EXPERIMENTAL STUDY OF ABSOLUTE INSTABILITY OVER A ROTATING DISK

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by

Hesham Abdel Ghafar Othman Bekhit, B.S., M.S.

Thomas C. Corke, Director

Graduate Program in Aerospace and Mechanical Engineering Notre Dame, Indiana

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Abstract

by

Hesham Abdel Ghafar Othman Bekhit

A series of experiments were performed to study the absolute instability of Type I traveling cross-flow modes in the boundary layer on a smooth disk rotating at constant speed. The basic flow agreed with analytic theory, and the growth of natural disturbances matched linear theory predictions. Controlled temporal disturbances were introduced by a short-duration air pulse from a hypodermic tube located above the disk and outside the boundary layer. The air pulse was positioned just outboard of the critical radius for Type I cross-flow modes. A hot-wire sensor primarily sensitive to the azimuthal velocity component, was positioned at different spatial locations on the disk to document the growth of disturbances produced by the air pulses. Ensemble averages conditioned on the air pulses revealed wave packets that evolved in time and space. Two amplitudes of air pulses were used. The lower amplitude produced wave packets with linear amplitude characteristics that agreed with linear-theory wall-normal eigenfunction distributions and spatial growth rates. The higher amplitude pulse produced wave packets that had nonlinear amplitude characteristics. The space-time evolution of the leading and trailing edges of the wave packets were followed well past the critical radius for the absolute instability based on Lingwood (1995). With the linear amplitudes, the absolute
instability was dominated by the convective modes, agreeing with the linear DNS simulations of Davies and Carpenter (2003). With the nonlinear amplitudes, larger temporal growth of the wave packets existed which supports the finite amplitude analysis of Pier (2003), and more closely resembles the wave packet evolution in the experimental study of Lingwood (1996). This suggests that the disturbance levels in the experiment that was intended to demonstrate the linear analysis, were likely finite.
To...

My wife Rehab and our children Hazem and Noha,

My parents, Mr. Ibrahim Taha, Mr. Mohammad Osman,

Salah, Iman, Mohammad, and Sherif.
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SYMBOLS

$P$ Pressure

$R$ Reynolds number

$T$ Temperature and time period of one rotation

$V_{exit}$ Exit velocity from the calibration nozzle

$n$ Azimuthal mode number

$r_{obs}$ Critical radius of absolute instability

$r_{cross}$ Critical radius of cross flow instability

$u_\theta$ Azimuthal velocity

$z^*$ Dimensionless wall-normal distance above the surface of the disk

$\beta$ Proportional constant

$\rho$ Air density

$\psi$ Spiral angle

$\omega$ Angular velocity

$\nu$ Kinematic viscosity

$r, \theta, z$ Cylindrical polar coordinates
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CHAPTER 1

INTRODUCTION AND OBJECTIVES

1.1 Introduction

The development of instability waves and the mechanisms through which they lead to transition to turbulence in three-dimensional boundary layer flows are of fundamental importance in hydrodynamic stability theory. Besides the intrinsic mathematical and experimental challenges that characterize stability work, the main driving force for the study of the boundary layer stability is the understanding, prediction, and the control of transition to turbulence. The origins of turbulent flow and transition to turbulence remain the most important unsolved problems in fluid mechanics and aerodynamics. The state of the three-dimensional boundary layer flow on the wings of an aircraft determines the viscous drag contribution to the total drag of the aircraft. The viscous drag, which depends on the flow state, can equal 40% - 50% of the total drag (Bushnell, et al. (1977) [7]). Any decrease in the viscous drag can lead to reduced fuel expenditures. Because laminar flow yields less viscous drag than turbulent flow, laminar flow is preferable, and results in a net fuel savings in comparison with the fuel expenditures of an aircraft with turbulent flow. This fuel savings can translate directly into reduced operating costs for the industry in terms of millions (if not billions) of dollars.

A substantial amount of stability work is devoted to three-dimensional boundary layers to obtain drag reduction by increasing the surface area of the wing over
which the flow is laminar, e.g. Cebeci and Stewartson (1980) [8], Mack (1984) [38],
and Malik, Li, and Chang (1994) [40]. It is preferable to maintain the laminar
boundary layer to reduce the total skin-friction drag since about 60% of the drag
on an aeroplane during flight is due to skin-friction drag (Reshotko (1984) [51]). To
maintain the laminar flow, two control methods could be used; active control such
as surface suction and surface cooling, and passive control where the surfaces are
covered by complaint coatings.

While the laminar flow transition of two-dimensional boundary layers is gov-
erned initially by plane Tollmien-Schlichting waves, three-dimensional boundary
layers are dominated by cross-flow instabilities. One fully three-dimensional flow
useful in studying transition phenomena is that the flow over a flat disk which ro-
tates about an axis perpendicular to its plane with a uniform angular velocity, ω, in
a fluid otherwise at rest. This particular boundary layer flow represents a canonical
three-dimensional flow which exemplifies the cross flow instability. cross-flow insta-
bility often results in the formation of the stationary co-rotating vortices, commonly
called cross-flow vortices. This phenomena is also observed in other flow geometries
such as the swept wings and rotating cones. cross-flow vortices arise as a result of
an inviscid instability of the inflectional cross-flow velocity profile produced by the
three-dimensionality of the mean flow field. The cross-flow instability is one type of
convective instability that could appear over a rotating disk. The second type is an
absolute instability in which the disturbances grow in time at every fixed point in
space. The implication of the absolute instability is that the steady, laminar basic
state cannot exist beyond the critical Reynolds number of absolute instability.

The boundary layer instability mechanism in the flow over a rotating disk is
similar to that exhibited in the leading edge region of a highly sweptback wing. Both
have inflectional cross-flow velocity profiles resulting in the cross flow instability. These instabilities appear as outward spiraling waves over the rotating disk surface.

The reasons for investigating the three-dimensional boundary layer on a rotating disk rather than on a swept wing are the following: there exists an exact numerical (mean laminar flow) solution to the Navier-Stokes equations (von Kármán (1921) [54]) and the velocity distribution of the laminar flow has been calculated by Cochran (1934) [10]. Moreover, the shape of the laminar velocity profiles and the thickness of the boundary layer are independent of the radius. The flow is thus much simpler to describe than that over a swept wing, where the displacement thickness and components of velocity vary along the streamlines because of the pressure gradients. There is also no need to change the sweep angle as in the case of the swept wing. Another advantage of studying three-dimensional boundary layers on a rotating disk is space consideration.

Most of the past researches studied the details of the development of the cross-flow and it is well understood (Reed and Saric (1989) [50] and Saric et al. (2003) [52]). On the other hand, there are only a few studies on the recently discovered absolute instability over a rotating disk, and it is still not fully examined. One of the important first contributions related to the absolute instability comes from Lingwood (1995) [35]. She theoretically studied the linear inviscid stability and the linear stability with viscous, Coriolis and streamline curvature effects, and concluded that the flow was absolutely unstable for Reynolds numbers above 510. Lingwood (1997) [37] corrected this value to be 507.3 instead. Lingwood (1996) [36] experimentally studied the unsteady disturbances over a rotating disk. She used a hole through the thickness of the disk to create time-dependent disturbances.

Lingwood's linear stability analysis is correct and has been substantiated re-
cently in a subsequent linearized Navier-Stokes simulation by Davies and Carpenter (2003) [17]. However in Lingwood’s (1996) [36] experiment, she predicts that transition to turbulence on the rotating disk was due to the absolute instability, and further that it accounted for the relatively close occurrence of transition Reynolds numbers of a number of past experiments. However, the linear simulation of Davies and Carpenter (2003) indicates that the absolute instability is localized in Reynolds number and is dominated by the convective instability, which then dictates where transition occurs.

1.2 Objectives

The studies of Lingwood (1995) [35] and Lingwood (1996) [36] and the simulation by Davies and Carpenter (2003) [17] have motivated us to re-investigate the role of the absolute instability on transition to turbulence of the flow on a rotating disk. So, the objective of this research is to study convective and absolute instabilities of the incompressible boundary layer flow over a disk that rotates in still air. The approach taken was to experimentally investigate the development of disturbance wave packets produced by controlled time-dependent disturbances. The experiment would follow that previously performed by Lingwood (1996) [36]. However, since it was motivated by linear stability analysis, documentation of the disturbances would be made to ensure that they agreed with linear theory predictions, and therefore satisfied the original theoretical assumptions.

Rather than use a hole through the disk to introduce time-dependent disturbances, we would develop a new technique that would produce short-time disturbances above the disk, outside the boundary layer. The advantages of this approach would be to eliminate stationary disturbances coming from the presence of the hole in the disk surface, and to have the ability to create disturbances anywhere over the
surface of the disk.

During the initial course of this research, two important papers were published. As mentioned, Davies and Carpenter (2003) suggested based on a linear simulation, that the convective instability dominates at all the Reynolds numbers investigated even for the strongly absolutely unstable region. Pier (2003) [48] numerically computed fully saturated primary finite-amplitude waves and the associated nonlinear dispersion relation using a local parallel flow approximation. He concluded that transition to a turbulent flow is triggered by a secondary absolute instability while the transition itself is controlled by primary absolute instability. Therefore another objective of this research was to investigate the effect of both linear and finite-amplitude disturbances on the absolute instability mechanism.

1.3 Road Map

This dissertation consists of eight chapters. Chapter 1 presents a motivation for this research and the objectives of this study. A literature review is found in Chapter 2, where the previous studies on the cross-flow and absolute instabilities were reviewed. In Chapter 3, a detailed description of the experimental setup is presented. The construction of the disturbance generator is also presented. Chapter 4 presents a complete explanation of experimental conditions and testing procedures. Information about the softwares that used to control the motion of the hot-wire probe and to acquire data are presented. The results are presented in three chapters. Chapter 5 shows the results and discussions of the base flow condition that is used to document the flow over the disk. Chapter 6 shows the results and discussions of the linear disturbances cases. Results and discussions of the finite-amplitude disturbances are found in Chapter 7. This is followed with concluding remarks in Chapter 8. All of the documentations of the development of the disturbance wave
packets and spectral of the velocity fluctuations analysis for the linear case are found in Appendices B and C. Appendices D and E contain similar results for the finite-amplitude disturbance case.
CHAPTER 2

BACKGROUND

2.1 Introduction

Linear stability theory has been extremely successful in predicting many essential features of the initial development of a variety of flow instabilities. In any such comparison, there is a choice between temporal theory and spatial theory. Recently there has been renewed interest in whether three-dimensional boundary layer flows are absolutely or convectively unstable. The absolute or convective nature of a linearly unstable flow determines how a disturbance, which introduced into the boundary layer and initially small, will evolve in time and space. Temporal stability theory assumes that a disturbance will evolve in time from some initial spatial distribution, and so the complex frequency has to be determined as a function of the real wave number. In spatial theory, it is assumed that a disturbance will evolve as it moves downstream, that is the complex wave number has to be determined as a function of the real frequency. An unstable flow is classified as being absolutely unstable if its response to an impulse in time and space amplifies unboundedly everywhere in space for large time. If, on the other hand, the impulse response at a fixed location in space grows and then ultimately decays after some time, then the flow is classified as being convectively unstable. Figure 2.1 shows the difference between types of instabilities. The impulse response of a flow is first defined as the instability wave field generated by a Dirac delta function in space and time. Here $t$
is the time, \( r \) is the space, and \( r_s \) is the source of disturbances. In the convectively unstable flow, the disturbances propagate away from the source as it grows, leaving the source area undisturbed (Figure 2.1a). Plane-Poiseuille flow and mixing layer flow are examples of convective instability. In contrast, a flow is absolutely unstable if its impulse response spreads into regions on both sides of the source, so that the disturbance grows in time at fixed spatial positions (Figure 2.1b). Examples of such flows are closed flow systems such as Taylor-Couette flow and Benard convection. Figure 2.1c shows the case where the disturbances are initially convective and at certain Reynolds number, they become absolutely unstable and the trailing edge of the wedge of instability asymptotes towards the vertical, which implies that beyond this radius the disturbance grows in time at all fixed radial locations, Lingwood (1996) [36].

Therefore, with the above consideration, in absolutely unstable flow any transient disturbance will grow exponentially large everywhere contaminating the flow upstream and downstream of its point of origination. The assumption of linear stability will break down at some point and nonlinear effects will become important. In convectively unstable flow, a transient disturbance will be convected away as it grows, eventually leaving the basic flow undisturbed. Then, it can be seen that spatial stability theory is only appropriate if the flow is convectively unstable. For absolute instability, on the other hand, a combined spatio-temporal analysis is required.

The convective-absolute classification of an instability is an important issue with regard to flow control. If the flow is convectively unstable, then in order to influence its spatial development localized forcing will be effective. On the contrary, the self excited growing instability in an absolutely unstable flow will overcome the
Figure 2.1. Sketches of typical impulses responses; (a) absolutely unstable, (b) convectively unstable, and (c) initially convectively but becoming absolutely unstable.
forcing. Therefore, it is extremely useful to define the parameter range where the flow is either convectively or absolutely unstable. Reed and Saric (1989) [50] and Saric et al. (2003) [52] reviewed the progress on the rotating disk problem up to date.

Figure 2.2 shows a sketch of the locations of critical radii. In this figure, \( r_{cross} \) refers to the critical radius of cross flow instability, \( r_{abs} \) refers to the critical radius of absolute instability, and \( r_{tr} \) refers to the radius at which the transition occurs. If there is an absolute instability, then the disturbances should grow temporally to large amplitudes immediately when they reached the critical radius of the absolute instability. In other words, the critical radius of transition should be less than the critical radius of absolute instability or, at most, they should be identical. In the following sections, the cross-flow and absolute instabilities will be reviewed.

![Figure 2.2. A sketch to demonstrate the relationships between different critical radii.](image-url)
2.2 Cross-flow Instability

Boundary layer transition on a rotating disk was first studied by Smith (1946) [55] using hot-wire techniques. He observed that sinusoidal disturbances appeared in the disk boundary layer at sufficiently large Reynolds numbers. Approximately 32 oscillations were observed within one revolution of the disk. Analysis of these data indicated that the disturbances propagate at an angle of approximately 14° (measured between a normal to the vortex axis and the radius). Figure 2.3 shows the description of the spiral angle.

Later, in a flow visualization study using the china-clay technique, Gregory et al. (1955) [21] observed 28-31 vortices spiraling outward over the disk at an angle of about 14°. The vortices, which appeared stationary relative to the disk, were first observed at a Reynolds number of $R \approx 430$. Transition to turbulence occurred at $R \approx 530$. The stationary-vortex flow established in the rotating disk flow was subsequently studied by Fedorov et al. (1976) [19], Kobayashi et al. (1980) [30], Clarkson et al. (1980) [9] and Malik et al. (1981) [42]. A phenomenon was observed by Fedorov et al. (1976) operating at generally higher rotational speeds and using visual and acoustic techniques. In addition to the 30 or so vortices observed by others, they also noted a region as low as $R \approx 245$ containing 14-16 vortices propagating at an angle of about 20°, which is later defined as Type II cross-flow instability.

Secondary vortex formation has also been documented. Clarkson et al. (1980) [9] using injected dye and high speed photography with a disk rotating in water, observed secondary vortices occurring between the primary (inflectional instability) vortices which ultimately led to breakdown to turbulence. This has also been observed by Kobayashi et al. (1980) [30] in air using titanium tetrachloride flow visualization.
Figure 2.3. A schematic diagram showing the coordinate system and a description of the spiral angle.

Figure 2.4. A schematic diagram showing the flow pattern on a rotating disk at 1800 rpm (Kobayashi et al. (1980)) [30].
Figure 2.4 shows the visualized flow pattern of a boundary layer flow on a disk rotating clockwise at a speed of 1800 revolutions per minute. It can be distinctly observed that two critical radii are indicated separating radially laminar, transition, and turbulent regimes, and that spiral vortices appear in the transition regime. The white lines drawn radially were used in order to synchronize the stroboscope (Kobayashi et al. (1980) [30]).

Kobayashi et al. (1980) and Malik et al. (1981), both using hot-wire techniques, estimated the critical Reynolds numbers to be 297 and 294, respectively, which tends to confirm the theoretical prediction of 287 by Malik et al. (1981). The discrepancy between the values of the critical Reynolds numbers obtained from hot-wire studies and the earlier relatively high values, $R \approx 400$, obtained by visual techniques clearly results from the insensitivity of the visual techniques to very small disturbances.

In the experiments reported by Malik et al. (1981) [42], the circumferential velocity fluctuations were measured to Reynolds numbers as low as $R \approx 125$ to test the validity of the linear stability theory calculations. It was shown that the experimental data supported the theoretical predictions when the effect of Coriolis forces and streamline curvature were included in the theory. Kohama (1984) [33] performed an experimental study in which he found that the spiral vortices being generated in the transition regime of a rotating disk were co-rotating vortices with neighboring ones; and the structure was essentially the same as the spiral vortices appearing on a rotating sphere. He also concluded that the phase velocity of the spiral vortices were zero which means that the generated vortices were fixed to a rotating disk.

Kohama (1987) [34] performed another experiment in which he proved that the number of the primary vortices was increased with a square root of Reynolds
number with a constant of proportionality of the range of 1.3–1.4. This result makes a contradiction with the result of Malik et al. (1981) who concluded that the relation between the primary vortices and Reynolds number is linear. On the other hand, Kohama (1987) showed that the secondary instability had the prominent role for the turbulent transition process than the primary instability.

The effect of the streamline curvature and Coriolis forces had been studied by Malik (1986) [41], Itoh (1994) [26], Itoh (1996) [27], Itoh (1997) [28], Takagi et al. (1998) [58], and Buck et al. (2000) [6]. Malik (1986) calculated the neutral curve for stationary disturbances including the effect of streamline curvature and Coriolis forces. He found that the minimum critical Reynolds number of 285.36, in agreement with the results of Malik et al. (1981) [42], and the vortex angle of 11.4° at the critical point. He also noted that a second minimum associated with a vortex angle of 19.45°, similar to that of Fedorov et al. (1976) [19]. Itoh (1994) formulated the linear stability problem to explicitly include the curvature of the inviscid streamline at the boundary layer edge. He found that strongly curved inviscid streamlines produced a centrifugal type instability similar to the Görtler instability on concave walls (Saric (1994) [53]).

Because streamlines are highly curved near the symmetry axis in rotating disk flow, Itoh (1996) [27] investigated whether the streamline curvature instability could occur in these boundary layer. He found that the linear stability neutral curve had two lobes: one for the cross-flow instability and the other for streamline curvature instability. The streamline curvature instability was unstable at much lower Reynolds numbers than cross-flow, and the range of unstable wave numbers decreased as Reynolds number grew. Cross-flow unstable wave number range was increased with increasing Reynolds number. He concluded that the streamline curvature instability
was the source of the lower branch of the neutral curve.

Although the streamline curvature instability was destabilized at a much lower Reynolds number than the cross-flow instability, in most experiments the cross-flow instability was observed. This discrepancy is likely a result of either smaller growth rates or smaller receptivity of the streamline curvature instability versus the cross-flow instability. To address the growth rate question, Itoh (1997) [28] calculated that, in a region unstable to both both instabilities, the growth of streamline curvature modes was larger than that of cross-flow modes. In a subsequent experiment, Takagi et al. (1998) [58] confirmed the predictions by Itoh (1997) at least qualitatively.

