EXPERIMENTAL INVESTIGATION OF THE CAVITATION OF AVIATION FUEL IN A CONVERGING-DIVERGING NOZZLE

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Abstract

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A cavitating nozzle flow was studied experimentally with high-speed video, pressure, and void fraction measurements. Results were obtained with both aviation fuel (JP-8) and water. Three different flow regimes were examined: a single-phase liquid flow where no cavitation occurred, fully developed cavitation originating from a pure liquid flow at the nozzle inlet, and a two-phase nozzle flow where gas bubbles were injected in to the nozzle inlet. The majority of the study focused on the fully developed cavitation, where below a certain nozzle back pressure, both fluids obtained a limiting mass flow rate through the nozzle test section as the flow became choked. High-speed video showed an abrupt region of bubble collapse (a bubbly shock) in the diffuser section of the nozzle for water cases. In fuel cases, bubbles persisted through the entire length of the nozzle with no obvious collapse region. The differences in the cavitation for fuel and water were attributed to the multi-component composition of the aviation fuel, resulting in a spatially distributed bubbly shock that was not visually obvious. Bubble injection experiments revealed a strong sensitivity of both the bubble shock location and thickness to the void fraction in the nozzle inlet for both fuel and water.
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CHAPTER 1

INTRODUCTION

The multi-phase, cavitating flow through a converging diverging nozzle is a complex problem with important impacts on internal flow engineering applications. Constricted flow geometries are ubiquitous in common devices such as pumps, valves, flow controllers, pipe diameter reductions/increases, heat exchangers, etc. Cavitation remains a persistent problem for internal flow devices such as these due to both performance reductions and the ability of cavitation bubbles to cause significant component surface damage during bubble collapse. Further, the introduction of a gas phase results in a compressible working fluid. Consequently, choked conditions can exist at locations of area constriction. Frequently these devices will fail to reach their design mass flow rates due to cavitation bubbles causing choked conditions.

In modern aircraft fuel systems, the aviation fuel is often used as a hydraulic fluid as well as a coolant for avionics. High-temperature, high pressure fuel is cycled throughout the airframe by various pumps, valves, and flow controllers, where cavitation can occur. Cavitation of fuel presents additional complexities (as compared to water) because fuel is a multi-component mixture. With over 250 different hydrocarbons present in the fuel, each with a unique vapor pressure, individual components may produce bubbles and collapse at different pressures and locations in these devices. Furthermore, bubble nucleation in the fuel is enhanced by the large quantities of contaminant microparticles that are present in JP-8.
For this investigation, a converging-diverging nozzle was chosen to study the basic aspects of fuel cavitation and to compare the results to more familiar behavior of cavitation in water. The nozzle geometry is simple with sufficient access for diagnostics. Furthermore, several models exist in the literature for two-phase nozzle flow \[7, 10, 26\]. However, little, if any, experimental data exists for comparison and validation of these models. The few experimental studies that exist have focused on bubbly nozzle flows (flows with a significant initial void fraction) rather than cavitating flows \[19, 17\]. All studies (both experimental and computational) have involved water, leaving the problem of fuel cavitation for this geometry unexamined. This study intends to provide a comprehensive experimental set of data for the converging-nozzle geometry, for both fuel and water cavitation.

The data that will be acquired include axial pressure profiles, nozzle flow rate, high-speed digital images of the cavitating region, and axial void fraction profiles. Pressure and flow rate data will be used to determine the pressure ratios and limiting mass flow rates when the nozzle becomes choked. The high speed digital imaging provides visualization of the flow as well as forming the basis for void fraction and velocity measurements.

1.1 Background

The following section will provide the motivation for this proposal, some background information on the type of fuel that will be studied, and a review of the relevant literature. The literature review will concentrate on the effects of microparticles on nucleation and the state of the art in modeling bubbly flows in converging-diverging nozzles.
1.1.1 Motivation

In nearly all cases bubble formation in the fluid is undesirable. As fluid is routed through reduced areas, if the pressure decrease is sufficient, bubbles may form. When the pressure recovers the bubbles collapse, and often violently. The pressure generated during collapse can be very high and cause erosion and pitting damage to the devices. Figure 1.1 is an example of this type of pitting damage. The picture shown is of a port plate for a piston fuel pump manufactured by the Honeywell Corporation. The surface damage seen around the orifice is the result of collapsing bubbles.

Figure 1.1. Pump port plate showing cavitation damage (courtesy Honeywell Corp.)

Another deleterious effect of cavitation in internal devices is a limiting mass flow rate. Once gas and/or vapor bubbles are formed, the resulting two phase mixture behaves as a compressible fluid. If the pressure ratio through any restrictions is high enough, the flow can become choked (in an analogous manner to gas flows) and bubbly shocks may form. For the design engineer not anticipating cavitation, the
achievable mass flow rate through a given device can be substantially less than what was expected. Tools and models that can accurately predict cavitation inception, the onset of choking, and what the limiting mass flow rate is under choked conditions are all needed for accurate performance estimates to be made.

1.1.2 Fluid Characterization

The specific fuel that will be used in this investigation is a kerosene based aviation fuel designated JP-8. It is used as the primary fuel for the USAF. Commercially available Jet-A (used worldwide as the primary fuel for commercial aviation) is identical to JP-8 with the exception of additives for anti-icing, corrosion resistance, and static dissipation [11]. Kerosene is composed of a complex mixture of hydrocarbons distilled from crude oil including saturated hydrocarbons, aromatics, paraffins, and olefins. The ratios of specific hydrocarbons can vary from sample to sample, and as a result property information is typically expressed in a range of values. Because each subcomponent of JP-8 has its own unique vapor pressure cavitation of JP-8 is more difficult to predict and characterize than water.

1.1.3 Nucleation and Particle Effects

Much like in nucleate boiling, bubbles need a nucleation site to catalyze the bubble growth process. The nucleation of bubbles into gaseous or vaporous cavities can be classified in to four different types of events [12]. The first two classifications, types I and II, are results of classical homogeneous and heterogeneous nucleation theory, where bubbles are formed either in the bulk or on solid surfaces where no initial amount of vapor of gas is present to initiate bubble growth. Both of these scenarios require extremely large decreases in pressure to create gas cavities. These theoretical predictions for the tensile strength of water are never achieved in practical situations and require an ultra-pure liquid sample. The third and fourth types of
nucleation events, types III and IV, presuppose some initial quantity of gas or vapor present to initiate the growth process. There is ample evidence in the literature that these catalysts are responsible for cavitation at pressures seen in many flow devices. A common example of type IV nucleation in a gaseous cavitation regime would be the streaming of bubbles from a beer or champagne glass [20]. In such cases cellulose fibers stuck on the wall act as nucleation sites, and bubble trains can be seen emanating from discrete locations on the glass [14].

In most practical engineering environments, cavitation events occur at pressures at or above the vapor pressure of the liquid, and would be classified as type III or type IV nucleation events. This phenomena has long been attributed to the presence of impurities in the liquid acting as nucleation sites. The size distribution and amount of solid impurities in a liquid sample, as well as its handling history and amount of dissolved gas content, have been used to explain the difficulty in predicting cavitation inception [2]. Potential nucleation sites include all solid surfaces (both microparticles in the liquid and container walls) as well as latent gas bubbles. Gas bubbles have been shown to quickly dissolve in to solution [8], so that with the exception of freshly drawn liquid samples, the majority of potential nucleation sites are gas pockets stabilized on solid surfaces [1]. The filtering of water to remove all solid particles greater than 0.2 µm increases the tensile strength of water and delays cavitation inception in acoustic cavitation experiments [9]. This supports the theory that trapped gas cavities play a role in cavitation inception. It further suggests that the size range of the impurities present is important as well. It has been shown that by subjecting a liquid sample to an ultrasonic pulse to break up and reduce the size of potential nuclei, the severity of cavitation can be reduced [6].

In addition to microparticle size, the surface geometry of solid surfaces acting as nucleation sites has been shown to be of importance. Irregular, concave surface
structures such as cracks and pits are necessary to stabilize the gas pockets thought responsible for cavity growth [22], [16]. Seeding filtered water with microparticles that are nearly spherical in shape had little effect on cavitation inception in vortex induced cavitation experiments [15]. Indeed, it can be shown that it is the characteristic size of the concave surface structures that is the most important parameter in determining whether trapped gas cavities will act as sources of bubble nucleation.

1.1.4 Previous Experimental Work

There are very few experimental investigations of cavitation for a converging-diverging nozzle geometry. A study of choked-foam flows [19] measured inlet, throat, and exit pressures for a water/air foam mixture flowing through a nozzle with an area contraction ratio of 2.75. Air and a foaming agent (to prevent bubble coalescence) were injected into the nozzle inlet over an initial void fraction range of $\alpha_o = 5\%$ to $\alpha_o = 61\%$. Departures from the homogeneous theory were found to be largest for lower initial void fractions (values that, while still an order of magnitude higher, are closest to those found in the present study).

Another experiment [23] examined air-water bubbly flows with an initial void fraction range of $\alpha_o = 20\%$ to $\alpha_o = 60\%$ with nozzle contraction ratios from 3.16 to 7.11. As with the previous study, inlet, throat, and exit pressure measurements along with gas and liquid flow rates comprised the data set. Results were compared with a homogeneous mixture model that accounted for slip between the liquid and gas phases. The developed theory agreed well with the data in the contracting section of the nozzle, but faired poorly in the divergent section.

The last experimental investigation for a nozzle geometry found in the literature was for a nozzle with an area contraction ratio of 2.7 seeded with dilute nitrogen bubbles and the pressure and void fraction along the nozzle axis were measured [10].
Void fractions for such a gentle contraction (and small flow acceleration) were small (∼ 5 %) throughout the entire nozzle and were used for comparison with solutions from a quasi-1D flow model that included bubble dynamic effects.

None of the flows in the aforementioned studies is a cavitating flow (in the sense that bubble nucleation is occurring - the bubble source was supplied artificially upstream). All of the experiments were conducted with water, and the spatial resolution of the measurements was not sufficient to provide a complete axial profile of pressure or void fraction along the nozzle.

1.1.5 Previous Modeling Efforts

Initial attempts at modeling a two-phase flow through a nozzle assumed that the fluid pressure is only a function of fluid density. This barotropic model [3] assumes that the only effect of the bubbles is to add compressibility to the liquid, and the two-phase system is modeled as a single phase. The barotropic relation allows the calculation of a sound speed of the mixture (which will be a function of the gas void fraction). Inputs to the model are the single-phase fluid properties and the void fraction, and outputs include the mixture sonic speed and the critical throat-to-inlet pressure ratio at which the nozzle becomes choked.

In practical nozzle flows such as are seen in fuel pumps and valves, the bubble dynamics (which are excluded from the barotropic model) are important. The pressure decrease and increase caused by the flow acceleration result in bubbles growing and collapsing, giving a void fraction profile along the axis of the nozzle. A steady model [26] was developed that couples a one-dimensional flow with area change with the Rayleigh-Plesset equation (which governs the dynamics of a single spherical bubble). Only for very small initial void fractions (or large cavitation numbers) could physical solutions be obtained; above a certain initial bubble population (or flow
tension) the solutions become statically unstable with the flow “flashing” entirely to
vapor and fluid velocities increasing without bound. An unsteady model [18] was able to show
that the unstable “flashing” solutions of the steady model are captured as unsteady bubblly shock waves propagating in the nozzle.

The previous models mentioned do not take into account bubble nucleation. They all require an initial bubble size distribution; by adding an equation for the bubble number density flux with a source term based on homogeneous nucleation theory, the range of inlet conditions giving stable solutions for the steady model can be extended [7]. While unstable solutions are still possible, the model with nucleation is capable of predicting a stable flow solution with a stationary bubbly shock.

The commercially available CFD packaged FLUENT has incorporated a cavitation model which uses the Rayleigh-Plesset equation to capture the bubble dynamics [21]. Liquid-phase velocity and pressure information are used to calculate the gas phase convection and vapor void fraction (from the bubble radius). Several empirical constants are used in the coupling between the two phases, and no nucleation model is included.

