THE DESIGN, VALIDATION, AND APPLICATION OF
AN INVERSE HEAT TRANSFER MEASUREMENT TECHNIQUE

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Abstract

by

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A heat transfer measurement technique based on the solution of an inverse conduction problem has been developed, validated, and applied in transonic turbine casings with variable surface roughness. A major advantage of the method is that it can be implemented for irregular surface geometries, as opposed to contact sensors which are generally not applicable in these types of situations. Additionally, the presence of contact sensors, e.g. heat flux gauges, is known to disturb the near wall flow field and modify thermal boundary conditions in many applications.

The inverse measurement technique requires an array of thermocouples to be embedded in machined holes within metal hardware. An internal sensor spacing criteria, i.e. the axial distance separating measurements and the appropriate wall normal distance between a thermocouple and an estimation boundary, is identified and evaluated for accurate inverse solutions. The measurement technique is theoretically and experimentally validated for various boundary conditions. Heat transfer measurements in over-rotor casings of a transonic turbine rig provide a practical demonstration of the method. A commercially available CFD code is validated based on the experimental turbine data and gives added confidence in the measurement technique.
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SYMBOLS

a Wavelet scale
b Wavelet positional shift
c Axial rotor chord length
c_p Specific heat capacity at constant pressure
D Diameter
E “Energy” metric based on wavelet coefficients
f Probability density function
h Heat transfer coefficient
I Number of available internal measurement
k Thermal conductivity
ℓ Polynomial window length
L Characteristic axial length
J_{i,j} Sensitivity coefficient
K Iteration counter
K Scaling parameter for test boundary conditions
m Number of axial grid points
n Number of spline intervals
N Number of unknown parameters
P_j Unknown boundary parameters
St Stanton number
q'' Heat flux
t time
\( T \)  Temperature

\( u \)  Flow speed

\( w_e \)  Endpoint of spline interval

\( x, y, z \)  Cartesian coordinate system

\( X \)  Wavelet coefficient

\( Y_i \)  Internal measurements

\( \alpha \)  Thermal diffusivity

\( \Gamma \)  Closed surface of \( \Omega \)

\( \delta \)  Length scale to parametrize test boundary conditions

\( \Delta x, \Delta y \)  Internal sensor spacing

\( \epsilon \)  Inverse solver error metric

\( \zeta \)  Damping matrix for inverse algorithm

\( \theta \)  Circumferential coordinate

\( \kappa \)  Wavenumber

\( \lambda \)  Shape parameter for test boundary conditions

\( \mu \)  Positive damping parameter for inverse algorithm

\( \sigma \)  Standard deviation

\( \pi \)  Pressure ratio

\( \rho \)  Density

\( \tau \)  Nondimensionalized temperature

\( \phi_q \)  Nondimensionalized heat flux

\( \varphi \)  Trial function(s)

\( \Phi \)  Shift parameter for test boundary conditions

\( \chi \)  Extrapolation distance

\( \Omega \)  Domain
SUBSCRIPTS

\( aw \) Adiabatic wall

\( T \) Total quantity

\( w \) Wall

1 Metal

2 Epoxy

SUPERSCRIPTS

\(-\) Average

\(*\) Complex conjugate

\(+\) Normalized turbulent wall coordinate
CHAPTER 1

BACKGROUND

1.1 Motivation

The focus of this work is the development, validation, and demonstration of a surface heat transfer measurement technique. The method is based on the solution of an inverse conduction problem, where thermal boundary conditions are inferred using data from embedded temperature sensors. The technique is an alternative to traditional methods such as heat flux gauges and other contact sensors, the presence of which may lead to inaccurate measurements as a boundary deviates from the adiabatic condition [71].

The inverse method was developed for use in the over-rotor casing of a transonic turbine, but is not limited to this application. Protecting components under severe thermal load is a major concern in turbomachinery and cooling designs depend on experimental correlations [41]. The hot section of gas turbine engines is of particular relevance. Measurements in turbine rigs at engine representative flow conditions provide boundary conditions for computational tools and correlations that are regularly used to make design decisions.
1.2 Surface Heat Transfer Measurement

Classical techniques for surface heat transfer measurement involve the use of heat flux gauges (HFGs) under transient conditions. Taler [78] reviewed the three basic gauge geometries. Heat flux is measured indirectly from the temperature response brought about by its effect; and thus HFGs typically require temperature instrumentation to be installed directly on the hardware surface. For example, Collins et al. [21] used a laser ablation process to print thin-film resistance thermometers onto a sample. Data obtained from HFGs are commonly interpreted with 1-D heat transfer models. To justify this assumption, the gauge must be well insulated from the surrounding part, e.g. the Pyrex® “button” gauges developed by Dunn [27]. Alternatively, the surface may be wrapped in an insulating material like Kapton® or constructed from a ceramic such as MACOR™ [80]. HFGs can be multi-layered as in Epstein et al. [32], consisting of an additional temperature measurement between a substrate and the surface.

HFGs are applicable to a limited type of geometries due to their intrusive nature. For example, it is challenging to study the heat transfer effects of surface roughness, e.g. in [39], using these devices. HFGs, such as button type gauges that are large in diameter, may disturb the near wall flow field and thermal boundary conditions, i.e. “heat island effect” [66]. The greatest difficulty in surface temperature measurement, as stated in [14], is ensuring that the sensor is reading as close as possible to the undisturbed temperature (in which the sensor is not present). A reasonable accuracy (< 5 K) is unlikely with contact methods (i.e. placing a sensor on the surface) if a large temperature gradient exists between the surface and surrounding fluid [14]; this is a consequence of the heat transfer through the sensor or other accessories (e.g. potting epoxy). Schultz and Jones [74] reviewed optical, non-contact techniques for surface temperature measurement, including temperature sensitive paints, phase-change coatings, liquid crystals, thermographic phosphors, and infrared emission.
Optical measurements often require complex calibration procedures, unlike the thermocouple and thin-film resistance thermometer that remain the most common forms of surface temperature instrumentation.

The appropriate rate equation for a convective process, known as Newton’s law of cooling, is

\[ q'' = h(T_w - T_{ref}), \]  

where \( q'' \) is the convective heat flux, proportional to the difference between a wall and reference temperature. The proportionality constant, \( h \), is termed the convective heat transfer coefficient. Eckert [31] observed that the heat transfer coefficient (\( h_{aw} \)) is independent of thermal boundary conditions when calculated based on a reference adiabatic wall temperature (\( T_{aw} \)). It is an invariant descriptor of the convective process: a function of only the geometry, flow field, and fluid properties [62]. An indirect method as applied by Lee and Settles [52], Crafton [22], Thorpe et al. [80], and many others is routinely used to determine \( T_{aw} \) (and \( h_{aw} \)). A direct method is one in which \( T_{aw} \) is measured on an insulated (\( q'' = 0 \)) model after thermal equilibrium is reached. Conversely, the indirect or extrapolation method involves making several heat flux measurements at different wall temperatures (e.g. by varying heater power level); a first order fit of these data can be extrapolated to the temperature at which \( q'' = 0 \), \( T_{aw} \), with the slope interpreted as \( h_{aw} \). Lavagnoli et al. [51] confirmed the linearity of the energy equation for a range of surface-to-adiabatic wall temperature ratios: \( T_w/T_{aw} > 0.8 \). As the ratio drops below 0.8, alterations of the local fluid properties (viscosity, thermal conductivity, and specific heat) result in boundary layer modifications that cause \( q'' \) vs. \( T_w \) data to deviate from a first order relation. A correction for this effect is given in Kays et al. [46].
1.3 Inverse Heat Conduction Problems

Estimation of surface heat transfer as well as thermophysical properties, energy generation, geometric parameter, etc. is challenging for structures that operate under strong thermal effects. Such parameters can, however, be predicted by solution of the inverse heat conduction problem (IHCP). This field has been investigated since the early 1950s. Most notably, the space program gave considerable impetus to the study of the IHCP [11]. For example, a direct measurement of surface temperature is almost impossible during space vehicle reentry and hence inverse methods must be considered.

Direct conduction problems use heat flux and/or temperature histories defined on all boundaries (in addition to model parameters, e.g. thermal conductivity) as functions of space and/or time to calculate an internal temperature distribution. In an inverse problem, for example, boundary conditions are determined from temperature measurements at one or more interior locations.

IHCPs are mathematically ill-posed, unlike direct problems which are well-posed in the Hadamard sense [40]. The existence of an inverse solution is assured by physical reasoning, but uniqueness can be proved for only some special cases [11]. An inverse solution may be highly sensitive to changes in the measured internal temperature data. A successful solution of an IHCP involves its reformulation as an approximate well-posed problem through regularization. In the direct problem, unphysical fluctuations (in space and/or time) of surface heat flux are damped out due to the diffusive nature of the heat conduction process. The opposite is true in the IHCP where noise in temperature measurements is amplified in its projection to the surface. As a result, oscillations in the computed boundary conditions may occur. Techniques such as future-sequential regularization [10], data filtering [65], Tikonov regularization [82], and hyperbolic regularization [85] add smoothing terms to a given cost functional to reduce the unstable effects of measurement errors.
Several numerical and analytical approaches for solution of the IHCP are summarized in References [5], [11], and [68]. The goal of most methods is to find the vector of unknown boundary parameters, $P$, which minimizes an ordinary least squares norm,

$$S(P) = \sum_{i=1}^{I} (Y_i - T_i(P))^2.$$  \hspace{1cm} (1.2)

Here, $T_i(P)$ is the estimated temperature obtained from solving the direct problem based on an initial guess for $P$; $I$ is the number of internal temperature measurements, $Y_i$, $i = 1:I$. For the steady problem, $I$ represents the number of available measurements in space used to estimate the unknown parameters (boundary conditions).

