AN INDIRECT STUDY OF THE ASTROPHYSICAL $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ REACTION
AND ITS INFLUENCE ON TYPE-1 X-RAY BURST LIGHT CURVES

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Abstract

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Shortly after their discovery in 1976, x-ray bursts were determined to be thermonuclear runaways occurring on the surface of neutron stars in binary systems with H/He rich low-mass companion stars. During these explosive events the sudden release of nuclear energy heats up the atmosphere, causing its luminosity to rapidly increase by more than an order of magnitude within a few seconds. Over the past three decades, thousands of bursts from almost a hundred systems have been observed, revealing a rich diversity of bursting characteristics. One of the more interesting characteristics observed is the double-peak structure in the luminosity curve seen in a handful of x-ray burst events. Through modeling and simulations of these exotic events, it was found that this double-peaked structure can arise in systems where a thermonuclear runaway is triggered within pure He layer below a mixed H/He layer. In this scenario, the rapid consumption of the pure He layer is seen to be the source of the first peak, while the second peak occurs due to the burning of the mixed H/He layer with the $\alpha$-process and the rp-process. Current x-ray burst sensitivity studies have revealed that certain ($\alpha$,p) reactions along the $\alpha$-process have a direct influence on the early rise-time structure of x-ray burst light curves originating from mixed H/He burning. More notably, results from the sensitivity study of Fisker et al., showed the importance of $^{34}$Ar as one possible waiting point in the rp-process
given its relatively long $\beta$-decay half-life and low Q-value for the $^{34}\text{Ar}(p,\gamma)$ reaction. The $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ reaction within the $\alpha$-process may act as a bypass for this waiting point, depending on its strength. Fisker et al. showed that by varying the strength of the $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ reaction, which was based on Hauser Feshbach (HF) prediction, a possible double peaked light curve would emerge in particular simulation for lower reaction strength values. This suggest that nuclear impedance by a possible waiting point in $^{34}\text{Ar}$ could contribute to the dipping structure observed in double peak light curves.

It has been augured that a HF predicted rate is most likely not valid for the $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ reaction based on level density considerations in the compound nucleus $^{38}\text{Ca}$. Given proximity of $^{38}\text{Ca}$ to the $N = Z = 20$ shell closure and it’s low $\alpha$-threshold, the number of levels available to participate at the relative bombarding energies may not be high enough to satisfy the statistical approach of a HF model. Instead this reaction may possibly proceed via a handful of strong $\alpha$-cluster resonances located within the relevant energy window, and because of this, any HF prediction would grossly mis-predict the $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ reaction rate.

The predictive power of current x-ray burst models depends critically on the accuracy of the many reaction rates involved. Therefore, to use these models to explore other parameters relevant to the double-peak bursting behavior, such as accretion rates and metallicities, this large uncertainty in the $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ reaction rate must be significantly reduced.

With this in mind, the $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ reaction was indirectly studied using the $^{40}\text{Ca}(p,t)^{38}\text{Ca}$ to study possible $(\alpha,p)$ resonances in the compound nucleus $^{38}\text{Ca}$. This experiment was performed at iThemba LABS using 100 MeV, dispersion matched proton beam and the K=600 spectrograph. Given the information collected on the triton reaction products in the focal plane detectors of the K=600 spectrograph, excitation energies of levels populated in the recoil nucleus $^{38}\text{Ca}$ were determined.
with uncertainties within 10 keV. From this experiment, 45 states were identified in $^{38}$Ca, of which 33 states above the $\alpha$-threshold, that could act as possible ($\alpha$,p), were identified.

With precise energy information on possible resonances taken from this work, along with model based assumptions to fill in the remaining unknown resonances parameters, a Monte Carlo calculation was performed based on narrow resonance formalism to generate distributions of the $^{34}$Ar($\alpha$,p)$^{37}$K reaction rate over a range of astrophysical temperatures relevant to XRB’s. From these rate distributions, the $^{34}$Ar($\alpha$,p)$^{37}$K reaction rate was found to be significantly lower than the corresponding HF predicted rate used in X-ray burst models. This lower ($\alpha$,p) rate implies that $^{34}$Ar is most likely a waiting point nuclei and a possibly impedance source that will contribute to the structure of the dip and second observed peak.

Additionally, to investigate the effects of a lower $^{34}$Ar($\alpha$,p)$^{37}$K reaction rate on x-ray burst light curves, simulations were performed using the single-zone self-consistent model, ONEZONE. These initial studies using the ONEZONE model show that this lower $^{34}$Ar($\alpha$,p)$^{37}$K rate delays the peak of the burst by roughly half a second, but further studies with more sophisticated models need to be done before any thing conclusive can be reached.
To my mother, Tamara Geltmaker.
Gone is the classical conception of the Universe as a serene and majestic ensemble whose slow evolution is regulated by the consumption of nuclear fuel. The Universe we know today is pervaded by the echoes of enormous explosions and rent by abrupt changes of luminosity on large energy scales. From the initial explosion to formation of galaxies and clusters, from the birth to the death of stars, high-energy phenomena are the norm and not the exception in the evolution of the Universe.

CHAPTER 1: INTRODUCTION

1.1 X-ray Binaries
  1.1.1 High Mass X-ray Binaries
  1.1.2 Low Mass X-ray Binaries

1.2 Observation of Bursting Behavior in LMXBs

1.3 XRB's and the Thermonuclear Runaway Model
  1.3.1 Breakout of the $\beta$-Limited CNO Cycle
  1.3.2 The rp-Process
  1.3.3 The $\alpha p$-Process

1.4 XRB Models

1.5 Influence of Reaction Rate Uncertainties on Simulated Light Curves

1.6 Nuclear Uncertainties within the $\alpha p$-Process

1.7 Sensitivity Studies of Double-Peaked XRB’s

1.8 PR137: Experimentally Measuring the $^{34}$Ar($\alpha,p)^{37}$K Reaction
  1.8.1 Advantages of the ($p,t$) Reaction

CHAPTER 2: PR137: EXPERIMENTAL SETUP

2.1 Brief Overview of PR137 at iThemba LABs

2.2 The K=600 Magnetic Spectrograph

2.3 The Focal Plane Detector System
  2.3.1 Plastic Scintillators
  2.3.2 Vertical Drift Chambers

2.4 100 MeV Proton Beam

2.5 Dispersion Matching Techniques

2.6 Faint Beam Method at iThemba LABS

2.7 Off-Focus Mode

2.8 Targets

2.9 Settings for the ($p,t$) Reaction
FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>A face-on schematic cartoon of a low-mass X-ray binary system.</td>
<td>3</td>
</tr>
<tr>
<td>1.2</td>
<td>Example XRB events observed by X-ray bursts</td>
<td>4</td>
</tr>
<tr>
<td>1.3</td>
<td>Diagram depicting a typical X-ray burst event</td>
<td>7</td>
</tr>
<tr>
<td>1.4</td>
<td>Possible reaction flows occurring during an XRB</td>
<td>9</td>
</tr>
<tr>
<td>1.5</td>
<td>Possible reaction flow through the sd-shell</td>
<td>12</td>
</tr>
<tr>
<td>1.6</td>
<td>X-ray bursts: Comparisons between models and observation</td>
<td>17</td>
</tr>
<tr>
<td>1.7</td>
<td>Simulated XRB Light Curves from Fisker et al. [19]</td>
<td>22</td>
</tr>
<tr>
<td>1.8</td>
<td>Simulated XRB Light Curves from Lampe et al. [56]</td>
<td>25</td>
</tr>
<tr>
<td>2.1</td>
<td>Layout of the experimental facility at iThemba LABS.</td>
<td>30</td>
</tr>
<tr>
<td>2.2</td>
<td>The K=600 spectrograph.</td>
<td>32</td>
</tr>
<tr>
<td>2.3</td>
<td>The focal plane setup of the K=600.</td>
<td>35</td>
</tr>
<tr>
<td>2.4</td>
<td>Schematics of the focal plane’s VDC detectors</td>
<td>37</td>
</tr>
<tr>
<td>2.5</td>
<td>Schematic of the beam transformation at the target</td>
<td>41</td>
</tr>
<tr>
<td>2.6</td>
<td>Lateral and angular dispersion matching</td>
<td>44</td>
</tr>
<tr>
<td>2.7</td>
<td>Faint beam observations in the focal plane</td>
<td>47</td>
</tr>
<tr>
<td>2.8</td>
<td>Raw focal plane spectra given each experimental setting.</td>
<td>53</td>
</tr>
<tr>
<td>3.1</td>
<td>Background reduction in focal plane spectra</td>
<td>55</td>
</tr>
<tr>
<td>3.2</td>
<td>Angle calibration using pepper pot.</td>
<td>57</td>
</tr>
<tr>
<td>3.3</td>
<td>Offline corrections to θ vs. x plots</td>
<td>60</td>
</tr>
<tr>
<td>3.4</td>
<td>Fitted spectrum of $^{22}Mg$.</td>
<td>63</td>
</tr>
<tr>
<td>3.5</td>
<td>Focal plane spectrum of $^{22}Mg$.</td>
<td>66</td>
</tr>
<tr>
<td>3.6</td>
<td>Residual energy plot from focal plane calibration</td>
<td>68</td>
</tr>
<tr>
<td>3.7</td>
<td>$B\rho$ map of focal plane</td>
<td>70</td>
</tr>
<tr>
<td>4.1</td>
<td>Final spectra of $^{38}Ca$ at $\theta_{lab} = 8^\circ$</td>
<td>73</td>
</tr>
<tr>
<td>5.1</td>
<td>Distributions of the $^{34}Ar(\alpha,p)^{37}K$ rate at selected temperatures.</td>
<td>85</td>
</tr>
<tr>
<td>5.2</td>
<td>The stellar $^{34}Ar(\alpha,p)^{37}K$ rate</td>
<td>87</td>
</tr>
</tbody>
</table>
5.3 Level Diagram of $^{38}$Ca along with selected Gamow windows . . . . . . 88
5.4 The stellar $^{34}$Ar($\alpha,p$)$^{37}$K rate including one possible $\alpha$-cluster state . 90
6.1 Time Integrated Reaction Flow during XRB simulation . . . . . . . . 98
6.2 Simulated Light Curves From ONEZONE XRB model . . . . . . . . 100
A.1 Types of Capture Reactions . . . . . . . . . . . . . . . . . . . . . . 109
A.2 The Gamow Window . . . . . . . . . . . . . . . . . . . . . . . . . . . 112
B.1 Cartoon of reaction kinematics . . . . . . . . . . . . . . . . . . . . . . 123
D.1 Spin Distributions based on FBGM . . . . . . . . . . . . . . . . . . . 133
D.2 The Rejection Method . . . . . . . . . . . . . . . . . . . . . . . . . . . 134
TABLES

1.1 THE GAMOW WINDOW FOR $^{34}$Ar(α,p)$^{37}$K GIVEN SELECTED XRB’S TEMPERATURES. ................................................. 28
2.1 ION-OPTIC PROPERTIES OF K=600 SPECTROGRAPH. ........ 33
2.2 LIST OF TARGET USED IN PR137. ........................................ 50
3.1 LIST STATES IN $^{22}$Mg USED FOR CALIBRATION OF FOCAL PLANE ................................................................. 67
4.1 LIST OF STATES IN $^{38}$Ca BELOW THE α-THRESHOLD ......... 74
4.2 LIST OF STATES IN $^{38}$Ca ABOVE THE α-THRESHOLD ........... 77
5.1 LIST OF TOTAL $^{34}$Ar(α,p)$^{37}$K REACTION RATES $N_A \langle \sigma v \rangle$ FROM THIS WORK ................................................................. 93
6.1 IGNITION CONDITIONS FOR ONEZONE SIMULATIONS. ...... 97
C.1 LIST OF STATES IN $^{38}$Ca GIVEN EACH EXPERIMENTAL SETTING ................................................................. 128
E.1 LIST OF PROTON SINGLE PARTICLE WIDTHS .................. 138
E.2 LIST OF α SINGLE PARTICLE WIDTHS ............................. 140
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CHAPTER 1

INTRODUCTION

1.1 X-ray Binaries

Since their first discovery in 1962 [33], X-ray binaries have captured our curiosity and have motivated a whole new field of observational astronomy. These systems dominate the night time sky within the x-ray regime, and display dramatic variations in luminosity on timescales from milliseconds to years. Soon after their discovery, it was recognized that the nature of these sources came from matter being transferred from the outer most layers of a normal star (donor) onto a compact companion (white dwarf, neutron star, or black hole). As this material reaches the compact object’s surface it releases a great amount of gravitational energy, which is quickly thermalized and radiated away in the x-ray regime [16].

With observations of more x-ray binary systems by satellites like Vela and Uhuru [23, 24, 34, 44], it was soon realized that there are two main processes, depending on the mass of the donor star, through which material can be transferred from a normal star onto its compact companion. These two sub-groups were aptly named High-mass X-ray Binaries (HMXBs) and Low-mass X-ray Binaries (LMXBs).

1.1.1 High Mass X-ray Binaries

For binary systems with more massive stars ($\geq 10 \text{ M}_\odot$), the outer layers of the donor star can be lost through significantly strong stellar winds. Some of this material ($\sim 0.1\%$) is subsequently caught in the strong gravitational potential well of the
compact companion through Bondi-Hoyle accretion [10]. The energy released as the matter falls into the compact companion’s potential well is converted into a strong flux of x-rays. The donors in these systems are all massive young stars, therefore the optical luminosity dominates the total emission of the binary system, making them easy to identify once their x-ray components are observed.

1.1.2 Low Mass X-ray Binaries

If the donor star is of low mass (less than or equal to the neutron star), it cannot produce strong enough stellar winds to power the x-ray source in the same way as a HMXB system. Instead, as the donor star evolves its outer envelope expands past the Roche lobe causing some of this material to transfer through the inner Lagrangian point onto the compact companion. This material has some angular momentum as it is transfered, and therefore does not fall directly on the the compact companion but instead forms a large accretion disk, as seen in fig. 1.1. As the material falls down through the accretion disk, much of the released gravitational energy (≈200 MeV/nucleon) goes into heating the inner regions of the accretion disk and the outer envelope of the compact object to the point where it glows in the x-ray regime. These systems are usually associated with older population II donor stars, and therefore the accreted matter will mostly consist of H and He, along with very small amounts of heavier elements [54]. In contrast to HMXB, these donor stars will often go undetected due to their size and faint optical signal.

1.2 Observation of Bursting Behavior in LMXBs

Within the class of LMXBs exists a special case of accreting binaries that display recurring bursts of x-rays that can last anywhere from a seconds to minutes, and releases typically $10^{39}$ ergs of energy during the event. For comparison, it would take the Sun’s corona about 3000 years to emit as much energy [25]. This phenomena,
Figure 1.1. A face on schematic cartoon of a low-mass X-ray binary system. Here, material is transferred from a low mass companion star to the neutron star through Roche lobe overflows. Due to the material having some angular momentum, an accretion disk forms around the compact companion (here a neutron star).

Later termed a type 1 X-ray burst (XRB), is seen as a sudden increase in the x-ray flux by usually more than an order of magnitude, followed by a slow decay, that sits on top of the already existing persistent flux from the inner accretion disk. This variation in the x-ray flux as a function of time during the bursting events is commonly referred to as the XRB’s light curve. These dramatic events typically occur in LMXB systems containing neutron stars as the compact companion. Since their first discovery in 1976 by Grindlay et al [35] and Belian et al [6], XRB’s have garnered a lot of attention within the x-ray astronomy community. To date over 6000 light curves from individual burst events in 84 LMXB systems have been observed [31]. Given the abundance of observations, a diverse range of bursting behaviors can be seen to emerge (see Figure 1.2 for selected examples of XRB profiles). For the most part these systems exhibit recurrent burst periods ranging from hours to days, burst timescales on the order of seconds to minutes, and regularly achieve peak luminosities of $\sim 10^{38}$ ergs s$^{-1}$.
Figure 1.2. Selected light curves of LMXB systems exhibiting bursting behavior.  

a. Figure taken from [32], showing single-peaked bursts observed by RXTE in systems GS 1826-24, 4U 1728-34, and , 4U 2129+12. The burst light curve from 4U 2129+12 in the bottom right panel shows evidence of PRE. 

b. Double-peaked light curve observed in 4U 1636-53 by EXOSAT [87].

c. Triple-peaked light curve observed in 4U 1636-53 also by EXOSAT [92].

d. Double-peaked light curve observed in MXB 1730-3354U by RXTE [5].
The majority of observed XRB light curves follow this theme of fast rise times followed by slow decays within some variation, however in a handful of systems have been seen to exhibit exotic multi-peak structures within the x-ray flux during a single XRB event. These types of exotic events have been observed in systems such as 4U 1608-52 [72, 73], GX 17+2 [53], 4U 1709-267 [45], and 4U 1636-53 [87, 92] (which has shown double- and triple-peaked bursts). Initially these structures were thought to originate from bursts undergoing Photospheric Radius Expansion (PRE), where the luminosities during the burst reach super-Eddington values causing the envelope to expand and thus temperatures to cool. Observationally this is seen as a drop in the x-ray flux or light curve as the distribution of the black body radiation shifts during this cooling phase. An example of a PRE-burst is given in bottom right panel of Figure 1.2a with 4U 2129+12. But all the these systems listed above show peak luminosities that are a factor of 2 to 3 below the Eddington limit, and therefore are considered to be non-PRE bursts. For the most part the burst have double- or triple-peaked structures that are separated by roughly 4 to 7 seconds, and generally have bursting rise times around 2 to 4 seconds for all peaks. Examples of double- and triple-peaked bursts observed in 4U 1636-53 by [87] and [92], and in MXB 1730-335 by [5], are shown in Figures 1.2b, 1.2c, and 1.2d, respectively.

Currently the time-structure of the x-ray flux, or light curve, is the only observable in these types of bursting LXMB systems, therefore in order to explore the underlying processes that lead to this behavior, we have to heavy rely on modeling and simulations.

1.3 XRB’s and the Thermonuclear Runaway Model

One year before the discovery of XRB’s, it was shown by Hasen and van Horn that radially thin shell burning of the accumulated H and He on a neutron star is highly susceptible to “thin shell” thermal instabilities under degenerate conditions
[37]. It was not long before these newly observed x-ray phenomena were associated with thermonuclear runaway processes on the neutron star’s surface triggered by “thin shell” instabilities within the H/He accreted material [58, 99].

Within this thermonuclear runaway framework, accreted material accumulates on the surface of the neutron star. During this accumulation period, temperatures are high enough that H will burn through the CNO cycles (or the $\beta$-limited CNO cycles at higher temperatures), while He will burn via the triple $\alpha$-process. After some time (hours to days) a thin shell instability develops, either through H or He burning, triggering a thermonuclear runaway that consumes the rest of the accreted material within a matter of seconds. The energy released by the rapid consumption of the H/He fuel is quickly thermalized (the photon mean-free-path is on the order of $10^{-5}$ cm) and transported to the surface where it heats the photosphere of the neutron star, which subsequently emits thermal x-rays with a spectrum resembling roughly a black body. Since this process is not cataclysmic, and the accretion of material from the donor star is continual, the ashes from this event are forced deeper and deeper into the neutron star’s ocean and crust, while the continued accretion process results in the cycle beginning again, triggering another burst after some time.

A diagram depicting a typical x-ray burst event, along with the overall gross features observed in the x-ray flux, is illustrated in Figure 1.3. Here, the bottom panel shows the structure of the accreted envelope above the neutron star before, during, and after the explosion. The top panel shows the overall x-ray flux exhibited during this event, where a sharp rise followed by a slow decay can be observed on top of the already persistent x-ray flux from the accretion disk.

Early dimensional analysis work by Joss [46], along with Lamb and Lamb [55], were able to show that this thermonuclear runaway model could roughly account for some of the gross observations of XRBs, such as recurrence times (on order of $10^4$ seconds) and overall energy released ($\leq 10^{39}$ ergs).
Figure 1.3. Bottom panel: the structure of the accreted layers (usually H and He) before, during, and after a typical x-ray burst event. Top panel: The overall x-ray flux observed during this event. The signature of an x-ray burst is a sudden increase and subsequently slow decay of the x-ray flux on top of an already existing persistent flux coming from the accretion disk.

Shortly after, the first numerical models employing this runaway framework were able to reproduce more detailed observational features seen in XRBs: such as the time structure of the x-ray flux during the event, the range of recurring time scales for burst cycles, and the ratio of energy released during accretion versus that released during the burst (see [4, 47] and reference therein). Further model studies by Fushiki & Lamb [30] showed that the overall burst behavior depends on three main parameters, the composition of the accreting matter, the rate of accretion, and the central temperature of the neutron star.

In 1981 Wallace and Woosley, using such numerical models, pointed out that in hot and dense environments containing a mixture of H and He, it is possible for material to leak out of the $\beta$-limited CNO cycles, upon which successive proton captures along with fast $\beta^+$ decays could rapidly process the material up to, and beyond, the iron group along the proton drip line, and furthermore, this process (deemed the
rp-process for it analogy to the r-process) could account for the exceptionally high energy production rate observed during XRBs events [94]. In addition to introducing the rp-process, Wallace and Woosley further showed that at higher temperatures, \((\alpha,p)\) reactions could possibly bypass the slower \(\beta^+\) decays of lighter mass nuclei within rp-process. This process would later be called the \(\alpha p\)-process.

The \(\alpha p\)- and rp-process, along with the breakout reaction of the \(\beta\)-limited CNO cycles, play crucial roles in determining some of the major characteristic observables in the overall structure of XRB light curves. An example of the reaction flow taken from a particular XRB simulation with a given set of ignition conditions [82] is shown in Figure 1.4 to illustrate the nucleosynthesis during an XRB event. Here the path of the \(\alpha p\)- and rp-process, along with the \(\beta\)-limited CNO cycles and breakout reactions, are labeled in green, purple, orange and blue, respectively. In the following subsections the \(\beta\)-limited CNO breakout reactions, the rp-process, and the \(\alpha p\)-process and their influence on XRB light curves are further discussed.

1.3.1 Breakout of the \(\beta\)-Limited CNO Cycle

As instabilities develop in either H or He burning, temperatures within the accreted envelope begin to increase rapidly. With increasing temperatures, additional charged particle reaction channels begin to open up within the \(\beta\)-limited CNO cycles. Once temperatures reach above \(T \approx 5 \times 10^8\) K, a secondary loop in the cycle opens up at \(^{14}\text{O}\) with \(^{14}\text{O}(\alpha,p){^{17}\text{F}(p,\gamma){^{18}\text{Ne}(\beta^+\nu){^{18}\text{F}(p,\alpha){^{15}\text{O}}, bypassing the slow \(\beta\)-decay}^{14}\text{O}(\beta^+\nu){^{14}\text{N}. This secondary loop can increase the rate of the \(\beta\)-limited CNO cycle by a factor up to 1.6 (given that \(^{14}\text{O}(\beta^+\nu){^{14}\text{N is totally bypassed) [94], and effectively shifts the equilibrium distribution towards \(^{15}\text{O}. At the same time, with the increase in temperatures, the \(^{15}\text{O}(\alpha,\gamma){^{19}\text{Ne reaction begins to allow material to breakout of the \(\beta\)-limited CNO cycle. Once out of the \(\beta\)-limited CNO cycle, the reaction flow can proceed via the \(\alpha p\)-process and the rp-process.\)
Figure 1.4. Possible reaction flows occurring during an XRB include the following: The triple alpha process (labeled in red), the $\beta$-limited CNO cycles (labeled in orange), along with their breakout reactions $^{15}\text{O}(\alpha,\gamma)^{19}\text{Ne}$ (labeled in blue) and $^{18}\text{Ne}(\alpha,p)^{21}\text{Na}$ (labeled in light blue), the $\alpha p$-process (main path labeled in in light green, while other paths are in dark green), and finally the rp-process (labeled in purple). In addition an insert is include to highlight where within the light curve each process is most influential. For more information on each process, see their respective sections.
Recent sensitivity studies have revealed that the $^{15}\text{O(}\alpha,\gamma)^{19}\text{Ne}$ reaction plays an integral role in determining the initial conditions of the thermonuclear runaway [20, 22]. Most notably these studies have shown that its strength not only effects the ignition point of the runaway, but also the recurrence time between subsequent bursts along with the transition between stable and unstable burning.

Additionally, the $^{18}\text{Ne(}\alpha,p)^{21}\text{Na}$ reaction has been identified as secondary break-out reaction for the $\beta$-limited CNO cycle, but only becomes important later at higher temperatures after the thermonuclear runaway has developed.

