AGB STAR NUCLEOSYNTHESIS OF THE MAGNESIUM ISOTOPES

Abstract

by

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$^{28}\text{Si}$ is formed by successive alpha captures on $^{12}\text{C}$ during the AGB (asymptotic giant branch) stage of stellar burning. The last reaction in this series, $^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$, has not been examined with sufficient sensitivity at alpha energies below 1.5 MeV. Several $^{28}\text{Si}$ states appear favorable for formation by this reaction in the alpha energy range of 1.0 - 1.5 MeV, motivating a new study of this reaction at the University of Notre Dame. To maximize experimental sensitivity, a high efficiency coincidence detection system was developed. Several previously unknown resonances were observed between 1.1 and 1.5 MeV, and an upper limit for any lower energy resonances was obtained. Resonance parameters were determined and reaction rates were calculated.

Additionally, neutron capture reactions on the stable magnesium isotopes were studied at the n_TOF facility. The data were fit using the R-matrix code SAMMY; newly calculated resonance parameters are presented.
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ACKNOWLEDGMENTS

This work was made possible by the help and support of many people. Thank you to my parents, Mike and Heather McNassar, for giving me a strong foundation and supporting all of my endeavors, scientific and otherwise. Thanks also to my siblings Anne and Patrick for your love and friendship. My husband Ian has kept me going when I wanted to give up, and helped me to achieve what I once thought was impossible.

Several people were instrumental in my decision to pursue a degree in physics research. My high school physics teacher Dr. Walter Hellman ignited my interest in physics with his quirky and innovative approach to teaching high school students. Dr. John Dash introduced me to experimental physics in his laboratory at Portland State University, where most of the research was conducted by high school interns. In college, I was assisted and encouraged by my advisor Dr. Fred Moore, who in particular helped me navigate the grad school application process. Thank you all for helping me to find my vocation.

Thank you to my advisor Dr. Michael Wiescher for giving me the opportunity to complete my own research project in nuclear astrophysics. I have enjoyed being a part of Michael’s research group, where the students are allowed to develop their own ideas and experiments, and act as mentors to each other. Thank you for teaching us to think big Michael!
Dr. Joachim Görres is an expert in experimental nuclear astrophysics, and he lends his knowledge to all of the group’s projects. Without him, I would have been lost in the lab and in my analysis many times. Thank you for putting up with us Joachim!

Thank you to the staff of the Nuclear Structure Lab for all of your help: to Dr. Larry Lamm for teaching me about the accelerators and vacuum systems, and many discussions about how to repair a decrepit low-energy accelerator; to Brad Mulder for getting the KN run the best it ever has (using a conveyer belt designed to transfer food, not charge, no less!); to Jim Kaiser for his help with anything electronic; and to Jerry Lingle for helping with all mechanical problems.

My fellow students have helped me in many ways, with their time, knowledge and friendship. Thank you to Dr. Ed Stech, Dr. Claudio Ugalde and Dr. Aaron Couture for being the “senior students” and teaching me about life in Michael’s group, as well as your generosity with your time and advice. Aaron in particular helped to make this experiment a success with his work on the detection system. Thank you to all the people who helped with the running of the experiments: Ed, Claudio, Aaron, Dr. Manoël Couder, Dr. Heide Costantini, Dr. Wanpeng Tan, Dr. Hye Young Lee, Annalia Palumbo, Mary Beard, Shawn O’Brien and P.J. LeBlanc. I am especially grateful for the support of Hye Young, both professional and personal.

This dissertation also benefited from the efforts of people outside the Nuclear Structure Lab. Dr. Kent Scheller, professor at the University of Southern Indiana and one of Michael’s first students, laid the foundation for the $^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$ experiment before I joined the group, and he continued to help through the end of the project. The n_TOF collaboration at CERN designed and ran the neutron capture experiment, and assisted with the data analysis. Special thanks to Dr. Paul Koehler for sharing his knowledge of SAMMY.

This work was financially supported by the University of Notre Dame and the National Science Foundation.
1.1 Stellar Evolution

Stars begin as clouds of gas, composed mainly of hydrogen and a small amount of helium, both produced in the Big Bang. Additionally, stars contain traces of other elements that were produced by previous generations of stars. If a gas cloud grows large enough, gravitational potential energy exceeds gas pressure and the cloud begins to contract. At first the heat generated by the collapse is radiated away, but as the cloud grows denser, the opacity increases and energy can no longer escape by radiation. The core of the cloud is heated by the continued collapse, until it reaches a temperature of $10^6$ K. At this point, protons in the core begin fusing to form helium, through a sequence of reactions called the proton-proton chain. The reactions of the proton-proton chain are detailed below:

\[ ^1\text{H} + ^1\text{H} \rightarrow ^2\text{H} + e^+ + \nu_e \]
\[ ^2\text{H} + ^1\text{H} \rightarrow ^3\text{He} + \gamma \]
\[ ^3\text{He} + ^3\text{He} \rightarrow ^4\text{He} + ^1\text{He} + ^1\text{He} \]

The energy produced by these nuclear reactions balances the gravitational energy to halt the collapse, and cloud has now become a stable, hydrogen burning star.

At higher temperatures, hydrogen burning may proceed by the CNO cycle, through which helium is formed using $^{12}\text{C}$ as a ‘catalyst’: 
\[ ^1\text{H} + ^{12}\text{C} \rightarrow ^{13}\text{N} \rightarrow ^{13}\text{C} + \text{e}^+ + \nu_e \]
\[ ^1\text{H} + ^{13}\text{C} \rightarrow ^{14}\text{N} \]
\[ ^1\text{H} + ^{14}\text{N} \rightarrow ^{15}\text{O} \rightarrow ^{15}\text{N} + \text{e}^+ + \nu_e \]
\[ ^1\text{H} + ^{15}\text{N} \rightarrow ^4\text{He} + ^{12}\text{C} \]

A star that is burning hydrogen in its core is said to be on the main sequence of the Hertzsprung-Russell (H-R) diagram. The H-R diagram plots stars’ surface temperature versus luminosity, and the main sequence is a distinct band formed by stars in the core hydrogen burning stage; The H-R diagram is shown in Figure 1.1. This is the longest lasting stage of a star’s life; however, the length varies depending on the mass of the star—a larger star requires more energy to maintain stability against gravity. Very small stars may remain on the main sequence for many billions of years, while the largest stars leave the main sequence after only a few tens of millions of years; the sun will spend about 10 billion years on the main sequence.

During the main sequence stage, hydrogen in the stellar core is being constantly converted into helium. The amount of hydrogen decreases, leading to a decrease in energy production, and eventually gravitational collapse resumes and the star leaves the main sequence. The temperature in the stellar interior begins to increase again, until hydrogen burning begins in a shell surrounding the helium core. Helium continues to accumulate in the core, fed by the hydrogen burning shell. When the temperature of the core reaches \(10^8\) K, helium burning through the triple-alpha process begins. The triple-alpha process is the fusion of three alphas to form \(^{12}\text{C}\). The energy produced by nuclear burning in the helium core and hydrogen shell causes the upper layers of the star to expand and cool, and the star can be classified as a red giant. During this stage the star may suffer from an outflowing of matter, called stellar winds.

Further evolution of a star depends on its initial mass. The reactions studied in this project take place in relatively low mass stars, and the description of further stellar
Figure 1.1 The H-R diagram is shown above. The band containing most of the points is the main sequence. The red giant branch is made of stars that have evolved off the main sequence, and the white dwarf region contains stars that have ceased nuclear fusion. The figure is courtesy of Rolfs and Rodney [36].

evolution will be limited to this mass range. Below $M = 8 \text{M}_\odot$, nuclear burning in the core will not proceed beyond the triple-alpha process. Similar to the end of core hydrogen burning, helium burning continues in an outward moving shell and the star enters the asymptotic giant branch (AGB) phase. Figure 1.2 illustrates the path a star takes on the H-R diagram from the main sequence to the asymptotic giant branch. An AGB star consists of a carbon-oxygen core, a thin helium burning shell, a helium intershell region, a thin hydrogen burning layer and a convective envelope. Burning in the shells is not stable or constant but cyclic in nature; instability is caused by the temperature sensitivity of the triple-alpha process. Additionally, electron degeneracy, which leads to decoupling of temperature and pressure in the helium shell, allows for greater temperature fluctuations and increased instability. The cyclic burning is called
Figure 1.2  The schematic evolution of a star from the main sequence to the asymptotic giant branch is shown above, for a star of mass $M \sim 1 \, M_\odot$. This figure is adapted from reference [24].

thermal pulsing, and it consists of four stages: off, on, power down and dredge up. During the off stage, energy is produced mainly by the hydrogen shell, and the helium intershell grows. The added material leads to increased burning in the helium shell, beginning the on stage. The energy produced by helium fusion pushes the hydrogen shell out, decreasing the temperature of the shell and causing hydrogen burning to cease. During the on stage, a convective region develops in the intershell and the products of helium burning ($^{12}$C and $^{16}$O) are mixed to the top of the intershell. During the power down stage, helium in the shell is exhausted, helium burning decreases and
the intershell convective region disappears. The decrease in energy production causes the outer layers to contract. The convective envelope moves inward and material from the intershell is mixed to the surface of the star. This mixing is called the third dredge up. Eventually the hydrogen shell will reignite, beginning the cycle again. Figure 1.3 shows the interior evolution of a thermally pulsing AGB star.

Figure 1.3 The evolution of the stellar interior is illustrated above, for one thermal pulse (top) and two consecutive pulses (bottom). This figure is adapted from reference [24].
The topic of this thesis, the $^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$ reaction, is relevant during the AGB phase. This reaction proceeds through a series of alpha captures on a seed of $^{12}\text{C}$; both $^4\text{He}$ and $^{12}\text{C}$ are abundant in AGB stars. The convective zone that develops during thermal pulses mixes $^{12}\text{C}$ from the core into the helium intershell, at temperatures around $T_9 = 0.1$. At this relatively low temperature, low energy resonances ($E_\alpha \sim 1$ MeV) will make important contributions to the reaction rate; thus, this experiment was completed to measure these resonances.

In particular, better characterization of the $^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$ reaction may lead to a better understanding of the silicon isotopic abundances observed in silicon carbide (SiC) meteorite grains, which are formed in the atmospheres of AGB stars [26]. The grains were expelled from previous generations of AGB stars by stellar winds, and were subsequently present in the material from which the solar system formed. Some of the grains were incorporated into meteors, without change of the grains’ composition, thus they are a unique source of information about AGB nucleosynthesis. The abundances of the $^{29}\text{Si}$ and $^{30}\text{Si}$ isotopes observed in these grains are higher than can be explained by predictions of neutron capture reactions in ABG stars [16]. To understand this discrepancy, rates for all reactions that produce and destroy the silicon isotopes must be known; this includes $^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$.

1.2 Stellar Reaction Rates

Stars can be thought of as “factories” for producing heavy elements from the building blocks of hydrogen and helium. In order to quantify these processes, the relevant quantities must first be defined. Then processes observed in laboratory experiments must be translated to the stellar environment.

Perhaps the most basic question to be answered about any nuclear reaction is: how often does this reaction occur in the stellar environment of interest? The reaction rate depends on the probability of the reaction to occur, a quantity known as the cross
section $\sigma$. This can be understood by the simple analogy of a dart board—the bigger the
board, the more likely it is to hit it with a dart. In fact, the cross section is given
classically by the size of the interacting particles as $\sigma = \pi (R_{\text{projectile}} + R_{\text{target}})^2$. However,
interacting nuclei are a quantum mechanical system, which introduces an energy
dependence (or, equivalently, a velocity dependence), $\sigma = \pi \hbar^2$. $\hbar$ is the de Broglie
wavelength of the reaction, defined as

$$\hbar = \frac{m + M}{M} \frac{h}{\sqrt{2mE}}, \quad (1.1)$$

where $m$ and $M$ are the mass of the projectile and the target, respectively, and $E$ is the
energy of the projectile. Other factors such as repulsion of like charges and the nature
of the force involved can also affect the cross section. Still, the larger the cross section,
the more likely a reaction is to occur. Cross section is usually given in units of barns (1
$b = 10^{-24}$ cm$^2$).

The reaction rate depends not only on the probability of the reaction, but also on
the number of particles present. Consider a gas of particle types $X$ and $Y$, with $N_X$ and
$N_Y$ particles per cm$^3$, and a relative velocity $v$ between the particles. We can arbitrarily
choose type $X$ to be targets at rest, and type $Y$ to be moving with velocity $v$. Then the
reaction rate is given by the number of type $X$ times the flux of type $Y (vN_Y)$ times the
reaction probability:

$$r = N_X N_Y v \sigma(v). \quad (1.2)$$

In a star, the velocity will not be a constant, but a distribution described by the function
$\phi(v)$. The rate equation becomes

$$r = \int_0^\infty N_X N_Y \phi(v) v \sigma(v) dv \equiv N_X N_Y \langle \sigma v \rangle. \quad (1.3)$$

The quantity $\langle \sigma v \rangle$ is called the reaction rate per particle pair.

The velocity distribution $\phi(v)$ of particles in a gas at temperature $T$ is given by a
Maxwell-Boltzmann distribution; the generic form is
Each type of reacting particle has its own velocity distribution, described by equation (1.4), and \(<\sigma_v>\) becomes a double integral over both distributions. For particle types \(X\) and \(Y\), with masses \(m_X\) and \(m_Y\) and velocity functions \(\phi(v_X)\) and \(\phi(v_Y)\), a transformation is made to the center-of-mass frame. The relevant quantities become the reduced mass \(\mu = m_Xm_Y/(m_X + m_Y)\), the total mass \(M = m_X + m_Y\), the center of mass velocity \(V\) and the relative velocity \(v\). The two velocity distributions are given by

\[
\phi(V) = 4\pi V^2 \left(\frac{M}{2\pi kT}\right)^{3/2} \exp\left(-\frac{MV^2}{2kT}\right)
\]

and

\[
\phi(v) = 4\pi v^2 \left(\frac{\mu}{2\pi kT}\right)^{3/2} \exp\left(-\frac{\mu v^2}{2kT}\right).
\]

The reaction rate per particle pair is

\[
<\sigma v> = \int_0^\infty \int_0^\infty \phi(V)\phi(v)v\sigma(v)dVdv.
\]

The integration over \(V\) can be immediately carried out, since the cross section depends only on \(v\). Inserting equation (1.6) for \(\phi(v)\) and substituting the center-of-mass energy \(E = \frac{1}{2}\mu v^2\) gives

\[
<\sigma v> = \left(\frac{8}{\pi\mu}\right)^{1/2} \left(\frac{1}{kT}\right)^{1/2} \int_0^\infty \sigma(E)E \exp\left(-\frac{E}{kT}\right)dE.
\]

### 1.3 Determining Reaction Rates from Laboratory Measurements

The final task remaining in the pursuit of the reaction rate is to determine the cross section \(\sigma(E)\). The form of \(\sigma(E)\) depends on the type of reaction being examined. Several different types of nuclear reactions are possible between two charged nuclei (reactions involving neutrons will not be considered now). These reactions include
scattering (elastic and inelastic), direct capture and resonance reactions. In order to fuse, two charged particles must overcome the Coulomb repulsion between them. For the \(^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}\) reaction the Coulomb barrier height is nearly 6 MeV. However, the energy \(E = kT\) corresponding to an AGB thermal pulse is around 10 keV; thus the reaction proceeds only through quantum mechanical tunneling. This is the case for all fusion reactions during quiescent stellar burning. In fact, the improbable nature of stellar fusion allows stars to live for billions of years—if nuclei were energetic enough to react without tunneling, the entire core would react instantaneously and destroy the star!

The \(^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}\) reaction is a resonance reaction in the alpha energy range studied in this experiment. Unlike scattering and direct capture, resonance reactions occur only at certain energies, where the energy of the reacting particles and the reaction Q-value add up to the energy of an excited state in the final nucleus. \(\sigma(E)\) varies smoothly with \(E\) in a direct reaction, but changes rapidly (and sometimes becomes very large) in a resonance reaction. Reactions may have both direct and resonance components, but in the case of \(^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}\) at \(E_{\alpha} = 1.0-1.5 \text{ MeV}\), only resonance features were observed. Therefore, the following derivation of \(\sigma(E)\) will be limited to resonance reactions.

Resonance reactions are two-step processes; the first step is fusion of the projectile and target nuclei to form an excited compound nuclear state, and the second step is the decay of the compound nucleus. Cross sections of resonance reactions are given by the Breit-Wigner formula:

\[
\sigma_{\text{BW}}(E) = \pi \hat{\lambda}^2 \frac{2J + 1}{(2J_1 + 1)(2J_2 + 1)} \frac{\Gamma_a \Gamma_b}{(E - E_R)^2 + (\Gamma/2)^2}.
\]

The \(\pi \hat{\lambda}^2\) term comes from the basic geometrical definition of \(\sigma\). The next term, which is abbreviated as \(\omega\), is a statistical factor accounting for the angular momentums of all the particles involved \((J, J_1 \text{ and } J_2\) correspond to the excited compound nucleus,
projectile and target, respectively). The final term is a response function that describes the resonant nature of the system. $\Gamma_i$ is known as the partial width, and the total width $\Gamma = \sum \Gamma_i$.