The IHCP has been numerically treated and extended to multiple dimensions with the help of computing architecture and improvements in computer capacities [64]. The data reduction technique is an optimization process based on an iterative algorithm. Initially, boundary conditions and other required parameters are assumed for the numerical solution of the direct problem. For example, Beck [10] employed a finite difference method, Blackwell [17] used finite elements, and Kurpisz and Nowak [50] applied a boundary element approach. Computed internal temperatures are then compared to measured results. The boundary conditions are adjusted iteratively such that the computed temperatures approach the measured values. Gradient based schemes like the conjugate gradient method (CGM) and Levenberg-Marquardt (L-M) method [53], [58] are commonly chosen to carry out the optimization. The L-M method has been used widely the literature, e.g. [43], [72], and [83].

Surface temperature or heat flux variations are frequently constrained by trial functions in order to limit the number of unknown parameters. This type of analysis is known as function specification. For example, Frank [36] approximated surface heat flux with a polynomial expression and determined the coefficients by the method least squares. Functions can be a sequence of constant segments, straight line segments,
parabolas, cubics, exponentials, etc. [11]. More recently, Kim et al. [48] used a cubic spline function in space and a linear function in time to retrieve heat flux on the surface of a three-dimensional slab.

Of the large body of literature on the IHCP, few publications have dealt with the steady-state problem directly. When encountered, the steady problem is most typically posed as a parameter estimation of constants or physical properties. Examples of this include thermal conductivity and heat transfer coefficient determination as in [9], [60], and [63]; as well as heat source and boundary shape identification, e.g. [26] and [59]. The Cauchy problem [13] arises frequently in the literature ([7], [49], [57], etc.), where over-specified data (Dirichlet and Neumann conditions) available on part of the boundary are used to estimate surface conditions over the rest of the domain. This “data completion” problem is not a traditional inverse problem as defined in [11] because its solution requires boundary data rather than internal measurements.

A select number of reports are available in which steady-state heat transfer is estimated on a two-dimensional domain, generally using simulated internal measurements from exact solutions of the heat equation. For example, Fang et al. [34] determined steady heat flux on a boundary using the linear superposition theorem. Al-Najem et al. [4] used both a least-squares minimization approach and a boundary element method to predict surface temperature distribution. Intelligent methods have recently received some attention in the field of steady IHCPs. For example, Deng and Hwang [23] applied a neural network and Wang et al. [83] validated a fuzzy inference method with a L-M analysis.

Uncertainty quantification in the IHCP can be divided into deterministic and probabilistic methods. Deterministic methods are most often executed by calculating the sensitivities of unknown parameters to changes in temperature measurements (e.g. numerically or through direct differentiation of an analytical solution). The sensitivities are then input to a standard uncertainty propagation equation,
\[ \sigma^2_{P_j} = \sum_{i=1}^{I} \left( \frac{\partial P_j}{\partial Y_i} \right)^2 \sigma^2_{Y_i}. \]  

(1.3)

Here, an assumption of uncorrelated errors is made as in Coleman and Steele [20], the variance of an unknown parameter is \( \sigma^2_{P_j} \), and the variance of an internal temperature measurement is \( \sigma^2_{Y_i} \). Blackwell and Beck [16] performed a similar analysis for a 1-D planar body and considered the sensitivities to initial condition, thermal conductivity, and volumetric heat capacity, in addition to the internal temperature measurements.

A statistical approach based on the maximum likelihood estimator (MLE) can be applied to IHCPs. A Bayesian framework provides a solution to the IHCP by formulating a complete probabilistic description of the unknowns and uncertainties given temperature data. The goal is to find a solution for the vector of unknown parameters, \( P \) which maximizes the probability of the observations (temperature measurements, \( Y \)) using relevant background information. Bayes’ formula [76] is the foundation of Bayesian inference,

\[ f(P|Y) \propto f(Y|P)f(P), \]  

(1.4)

where the proportionality indicates the absence of a normalizing constant, \( 1/f(Y) \). The posterior probability density function (PDF) is \( f(P|Y) \), the likelihood function is \( f(Y|P) \), and the prior PDF is \( f(P) \). Confidence intervals on \( P \) and other statistics are determined by computing the posterior PDF. Regularization is added through the prior PDF. Fadale et al. [33] and Wang et al. [84] present a more comprehensive review as well as examples of probabilistic methods for uncertainty quantification in inverse problems.
1.4 Aerothermal Aspects of Turbine Casing Heat Transfer

Research focused on turbine heat transfer has been historically less prominent than that on the associated aerodynamics. Early designers relied on physical evidence (partially melted, totally destroyed, or missing pieces) rather than predictions to reveal components with heat transfer problems [29]. It is well-known that gas turbine engine performance can be improved by raising turbine inlet temperature [41] (at the cost of increased cooling requirements [47]); and as the turbine designer is continually motivated to do so, advancements in experimental techniques (in conjunction with predictive tools) are necessary to understand the more complicated flow environment. Flat plate and cascade facilities are credited with initial developments, which have since progressed to rotating rigs.

Turbine casing (i.e. a stationary outer component which bounds the rotating blades) heat transfer is an aerothermal problem dominated by convection between the over-tip leakage flow and the solid. A small gap between the rotor blade tip and casing wall is unavoidable for unshrouded turbomachinery. Its purpose is to prevent the rotor blades from contacting the casing (termed a “blade rub”), limiting the potential for catastrophic engine failure. A shrouded rotor eliminates the tip gap, but is not practical for modern high-pressure turbine (HPT) designs due to additional weight, stress, and cooling requirements [75].

A strong pressure gradient is present across the clearance gap which creates a leakage flow path from the pressure-side (PS) of the blade to the suction-side (SS). Bindon [15] illustrated some important tip gap flow features, shown in Figure 1.1. The flow entering the tip gap separates at the PS edge and contracts to a jet. A vena contracta is evident after the fluid is accelerated into the gap. The gas exits the gap with velocity vectors that are misaligned with the mainstream fluid and a shear layer results in the creation of the tip leakage vortex (TLV). The mixing of the TLV accounts for approximately $1/2$ of the total losses in the region [15]. High momentum
Leakage flow reattaching over
the bubble forms vortex with
relatively little loss

Flow established within
separation bubble due to
negative pressure gradient

Bubble flow grows in size
and separates as gradient
falls away

Leakage jet mixes with stagnant
bubble flow to eject it from the
high pressure suction side of gap

Clearance flows here warrant
further study

Rapid increase in size due to loss
core separation in positive
suction corner pressure gradient

Figure 1.1. 3-D representation of tip leakage flow field [15]

and temperature fluid is transported into the boundary layer where kinetic energy
is dissipated without work extraction. The casing is thus exposed to high levels of
convective heat transfer, complicated by the unsteadiness of periodic blade-passing
events. Further information regarding the aerodynamics of turbine leakage flows is
available in References [24], [69], and [79].

Dunn [28] acquired time-resolved measurements of unsteady casing heat transfer
and pressure in a HPT. His group is responsible for some of the earliest work regarding
gas turbine heat transfer in rotating rigs, e.g. in [30]. Sample heat flux and surface
pressure histories are given in Figure 1.2 and were shown to be in phase. The casing
is exposed to a higher velocity and convection rate as the leakage flow overtakes and
passes the blade tip.
Thorpe et al. performed time-mean [80] and time-resolved [81] heat transfer measurements on the over-tip casing wall of a transonic axial flow turbine. Heat flux and adiabatic wall temperature were shown to decrease with axial position, primarily associated with the reduction in flow stagnation temperature through the blade row. Circumferential variations in heat transfer rate were small, implying that the nozzle vanes do not significantly influence casing boundary layer properties. As in the earlier work of Dunn [28], a large unsteady component of time-resolved heat flux was observed due to blade-passing events.

If time-resolved information is available, the total casing heat load may be separated into two parts originating from: the over-tip leakage flow and the passage flow. This type of analysis was applied in [81] and is summarized in Figure 1.3. Here, the heat flux characteristic of each flow regime is plotted as a function of axial position. Upstream of the rotor, the passage flow accounts for the entirety of casing heat transfer. As axial position is increased, the portion of the overall heat load due
to the passage flow is diminished. The leakage flow, however, augments casing heat transfer, altering the total rate of its positional decay from that of the passage flow contribution. At 40% rotor chord, half of the casing heat transfer rate was a result of the over-tip leakage flow for the conditions in [81].

Casing adiabatic wall temperatures were measured in [80] as greater than those at the stage inlet. Accordingly, the leakage fluid was shown to have an absolute circumferential velocity above the turbine inlet value, indicative of work addition to the flow by the rotor. The gas experiences a positive temporal static pressure gradient as flow enters the tip gap, consistent with a rise in stagnation enthalpy in the casing reference frame [38].

Thorpe et al. [81] concluded that the rotor tip gap resembles a “super” high-pressure compressor stage (known as the “rotor compressive heating” phenomenon). The design of a blade, and the incorporation of blade-tip or casing treatments, can influence the magnitude and direction of the tip leakage velocity. A design should be avoided if it is likely to produce strongly circumferential tip leakage flows, since
this will yield high recovery temperatures on the casing wall. One such geometric modification to the casing for “tip leakage desensitization” was studied by Ameri et al. [6], where a marginal reduction in heat transfer was observed with a recessed casing treatment.