1.2 Summary

To summarize, presently there are no published experimental data for cavitating nozzle flows with sufficient spatial resolution for meaningful comparisons with the current models. The experimental data for such flows are scarce, limited to cases where bubble nucleation is not a primary source of the two-phase mixture, and only exist with water as the liquid studied. The purpose of this research is to provide pressure, void-fraction, and bubble velocity measurements in both fuel and water for comparison with existing models. In addition, the experimental data will give
insight in to what are the governing physics for the different cavitating regimes and highlight which models are appropriate for each regime.
CHAPTER 2

DIAGNOSTICS

In this chapter a complete description of the experimental apparatus and the diagnostic tools used to acquire data is described. The nozzle test section was characterized experimentally by making measurements of pressure, flow rate, void fraction, and velocity, where possible.

2.1 Experimental Setup

Cavitation experiments typically require large flow rates and small geometries in order to achieve pressures low enough to observe cavitation. With JP-8 as the working fluid, large flow rates are undesirable due to the difficulty in handling large quantities of fuel. Concomitant with lower flow rates, small geometries are necessary to observe cavitation, which can be difficult to instrument. As a compromise, a reduced-pressure facility was designed so that reasonable amounts of fuel (∼1 gpm) could be used in a test section that was large enough to employ diagnostics. By biasing the reference pressure towards vacuum in a blow-down setup, these goals were achieved.

A photograph and schematic of the facility is shown in Figure 2.1. Two stainless steel, 38 L tanks are used as liquid reservoirs. First the upstream tank is filled with the fluid. Next, the downstream tank is isolated from the system with a valve and is pumped down by a vacuum pump to the desired pressure. The pressure difference
between the upstream tank and the downstream tank drives the flow. The flow is initiated when the valves between the two tanks are opened and the upstream tank empties through the test section into the downstream tank.

Figure 2.1. Schematic and image of blow-down facility

The test section is a 1.6 mm depth channel with a rectangular cross section machined out of Plexiglas in the shape of a converging-diverging nozzle. A cover plate along with a rubber O-ring seal the flow. The nozzle contour was designed using a fifth-order polynomial fit to ensure zero-slope conditions at the inlet, exit, and throat transitions. The diffuser length was chosen so that the maximum slope
of each wall is 7° to reduce flow separation. The overall length of the nozzle, \( L \), is 127 mm, the inlet and exit heights, \( h \), are both 19 mm, and the throat depth in the \( z \)-direction, \( d \) is 1.58 mm. This gives the nozzle an area ratio of 12:1. An image and schematic of the nozzle geometry is shown in Figure 2.2.

![Nozzle Geometry Diagram](image_url)

**Figure 2.2. C/D Nozzle Test Section Geometry**

2.2 Pressure Measurements

The axial pressure distribution in the nozzle is measured using Setra model 209 pressure transducers. Static pressure taps were drilled normal to the test section coverplate along the centerline of the nozzle and were connected to the transducers using plastic tubing. The taps were spaced at intervals of 3.2 mm over the 12.7 cm length of the nozzle, resulting in 40 measurement locations. The pressures were acquired via a PC running LabView at a sampling frequency of 20 kHz. With
upstream and downstream reservoirs of 38 L, the tank back pressure and all pressure measurements can be considered constant over the time length of data acquisition (typically on the order of 30 s). Pressure data at several different back pressures (both cavitating and pure liquid flow) were acquired.

Pressure data for a typical run for water are presented in Figure 2.3. Shown are the inlet, throat, and exit pressures as a function of time. The valve between the test section and downstream tank (pumped down to ∼10 kPa) is opened at \( t = 5 \) s. The exit pressure (dotted line) slowly rises as the downstream tank fills. Inlet pressure (solid line) stays relatively constant (there is a slight decrease in pressure due to the head associated with the filled upstream tank). The pressure in the throat also remains steady for the duration of the run. Typical run times are on the order of 120 s, which is seen from the constant throat to inlet pressure ratio. This also indicates that there is a choked flow condition at the throat; indeed, the flow meter indicates a constant flow rate of 1 gpm until \( t = 125 \) s, even though the pressure difference between inlet and exit is steadily decreasing during the run.

2.3 High-Speed Photography

In addition to pressure data, high-speed digital photography is used to characterize the structure of the flow and provide flow visualization. A FASTCAM-ultima APX model high-speed video acquisition system (manufactured by PHOTRON Limited) is used to capture the high speed images. The system is capable of frame rates from 2000 frames per second (fps) at full 1024x1024 resolution to as fast as 120,000 fps at reduced resolution (128x16). The shutter speed can be set independently of frame rate to any value from the inverse of the frame rate to as short as 4 \( \mu s \). Typical camera settings that were used while photographing the cavitating flow were a resolution of 1024x256, frame rate of 8,000 fps (the maximum possible frame rate for
the 1024x256 resolution setting), and a shutter speed of 8 µs. At these settings, the maximum length of a video record that can be acquired is slightly over 1 s. Sample images for the cavitating nozzle flow with both water and JP-8 as the working fluid can be seen in Figure 2.4, taken with the high speed video camera at a shutter exposure time of 6.7 µs.

In addition to providing visualization of the flow, the high-speed imaging was used to make quantitative experimental measurements of both velocity and void fraction.

2.4 Void Fraction Measurements

A thorough characterization of the multiphase flow in the nozzle requires measured profiles of the local void fraction as a function of axial distance along the centerline of the nozzle (referred to as the $x$-direction).

The nature of this particular flow makes it difficult to indentify a void fraction
measurement technique that is accurate in all of the multi-phase flow regimes present in the nozzle. Consider the high-speed image of the nozzle cavitating with water in Figure 2.4. The image shows the evolution of the cavitating flow in the following sequence: pure liquid upstream of the throat, cavitation initiation, bubble growth in to slug-like gas structures, near pure gas phase just upstream of a bubbly shock, violent bubble collapse followed by a region of dilute spherical bubbles. Therefore, an appropriate void fraction diagnostic technique must be able to accurately measure void fraction from a vapor volume fraction of $\alpha = 0$ (pure liquid) all the way to a vapor volume fraction of $\alpha = 1$ (pure gas). In addition, the flow is moving at speeds of up to 20 m/s, with bubbles as small as 100 $\mu$m, requiring a frequency response of up to 200 kHz.

A key tenet of all of the void fraction measurement techniques that will be examined for this study is the concept of a two-state local phase density, $X(x,t)$ [4]. This method will be employed for impedance based, heat transfer based, and
optically based sensors. At any location, \( x \), at any instant in time, \( t \), the local phase density will either be pure gas \( X(x, t) = 1 \), or pure liquid \( X(x, t) = 0 \). Measurements of \( X(x, t) \) are made by a point sensor, whose size must be sufficiently small such that it is smaller than the smallest gas bubble in the flow.

To convert the local phase density \( X(x, t) \) to a local void fraction measurement, \( \alpha(x) \) that is independent of time, a running average is needed. That is, assuming that the time over which the phase density is averaged is of sufficient length, the vapor volume fraction \( \alpha(x) \) will be the percentage of time, over the sample record, that the local phase density \( X(x, t) = 1 \).

Any snapshot of the flow, such as Figure 2.4, will show the flow to be unsteady; individual bubbles are created, grow rapidly, and collapse as they move through the nozzle. Taken over a long enough time scale, however, the flow achieves a quasi-steadiness. Consider Figure 2.5, which is an image taken of the cavitating nozzle flow where the shutter length has been increased to 33 ms, which is approximately the time scale over which the naked eye can process.

Figure 2.5. Image of cavitating nozzle flow with increased shutter exposure time. The flow is from left to right.

Individual bubbles disappear, and the cavitation zone appears as a milky-white cloud. The bubbly shock has a well defined boundary, and it is difficult to detect any remaining bubbles in the downstream portion of the nozzle. In this image, at
the longer time scales, the void fraction along the centerline of the nozzle moving from left to right is steady. The only difference between the images in Figure 2.4 and Figure 2.5 is the length of the shutter exposure; all experimental conditions (flow rate, pressure difference across the nozzle inlet and exit, and so forth) are identical.

Several techniques to measure the void fraction in the nozzle were attempted. Probe based measurements, including both hot-film anemometry and a resistivity based needle probe, were discarded due to their invasive nature and inability to distinguish between the gas and liquid phases with sufficient sensitivity. Both probes were unable to provide a reliable phase-density signal for bubbles less than one centimeter in diameter. More detail on their implementation for this flow is provided in the appendix.

2.5 Optical Techniques

Once it was determined that probe-based measurements were insufficient, optical techniques were explored. The main advantage of an optical technique is its non-intrusiveness. No probe or device need be placed in the nozzle test section. Two different optical techniques were assessed for this study: the scattered light from a laser source received by a lateral effect detector, and an image processing technique utilizing high speed imaging.

2.5.1 Void Fraction from Scattered Laser Light Source

A schematic of the technique is displayed in Figure 2.6. A HeNe laser was passed through the test section to an optical lateral effect detector (OT-301 DL Dual Axis Lateral Effect Detector, Photonics, Inc). The signal strength for a pure liquid flow was recorded. As gas filled cavitation bubbles passed through the beam, the laser light was scattered away from the photodetector’s sensing region, resulting in a loss of signal. By setting a threshold on the output voltage, the two-state phase density
is obtained for the flow. As described above, if sufficient length time records are acquired, the time-averaged local void fraction will converge to a steady state value.

Figure 2.6. Schematic of void fraction measurements with laser and photodiode detector

One of the main disadvantages of this technique is that the scattered light signal received by the detector will be a ‘line-of-sight’ signal. As can be seen from Figure 2.6, the test section has a finite depth, and the laser beam will be scattered by any gas void in the beam path. The void fraction measured as a result will be an upper bound on the true value, as it cannot be assumed that all gas voids are of sufficient size to fully fill the test section in the direction of the beam path. That is, it is possible to have multiple light scatterers (bubbles) in the beam direction, which will detected the same as one single large bubble encompassing the entire channel depth.

A sample signal output from the photodetector is shown in Figure 2.7. This signal was acquired with fuel as the sample liquid at an axial location of $x/L = 0.4$. A voltage threshold of 2.0 V was chosen as the dividing line between indications of liquid and gas. That is, voltage values below the threshold indicate a gas phase in the measurement volume, while values above the threshold indicate liquid. The single-
threshold level signal processing technique has been demonstrated as an effective means of calculating void fraction for probes emitting a two-state signal based on the phase surrounding the probe sensing element[4]. By performing a sensitivity study to the threshold level, it was found that the calculated void fraction value was insensitive to threshold levels in the range of 1.9 V to 2.3 V. It should be noted that the resulting data is only proportional to the actual gas void fraction in the flow; calibration is required to relate the laser-scattering data to the actual magnitude of the void fraction.

![Photodiode Output, x/L = 0.4](image)

Figure 2.7. Raw voltage output from lateral effect detector at x/L = 0.4 (JP-8).

2.5.2 Void Fraction from High Speed Video

Void fraction estimates were made using high speed digital imaging by exploiting the difference in pixel intensities between the two phases. By lighting the apparatus from behind with a diffuse light source (such that the camera received forward scattered light), the contrast between the gas and liquid phases was increased. Like
the laser light scattering technique, this was also a ‘line-of-sight’ measurement. The three dimensional test section is projected on to a two dimensional image, and the resulting void fraction measurement will reflect that. A sample image, taken at a frame rate of 8,000 frames per second with a shutter length of 6.7 $\mu$s, is presented in Figure 2.8. As a reminder of the coordinate system being used, Figure 2.1 is repeated as well.

![Figure 2.8. Backlit high speed image of cavitating nozzle. The flow is from left to right. Bubbles appear as dark structures, while the liquid phase is represented by the bright regions.](image)

The void fraction at a particular $x$ location is calculated from its time averaged pixel intensity. A square pixel interrogation region, chosen such that the number
of pixels inside the square is smaller than the smallest bubbles, is centered at the desired $x$ location. For any given instant in time (or video frame), a bubble may or may not be present in the interrogation region. However, by averaging over a large enough number of frames, the cavitating flow becomes quasi-steady.