For further discussions on these breakout reactions, and their overall effects on bursting behaviors in XRB models, see [20, 98] and references therein.

1.3.2 The rp-Process

The rp-process can be described as a series of proton captures followed by $\beta^+$ decays. Here, successive proton captures occur until the flow reaches either a nucleus $(A, Z)$ that is in $(p,\gamma)$-$(\gamma,p)$ equilibrium with the the nucleus above it $(A + 1, Z + 1)$, meaning photo-disintegration happens just as quickly as proton capture, or the proton drip line, meaning $(A + 1, Z + 1)$ is proton unbound. At this point the flow has to wait to $\beta^+$-decay to $(A, Z - 1)$, upon where the process of successive proton captures occurs again until the flow encounters another nucleus in $(p,\gamma)$-$(\gamma,p)$ equilibrium or the proton drip line. Nuclei where the flow stalls and waits to $\beta^+$-decay are referred to as waiting points. The main flow of the rp-process rides along the proton rich side of the valley of stability starting at $^{19}\text{Ne}$ and, depending on peak temperature, can proceed all the way up to it’s natural end point of the SnSbTe cycle [83]. The actual endpoint of the rp-process in a specific burst depends on the amount of H available that accumulated during the accretion process. Therefore, the endpoint of the rp-process can vary from system to system, and even burst to burst within the same system.
Another important aspect of the rp-process that influences the resultant light curve is the individual $\beta^+$-decay half-lives of the waiting point nuclei, as their sum determines the overall bursts timescale [93]. In this sense, the decay structure of the XRB light curve depends critically on the path of the rp-process and which waiting point nuclei are encountered. For a more detailed review of the rp-process and all the possible waiting points please see [18, 21, 84].

1.3.3 The $\alpha p$-Process

The $\alpha p$-process is characterized as a series of ($\alpha$,p) and (p,$\gamma$) reactions beginning with the breakout reaction $^{18}\text{Ne}(\alpha,p)^{21}\text{Na}$ proceeding all the way up to possibly the mass region $A = 42$ (depending on the peak burst temperatures). For the most part, the $\alpha p$-process runs along the $T_z = -1$ nuclei through the sd-shell, but parallel ($\alpha$,p)-(p,$\gamma$) reaction chains can branch off and develop along $T_z = -1/2$ and $-3/2$ nuclei as well. Unlike the rp-process, the He burning through the $\alpha p$-process is highly temperature dependent and typical timescales for the $\alpha p$-process is on the order of a fraction of a second [18]. At some point, as the reaction flow increases in mass, further ($\alpha$,p) reaction are hindered by an increasing Coulomb barrier and the main reaction flow transitions from the $\alpha p$-process to the rp-process. The actual transition point between the $\alpha p$-process to the rp-process in a particular burst depends critically on the peak temperatures reached and the strengths of the individual ($\alpha$,p) reactions. The typical reaction flow through the sd-shell during a XRB event is highlighted in Figure. 1.5.

The main $\alpha p$-process sequence, starting from $^{18}\text{Ne}$ and going up to $^{42}\text{Ti}$, can be written as $^{18}\text{Ne}(\alpha,p)^{21}\text{Na}(p,\gamma)^{22}\text{Mg}(\alpha,p)^{25}\text{Al}(p,\gamma)^{26}\text{Si}(\alpha,p)^{29}\text{P}(p,\gamma)^{30}\text{S}(\alpha,p)^{33}\text{Cl}(p,\gamma)^{34}\text{Ar}(\alpha,p)^{37}\text{K}(p,\gamma)^{38}\text{Ca}(\alpha,p)^{41}\text{Sc}(p,\gamma)^{42}\text{Ti}$. This main flow is highlighted in bright green in Figure. 1.5.

It is important to note that the $\alpha p$-process competes with the rp-process within
Figure 1.5. Possible reaction flows occurring during an XRB through the sd-shell. This includes the breakout reactions $^{15}$O($\alpha$, $\gamma$)$^{19}$Ne (labeled in blue) and $^{18}$Ne($\alpha$,p)$^{21}$Na (labeled in light blue), the $\alpha$-p-process (main path labeled in in light green, while other paths are in dark green), and finally the rp-process (labeled in purple). In this region the $\alpha$-p-process competes with the rp-process, and the times scales of the energy generation rate depends critically on which process dominates the reaction flow.
the sd-shell region, as \((\alpha, p)\) reactions on even-\(Z\), \(T_z = -1\) nuclei can bypass the slower \(\beta^+\)-decay waiting points. Once \((\alpha, p)\) reactions in the \(\alpha\)p-process begin to bypass these waiting point nuclei, the flow through the sd-shell is increased considerably. Due to the fact that these \((\alpha, p)\) and \((p, \gamma)\) reactions are not hindered by weak processes, the energy generation rate is considerably increased as material is processed quickly through the sd-shell. In this sense, the \(\alpha\)p-process will have a direct influence on the early rise time structure of the XRB light curve.

1.4 XRB Models

Due to the extreme gravitational fields on the surface of neutron stars, it is likely that little-to-no material will be ejected during these explosive events. Therefore, the primary observables of XRB systems are restricted to the characteristics of the observed light curve. Because of this, X-ray burst simulations become invaluable tools in the process of exploring the underlying mechanisms driving these events. Only through comparisons between simulated light curves and the observed light curve can estimates on the physical environment of a particular LMXB system be made.

Unlike other stellar explosive events, such as nova and supernova, XRB simulations pose a challenge because the thermodynamic trajectories during the burst are determined by the energy generated by many possible reactions within the \(\alpha\)p-process and rp-process. To accurately model all of the relevant details of XRBs, and use the simulated light curves to explore the underlying physics, massive nucleosynthesis networks closely coupled with multi-dimensional hydrodynamic models that self-consistently evolve thermodynamic trajectories are needed.

Unfortunately, the computational power required to capture all of these relevant processes in a single simulation is not presently available. To get around this, simulations focus on detailed analyses of one particular aspect of the burst while employing
approximation and simplification for the rest. Over the past three decades, XRB simulations have followed mainly two different approaches: either they employ an extensive reaction network along with simplified hydrodynamic models to characterize the energy generation that drives the burst, or they utilize state of the art hydrodynamic models along with small reaction networks to characterize the initial condition that triggers the burst and the response of the neutron star’s envelope/atmosphere once the burst develops. Both approaches have their advantages and disadvantages. The simplifications made by each approach reduces the interpretative power of the given simulation, but on the other hand, through smaller more detailed studies, the major uncertainties within a particular process (along with it’s possible effect on the simulated light curve) can be identified and further explored.

Detailed nucleosynthesis studies of XRBs require extensive nuclear reaction networks, with hundreds of nuclei, ranging from stable isotopes to the proton drip line, along with thousands of nuclear interactions. In order to accommodate a full size reaction network, simulations usually employ a parameterized one-zone one-dimensional model using simple prescriptions to evolve temperatures and densities, either through semi-analytical hydrodynamic models [36, 48, 49, 81, 83, 94], or pre-defined T-\(\rho\) profiles extracted from more sophisticated 1-D multi-zone hydrodynamic models [43, 69, 70]. This simplification of the hydrodynamics makes these types of simulations computationally less demanding, therefore many simulations can be done in a relatively short period of time. However they have the disadvantage of assuming a single-zone for the burning, and therefore the energy generated in this one zone is directly converted to a light curve. By design these one-zone models do not take into account any sort of sedimentation, convective mixing, or radiation transport, all of which can have significant effects the resultant light curve. Because of this, light curves from these types of simulations are usually only compared relative to each other, and rarely to actual observed light curves. The major advantage of these
types of one-zone simulations is that variations in the nuclear physics inputs (such as masses or reaction rates) for thousands of isotopes, and their effect on the simulated light curve, can be examined. Therefore, through the comparison of light curves from thousands of simulations with individually varied nuclear inputs, the major uncertainties within the nuclear parameters can be explored and a hierarchy of the most influential nuclear inputs for simulated light curves can be established.

Similarly, in order to study some of the hydrodynamics processes that occur during XRBs, simulations use multi-zone hydrodynamic codes accompanied by small nuclear reaction networks are utilized. These types of simulations aim to understand burst aspects such as the ignition condition’s dependence on accretion rate and metallicity [65, 88, 89, 100], convective mixing [14, 57, 96], and flame propagation [13, 86]. For the most part these multi-zone hydrodynamic models are 1-D, using the underlying assumption of spherical symmetry (with the exception of a few two-dimensional models [13, 57, 86]). As stated before, these models use relatively smaller nuclear reaction networks and therefore do not capture the proper rate of nuclear energy released during the burst. Therefore, simulated light curves from these types of models are rarely compared to observed light curves. Instead, these models are utilized to explore the thermodynamic conditions that lead to bursting behavior.

More recently, a small number of multi-zone one-dimensional models have been able to incorporate the extensive nuclear reaction networks needed to achieve more realistic simulations. However, due to the time limitations of running such computationally intensive simulations, studies have so far been restricted to examining the influence of a handful of reactions or metallicities in the accreted material. Most studies utilize a multi-zone 1-D hydrodynamic codes with a detailed reaction network. Thielemann et al. [91], demonstrated the importance of select proton capture rates on $^{27}\text{Si}$, $^{31}\text{S}$, and $^{35}\text{Ar}$ and their influence on the resultant light curve; Fisker et al. [19], who used a network of 300 isotopes (up to $^{107}\text{Te}$) on top of a dynamic
multi-zone 1-D model, varied two important \((\alpha, p)\) reactions within the \(\alpha p\)-process demonstrating their possible influence on double peaked light curves. Later, using the same model, Fisker et al. [22] varied the \(^{15}\text{O}(\alpha, \gamma)\) rate, showing the importance of this reaction in determining ignition conditions in XRBs. Woosely et al. [101], using an adaptive network of 1300 isotopes with a multi-zone 1-D model, showed that effects of waiting point nuclei have on the resulting light curve by globally varying \(\beta^+\)-decay rates for nuclei above \(A = 59\).

The first quantitative comparison of simulated light curves was preformed by Heger et al. [40] using a multi-zone one-dimensional model [101] and the “textbook” burster GS 1826-24 [32], which is known to exhibit extremely regular burst with a period of 3-6 hours. Heger et al. found that, for a particular metallicity of \(Z_{\text{CNO}} = 0.02\), the mean simulated light curve agrees remarkably well with the observed mean light curve from GS 1826-24 (taken over a year). Additionally, the simulated bursts showed similar recurrence times around \(\sim 4.1\) hours.

The remarkable ability of the Heger study [101] to correctly match the observed light curve of GS 1826-24 is definitely promising. However, GS 1826-24 is believed to be an exceptional case (hence the name “textbook burster”), and if these models are going to be used to interpret other observed light curves, ones that are less regular than the “textbook burster”, such as the double peak light curve of 4U 1636-53 [92], then reliable nuclear physics inputs are essential in increasing the predictive power of their simulated light curves.

Given that single multi-zone one-dimensional model simulations utilizing large reaction networks require a large amount of computing time, comprehensive sensitivity studies of nuclear inputs (hundreds of isotopes and thousands of reactions) are not yet realistic. Instead most systematic sensitivity studies of the various nuclear inputs have been made with either one-zone or post-processing models. Even through these simplified models neglect crucial hydrodynamic effects, they play an important role
Figure 1.6. Figure directly taken from [40]. Comparison of observed (histogram) and two simulated light curves (solid and dashed line). The histogram is taken as the average light curve from GS 1826-24 during the year 2000 (error bars are $1\sigma$ variations from burst to burst). The solid and dashed line are the average simulated light curves from models A3 ($Z_{\text{CNO}} = 0.02$) and B3 ($Z_{\text{CNO}} = 0.001$) taken from [101]. The Grey bands represent the $1\sigma$ variation from simulated bursts.
in identifying nuclear input uncertainties that have the biggest impact on XRB light
curves. Once the most critical uncertainties have been determined through these
one-zone models, further studies can then be done using multi-zone one-dimensional
models.

1.5 Influence of Reaction Rate Uncertainties on Simulated Light Curves

As stated in section 1.4, the structure of the light curve is to a large extent gov-
erned by the nuclear processes occurring during the burst. Any sort of uncertainties
in the nuclear inputs taken in a single XRB model, such as masses or reaction rates,
can directly influence the shape of the simulated light curves. Therefore it is crucial
to reduce the most dominant nuclear uncertainties in an attempt to strengthen the
predictive power of simulated light curves of XRB models.

Most of the nuclei that participate in the rp-process are unstable, located between
just off the valley of stability up to the proton drip line. Among the unstable nuclei
that participate in the αp- and rp-process are ∼1000 plus reactions that could possibly
have some influence on the final abundances and light curves for a particular XRB
event. In an attempt to model these large reaction networks, of which the vast
majority are not experimentally known, theoretical predictions are used to estimated
the astrophysical rates. The most widely used theoretical nuclear reaction model is
based on the statistical Hauser-Feshbach theory [39].

The statistical Hauser-Feshbach (HF) model has proven to be a very useful tool for
these types of XRB simulation, as it requires very few ingredients (such as transmis-
sion coefficients, level densities, partition-functions, and stellar enhancement factors)
for predicting astrophysical cross sections and therefore subsequent reaction rates.
The HF model implicitly assumes that there are enough resonances at the appropri-
ate bombarding energies, such that the cross section can be regarded as an average
over many resonances. Given this assumption, the validity of a particular HF pre-
dicted cross section or reaction rate depends critically on the level densities in the compound nucleus involved. This condition may not always be met in the many proton or alpha capture reactions occurring within the αp- and rp-process, especially around closed shell nuclei, where the level density decreases, or around the proton drip line, where the Q-values are smaller. Work by Rauscher et al.\cite{77} has shown that if this condition is not met then a global HF model predictions can grossly over-estimate the cross section. Therefore, HF predictions for reaction rates used in XRB sensitivity studies are usually quoted with an order of magnitude uncertainty (sometimes two orders of magnitude!).

Given the sheer amount of reactions (and the fact that most of these nuclei are unstable), it is almost impossible to measure every single reaction in the αp- and rp-process in the effort to reduce uncertainties experimentally. Therefore a reaction hierarchy must be establish using sensitivity studies that could identify the most influential reactions (meaning their uncertainties have a large impact on nucleosynthesis within their respective model) during XRBs. Once key reactions are identified, experimentalist can then attempt measure the reaction in the lab, either through direct or indirect methods, in order to reduce the unwanted uncertainties.

Many of the sensitivity studies mentioned in section 1.4 have investigated the impact of individual reaction rate uncertainties on the predicted light curves. The results of these studies have spurred many more experiments attempting to either measure the reaction for the first time, or reduce the uncertainty further. For a review on what key reactions have been identified in XRB nucleosynthesis, along with the experimental efforts to measure them please see \cite{80} and \cite{71}, and references therein.
1.6 Nuclear Uncertainties within the $\alpha$-p-Process

The $\alpha$-p-process plays an important role in the nucleosynthesis during XRBs, as the various ($\alpha$,p) reaction within the sd-shell may act as bypasses to possible waiting point nuclei in the rp-process. The ability to bypass one-or-more rp-process waiting points would significantly accelerate the nuclear flow through the sd-shell, thus increasing the energy generation rate. This sort of change in the energy generation by either the acceleration (bypass) or impedance (waiting point) of the nuclear flow through the sd-shell is believe to be directly imprinted on the early rise-time structure observed in XRB light curves. The possible waiting points in the rp-process along the sd-shell have been identified along the $T_z = (N - Z)/2 = -1$ chain as $^{22}$Mg, $^{26}$Si, $^{30}$S, $^{34}$Ar, and $^{38}$Ca [19]. All of these nuclei have relatively long $\beta$-decay half-lives compared to XRB timescales, and small (p,$\gamma$) Q-values that results in (p,$\gamma$)-(\gamma,p) equilibria blocking nuclear flow at relatively low temperatures. Therefore, whether or not the nuclear flow is impeded at one of these possible waiting point nuclei is predominantly determined by the strength of the ($\alpha$,p) reaction at the respective XRB temperatures. If a particular ($\alpha$,p) reaction is relatively strong then the nuclear flow can move unhindered through the respective waiting point nuclei. On the other hand, if the ($\alpha$,p) reaction is relatively weak then the nuclear flow is stalled as it waits to $\beta$-decay, which can be on the order of seconds. The latter scenario results in a dip in the nuclear energy generation rate thus causing the rise time of the XRB light curve to drop.

Contrary to their importance, many of these ($\alpha$,p) reactions within the $\alpha$-p-process are experimentally unknown, and therefore XRB simulation use HF predicted rates to fill in this missing information within their nuclear reaction networks. In this region, the reliability of a HF model to accurately predict an ($\alpha$,p) reaction is questionable when taking into consideration the level density, along with possible $\alpha$-clustering structure, of nuclei within the sd-shell. As pointed out in section A.5, if the level
density is too low for the statistical approach used by HF models then the predicted
cross-section and subsequent reaction rate maybe grossly over-estimated. Additionally, if there are any strong \( \alpha \)-cluster states, the statitical approach of HF will most likely underestimate the cross-section at the corresponding energies. Due to the uncertainties that comes with applying a HF model to predict these \((\alpha,p)\) rates, it is difficult to determine which nuclei will act as waiting points.

1.7 Sensitivity Studies of Double-Peaked XRB’s

It was first pointed out by Fisker \textit{et al.} [19] that the double-peak structure observed in systems like [45, 73, 87] can arise when a thermonuclear runaway is triggered within pure He layer below a mixed H/He layer. In this scenario, the rapid consumption of pure He layer is seen to be the source of the first peak, while the second peak occurs due to some waiting point impedance in the mixed H/He burning within the rp-process. With the idea that wainting points are determined by the strength of the corresponding \((\alpha,p)\) reaction in the \(\alpha\p\)-process and the fact that the HF predicted rates used for these highly influential \((\alpha,p)\) reactions come with large uncertainties, Fisker \textit{et al.} performed a sensitivity study of \((\alpha,p)\) reactions along the \(\alpha\p\)-process. The motivation of this study was to determine if \((\alpha,p)\) reactions within the \(\alpha\p\)-process could act as a nuclear waiting point impedances, and if so could they possibly account for the double peak structure observed in several XRB light curve sources.

The model used by Fisker \textit{et al.} was a self-consistent multi-zone one-dimensional model, taking into account turbulent mixing and heat transport [19], and included a reaction reaction network consisting of 298 isotopes. For this study Fisker \textit{et al.} choose a relatively slower accretion rate of \( \dot{M} = 5 \times 10^{16} \text{ g s}^{-1} \), allowing for a highly unstable helium base to accumulate at the bottom of the accreted layer. Within half a day of accumulation, this He layer ignites in a pure He flash completely consuming
Figure 1.7. Figure taken directly from [19]. Simulated light curves plotted for three cases. The solid black lines represent simulation using the accepted HF predicted rates of NON-SMOKER [76], labeled as “unaltered”. Two additional simulation were done, each adjusting the \( ^{30}\text{S}(\alpha,p)^{33}\text{Cl} \) (dotted line) and \( ^{34}\text{Ar}(\alpha,p)^{37}\text{K} \) (dashed line) rate by a factor of 100.

the base He layer within tenths of a second. The He flash creates a large temperature gradient across the accreted envelope allowing for highly efficient heat transport to the surface. This He flash is the origin of the first peak which is observed as a sharp increase in the of the x-ray luminosity over the period of less than a second. The heat generated by this He flash is quickly transported to the H/He shell above causing a second mixed H/He flash where material is consumed via the \( \alpha p \)- and rp-process. It is the detailed interplay between the rp-process waiting points and the \( \alpha p \) reaction of the \( \alpha p \)-process in the sd-shell that determine the over-all rise-time slope of the second peak and the time between peak luminosities.

Within this study, Fisker et al. determined that \( ^{30}\text{S} \) and \( ^{34}\text{Ar} \) were the best
possible candidates for waiting point nuclei in the sd-shell. XRB simulation were then performed assuming the predicted HF rates of $^{30}\text{S}(\alpha,p)^{33}\text{Cl}$ and $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ could be off by a factor of 100. The resultant light curve from this sensitivity study is given in Figure 1.7. Here the solid line is the “unaltered” light curve which was calculated using the accepted HF predicted rates of NON-SMOKER [76]. While the dotted line is the result of increasing the $^{30}\text{S}(\alpha,p)^{33}\text{Cl}$ by a factor of 100, and the dashed line is the result of increasing the $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ by a factor of 100. From Figure 1.7, the influence these reaction have on the structure of the double peaks is evident. If the $^{30}\text{S}(\alpha,p)^{33}\text{Cl}$ reaction rate were faster, the dip seen in the “unaltered” light curve would decrease or disappear all together, while increasing the $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ would create a more pronounced observed dip between peaks. One thing that should be noted when comparing these simulated light curves of Fisker et al. to observation is that the dip between the peaks is not nearly as pronounced as what has been observed in most double peak sources.

Fisker et al. goes on to show that the application of a statistical HF model for determining these two rates may not be valid when taking into account the level density and nuclear structure in the compound nuclei, $^{34}\text{Ar}$ and $^{38}\text{Ca}$, respectively. Looking at $\alpha$-capture on mirror nuclei, Fisker suggests that the reactions that the $^{30}\text{S}(\alpha,p)^{33}\text{Cl}$ and $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ reactions are most likely dominated by pronounced, natural parity, $\alpha$-cluster resonances.

In addition to the nuclear waiting point impedance suggest by Fisker et al. being the possible source of the observed double peak structure in XRB light curves, several other mechanisms have been proposed to explain this behavior, including: A multi-step process in the release of thermonuclear energy based on dynamical explosion mechanisms [26, 72, 87]; Fujimoto et al. suggest that shear instabilities due to differential rotation between layers would mix unburnt material from upper layers into burning regions [27]; Another mechanisms proposed by Bhattacharyya and
Strohmayer [7, 8] suggests that if ignition occurred at either the neutron star rotational pole or at high latitudes, the burning front would stall as it propagates in the opposite direction of the transferred material of the accretion disk around the equator. Once past the equator, the burning front would propagate more rapidly towards the opposite pole, thus resulting in a second peak.

More recently, a study by Lampe et al. [56], using the self-consistent multi-zone one-dimensional model of [101], found a double-peak structure result for simulations with certain accretion rates and metallicities. The resultant double-peak light curves from simulations given these particular set of parameters are shown Figure 1.8. Lampe et al. also suggests that second peak is most likely due to a nuclear waiting point impedance, but found that the structure of the two peaks along with the dip in the middle was highly depended on the selection of the accretion rate and metallicity of the accreted material. From this study it is seen that a double peak structure is more prominent at higher metallicities, which may be unrealistic given the usual type of Pop. II donor star in LMXBs. Lampe et al. goes on to conclude that even if the condition that would lead to a double-peaked burst exists in a particular system, its subsequent observation is not always guaranteed, as the structure might be smeared out due to the way that the burning front propagates. In this sense, the Lampe et al. study further emphasizes that there are many components of that can influence the structure of an observed double-peaked burst.

At the moment it is not possible to discern which mechanism is most likely accountable for the observed double peak structure in some XRB systems due to the many uncertainties that exist in even the most state of the art simulations. Furthermore given the lack of knowledge concerning the dense neutron star environment, many of these physical hydrodynamic uncertainties within XRB simulation will be extremely difficult to reduce. On the other hand it is possible to reduce the uncertainties in the \((\alpha,p)\) reaction rates that have been proposed by Fisker et al. as one
Figure 1.8. Figure taken and modified from [56]. Simulated light curves plotted various accretion rates and metallicities. Left Panel: Simulated light curves for various accretion rates given a metallicity of $Z = 0.04$. Here the He burst peak is observed as a small shoulder to the much bigger H/He burst. Right Panel: Simulated light curves for various accretion rates given a metallicity of $Z = 0.1$. Here a more pronounced double peak structure can be seen, and the He burst peak and the dip increase with lower accretion rates.

possible mechanism behind double peak light curves. In this sense, by experimentally measuring these ($\alpha,p$) cross-sections or reaction rates (directly or indirectly), and thus reducing the main nuclear uncertainties, it is possible to conduct more detailed investigations of the physical hydrodynamic uncertainties within the double-peaked XRB simulation.

