Hidden Variables and Quantum Uncertainty
(Written Symposium, 9th Issue)

Variables cachées et indéterminisme quantique
(Symposium écrit, 9ème livraison)

Verborgene Parameter und Quanten-Unbestimmtheit
(Schriftliches Symposium, 9. Heft)

march 1976 mars

Contents

16.0 M. Mugur-Schächter - What is at Stake in the experiments on Bell's Inequality? 1
17.0 J.S. Bell - The Theory of Local Beables 11
14.3 O. Costa de Beauregard - Remarques à propos de 14.1 et 14.2 25
14.4 F. Bonsack - Réponse à 14.3 27
18.0 B. Hoffmann - Quelques remarques 28
What is at Stake in the experiments on Bell's Inequality?

The physical content of Bell's inequality is expressed directly in terms of isolation and separability, instead of locality, and the specific questions raised by this segregation are briefly investigated. This is realized via a comment on a recent similar attempt by B. d'Espagnat.

I. d'Espagnat's treatment

Concepts and assertions. - As regards the concepts it is merely assumed that the words "system", "isolated system" and "proposition" can be used in the usual way. In particular, a system is considered "isolated" if it lies arbitrarily far from or outside the light cones of all other systems.

By means of two definitions and four assumptions (D+A) a description is built up - in conditional and operational terms - for the conception according to which an isolated system possesses intrinsic and persistent properties. The definition 2 concerns an isolated system and the assumption 2 (2, pg. 1426) is a conditional proposition, asserted to be true for a system if this system is isolated.

Consequences and inequalities. - The particular experiment envisaged by Bell is then considered: a spin-zero particle decays into two particles U and V of equal spin, by a spin-conserving interaction. We denote by \( E_{U+V} \) an ensemble of \( N \) composite systems \( U+V \) all of the same type and identically prepared. It is shown that the strict spin-correlation together with the assumption that (D+A) is applicable to the U and the V of any composite system \( U+V \in E_{U+V} \), entail for \( E_{U+V} \) the well known inequality of Bell, as well as other similar inequalities.

*) D for Definition, A for Assumption, see appendix
Discussion and conclusion. - The established results are confronted with quantum mechanics, with (D+A) and with experiment. It is asserted that since quantum mechanics is in contradiction with all the demonstrated inequalities - if quantum mechanics were found true by experiment the conception according to which "when a proposition is true on a system S this constitutes an intrinsic property of S, which can be neither imparted to S nor withdrawn from S as long as S is isolated" (i.e. (D+A)) would have to be dropped (the summary). Furthermore, Einstein's principle of separability is recalled: if S₁ and S₂ have interacted in the past but are now arbitrarily far from one another "the real, factual situation of system S, does not depend on what is done with system S₂ which is spatially separated from the former". It is asserted that the truth of this principle can be decided by the same experimental tests which are able to decide on the truth of (D+A), namely those on Bell's or similar inequalities.

II. Comment

We consider the propositions:

\[ p₁ : S \text{ is isolated} \]
\[ p₂ : S \text{ possesses intrinsic persistent properties, which together with any given apparatus used for some measurements on } S, \text{ determine entirely the individual results of these measurements, independently of the state of any other system or other apparatus.} \]
\[ p₃ : \text{Bell's inequalities are verified.} \]

The notations \( x, \bar{x}, \Rightarrow, \land, \lor \) mean respectively: the proposition \( x \) is true, the proposition \( x \) is false, and the logical implication, conjunction, disjunction. When \( p₁, p₂ \) or \( p₃ \) concerns exclusively a system \( S \subseteq \{U, V\} \) where \( U \) and \( V \) are the two parts of a composite system \( U+V \subseteq EU+V \) we write respectively \( p₁(U,V), p₂(U,V) \) or \( p₃(U,V) \).

The object of study in d'Espagnat's work con-
sists in the consequences on (D+A) of the hypotheses \( \overline{p}_3(U,V) \) and the asserted conclusion seems to be \( \overline{p}_3(U,V) \Rightarrow (D+A) \). Now:

The Assumption 2 contained in (D+A) is a conditional proposition, asserted to be true for \( S \) if \( S \) is isolated. Therefore, (a) (D+A) admits the representation \( (p \Rightarrow p) \) so that the asserted conclusion can be written \( (\overline{p}_3(U,V) \Rightarrow (p \Rightarrow p)) \), and (b) each time that the Assumption 2 has been applied to an \( S \in \{U,V\} \) it has been admitted that this \( S \in \{U,V\} \) is isolated; hence the realized demonstration is \( \{ p_1(U,V) \land (p \Rightarrow p) \Rightarrow p_3(U,V) \} \), which entails any one of the following equivalent propositions:

\[
(1) \quad \overline{p}_3(U,V) \Rightarrow [p_1(U,V) \land (p \Rightarrow p_2)]
\]

or

\[
\overline{p}_3(U,V) \Rightarrow \overline{p}_1(U,V) \lor (p \Rightarrow p_2)
\]

or

\[
\overline{p}_3(U,V) \land p_1(U,V) \Rightarrow (p \Rightarrow p_2)
\]

where the argument \( (U,V) \) has to be taken into account. Consequently our comments are these:

I. - \( \overline{p}_3(U,V) \Rightarrow (p \Rightarrow p) \) follows from the realized demonstration, not as a necessity but merely as a possibility which coexists with the alternative possibilities \( (p \Rightarrow p_2) \land \overline{p}_1(U,V) \)

or \( (p \Leftrightarrow p_2) \land \overline{p}_1(U,V) \) while \( \overline{p}_3(U,V) \).