Beginning with Nikuradse’s [67] work in 1933, a considerable increase in heat transfer due to surface roughness has been reported in the literature. The effects of elevated levels of surface roughness on turbomachinery performance have since been studied at all practical levels: fundamental flat plate wind tunnel research, multi-blade cascade facilities [2], and full-up system level tests [18]. These studies support the expected result that roughness increases surface drag and heat transfer. For turbomachinery, this translates to higher heat loads, accelerated part degradation, and lower stage efficiencies. Turbine surfaces experience significant degradation with service; an order of magnitude increase in rms roughness is typical due to the harsh operating environment [19].

In summary, two major contributors to casing heat transfer have been identified in the literature:

• *The higher relative total temperature of the fluid near the casing wall:* the leakage fluid does not participate in work extraction in the same manner as the passage flow,

• *The rotor compressive heating phenomenon:* work is input to the leakage fluid by the rotor.

Other contributors include elevated surface roughness and hot streak migration [25].
1.5 Document Overview

A need for an alternative to heat flux gauges and other similar instrumentation has been identified through a review of the more traditional methods for surface heat transfer measurement. The limitations of these available techniques are particularly apparent in high heat flux situations and when complex geometries are to be studied. Inverse measurements, although challenging, provide an opportunity to quantify thermal boundary conditions in a minimally intrusive manner, and are thus applicable to a wider range of research than is possible with contact sensors.

The present work is based on the development and validation of an inverse technique for heat transfer measurements in turbine casings. The method uses data from hardware embedded thermocouples to estimate surface heat transfer. Chapter 2 is focused on the design of an internal thermocouple array for the inverse method. Here, the accuracy of the inverse solver is evaluated for both fine and coarse sensor grids while considering experimental constraints. Chapter 3 is concerned with the implementation of the inverse technique in a physical model, aspects of which require theoretical and experimental validations which are subsequently presented. Chapter 4 shows a demonstration of a validated inverse technique in both a smooth wall turbine casing and a component with elevated surface roughness. Comparisons of the results of smooth wall experiments to the output of a commercially available CFD code are used to validate the simulations and impart further confidence in the inverse measurement technique. Overall conclusions are offered in Chapter 5.
CHAPTER 2

DESIGN OF AN EMBEDDED THERMOCOUPLE ARRAY

2.1 Introduction

The inverse conduction problem studied here involves the estimation of boundary conditions using temperature information at several interior locations within a domain. The axial distance required between measurements and their appropriate wall normal distance from a boundary, termed the “sensor spacing” for the remainder of this work, is unknown and to be determined.

The purpose of this chapter is to identify and evaluate an internal sensor spacing criteria for accurate inverse solutions. Several references in the literature conclude (a somewhat obvious result) that sensors should be placed at locations that allow for the maximum information to be inferred about the unknown boundary conditions, *e.g.* [11]. The “optimal” locations occur in regions where the measurements are most sensitive to unknown parameters [12], coinciding with the intuition that the largest number of available measurements should be positioned as near as possible to an estimation boundary [54].

As the field of IHCPs is largely theoretical, optimal sensor spacings are often assigned in the absence of physical restrictions, *i.e.* in [23]. Prediction errors are often reported as being well within experimental uncertainties with little sense of the degradation in more practical situations (*i.e.* as the amount of available measurements decreases). The goal of the present work, however, is to apply the inverse measurement technique in experiments. The hardware must be built under constraints of cost,
machining limitations, etc. and it thus may be impractical or unnecessary (in terms of allowable uncertainties) to produce an idealized grid of internal measurements.

The following analysis aims to quantify IHCP errors for a variable (i.e. both coarse and fine) array of embedded thermocouples. The results will aid in the design of hardware used for experiments detailed in Chapters 3-4.

Additionally, the boundary conditions considered are characterized by a higher wavenumber content than some of the test cases in the literature, e.g. [4]. Although the focus of this chapter is on the accuracy of the prediction corresponding to the physical surface (e.g. the flow path surface of a turbine casing), the entirety of boundary conditions were treated as unknowns. This is in contrast to separate studies where two [23] or three [4] of the boundaries on a rectangular domain were known (insulated) and the remaining were estimated. Lastly, the steady IHCP was solved, rather than the common transient case. Here, since local variations in the unknown surface boundary condition may exist over a small spatial domain, the steady problem becomes more difficult as it is limited by the sensor spacing as opposed to the sampling frequency of the measurements (Hz).

2.2 Sample Geometry with Regular Sensor Array

A 2-D model of a regular grid of thermocouples embedded in a rectangular solid was considered, Figure 2.1 (top). The variable sensor array and temperature boundary conditions are described in the following section. The corners of the domain are labeled as A, B, C, and D, respectively. Here, the simulated measurement locations are represented by orange circles placed $\Delta y$ from the $(x, 0)$ boundary and spaced $(\Delta x, \Delta y)$ apart. The parameter $\chi$ corresponds to the distance along $x$ or $y$ between the $(0, y)$, $(x, y_{max})$, and $(L, y)$ boundaries and the nearest measurement. The $(0, y)$, $(x, y_{max})$, and $(L, y)$ boundaries are marked by dashed red lines and are located within the solid (i.e. the boundaries are not defined by the physical surfaces of the part).
The parameter $\chi$ is shown as smaller than $\Delta y$ because the “surface”, $(x, 0)$, condition is the main prediction of interest and therefore marks a physical boundary in which machining limitations preclude $\Delta y \to 0$. In practice, $\chi$ should be made as small as possible to reduce the ill-posedness of the IHCP.

The approach is to assign test boundary conditions on the entire domain for the direct numerical solution of the steady, linear heat equation. Direct solutions are then sampled at discrete points to generate a grid of internal “measurements” and these data are input to an IHCP solver. In this method, temperature predictions for a given measurement array and boundary condition are compared with the known “exact” solutions to quantify uncertainty.

The form of the test surface temperature boundary condition is,

$$T(\tilde{x}) = K \left( -\text{atan}(\lambda \{ \tilde{x} + \Phi \}) + \text{atan}(\lambda \{ L/2 + \Phi \}) \right) + T_B, \quad (2.1)$$

where $-L/2 \leq \tilde{x} \leq L/2$ and $x = \tilde{x} + L/2$. Term I. is an inverse tangent function, with
$T_A > T_B$, shifted in the negative $\tilde{x}$ direction by $\Phi$. The shift, $\Phi$, was implemented to avoid any simplifications in the IHCP associated with symmetry. The $\lambda$ parameter controls the shape of the trigonometric function. As $\lambda \to 0$, the temperature profile is close to a linear function of $x$. Whereas for $\lambda \to \infty$, a temperature discontinuity is approached at $x = L/2 - \Phi$ (i.e. the temperature profile approaches a step function). The magnitude of Term I. is bounded by $\pi/2$ and $-\pi/2$. Term II. is an offset such that the lower bound becomes 0. The constant,

$$K = \frac{T_A - T_B}{\text{atan}(\lambda\{L/2 + \Phi\}) - \text{atan}(\lambda\{-L/2 + \Phi\})},$$  

(2.2)

scales the temperature to match the desired $\Delta T = T_A - T_B$, and $T_B$ is a final shift for $T(x = L) = T_B$.

A normalized temperature,

$$\tau = \frac{T(x) - T_B}{T_A - T_B},$$  

(2.3)

is plotted in Figure 2.1 (bottom) as a function of $x/L$. An additional variable,

$$\delta = \frac{T_A - T_B}{\frac{dT}{dx}}(x = \frac{L}{2} - \Phi),$$  

(2.4)

was defined as a length to scale parametrize the highest wavenumber content of the temperature profile. The choice ($\delta$) is an alternative to $\lambda$ that is not limited to test cases defined by Equation 2.1 and offers a convenient visualization of the length scale. The $\delta$ parameter is of value for its ease of calculation and application to boundary conditions of unknown functional forms (that will be encountered in experiments described in Chapters 3-4). A line was fit to the surface temperature distribution in the vicinity of $x/L = 1/2 - \Phi/L$ (dashed red) and $\delta/L$ is depicted as the $x/L$ distance required to achieve the $\Delta \tau = \tau_A - \tau_B$ of the given boundary condition. The form of the surface boundary condition is representative of the time-mean axial variation of
Figure 2.2. Direct temperature fields: (a.) \((x,0)\) boundary conditions, and solution for (b.) high \(\delta\) case, (c.) moderate \(\delta\) case, (d.) low \(\delta\) case

casing wall temperature as shown for example in [80]. Linear temperature profiles were assigned between points \(B\) and \(C\), \(C\) and \(D\), and \(D\) and \(A\), consistent with the experimental temperature gradients expected in later work.

Three sample \(T(x,0)\) conditions are displayed in Figure 2.2 (a.). As \(\delta \to L\) (gray), the \(y = 0\) profile approaches a line. Contours of the direct internal temperature fields are provided in (b.): high \(\delta\) case (from gray profile), (c.): moderate \(\delta\) case (from black profile), and (d.): low \(\delta\) case (from red profile). Here, \(x/L\) defines the abscissa and \(y/L\) is on the ordinate. The contours are observed to bend inward near the surface as \(\delta\) is decreased, forming something that can be compared to a “thermal boundary layer”. The high gradient regions of the solution increase the difficulty of low \(\delta\) IHCPs in comparison to high \(\delta\) cases.
2.3 Criteria for Sensor Spacing based on a Wavelet Transformation

A wavelet analysis [37], applied to the family of test functions described above, is proposed as a means of developing a sensor spacing criteria for \( \Delta x \). A link between the spatial length scales of the boundary temperature (i.e. \( \delta \)) and the \( \Delta x \) spacing required for its accurate inverse reconstruction is identified.