Buck et al. (200) [6] demonstrated that for point-source forcing early growth of streamline curvature disturbances could suppress the appearance of cross-flow disturbances that were observed under zero-forcing conditions. However, they suggested that the suppression might not prevent or delay transition because the large-amplitude streamline curvature disturbances may lead to turbulence.

Hall (1986) [22] investigated the stationary instabilities asymptotically. He found that in addition to the inviscid modes found by Gregory et al. (1955) [21] a stationary short wavelength mode whose structure was fixed by a balance between viscous and Coriolis forces and could not be described by an inviscid theory. The non-parallel effects had been taken into consideration in his procedures. He found good agreement with Malik (1986) [41] in high Reynolds number limit and concluded that his theory could be useful tool in finding structures in general three-dimensional boundary layer.

A major contribution to the understanding of the rotating disk flow was that of Mack (1985) [39]. He followed Gaster (1975) [20] to theoretically study the
stability characteristics of the fixed vortices by assuming a white-spectrum, zero-frequency source distribution over Wilkinson and Malik’s (1983 [63], 1985 [64]) area of roughness. He then let the disturbances differential equations filter and amplify the spectrum into a wave-interference pattern that turned out to be very similar to the pattern observed by Wilkinson and Malik. With curvature, a critical Reynolds number was found to be 287 and the detailed characteristics of the results were in excellent agreement with experiments.

Wilkinson and Malik (1985) [64] conducted an experimental study for a clean disk and a disk with a single, isolated roughness element. They concluded that the stationary vortex flow field observed on the disk prior to transition was originated at discrete, isolated disturbance sites. These sites were apparently randomly situated atmospheric dust motes. They reported that the disturbance generated by a single, three-dimensional surface roughness evolved spatially as a wave packet. The wave packet spread rapidly around the disk, merged with each other, eventually filled the entire circumference of the disk. They have also observed that the stationary, secondary instabilities were invariably the final stage of the transition process prior to turbulent breakdown. The integrated-growth factor, measured from initiation of the disturbance to turbulent breakdown, for the wave packet resulting from the dust motes was in the range $e^9$ to $e^{10}$. They found that the transition Reynolds number based on hot-wire evidence of turbulent breakdown was in the range $543 < R < 556$ for the clean disk and $521 < R < 530$ for the disk with a single, three-dimensional roughness element.

The characteristics of the instabilities of the von Kármán (rotating disk) boundary layer have been numerically computed by Faller (1991) [18] for a large range of the parameters; Reynolds number, wavelength and angle. The results included
growth rates, phase speeds, group velocities, the ratio of the energy of the longitudinal fluctuations to that of the overturning cells and the cell structures. The computed results were in excellent agreement with laboratory observations.

The study of the sensitivity of the Type I cross-flow instability to patterned distributed roughness was done experimentally by Corke and Knasiak (1994) [14]. They used the same rotating disk facility used previously by Wilkinson and Malik (1985) [64], Malik et al. (1981) [42], and Corke (1992) [13]. They found that the amplified azimuthal wave number was varied linearly with the Reynolds number with a constant of proportionality of 0.0484 for supercritical radii of Type I cross-flow mode. This linear variation was valid for stationary and non-stationary disturbances. Although the time-averaged velocity profiles had an inflexion, they did not observe high frequencies at transition associated with an inviscid secondary instability. Möller and Bippes (1989) [45] used the hot-wire measurements to examine the development of stationary and traveling waves in a three-dimensional boundary layer. They suggested the importance of the traveling modes by showing that when traveling and stationary modes existed together, a transition Reynolds number has been reduced.

The stability curves for traveling disturbances have been computed numerically by Balakumar and Malik (1990) [2]. They showed that the existence of a critical angle of approximately -35.34°, below which all the waves were linearly damped. A secondary instability analysis of the rotating disk boundary layer flow has been performed by Balachandar et al. (1992) [1]. They found that secondary disturbances evolved with a quite large growth rate compared to that of primary disturbances.

rotating disk. They studied three cases of Type I cross-flow instability using hot-wire techniques. The surface roughness patterns consisted of ink dots which were applied to a kapton film that was bounded to the disk surface. These three cases were 27-dots spaced equally around the circumference at a radius which was supercritical to the growing instability, 19 logarithmic spirals formed by a series of five dots per spiral and the reference case which was the smooth (or clean) disk. They documented that the mean flow was unchanged by the roughness of the surface. They also observed inflexional profiles developed after the linear region where these profiles came out as a result of a mean flow distortion caused by the large amplitudes produced by the cross-flow vortices. They found that the linear relation between the wave number and Reynolds number was valid for the three cases. A significant growth of very low wave number stationary waves was found that were not growing according to linear theory. The amplification rates for the stationary modes was agreed well with those found in the literature for the computational results of Mack (1985) [39] and the experimental results of Corke and Knasiak (1994) [14], while the amplification rates for the traveling modes were slightly higher than the stationary modes which agrees with the linear theory.

2.3 Absolute Instability

The concept of absolute versus convective instability has been introduced by Briggs (1964) [5] and Bers (1975) [3] in plasma physics. A good review of some topic relating to fluid flows may be found in Huerre and Monkewitz (1990) [25].

Singularities arising in the dispersion relationship enable the nature of the instability to be determined. The variation of the Reynolds number can cause such points to occur in the flow field and thus change the behavior of a flow from a convectively unstable state to an absolutely unstable one. These points form whenever
modes associated with waves propagating in opposite or same directions coalesce. If the coalescing branches originate from the waves propagating in opposite directions, the singularity which cause resonance is said to be of pinch-type. Examples of flows demonstrating this phenomenon are near-wake flows (Betchov and Criminala (1966) [4] and Monkewitz (1988) [46]) and mixing layer flows with backflow (Huerre and Monkewitz (1985) [24]). On the other hand, if the two coalescing modes originate from wave propagating in the same direction, then the corresponding singularity is of the double-pole type. When these coalescing modes are nearly neutral, the damping rates are very small and thus a resulting short term algebraic growth for small times or short distances may carry the whole system into nonlinear stage long before the exponentially growing mode does. Koch (1986) [32] addressed this case for Plane-Poiseuille flow and Blasius boundary layer flow.

In the case of the rotating disk, the pinch in the radial wavenumber plane is sufficient for an absolute instability to occur, as the periodicity of the model requires an integer circumferential wavenumber. In the case of the swept wing, both wavenumbers may be complex, implying that a pinch in one wavenumber plane only does not lead to an absolute instability [59].

The points belong to the first class given above have been studied by Lingwood (1995) [35] and Lingwood (1996) [36] for the boundary layer over a rotating disk. Using Briggs criterion and assuming that the flow was parallel, Lingwood (1995) concluded that the absolute instability mechanism found in this flow might cause the disturbances to grow exponentially temporarily at a fixed radius that corresponding to a Reynolds number of 510, leading to an unbounded linear response that would promote nonlinearity followed by transition. Lingwood (1997) [37] corrected this value to be 507.3 instead. Incidentally, the critical Reynolds number for the flow to
undergo an absolutely unstable stage has been found in her study very close to the experimentally observed critical Reynolds number for the flow to transitional. The results from the inviscid analysis showed that the absolute instability was not caused by Coriolis effects nor by streamline curvature effects and the onset of absolute instability was consistent with the experimental observations of the critical Reynolds number for the onset of transition (≈ 513). This indicated that the same mechanism may be possible on swept wings where the Coriolis effects are not present but the boundary layers are otherwise similar.

Lingwood (1996) [36] performed an experimental study designed to capture the temporal growth associated with the absolute instability. This involved introducing unsteady disturbances into the boundary layer and following their development in space and time. The unsteady disturbances were a short duration air pulse that emanated from a hole in the disk surface. The pulse occurred once every disk rotation. The location of the pulse was just outboard of the minimum critical radius for Type I cross-flow modes. Lingwood followed the evolution of the azimuthal velocity fluctuations with a hot-wire sensor placed at different radial and azimuthal distances from the air pulse. Ensemble averages of the time series, correlated with the azimuthal position of the air pulse, revealed wave packets. When the leading and trailing edges of the wave packets were presented in terms of their Reynolds number (radius) and time (azimuthal position with respect to the disk rotation speed) they revealed a tendency for an accelerated advancing of the trailing edge. Unfortunately Lingwood’s measurements stopped short of the critical radius, with the observation that the well defined structure of the wave packet disintegrated when the critical Reynolds number was reached. The assumption was that this was due to the absolute instability which caused the onset to turbulence.
Lingwood (1997) [37] focused on the possibility of an absolute instability occurring in some parameter range for the Bödewadt, the Ekman and the von Kármán boundary layers; BEK system. These flows have been found to be absolutely unstable in the direction normal to the free-stream velocity (the radial direction in the BEK system) above particular Reynolds numbers and for certain frequencies and wave-numbers of disturbances. For all flows but the Ekman layer, which is parallel in the strict sense, the parallel flow approximation had been used in the viscous analysis. Inviscid results were shown that in all cases, the absolute instability persisted in the limit of large Reynolds number. This means that the viscous and Coriolis effects were not primary in causing the absolute instability. She found that the general absolute instability characteristics were relevant to the physical behavior of the flows which had been shown for the particular case of the von Kármán flow (Lingwood (1995) [35] and Lingwood (1996) [36]) where the onset of transition and absolute instability appeared to coincide. She found that the onset of absolute instability occurred at about Reynolds number of 507.

As mentioned by Lingwood (1995) [35], the velocity profiles for the rotating disk boundary layer exhibit reverse flow when resolved in a range of directions between the radial and azimuthal directions. Since absolute instability requires upstream propagation, and a reverse undisturbed flow facilitates upstream propagation, it is found in many examples of absolutely unstable flow. Examples are given by Huerre and Monkewitz (1990) [25]. These examples include the wakes behind circular cylinders (Mathis et al. (1984) [43], Koch (1985) [31], Triantafyllou et al. (1986) [61], Monkewitz (1988) [46], and Strykowski and Sreenivasan (1990) [57]), the floating cylinder (Triantafyllou and Dimas (1989) [60]), the countercurrent mixing layers in circular jets (Strykowski and Niccum (1991) [56]), and blunt bodies (Hannemann and Oertel (1989) [23] and Oertel (1990) [47]). But reverse flow certainly does not
guarantee absolute instability, nor is it always necessary.  

Cooper and Carpenter (1997a) [11] studied the effect of the wall compliance on the boundary layer instability over a rotating disk. They showed that wall compliance has a substantial stabilizing effect on the upper branch instability of Gregory et al. (1955) [21]. The effect on the viscous lower branch is found to be strongly destabilizing. Cooper and Carpenter (1997b) [12] showed that the presence of the wall compliance suppresses one of the coalescing eigenmodes postponing the absolute instability at least to a higher Reynolds number. Beyond a critical level of wall compliance the results suggested that complete suppression of the absolute instability was possible, removing a major route to transition in the boundary layer flow over a rotating disk.  

Recently, Davies and Carpenter (2003) [17] numerically studied the linear global behavior corresponding to the absolute instability of the boundary layer flow over a rotating disk. This involved solving the fully linearized Navier-Stokes equations for conditions of the rotating disk flow. In their simulation, they introduced impulse-like disturbances that led to the growth of wave packets. The results were found to reproduce the behavior observed by Lingwood (1996) [36]. In particular there was close agreement to the space-time development of the leading and trailing edges of the wave packets found in the experiment. However, in the absolutely unstable region, the strong temporal growth and upstream propagation was not sustained for long times, and the convective instabilities eventually dominated. Thus they concluded that the absolute instability of the rotating disk boundary layer does not produce a linear amplified global mode, but seems to be associated with a transient temporal growth. As such, it cannot explain or solely account for the turbulence transition locations observed in experiments.
An answer to this is suggested by Pier (2003) [48] who examined the secondary instability of finite amplitude waves of the rotating disk flow. The analysis revealed that the primary saturated waves initiated at the critical radius of the absolute instability are already absolutely unstable with respect to secondary perturbations. In this scenario, the primary nonlinear waves are a prerequisite for the development of the secondary instability that leads to transition to turbulence. The primary waves in this case are traveling with respect to the disk frame and have an azimuthal wave number \( n \) of 68. According to Lingwood’s (1995) [35] analysis, \( n=75 \) should be absolutely unstable for a Reynolds number beyond 520. This is a potential problem since it is well documented by theory and experiments that the most amplified (linear) range of azimuthal wave numbers of Type I modes is \( 24 \leq n \leq 45 \). Thus there is expected to be little energy in this range of higher azimuthal wave numbers due to linear convected modes.
CHAPTER 3

EXPERIMENTAL SETUP

This setup was initially designed at the Illinois Institute of Technology (IIT). It was primarily used by Knasiak (1996) [29], Matlis (1997) [44], and Corke and Matlis (2004) [16]. It is also similar to that used by Wilkinson and Malik (1985) [64], Malik et al. (1981) [42], and Corke (1992) [13]. In this Chapter, the experimental setup will be described in detail, including the previous setup and the modifications that were done for the current research. This setup description will be divided into the following parts: base and spindle, spindle controller, rotating disk, traversing mechanisms to control the hot-wire position, traversing mechanism to control the air pulse generator relative position with respect to the hot-wire, construction of the air pulse generator, and construction of the hot-wire probe.

3.1 Base and Spindle

A schematic of the rotating disk setup is shown in Figure 3.1. The disk is located at the top of the spindle of a motorized air bearing. The rotating disk assembly sits upon the base. This base is a large steel flange that is of a configuration that easily lends itself to stable mounting and ventilation of the disk assembly, as well as allowing relatively easy access. The bottom face of the flange is mounted to the floor using three expansion nuts and studs spaced 120° apart at a diameter of 14.5 in. (36.83 cm). On each stud, a nut was placed and the flange was then placed over the studs and on the nuts.

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Figure 3.1. A sketch of the overall experiment setup without the air pulser mechanism.
The air bearing spindle was manufactured by Dover Instrument Corporation located in Westboro, MA. The model number is SP-635-RF-BDC-HH-14345-01. The spindle digital feedback controller is model number DMM-2001. An operating pressure of 90 psig and a flow rate of 1.78 CFM are needed for the air bearing. The pressure could be adjusted through a regulating valve with a gage inserted in the pressure line. The compressor is a product of Thomas pumps & compressors company, model T-35HD. Some of its specifications are: air displacement is 4.5 CFM, tank size is 3 Gal., and automatic control stops at 125 psig.

There is no mechanical connection to the disk spindle since the pressure and flow rate allow the spindle rotating surface to spin freely without allowing the air bearing surfaces to touch. The advantages of air bearing are to reduce the vibration to the lowest possible level, and to give a long life for components since there are no parts to wear out. The air supply to the spindle needs to be free of moisture and particulate matter. Therefore, two filters were inserted in the air supply line. The first filter in this series is a porous element filter with a 5 µm rating which was intended to remove most of the particulate matter. The second filter is a coalescing filter with a 0.1 µm rating which removes the remaining solid particles as well as any oil, water, and any other liquids. To make sure that there is always a pressurized air supply to the air bearing when operating, there is a connection between the air supply line and the spindle controller. If a pressure loss to the air bearing occurs, a normally open pressure switch closes at a preset pressure level, sending a discrete ground to the spindle controller that causes immediate deceleration of the spindle rotation.
3.2 Spindle Controller

The DMM-2001 air spindle controller is an integrated single axis machine controller that allows feedback from rotary or linear encoders. The feedback is used for controlling position and speed. The DMM-2001 is a completely self contained 19 inch rack mountable enclosure. All power supplies, amplifiers, cables, and connectors are included. The velocity stability of the spindle while under control is ±0.003 % of the input rotational velocity. Complete details of the spindle controller may be found in Appendix A.

As the spindle rotates, an optical encoder senses the passage of an etched line and sends a square-wave signal to the spindle controller. The controller uses these signals for feedback control of the spindle rotation rate. The back of the DMM-2001 controller uses a 9-pin connector which provides an output for the two needed signals.

3.3 Rotating Disk

The disk was designed and fabricated by Knasiak (1996) [29]. The surface of the disk used for this experiment was hardened through an anodizing process and then precisely ground to provide a smooth disturbance-free surface. The disk was 18 in. (45.72 cm) diameter aluminum plate. The disk thickness was 1.00 in. (2.54 cm). It was diamond lapped to be flat and parallel to within 0.0038 mm. The surface of the disk was polished to a 2 μm finish. Through the tight control of tolerances during machining, the disk is well balanced.

3.4 Traversing Mechanisms to Control the Hot-wire

To acquire data at any point of space, a three-dimensional traversing mechanism was used for this experiment. This mechanism is based upon a cylindrical
coordinate system. The horizontal mechanism is used for radial motion over the
disk. The vertical mechanism is used to move the hot-wire in the wall normal di-
rection above the disk surface. The vertical and horizontal mechanisms are used
to place the probe in the desired location in the space. The traversing mechanisms
were also designed and fabricated by Knasiak (1996) [29]. Some modifications were
made for this research.

3.4.1 Horizontal Traversing Mechanism

The horizontal traversing mechanism was designed to minimize the interference
to the flow over the disk surface. Therefore, this mechanism was mounted high over
the disk surface by a fairly good distance. The vertical distance between the disk
surface and the horizontal mechanism was about 11.81 in. (30 cm). The mounting
pads for the vertical support of the horizontal superstructure were placed 180° apart
on the spindle mounting platform. These pads were screwed into two pairs of the
mounting holes on the platform. Each of these pads accepted two, one-half inch
diameter steel rods. The steel rods were secured to the mounting pads using set
screws. The material and diameter of these supporting rods were chosen so that
vibration was minimized.

Figure 3.2 shows the detail of the horizontal traversing mechanism. Two mech-
anisms were mounted at the top of the vertical support bars. The uppermost piece
was a leveling mount. This was used to help stabilize the horizontal superstructure,
as well as to assist in the procedure that ensures the disk surface and horizontal
superstructure were parallel. The second piece was a support block. This was
mounted just below the leveling mount and served as the mounting point for the
horizontal superstructure and linear bearings, the horizontal drive screw, and the
horizontal stepper motor. Two parallel clearance holes were drilled through each
Figure 3.2. Horizontal traverse apparatus and support with leveling mount.
support block so that they could slide freely on the vertical support bars. A hole was drilled through the leveling mount for clearance of a 10-32 screw. The screw fitted into a tapped hole in the top surface of the support block. A spring was compressed between the leveling mount and the support block, ensuring that tension was always applied to this screw. The screw served two purposes: vertical support of the support blocks and, by turning this screw, controlled vertical movement of the support blocks. The vertical movement of the support blocks was what allows for precise leveling of the horizontal superstructure relative to the plane of the disk surface.