The reaction rate per particle pair can now be written as

$$\langle \sigma v \rangle = \left( \frac{8}{\pi \mu} \right)^{1/2} \left( \frac{1}{kT} \right)^{3/2} \int_0^\infty \sigma_{\text{BW}}(E) E \exp\left( \frac{-E}{kT} \right) dE.$$  \hspace{1cm} (1.10)

For a narrow resonance ($\Gamma \ll E_R$) the Maxwell-Boltzmann distribution is essentially constant over the energy range where $\sigma$ is non-zero, so it can be moved outside the integral. This fact is illustrated in Figure 1.4. Integrating the Breit-Wigner cross section leads to

$$\langle \sigma v \rangle = \left( \frac{2\pi}{\mu kT} \right)^{3/2} \hbar^2 (\omega \gamma) \exp\left( \frac{-E_R}{kT} \right),$$  \hspace{1cm} (1.11)

with $\omega$ as defined above and $\gamma = \Gamma_a \Gamma_b / \Gamma$. The quantity $\omega \gamma$ is called the resonance strength, and it is this quantity which was extracted from the experimental data to determine the reaction rate in this project.

It is evident from the $\exp(-E_R/kT)$ term that resonances at $E_R$ much higher than $kT$ will usually not have a large effect on the reaction rate, and resonances near $kT$ dominate the rate. This is the reason that nuclear astrophysicists often study reactions in an energy range that may be considered low in comparison to other nuclear physics experiments. The need to probe low energies complicates the nuclear astrophysicist’s task: lower projectile energy equates to lower probability for the projectile to tunnel through the Coulomb barrier. In fact, the cross section decreases exponentially as the projectile energy falls below the Coulomb barrier. Thus experiments may require high beam currents (and targets that are durable enough to withstand the high currents), long measuring times and complicated coincidence detection setups. Even using all of these
methods, it is usually not possible to measure reactions in the stellar energy range, and the cross section must be extrapolated.

Finally, the resonance parameters described above must be related to some quantities that can be directly measured. For the $^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$ experiment, the measured quantity was the yield, which is defined as the number of reactions per incident alpha. Therefore, the yield depends on the cross section and the number of target particles; for a thick target, where the target thickness is much greater than the width of the resonance, the yield is given by

$$Y(E_0) = \int \sigma(E) ndx = \int_{E_0-\Delta}^{E_0} \frac{\sigma(E)}{\varepsilon(E)} dE . \quad (1.12)$$

$E_0$ is the incident beam energy, and $\Delta$ is the target thickness. $\alpha(E)$ is called the stopping cross section, and it depends on the target material. $\sigma(E)$ is the Breit-Wigner cross section.
section. For a narrow resonance, the quantities $\vec{\chi}^2$, $\Gamma_i$, $\Delta$ and $\epsilon$ are approximately constant over the energy range of the resonance, and the yield becomes

$$Y(E_0) = 2\pi\vec{\chi}^2 \omega \gamma \frac{1}{\epsilon} \left[ \arctan \left( \frac{E_0 - E_E}{\Gamma/2} \right) - \arctan \left( \frac{E_0 - E_E - \Delta}{\Gamma/2} \right) \right].$$  (1.13)

The maximum of this function, which corresponds to the height of the Breit-Wigner curve plateau, is

$$Y_{\infty} = \frac{2\pi^2 \vec{\chi}^2}{\epsilon} \omega \gamma.$$  (1.14)

$Y_{\infty}$ is called the thick target yield. From this expression, it is evident that if the plateau height is measured, the resonance strength, and, ultimately, the reaction rate, can be calculated using equation (1.11).

1.4 Properties of the $^{24}$Mg($\alpha,\gamma$)$^{28}$Si Reaction

The $^{24}$Mg($\alpha,\gamma$)$^{28}$Si reaction has a Q-value of 9.984 MeV; alpha energies in the region of 1 MeV form $^{28}$Si nuclei with excitation energies of around 11 MeV. These nuclei decay by gamma emission to the stable ground state of $^{28}$Si. The level density in $^{28}$Si at energies near 11 MeV is around 20 levels per MeV. However, there is a restriction on which levels can be formed by the $^{24}$Mg($\alpha,\gamma$)$^{28}$Si reaction. $^{24}$Mg and $^4$He both have spin angular momentum $s = 0$ and parity $\pi = +$. In general, the total angular momentum of a reaction product is given by $J = s_1 + s_2 + l$. $s_1$ and $s_2$ are the spins of the reacting particles, $l$ is the orbital angular momentum of the reaction, and the bold type indicates that the two spins add vectorially: $|s_1 - s_2| \leq J \leq s_1 + s_2$. In the particular case of $^{24}$Mg($\alpha,\gamma$)$^{28}$Si, the spin of the $^{28}$Si nucleus can have only one value, equal to the orbital angular momentum $l$. Additionally, the parity of a reaction product is given by $(-1)^l$. Thus in the $^{24}$Mg($\alpha,\gamma$)$^{28}$Si reaction, only so-called natural parity states can be formed: the compound nucleus must have $J^\pi = 0^+, 1^-, 2^+, 3^-$, etc...
If the spins and parities of $^{28}\text{Si}$ levels with excitation energies around 11 MeV are known, then it is possible to calculate alpha energies that may form resonances. Indeed, most of the levels in the energy range of interest have known spin and parity. This information has been determined from other reactions with higher cross sections, such as $^{27}\text{Al}(p,\gamma)^{28}\text{Si}$ and $^{28}\text{Si}(p,p')$. TABLE 1.1 summarizes the most recent determination of level energies and $J^\pi$'s [14]. Also shown are the alpha energies required to form these levels, and a prediction of whether the level is likely to form a resonance.

### 1.5 Previous Measurements of $^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$

The first experiment to explore the $^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$ reaction was performed in 1962 by Smulders and Endt [37]. The goal of the experiment was to use alpha capture as a spectroscopic tool, and the $^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$ reaction was chosen because it has a high Q-value, and thus produces gamma rays with energies high enough to be seen above the neutron background produced by the $^{13}\text{C}(\alpha,n)^{16}\text{O}$ reaction. A 10x10 cm NaI detector was used, with beam currents ranging from 3 to 8 $\mu$A impinging on evaporated Mg targets. Angular distributions were examined using one fixed and one rotating counter, in order to determine the spin and parity of the $^{28}\text{Si}$ levels. The lowest energy resonance was observed at 1529 keV, with $\omega\gamma = 0.11$ eV and $J^\pi = 1^-$. Nineteen other resonances were observed in the alpha energy range from 1.5-3.2 MeV, most of which were determined to correspond to states previously known from the $^{27}\text{Al}(p,\gamma)^{28}\text{Si}$ reaction.

In 1969, Lyons investigated the $^{28}\text{Si}$ nucleus using both $(p,\gamma)$ and $(\alpha,\gamma)$ reactions [28],[29]. Unlike the Smulders and Endt experiment, this study had an astrophysical motivation: determination of the photodisintegration cross section of $^{28}\text{Si}$. This photodisintegration is thought to occur after oxygen burning in massive stars, and produce alphas and other nuclei that fuse to form iron group elements. The gamma ray yield was measured with 2 NaI detectors, placed perpendicular to the beam on opposite
<table>
<thead>
<tr>
<th>$E_{\text{ex}}$ (keV)</th>
<th>$J^\pi$</th>
<th>$E_\alpha$ (cm)</th>
<th>$E_\alpha$ (lab) (keV)</th>
<th>Favorable</th>
</tr>
</thead>
<tbody>
<tr>
<td>10181.60</td>
<td>3-</td>
<td>197.4</td>
<td>230.3</td>
<td>yes</td>
</tr>
<tr>
<td>10189.59</td>
<td>5-</td>
<td>205.4</td>
<td>239.6</td>
<td>yes</td>
</tr>
<tr>
<td>10209.01</td>
<td>3+</td>
<td>224.8</td>
<td>262.3</td>
<td>no</td>
</tr>
<tr>
<td>10272.3</td>
<td>0+</td>
<td>288.1</td>
<td>336.1</td>
<td>no</td>
</tr>
<tr>
<td>10310.92</td>
<td>4+</td>
<td>326.7</td>
<td>381.2</td>
<td>yes</td>
</tr>
<tr>
<td>10376.24</td>
<td>3+</td>
<td>392.0</td>
<td>457.3</td>
<td>no</td>
</tr>
<tr>
<td>10418.25</td>
<td>5+</td>
<td>434.1</td>
<td>506.5</td>
<td>no</td>
</tr>
<tr>
<td>10514.1</td>
<td>2+</td>
<td>529.9</td>
<td>618.2</td>
<td>yes</td>
</tr>
<tr>
<td>10541.0</td>
<td>3-</td>
<td>556.8</td>
<td>649.6</td>
<td>yes</td>
</tr>
<tr>
<td>10596.18</td>
<td>1+</td>
<td>612.0</td>
<td>714.0</td>
<td>no</td>
</tr>
<tr>
<td>10668.05</td>
<td>3+(2+)</td>
<td>683.9</td>
<td>797.9</td>
<td>---</td>
</tr>
<tr>
<td>10668.34</td>
<td>4+</td>
<td>684.1</td>
<td>798.1</td>
<td>yes</td>
</tr>
<tr>
<td>10724.7</td>
<td>1+</td>
<td>740.5</td>
<td>863.9</td>
<td>no</td>
</tr>
<tr>
<td>10778</td>
<td>1+ - 5+</td>
<td>793.8</td>
<td>926.1</td>
<td>---</td>
</tr>
<tr>
<td>10805.5</td>
<td>2+</td>
<td>821.3</td>
<td>958.2</td>
<td>yes</td>
</tr>
<tr>
<td>10883.45</td>
<td>2+(2-,3+)</td>
<td>899.3</td>
<td>1049.1</td>
<td>---</td>
</tr>
<tr>
<td>10900.42</td>
<td>1+</td>
<td>916.2</td>
<td>1068.9</td>
<td>no</td>
</tr>
<tr>
<td>10915.7</td>
<td>3-</td>
<td>931.5</td>
<td>1086.8</td>
<td>yes</td>
</tr>
<tr>
<td>10944.0</td>
<td>4+</td>
<td>959.8</td>
<td>1119.8</td>
<td>yes</td>
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<tr>
<td>10952.8</td>
<td>2+</td>
<td>968.6</td>
<td>1130.0</td>
<td>yes</td>
</tr>
<tr>
<td>10994</td>
<td>1+(1-,2+)</td>
<td>1099.8</td>
<td>1178.1</td>
<td>---</td>
</tr>
<tr>
<td>11078.52</td>
<td>3-</td>
<td>1094.3</td>
<td>1276.7</td>
<td>yes</td>
</tr>
<tr>
<td>11100.0</td>
<td>6+</td>
<td>1115.8</td>
<td>1301.8</td>
<td>yes</td>
</tr>
<tr>
<td>11142</td>
<td>2+</td>
<td>1157.8</td>
<td>1350.8</td>
<td>yes</td>
</tr>
<tr>
<td>11195.22</td>
<td>4+</td>
<td>1211.0</td>
<td>1412.9</td>
<td>yes</td>
</tr>
<tr>
<td>11242</td>
<td>---</td>
<td>1257.8</td>
<td>1467.4</td>
<td>---</td>
</tr>
<tr>
<td>11265.0</td>
<td>3-</td>
<td>1280.8</td>
<td>1494.3</td>
<td>yes</td>
</tr>
<tr>
<td>11295.4</td>
<td>1-</td>
<td>1311.2</td>
<td>1529.7</td>
<td>yes</td>
</tr>
</tbody>
</table>

NOTE: Level energies and spin-parity assignments are taken from reference [14]. Favorability determination is based on spin-parity, as the $^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$ reaction may only form states with natural parity.
sides of the target chamber. The $^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$ reaction was observed at alpha energies below 2.8 MeV, using evaporated Mg targets and beam currents of 2-6 $\mu$A. Above 1.5 MeV, the yield curve agreed with the data obtained by Smulders and Endt, but an additional feature was discovered at $E_\alpha \approx 1.35$ MeV. Further study suggested a resonance at this energy, but the weakness of the resonance precluded more thorough characterization. Lyons settled on a resonance energy of $1358 \pm 7$ keV and a resonance strength of 0.0019 eV. (For the 1530 resonance of Smulders and Endt, Lyons gave a resonance energy of 1533 keV and a strength of 0.081 eV.) The photodisintegration rates through the $p_0$ and $\alpha_0$ channels were calculated for $T_9 = 1.5-3.0$.

Another study of the $^{28}\text{Si}$ nucleus by $(p,\gamma)$ and $(\alpha,\gamma)$ was done by Maas et al in 1978 [30]. This experiment improved upon the previous work by replacing the NaI crystals with high-resolution Ge(Li) detectors. The strengths and branching ratios of the resonances were measured, and $(p,\gamma)$ and $(\alpha,\gamma)$ resonances were compared to more precisely determine the energies of the corresponding $^{28}\text{Si}$ levels. Unfortunately, the low-energy resonance found by Lyons at 1358 keV was not examined. However, the Smulders and Endt value of $\omega\gamma = 0.11$ eV for the 1530 keV resonance was confirmed. Other groups have since examined the $^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$ reaction further, as a spectroscopic tool, but none have studied alpha energies below 1.5 MeV, to confirm the resonance found by Lyons and to search for new resonances. Lyons’ NaI experiment could be improved upon using modern germanium detectors. The published values for the lowest energy resonance parameters are listed in TABLE 1.2.
### TABLE 1.2

$^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$ LOW-ENERGY RESONANCE PARAMETERS

<table>
<thead>
<tr>
<th>$E_\alpha$ (keV)</th>
<th>$E_{ex}$ (keV)</th>
<th>$\omega\gamma$ (eV)</th>
<th>$J^\pi$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1358.7</td>
<td>11148.7</td>
<td>0.00196</td>
<td>---</td>
<td>Lyons [29]</td>
</tr>
<tr>
<td>1530.03 15</td>
<td>11295.4 4</td>
<td>0.112</td>
<td>1$^-$</td>
<td>Maas et al [30]</td>
</tr>
<tr>
<td>1786.60 20</td>
<td>11515.3 4</td>
<td>0.061</td>
<td>2$^+$</td>
<td>Maas et al [30]</td>
</tr>
</tbody>
</table>

NOTE: The alpha and level energies listed in this table are taken from the listed references, and may vary from the most recent values, listed in TABLE 1.1.
CHAPTER 2
EXPERIMENTAL PROCEDURES

The $^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$ experiment consisted of two separate runs, each approximately two weeks long and performed one year apart. The entire experiment, including target preparation, was carried out in the Nuclear Structure Laboratory at the University of Notre Dame. Both runs utilized the KN accelerator and a detection system comprised of four sodium iodide detectors and one high purity clover germanium detector, though the detector arrangement differed for the two runs. The first run was mainly a test of the setup, to determine its sensitivity to observe weak resonances. This preliminary work revealed new features in the $^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$ yield curve. Using these observations as a guide, experimental parameters were optimized in an attempt to fully characterize the new features in the second run.

In both runs, a coincidence detection scheme was used in order to reduce background in the gamma ray spectrum. The $^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$ reaction produces $^{28}\text{Si}$ in an excited state, at around 11 MeV. The nuclei then decay by emitting one or more gamma rays, as illustrated in Figure 2.1. For all $^{28}\text{Si}$ states produced in this experiment, at least some percentage of the decays include the transition from the first excited state at 1.779 MeV to the ground state. A coincidence was formed by observing a high energy (>3 MeV) gamma ray in the sodium iodide detectors and the corresponding 1.779 MeV gamma in the clover. By only counting events where this coincidence was observed, the background around the 1.779 line in the clover is reduced up to three orders of magnitude.
Figure 2.1 Examples of decays from an excited $^{28}\text{Si}$ state are shown above. Most decay cascades include the 1779 keV gamma ray.

2.1 Accelerator and Beam Line

The alpha beam was produced by the KN model Van de Graaff accelerator. This machine is capable of producing ion beams with energies from 1.0 to 3.5 MeV, and currents up to 150 $\mu$A on target. In this experiment, the highest alpha energy needed was 1.6 MeV, but the maximum current was often required. The energy resolution of the accelerator was measured to be 2.4 keV for alphas.

The KN accelerator is based on the principle of the Van de Graaff generator. The basic parts are a charging supply, a high voltage terminal, and an insulating belt that runs between pulleys from the charging supply to the terminal. Positive charge is transferred from the charging supply to the belt by coronal discharge from a set of sharp
comb-like points. The belt physically carries the charge to the terminal, which is a hollow conducting shell. The terminal contains another set of points. The positive charge on the belt induces a negative charge on the terminal points, and the charge from the belt travels through the points to the surface of the terminal. The result is a relatively simple system that is capable of producing potentials of millions of volts. A schematic drawing of a Van de Graaff generator can be seen in Figure 2.2.

Figure 2.2 The basic concept of a Van de Graaff generator is illustrated above. Charge is transported from the charging supply to the high voltage terminal by an insulating belt. This figure is taken from reference [39].
A positive ion source is placed inside the terminal shell. The KN accelerator employs an RF ion source, which consists of a small glass bottle and RF electrodes. The RF field causes electrons inside the bottle to oscillate, and they collide with and ionize source gas atoms. The ions are repelled out of the terminal shell and down the accelerator tube. A series of electrodes separated by resistors connects the terminal back to neutral ground, providing a uniform voltage gradient which accelerates the ions evenly along the length of the tube, and also focuses the ion beam.

After exiting the accelerator, the ion beam is focused and steered by various electrostatic and magnetic elements. To filter out contaminant ions and to select the exact energy of the beam, it is passed through an analyzing magnet, which takes advantage of the fact that charged particles have a curved trajectory through magnetic fields. Specifically, the radius of the trajectory is given by

\[ r = \frac{\sqrt{2mE}}{qB}. \]  

The radius \( r \) is fixed by the geometry of the magnet, and \( B \) is adjusted to allow particles of the desired mass and charge state to have the proper trajectory to pass through the magnet.