1.8 PR137: Experimentally Measuring the $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ Reaction

Presently, direct experimental studies of the $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ reaction are not feasible at the relative astrophysical energies, due to the requirement of radioactive beam with intensities around $10^8$ particles s$^{-1}$. Therefore, in the attempt to reduce its nuclear uncertainties illustrated in the sensitivity work by Fisker et al., the experiment PR137 was proposed at iThemba LABS to indirectly study the $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ reaction by precisely measuring the excitation energies $\alpha$-unbound natural parity states in $^{38}\text{Ca}$. 

25
It is these natural parity states in $^{38}$Ca that will act as resonances in the $^{34}$Ar($\alpha$,p)$^{37}$K reaction. Therefore, by determining precise resonance energies and using modest nuclear structure assumptions, the reaction can be experimentally determined, albeit indirectly, for the first time.

Currently there is only a handful of states known above the $\alpha$-threshold in $^{38}$Ca at $E_\alpha = 6105.12$ keV (see Table 4.2) [1, 51, 68], of which no spin assignments are known. It should be noted that a similar (p,t) experiment investigating $\alpha$-unbound natural parity states in $^{38}$Ca was performed by O’Brien et a.[67], but results were never published (for completeness, these states are included in Table 4.2).

With this lack of information in mind, the main goal of PR137 was to identify all possible ($\alpha$,p) resonances in $^{38}$Ca, by precisely measuring natural parity states above the $\alpha$-threshold using the two neutron transfer reaction, $^{40}$Ca(p,t)$^{38}$Ca. With precise resonances energies experimentally measured, the $^{34}$Ar($\alpha$,p)$^{37}$K reaction rate can be more accurately determined using the formalism described in Appendix A.

1.8.1 Advantages of the (p,t) Reaction

The two neutron transfer (p,t) reaction was chosen to study states above the $\alpha$-threshold in $^{38}$Ca for two main reasons: First, at high incoming proton energies and at very forward scattering angles, the (p,t) reaction on $^{40}$Ca offers the selectivity of predominately populating natural parity states in $^{38}$Ca. At higher proton energies ($\sim$ 100 MeV for PR137), the (p,t) reaction is thought to be dominated by a one-step two-particle spin-zero transfer process [90]. In this case, given the spin of $^{40}$Ca is $J^\pi = 0^+$, and the spins of the transferred neutrons couple to $S = 0$, the spin and parity of the states in the recoil $^{38}$Ca, are solely determined by the orbital angular momentum transfer, thus only natural parity states will be populated through this direct mechanism. Evidence of the selectivity of the (p,t) reaction at 100 MeV proton beam energies has been observed in previous experiments with well known
natural parity states being strongly favored \([3, 59–61]\). Second, energy resolution limitations due to target thickness are minimal compared to other transfer reactions, such as \((^4\text{He},^6\text{He})\). Therefore, when coupled with a high resolution magnetic spectrograph setup, the \((p,t)\) allows for high energy resolution measurements in which closely spaced levels at higher energies in \(^{38}\text{Ca}\) can be resolved and identified. Using the Gamow window approximation described in section A.3, the energy range where states in \(^{38}\text{Ca}\) will be the most influential in governing the \(^{34}\text{Ar}(\alpha,p)^{37}\text{K}\) reaction for selected XRB temperatures is shown in Table 1.1. From this, it can be seen that the excitation energy range of interest in \(^{38}\text{Ca}\) for PR137 starts slightly above the \(\alpha\)-threshold at 7116.12 keV (lower energy range for \(T = 0.5\) GK) and spans up to 10198.32 keV (upper energy range for \(T = 2\) GK). Given that this range of interest spans up to \(\sim 10\) MeV in excitation energy, high energy resolution techniques are needed to resolve possible closely spaced states. In order to attain the energy resolution needed to resolve high lying excited states in \(^{38}\text{Ca}\), the \(^{40}\text{Ca}(p,t)\) reaction was performed at iThemba LABS using the K=600 spectrograph along with dispersion matching techniques. Ultimately, with this setup an energy resolution of \(\sim 35\) keV was achieved in the focal plane of the K=600 spectrograph.
TABLE 1.1

THE GAMOW WINDOW FOR $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ GIVEN SELECTED XRB’S TEMPERATURES.

<table>
<thead>
<tr>
<th>Temperature [GK]</th>
<th>Gamow Window [keV]</th>
<th>Excitation Range in $^{38}\text{Ca}$ [keV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>1011 - 1553</td>
<td>7116 - 7658</td>
</tr>
<tr>
<td>0.8</td>
<td>1352 - 2155</td>
<td>7457 - 8260</td>
</tr>
<tr>
<td>1.0</td>
<td>1552 - 2519</td>
<td>7657 - 8624</td>
</tr>
<tr>
<td>1.5</td>
<td>1989 - 3345</td>
<td>8094 - 9450</td>
</tr>
<tr>
<td>2.0</td>
<td>2369 - 4093</td>
<td>8474 - 10198</td>
</tr>
</tbody>
</table>
CHAPTER 2

PR137: EXPERIMENTAL SETUP

The use of forward-angle, high-energy resolution, (p,t) measurements to investigate possible (\(\alpha,p\)) resonances in \(T_z = -1\) nuclei were first realized during the early 2000’s at the Research Center for Nuclear Physics (RCNP) using the Grand Raiden magnetic spectrograph, with several successful experiments that indirectly determined (\(\alpha,p\)) reaction rates by investigating natural parity states in the compound nucleus [59–61]. With the success of these first (p,t) measurements at RCNP we proposed a similar (p,t) experiment at iThemba LABS using the K=600 spectrograph to study natural parity states \(^{38}\text{Ca}\). The development of a high energy-resolution zero-degree facility used to perform such (p,t) reaction measurements at iThemba LABS is well documented in Ref. [66]. The experimental setup and techniques used in PR137, such as beam lines, the K=600 spectrograph, focal plane detectors, Dispersion matching techniques, targets, and other various experimental settings are all reviewed in this chapter.

2.1 Brief Overview of PR137 at iThemba LABs

The layout of the experimental facility at iThemba LABS is illustrated in Fig. 2.1. For PR137, a proton beam from the MINIMAFIOS ECR ion source was extracted by the Solid Pole Injector Cyclotron 2 (SPC2) and injected into the Separated Sector Cyclotron (SSC) for further acceleration up to 100 MeV. The proton beam was then transported to the K=600 vault through the X, P1, P2, and S beam lines where it impinged upon a target within the scattering chamber in front of the K=600
Figure 2.1. Layout of the experimental facility at iThemba LABS. Protons were produced with an ECR source and extracted using the Solid Pole Injector Cyclotron 2 (SPC2). A 100 MeV proton beam was then delivered by the Separated Sector Cyclotron (SSC) to the scattering chamber located in front of the K=600 magnetic spectrograph.
magnetic spectrograph. After hitting the target the proton beam was collected in a Faraday cup located inside the K=600, while the reaction products (mostly tritons) were momentum analyzed by the K=600 spectrograph, and transported to the focal plane detector system, which consisted of two vertical drift chambers and two plastic scintillators.

2.2 The K=600 Magnetic Spectrograph

The K=600 magnetic spectrograph is classified as a QDD spectrograph and consists of 5 active elements: a quadrupole magnet, two dipole magnets and two trim coils (K and H). A schematic of the K=600 spectrograph system is shown in Fig. 2.2. The H and K coils are shaped pole-face current windings used to make final focusing adjustments in the focal plane. The K-coil, a triangular shaped quadrupole element, is used to correct out first order kinematic effects in the horizontal scattering angle as a function of momentum, or the \( x|\theta \) component in ion-optics matrix terms. The H-coil, a circular shaped hexapole element, correct out higher order aberrations in the horizontal scattering angle as a function of momentum, \( x|\theta^2 \).

By varying the ratio of the dipole magnets, D1 and D2, the momentum dispersion \( x/\Delta p/p \) of the K=600 system can be varied between the values of -6.2 cm/% and -9.8cm%. Given this range of momentum dispersion values, three different focal plane positions can be used for a given experiment, as seen in Fig. 2.2. For PR137, the medium-dispersion plane was used with a momentum dispersion value of -8.4 cm/\%. The characteristic ion-optics of the K=600 spectrometer in vertical focus mode for the medium- and high-dispersion plane are listed in Table. 2.1. Given that the momentum acceptance of the K=600 in the medium-dispersion plane is 9.7%, two slightly different K=600 field settings were used to cover the full momentum range required in this experiment (further discussed in section 2.9).
Figure 2.2. A schematic overview of the K=600 spectrograph taken from [66]. The K=600 system consist of 5 active elements: a quadrupole magnet, two dipole magnets and two trim coils (K and H).
## Table 2.1

The calculated ion-optic matrix elements of the K=600 spectrograph for the medium- and high-dispersion focal plane.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Medium Dispersion Matching</th>
<th>High Dispersion Matching</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix Element</td>
<td>R = 1.00&lt;sup&gt;a&lt;/sup&gt;</td>
<td>R = 1.49&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>$(x</td>
<td>x)$</td>
<td>-0.52</td>
</tr>
<tr>
<td>$(\theta</td>
<td>\theta)$</td>
<td>-1.89</td>
</tr>
<tr>
<td>$(y</td>
<td>y)$</td>
<td>-5.45</td>
</tr>
<tr>
<td>$(\phi</td>
<td>\phi)$</td>
<td>-0.20</td>
</tr>
<tr>
<td>$(x</td>
<td>\Delta p)$</td>
<td>-8.4 cm/%</td>
</tr>
<tr>
<td>$p_{max}/p_{min}$</td>
<td>1.097</td>
<td>1.048</td>
</tr>
</tbody>
</table>

<sup>a</sup> Ratio of the two magnetic field of the dipole magnets $R = B(D1)/B(D2)$. 
The solid angle acceptance of the K=600 spectrograph can be defined further upstream by six possible collimators housed in a collimator carousel. In an effort to optimize ratio of counts-to-background, a setup with no collimator was used allowing for the full vertical and horizontal acceptance of the K=600 spectrograph ($\pm 2.51^\circ$ around the central ray) to be utilized.

At the K=600’s turning axis, is a 524 mm diameter scattering chamber. Within this scattering chamber is a target ladder capable of holding six targets along with a small brass Faraday cup used to stop the beam when the K=600 spectrograph is used at angles above $5^\circ$.

After the 100 MeV proton beam impinged upon the target located inside the scattering chamber, the reaction products were momentum analyzed by the K=600 spectrograph and subsequently observed in the focal plane detector system located at the medium dispersion plane. The proton beam is stopped in either an L-shaped brass beam-stop inside the dipole magnet D1, or a square shaped brass beam-stop inside the scattering chamber, depending on the angle of the K=600.

2.3 The Focal Plane Detector System

The focal plane detector system consists of two vertical drift chambers (VDC) and two plastic scintillators, all positioned at an angle of $35.7^\circ$ relative to the central ray of the medium dispersion focal plane, illustrated in Figure 2.3. The position-sensitive VDCs are used to determine the momentum and scattering angle of all reaction products through the reconstruction of their horizontal and vertical track components through the focal plane, while the plastic scintillators serve as a master trigger of the focal plane system in addition to aiding in particle identification of reaction products. A third plastic scintillator was added on as a veto for background consisting of lighter reaction products, but in the end it was not needed.
Figure 2.3. The focal plane detector system consisted of two vertical drift chambers followed by two plastic scintillators (referred to here as paddles). A third plastic scintillator was added later to act as a veto.

2.3.1 Plastic Scintillators

The downstream half of the focal plane system includes two rectangular (1219 x 102 mm$^2$) plastic scintillating detectors. For PR137, the thickness of the scintillators, 3.175 and 6.35 mm respectively, were chosen such that the highest energy tritons would stop in the second scintillator. At each end of the two scintillators are photomultiplier tubes (PMTs) converting the generated light output from the scintillating material into electronic signals. The PMTs were gain matched so that the signal output was symmetric for reaction products striking the center of the scintillator. Electronic signals from each PMT are digitized with a 32 channel Charge-to-Digital Converter (QDC: CAEN model V792N). These plastic scintillators play an integral part in the operation of the focal plane detector system as they serve as the overall trigger of the focal plane system. In addition to acting as the master trigger, these plastic scintillator detectors offer vital information on particle identification.
through energy loss and time-of-flight determination. A time-of-flight for each reaction product is taken as the relative time between a coincidence hit in the two plastic scintillators and a common stop generated by the SSC’s Radio Frequency (RF), while an energy-loss signal is recorded as the energy deposited in each scintillators by an incident reaction product traveling through the focal plane. Finally, the overall trigger of the focal plane system is then a proper coincidence signal between the two plastic scintillators.

2.3.2 Vertical Drift Chambers

The upstream half of the focal plane system consists of two Vertical Drift Chambers (VDCs). Each VDC houses two gold-plated tungsten wire-planes in a UX configuration with respect to each other. The X wires are mounted perpendicular to the horizontal scattering plane, while the U wires are mounted at 50° with respect to the horizontal scattering plane (illustrated in the bottom panel of Figure 2.4).

The X-wire plane is composed of 198 signal wires along with 199 field shaping wires interspersed every 2 mm. The U-wire plane is composed of 143 signal wires along with 144 field-shaping wires interspersed every 2 mm. For both wire planes, the signal wires are 20 µm in diameter, while the field shaping wires are 50 µm in diameter. As seen in the top panel of Figure 2.4, the wire planes are sandwiched between three 20 µm thick aluminum foil cathode planes. Each cathode plane is spaced 8 mm apart from the adjoining wire plane. Beyond the outer two cathodes, two 25 µm thick Mylar planes are used to separate the interior of each VDC from atmosphere.

During operation, the active volume of the VDC between the two Mylar windows is continuously filled with a gas mixture of 90% Ar and 10% CO₂, the field shaping wires are biased at -0.5 kV, while the aluminum cathode planes are biased to ∼ -3.6 kV. Ionized reaction products that traverse the active region create a trail of ion pairs.
Figure 2.4. Top panel: A top-down view of one VDC. An ionized reaction product traverse the gas filled active region, creating a trail of ion pairs. The electrons from the ion pairs are collected by the external electric field in the signal wires. Bottom panel: A face-on schematic of the wire-planes in one VDC. The X-wires are aligned perpendicular to the horizontal scattering plane while the U-wires are places at 50° with respect to the horizontal scattering plane. This configuration allows for the track reconstruction of the reaction products through the VDCs.
(Ar\(^+\) + electrons). The highly mobile electrons from these ion pairs (1000 greater mobility than their Ar\(^+\) partners) drift in the electric field created between the field shaping wires and the cathode planes. Closer to the wire plane, where electric field gradients are strongest, Townsend avalanching occurs resulting in a large electric signal collected by the signal wires. A 10\% CO\(_2\) mixture was added to the Ar gas as a quenching agent to absorb high energy photons coming directly from Ar nuclei that were excited by incoming reaction products.

The physical quantity recorded by the VDCs is the amount of time it takes for the electrons in the gaseous medium to drift from their point of origin to the signal wires. These drift times are measured relative to a start signal generated by a coincidence event in the two plastic scintillators. For one event, multiple signal wires will fire and record various drift timings according to the ion’s track (angle and wire-plane crossing) through the VDC. With the use of look up tables, these drift times are converted to drift lengths and the angle and wire-plane crossing point of the ion can be determined. These drift tables are calibrated using a spectrum of reaction products with a continuum of momenta, allowing for all signal wires in both planes to fire.

Signals from each wire plane are passed through sets of 16-channel combined pre-amplifier and discriminator cards (Model P-TM 005 manufactured by Technoland Corporations). The 198 X-wires are grouped into 13 16-channel electronic cards, while the 143 U-wire are grouped into 9 16-channel electronic cards. After pre-amplification and discrimination, the timing signals are recorded using a 128 channel multi-event Time-to-Digital Converter (TDC: CAEN model V1190A). Finally, these timing signals are compared to the start-time signal generated by a coincidence event in the plastic scintillators to get drift times.
2.4 100 MeV Proton Beam

A beam energy of 100 MeV was chosen for PR137 as this was the beam energy used in previous (p,t) measurements at RCNP[59–61]; and given that PR137 already shares many similarities with these past RCNP measurements, using the same beam energy of 100 MeV allowed us to make direct comparisons when benchmarking PR137.

Initially a high-quality achromatic beam must be established in order to achieve the best possible results with dispersion matching techniques (covered in Sec. 2.5). An achromatic beam is usually seen as a small beam spot on the target where the position and angle of the beam on the target is independent of its momentum spread. To establish a good achromatic beam tune, the beam transmission through the SSC was maximized to \( \approx 95\% \), which in turn minimized possible sources of beam halo. Once a high quality achromatic beam tune was establish with an optimal transmission through the SSC, dispersion matching techniques could be utilized to achieve a better resolution than the energy spread of the beam.

2.5 Dispersion Matching Techniques

At iThemba LABS, the energy spread of the 100 MeV beam coming from a single turn extraction out of the SSC is on the order of \( \sim 100 \) keV, or \( \sim 0.1\% \) of the beam energy. In general, this inherent energy spread of the proton beam will limit the achievable energy resolution seen in the focal plane of the K=600 spectrograph. With the goal of identifying possible high lying (Ex. Energy = 7 - 13 MeV), closely spaced \( (D_{Ex} = 100 - 30 \) keV) excited states, dispersion matching techniques between the beam line and K=600 spectrograph were utilized to ultimately increase the focal plane’s energy resolution from \( \sim 100 \) keV to \( \sim 35 \) keV. Following the techniques laid out by Fujita et al. [28], the basics of dispersion matching are reviewed here using the TRANSPORT notation adopted in [29].

39
To begin with, a source point was created using slit-9X located right after extraction of the SSC on the X line (see Figure. 2.1). As the beam passes through these slits, each particle can be represented by the coordinates \( \vec{X}_0 = (x_0, \theta_0, \delta_0) \), where \( x_0 \), \( \theta_0 \), and \( \delta_0 \) represents its horizontal position, horizontal angle, fractional momentum deviation (all relative to the central ray’s trajectory through the ion-optic system).

As the beam is transported from the source point to the target, it’s original coordinates are transformed by the reduced, horizontal \( 3 \times 3 \) matrix, \( \hat{\mathbf{B}} = \{b_{ij}\} \). Here \( \hat{\mathbf{B}} \) represents the X, P1, P2, and S beam lines. Defining \( \vec{X}_0 \) as the source coordinates and \( \vec{X}_1 \) as the target coordinates, the transformation from \( \vec{X}_0 \) to \( \vec{X}_1 \) can be written in vector form as:

\[
\begin{bmatrix}
  x_1 \\
  \theta_1 \\
  \delta_1
\end{bmatrix} =
\begin{bmatrix}
  b_{11} & b_{12} & b_{16} \\
  b_{21} & b_{22} & b_{26} \\
  0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  x_0 \\
  \theta_0 \\
  \delta_0
\end{bmatrix}.
\]  

(2.1)

A critical transformation occurs as the beam impinges and reacts with the target. This transformation can be written as \( \vec{X}_2 = T\vec{X}_1 \), where \( \vec{X}_2 \) represents the post reaction coordinates at the target position and the target function, \( T \), is defined as:

\[
T = \frac{\cos(\alpha - \Phi_T)}{\cos(\Phi_T)}. 
\]  

(2.2)

Here, \( \alpha \) is the scattering angle of the central ray in the laboratory frame (referred to as the “nominal” scattering angle) and \( \Phi_T \) is the tilting angle of the target. The true scattering angle of a specific beam particle is then given as \( \beta = \alpha + \theta_2 - \theta_1 \). The relationship between \( \vec{X}_1 \) and \( \vec{X}_2 \) through these quantities is shown in Figure. 2.5. An “effective” scattering angle can be defined as \( \Theta = \theta_2 - \theta_1 \), representing how much the true scattering angle of a particle differs from the nominal scattering angle. Since the beam interacts with the target, there is a change in the relative momentum from...
Figure 2.5. Schematic of an incident beam particle relative to the central beam ray interacting with the target and scattering away as a reaction product particle relative to the central spectrograph ray. Figure was taken from [28].

\[ \delta_2 = K \Theta + C \delta_1. \]  

(2.3)

Here, K is the kinematic factor defined as \((1/p_{out})(\partial p_{out}/\partial \alpha)\) and C is the relative momentum ratio defined as \((p_{in}/p_{out})(\partial p_{out}/\partial p_{in})\), where \(p_{in}\) is the momentum of the incoming particle at the target position, and \(p_{out}\) is the momentum of the outgoing particle at the target position.

As reaction products are transported through the K=600 spectrograph to the focal plane, their coordinates are transformed again from the target position, \(\vec{X}_2\), to the focal plane, \(\vec{X}_{fp}\), by another reduced, horizontal 3×3 matrix, \(\vec{S} = \{s_{ij}\}\). The
relationship between $\vec{X}_2$, $\vec{X}_{fp}$, and $\hat{\mathbf{S}}$ can be written as:

$$
\begin{bmatrix}
  x_{fp} \\
  \theta_{fp} \\
  \delta_{fp}
\end{bmatrix} =
\begin{bmatrix}
  s_{11} & s_{12} & s_{16} \\
  s_{21} & s_{22} & s_{26} \\
  0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  x_2 \\
  \theta_2 \\
  \delta_2
\end{bmatrix}
$$

(2.4)

Combining the $\hat{\mathbf{S}}$, $T$, and $\hat{\mathbf{B}}$ transformations (Eq. 2.1, 2.2, and 2.4), results in the following equations relating the source point coordinates $\vec{X}_0$ to the focal plane coordinates $\vec{X}_{fp}$,

$$
x_{fp} = x_0(s_{11}b_{11}T + s_{12}b_{21})
+ \theta_0(s_{11}b_{12}T + s_{12}b_{22})
+ \delta_0(s_{11}b_{16}T + s_{12}b_{26} + s_{16}C)
+ \Theta(s_{12} + s_{16}K),
$$

(2.5)

$$
\theta_{fp} = x_0(s_{21}b_{11}T + s_{22}b_{21})
+ \theta_0(s_{21}b_{12}T + s_{22}b_{22})
+ \delta_0(s_{21}b_{16}T + s_{22}b_{26} + s_{26}C)
+ \Theta(s_{22} + s_{26}K),
$$

(2.6)

and

$$
\delta_{fp} = K\Theta + C\delta_0.
$$

(2.7)

It can be seen that to achieve a minimal size of $x_{fp}$ and get the best possible spatial resolution in the focal plane, the last three coefficients in Eq. 2.5 must be minimized, effectively canceling out $\theta_0$, $\delta_0$, and $\Theta$’s influence on $x_{fp}$. First, through kinematic corrections, the coefficient of the $\Theta$ term in Eq. 2.5 can be eliminated by setting

$$
s_{12} = -s_{16}K.
$$

(2.8)
Next, the coefficient of $\theta_0$ can be set to zero by shifting the focus on the target slightly downstream of the target position, known as focus matching, where

$$\frac{b_{12}}{b_{22}} = -\frac{s_{12}}{s_{11}} T. \tag{2.9}$$

Finally, minimizing the the coefficient of the $\delta_0$ term in Eq. 2.5 is slightly more complex. The coefficients of the $\delta_0$ terms in both Eqs. 2.5 and 2.6 are set to zero and the beam transfer elements $b_{26}$ and $b_{16}$ are solved for simultaneously. The resulting relations for $b_{26}$ and $b_{16}$ are:

$$b_{26} = (s_{21}s_{16} - s_{11}s_{26})C, \tag{2.10}$$

and

$$b_{16} = \frac{s_{16}}{s_{11}}(1 + s_{11}s_{26}K - s_{21}s_{16}K)\frac{C}{T}. \tag{2.11}$$

The elimination of $\delta_0$ in Eq. 2.5 is known as lateral dispersion matching, while the elimination $\delta_0$ in Eq. 2.6 is known as angular dispersion matching. Once the last three terms in Eq. 2.5 are adjusted to zero the remaining term of $x_0(s_{11}b_{11}T + s_{12}b_{21})$ defines the smallest possible image in the focal plane and thus the focal plane’s resolution. It should be noted that Eq. 2.5 also depends on the coordinates of $\vec{X}_0$, and therefore great care was taken to establish a small, focused, and well defined object point at slit-9X in order to minimize the contributions from $x_0, \theta_0$, and $\delta_0$.