The horizontal superstructure consisted of a bearing mount plate, two linear bearings with mounting pads, a horizontal drive screw, a drive block, and a primary vertical traversing mechanism mounting plate. The bearing mounting plate, horizontal linear bearings, and horizontal drive screw can be seen in Figure 3.2. The bearing mounting plate was secured to the two support blocks mounted on the vertical support bars. Mounted horizontally along the length of this plate were two NSK linear bearings with two mounting pads (NSK is a leading global supplier of bearings, linear motion equipments, drives and mechatronic systems). An aluminum plate mounted to the two mounting pads. This was the primary vertical traversing mechanism mounting plate. This plate allowed for the attachment of the secondary vertical traversing mechanism mounting plate and a smaller aluminum block that has been drilled and tapped. This block was the horizontal drive block. The horizontal drive screw passed through the threaded hole in the block.

The horizontal movement of the vertical traversing apparatus was driven by a stepper motor manufactured by the Bodine Electric Company of Chicago. The stepper motor model number is 200EK0010-23T2BEHD. This model provided 200
steps per revolution when operated in a full-step mode or 400 steps per revolution in half-step mode. During this experiment, the stepper motor was used in half-step mode. The stepper motor was mounted to a support block. The horizontal drive screw was connected to the stepper motor by a slide fit collar with set screws. The drive screw passed through the drive block and was supported on the other end by the other support block with a rotary bearing press fit into it. The drive screw had a thread pitch of 20 threads per inch. Using the thread pitch and number of steps per revolution, there were 8000 steps per inch. The overall horizontal movement of the center of the vertical traversing mechanism mounting plate was about ±10.75 in. (±27.31 cm) from the center of the disk. The accuracy of the movement was ±0.025 mm.

3.4.2 Vertical Traversing Mechanism

As mentioned before, the vertical traversing mechanism was used to move the hot-wire probe above the disk surface in the wall-normal direction. The vertical traversing apparatus consisted of a secondary vertical mounting plate, motor/screw mounting tower, two linear bearings, a slide block, a Linear Variable Differential Transformer (LVDT) with connecting linkage, a stepper motor, a block to hold the hot-wire, and a drive screw, as shown in Figure 3.3. The vertical traverse could also hold an inking pen that was used in the initial research to place dots on the disk surface. The dots were not used in the final experiments.

The secondary mounting plate was machined so that the motor/screw mounting tower, the two linear bearings, and the mounting block for the LVDT could be attached to it. The secondary mounting plate was then attached directly to the primary mounting plate. The stepper motor for the vertical traversing apparatus was the same type as the one used for the horizontal traversing apparatus. The
Figure 3.3. Vertical traverse apparatus and LVDT with its linkage.
vertical stepper motor was mounted at the top of the motor/screw tower with the drive screw connected in the same manner as the horizontal drive screw. The vertical slide block was drilled and tapped so that the drive screw could be run through it. The slide block was also attached to the vertical linear bearing mounting pads. This was done to minimize any lateral movement of the slide block within the motor/screw mounting tower. To be more accurate in the wall normal motion, we used a different drive screw but same stepper motor. The vertical drive screw had a thread pitch of 32 threads per inch and, on the half-step mode, 12,800 steps were needed to move one inch vertically.

Since there was no feedback to the stepper motors, they might move in the downwards direction while they supposed to move upwards. If this happened, the hot-wire probe could hit the surface of the disk. To enhance the placement accuracy of the vertical traverse apparatus, a linear variable differential transformer (LVDT) was used to supply feedback to the digital acquisition and control computer (DAC). This allowed the slide block and, hence, the hot-wire probe to be moved to positions with an accuracy in the range of the LVDT’s accuracy. The accuracy of the LVDT was $\pm 50 \, \mu m$ when moved to an absolute reference position. For movement relative to this reference position, the LVDT’s accuracy was $\pm 100 \, \mu m$. The absolute reference position was chosen to be the lowest point that the hot-wire probe would reach.

The LVDT was manufactured by Schaevitz Engineering, and supplied by Lucas Control System Products. The LVDT was mounted to the side of the motor/screw tower. The core of the LVDT was connected to an adjustable linkage, which, in turn, was mounted to the slide block. The linkage was adjusted so that after a reference voltage was obtained from the LVDT, the lowest absolute reference height could be set.
Both regular movement of the vertical traversing mechanism and setting of the reference position were accomplished using the computer digital to analog converter (DAC) and software control programs. The motion calibration given for the LVDT was 32.95 Volts per cm. The maximum range of movement provided by the LVDT in its linear range was 6.35 mm. The minimum and maximum voltages output by the LVDT at its lower and upper ranges are -10.221 and +10.183 Volts, respectively. Since the limits of voltage input to the DAC are ±10 Volts, the effective usable movement of the LVDT was then 5.59 mm. This was more than enough vertical movement since the total movement of the hot-wire probe during the data acquisition was expected to be less than 1.638 mm.

3.5 Traversing Mechanism to Control the Air Pulse

Figure 3.4 shows the overall setup when using a traversing mechanism to control the relative position of the air pulse with respect to the hot-wire. The design of the traversing mechanism has been made in such a way to create an air disturbance at any desired position above the disk. It consisted of a horizontal metal bar with scale. This bar was connected to the horizontal traversing mechanism through a special holder that allows two types of motions; upwards and downwards with respect to the disk surface and a rotation motion about the vertical axis. On the same horizontal bar, another holder was used to hold the air-jet tube that was connected to the air pressure line through two solenoid valves. The air-jet tube itself could be moved upwards and downwards to control its vertical distance above the disk surface. The air pulse intensity could be controlled by moving the probe up and down and/or by change the pressure supply through a pressure regulator valve that was located upstream of the solenoid valves.

After several attempts, two solenoid valves were used instead of one. We had
Figure 3.4. A sketch of the overall experiment setup showing the traversing mechanism to control an air pulse.
tried only one normally-closed solenoid valve and it worked fine but the air pulse duration was too large. The reason for that was due to mechanical limitation of the solenoid valve. This type of solenoid valve uses a spring to return back to its original (closed) position. So, it would not generate a pulse of very short duration. The basic idea of using two solenoid valves was that they could be energized independently of the other. We chose the first solenoid valve to be a normally-closed type and the second one to be a normally-open type. A signal was sent to open the first valve and followed by another signal to close the second valve. The time between these two signals was controlled to give the required air pulse duration.

A typical velocity time series coming from the air-jet is shown in Figure 3.5 for four disk rotations. The solid line represents the ensemble average while the dashed line represents the time series. A closer view of the ensemble average pulse duration for one disk rotation is shown in Figure 3.6. Here $T$ is the time for one rotation of the disk, $t$ is the time, $A$ is the velocity amplitude, and $A_{\text{max}}$ is the maximum velocity amplitude. As shown in the figure, the start of the air pulse was almost vertical, but the end was not. The pulse duration was about 20% (12 msec) of the time needed for one rotation of the disk. However, the majority of the signal lied within only 10% of the disk rotation (about 6 msec). One can notice that there was a time delay between the electronic signal and the valve response. This time delay had been taken into account during the data processing.

The two solenoid valves were connected to the pressure regulator valve through a flexible tube that made it easy to adjust the whole mechanism while positioning the air pulse jet around the disk. Special care was taken to minimize any vibration in the assembly during operation.

All of the previous researchers (e.g., Lingwood (1996) [36] and Wilkinson et
Figure 3.5. A time series pulse duration compared with the ensemble average.

Figure 3.6. A closer view of the ensemble average pulse duration for one rotation.
al. (1989) [62]) had used a hole through the disk surface to create time-dependent disturbances. This approach has two disadvantages; the first is that the location of the disturbances can not be changed. Second, stationary disturbances can be created as shown in Figure 3.7. Our approach has a number of advantages. The first is that time-dependent disturbances could be generated anywhere above the disk surface. The second is that the disk surface was left smooth which is very important for this study because the instability modes are very sensitive to surface imperfections.

A disadvantage of using the solenoid valves was that it was very difficult to have precise, repeated actions. To overcome this, the number of ensemble averages was increased.

![Image](image_url)

Figure 3.7. Stationary disturbances coming from the hole as shown by Wilkinson et al. (1989) [62].

3.6 Construction of the Air Pulser Jet

Figure 3.8 shows the detail of the air pulser jet assembly. The basic idea is to minimize the volume of compressed air downstream the solenoid valves. To construct the air pulser, the following components were used as numbered in the
right hand side of Figure 3.8. First, one end of a brass pipe fitting (number 2) was glued to a 1/8 in. hollow brass tube (number 3). This was then put through a 1/4 in. hollow brass tube (number 5). On the other hand, a 0.2 mm diameter brass tube (number 4) was inserted through a plastic hub (number 6). This plastic hub had a stainless steel hypodermic tube with inside/outside diameters of 0.2032/0.4064 mm. The 0.2 mm diameter brass tube was glued to the hypodermic tube so that they became one piece. Here, a special care was taken to ensure that the glue did not block the tube and/or the hypodermic tube. The 0.2 mm tube was carefully inserted through the 1/8 in. hollow brass tube (number 3) so that it was glued from one side to the hypodermic tube and the other side was extended from the brass pipe fitting (number 2). Before connected the normally-open solenoid valve exit (number 1) to the brass pipe fitting (number 2), a piece of rubber (number 7) was inserted around the 0.2 mm tube to minimize the volume of compressed air.

3.7 Construction of the Hot-wire Probe

A single hot-wire was used to measure the velocity fluctuations. It was made from tungsten with a 0.00015 in. diameter and the length-to-diameter ratio of about 450. The hot-wire probe was constructed of 1/4 in. (6.35 mm) diameter stainless steel tube. According to the dimensions of the traverse mechanisms, the probe length was 12.99 in. (33 cm) so that after it was mounted to the vertical traverse, it could reach the disk surface. The two 1.5 in. long broaches were glued into one end of the tubing. Electrical connections were made to the broaches with lengths of wire passed through the tubing. After ensuring that there was no current path between each of the lead wires and between each lead wire and the probe body, a hot-wire was soldered to the tip of the broaches. The lead wires were connected to a Constant Temperature Anemometer (CTA), making sure that these connections
Figure 3.8. air pulser details.
were insulated from earth ground. The anemometer was a DANTEC, model 56C17 CTA.

The output signal from the CTA was split out into two signals; the DC-contained and the DC-removed signals. The DC-contained was connected directly to the DAC, while the DC-removed was connected to an amplifier and a filter before connected to the DAC. In this experiment, a Stanford Research Systems (SRS), model SR560, low-noise preamplifier and filter combination was used. A high pass filter cutoff was set at 250 Hz while the low pass filter cutoff was set at 1,250 Hz. The signal was amplified by a factor of 10. This allows the DAC to acquire the AC signal data with better resolution.
CHAPTER 4

EXPERIMENTAL CONDITIONS AND TESTING PROCEDURE

In this chapter, a detailed description of the experimental conditions and testing procedures, including information on the data acquisition are presented.

4.1 Experimental Conditions

In this section, the criteria for selecting the positions of velocity measurements in the radial and the wall normal directions are presented. Table 4.1 shows the values of the experimental parameters and constants used during the coarse of the study.

4.1.1 Velocity Measurement Positions: Base Flow Case

As part of the validation of the current measurements, comparisons were made with previous studies. In addition, there is an exact similarity solution of the Navier-Stokes equations for the mean flow (von Kármán (1921)) for an infinite diameter disk that allows a direct comparison.

Measurement locations for the base condition documentation are listed in Table 4.2. This consists of 22 radial positions. These positions are given in terms of their absolute radii as well as the corresponding non-dimensionalized values with respect to both cross-flow and absolute instabilities critical radii. Where $r_{cross}$ refers to the Type I cross-flow instability critical radius, and $r_{abs}$ refers to absolute instability critical radius. Reynolds number ($R$) is defined as $R = r(\omega/\nu)^{1/2}$, where $r$
TABLE 4.1

EXPERIMENTAL CONDITIONS.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disk rotation</td>
<td>1000 RPM</td>
</tr>
<tr>
<td>$\omega$</td>
<td>104.7 Rad/sec</td>
</tr>
<tr>
<td>Air kinematic viscosity</td>
<td>$1.5882 \times 10^{-5}$ m$^2$/sec</td>
</tr>
<tr>
<td>Critical Reynolds number (cross-flow)</td>
<td>287 (theory) [64]</td>
</tr>
<tr>
<td>Critical radius (cross-flow)</td>
<td>4.40 in. (11.18 cm)</td>
</tr>
<tr>
<td>Critical Reynolds number (absolute instability)</td>
<td>507.3 (theory) [35]</td>
</tr>
<tr>
<td>Critical radius (absolute instability)</td>
<td>7.78 in. (19.76 cm)</td>
</tr>
<tr>
<td>Sampling rate</td>
<td>2500 Hz</td>
</tr>
</tbody>
</table>

is the radius, $\omega$ is the angular velocity, and $\nu$ is the kinematic viscosity. So, based on the disk rotation of 1,000 revolutions per minute, $r_{\text{cross}}$ is 4.40 in. (11.18 cm) while $r_{\text{abs}}$ is 7.78 in. (19.76 cm). The selection of these radial positions was based on having a close increment near the absolute instability critical radius so that the growth of the absolute instability could be captured.

At each radial position, there were 24 measurement positions taken in the wall normal direction. These values are shown in Table 4.3 in inch and in millimeter units. The table also shows the corresponding non-dimensionalized wall-normal distances, $z^*$, where it is defined as $z^* = z(\omega/\nu)^{1/2}$. The vertical datum position of the hot-wire was 0.0059 in. (0.149 mm) above the disk surface. This then represents the lowest height to be acquired. It corresponds to $z^* = 0.3826$.

The increment in the wall normal direction was not constant. This is because better resolution of the boundary layer velocity profile is required close to the disk surface, where is the largest mean velocity variation. On the other hand, away from the disk surface, less resolution was needed. Finer spatial resolution was also needed
TABLE 4.2

MEASUREMENT RADIAL POSITIONS FOR BASE FLOW CONDITION.

<table>
<thead>
<tr>
<th>Position #</th>
<th>Radius in in. (cm)</th>
<th>(\frac{r}{r_{cross}})</th>
<th>(\frac{r}{r_{abs}})</th>
<th>R</th>
<th>Position #</th>
<th>Radius in in. (cm)</th>
<th>(\frac{r}{r_{cross}})</th>
<th>(\frac{r}{r_{abs}})</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.7559 (7.00)</td>
<td>0.6261</td>
<td>0.3543</td>
<td>180</td>
<td>12</td>
<td>7.0866 (18.00)</td>
<td>1.6100</td>
<td>0.9109</td>
<td>462</td>
</tr>
<tr>
<td>2</td>
<td>3.1496 (8.00)</td>
<td>0.7156</td>
<td>0.4049</td>
<td>205</td>
<td>13</td>
<td>7.2835 (18.50)</td>
<td>1.6547</td>
<td>0.9362</td>
<td>475</td>
</tr>
<tr>
<td>3</td>
<td>3.5433 (9.00)</td>
<td>0.8050</td>
<td>0.4555</td>
<td>231</td>
<td>14</td>
<td>7.4803 (19.00)</td>
<td>1.6995</td>
<td>0.9615</td>
<td>488</td>
</tr>
<tr>
<td>4</td>
<td>3.9370 (10.00)</td>
<td>0.8945</td>
<td>0.5061</td>
<td>257</td>
<td>15</td>
<td>7.6772 (19.50)</td>
<td>1.7442</td>
<td>0.9868</td>
<td>501</td>
</tr>
<tr>
<td>5</td>
<td>4.3307 (11.00)</td>
<td>0.9839</td>
<td>0.5567</td>
<td>282</td>
<td>16</td>
<td>7.7756 (19.75)</td>
<td>1.7665</td>
<td>0.9995</td>
<td>507</td>
</tr>
<tr>
<td>6</td>
<td>4.7244 (12.00)</td>
<td>1.0733</td>
<td>0.6073</td>
<td>308</td>
<td>17</td>
<td>7.8740 (20.00)</td>
<td>1.7889</td>
<td>1.0121</td>
<td>513</td>
</tr>
<tr>
<td>7</td>
<td>5.1181 (13.00)</td>
<td>1.1628</td>
<td>0.6579</td>
<td>334</td>
<td>18</td>
<td>7.9724 (20.25)</td>
<td>1.8113</td>
<td>1.0248</td>
<td>520</td>
</tr>
<tr>
<td>8</td>
<td>5.5118 (14.00)</td>
<td>1.2522</td>
<td>0.7085</td>
<td>359</td>
<td>19</td>
<td>8.0709 (20.50)</td>
<td>1.8336</td>
<td>1.0374</td>
<td>526</td>
</tr>
<tr>
<td>9</td>
<td>5.9055 (15.00)</td>
<td>1.3417</td>
<td>0.7591</td>
<td>385</td>
<td>20</td>
<td>8.2677 (21.00)</td>
<td>1.8784</td>
<td>1.0628</td>
<td>539</td>
</tr>
<tr>
<td>10</td>
<td>6.2992 (16.00)</td>
<td>1.4311</td>
<td>0.8097</td>
<td>411</td>
<td>21</td>
<td>8.4647 (21.50)</td>
<td>1.9231</td>
<td>1.0881</td>
<td>552</td>
</tr>
<tr>
<td>11</td>
<td>6.6929 (17.00)</td>
<td>1.5206</td>
<td>0.8603</td>
<td>437</td>
<td>22</td>
<td>8.6614 (22.00)</td>
<td>1.9678</td>
<td>1.1134</td>
<td>565</td>
</tr>
</tbody>
</table>
in the approximate mid-height of the boundary layer, where the peak amplitudes of the velocity fluctuations were expected.

4.1.2 Velocity Measurement Positions: Disturbance Cases

In order to compare the current results with Lingwood (1996) [36], the air pulser was located at the same Reynolds number, \( R=311 \) (radius=12.1 cm). Hot-wire measurements were then taken at different azimuthal and radial positions relative to the air pulser location. The radial positions for the velocity measurements are given in Table 4.4. The relative azimuthal angles of the velocity measurements are given in Table 4.5.

In a majority of the measurements with the air pulse disturbances, the velocity was measured at only one height above the disk surface. This was at \( z=0.5\text{mm} \), which corresponded to \( z^*=1.284 \), and is the height of the maximum amplitude in the eigenfunction of the linear cross-flow modes.

In some of the pulse disturbance cases, wall normal measurements were taken in order to document the wall-normal amplitude distribution of the wave packets and compare them to the linear theory eigenfunction. The number of wall normal points was usually 6–8, and approximately evenly spaced.

4.1.3 Air-Pulse Disturbance Conditions

The parameters available for adjusting the air pulse generator were the distance from the wall, source pressure level, and pulse duration. Generally, we wanted the jet exit and hypodermic tubing out of the boundary layer so that it would not produce a stationary disturbance. After several measurements, the air pulser was placed 4 mm (\( z^*=10.272 \)) above the disk surface.
TABLE 4.3
MEASUREMENT WALL NORMAL POSITIONS FOR BASE FLOW CONDITION.