The transform is based on taking inner products to measure the similarity between a “signal”, \( T(x) \) in this case, and a particular analyzing wavelet function, \( \psi \),

\[
X_{a,b} = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} T(x) \psi^* \left( \frac{x-b}{a} \right) dx.
\]

Wavelet coefficients are represented by \( X \) and the \( * \) symbolizes the complex conjugate. The real-valued Morlet wavelet was used in this work with the built-in \texttt{cwt}\(^\text{\textregistered}\) code in MATLAB\(^\text{\textregistered}\) R2017a. The algorithm compares \( T(x) \) to shifted and compressed/stretched versions of the wavelet (\( \psi \)) to obtain \( X \), a function of two variables: scale, \( a \) and position, \( b \). The Morlet wavelet is defined as,

\[
\psi \left( \frac{x-b}{a} \right) = e^{-\left( \frac{x-b}{a} \right)^2/2} \cos \left( \frac{5(x-b)}{a} \right),
\]

and shown in Figure 2.3 as a blue curve for \( a = 1 \) m and \( b = 0 \).

A wavelet transform, as opposed to a Fourier transform, allows for the wavenumber components of the temperature profile to be decomposed as a function of space. The dominant wavenumber content of the wavelet, \( \kappa \), at a given scale was determined by fitting a purely periodic signal to the center length scale of \( \psi \) [61]. For example, at \( a = 1 \) m, \( \kappa = 13/16 \) m\(^{-1}\), as visualized by the dashed red cosine wave in Figure 2.3.

The wavelet transform of the temperature profile in Figure 2.4 (a.) is shown below in (b.). A standard procedure known as “signal following”, where the boundary condition is extrapolated beyond the domain of interest, was implemented to avoid
Figure 2.3. Morlet wavelet (blue) and a cosine fit (dashed red) to find \(\kappa\)

“edge effects” \[3\]. The spatial coordinate, \(x/L\), was used to define the abscissa and the wavenumber, \(\kappa\), was plotted on the ordinate. Contours of “energy” contribution,

\[
E_{a,b} \equiv \frac{X_{a,b}}{\int_{\kappa_{\text{min}}}^{\kappa_{\text{max}}} \int_{0}^{L} X_{a,b} d\kappa \, dx},
\]

are displayed as the natural logarithm of \(E_{a,b}\). The energy at a fixed scale and position is defined as the ratio of the wavelet coefficient, at that particular \(a\) and \(b\), and the sum of all wavelet coefficients (at all scales and positions). Here, \(\kappa\) is the vector of wavenumbers contained in the temperature distribution, \([\kappa_{\text{min}}, \kappa_{\text{max}}]\).

The larger the value of a contour in Figure 2.4 \((b.)\), the greater similarity between the temperature distribution and the wavelet. The regions of greatest similarity correspond to low wavenumbers for this temperature profile and are in red. Near \(x/L = 0.4\), where \(\partial T/\partial x\) is highest, the correlation between the temperature and \(\psi\) at high wavenumber scales is greatest. This region contains the highest wavenumber content in the profile. For \(x/L > 0.8\), the temperature is nearly constant and thus dominated by lower wavenumbers. The wavenumber, \(\kappa\), is similar to the \(\Delta x\) sensor spacing. This suggests that a denser array of thermocouples is required to estimate the temperature in regions of high spatial gradients, as opposed to a coarser spacing in lower gradient regions. A salient advantage of the above analysis is, therefore, the
ability to determine the spatial wavenumber content of a test function and apply this information in the design of non-uniform internal sensor grids.

The present analysis is restricted to measurement grids of constant $\Delta x$ and thus a spatially averaged $E_{a,b}$ (i.e. as a function of only $\kappa$),

$$\bar{E}_a = \int_{\kappa_{\min}}^{\kappa_a} \int_0^L E_{a,b} \, d\kappa \, dx,$$

(2.8)

is a useful measure of the correlation between the temperature profile and wavelets of $\kappa_{\min} \leq \kappa \leq \kappa_a$. An internal sensor array with irregular spacing was neither required to achieve accurate inverse solutions under the constraints of this work (and considering the high $\delta$ boundary conditions in Chapters 3-4) nor practical in terms of the small overall length of the domain, $L$.

The wavelet coefficients were calculated for several $\delta$ boundary condition cases.
For each $\delta$, $X_{a,b}$ results were integrated to determine $\bar{E}_a$. The curves for all test boundary conditions collapse if plotted against $(\kappa_a \delta)^{-1}$. A single black curve is shown in Figure 2.5 where $1 - \bar{E}_a$ defines the ordinate. The curve is near 0 for $(\kappa_a \delta)^{-1} < 0.4$, where $\kappa_a$ is close to $\kappa_{max}$. The wavelet coefficients required to completely reconstruct the temperature distribution via an inverse transform are thus bounded by $\kappa_a \approx 2.5/\delta$. As $\kappa_a$ is reduced below $2.5/\delta$, the curve rises. An attempt at reconstructing $T(x)$ for $\kappa_a < 2.5/\delta$ would yield an inaccurate profile as the wavelet coefficients needed to capture the high spatial gradients would be omitted. A critical scale is therefore defined as $\kappa_{critical} \equiv 2.5/\delta$. A larger $\delta$ means that a greater correlation exists between the temperature and wavelets at higher $a$.

The motivation is to use the above information to deduce an internal sensor spacing criteria. The wavenumber, $\kappa$, defined as proportional to the center length scale of the wavelet, is treated as $(\Delta x)^{-1}$ to do so. Physically, an accurate inverse solution is unlikely if $\Delta x > 1/\kappa_{critical}$. A spacing criteria of $\Delta x/\delta \approx 0.4$ is thus suggested, which is simply the Nyquist criteria for spatial measurements. As $\delta$ decreases, the temperature is associated with higher wavenumber content and a smaller $\Delta x$ is necessary for accurate predictions. A design in which $\Delta x/\delta \to 0$ would yield extra cost with little added benefit (i.e. assuming reasonable measurement errors and sufficient $\Delta y$).

2.4 Impact of Trial Function on Sensor Spacing

A $\Delta x$ spacing criteria was proposed in the previous section. The current section will review the basic methodology used to solve the IHCP in terms of cubic spline trial functions. The effect of both $\Delta x$ and $\Delta y$ on inverse solver error will be elucidated by examining the sensitivity coefficients of the chosen functional form.

The gradient based Levenberg–Marquardt method was applied to solve the IHCP. Although the algorithm is suitable for both steady and transient IHCPs,
steady-state heat transfer is the focus of this work. The approach, as described in Chapter 1, is to find the vector of unknown boundary parameters, \( P \), which minimizes an ordinary least squares norm,

\[
S(P) = \sum_{i=1}^{I} [Y_i - T_i(P)]^2. \tag{2.9}
\]

The unknown parameters are updated at iteration \( K + 1 \) via

\[
P^{K+1} = P^K + [(J^K)^T J^K + \mu^K \zeta^K]^{-1} (J^K)^T [Y - T(P^K)], \tag{2.10}
\]

where the sensitivity or Jacobian matrix, \( J \) is determined by

\[
J = \left[ \frac{\partial T^T(P)}{\partial P} \right]^T. \tag{2.11}
\]

Here, \( T \) is the vector of estimated internal temperatures at each measurement location. The damping parameter, \( \mu^K \), is a positive scalar, and \( \zeta^K = \text{diag}[(J^K)^T J^K] \). The damping parameter was made large (i.e. 1,000) at the beginning of iterations and the solution tended toward the steepest descent method. As the solution advanced, the parameter \( \mu^K \) was reduced to speed up convergence. More details regarding the
L-M method are provided in [68].

The IHCP was posed as a function specification problem as in [36] due to the known challenges associated with unconstrained inverse solutions, e.g. [42], [84]. Here, \( T(x, 0) \equiv \varphi(x) \), where \( \varphi(x) \) is a spline [35] of the third degree on \([0, L]\),

\[
\varphi(x) = \left\{ \begin{array}{ll}
\varphi_0(x) = A_0 x^3 + B_0 x^2 + C_0 x + D_0, & w_0 \leq x \leq w_1 \\
\vdots & \\
\varphi_{n-1}(x) = A_{n-1} x^3 + B_{n-1} x^2 + C_{n-1} x + D_{n-1}, & w_{n-1} \leq x \leq w_n,
\end{array} \right.
\tag{2.12}
\]

\( \varphi(x) \in C^2[0, L] \), \( 0 = w_0 < w_1 < \ldots < w_n = L \), and \( \varphi_e(x) \in \mathbb{P}^3 \). There are 4n degrees of freedom or coefficients to be determined \((A_e, B_e, C_e, D_e)_{e=0}^{n-1}\). Since a cubic spline must satisfy,

\[
\begin{aligned}
\varphi_{g-1}(x_g) &= \varphi_g(x_g) \\
\varphi(x) : \varphi'_{g-1}(x_g) &= \varphi'_g(x_g) \\
\varphi''_{g-1}(x_g) &= \varphi''_g(x_g)
\end{aligned}
\]

\( 3(n - 1) \) equations are available to calculate the unknown coefficients. In addition, \( \varphi(x_e) \equiv T(x_e) \), \( e = 0, \ldots, n \), results in \( n + 1 \) equations. The remaining 2 equations necessary to solve for all unknown coefficients are \( \varphi''(w_0) = \varphi''(w_n) = 0 \), under the natural cubic spline constraint. A schematic is shown in Figure 2.6 to clarify the salient of the cubic spline parameters. A key parameter to be referenced later is the length of the polynomial window or spline interval, \( \ell = w_{e+1} - w_e \).