Examining equations 2.10 and 2.11, it can be seen that the beam line parameters, $b_{26}$ and $b_{16}$, should be adjusted in accordance with the changes made to the spectrometer parameters $s_{(i,j)}$ during the kinematic corrections (equation. 2.8). With the ratio of $s_{12}$ and $s_{16}$ depending on the kinematic factor (which changes as a function of angle), it should be noted that dispersion matching conditions for any two given angles will not be the same.
Figure 2.6. Schematic illustrating ion trajectories for different relative momenta, $\delta$, given the matching conditions between a beam line and a magnetic spectrograph for a) focus matched achromatic conditions, b) focus and lateral dispersion matched conditions, and c) focus, lateral, and angular dispersion matched conditions.
The concept of lateral and angular dispersion matching a spectrograph and beam line is illustrated in Figure. 2.6 where the trajectories of beam particles with three different relative momenta ($\delta_0 = 0, +\Delta p/p, \text{ and } -\Delta p/p$) are shown. In Figure. 2.6a, the initial condition of double-achromatic has been achieved and focusing matching has already been established. Here, $b_{16} = 0$ and $b_{26} = 0$, meaning that the position and angle of the beam on the target is independent of the momentum spread of the beam. In this condition, particles with different $\delta_0$ values will be momentum analyzed and dispersed as they travel through the spectrograph. It is clear from Figure. 2.6a that the energy resolution of the focal plane is limited to the momentum (energy) spread of the beam. In Figure. 2.6b, the different $\delta_0$ beam trajectories are shown when lateral dispersion matching conditions are realized. Here, $b_{16}$ is matched and $b_{26} = 0$, meaning that the position the beam on the target is now dependent on the momentum spread of the beam, but the angle is not. In these conditions, the dispersion at the target is now compensated by the spectrograph, resulting in a lateral focus in the focal plane, but particles with different $\delta_0$ values strike the focal plane with different angles. Finally, in Figure. 2.6c, by adjusting the angular dispersion of beam particles with different $\delta_0$ values on the target, their trajectories not only hit the focal plane at the same position but also with the same angle. With both $b_{16}$ and $b_{26}$ matched, an achromatic focus is achieved at the focal plane and the beam line is now considered fully dispersion matched with the magnetic spectrograph.

Through this dispersion matching technique, the achromatic focus of the ion-optic system is moved from the target to the focal plane of the spectrograph and the spectral resolution of the focal plane is no longer limited by the energy spread of the beam, but now by the resolving power of the spectrograph, which is given as:

$$R = \frac{1}{2x_0} \frac{s_{16}}{M_x}.$$ (2.12)
Here, $M_x$ is the $x_0$ coefficient in equation 2.5, and is considered as the overall magnification in $x$ from $\vec{X}_0$ to $\vec{X}_{fp}$.

2.6 Faint Beam Method at iThemba LABS

In practice, dispersion matching the K=600 spectrograph and the X, P1, P2, and S beam lines is done using an attenuated beam ($\sim 10^3$ particles/second) going directly through the spectrograph at $0^\circ$ and into the focal plane detectors. This method of directly observing a faint beam in the focal plane was first developed at RCNP with the Grand Raiden spectrograph to realize the dispersion matching conditions for K = 0. By observing the beam image through $\theta_{fp}$ vs $x_{fp}$ 2-dimensional histograms, various matching conditions can be realized. Starting with a double-aromatic beam at the target and only focus matching realized, beam particles with different relative momenta ($\delta$) will hit the focal plane at different positions and horizontal angles, and the focal plane image will look similar to that of Figure. 2.7a. Once lateral dispersion matching is realized, then beam particles with different relative momenta will hit the focal plane at the same positions but with different horizontal angles. The beam image given these conditions will look similar to Figure. 2.7b, where a high energy resolution is achieved, but any horizontal angle information cannot be determined. Once full dispersion matching is realized (focus, lateral dispersion and angular dispersion), then beam particles with different relative momenta will hit the focal plane at the same x-position and horizontal angle. With the fully matched conditions the image of the beam is completely minimized within $\theta_{fp}$ and $x_{fp}$ at the focal plane. The resulting $\theta_{fp}$ vs $x_{fp}$ plot given with full dispersion matching between the K=600 and beam lines is displayed in Figure. 2.7c.
Figure 2.7. Cartoon of the faint beam images as seen in the focal plane through $\theta_{fp}$ vs $x_{fp}$ scatter plots for different matching conditions. Beam images are shown for the following conditions: a) focus matched b) focus and lateral dispersion matched, and c) full (focus, lateral, and angular) dispersion matched.

2.7 Off-Focus Mode

In order to precisely calculate the energy of excited states in the recoil nucleus, the scattering angle of the outgoing tritons must be accurately determined. For measurements near 0°, the scattering angle depends equally on both the horizontal and vertical components. Through angular dispersion matching in the horizontal plane, the horizontal scattering angle, $\theta_{\text{scat}}$, can be accurately reconstructed using the horizontal angles of the reaction products recorded in the focal plane, $\theta_{fp}$. As for the vertical scattering angle, since there is no angular dispersion matching in the vertical plane and the overall angle magnification of the K=600 is relatively small (reported in [66] as $s_{44} = 0.2$), it becomes extremely difficult to determine the vertical angle in the focal plane and therefore the vertical scattering angle. To improve the vertical angular resolution in the focal plane, the K=600 spectrograph is operated in off-focus mode, which takes advantage of its vertical ion-optical properties.

In terms of the first order ion-optics, the transformation in the vertical plane of
the reaction products from the target to the focal plane by the reduced, vertical 3×3 matrix, \( \hat{S} = \{s_{ij}\} \) is given as:

\[
\begin{bmatrix}
y_{fp} \\
\phi_{fp} \\
\delta_{fp}
\end{bmatrix} =
\begin{bmatrix}
s_{33} & s_{34} & s_{36} \\
s_{43} & s_{44} & s_{46} \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
y_2 \\
\phi_2 \\
\delta_2
\end{bmatrix}
\] (2.13)

where \( Y_2 = (y_2, \phi_2, \delta_2) \) and \( Y_{fp} = (y_{fp}, \phi_{fp}, \delta_{fp}) \) are the vertical coordinates at the target and focal plane.

Solving for \( y_{fp} \) in equation 2.13 we get the following relation,

\[
y_{fp} = s_{33}y_2 + s_{34}\phi_2 + s_{36}\delta_2.
\] (2.14)

It can be seen from equation 2.14 that information of the vertical scattering angle, \( \phi_2 \), can be extracted from the measured vertical position in the focal plane, \( y_{fp} \), if two things are realized: A minimization of it’s first term, \( s_{33}y_2 \), and an overall increase in magnitude of the second term, \( s_{34}\phi_2 \). The first term in equation 2.14 is the overall vertical magnification, \( s_{33}y_2 \), and can be minimized by reducing the vertical width of the beam image on the target as much as possible, effectively setting \( y_2 \approx 0 \). This minimization of the vertical beam image at the target position reduced the ambiguity of \( y_{fp} \) on \( y_2 \), allowing equation 2.14 to predominately depend on just \( \phi_2 \).

The second term is the vertical position’s dependence on the scattering angle, \( s_{34}\phi_2 \) and by increasing the magnitude of \( s_{34} \), reaction products with different outgoing vertical scattering angles will hit the focal plane at different vertical positions. The results of these two steps make it possible to determine \( \phi_2 \) (or \( \phi_{scat} \)) from the measured vertical position in the focal plane, \( y_{fp} \). This mode of operation has the overall effect of moving the focus of the spectrograph away from the focal plane, and is known as off-focus mode.
With dispersion matching and operating the K=600 in off-focus mode, the scattering angle of the outgoing tritons could be properly reconstructed using the $y_{fp}$ and $\theta_{fp}$ information in the focal plane.

2.8 Targets

Several targets were used during this experiment, either for calibration and diagnostic purposes or as targets to investigate the states of interest (listed in Table 2.2). A $^{12}$C (1.05 mg/cm$^2$) target was used to check the initial settings of the focal plane detectors along with identifying the position of possible contamination peaks in the focal plane coming from $^{12}$C. A 8 $\mu$m thick Kapton target was used to verify the initial field settings of the K=600 spectrograph. A 0.2 mg/cm$^2$ $^{197}$Au target was used to investigate and minimize background (halo) with the achromatic beam. While a 4 mg/cm$^2$ $^{197}$Au was used for establishing off-focus mode, producing a white spectrum for the drift time calibration of protons in the focal plane, and calibrating the reaction product scattering angle. A 2.10 mg/cm$^2$ Mylar target was used to identify possible contamination peaks in the focal plane coming from $^{16}$O. A 2.10 mg/cm$^2$ $^{24}$Mg target was used as a magnetic rigidity calibration of the focal plane. Finally, a 2.10 mg/cm$^2$ isotopically pure $^{40}$Ca target was used to populated states in $^{38}$Ca.

All target thickness were determined using a setup consisting of a small silicon surface barrier detector and a $^{228}$Th alpha particle source. By looking at the energy loss of various alphas coming from the alpha decay chain of $^{228}$Th through each target, the thickness of that target can be determined to within 10%.

The energy resolution limitation due to target thickness is taken as the energy difference between a (p,t) reaction occurring a the front of the target vs the back. For the 2.10 mg/cm$^2$ $^{40}$Ca target, $\Delta E = 20.9$ keV.
<table>
<thead>
<tr>
<th>Target</th>
<th>Thickness</th>
<th>Purpose of target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibration and diagnostic targets</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{12}$C</td>
<td>1.05 mg/cm$^2$</td>
<td>Focal plane diagnostics and identifying contamination peaks coming $^{12}$C</td>
</tr>
<tr>
<td>Mylar ($C_{10}H_{8}O_4$)</td>
<td>2.10 mg/cm$^2$</td>
<td>Identifying contamination peaks coming $^{16}$O</td>
</tr>
<tr>
<td>Kapton ($C_{22}H_{10}N_2O_5$)</td>
<td>8 µm</td>
<td>Focal plane diagnostics</td>
</tr>
<tr>
<td>$^{24}$Mg</td>
<td>2.10 mg/cm$^2$</td>
<td>Magnetic rigidity calibration of focal plane</td>
</tr>
<tr>
<td>$^{197}$Au</td>
<td>0.2 mg/cm$^2$</td>
<td>Achromatic beam diagnostics</td>
</tr>
<tr>
<td>$^{197}$Au</td>
<td>4 mg/cm$^2$</td>
<td>Proton drift-time calibration, scattering angle calibration, and achieving off-focus mode</td>
</tr>
<tr>
<td>Target to investigate states of interest</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{40}$Ca</td>
<td>2.10 mg/cm$^2$</td>
<td>Investigating states in $^{38}$Ca</td>
</tr>
</tbody>
</table>
2.9 Settings for the (p,t) Reaction

As discussed in the beginning of this chapter, the goal of this experiment was to obtain high-resolution, near zero-degree, (p,t) spectra of $^{38}$Ca covering an excitation energy range starting from the ground states all the way up to roughly 13 - 15 MeV. With these requirements, focal plane spectra of the various (p,t) reactions were taken at four different experimental settings, including two angles ($\theta_{lab} = -1.2^\circ$ and $8^\circ$) and two magnetic field settings of the K=600. As an example, raw focal plane spectra for the $^{40}$Ca(p,t)$^{38}$Ca reaction is given for the four different (p,t) reaction settings in Figure. 2.8.

The near zero-degree angle of $\theta_{lab} = -1.2^\circ$ was chosen due to the presence of substantial background at $\theta_{lab} = 0^\circ$ coming from the D1 Faraday cup. By changing the K=600 angle to $\theta_{lab} = -1.2^\circ$, this background was significantly reduced and $\theta_{lab} = 0^\circ$ was still covered by the angle acceptance of the spectrograph. An additional angle, $\theta_{lab} = 8^\circ$, was taken to aid in the identification of target contamination peaks in the focal plane. The main target contamination concerns were $^{12}$C(p,t) and $^{16}$O(p,t), seen as $^{10}$C and $^{14}$O in the focal plane. By measuring at two different angles the contaminants could be identified by their kinematic shifts, as their shifts would be different than the that of the peaks of interest.

As stated in section 2.2, the momentum acceptance of the K=600 in the medium-dispersion plane is 9.7%, which translates to roughly a 19% acceptance in energy, or 15.2 MeV given 80 MeV tritons. In reality, the energy acceptance of the K=600 at the medium-dispersion plane is slightly smaller due to edge effects and the geometry of the focal plane detectors. During this experiment is was measured to be around 17%, meaning an energy window of ~13 MeV for 80 MeV tritons. Therefore, to cover the full excitation energy (or momentum) range from ground state to ~14 MeV in $^{38}$Ca, two slightly different K=600 B-field settings were used during this experiment. The choice of each B-field setting is further discussed in section. 3.6.
To indicate which (p,t) reaction settings were used for a particular measurement, all spectra in this work are labeled with the reaction, the angle of the K=600 ($\theta_{K600} = -1.2^\circ$ or $\theta_{K600} = 8^\circ$), and the B-field settings of the K=600 (B$_1$ or B$_2$).
Figure 2.8. Raw spectra at the four given experimental settings, $\theta_{K600} = -1.2^\circ$ and $8^\circ$, along with B1 and B2. For the most part, substantial background was observed at $\theta_{K600} = -1.2^\circ$ coming from protons scattering off of the D1 Faraday cup. This background was not seen at $\theta_{K600} = 8^\circ$, due to the beam stopping in the scattering chamber.
Beginning with the raw spectrum recorded by the focal plane detectors, a few procedures need to be applied before excitation energy in the recoil nucleus can be extracted. These procedures include: particle identification and background reduction of the focal plane, off-line software corrections to increase the energy resolution, fitting procedures to obtain important parameters of peaks (centroids and FWHM), calibration of the scattering angle and energy of the outgoing reaction products (tritons), and overlap matching techniques to extend the energy calibration to higher excitation energies. All of these procedures and techniques were done with the help of the data analysis software, ROOT [11], and are covered in detail in the following sections.

3.1 Background Identification and Reductions

The first of these procedures is reducing the background in all focal plane spectra through particle identification cuts (energy-loss and time-of-flight). These cuts, along with their effect on the x-position focal plane spectrum, are illustrated in Figure. 3.1. The energy-loss cut (as seen in the top left panel of Figure. 3.1) is made graphically on a two-dimensional energy-loss histograms created by the energy-loss signals coming from the two plastic scintillator detectors. Additionally, a time-of-flight cut was made using two-dimensional histograms containing the time-of-flight signal on the horizontal axis and the horizontal angle in the focal plane on the vertical axis (top right panel of Figure. 3.1). The effects of these two cuts on reducing the
Figure 3.1. Top-left: $\Delta E_1$ vs. $\Delta E_2$ plot, along with the energy-loss graphical cut used for background reduction. Top-right: Time-of-flight vs. focal plane horizontal angle plot, along with the time-of-flight graphical cut used for background reduction. Bottom: Sum of spectra taken for $^{40}\text{Ca}(p,t)^{38}\text{Ca}$ at $\theta_{K600} = -1.2^\circ$ with the 1st B-field settings of the K=600. The raw spectrum is given in red, while a spectrum with just the time-of-flight cut is given in orange, and finally a spectrum with both time-of-flight and energy-loss cuts is shown in dark blue.
background in the focal plane can be seen in the bottom half of Figure. 3.1) with an example using an x-position spectrum from $^{40}\text{Ca}(p,t)^{38}\text{Ca}$ at $-1.2^\circ$ and B1. Here, an initial raw spectrum is plotted in red. The raw spectrum plus with time-of-flight cut applied is plotted in orange, and raw spectrum plus with both time-of-flight and energy-loss cuts applied is plotted in dark blue.

Near $\theta_{K600} \approx 0^\circ$, the majority of the background observed in the focal plane originated from scattering of the beam off of the (p,t) Faraday cup located inside D1 (see Figure. 2.2). These protons would scatter off of the D1 Faraday cup at angles and momenta that allowed them to reach the focal plane detectors. This background was substantially reduced using either a time-of-flight or energy-loss cut. As seen in the top-right panel of Figure 3.1, protons that scatter off the D1 Faraday cup come into the focal plane across a range of angles that extend past the horizontal angle acceptance of the K=600 ($\pm 2.5^\circ$) and have broad range of flight times. It should be noted that this background was only a problem at $\theta_{K600} = -1.2^\circ$, and more specifically with the 2nd B-field settings of the K=600. Examining the focal plane spectrum in the second to the top panel of Figure. 2.8 shows that the background from the D1 Faraday cup is substantial at this settings. At $\theta_{K600} = 8^\circ$ the beam was stopped with a Faraday cup located inside the scatting chamber and no substantial background was observed in the focal plane at either B-field setting. For spectra taken at $\theta_{K600} = 8^\circ$, a single time-of-flight cut was used to clean up any background observed in the focal plane. Figure. 2.8 shows raw spectra for all four settings, illustrating the point that background was only a major problem near $0^\circ$.

3.2 Scattering Angle Calibration

As previously mentioned in sections 2.5 and 2.7, to reconstruct the vertical and horizontal components of the scattering angle, measurements were performed with the K=600 in off-focused mode and dispersion matching was realized. To find the
Figure 3.2. Top-left: Elastic scattering peak of $^{197}$Au(p,p) at four different position in the focal plane. Top-right: Schematic drawing of the multi-hole collimator used for the scattering angle calibration (all measurements given in mm). Bottom-left: Raw $Y_{fp}$ vs. $\theta_{fp}$ plot with each spectra taken at various $x_{fp}$. Bottom-right: Resultant reconstruction of vertical and horizontal scattering angle after calibration.
relationship between the scattering angle and the corresponding angle recorded in the focal plane, a calibration was performed using a multi-hole collimator, called a pepper pot, located within the collimator carousel. A schematic of the pepper pot is given in the top-right of Figure 3.2. For this procedure the K=600 was set to $\theta_{K600} = 10^\circ$ and the beam was stopped in the Faraday cup inside the scattering chamber. With the 4 mg/cm$^2$ $^{197}$Au target, the magnetic elements of the K=600 were scaled such that the proton elastic scattering peak was recorded at four different x-positions across the focal plane. The positions of the proton elastic peak given the four field settings were $x_{fp} = 159$ mm, $x_{fp} = 294$ mm, $x_{fp} = 589$ mm, and $x_{fp} = 691$ mm (shown in the top-left of Figure 3.2). With the use of well defined holes given by the pepper pot, a relation between the the horizontal and vertical focal plane coordinates and the horizontal and vertical scattering angles can be recognized. The raw image of the pepper pot in the focal plane for each x-position is displayed in the bottom-left of Figure 3.2. From this plot a clear dependence on the $x_{fp}$ can be seen in both $\theta_{fp}$ and $y_{fp}$ (related to $\phi_{fp}$ in off-focus mode). The relationship between the horizontal scattering angle ($\theta_{scat}$) and the horizontal focal plane angle ($\theta_{fp}$) can be written as:

$$\theta_{scat} = \theta_{fp} \sum_{i=0}^{2} a_i x_{fp}^i + \sum_{i=0}^{2} b_i x_{fp}^i.$$  \hfill (3.1)

While the relationship between the vertical scattering angle ($\phi_{scat}$) and the vertical position seen of the focal plane ($y_{fp}$) can be written as:

$$\phi_{scat} = y_{fp} \sum_{i=0}^{3} \sum_{j=0}^{2} c_{ij} x_{fp}^j \theta_{fp}^i + \sum_{i=0}^{3} \sum_{j=0}^{2} d_{ij} x_{fp}^j \theta_{fp}^i.$$  \hfill (3.2)

Through this calibration with the pepper pot image, the parameters in Equations 3.1 and 3.1 ($a_i$, $b_i$, $c_{ij}$, and $d_{ij}$) can be determined with a $\chi^2$ minimization. With the parameters in equations 3.1 and 3.2 known, the scattering angles of the reaction...
products can be reconstructed from the information recorded in the focal plane ($\theta_{fp}$ and $y_{fp}$). The results of this angle calibration procedure is illustrated in the bottom-right panel of Figure. 3.2 with a plot of the reconstructed horizontal and vertical scattering angle.

With the ability to properly reconstruct the scattering angles, the kinematics of the reaction at the target can be accurately calculated, allowing for the precise determination of the excitation energy of states populated in the recoil nucleus.

3.3 Off-line Corrections

The goal of dispersion matching is to eliminate the first order dependence of the focal plane’s resolution on the momentum dispersion of the beam. But even with a fully dispersion matched beam, higher-order magnetic field aberrations, along effect due to reaction kinematic, can still limit the observed energy resolution in the focal plane. These aberration and kinematic effects usually have a strong dependence on scattering angles $\theta_{scat}$ and $\phi_{scat}$, and their limitation on the energy resolution becomes evident upon examining either $\theta_{scat}$ vs. $x_{fp}$, or $\phi_{scat}$ vs. $x_{fp}$, in the focal plane. An example $\theta_{scat}$ vs. $x_{fp}$ plot from $^{40}\text{Ca}(p,t)^{38}\text{Ca}$, ranging from $x_{fp} = 500 - 680$ mm, is given in the top panel of Figure. 3.3, where the aberrations and kinematic effect can be clearly seen within the discrete states coming from $^{38}\text{Ca}$. Here, two states are highlighted with orange dashed lines, one from $^{38}\text{Ca}$ and one from $^{14}\text{O}$. Through the projection of these lines down onto the x-position axis, it becomes apparent that this curvature in $x_{fp}$ will affect the width of the corresponding peak, and therefore the capacity to resolve, and identify, closely spaced peaks.

Fortunately with the ability to reconstruct the scattering angles as discussed in Section. 3.2, these effects can be amended by subtracting out all $\theta_{scat}$ and $\phi_{scat}$ dependences in $x_{fp}$. The result of this method is that all reaction products for a particular state in the recoil nucleus will be corrected around the value of the central
Figure 3.3. Example of off-line correction performed on a subsection of a $^{40}\text{Ca}(p,t)^{38}\text{Ca}$ spectrum taken at $\theta_{K=600} = 8^\circ$ and B2. Here, the top panel shows the raw $\theta_{\text{scat}}$ vs. $x_{fp}$ plot, the middle panel shows the corrected $\theta_{\text{scat}}$ vs. $x_{fp}$ plot, and the bottom panel shows the difference between the uncorrected and corrected in the focal plane’s x-position. See text for more details.
ray, \( x_{fp}(\theta = 0) \), thus straightening out all lines from a particular recoil nucleus observed in either \( \theta_{scat} \) vs. \( x_{fp} \), or \( \phi_{scat} \) vs. \( x_{fp} \). First, corrections in \( \phi_{scat} \) can be made with the equation

\[
x'_{fp} = \sum_{i=0}^{4} \sum_{j=0}^{2} a_{ij} x'_{fp} \phi_{scat}^i.
\] (3.3)

Following the \( \phi_{scat} \) corrections, additional corrections in \( \theta_{scat} \) can be done with the equation

\[
x''_{fp} = \sum_{i=0}^{6} \sum_{j=0}^{2} b_{ij} x'_{fp} \theta_{scat}^i.
\] (3.4)

In an attempt to reduced the complexity of finding the \( b_{ij} \) coefficients in equation 3.4 globally for the full focal plane, focal plane spectra were split into small subsections and the coefficients were found locally. This approach allowed for higher order terms in \( \theta_{fp} \) to be corrected out, resulting in slightly better energy resolution in the final focal plane spectra. As an example, corrections were made to the \( \theta_{scat} \) vs. \( x_{fp} \) plot (from \( x_{fp} = 500 - 680 \) mm.) in the top panel of Figure 3.3. Here, higher order aberration, along with the kinematic effects seen in \(^{38}\text{Ca}\), were corrected out. The results of these corrections are shown in the middle panel Figure 3.3, where all lines coming from \(^{38}\text{Ca}\) have been completely straighten out with respect to \( \theta_{scat} \).

To emphasize the difference that these off-line correction make in the focal plane’s x-position, the bottom panel of Figure 3.3 shows the x-position histograms before (red) and after (blue) off-line corrections. Looking at the region between \( x_{fp} = 560 \) and 580 mm, it can be seen that four peaks (blue) are resolved from an original structure that looks like two peaks (red).

After all off-line corrections are made locally, the resultant \( x''_{fp} \) histograms are combined back into a full focal plane spectrum. Through these corrections, a final energy resolution of \( \sim 35 \text{ keV} \) (taken as the full-width at half-max of the ground state of \(^{38}\text{Ca}\)) was achieved in the focal plane.