The absence of these alternative possibilities in the formulation given to the conclusion seems to indicate that it has been admitted - not as a tentative hypothesis, but as an evident fact - that \( p_1 \) is true for \( S \in \{U,V\} \). (This same assumption has been made by other authors \(^3\), \(^4\), etc.. and seems to be attributed also to Bell, who in fact remains non-committal, as it appears below). Now, in the experiments envisaged here the two apparatuses used respectively for measurements on \( U \) and on \( V \) can (in principle) be placed indeed arbitrarily "far" from one another. But whether \( U \) and \( V \) also are therefore "far" from one another is neither an a priori conceptual evidence nor an established experimental fact: this depends on the geometrical characteristics of what is denominated \( U \) and \( V \).
Future investigations might establish that $p_1$ is true indeed for $S \subseteq \{U,V\}$. Meanwhile it is acceptable to postulate this truth tentatively and to investigate on the consequences. But as long as $p_1(U,V)$ has neither been established nor explicitly postulated, the a priori elimination of the possibility $\overline{p}_1(U,V)$ is logically arbitrary.

(II) Einstein's principle of separability is essentially equivalent to $(D+A)$ hence it admits the same representation $(p_1 \Rightarrow p_2)$ and can be studied indeed by the same experimental tests. But the form (1) of the actually demonstrated conclusion and the comment (I) show that the experimental tests on Bell's (or similar) inequalities cannot suffice for deciding on the truth of $(p_1 \Rightarrow p_2)$: if the chosen aim is to obtain information about $(p_1 \Rightarrow p_2)$, such tests have to be conjugated with tests concerning independently $p_1(U,V)$; the semantics under the terms "isolated system", hence also under the word system, has to be elucidated.

To conclude, we regard (1) - not $\{\overline{p}_3(U,V) \Rightarrow (p_1 \Rightarrow p_2)\}$ - as the result actually established by d'Espagnat.

III. Locality in Bell's Sense, Isolation and Separability

On the basis of the preceding critical comments it is now easy to express the physical content of Bell's inequality directly in terms of isolation and separability:

The aim of Bell's demonstration has been to show that the statistical predictions of quantum mechanics are not compatible with the additional variables conceived in the EPR paradox, which "were to restore to the theory causality and locality". The minimal assumption sufficient for this aim is that of the possibility for $S \subseteq \{U,V\}$ of hidden variables "local" in the Bell-Wigner sense. This is equivalent to $p_2(U,V)$. Bell's theorem is $p_2(U,V) \Rightarrow p_3(U,V)$. This entails $\overline{p}_3(U,V) \Rightarrow \overline{p}_2(U,V)$, which leaves undefined the truth-values of both
p,(U,V) and (p, \implies p_2): Bell's segregation does not individualize the problems of isolation and of separability.

If however it is wanted to analyze the physical content of the hypothesis \( \overline{p}_3(U,V) \) precisely in terms of the concepts of isolation and separability, then Bell's theorem together with \( ((p, \implies p_2) \land p,(U,V)) \implies p_3(U,V) \), lead to the propositions (1). It follows that the experimental investigations on Bell's inequality - if they are done with the aim to obtain information not exclusively on locality in Bell's sense versus the truth of the quantum mechanical statistical predictions, but also on individual isolation and separability - require independent research on isolation. Thus we rejoin the remarks which constitute our critical comments on d'Espagnat's treatment.

To see this more detailedly let us examine first the available "explanations" in terms of isolation and separability, if \( p_3(U,V) \) were found to be violated always (for any type of initial decaying system, any direction and any distance):

(a) \( p,(U,V) \land (p, \implies p_2) \), i.e. what is named U and V become space-like separated objects, but their "properties" do never separate, remaining connected by non-relativistic signals,

(b) \( \overline{p}_3(U,V) \land (p, \implies p_2) \), i.e. the "parts" denominated U and V either do never separate space-like, or do not even separate spatially so that they communicate respectively by relativistic signals, or by "influences" which need not possess enough structure to be signals able to carry information so that they are not submitted to limitation of speed (Bohm and Hiley, "On the intuitive understanding of non-locality as implied by quantum theory", pg. 26-27 of the manuscript). With \( (U,V) \) it is obviously adequate to consider U+V as one single system, and a pair of a measurement on what is named U and a measurement on what is named V as a pair of two distinct but interacting tests on two different but "compatible" (opera-
tionally) qualities of U+V. Such a view raises the problem of the inner space-time organization of U+V. In particular, can a time-order be defined between interactions taking place at different space-points of U+V, and, if this can be done in some consistent way, in what does the definition consist? With respect to such questions the problem of the reduction of the quantum mechanical state vector could acquire new aspects, essential for the estimation of the hidden-variables attempts. The problem of the space-time laws interior to what is named one system (in some given theoretical segregation), falls probably outside the domain of relativistic mechanics, thus yielding a bias for interaction between quantum mechanics and the relativistic conceptions, even though quantum mechanics is a newtonian theory inasmuch as it is a mechanics. Approaches like those of Bohm, of Bohm and Hiley and of David Finkelstein seem particularly appropriate for dealing with the possibility \( p_1(U,V) \land (p_1 \Rightarrow p_2) \). \( (c) \ p_1(U,V) \land (p_1 \Rightarrow p_2) \). It would be difficult to maintain that (a) is less strange than (b) or (c) from the standpoint of the present-day knowledge. Anyhow the point is that only independent conceptual and experimental researches on the semantics under the terms isolated system and system could distinguish between (a) and (b) or (c), while (b) and (c) would still remain undistinguished (however if \( p_1(U,V) \) were established in some theoretical structure, \( (p_1 \Rightarrow p_2) \) could be postulated in that structure). We note that a null spatial extension of what is named U and V (irreconcilable with the "wave-like manifestations" of microsystems) would insure (a), while an infinite spatial extension (which would efface the concept of "system") would insure (b) or (c): the spatial extension of the objects named U and V plays a central role in the understanding of the physical content of Bell's theorem in terms of isolation and separability.