Just before arriving at the target, the beam passed through a liquid-nitrogen-cooled copper pipe. This cold finger removed any carbon that was traveling with the beam. The beam was wobbled to produce a beam spot size of 1.6 x 1.0 cm on the target. The wobbler consists of two steerers (horizontal and vertical) which sweep the beam across the surface of the target to prevent localized overheating by the highly-focused beam. The target was mounted on a brass target holder, which allowed for water cooling of the target. The target holder was electrically isolated from the rest of the beam line, and the charge deposited by the beam was collected in order to determine the total number of alphas delivered to the target. To prevent electrons from escaping
from the target, and interfering with the charge measurement, the cold finger was biased with a negative potential of 300 volts.

2.2 Magnesium Targets

Significant work was done to produce magnesium targets. Due to the low cross section of the $^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$ reaction in the alpha energy range of interest, the experiment required targets that would remain stable at alpha beam currents over 100 $\mu$A, for a reasonable amount of time (i.e. 5-6 C of accumulated charge). Several target productions methods were tested: sputtering, implantation and evaporation. A satisfactory combination of plateau height and target profile was obtained only with evaporated targets.

Often for $(p,\gamma)$ and $(\alpha,\gamma)$ experiments requiring a beam-stop target, tantalum backings are used because tantalum does not react with the beam. Initially tantalum backings were tested, but were found to be unusable due to blistering of the surface by the high current alpha beam. In this experiment, because the alpha energy range was quite low, it was possible to substitute oxygen-free copper without observing reactions between the backing and the beam. The copper showed no surface damage, provided it was no thicker than 0.02 in.

In initial tests with copper, however, only 20 percent of the copper backings produced acceptable evaporated targets. This problem was traced to the cleanness of the copper surface. A common cleaning process for tantalum target backings is ‘baking:’ passing an electrical current through the backing, heating it to red-hot and burning off surface dirt and oils. This method can not be applied to copper, due to its high electrical conductivity. Etching the copper surface slightly by soaking overnight in a 5% acetic acid solution increased the percentage of useable targets to 90%. After soaking in the acid, the backings were rinsed with deionized water, acetone and ethanol
to remove any debris and water spots, then dried with a heat gun. The backings were then placed in the evaporator to be used immediately.

The evaporation source material was 99.99% pure natural magnesium pieces; natural magnesium is composed of 78.99% $^{24}\text{Mg}$, 10.00% $^{25}\text{Mg}$ and 11.01% $^{26}\text{Mg}$. The backings were placed approximately 10 cm above the tantalum evaporation boat, and the evaporator bell jar was pumped down to $\sim 2 \times 10^{-6}$ T. A liquid nitrogen baffle was used to prevent backstreaming of oil from the diffusion pump. Once the operating vacuum was reached, the current through the boat was slowly increased until evaporation began. The thickness of the evaporated film was measured by a quartz crystal monitor. The current was increased as needed to maintain evaporation, until the desired thickness was deposited; the evaporation typically took less than 10 minutes.

After the evaporation was complete, the targets were allowed to cool for at least 15 minutes, then the bell jar was vented with argon. The targets were immediately transferred to an argon-filled container, where they were stored until they were used. The targets were stored in argon to prevent oxidization of the magnesium coating. Even in the argon environment, targets were found to oxidize over a period of several months, so targets were generally used within one week of production.

### 2.3 Detection System

The detection system used for both experiments consisted of four thallium-doped sodium iodide (NaI(Tl)) detectors and a high-purity germanium (HPGe) clover detector. These two detector types operate on different principles, and have different properties which make their use advantageous in different situations. The detectors will be described in generalized terms, followed by a discussion of the specific setups used in the $^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$ experiments.

The purpose of all gamma ray detectors is to convert incident gamma rays into electrical signals that can be analyzed as desired by the experimenter. The manner in
which the conversion occurs is particular to the type of detector. However, two features are often required in gamma ray spectroscopy experiments: information on the energy of the gamma ray and the time at which it was detected. The first requirement leads to the choice of a detector material that produces an electrical pulse with magnitude proportional to the incident gamma energy. The second constraint requires a material in which the electrons produced can be gathered quickly into the output signal. Additionally, the detector should be as efficient as possible at converting incident gammas into output pulses.

2.3.1 Detector Types

NaI(Tl) detectors operate on the principle of scintillation. When a gamma ray enters a scintillation detector, it excites electrons in the scintillator molecules. These excited electrons promptly decay by photon emission, through a process known as fluorescence. In a NaI(Tl) crystal, the light output is linearly related to the incident gamma energy above ~400 keV [25]. These photons are collected by a cathode made from photosensitive material, which produces electrons through the photoelectric effect. The photoelectrons are then multiplied into an output pulse by a photomultiplier. A photomultiplier consists of a chain of electrodes, called dynodes, to which a high voltage is applied. A photoelectron impacts the first dynode, and the energy from the original electron allows many (~30-50) electrons to escape from the dynode. Some of these electrons strike the next dynode, again multiplying the original signal, and so on, until a large number (~10^7) of electrons are collected at the final electrode, forming an electrical pulse that is then recorded and analyzed by the experimenter.

HPGe detectors are semiconductors, rather than scintillators. A semiconductor is a material that has a small (~1 eV in germanium) energy gap between the valence band and the conduction band (for contrast, an insulator has a larger band gap, while in a conductor the valence and conduction bands overlap). When a gamma ray passes
through a semiconductor, it excites electrons from the valence band to the conduction band, and leaves holes in the valence band. A bias voltage is applied across the crystal to sweep the charge carriers to the collection electrodes. To avoid thermal excitation of electrons across the band gap, the detector crystal is cooled to liquid nitrogen temperature.

Scintillator and semiconductor detectors have different properties, which make them useful for different purposes. Scintillators, NaI(Tl) in particular, have a much higher efficiency than semiconductors. Additionally, scintillators are much less expensive than semiconductors, and are available in larger sizes. These effects are due to the high purity necessary for semiconductor detectors, making their manufacturing more difficult and costly. Scintillators also have a faster response time, making them preferable for some high-rate experiments. They are also simpler to use, because they do not require cooling.

Semiconductor detectors are favored for their superior energy resolution, which is easily ten times better than scintillators. This is due to the fact that the number of electron-hole pairs produced by a gamma ray is nearly two orders of magnitude larger that the number of photoelectrons produced in a scintillator. The limitation on crystal size can be compensated for by mounting several crystals in the same housing; a clover detector consists of four crystals, arranged such that the cross section of the detector resembles a four-leaf clover. In low-rate experiments, such as $^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$, the longer response time of a semiconductor does not negatively affect the detection of gamma rays. Properties of the detectors used in this experiment are listed in table TABLE 2.1.

2.3.2 Interaction of Gamma Rays with Detectors

Ideally, all gamma rays that enter a detector would be fully absorbed, leading to a gamma ray spectrum consisting of a single sharp peak at the gamma ray energy. In reality, there are three mechanisms through which a gamma ray may interact with
TABLE 2.1
PROPERTIES OF DETECTORS USED IN $^{24}$Mg($\alpha,\gamma$)$^{28}$Si EXPERIMENT

<table>
<thead>
<tr>
<th>Detector Type</th>
<th>Active Volume (cm$^3$)</th>
<th>Resolution at 1332 keV (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NaI(Tl)</td>
<td>4942 (8” diameter x 6” length)</td>
<td>64</td>
</tr>
<tr>
<td>HPGe Clover (add-back)</td>
<td>~650 (~4.5cm diameter x 8cm length)</td>
<td>3.0</td>
</tr>
</tbody>
</table>

 detector material: photoelectric absorption, Compton scattering, and pair production. In photoelectric absorption, the gamma ray energy is fully absorbed by a detector atom. A photoelectron is released, with energy equal to the gamma ray energy minus the electron binding energy. The energy of the photoelectron is absorbed by the surrounding material, and the full energy of the gamma ray is registered by the detector, leading to the desired sharp peak at that energy. This effect is dominant at low gamma energies (below 1 MeV).

In Compton scattering, a gamma ray imparts some of its energy to an electron, resulting in a less energetic photon and a scattered electron with energy equal to the energy lost by the photon. The photon can then undergo further scattering, photoelectric absorption, or it can escape from the detector. The amount of energy deposited in the detector can range from the full energy of the incident photon down to essentially zero. These interactions lead to a broad feature known as the Compton continuum in the gamma ray spectrum. Compton scattering is the most likely interaction in the mid-energy range, from ~ 500 keV to 5 MeV.

In pair production, a gamma ray is converted to an electron-positron pair in the Coulomb field of a detector material nucleus. This process requires that the incident gamma have energy at least equal to the mass of the electron-positron pair, 1022 keV; pair production is the dominant interaction above $E_\gamma = 5$ MeV. In Figure 2.3, the energy ranges of all three interaction types can be seen. The electron and positron carry the
gamma ray energy minus the 1022 keV required to produce the pair. If both particles are absorbed by the detector, a signal at the full gamma ray energy is formed. However, one or both of the particles may escape from the detector, leading to peaks at 511 keV and 1022 keV below the full energy. These are called the first and second escape peaks.

An example of a gamma ray spectrum can be seen in Figure 2.4. Different areas of the spectrum are used to determine the different types of efficiencies. The total efficiency of a detector is defined as the ratio between the total number of counts in the spectrum and the number of photons emitted from the gamma ray source. The photopeak efficiency is the ratio between the number of counts in the photopeak and the number of gamma rays with energy equal to the photopeak energy emitted. The calculation of these efficiencies will be described in the following chapter.
Figure 2.4  A model detector spectrum for monoenergetic gamma rays can be seen above. The figure is courtesy of Krane [20].

2.3.3 Detector Array

In both $^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$ experimental runs, a coincidence detection scheme was used in order to reduce background in the gamma ray spectrum. The four NaI(Tl) detectors were mounted in a common stand, and each was rotated 45 degrees in polar and azimuthal angles, with respect to the beam. Additionally, each detector was shifted ‘clockwise’ 1.5”, so that the line perpendicular to each detector face was not pointing directly to the target. This allowed the detectors to be moved 1.0” closer to the target, thus increasing the solid angle coverage. This arrangement is illustrated in Figure 2.5. The HPGe clover detector was placed behind the target, with its face parallel to the target.

The configuration of the NaI(Tl) detectors, the clover and the target was different for the two experiments. For the first experiment, a 0° setup was used. The
Figure 2.5  The offset configuration of the NaI(Tl) detectors is shown, with the NaI(Tl) crystals in green and the target holder in blue. The direction of the beam is nearly normal to the page, toward the reader. The figure is courtesy of A. Couture [8].

NaI(Tl) stand was placed perpendicular to the beam, as was the target. The clover detector was placed with its face against the back of the target holder. The target holder was specially designed with the water cooling lines exiting the front of the holder, so that the back was smooth and the clover could be placed as close as possible to the target. A thin Teflon sheet was placed between the target holder and the detector to maintain electrical isolation of both the detector and the target chamber assembly. The face of the detector housing was 13 mm away from the target.

For the second experimental run, the setup was modified to a 45° arrangement. The NaI(Tl) stand was rotated to 59° from the beam axis—this was the smallest angle that could be achieved before the beam line and the detector stand collided. The two detectors on the beam-right side of the stand were moved 1.0” further from the target to facilitate the rotation. A 45° target chamber was used, and the clover detector was positioned with its face parallel to the target. The face of the detector housing was 19
Figure 2.6 The two detector setups are shown. The 0° configuration is on the left; the beam is moving from right to left across the photo. The 45° configuration is on the right; the beam direction is normal to the page.

mm away from the target—the distance was larger than for the first experiment because the 45° target holder had water cooling lines on the back. Photographs of the 0° and 45° setups can be seen in Figure 2.6.

The 45° setup was used to reduce the effect of angular dependence on the observed gamma rays. Gamma decays can be classified by their multipole order $2L$, where $L \hbar$ is the angular momentum carried by the photon. Conservation of angular momentum gives these selection rules:

$$I_i = L + I_f$$

and

$$|I_i - I_f| \leq L \leq I_i + I_f.$$  

The most common gamma transitions are dipole, which have $L = 1$ ($2L = 2$). In fact, an increase in $L$ by one reduces the transition probability by a factor of approximately $10^{-5}$ [20]. The angular distribution of radiation is governed by the Legendre polynomial. For the case of dipole radiation, the Legendre polynomial is given by

$$P_2 = \frac{1}{2}(3 \cos^2 \theta - 1).$$

This function is minimized at $\theta = 55^\circ$. Thus the different
intensities of different gamma rays observed at 55° are due only to their different transition probabilities, and not to any angular effects.

This effect was approximated in the second experiment by observing at 45°. This angle was sufficient due to the size of the detectors used—the large size meant that gamma rays from a range of angles were observed, and any angular effects were integrated over this range. Additionally, the construction of a 45° target chamber was simpler.

2.4 Electronics

As stated previously, the NaI(Tl) detectors were used to observe primary gamma rays (decays of the initial $^{28}$Si excited state $E_{ex}$ to the first excited state), and the clover detected the corresponding transitions from the first excited state to the ground state ($E_\gamma = 1779$ keV). This mode of operation took advantage of the different strengths of the two detector types used: the high efficiency of the NaI(Tl) crystals and the superior energy resolution of the HPGe clover. An overview of the data acquisition electronics will be given, followed by a description of the software coincidence requirements.

The trigger for the data acquisition was germanium singles, which was an event in any of the clover crystals, above the threshold of approximately 100 keV. This essentially created a coincidence requirement for the NaI(Tl) detectors, as their signals would only be acquired if they came in coincidence with a clover signal. This hardware cut was necessary to achieve a reasonable dead time in the data acquisition; because of the large size and efficiency of the NaI(Tl) detectors, operating the four NaI(Tl) detectors in singles mode produced a dead time of over 50% when observing only room background!

The germanium singles trigger was used as a gate for the analog-to-digital converters (ADC’s), as well as for a NIM register. The ADC’s received energy information from the detectors, and converted it to a digital string that could be read by
the data acquisition system. The information from the NIM register was used to
determine which detectors fired during each event within a 200 ns window, allowing for
rejection of random coincidences. Additionally, a time-to-amplitude converter (TAC)
was used to further reduce random coincidences. A TAC produces an analog output
pulse whose amplitude is proportional to the time delay between two input signals. The
TAC gate was set to maximize the peak-to-background ratio for the 1779 keV gamma
line. A schematic of the timing electronics is seen in Figure 2.7. Additionally, a
complete block diagram of all of the electronics, and a list of the modules used, can be
found in Appendix A.

The final, but most important, coincidence condition was an event above 2.7
MeV in any of the NaI(Tl) detectors. This energy was chosen to eliminate most of the
environmental background, but to include the transition from the second excited state to
the first excited state, at $E_\gamma = 2839$ keV. This and the other coincidence conditions led
to a dramatic reduction in background at 1779 keV, and thus greatly increased
experimental sensitivity. This is illustrated in Figure 2.8. These spectra are from a
weak resonance in $^{24}$Mg($\alpha,\gamma$)${}^{28}$Si. While the absolute number of counts is reduced by
applying the coincidence requirements, the background continuum is reduced by a
factor of $\sim 100$, and gamma peaks from room background are eliminated, making the
1779 keV peak clearly visible.

The energy signals from all of the detectors were recorded at each trigger.
Because the clover is composed of four germanium crystals, it can be operated in two
different modes: direct and add-back [11]. In direct mode, each crystal is treated as a
separate detector. If a summed spectrum is constructed, the number of counts in a peak
will be equal to the sum of the counts in each individual crystal. In add-back mode,
coincident events in the separate crystals are summed. In the case that a gamma ray
deposits some energy in more than one crystal (through Compton scattering or pair
Figure 2.7 A schematic illustration of the timing electronics can be seen above. The trigger for the data acquisition is generated from an event in any crystal of the clover detector.
Figure 2.8  The 1779 keV line is shown in these clover spectra. On the left is the ungated spectrum, with the 1779 keV line circled. The other peaks in the spectrum are from environmental background. In particular, the largest peak, which has the potential to obscure the 1779 peak, is from the decay of $^{214}$Bi. On the right is the coincidence spectrum. The only peak visible is the 1779 peak, and it is clearly visible above the remaining background.

production), the full energy of the gamma ray can be reconstructed. The add-back factor is equal to add-back counts divided by direct counts; for the 662 keV line of $^{137}$Cs (single $E_\gamma$ source) it was observed to be 1.2. Add-back mode was used exclusively in this experiment.

2.5 January 2005 Experimental Run

The first experimental run took place in January of 2005. The goal of this experiment was to obtain a yield curve from ~1.0 to 1.5 MeV and to determine previously unknown resonance energies. This experiment utilized the $0^\circ$ detector setup. Evaporated magnesium targets of thickness ranging from 1000 to 4000 Å were used.

The yield curve can be seen in Figure 2.9. The resonances at $E_\alpha = 1530$ and 1350 keV were previously known [29]. A new resonance can be clearly seen at $E_\alpha =$
Figure 2.9 The yield curve from the January 2005 experiment is seen above. Previously known resonances are seen at 1530 and 1350 keV. A new resonance is clearly seen at 1277 keV, and hints of other new resonances are seen at 1178 and ~1100 keV.

1277 keV. Additionally, there is evidence of two other new resonances at 1178 keV and around 1100 keV. The yield at the points around 1100 keV is nearing the lower limit of sensitivity.

The maximum beam current on target during this experiment was approximately 60 μA; the average beam current was 45 μA. Each target was exposed to approximately 4 C of accumulated charge, which corresponded to about one day of running. With this beam current and accumulated charge, no deterioration of the yield on top of the 1530 keV resonance was observed.