An additional advantage from off-line corrections is that contamination peaks
within focal plane spectra can now be more easily identified. This is illustrated in the top two panels of Fig. 3.3, where two states are highlighted with orange dashed lines, one from $^{38}\text{Ca}$ and one from $^{14}\text{O}$. Due to the differences in the reaction kinematics, the kinematic effects observed in these two lines are slightly different. If off-line corrections are performed to straighten out all states coming from $^{38}\text{Ca}$, then lines from contaminants, like $^{14}\text{O}$, will have residual kinematic effects in the final $\theta_{\text{scat}}$ vs. $x''_{fp}$ plot, allowing for them to be easily identified.

3.4 Peak Fitting

In order to calculate the excitation energy of all the states populated in a particular recoil nucleus, the centroid of each peak observed in its corresponding focal plane spectrum must be identified. Following off-line corrections, the centroid of all peaks observed in the focal plane were found by fitting them with a symmetric Gaussian distribution,

$$f(x) = [a]\exp\left(\frac{-(x - [b])^2}{2[c]^2}\right),$$

(3.5)

where, $[a]$ is the peak height, $[b]$ is the peak’s centroid, and $[c]$ is related to the Full Width at Half Maximum (FWHM) of the peak through the equation, FWHM = $2[c]\sqrt{2\ln(2)}$.

Isolated peaks, mostly low lying excited states, were fitted individually with this Gaussian distribution, while closely spaced, non-isolated, peaks were fitted together within a chosen subrange of the focal plane. Initially this given subrange in the focal plane is scanned for any possible peaks using the TSpectrum class provided by the data analysis software, ROOT [11]. Once a possible peak is identified within this subrange, initial guesses for it’s parameters, $[a]$, $[b]$, and $[c]$, are generated with a symmetric Gaussian fit locally around it’s position (usually $\pm 2$ mm). Finally, with initial values for all parameters of each peak determined, a global fit is performed.
across the entire subrange. It should be noted that all parameters were allowed to vary within 50% of its initial guess value during each global fit.

An example of this fitting procedure is shown for a subrange, \( x = 175 - 300 \) mm, in the \(^{24}\text{Mg}(p,t)^{22}\text{Mg}\) spectrum, at \( \theta_{K=600} = -1.2^\circ \). For the most part the global fit across this subrange in \(^{24}\text{Mg}(p,t)^{22}\text{Mg}\) does well in replicating the peak structure. From this fit, the centroid, peak height, and FWHM are all recorded for each peak for further analysis.

In this example, one peak around \( x = 228 \) mm, indicated with a small arrow in Figure. 3.4, was fitted improperly and therefore not recorded. In a situations like this, the subrange would be shorted further and refitted. If the fit still does not satisfactorily replicate the given spectra and small background function is then added, usually a second order polynomial. The main source of improper fitting comes from \(^{10}\text{C}\) and \(^{14}\text{O}\) contaminate peaks that contain residual kinematics shifts after the off-line corrections. An example of this is highlighted in Figure. 3.3 after off-line
corrections. Here all the $^{38}$Ca peaks are completely strait, forming nice Gaussian distributions when projected onto the x-position axis, but an $^{14}$O contamination peak, around $x = 630$ mm, contains residual kinematic effects and therefore will not form a Gaussian distribution when project onto the x-position axis.

After each peak was properly fitted in the focal plane and its parameters extracted from the corresponding Gaussian distribution, the area under each peak was taken using two different methods depending on how isolated the peak was. For completely isolated peaks, the area was taken as the number of counts contained in the peak. For closely-spaced peaks, the area was taken as $A_i = \frac{a_i}{c_i} \sqrt{2\pi}$, where, $[a_i]$ and $[c_i]$ are the parameters extracted from its Gaussian distribution.

Ultimately when reporting the final peak position from a given fit, two different uncertainties must be taken into consideration. First is a systematic uncertainty, $\delta x_{sys}$, in the fitting routine itself. This uncertainty is usually quite small, around a few percent of a mm in the focal plane, and is part of the output of the fitting routine. The second uncertainty is a statistical uncertainty due to the finite number of counts under a given peak. The statistical uncertainty, $\delta x_{stat}$, is given as

$$\delta x_{stat} = \frac{\text{FWHM}}{\sqrt{A}},$$

where $A$ is the area under the peak. The final uncertainty in the position of the peak is then taken as the sum of the systematic and the statistical uncertainty,

$$\delta x_{tot} = \delta x_{sys} + \delta x_{stat}.$$
3.5 Energy Calibration

The K=600 spectrograph separates particles based on their magnetic rigidity, which is related to the particle’s momentum through the Lorentz force equation:

\[ B\rho = \frac{p}{q}, \]  

(3.8)

where \( p \) is the particle’s momentum and \( q \) is the particle’s charge state. Therefore by measuring the particles x-position in the focal plane, \( \rho \), the momentum of the particle can be determined through equation 3.8. With most magnetic spectrographs, the relationship between the particle’s x-position in the focal plane and it’s momentum is found to be, in first order, linear in momentum. A small quadratic term is added to account for all the higher order terms. This relationship can be written as

\[ B\rho = ax_c^2 + bx_c + c. \]  

(3.9)

Here, \( x_c \) is the corrected x-position in the focal plane (as discussed in section 3.3). In order to find the coefficients in equation 3.9, a calibration of the focal plane was done using well-known low-lying states in \(^{22}\)Mg populated by the \(^{24}\)Mg(p,t) reaction. This procedure of calibrating the focal plane of the K=600 spectrograph using states in \(^{22}\)Mg follows the same procedures used in previous high energy-resolution (p,t) experiments at RCNP with the Grand Raiden spectrograph \([2, 59, 61]\). With the excitation energy in the recoil nucleus known, the momentum of the outgoing tritons can be precisely determined using equation B.14 derived in Appendix B. With the tritons momentum determined, their magnetic rigidity can be calculated using equation 3.8.

For this calibration, the K=600 was set to \( \theta = -1.2^\circ \) and a 2.10 mg/cm\(^2\) \(^{24}\)Mg target was used. As seen in Fig. 3.5, outgoing tritons corresponding to several
low-lying states in $^{22}\text{Mg}$ fully covered the focal plane. The excitation energies for these states in $^{22}\text{Mg}$ were taken from Matic et al. [59], which studied levels in $^{22}\text{Mg}$ using the $^{24}\text{Mg}(p,t)^{22}\text{Mg}$ reaction with the Grand Raiden spectrograph at RCNP. Highlighted by oranges dots in Figure. 3.5, 7 states ranging from the g.s. to 6.226 MeV were chosen for the focal plane calibration. The reference values, calculated magnetic rigidities, and position in the focal plane are listed in Table. 3.1.

With the magnetic rigidity known and the x-position measured in the focal plane (last two columns Table. 3.1), the coefficients of equation 3.9 were found to be

$$
\begin{align*}
    a &= -1.72986675 \times 10^{-08} \pm 2.2538884 \times 10^{-11} \\
    b &= 2.66671595 \times 10^{-04} \pm 2.26697901 \times 10^{-07} \\
    c &= 2.0535122 \pm 5.498097 \times 10^{-05},
\end{align*}
$$

The goodness of fit metric for this calibration is given as $\chi^2_{\text{red}} = 1.0008$ and a residual
TABLE 3.1

INFORMATION ON STATES IN $^{22}$MG USED FOR CALIBRATION OF FOCAL PLANE

<table>
<thead>
<tr>
<th>Excitation Energy (MeV) [59]</th>
<th>Magnetic Rigidity (T·m)</th>
<th>Focal Plane Position (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>g.s.</td>
<td>2.217820</td>
<td>642.95</td>
</tr>
<tr>
<td>1.24718(3)</td>
<td>2.200489</td>
<td>572.41</td>
</tr>
<tr>
<td>3.30821(6)</td>
<td>2.171532</td>
<td>456.06</td>
</tr>
<tr>
<td>4.4020(3)</td>
<td>2.156000</td>
<td>394.32</td>
</tr>
<tr>
<td>5.0354(5)</td>
<td>2.146940</td>
<td>358.71</td>
</tr>
<tr>
<td>5.9538(8)</td>
<td>2.133761</td>
<td>307.08</td>
</tr>
<tr>
<td>6.0362(8)</td>
<td>2.132573</td>
<td>302.41</td>
</tr>
<tr>
<td>6.2261(10)</td>
<td>2.129834</td>
<td>291.73</td>
</tr>
</tbody>
</table>
Figure 3.6. Residual energy plot from focal plane calibration using well known low lying states in $^{22}$Mg. The goodness of fit is $\chi^2_{red} = 1.0008$. Error bars taken from [59].

energy (excitation energy) plot is shown in Figure 3.6. It should be noted that these parameters are not independent from one another and therefore when calculating the uncertainty of equation 3.9, the covariance error matrix of the parameters should be taken into account. Including the error in determining the centroid of a peak (equation 3.7), the final uncertainty in magnetic rigidity is calculated as

$$
\delta B\rho^2 = \left( \frac{\partial B\rho}{\partial x} \right)^2 (\delta x)^2 + \left( \frac{\partial B\rho}{\partial a} \right)^2 (\delta a)^2 + \left( \frac{\partial B\rho}{\partial b} \right)^2 (\delta b)^2 + \left( \frac{\partial B\rho}{\partial c} \right)^2 (\delta c)^2 + \frac{\partial B\rho}{\partial a} \frac{\partial B\rho}{\partial b} \delta a \delta b + \frac{\partial B\rho}{\partial a} \frac{\partial B\rho}{\partial c} \delta a \delta c + \frac{\partial B\rho}{\partial b} \frac{\partial B\rho}{\partial c} \delta b \delta c.
$$

(3.11)

Here, all uncertainties are added quadratically and the final three terms in equation 3.11 are the cross terms in the $3 \times 3$ covariance error matrix.

Finally, with the relationship between the horizontal position in the focal plane and the outgoing triton’s magnetic rigidity (momentum) known, equation B.14 can be used to calculate the energy of the corresponding excited state populated in the recoil nucleus. The final uncertainty quoted in the excitation energy calculated using equation B.14 contains the following, the uncertainty in the target thickness (taken
generally as 10%), uncertainties in energy losses of incoming protons and outgoing tritons in the targets (taken as 5%) [103], uncertainty the reaction-angle determination ($\pm 0.05^\circ$), uncertainties in masses of all the reaction products involved [95], and the uncertainty in the tritons outgoing momentum calculated with equation 3.9.

3.6 Overlap Matching Techniques

As explained in section 2.2, to cover the full excitation energy range, from ground state to $\sim$13 MeV in $^{38}$Ca, two slightly different K=600 settings were used during this experiment. To illustrate why two different field settings are needed, Figure 3.7 shows the magnetic rigidity of tritons coming from selected ground, and excited, states in different recoil nuclei. In addition, the momentum acceptance of the focal plane in terms of magnetic rigidity is given for two different K=600 B-field settings (gray bars). By plotting the magnetic rigidities of tritons populating various states in the recoil nuclei, the positions of peaks with respect to each other in the focal plane for a given B-field setting can be predicted. Looking at this figure, the first K=600 B-field setting (aptly nicknamed $B_1$) was selected such well-known low-lying states in $^{22}$Mg would fall right in the middle of the focal plane. The second K=600 B-field setting ($B_2$) was then chosen such that the regions of interest (labeled as ROI in Figure 3.7) in $^{38}$Ca then fall within the middle of the focal plane which was previously calcibrated by states in $^{22}$Mg.

As discussed in section 3.5, the calibration of the focal plane was performed using states in $^{22}$Mg. This calibration was done given the first B-field setting of the K=600 ($B_1$), and can only be applied to focal plane spectra taken with this same setting. Fortunately, this focal plane calibration taken at $B_1$ can be scaled to the secondary focal plane, $B_2$, by including a linear factor that goes as the ratio of the field strengths
Figure 3.7. Magnetic rigidities ($B\rho$) of tritons coming from selected ground, and excited, states in different recoil nuclei. The gray bars represent the momentum acceptance of the focal plane given the two different B-field settings of the K=600 spectrograph. The regions labeled ROI represent the regions of interest in the respective nucleus.
between the two different settings,

\[ B\rho = \frac{B_2}{B_1}(ax_c^2 + bx_c + c). \]  

(3.12)

Here, \( B_1 \) is related the field strength of the K=600 dipoles during the calibration, and \( B_2 \) is related the field strength of the K=600 dipoles at the second setting. Due to differential hysteresis, it is not practical to use an NMR reading from one of the K=600 dipole magnets to calculate \( \frac{B_2}{B_1} \). Instead an overlap matching technique was used to find \( \frac{B_2}{B_1} \). Here, a few states are measured and identified in the two different B-field settings, then \( \frac{B_2}{B_1} \) is minimized such the momentum from the states seen with the new settings (\( B_2 \)) would match their corresponding momentum measured with the original settings (\( B_1 \)). Through this overlap matching technique the calibration of the focal plane could be extended beyond it’s initial settings.
CHAPTER 4

STATES IDENTIFIED IN $^{38}\text{Ca}$

As stated in section 2.9, focal plane spectra of $^{38}\text{Ca}$ was taken at four different experimental settings (two magnetic field setting and two lab angles). With the use of an overlapping technique, these four settings allowed us to possibly identify excited states in $^{38}\text{Ca}$ up to 15 MeV at two different angles. For the most part states are quoted in the final result only if they were identified at both angles, $-1.2^\circ$ and $8^\circ$, and if states identified in multiple settings, then the final excitation energy is given as the weighted mean of the individual values observed at each focal plane setting (as described in section C). In this chapter the resultant excitation energies of states in $^{38}\text{Ca}$ observed in this work are presented along with states published in previous works for comparison. Any discrepancies found between identified states from this work and previous works are discussed further at the end of this chapter.

4.1 Level Structure of $^{38}\text{Ca}$

In total, 45 states were identified in $^{38}\text{Ca}$ from this work, 4 states below the proton threshold, $E_x = 4547.27(22)$ keV [95], 8 states between the proton and $\alpha$-threshold, $E_x = 6105.12(21)$ keV [95], and 33 states above the $\alpha$-threshold up to 12 MeV in excitation energy. It should be noted that states were identified only if they were confirmed at both angles, $\theta_{lab} = -1.2^\circ$ and $8^\circ$, with the exception of 9 states that showed strong signals in the $\theta_{lab} = 8^\circ$ spectra but were difficult to identify at $\theta_{lab} = -1.2^\circ$ due to background coming from the D1 Faraday cup (see section 3.1 for more details).
All excited states identified in $^{38}\text{Ca}$ from this work are displayed in Figure 4.1, with the final focal plane spectra of $^{38}\text{Ca}$ at $\theta_{\text{lab}} = 8^\circ$.

Figure 4.1. Final spectra of $^{38}\text{Ca}$ at $\theta_{\text{lab}} = 8^\circ$ with background reductions, dispersion matching techniques, and offline correction applied, for both K=600 field setting. Here, states labeled with blue have been observed in previous works, while states observed in orange are reported for the first time in this work.
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4.1.1 States below the $\alpha$-Threshold

Prior to this work, several experiments probed excited states in $^{38}\text{Ca}$ [1, 38, 51, 68, 85]. States identified in this work below the $\alpha$-threshold from the $^{40}\text{Ca}(p,t)^{38}\text{Ca}$ reaction are given in Table 4.1, along with previous measurements. The well-known g.s., 2214.8 keV, 4387.1 keV, and 4903.5 keV states were all used to match the absolute calibration to the $^{38}\text{Ca}$ spectra at both angles. Of the states below the $\alpha$-threshold reported here, most agree well with previous works with the exception of the 5832(8) keV state that is slightly higher than the values of 5819.8(17) keV, 5810(5) keV, and 5790(40) keV that was previously reported by O’Brien et al. [67], Paddock et al. [68] and Alford et al. [1], respectively.

4.1.2 States above the $\alpha$-Threshold

These $\alpha$ unbound states in $^{38}\text{Ca}$ are expected to contribute as natural parity resonances to the cross section of $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$. Prior to this work, only 8 states were experimentally known above the $\alpha$-threshold of 6105.12(21) keV, as states from O’Brien et al. were never published (discussed further in section 4.2). Combining the results from this work and that of O’Brien et al., a total of 31 new states have been identified in $^{38}\text{Ca}$ up to $\approx 12$ MeV in excitation energy. In this work, a total of 33 states above the $\alpha$-threshold up to $\sim 12$ MeV were observed. All states identified in this work, along with previous (p,t) and ($^3\text{He},n$) measurements are reported in Table 4.2. It should be noted that 11 states were strongly identified at $\theta_{\text{lab}} = 8^\circ$, but could not be confidently identified at $\theta_{\text{lab}} = -1.2^\circ$ due to high background from beam scattering coming from the Faraday cup inside dipole D1. These 11 states (displayed with an asterisk * in Table 4.2) were included in the final results because they displayed the same kinematic shift over the horizontal angle acceptance of the K=600 spectrograph at $\theta_{\text{lab}} = 8^\circ$ ($\pm 2.5^\circ$) as observed for other $^{38}\text{Ca}$ states.
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78
4.2 Discrepancies between this Work and O’Brien et al.

For the most part, states identified in this work agree well with previous works (within the errors quoted), with the exception of the work by O’Brien et al. [67]. A more thorough comparison between the states identified here and the states of O’Brien et al. shows a systematic difference in excitation energies developing above \( \sim 5.7 \) MeV. On average, the excitation energy for a particular state quoted by O’Brien et al. is 17 keV lower than the corresponding excitation energy quoted in this work.

This discrepancy is concerning as the experiment performed by O’Brien et al. is almost identical to experiment of this work. The experiment of O’Brien et al. was performed at RCNP using the Grand Raiden spectrograph along with a similar focal plane detector system and proton beam energy. In addition, similar experimental techniques were used, including dispersion matching, angular calibrations, and energy calibration.

Although there was a similarity in experimental setup and technique, some parts of the analysis of the two works differ slightly. In this work, corrections of kinematic and higher order aberrations observed in the focal plane was performed locally using a 6\textsuperscript{th} order polynomial, while O’Brien et al. used a single 2\textsuperscript{nd} order polynomial for a global correction (across the entire focal plane). By comparing the results of these different correction methods, Figure 3.3 in this work and Figures 5.7 and 5.8 in [67], it can be seen the corrected peaks in O’Brien et al. have substantial residual curvature. As seen in Figure 5.8 of O’Brien et al., this residual curvature greatly affects the shape of the peak seen in the focal plane’s x-position spectrum. For O’Brien et al. this asymmetric shape proved difficult to fit with a Gaussian distribution, therefore multiple Gaussian distribution were fitted with various subranges of the peak and the peak position was taken as an average of the centroids. Unfortunately the resultant Gaussian fits were not shown in [67], therefore it is difficult to see how effective this average fit was. If it was not, and this residual curvature exists in all of the states
identified by O’Brien et al. it could possibly account for some of the observed shift in the quoted excitation energy.

Additionally, since the average Gaussian fitting method was used in determining the centroids for states in $^{22}$Mg, this observed residual curvature could alter the calibration function slightly. This would in turn impact the ability of the calibration function to accurately reconstruct the reaction product’s magnetic rigidy given its position in the focal plane.

Finally, another possibility is that since this shift is only observed at higher excitation energy, this discrepancy could originate from improper overlap matching techniques (discussed in section 3.6). This would affect the ability to extend the calibration function to higher excitation energy. A matching factor that is slightly off would result in considerably shifted excitation energies calculated for peaks identified in focal plane spectra with lower momentum bytes.

Ultimately, it is difficult to tell where exactly this systematic difference observed in the quoted excitation energies originate from. For now, the subsequent calculation of the $^{34}$Ar($\alpha$,p)$^{37}$K reaction rate will be done only using the states identified in this particular work, PR137, and exclude the states identified by O’Brien et al. Time permitting, a re-analysis of the O’Brien et al. work could shed more light on the observed discrepancies, allowing for the states in the more states O’Brien et al. work to be included.
CHAPTER 5

ASTROPHYSICAL REACTION RATE

In this work a total of 45 states in $^{38}\text{Ca}$ were identified, of which 33 states were observed above the $\alpha$-threshold of $E_x = 6105.12(21)$ keV $^{95}$, up to 12000 keV in excitation energy. In order to calculate the $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ reaction rate given the this information, it is assumed that every state identified in this work will participate as a resonances in the $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ reaction. This assumption is considered to be valid given that these states were populated using a two neutron transfer ($p,t$) reaction at high energies (\sim 100 MeV) and very forward angles, where low-spin natural parity states are thought to be heavily favored. With the assumption that states identified in this work will act as strong resonances in the $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ reaction, their information can be used to determine its reaction rate at XRB temperatures for the first time.

5.1 Justification for using Narrow Resonance Formalism

By examining the single particle widths calculated in Appendix E for excited states in $^{38}\text{Ca}$, two important conditions can be realized for calculating the $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ reaction rate. First, the proton partial width will make up the majority of the total width for each resonance. This comes from that the observation that the calculated proton single particle widths in table E.1 are much greater than the corresponding calculated $\alpha$ single particle widths listed in table E.2, ($\Gamma_p^{sp} \gg \Gamma_{\alpha}^{sp}$). In addition, evaluating gamma widths calculated using the technique described in $^{97}$, show that corresponding gamma widths are on the order of eV. Therefore, given a reasonable
choices of the proton and α-spectroscopic factors, the the proton partial widths will dominate the total widths for most resonances,

$$\Gamma_p = S_p \Gamma_p^\text{sp} \approx \Gamma_{\text{tot}}. \quad (5.1)$$

The second important condition is that all of these resonances can be considered narrow, meaning they meet the condition $\Gamma_{\text{tot}}/E_{\text{res}} \ll 10\%$. This comes from the basis that, on average, proton partial widths are much smaller than their respective single particle widths, $\Gamma_p \ll \Gamma_p^\text{sp}$, and spectroscopic factors are usually on the order of $S_i \sim 0.1 - 0.01$. Taking into account this typical range of spectroscopic factors, and that the proton width dominates the total width, a narrow resonance treatment of the reaction rate can be justified.

Therefore, to calculate the $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ reaction rate, given the information from this work, the narrow resonances formalism describe in section A.4 can be adopted, where each resonances contribution at a particular temperature is added incoherently using the equation A.22. From this equation the total reaction rate per mole as a function of temperature can be written out numerically as

$$N_A \langle \sigma \nu \rangle = 1.54 \times 10^{11} (\mu T_9)^{-3/2} \sum_i (\omega \gamma)_i \text{Exp} \left( \frac{-11.605 E_i}{T_9} \right), \quad (5.2)$$

with $\mu$ being the reduced mass (amu), $T_9$ the temperature ($10^9$ K), $(\omega \gamma)$ the resonance strength (MeV), $E_i$ the resonance energy in the center-of-mass system (MeV), and $N_A$ as Avogadro’s number ($N_A = 6.0221409 \times 10^{23}$). Given that the proton partial width dominates the total width for each resonance, the individual resonances strengths, $(\omega \gamma)_i$ can be written as

$$(\omega \gamma)_i = \frac{2J_i + 1}{(2j^{4He} + 1)(2j^{34Ar} + 1)} \cdot \frac{\Gamma_\alpha \Gamma_p}{\Gamma_{\text{tot}}} \approx (2J_i + 1) \cdot \Gamma_\alpha. \quad (5.3)$$
Here, the spins of both $^4$He and $^{34}$Ar are zero ($j_{^4He} = j_{^{34}Ar} = 0$). Finally, the $\alpha$ partial width is given as

$$\Gamma_\alpha = S_\alpha \cdot \Gamma^{sp}_\alpha,$$

(5.4)

where, $\Gamma^{sp}_\alpha$ is the $\alpha$ single particle width, and $S_\alpha$ is the $\alpha$ spectroscopic factor.

All single particle widths discussed in this work were calculated using the BIND sub-routine in the DWUCK4 code [52], which calculates single particle widths by integrating radial wave functions based on the solution to the Schrödiger equation with a real potential and a given set of quantum numbers. A brief overview of the DWUCK4 code, along with how a set of quantum numbers was determined for a given spin, is given in Appendix E.

5.2 Calculating the $^{34}$Ar($\alpha$,p)$^{37}$K Reaction Rate

From this work, only the resonance energies are known, and therefore two additional assumptions must be made concerning spins and spectroscopic factors in order to calculate the $^{34}$Ar($\alpha$,p)$^{37}$K reaction rate using equation 5.2.