*) the quoted manuscript.
If, conversely, \( p_3(U,V) \) were found not to be violated in certain circumstances (certain types of initial decaying system, some particular directions, or beyond certain distances) then Bell's implication \( p_2(U,V) = p_3(U,V) \) would leave open the two possibilities (a') \( p_2(U,V) \) and (b') \( p_3(U,V) \) and (b') would bring into consideration the possibility \( p_1(U,V) \) as an "explanation". Then the necessity of an independent research on \( p_1(U,V) \) would arise again.

IV. Isolation and topological properties

As soon as a "system" is conceived to be "spatially separated" from all "other systems" and arbitrarily "for" from their light cones, inescapably assumptions are introduced, explicitly or implicitly, concerning topological properties of what is called a system: space-like extension and form as a function of time. For systems \( S \in \{U,V\} \) such properties might a priori depend on the type of initial decaying system used, on the duration of the process of emission, on (anisotropic) characteristics of this process, or on other conceivable parameters. If confronted with the two-systems formalism of quantum mechanics such assumptions would probably raise difficult conceptual questions (for instance, what could be the operational orthodox definition of the "spatial separation" and the "distance" between two "systems" to which do not even correspond individual wavefunctions?). Obviously such questions cannot be answered on a purely theoretical level. Therefore we call attention to the fact that this writer and two collaborators happen to have defined - in an entirely different context - a conceptual and experimental approach which could yield a first orientation concerning the topological properties of what would adequately admit the denomination of one microsystem. This approach is based on the phenomena of corpuscular interference.
Conclusion

Bell's theorem can be viewed as a confrontation between the statistical predictions of quantum mechanics, relativity, the concept of system and the concept of isolated system. When referred to this view locality in Bell's sense appears as a synthetic concept where aspects calling into play relativistic considerations and the physical significances assigned to the terms system and isolated system are melted together yielding one single opponent to the statistical predictions of quantum mechanics. Therefore the tests on Bell's inequality cannot suffice for a progress in our knowledge concerning neither the concept of individual isolation nor that of individual separability (where isolation is included as an explicit condition). Such progress - if it is researched - could be obtained only by combining the tests on Bell's inequality with independent investigations on the concept of isolation, and this would demand an inquiry on the inner space-time structure of the object to be named one microsystem. When entering upon such an investigation it seems methodologically efficient to assume that each one of the entities concerned will undergo transformation, quantum mechanics as well as relativity as well as our conceptions on micro-objects; the most fertile religion seems to be the absence of any religion.

Acknowledgments

I am indebted to Dr. D. Evrard for a very clarifying discussion. I also thank F. Thieffine and Dr. D. Evrard for having checked the manuscript.

M. Mugur-Schächter
Laboratoire de Mécanique Quantique
Faculté des Sciences, Reims
Assumption 1.

It is meaningful to associate to any proposition a defined on a type $T$ of systems a family $F(a)$ of systems $S$ of the type $T$, $F(a)$ being defined by the two following conditions: (i) The systems $S$ that belong to $F(a)$ are those and only those that are such that if $a$ were measured on $S$ by any method the result would necessarily be obtained, and (ii) the fact that a given $S$ belongs to $F(a)$ is an intrinsic property of $S$ (i.e., it does not depend on whether or not $S$ will interact with some instrument devised so as to measure $a$).

Definition 1.

Iff (if and only if) $S$ belongs to $F(a)$, $a$ is said to be true on $S$.

Definition 2.

Let a system $S$ be isolated between times $t_a$ and $t_b$. $a$ is said to be persistent on $S$ between $t_a$ and $t_b$ iff the condition that $a$ is true at time $t$, entails that it is true at time $t_2$, for any $t$, and $t_2$ satisfying

$$t_a < t_1 < t_2 < t_b.$$ 

Assumption 2.

Let $t_a < t_1 < t_2 < t_b$, let $S$ be isolated between $t_a$ and $t_b$, and let $a$ be persistent between $t_1$ and $t_b$. Then if $a$ is true at $t_2$, it is also true at $t_1$.

Assumption 3.

Let $C$ be a set of general experimental conditions and let $C'$ be conditions obtained from $C$ by changing only the experimental devices with which $S$ will interact after time $t$. Then if $C$ entails that $a$ is true on $S$ at time $t$, $C'$ also entails that $a$ is true on $S$ at time $t$. 
Assumption 4.

If $a$ is true on $S$, then it is also true on any system $S + S'$ of which $S$ is a part. Conversely, if $a$ is a proposition defined on systems of the type of $S$, if it bears on $S$, and if it is true on $S + S'$, then it is true on $S$.

BIBLIOGRAPHY

1) J.S. BELL, Physics, 1, 195 (1964).
4) A. HORNE, A. SHIMONY, Epistemological Letters, Ass. F. Gonseth, Nov. 73.
Abstract: An attempt is made to formulate more explicitly a notion of "local causality": correlations between physical events in different spacetime regions should be explicable in terms of physical events in the overlap of the backward light cones. It is shown that ordinary relativistic quantum field theory is not locally causal in this sense, and cannot be embedded in a locally causal theory.

Introduction - The Theory of Local Beables

This is a pretentious name for a theory which hardly exists otherwise, but which ought to exist. The name is deliberately modelled on "the algebra of local observables". The terminology, be-able as against observ-able, is not designed to frighten with metaphysics those dedicated to real physic. It is chosen rather to help in making explicit some notions already implicit in, and basic to, ordinary quantum theory. For, in the words of Bohr, "it is decisive to recognize that, however far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms". It is the ambition of the theory of local beables to bring these "classical terms" into the mathematics, and not relegate them entirely to the surrounding talk.