The data collected in this first experiment gave information about the existence of new resonances, but was not sufficient to satisfactorily characterize the new features.
Therefore, a second experiment was planned, and several improvements were made to the setup. One improvement was the development of the target cleaning procedure described in section 2.2. The rest of the changes involved the accelerator and associated beam optics.

Two important modifications were made to the KN accelerator. The first was a change in the material used for the charging belt. Originally, the accelerator ran with specially manufactured charging belts from High Voltage Engineering. These belts are no longer available, and labs must either employ previously used HVE belts or find a suitable replacement. One replacement option is plastic-coated conveyer belts, and such a belt was used during the first experiment. However, this belt was far from ideal. The belt material did not accept the charge as readily as the original HVE material. Also, the belt was quite thick, which caused it to hit the belt guides as it ran. Charge leaked from the belt to the guides, leading to unstable charging of the terminal. Additionally, the charge that came off of the belt could accumulate on the acceleration planes, which caused further perturbations in the beam energy and profile. The end result of all of these instabilities was a low and unsteady beam current on target.

Between the first and second $^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$ experiments, a different type of conveyer belt was installed in the accelerator. The new belt was thinner, and had better charging properties. The thinner belt did not hit the belt spacers, which improved the beam stability.

The second change to the accelerator was the means by which the charge is transferred to the belt. During the first experiment, the charging ‘comb’ described in section 2.1 was actually a very thin steel sheet. The moving belt generated a draft which pushed the sheet too far away from the belt, making the coronal discharge less stable. After the first experiment, the steel sheet was replaced by a brass screen. The screen was not affected by the air current, and the better contact between the screen and
the belt lead to more stable charging and better charging efficiency. The terminal voltage was steadier, and thus so was the beam.

Finally, modifications to the beam focusing procedure were adopted in order to ensure that the beam spot on the target was the same shape and at the same position for the entire experimental energy range. This was important because the 1530 keV resonance was used to monitor the yield of the targets, and because the resonance is relatively strong, a shift in the position of the beam could mask target deterioration from lower energy running. The beam focusing element closest to the target is a set of magnetic quadrupole lenses. The power supplies for these magnets are linked to the master reference control; the master reference scales the current of the controlled beam transport elements to the energy of the beam. It was found, however, that the coupling between the master reference and the quads was not able to maintain a constant beam spot over the experimental energy range. Additional adjustments to the quads’ power were necessary, so the adjustment amounts were measured and used to construct a linear curve for continuous adjustment.

2.6 January 2006 Experimental Run

After completing the setup optimizations described in the previous section, the second $^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$ experimental run was performed. The goal of this experiment was to obtain a more complete yield curve, and to accumulate statistics at the new resonance energies, so that the resonances could be characterized with regard to resonance strength and branching ratios. During this run, the 45° detector configuration was used. Targets were all intended to have approximately the same thickness, 1000 Å. In actuality, problems with the evaporation thickness monitor led to variations of up to a factor of 2 in the measured target thickness.

The yield curve from the second run can be seen in Figure 2.10. The amount of data collected in this run was significantly more than in the previous run, thanks to the
accelerator improvements which led to an increase in beam current. The resonance seen at 1277 keV in the first experiment was confirmed, as was the possible resonance at 1178 keV. Additionally, a new resonance was observed at 1413 keV. The possible resonance at around 1100 keV could not be confirmed, but data was collected for the determination of an upper limit. The lowest energy point of the yield curve, at 1065 keV, was the result of a very long run (>20 C accumulated charge). The target was very thick, so that the beam would be stopped in the magnesium, leading to an upper limit for all resonances below the beam energy.

The beam current on target ranged from 100 to 150 μA, a factor of 2 to 3 more than during the first experiment; the data collection rate was increased by the same factor. The targets were again each used for about one day, but with the increase in
beam current, one day of running corresponded to approximately 7 C of accumulated charge. This combination of high beam current and long running times did lead to some deterioration of the target yield. This effect, and correction of the data, will be discussed further in the next chapter.
The January 2006 $^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$ experiment produced a data set consisting of several hundred gamma ray spectra at alpha energies in the 1.0-1.5 MeV range. The raw data was gated for coincidences, as described in the previous chapter. The resulting peak at 1779 keV in the clover spectra was then analyzed to extract the yield at each experimental energy. To determine the area of the peak, the background was fit with a second order polynomial function, and subtracted from the raw area. The resulting net area $I$ was used to determine the number of $^{28}\text{Si}$ nuclei formed by each incoming alpha:

$$Y_\alpha = \frac{I \cdot e}{LQ \cdot \eta \cdot BR}.$$  

(3.1)

$e$ is the electron’s charge, $LQ$ is the live charge collected during the run (which accounts for the time the data acquisition CPU was busy), $\eta$ is the probability to detect the 1779 keV gamma ray, and $BR$ is the probability that the decay of the excited $^{28}\text{Si}$ nucleus includes the 1179 keV transition from the first excited state to the ground state. The yield calculated from the equation above can then be used to determine the resonance strength, using equation (1.14):

$$Y_\omega = \frac{2\pi^3 \lambda^2}{\epsilon} \omega \gamma.$$
3.1 Experimental Systematics

In order to properly extract the yield from the experimental data, the detection probability (referred to as detection efficiency) $\eta$ must be known. This quantity is particular to the detection scheme of the experiment. Additionally, the peak area $I$ must be corrected for target deterioration due to accumulated charge. These experimental systematics will be described in the following section.

3.1.1 Energy Calibration

The first analysis task was energy calibration of the detectors. For the clover detector, the analysis mainly focused on the 1779 keV gamma, so precise energy calibration over the entire range was not critical. However, a reliable calibration was needed at lower energies to have good resolution in the add-back spectrum.

First a $^{60}$Co source ($E_{\gamma} = 1173$ and 1332 keV) was used to set the gain of all the detectors so that the full scale was approximately 16 MeV, giving a rough calibration of 2 keV per channel. Additionally, the fine gain was adjusted so that the $^{60}$Co peaks were in nearly the same channels for all the clover crystals and all the NaI(Tl) detectors. Then the exact energy calibration was determined for each detector. For the lower energy region (< 1.5 MeV), radioactive sources, including $^{60}$Co, $^{22}$Na ($E_{\gamma} = 511$ and 1275 keV) and $^{137}$Cs ($E_{\gamma} = 662$ keV) were used. For higher energies, the $E_{p} = 992$ keV resonance of the $^{27}$Al(p,$\gamma$)$^{28}$Si reaction was measured; the gammas used for calibration were $E_{\gamma} = 1779, 2839, 4743, \text{and } 6020$ keV. The higher energy gammas produced by the reaction were not used, because they were Doppler-shifted, and because a precise calibration was not needed for higher energies. A linear curve fit was made for each detector, in the form of $E_{\gamma}$ [keV] = $a \times$ Channel # + $b$. The coefficient $a$ was approximately 2, as specified by the amplifier gain, and $b$ was equal to the offset (ideally the offset would be zero, but it was measured to be between 5 and 20). Once the detectors were calibrated, the add-back spectra could be formed. The calibrations
were checked at several points during the experimental run, but were found to be stable for the clover, and variations were within the energy resolution for the NaI(Tl) detectors.

### 3.1.2 Detection Efficiency

The analysis of the $^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$ data required the calculation of three types of detector efficiency: total efficiency (denoted $\eta^T$), photopeak efficiency (denoted $\eta$) and coincidence efficiency (denoted $\eta_{\text{coin}}$). In general, efficiency is the ratio between the number of gamma rays detected and the number emitted by the radiation source. The efficiency depends on the detector used and the source-detector configuration, so it must be measured for each particular experimental setup. In order to have a known gamma ray emission rate, radioactive sources of known activity and resonances with well known resonance strengths and decay schemes are used.

**Total Efficiency**

In the model spectrum shown in Figure 2.4, calculation of the total efficiency would be quite simple, because the gamma rays are monoenergetic and the spectrum extends down to $E_\gamma = 0$. In a real spectrum, there are often several gamma ray energies present, from the radiation source and from the background. Additionally, thresholds must usually be set above $E_\gamma = 0$ to avoid acquisition dead time due to the large number of low energy signals and electronic noise. These effects complicate the measurement of total efficiency considerably.

It is possible, however, to estimate the expected total efficiency for a known detector configuration. The probability per unit length for a photon to be absorbed is called the attenuation coefficient (denoted by $\mu$). This quantity depends on the absorbing material, and the energy of the photon. Values of $\mu$ for a variety of detector materials over an energy range of $E_\gamma = 1 \text{ keV}$ to $20 \text{ MeV}$ are given in reference [10]. The probability for interaction along a path length $x$ inside the detector is $1 - e^{-\mu x}$. The
total efficiency is then given by this quantity integrated over the solid angle of the detector:

\[ \eta^T = \frac{1}{4\pi} \int (1 - e^{-\mu}) d\Omega. \]  

(3.2)

This equation and the attenuation coefficients from reference [10] were used to calculate the total efficiency for the clover detector.

For the calculation, the germanium crystals were assumed to have a square cross section. In the clover detector, this is not exactly the case. Germanium crystals are grown in a cylindrical shape, then shaved down to facilitate close packing into the clover configuration. The actual crystal cross section can be seen in Figure 3.1. The cross sectional area of each crystal was computed, and a square cross section of the same area was used in the efficiency calculation. Total efficiency values were calculated for the entire energy range of interest, from less than 1 MeV to 11 MeV. An error of 3% was assigned to these efficiencies to account for uncertainty in the source-detector distance.

The calculated values were assumed to be correct relative to each other, but it was desirable to compare the calculation to a measured value. A \(^{137}\)Cs source was used for the measurement—\(^{137}\)Cs produces monoenergetic gamma rays of \(E_\gamma = 662\) keV. To extrapolate the counts down to \(E_\gamma = 0\), two possibilities were considered, as illustrated in Figure 3.2. The number of counts in the extrapolated region was taken to be the average of the two extremes, and the error was taken as half of the difference. The estimated number of counts was added to the number observed in the rest of the spectrum to obtain the total number of events observed \(I\). The number of decays that occurred in the counting time was calculated from the basic exponential law of radioactive decay: \(A(t) = A_0 e^{-\lambda t}\). \(A\) is the source activity at the time of the measurement, \(A_0\) is the source activity measured at a known time, \(\lambda\) is the decay constant (equal to the natural logarithm of 2 divided by the half-life), and \(t\) is the time.
Figure 3.1 This is a drawing of the clover detector cross section, based on information from the manufacturer. The gray outline represents the aluminum detector housing; the area inside the housing is kept at a vacuum. The modified cylindrical shape of the germanium crystals can be seen.

between the measurement of $A_0$ and $A$. The total number of decays is then given by $N = A(t)\Delta t$, with $\Delta t$ equal to the measurement time. Finally, the total efficiency $\eta^T = I/N$ was obtained, and the normalization factor between the calculated and measured efficiency values was found to be 0.82. An error of 7% was assigned to the normalized total efficiencies. The efficiency values ranged from approximately 18% at $E_\gamma = 662$ keV to 12% at 11 MeV.

The total efficiency values obtained in this manner were not exact, due to the simplification of the clover geometry for purposes of the calculation, and the small overlap between calculated and measured values. However, the clover total efficiencies were not used directly in the calculation of resonance strengths, rather they were needed for making corrections to the photopeak efficiencies. The corrections were usually small, and in all cases the largest uncertainty was from another source. Therefore,
Figure 3.2 The $^{137}$Cs spectrum used to measure the total efficiency of the clover detector is shown above. The two extreme values that were considered for the extrapolation down to $E_\gamma = 0$ are shown with dashed lines.

although the total efficiencies may have been inexact, they were sufficiently accurate for the purposes for this analysis method.

**Photopeak Efficiency**

The photopeak efficiency of the clover detector was determined for the entire energy range of interest, from 1 to 11 MeV. The ideal method for this calculation would be to use standard gamma-ray sources, with precisely measured activities. However, sources typically produce gamma rays with energies below 3 MeV. For the low energy range, a $^{60}$Co source was used ($E_\gamma = 1173$ and 1332 keV). For higher energies, well-known $^{28}$Si excited states were used; the excited states were produced by the $^{27}$Al(p,$\gamma$)$^{28}$Si reaction. Two resonances were used, at $E_p = 992$ and 679 keV. These resonances have well-known strengths and decay schemes, so the number of gamma rays of a given energy can be reliably calculated. An additional advantage was that these reactions produced the same nucleus as the experimental reaction, so the gamma
ray energies were entirely relevant, and extrapolation to experimental energies would not be necessary.

The photopeak efficiency is determined by dividing the number of gamma rays observed in the photopeak by the total number of gamma rays emitted. In the case of the $^{60}$Co source, the total number of gammas emitted was determined using the equation $N = A(t)\Delta t$ and the exponential law of decay, as described in the previous section. For the $^{27}$Al(p,$\gamma$$^{28}$Si) reactions, the thick target yield (reactions per incident proton) was calculated from the known resonance parameters using equation (1.14):

$$Y_\infty = \frac{2\pi^2 \kappa^2}{\varepsilon} \omega \gamma .$$

$\kappa$ was calculated using equation (1.1), and the stopping power $\varepsilon$ was taken from SRIM 2003 [41]. Relative values for the resonance strengths $\omega \gamma$ were taken from reference [14], and were scaled to the 992 keV resonance strength given in reference [33]. The values used for $\kappa$, $\varepsilon$, $\omega \gamma$ and $Y_\infty$ can be found in Appendix B.

For both the $^{60}$Co source and the $^{27}$Al(p,$\gamma$$^{28}$Si) reactions, more than one gamma ray was emitted in the decay of the excited nucleus. If two gammas enter the detector at the same time (within the time resolution of the detection system), a summed signal could be produced at an energy not equal to either of the gamma energies. The formation of a summed signal means that the event was lost from the full energy peak of either gamma. This problem is particularly prominent for large detectors and small source-detector distances; both of these factors were present in the $^{24}$Mg($\alpha$$,\gamma$$^{28}$Si experiment. Therefore, it was important to make corrections for these “summing out” effects when calculating the clover photopeak efficiency.

The correction procedure will be described for the simple decay of $^{60}$Co. $^{60}$Co decays by beta minus emission to an excited state of $^{60}$Ni, which then decays by emission of two subsequent gammas of energy 1173 and 1332 keV, as illustrated in Figure 3.3. Suppose the first gamma deposits all of its energy in the detector. If the
second gamma ray does not interact with the detector, this event will be properly registered in the 1173 keV photopeak. The number of events observed in the photopeak of the first gamma would be \( I_{\text{real}} = N \cdot \eta(1173) \) where \( N \) is the number of decays and \( \eta(1173) \) is the photopeak efficiency for \( E_\gamma = 1173 \text{ keV} \). Now consider the possibility that the second gamma also deposits some energy in the detector—the resulting signal will no longer be at 1173 keV. The second gamma need not deposit its full energy to disturb the detection of the 1173 gamma, as any excess energy will remove the event from the 1173 keV photopeak. Thus the probability for a summation event to occur depends on the total efficiency for detecting the second gamma, \( \eta^T(1173) \). Accounting for this summing, the number of events observed in the 1173 keV photopeak is given by

\[
I_{\text{obs}} = N \cdot \eta(1173) \cdot [1 - \eta^T(1332)],
\]

(3.3)

and the real number of counts can be calculated:
\[ I_{\text{real}} = N \cdot \eta(1173) = \frac{I_{\text{obs}}}{1 - \eta^T(1332)}. \] (3.4)

In the general case of a gamma ray \( i \) in a cascade of any number of gamma rays, the observed number of events must be corrected as

\[ I_{\text{real}} = \frac{I_{\text{obs}}}{1 - c_i}, \] (3.5)

where \( c_i \) is the probability that any other gammas were observed in addition to \( i \). To determine the correction factor \( c_i \), some definitions are needed. \( E_R \) denotes the initial excited state, and \( E_i \) denotes intermediate excited states. \( P_i \) is the probability that \( E_i \) will decay by emitting a particular gamma ray \( \gamma_i \). The branching ratio \( B_i \) is the probability that the decay of \( E_R \) will include \( \gamma_i \). The correction factor can then be written as

\[
c_i = \frac{1}{B_i} \sum_{\alpha_i} \left[ \left( \prod_{a(\alpha_i)} P_{a(\alpha_i)} \right) \left( -\eta_{T,i} + \sum_{a(\alpha_i)} \eta_T^{a(\alpha_i)} \right) \right], \] (3.6)

where

\[
B_i \equiv \sum_{\alpha_i} \left( \prod_{a(\alpha_i), \gamma_i} P_{a(\alpha_i)} \right) [8]. \] (3.7)

The sum over \( \alpha_i \) is a sum over all cascades that include \( \gamma_i \), and \( a(\alpha_i) \) is an index including all gamma rays in cascade \( \alpha_i \). Note that in the calculation of \( c_i \), the product is over all gamma rays in the cascade \( \alpha_i \), while for \( B_i \) it includes only gammas that precede \( \gamma_i \). Calculations of \( B_i \) and \( c_i \) for \( ^{60}\text{Co} \) and the two \( ^{27}\text{Al}(p,\gamma)^{28}\text{Si} \) reactions can be found in Appendix C.