Let $\bar{I}_{x,y,t}$ be the average intensity of all the pixels within the square interrogation box at a particular location $(x, y)$ location, and time, $t$. A threshold filter is then applied so that the local phase density $X_{x,y,t}$ for that location and video frame is determined by

$$X_{x,y,t} = \begin{cases} 
1 & (\bar{I}_{x,y,t} > P) \\
0 & (\bar{I}_{x,y,t} < P)
\end{cases},$$

(2.1)

where $P$ is a threshold pixel intensity used to distinguish between the two phases. Thus, for a given frame at a given location, the void fraction is considered to either be pure gas ($X_{x,y,t} = 1$), or pure liquid ($X_{x,y,t} = 0$).

A two-frame image sequence demonstrating this calculation is presented in Figure 2.9. The two figures are a magnified region of the downstream portion of the nozzle, taken from the square shown in the image from Figure 2.8. The interrogation region is shown as a 3x3 pixel box in each figure. The average pixel intensity, $\bar{I}_{x,y,t}$, would be calculated as the average of the 9 pixels in the box for each frame. For the frame on the left hand side, a dark bubble is clearly within the interrogation box. For this frame, the $\bar{I}_{x,y,t}$ is greater than the intensity threshold, $P$, and the phase density for this frame at this location would be recorded as $X_{x,y,t} = 1$, or pure gas. For the figure on the right (which is the next frame in the video image sequence), the bubble has convected downstream, such that the void fraction is calculated as $X_{x,y,t} = 0$, or pure liquid.

The average phase density at a given location in the $x$-$y$ plane, $\bar{X}_{x,y}$, is deter-
mined from

\[ \tilde{X}_{x,y} = \frac{1}{T} \int_{t=0}^{T} X_{x,y,t} dt, \quad (2.2) \]

where \( T \) is the total time record length in the video record. A large enough number of frames must be processed such that the flow is quasi-steady and the average phase density converges. Figure 2.10 is a plot of the calculated average phase density for the nozzle location shown in Figure 2.9 as a function of the total number of frames, \( N \), used in the calculation. From the figure it can be seen that the void fraction for this location converges after approximately 400 frames, which is the equivalent of 27 ms.

To convert the local phase density at a point in the \( x-y \) plane in to a volume based void fraction measurement, the following is done. At a given \( x \) location, the local phase density \( X_{x,y} \) is computed. This phase density is then integrated in the
Figure 2.10. Average phase density as a function of total frames $N$ for the axial location shown in the images of the previous figure.

$y$-direction and divided by the local nozzle height to provide the $(x, y)$ projection of the local void fraction:

$$
\bar{X}_x = \frac{1}{h} \int_{y=0}^{h} \bar{X}_{x,y} dy.
$$

(2.3)

Since no video information is available in the $y-z$ plane, some estimation of the phase density distribution in the $z$ direction must be made. The average bubble size in the inlet/throat area of the nozzle is on the order of 60% of the depth of the nozzle ($d = 1/16$ in.) in the $z$-direction. It is therefore assumed that no more than a single gas cavity exists in any slice taken over the $z$-dimension. Finally, the $z$ variation in phase density is simply assumed to be the projection of a sphere, centered in the nozzle, with radius equal to that of the mean bubble radius (determined to be $\bar{r} = 1$ mm). This gives the final value for the void fraction, $\alpha$, at any $x$ location:
\[ \alpha(x) = \frac{1}{hd} \int_{z=0}^{d} \int_{y=0}^{h} \tilde{X}_{x,y} X_{z} dy dz, \]  

(2.4)

where \( d \) is the nozzle depth \( (d = 1/16 \text{ in.}) \) and \( X_{z} \) is the assumed \( z \)-direction variation in phase density.

2.5.3 Calibration of Void Fraction Measurements

In order to calibrate the high-speed video void fraction method, a flow with a known initial void fraction was necessary. This was accomplished by injecting gas bubbles directly into the inlet of the nozzle using hypodermic needles. The gas flow rate supplied to the needles prior to injection was measured with a bubble flowmeter. The initial void fraction could then be calculated directly with knowledge of the liquid flow rate from

\[ \alpha_{o} = \frac{Q_{\text{gas}}}{Q_{\text{liq}} + Q_{\text{gas}}}, \]  

(2.5)

where \( Q_{\text{gas}} \) and \( Q_{\text{liq}} \) are the measured gas and liquid flow rates, respectively. Using this bubble injection technique, flows with an initial void fraction range of \( \alpha_{o} = 0.5 \% \) to \( \alpha_{o} = 2.5 \% \) were possible.

An example of how the void fraction was calculated for the bubble injection is outlined in Figure 2.11. The top picture in the figure is an image of the nozzle inlet covering the five needle injection locations. The flow is from left to right, in the positive \( x \)-direction. A slice is outlined in blue for the \( x \)-location at which the void fraction was computed.

The local phase density, \( X_{x,y} \) is calculated as described earlier for each \( y \)-location, and its variation as a function of \( y \) is presented in the middle plot. Two peaks are seen in the function at the locations of the bubble trains emanating from the needles. Finally, the \( z \)-variation of phase density, \( X_{z} \), which is the projection of a bubble of
Figure 2.11. Calculation of volumetric void fraction; the phase density variations in the $y$ and $z$ directions are integrated to give the total void fraction for the $x$-location pictured.

mean radius in the $z$-direction, is presented in the plot on the right. The final void fraction at this location is then calculated from Equation 2.4.

The void fraction was calculated in this manner for several $x$-locations from the image in Figure 2.11. The initial void fraction for the image, as calculated from Equation 2.5, was $\alpha_o = 1.05 \%$. The results are presented in Figure 2.12.

The image-based void fraction measurement predicts a value of $\alpha = 1.3$ for the axial location just downstream of all five needle injection locations. This value is
roughly 20 % higher than the value computed from the actual gas and liquid flow rates. The average void fraction is then calculated at numerous $x$ locations along the centerline of the nozzle to produce the axial void fraction profile.

Due to its non-invasiveness, compatibility with both fuel and water, and the ability to clearly distinguish between the gas and liquid phases (and therefore providing accurate estimates of the local phase density $X(x, t)$, high-speed video was determined to be the most promising technique for measuring the void fraction distribution in the nozzle flow.
CHAPTER 3

EXPERIMENTAL RESULTS

This chapter summarizes and presents the data for all experimental measurements made for this cavitation study. Measurements of pressure, void fraction, and velocity, using the diagnostic techniques described in the previous chapter, were made. In addition to these measurements, extensive visualization of the flow was conducted using high speed digital imaging.

While the focus of this investigation and the majority of the data that was taken was for a converging-diverging nozzle geometry, two ancillary geometries were studied as well. The first of these was a radial disk geometry, which provided a higher strain rate (as compared to the nozzle). The second was a ball valve model, which was examined to observe cavitation in an application-based test section. The measurements that were made for the radial disk geometry can be found in Appendix A.

Three different working fluids were chosen for this investigation: aviation fuel, water, and dodecane ($C_{12}H_{26}$). Aviation fuel, specifically JP-8, was the fluid of primary interest. However, because water has been studied extensively in the literature, a complete characterization of water was made as well. This was done both to provide a comparison with JP-8 as well as to establish an archival set of water data for comparison with the limited experiments in the literature and the many modeling studies that have been performed with water for a nozzle geom-
etry. Cavitation of fuel presents additional complexities (as compared to water) because fuel is a multi-component mixture. With over 250 different hydrocarbons present in the fuel, each with a unique vapor pressure, individual components may produce bubbles and collapse at different pressures and locations in these devices. Reliable information on fuel properties also is difficult to obtain, as property values can vary significantly between supply sources. Furthermore, bubble nucleation in the fuel is enhanced by the large quantities of contaminant microparticles that are present in JP-8. Initial data sets with JP-8 suggested that the fact that fuel is a multi-component mixture was important to the cavitation physics. To investigate this aspect further, data sets using dodecane (C\textsubscript{12}H\textsubscript{26}) were taken. Dodecane is the largest constituent of JP-8 by mass. This provided a data set of a single hydrocarbon component for comparison.

As mentioned in the introduction, the cavitation nucleation process has a profound influence on the resulting flow. The size, number density, and location of trapped gas nucleii (whether they be on the walls of the device or trapped on microparticle impurities in the liquid) have the potential to effect the evolution of the gas phase in cavitating flows. In addition to studying different internal geometries and different working fluids, the effect of initial void fraction (manifested by the nucleii distribution) was explored. Three separate experiments, each performed with the converging/diverging nozzle geometry, were conducted. First, the dissolved air content in water was varied. Second, iron oxide microparticles extracted from fuel samples were added to the sample liquid in varying concentrations. Both of these experiments modified the initial void fraction in a way that was difficult to quantify. Finally, in an attempt to specifically determine the initial void fraction present in the inlet of the nozzle, direct air bubble injection through needles places in the inlet was performed.
Previous studies that included experimental measurements typically only reported pressure values at the inlet and throat locations. The spatial resolution of the measurements in the current study was much higher and allowed for a detailed examination of the manner in which pressure and void fraction were evolving along the nozzle centerline. Also, for the bubble injection experiments, the initial void fraction in the nozzle is a measured quantity, another piece of information that is lacking from the few reported experimental studies in the literature. As a result, the current data set is amenable to comparison with the latest state of the art cavitation models due to its high spatial resolution and measurement of the nozzle inlet conditions.

3.1 Overview of Operating Conditions

Table 3.1 summarizes the different operating conditions. For all experiments, the upstream tank was open to the atmosphere. The downstream tank back pressure, $P_b$, was pumped down to the values shown in the table. The throat velocity, $u_t$, was measured for one cavitating case for fuel and one cavitating case for water by optically zooming in on the throat region right at the location of initial bubble formation. Tracking these bubbles frame by frame allowed for estimates of the velocity to be calculated.

Volumetric flowrates, $Q$, were measured using a magna-helix flowmeter placed upstream of the nozzle test section. All cases with a tank pressure above $P_b = 40$ kPa were single-phase (pure liquid) experiments. For all fully cavitating flows that were studied, the flow rate through the nozzle was constant regardless of the tank back pressure, indicating a choked flow. Similarly, the throat velocity (and pressure, as will be shown when discussing the pressure measurements) was constant. The measured throat velocities for choked conditions (which would presumably be a
TABLE 3.1

SUMMARY OF EXPERIMENTAL OPERATING CONDITIONS

<table>
<thead>
<tr>
<th>Case</th>
<th>Fluid</th>
<th>$P_b$ (kPa)</th>
<th>$Ca$</th>
<th>$C_{pt}$</th>
<th>$Re_t$</th>
<th>$Q$ (L/min)</th>
<th>$u_t$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>JP-8</td>
<td>60</td>
<td>220</td>
<td>$-100 \pm 5$</td>
<td>-</td>
<td>2.3</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>JP-8</td>
<td>40</td>
<td>220</td>
<td>$-200 \pm 20$</td>
<td>-</td>
<td>3.8</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>JP-8</td>
<td>20</td>
<td>220</td>
<td>$-205 \pm 25$</td>
<td>14 000</td>
<td>4.2</td>
<td>20 ± 2</td>
</tr>
<tr>
<td>4</td>
<td>JP-8</td>
<td>15</td>
<td>220</td>
<td>$-205 \pm 25$</td>
<td>14 000</td>
<td>4.2</td>
<td>20 ± 2</td>
</tr>
<tr>
<td>5</td>
<td>H$_2$O</td>
<td>60</td>
<td>175</td>
<td>$-100 \pm 5$</td>
<td>-</td>
<td>1.9</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>H$_2$O</td>
<td>40</td>
<td>175</td>
<td>$-150 \pm 10$</td>
<td>-</td>
<td>3.4</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>H$_2$O</td>
<td>20</td>
<td>175</td>
<td>$-160 \pm 15$</td>
<td>21 000</td>
<td>3.8</td>
<td>14 ± 1</td>
</tr>
<tr>
<td>8</td>
<td>H$_2$O</td>
<td>15</td>
<td>175</td>
<td>$-160 \pm 15$</td>
<td>21 000</td>
<td>3.8</td>
<td>14 ± 1</td>
</tr>
</tbody>
</table>

measure of the sonic speed of the two-phase mixture) was $u_t = 20$ m/s for fuel and $u_t = 14$ m/s for water. Both of these values are an order of magnitude less than the sonic speeds for the corresponding liquid and gas phases.