The concept of "observable" lends itself to very precise mathematics when identified with "self-adjoint operator". But physically, it is a rather wooly concept. It is not easy to identify precisely which physical processes are to be given the status of "observations" and which are to be relegated to the limbo between one observation and another. So it could be hoped that some increase in precision might be possible by concentration on the beables, which can be described in "classical terms", because they are there. The
beables must include the settings of switches and knobs on experimental equipment, the currents in coils, and the readings of instruments. "Observables" must be made, somehow, out of beables. The theory of local beables should contain, and give precise physical meaning to, the algebra of local observables.

The word "beable" will also be used here to carry another distinction, that familiar already in classical theory between "physical" and "non-physical" quantities. In Maxwell's electromagnetic theory, for example, the fields $\mathbf{E}$ and $\mathbf{H}$ are "physical" (beables, we will say) but the potentials $\Phi$ and $\mathbf{A}$ are "non-physical". Because of gauge invariance the same physical situation can be described by very different potentials. It does not matter that in Coulomb gauge the scalar potential propagates with infinite velocity. It is not really supposed to be there. It is just a mathematical convenience.

One of the apparent non-localities of quantum mechanics is the instantaneous, over all space, "collapse of the wave function" on "measurement". But this does not bother us if we do not grant beable status to the wave function. We can regard it simply as a convenient but inessential mathematical device for formulating correlations between experimental procedures and experimental results, i.e., between one set of beables and another. Then its odd behaviour is as acceptable as the funny behaviour of the scalar potential of Maxwell's theory in Coulomb gauge.

We will be particularly concerned with local beables, those which (unlike for example the total energy) can be assigned to some bounded space time region. For example, in Maxwell's theory the beables local to a given region are just the fields $\mathbf{E}$ and $\mathbf{H}$, in that region, and all functionals thereof. It is in terms of local beables that we can hope to formulate some notion of local causality.
Of course we may be obliged to develop theories in which there are no strictly local beables. That possibility will not be considered here.

1) Local determinism

In Maxwell's theory, the fields in any space-time region $1$ are determined by those in any space region $V$, at some time $t$, which fully closes the backward light cone of $1$:

Because the region $V$ is limited, localized, we will say the theory exhibits local determinism.

We would like to form some notion of local causality in theories which are not deterministic, in which the correlations prescribed by the theory, for the beables, are weaker.

2) Local causality

Consider a theory in which the assignment of values to some beables $\Lambda$ implies, not necessarily a particular value, but a probability distribution, for another beable $A$. Let

$$\{A | \Lambda \}$$

denote the probability of a particular value $A$ given particular values $\Lambda$. Let $A$ be localized in a space-time region $1$. Let $B$ be a second beable localized in a second region $2$ separated from $1$ in a spacelike way:
Now my intuitive notion of local causality is that events in 2 should not be "causes" of events in 1, and vice versa. But this does not mean that the two sets of events should be uncorrelated, for they could have common causes in the overlap of their backward light cones. It is perfectly intelligible then that if $\Lambda$ in (1) does not contain a complete record of events in that overlap, it can be usefully supplemented by information from region 2. So in general it is expected that

$$\{A | \Lambda, B\} \neq \{A | \Lambda\}$$

However, in the particular case that $\Lambda$ contains already a complete specification of beables in the overlap of the two light cones, supplementary information from region 2 could reasonably be expected to be redundant. So, with some change of notation, we formulate local causality as follows:

Let $N$ denote a specification of all the beables, of some theory, belonging to the overlap of the backward light cones of spacelike separated regions 1 and 2. Let $\Lambda$ be a specification of some beables from the remainder of the backward light cone of 1, and $B$ of some beables in the region 2. Then in a locally causal theory

$$\{A | \Lambda, N, B\} = \{A | \Lambda, N\}$$

whenever both probabilities are given by the theory.

3) Quantum mechanics is not locally causal

Ordinary quantum mechanics, even the relativistic quantum field theory, is not locally causal in the sense of (2). Suppose, for example, we have a radioactive nucleus which can emit a single $\alpha$-particle, surrounded at a considerable distance by $\alpha$-particle counters. So long as it is not specified that some other counter registers, there is a chance for a particular counter that it registers. But if it is specified that some other counter does re-
register, even in a region of space-time outside
the relevant backward light cone, the chance that
the given counter registers is zero. We simply do
not have (2). Could it be that here we have an in-
complete specification of the beables N? Not so
long as we stick to the list of beables recognized
in ordinary quantum mechanics - the settings of
switches and knobs and currents needed to prepare
the initial unstable nucleus. For these are comple-
tely summarized, in so far as they are relevant for
predictions about counter registering, in so far
as such predictions are possible in quantum mecha-
nics, by the wave function.

But could it not be that quantum mechanics is
a fragment of a more complete theory, in which
there are other ways of using the given beables,
or in which there are additional beables - hither-
to "hidden" beables? And could it not be that
this more complete theory has local causality?
Quantum mechanical predictions would then apply
not to given values of all the beables, but to
some probability distribution over them, in which
the beables recognized as relevant by quantum me-
chanics are held fixed. We will investigate this
question, and answer it in the negative.