Using the summing correction factors, the “real” number of counts was obtained for each photopeak. Using this number and the thick-target yield from equation 1.14, the photopeak efficiency could be calculated from equation 3.1:

\[
Y_x = \frac{I \cdot e}{LQ \cdot \eta \cdot BR} \Rightarrow \eta = \frac{I_{\text{real}} \cdot e}{Y_x \cdot LQ \cdot BR}.
\]
The $E_p = 992$ keV resonance is more fully characterized than the $E_p = 679$ keV resonance, so the efficiency values obtained from the $E_p = 679$ keV resonance were normalized to the $E_p = 992$ keV values. This was accomplished by examining 4 gammas rays over the entire range of energies which had the same or similar energies for both resonances. The average normalization factor was determined, and several efficiencies from the $E_p = 679$ keV resonance were corrected and included in the final efficiency curve. The clover photopeak efficiency can be seen in Figure 3.4.

![Clover Addback Photopeak Efficiency--With Fit](image)

Figure 3.4 The calculated photopeak efficiencies for the clover detector, and the fits to those values, can be seen above.
The measured efficiencies were fit in two regions, below and above 3 MeV, as shown in Figure 3.4. The fitting function was of the form

\[ \eta(E_\gamma) = A_0 + \frac{A_1}{E_\gamma} + \frac{A_2}{E_\gamma^2} \].

(3.8)

The fits were purely phenomenological, but they were useful because they accurately reproduced the measured efficiency values. The fits were used to calculate the photopeak efficiencies for gamma rays that were observed in the $^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$ runs which did not correspond to the calibration points.

**Coincidence Efficiency**

The coincidence efficiency was calculated using three $^{27}\text{Al}(p,\gamma)^{28}\text{Si}$ resonances, at $E_p = 992, 760$ and 679 keV, with decay schemes similar to the $^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$ resonances studied. In this case, the coincidence efficiency is equal to the product of the 1779 keV photopeak efficiency in the clover detector and the “total” efficiency to detect the primary gamma rays in the NaI(Tl) detectors (this efficiency is not quite the same as the total efficiency defined previously, because only gammas with energy above 2.7 MeV were used). While the 1779 keV efficiency is a constant, the primary efficiency varies depending on the energy of the primary, leading to the choice of $^{27}\text{Al}(p,\gamma)^{28}\text{Si}$ resonances with decay schemes similar to the $^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$ resonances. Again equation (3.1) was used, with the feeding probability of the 1779 keV gamma used for the branching ratio. (The feeding probability is defined as the percentage of decays from the excited state that include the 1779 keV transition.) The feeding probabilities and resulting coincidence efficiencies are listed in TABLE 3.1.

The coincidence efficiency for a general decay was taken to be the weighted average of the three values [9]. In the case of values with known error, the weights can be expressed as
TABLE 3.1
COINCIDENCE EFFICIENCY FOR $^{27}$Al(p,γ)$^{28}$Si RESONANCES

<table>
<thead>
<tr>
<th>$E_p$ (keV)</th>
<th>Feeding Probability (%)</th>
<th>$\eta_{\text{coin}}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>992</td>
<td>0.93 ± 0.03</td>
<td>0.41 ± 0.03</td>
</tr>
<tr>
<td>760</td>
<td>0.97 ± 0.03</td>
<td>0.20 ± 0.07</td>
</tr>
<tr>
<td>679</td>
<td>0.76 ± 0.05</td>
<td>0.29 ± 0.10</td>
</tr>
</tbody>
</table>

$$w_i = \frac{1}{\delta \eta_{\text{coin}}^2},$$ \hspace{1cm} (3.9)

so that the most precise value has the largest weight. The weighted average is given by

$$\bar{\eta}_{\text{coin}} = \frac{\sum_i \eta_{\text{coin}} w_i}{\sum_i w_i},$$ \hspace{1cm} (3.10)

with error equal to

$$\delta \bar{\eta}_{\text{coin}} = \sqrt{\frac{1}{\sum_i w_i}}.$$

The result of the averaging was \(\bar{\eta}_{\text{coin}} = 0.37 \pm 0.03\%\).

3.1.3 Yield Correction

After accumulating a charge of around 7 C, all of the targets showed some loss of yield, in the range of 10 to 20%. This effect is illustrated in Figure 3.5. The reduction in yield was caused by the beam damaging the surface of the target, through sputtering and localized heating. Although the change in yield is not dramatic, it must be accounted for in order to properly calculate the resonance strength. The profile of the 1530 keV resonance was measured for each target after it was mounted on the beam line, and again before it was removed.
Figure 3.5 The profile of target 27 at the 1530 keV resonance is shown before and after accumulating 6 C. The yield on the plateau is reduced by approximately 15%, while the width remains the same.

The plateau height was parameterized as a function of accumulated charge for all targets. The height was determined by fitting the front edge and plateau of the target profile curve with the arctangent form of the yield, given in equation (1.13). The fitting function was

$$ Y(E) = \frac{H}{\pi} \arctan \left( \frac{E - E_R - \delta}{\Gamma/2} \right). $$  (3.9)

The constants $E_R = 1503.03$ keV and $\Gamma \approx 0.15$ keV were taken from reference [13]. The parameter $\delta$ was allowed to vary, to account for shifts in the front edge of the profile, caused by hysteresis in the analyzing magnet. $H$ was the fit parameter that
Figure 3.6 A plot of the relative loss of yield for all targets is shown above. The linear fit of the points was used to correct the data for yield loss as a function of accumulated charge.

corresponded to the plateau height. Values of $H(Q)/H(0)$ were plotted for all targets, and a universal linear fit was made. The data points and fit can be seen in Figure 3.6. The resulting correction is

$$I_{corrected} = \frac{I(Q)}{-0.0107Q + 0.9333},$$

(3.10)

with $Q$ equal to the charge accumulated on target until half way through the run in question.

Another correction was made for variation in the targets’ initial plateau heights. The variations may have been due to contaminants in the evaporated magnesium film, though no unidentified gamma rays were seen in the spectra. The targets were also observed to oxidize to varying degrees, which could have affected the plateau height.
The theoretical yield for a pure natural magnesium target as calculated from equation (1.14) is $2.41 \times 10^{-12}$ $^{28}\text{Si}/\alpha$, for $E_{\alpha} = 1530$ keV. The target with the highest initial plateau height (target #28) produced a yield of $2.30 \times 10^{-12}$ $^{28}\text{Si}/\alpha$, which is a good agreement with the theoretical value. The raw yield of target with the lowest initial plateau height was about 30% lower than this value. The targets were normalized to the height $H(0)_{\text{max}}$ of target #28, so that the fully corrected counts were given by

$$ I_{\text{final}} = \frac{I_{\text{corrected}}}{H(0)} \times H(0)_{\text{max}}. $$

(3.11)

### 3.2 Branching Ratios

In addition to the detection efficiency and the “real” number of counts in the photopeak, there is one remaining element that must be known in order to calculate the yield: the decay scheme of the excited $^{28}\text{Si}$ state. For several of the $^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$ resonances measured in this experiment, the decay schemes were completely unknown. Therefore, it was necessary to calculate the branching ratios for these resonances. Additionally, several previous branching ratio measurements were confirmed.

The method of calculation was based on equation (3.1):

$$ Y_{\infty} = \frac{I \cdot e}{LQ \cdot \eta \cdot BR}. $$

Consider a $^{28}\text{Si}$ decay in which three gamma rays are observed: 1) the transition from the excited state to the ground state; 2) the transition from the excited state to the first excited state; 3) the transition from the first excited state to the ground state. If the efficiencies and branching ratios are known, each of the gamma rays could be used to calculate the thick target yield $Y_{\infty}$, and the right hand side of equation (3.1) can be equated for the three gamma rays:

$$ \frac{I_1}{\eta \cdot BR_1} = \frac{I_2}{\eta \cdot BR_2} = \frac{I_3}{\eta \cdot BR_3}. $$
In the simple decay under consideration, $BR_2 = BR_3$ and $BR_1 + BR_2 = 1$. This system of equations can be solved for $BR_1$ and $BR_2$. However, the gamma ray yields $I_1$ and $I_2$ must be corrected for summing, as described in section 3.1.2.

The details of the branching ratio calculations can be followed in Appendix D, and the results are summarized in TABLE 3.2. The $^{24}$Mg($\alpha,\gamma)^{28}$Si resonance formed at $E_\alpha = 1178$ keV leads to the $^{28}$Si excited state $E_{ex} = 10.994$ MeV. According to reference [14], this state decays 100% directly to the ground state. However, in the present experiment a branching to the first excited state was observed in addition to the ground state transition, and new branching ratios were calculated. Branching ratios were not calculated for the resonance at $E_\alpha = 1277$ keV ($E_{ex} = 11.079$ MeV) because the gamma rays could not be discerned from the clover spectrum. The values given in reference [3] were adopted. For the resonance at $E_\alpha = 1350$ keV, leading to $E_{ex} = 11.142$ MeV, there were no previously measured branching ratios. These were easily calculated from the current data, owing to the relatively high resonance strength at this alpha energy. At $E_\alpha = 1413$ keV ($E_{ex} = 11.195$ MeV) the situation was the same as at 1277 keV. The values from reference [13] were used. Finally, the branching ratios for the well-known $E_\alpha = 1530$ keV ($E_{ex} = 11.295$ MeV) were calculated, and found to be in agreement with the branching ratios given in reference [30].

### 3.3 Resonance Strengths

Finally the resonance strengths could be calculated for each of the observed resonances by combining the two expressions for the thick target yield (equations (1.14) and (3.1)):

$$
\omega \gamma = \frac{I_{\text{coin}} \cdot e \cdot e}{2\pi^2 \hbar^2 \cdot \eta_{\text{coin}} \cdot BR \cdot LQ}.
$$

(3.12)
<table>
<thead>
<tr>
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<th>4.98</th>
<th>6.28</th>
<th>6.69</th>
<th>6.88</th>
<th>7.38</th>
<th>7.42</th>
<th>8.26</th>
<th>9.32</th>
<th>9.38</th>
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</thead>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td>12  2</td>
</tr>
<tr>
<td>1120</td>
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<td>53 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>11  2</td>
<td>22  3</td>
<td>14  2</td>
<td></td>
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<td></td>
</tr>
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<td>100</td>
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<tr>
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<td>10994</td>
<td>85 8</td>
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<td>11079</td>
<td>35 1</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>1350</td>
<td>11142</td>
<td>44 4</td>
<td>10  1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>32  3</td>
<td>14  2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1413</td>
<td>11195</td>
<td>39 6</td>
<td>29  5</td>
<td></td>
<td></td>
<td>10  2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1530</td>
<td>11295</td>
<td>73 5</td>
<td>19  2</td>
<td></td>
<td></td>
<td></td>
<td>2.5 1.0</td>
<td>2.2 1.0</td>
<td>3.0 1.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>75 8</td>
<td>21 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>&lt;3</td>
<td>2   1</td>
<td>2   1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 3.2**
BRANCHING RATIOS
The stopping power \( \varepsilon \), de Broglie wavelength \( \hbar \) and 1779 keV feeding probability for each resonance are listed in TABLE B.2. First the resonance strength of the 1530 keV resonance was calculated: the value was found to be 94 ± 14 meV. Then the strengths of the other resonances were calculated relative to this value by making a ratio using equation (3.12); the resulting formula is

\[
\omega(\varepsilon(E_R)) = \omega(1530) \frac{E_R \cdot I_{\text{coin}}(E_R)}{1530 \cdot I_{\text{coin}}(1530)} \cdot \varepsilon(E_R) \cdot LQ(1530) \cdot BR(1530)
\]

The resulting resonance strengths are listed in TABLE 3.3.

<table>
<thead>
<tr>
<th>( E_R ) (keV)</th>
<th>( \omega )--this experiment (meV)</th>
<th>( \omega )--previous value (meV)</th>
<th>( \omega )--adopted value (meV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1413</td>
<td>0.21 ± 0.04</td>
<td>---</td>
<td>0.22 ± 0.04</td>
</tr>
<tr>
<td>1351</td>
<td>1.9 ± 0.3</td>
<td>1.9 ± 0.6 [29]</td>
<td>2.0 ± 0.3</td>
</tr>
<tr>
<td>1277</td>
<td>0.058 ± 0.011</td>
<td>---</td>
<td>0.062 ± 0.011</td>
</tr>
<tr>
<td>1178</td>
<td>0.22 ± 0.06</td>
<td>---</td>
<td>0.23 ± 0.06</td>
</tr>
<tr>
<td>1120/1130</td>
<td>&lt; 0.016</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>1087</td>
<td>&lt; 0.014</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>&lt;1060</td>
<td>&lt; 0.002</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>

The value for \( \omega(1530) \) agrees within errors with the previous value of 110 ± 20 meV from Maas et al [30], so a weighted average was taken, resulting in the adopted value of 99 ± 11 meV. The other resonance strengths were scaled to this average to determine the adopted values given in TABLE 3.3.
3.3.1 Resonance Strength Upper Limits

Resonance strength upper limits were calculated for two energies where levels exist in the $^{28}\text{Si}$ nucleus, even though resonances were not clearly observed in the yield curve. The energies are $E_\alpha = 1120/1130$ keV and 1087 keV, corresponding to $E_{ex} = 10944/10953$ keV and 10916 keV. (The thickness of the targets prohibited differentiation between the possible resonances as 1120 and 1130 keV.) In both cases the feeding probability was set to the maximum value of 1.00; the branchings are listed in TABLE 3.2. The calculated resonance strength upper limits are $\omega \gamma(1120/1130) < 0.016$ meV and $\omega \gamma(1087) < 0.014$ meV.

3.3.2 Very Thick Target

A target of sufficient thickness to stop the beam in the magnesium layer was used for over 22 C of accumulated charge at a beam energy of 1060 keV in order to obtain an upper limit on the strengths of all resonances below this energy. The yield for this run is represented by the lowest energy point in Figure 2.10, and is seen to be quite low compared to the other long run yields, suggesting that there are no strong resonances at energies below 1060 keV. Assuming a feeding probability of 1.00, the resonance strength upper limit was found to be $\omega \gamma(E_\alpha < 1060) < 0.002$ meV. This value could be used to estimate requirements on beam current and detection efficiency for future experiments probing lower alpha energies.

3.4 1178 keV Resonance

In the case of the $^{28}\text{Si}$ level formed at $E_\alpha = 1178$ keV ($E_{ex} = 10994$ keV), the spin has not been precisely determined by previous experiments—a value of 1+ is suggested, with 1- and 2+ also possible [14]. The fact that this level is formed by the $^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$ reaction shows that the level cannot have spin and parity 1+, because only natural parity levels can be formed by alpha capture on a spin 0 nucleus. An
attempt was made to determine the spin by comparing the systematics of other levels in the same energy range.

The resonance strength can be written as

\[ \omega \gamma = \frac{2J + 1}{(2j_1 + 1)(2j_2 + 1)} \frac{\Gamma a \Gamma b}{\Gamma}, \tag{3.14} \]

which can be thought of as an integrated cross section. In the case of \(^{24}\text{Mg} + \alpha\), both \(j_1\) and \(j_2\) are zero, simplifying the equation to

\[ \omega \gamma = (2J + 1) \frac{\Gamma a \Gamma b}{\Gamma}. \tag{3.15} \]

If it is assumed that \(\Gamma \gamma\) is much larger than \(\Gamma \alpha\), the resonance strength can be estimated as

\[ \omega \gamma \approx (2J + 1) \Gamma a, \tag{3.16} \]

giving

\[ \Gamma a \approx \frac{\omega \gamma}{2J + 1}. \tag{3.17} \]

It is not clear that \(\Gamma \gamma\) is in fact larger than \(\Gamma \alpha\), but the available evidence seems to support this assertion: out of the five nearest \(^{28}\text{Si}\) levels for which \(\Gamma \gamma\) and \(\Gamma \alpha\) are known, four have larger gamma widths. These levels are shown in TABLE 3.4.

Assuming that equation (3.17) is valid for this level, alpha widths can be calculated for the two possible \(J\) values. The width can be expressed as a function of the penetrability \(P\), the spectroscopic factor \(\theta^2\), and the radius of the compound nucleus \(R_n\):

\[ \Gamma = \frac{2\hbar}{R_n} \left( \frac{2E}{\mu} \right)^{1/2} P(E, R_n) \theta^2. \tag{3.18} \]
\begin{table}
\centering
\caption{PARTIAL WIDTHS OF SOME $^{28}\text{Si}$ LEVELS}
\begin{tabular}{|c|c|c|c|c|}
\hline
$E_{ex}$ (keV) & $E_{\alpha}$ (lab) (keV) & $\Gamma$ (eV) & $\Gamma_\gamma$ (eV) & $\Gamma_\alpha$ (eV) \\
\hline
11657 & 1952 & 0.18 & 0.035 & 0.14 \\
11669 & 1966 & 0.46 & 0.28 & 0.18 \\
11779 & 2094 & $<5$ & $>5$ & $6 \times 10^{-3}$ \\
11867 & 2197 & $<5$ & $<5$ & $<2 \times 10^{-3}$ \\
11900 & 2235 & 0.3 - 40 & 0.3 - 40 & $7 \times 10^{-4}$ \\
\hline
\end{tabular}
\end{table}

NOTE: The total widths are taken from reference [14], and the partial widths are taken from reference [13].

The penetrability is equal to the ratio of the amplitudes of the alpha wave functions inside and outside the compound nucleus. This value was calculated in terms of the Coulomb wave function for both $J$ values.