The polynomial window length is a variable in the function estimation problem. An analysis was performed to understand the effect of this parameter on an inverse solution. The inverse algorithm was tested for a \( \Delta x/\delta = 0.5 \) boundary condition and a sensor grid corresponding to \( \Delta x/L = 0.2 \) and \( \Delta y/L = 0.1 \). The exact solution is plotted as a function of \( x/L \) in Figure 2.7 (a.) with a solid blue curve. It is
compared to inverse solutions of varying polynomial window length, $\ell$, holding all other parameters constant. The purple curve was calculated by enforcing a spline constraint with $\Delta x/\ell = 0.7$. Similarly, the orange curve is for a case in which $\Delta x/\ell = 1$ and the red curve corresponds to $\Delta x/\ell = 1.9$. Setting $\ell$ as larger than $\Delta x$ has a smoothing effect on the inverse solution and is similar to fitting a polynomial of an insufficient order to describe the exact distribution. When $\Delta x/\ell = 1$, the inverse and exact solutions are in excellent agreement, as expected because $\Delta x/\delta$ is near 0.4. Further decreasing $\ell$ results in unphysical oscillations due to instabilities associated with the IHCP. The above provides a qualitative argument for setting $\ell = \Delta x$.

An error metric was defined as,

\[
\epsilon = \sqrt{\frac{1}{m_x} \sum_{i=1}^{m_x} \left( T_{\text{exact}}(x_i) - T_{\text{IHCP}}(x_i) \right)^2},
\]

to quantify the observations in Figure 2.7 (a.). The exact and inverse solutions were evaluated at $m_x = 100$ equally spaced points is $x$. The error metric was computed for several $\Delta x/\ell$ cases, nondimensionalized by $\Delta T = T_A - T_B$, and plotted as function of $\Delta x/\ell$ in Figure 2.7 (b.). A spline was fit through the discrete points for illustration purposes. The colored points in (b.) correspond to the temperature solutions of the
same color in (a.). For small $\Delta x/\ell < 1$, the error is large ($> 10\% \Delta T$) and decreases to a minimum value at $\Delta x/\ell = 1$. For $\Delta x/\delta > 1$, the errors are observed to increase at a lower slope (than for $\Delta x/\ell < 1$) due to oscillations in the computed boundary condition. Although the error is $\sim 5\% \Delta T$ for $\Delta x/\ell \approx 4$, the effect of the oscillations is magnified when calculating the heat flux from the temperature field. Large $\Delta x/\ell$ values should thus be avoided. A $\Delta x/\ell > 1$ is analogous to using wavelets of a center length scale narrower than $\Delta x$ to estimate the temperature profile. Overall, a $\ell = \Delta x$ should be specified for the cubic spline function estimation approach to the IHCP. The general form of $\epsilon/\Delta T$ as a function of $\Delta x/\ell$ is preserved for different $\Delta x$, $\Delta y$, and $\delta$ values, with offsets in the magnitudes of minimum error and changes in slopes for $\Delta x/\ell \neq 1$.

Sensitivity coefficients are defined as the first derivative of the estimated internal
temperature, $T_i$, with respect to the unknown spline parameters, $P_j$, $j = 1, \ldots, N,$

$$J_{i,j} = \frac{\partial T_i}{\partial P_j}.$$ \hspace{1cm} (2.15)

The values represent the components of the sensitivity matrix in Equation 2.11. The sensitivity coefficients for the cubic spline functional form were examined in order to confirm the $\ell$ criteria and recommend a $\Delta y$ criteria. In this case, the unknown parameters are defined as the boundary temperatures at the endpoints of the spline intervals. The $\mathbf{P}$ vector was input to an algorithm that enforced the continuity of the inverse temperature solution, and its first and second derivatives on each interval (and the natural constraint) at all iterations.

The sensitivity coefficients for an unknown spline parameter, $P_j$, are displayed as contours in Figure 2.8. The ratio of $x - x_e$ (where $x_e$ is the grid point corresponding to the endpoint of a spline interval) and $\ell$ defines the abscissa. The distance from the $(x,0)$ surface is plotted as $y/\ell$ on the ordinate. Sensitivity coefficients were calculated for variable $\ell$. The results were self-similar in that the coefficients scaled with the width of the polynomial window, \textit{i.e.} estimated internal temperatures were sensitive to the unknown parameter over a wider spatial extent for larger $\ell$, and vice versa. Sensitivity coefficients were independent of $\delta$ due to the linearity of the inverse problem [11].

Regions of highest sensitivity coefficient are shown in red; $J_{i,j}$ is effectively 0 in white areas, and light blue corresponds to negative sensitivities. Ideally, a sensor would be placed where the sensitivity is 1: this occurs at $x_e$ and $y/\ell = 0$. Here, if $P_j$ is increased by a single unit, the direct solution at the measurement location will be increased by the same amount. Sensitivity coefficients decay with increasing distance from $x_e$ and increasing $y/\ell$. This decay is particularly apparent on the abscissa beyond $\pm 1/2$. The positional rate of $J_{i,j}$ decay is slower in $y$ than in $x$. The negative regions are an artifact of the definition of the trial functions. Overall,
the sensitivity coefficients confirm that a sensor grid should be constructed such that \( \ell \approx \Delta x \). It appears that a \( \Delta y/\ell < 1.5 \) will yield an acceptable solution as the sensor would remain in a relatively high sensitivity region of approximately 0.2.

2.5 Monte Carlo Simulations to Evaluate Sensor Spacing Criteria

A \( \Delta x \) spacing was recommended in Section 2.3 followed by one for \( \ell \) and \( \Delta y \) in Section 2.4. A series of Monte Carlo simulations were performed to test these criteria. The results are summarized in this section.

First, the inverse algorithm was executed for several \( \delta \) cases represented by the colors in Figure 2.9 (a.) (\( \delta/L \) value corresponding to each color provided in top-left of figure). The simulations were run with variable \( (\Delta x, \Delta y) \) sensor grids and \( \ell = \Delta x \). The ratio \( \epsilon/\Delta T \) was computed based on the known test functions and plotted against \( \Delta x/L \). This solver error is shown to increase with \( \Delta x \), as expected. The 4 symbols represent different \( \Delta y \) cases. All cases are explained by \( \Delta y/\ell < 1.5 \) and the observation that the output for a fixed \( \Delta x \) and \( \delta \) collapses at the various \( \Delta y \) conditions provides confidence in the spacing criteria: the inverse solution is...
independent of $\Delta y$ within this range for the test conditions considered. For $\Delta x \to 0$, $\epsilon/\Delta T$ is near 0 for all $\delta$ cases considered. The IHCP becomes more difficult when $\Delta x$ and $\delta$ are increased (i.e. higher $\epsilon/\Delta T$). The variations in the inverse solver error are greater between the low $\delta$ and high $\delta$ cases as $\Delta x$ is increased.

![Graph](image)

Figure 2.9. Normalized inverse solver error as a function of axial measurement spacing (a.), and boundary condition, $\Delta x/\delta$ (b.)

The 4 $\Delta y$ points were replaced by a single average point for each $\delta$-$\Delta x$ combination and shown as a function of $\Delta x/\delta$ in Figure 2.9 (b.). The results collapse and are fit with a polynomial of the form $\Lambda(\Delta x/\delta)^2$. The fit (dashed black curve) is consistent with second order errors encountered when solving the heat equation. The trend is analogous to the Figure 2.5 curve and the same $\Delta x/\delta$ criteria of 0.4 may be concluded. A $\Delta x/\delta > 1$ would result in unacceptable errors. It should be noted that the wavelet analysis comes at significantly less computation expense than the Monte Carlo approach while yielding the same conclusion.

Next, some cases where $\Delta y/\ell > 1.5$ were considered in an attempt to violate the
spacing criteria. This analysis was conducted to determine the increase in $\epsilon/\Delta T$ for large $\Delta y/\ell$. The inverse error is again plotted as a function of $\Delta x/\delta$ in Figure 2.10 with the results of $\Delta y/\ell = 1.7$ simulations marked as blue squares and $\Delta y/\ell = 2.3$ trials shown as red circles. The baseline solver error calculated from Figure 2.9 simulations is re-plotted as a dashed black curve. Increasing $\Delta y/\ell$ past 1.5 will produce a family of error curves shifted above the baseline. The higher the $\Delta y/\ell$ is set above 1.5, the greater the upward shift in the $\epsilon/\Delta T$ curve. Interestingly, an extrapolation of the discrete points at both $\Delta y/\ell > 1.5$ values suggests that the solver error still approaches 0 as $\Delta x/\delta \to 0$. The error levels associated with the additional test cases (blue/red) are still within an acceptable range, however.

Figure 2.10. Examples of Insufficient $\Delta y$: $\Delta y/\ell = 1.7$ for blue squares and $\Delta y/\ell = 2.3$ for red circles

It is recognized that an inverse solution is subject to random errors that will augment the above $\epsilon/\Delta T$ calculations. Such errors include measurement noise and positional uncertainties. The addition of random errors to the simulated data will undoubtedly drive the $\Delta y$ criteria closer to the wall as such errors will be amplified in their projection to the surface. The amount that $\Delta y$ must decrease in the presence of random error was determined as follows.

