1 OF 1

UNLESS OTHERWISE SPECIFIED
DIMENSIONS ARE IN INCHES
ANGLES = 0.5
2 PL = 0.01 3 PL = 0.005

OTHER HOLES SAME PATTERN AS MOTOR_PULLEY_BASE

- #4-40 TAP THRU
- \( \phi 1.900 \text{ BC} \) (2) HOLES

229
UNDIMENSIONED ARCS ARE NON-CRITICAL

CLAMP AROUND 45MM MOTOR AND 2x .06 RUBBER DAMPER
(Ø 1.892) (4) PLACES

#6-32 TAP .50
(5) HOLES

#6-32 TAP THRU

R1.000

54.0°
CLAMP AROUND Ø45MM MOTOR AND 2x .06 RUBBER DAMPER (Ø 1.892) (4) PLACES

UNDIMENSIONED ARCS ARE NON-CRITICAL

#6-32 CLEARANCE THRU

#6-32 TAP ¶ .50 (3) HOLES

R1.000

54.0°

5.125

.190

.875

.125

.125

.250

.250

.250

.250

.250

.250

.250