Currently, there is no experimental information known on $\alpha$ spectroscopic factors for any state in $^{38}$Ca. To overcome this lack of information, a global $\alpha$ spectroscopic factor of $S_\alpha = 0.01$ is adopted, meaning that all partial widths ($\Gamma_\alpha$) are about 1% of the total single particle widths ($\Gamma^{sp}_\alpha$). This value, which represents a standard small $\alpha$-cluster configuration, has been used in similar types of ($\alpha$,p) reaction calculations within the sd-shell [2, 61].

Additionally, no experimental information exists for spins on states above the $\alpha$-threshold in $^{38}$Ca. Therefore the spin of each state in this calculation is determined by randomly sampling a spin distribution function based on a similar spin distributions used in the Back-Shifted Fermi Gas level density model. A review of these spin distribution functions and how they are used to determine the spin for a given reso-
nances is reviewed in Appendix D.1. Once the spin is selected for a particular state, a single particle width is then calculated for that given spin, upon which equation 5.4 is then used to determine the state’s resonance strength.

In order to avoid making further assumptions about spins of individual states on a case by case basis, a Monte Carlo approach is adopted for calculating the $^{34}$Ar($\alpha$,p)$^{37}$K reaction rate. In this narrow resonance calculation, resonances strengths for each level are determined by randomly sampling spins from the spin distributions given in section D.1, and calculated based on equation 5.3 (here the corresponding $\alpha$-partial widths are determined using table E.2 and equation 5.4). Once all the resonances strengths are determined for a given set of spins, equation 5.2 is used to calculate the total reaction rate as a function of temperature. This process of generating random spins sets, determining resonances strengths and calculating the subsequent total reaction rate is repeated $10^6$ times in order to obtain distributions of reaction rates a specific XRB temperatures. The distributions of the $^{34}$Ar($\alpha$,p)$^{37}$K reaction rate at selected temperatures relevant to XRB’s is given in Figure 5.1. Using this Monte Carlo approach for determining spins, coupled with a narrow resonance reaction rate calculation, allows for the uncertainties in not knowing the spins of individual resonances to be quantified. At each temperature, from the given distribution, the median rate is determined by calculating the 0.5 quantile, the 68% and 95% limits are determined by calculating the 0.16 to 0.84 and 0.025 and 0.975 quantiles receptively. The 68% and 95% limits are labeled blue and green in Figure 5.1.

An interesting structure can be seen in the rate distributions at lower temperatures that eventually diffuses into a Gaussian distribution at high temperatures ($T \geq 1.0$ GK). This structure at lower temperature distributions is believed to originate from the fact hat only a handful of resonances are governing the reaction rate at these temperatures. Given that sampling from a discrete spin distribution yields discrete resonance strength values, the sum of resonances contributions based on a
Figure 5.1. Distributions of the $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ reaction rate at selected temperature relevant to XRB’s. These distributions are generated by random sets of spin assignments for resonances in a reaction rate calculation based on a narrow resonances formalism. The regions highlighted in blue and green represent the 68% and 95% limits, based on the 0.16 and 0.84 quantiles, and 0.025 and 0.975 quantiles, respectively. A median rate is then taken from the 0.50 quantile of each rate distribution.
few resonances at a particular temperature will then be semi-discrete. This effect is observed to disappear once temperatures are high enough that more resonances begin to contribute to the reaction rate, and therefore the effects of discrete sampling is smeared out, resulting in a Gaussian distribution.

Finally, given the various calculated quantiles for the resultant rate distributions, a total reaction rate can be plotted as a function of temperature, as shown in Figure 5.2. Here, the median rate, the 68%, and 95% limits are plotted along side two HF model predictions from NON-SMOKER\textsuperscript{WEB} v5.0w [76] and Talys 1.6 [50]. The orange shaded region in Figure 5.2 represents the region between the highest and lowest rates calculated through this Monte Carlo treatment of level spins, while the blue shaded region represents the order of magnitude uncertainty that is usually quoted with HF rates.

For completeness, it should be noted that the NON-SMOKER\textsuperscript{WEB} v5.0w rate was quoted directly from the reaction rate database, library JINA REACLIB [15]. Theoretical rates in this library are calculated based on a modified version of the NON-SMOKER code et al.[76], and the relevant input parameters used to determine this predicted rate were: global $\alpha$-optical model potentials taken from McFadden and Satchler [62], level density models taken as a Back-shifted Fermi Gas model with parameters from Rauscher et al. [77], and theoretical masses taken from Möller et al. [64]. The Talys 1.6 predicted rate was performed using a more sophisticated reaction code by Koning et al.[50]. For this comparison, the same input parameters as NON-SMOKER\textsuperscript{WEB} v5.0w were chosen for the Talys 1.6 calculation, with the exception of the level density model which did not include the Rauscher model. Instead a constant temperature + Fermi gas model was used [50].

As seen in Figure 5.2, the reaction rate from this work is lower than both NON-SMOKER\textsuperscript{WEB} v5.0w and Talys 1.6 HF predictions across all temperatures. The temperature range where the $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ reaction will be most influential on XRB
Figure 5.2. The $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ reaction rate as a function of stellar temperature as calculated using the information from this work. The median rate is plotted along with the 68% and 95% limits. In addition, two statistical model predicted rates, NON-SMOKER$^{WEB}$ v5.0w and Talys 1.6, are plotted. In the bottom panel, all rates are normalized to the NON-SMOKER$^{WEB}$ v5.0w rate. See text for more information.
light curves starts at $T \approx 0.7$ GK and extends up to peak burst temperatures of $T \approx 2.0$ GK. Through out this range, the mean $^{34}$Ar($\alpha$,p)$^{37}$K rate is substantially lower than both the HF predicted rates across all temperatures relevant for XRBs. This discrepancy is most likely due to the level density in $^{38}$Ca not being high enough to meet the criterion needed to reliably apply a statistical model to predict the $^{34}$Ar($\alpha$,p)$^{37}$K cross section and subsequent reaction rate. Therefore, it seems that the statistical approach of a HF model for this rate is not valid for most temperatures observed in XRBs, and this reaction is most likely driven by sparse isolated narrow resonances corresponding to levels located at the relevant energy range within $^{38}$Ca.

To further illustrate this condition of sparse levels governing $^{34}$Ar($\alpha$,p)$^{37}$K rate, a level diagram of $^{38}$Ca along with Gamow window approximations is shown in Figure 5.3. With the approximate Gamow window displayed for selected temperatures, it can be seen that the amount of resonances within the relevant astrophysical energies is relatively low for most XRB temperatures, especially at temperature ranges below $T = 1$ GK, where at most $\sim 7$ states are observed to lie within the Gamow window.

It should be noted that the 68% and 95% limits displayed in Figure 5.2, only represent the approximated uncertainty associated with not knowing the individual spins for each possible resonance used in this calculation. An additional uncertainty
arising from not knowing the spectroscopic factor for each of the resonances is not represented in Figure 5.2 because of the simplistic treatment of assigning a global value of $S_\alpha = 0.01$, which was meant to represent small $\alpha$-cluster configuration within the observed states. However, if any $\alpha$-cluster states exist in $^{38}$Ca within the relevant energy range, then there would be a direct enhancement of the $^{34}$Ar($\alpha$,p)$^{37}$K reaction rate within a relative narrow temperature range corresponding to the resonance energy of the particular $\alpha$-cluster state (this is further discussed in section 5.3). From this it can be seen that the discrepancy between this work and the HF predicted rates within the temperature ranges plotted in Figure 5.2 are directly dependent on the assumptions made when assigning $\alpha$-spectroscopic factors to each of the states within the Gamow window.

5.3 Possible $\alpha$-Clustering Effects on the $^{34}$Ar($\alpha$,p)$^{37}$K Rate

In the past, $\alpha$-transfer measurements within this mass region ($A = 40$) have shown strong $\alpha$-clustering effects [12], with spectroscopic factors determined to be as high as $S_\alpha \sim 0.5$ ($E_x = 9.14$ MeV in $^{40}$Ca [102]). In order to investigate the effects of possible $\alpha$-cluster states on the $^{34}$Ar($\alpha$,p)$^{37}$K reaction rate, an additional rate calculation was performed where the $\alpha$-spectroscopic factor of a single resonance was increased by an order of magnitude. This directly increases the resonance strength of the particular resonance and effectively enhances the reaction rate in a local temperature range. Similar to the previous calculation, resonances strengths for each level are determined by randomly sampling spins from the spin distributions given in section D.1, and calculated based on equation 5.3. However, in this calculation one resonance is selected from a uniform distribution and is given a spectroscopic factor of $S_\alpha = 0.1$ (representing a strong $\alpha$-cluster state), while the rest of the resonances are assigned a spectroscopic factor of $S_\alpha = 0.01$. Once all resonance strengths are determined for the given spin set, the total reaction rate is then calculated using equation 5.2. This
Figure 5.4. The $^{34}$Ar($\alpha$,p)$^{37}$K reaction rate as a function of stellar temperature calculated similarly to that shown in Figure 5.2. With the exception that one resonance is selected out of a uniform distribution and given a spectroscopic factor of $S_\alpha = 0.1$, representing an $\alpha$-cluster state. Labels are similar to that in Figure 5.2.
calculation is then repeated $10^6$ times in order to obtain distributions of reaction rates at selected XRB temperatures. The results of this calculation are shown in Figure 5.4, where the total reaction rate is plotted as a function of temperature, along with the two same HF model predictions used in Figure 5.2, NON-SMOKERWEB v5.0w [76] and Talys 1.6 [50]. Similarly, the median rate, along with the 68% and 95% limits, from the reaction rate distributions generated in this calculation are plotted in Figure 5.4. The orange shaded region in Figure 5.4 represents the region between the highest and lowest rates calculated through this Monte Carlo treatment of level spins, while the blue shaded region represents the order of magnitude uncertainty that is usually taken with HF rates. From Figure 5.4, it can be seen that the main effect from including one possible $\alpha$-cluster state among any of the possible resonances is an overall increase in the upper 68% and 95% limits, but the median rate remains unchanged from the original calculation performed with no $\alpha$-clustering taken into account. In this calculation there is some overlap at all temperatures between the uncertainties quoted with the HF predicted rates and the 95% limit calculated from this exercise. This overall increase in the upper limits, while the mean rate remains unchanged, originates from the fact that the resonance strength for only one resonance is enhanced by an order of magnitude, and therefore the reaction rate is increased in a limited temperature range corresponding to the resonance energy, while the rate at all other temperatures remain unchanged. This has the effect of increasing the high value shoulder in each reaction rate distribution at each temperature. If more than one resonance was given a stronger spectroscopic factor then there would be a definite increase in the mean rate. In this sense the mean rate quoted in this work (calculated assuming very little $\alpha$-clustering) can be taken as an overall lower limit for the $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ reaction rate across all relevant XRB temperatures, and it is emphasized that any $\alpha$-clustering should enhances the reaction rate only within a narrow temperature range. All rates calculated in this section, along with the two
HF model predictions, are shown in Table 5.1 for comparison at XRB temperatures ranging from 0.1 to 10 GK. All total rates listed in table 5.1 are given in units of $[\text{cm}^3 \text{ mole}^{-1} \text{ sec}^{-1}]$. 
TABLE 5.1

LIST OF TOTAL REACTION RATES $N_A \langle \sigma v \rangle$ AT SELECTED X-RAY BURST TEMPERATURES.

<table>
<thead>
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<th>Temperature [GK]</th>
<th>NON-SMOKER</th>
<th>TALYS 1.6</th>
<th>Lower 95%</th>
<th>Lower 68%</th>
<th>Median</th>
<th>Upper 68%</th>
<th>Upper 95%</th>
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TABLE 5.1

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CHAPTER 6

ASTROPHYSICAL IMPLICATIONS

In the previous section the $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ reaction rate at XRB’s temperatures was indirectly determined based on experimental results of measured excitation energies of levels above the $\alpha$-threshold in $^{38}\text{Ca}$. For the most part this rate was found to be significantly lower than HF predictions ($\sim 0.05$ times lower across all temperatures). Using the mean rate value for the $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ reaction determine in section 5.3, a single-zone one-dimensional XRB simulation can be done to investigate the effects of a lower rate would have on simulated light curves. For this exercise the single-zone model ONEZONE [83] is used with four different rates for the $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ reaction. Examining Figure 5.4, the four rates can approximately be taken as the NON-SMOKER$^{W_{EB}}$ v5.0w (NS) rate scaled by different constants $N_A \langle \sigma v \rangle = f \times N_A \langle \sigma v \rangle_{NS}$, where $f$ is selected to be 0.5, 0.1, 0.05, and 0.01, respectively. Here, the value $f = 0.05$ best represents the mean rate found in this work.

6.1 Single-Zone XRB Model: ONEZONE

The ONEZONE model, developed by Schatz et al. [83], shares many similarities with the single zone model used by [48], and was used to extensively to explore the end point of the rp-process. Within this model, the thermodynamic conditions and composition of a single zone is evolved self consistently, taking into account the energy generated during the thermonuclear runaway. This model assumes nuclear burning at a constant pressure, $P$, which is solely determined by the surface gravity of the
neutron star, $g_s$, along with the mass of the accumulated layer in the zone, $\Delta M$ (see [48] for more details). The evolution of the temperature is then taken as

$$c_P \frac{dT}{dt} = \dot{\varepsilon}_{\text{nuc}} - \dot{\varepsilon}_\nu - \dot{\varepsilon}_{\text{cool}},$$

(6.1)

where $c_P$ is the specific heat at a constant pressure, $\dot{\varepsilon}_{\text{nuc}}$ is the nuclear energy generation rate, and $\dot{\varepsilon}_\nu$ is the energy loss rate due to neutrino emission from $\beta$-decays, and $\dot{\varepsilon}_{\text{cool}}$ is the radiative cooling loss, which is given as

$$\dot{\varepsilon}_{\text{cool}} = \frac{a c T^4}{3 \kappa y^2}.$$

(6.2)

Here, $\kappa$ is the opacity within the neutron star’s atmosphere (see [82] for in-depth discussion of its treatment in ONEZONE), $a$ is the radiation constant, $c$ is the speed of light, and $y$ is the column density. The column density is determined by $y = P/g_s$.

A reaction rate network including 688 nuclei from hydrogen to tellurium is coupled to the thermodynamic equations through the nuclear energy generation rate, $\dot{\varepsilon}_{\text{nuc}}$. During each time step, $dt$, $\dot{\varepsilon}_{\text{nuc}}$ is calculated by taking the change in the abundance for each species, $dY_i$, and multiplying it by its respective atomic mass excess, $\Delta_i$. Summing over all nuclei, yields the total nuclear generation rate per time step

$$\dot{\varepsilon}_{\text{nuc}} = \sum_i dY_i \Delta_i.$$

(6.3)

It should be noted that because this model only concerns a single zone, any sort gradients in temperature, density, and composition (which are critical for convective mixing and radiative transport) are ignored. This simplification of energy transport makes the direct comparison of ONEZONE simulated light curve to observed light curves impractical. But given the self-consistent coupling between the reaction network and energy generation, the ONEZONE model is a optimal candidate for
IGNITION CONDITIONS FOR ONEZONE SIMULATIONS.

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<th>Input Parameters</th>
<th>Values</th>
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<td>Pressure</td>
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<td>Helium Mass Fraction</td>
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exploring the sensitivity of simulated light curves to changes in a particular nuclear reaction rate.

The initial conditions for this simulation were taken to imitate an highly energetic burst, thus producing a full rp-process all the way up to the SnSbTe cycle [83]. The main conditions for this ONEZONE simulation are displayed in Table 6.1. In addition to a full rp-process, these conditions also allow for a very strong $\alpha$-p-process with multiple parallel $(\alpha, p)$-chains accelerating the flow through the sd-shell. Given these initial conditions, the peak burst temperature reached during the simulation was $\sim 1.8$ GK. These conditions are therefore ideal for testing the sensitivity of simulated light curves to the $^{34}\text{Ar}(\alpha, p)^{37}\text{K}$ reaction rate strength, as $\alpha$-captures in this mass range are usually hindered by the Coulomb barrier at lower temperatures.

The time-integrated reaction flux during this simulation (for the original NS rate) is displayed in Figure 6.1, where, the time-integrated reaction flux is taken as the net number of reactions between two isotopes i and j integrated over some time interval
Figure 6.1. Nuclear Flow during XRB simulation in ONEZONE model through the sd-shell. The different lines represent the reaction flow relative to the triple-\(\alpha\) reactions. The think solid lines illustrate relative flow of 50\% or more, the thin solid lines illustrate relative flow between 50\% - 10\%, and the thin dashed lines illustrate relative flow 10\% - 1\%.
\[ f_{ij} = \int_{t_0}^{t_1} \left[ \left( \frac{dY_i}{dt} \right)_{j \rightarrow i} - \left( \frac{dY_j}{dt} \right)_{i \rightarrow j} \right] dt, \quad (6.4) \]

Here, \( Y_i \) is the isotopic abundance given as the mass fraction divided by mass number, \( Y_i = X_i / A_i \). The different lines in Figure 6.1 represent the reaction flux relative to the triple-\( \alpha \) reactions, of which all of the reaction flux traverses. The thin solid lines illustrate relative flux of 50\% or more, the thin solid lines illustrate relative flux between 50\% - 10\%, and the thin dashed lines illustrate relative flux 10\% - 1\%. From Figure 6.1, it can be seen that the majority of the time-integrated reaction flux goes through \( ^{34}\text{Ar} \), with some flux flowing around \( ^{34}\text{Ar} \) via \( ^{33}\text{Ar}(\alpha,p) \) (line solid line) and \( ^{32}\text{Ar}(\alpha,p) \) (thin dashed line). Once the nuclear flow reaches \( ^{34}\text{Ar} \) there are several reaction pathways that open up, including the \( ^{34}\text{Ar}(\alpha,p)^{37}\text{K} \) reaction. However, it should be noted that these line represent time-integrated flux and therefore detailed information, such as which reaction dominates when, is somewhat lost using this quantity.

6.2 Results using ONEZONE

With these initial conditions, ONEZONE simulations were performed using the four different scaled HF predicted rates for the \( ^{34}\text{Ar}(\alpha,p)^{37}\text{K} \) reaction \((f = 0.5, 0.1, 0.05, \) and \( 0.01) \). In addition, in an attempt to compare to Fisker et al. \([19]\) (see Figure 1.7), one simulation was perform using \( f = 100 \). The resultant light curves from these simulations on presented in Figure 6.2. Here, each solid color line illustrates the light curve of a particular simulation given a varied rate of the \( ^{34}\text{Ar}(\alpha,p)^{37}\text{K} \) reaction, while the dashed line is the light curve given the original NS rate. The bottom panel of Figure 6.2 displays each simulated light curve based on the varied \( ^{34}\text{Ar}(\alpha,p)^{37}\text{K} \) rates normalized to the light curve based on the original NS rate. Additionally, an insert of a zoomed-in region is included to show the structure of each light curve peak.
Figure 6.2. Simulated light Curves from the ONEZONE model for given variations in the $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ rate. The original rate, whose light curve is displayed with a dash line, was a HF predicted rate (see text), while the other light curves, shown in colored solid lines, are calculated based on a scaled version of the HF prediction. The rate that most agrees with the determined rate in this work is indicated with the red arrow ($f = 0.05$). The bottom panel shows the ratio between light curve based on the HF prediction and the light curves based on the varied rate.
for a given simulation.

From Figure 6.2 it is evident that the strength of the $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ reaction influences the resulting simulated light curve mainly through the slope or rise-time structure of the peak. Here it can be seen that a reduced $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ rate has the effect of decreasing the slope in the upper half of the rise-time peak structure, which in turn slightly delays the peak of the burst (as much as $\sim 1$ sec for $f = 0.01$), and vise-versa with the increase in the $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ rate. In addition, it can be seen that the decay of the peak is slightly delayed by a smaller $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ rate. Looking at the $f = 0.05$ rate (indicated with the red arrow), which most closely matches the $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ reaction rate determined in this work, the slope in the upper half of the rising peak is decreased enough to delay the peak luminosity by $\sim 0.6$ sec (as seen in the insert of Figure 6.2), which can be considerable compared to burst rise-times, and the observed peak-to-peak time of 4 - 7 seconds mentioned in section 1.2. This observed small delay in the ONEZONE model hints that the strength of $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ determined in this work is weak enough to possibly contribute to the double peak structure, depending on the ignition conditions of the burst.

At this point it should be emphasized that it is difficult to compare this particular ONEZONE simulation to observations, or even in detail to other multi-zone models, due to the simplifications made in these types of single-zone models. However, the behavior observed in the rise-time slope is consistent with the that observed in the work of Fisker et al. [19]. Looking at Figure 1.7, an increase in the slope of the second peak (the mixed H/He burst) is observed when the strength of $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ reaction is increased by a factor of 100 (dashed line in Figure 1.7 and solid pink line in 6.2). However, the overall peak luminosity is observed to decrease in this work given the increase in the $^{34}\text{Ar}(\alpha,p)^{37}\text{K}$ rate, while the opposite effect is seen in the work by Fisker et al.. Given the resultant light curve from this particular ONEZONE simulation, it is difficult to draw definite conclusions on how much of
a role the $^{34}$Ar($\alpha$,p)$^{37}$K reaction plays in the observed XRB systems that exhibit a double peak structure. Clearly further investigation is needed using multi-zone models, similar to the models used by [18, 19, 101]. Utilizing a multi-zone model with the $^{34}$Ar($\alpha$,p)$^{37}$K reaction determined here, which is unfortunately beyond the scope of this work, would allow for a more direct comparison to the studies by Fisker et al. [19] and Lampe et al. [56].
CHAPTER 7

DISCUSSIONS AND CONCLUSIONS

The majority of this work was motivated by the Fisker et al. study [19], who showed that the uncertainties in the $^{34}$Ar($\alpha$,p)$^{37}$K HF predicted rate is clearly visible in the simulated light curves of double-peaked XRB models. Therefore, in the attempt to reduce its nuclear uncertainties illustrated by Fisker et al., the experiment PR137 was proposed to indirectly study the $^{34}$Ar($\alpha$,p)$^{37}$K reaction by precisely measuring the excitation energies $\alpha$-unbound natural parity states in $^{38}$Ca. In this experiment, excitation energies of $\alpha$-unbound levels in the compound nucleus, $^{38}$Ca, were measured using the two neutron transfer $^{40}$Ca(p,t)$^{38}$Ca reaction at very forward angles. This experiment was performed at iThemba LABS using 100 MeV, dispersion matched proton beam and the K=600 spectrograph.

With precise energy information on possible resonances taken from this work, along with model based assumptions to fill in the remaining unknown resonances parameters, a Monte Carlo calculation was performed based on narrow resonance formalism, presented in section A.4, to generate distributions of the $^{34}$Ar($\alpha$,p)$^{37}$K reaction rate over a range of astrophysical temperatures relevant to XRBs. From these distributions, a median rate, along with upper and lower 68% and 95% limits were determined for the $^{34}$Ar($\alpha$,p)$^{37}$K reaction rate. This rate was then compared to two HF model $^{34}$Ar($\alpha$,p)$^{37}$K predictions commonly used in XRB models (NON-SMOKE$^{WMB}$ v5.0w [76] and Talys 1.6 [50]), and found to be significantly lower across the full range of XRB temperatures (see Figures 5.2 and 5.4). The lower rate found in this work suggest that $^{34}$Ar could possible act as a nuclear waiting point.
impedance in the mixed H/He burst and therefore contribute to the double-peak structure observed in some XRB light curves.

Using the $^{34}$Ar($\alpha,p$)$^{37}$K reaction rate determined from this work, initial sensitivity studies utilizing the single-zone XRB model, ONEZONE, were performed. The initial conditions of the simulation (listed in table 6.1) were chosen simulate an highly energetic mixed H/He burst, thus ensuring an extended $\alpha$-process up to $^{38}$Ca. Results of this particular simulation show that the peak of light curve given the $^{34}$Ar($\alpha,p$)$^{37}$K rate determined from this work is delayed by roughly half a second relative to the peak of the light curve given the NON-SMOKER rate. The results of this study suggest that a lower $^{34}$Ar($\alpha,p$)$^{37}$K rate will contribute to a more pronounced double peak structure but cannot account for the observed impedance solely by itself.

As mentioned in the beginning of Chapter 2, several indirect studies of ($\alpha,p$) reactions along the $\alpha$-process similar to this work have been performed in the past at RCNP[2, 59, 61]. The majority of these works find similar results in that the experimentally determined ($\alpha,p$) rate is substantially lower relative it’s corresponding HF prediction. The lower rates found in these works suggest that an impedance may not come from single waiting point with one very weak ($\alpha,p$) reaction, but from a combination of multiple waiting points that sum together to yield a greater impedance.