Equation (3.18) was used to determine the spectroscopic factor for each possible $J$, and the results were each compared with the spectroscopic factors of other resonances with the same $J$; the spectroscopic factors were obtained from reference [17]. The calculated spectroscopic factors for the 1178 keV resonance were 0.043 for $J = 1$ and 0.078 for $J = 2$. The range for the spectroscopic factors of other $J = 1$ resonances is 0.0050 to 0.11, while the range for $J = 2$ is 0.000025 to 0.044. The value calculated for $J = 1$ falls within the range of the other spectroscopic factors, while the value for $J = 2$ does not, suggesting that the spin of the 10994 keV level is 1, with negative parity.
CHAPTER 4
RESULTS AND CONCLUSIONS

4.1 Reaction Rates

The reaction rate was calculated over a temperature range of 0.1 to 10 GK, including all resonances up to \( E_a = 3792 \) keV, using equation (1.11) in summed form:

\[
\langle \sigma v \rangle = \left( \frac{2 \pi}{\mu kT} \right)^{3/2} \hbar^2 \sum_i (\omega \gamma_i) \exp \left( -\frac{E_i}{kT} \right).
\] (4.1)

A list of the levels and resonance strengths used can be found in TABLE 4.1.

The reaction rate was compared to several other reaction rate calculations to determine the effect of the newly measured resonance strengths. The rates were taken from Caughlan and Fowler’s well-known 1988 paper (referred to as CF88) [5], and a paper by Rauscher, Thielemann, Görres and Wiescher (referred to as RTGW) [35], which deals specifically with the capture of alpha particles by \( N = Z \) nuclei. The rate of CF88 is derived from measured resonances and suspected resonances in the compound nucleus, as well as non-resonant continuum features [15]. The rates were fit over a temperature range of \( T_9 = 10^{-3} - 10 \) by least-squares analysis. There are three different rates given by RTGW: the rate calculated using the NON-SMOKER Hauser-Feshbach code, a modified NON-SMOKER rate which includes isospin effects (referred to as the empirical rate), and a rate calculated from experimental data available at the time. A comparison of the current experimental rate with these previously calculated rates is shown in Figure 4.1.
### TABLE 4.1
RESONANCE LEVELS USED FOR RATE CALCULATION

<table>
<thead>
<tr>
<th>$E_\alpha$ (keV)</th>
<th>$E_{ex}$ (keV)</th>
<th>$\omega\gamma$ (eV)</th>
<th>$E_\alpha$ (keV)</th>
<th>$E_{ex}$ (keV)</th>
<th>$\omega\gamma$ (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1178</td>
<td>10994</td>
<td>0.00023</td>
<td>2703</td>
<td>12301</td>
<td>0.020</td>
</tr>
<tr>
<td>1277</td>
<td>11079</td>
<td>0.00006</td>
<td>2866</td>
<td>12440</td>
<td>1.0</td>
</tr>
<tr>
<td>1351</td>
<td>11142</td>
<td>0.0020</td>
<td>2906</td>
<td>12474</td>
<td>1.5</td>
</tr>
<tr>
<td>1413</td>
<td>11195</td>
<td>0.00022</td>
<td>2921</td>
<td>12488</td>
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<td>11295</td>
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<td>2994</td>
<td>12550</td>
<td>0.7</td>
</tr>
<tr>
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<td>12725</td>
<td>3.6</td>
</tr>
<tr>
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<td>0.04</td>
<td>3291</td>
<td>12805</td>
<td>0.21</td>
</tr>
<tr>
<td>1952</td>
<td>11657</td>
<td>0.14</td>
<td>3303</td>
<td>12815</td>
<td>0.20</td>
</tr>
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<td>1966</td>
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<td>3429</td>
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<td>0.09</td>
<td>3792</td>
<td>13235</td>
<td>1.3</td>
</tr>
</tbody>
</table>

NOTE: The level energies are taken from reference [13]. The resonance strengths for the first five levels are from the current experiment, and the rest are from reference [13].

Although the rate was calculated for $T_\theta = 0.1 - 10$, only the region below $T_\theta = 1.5$ is shown in Figure 4.1, because the rates are approximately the same over the rest of the temperature range. This is because the higher temperature region is dominated by higher energy resonances, which have well-determined energies and strengths. The new resonances contribute in the lowest temperature region, as seen in the comparison between the old and new experimental rates. The discrepancy between the experimental rate and the theoretical rates (CF88, NON-SMOKER and empirical) below $T_\theta = 0.3$ is due to the inclusion of several resonances which are presumed to be present at energies below the range of this experiment. A tabulation of all of the rates over the entire temperature range can be found in Appendix E.
Comparison of Reaction Rates

Figure 4.1 The current experimental rate is shown, with other calculated rates for comparison. Above $T_0 = 1.5$, all of the rates are approximately the same.

The contribution of each of the measured resonances to the total experimental rate was calculated using equation (1.11); the results can be seen in Figure 4.2. Above $T_0 \sim 0.6$ the 1530 keV resonance dominates the rate, while below 0.6 the 1178 keV resonance gives the largest contribution. It may seem surprising that the 1413 and 1351 keV resonances do not dominate the rate in any temperature region, since their strength is the same as and an order of magnitude larger than the 1178 keV resonance strength, respectively (see TABLE 3.3). However, the rate depends not only on the strength of the resonance, but also on the resonance energy.

In addition to the reaction rates of the measured resonances, the theoretical contributions from possible lower energy resonances were examined. The candidate $^{28}$Si levels were chosen based on a favorable spin assignment (low $J$ with natural parity); the levels are listed in TABLE 4.2. The partial widths for the levels were
Contributions of Individual Resonances

Figure 4.2  The rates of each of the measured resonances are shown above.

### TABLE 4.2

**POSSIBLE LOW ENERGY $^{24}$Mg($\alpha,\gamma$)$^{28}$Si RESONANCES**

<table>
<thead>
<tr>
<th>$E_{ex}$ (keV)</th>
<th>$J^\pi$</th>
<th>$E_\alpha$ (cm) (keV)</th>
<th>$E_\alpha$ (lab) (keV)</th>
<th>$\omega_\gamma$ (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10514.1</td>
<td>2+</td>
<td>530</td>
<td>618</td>
<td>$2.83 \times 10^{-12}$</td>
</tr>
<tr>
<td>10805.5</td>
<td>2+</td>
<td>822</td>
<td>959</td>
<td>$3.55 \times 10^{-7}$</td>
</tr>
<tr>
<td>10883.5</td>
<td>2+</td>
<td>899</td>
<td>1049</td>
<td>$2.95 \times 10^{-6}$</td>
</tr>
</tbody>
</table>
calculated using equation (3.18). To estimate a value for $\theta^2$, which is generally determined experimentally, known values for nearby resonances with the same spin were averaged, leading to a value of 0.013 (the range of values was 0.000025 to 0.044).

The resonance strengths were then calculated from equation (3.17), and used to determine the reaction rate for each theoretical resonance. The results are shown in Figure 4.3. For comparison, the current experimental rate and the empirical rate are shown. The 1049 keV resonance appears to be too weak to contribute significantly to the reaction rate. The 958 keV resonance may be the dominant resonance for a small portion of the temperature range. In the lowest part of the temperature range (below $T_9 \sim 0.25$) the 618 keV resonance is likely the largest contributor.

**Maximum Contributions of Possible Resonances**

![Graph showing contributions of possible resonances](image)

Figure 4.3 The theoretical contributions of several possible low energy resonances are shown above. The theoretical rate shown in the plot is the empirical rate.
4.2 Conclusions

This measurement of the $^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$ reaction led to the discovery and characterization of several previously unknown resonances. These new resonances help to lessen the difference between the experimental and theoretically calculated reaction rates in the energy range relevant to AGB nucleosynthesis. Further, it has been shown that resonances at energies below the range of this experiment can make up the rest of the difference between observation and theory. Therefore, it can be concluded that the current theoretical models of this reaction are reasonable, and can appropriately be employed by nucleosynthesis simulations.
In addition to alpha capture, $^{24}\text{Mg}$ may also undergo neutron capture in AGB stars. This reaction, along with neutron capture on $^{25}\text{Mg}$ and $^{26}\text{Mg}$, was measured at the Neutron Time-of-Flight (n_TOF) facility at CERN in June 2003. First the motivation for this experiment will be examined. The experiment setup will then be described, followed by details of the data reduction and analysis, and a discussion of the results.

In the AGB stellar environment, neutron capture on $^{24}\text{Mg}$ (and subsequently on $^{25}\text{Mg}$ and $^{26}\text{Mg}$) has a role in establishing the magnesium isotopic abundance. In AGB stars, neutron capture proceeds through the slow neutron capture process, or $s$-process. Consider a nucleus with proton number $Z$ and atomic mass $A$ capturing a neutron to form the heavier isotope ($Z, A + 1$). If this isotope is stable, it may capture another neutron, and so on, until the resulting nucleus is unstable. In a low neutron flux environment, like an AGB star, the unstable nucleus will likely beta decay to a stable nucleus before capturing additional neutrons. Therefore, the $s$-process will follow the valley of stability on the chart of the nuclides, which can be seen in Figure 5.1. The $s$-process creates nuclei up to $^{209}\text{Bi}$, which is the heaviest stable nucleus.

SiC meteorite grains show predicted $s$-process abundances, especially of the noble gasses, indicating that these grains were indeed formed in AGB stars [26]. If the magnesium isotopic abundances observed in the grains were due solely to neutron capture, the abundances could be calculated from the neutron capture cross sections.
Figure 5.1 The chart of the nuclides is shown above. The black squares indicate stable nuclei; the $s$-process is confined to these nuclei. The tan area consists of unstable but known nuclei, while the blue area indicates nuclei that are theorized to exist but have not yet been observed. The figure is taken from reference [5].

Variations from the predicted values would indicate the presence of other reactions involving the magnesium isotopes.

One such reaction may be a neutron source in AGB stars: $^{22}\text{Ne}(\alpha,n)^{25}\text{Mg}$. This reaction was once thought to be the dominant neutron source in AGB stars. $^{14}\text{N}$ is abundant in the intershell region due to CNO cycling, and $^{22}\text{Ne}$ is produced during thermal pulses by the reaction sequence $^{14}\text{N}(\alpha,\gamma)^{18}\text{F}(\beta^+\nu)^{18}\text{O}(\alpha,\gamma)^{22}\text{Ne}$. If the temperature is above 300 million K, then the $^{22}\text{Ne}(\alpha,n)^{25}\text{Mg}$ reaction occurs readily [24]. However, this reaction must produce an enhancement in $^{25}\text{Mg}$ above the neutron capture abundance, which is not clearly observed. Another source of neutrons is the $^{13}\text{C}(\alpha,n)^{16}\text{O}$ reaction, which requires temperatures of only 100 million K. During the third dredge-up phase, protons from the convective envelope can mix down into the
intershell, where they combine with $^{12}\text{C}$ to produce $^{13}\text{C}$ via the reactions

$^{12}\text{C}(p,\gamma)^{13}\text{N}(\beta^+\nu)^{13}\text{C}$.

Precise knowledge of the magnesium neutron capture cross sections is needed to determine the amount of $^{25}\text{Mg}$ enhancement, if any, in SiC meteorite grains. The relative strength of the two neutron sources can be derived from the level of enhancement observed. Additionally, knowledge of the $^{26}\text{Mg}$ level structure can help in the determination of resonance parameters for the $^{22}\text{Ne}(\alpha,n)^{25}\text{Mg}$ reaction. However, neutron capture cross sections for the magnesium isotopes are quite small, and dominated by resonances, and most previous measurements suffer from large systematic uncertainties due to unrecognized background components [7]. To improve on this data, the cross sections were measured at the n_TOF facility.

5.1 Experimental Procedures

The neutron capture experiment was carried out at the n_TOF facility at CERN in June 2003. n_TOF utilizes the 20 GeV proton beam from the proton synchrotron injector. Neutrons are produced by spallation of the proton beam on an 80 cm x 80 cm x 60 cm lead target. The target is surrounded by water for cooling, and also for moderation of the neutrons. The resulting pulsed neutron flux is intense (on the order of $10^5$ neutrons/cm$^2$/pulse at the sample) and has a wide energy range: 1 eV to 250 MeV, with 2/3 of the flux in the 1 eV to 1 MeV range. The neutron beam travels 187.5 m from the spallation target to the n_TOF experimental hall, where the neutron energy is determined by the time of flight. The energy resolution is very good: $3 \times 10^{-4}$ at $E_n \sim 3$ eV and $1.5 \times 10^{-3}$ at $E_n \sim 30$ keV. To monitor the neutron beam, a Mylar foil with a $^6\text{Li}$ deposit is placed in the beam. An array of Si(Li) detectors observe the $^6\text{Li}(n,\alpha)$ reaction to determine the neutron flux on a pulse-by-pulse basis.

The experimental area is heavily shielded, by concrete to minimize neutrons and gamma rays and by iron to block muons, in an effort to achieve low ambient
Gamma rays from the neutron capture events are measured by a pair of C_6D_6 liquid scintillator detectors. This type of detector has a low sensitivity to neutrons in the energy range studied. Additionally, the detector housings were changed from aluminum to carbon fiber to further reduce neutron sensitivity. This was necessary because the neutron scattering cross section may be larger than the neutron capture cross section, especially for low mass targets such as magnesium. More details on the optimized C_6D_6 detectors can be found in reference [34]; a photograph of the target chamber and detectors can be seen in Figure 5.2. A new data acquisition system was designed for n_TOF, based on high-frequency Flash ADC’s, because the high neutron flux would lead to unacceptable pile-up with standard data acquisition systems.

The magnesium samples were in the form of small disks. The first sample was natural magnesium metal (79% ^{24}\text{Mg}, 10% ^{25}\text{Mg} and 11% ^{26}\text{Mg}), with a diameter of 2.2 cm and a thickness of 0.8 cm. Enriched magnesium oxide, enclosed in aluminum cans, was used for the ^{25}\text{Mg} and ^{26}\text{Mg} samples; the oxide samples had a diameter of 2.2 cm and a thickness of 0.2 cm. These samples were thin compared to those used in the most recent experiment [40]. Thinner samples were used in order to reduce the target corrections necessary for multiple scattering and self shielding. All of the samples were mounted in air on a carbon fiber sample changer.

5.2 Data Reduction

The raw signal from the n_TOF neutron capture experiment consists of the energy deposited in the detectors versus neutron time of flight. The neutron time of flight was converted to neutron energy, employing special relativity for energies above 100 keV. The first step in the data reduction is to correct for detector efficiency. This is accomplished through the pulse height weighting technique. The resulting yield must next be corrected for the background. Finally, the yield is normalized to the well
known yield of gold in order to correct for the neutron flux. These steps will be described in more detail in this section.

The low efficiency and small solid angle coverage of the $^6\text{C}_\text{D}_6$ detector allowed the use of the pulse height weighting technique in the data analysis. In this type of experiment, most gamma rays are detected with a multiplicity of one, that is, only one gamma ray of a decay cascade is detected. The pulse height weighting technique is used to make the detection efficiency dependent on the total energy of the excited state instead of the energy of the detected gamma. The total energy is easily calculated as the neutron energy plus the neutron separation energy of the target nuclei. Consider a nucleus with excitation energy $E_c$, which is equal to the sum of the energies of the cascade gammas:

$$E_c = \sum_{i=1}^{n} E_{\gamma_i}. \quad (5.1)$$

The detection efficiency for the cascade is given by:
\[ \eta_c = 1 - \prod_{i=1}^{n} \left(1 - \eta_{\gamma_i}\right), \]  

(5.2)

where \( \eta_{\gamma_i} \) is the detection efficiency for gamma ray \( i \). If the detection efficiency for each gamma ray is small \( (\eta_{\gamma_i} \ll 1) \), then the cascade efficiency can be approximated as

\[ \eta_c \approx \sum_{i=1}^{n} \eta_{\gamma_i}. \]  

(5.3)

If the efficiency of the detector is proportional to the gamma ray energy \( (\eta_{\gamma_i} = k \times E_{\gamma_i}) \), then using equation 5.3 the cascade efficiency would be proportional to the cascade energy:

\[ \eta_c = \sum_{i=1}^{n} \eta_{\gamma_i} = k \times \sum_{i=1}^{n} E_{\gamma_i} = k \times E_c. \]  

(5.4)

A linear relation between efficiency and gamma ray energy is not generally a feature of gamma ray detectors, but the detector response can be manipulated in order to impose this condition. The detector response function \( R_\gamma(E) \) describes the probability distribution for energy \( E \) deposited in the detector by a gamma ray of energy \( E_{\gamma_i} \), as illustrated in Figure 5.3. The response is related to the efficiency by

\[ \eta_{\gamma} = \int R_\gamma(E) dE. \]  

(5.5)

A weighting function \( W(E) \) can be introduced so that the proportionality condition is satisfied:

\[ \eta_{\gamma} = \int W(E) R_\gamma(E) dE = k \times E_{\gamma}. \]  

(5.6)

The detector response functions were determined by Monte Carlo simulations over a wide energy range. Special care was taken to include all features of the experimental setup, including the geometry of each separate sample. The weighting function was then determined by a least-squares fit. The method was verified by comparing calculated values to measured values for the 1.15 keV resonance in \(^{56}\text{Fe} \), and the
accuracy was found to be 2%. More details on the pulse height weighting technique can be found in reference [1].

The next step in the analysis is to subtract the background from the yield versus neutron energy spectrum. The first background component was stray gamma rays produced in the experimental area, through a number of different processes. These include photons produced in the spallation target, decay of charged particles (especially muons) produced in the spallation target, and capture of neutrons in the experimental area. To quantify these effects, the gamma yield was measured with no target in place.

The second background component in the magnesium experiment was neutrons scattering from the sample into the detectors. Although the detectors were designed to
have low neutron sensitivity, it is possible for neutrons to deposit energy in the
detectors by scattering on the deuterons. To account for this effect, the yield was
measured with a carbon sample in place; carbon has very few neutron capture
resonances so observed events would be from scattered neutrons. Additionally, carbon
was the best choice in the case of magnesium because of the relatively similar $Z$ values.