Locality inequality

Consider a pair of beables A and B, belong-
ing respectively to regions 1 and 2 with spa-
elike separation, which happen by definition to
have the property

$$|A| \leq 1 \quad |B| \leq 1$$  \hspace{1cm} (3)

Consider the situation in which beables \(\Lambda, M, N\)
are specified, where N is a complete specifica-
tion of the beables in the overlap of the light
cones, and \(\Lambda\) and M belong respectively to the
remainders of the two light cones.
Consider the joint probability distribution
\[ \{ A, B | \Lambda, M, N \} \]
(4)

By a standard rule of probability, it is equal to
\[ \{ A | \Lambda, M, N, B \} \{ B | \Lambda, M, N \} \]
(5)

which, by (2), is the same as
\[ \{ A | \Lambda, N \} \{ B | M, N \} \]
(6)

This says simply that correlations between \( A \) and \( B \) can arise only because of common causes \( N \).

Consider now the expectation value of the product \( AB \)
\[ p(\Lambda, M, N) = \sum_{A,B} A \cdot B \cdot \{ A | \Lambda, N \} \{ B | M, N \} \]
(7)

(where the summation stands also, if necessary, for integration)
\[ = A(\Lambda, N) \cdot B(M, N) \]
(8)

where \( A \) and \( B \) are functions of the variables indicated, and
\[ |A| \leq 1 \quad |B| \leq 1 \]
(9)

for all values of the arguments. Let \( \Lambda' \) and \( M' \) be alternative specifications, of the same regions, to \( \Lambda \) and \( M \).

\[ p(\Lambda, M, N) + p(\Lambda', M', N) = A(\Lambda, N) \left[ B(M, N) \pm B(M', N) \right] \]
(10)

\[ p(\Lambda', M, N) + p(\Lambda', M', N) = A(\Lambda', N) \left[ B(M, N) \pm B(M', N) \right] \]

whence, using (9),
\[ |p(\Lambda, M, N) \pm p(\Lambda, M', N)| \leq |B(M, N) \pm B(M', N)| \]
(11)

\[ |p(\Lambda', M, N) \pm p(\Lambda', M', N)| \leq |B(M, N) \pm B(M', N)| \]

so that finally, again invoking (9), and \(|a+b| + |a-b| \leq 2 \text{ Max } (|a|, |b|)\)
\[ |p(\Lambda, M, N) \pm p(\Lambda, M', N)| + |p(\Lambda', M, N) \pm p(\Lambda', M', N)| \leq 2 \]
(12)
Suppose now the specifications $\Lambda$, $M$, $N$ are each given in two parts

$\Lambda \equiv (a, \lambda)$  
$M \equiv (b, \mu)$  
$N \equiv (c, \nu)$

where we are particularly interested in the dependence on $a$, $b$, $c$, while $\lambda, \mu, \nu$, are averaged over some probability distributions - which may depend on $a$, $b$, $c$. In the comparison with quantum mechanics, we will think of $a$, $b$, $c$, as variables which specify the experimental set-up in the sense of quantum mechanics, while $\lambda, \mu, \nu$, are in that sense either hidden or irrelevant.

Define

$$P(a, b, c) = p\left((a, \lambda), (b, \mu), (c, \nu)\right)$$  

(13)

where the bar denotes the averaging over $(\lambda, \mu, \nu)$ just described. Now applying again the locality hypothesis (3), the distribution of $\lambda$ and $\nu$ must be independent of $b, \mu$ - the latter being outside the relevant backward light cones.

$$\left|P(a, b, c) \mp P(a, b', c)\right| \leq \left|p((a, \lambda), (b, \mu), (c, \nu)) \mp p((a, \lambda), (b', \mu'), (c, \nu))\right|$$

(14)

because the mod of the average is less than the average of the mod. In the same way

$$\left|P(a', b, c) \mp P(a', b', c)\right| \leq \left|p((a', \lambda'), (b, \mu), (c, \nu)) \mp p((a', \lambda'), (b', \mu'), (c, \nu))\right|$$

(15)

Finally then, from (14), (15) and (12),

$$P(a, b, c) \mp P(a, b', c) \mp P(a', b, c) + \left|P(a, b, c) \mp P(a', b', c)\right| \leq 2$$

(16)

Quantum mechanics

Quantum mechanics, however, gives certain correlations which do not satisfy the locality inequality (16).
Suppose, for example, a neutral pion is produced, by some experimental device, in some small space-time region 3. It quickly decays into a pair of photons. Suppose we have photon counters in space-time regions 1 and 2 so located with respect to 3 that when one photon falls on 1, the second falls (or nearly always does) on 2. If the $\pi^0$ is at rest the counters must be equally far away in opposite directions and their sensitive times appropriately delayed. Of course, both photons will often miss both counters. Suppose finally that both counters are behind filters which pass only photons with specified linear polarization, say at angles $\theta$ and $\phi$ respectively to some plane containing the axis joining the two counters.

Let us calculate according to quantum mechanics the probability of the various possible responses of the counters. If $|\theta\rangle$ denotes a photon linearly polarized at an angle $\theta$, then for the photons going towards the counters the combined spin state is

$$|s\rangle = \frac{1}{\sqrt{2}} |0\rangle |\pi/2\rangle - \frac{1}{\sqrt{2}} |\pi/2\rangle |0\rangle$$

(17)

where first and second kets in each term refer to the photons going towards regions 1 and 2, respectively. This form is dictated by considerations of parity and angular momentum. The probability that such photons pass the filters is then proportional to

$$\frac{1}{2} |\langle \theta | 0 \rangle \langle \phi | \pi/2 \rangle - \langle \theta | \pi/2 \rangle \langle \phi | 0 \rangle|^2 = \frac{1}{2} |\cos \theta \sin \phi - \sin \theta \cos \phi|^2$$

(18)

The corresponding factor for photon 1 to pass and photon 2 not is

$$\frac{1}{2} |\langle \theta | 0 \rangle \langle \phi + \pi/2 | \pi/2 \rangle - \langle \theta | \pi/2 \rangle \langle \phi + \pi/2 | 0 \rangle|^2 = \frac{1}{2} |\cos (\theta - \phi)|^2$$