Also included in the table are the cavitation number, throat pressure coefficient, and throat Reynolds numbers, whose definitions are

$$Ca = \frac{P_{\infty} - P_v}{\frac{1}{2} \rho_l U_{\infty}^2},$$

$$C_{pt} = \frac{P_t - P_{\infty}}{\frac{1}{2} \rho_l U_{\infty}^2},$$

$$Re = \frac{\rho_l U_{\infty} h}{\mu_l}.$$  

When $-Ca = C_{pt}$, then the throat pressure is equal to the vapor pressure. Comparing these two values provides a rough estimate of when cavitation is likely to occur. It is possible, however, for cavitation to initial at pressure above the vapor pressure. Possible mechanisms for this include impurities and other available nucleation sites, in addition to pressure fluctuations below the mean pressure. The
throat Reynolds number is high enough for both liquids to expect turbulent flow, and the pressure at the core of turbulent eddies could be below the vapor pressure even when the mean pressure is above.

3.2 Flow Visualization

In this section, high-speed digital images of the cavitating flow are presented. For all images shown in this section, the camera frame rate was set to 15 000 frames/s, with a shutter length of 8 µs. Figures 3.1, 3.2, and 3.3 present representative high-speed images of the cavitating flows for the three liquids.

An image of the cavitating flow for the cases where $P_b = 20$ kPa (case number 3 from Table 3.1) is displayed in Figure 3.1. The flow is from left to right and the image captures an area from $x/L = 0.2$ to $x/L = 0.8$. As the flow accelerates towards the throat, the pressure is reduced until bubbles begin to form at $x/L \approx 0.25$. The initially spherical bubbles grow rapidly downstream of the throat and distort into a slug-like gas void. At an axial location of $x/L \approx 0.6$, there is an abrupt change in the flow structure. The large gas voids collapse over a small region and the void fraction is greatly reduced. This bubble collapse zone is referred to as a bubbly-shock, and exhibits many of the characteristics of shock waves in gas dynamics flows. Downstream of the shock, the flow is characterized by a dilute two-phase mixture of spherical bubbles.

Figure 3.2 is a high-speed digital image of JP-8 cavitating in the nozzle test section. The tank back pressure was $P_b = 20$ kPa, which corresponds to case 7 from Table 3.1. In fuel, the bubbles remain mostly as individual spherical bubbles. In water, the bubble growth phase has initially spherical bubbles that grow quickly and coalesce into gas slugs. The most obvious difference between the two liquids is the presence of an abrupt bubble breakdown zone in water, with a corresponding
Figure 3.1. Image of H$_2$O cavitating mixture, $P_b = 20$ kPa.

Figure 3.2. Image of JP-8 cavitating mixture, $P_b = 20$ kPa.

Figure 3.3. Image of dodecane cavitating mixture, $P_b = 20$ kPa.
nearly discontinuous change in bubble size and number. This bubbly shock structure is not visible in fuel, and the initial bubble growth phase is followed by a nearly homogeneous bubbly flow with much smaller bubble sizes than in water. Bubbles remain in the fuel throughout the diffuser section of the nozzle, and never disappear. In water flow, downstream of the bubbly shock, a dilute mixture of spherical bubbles is present.

Also included for comparison is an image taken of dodecane for the same operating conditions as the fuel and water images (Figure 3.3). The image appears very similar to the Figure 3.2, the fuel case, with some subtle differences. There is no visible bubbly shock, yet the bubble size distribution in the diffuser section of the nozzle is different than the fuel case. The smallest bubbles that were present in the fuel appear to be gone for dodecane, and the bubble sizes appear much larger. It can be imagined that the JP-8 image is the same as the dodecane, with a population of tiny bubbles (with radii less than 10 µm) superimposed on top of the dodecane bubble population.

An image taken at higher magnification levels (3x) to highlight the bubbly shock structure in water is presented in Figure 3.4. Just upstream of the shock is a high void fraction, complex gas slug, with no spherical bubbles remaining. As this gas slug encounters the large pressure rise of the shock, smaller slugs and bubbles are expelled and collapse such that there is a very large reduction in gas content over a very small length of the nozzle. The pressure and void fraction data show an abrupt increase in pressure along with an abrupt decrease in void fraction.

3.3 Pressure Data

Pressure data are presented in this section for all of the cases listed in Table 3.1. In addition, comparison plots between the different liquids are included. All
pressure data are normalized by the nozzle inlet pressure such that the range is from 0 to 1 and non-dimensional.

3.3.1 Water

Pressure measurements (normalized by the inlet pressure, $P_o$) along the nozzle centerline are plotted versus axial position for all cases (cases 1-4 from Table 3.1) in Figure 3.5. For reference, the nozzle geometry is superimposed on each figure.

The profiles for the $P_b = 60$ kPa and $P_b = 40$ kPa tank back pressure cases (cases 1 and 2) are single-phase (no cavitation). They are presented here for reference. The two cases of interest are the ones with $P_b = 20$ kPa and $P_b = 15$ kPa (cases 3 and 4), which are both fully cavitating flows. The pressure through the inlet and throat of the nozzle are nearly identical for these cases, even though the back pressure is 33 % lower for case 4. This pressure behavior is a hallmark of a supersonic flow, as pressure information in the downstream section of the nozzle is not able to propagate upstream.

The bubbly shock is clearly present in the water data, with a sharp pressure
rise occurring at the shock location \((x/L = 0.45\) and \(x/L = 0.6\) for cases 3 and 4, respectively). The position of the shock moves upstream towards the throat as the back pressure is increased. Eventually the back pressure becomes high enough at which cavitation ceases and evidence of a shock disappears \((P_b = 40 \text{ kPa}, \text{ case 2})\).

### 3.3.2 Fuel

The axial pressure variation in fuel for cases 5 through 8 from Table 3.1 is displayed in Figure 3.6.

Similar to the pressure data for water, the curves for the back pressures of \(P_b = 60 \text{ kPa}\) and \(P_b = 40 \text{ kPa}\) were single-phase with no cavitation. Cases 7 and 8 both exhibited fully cavitating flows. The pressure data over the nozzle inlet is nearly identical for the cavitating cases; from \(x/L = 0\) to \(x/L = 0.3\) the pressure data for cases 6, 7, and 8 are the same. This indicates that the nozzle must be choked, and supersonic conditions must be present at the nozzle throat. For the
Figure 3.6. Axial pressure distribution in JP-8.

fuel pressure data, there is no sharp pressure rise indicative of shock behavior. The pressure rises gradually from a minimum value of $P/P_o = 0.1$ at $x/L = 0.3$ up to the nozzle exit pressure. It is difficult to reconcile the lack of an abrupt pressure rise, a clear indication of a shock, with the conclusion that the flow is supersonic and choked. This apparent contradiction will be addressed in later sections. It is hypothesized that there is a shock present in the fuel, but it is spread spatially as a result of fuel being a multi-component mixture.

3.3.3 Fuel/Water Comparisons

To highlight the differences between the fuel and water data, Figure 3.7 plots both cases together for $P_o = 20$ kPa. Here, the differences in the pressure increase across the shock are readily apparent. The bubbly shock for the water data is clear, and centered at a location of $x/L = 0.42$. For fuel, the pressure increase attributed to the bubbly shock occurs between $x/L = 0.35$ and $x/L = 0.5$. 

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Figure 3.7. Axial pressure distributions in H$_2$O, JP-8, and C$_{12}$H$_{22}$, $P_b = 20$ kPa.

Also included in the figure is a pressure distribution for dodecane as the working fluid. The JP-8 and dodecane curves are similar everywhere except in the region of the bubbly shock. While the minimum pressure for both occurs at the same location ($x/L = 0.35$), the dodecane data indicates a much sharper increase in pressure across the shock.

3.4 Void Fraction

Void fraction data are presented here using the high-speed imaging based technique described in the previous chapter. The void fraction profiles for all three liquids at a back pressure of $P_b = 20$ kPa are presented. At the end of this section, comparison plots are presented to highlight the differences between the liquids.
3.4.1 Water

The axial distribution of void fraction for water at a back pressure of $P_b = 20$ kPa (case 3) is shown in Figure 3.8. Along with void fraction, the pressure data is included in the plot to help illustrate the strong correlation between the two. Also included in the figure is the nozzle contour.

![Axial Pressure and Void Fraction Distribution, H2O](image)

Figure 3.8. Axial void fraction distribution in H$_2$O, $P_b = 20$ kPa.

As the pressure decreases in the nozzle and bubbles start to form, the void fraction increases abruptly from zero at $x/L = 0.2$. The void fraction quickly reaches a maximum of $\alpha = 0.8$ at $x/L = 0.25$. The bubbly shock location is clearly evident in the void fraction profile, as the void fraction rapidly decreases at the shock location of $x/L = 0.45$. Downstream of the shock, the void fraction is roughly constant and is approximately 5%. The image shows this region of the nozzle to contain a dilute mixture of bubbles.

The connection between pressure and void fraction is apparent; once the pressure
decreases to a value low enough to initiate cavitation, the void fraction becomes non-
zero before increasing rapidly to its maximum value at the location of minimum
pressure. The shock location is easily identifiable from either set of data, as the
sharp decline in void fraction occurs at the same axial location as the pressure
increase due to the bubbly shock.

3.4.2 Fuel

Figure 3.9 plots the void fraction along the centerline of the nozzle for a back
pressure of $P_b = 20$ kPa. This corresponds to case number 7 from Table 3.1. The
axial pressure distribution is included to highlight the relationship between pressure
and void fraction.

![Figure 3.9. Axial void fraction distribution in JP-8, $P_b = 20$ kPa.](image)

In a similar fashion to the water data, the void fraction in fuel is near zero at the
nozzle inlet, and at the throat increases rapidly to its maximum value of $\alpha = 0.4$
at an axial location of $x/L = 0.25$. Unlike the water data, there is no dramatic decrease in void fraction indicating the presence of a bubbly shock. Instead, the void fraction decreases gradually as the pressure increases. As was mentioned in the previous section when discussing the fuel pressure data, it is thought that a bubbly shock (or multiple weak shocks) are present, and that the slower rate of decrease in void fraction as compared to water is a consequence of fuel being composed of many different components, each with unique properties.

3.4.3 Dodecane

Void fraction data for dodecane is shown in Figure 3.10, for a back pressure of $P_b = 20$ kPa. As for the previous liquids, the pressure data for the same operating conditions is included. Unlike the fuel data, there is clear evidence of a bubbly shock in the dodecane data. At $x/L = 0.35$, there is an indicative decrease in void fraction with the concomitant increase in pressure. It is notable that the shock evidence is so clear in the pressure and void fraction data, while visual evidence (from Figure 3.3) is inconclusive. Clearly, imaging alone is not sufficient to determine the presence of a bubbly shock.

3.4.4 Fuel/Water Comparisons

For purpose of comparison, the void fraction profiles for all three liquids presented above are plotted together in Figure 3.11. All three liquids appear to initiate cavitation at the same axial location, right at the start of the nozzle throat, at $x/L \approx 0.2$. Fuel appears to start cavitating slightly farther upstream, but it is doubtful that this finding is significant due to the spatial uncertainty in the measurements. Water achieves the largest void fraction ($\alpha_{max} = 0.8$), followed by dodecane ($\alpha_{max} = 0.55$), and then fuel ($\alpha_{max} = 0.4$). The presence of a bubbly shock is clear for water (at $x/L = 0.4$) and dodecane (at $x/L = 0.35$). For fuel, any
Figure 3.10. Axial void fraction distribution in $C_{12}H_{22}$, $P_0 = 20$ kPa.