<table>
<thead>
<tr>
<th>Wall normal Position #</th>
<th>Distance in in. (mm)</th>
<th>z*</th>
<th>Wall normal Position #</th>
<th>Distance in in. (mm)</th>
<th>z*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0059 (0.1490)</td>
<td>0.3826</td>
<td>13</td>
<td>0.0309 (0.7840)</td>
<td>2.0130</td>
</tr>
<tr>
<td>2</td>
<td>0.0074 (0.1871)</td>
<td>0.4804</td>
<td>14</td>
<td>0.0334 (0.8480)</td>
<td>2.1773</td>
</tr>
<tr>
<td>3</td>
<td>0.0089 (0.2252)</td>
<td>0.5782</td>
<td>15</td>
<td>0.0364 (0.9240)</td>
<td>2.3723</td>
</tr>
<tr>
<td>4</td>
<td>0.0109 (0.2760)</td>
<td>0.7086</td>
<td>16</td>
<td>0.0394 (0.9999)</td>
<td>2.5673</td>
</tr>
<tr>
<td>5</td>
<td>0.0129 (0.3271)</td>
<td>0.8397</td>
<td>17</td>
<td>0.0423 (1.0748)</td>
<td>2.7597</td>
</tr>
<tr>
<td>6</td>
<td>0.0149 (0.3781)</td>
<td>0.9708</td>
<td>18</td>
<td>0.0454 (1.1520)</td>
<td>2.9579</td>
</tr>
<tr>
<td>7</td>
<td>0.0168 (0.4279)</td>
<td>1.0986</td>
<td>19</td>
<td>0.0484 (1.2290)</td>
<td>3.1556</td>
</tr>
<tr>
<td>8</td>
<td>0.0189 (0.4789)</td>
<td>1.2297</td>
<td>20</td>
<td>0.0524 (1.3301)</td>
<td>3.4151</td>
</tr>
<tr>
<td>9</td>
<td>0.0209 (0.5300)</td>
<td>1.3608</td>
<td>21</td>
<td>0.0564 (1.4317)</td>
<td>3.6760</td>
</tr>
<tr>
<td>10</td>
<td>0.0234 (0.5940)</td>
<td>1.5251</td>
<td>22</td>
<td>0.0604 (1.5330)</td>
<td>3.9362</td>
</tr>
<tr>
<td>11</td>
<td>0.0259 (0.6570)</td>
<td>1.6869</td>
<td>23</td>
<td>0.0654 (1.6600)</td>
<td>4.2623</td>
</tr>
<tr>
<td>12</td>
<td>0.0284 (0.7210)</td>
<td>1.8512</td>
<td>24</td>
<td>0.0704 (1.7870)</td>
<td>4.5884</td>
</tr>
</tbody>
</table>

TABLE 4.4
MEASUREMENT RADIAL POSITIONS IN CASE OF DISTURBANCE EXCITED FLOW.

<table>
<thead>
<tr>
<th>Radial Position #</th>
<th>Radius in in. (cm)</th>
<th>( R )</th>
<th>Radial Position #</th>
<th>Radius in in. (cm)</th>
<th>( R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.72 (12.00)</td>
<td>308</td>
<td>7</td>
<td>7.09 (18.00)</td>
<td>462</td>
</tr>
<tr>
<td>2</td>
<td>5.12 (13.00)</td>
<td>334</td>
<td>8</td>
<td>7.48 (19.00)</td>
<td>488</td>
</tr>
<tr>
<td>3</td>
<td>5.51 (14.00)</td>
<td>359</td>
<td>9</td>
<td>7.87 (20.00)</td>
<td>513</td>
</tr>
<tr>
<td>4</td>
<td>5.91 (15.00)</td>
<td>385</td>
<td>10</td>
<td>8.27 (21.00)</td>
<td>539</td>
</tr>
<tr>
<td>5</td>
<td>6.30 (16.00)</td>
<td>411</td>
<td>11</td>
<td>8.66 (22.00)</td>
<td>565</td>
</tr>
<tr>
<td>6</td>
<td>6.69 (17.00)</td>
<td>437</td>
<td>12</td>
<td>8.86 (22.50)</td>
<td>578</td>
</tr>
</tbody>
</table>
TABLE 4.5

RELATIVE ANGLES BETWEEN THE HOT-WIRE AND THE AIR PULSER.

<table>
<thead>
<tr>
<th></th>
<th>Angle in (°)</th>
<th>Angle in (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>70</td>
<td>11</td>
</tr>
<tr>
<td>5</td>
<td>100</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>130</td>
<td>13</td>
</tr>
<tr>
<td>7</td>
<td>160</td>
<td>14</td>
</tr>
</tbody>
</table>

As had been mentioned in Chapter 3, it was not guaranteed that every jet pulse was identically the same. Therefore, typically at each measurement location 1000 records were acquired for ensemble averaging. Also, the ensemble averages were increased to further reduce the effect of stationary disturbances.

A two second pause was needed between air pulses to allow the solenoid valves to settle down. This increased the data acquisition time to about one hour at each location. To use the time wisely, the ensemble average was also compiled in real-time on an oscilloscope that used the solenoid pulse as a trigger. The real-time ensemble provided a guide if a wave packet was to be formed at particular angles relative to the pulsed-air generator. If not, valuable acquisition time would not be wasted. Figures 4.1 shows a schematic representation of the instrumentation.

There were two cases to study: linear amplitude and finite-amplitude disturbances. It will be shown later how these two cases were determined. Examples of the effect of the pulse generator pressure are shown in Figures 4.2 through 4.4. The data shown in Figures 4.2 and 4.3 were measured at a Reynolds number of 347
Figure 4.1. A schematic representation of the instrumentation.
and a relative angle of 20° while those in Figures 4.4 were measured at a Reynolds number of 334 and a relative angle of 10°. In fact, these results were not sensitive to the difference in the measurement position since both were in the vicinity of the air pulser jet and the difference in $\theta$ was very small.

Figure 4.2 shows the ensemble average of velocity fluctuations triggered on the pulse solenoid valve signal. It is clear that the higher pressure produce larger velocity fluctuations.

Spectra of the velocity fluctuations in Figure 4.2 are shown in Figure 4.3. Amplitudes corresponding to spectral peaks were converted to root mean square by taking the areas under the peaks and normalized them by one Hertz frequency band width. As the pressure was increased, the amount of energy in the spectrum increased, and the band width increased. Since the absolute instability was for traveling cross-flow modes with azimuthal mode number of 68 and higher, it was important that the air pulser put energy into higher frequencies as well.

Figure 4.4 shows the velocity fluctuation and the corresponding spectra with a supply pressure of 80 psi. The above results were measured in the vicinity of the disturbances to make sure a wide band of frequencies was excited near the source of disturbances in order to excite a broad range of cross-flow modes.

4.2 Testing Procedure

This section presents the experimental procedure and the data analysis methods. A National Instruments AT-MIO-16E-1 data acquisition card was used to sample voltages. It is a 12-bit A/D card. It has 3 sub-devices; an analog input (8 differential channels), an analog output (2 channels), and a digital input and output (8 channels). For data acquisition, a COntrol and MEasurement Device Interface
Figure 4.2. Velocity fluctuations at selected pressures at \( R=347 \) and \( \theta=20^\circ \).
Figure 4.3. Spectra of velocity fluctuations corresponding to cases shown in Figure 4.2.
Figure 4.4. Velocity fluctuation and the corresponding spectra at pressure=80psi, \(R=334\) and \(\theta=10^\circ\).
(COMEDI) software was used under a LINUX operating system. The COMEDI’s version was COMEDI-0.7.66 with COMEDILIB-0.7.21.

Before starting data acquisition, there were some precautions that were taken into consideration. All near air ventilation grills (air vents) in the laboratory were turned off so that there were no external sources of air motion. This air draft was found to have a great effect on the results, especially at far vertical distances from the disk surface, where the mean flow velocity is small. Also, it was important to wipe the disk surface to remove any dust particles. Non-abrasive and lint free wipes were used for this.

4.2.1 Software for Control and Data Acquisition

There were a number of software programs that were used in this experiment. The first one was used during the calibration of the hot-wire. In this program, two channels corresponding to voltages from a pressure transducer connected to a Pitot-static probe, and to the output from a CTA were simultaneously acquired. These were sampled to determine time-averaged quantities.

Another program was used to move the horizontal and vertical traversing mechanisms an exact number of steps. This program was essential for precisely locating the hot-wire sensor above the disk at different radii.

There were two main programs used to control time-series data acquisition. One was for the base flow case, and the other was for the pulsed disturbance cases. They are similar in the main part but different in the way the traverse mechanisms moves. Before running the main program, the probe was moved to the edge of the disk and at the designated datum above the disk surface. To ensure that the datum height was correct, an optical telescope (cathetometer) was used to measure the distance
between the hot-wire and its reflection on the disk surface. Figure 4.5 shows how
the view looked when someone looked through the cathetometer. Dividing this
distance by two gave the actual datum height. This distance was measured near the
disk edge and near the disk center to ensure that the disk surface was parallel with
the horizontal traverse mechanism.

The main data acquisition program for the base line measurements read all
of the needed information from a separate input file that included the starting
radial position, the number of radial positions and their locations, the number of
wall normal positions and their locations, the number of channels to be acquired,
number of sampled time-series points, and the sampling rate. Only the hot-wire
output signal was acquired. When starting the data acquisition program, the probe
would move to the starting radial location and then acquire time-series data from

Figure 4.5. A sketch to show how to determine the datum height.
the first wall-normal location. According to the experimental parameters, the length of the time-series corresponded to nine disk rotations would be acquired. The total time-series corresponded to a data record. A total of 25 records were acquired at each wall-normal position.

When the hot-wire sensor was traversed in the wall-normal direction, the voltage coming from the LVDT was checked after each step and a comparison between each two successive steps was made to ensure that the vertical stepper motor was moving in the correct direction. Otherwise, the program was stopped. In addition, the LVDT’s voltage was checked with the datum distance, which was represented by a voltage. For accuracy, an average of 100 samples were taken to represents the voltage at each location. After completing a wall-normal survey, the probe was moved horizontally to the next radial position. It was then moved down to the datum vertical position. The LVDT’s voltage was checked during the downward motion to ensure that the hot-wire would not touch the disk surface.

The combination of the upward vertical, horizontal, and downward vertical motions was called a cycle. A number of cycles were repeated, according to the input data file, to cover the radial spatial samples. The radial increments were controlled from the input data file.

There were a number of output files saved during the data acquisition. These included the raw voltages from the CTA and absolute \((r, \theta, z)\) locations of the sensor. These were written after every record so if a problem occurred, there was no loss of data or the absolute position of the sensor at the last measurement.

The main program of excited flow was different from that of base flow case. In addition to the hot-wire output signal, another signal that was sent to trigger the
solenoid valves was also acquired. This signal was saved in the output files to used later as the starting point of the ensemble average of the time series of the velocity fluctuations. The number of records has been increased to 1000 records to overcome the disadvantage of non-guaranteed repeated responses from the solenoid valve.

4.2.2 Data Processing

In the this paragraph, a description of data processing for the base flow case will be given. The output signal coming from the CTA was split into two signals: DC-contained and DC-removed. The DC-contained signal went directly to the A/D card. The DC-removed was filtered and amplified by a factor of 10 before went the A/D card. To get the corresponding velocity, the DC-removed signal was divided by the gain and then added to the DC-contained signal. A calibration equation was applied to the resultant voltage to get the corresponding total velocity including the mean. Then, the mean velocity was subtracted from the total to get the velocity fluctuations. These velocity fluctuations included stationary and traveling waves.

In cases of applying time-dependent disturbances, the signal that used to trigger the solenoid valve was acquired. The two signals coming from the hot-wire were simultaneously acquired and then a pulse signal was introduced to the solenoid valve. Figure 4.6 shows a sketch of the three signals. Here channels 0 and 1 were used for the hot-wire signals while channel 2 was used for the trigger signal. This ensured that the starting of the pulse signal was always acquired. Later, the pulse signal used as the starting of each record (represented by a dashed line in the figure). The data acquired before introducing the signal were discarded. This caused each record to be consisted of eight complete rotations. In the presentation of the results in the coming chapters, only four rotations were used to present the time series.
Figure 4.6. A sketch showing the timing for the acquired signals for the time-dependent disturbance cases.
4.3 Amplitude Envelope Determination

The application of the time-dependent disturbances resulted in the growth and development of wave packets. The analysis of these is the main topic of Chapters 6 and 7. Part of the analysis was to determine the leading and trailing edges of the wave packets. This necessitated the ability to define the amplitude envelope of the wave packet.

The standard approach used for defining the envelope of an amplitude modulated time-series is through Hilbert transform (Rabiner and Gold (1975) [49]). The process of using a Hilbert filter to obtain the amplitude envelope of the modulated time-series obtained in the experiment is shown in the schematic in Figure 4.7.

The process of determining the amplitude envelope started with the ensemble averaged time series. The Hilbert filter is a 90° phase shifting filter. Normally, it is applied to a single frequency component, which requires applying a narrow-band filter that will pass the frequency of interest. In our case, we found that a band-pass filter with cutoff frequencies that encompassed the range of linear amplified frequencies produced a good result. In practice, two digital filters were used. The spectra of the filters are shown in Figure 4.8. Here, the abscissa represents the frequency normalized by the sampling rate, $f_s$, which is 2500 Hz. One filter was designed azimuthal mode numbers between 22 and 40. This was the full range of linearly amplified modes for the range of radii surveyed. the other filter was designed to pass azimuthal mode number from 22 to 75. The higher mode numbers in this case are in the range (67–75) that are expected to be absolutely unstable.

The Hilbert filter was applied to the filtered time series, $u(t)$, to obtain a 90° phase shifted version of the original, $\hat{u}(t)$. From this, the analytic time series was
constructed, namely $u(t) + i\bar{u}(t)$. Finally, the amplitude of the analytic time series was obtained by using the formula $|U(t)| = |u^2(t) + i\bar{u}^2(t)|^{1/2}$.

Figure 4.9 shows an example of using the band-pass filter and Hilbert transform to obtain the amplitude envelope of a wave packet. This corresponds to an air pulser pressure of 28 psi, and Reynolds number of 462 (radius of 18 cm) with an angle between the hot wire sensor and the air pulser jet of 70°. The bottom plot shows the original signal as the solid curve. The band-pass filtered time series is shown as the dashed curve. This is a particularly difficult case because the location was close to the source of disturbance, and it generated a broad range of frequencies that have not yet been selectively amplified by the flow.

The top plot of Figure 4.9 shows the filtered wave packet with its corresponding envelope obtained from the Hilbert filtering. The vertical axis of the top plot is different from that of the bottom one since the amplitude of the filtered packet is considerably smaller than that of the unfiltered time series.

Figure 4.10 shows another example of obtaining the amplitude envelope using the Hilbert Filtering. For this figure, the parameters are the same as those of Figure 4.9 except the angle between the pulsed jet and the hot-wire was increased to 280°. In this case, the original wave packet is very clear, and it is only slightly changed by the band-pass filter. The amplitude envelope is easily constructed in this case, leading to an easy identification of the wave packet leading and trailing edges.
Figure 4.7. Hilbert filter flow chart used in obtaining amplitude envelope of wave packets observed in the experiment.
Figure 4.8. Spectra of digital filters used with Hilbert transform to obtain amplitude envelope of wave packets. Passed mode numbers of 22–40 (top) and 22–75 (bottom).
Figure 4.9. Time series and the corresponding wave packet envelope at $R=462$ ($r=18.0$ cm), $\theta =70^\circ$, pressure=28psi.
Figure 4.10. Time series and the corresponding wave packet envelope at $R=462$ ($r=18.0$ cm), $\theta =280^\circ$, pressure=28 psi.
CHAPTER 5

RESULTS AND DISCUSSIONS: BASE FLOW CONDITIONS

During the course of the current study, the absolute instability in the flow over a rotating disk was investigated for two different cases of linear disturbances and finite-amplitude disturbances. Before introducing the disturbances, measurements for the case of basic flow were performed to verify that the flow represented the canonical conditions. The purpose of this chapter is to present the results of these measurements that document the flow conditions on the disk prior to adding temporal disturbances. These will be derived from measurements of the time-averaged (mean) and fluctuating azimuthal velocities in the boundary layer on the disk. Comparison will be made to previous calculations by Mack (1985) [39].

5.1 Basic Flow

Figure 5.1 shows the profiles of the mean azimuthal velocity component at different radial locations on the disk. The profiles are shown with similarity wall distance, \( z^* = z(\omega/\nu)^{1/2} \), and the similarity azimuthal velocity, \( u^* = u_\theta/(r\omega) \). The solid curve is the similarity solution obtained by von Kármán (1921) [54] for an infinite diameter disk. The profile covered the radial direction that is inboard and outboard of both Type I cross flow and absolute instability critical Reynolds numbers. The Reynolds number of the Type I cross flow instability is 287 while it is 507.3 for the absolute instability.
Figure 5.1. Comparison of the wall-normal azimuthal velocity distribution for the basic flow at different radii with that of von Kármán (1921).
Figure 5.1 is divided into two plots for better presentation. As shown in the lower plot, the velocity profiles collapse onto a single curve for different radii, and these are in very good agreement with the theoretical profile. In the top plot, the velocity profiles corresponding to Reynolds numbers up to 526 also collapsed onto the theoretical profile. If we define the transition location to be where the mean velocity profile deviates from the laminar mean velocity profile, then this occurs in our experiment at $R=539$. This result is very close to that of Wilkinson and Malik (1985) [64], who found the transition Reynolds number was in the range $543<R<556$. Note that all these are well outboard of the absolute instability critical Reynolds number.

Figure 5.2 shows the mean velocity profiles at various Reynolds numbers as measured by Lingwood (1996) [36]. With regard to cases (a) and (c) that represent the azimuthal velocity profiles, there is an excellent agreement with our current experiment. According to Lingwood’s results, the flow was still laminar up to $R=471$ and began to deviate from the laminar profile at $R=555$. Lingwood skipped profiles in the range of Reynolds numbers between 471 to 555. Our transition Reynolds number lies in this range. The fact that it is well past the absolute instability critical Reynolds number suggests that the absolute instability is not a global mode that is responsible for transition as Lingwood (1996) suggested.

5.2 Basic Flow Velocity Time Series

In this section, examples of the time series of velocity fluctuations are shown. For this, various wall-normal positions are presented at $R=501$ ($r/r_{cross} = 1.7442$) as shown in Figure 5.3. A total of six wall-normal heights are shown corresponding to $z^*$ of 0.709, 1.099, 1.818, 2.144, 3.286, and 4.393. Note that the amplitude scale changes with respect to the $z^*$ of the measurement to reflect the changing amplitude
Figure 5.2. Mean velocity profiles of Lingwood (1996) [36], figure 4. a) azimuthal velocities, b) radial velocities, and c) azimuthal velocities into turbulence regime. The solid lines indicate the theoretical profiles.
of the velocity fluctuations. According to the linear theory, for the Type I cross-flow mode the amplitude maximum is at approximately $z^* = 1.4$.

The time series show quasi-periodic fluctuations that are somewhat amplitude modulated. If we count the number of peaks (periods) in one disk rotation, the average number is about 32. This agrees well with linear theory estimates for azimuthal mode number at this Reynolds number.

5.3 Basic Flow Velocity Spectra

Figure 5.4 shows the corresponding amplitude spectra of the velocity fluctuations shown in Figure 5.3. The spectra have been generated by applying a 128 point Fast Fourier Transform (FFT), with a Hanning window. The spectra correspond to 200 averages.