(19)
and so on. The probabilities for the various possible counting configurations are then

\[ p(\text{yes, yes}) = \frac{x \sin(\theta - \phi)}{2} \]

\[ p(\text{yes, no}) = \frac{x \cos(\theta - \phi)}{2} \]

\[ p(\text{no, yes}) = \frac{x \cos(\theta - \phi)}{2} \]

\[ p(\text{no, no}) = \frac{x \sin(\theta - \phi)}{2} \]  

(20)

\[ p(\text{no, no}) = \frac{x \sin(\theta - \phi) + x(1 - \frac{\Omega}{4\pi}) + (1-x)}{2} \]

where \( x \) is the probability that the \( \pi^0 \) production mechanism actually works, \( \Omega \) the (small) solid angle subtended by each counter at the production point, and no allowance has been made for bad timing, bad placing, or inefficient counting.

Now let us count \( A = \pm 1 \) for (yes/no) at 1 and \( B = \pm 1 \) for (yes/no) at 2. Then the quantum mechanical mean value of the product is

\[ P(\theta, \phi) = p(\text{yes, yes}) + p(\text{no, no}) - p(\text{yes, no}) - p(\text{no, yes}) \]

\[ = 1 - \frac{x \Omega}{4\pi} \left( 1 + \cos 2(\theta - \phi) \right) \]  

(21)

so that

\[ P(\theta, \phi) - P(\theta', \phi') + P(\theta', \phi) + P(\theta', \phi') - 2 = \frac{x \Omega}{4\pi} \left\{ \cos 2(\theta - \phi) - \cos 2(\theta - \phi') \right\} - \cos 2(\theta' - \phi) - \cos 2(\theta' - \phi') - 2 \]  

(22)

the right-hand side of this expression is sometimes positive. Take in particular

\[ \phi = 0, \quad 2\theta = \frac{\pi}{4}, \quad -2\phi' = \frac{\pi}{2}, \quad 2\theta' = \frac{3\pi}{4} \]  

(23)

in which case the factor in curly brackets is

\[ \left\{ \cos 2(\theta - \phi) - \cos 2(\theta - \phi') \right\} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 2 = +2(\sqrt{2} - 1) \]  

(24)

but if quantum mechanics were embeddable in a locally causal theory (16) would apply, with \( a \rightarrow \theta, \rightarrow \phi \), and \( c \) the implicit specification of the production mechanism, held fixed in (22). The right-hand side of (22) should then be negative.

So quantum mechanics is not embeddable in a locally causal theory as formulated above.
6) Experiments

These considerations have inspired a number of experiments. The accuracy of quantum mechanics on the atomic scale makes it hard to believe that it could be seriously wrong on that scale in some hitherto undiscovered way. The ground state of the helium atom, for example, is just the kind of correlated wave function which is embarrassing, and its energy comes out right to very high accuracy. But perhaps it is sensible to verify that these curious correlations persist over macroscopic distances.

Experiments so far performed do not at all approach the ideal in which the settings of the instruments are determined only while the particles are in flight. When they are decided in advance, in space time regions projecting into the overlap of the backward light cones, (16) does not follow from (12). For it was supposed in (12) that the complete specification $N$ of the overlap is the same for the various cases compared. So one can imagine a theory which is locally causal in our sense but still manages to agree with quantum mechanics for static instruments. But it would have to contain a very clever mechanism by which the result registered by one instrument depends, after a suitable time lapse, on the setting of an arbitrarily distant instrument. So static experiments are also quite interesting.

Practical experiments are far removed from the ideal in other directions also. Geometrical and other inefficiencies lead to counters registering (no, no) with overwhelming probability, (yes, yes) very seldom, and (yes, no) and (no, yes) with probabilities only weakly dependent on the settings of the instruments. Then from (21)

$$P = 1 - \varepsilon^2$$

with $\varepsilon^2$ weakly dependent on the variables, so that (16) is trivially satisfied. The authors in general...
make some more or less and hoc extrapolation to connect the results of the practical with the result of the ideal experiment. It is in this sense that the entirely unauthorized "Bell's limit" sometimes plotted along with experimental points has to be understood. But such experiments also are of very high interest. For if quantum mechanics is to fail somewhere, and in the absence of a monstrous conspiracy, this should show up at some point on this side of the ideal gedanken experiment.

Several of these experiments show impressive agreement with quantum mechanics, and exclude deviations as large as might be suggested by the locality inequality. Another experiment, very similar to one of those quoted, is said to be in agreement with it and yet in dramatic disagreement with quantum mechanics! And another experiment disagree significantly with the quantum prediction. Of course any such disagreement, if confirmed, is of the utmost importance, and that independently of the kind of consideration we have been making here.

1) Messages

Suppose that we are finally obliged to accept the existence of these correlations at long range, and the gross non-locality of nature in the sense of this analysis. Can we then signal faster than light? To answer this we need at least a schematic theory of what we can do, a fragment of a theory of human beings. Suppose we can control variables like a and b above, but not those like A and B. I do not quite know what "like" means here, but suppose that variables somehow fall into two classes, "controllables" and "uncontrollables". The latter are no use for sending signals, but can be used for reception. Suppose that to A corresponds a quantum mechanical "observable", an operator $a$. Then if

$$\delta a / \delta b \neq 0$$
we could signal between the corresponding space
time regions, using a change in \( b \) to induce a
change in the expectation value of \( \alpha \) or of some
function of \( \alpha \).