Figure 3.11. Axial void fraction distribution for $H_2O$, JP-8, and $C_{12}H_{22}$. 

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bubbly shock is distributed spatially and is not obvious. In the nozzle diffuser, the void fraction levels are lowest for water, and are very similar between dodecane and JP-8. This observation is consistent with the flow visualization, as the water images show a dilute bubbly flow in this region while the fuel and dodecane exhibit much larger bubbly populations. It is evident from the void fraction data that the multi-component composition of fuel is effecting the cavitation physics.

3.5 Manipulation of Initial Void Fraction

In this section, data for the three experiments that attempted to modify the initial void fraction in some manner are presented. This includes the effects of degassing the liquid, of microparticle concentration, and of direct bubble injection through hypodermic needles.

3.5.1 Degassing

Often for cavitation studies in external flows it is common to include some information as to the amount of dissolved air that is present in the water. It is well known that the amount of gas dissolved in water can have a profound effect on the initiation of cavitation. To determine whether or not dissolved air was effecting the cavitating nozzle flows, pressure distributions for liquid samples that had been exposed to vacuum pressures (and thus removed of their dissolved air content) were measured.

The results of these experiments are presented in Figure 3.12. Three different pressure distributions are shown; a control in which the sample liquid was taken directly from the tap source, a sample in which the water was exposed to vacuum for four hours, and a sample where the water was exposed to vacuum for 16 hours. No differences between the three data sets to within the experimental uncertainty were found. All three cases exhibited similar strain rates through the nozzle contraction,
and all three had their bubbly shocks located at $x/L = 0.55$.

For the nozzle operating at a back pressure where a standing bubbly shock occurs, pressures low enough to initiate cavitation are easily achieved. The effect of dissolved air content is primarily a diffusion process. For this experiment with these operating conditions, the residence time of any microbubble dissolved in the liquid through the nozzle (approximately 10 ms) is substantially larger than the time for a bubble to grow to visible size. Therefore, it is not surprising that the amount of dissolved air content in the liquid sample has no effect on the pressure distribution in the nozzle (the time for a 1 $\mu$m diameter bubble to grow to the size of 1 mm exposed to similar conditions as to those experienced in the nozzle experiments was estimated to be greater than 100 s).
3.5.2 Microparticle Effects

Another attempt to modify the source of potential nucleation sources (and therefore the initial void fraction) was made through the addition of solid impurities into the liquid. Different amounts of microparticles (iron oxide particles filtered from a sample of JP-8) were mixed with samples of water, and their effect on the cavitating flow was observed.

Figure 3.13 shows two images of the C-D nozzle cavitating at identical flow rates and pressures. In the top image, the fluid is distilled water. These images were taken at a frame rate of 30 fps, so that the resulting pictures represent a time average of the flow (as compared to the high-speed images at 15,000 fps that were shown earlier). In the bottom image, the distilled water has been loaded with a concentration of 0.4 g/L of the filtered microparticles. The location of the bubble collapse was moved farther back in to the nozzle diffuser by the presence of the microparticles. It was also observed that when microparticles were not present, the cavitation originated from the nozzle walls, which acted as the major source of nucleation. With particles present, bubble nucleation sites were ubiquitous in the fluid, and the cavitation initiated across the entire nozzle throat. This was further qualitative evidence that microparticles play a major role in bubble nucleation.

To quantify these observations, void fraction profiles for different microparticle concentrations were acquired. The results are presented in Figure 3.14. All four plots shown are for water at a back pressure of \( P_b = 20 \) kPa. The concentration of iron oxide microparticles was varied from 0 g/L (no added particles) up to a concentration of 0.55 g/L. At higher microparticle concentrations, the decrease in void fraction indicative of the bubbly shock location was larger than for the lower microparticle concentrations. The microparticles had the effect of resolving the shock spatially. As was seen in the images of Figure 3.13, the presence of particles
Figure 3.13. Images of water both with (bottom) and without (top) microparticles

Figure 3.14. Axial void fraction distribution for H\textsubscript{2}O showing particle effects, \( P_b = 20 \) kPa.
caused cavitation to occur in the bulk of liquid, as opposed to primarily of the nozzle walls. The effect on the shock structure was seen as well; cavitation off of a wall lead to an angled, irregular shock. High-speed video of these flows showed that the wall nucleation was intermittent, and that the unsteadiness that resulted caused the shock position to oscillate slightly about its mean value. The end result was that by providing a constant source of nuclei in the bulk, the microparticles had the effect of reducing the unsteadiness in the flow and caused the shock to be stationary, resulting in a sharper decrease in the void fraction profile.

The microparticle experiments demonstrated a clear connection between initial void fraction, available cavitation nuclei, and shock behavior. Unfortunately, it is extremely difficult to estimate the initial void fraction changes caused by increasing the microparticle concentration. For modeling purposes, quantifying the initial void fraction is important. This leads to the final experiment that was performed to investigate initial void fraction effects.

3.5.3 Bubble Injection

The cavitation for the results presented thus far is due solely to naturally occurring bubble nuclei. The cavitation is initiated primarily from the walls of the nozzle test section, where micro-pits on the surface of the walls are providing the nuclei necessary to form bubbles. Without a method to quantify the initial amount of gas present in the flow (that is, the initial void fraction for the problem), modeling is very difficult. It will be shown in the modeling chapter that the majority of the models for cavitating nozzle flow that are present in the literature demonstrate a large sensitivity to the initial conditions chose, primarily the initial void fraction. Obtaining an experimental value of this initial void fraction is difficult for cavitation initiated by naturally occurring nuclei.
To overcome this difficulty, experiments were preformed where a known quantity of gas was injected into the nozzle inlet through 100 µm-diameter needles. The flow rate of the injected gas was measured with the bubble flow meter described in the previous chapter. By also measuring the flow rate of the liquid/gas mixture, the initial void fraction injected into the nozzle was calculated. Through metering valves the amount of gas being injected could be controlled such that the initial void fraction range of $\alpha_o = 0.5\%$ to $\alpha_o = 2.5\%$ could be studied.

Figure 3.15 plots the axial location of the bubbly shock, determined visually, as a function of the initial void fraction, $\alpha_o$, being injected into the nozzle, for three different back pressure conditions ($P_b = 10$ kPa, $P_b = 15$ kPa, and $P_b = 20$ kPa). The shock location varied from $x/L = 0.4$, to $x/L = 1.0$ (a condition where the shock was seen to beyond the nozzle exit) as the initial void fraction ranged from $\alpha_o = 0$ (corresponding to the “natural” cavitation condition) to $\alpha_o = 2\%$. The shock location was more sensitive to $\alpha_o$ than it was to the back pressure.

Figure 3.15. Shock location as a function of initial void fraction in H$_2$O

Next, the axial pressure distribution for different injection amounts was mea-
sured, all at a back pressure of $P_b = 20 \text{ kPa}$. The results are shown in Figure 3.16. Immediately below the pressure data is a high-speed image of the bubble injection for the case where $\alpha_o = 1.0 \%$. Four plots are presented in the figure, which are the pressure distributions for $\alpha_o = 0 \%$, $\alpha_o = 0.5 \%$, $\alpha_o = 1.0 \%$, and $\alpha_o = 1.5 \%$. The effect of initial void fraction on the location of the bubbly shock (indicated by the sharp pressure rise) is evident in the pressure data. In addition to its effect on shock location, the initial void fraction has an effect on the shape of the pressure profile in the nozzle inlet. As $\alpha_o$ increases, the derivative $dP/dx$ in the nozzle inlet also increases. As more gas is injected in the inlet, the effective area is decreased. This causes the flow to accelerate faster for higher initial void fraction values. As the flow accelerates, the pressure decreases over a smaller axial distance, resulting in the steeper shape to the pressure data in the nozzle inlet.

![Figure 3.16](image_url)

Figure 3.16. Effect of initial void fraction on the axial pressure distribution in H$_2$O

The axial pressure distribution in fuel with bubble injection is presented in Figure
3.17. The back pressure for all profiles was $P_b = 20$ kPa. Similar to the water data in Figure 3.16, four profiles are plotted, covering an initial void fraction range of $\alpha_o = 0 \%$ to $\alpha_o = 1.5 \%$. Also pictured is an image of the $\alpha_o = 1 \%$ injection case. Compared with the pressure data from Figure 3.6 with no bubble injection, the pressure recovery in the diffuser section is more abrupt for the cases with injection. The inclusion of bubbles in the nozzle inlet has the effect of organizing the bubbly shock structure. While still more spread spatially than in water, the presence of a shock in fuel is more evident when bubble injection is included. As the initial void fraction is increased, the location of the bubbly shock in the fuel moves downstream towards the nozzle exit.

![Figure 3.17. Effect of needle injection in JP-8.](image)

In order to highlight and summarize the differences in the pressure data for the two different liquids, the axial pressure distributions for both fuel and water at a
tank back pressure of $P_b = 20$ kPa and $\alpha_0 = 1 \%$ are plotted in Figure 3.18.

Figure 3.18. Velocity and pressure in both JP-8 and H$_2$O.

Also included in Figure 3.18 are the velocities of the injected bubbles from the nozzle inlet until the throat (where the bubbles grow very rapidly and become difficult to track). From the pressure data, the differences in the spatial resolution of the shock are evident. While the shock for both fuel and water are located nominally at $x/L = 0.55$, the pressure increase for water occurs over a distance of approximately $x/L = 0.05$, where as the fuel data pressure increase requires more than twice the $x/L$ distance. The minimum pressure achieved for water is lower than that for fuel ($P/P_o = 0.05$ for water vs. $P/P_o = 0.12$ for fuel). Finally, the strain rate ($dU/dx$) in the contracting portion of the nozzle is higher for water than for fuel.
3.6 Summary of Experimental Data

Based on all of the experimental results presented in the current chapter, the following conclusions are drawn:

- For both fuel and water, the pressure and void fraction data are closely linked. That is, a decrease in pressure results in a corresponding increase in void fraction, and vice versa. This suggests that a modeling approach where pressure is solely a function of the mixture density \((P = f(\rho))\) might be appropriate.

- For cavitating flows for either liquid, the flow was found to be supersonic based on a choked mass flow rate.

- A clear, visible, bubbly shock was observed for cavitating flows for water. The pressure and void fraction data confirmed this observation. As the back pressure was decreased, the location of the shock moved downstream.

- The presence of a bubbly shock in cavitating fuel flows was inconclusive based on high speed digital imaging. A gradual pressure increase and corresponding void fraction decrease was observed for these flows, but were much less spatially resolved than the water data. Decreasing the back pressure shifted this pressure recovery downstream, suggesting the presence of a shock that was spread spatially. It was hypothesized that the reason for this spatial distribution was the multi-component composition of the fuel.

- Injecting a known quantity of gas in the nozzle inlet allowed the initial void fraction to be determined.

- The location of the bubbly shock was found to be very sensitive to the initial void fraction for the bubble injection experiments in both fuel and water.
CHAPTER 4

MODELING

In this section, some of the modeling efforts described in the introduction are compared with the experimental measurements of the previous chapter. A barotropic model, in which a cavitating mixture is treated as homogeneous with constant properties, and a quasi-1D model that incorporates bubble dynamics, are the two models of interest. A discussion of the Raleigh-Plesset equation and its physics are included and bubble dynamic trends compared with the experimental data. Since the initial bubble size distribution is identified as a critical parameter, a characterization of the surface geometry of the microparticles filtered from the JP-8 is considered. The effect of the resulting size distribution on the cavitating flow is discussed.