Simulated data were corrupted with samples from the 3 probability density func-
tions in Figure 2.11. Here, the magnitude of the random error is on the abscissa and the probability density is on the ordinate. The probability density functions were randomly sampled for each internal sensor location. The exact temperature at that location ($\sigma = 0$) was offset by the resulting measurement noise and a positional component based on the temperature difference between the intended ($\sigma = 0$) and “actual” (i.e. shifted by positional errors) location. A Gaussian distribution (a.) was used to model measurement noise, as is standard practice. A single-sided truncated normal distribution was chosen to account for depth error (b.). This choice will be clarified in the Chapter 3 and is due to the construction of the physical hardware. The maximum error in $x$-position was limited to the diameter of a sensor hole (i.e necessary to embed a thermocouple in a body) by assuming a truncated normal distribution (c.).

![Figure 2.11](image)

**Figure 2.11.** Sources of random error and assumed probability density functions: (a.) measurement, (b.) depth ($y$), and (c.) in $x$ location

Simulations were re-run using corrupted “measurements” at 4 $\Delta y/\ell$ conditions. The results are summarized in Figure 2.12. Here, solver error is plotted as a function of $\Delta x/\delta$ as before. The dashed black line is the baseline reference for the $\sigma = 0$ random error cases that obeyed the current spacing criteria. The pink squares
represent inverse output using corrupted data at $\Delta y/\ell = 0.2$. Similarly, blue triangles are for $\Delta y/\ell = 0.4$, red circles for $\Delta y/\ell = 1.2$, and gold crosses for $\Delta y/\ell = 2.1$. The level of random error tested, considered as representative of that in the Chapter 4 turbine casing experiments, has little effect on the inverse solution for the two lowest $\Delta y/\ell$ cases. For $\Delta y/\ell = 1.2$ and $\Delta x/\delta < 0.3$, the addition of random error is characterized by small differences from the baseline. Beyond $\Delta x/\delta \approx 0.5$ (for $\Delta y/\ell = 1.2$), the effect of the random error becomes significant. The result is in agreement with the baseline for the highest $\Delta x/\delta$ case. Here, the solver error for $\sigma = 0$ is large and dominates the inverse output. The $\Delta x/\delta$ criteria shifts to the right ($< 0.2$) when $\Delta y/\ell$ is further increased to 2.1. In theory, $\epsilon \to 0$ as $\Delta x/\delta \to 0$ for any error level, assuming uncorrelated errors. The new criteria may be accompanied by an experimental impossibility, as decreasing $\Delta x$ while constraining $\ell = \Delta x$ comes at the cost of decreasing $\Delta y$ to maintain a fixed $\Delta y/\ell$ ratio. For large $\Delta x/\delta$ and $\Delta y/\ell$ (i.e. 2.1), the converged inverse solutions are close to the initial guess as the optimization problem becomes severely difficult. Overall, the recommendation is to adjust the $\Delta y/\ell$ criteria to $< 0.5$ in the presence of this level of random error.

![Figure 2.12. Effect of random error on inverse solver error for variable $\Delta y/\ell$](image)

Figure 2.12. Effect of random error on inverse solver error for variable $\Delta y/\ell$
2.6 Design Procedure Summary

The design procedure is summarized in the Figure 2.13 flowchart. Here, inputs are located left, required calculations are located middle, and decisions are located right. First, a domain and family of representative test functions is determined. A criteria is defined based on the wavenumber content of the temperature profiles and tested using a wavelet analysis to select $\Delta x$. A solution of the IHCP is not required for this portion of the procedure and the general Nyquist criteria for spatial sampling is recovered. Trial functions are chosen for the function estimation problem and a sensitivity analysis is performed, for example, to set the width of the polynomial window corresponding to these functions (i.e. conclude $\ell = \Delta x$). This analysis yields an initial recommendation for $\Delta y$. Finally, a Monte Carlo analysis is undertaken to test the spacing criteria and update it considering random error.

The calculations detailed in this chapter provide an estimation of inverse solver error for $\sigma = 0$ as a function of the internal sensor spacing. This quantity is defined as the error associated with an inverse solution obtained from “exact measurements”, i.e. no noise or positional errors; and is due to internal sensor configurations which deviate from $\Delta x$ and $\Delta y \rightarrow 0$. Typically, $\sigma = 0$ solver error is assumed negligible in the largely theoretical IHCP literature, where most optimal experimental designs are possible. The $\epsilon/\Delta T$ curve from this analysis is a valuable tool in the design of embedded thermocouple arrays when cost restrictions are considered. In addition, the effect of random errors (i.e. generally the only error component considered in the literature and not separated from $\sigma = 0$ solver error) was studied and the results should be used in combination with the $\sigma = 0$ conclusions when estimating overall uncertainties.
Figure 2.13. Flow chart as summary of design procedure
CHAPTER 3
IMPLEMENTATION AND VALIDATION

3.1 Introduction

This chapter details the implementation of the inverse measurement technique in physical hardware as well as a series of theoretical and experimental validations designed to evaluate its efficacy.

The proposed measurement technique involves inserting thermocouple sensors (TCs) within a solid structure and using these data to estimate the unknown boundary conditions by an IHCP. A schematic cross-section of a single thermocouple immersed in a solid is shown in Figure 3.1. The thermocouple sensor is installed in a tapered hole within a metal part, permanently bonded with epoxy. The taper provides a means to locate the measurement (along the $x$ coordinate). The depth error may only be in a single direction, i.e. if the sensor was not in contact with the bottom of the hole. The single sensor shown here represents one element in an array of several embedded thermocouples required for an accurate inverse solution.

From a mathematical perspective, the metal and epoxy are viewed as two sub-regions $\Omega_1$ and $\Omega_2 \subseteq \mathbb{R}^3$. The closed surface of $\Omega_1$ is denoted by $\Gamma_1$ and marked with a dashed blue line in Figure 3.1. The connecting interface between $\Omega_1$ and $\Omega_2$ is defined as $\Gamma_2$ and highlighted in orange. A two material system may be assumed if small diameter thermocouples are used.
Figure 3.1. Hardware integrated thermocouple concept: TC is shown surrounded by material #2 in a hole within material #1

Neglecting the presence of the embedded thermocouple, the governing “heat diffusion equation” in each constant thermal conductivity domain is

\[
\frac{\partial T_1}{\partial t} = \alpha_1 \nabla^2 T_1, \quad \text{in } \Omega_1, \quad t \in [0, t_{max}], \quad (3.1)
\]

\[
\frac{\partial T_2}{\partial t} = \alpha_2 \nabla^2 T_2, \quad \text{in } \Omega_2, \quad t \in [0, t_{max}], \quad (3.2)
\]

where the thermal diffusivities are

\[
\alpha_1 = \frac{k_1}{\rho_1 c_{p,1}}, \quad \text{and} \quad \alpha_2 = \frac{k_2}{\rho_2 c_{p,2}}, \quad (3.3)
\]

and the Dirchlet and Neuman boundary conditions at the two material interface are

\[
T_1(\hat{x}, t) = T_2(\hat{x}, t), \quad \text{and} \quad k_1 \frac{\partial T_1(\hat{x}, t)}{\partial \hat{n}} = k_2 \frac{\partial T_2(\hat{x}, t)}{\partial \hat{n}}, \quad \text{on } \Gamma_2, \quad t \in [0, t_{max}]. \quad (3.4)
\]
The \( \hat{\text{v}} \) is used to symbolize vector quantities. The ability to treat the two material system (neglecting the thermocouples) as homogeneous is ideal in the sense that it reduces the complexity of an already ill-posed IHCP. A homogeneous domain would result if \( \alpha_1 = \alpha_2 \) and \( k_1 = k_2 \) under transient conditions. It is sufficient to match the thermal conductivity of the epoxy and metal for steady conditions. This can be concluded after examining Equations 3.1-3.2, coupled with the Neumann boundary condition, Equation 3.4.

3.2 Experimental Design

A summary of the implementation of the inverse measurement technique, followed by a description of a set of experiments designed to validate the method is presented in the following section.

3.2.1 General Application of the Inverse Measurement Technique

The general principal of the inverse technique is to estimate surface temperature (and heat flux) distributions from internal measurements. A major challenge is the difficulty of incorporating a set of temperature sensors in a solid material with minimal disruption to the heat transfer field. This is reflected in the IHCP literature, where internal temperature "measurements" are most often simulated from exact solutions of the heat equation, as in Imber and Khan [44].

A design that allows for metal components is critical in environments subject to high thermal stresses (such as those found in turbomachinery). This differs from the few experimental IHCP in the literature that typically deal with low-conductivity models, \textit{i.e.} in [86]. The concept is illustrated in Figure 3.1 where a single thermocouple sensor is installed in a tapered hole within a metal (material #1) and potted with epoxy (material #2). Data from the thermocouple may be used to estimate the surface temperature on some portion of \( \Gamma_1 \), for example. It is important to note that
Figure 3.1 is an idealized schematic and several internal temperature measurements are generally required to accurately estimate an unknown boundary condition.