The time series shown in Figure 5.3 are expected to contain both stationary and traveling modes. However, the horizontal axis can be represented by the mode number ($n$) since the traveling modes speed is about 85% of the local disk speed. The spectra indicate that the time series contain a band of frequencies that have corresponding mode numbers ranging from approximately 25 to 40, with the highest amplitude being at approximately $n = 30$. These are well within the range of the amplified mode numbers predicted from linear theory for the disk boundary layer.

The wall-normal amplitude distribution of the fluctuations of mode $n = 28$ are shown in Figures 5.5 and 5.6. These correspond to eight different radial locations of $R = 462, 475, 488, 501, 507, 513, 520$, and $526$. Note that these are inboard, exactly at, and outboard of the critical Reynolds number of the absolute instability. Based on the spectra, $n = 28$ is representative of the energy in cross-flow modes since it is in the center of the band of amplified modes over the range of radii presented. Drawn
Figure 5.3. Time series of velocity fluctuations at $R=501$ for $z^*=0.709, 1.099, 1.818, 2.144, 3.286,$ and $4.393$. 
Figure 5.4. Amplitude spectra of time series of velocity fluctuations at $\text{Re}=501$ for $z^*=0.709, 1.099, 1.818, 2.144, 3.286, \text{and } 4.393$. 
Figure 5.5. Normalized wall-normal distributions of azimuthal velocity fluctuations for \( n=28 \) at \( r/r_{cross}(R) = 1.6100(462), 1.6547(475), 1.6995(488) \), and 1.7442(501). Line corresponds to linear theory eigenfunction.
Figure 5.6. Normalized wall-normal distributions of azimuthal velocity fluctuations for n=28 at \( r/r_{cross}(R) = 1.7665(507), 1.7889(513), 1.8113(520), \) and 1.8336(526). Line corresponds to linear theory eigenfunction.
for comparison is the linear eigenfunction for a Type I cross-flow mode. In all of
the radii, the measured amplitude distribution agrees well with the linear theory
distribution. This is a strong indication that the velocity fluctuations are due to
linear cross-flow modes.

Now, that we verified that the mean velocity profiles were in a good agreement
with laminar flow theory, and proved that the wall-normal velocity distributions
are consistent with the linear theory eigenfunctions of cross-flow modes, the next
check is to verify the radial growth rate of the modes. To do so, amplitude values
of different mode numbers were documented versus the radial position. These were
derived from the spectra such as shown in Figures 5.7 and 5.8 which correspond
to \( R=462, 475, 488, 501, 507, 513, 520, \) and 526. The spectra shown in the figures
were taken at the boundary layer height at which the amplitudes of the velocity
fluctuations were maximum.

Linear theory predicts a band of most amplified mode numbers would occur
for a given Reynolds number. As the radius increases, the most amplified band
shifts towards higher frequencies. Therefore, we expect a pattern of growth in
amplitude of higher frequencies with increasing Reynolds numbers. The spectra
in Figures 5.7 and 5.8 substantiate this pattern. For example at \( R=462, \) the
maximum amplitude was at \( n=28, \) while at \( R=501 \) it was at approximately \( n=30. \)
At the highest Reynolds number shown (526), the largest amplitude occurred at
\( n=33. \) All are predicted by linear theory.

One can notice that past \( R=507, \) there were some mode numbers in the range
of 62–68 that have growing amplitudes. These mode numbers are in the range of fre-
quencies of traveling modes that are predicted to be absolutely unstable (Lingwood
(1995) [35]).
Figure 5.7. Spectra at $r/r_{\text{cross}}(R) = 1.6100(462), 1.6547(475), 1.6995(488), \text{ and } 1.7442(501)$, for the basic flow condition.
Figure 5.8. Spectra at $r/r_{\text{cross}}(R) = 1.7665(507), 1.7889(513), 1.8113(520),$ and $1.8336(526),$ for the basic flow condition.
Figure 5.9 shows the radial growth in amplitude of selected modes taken from the previous spectra. These were constructed by plotting the maximum amplitudes at $z^*$=1.4, for $n$=28, 30, 32, and 34, at different radial locations. In order to show the exponential growth, a log scale is used for the amplitude axis. The amplitude has also been normalized by the local disk surface velocity, $U_{\text{max}} = r \omega$. This representation is the same as used by Knasiak (1996) [29] and Matlis (1997) [44].

Figure 5.9 illustrates that the selected modes initially grow exponentially. The line through the points corresponds to a radial growth rate $\alpha_r = 0.043$, which is the theoretical maximum growth rate of a pure wave given by Mack (1985) [39]. The measured growth rates for the different modes are represented well on average, by the theory. The end of the exponential growth indicates the beginning of the non-linear region of the mode development. This agrees well with the deviation of the mean velocity profiles from the laminar solution that was shown in Figure 5.1.
Figure 5.9. Radial growth of the disturbance amplitudes at azimuthal mode numbers of 28, 30, 32, and 34. Line corresponds to maximum growth rate of a pure wave based on theory by Mack (1985) [39].
CHAPTER 6

RESULT AND DISCUSSIONS: LINEAR TEMPORAL DISTURBANCES

Once the basic flow was documented, the effect of introducing time-dependent disturbances using the pulsed air jet was studied. A detailed description of the pulsed air jet and the selection of the operation conditions was presented in Chapter 4.

Two conditions of disturbance amplitudes were investigated: linear and finite-amplitude. In both cases, we documented that a wide band of frequencies were generated. This chapter will deal with the results for the lower amplitude linear disturbance case.

For all cases, the air pulser jet was placed at a fixed Reynolds number of 311 (radius of 12.1 cm), which was the same as that used by Lingwood (1996) [36]. The hot-wire was placed at different radial and azimuthal locations relative to the air pulser jet. A schematic of this is shown in Figure 6.1. The output of the hot-wire anemometer was acquired along with the signal used to trigger the air pulse. In post-processing, the time series were ensemble averaged with respect to the time instant when the air pulse was initiated. Typically 1000 averages went into the ensemble. The analysis of the ensemble averaged velocity time series is the emphasis of this chapter.

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Figure 6.1. A sketch of relative positions of hot wire and air pulser jet (Not to scale).
6.1 Wave Packet Development

An example of ensemble averaged velocity time series are shown in Figure 6.2. These correspond to different wall-normal distance measured at \( R=437 \) and \( \theta=220^\circ \). In this and other plots like it, the abscissa represents the time, \( t \), required for the wave packets to reach the hot wire sensor. It is normalized by the time required for the disk to spin one rotation, \( T \). The vertical axis is the amplitude. In this figure, the amplitude scale has been kept the same for better comparison.

The purpose of Figure 6.2 is to illustrate the wall-normal amplitude distribution of the wave packet. We observe that the maximum amplitude occurs at \( z^*=1.33 \), which is close to where we expect the maximum to be based on the linear theory eigenfunction for a Type I cross-flow mode.

A similar set of ensemble averaged time series are shown in Figure 6.3. These correspond to different wall-normal heights for \( R=513 \) and \( \theta=280^\circ \). These again show a maximum near the theoretical eigenfunction maximum at \( z^*=1.38 \). Also comparing the maximum amplitude levels for the two Reynolds numbers, one can notice a large increase from about 0.06 at \( R=437 \) to 0.5 at \( R=513 \). This suggests the type of large amplification occurring in the wave packet development.

From Figures 6.2 and 6.3, we selected the maximum values of the wave packet envelopes and then plotted their wall-normal distributions. These are shown with error bars in Figure 6.4. Also plotted for comparison is the linear theory eigenfunction. The abscissa is normalized by the maximum amplitude at each radial location. From this figure, we notice that at \( R=437 \), the amplitude distribution matches very well with the linear theory eigenfunction. This is an indication that the wave packet amplitude is linear. At \( R=513 \), the amplitude distribution does not match linear theory quite as well, but would still be considered to be linear.
Figure 6.2. Wave packets at different wall-normal positions for \( R=437 \) (\( r=17.0 \) cm), \( \theta =220^\circ \), pressure=28psi.
Figure 6.3. Wave packets at different wall-normal positions for $R = 513$ ($r = 20.0$ cm), $\theta = 280^\circ$, pressure=28 psi.
The effect that the wave packets have on the mean velocity is presented in Figure 6.5 for the conditions in Figure 6.4, and an additional higher Reynolds number of 565. These were obtained by considering only one disk rotation that encompassed the wave packet location in time.

Figure 6.4. Wall-normal amplitude distribution of wave packets measured at $R=437$, $\theta=220^\circ$ and $R=513$, $\theta=280^\circ$. Pulser pressure=28psi.

The mean velocity profiles at $R=437$ and 513 collapsed very well onto the laminar velocity profile. This again suggests that the boundary layer disturbances
Figure 6.5. Comparison of the mean velocity in the azimuthal average. For linear disturbance case measured at $R=437, 513,$ and 565.
are linear. A mean flow distortion was observed at $R=565$ in Figure 6.5. This was also observed in the case without the disturbance generator that was shown in Figure 5.1.

Having shown that the initial development of the disturbance wave packets were linear, the following figures document the development of the wave packets at different Reynolds number and relative angles between the air pulser jet and hot-wire. A selection of these are presented in this chapter. The complete development of wave packets at all radial locations and all angles for the infinitesimal disturbances case may be found in Appendix B.

Figure 6.6 shows the ensemble averaged time series at $R=308$ (radius of 12.0 cm). Note that this is slightly inboard of the radius where the air pulser was placed (12.1 cm). One can notice that at this small radius, the wave packet is only evident at the smaller angles. At the larger angles only the background, low level fluctuations are evident.

It is important to point out that the time delay between the electronic signal from the computer and the activation of the solenoid valves was found to be approximately $0.18 \ t/T$. This time delay was not removed from the ensemble averaged time series such as shown in Figure 6.6.

Figures 6.7 and 6.8 show the ensemble averaged time series at $R=359$ (radius of 14.0 cm), and relative angles of $10^\circ$, $20^\circ$, $25^\circ$, $40^\circ$, $70^\circ$, $100^\circ$, $130^\circ$, $160^\circ$, $190^\circ$, and $220^\circ$. A traveling wave packet is evident at the smaller angles presented in Figure 6.7. At $\theta=130^\circ$ and larger in Figure 6.8, the wave packet is no longer visible within the background fluctuations. Note that the scale in these plots has been expanded by 40 times compared to $\theta=40^\circ$ in Figure 6.7.

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Figure 6.6. Ensemble averaged time series at $R=308$ ($r=12.0$ cm) for different angles, $\theta$ =10°, 20°, 25°, 40°, 100°, and 160°, pressure=28 psi.

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Figure 6.7. Ensemble averaged time series at $R=359$ ($r=14.0$ cm) for different angles, $	heta =10^\circ$, $20^\circ$, $25^\circ$, $40^\circ$, $70^\circ$, and $100^\circ$, pressure=28psi.
Figure 6.8. Ensemble averaged time series at $R=359$ ($r=14.0$ cm) for different angles, $\theta = 130^\circ$, $160^\circ$, $190^\circ$, and $220^\circ$, pressure=28psi.
Referring back to Figure 6.1, the relative angle between the air pulser and the hot-wire (θ) is equivalent to the development time for the disturbances to grow and evolve. For example in Figure 6.7, at θ=20° the disturbance appeared as a wave packet centered at approximately t/T=0.5. As the measurement angle was increased, the wave packet was observed to grow in amplitude and spread out in time. Note that for time advancing to the right in the figure, the leading edge of the wave packet is on the left, and the trailing edge is on the right.

Figures 6.9 through 6.11 show the ensemble averaged time series at R=513 (radius of 20.0 cm) for angles of 10°, 20°, 25°, 40°, 70°, 100°, 130°, 160°, 190°, 220°, 250°, 280°, 310°, 340°, and 355°. Figure 6.9 shows the smaller angles from 10 to 70 degrees. At the smaller angles, an identifiable wave packet was only evident with the sensor at θ=10°. The center of the wave packet was at t/T=2.2. This means that this wave packet has traveled twice around the disk before it was detected in the ensemble average.

At other angles in Figure 6.9, it is difficult to identify an isolated wave packet from the background fluctuations. However, as the relative angle of the hot-wire increases past θ=130°, in Figure 6.10, isolated wave packets were again evident. This remains the case to the highest relative angles shown in Figure 6.11. The ratio of the maximum wave packet amplitude to the local disk speed is about 3.2%. At highest angles, the wave packet was traveling twice around the disk when it was detected by the hot-wire sensor. Of particular interest to the role of the absolute instability is the time between the leading and trailing edges of the wave packet. An analysis of that is forthcoming.

The linear theory of Lingwood (1995) [35] predicts that an absolute instability exists for traveling cross-flow modes above R=507.3. The evidence of the absolute in-
Figure 6.9. Ensemble averaged time series at \( R=513 \) (\( r=20.0 \) cm) for different angles, \( \theta =10^\circ, 20^\circ, 25^\circ, 40^\circ, \) and \( 70^\circ, \) pressure=28psi.
Figure 6.10. Ensemble averaged time series at $R=513$ ($r=20.0$ cm) for different angles, $\theta = 100^\circ$, 130$^\circ$, 160$^\circ$, 190$^\circ$, and 220$^\circ$, pressure=28 psi.
Figure 6.11. Ensemble averaged time series at $R=513$ ($r=20.0$ cm) for different angles, $\theta = 250^\circ$, 280°, 310°, 340°, and 355°, pressure=28 psi.
stability would be temporal growth of the disturbance wave packet as it approached
the critical Reynolds number. One evidence of this would be the spreading of the
leading and trailing edges of the disturbance wave packet.

Two methods were used in determining the locations of the leading and trail-
ing edges of the wave packets. The first was by visual inspection of the ensemble
averaged time series. The second was by an automated software that was written
for the post processing of the data.

In either approach, only \((R, \theta)\) combinations that showed identifiable isolated
wave packets were used to define the leading and trailing edges. Examples of those
included would be any of those in Figure 6.11. An example of one not included
would be that at \(\theta=100^\circ\) in Figure 6.10.

Determining the edges of the wave packet visually might appear to be a bit
more arbitrary. In the case of the automated approach, the edges were defined
as the points that were before and after the maximum in the amplitude envelope
that were equal to the average of the background fluctuations. The background
fluctuation average was computed over a portion of time away from the wave packet,
for example from \(t/T\) of 2.5 to 4.0 in the ensemble average in Figure 6.11.

The results of the automated determination of the leading and trailing edges of
the wave packets are shown in Figure 6.12. The top plot corresponds to ensemble
average amplitude envelope that was obtained by band-pass filtering between mode
numbers of 22–40. The bottom part of the figure corresponds to filtering that passed
modes from 22–75. The solid curves represent the Lingwood’s (1996) results. The
dashed line denotes the Reynolds number of absolute instability, \(R=507.3\). The
difference between the two plots is only slight. It is clearly shown that the wave
packets grown and convected away from the source disturbance.
Figure 6.12. Leading and trailing edges of the wave packets corresponding to band-passed mode number between 22–40 (top) and 22–75 (bottom) for pressure=28psi, based on automated detection.
What is important with the data in Figure 6.12 is the envelope of the extremes of the leading and trailing edge times at a given Reynolds number. The values for the leading edges fall a little below those of Lingwood, but the trend with increasing Reynolds number is very similar.

Figure 6.13. Leading and trailing edges of the wave packets based on visual interpretation of ensemble averaged time series. Pulser pressure=28psi.

The trend for the trailing edge initially followed that of Lingwood. However, as the absolute instability Reynolds number is approached, the trailing edge expansion appears to asymptote. We note that the part of Lingwood’s curve for $t/T>2.0$ was not based on measurement, but was an extrapolation of the expected trend. The asymptoting behavior of the expansion of the wave packet observed for the linear disturbance agrees with the linear simulation of Davies and Carpenter (2003) [17].
Their conclusion is that for linear disturbances, the absolute instability has a finite duration and is thereby not a global mode that would dominate transition to turbulence. As a result, the convective instability dominates.

The results of the visual determination of the leading and trailing edges of the disturbances wave packets are shown in Figure 6.13. It agrees very well with the automated approach and leads to the same conclusion.

The spatio-temporal development of the disturbances are further illustrated in Figure 6.14 which shows contours of the peak amplitude of the wave packets. The amplitudes have been normalized by the minimum value and displayed on a log scale to highlight linear growth. The heavier solid lines represent propagation boundaries with spreading rates taken from the linear simulations of Davies and Carpenter (2003). The dashed vertical line represents the critical Reynolds number of absolute instability. They were found to encompass well the disturbance amplitude evolution and further suggest a convective nature even well past the absolute instability Reynolds number.

The radial trajectories of the amplitude maxima and their temporal growth taken from Figure 6.14, are shown in Figure 6.15. In the top plot, the solid curve is a spline fit to the $R_{Amaz}$ distribution. The dashed line represents the critical Reynolds number for the absolute instability. In the lower plot, the solid line is a linear fit to the exponentially growing amplitudes.

Figure 6.15 reveals that the wave packet amplitude initially grows exponentially in time but eventually saturates and decays, presumably due to nonlinear effects. While the amplitude grows, the radius of the amplitude maximum increases. The radius location maximum coincides with time of the amplitude maximum, and then
Figure 6.14. Contours of constant log($A/A_{\text{min}}$) of the disturbance wave packet maximum envelope amplitude for pressure=28psi.
Figure 6.15. Radial locations of maximum amplitude versus time (top) and temporal growth of maximum amplitude (bottom) for disturbance wave packets at pressure=28psi.
moves to smaller radii as the amplitude decreases. As well be shown in the next section, the location of the nonlinear amplitude saturation of the most amplified mode \((n=32)\) coincides with location of the temporal amplitude maximum. This again suggests that the dominant instability is convective in this case.

6.2 Spectral Analysis of Disturbance Wave Packets

Spectral analysis was applied to the time series to determine the frequency content of the disturbances produced by the air pulse. This allowed us to focus on the amplitude of particular azimuthal mode number, \(n\). A complete set of the spectra at all radial and angle locations for the linear disturbance cases may be found in Appendix C. This section will focus on the specific cases for which ensemble averages time series were presented in previous section.

Figure 6.16 shows the amplitude spectra of disturbance time series measured at \(R=308\) (radius=12.0 cm) for the same angles shown in Figure 6.6. The abscissa represents the azimuthal mode number which is found by dividing the frequency by the disk rotation frequency. At this small radius (Reynolds number) there is no dominant band of frequencies that has been amplified in the boundary layer.

The amplitude spectra corresponding to the ensemble averaged time series shown in Figures 6.7 and 6.8 at \(R=359\) are presented in Figures 6.17 and 6.18. This shows the evolution of energy going from broad band to a band between \(n\) of 20 to 40 that is expected to be amplified based on linear theory. This band is only evident at \(\theta=70^\circ\) which is the approximate angle at which an isolated wave packet was identifiable.