Finding a model to adequately predict pressure and void fraction over the entire length of the nozzle is difficult. The nozzle flow evolves from a low void fraction ($\alpha < 1\%$), spherical bubble flow in the inlet and throat, to a very high void fraction ($\alpha > 50\%$), slug-flow upstream of the shock, to a nearly homogenous two-phase mixture in the nozzle diffuser. Certain models proved capable of reasonable predictions in one flow regime only to fail in others.

To illustrate further the complex nature of the flow, Figure 4.1 is two images of the C-D nozzle (the top image is water, the bottom image is fuel) under identical lighting and operating conditions. For both cases, the throat-to-inlet pressure ratio and flow rate are constant, indicating that the nozzle is operating under choked
conditions. The different regimes of the complex cavitating flow have been outlined.

Figure 4.1. Images of H₂O (top) and JP-8 (bottom) cavitating mixtures

For both liquids, bubble formation begins slightly aft of the smallest throat area. Upon bubble formation, both liquids exhibit a rapid bubble growth region. As the pressure in the nozzle starts to recover in the diffuser, bubble breakdown occurs. For water, this bubble breakdown happens over a very narrow region in the form of a bubbly shock. Behind this shock, nearly all gas voids have disappeared, and only dilute large \( r \approx 1 \text{ mm} \) bubbles are present. In fuel, the bubble breakdown region is extended over a length of 1 cm, after which appears a homogeneous, two-phase mixture of much smaller \( r \approx 100 \mu\text{m} \) bubbles. It is obvious from the images that a single model for all regions of two-phase flow that occur in the nozzle would be
difficult to develop.

4.1 Barotropic Model

A goal of the barotropic model is to determine properties for a gas-liquid mixture based on the properties of the individual components. The resulting mixture then is analyzed as a single-component fluid, and the pressure in the fluid is a function of the mixture density alone. A thorough review of this model is presented in References [3] and [24]. The conditions necessary for this analysis are

\[ \rho_L c_L^2 \gg \rho_V c_V^2, \]  
(4.1)

and

\[ \rho_L \gg \rho_V, \]  
(4.2)

in which \( c \) is the speed of sound, \( \rho \) is the density, and the subscripts \( L \) and \( V \) refer to liquid and vapor, respectively. These conditions are met for cavitating mixtures of either JP-8 or of water, whether the gas phase is air or vapor bubbles.

4.1.1 Sonic Velocity

The expression for the sonic velocity of the two-phase mixture is

\[ c_{mix} = \sqrt{\rho_L \frac{c_V^2}{\rho_L (1 - \alpha_V)}}, \]  
(4.3)

where \( \alpha_V \) the void fraction of the vapor. For JP-8 liquid-vapor mixture calculations, the following values are assumed: \( \rho_L = 800 \text{ kg/m}^3 \) (JP-8), \( \rho_V = 5.5 \text{ kg/m}^3 \) (JP-8), \( c_L = 1174 \text{ m/s} \) (JP-8), and \( c_V = 185 \text{ m/s} \) (methane). The mixture sonic velocities for the mixture of JP-8 liquid and vapor, and for the mixture of water liquid and vapor, are shown in Figure 4.2. For either case, the mixture sonic velocity
is relatively constant over the vapor void fraction range from approximately 0.2 to 0.8. Both mixtures exhibit sonic velocities that are well below their constituent values, which is characteristic of a eutectic mixture.

Experimental values for the sonic velocities were obtained by high speed imaging. The camera shutter time was chosen so that small bubbles entrained from the upstream holding tank would appear as streaks in the still frames. By measuring the lengths of these bubble streaks using image processing software (after calibrating against the known throat height), the throat velocity was measured. Figure 4.3 is an image of the cavitating C-D nozzle (with fuel). The shutter speed was set to 67 $\mu$s, and trace lengths were typically on the order of 1.5 mm. After analyzing several bubble traces for both fuel and water, the throat velocities were determined to be
Comparing the measured values to Figure 4.2, the trend was as expected (water has a lower sonic velocity than JP-8). For water, over the flat portion of the curve (0.2 ≤ α ≤ 0.8) the sonic velocity is 15 m/s, which is in good agreement with the measured value. For fuel, the measured value is approximately 75\% lower than theory. One possible explanation for the discrepancy is the uncertainty in the fuel property values, particularly the specific heat of the fuel vapor. In calculating Figure 4.2, methane properties were used due to the lack of availability of $c_V$ for JP-8. Estimates of $c_V$ for JP-8 could be obtained by varying $c_V$ in Equation 4.3 to match the experimentally determined sonic velocities.

### 4.1.2 Compressible Bulk Modulus

The expression for the mixture theoretical bulk modulus is

$$E_{mix} = \frac{1}{(1 - \alpha_V/E_L + \alpha_V/E_V)}.$$  \hfill (4.6)

The bulk moduli of the two mixtures are displayed in Figure 4.4. Both mixtures have approximately the same values, which also are relatively independent of the vapor void fraction for values ranging from approximately 0.2 to 1.0, where the
mixture bulk modulus equals approximately 0.2 MPa for either mixture.

An alternative expression for the bulk modulus assuming the mixture to be a compressible gas can be defined as

\[ E_{\text{mix}} = kp, \]  

(4.7)

in which \( k \) is the compressible mixture’s ratio of specific heats and \( p \) is the local pressure. Using this expression, an experimental value for the bulk modulus is obtained through pressure measurements.

Measured throat-to-inlet pressure ratio, \( p^* \), values versus time are shown for the mixture cases of JP-8 and of water in Figure 4.5. The pressure data clearly establish
that the nozzle is choked and that there is a critical throat-to-inlet pressure ratio for each mixture (0.29 for JP-8 and 0.09 for water). Treating the mixture as a compressible substance, where

$$p^* = p_{th}/p_i = \left[\frac{2}{k+1}\right]^{k/(k-1)}, \quad (4.8)$$

allows $k$ to be determined for each mixture case. Using these values, the mixture bulk modulus at the nozzle throat can be computed from Equation 4.7, in which $p = p_{th} = p^* p_i$. The resulting values are 0.1 MPa for the JP-8 mixture and 0.2 MPa for the water mixture. Both these values compare very well with that determined using the mixture model (Equation 4.6) of 0.2 MPa.
4.1.3 Choked Mass Flux

Finally, the experimentally determined mass flux under choked conditions can be used to identify the maximum volumetric flow rates achievable for a given minimum flow cross-sectional area assuming similar, fully choked flow conditions. These values are presented in Figure 4.6.

![Figure 4.6. Maximum volumetric flow rates versus minimum flow cross-sectional area for two cases: JP-8 and water.](image)

Figure 4.6. Maximum volumetric flow rates versus minimum flow cross-sectional area for two cases: JP-8 and water.

A summary of all the flow parameters that were obtained experimentally for comparison with the barotropic model is presented in Table 4.1

4.2 Rayleigh-Plesset Equation

The dynamics of a spherical bubble in an infinite medium are governed by the Raleigh-Plesset equation, which presented in non-dimensional form is:
In Equation 4.9, \( R \) is the non-dimensional bubble radius as a function of time. The length and velocity scales used to non-dimensionalize the problem are the initial bubble radius, \( R_o \), and the inlet velocity of the nozzle, \( U_\infty \). Three non-dimensional groups are present in the equation:

\[
R \ddot{R} + \frac{3}{2} (\dot{R})^2 = - \left( \frac{C_p}{2} + \frac{C_a}{2} (1 - R^{-3k}) + \frac{2}{We} (R^{-1} - R^{-3k}) + \frac{4 \dot{R}}{Re \dot{R}} \right).
\]  

(4.9)

These three parameters are the cavitation number, Weber number, and Reynolds number (based on fluid properties), respectively.

The bubble volume is driven by the pressure term \( C_p \) on the right hand side, which is defined as

\[
C_p = \frac{P(t) - P_\infty}{\frac{1}{2} \rho_f U_\infty^2}.
\]  

(4.13)
In this equation, \( P(t) \) represents the local time varying static pressure that bubble experiences. When the cavitation number, \( Ca \), is lower than the minimum value of \( -C_p(t) \), the pressure is below vapor pressure.

4.2.1 Effect of Initial Bubble Radius

Using a sinusoidal pressure variation that cycles from atmospheric pressure to the vapor pressure of the liquid (at which \( Ca = -C_{p_{min}} \)), the effect of the initial bubble size, \( R_o \), was examined. Figure 4.7 displays solutions to the Raleigh-Plesset equation (in water) for \( R_o = 1 \) µm, 10 µm, and 100 µm (top plot). The sinusoidally varying pressure history is shown on the bottom plot of the figure. The time axis is scaled such that \( t = 1 \) corresponds to the amount of time it would take for a bubble traveling with the flow to transit the length \( (L = 10 \text{ cm}) \) of the experimental nozzle. The viscosity is artificially increased (to \( \mu_f = 0.03 \text{ N} \cdot \text{s/m}^2 \)) to include bubble damping mechanisms [5].

The effect of initial radius \( R_o \) is manifested in the \( 2/We \) term on the right hand side of Equation 4.9. Smaller initial radii increase the restoring force generated by surface tension \( (\approx 2\sigma/R_o) \), and decreases the maximum bubble size. As \( R_o \) increases, the bubble dynamics show a violent collapse and rebound. The profiles increase in amplitude and are delayed in time as the initial bubble grows.

In an actual flow device, there is a range of initial radii present in the form of trapped gasses on solid surfaces and microparticles. Figure 4.7 emphasizes the need for detailed knowledge of the characteristic sizes of the nuclei present in the C-D nozzle experiment. Detailed analysis of the microparticles present in the fuel used in the experiments will be presented in a later section.
Figure 4.7. Solutions to Raleigh-Plesset equation for different $R_0$ (top), pressure forcing (bottom)
4.2.2 Fuel vs. Water

Surface tension effects are explored further by solving the Raleigh-Plesset equation for fuel and for water and then comparing the results. Figure 4.8 shows the bubble size time histories for identical pressure forcing shown in Figure 4.7. To isolate surface tension effects, $R_o$, $Ca$, and $Re$ are held constant for the two solutions.

![R-P solutions for fuel and water](image)

Figure 4.8. R-P solutions for fuel ($Ca = 0.98$, $Re = 33$, $We = 352$) and water ($Ca = 0.98$, $Re = 33$, $We = 139$)

The fact that JP-8 has a surface tension approximately 2.5 times that of water results in similar-sized initial bubbles that continue to behave as smaller bubbles in water. Fuel bubbles do not grow as large and have a more delayed collapse than is seen for water bubbles.

Returning to the images at the beginning of this section (Figure 4.1), this effect of
surface tension possibly explains the differences seen in the cavitating regions for fuel and water. The water image shows an abrupt collapse in the bubbles in the form of a bubbly shock, which could be explained from the violent collapse predicted in the Raleigh-Plesset solutions. Conversely, the fuel image has a homogenous collection of very small bubbles in the diffuser. This is perhaps the result of the relaxed bubble dynamics solutions seen in Figure 4.8. Since fuel is a multi-component mixture, it is also possible that the absence of a single, abrupt bubble collapse is the result of different fuel components (with different vapor pressures) nucleating and collapsing in different sections of the nozzle.

4.2.3 Critical Radius of a Gas Cavity

The Rayleigh-Plesset equation does not model nucleation; that is, it assumes that bubbles of a given initial size are present. It has been argued that cavitation bubbles originate from trapped pockets of gas on solid surfaces. The characteristic size of these cracks and pits will determine at what pressures a nucleation site will become active. As seen above in Figure 4.7, the initial bubble size is an important parameter when analyzing the bubble dynamics present in a cavitating flow.