Steady-state heat transfer is the focus of this work and an epoxy that yields $k_2 \approx k_1$ is desired. A thermally conductive epoxy is well-suited for potting a thermocouple in metal. Metals such as aluminum ($k \approx 170$ W/[m-K]) or stainless steel ($k \approx 16$ W/[m-K]) are natural choices for components in a turbine test facility. Conductive epoxies can have $k$ as high as 25 W/[m-K], and thus stainless steel (or steel of similar $k_1$) is suggested for a thermal conductive ratio of $k_2/k_1 \approx 1$. The Epoxy Technology EK1000™ compound [1] was found with $k$ roughly equal to the corresponding stainless steel value after the appropriate cure schedule (i.e. temperature vs. time required for epoxy to harden) was implemented. This material was selected for the measurement technique along with 17-4 PH H1150 stainless steel. Cure schedule significantly affects the thermal conductivity of a material. The physics of this phenomenon are complicated as proper cure conditions are dependent on the chemistry of the adhesives. Manufacturer recommendations were followed in this case such that the part cured at 423 K for 1 hour and 448 K for an additional hour.

The specific heat and density of stainless steel and the EK1000™ epoxy are listed in Table 3.1. The metal and epoxy have comparable $c_p$ values; however, the epoxy is less than half of the density of stainless steel. Therefore, if $k$ is matched, the ratio of $\alpha_2/\alpha_1 \approx 2.8$. An epoxy-metal combination with $\alpha_2 = \alpha_1$ was not found, but an $\alpha$ mismatch was not important since this research is concerned with steady-state heat transfer.

3.2.2 Description of Validation Experiments

A set of steady validation experiments were completed on a stainless steel ring with embedded thermocouples. This geometry was chosen for two reasons: (1.) the
TABLE 3.1
THERMAL PROPERTIES OF STAINLESS STEEL AND THE SELECTED EPOXY

<table>
<thead>
<tr>
<th>Material</th>
<th>(c_p) [J/kg-K]</th>
<th>(\rho) [kg/m(^3)]</th>
<th>(k) [W/(m-K)]</th>
<th>(\alpha) [mm(^2)/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>17-4 PH Steel (Metal)</td>
<td>460</td>
<td>7,820</td>
<td>16.4</td>
<td>4.56</td>
</tr>
<tr>
<td>EK1000(^{\text{TM}}) (Epoxy)</td>
<td>389</td>
<td>3,340</td>
<td>(k_2 \approx k_1)</td>
<td>12.6</td>
</tr>
</tbody>
</table>

annular nature of the part eliminated 3-D edge effects, and (2.) the hardware was intended for use as an over-rotor turbine casing in a Chapter 4 study. The internal sensors were used to solve the IHCP and these results were compared to independent surface temperature measurements.

A schematic cross-sectional view of the hardware is shown in Figure 3.2 (a.). Here, \(L\) defines a characteristic axial length. Data from a plane of 20, Type-K, fine-wire \((D/L = 3.1(10)^{-3})\) thermocouples were available for solution of the IHCP. The thermocouples (and data acquisition system) were calibrated prior to their installation in a variable temperature water bath using an Omega\(^{\circledR}\) 450-ATH thermistor. The maximum difference between the thermistor and a thermocouple reading was less than 0.3 K. The thermistor accuracy is \(\pm (0.25 \text{ K} + 0.1\% \text{ of the reading})\). The array was designed for \(\Delta y/\ell = 0.3\), assuming \(\ell = \Delta x\). This is within the spacing criteria suggested in Chapter 2.

The following experiments were considered 2-D due to symmetry: heat transfer variation was important in the \(x\) and \(y\) coordinates only. Thermocouples were installed via a circumferential cavity in the outer diameter (OD) of the ring. Figure 3.2 (b.) provides an illustration of the TC installation process. Note the slight curvature of the annulus. Each column of temperature sensors (i.e. along the vertical...
dashed line) in (a.) was inserted within a set of $D/L = 4.1(10)^{-2}$ holes. These holes were 5-axis machined to terminate at a constant circumferential plane ($z = 0$) and tapered in order to locate the measurement positions. The embedded thermocouples were potted using EK1000™ epoxy with the suggested cure schedule to match the $k$ of stainless steel. Additional thermocouples were attached in close contact with the ring surface along $(x, 0, 0)$, as well as the other faces of the part marked by orange circles, and on the floor of the OD cavity using an OMEGABOND® paste (OB-101, $k \approx 1 \text{ W}/[\text{m-K}]$). An active cooling system was set in the cavity after all internal temperature sensors were embedded and the EK-1000™ compound was hardened; and then potted with a high $k$ epoxy (Epoxies, ETC® 50-3100). A temperature controlled (0.5 K increments) chiller was capable of circulating a 50% glycol and 50% water mixture through the copper loop to modulate $\partial T/\partial y$.

Three different boundary conditions were considered: a $\partial T/\partial x$ dominant case, a $\partial T/\partial y$ dominant case, and a case with both $x$ and $y$ temperature gradients (i.e. “turbine-like”), as summarized in Figure 3.3. A dominant temperature gradient in $x$ was imposed for the first case (a.), where a heat source (from Varitemp Heat Gun®, VT-750C) was applied to one side of the part and a fan circulated cooling
air on the opposite face. The \((x,0,0)\) location was chosen to validate the inverse technique. Here, the embedded thermocouple array was deemed to be sufficiently close to the surface \((\Delta y/\ell = 0.3)\) such that the IHCP could be solved accurately (based on theoretical calculations in Chapter 2). For case (a.), the inner diameter (ID) of the hardware was insulated. The nearly adiabatic surface resulted in accurate measurements from the contact thermocouples that were then used to validate inverse output.

The second experiment (b.) was designed to produce a large temperature gradient in the \(y\) direction. A heat source was directed on the ID of the ring and cooling fluid was circulated through the OD cavity. The \(x = x_{\text{min}}\) and \(x = x_{\text{max}}\) surfaces of the metal were insulated as shown in the schematic. The large heat flux normal to the surface TCs resulted in high errors in the contact measurements. These errors were unacceptable for validation purposes. An attempt was made to withdraw the true undisturbed surface temperatures from the data; however, the sensors were installed at an unknown, non-constant (bent sensing elements) distances above the surface that (although small, \(y/L \sim -2.0(10)^{-2}\)) were difficult to quantify.

An alternative, non-contact surface temperature measurement method was employed for this case using radiation thermometry. A FLIR® model E40 infrared camera was directed at the ID surface to acquire validation data. Thermal techniques are not without challenges as the radiation measured by the camera is a function of the object temperature as well as the emissivity. Radiation also originates from the surroundings and is reflected in the object. The radiation from the object and the reflected radiation are functions of the absorption of the atmosphere. The FLIR® system is able to compensate for the effects of a number of different radiation sources if the appropriate object parameters are supplied: object emissivity, reflected apparent temperature, distance between the object and camera, relative humidity, and atmospheric temperature. The data sheet reports an accuracy of \(\pm 2\) K, although
Figure 3.3. Experimental boundary conditions: (a.) $\partial T/\partial x$ dominant, (b.) $\partial T/\partial y$ dominant, (c.) a “turbine-like” case

as quoted in FLIR® Research & Science: *with proper calibration the possible margin of error can be less than 1 K*. This level of accuracy is also demonstrated in the literature [56].

The third experiment (c.) was meant to simulate a “turbine-like” heat transfer field characterized by both axial ($\partial T/\partial x$) and radial ($\partial T/\partial y$) temperature gradients. The heat source and cooling air were applied where marked in the sketch and a splitter plate was used to separate the “hot” and “cold” air streams. Two separate IR images were taken on each side of the splitter plate for validation. Here, the surface was at a much greater temperature rise above the background ambient value than in the second experiment and because of this was more difficult to measure optically [56]. A strip of Scotch™ Brand 88 black vinyl electrical tape, with a well-known $\epsilon = 0.96$ was fixed to the hardware as a target. This was done at the recommendation of FLIR® to increase $\epsilon$ and thus measurement accuracy. The small thickness of the
tape \((y/L = -4.1(10)^{-3})\) was assumed to have an insignificant effect on the surface temperature measurement. It is common practice to increase the target emissivity of a metal object with tape, paints, coatings, etc. to improve the quality of an infrared measurement.

Approximate internal temperature fields are shown for each experimental boundary condition in Figure 3.4. These data were generated by solving the direct heat condition problem based on experimentally measured boundary conditions. Here, the black circles represent the locations of internal TCs. A maximum temperature difference of 25 K over the domain was typical for case (a.); a \(\Delta T \approx 20\) K was achievable in the second experiment. The third experiment was characterized by \(\Delta T \approx 30\) K with the OD cooling flow set to “OFF” and a \(\Delta T \approx 40\) K with the cooling “ON”. The results are termed approximate for three reasons. First, it was not feasible to measure the boundary conditions along a closed surface due to the geometry of the part; three embedded thermocouples potting in high \(k\) epoxy on the floor of the OD cavity were used as boundary conditions. Second, discrete temperature measurements were fit with natural cubic spline constraints to estimate continuous distributions. It is possible that a select thermocouple density was insufficient (especially along the cavity floor) to produce a true representation of the boundary condition. Lastly, boundary thermocouples may not have been in perfect contact with surface, augmenting measurement errors proportional to the incident heat flux. For example, a constant surface temperature distribution (based on the trend captured by the IR camera) was used to produce the case (b.) map because the surface thermocouples were unreliable.

3.3 Validation Results

The following section contains two parts. First, a direct theoretical model was used to demonstrate the effect of the TC potting epoxy on the internal measurements. Second, experimental validations are presented for the three classes of boundary
Figure 3.4. Approximate internal temperature fields [K] for (a.) $\partial T/\partial x$ dominant, (b.) $\partial T/\partial y$ dominant, (c.) a “turbine-like” case conditions described in Figure 3.3.