Given the results from this work with possible resonances energies identified, future experiments should focus on reducing the uncertainties associated with unknown spins and spectroscopic information for levels above the $\alpha$-threshold. Comparing the reaction rate results from Figures. 5.2 and 5.4, the substantial increase in the rate’s uncertainty stemming from the inclusion of possible $\alpha$-cluster states emphasizes the need to for experimental investigations of the $\alpha$-strengths of all states above the $\alpha$-threshold in $^{38}$Ca, as the $\alpha$-spectroscopic factor for each one of these states will ultimately determine how much of an influence it has on the $^{34}$Ar($\alpha,p$)$^{37}$K reaction
rate within the corresponding temperature range. Additionally, future experiments should investigate the spin parities of levels identified in this work as this has a great effect on the penetrability and therefore partial widths for a given resonance. Depending on the spin of a state, the penetrability can vary by roughly two orders of magnitude at lower center-of-mass energies, and by about one order of magnitude at higher energies.

Finally, further sensitivity studies should be performed using XRB models given the experimental results of this work. In this sense, more detailed studies similar single-zone models to ONEZONE can be performed utilizing a parameterized set of ignition conditions. The ignition conditions used in this work (see table 6.1) may not represent the best conditions to study the influence of the $^{34}$Ar($\alpha$,p)$^{37}$K reaction on single-zone light curves. This is evident in Figure 6.1, where the time-integrated reaction flow path at one point goes around the $^{34}$Ar($\alpha$,p)$^{37}$K reaction. A parameterize study using the ONEZONE model of a mixed H/He burst may reveal a set realistic ignitions conditions that could lead to a lower $^{34}$Ar($\alpha$,p)$^{37}$K reaction rate greatly influencing the resultant light curves. In addition to single-zone model studies, multi-zone models (similar to that of [19, 101]) can be used given the experimental results of this work. These types of studies would offer the ability to further constrain other important parameters within these XRB systems that exhibit double peak bursting behavior, such as metallicities and accretion rates.
A.1 Stellar Reaction Rates

The reaction rate in a particular stellar environment depends predominantly the number of nuclei available for the given reaction, the relative velocity between nuclei, and the probability that the reaction will occur at that given velocity. Considering some volume containing two species of particles at some temperature, the reaction rate can be taken as

\[ r_{ij} = n_i n_j v_{ij} \sigma(v_{ij}), \]

(A.1)

where \( n_i \) and \( n_j \) are the number densities of the two nuclei within the volume, \( v_{ij} \) is the relative velocity between the interacting nuclei and \( \sigma(v_{ij}) \) is the probability of the reaction occurring at that relative velocity (this is also called the cross-section for classical reasons). For stellar environments in thermodynamic equilibrium, the velocity of particles moving around in the ionized plasma can be represented by a Maxwell-Boltzmann distribution,

\[ \phi(v) = 4\pi v^2 \left( \frac{m}{2\pi kT} \right)^{3/2} \exp \left( -\frac{mv^2}{2kT} \right), \]

(A.2)

where \( k \) is the Boltzmann constant, \( T \) is the temperature of the plasma. The reaction rate between the two species within the volume for all relative velocities can then be taken as a double integral over the velocity distributions of both nuclei. In the center
of mass frame, this can be written as

$$r_{ij} = n_i n_j \int_{V=0}^{\infty} \int_{v=0}^{\infty} \phi(V) \phi(v_{ij}) v_{ij} \sigma(v_{ij}) dV dv_{ij}, \quad (A.3)$$

where \( V \) is the center of mass velocity, \( V = (m_i v_i + m_j v_j)/(m_i + m_j) \). Using the condition

$$\int_{V=0}^{\infty} \phi(V) = 1, \quad (A.4)$$

the reaction rate is rewritten as

$$r_{ij} = n_i n_j \langle \sigma v \rangle_{ij}. \quad (A.5)$$

Here, \( n_i n_j \) can be considered the total number density of particle pairs, and \( \langle \sigma v \rangle_{ij} \) is the reaction rate per particle pair,

$$\langle \sigma v \rangle_{ij} = \int_{v=0}^{\infty} \phi(v_{ij}) v_{ij} \sigma(v_{ij}) dv_{ij}. \quad (A.6)$$

Using the kinetic energy relation \( E = \mu_{ij} \frac{v_{ij}^2}{2} \), where \( \mu_{ij} \) is the reduced mass, the velocity distribution can be converted into a center-of-mass energy distribution, and the reaction rate per particle pair can be rewritten as

$$\langle \sigma v \rangle_{ij} = \left( \frac{8}{\pi \mu_{ij}} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_{0}^{\infty} E \sigma(E) e^{-E/kT} dE, \quad (A.7)$$

In this sense, if the cross-section for a given reaction can be determined as a function of energy, either through experiment or modeling, then the reaction per particle pair can be easily calculated by numerically solving equation A.7.
A.2 Stellar Cross Sections and The Astrophysical S-factor

For charge particle reactions in stellar environments, the cross section can be factorized into three main terms,

\[ \sigma = \frac{1}{E} e^{-2\pi\eta} S(E). \]  

(A.8)

Here, the term \(1/E\) represents the quantum mechanical nature of the interaction with the deBroglie wavelength \(\lambda^2 \propto 1/E\), the \(e^{-2\pi\eta}\) term represents the probability of the two particles tunneling through their respective coulomb barrier, and \(S(E)\) basically represents everything else (otherwise known as nuclear physics!). This term, referred to as the astrophysical S-factor, represents the probability of the two particles “sticking together” once they overcome the coulomb barrier. This S-factor can sometimes vary slowly as a function of energy, and other times it can be quite a complicated function of energy, depending on the energetics of the reaction and the nuclear structure of the nuclei involved. To illustrate the advantage of using the astrophysical S-factor and its relation to the cross section, the astrophysical S-factor along with the cross section are plotted as a function of energy for some charge particle induced reaction in Figure A.1. This reaction, as shown here, is governed by two different components. At low energies it is determined by a direct capture component, while at higher energies it is determined by a resonance capture component.

A.2.1 Direct or Non-Resonant Capture

As seen in Figure A.1, the cross section for charged particle induced nuclear reaction drops rapidly by orders of magnitude with decreasing center-of-mass energy. This effect is due to the exponential term of the tunnel probability. Using equation A.8, the tunnel probability, along with the \(1/E\) dependence from the deBroglie wavelength, can be factored out, leaving just \(S(E)\). If there are no excited states in the
Figure A.1. **Top:** The cross section and S-Factor as a function of energy for some charge-particle induced reaction. This reaction, as shown here, is governed by two different components. At low energies it is determined by a direct capture component, while at higher energies it is determined by a resonance capture component. **Bottom:** A schematic sketch of a nuclear potential well plus coulomb barrier, along with an excited state located at $E_{ex}$ with a width $\Gamma_{tot}$. For an initial particle energy $E_1$, the reaction will proceed through non-resonant (or direct) capture, while for $E_2$, the reaction proceed via resonance capture.
compound nucleus around the energy corresponding to the incoming energy of the particle (labeled as $E_1$ in Figure. A.1), then the S-Factor will vary smoothly as a function of energy. This type of behavior in the S-factor can be characterized as a direct (or non-resonant) capture.

Given that the S-factor varies very little over this lower energy range, it is sometimes possible to approximate this S-factor as a constant, such that $S(E) \approx S(E_0)$ (represent as the dashed red line in Figure. A.1). Using this approximation allows for a more reliable extrapolation down to energies where the measurements of the reaction cross section are not feasible, as illustrated with the red dashed line.

A.2.2 Resonance Capture

For the case of non-resonance reactions, the probability of the two nuclei combining (once past the coulomb barrier) was assumed to vary slowly with the center of mass energy. Now suppose that the particle has penetrated the coulomb barrier at the exact energy corresponding to an excited state in the compound nucleus that is formed (labeled as $E_2$ in Figure. A.1). If this exited state shares similar quantum numbers to that of the combining incoming system (known as the entrance channel), then this state will act as a doorway to the compound nucleus formation, thus substantially increasing the probability of the reaction to occur, as seen in the cross section and S-factor plotted in Figure. A.1. These sharp increases in the cross section are referred to as resonances within the reaction, and occur at center-of-mass energies corresponding to $E_r = E_{ex} - Q$, where $E_{ex}$ is the energy of the excited state and $Q$ is the Q-value of the reaction. In the case illustrated in Figure. A.1 this resonance occurs at $E_2$.

In the case of a resonance, the cross section as a function of energy can be described
using the Breit-Wigner formula,

\[ \sigma(E) = \pi \lambda^2 \frac{(2J + 1)(1 + \delta_{12})}{(2J_1 + 1)(2J_2 + 1)} \frac{\Gamma_a \Gamma_b}{(E - E_r)^2 + \left(\Gamma_{\text{tot}}/2\right)^2}. \]  

(A.9)

Here, \( J \) and \( E_r \) are the spin and energy of the resonances, respectively. \( J_1 \) and \( J_2 \) are the spins of the two particles, in this case projectile and target and \( \lambda \) is de Broglie wavelength. Finally, \( \Gamma_a \) and \( \Gamma_b \) are referred to as the partial widths of the incoming and outgoing channels, and \( \Gamma_{\text{tot}} \) is the total width of the resonance, taken as the sum of the widths of each channel

\[ \Gamma_{\text{tot}} = \sum_i \Gamma_i = \sum_i \frac{\hbar}{\tau_i}, \]  

(A.10)

Here, the widths \( \Gamma_i \) are related to the mean lifetime \( \tau_i \) of the state to some emission process \( i \) by the Heisenberg uncertainty principle \( \Gamma_i = \hbar/\tau_i \).

A.3 Non-Resonances Reaction and The Gamow Window Approximation

At low bombarding energies, far away from possible reaction resonances, the astrophysical S-factor is a slowly varying function and therefore can be approximated to be constant over the energy range considered, \( S(E) \approx S(E_0) \). Using this approximation, plugging equation A.8 into A.7 becomes,

\[ \langle \sigma v \rangle_{ij} = \left( \frac{8}{\pi \mu_{ij}} \right)^{1/2} \frac{1}{(kT)^{3/2}} S(E_0) \int_0^\infty e^{\frac{-E}{kT}} e^{-\frac{b}{E^{1/2}}} dE. \]  

(A.11)

where \( b = \pi e^2 Z_i Z_j \sqrt{2\mu_{ij}}/\hbar \). Within the integrand, two competing functions combine to determine the energy range that is most likely to contribute to the reaction rate per particle pair. This is illustrated in Figure. A.2, where the Gamow factor term approaches zero for small energies, while the \( e^{-E/kT} \) factor from the Maxwell-Boltzmann distribution approaches zero for large energies. By taking the first derivative of the
Figure A.2. This figure was taken, and modified, from [42]. **Left:** The Maxwell-Boltzmann factor (dashed line) and the Gamow factor (dot-dashed line) plotted, along with their product (solid line), as a function of center-of-mass energy for the $^{12}$C($\alpha$, $\gamma$)$^{16}$O at $T = 2 \times 10^8$ K. **Right:** The Gamow peak plotted on a linear scale (solid line), along with a scaled Gaussian distribution (dashed line) to illustrate the similarities.

Integrand with respect to $E$ and setting it equal to zero, the maximum of the integrand can be found, and the energy at which most of the reactions will occur, $E_0$ can be determined. This result of this operation yields,

$$E_0 = \left( \frac{\mu_{ij}}{2} \right)^{1/3} \left( kT \right)^{2/3}.$$  \hspace{1cm} (A.12)

$E_0$ is referred to as the Gamow energy and represents the most effective energy at which non-resonant reactions occur for a given temperature $T$.

From Figure. A.2, it is clear that the shape of the Gamow peak can be approximated using a Gaussian function. Following [42], the Gamow peak can be approximated in Gaussian form as

$$\exp \left( -\frac{E}{kT} - \frac{b}{E^{1/2}} \right) \approx \exp \left( -3E_0 \right) \exp \left[ - \left( \frac{E - E_0}{\Delta/2} \right)^2 \right].$$  \hspace{1cm} (A.13)
where, $\Delta$ is the effective width, taken as

$$\Delta = \frac{4}{3^{1/2}} \left( E_0 kT \right)^{1/2}. \quad (A.14)$$

$\Delta$ is referred to as the Gamow window and represents the energy range wherein most of the non-resonant reactions take place. For most thermonuclear reactions, the Gamow energy ($E_0$) and window ($\Delta$) can be found using equations A.12 and A.14. This Gamow window approximation is an important tool as it allows someone to identify the energy range in the cross section that is most influential to stellar burning. It should be noted, as pointed out by Rauscher et al.[75], that this derivation of the Gamow window implicitly assumes that the energy dependence of the S-factor in equation A.8 is dominated by the direct capture component. If there are any resonances, then this might change the energy range of the effective Gamow window according to the energy dependence of the averaged widths of the resonances.

The non-resonance reaction rate per pair can be calculated by substituting equation A.13 into equation A.11 yielding

$$\langle \sigma v \rangle_{ij} \approx \left( \frac{8}{\pi \mu_{ij}} \right)^{1/2} \frac{S(E_0)}{(kT)^{3/2}} \exp \left( -\frac{3E_0}{kT} \right) \int_0^\infty \exp \left[ -\left( \frac{E - E_0}{\Delta/2} \right)^2 \right]. \quad (A.15)$$

As shown in [42], the lower integration limit can be extended to $-\infty$ without introducing significant error. Therefore, the integral over the Gaussian becomes $\sqrt{\pi} \Delta/2$, and the reaction rate can be written as

$$\langle \sigma v \rangle_{ij} \approx \left( \frac{2}{\mu_{ij}} \right)^{1/2} \frac{S(E_0)}{(kT)^{3/2}} \exp \left( -\frac{3E_0}{kT} \right). \quad (A.16)$$

Using equation A.16, the reaction rate per particle pair can be calculated as long as the astrophysical S-factor is known, and it is slowly varying such that it can be approximated by $S(E) = S(E_0)$. For further discussions on non-resonant reaction
A.4 Reactions through Narrow and Isolated Resonances

Considering the case illustrated in Figure A.1 where a capture reaction occurs at the same center of mass energy corresponding to a single isolated excited state in the compound nucleus. This state acts as a resonance in the reaction, and in turn the cross section is enhanced by several orders of magnitude. As stated in section A.2.2 the cross section for this type of resonance capture can be described using equation A.9. Plugging this into equation A.7 yields the following.

\[
\langle \sigma v \rangle_{ij} = \left( \frac{8}{\pi \mu_{ij}} \right)^{1/2} \frac{1}{(kT)^{3/2}} \omega \int_{0}^{\infty} E \pi \lambda^2 \frac{\Gamma_a \Gamma_b}{(E - E_r)^2 + (\Gamma_{\text{tot}}/2)^2} e^{-E/kT} dE, \quad (A.17)
\]

where \( \omega \equiv (2J + 1)(1 + \delta_{ij})/[(2J_i + 1)(2J_j + 1)] \). This can be further simplified to

\[
\langle \sigma v \rangle_{ij} = \frac{\sqrt{2\pi \hbar^2}}{(\mu_{ij} kT)^{3/2}} \omega \int_{0}^{\infty} \frac{\Gamma_a \Gamma_b}{(E - E_r)^2 + (\Gamma_{\text{tot}}/2)^2} e^{-E/kT} dE. \quad (A.18)
\]

For sufficiently narrow resonances, the Maxwell-Boltzmann term \( e^{-E/kT} \) and the partial widths (\( \Gamma_a \) and \( \Gamma_b \)) can both be approximated as a constant over the total width of the resonance. This approximation can usually be applied if the resonance width is much smaller than the resonance energy, \( \Gamma_{\text{tot}} \ll E_{\text{res}} \). In [79] a quantitative criterion is taken as \( \Gamma_{\text{tot}}/E_{\text{res}} < 10\% \). In this case, the Maxwell-Boltzmann term along with the partial widths can be replaced by their value at \( E_{\text{res}} \),

\[
\langle \sigma v \rangle_{ij} = \frac{\sqrt{2\pi \hbar^2}}{(m_{ij} kT)^{3/2}} e^{-E/kT} \omega \frac{2\Gamma_a \Gamma_b}{\Gamma_{\text{tot}}} \int_{0}^{\infty} \frac{\Gamma_{\text{tot}}/2}{(E - E_r)^2 + (\Gamma_{\text{tot}}/2)^2} dE, \quad (A.19)
\]

and the remaining integral can be solved analytically

\[
\langle \sigma v \rangle_{ij} = \frac{\sqrt{2\pi \hbar^2}}{(m_{ij} kT)^{3/2}} e^{-E/kT} \omega \frac{2\Gamma_a \Gamma_b}{\Gamma_{\text{tot}}} \cdot \pi. \quad (A.20)
\]
Finally this can be rewritten as

\[ \langle \sigma v \rangle_{ij} = \left( \frac{2\pi}{m_{ij}kT} \right)^2 \hbar^2 e^{-E/kT} \omega \gamma, \]  

(A.21)

where, \( \gamma \equiv \Gamma_a \Gamma_b / \Gamma_{tot} \). The quantity \( \omega \gamma \) is referred to as the resonances strength and is proportional to the area under the resonance cross section, or equivalently, to the product of the maximum cross section, \( \sigma(E_{res}) = \left( \lambda_{res}^2 / \pi \right) \omega \Gamma_a \Gamma_b / \Gamma_{tot}^2 \), and the total width, \( \Gamma_{tot} \).

If several narrow isolated resonances are contributing to the cross section, then their contributions can be added incoherently, such that

\[ \langle \sigma v \rangle_{ij} = \left( \frac{2\pi}{m_{ij}kT} \right)^2 \hbar^2 \sum_i (\omega \gamma)_i e^{-E_i/kT}, \]  

(A.22)

where, \( E_i \) and \( (\omega \gamma)_i \) are the resonance energies and strengths of each individual resonance being summed over.

A.5 Reactions through Many Resonances: the Statistical Model

As discussed in section. A.2.2, and subsequently through A.4, when a reaction occurs at a particular energy corresponding to an excited state (or level) in the compound nucleus, this state can act as resonance in the reaction, thus strongly enhancing the cross section at a particular energy \( E_r \). Now consider the case where the number of states \( a \) at a given bombarding energy in the compound nucleus is increased, each one participating as a resonances in the reaction. As the number of resonances increases the reaction cross section becomes ever more complicated. Eventually the number of states in the compound nucleus becomes so high that the reaction mechanism can be treated as a statistical process and averages quantities can begin to be used in describing the reaction cross section. This is the basis for
Hauser-Feshbach theory, which calculates an energy averaged cross sections using average transmission functions.

Given a fusion reaction \(A + a\) proceeds through many resonances of spin \(J\) in the compound nucleus \(C^*\), the cross section can be taken as a sum over these resonances using the Breit-Wigner formula (equation A.9). The energy averaged fusion cross section, if taken over a small energy window \(\Delta\), can be written such that

\[
\langle \sigma_J^a(E) \rangle = \omega_J^a \frac{\pi}{k^2} \sum_i \frac{\Gamma_{a,i} \Gamma_{\text{tot},i}}{(E - E_i)^2 + (\Gamma_{\text{tot}}/2)^2} dE = \omega_J^a \frac{\pi}{k^2} \frac{2\pi}{\Delta} \sum_i \Gamma_{a,i}. \tag{A.23}
\]

Here, the deBroglie wavelength is written in terms of the k vector \(k = 2\pi/\lambda\). The index \(a\) represents the incoming formation channel, and indices \(i\) represent the resonances participating in the reaction. \(\omega_J^a\) is the statistical factor given as \((2J + 1)(1 + \delta_{aA})/[(2J_a + 1)(2J_A + 1)]\). Furthermore, \(\Gamma_{a,i}\) is the partial width for channel \(a\), and \(\Gamma_{\text{tot},i}\) is the total width of each resonance.

Given that the number of levels within this energy region is \(\Delta/D\), where \(D\) is the average level spacing, a mean partial width can be defined as

\[
\langle \Gamma_a \rangle = \frac{D}{\Delta} \sum_i \Gamma_a^i. \tag{A.24}
\]

Now using equation A.24, the averaged fusion or compound nucleus formation cross section can then be simplified such that

\[
\langle \sigma_J^a(E) \rangle = \frac{\omega_J^a \Gamma_a}{k^2} \frac{2\pi}{\Delta} \frac{\langle \Gamma_a \rangle}{D}. \tag{A.25}
\]

Here, the quantity \(\langle \Gamma_a \rangle / D\) is known as the strength function. Now similarly, the optical model can also be used to calculate energy-averaged reaction cross sections that are calculated using transmission coefficients based on absorption in a complex
potential. This scan be written as,

\[ \langle \sigma^J_a(E) \rangle = \frac{\omega_{\sigma}^J \pi}{k_a^2} T^J_a. \]  

(A.26)

Comparison between equations A.25 and A.26, reveals a crucial aspect in Hauser-Feshbach theory, which is that in the formation of the compound nucleus, the strength function, \( \langle \Gamma_a \rangle / D \), can be coupled to an optical model transmission coefficient,

\[ T^J_a = 2\pi \frac{\langle \Gamma_a \rangle}{D}. \]  

(A.27)

This is a key aspect of Hauser-Fesbach theory, if the level density if high enough in the compound nucleus, either with many overlapping resonances and many many narrow isolated resonances, then the energy averaged fusion cross section can be calculated by solving for transmission coefficients for some given nuclear optical model potential (using equation A.26).

The results of equation A.26 can be taken further, and the cross section for some compound reaction, say \( A(a,b)B \), that proceed through these many resonance discussed here can be determined. For simplicity, the following derivation is done with all the reaction constituents taken as spin-zero particles. Using Bohr’s independences hypothesis and the principle of detailed balance, the partial cross section for compound nucleus formation of some spin \( J \) through channel \( a \), and subsequent decay through channel \( b \) can be written as

\[ \sigma^J_{a\rightarrow b}(E) = \sigma^J_{F, a}(E) G^J_b(E) = \sigma^J_{F, a}(E) \frac{k_b^2 \sigma^J_{F, b}(E)}{\sum_c k_c^2 \sigma^J_{F, c}(E)}. \]  

(A.28)

Here the index \( \sigma_{F,i} \) represent the fusion cross section for some channel \( i \), \( G^J_b(E) \) represent the decay probability of the compound nucleus through channel \( b \). Using the principle of detailed balance, \( G^J_b(E) \) was rewritten as \( k_b^2 \sigma^J_{F, b}(E) / \sum_c k_c^2 \sigma^J_{F, c}(E) \).
where the numerator is the fusion cross section for channel \( b \) and the denominator is sum of fusion cross-section for all open channels \( c \). Now taking an energy average of this partial cross section yield the following result,

\[
\langle \sigma^J_{a \rightarrow b}(E) \rangle = k_b^2 \left\langle \frac{\sigma^J_{F:a}(E)\sigma^J_{F:b}(E)}{\sum_c k_c^2 \sigma^J_{F:c}(E)} \right\rangle = k_b^2 \frac{\left\langle \sigma^J_{F:a}(E) \right\rangle \left\langle \sigma^J_{F:b}(E) \right\rangle}{\sum_c k_c^2 \left\langle \sigma^J_{F:c}(E) \right\rangle} W_{ab} \quad (A.29)
\]

Here, \( W_{ab} \) is a correction factor defined as later as a fluctuation factor. Using the all important equation A.26, and summing all spins \( J \) and parities \( \Pi \), the energy-averaged cross section for the reaction \( A(a,b)B \) can be expressed as

\[
\sigma^{HF}_{a \rightarrow b}(E) = \sum_{J,\Pi} \langle \sigma^J_{a \rightarrow b}(E) \rangle = \frac{\pi}{k_a^2} \sum_{J,\Pi} (2J + 1) \frac{T_a^J T_b^J}{\sum_c T_c^J} W_{ab}. \quad (A.30)
\]

This is the Hauser-Fesbach formula of the cross section of for the reaction \( A(a,b)B \) where all the constituents are spin zero and the, this can be further genericize by summing over all the possible channel spins \( S_i \) and orbital angular momentum \( l \) couplings, \( | S_i + l \rangle \), along with summing over all possible states populated in the final nucleus \( B^* \),

\[
\sigma^{HF}_{(a:S_a,l) \rightarrow (b:S_b,l')}^{(S_a+l')} (E) = \frac{\pi}{k_a^2} \sum_{J,\Pi} \sum_{|S_a+l\rangle} (2J + 1) \frac{T_a^J \sum_{S_B+l'} \langle T_b^J \rangle}{(2I_A + 1)(2I_B + 1) \sum_c \sum_{S_c+l''} \langle T_c^J \rangle} W_{ab}. \quad (A.31)
\]

In this equation the outgoing transmission coefficients are taken as

\[
\langle T_i^J \rangle = \sum_i T_i(E_B^i, J_B, \Pi_B) \quad + \int_{E_B^n}^{E-B-Q_B} \sum_{J_B \Pi_B} T(E_B, J_B, \Pi_B) \rho(E_B, J_B, \Pi_B) dE_B, \quad (A.32)
\]

where, the first term is the sum of transmission coefficients to known states in the final nucleus \( B \) at excitation energies \( E_B^i \). Once the last known excitation energy in \( B \) is reached, \( E_B^n \), an energy averaged transmission coefficient times the level density
at an excitation energy $\bar{E}_B$ is integrated from $E_B^n$ all the way up to $E_x = E - Q_{b,B}$ (where $Q_{b,B}$ is the separation energy in channel $b$).