Consider a force balance on a microscopic bubble trapped on a solid surface in equilibrium with its surroundings. Let $P_b$ be the pressure inside the bubble, and let $P_\infty$ be the pressure in the far field of the surrounding liquid. For the bubble to grow,

$$P_b > P_\infty.$$  \hspace{1cm} (4.14)

The contents of the bubble include the partial pressure of whatever gasses are dissolved in the bulk, $P_{G_0}$, a restoring force due to surface tension $2\sigma/R_0$ (where $\sigma$ is the surface tension in the liquid and $R_0$ is the initial bubble radius), and the vapor
pressure of the liquid, $P_v$. Thus, at equilibrium,

$$P_\infty = P_v + P_{G_o} + \frac{2\sigma}{R_o}. \tag{4.15}$$

If the far field pressure is now lowered from its equilibrium value, either hydrodynamically or acoustically, the bubble will grow. It can grow either by the diffusion of dissolved gas from the bulk into the bubble (reducing the pressure sets up a concentration gradient across the bubble interface where the concentration of the gas dissolved in bulk will be higher than in the bubble) or by the local static pressure being reduced to the vapor pressure. Equation 4.15 can be solved for the critical bubble radius that will grow when exposed to a pressure of $P_\infty$:

$$R_o = \frac{2\sigma}{P_{G_o} + P_v - P_\infty}. \tag{4.16}$$

If the assumption is made that a gas microbubble stabilized on a solid surface will have a radius on the order of the characteristic dimension of the crack or pit in which it is located, Equation 4.16 provides an estimate of the size of gas cavity that will be activated and nucleate bubbles for a given $P_\infty$. It should be noted that Equation 4.16 is a static analysis; the dynamics of a bubble of initial size $R_o$ will be governed by the Raleigh-Plesset equation.

### 4.3 Microparticle Surface Characterization

Because $R_o$ is such an important parameter, it is necessary to estimate the range of potential nucleation site sizes present in the C-D nozzle experiment. Unfiltered jet fuel contains a measurable amount of solid particles. Data provided by the Honeywell Corporation shows a particle mass concentration of approximately 2 mg/L, and a number concentration of $10^7$ particles/L. Microscopic cracks and pits in these microparticles act as nucleation sites for bubble growth. Their size and number
density per unit volume of liquid must be determined to estimate the initial bubble sizes and compute an initial void fraction.

The nucleation sites of these microparticles were fully characterized by using microscopic images. Several particles were photographed with a microscope under 10X magnification. Sample photographs from this procedure are presented in Figure 4.9.

Two site characteristics were taken from these photographs: the size distribution of potential nucleation sites, and their area density (number of sites per unit area). In addition to the size range of nucleation sites, the total amount of gas initially present per unit volume of liquid (initial void fraction) needs to be determined. To do so, a simplifying assumption is made that each pit that contains some trapped gas will be modeled as a spherical bubble of radius equal to the size of the nucleation site. Thus, the size distribution of nucleation sites provides an initial population of
microbubbles whose volume can be summed up to provide that total amount of gas present.

The measured size distribution, take from a series of photographs similar to Figure 4.9, is presented in Figure 4.10. In order to use Figure 4.10 to calculate the total gas trapped on a single particle, the total number of active sites on a microparticle must be determined. The total number of active nucleation sites can be calculated by multiplying the particle surface area by the nucleation site area density, which is given by:

\[ N_s = 4\pi r_p^2 \beta p_1, \]  

(4.17)

where \( N_s \) is the total number of active nucleation sites, \( r_p \) is the radius of the microparticle, \( \beta \) is the nucleation site area density, and \( p_1 \) is the probability that a given site contains a microbubble. A value of \( p_1 = 1 \) implies that all pits and cracks on the particle surface contain trapped gas. The nucleation site area density, \( \beta \), was determined from the images of the microparticles. This value was estimated as

\[ \beta = 3600 \pm 500 \text{ sites/mm}^2. \]  

(4.18)

Knowing the total number of microbubbles present on a given particle, the site size distribution can be used to sum up the total bubble volume for a single particle. Let \( f_i \) be the frequency of occurrence for initial bubble size \( r_{bi} \) (taken from Figure 4.10), and let \( K_b \) be the total number of bins in the frequency distribution. Summing over all bins gives the total volume of gas for a single particle, \( V_p \):
Figure 4.10. Size distribution of nucleation sites on microparticles

\[ V_p = \frac{K_b}{3} \sum_{i=1}^{K_b} \left( \frac{4}{3} \pi r_{bi}^3 f_i N_s \right), \quad (4.19) \]

\[ = \frac{16}{3} \pi r_p^2 \beta_p \sum_{i=1}^{K_b} (f_i r_{bi}^3). \quad (4.20) \]

There are two additional factors to consider when calculating the total gas trapped on a particle. Obviously, it is impossible for a microparticle to be smaller than a given nucleation site. Thus, in performing the summation in Equation 4.20, only terms for which the following condition is met should be included in the summation:

\[ r_{bi} \leq k_1 r_p. \quad (4.21) \]

The constant \( k_1 \) is yet to be determined, but for now, a value of \( k_1 = 0 \) will be used.
(that is, no nucleation site can be larger then half the size of the microparticle).

The other consideration is less obvious. Below some critical radius $R_o$, surface
tension forces will dominate such that bubbles will not grow no matter how low a
pressure they are exposed to. Thus, nucleation sites with trapped gas of size smaller
than $R_o$ will not be active and will not contribute to the amount of gas evolving in
the flow. As a result, the summation in Equation 4.20 should only include terms
where

$$r_{b_i} \geq R_o,$$

(4.22)

where $R_o$ is the same critical radius of a gas cavity that was developed in Equation
4.16.

If the size distribution of microparticles were uniform, the total gas volume
per unit liquid volume could be determined simply by multiplying Equation 4.20
(modified by the rules given in Equations 4.21 and 4.22) by the total number of
particles per unit volume. Of course, this distribution is not uniform and there is a
wide range of particle sizes present in the liquid. In a manner similar to that which
was employed in calculating the total gas volume per particle, $V_p$, the particle size
distribution can be used to sum up the total volume over all particles. Let $f_j$ be
the frequency of occurrence of a microparticle of size $r_{p_j}$ (with total bins $K_p$, and
let $N_p$ be the total number of particles per unit volume. The total bubble volume
per unit liquid volume is calculated as
\[ V_t = \sum_{j=1}^{K_p} V_{pj} f_j N_p, \quad (4.23) \]
\[ = \sum_{j=1}^{K_p} f_j N_p \sum_{i=1}^{K_b} \left( \frac{4}{3} \pi r_{bi}^3 f_i N_{s_j} \right), \quad (4.24) \]
\[ = \sum_{j=1}^{K_p} f_j N_p \sum_{i=1}^{K_b} \left( \frac{4}{3} \pi r_{bi}^3 f_i \left( 4\pi r_{pj}^2 \beta_k \right) \right), \quad (4.25) \]
\[ V_t = \frac{16}{3} \pi^2 N_p \beta p_1 \sum_{j=1}^{K_p} \sum_{i=1}^{K_b} \left( f_j r_{pj}^2 f_i r_{bi}^3 \right). \quad (4.26) \]

Finally, the initial void fraction is calculated as the ratio of the volume of gas to the total volume:

\[ \alpha_i = \frac{V_t}{1+V_t}. \quad (4.27) \]

For the microparticles filtered from the JP-8, estimates of the range of \( \alpha_i \) were calculated to be \( (0.001 \% \leq \alpha_i \leq 0.0035 \%) \). These values are used as input to the one-dimensional C-D nozzle model [26] that calculates the void fraction as a function of distance along the nozzle, and will be described in the next section.

4.4 Quasi-1D Bubbly Flow in a Converging-Diverging Nozzle

To investigate possible explanations for the discrepancies between the fuel and water behavior, a steady, quasi-one dimensional model was utilized. This model is given by the system of algebraic-differential equations presented below. All variables have been non-dimensionalized by their values at the nozzle inlet, where \( R_o \) is the inlet bubble radius, \( u_o \) is the inlet axial velocity, \( \rho_o \) is the inlet density, \( P_o \) is the inlet pressure, \( A_o \) is the inlet nozzle area, and \( \alpha_o \) is the inlet void fraction.

\[ (1-\alpha)u A(x) = (1-\alpha_o), \quad (4.28) \]
\[
\frac{du}{dx} = -\frac{1}{2(1-\alpha)} \frac{dC_p}{dx}, 
\]  
(4.29)

\[
R \left( u^2 \frac{d^2R}{dx^2} + u \frac{du}{dx} \frac{dR}{dx} \right) + \frac{3u^2}{2} \left( \frac{dR}{dx} \right)^2 = 
- \left[ \frac{\sigma}{2} \left(1 - R^{-3\gamma}\right) + \frac{2}{Re} \frac{dR}{dx} + \frac{2}{We} (R^{-1} - R^{-3\gamma}) + \frac{1}{2} C_p \right]. 
\]  
(4.30)

The following non-dimensional groups are defined:

\[
\sigma = \frac{P_o - P_v}{\frac{1}{2} \rho_o u_o^2}, 
\]  
(4.31)

\[
C_p = \frac{P - P_o}{\frac{1}{2} \rho_o u_o^2}, 
\]  
(4.32)

\[
Re = \frac{\rho_o u_o R_o}{\mu}, 
\]  
(4.33)

\[
We = \frac{\rho_o u_o^2 R_o}{S}, 
\]  
(4.34)

where \( P_v \) is the liquid vapor pressure, \( \mu \) is the liquid viscosity, and \( S \) is the liquid surface tension. All flow variables are a function of the spatial coordinate \( x \) only. The polytropic index \( \gamma = 1 \) is used to model isothermal bubble compression. It has been shown experimentally that an isothermal polytropic process is appropriate in fuel and water [13]. Mass and momentum conservation for the flow are modeled by Equations 4.28 and 4.29, respectively. The bubble dynamics are modeled by Equation 4.30, which relates the bubble radius to the local pressure. This is then coupled back to the continuity and momentum equations through the void fraction, which is determined from the bubble radius by

\[
\alpha = \frac{\frac{4}{3} \pi \eta R^3}{1 + \frac{4}{3} \pi \eta R^3}, 
\]  
(4.35)
where $\eta$ is the initial bubble population per unit volume.

Equations 4.28 through 4.30, along with Equation 4.35 constitute four equations for the four unknowns $C_p$, $u$, $R$, and $\alpha$, with initial conditions $C_p(x = 0) = 0$, $u(x = 0) = 1$, and $R(x = 0) = 1$, and $\alpha(x = 0) = \alpha_o$. These equations were solved using a 4th-order Runge-Kutta integration algorithm and the upstream conditions $u_o = 10 \text{ m/s}$, $R_o = 100 \mu\text{m}$, and $\sigma = 0.9$, and $Re = 33$. For fuel simulations, $We = 352$, whereas for water, $We = 139$. Figure 4.11 shows sample solutions of the void fraction distribution for water for a range of initial void fractions from $\alpha_o = 0.001 \%$ to $0.0035 \%$.

![Figure 4.11. Numerically determined void fraction profile](image)

These void fraction profiles are shown to highlight the extreme sensitivity of the solution to the initial void fraction. As the flow accelerates through the throat of the nozzle, the pressure decreases and bubbles form. As $\alpha_o$ is increased, the peak
void fraction in the nozzle increases and the location of peak void fraction moves downstream. Above a critical $\alpha_o$, a bifurcation occurs and the solution becomes unstable, as the void fraction rapidly increases to a value of 100% (flashing to vapor). While the assumption of a uniform initial bubble size distribution is not realistic in the experiment, the effects of a non-uniform initial bubble size distribution on the nature of the flashing instability were found to be minor [25].

This flashing solution is not physically realizable, and is indicative of a change in the nature of the flow dynamics. It has been shown that the flashing instability for steady-state flow regimes correspond to unsteady shocked solutions when time dependent terms are included in the model [18]. The critical $\alpha_o$ value that results in the flashing solution can be used as an indicator of when shocked solutions might be expected. Figure 4.12 is a plot of the cavitation number, $\sigma$, and a function of the $\alpha_o$ that resulted in unstable solutions to the steady model presented above.