The yield with no target was subtracted from the carbon yield to determine the
yield due to scattering only. This yield was scaled point wise by the ratio of the
magnesium scattering cross section to the carbon scattering cross section to establish
magnesium scattering yield. The scattering cross sections were taken from the JENDL
database [18]. The no target yield was then added back to the scattering yield to give
the total background, accounting for both beam induced and target induced gammas.

An additional background component was present in the $^{25}\text{Mg}$ and $^{26}\text{Mg}$ yields, due to
the aluminum can used to hold the samples. In these cases, the yield from the empty
can was added to the background yield.

The background yield was subtracted from the measured yield for each sample
to obtain the neutron capture yield. To determine the capture cross section, the yield
was normalized to the neutron flux at the target. The target size was smaller than the
neutron beam spot, so the useable percentage of the beam was ascertained by measuring
a well-known resonance in $^{197}\text{Au}$. The gold target was the same size as the magnesium
samples and was placed in the same position to ensure that the fraction of the beam
impinging on it would be the same as for the magnesium targets. By comparing the
known cross section and the measured yield, the flux was determined, and the
magnesium capture cross sections were calculated.

5.3 R-Matrix Fitting

To parameterize the magnesium neutron capture cross sections in terms of
resonance energies and widths, the data was fit with the R-matrix code SAMMY. R-
matrix theory describes the interaction of nuclei in terms of observable quantities, rather than the physics of the interaction. The theory divides configuration space into two parts: the internal region, which corresponds to the compound nucleus, and the external region, where particle pairs, called channels, enter and exit the internal region. This is illustrated in Figure 5.4. The complete R-matrix theory is described by Lane and Thomas in reference [21], and its implementation in SAMMY is detailed in reference [22].

![Figure 5.4](image)

Figure 5.4 A schematic drawing of a compound nucleus reaction is seen above. Entrance channels can also appear as exit channels, but some exit channels cannot be entrance channels (e.g. fission channels). No assumptions are made about the nature of the interaction inside the internal region.

The cross sections calculated with the R-matrix theory cannot completely describe experimentally observed quantities, since there are always other effects than the interaction of a single neutron with a single nucleus. These effects include beam and sample size, impurities in the target, thermal motion of target nuclei, uncertainty in the neutron energy, and detector efficiency. All of these effects can be accounted for in the SAMMY code, in order to make a meaningful comparison between measured and calculated quantities.

SAMMY fits the data by means of Bayes’ method (also called generalized least squares). Bayes’ method requires that starting values be provided for all parameters to
be fit, but unlike the least squares method, the uncertainties for the parameters and the correlations between them are also known. Thus, results from a previous fitting can be used as input to a new analysis, and several sets of data can be fit sequentially, rather than simultaneously.

The initial parameters for resonance energies and widths were taken from references [19] and [40]. The general procedure for using SAMMY, as described in the latest version of the SAMMY manual (reference [23]), will be outlined. The first step is to use the best data available (generally high-resolution transmission data) to make a test of the input files without varying any of the parameters. Once the input files are deemed to be satisfactory, fits can be performed on the experimental data files—the parameters of large resonances are allowed to vary in order to generate a rough fit. This step is repeated, examining smaller and smaller regions until the fit is good for each data set. The final analysis is completed by allowing all relevant parameters to vary, and using the output parameters and covariances from one run as the input of the next run. First the energy parameters were allowed to vary, to fix the correct position for each resonance. Then the gamma and neutron widths were varied. The sequence of analysis was the reference data from natural Mg transmission, followed by the n_TOF natural Mg capture file (iterated several times), transmission followed by $^{25}\text{Mg}$ (iterated) and finally transmission followed by $^{26}\text{Mg}$ (iterated). The transmission data and fit can be seen in Figure 5.5.

The fitted resonance parameter values for $E_n = 0 – 5 \text{ MeV}$, along with the values from a recent Oak Ridge analysis (reference [19]), are listed in TABLE 5.1, and fits for prominent resonances can be seen in Figure 5.6 ($^{25}\text{Mg}$), Figure 5.7 ($^{24}\text{Mg}$), and Figure 5.8 ($^{26}\text{Mg}$). In the Oak Ridge analysis, the gamma widths of several resonances were assumed to be 3.000 eV—the real widths were not observed due to a lack of experimental sensitivity. The n_TOF analysis produced experimental values for these gamma widths. Additionally, several neutron widths were found to be incorrect, most
notably those of the broad resonances at 19.9 keV ($^{25}\text{Mg}$) and 83.1 keV ($^{24}\text{Mg}$). It was expected that the n_TOF data would provide a better characterization for broad resonances in particular, because of the reduction in systematic uncertainties through the sample and detector choices.

### TABLE 5.1

Mg(n,\(\gamma\)) RESONANCE PARAMETERS

<table>
<thead>
<tr>
<th>(n) TOF Experiment</th>
<th>Koehler Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>(E_n) (eV)</td>
<td>(\Gamma_\gamma) (eV)</td>
</tr>
<tr>
<td>(^{25}\text{Mg})</td>
<td>19929</td>
</tr>
<tr>
<td></td>
<td>62777</td>
</tr>
<tr>
<td></td>
<td>79716</td>
</tr>
<tr>
<td></td>
<td>81212</td>
</tr>
<tr>
<td>(^{24}\text{Mg})</td>
<td>46372</td>
</tr>
<tr>
<td></td>
<td>83126</td>
</tr>
<tr>
<td></td>
<td>93606</td>
</tr>
<tr>
<td></td>
<td>156303</td>
</tr>
<tr>
<td></td>
<td>177003</td>
</tr>
<tr>
<td></td>
<td>188351</td>
</tr>
<tr>
<td></td>
<td>257241</td>
</tr>
<tr>
<td></td>
<td>311517</td>
</tr>
<tr>
<td></td>
<td>362052</td>
</tr>
<tr>
<td></td>
<td>474492</td>
</tr>
<tr>
<td>(^{26}\text{Mg})</td>
<td>68529</td>
</tr>
<tr>
<td></td>
<td>219394</td>
</tr>
</tbody>
</table>
Figure 5.5 The natural Mg transmission data and SAMMY fit can be seen above. The fit is acceptable for all observed resonances.
Figure 5.6 The n_TOF data and SAMMY fits for $^{25}$Mg(n,γ) are shown, focused on regions where resonances are observed. The x-axis is the neutron energy (keV) and the y-axis is the cross section (b).
Figure 5.7  The n_TOF data and SAMMY fits for \(^{\text{nat}}\text{Mg}(n,\gamma)\) are shown, focused on regions where \(^{24}\text{Mg}(n,\gamma)\) resonances are observed. The x-axis is the neutron energy (keV) and the y-axis is the cross section (b).

Figure 5.8  The n_TOF data and SAMMY fits for \(^{26}\text{Mg}(n,\gamma)\) are shown, focused on regions where resonances are observed. The x-axis is the neutron energy (keV) and the y-axis is the cross section (b).
APPENDIX A

ELECTRONICS

The block diagrams for the timing and energy processing electronics are found here. Following the figures is a key for the abbreviations used (TABLE A.1).
Figure A.1 (Next Page) The electronics for processing the clover timing signals are shown in this figure. This portion of the electronics created the gates for the data acquisition. Additionally, the signals from each detector were collected by a scaler and a register.
Figure A.2  The electronics for processing the NaI(Tl) timing signals are shown above. The signals were collected by the scaler and the register.
Figure A.3  The electronics which processed the clover energy signals are shown at the top, while the electronics for the NaI(Tl) energy signals are shown in the center. At the bottom are the TAC electronics.
# TABLE A.1

**ELECTRONICS MODULES USED IN THE $^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$ EXPERIMENT**

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAEN</td>
<td>High Voltage Power Supply</td>
</tr>
<tr>
<td>J 65</td>
<td>Jorway 12-Bit NIM Input Register</td>
</tr>
<tr>
<td>L 429A</td>
<td>LeCroy Quad Mixed Logic Fan-In/Fan-Out</td>
</tr>
<tr>
<td>L 4434</td>
<td>LeCroy 32 Channel Latching Scaler</td>
</tr>
<tr>
<td>L 4616</td>
<td>LeCroy 16 Channel ECL/NIM/ECL Converter</td>
</tr>
<tr>
<td>O AD413A</td>
<td>EG&amp;G Ortec CAMAC Quad 8K ADC</td>
</tr>
<tr>
<td>O 567</td>
<td>EG&amp;G Ortec Time-to-Amplitude Converter/Single-Channel Analyzer</td>
</tr>
<tr>
<td>O 572</td>
<td>EG&amp;G Ortec Spectroscopy Amplifier and Pile-Up Rejector</td>
</tr>
<tr>
<td>O 660</td>
<td>Ortec Dual 5-kV Detector Bias Supply</td>
</tr>
<tr>
<td>O 672</td>
<td>EG&amp;G Ortec Spectroscopy Amplifier</td>
</tr>
<tr>
<td>O 863</td>
<td>Ortec Quad Timing Filter Amplifier</td>
</tr>
<tr>
<td>O CF8000</td>
<td>EG&amp;G Ortec/ESN Octal Constant Fraction Discriminator</td>
</tr>
<tr>
<td>O GG8000</td>
<td>EG&amp;G-ESN Octal Gate Generator</td>
</tr>
<tr>
<td>PS 726</td>
<td>Phillips Scientific 16 Channel Logic Level Translator</td>
</tr>
<tr>
<td>PS 752</td>
<td>Phillips Scientific Quad Two-Fold Logic Unit</td>
</tr>
<tr>
<td>PS 755</td>
<td>Phillips Scientific Four Input Majority Logic Unit</td>
</tr>
<tr>
<td>PS 794</td>
<td>Phillips Scientific Gate/Delay Generator</td>
</tr>
</tbody>
</table>
APPENDIX B

RESONANCE PARAMETERS

The constants used to calculate the thick-target yields for $^{27}\text{Al}(p,\gamma)^{28}\text{Si}$ reactions, and the resulting yields, are listed in TABLE B.1. The de Broglie wavelength $\hbar$ is given by equation 1.1:

$$\hbar = \frac{m + M}{M} \frac{\hbar}{\sqrt{2mE}},$$

and the stopping power $\varepsilon$ is taken from SRIM 2003 (and converted from the lab to center-of-mass frame). The relative values for the resonance strengths $\omega\gamma$ are taken from reference [14], and are scaled to the 992 keV resonance strength given in reference [33]. The thick target yield was calculated according to equation 1.14:

$$Y_x = \frac{2\pi^2 \hbar^2}{\varepsilon} \omega\gamma.$$

The de Broglie wavelengths, stopping powers and 1779 keV feeding probabilities for the $^{25}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$ resonances are given in TABLE B.2.
### TABLE B.1

$^{27}$Al($p, \gamma$)$^{28}$Si RESONANCE PARAMETERS

<table>
<thead>
<tr>
<th>$E_p$ (keV)</th>
<th>$\lambda^2$ (cm$^2$)</th>
<th>$\varepsilon$ (eV·cm)</th>
<th>$\omega\gamma$ (eV)</th>
<th>$Y_\infty$ ($^{28}$Si/proton)</th>
</tr>
</thead>
<tbody>
<tr>
<td>992</td>
<td>$2.269 \times 10^{-25}$</td>
<td>$7.593 \times 10^{-15}$</td>
<td>$1.91 \pm 0.11$</td>
<td>$(1.13 \pm 0.07) \times 10^{-9}$</td>
</tr>
<tr>
<td>760</td>
<td>$2.962 \times 10^{-25}$</td>
<td>$8.765 \times 10^{-15}$</td>
<td>$0.18 \pm 0.06$</td>
<td>$(1.20 \pm 0.40) \times 10^{-10}$</td>
</tr>
<tr>
<td>679</td>
<td>$3.315 \times 10^{-25}$</td>
<td>$9.287 \times 10^{-15}$</td>
<td>$(61 \pm 20) \times 10^{-3}$</td>
<td>$(4.3 \pm 1.4) \times 10^{-11}$</td>
</tr>
</tbody>
</table>

### TABLE B.2

$^{24}$Mg($\alpha, \gamma$)$^{28}$Si RESONANCE PARAMETERS

<table>
<thead>
<tr>
<th>$E_\alpha$ (keV)</th>
<th>$\lambda^2$ (cm$^2$)</th>
<th>$\varepsilon_{CM}$ (eV·cm)</th>
<th>Feeding Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1530</td>
<td>$4.66 \times 10^{-25}$</td>
<td>$41.93 \times 10^{-15}$</td>
<td>$0.25 \pm 0.03$</td>
</tr>
<tr>
<td>1413</td>
<td>$5.04 \times 10^{-25}$</td>
<td>$43.07 \times 10^{-15}$</td>
<td>$0.94 \pm 0.09$</td>
</tr>
<tr>
<td>1351</td>
<td>$5.27 \times 10^{-25}$</td>
<td>$43.68 \times 10^{-15}$</td>
<td>$0.75 \pm 0.05$</td>
</tr>
<tr>
<td>1277</td>
<td>$5.58 \times 10^{-25}$</td>
<td>$44.42 \times 10^{-15}$</td>
<td>$0.84 \pm 0.02$</td>
</tr>
<tr>
<td>1178</td>
<td>$6.05 \times 10^{-25}$</td>
<td>$45.38 \times 10^{-15}$</td>
<td>$0.15 \pm 0.03$</td>
</tr>
<tr>
<td>1120/1130</td>
<td>$6.30 \times 10^{-25}$</td>
<td>$45.84 \times 10^{-15}$</td>
<td>$0.74/1.00$</td>
</tr>
<tr>
<td>1087</td>
<td>$6.55 \times 10^{-25}$</td>
<td>$46.25 \times 10^{-15}$</td>
<td>$1.00$</td>
</tr>
<tr>
<td>&lt;1060</td>
<td>$6.72 \times 10^{-25}$</td>
<td>$46.50 \times 10^{-15}$</td>
<td>---</td>
</tr>
</tbody>
</table>
APPENDIX C
SUMMING CORRECTIONS

The calculation of the summing corrections for determination of the clover photopeak efficiency is described in this appendix.

C.1 $^{60}$Co Summing Corrections

![Decay Scheme for $^{60}$Ni](image)

Figure C.1 The decay scheme for $^{60}$Ni (from beta-decay of $^{60}$Co) is shown above. Gamma rays are labeled for calculation of summing correction coefficients.

The values of the branching ratios and summing corrections for the decay of $^{60}$Ni are listed in TABLE C.1. The values were calculated using equations 3.6 and 3.7, as detailed following TABLE C.1.
TABLE C.1

$^{60}$Ni DECAY SUMMING PARAMETERS

<table>
<thead>
<tr>
<th>$i$</th>
<th>$E_i$</th>
<th>$E_f$</th>
<th>$E_\gamma$</th>
<th>$P_i$</th>
<th>$B_i$</th>
<th>$\eta^\gamma(E_\gamma)$</th>
<th>$c_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2505.7</td>
<td>1332.5</td>
<td>1173.2</td>
<td>1.00</td>
<td>1.00</td>
<td>0.1604</td>
<td>0.1560</td>
</tr>
<tr>
<td>2</td>
<td>1332.5</td>
<td>0.0</td>
<td>1332.5</td>
<td>1.00</td>
<td>1.00</td>
<td>0.1560</td>
<td>0.1604</td>
</tr>
</tbody>
</table>

NOTE: The energies and decay probabilities are taken from reference [38].

\[
B_1 = P_1 \\
B_2 = P_1 P_2 \\

\begin{align*}
    c_1 &= \frac{1}{B_1} \left( P_1 P_2 \eta_2^\gamma \right) \\
    c_2 &= \frac{1}{B_2} \left( P_1 P_2 \eta_1^\gamma \right)
\end{align*}

89
Figure C.2  The decay scheme of the $^{28}$Si level produced by the $^{27}$Al(p,γ)$^{28}$Si reaction at $E_p = 992$ keV is shown above, with gamma rays labeled for summing correction calculations. The decays marked with thicker blue arrows were used for the clover efficiency measurement.