When using a model to predict cross sections and subsequently reaction rates, Hauser-Feshbach theory and equation A.31 can prove to be a powerful tool. As long as the criterion of high level densities in the compound nucleus is met, the cross section can be described as an average over many resonances, and can be determined using transmission coefficients calculated via absorption by an imaginary part in the potential using optical models. With this use of transmission coefficients to obtain cross section predictions, only a handful of ingredients are needed, such as nuclear masses, global optical model parameters, level densities, and photon-strength functions. With this advantage of very few input parameters, stellar modelers usually employ Hauser-Feshbach predicted rates in their reaction rate networks when little-to-no information is known for a particular reaction. Generally, the use of Hauser-Feshbach theory to predict astrophysical reaction rates is acceptable for capture reaction on intermediate and heavy mass nuclei, while for light nuclei the level density might fall below the critical value where the statistical approach of Hauser-Feshbach is no longer valid. In the cases of lower level densities in the compound nucleus, if no information on the reaction exist, then resonances or direct capture calculations (covered in sections A.2.2 and A.2.1) utilizing parameters determined by the shell model might be more applicable.
APPENDIX B

REACTION KINEMATICS

The purpose of this appendix was to present the necessary kinematic equations needed in other chapters of this work. With an incoming proton beam energy of 100 MeV ($\beta = v/c = 43\%$), relativistic effects must be taken into account if various kinematic quantities are to be calculated accurately (such as excited states in recoil nuclei or momenta of reaction products). To simplify some of these kinematic calculations, a brief introduction of the momentum four vector is given, then the necessary relativistic kinematic equations are derived using this four vector formalism. As stated before only equations needed for this work will be derived and this is in no way a review. For a more thorough review of relativistic kinematics with the use of four vectors, please see [9, 78].

B.1 The Momentum Four Vector

In special relativity the energy and momentum of a particle can be expressed in four vector form, known as the momentum four vector

$$\vec{P} = (p_0, p_1, p_2, p_3) = \left( \frac{E}{c}, \vec{p} \right).$$

(B.1)

Here, $E/c$ is the time-like component, $p_0$, with $E$ being the relativistic energy of the particle, and $c$, the speed of light; and the space-like component is $\vec{p}$, the momentum three vector.
This momentum four-vector is a Lorentz vector and therefore a Lorentz invariant, \( s \), can be calculated by taking the scalar product of two four-vectors,

\[
s_{ij} = \vec{P}_i \cdot \vec{P}_j = \frac{-E_i E_j}{c^2} + \vec{p}_i \cdot \vec{p}_j,
\]

(B.2)

and for these two four-vectors, this Lorentz invariant, \( s_{ij} \), is the same in all inertial reference frames. For \( P_i = P_j \), calculating the Lorentz invariant becomes

\[
s_{ii} = \vec{P}_i \cdot \vec{P}_i = \frac{-E_i^2}{c^2} + p_i^2.
\]

(B.3)

Using the definition for total relativistic energy, \( E = \gamma mc^2 \), and relativistic momentum, \( p = \gamma mv \), equation B.3 can be modified to

\[
s_{ii} = -(\gamma m c)^2 + (\gamma m v)^2 = -\gamma^2 m_i^2 c^2 \left( 1 - \frac{v^2}{c^2} \right) = -m_i^2 c^2,
\]

(B.4)

where, \( \gamma = 1/\sqrt{1 - v^2/c^2} \) is the Lorentz factor. Equation B.4 shows that the quantity \(-m_i^2 c^2 \) is the Lorentz invariant for a given momentum four vector \( \vec{P}_i \). From equations B.3 and B.4, the familiar momentum-energy relation in special relativity is found,

\[
s_{ii} = \frac{-E_i^2}{c^2} + p_i^2 = -m_i^2 c^2 \rightarrow E_i^2 = (p_i c)^2 + (m_i c^2)^2
\]

(B.5)

Another important relation is the summing of four vectors, which is written as

\[
\vec{P}_i + \vec{P}_j = \left( \frac{E_i + E_j}{c}, \vec{p}_i + \vec{p}_j \right),
\]

(B.6)

and the Lorentz invariant can be calculated as

\[
s_{i+j} = (\vec{P}_i + \vec{P}_j)^2 = \left( \frac{E_i}{c} + \frac{E_j}{c} \right)^2 + (\vec{p}_i + \vec{p}_j)^2.
\]

(B.7)
Equation B.8 can further be expanded to

\[ s_{i+j} = (\vec{P}_i + \vec{P}_j)^2 = -(m_i c)^2 - (m_j c)^2 - \frac{E_i E_j}{c^2} + 2 \vec{p}_i \cdot \vec{p}_j. \]  

(B.8)

Finally a more general form of equation B.8 can be written as

\[ s = \left( \sum_i \vec{P}_i \right)^2 = - \left( \sum_i \frac{E_i}{c} \right)^2 + \left( \sum_i \vec{p}_i \right)^2. \]  

(B.9)

B.2 Relativistic Kinematics and Conservation of the Momentum Four-Vector

In Figure. B.1 is a generalized example of a reaction depicted in the lab frame. Here a collision between a projectile (\(a\)) and a stationary target nucleus (\(A\)) go on to produce a recoil nucleus (\(B\)) emitted at an angle \(\phi\) and an ejectile (\(b\)) emitted at an angle \(\theta\).

The kinematics of this reactions are mainly determined by two conservation laws: the conservation of energy and the conservation of linear momentum. These two conservation laws can be neatly wrapped up in the conservation of the total momentum four vector. Using the notation from Figure. B.1 (\(a + A \rightarrow B + b\)), the conservation of the momentum four vector can be written as

\[ \vec{P}_a + \vec{P}_A = \vec{P}_B + \vec{P}_b. \]  

(B.10)

For this reaction, the momentum four vectors for the beam, target, and reaction...
Figure B.1. Reaction $a + A \rightarrow B + b$ as observed in the lab reference frame. Here, a projectile ($a$) collides with a stationary target nucleus ($A$), producing a recoil nucleus ($B$) and an ejectile ($b$) emitted at an angles $\phi$ and $\theta$, respectively.

Products are given as

\[ \vec{P}_a \equiv (E_a, \vec{p}_a) \]
\[ \vec{P}_A \equiv (m_A c, 0) \]
\[ \vec{P}_B \equiv (E_B, \vec{p}_B) \]
\[ \vec{P}_b \equiv (E_b, \vec{p}_b). \]  

(B.11)

Usually during a typical nuclear experiment the recoil goes undetected, meaning that initially, information on $\vec{p}_B$ is not known, making it difficult to use equations B.10 and B.11 to reconstruct any kinematic information given the general form they are in. To get around this and eliminate the need to know $\vec{p}_B$, equation B.10 can be manipulated such that

\[ (\vec{P}_B)^2 = (\vec{P}_a + \vec{P}_A - \vec{P}_b)^2. \]  

(B.12)
The right hand side of B.12 is the Lorentz invariant for \( \vec{P}_B \), where \( \vec{P}_B^2 = -(m_B^*c)^2 \).

Using equation B.9, the left hand side can be expanded to

\[
(m_B^*c)^2 = (m_a^2c^2)^2 + (m_A^2c^2)^2 + (m_b^2c^2)^2 + 2E_a m_A c^2 \\
- 2E_a E_b - 2m_A c^2 E_b + 2\vec{p}_a \cdot \vec{p}_b c^2.
\]

(B.13)

Here, \( m_B^*c^2 \) is the mass of the recoil in a given excited state \( (m_Bc^2 + Ex_B) \), \( E_a \) and \( p_a \) are the total energy and momentum of the beam, and \( E_b \) and \( p_b \) are the total energy and momentum of the ejectile. Using equation B.5 and the dot product relation between two vectors \( (\vec{p}_b \cdot \vec{p}_b = p_a p_b \cos \theta_{ab}) \), equation B.13 can be rewritten as

\[
(m_Bc^2 + Ex_B)^2 = (m_a^2c^2)^2 + (m_A^2c^2)^2 + (m_b^2c^2)^2 + 2E_a m_A c^2 \\
- 2(E_a + m_A c^2)\sqrt{(m_b^2c^2)^2 + (p_b c)^2} \\
+ 2p_b c \sqrt{E_a^2 - (m_a^2c^2)^2 \cos \theta}.
\]

(B.14)

Examining equation B.14, given that the masses and the energy of the beam are all well known, the three unknown quantities remain, the momentum of the ejectile \( (p_b) \), the angle at which it’s emitted in the reaction plane \( (\theta) \), and the excited state in the recoil nucleus that was populated during the reaction \( (Ex_B) \). By setting one of these three quantities, then measuring another, the final quantity can be calculated using equation B.14 (heavy algebra notwithstanding). The easiest case (least amount of algebra) is to measure the energy (momentum) of the ejectile at a particular angle (set \( \theta \) and measure \( p_b \)), then use equation B.14 to solve for the excited state that was populated in the recoil nucleus.

B.3 Including Target Thickness Effects

As seen in Figure. B.1, target thickness effects will play a role in determining the incoming energy of the beam, along with the observed energy of the outgoing
ejectile. Realistically, the energy of the incoming beam will loss some amount of energy in the target before it reacts and the ejectile with also loss some amount of energy as it exits the target and is subsequently detected. Therefore certain terms in equation B.14 have to be adjusted to account for these energy loss effects. The total energy of the beam, \( E_a \), should be taken as

\[
E_a = m_a c^2 + T_a - \Delta E_a, \quad \text{(B.15)}
\]

where \( \Delta E_a \) is the energy loss of the beam in the target before the reaction. Additionally, the momentum (or energy) of the ejectile should be written as

\[
p_b c = \sqrt{(m_b c^2 + T'_b + \Delta E_b)^2 - (m_b c^2)^2}, \quad \text{(B.16)}
\]

where \( \Delta E_b \) is the energy loss of the ejectile in the target after the reaction, and \( T'_b \) is the energy of the ejectile observed in the focal plane detector. By plugging equations B.15 and B.16 into equations B.14 the excitation energy of a state populated in the recoil nucleus can be determined more accurately.

All energy loss terms for the incoming proton and the outgoing tritons in the various targets were taken from the sofware program SRIM[103], which calculates the stopping power of ions in matter.
APPENDIX C

IDENTIFYING STATES IN $^{38}$Ca AT $\theta_{\text{LAB}} = -1.2^\circ$ AND $8^\circ$

As stated in section 2.9, focal plane spectra of $^{38}$Ca were taken at four different experimental settings (two magnetic field setting and two lab angles). With the use of an overlapping technique, these four settings allowed us to possibly identify excited states in $^{38}$Ca up to 15 MeV at two different angles. In this appendix, the process of calculating the excitation energy of the final states given multiple measurements at two angle -1.2° and 8°, and magnetic field settings (B1 and B2).

C.1 Calculating the Weighted Mean

Once peaks were fitted in a focal plane spectrum for a given experimental setting and their centroids determined, the excitation energy in $^{38}$Ca and it’s corresponding error were determined using equations 3.12 and B.14. In the case that a peak is observed in multiple focal plane spectra, its final excitation energy is given as the weighted mean of the individual values observed at each focal plane setting. This can be written as

$$E_f = \frac{\sum_i E_i/\sigma_i^2}{\sum_i 1/\sigma_i^2},$$
\hspace{1cm} (C.1)

where $E_i$ is the excitation energy of the state identified at a given experimental setting, and $\sigma_i$ is its corresponding uncertainty. The final error is then given as the error of the weighted mean.

$$\sigma_f^2 = \frac{1}{\sum_i 1/\sigma_i^2}.$$  
\hspace{1cm} (C.2)
States identified in focal plane spectra at each experimental setting, along with the calculated weighted mean and error is given in table. C.1. The header of table C.1 includes the scattering angle and magnetic field setting used to obtain each particular focal plane spectrum. In addition, the scaling ratios used in equation 3.12 are listed in each header, along with the states used to find each ratio, denoted by their respective superscript. In total, all states listed in the final weighted mean column were observed at both angles, with the exception of 13 states that were only observed at one angle, but were included in final results because they either displayed the same kinematic shift over the horizontal angle acceptance of the K=600 spectrograph at $\theta_{lab} = 8^\circ \ (\pm 2.5^\circ)$ as observed for other $^{38}\text{Ca}$ states, or they matched $^{38}\text{Ca}$ states seen in previous works. These states are displayed with a * in table. C.1.
### TABLE C.1

**LIST OF STATES IN $^{38}$CA GIVEN EACH EXPERIMENTAL SETTING**

<table>
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<tr>
<th>$\theta_{lab} = -1.2$</th>
<th>$\theta_{lab} = 8$</th>
<th>$\theta_{lab} = -1.4$</th>
<th>$\theta_{lab} = 8$</th>
<th><strong>Final</strong></th>
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<tbody>
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<td>B2</td>
<td>B1</td>
<td>B2</td>
<td><strong>Weighted</strong></td>
</tr>
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<td>$f = 0.998425^a$</td>
<td>$f = 0.99997^b$</td>
<td>$f = 0.95879^c$</td>
<td>$f = 0.95935^d$</td>
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</tr>
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\hline
B1 & B2 & B1 & B2 & Weighted \\
\hline
f = 0.998425$^a$ & f = 0.99997$^b$ & f = 0.95879$^c$ & f = 0.95935$^d$ & Mean \\
(keV) & (keV) & (keV) & (keV) & (keV) \\
\hline
10121(22) & 10101(10) & 10104(9) & & \\
10414(19) & 10409(10) & 10410(9) & & \\
10563(15) & 10555(10) & 10557(8) & & \\
10946(11) & 10946(11) & 10946(11)$^\ast$ & & \\
11089(11) & 11089(11)$^\ast$ & & & \\
11189(13) & 11189(13)$^\ast$ & & & \\
11500(12) & 11500(12)$^\ast$ & & & \\
11861(11) & 11861(11)$^\ast$ & & & \\
\hline
\end{tabular}
\end{table}
SELECTING SPINS FROM FBGM MODEL DISTRIBUTIONS

Unfortunately, little-to-no information is known for the spins of levels identified above the alpha threshold in $^{38}$Ca in this work (See Chapter 4). Therefore, to calculate the $^{34}$Ar($\alpha$,p)$^{37}$K stellar reaction rate, certain models must be utilized to fill in this missing information.

We utilize the spin distribution function taken from the Fermi Back-Shifted Gas Model (FBGM). This model describes the level density in a nucleus based on treating the constituent nucleons as a system of weakly interaction fermions. In this framework the level density can be factorized into three functions based on excitation energy, spin, and parity,

$$\rho(E_{ex}, J, \Pi) = P(E_{ex}, J, \Pi)R(E_{ex}, J)\rho^{tot}(E_{ex}).$$  \hspace{1cm} (D.1)

Here, corresponds to the total number of levels per MeV, independent of spin and parity, around the excitation energy, $\rho^{tot}(E_{ex})$. Additionally, the distribution of a particular spin and parity around $E_{ex}$ can be found by multiplying the total level distribution by the spin distribution, $R(E_{ex}, J)$, and the parity distribution, $P(E_{ex}, J, \Pi)$.

In this appendix we will only focus on the spin distribution function $R(E_{ex}, J)$, for a detail review on the Fermi Back-Shifted Gas Model and how it is used to calculate level density please see [17].
D.1 Level Distributions as a function of Excitation Energy and Spin

In order to determine the probability of a level of a particular spin, $J$, being at a given excitation energy, $E_{ex}$ within the FBGM, the framework laid out in the TALYS 1.6 user manual is adopted. Starting with the overall spin distribution function,

$$R(E_{ex}, J) = \frac{2J + 1}{2\sigma^2} \exp \left[ -\frac{(J + 1/2)^2}{2\sigma^2} \right]. \quad \text{(D.2)}$$

Here, $\sigma$ is referred to as the spin cut-off parameter, which is a function of $E_{ex}$ and is given as

$$\sigma^2(E_{ex}) = (0.83A^{0.26})^2 + \frac{E_{ex} - E_d}{S_n - E_d} \left( 0.01389 \frac{A^{5/3}}{\tilde{a}} \sqrt{a(E_{ex})U} - (0.83A^{0.26})^2 \right). \quad \text{(D.3)}$$

Here, $A$ is the mass number, $a$ is the level density parameter (while $\tilde{a}$ is its asymptotic value taken at the neutron separation energy, $S_n$), $U$ is the shifted excitation energy ($U = E_{ex} - \Delta$), and $E_d$ is the energy in the middle of the discrete level region. The level density parameter $a(E_{ex})$ is taken as,

$$a(E_{ex}) = \tilde{a} \left( 1 + \delta W \frac{1 - \exp[-\gamma U]}{U} \right), \quad \text{(D.4)}$$

where $\delta W$ is the correction factor for shell effects, and $\gamma$ is the damping parameter that determine how fast $a(E_{ex})$ approaches $\tilde{a}$. All parameters listed in equations D.3 and D.4 were taken directly from the level density files included in TALYS 1.8, and subsequent spin distributions were then calculated using equation D.2. The spin distributions selected excitation energies are plotted in Figure. D.1.

In order to sample of the spin distribution for a given energy during a Monte Carlo reaction rate calculation, the Rejection method is utilized.
Figure D.1. Using equation D.2, distribution of spins for selected excitation energy in $^{38}$Ca are plotted. These spin distributions are based off the Fermi Back-Shifted Gas Model for level densities.

D.2 The Rejection Method

The rejection method is robust technique based on a simple geometrical argument for generating random deviates $x$, given some user defined probability function $P(x)$. Given some probability function, a two dimensional space is created around the function such that one dimension corresponds to the deviates within the range to be generated, while the other is independent, but it must contain the max and min values of $P(x)$. In the case of Figure. D.2, the deviants are spins, $J$, ranging from 0 to 6, the second deviants is arbitrary (labeled as $Y$), and the distribution that is being sampled is $P(J)$.

First a $J_i$ value is randomly sampled from a uniform distribution ranging from 0 to 6. Additionally, a $Y_i$ value is randomly sampled from a uniform distribution ranging from 0 to 0.5. If the point $(J_i, Y_i)$ falls within the user defined distribution, meaning $Y_i < P(J_i)$ then $J_i$ is accepted into sample (as seen with $J_1$ and $J_3$ in Figure.
Figure D.2. Diagram depicting the general framework of the Rejection Method, given three selected points. Here, $J_1$ and $J_3$ are accepted into the final sample, as both $Y_1$ and $Y_3$ are within the probability distribution $P(J)$, while $J_2$ is rejected from the final sample since $Y_2$ falls above $P(J_2)$. In this manner, by sampling $n$ number of $(J_i, Y_i)$ points and comparing it to the distribution, $P(J)$, the sample takes on the same distribution.
But if \((J_i, Y_i)\) falls outside the distribution, \(Y_i > P(J_i)\), then \(J_i\) is rejected from the sample and the process starts over again (as seen with \(J_2\) in Figure. D.2).

Through this process, sampling points \((J_i, Y_i)\) made up of deviants from two uniform distributions and only accepting the points that fall within the probability function \(P(J)\), the accepted \(J\) values will have the desired distribution \(P(J)\).

It should be noted that the efficiency of this method depends critically on the area taken around the distribution function. The smaller the area around the distribution function the smaller the probability of points being rejected for a given sample. For more information on the rejection method please see [74].
APPENDIX E

CALCULATING SINGLE-PARTICLE WIDTHS

Single-particle widths are calculated using the BIND subroutine in the Zero Range Distorted Wave Born Approximation (DWBA) code DWUCK4 [52]. This subroutine constructs a single-particle radial wave function \( u(r) \) based on the solution of the Schrödinger equation for a real potential and a given set of quantum numbers, \( J \), \( n \), and \( l \). Here, \( J \) is the spin of the resonances, \( n \) is the number of nodes in the radial wave function, and \( l \) is the orbital angular momentum. Essentially this program constructs a radial wave function \( u(r) \) corresponding to a resonance by setting the phase shift to \( \delta = \pi/2 \), matches logarithmic derivatives of this radial wave functions at some boundary by adjusting the potentials strength, and then the integrates from the origin out to some cut off radius where the nuclear potential can essentially can be set to zero. The single-particle particle width can then be determined by

\[
\frac{2}{\Gamma_{sp}} \approx \frac{2\mu_{ij}}{\hbar^2 k} \left[ \int_0^{R_{max}} |u(r)|^2 \, dr + \Delta_s \right], \tag{E.1}
\]

where, \( \mu_{ij} \) is the reduced mass, \( k \) is the wave number, and \( \Delta_s \) is a surface correction term to the volume integral. For a more detailed rewview of the BIND subroutine with the DWUCK4 code please see Appendix A. of [41].

Using DWUCK4, \( \alpha \) and proton single-particle width are calculated up to spin \( J = 4 \). Before each spin calculation, the set of quantum numbers \((J, n, l)\) used to constructs a single-particle radial wave function \( u(r) \) needs to be determined.
E.1 Determining Quantum Numbers

For calculating proton single-particle widths, $n$ and $l$ for a given $J$ are determined using the shell model. In this case, by taking into account the number of protons in the core ($^{37}$K), it is possible to determine the particular orbital ($s, p, d, f, ...$) that the valence proton needs to be in to contribute the desired orbital angular momentum to reach the assumed $J$ value. Once the proton orbitals are matched to their respective resonance spins, $n$ and $l$ values for that state are assigned based on the particular orbital and the appropriate proton single-particle width can be calculated using the DWUCK4 code.

Additionally, for calculating $\alpha$ single-particle widths, $n$ and $l$ for a given $J$ are determined using a similar prescription is given by Mohr et al. [63]. Here, the quantum numbers, $n$ and $l$, can be determined using the Wildermuth condition,

$$\sum_{i} 2n_i + l_i = Q = 2N + L. \quad (E.2)$$

Here $Q$ is the number summed of oscillator quanta, $N$ and $L$ are the number of nodes and angular momentum of the formed $\alpha$-particle, while $n_i$ and $l_i$ are the number of nodes and angular momentum of the constituent nucleons the make up the $\alpha$-particle. For the case of $^{34}$Ar + $\alpha$, the summed oscillator quanta, $Q$, is either 8 for even $J$ or 9 for odd $J$. Given that both $^{34}$Ar and $\alpha$ have $0^+$ ground states the $J$ is determined solely by angular momentum of the formed $\alpha$-particle, $L$. Once $Q$, $N$, and $L$ are determined the appropriate $\alpha$ single-particle width can be calculated using the DWUCK4 code.

E.2 Proton Single Particle Widths

For each state identified in this work, proton single particle widths are calculated for spin values ranging from $J = 0$ to $J = 4$. All five proton single particle width
values, along with the respective quantum numbers, are listed in table E.1 for each state. The quantum numbers are displayed as \((n, J, l)\).

**TABLE E.1**

**LIST OF PROTON SINGLE PARTICLE WIDTHS**

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<th>J = 3</th>
<th>J = 4</th>
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### TABLE E.1

Continued

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#### E.3 Alpha Single Particle Widths

For each state identified in this work, \( \alpha \)-single particle widths are calculated for spin values ranging from \( J = 0 \) to \( J = 4 \). All five \( \alpha \)-single particle width values,
along with the respective quantum numbers, are listed in table E.2 for each state. The quantum numbers are displayed as \((n, J, l)\).

**TABLE E.2**

**LIST OF \(\alpha\) SINGLE PARTICLE WIDTHS**

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BIBLIOGRAPHY


142