For cavitation numbers and initial void fractions below the plotted curves, stable solutions were found. At higher tensions and inlet void fractions, unstable solutions resulted. Thus, the two curves in the figure can be thought of as the dividing line between the absence and presence of bubbly shocks. The differences between the values between fuel and water are not significant and at first glance do not appear to offer any insight into why water flows include bubbly shocks and fuel flows do not. Since both the fuel and water experiments were conducted at the same back pressures (and thus similar cavitation numbers), the hydrodynamic factors were identical.

In the experiment, initial void fractions are manifested through the presence of trapped pockets of undissolved gas on microparticle impurities or cracks and pits along the walls of the test section apparatus. The role of solid impurities acting as cavitation nuclei has been studied extensively in the literature [1], [12], [14], [73].
Figure 4.12. Cavitation number ($\sigma$) as a function of critical initial void fraction ($\alpha_o$) for both JP-8 and H$_2$O.

[16], [15]. The total volume of gas present in these cavitation nuclei is strongly dependent on the ability of the liquid to wet all available surfaces. Liquids with high surface tensions (such as water) will be ineffective at wetting the concave surfaces structures associated with the nucleation sites, and will result in higher initial void fractions. Since the surface tension of jet fuel is three times lower than that of water, it should have a much smaller initial void fraction in the nozzle experiments. It is conjectured that the superior ability of fuel to reduce the number of potential nucleation sites through wetting could explain why the fuel experiments would be conducted in a flow regime where stable, unshocked flows are present. Conversely, the water experiments would be at initial void fractions large enough to produce unstable flows where shocks are present.
4.5 Conclusions

For identical back pressures, it was found that the nature of cavitation for jet fuel in a converging-diverging nozzle was drastically different than for the cavitation of water in the same nozzle. While both fluids became choked below a critical back pressure, cavitation bubbles for fuel persisted throughout the diffuser section of the nozzle, while a bubbly shock was observed for water. The presence of the bubbly shock in water (and the lack of one in fuel) was confirmed visually with high-speed video, and quantitatively with axial pressure and void fraction measurements.

By studying a quasi-one dimensional model for a cavitating nozzle flow, it was shown that the initial void fraction is an important parameter in determining the nature of the resulting solution. The model solutions were sensitive to the initial void fraction, becoming unstable and leading to a flashing solution above a critical value. Other models have shown that the flashing instability could be an indicator of flow solutions containing bubbly shocks when unsteady effects are included. It was hypothesized that the experimental differences between fuel and water could be attributed to a smaller initial void fraction in the fuel than in the water, due to the superior wetting capabilities of the fuel. This smaller initial void fraction could result in the fuel experiments being conducted in a regime of stable bubble growth. Alternatively, water is in an unstable flow regime, leading to the presence of bubbly shocks.
The cavitation of jet fuel in a converging-diverging nozzle was found to be a very complicated flow that featured several distinct two-phase flow regimes. For experiments with no artificial bubble injection, the void fraction fluid entering the inlet of the nozzle was low ($\alpha_o < 0.01\%$). As the flow accelerated, the pressure decreased to a value that was low enough to initiate cavitation just upstream of the nozzle throat. In the throat, spherical bubbles formed and grew explosively into large gas slugs. In water, these slugs collapsed over a very short distance. In fuel, the bubble collapse regime was distributed more spatially and the slugs broke down into a foam-like flow with a large number of small ($R < 100\,\mu m$) spherical bubbles. For comparison, water experiments revealed a noticeably different bubble collapse region, where the gas slugs collapsed abruptly to a dilute flow of larger ($R > 1\,mm$) bubbles.

Pressure and void fraction profiles of the cavitating flows revealed several important observations. For both jet fuel and water, the pressure and void fraction profiles were highly correlated spatially, where an increase in one resulted in a concomitant decrease in the other. Pressure data confirmed choked flow conditions for both fluids; the pressure values upstream of the bubble collapse region was completely unaffected by the back pressure at the nozzle exit. In addition, the location of the bubbly shock was found to move towards the nozzle exit as the nozzle back
pressure was lowered. The pressure increase (and corresponding void fraction decrease) attributed to the bubbly shock was found to be gradual for fuel and abrupt for water. Additional measurements with dodecane as the liquid exhibited a sharp pressure increase across the bubbly shock, highlighting the differences between fuel and water most likely were a consequence of the multi-component composition of the jet fuel.

Experiments were performed to determine the effect of the initial void fraction on the cavitating flow. The amount of dissolved gas in the liquid was found to have little effect on the pressure profiles or bubble shock location. Increasing the amount of natural nuclei in the liquid by adding iron oxide microparticles demonstrated a significant effect on the bubbly shock. Above a certain microparticle concentration threshold, cavitation initiation occurred in the bulk of the liquid (as opposed to solely from the nozzle walls). This resulted in a decrease in the thickness of the bubbly shock, determined by measuring the spatial \((x/L)\) distance over which the pressure increase from the shock was observed. These preliminary experiments in modifying the initial void fraction led to an additional series of experiments in which bubbles were injected directly in to the nozzle inlet, where the initial void fraction could be measured accurately, and varied over the range of \(\alpha_o = 0.5\%\) to \(\alpha_o = 2.5\%\). These bubble injection experiments revealed that the location of the bubbly shock (for both fuel and water) was extremely sensitive to the initial void fraction. Over the initial void fraction range studied, the bubbly shock could be moved from just downstream of the throat \((x/L = 0.45)\) to beyond the nozzle exit \((x/L > 1)\). Initial void fraction was found to have a stronger influence on shock location than nozzle back pressure.

A review of different models present in the literature determined that no single model could successfully predict all aspects of the cavitating flow. The close connec-
tion between pressure and void fraction that was observed in the experimental data suggested that a barotropic model, where the pressure in the nozzle is a function only of the void fraction, might be appropriate. It was found that for global flow parameters such as throat velocity, throat-to-inlet pressure ratio, and compressible bulk modulus, the barotropic model provided reasonable predictions. The barotropic model, however, was unable to predict bubbly shock behavior, and specifically the sensitivity of the position of the shock with initial void fraction that was found. Bubble dynamic effects, modeled by the Raleigh-Plesset equation, seemed consistent with portions of the experimental data. The Raleigh-Plesset equation exhibits a high sensitivity to initial bubble radius. Also, surface tension effects could possibly explain some of the differences between the bubbly shock behavior in fuel and water. Bubble collapse simulations revealed that fuel, with a surface tension 2.5 times less than water, should have bubble growth and collapse profiles that are more gradual than water. Bubble collapse simulations in water were found to be violent with a succession of bubble collapses and rebounds. Finally, a quasi-1D model coupling mass, momentum, and bubble dynamics again highlighted the extreme sensitivity of the flow on the initial void fraction. While direct comparisons of the pressure and void fraction profiles between experiment and model could not produce any reasonable agreement, the observation that both experiment and model feature a strong dependence on initial void fraction suggests that any successful model will need to include bubble dynamic effects.

This study has begun to fill the need for detailed experimental data for cavitating nozzle flows. The development of appropriate models depends on quality data, taken over a wide range of initial void fractions and nozzle back pressures. Modeling the cavitation of multi-component liquids such as jet fuel introduces an additional level of complexity that has not currently been addressed by any models. The results
presented here provide an archival set of measurements for both aviation fuel and water from which future models can be developed.
A radial disk geometry was studied to determine the effect of strain rate on the cavitation physics. Figures A.1 and A.2 are high-speed images of a radial disc flow experiment. The flow emerges out of the orifice at the center of the ring and is turned 90° to flow radially outward in a thin layer (∼250 µm) between two large circular disks. Higher strain rates than that found in the nozzle test section (and therefore smaller cavitation numbers) are caused by the rapid 90° turn. A circular bubbly shock (the white ring ending the cavitating region) can be seen in the photographs for both fuel and water. As opposed to the nozzle experiments, the shock is clearly visible in both liquids.

The pressure rise across the bubbly shock was measured for both liquids and plotted in Figure A.3. The spatial coordinate is normalized by the orifice diameter, and the pressure data is normalized by the pressure value upstream of the shock. Included on the figure is pressure data from the nozzle (where the spatial coordinate is normalized by the throat dimension). For both fuel and water, the pressure rise across the shock is greater, and the shock width is smaller for the disk experiment when compared to the nozzle. The higher strain rate of the turning flow generated a stronger, thinner shock.
Figure A.1. Cavitating radial disc flow with JP-8
Figure A.2. Cavitating radial disc flow with H$_2$O
Figure A.3. Pressure rise across bubbly shock.
APPENDIX B

PRESSURE MEASUREMENT UNCERTAINTY ANALYSIS

B.1 Design Stage Uncertainty

B.1.1 Zero-Order Uncertainty

A/D Resolution, \( u_0 \) (14-bit A/D with a 0 V to 10 V FSO):

\[
\begin{align*}
  u_0 &= \frac{Q}{2}, \\
  &= 0.5 \left( \frac{10 \text{ V}}{2^{14}} \right), \\
  &= 0.31 \text{ mV}.
\end{align*}
\]

B.1.2 Instrument Uncertainty

SETRA pressure transducer has stated accuracy of 0.25 % of full scale output. In volts, the transducer accuracy is:

\[
\begin{align*}
  u_1 &= 0.0025(10 \text{ V}), \\
  &= 25 \text{ mV}.
\end{align*}
\]

B.1.3 Summary

A summary of all design stage uncertainties is presented in Table B.1:
TABLE B.1

DESIGN STAGE UNCERTAINTY ESTIMATES

<table>
<thead>
<tr>
<th>$u_0$ (mV)</th>
<th>$u_1$ (mV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.31</td>
<td>25</td>
</tr>
</tbody>
</table>

The total design stage uncertainty, $u_d$, is the root sum square of the values in Table B.1:

$$u_d = \sqrt{u_0^2 + u_1^2},$$  \hspace{1cm} (B.6)

$$= \sqrt{0.31^2 + 25^2},$$  \hspace{1cm} (B.7)

$$= 25 \text{ mV.}$$  \hspace{1cm} (B.8)

B.2 Temporal Precision Errors

The temporal precision error, $u_t$, is found as a function of $x/L$ location from:

$$u_t = t_{\nu,P}S_x,$$  \hspace{1cm} (B.9)

where $S_x$ is the standard deviation of the voltage measurements taken from a time series of 100 ($\nu = 99$) data points. A value of 2 is used for the student-t factor. Results for selected axial locations (a bubbly shock exists at $x/L \approx 0.45$) are presented in Table B.2:
B.3 Total Uncertainty in Voltage Measurements

The total uncertainty in the voltage measurements, $u_V$, are calculated from the root sum square of the design stage uncertainty and the temporal precision error:

$$ u_V = \sqrt{u_d^2 + u_t^2}. \tag{B.10} $$

Results for the total voltage uncertainty are presented in Table B.3:

<table>
<thead>
<tr>
<th>$x/L$</th>
<th>0</th>
<th>0.3</th>
<th>0.4</th>
<th>0.45</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_v$ (mV)</td>
<td>31</td>
<td>33</td>
<td>47</td>
<td>221</td>
<td>155</td>
<td>118</td>
<td>124</td>
</tr>
</tbody>
</table>
presented in Table B.4:

<table>
<thead>
<tr>
<th>$x/L$</th>
<th>0</th>
<th>0.3</th>
<th>0.4</th>
<th>0.45</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_p$ (psi)</td>
<td>0.51</td>
<td>0.51</td>
<td>0.52</td>
<td>0.81</td>
<td>0.67</td>
<td>0.60</td>
<td>0.62</td>
</tr>
</tbody>
</table>

B.5 Pressure Ratio Uncertainty

\[ p^* = \frac{p}{p_o} \]  \hspace{1cm} (B.11)

\[ u_p^* = \sqrt{\left( \frac{\partial p^*}{\partial p} u_p \right)^2 + \left( \frac{\partial p^*}{\partial p_o} u_{p_o} \right)^2}, \]  \hspace{1cm} (B.12)

\[ = \sqrt{\left( \frac{u_p}{p_o} \right)^2 + \left( \frac{p}{p_o^2} u_{p_o} \right)^2}. \]  \hspace{1cm} (B.13)
BIBLIOGRAPHY