The values of the branching ratios and summing corrections for the decay of the excited $^{28}$Si state formed by $E_p = 992$ keV are listed in TABLE C.2. The values were calculated using equations 3.6 and 3.7, as detailed following TABLE C.2.
### TABLE C.2

$^{28}$Si DECAY ($E_{ex} = 12541$ keV) SUMMING PARAMETERS

<table>
<thead>
<tr>
<th>$i$</th>
<th>$E_i$</th>
<th>$E_f$</th>
<th>$E_\gamma$</th>
<th>$P_i$</th>
<th>$B_i$</th>
<th>$\eta^*(E_\gamma)$</th>
<th>$c_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12541</td>
<td>1779</td>
<td>10762</td>
<td>0.764</td>
<td>0.764</td>
<td>0.1230</td>
<td>0.1457</td>
</tr>
<tr>
<td>2</td>
<td>12541</td>
<td>4618</td>
<td>7923</td>
<td>0.0409</td>
<td></td>
<td>0.1206</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>12541</td>
<td>6276</td>
<td>6265</td>
<td>0.0215</td>
<td>0.0215</td>
<td>0.1204</td>
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NOTE: The energies and decay probabilities are taken from reference [12].
$B_1 = P_1$
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$B_6 = P_6$
$B_7 = P_7$
$B_8 = P_8$
$B_9 = P_9$
$B_{25} = P_6 P_{25} + P_6 P_{18} P_{25}$
$B_{36} = P_2 P_{36} + P_3 P_{36} + P_3 P_{36} + P_4 P_9 P_{36} + P_5 P_9 P_{36} + P_6 P_{27} P_{36} + P_6 P_{27} P_{36} + P_7 P_{22} P_{36}$
$B_{37} = P_5 P_{37} + P_5 P_{37} + P_5 P_{37} + P_6 P_{37} + P_6 P_{37} + P_6 P_{37} + P_6 P_{37} + P_7 P_{24} P_{37}$
$B_{38} = P_8 P_{38} + P_8 P_{38} + P_8 P_{38} + P_8 P_{38} + P_8 P_{38} + P_8 P_{38} + P_8 P_{38} + P_9 P_{28} P_{38}$

$$c_1 = \frac{1}{B_1} \left( P_{1} P_{37} \eta_{37}^T \right)$$

$$c_3 = \frac{1}{B_3} \left[ P_{33} P_{37} \left( \eta_{33}^T + \eta_{37}^T \right) + P_{34} P_{36} \eta_{36}^T + \eta_{37}^T \right]$$

$$c_6 = \frac{1}{B_6} \left[ P_{25} P_{37} \left( \eta_{25}^T + \eta_{37}^T \right) + P_{26} P_{36} \eta_{36}^T + \eta_{37}^T \right]$$

$$c_7 = \frac{1}{B_7} \left[ P_{20} P_{21} \eta_{20}^T + P_{21} P_{37} \left( \eta_{21}^T + \eta_{37}^T \right) + P_{22} P_{36} \eta_{36}^T + \eta_{37}^T \right]$$

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\[
\begin{align*}
c_8 &= \frac{1}{B_8} \left[ P_{P,14} P_{P,37} \left( \eta_{14}^T + \eta_{37}^T \right) + P_{P,15} P_{P,36} P_{P,37} \left( \eta_{15}^T + \eta_{36}^T + \eta_{37}^T \right) \right. \\
&+ P_{P,16} P_{P,33} P_{P,37} \left( \eta_{16}^T + \eta_{33}^T + \eta_{37}^T \right) + P_{P,17} P_{P,28} P_{P,37} \left( \eta_{17}^T + \eta_{28}^T + \eta_{37}^T \right) \\
&+ P_{P,18} P_{P,25} P_{P,37} \left( \eta_{18}^T + \eta_{25}^T + \eta_{37}^T \right) + P_{P,19} P_{P,20} \left( \eta_{19}^T + \eta_{20}^T \right) \\
&\left. + P_{P,21} P_{P,37} \left( \eta_{19}^T + \eta_{21}^T + \eta_{37}^T \right) + P_{P,22} P_{P,36} P_{P,37} \left( \eta_{19}^T + \eta_{22}^T + \eta_{36}^T + \eta_{37}^T \right) \\
&+ P_{P,23} P_{P,35} \left( \eta_{19}^T + \eta_{23}^T + \eta_{35}^T + \eta_{37}^T \right) + P_{P,24} P_{P,33} P_{P,37} \left( \eta_{19}^T + \eta_{24}^T + \eta_{33}^T + \eta_{37}^T \right) \\
&+ P_{P,27} P_{P,34} P_{P,37} \left( \eta_{19}^T + \eta_{27}^T + \eta_{34}^T + \eta_{36}^T + \eta_{37}^T \right) \right] \\

c_9 &= \frac{1}{B_9} \left[ P_{P,10} \eta_{10}^T + P_{P,11} P_{P,37} \left( \eta_{11}^T + \eta_{37}^T \right) + P_{P,12} P_{P,36} P_{P,37} \left( \eta_{12}^T + \eta_{36}^T + \eta_{37}^T \right) \right. \\
&\left. + P_{P,13} P_{P,35} P_{P,37} \left( \eta_{13}^T + \eta_{35}^T + \eta_{37}^T \right) \right] \\
c_{25} &= \frac{1}{B_{25}} \left[ P_{P,25} P_{P,37} \left( \eta_{25}^T + \eta_{37}^T \right) + P_{P,26} P_{P,25} P_{P,37} \left( \eta_{18}^T + \eta_{26}^T + \eta_{37}^T \right) \right. \\
&\left. + P_{P,27} P_{P,24} P_{P,37} \left( \eta_{18}^T + \eta_{27}^T + \eta_{34}^T + \eta_{37}^T \right) \right] \\
c_{36} &= \frac{1}{B_{36}} \left[ P_{P,32} P_{P,36} P_{P,37} \left( \eta_{32}^T + \eta_{37}^T \right) + P_{P,34} P_{P,36} P_{P,37} \left( \eta_{34}^T + \eta_{37}^T \right) \right. \\
&\left. + P_{P,35} P_{P,36} P_{P,37} \left( \eta_{35}^T + \eta_{37}^T \right) \right. \\
&+ P_{P,36} P_{P,36} P_{P,37} \left( \eta_{36}^T + \eta_{37}^T \right) + P_{P,37} P_{P,36} P_{P,37} \left( \eta_{37}^T \right) \\
&\left. + P_{P,38} P_{P,36} P_{P,37} \left( \eta_{38}^T + \eta_{37}^T \right) \right. \\
&+ P_{P,39} P_{P,36} P_{P,37} \left( \eta_{39}^T + \eta_{37}^T \right) + P_{P,40} P_{P,36} P_{P,37} \left( \eta_{40}^T + \eta_{37}^T \right) \\
&\left. + P_{P,41} P_{P,36} P_{P,37} \left( \eta_{41}^T + \eta_{37}^T \right) \right]
\end{align*}
\]
\[ c_{37} = \frac{1}{B_{37}} \left[ P_1 P_{37} \eta_{1}^T + P_2 P_{36} P_3 (\eta_{2}^T + \eta_{36}^T) + P_3 P_{33} P_{37} (\eta_{3}^T + \eta_{33}^T) \right. \\
+ P_4 P_{34} P_{36} P_{37} (\eta_{4}^T + \eta_{34}^T + \eta_{36}^T) + P_4 P_{31} P_{37} (\eta_{4}^T + \eta_{31}^T) \\
+ P_5 P_{32} P_{36} P_{37} (\eta_{5}^T + \eta_{32}^T + \eta_{36}^T) + P_5 P_{28} P_{37} (\eta_{5}^T + \eta_{28}^T) \\
+ P_5 P_{29} P_{36} P_{37} (\eta_{5}^T + \eta_{29}^T + \eta_{36}^T) + P_6 P_{26} P_{36} P_{37} (\eta_{6}^T + \eta_{26}^T + \eta_{36}^T) \\
+ P_6 P_{27} P_{33} P_{37} (\eta_{6}^T + \eta_{27}^T + \eta_{36}^T) + P_6 P_{27} P_{34} P_{36} P_{37} (\eta_{6}^T + \eta_{27}^T + \eta_{34}^T + \eta_{36}^T) \\
+ P_7 P_{21} P_{37} (\eta_{7}^T + \eta_{21}^T) + P_8 P_3 P_{36} P_{37} (\eta_{7}^T + \eta_{22}^T + \eta_{36}^T) \\
+ P_7 P_{23} P_{33} P_{37} (\eta_{7}^T + \eta_{23}^T + \eta_{33}^T) + P_7 P_{24} P_{33} P_{37} (\eta_{7}^T + \eta_{24}^T + \eta_{33}^T) \\
+ P_7 P_{24} P_{34} P_{36} P_{37} (\eta_{7}^T + \eta_{24}^T + \eta_{34}^T + \eta_{36}^T) + P_8 P_{14} P_{37} (\eta_{8}^T + \eta_{14}^T) \\
+ P_8 P_{15} P_{36} P_{37} (\eta_{8}^T + \eta_{15}^T + \eta_{36}^T) + P_8 P_{16} P_{33} P_{37} (\eta_{8}^T + \eta_{16}^T + \eta_{33}^T) \\
+ P_8 P_{16} P_{34} P_{36} P_{37} (\eta_{8}^T + \eta_{16}^T + \eta_{34}^T + \eta_{36}^T) + P_8 P_{17} P_{28} P_{37} (\eta_{8}^T + \eta_{17}^T + \eta_{28}^T) \\
+ P_8 P_{17} P_{29} P_{36} P_{37} (\eta_{8}^T + \eta_{17}^T + \eta_{29}^T + \eta_{36}^T) + P_8 P_{18} P_{25} P_{37} (\eta_{8}^T + \eta_{18}^T + \eta_{25}^T) \\
+ P_8 P_{18} P_{27} P_{33} P_{37} (\eta_{8}^T + \eta_{18}^T + \eta_{27}^T + \eta_{33}^T) \\
+ P_8 P_{19} P_{27} P_{34} P_{36} P_{37} (\eta_{8}^T + \eta_{19}^T + \eta_{27}^T + \eta_{34}^T + \eta_{36}^T) + P_9 P_{19} P_{21} P_{37} (\eta_{9}^T + \eta_{19}^T + \eta_{21}^T) \\
+ P_8 P_{19} P_{22} P_{33} P_{37} (\eta_{8}^T + \eta_{19}^T + \eta_{22}^T + \eta_{33}^T) + P_8 P_{19} P_{23} P_{35} P_{37} (\eta_{8}^T + \eta_{19}^T + \eta_{23}^T + \eta_{35}^T) \\
+ P_8 P_{19} P_{24} P_{33} P_{37} (\eta_{8}^T + \eta_{19}^T + \eta_{24}^T + \eta_{33}^T) \\
+ P_9 P_{19} P_{24} P_{34} P_{36} P_{37} (\eta_{8}^T + \eta_{19}^T + \eta_{24}^T + \eta_{34}^T + \eta_{36}^T) + P_9 P_{11} P_{37} (\eta_{9}^T + \eta_{11}^T) \\
+ P_9 P_{12} P_{36} P_{37} (\eta_{9}^T + \eta_{12}^T + \eta_{36}^T) + P_9 P_{13} P_{35} P_{37} (\eta_{9}^T + \eta_{13}^T + \eta_{35}^T) } \]
C.3 \(^{27}\text{Al}(p, \gamma)^{28}\text{Si}, E_p = 679 \text{ keV Summing Corrections}\)

Figure C.3  The decay scheme of the \(^{28}\text{Si}\) level produced by the \(^{27}\text{Al}(p, \gamma)^{28}\text{Si}\) reaction at \(E_p = 679 \text{ keV}\) is shown above, with gammas numbered for summing corrections. The decays marked with thicker blue arrows were used for the clover efficiency measurement, while the gammas marked with thicker red arrows were used to normalize the efficiency values to the \(E_p = 992 \text{ keV}\) resonance.

The values of the branching ratios and summing corrections for the decay of the excited \(^{28}\text{Si}\) state formed by \(E_p = 679 \text{ keV}\) are listed in TABLE C.3. The values were calculated using equations 3.6 and 3.7, as detailed following TABLE C.3.
### TABLE C.3

$^{28}\text{Si DECAY ($E_{ex} = 12239 \text{ keV}$) SUMMING PARAMETERS}$

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**NOTE:** The decay probabilities are taken from reference [31].

\[
B_1 = P_1 \\
B_2 = P_2 \\
B_3 = P_4 \\
B_{10} = P_4 P_{10} \\
B_{13} = P_2 P_{13} + P_3 P_9 P_{13} \\
B_{14} = P_1 P_{14} + P_2 P_{13} P_{14} + P_3 P_{12} P_{14} + P_4 P_{11} P_{14} + P_5 P_{10} P_{14} + P_6 P_9 P_{13} P_{14} + P_7 P_8 P_{14} + P_8 P_7 P_{14} \\
\]

\[
c_i = \frac{1}{B_1} \left( P_1 P_{14} \eta^T_{14} \right) \\
c_2 = \frac{1}{B_2} \left[ P_2 P_{13} P_{14} \left( \eta^T_{13} + \eta^T_{14} \right) \right]
\]
\[
c_4 = \frac{1}{B_4} \left[ P_{4} P_{10} \eta_{10} + P_{4} P_{11} P_{14} \left( \eta_{11} + \eta_{14} \right) \right]
\]

\[
c_{10} = \frac{1}{B_{10}} \left( P_{4} P_{10} \eta_{14} \right)
\]

\[
c_{13} = \frac{1}{B_{13}} \left[ P_{2} P_{13} P_{14} \left( \eta_{2} + \eta_{14} \right) + P_{5} P_{9} P_{13} P_{14} \left( \eta_{5} + \eta_{9} + \eta_{14} \right) \right]
\]

\[
c_{14} = \frac{1}{B_{14}} \left[ P_{1} P_{14} \eta_{1} + P_{2} P_{13} P_{14} \left( \eta_{2} + \eta_{13} \right) + P_{3} P_{12} P_{14} \left( \eta_{3} + \eta_{12} \right) + P_{4} P_{11} P_{14} \left( \eta_{4} + \eta_{11} \right) \right.
\]
\[
+ P_{5} P_{8} P_{14} \left( \eta_{5} + \eta_{8} \right) + P_{5} P_{9} P_{13} P_{14} \left( \eta_{5} + \eta_{9} + \eta_{13} \right) + P_{6} P_{7} P_{14} \left( \eta_{6} + \eta_{7} \right) \left. \right]
\]
APPENDIX D
BRANCHING RATIO CALCULATIONS

The calculations of the branching ratios for $^{28}\text{Si}$ levels formed by the $^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$ reaction are detailed in this appendix. In the equations, $P_i$ represents the probability for the level to decay by emitting gamma ray $i$. The systems of equations were solved symbolically.

D.1 $E_\alpha = 1178$ keV

The decay of the $^{28}\text{Si}$ level is illustrated in Figure D.1, and is followed by the system of equations used to calculate the branching ratios.
Figure D.1  The decay of the $E_{ex} = 10994$ state is shown above. The thick blue lines indicate the gammas that were used for the branching ratio calculations.

\[
\frac{I_1^{obs}}{\eta_1 P_1} = \frac{I_3^{obs}}{\eta_3 P_3} \left( \frac{1}{1 - \eta_2^P} \right)
\]

\[P_1 + P_3 = 1\]

**D.2 $E_\alpha = 1350$ keV**

The decay of the $^{28}$Si level is illustrated in Figure D.2, and is followed by the system of equations used to calculate the branching ratios.
The decay of the $E_{\text{ex}} = 11142$ state is shown above. The thick blue lines indicate the gammas that were used for the branching ratio calculations.

\[
\frac{I_1^{\text{obs}}}{\eta_1 P_1} \left( \frac{1}{1 - \eta_{10}^T} \right) = \frac{I_2^{\text{obs}}}{\eta_2 P_2} \left( \frac{1}{1 - \eta_9^T - \eta_{10}^T} \right) \\
\frac{I_2^{\text{obs}}}{\eta_2 P_2} \left( \frac{1}{1 - \eta_9^T - \eta_{10}^T} \right) = \frac{I_3^{\text{obs}}}{\eta_3 P_3} \left( \frac{1}{1 - \eta_7^T P_7 - \eta_8^T P_8 - \eta_{10}^T P_8} \right) \\
\frac{I_3^{\text{obs}}}{\eta_3 P_3} \left( \frac{1}{1 - \eta_7^T P_7 - \eta_8^T P_8 - \eta_{10}^T P_8} \right) = \frac{I_5^{\text{obs}}}{\eta_5 P_5} \left( \frac{1}{1 - \eta_4^T} \right) \\
P_1 + P_2 + P_3 + P_5 = 1
\]

D.3 $E_\alpha = 1530$ keV

The decay of the $^{28}\text{Si}$ level is illustrated in Figure D.3, and is followed by the system of equations used to calculate the branching ratios.
The decay of the $E_{\text{ex}} = 11295$ state is shown above. The thick blue lines indicate the gammas that were used for the branching ratio calculations.

\[
\begin{align*}
\frac{I_{1}^{\text{obs}}}{\eta_{1}P_{1}} &= \frac{I_{2}^{\text{obs}}}{\eta_{2}P_{2}} \left( \frac{1}{1 - \eta_{12}^T} \right) \\
\frac{I_{2}^{\text{obs}}}{\eta_{2}P_{2}} \left( \frac{1}{1 - \eta_{12}^T} \right) &= \frac{I_{3}^{\text{obs}}}{\eta_{3}P_{3}} \left( \frac{1}{1 - \eta_{10}^T - \eta_{12}^T} \right) \\
\frac{I_{5}^{\text{obs}}}{\eta_{5}P_{5}} \left( \frac{1}{1 - \eta_{10}^T - \eta_{12}^T} \right) &= \frac{I_{6}^{\text{obs}}}{\eta_{6}P_{6}} \left( \frac{1}{1 - \eta_{9}^T - \eta_{12}^T} \right) \\
\frac{I_{6}^{\text{obs}}}{\eta_{6}P_{6}} \left( \frac{1}{1 - \eta_{9}^T - \eta_{12}^T} \right) &= \frac{I_{7}^{\text{obs}}}{\eta_{7}P_{7}} \left( \frac{1}{1 - \eta_{8}^T P_{8} - \eta_{12}^T P_{8} - \eta_{12}^T P_{8}} \right) \\
P_{1} + P_{2} + P_{3} + P_{4} + P_{5} &= 1
\end{align*}
\]
APPENDIX E

TABULATED REACTION RATES

TABLE E.1

\(^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si} \) REACTION RATES FOR \( T_9 = 0.1 - 10 \)

<table>
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<tr>
<th>( T_9 )</th>
<th>Current Exp. Rate</th>
<th>CF88 Rate</th>
<th>Old Exp. Rate</th>
<th>NON-SMOKER</th>
<th>Empirical Rate</th>
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<td>0.00E+00</td>
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<td>1.41E+03</td>
<td>1.21E+03</td>
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</table>
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