Suppose next that what we do when we change \( b \)
is to change the quantum mechanical Hamiltonian
\( \mathcal{H} \) (say by changing some external field), so that
\[
\int dt \mathcal{H} = \mathcal{B} \delta b
\]
where \( \mathcal{B} \) is again an "observable" (i.e., an opera-
tor) localized in the region 2 of \( b \). Then it is
an exercise 3 in quantum mechanics to show that
if in a given reference system region (2) is en-
tirely later in time than region (1)
\[
\frac{\delta \alpha}{\delta b} = 0
\]
while if the reverse is true
\[
\frac{\delta \alpha}{\delta b} = [\alpha, -\frac{\hbar}{i} \mathcal{B}]
\]
which is again zero (for spacelike separation) in
quantum field theory by the usual local commutati-
vity condition.

So if the ordinary quantum field theory is em-
bedded in this way in a theory of beables, it im-
plies that faster than light signalling is not
possible. In this human sense relativistic quantum
mechanics is locally causal.

8) Reservations and acknowledgements

Of course the assumptions leading to (16) can
be challenged. Equation (22) may not embody your
idea of local causality. You may feel that only the
"human" version of the last section is sensible and
may see some way to make it more precise.

The space time structure has been taken as given
here. How then about gravitation?

It has been assumed that the settings of instru-
ments are in some sense free variables - say at the
whim of experimenters - or in any case not deter-
mined in the overlap of the backward light cones.
Indeed without such freedom I would not know how to
formulate any idea of local causality, even the mo-
dest human one.
This paper has been an attempt to be rather explicit and general about the notion of locality, along lines only hinted at in previous publications [Refs. 2, 4, 10, 19]. As regards the literature on the subject, I am particularly conscious of having profited from the paper of Clauser, Horne, Holt and Shimony, which gave the prototype of (16), and from that of Clauser and Horne. As well as a general analysis of the topic this last paper contains a valuable discussion of how best to apply the inequality in practice; I am indebted to it in particular for the point that in two-body decays (as compared with three-body) the basic geometrical inefficiencies enter in (22) in a relatively harmless way. I have also profited from many discussions of the whole subject with Professor B. d'Espagnat.

Presented at the sixth GIFT Seminar Jaca, 2-7 June 1975

J.S. Bell, CERN - Genève

REFERENCES

2) J.S. Bell, Physics 1, 195 (1965).
10) J.S. Bell, Science 177, 880 (1972).
17) M. Jammer, The Philosophy of Quantum Mechanics, Wiley (1974). See in particular references to T.D. Lee (p.308) and R. Friedberg (pp.244, 309, 324).
22) H.P. Stapp, Berkeley preprint (1975), LBL 3685.
24) A. Baracca, Firenze preprint (1975).
Cher Monsieur,

Dans les Lettres Epistémologiques de Novembre dernier, vous proposez une "issue de secours" au paradoxe EPR, qui en fait vous y piégez sans recours... Bell vous a d'ailleurs répondu par avance, juste après ses formules (2).

Vous proposez que le résultat de la mesure A, qui par hypothèse dépend de a et des \( \lambda \), dépend "en outre" d'autres variables aléatoires impliquées par l'interaction entre le système et les appareils de mesure en A et B. Mais ces "autres" variables ou bien dépendront de a et b, ou bien (cas particulier du précédent) n'en dépendront pas. Votre prétendue généralisation n'en est donc pas une, et vous restez soumis à l'alternative des formules (2) et (3) de Bell, c'est-à-dire au dilemme que vous savez: To be local or to be quantal, that is the question.

La "non-séparabilité" (pour parler comme d'Espagnat) des sous-systèmes observés en A et B consiste, selon moi, en le contact réel qu'ils ont dans leur commun passé I. Le paradoxe EPR consiste en ce que les deux mesures, A et B, produisent le même collapse du \( \psi \), en sorte qu'entre A et B il y a non seulement télédiction (comme c'était le cas en mécanique statistique classique) mais aussi téléaction (en un sens subtil). Il est donc inévitale (selon moi) de rejeter la majeure du syllogisme d'Einstein, Podolsky et Rosen "Si, sans aucunement perturber un système (disons celui de A), l'on peut cependant (par une mesure effectuée en B) télédiriger exactement...". De ce point de vue, la différence essentielle entre la mécanique statistique classique, et la statistique de la mécanique quantique, est que dans l'ancienne le dé était jeté en I, tandis que dans la nouvelle le dé est jeté en A et B.

Je pense que le paradoxe EPR est "le 3ème orage du XXe siècle" (les deux premiers étant ceux indiqués par Lord Kelvin dans une célèbre
lecture: le paradoxe de Michelson-Morley et le paradoxe de Lummer-Pringsheim). Le 3ème orage est celui qui menace depuis l'origine de la nouvelle mécanique quantique, ou mécanique ondulatoire.

La vérité, cependant, est que le 3ème orage est plus radical que celui d'une mécanique: c'est celui d'un calcul des probabilités. Le 3ème orage est inhérent à la nature même du nouveau calcul des probabilités qui est d'être un calcul ondulatoire des probabilités. Par lui, la symétrie de droit des deux faces de l'information d'Aristote (et de la cybernétique), acquisition de connaissance et pouvoir d'action ou d'organisation est projetée dans l'espace-temps comme symétrie de droit des ondes retardées et avancées.

C'est par suite d'un préjugé macroscopique erroné que l'Ecole de Copenhague pense que le collapse du \( \psi \) intéresse le futur seul, et non le passé. Elle projette indûment sur l'événement aléatoire individuel (la transition) la dissymétrie de fait (non de droit) liée à l'irréversibilité macroscopique. L'événement aléatoire individuel est, lui, strictement symétrique en avenir et passé (comme en mécanique statistique classique: paradoxes de Loschmidt et de Zermelo). Par exemple, la fonction propre d'une mesure de position \( x_0 \) d'espace-temps d'une particule relativiste de Klein-Gordon est le propagateur de Jordan-Pauli \( D(x - x_0) \), nul dans l'aillleurs de \( x_0 \). Si la particule est trouvée en \( x_0 \), il est certain qu'elle repart dans le demi cône futur et qu'elle est arrivée dans le demi-cône passé. Cette mesure de position affecte donc symétriquement le futur et le passé.