Figures 6.19 through 6.21 show the spectra at \(R=513\) which correspond to the ensemble averaged time series in Figures 6.9 through 6.11. Note that this location
Figure 6.16. Amplitude spectra of disturbance time series measured at $R=308$ ($r=12.0$ cm) for different angles, $\theta = 10^\circ$, $20^\circ$, $25^\circ$, $40^\circ$, $100^\circ$, and $160^\circ$, pressure=28psi.
Figure 6.17. Amplitude spectra of disturbance time series measured at $R=359$ ($r=14.0 \text{ cm}$) for different angles, $\theta =10^\circ$, $20^\circ$, $25^\circ$, $40^\circ$, $70^\circ$, and $100^\circ$, pressure=28psi.
Figure 6.18. Amplitude spectra of disturbance time series measured at $R=359$ ($r=14.0$ cm) for different angles, $\theta =130^\circ$, $160^\circ$, $190^\circ$, and $220^\circ$, pressure=28psi.
is outboard of the critical radius of the absolute instability location. At angles up to 160°, the spectra appeared as a relatively smooth band that brackets the mode numbers expected to be amplified based on linear stability theory. At relatively higher angles, where the isolated wave packets were well defined, the band increases in width particularly towards higher frequencies. The maximum amplitude in the disturbance time series remained close to \( n=32 \). The energy in higher modes such as 68 to 75 is considerably smaller by comparison.

As mentioned before, the spectral analysis allows us to focus on the amplitude of particular mode numbers. Figures 6.22 and 6.23 shows the wall-normal amplitude distribution for \( n=32 \) and 75 at \( R=437, 513, \) and 565. Note that this equivalent to Figure 6.4 for the lower two Reynolds numbers. The distributions at \( R=437 \) and 513 agree well with the shape of the linear eigenfunction, indicating that they are still in the linear region. At \( R=565 \), the amplitudes were no longer linear. This is consistent with the mean velocity profiles in Figure 6.5 that showed a large mean flow distortion at \( R=565 \).

Figure 6.23 shows the wall-normal distribution of \( n=75 \) for the same locations as Figure 6.22. None of the disturbances compare very well with the linear theory eigenfunction. The reason is likely due to the very low amplitude levels of this mode number, which is far from the most amplified linear modes.

The space-time development of the amplitude of a selected mode number is presented in Figures 6.24 through 6.26. This is shown as contours of constant levels of the \( \log(A/A_{\text{min}}) \), where \( A \) is the amplitude of the specific mode number, and \( A_{\text{min}} \) is the minimum amplitude in the data set. The solid curve in the figures represents a logarithmic spiral trajectory which would be representative of the path disturbances are expected to take over the disk. Note that \( t/T \) in these figures refers
Figure 6.19. Amplitude spectra of disturbance time series measured at $R=513$ ($r=20.0$ cm) for different angles, $\theta = 10^\circ, 20^\circ, 25^\circ, 40^\circ,$ and $70^\circ$, pressure=28psi.
Figure 6.20. Amplitude spectra of disturbance time series measured at $R=513$ ($r=20.0$ cm) for different angles, $\theta = 100^\circ$, $130^\circ$, $160^\circ$, $190^\circ$, and $220^\circ$, pressure=28 psi.
Figure 6.21. Amplitude spectra of disturbance time series measured at $R=513$ ($r=20.0$ cm) for different angles, $\theta = 250^\circ, 280^\circ, 310^\circ, 340^\circ, \text{ and } 355^\circ$, pressure=28psi.
Figure 6.22. Wall-normal amplitude distribution of the disturbances at \( n=32 \) for \( R=437, 513, \) and 565 for pressure=28 psi.
Figure 6.23. Wall-normal amplitude distribution of the disturbances at $n=75$ for $R=437, 513, \text{ and } 565$ for pressure=28 psi.
Figure 6.24. Contour plots of $\log(A/A_{\text{min}})$ for $n=32$ based on the spectra of disturbance time series for pressure=28psi.
Figure 6.25. Contour plots of $\log(A/A_{\min})$ for $n=67$ based on the spectra of disturbance time series for pressure=28psi.
Figure 6.26. Contour plots of $\log(A/A_{\text{min}})$ for $n=75$ based on the spectra of disturbance time series for pressure=28psi.
to the $\theta$ location of the measurements.

Figure 6.24 shows the development of $n=32$. For this, we observe that the trajectory of the maximum amplitude followed pretty well the theoretical logarithmic spiral. This suggests that the disturbance wave packets were developing as linear waves.

Figures 6.24 and 6.25 show the development of $n=67$ and 75. These are mode numbers that are in the range to be absolutely unstable. The amplitudes of each number normalized by their minimum amplitudes in the data set. With respect to their minimum levels, both modes exhibit a continuous exponential growth, although the growth rate is less than $n=32$. This is expected based on linear theory. As with $n=32$, the locations of the amplitude maxima track well with the theoretical logarithmic spiral.

Based on Figures 6.24 to 6.26, the maximum amplitudes were close to $t/T=0.44$. Therefore the radial growth of the three modes were plotted at the fixed time of $t/T=0.44$ in Figure 6.27. The amplitudes are shown as a log scale to emphasize exponential growth. The $n=32$ mode amplification rate was larger as expected from the linear theory. It also began to grow at a lower Reynolds number as expected.

The $n=67$ and 75 modes displayed linear growth that extended past the critical Reynolds number for absolute instability. Therefore in terms of their amplitude alone, there was no apparent effect of crossing this critical Reynolds number.
Figure 6.27. Amplitude growth of azimuthal mode numbers of 32, 67, and 75, for different Reynolds numbers at $t/T=0.44$ and the pressure=28psi.
CHAPTER 7

RESULTS AND DISCUSSIONS: FINITE-AMPLITUDE DISTURBANCES

In this chapter, the results based on finite-amplitude disturbance will be presented. All parameters are kept the same as the linear amplitude disturbance case except the air pulser pressure which was increased to 80 psi. In general, the presented figures in this chapter are similar to those shown in Chapter 6 for comparison. In following section, examples of the ensemble averaged time series will be shown. A complete set of the wave packets developments at all radial locations and all angles can be found in Appendix D.

7.1 Wave Packet Development

Wall-normal measurements were performed at selected radial locations to document the wall-normal amplitude distribution of the wave packets. Figure 7.1 shows the ensemble averaged velocity time series for the measurements taken at \( R=513 \) and \( \theta=340^\circ \). Multiple wave packets are observed at some wall-normal locations. This phenomena was not observed for the linear amplitude disturbance. As a result, the wave packets leading and trailing edges would extend to larger times compared with the linear disturbance case. As shown in the figure, the maximum amplitude occurred at \( z^*=1.78 \) which was far from that based on the linear theory eigenfunction for a Type I cross-flow mode \( (z^*=1.4) \).

Another set of ensemble averaged time series are shown in Figure 7.2 for the
Figure 7.1. Wave packets at different wall-normal positions for $R=513\ (r=20.0\ \text{cm}),\ \theta =340^\circ,\ \text{pressure}=80\text{psi}$.
data measured at $R=539$ and $\theta=340^\circ$. By noting that the vertical axis was kept the same for all plots, the amplitude maximum occurred at $z^*=1.805$. The values of amplitude maximum were increased by increasing Reynolds number. For example, it was about 0.45 at $R=513$ and $\theta=340^\circ$ and about 0.6 at $R=539$ and $\theta=340^\circ$. This change in amplitude maximum versus Reynolds number is small compared with that for the linear case.

Based on the results shown in Figures 7.1 and 7.2, the maximum amplitude of the wave packet at each wall-normal location was selected. The distribution of these maximum amplitude were plotted as shown in Figure 7.3. The solid curve represents the distribution of the linear theory eigenfunction. The abscissa was normalized by the maximum amplitude at each radial location. As shown in the figure, both of the distributions at the two radial locations deviated from the linear theory eigenfunction. This indicates that the wave packets amplitudes were not linear for this higher disturbance level.

By considering only one disk rotation that encompassed the wave packet location in time, the azimuthal mean velocity profiles were obtained. These are shown in Figure 7.4. The solid curve is the similarity solution obtained by von Kármán (1921) for an infinite diameter disk. In this case with the higher disturbance level, a mean flow distortion was observed at both locations. Comparing with mean velocity profiles for the base flow case, Figure 5.1, or that for the lower disturbance amplitude in Figure 6.5, we concluded that the wave packet amplitudes in this case were not linear. This is consistent with the fluctuation amplitude distributions shown in Figure 7.3.

The results shown in the previous figures indicate that the higher amplitude disturbance lead to nonlinear amplitudes, as intended. We then documented its
Figure 7.2. Wave packets at different wall-normal positions for $R=539$ ($r=21.0$ cm), $\theta =340^\circ$, pressure=80psi.
Figure 7.3. Wall-normal amplitude distribution of wave packets measured at $R=513$, $\theta=340^\circ$ and $R=539$, $\theta=340^\circ$. Pulser pressure=80psi.
Figure 7.4. Comparison of the mean velocity in the azimuthal average. For finite-amplitude disturbance case measured at $R=437$, 513 and 539.
effect on the wave packet development. For this, the velocity fluctuation ensemble averaged time series at selected radial locations, with different angles relative to the air pulser jet, will be presented. Figure 7.5 shows the ensemble averaged time series at $R=308$ (radius of 12.0 cm). As with the lower amplitude disturbances, the air pulser jet was placed at at $R=311$ (radius of 12.1 cm). Therefore the results shown in Figure 7.5 were measured at a radial location that was slightly inboard to that of the air pulser jet. At this small radius, the wave packet was only evident at smaller angles. By noting the change in the amplitude axis, the ensemble averaged time series at the two highest angles ($40^\circ$ and $70^\circ$) reflect only the low amplitude background fluctuation. We note that at $\theta=10^\circ$ in this case, the maximum amplitude of the wave packet is about 15 times of that for the linear disturbance case that was shown in Figure 6.6.

There was a time delay of about 0.18 $t/T$ between the electronic signal from the computer and the activation of the solenoid valves. This time delay was not removed in the previous ensemble averaged time series and others that are forthcoming.

Figures 7.6 and 7.7 show ensemble averaged time series at $R=359$ (radius of 14.0 cm), and relative angles of $10^\circ$, $40^\circ$, $70^\circ$, $100^\circ$, $130^\circ$, $160^\circ$, and $220^\circ$. A small amplitude disturbance wave packet was evident at the smaller angle ($\theta=10^\circ$). The wave packet maximum amplitude appears to peak at $\theta=40^\circ$ then decreases at larger angles from the disturbance generator. More importantly, multiple wave packets were observed at all angles. This is quite evident when comparing to ensemble averages at the same Reynolds number location with the smaller disturbance amplitude that were shown in Figures 6.7 and 6.8.

The data shown above were in the close vicinity of the disturbance source. The relative angle between the air pulser and the hot-wire ($\theta$) is equivalent to the
Figure 7.5. Ensemble averaged time series at $R=308 \ (r=12.0 \text{ cm})$ for different angles, $\theta = 10^\circ$, $40^\circ$, and $70^\circ$, pressure=80psi.
Figure 7.6. Ensemble averaged time series at $R=359$ ($r=14.0$ cm) for different angles, $	heta =10^\circ$, $40^\circ$, $70^\circ$, and $100^\circ$, pressure=80psi.
Figure 7.7. Ensemble averaged time series at $R=359$ ($r=14.0$ cm) for different angles, $\theta =130^\circ$, $160^\circ$, and $220^\circ$, pressure=80 psi.
development time for the disturbances to grow and evolve (Figure 6.1). So, for
larger angles, the wave packet was expected to spread out in time.

Figures 7.8 through 7.10 show the ensemble averaged velocity time series at
\( R=513 \) (radius of 20.0 cm). Note that this radius is just past the critical value for
the absolute instability. The disturbance wave packet was initially evident at the
smaller angle of 10° as shown in Figure 7.8. Note that at \( \theta=10^\circ \), the wave packet
center was at about \( t/T=2.1 \), which means that it was detected by the hot-wire
sensor in the second rotation around the disk. When increasing the angle between
the air pulser and the hot-wire sensor from 40 to 100 degrees, it was difficult to
identify isolated wave packets. However at \( \theta=130^\circ \), shown at the top of Figure 7.8,
an isolated wave packet was again evident.

Past \( \theta=130^\circ \), isolated wave packets were always evident. The maximum ampli-
tude of the wave packet increased with time until it reached its peak at \( \theta=220^\circ \) in
Figure 7.9. At higher angles, such as seen in Figure 7.10, secondary wave packets
were detected by the hot-wire sensor in the third rotation around the disk. This is
very important and indicates a dramatic spreading of the disturbance wave packets,
especially when compared to those for the lower disturbance amplitude at the same
Reynolds number in Figure 6.11.

Leading and trailing edges of the wave packets were selected from the ensemble
averaged time series based on the automated edge detection software and by visual
inspection. Only the identifiable isolated wave packets were used. For example in
Figure 7.8, the wave packets at \( \theta=70^\circ \) were not included while the wave packet at
\( \theta=10^\circ \) was included.

Figure 7.11 shows the leading and trailing edges of the wave packets based on
automated determination. This result corresponds to ensemble averaged time se-
Figure 7.8. Ensemble averaged time series at $R=513$ ($r=20.0$ cm) for different angles, $\theta = 10^\circ$, $40^\circ$, $70^\circ$, $100^\circ$, and $130^\circ$, pressure=80 psi.
Figure 7.9. Ensemble averaged time series at $R=513 \ (r=20.0 \ cm)$ for different angles, $\theta = 160^\circ, 190^\circ, 220^\circ, \ and \ 250^\circ$, pressure=80psi.
Figure 7.10. Ensemble averaged time series at \( R=513 \) \( (r=20.0 \text{ cm}) \) for different angles, \( \theta = 280^\circ, 310^\circ, 340^\circ, \) and \( 350^\circ \), pressure=80 psi.
ries based on bass-band filtering between mode numbers of 22-40 (top) and 22-75 (bottom). Lingwood’s (1996) results are presented as solid curves. The dashed line represents the critical Reynolds number of the absolute instability. A slight difference between the two plots is noted. Comparing with the linear amplitude disturbance case shown in Figure 6.12, the scattering near the disturbance source was greater. One reason may be that the air pulser jet was not perfectly perpendicular to the disk surface. The other is that the momentum coming from the air pulser jet was greater so that its effect spread further in space and time compared to the lower amplitude case.

Figure 7.12 shows the leading and trailing edges of the wave packets based on visual determination. The linear amplitude disturbance case was shown in the top plot for comparison. Both methods of determining the leading and trailing edges lead to the same result. Focusing on the finite-amplitude disturbance case, the trailing edge followed the solid curve even at higher Reynolds numbers and times. However, there were lack of results in the region of $R>513$ and $t/T>2.1$. The reason is that it became difficult to define the leading and trailing edges of isolated wave packets that were above the background disturbances in this region. This was not a problem at the lower disturbance amplitudes.

Figure 7.13 shows the spatio-temporal development of the disturbances. The contours correspond to the peak amplitude of the wave packets. The amplitudes have been normalized by the minimum value and displayed on a log scale to emphasize the linear growth. The heavier solid lines represent propagation boundaries with spreading rates taken from the linear simulations of Davies and Carpenter (2003). We observe that the peak amplitude immediately passing the critical Reynolds number of absolute instability (shown as dashed vertical line) grew rapidly in time,
Figure 7.11. Leading and trailing edges of the wave packets corresponding to band-passed mode number between 22–40 (top) and 22–75 (bottom) for pressure=80psi, based on automated detection.
Figure 7.12. Leading and trailing edges of the wave packets based on visual interpretation of ensemble averaged time series at pressures=28psi (top) and 80psi (bottom).
greatly exceeding the linear spreading rate. This is quite evident in comparing with the linear disturbance amplitude distribution of the wave packets that was shown in Figure 6.14. The fact that this closely coincides with the critical Reynolds number of the absolute instability suggests it is associated with a global mode growth.

Figure 7.14 shows the radial trajectories of the amplitude maxima and their temporal growth taken from Figure 7.13. In the top plot, the dashed line represents the critical Reynolds number for the absolute instability. The solid curve is a spline fit to the $R_{A_{max}}$ distribution. We observe that the radial location of the maximum amplitude was increasing with time until it asymptoted to $R_{A_{max}} \approx 530$ at about $t/T = 1.8$. In the lower plot, the temporal growth of the wave packet amplitude was not linear. This was expected for the finite-amplitude disturbances case and can be compared to Figure 6.15 for the smaller disturbance amplitude.

Figure 7.15 shows a comparison of the radial amplitude locations and temporal growth for the linear amplitude and finite-amplitude disturbances cases. The trend of radial trajectories of both cases are similar, although the radial locations of the maximum amplitudes for linear disturbance case was greater. Neither of the two cases showed $R_{A_{max}}$ to be at $R_{ubs}$. In the temporal growth plot, the peak values of the maximum wave packet amplitudes were slightly larger for the finite-amplitude disturbance case. More importantly the maximum value was reached at a shorter $t/T$, and the amplitude remained relatively large at larger $t/T$ compared to the smaller amplitude disturbance.

7.2 Spectral Analysis of Disturbance Wave Packets

In the previous section, the ensemble averaged time series were presented. This included all mode numbers. To focus on the amplitude of particular azimuthal mode numbers, $n$, a spectral analysis was applied to the time series to determine
Figure 7.13. Contours of constant log($A/A_{\text{min}}$) of the disturbance wave packet maximum envelope amplitude for pressure=80psi.
Figure 7.14. Radial locations of maximum amplitude versus time (top) and temporal growth of maximum amplitude (bottom) for disturbance wave packets at pressure=80psi.
Figure 7.15. Comparison of radial locations of maximum amplitude versus time (top) and temporal growth of maximum amplitude (bottom) for disturbance wave packets at pressures=28 and 80 psi.
the frequency content of the disturbance wave packets. In this section, examples of the spectra at selected radial and angle locations are presented. A complete set of the spectra at all radial and angle locations for the finite-amplitude disturbance cases can be found in Appendix E.

Figure 7.16 shows the amplitude spectra of disturbance time series measured at \( R=308 \) (radius=12.0 cm) for \( \theta =10^\circ, 40^\circ, \) and \( 70^\circ \). The abscissa represents the azimuthal mode number which was found by dividing the frequency by the disk rotation frequency. At this small Reynolds number, we observe energy in a wide band of frequencies when measured close to the air pulser jet, at \( \theta=10^\circ \). Referring to Figure 7.5, multiple wave packets were evident at this angle. At this close spacing, the boundary layer has yet to selectively amplify the narrow band of Type I cross-flow modes. The power spectra that was in Figure 4.4 is very similar.

Figures 7.17 and 7.18 show the amplitude spectra at \( R=359 \). This corresponds to the ensemble averaged time series that were shown in Figures 7.6 and 7.7. In this case, the broad band of frequencies is beginning to be selectively amplified about a center frequency of approximately \( n=30 \). This grows up to \( \theta=100^\circ \) and decays at larger angles.