Je ne discuterai pas ici les implications du 3ème coup de foudre du XXe siècle, l'ayant déjà fait ailleurs. Elles seront au moins aussi com- motionnantes que celles des deux coups de foudre précédents...

Bien à vous.

O. Costa de Beauregard
Institut Henri Poincaré
Paris.
J'ai développé en détail le formalisme quantique totalement covariant lié à ce genre de problèmes. Le problème ici est celui de la probabilité de passage de la particule à travers un élément $d\sigma$ d'hypersurface $\sigma$ du genre espace; l'ensemble orthogonal des fonctions propres correspondantes est celui des $D(x - x_0)$ ayant leurs sommets $x_0$ sur $\sigma$. Le développement du $\psi(x)$ sur les $D$ est la formule résolvant le problème de Cauchy, les coefficients du développement étant les $\psi(x_0)$. (O. Costa de Beauregard, Précis de Mécanique Quantique Relativiste, Monographies Dunod, Paris, 1967).

14.4. F. Bonsack - Réponse à 14.3

La question que j'ai posée était la suivante: (formulée de façon précise):

Peut-on, étant donné un appareil de mesure $\lambda$, attribuer à chaque point de l'espace des $\lambda$ un résultat de mesure défini $A = +1$ ou $A = -1$? (sauf éventuellement à des points frontières entre deux domaines, l'ensemble de ces points étant de mesure nulle).

C'est dans cette hypothèse qu'avait été faite la démonstration primitive de Bell, c'est aussi ce que suggère l'écriture

$$A(\lambda)$$

à étant p.ex. une direction de polarisation "subject to experimental manipulation" écrit Bell, et non une variable aléatoire. Sinon on écrirait

$$A(\mu \lambda)$$

Dans l'hypothèse de localité, le théorème de Bell repose essentiellement sur le fait qu'on peut choisir de "filtrer" la même distribution des $\lambda$, sans la modifier, avec différents discriminateurs $\lambda, \lambda', \beta, \beta'$ etc. Une variation aléatoire de l'état initial de l'appareil de mesure ne peut donc pas non plus être comprise dans le $\lambda$.

Je conteste donc que Bell ait exclu par avance,
dans son article 14.1, l'hypothèse que j'envisageais. D'ailleurs, ce ne semble pas être l'avis de Bell puisqu'il m'a envoyé un second article avec la remarque: "This may be relevant to the question you raise in the latest epistemological letters".

Ceci dit, il est fort possible que Clauser et Horne, ou que Bell, dans d'autres papiers, aient démontré une version perfectionnée du théorème avec des suppositions du type $A(\mu, \lambda)$, $\mu$ étant une variable aléatoire cachée décrivant l'état initial du discriminateur. Mais je n'ai malheureusement pas encore eu tout le loisir d'étudier ces démonstrations avec l'attention qu'elles méritent.

F. Bonsack

18.0 B. Hoffmann - Quelques remarques

Dear Sir, In reading the various discussions about the interpretation of quantum mechanics, and particularly those concerning the EPR argument, locality, and Bell's Theorem, I cannot help feeling that the following remarks of P.W.Bridgman are not wholly irrelevant. They appear on page 77 of his book A Sophisticate's Primer of Relativity (Wesleyan University Press, Middleton, Conn. 1962), and their extension to the angular momenta of particles is immediate.

Having discussed the setting up of an inertial frame, Bridgman considers two particles initially at rest in it, and imagines that a repulsive force is briefly applied between them. He goes on to say (original emphasis):

"After a suitable relative velocity has been built up, we remove the force. The bodies continue to separate for all future time at the same relative velocity. This . . . is a veritable miracle. For in preserving a constant relative velocity, the bodies preserve a constant relation to each other. But how can such a relation be maintained? By hypotheses, the bodies are force-free and therefore
without mutual interaction. Furthermore, they are, by hypothesis, moving through empty space where there is nothing which conceivably might mediate distant connection. Yet somehow concealed in these particles there is a memory of their past connection and a prediction for the future of their coming relation. How can particles do so much, and what is the conceivable mechanism by which they might do it?"

Sincerely yours,

Banesh Hoffmann
Dept of Mathematics
Queens College of the City University of New York, Flushing
"Epistemological Letters" are not a scientific journal in the ordinary sense. They want to create a basis for an open and informal discussion allowing confrontation and ripening of ideas before publishing in some adequate journal.

Les "Lettres épistémologiques" ne voudraient pas être un périodique comme les autres. Elles désirent instaurer un mode de discussion libre et informel, permettant de confronter les idées, de les faire mûrir, avant leur éventuelle publication définitive dans une véritable revue.

Die "Epistemologischen Briefe" sollten keine wissenschaftliche Zeitschrift im üblichen Sinne sein. Sie möchten eher Gelegenheit bieten, frei und formlos Ideen auszutauschen und reifen zu lassen, welche dann in einer eigentlichen Fachzeitschrift veröffentlicht werden könnten.

Contributions, remarks, objections, answers should be sent to:

Les contributions, remarques, objections, réponses sont à envoyer:

Beiträge, Bemerkungen, Einwände, Antworten sind zu richten an:

ASSOCIATION FERDINAND GONSETH
CASE POSTALE 1081
CH - 2501 BIENNE.

Nouvelle adresse du secrétaire:

François Bonsack
Av. de Cour 155
CH - 1007 Lausanne.