Figures 7.19 through 7.21 show the amplitude spectra at \( R=513 \). This corresponds to the ensemble averaged time series shown in Figures 7.8 through 7.10. This location is just outboard of the critical radius of absolute instability. At the smaller angles at this radius, we observe a large relatively narrow peak centered at about \( n=30 \). In addition we observe energy at \( n=60 \) which could be a harmonic. At the larger angles in Figure 7.20, we observe a rapid broadening of peak at \( n=30 \). This eventually expands to higher mode numbers as seen in Figure 7.21.
Figure 7.16. Amplitude spectra of disturbance time series measured at $R=308$ ($r=12.0$ cm) for different angles, $\theta = 10^\circ$, $40^\circ$, and $70^\circ$, pressure=80psi.
Figure 7.17. Amplitude spectra of disturbance time series measured at $R=359$ ($r=14.0 \text{ cm}$) for different angles, $\theta = 10^\circ$, 40$^\circ$, 70$^\circ$, and 100$^\circ$, pressure=80psi.
Figure 7.18. Amplitude spectra of disturbance time series measured at \( R=359 \) (\( r=14.0 \) cm) for different angles, \( \theta =130^\circ, 160^\circ, \) and \( 220^\circ \), pressure=80psi.

The wall-normal amplitude distributions for \( n=32 \) and 75 at \( R=513 \) and 539 are shown in Figures 7.22 and 7.23. For the \( n=32 \) mode, at both radial locations there was a deviation from the amplitude distribution from the linear theory eigenfunction. This substantiates the earlier results that the disturbance amplitudes are no longer linear.

At \( n=75 \), the amplitude distribution agrees fairly well with the linear eigenfunction. This was not the case with the previous low amplitude disturbances. As shown in Figure 6.23, the amplitude level in that case at \( n=75 \) was very low and the comparison with the linear theory eigenfunction was poor.
Figure 7.19. Amplitude spectra of disturbance time series measured at \( R=513 \) (\( r=20.0 \) cm) for different angles, \( \theta =10^\circ, 40^\circ, 70^\circ, 100^\circ, \) and \( 130^\circ, \) pressure=80psi.
Figure 7.20. Amplitude spectra of disturbance time series measured at \( R=513 \) (\( r=20.0 \) cm) for different angles, \( \theta = 160^\circ, 190^\circ, 220^\circ, \) and \( 250^\circ \), pressure=80psi.
Figure 7.21. Amplitude spectra of disturbance time series measured at $R=513$ ($r=20.0$ cm) for different angles, $\theta = 280^\circ$, $310^\circ$, $340^\circ$, and $350^\circ$, pressure=80psi.
Figure 7.22. Wall-normal amplitude distribution of the disturbances at $n=32$ for $R=513$ and 539 for pressure=80psi.
Figure 7.23. Wall-normal amplitude distribution of the disturbances at \( n=75 \) for \( R=513 \) and 539 for pressure=80psi.

Figures 7.24 through 7.26 show the space-time development of the wave packet amplitude at selected mode numbers. The contours had constant levels of \( \log(A/A_{\min}) \), where \( A \) is the amplitude of the specific mode number, and \( A_{\min} \) is the minimum amplitude in the data set. The solid curve is a logarithmic spiral trajectory based on linear theory.

For \( n=32 \), the maximum amplitude trajectory still follows the logarithmic spiral
Figure 7.24. Contour plots of $\log(A/A_{\text{min}})$ for $n=32$ based on the spectra of disturbance time series for pressure=80psi.
Figure 7.25. Contour plots of $\log(A/A_{\text{min}})$ for $n=67$ based on the spectra of disturbance time series for pressure=80psi.
Figure 7.26. Contour plots of $\log(A/A_{min})$ for $n=75$ based on the spectra of disturbance time series for pressure=80psi.
fairly well as shown in Figure 7.24. The contour plots for \( n=67 \) and 75 in Figures 7.25 and 7.26 are almost the same. The amplitude maximum also follows the spiral trajectory fairly well.

![Graph showing amplitude growth](image)

Figure 7.27. Amplitude growth of azimuthal mode numbers of 32, 67, and 75, for different Reynolds numbers at \( t/T=0.44 \) and the pressure=80psi.

Figure 7.27 shows the radial growth of the maximum amplitudes at mode numbers of 32, 67, and 75. This obtained from Figures 7.24 through 7.26 at \( t/T=0.44 \). Note that the vertical axis is log to highlight the linear growth. The two solid lines represent the exponential fit for the linear disturbance cases that were presented in Figure 6.27.
For \( n=67 \) and \( 75 \), the radial growth was almost linear and just shifted compared to the lower amplitude case. The radial growth of \( n=32 \) mode is clearly not linear with the higher initial amplitude. This was of course the basis for selecting the higher disturbance level, but the growth rate of the \( n=32 \) mode substantiates this fact.

Figure 7.28 shows the comparison of maximum amplitude growth in time for azimuthal mode numbers of 32 (the most amplified by the linear theory) and 68 (where the absolute instability exists) for the linear amplitude and finite-amplitude cases. Here, the amplitude represented in log scale to emphasis the exponential growth. The vertical dashed line represents the critical Reynolds number of absolute instability. For the mode number of 32, the initial growth was linear for both cases and the amplitude of the nonlinear disturbance case was greater than that of the linear disturbance as expected. The slope was increased when just passing the critical Reynolds number of absolute instability and they saturated at \( R \approx 555 \). For the mode number of 68, the initial growth was also linear but the slope was less than that of \( n=32 \). By just passing the critical Reynolds number of absolute instability, the slope was also increased but to less degree compared with \( n=32 \). The growth continued to increase and did not saturate. Probably, more space was needed to saturate. This behavior could not be explained by either Lingwood’s (1995) theory or Pier’s (2003) scenario of transition.
Figure 7.28. Maximum amplitude growth in time of azimuthal mode numbers of 32 and 68, for different Reynolds numbers at pressure=28 and 80psi.
CHAPTER 8

CONCLUDING REMARKS AND FUTURE WORK

8.1 Conclusions

In this work, convective and absolute instabilities were experimentally investigated. This was done by documenting the evolution of time-dependent disturbances of the incompressible boundary layer flow over a rotating disk in an otherwise still air. The disturbances were introduced from a hypodermic tube located outside the boundary layer above the disk surface. This was a new technique to introduce the disturbances. The designed air pulse generator produced controlled and short-duration pulses. The velocity fluctuations were measured using hot-wire anemometry techniques. The hot-wire sensor was primarily sensitive to the azimuthal velocity component. Ensemble averaged velocity fluctuation time series were measured at different radial locations. The angle between the air pulser jet and the hot-wire was changed to cover the whole circumference.

The base flow case was studied to verify that the flow represented the canonical conditions. The mean azimuthal velocity profiles were in excellent agreement with the similarity solution obtained by von Kármán (1921) for an infinite diameter disk. The wall-normal distribution of the azimuthal velocity fluctuations agreed well with the linear theory eigenfunction at radial locations inboard and outboard.
of the critical radius for the absolute instability. Radial growth of the disturbance amplitudes at azimuthal mode numbers of 28, 30, 32, and 34 matched very well with the theoretical maximum growth rate of a pure wave given by Mack (1985).

Two cases of controlled disturbances were studied, one with a lower amplitude that satisfied linear theory assumptions, and the other with a higher finite-amplitude level. In both cases, a wide range of frequencies near the disturbance source were generated. The wave packets produced by the lower amplitude disturbance had a wall-normal amplitude distribution that agreed well with the linear theory eigenfunction. Those from the higher amplitude disturbance deviated from linear theory.

For the linear disturbance case, the initial amplitude distribution of the wave packet was linear. Comparing with Lingwood’s (1996) results, the temporal growth of the wave packet trailing edge asymptoted. The flow did not transitioned to turbulence to well past the absolute instability critical radius. This agrees well with the linear direct numerical simulations of Davies and Carpenter (2003), that indicated that in this instance the transition is dominated by the convective instability.

For the finite-amplitude disturbance case, multiple wave packets were observed. This indicated a dramatic spreading of the disturbance wave packets compared to those for the linear disturbance amplitude. In this case, the temporal growth of the trailing edge of wave packets agreed with Lingwood’s (1996) predictions. However, the amplitude of the higher mode numbers \( n=68-75 \) was still less than the most amplified convective modes. This partially confirms the analysis of Pier (2003), but failed to show the dominant global mode behavior predicted for \( n=68 \).
8.2 Recommendations

According to Pier (2003), the primary nonlinear waves are a prerequisite for the development of the secondary instability that leads to transition to turbulence. The primary waves travel with respect to the disk frame and have an azimuthal wave number \( n \) of 68.

Following the same techniques that were used before by Corke and Knasiak (1998), roughness pattern consisted of ink dots applied to the disk surface, could be used to excite an azimuthal mode number of 68. In addition to the spatial disturbance, a time-dependent disturbance could also be applied. In this way, high amplitude disturbances at large mode numbers could be created that possibly lead to conditions more suitable to Pier’s analysis.

By increasing the disk speed, time-dependent disturbances could be excited outboard from the critical radius of absolute instability, say at \( R=550 \). For this case, measurements at the critical Reynolds number for absolute instability will reveal whether or not the transition to turbulence is dominated by absolute instability.
APPENDIX A

In this Appendix, descriptions of the spindle controller, pulse conditioning circuit, and calibration of the hot-wire are given.

A.1 Spindle Controller

When first powered up, the DMM-2001 waits for 5 seconds for the internal power in the controller to stabilize, goes through a brushless motor commutation function, then kills spindle axis so it is free to rotate. In case of a pressure loss, the servo loop will open and kill the drive. If the spindle is running at the time of the loss of air pressure, the drive will first decelerate to a stop and then kill as it would during a stop condition. The front panel is an active and includes all switches for manual control. It contains Run/Stop toggle switch, CW/CWW toggle switch, Local/Remote toggle switch, 5-Digit speed thumbwheel (rpm’s), Speed enter pushbutton, 5-Digit acceleration time thumbwheel (rpm’s/sec), Acceleration/Deceleration enter pushbutton, and 2 line display for spindle speed, acceleration/deceleration and faults. The Local/Remote switch in the local position allows operation of the spindle from the front panel only. The default spindle speed is set to zero at power up in the event that the Run/Stop switch is in the run position. In order to run at any speed, the operator must first press the Enter Speed pushbutton to enter in the speed on the thumbwheel. An acceleration rate is then selected and entered into the controller using the Acceleration/Deceleration thumbwheel and Enter button. The
operator may select either CW or CCW direction. The Run/Stop toggle switch is then moved to the run position. The spindle will begin to accelerate to speed and will then lock in at the speed selected on the thumbwheel. To stop the spindle, move the Run/Stop toggle switch to the stop position. The spindle will decelerate to a stop and will be free to move by hand. If while running, the operator wishes to change direction, change the CW/CCW toggle switch to the opposite position. The spindle will decelerate to a stop and will immediately accelerate to speed in the opposite direction. If a change of speed is required while running, change the Speed thumbwheel to the desired speed and press the Enter button. The spindle will accelerate or decelerate to the new speed.

The Local/Remote toggle switch allows the operator to notify the disk controller whether the front panel will be used for control of the spindle rotation, the Local/Remote toggle switch is on the Local position, or is the disk controller is to be operated from a remote site through the use of the controller’s many input/output (I/O) ports, the Local/Remote toggle switch is on the Remote position. These many I/O ports allow for programmable movement and position tracking of the spindle. In the present work, the operation of the disk will be done locally.

A.2 Pulse Conditioning Circuit

Two signals are generated from disk controller. The first signal is a single pulse per one complete rotation of the disk. The second signal is a pulse per 0.5°, i.e., 720 pulses per one complete rotation. Theses pulses are generated by two etched glass disks internal to the spindle, with each glass disk having one and 720 etched lines respectively. The single pulse signal provides a Transistor-Transistor-Logic (TTL) pulse train that can be digitized and acquired simultaneously with the data. This timing signal was used to define the starting of a rotation for later data processing.
By this way, it is guaranteed that for each rotation, the data was acquired from the same azimuthal location of the disk. The second signal, 720 pulses per rotation, was used at an early point of the research to control the rotational position of the disk while placing the dots on the disk surface using the azimuthal stepper motor. The dots were not used in the final experiments.

Figure A.1 shows a schematic diagram of the pulse conditioning circuit. This circuit was repeated twice so that we can use the two signals (the one per complete rotation and the one per half degree) simultaneously during the application of the dots. In both cases, the width of the pulses were too short to be detected during data acquisition. So, the pulse conditioning circuit was used to increase the width of the pulses. The circuit consists of a voltage level detector followed by a timing circuit. The level indicator was constructed from an LM-322 timing chip. With a value of 4.7 kΩ for R3, it outputs a 5 volt pulse if the input signal voltage exceeds a set threshold voltage. The threshold voltage was set by the potentiometer in the bottom LM-322 circuit as seen in Figure A.1. The second circuit elongates the short pulse sent to it by the level detector. The width of the pulse was set by the potentiometer in the RC network shown in the top part of Figure A.1. The criteria for the width of the 0.5° pulse was that it corresponded to two sample points at the acquisition rate used in the experiment. This ensured that when the pulse was digitized, it would not be missed. For the pulse corresponding to a complete rotation of the disk, the pulse width was expanded to a time duration equal to roughly one-half of the disk rotation. In the data processing, the leading edge of this pulse was used as a reference to indicate the beginning of each disk rotation.
Figure A.1. Pulse conditioning circuit for both 360° and 0.5° pulses from the disk controller.
A.3 Calibration

To calibrate the hot-wire, a known velocity must be available as in the case of wind tunnel. For rotating disk, the only known velocity is the velocity of the disk surface itself and, hence, the flow velocity at the disk surface provided that the rotation speed should be given. Obviously, the needles can not touch the disk surface, so this known velocity is useless in calibration and, by then, there are two alternatives. Either removing the hot-wire from its place and calibrate it in a wind tunnel, as an example, or making a design for a device which could be used to calibrate the hot-wire in its place.

The first method has some disadvantages. First, this includes frequent removal of the hot-wire from its place. Second, this needs careful readjustment with the radial direction each time the probe is removed and replaced. Also, the datum height above the disk surface must be remeasured. Third, during the process of removal and replace the risk of damaging the hot-wire will be increased. Fourth, it is recommended that the hot-wire is calibrated in its place so that the calibration accuracy will be increased.

For the reasons mentioned above, the second alternative was chosen. Figure A.2 shows a special design nozzle which used to calibrate the hot-wire. As shown in the figure, the air supply coming from the same compressor that used to supply the air bearing is entering from the nozzle back and before reaching the nozzle exit, there are a number of parallel tubes. The purpose of these tubes is to prevent the air swirling. The air then passes through a side opening which used to measure the static pressure and finally it exits from an one inch nozzle. The side opening is connected with a pressure transducer through a flexible tube. The calibration procedure will be done on two steps; first the side opening will be calibrated against
a Pitot tube which placed at the center of the nozzle exit and then using this calibration to calibrate the hot-wire itself.

A.3.1 Calibration of Static Pressure Tube

The side opening will be used to measure the static pressure. The calibration procedure will be as following

- Place the Pitot tube at the center of the nozzle exit. Since we have a turbulent flow, then if the Pitot tube was shifted a little bit from the nozzle center, the results would not be affected.

- Connect the Pitot tube with a pressure transducer which converts the pressure value to voltage by using a calibrated relation between the reading voltage and the corresponding pressure.

- Connect the side opening flexible tube with another pressure transducer.

- Wait for some time to reach a steady state conditions and record the zero error in both pressure transducers.

- Wait until the compressor pressure output reaches 120 psig and, then, turn the compressor off.

- Open the pressure valve, which connected to the nozzle inlet, to its full opening and start recording the output for both static pressure and total pressure.
simultaneously, by using the DAC. By time, the pressure value coming from
the compressor will be decreased so that we can acquire different pressure
values, which means different velocity values, from both pressure transducers.

• After we got all possible range of pressures, we use these data to convert the
voltage values of the Pitot tube into the corresponding velocities.

• Recording the atmospheric pressure using a barometer and atmospheric tem-
perature using a thermometer, the air density could be calculated by applying
the equation of state,

\[ \rho = \frac{P}{RT}, \tag{A.1} \]

where \( P \) is the atmospheric pressure in Pa, \( R \) is the air constant, \( R = 287 J/KgK \), \( T \) is the atmospheric temperature in K, and \( \rho \) is the air density in \( Kg/m^3 \).

• By applying Bernoulli’s equation to the Pitot tube,

\[ V_{exit} = \sqrt{\frac{2\Delta P}{\rho}}, \tag{A.2} \]

where \( V_{exit} \) is the exit velocity from the nozzle, we could calculate the velocities
 corresponding to the total pressure values.

• Then, at this stage, we have voltage values recorded at the static pressure tube
and the corresponding velocity values at the nozzle exit. There is no relation
between the static tube pressure and the total pressure measured by the Pitot
tube since we do not know the pressure drop between them. In fact, the only
information needed is to get a relationship between the static pressure value
in voltage and the corresponding velocity at the nozzle exit in \( m/s \).

• By applying a third or fourth order polynomial equation, we got the following
relationship:

\[ P_{static} = f(V_{exit}). \tag{A.3} \]

where \( P_{static} \) is the static tube pressure and \( V_{exit} \) is the nozzle exit velocity.

A.3.2 Calibration of Hot-wire

The second step is using the calibrated static pressure tube to calibrate the
hot-wire itself, since the static pressure tube, by then, represents the desired known
velocity which is in an analogy to the free stream velocity in wind tunnel. The
following steps show the calibration procedure.

• Recharge the compressor up to 120 \textit{psig} and then turn it off.

• Remove the Pitot tube from the center of the nozzle exit and place the hot-wire
instead.

• Open the pressure valve to its full opening.
• Record the voltage from static pressure tube and the hot-wire simultaneously.

• Apply the calibration equation, Equation A.3, to convert the static pressure reading voltages into the corresponding velocities.

• By then, we have the output voltage values and the corresponding velocity values.

• By applying a third or fourth order polynomial equation, we got the desired relationship:

\[ Volt = f(V_{exit}). \]  \hspace{1cm} (A.4)

where \( Volt \) is the voltage.

Figure A.3 shows a typical calibration curve for the hot wire. In general, the calibration equations during the course of this study were either polynomial of third order or fourth order. For the data shown in Figure A.3, the calibration equation was

\[ Velocity = (-1.71574 \ast volt^3 + 9.71543 \ast volt^2 - 13.02784 \ast volt + 4.79207)^2 \]  \hspace{1cm} (A.5)

The average error for this calibration equation is 0.03477 while the standard deviation is 0.045929. To check the calibration equation, we measure the circumferential velocity at different heights above the surface of the disk. Figure A.4 shows the comparison of such measurements with laminar flow solution. For this comparison, we select a radial location in the linear region, i.e., radius=17cm (\( Re = 437 \)). As shown in the figure, the results are in an excellent agreement with theory.
Figure A.3. A typical calibration curve.

Figure A.4. Check calibration equation against the theory.
APPENDIX B

REST OF RESULTS: ENSEMBLE AVERAGED TIME SERIES OF LINEAR TEMPORAL DISTURBANCES

Figure B.1. Ensemble averaged time series at $R=282$ ($r=11.0$ cm) for different angles, $\theta =20^\circ$ and $25^\circ$, pressure=28psi.
Figure B.2. Ensemble averaged time series at $R=334$ ($r=13.0$ cm) for different angles, $\theta = 10^\circ$, $20^\circ$, $25^\circ$, and $40^\circ$, pressure=28psi.
Figure B.3. Ensemble averaged time series at $R=334$ ($r=13.0$ cm) for different angles, $\theta = 70^\circ$, 100$^\circ$, 130$^\circ$, and 160$^\circ$, pressure=28psi.
Figure B.4. Ensemble averaged time series at $R=385$ ($r=15.0$ cm) for different angles, $\theta =10^\circ$, 20$^\circ$, 25$^\circ$, 40$^\circ$, 70$^\circ$, and 100$^\circ$, pressure=28 psi.
Figure B.5. Ensemble averaged time series at $R=385$ ($r=15.0$ cm) for different angles, $\theta =130^\circ$, $160^\circ$, $190^\circ$, $220^\circ$, and $250^\circ$, pressure=28psi.
Figure B.6. Ensemble averaged time series at $R=411$ ($r=16.0$ cm) for different angles, $\theta = 10^\circ$, $20^\circ$, $25^\circ$, $40^\circ$, $70^\circ$, and $100^\circ$, pressure=28 psi.
Figure B.7. Ensemble averaged time series at $R=411 \ (r=16.0 \ \text{cm})$ for different angles, $\theta = 130^\circ$, 160$^\circ$, 190$^\circ$, 220$^\circ$, 250$^\circ$, and 280$^\circ$, pressure=28psi.
Figure B.8. Ensemble averaged time series at $R=437$ ($r=17.0$ cm) for different angles, $\theta =20^\circ$, 25$^\circ$, 40$^\circ$, 70$^\circ$, 100$^\circ$, and 130$^\circ$, pressure=28psi.
Figure B.9. Ensemble averaged time series at $R=437 \ (r=17.0 \text{ cm})$ for different angles, $\theta = 160^\circ, 190^\circ, 220^\circ, 250^\circ, 280^\circ, \text{ and } 310^\circ$, pressure=28 psi.
Figure B.10. Ensemble averaged time series at $R=462$ ($r=18.0$ cm) for different angles, $\theta = 25^\circ$, $40^\circ$, $70^\circ$, $100^\circ$, $130^\circ$, and $160^\circ$, pressure=28psi.
Figure B.11. Ensemble averaged time series at $R=462$ ($r=18.0$ cm) for different angles, $\theta = 190^\circ$, $220^\circ$, $250^\circ$, $280^\circ$, $310^\circ$, and $340^\circ$, pressure=28psi.
Figure B.12. Ensemble averaged time series at $R=488$ ($r=19.0$ cm) for different angles, $\theta = 40^\circ$, $70^\circ$, $100^\circ$, $130^\circ$, $160^\circ$, and $190^\circ$, pressure=28psi.
Figure B.13. Ensemble averaged time series at $R=488$ ($r=19.0$ cm) for different angles, $\theta = 220^\circ, 250^\circ, 280^\circ, 310^\circ, 340^\circ$, and $355^\circ$, pressure=28 psi.
Figure B.14. Ensemble averaged time series at $R=539$ ($r=21.0$ cm) for different angles, $\theta = 10^\circ, 20^\circ, 25^\circ, 70^\circ$, and $100^\circ$, pressure=28psi.
Figure B.15. Ensemble averaged time series at $R=539$ ($r=21.0$ cm) for different angles, $\theta = 130^\circ$, $160^\circ$, $190^\circ$, $220^\circ$, and $250^\circ$, pressure=28 psi.
Figure B.16. Ensemble averaged time series at $R=539$ ($r=21.0$ cm) for different angles, $\theta = 280^\circ$, $310^\circ$, $340^\circ$, and $355^\circ$, pressure=28 psi.
Figure B.17. Ensemble averaged time series at $R=565$ ($r=22.0$ cm) for different angles, $\theta = 10^\circ, 20^\circ, 25^\circ, 130^\circ, 160^\circ$, and $190^\circ$, pressure=28psi.
Figure B.18. Ensemble averaged time series at $R=565$ ($r=22.0$ cm) for different angles, $\theta = 220^\circ$, $250^\circ$, $280^\circ$, $310^\circ$, $340^\circ$, and $355^\circ$, pressure=28psi.
Figure B.19. Ensemble averaged time series at $R=578$ ($r=22.5$ cm) for different angles, $\theta = 10^\circ$, $20^\circ$, $25^\circ$, $160^\circ$, $190^\circ$, and $220^\circ$, pressure=28psi.
Figure B.20. Ensemble averaged time series at $R=578$ ($r=22.5$ cm) for different angles, $\theta = 250^\circ$, 280$^\circ$, 310$^\circ$, 340$^\circ$, and 355$^\circ$, pressure=28psi.
Figure B.21. Wave packets at different wall-normal positions for $R=565$ ($r=22.0$ cm), $\theta = 280^\circ$, pressure=28 psi.
APPENDIX C

REST OF RESULTS: AMPLITUDE SPECTRA OF LINEAR TEMPORAL DISTURBANCES

![Graphs showing amplitude spectra at different angles](image)

Figure C.1. Amplitude spectra of disturbance time series measured at $R=282$ ($r=11.0$ cm) for different angles, $\theta = 20^\circ$ and $25^\circ$, pressure=28psi.
Figure C.2. Amplitude spectra of disturbance time series measured at $R=334$ ($r=13.0$ cm) for different angles, $\theta =10^\circ$, 20°, 25°, and 40°, pressure=28psi.
Figure C.3. Amplitude spectra of disturbance time series measured at $R=334$ ($r=13.0$ cm) for different angles, $\theta = 70^\circ$, $100^\circ$, $130^\circ$, and $160^\circ$, pressure=28psi.
Figure C.4. Amplitude spectra of disturbance time series measured at $R=385$ ($r=15.0$ cm) for different angles, $\theta = 10^\circ, 20^\circ, 25^\circ, 40^\circ, 70^\circ$, and $100^\circ$, pressure=28 psi.
Figure C.5. Amplitude spectra of disturbance time series measured at $R=385$ ($r=15.0$ cm) for different angles, $\theta =130^\circ$, $160^\circ$, $190^\circ$, $220^\circ$, and $250^\circ$, pressure=28psi.
Figure C.6. Amplitude spectra of disturbance time series measured at $R=411$ ($r=16.0$ cm) for different angles, $\theta = 10^\circ$, $20^\circ$, $25^\circ$, $40^\circ$, $70^\circ$, and $100^\circ$, pressure=28psi.
Figure C.7. Amplitude spectra of disturbance time series measured at $R=411$ ($r=16.0$ cm) for different angles, $\theta =130^\circ$, $160^\circ$, $190^\circ$, $220^\circ$, $250^\circ$, and $280^\circ$, pressure=28psi.
Figure C.8. Amplitude spectra of disturbance time series measured at \( R=437 \) (\( r=17.0 \) cm) for different angles, \( \theta =20^\circ, \ 25^\circ, \ 40^\circ, \ 70^\circ, \ 100^\circ, \) and \( 130^\circ \), pressure=28psi.
Figure C.9. Amplitude spectra of disturbance time series measured at $R=437$ ($r=17.0$ cm) for different angles, $\theta = 160^\circ$, $190^\circ$, $220^\circ$, $250^\circ$, $280^\circ$, and $310^\circ$, pressure=28psi.
Figure C.10. Amplitude spectra of disturbance time series measured at $R=462$ ($r=18.0$ cm) for different angles, $\theta = 25^\circ$, $40^\circ$, $70^\circ$, $100^\circ$, $130^\circ$, and $160^\circ$, pressure=28psi.
Figure C.11. Amplitude spectra of disturbance time series measured at $R=462$ ($r=18.0$ cm) for different angles, $\theta = 190^\circ, 220^\circ, 250^\circ, 280^\circ, 310^\circ$, and $340^\circ$, pressure=28psi.
Figure C.12. Amplitude spectra of disturbance time series measured at $R=488$ ($r=19.0$ cm) for different angles, $\theta =40^\circ$, 70$^\circ$, 100$^\circ$, 130$^\circ$, 160$^\circ$, and 190$^\circ$, pressure=28psi.
Figure C.13. Amplitude spectra of disturbance time series measured at $R=488$ ($r=19.0$ cm) for different angles, $\theta = 220^\circ$, $250^\circ$, $280^\circ$, $310^\circ$, $340^\circ$, and $355^\circ$, pressure=28psi.
Figure C.14. Amplitude spectra of disturbance time series measured at $R=539$ ($r=21.0$ cm) for different angles, $\theta =10^\circ$, $20^\circ$, $25^\circ$, $70^\circ$, and $100^\circ$, pressure=28psi.
Figure C.15. Amplitude spectra of disturbance time series measured at $R=539$ ($r=21.0$ cm) for different angles, $\theta = 130^\circ$, $160^\circ$, $190^\circ$, $220^\circ$, and $250^\circ$, pressure=28psi.
Figure C.16. Amplitude spectra of disturbance time series measured at $R=539$ ($r=21.0$ cm) for different angles, $\theta = 280^\circ$, 310$^\circ$, 340$^\circ$, and 355$^\circ$, pressure=28psi.
Figure C.17. Amplitude spectra of disturbance time series measured at $R=565$ ($r=22.0$ cm) for different angles, $\theta=10^\circ$, $20^\circ$, $25^\circ$, $130^\circ$, $160^\circ$, and $190^\circ$, pressure=28psi.
Figure C.18. Amplitude spectra of disturbance time series measured at $R=565$ ($r=22.0$ cm) for different angles, $\theta = 220^\circ$, 250$^\circ$, 280$^\circ$, 310$^\circ$, 340$^\circ$, and 355$^\circ$, pressure=28psi.
Figure C.19. Amplitude spectra of disturbance time series measured at $R=578$ ($r=22.5$ cm) for different angles, $\theta =$10°, 20°, 25°, 160°, 190°, and 220°, pressure=28psi.
Figure C.20. Amplitude spectra of disturbance time series measured at $R=578$ ($r=22.5$ cm) for different angles, $\theta = 250^\circ$, 280°, 310°, 340°, and 355°, pressure=28psi.
APPENDIX D

REST OF RESULTS: ENSEMBLE AVERAGED TIME SERIES OF FINITE-AMPLITUDE DISTURBANCES
Figure D.1. Ensemble averaged time series at $R=334 \ (r=13.0 \ cm)$ for different angles, $\theta = 10^\circ, 40^\circ, 70^\circ, \text{ and } 100^\circ$, pressure=80psi.
Figure D.2. Ensemble averaged time series at $R=385$ ($r=15.0$ cm) for different angles, $\theta = 10^\circ$, 40°, 70°, 100°, 130°, and 160°, pressure=80psi.
Figure D.3. Ensemble averaged time series at $R=385$ ($r=15.0$ cm) for different angles, $\theta = 190^\circ$, $220^\circ$, $250^\circ$, $280^\circ$, and $310^\circ$, pressure=80 psi.
Figure D.4. Ensemble averaged time series at $R=411$ ($r=16.0$ cm) for different angles, $\theta =10^\circ$, $40^\circ$, $70^\circ$, $100^\circ$, $130^\circ$, and $160^\circ$, pressure=80psi.
Figure D.5. Ensemble averaged time series at $R=411$ ($r=16.0$ cm) for different angles, $\theta = 190^\circ$, 220$^\circ$, 250$^\circ$, 280$^\circ$, and 310$^\circ$, pressure=80psi.
Figure D.6. Ensemble averaged time series at $R=437$ ($r=17.0$ cm) for different angles, $\theta =10^\circ$, 40$^\circ$, 70$^\circ$, 100$^\circ$, 130$^\circ$, and 160$^\circ$, pressure=80psi.
Figure D.7. Ensemble averaged time series at $R=437$ ($r=17.0$ cm) for different angles, $\theta = 220^\circ$, $250^\circ$, $280^\circ$, $310^\circ$, $340^\circ$, and $350^\circ$, pressure=80 psi.
Figure D.8. Ensemble averaged time series at $R=462 \ (r=18.0 \text{ cm})$ for different angles, $\theta =10^\circ$, $40^\circ$, $70^\circ$, $100^\circ$, $130^\circ$, and $160^\circ$, pressure=$80$psi.
Figure D.9. Ensemble averaged time series at $R=462$ ($r=18.0$ cm) for different angles, $\theta = 220^\circ$, $250^\circ$, $280^\circ$, $310^\circ$, $340^\circ$, and $350^\circ$, pressure=80psi.
Figure D.10. Ensemble averaged time series at $R=488$ ($r=19.0$ cm) for different angles, $\theta =10^\circ$, $40^\circ$, $70^\circ$, $100^\circ$, $130^\circ$, and $160^\circ$, pressure=80psi.
Figure D.11. Ensemble averaged time series at $R=488$ ($r=19.0$ cm) for different angles, $\theta =220^\circ$, $250^\circ$, $280^\circ$, $310^\circ$, $340^\circ$, and $350^\circ$, pressure=80 psi.
Figure D.12. Ensemble averaged time series at $R=539$ ($r=21.0 \text{ cm}$) for different angles, $\theta = 10^\circ$, 70$^\circ$, 100$^\circ$, 130$^\circ$, 160$^\circ$, and 190$^\circ$, pressure=80psi.
Figure D.13. Ensemble averaged time series at $R=539$ ($r=21.0$ cm) for different angles, $\theta = 220^\circ$, 250\textdegree, 280\textdegree, 310\textdegree, 340\textdegree, and 350\textdegree, pressure=80psi.
Figure D.14. Ensemble averaged time series at $R=565$ ($r=22.0$ cm) for different angles, $\theta = 10^\circ$, $130^\circ$, $160^\circ$, $190^\circ$, and $220^\circ$, pressure=80psi.
Figure D.15. Ensemble averaged time series at $R=565$ ($r=22.0$ cm) for different angles, $\theta = 250^\circ, 280^\circ, 310^\circ, 340^\circ, \text{ and } 350^\circ$, pressure=80 psi.
Figure D.16. Ensemble averaged time series at $R=578$ ($r=22.5$ cm) for different angles, $\theta =10^\circ$, 220$^\circ$, 250$^\circ$, 280$^\circ$, 310$^\circ$, and 340$^\circ$, pressure=80psi.
Figure D.17. Ensemble averaged time series at $R=578$ ($r=22.5$ cm) for different angles, $\theta = 220^\circ$, 250$^\circ$, 280$^\circ$, 310$^\circ$, 340$^\circ$, and 350$^\circ$, pressure=80psi.
APPENDIX E

REST OF RESULTS: AMPLITUDE SPECTRA OF FINITE-AMPLITUDE DISTURBANCES
Figure E.1. Amplitude spectra of disturbance time series measured of disturbance time series measured at $R=334$ ($r=13.0$ cm) for different angles, $\theta =10^\circ$, $40^\circ$, $70^\circ$, and $100^\circ$, pressure=80psi.
Figure E.2. Amplitude spectra of disturbance time series measured at $R=385$ ($r=15.0$ cm) for different angles, $\theta =10^\circ$, $40^\circ$, $70^\circ$, $100^\circ$, $130^\circ$, and $160^\circ$, pressure=80psi.
Figure E.3. Amplitude spectra of disturbance time series measured at $R=385$ ($r=15.0$ cm) for different angles, $\theta = 190^\circ, 220^\circ, 250^\circ, 280^\circ,$ and $310^\circ$, pressure=80psi.
Figure E.4. Amplitude spectra of disturbance time series measured at $R=411$ ($r=16.0 \text{ cm}$) for different angles, $\theta =10^\circ$, $40^\circ$, $70^\circ$, $100^\circ$, $130^\circ$, and $160^\circ$, pressure=$80\text{ psi}$. 
Figure E.5. Amplitude spectra of disturbance time series measured at $R=411$ ($r=16.0$ cm) for different angles, $\theta = 190^\circ$, 220$^\circ$, 250$^\circ$, 280$^\circ$, and 310$^\circ$, pressure=80psi.
Figure E.6. Amplitude spectra of disturbance time series measured at \( R=437 \) (\( r=17.0 \text{ cm} \)) for different angles, \( \theta = 10^\circ, 40^\circ, 70^\circ, 100^\circ, 130^\circ, \) and \( 160^\circ \), pressure=80psi.
Figure E.7. Amplitude spectra of disturbance time series measured at $R=437$ ($r=17.0$ cm) for different angles, $\theta =220^\circ$, 250$^\circ$, 280$^\circ$, 310$^\circ$, 340$^\circ$, and 350$^\circ$, pressure=80psi.
Figure E.8. Amplitude spectra of disturbance time series measured at \( R=462 \) (\( r=18.0 \) cm) for different angles, \( \theta =10^\circ, 40^\circ, 70^\circ, 100^\circ, 130^\circ, \) and \( 160^\circ \), pressure=80psi.
Figure E.9. Amplitude spectra of disturbance time series measured at $R=462$ ($r=18.0 \text{ cm}$) for different angles, $\theta =220^\circ$, $250^\circ$, $280^\circ$, $310^\circ$, $340^\circ$, and $350^\circ$, pressure=80psi.
Figure E.10. Amplitude spectra of disturbance time series measured at $R=488$ ($r=19.0$ cm) for different angles, $\theta = 10^\circ$, 40$^\circ$, 70$^\circ$, 100$^\circ$, 130$^\circ$, and 160$^\circ$, pressure=80psi.
Figure E.11. Amplitude spectra of disturbance time series measured at $R=488$ ($r=19.0$ cm) for different angles, $\theta = 220^\circ$, $250^\circ$, $280^\circ$, $310^\circ$, $340^\circ$, and $350^\circ$, pressure=80psi.
Figure E.12. Amplitude spectra of disturbance time series measured at $R=539$ ($r=21.0$ cm) for different angles, $\theta = 10^\circ$, $70^\circ$, $100^\circ$, $130^\circ$, $160^\circ$, and $190^\circ$, pressure=80psi.
Figure E.13. Amplitude spectra of disturbance time series measured at $R=539$ (r=21.0 cm) for different angles, $\theta = 220^\circ$, 250°, 280°, 310°, 340°, and 350°, pressure=80 psi.
Figure E.14. Amplitude spectra of disturbance time series measured at $R=565$ ($r=22.0$ cm) for different angles, $\theta =10^\circ$, $130^\circ$, $160^\circ$, $190^\circ$, and $220^\circ$, pressure=80psi.
Figure E.15. Amplitude spectra of disturbance time series measured at $R=565$ $(r=22.0$ cm) for different angles, $\theta = 250^\circ$, $280^\circ$, $310^\circ$, $340^\circ$, and $350^\circ$, pressure=80psi.
Figure E.16. Amplitude spectra of disturbance time series measured at $R=578$ (r=22.5 cm) for different angles, $\theta = 10^\circ$, 220$^\circ$, 250$^\circ$, 280$^\circ$, 310$^\circ$, and 340$^\circ$, pressure=80psi.
Figure E.17. Amplitude spectra of disturbance time series measured at $R=578$ (r=22.5 cm) for different angles, $\theta = 220^\circ$, 250$^\circ$, 280$^\circ$, 310$^\circ$, 340$^\circ$, and 350$^\circ$, pressure=80psi.
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