3.3.1 Direct Theoretical Model of a Non-homogeneous Domain

Embedded thermocouple holes (filled with epoxy) were treated as regions of constant thermal conductivity, $k_2$, within a $k_1$ (metal) domain. The two-material direct problem was solved using Equations 3.1-3.2 in the appropriate epoxy-metal domains. Steady-state heat transfer was assumed. The Equation 3.4 boundary conditions were imposed at the epoxy-metal interfaces. The same surface boundary conditions in Figure 3.4 (i.e. in which only a single material was considered) were specified. A 2nd order discretization was implemented in an iterative successive over-relaxation (SOR) algorithm. Internal temperature field comparisons between the non-homogeneous and homogeneous models provided a metric to evaluate the effect of the epoxy on the internal measurements.

Difference contours are plotted in Figure 3.5 for boundary condition set #1: the $\partial T/\partial x$ dominate case. The abscissa was defined by $x/L$ and the ordinate is shown as $y/L$. The contours in (a.) represent the difference between the direct problem solution on a non-homogeneous domain in which $k_2/k_1 = 0.75$, and the homogeneous
result in Figure 3.4 (a.). A similar calculation was performed to produce Figure 3.5 (b.); however, the epoxy was set at a greater thermal conductivity value than the metal, $k_2/k_1 = 1.25$.

A small positive bias dominates near the column of internal measurements (shown as black circles) closest to the heat source when the epoxy $k$ is less than that of the metal. Here, the lower $k$ epoxy acts to resist the transfer of heat along the $x$ direction. The opposite is true for the case where $k_2 > k_1$, although the dominate negative bias is smaller than the dominate positive bias observed in (a.). The temperature biases are largest outside of the hole locations. This is an effect of the diffusive nature of the heat equation and is beneficial in terms of the accuracy of an inverse solution (i.e. for $k_2 \neq k_1$ via a homogeneous direct solver). These results demonstrate that the impact of the potting epoxy yields internal temperature differences that are an order of magnitude less than the thermocouple measurement uncertainty for this class of boundary conditions. Furthermore, the calculations suggest that the $k$ of the epoxy and metal do not need to be matched exactly in order to produce accurate inverse measurements while assuming a homogeneous domain.

Figure 3.6 shows some difference statistics for each of the three classes of boundary conditions. The dashed curves indicate the maximum difference, $\Delta$, between the
non-uniform and uniform models on the complete domain. The solid curves depict this maximum difference over the 20 internal measurement locations as a function of \( k_2/k_1 \). A positive bias results when \( k_2 < k_1 \) and a negative bias dominates for \( k_2 > k_1 \).

The biases at the measurement locations are less than the maximum differences over the entire domain. Here, the “turbine-like” class of boundary conditions exhibits the largest bias, as it is characterized by the greatest hardware \( \Delta T \) among the three configurations.

![Graph showing difference statistics for three test boundary conditions](image)

**Figure 3.6. Non-uniform/uniform 2-D solver difference statistics for three test boundary conditions**

It is recognized that the thermocouple installation process, represented schematically in Figure 3.2(b.), introduces a three-dimensionality that may need to be modeled for accurate inverse solutions. To quantify this effect, a 3-D grid was generated to account for the thermocouple hole geometry. A cross-section of the model is provided in Figure 3.7(a.), where the collocation points in the epoxy (\( k_2 \)) region are shown as green circles; regions of \( k_1 \) (metal) are black. The view is that of an \( x \) slice through the center of the TC holes. A similar numerical solution to the above 2-D analysis was
computed for 3-D. The difference statistics for the 2-D and 3-D solver \((z = 0 \text{ plane})\) are plotted in \((b.)\) for the “turbine-like” case. The results demonstrate that a 2-D solver offers \(\Delta\) statistics that are conservative and very close in magnitude to the 3-D calculations (black). It can be concluded that a 2-D homogeneous solver is sufficient for this geometry. Again, a favorable result considering the added computational expense due to calculating over a third dimension.

Figure 3.7. 3-D model: \((a.)\) \(x\) slice through computational domain at TC hole center (regions of \(k_2\) in green), and \((b.)\) comparison of 2-D/3-D difference statistics for B.C. \#3

3.3.2 Comparisons of Inverse Technique to Independent Surface Measurements

First, the temperature differences between internal measurements and the inverse solutions are shown in Figure 3.8 for the three classes of experimental boundary conditions. The domain for the IHCP was bounded by the dashed red lines and solid blue curve. The circles mark the sensor locations and are colored by the difference between the measurement and IHCP output at that point in the domain. The differences are small \((i.e. \text{ bounded by } \pm 0.35 \text{ K and near 0 at many locations})\) for each
Figure 3.8. Difference between internal temperature measurements and IHCP solution [K] for each experimental boundary condition of the three cases which is a validation of the existence of the inverse solutions.

The results of the experimental validations at the physical (solid blue) surface are given in Figure 3.9. The solid red curves are the solutions of the IHCP for each case based on internal temperature data. These results are provided $\chi/L = \pm 4.1(10)^{-2}$ beyond the $x$ extent of the embedded measurement grid. This corresponds to a $\chi/\ell = 0.34$. An extrapolation beyond this point is difficult due to the ill-posedness of the IHCP. The independent surface temperature measurements used to validate the inverse technique were available along the entire $(x, 0, 0)$ domain, except where optical access was blocked by the splitter plate in boundary condition set #3.

Figure 3.9 (a.) shows the surface temperature plotted as a function of $x/L$ for boundary condition set #1. Insulated contact thermocouple measurements are marked by light blue circles and fit with a natural cubic spline (dashed black curve). The inverse output is in agreement with the contact TCs within the measurement uncertainty (estimated as $\pm 0.6$ K for this condition). The two means of quantifying boundary temperatures yield a similar trend. However, the contact TC results appear to exhibit a bias of $-0.3$ K where the surface deviates from the adiabatic condition; it is suspected that the TCs are located slightly off of the surface.

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A boundary condition set #2 experiment is shown in Figure 3.9 (b). Here, the blue “X” symbols are measurements from the IR camera. The surface condition is at a nearly constant temperature. The trend is a result of the geometry of the cavity cooling loop. The copper tubes are not in contact with the center of the cavity floor creating a slightly hotter surface profile around this x location. The two measurement methods again produce a similar trend with differences that are smaller than the measurement uncertainty (±1 K).

The results of two separate boundary condition #3 experiments are given in Figure 3.9 (c)-(d). Here, Δx/δ is approximately 0.2. The cavity cooling was “OFF” for
case (c.) and “ON” for case (d.). The IR measurements compare within uncertainty to the inverse result for both cases. These results will provide confidence in the work presented in Chapter 4.

3.4 Conclusions

A heat transfer measurement technique was implemented by embedding thermocouples in machined holes within metal hardware and using these data to estimate the unknown surface boundary conditions. This type of inverse analysis was proposed as a non-intrusive alternative to HFGs. Many heat flux gauges require the direct measurement of boundary temperatures and are thus inherently intrusive to the surface. Such contact measurements may be unreliable, especially when large heat fluxes are present. The inverse technique was developed for use in the over-rotor casing of a turbine rig, where optical access was limited. The approach has a wide range of applications with the major requirement being the ability to install an array of temperature sensors beneath the surface using an epoxy with thermal properties that are reasonably close to the base material.

The internal thermocouples and potting epoxy introduce a non-homogeneity in the test component. Validations were necessary to determine the effect of this non-uniformity on the thermal field. The concept was validated in 1-D in a previous work [42] and now for a much greater thermocouple density in 2-D. Theoretical models (2-D/3-D) were used to verify that the two-material, epoxy-metal combination required to apply the inverse method could be treated as homogeneous; and that the 3-D features added by the TC holes could be neglected. This was demonstrated for three boundary conditions at steady-state over a large range of $0.75 \leq k_2/k_1 \leq 1.25$. Experiments were performed which confirmed the theory. Overall, the inverse technique provides a means to accurately estimate surface boundary conditions in a non-contact manner when optical techniques are not feasible.
CHAPTER 4
HEAT TRANSFER IN TURBINE CASINGS

The inverse measurement technique was used to quantify heat transfer in the over-rotor casing of a transonic turbine. Both experimental data and computational results were acquired to study casing heat transfer. Experimental data are available for a smooth wall casing and a geometry with elevated surface roughness. The numerical analysis was restricted to the smooth wall casing. This chapter is organized as follows: the experimental and computational setups are described, followed by a comparison of the results from each investigation, and conclusions.

4.1 Experimental Setup

The following section provides an overview of the research facility, aspects of the turbine stage, and the primary instrumentation required for the casing heat transfer experiments.

4.1.1 ND-TRT Facility

Experiments pertaining to the current work were completed at the Notre Dame Transonic Research Turbine (ND-TRT) facility. The continuous, high-speed turbine rig was designed for component level testing at engine representative flow conditions [55]. The key features of the facility are summarized below; and a more detailed discussion is found in Schmitz [73].

A schematic of ND-TRT is shown in Figure 4.1. Moderately hot (∼350 K) fluid enters the turbine stage (nozzle/rotor as in Figure 4.2) at station 1 and exits as cold
fluid (∼ 285 K), kept above the dew point and freezing temperature. The air passes through a venturi (station 8) for mass flow rate measurement and enters a centrifugal compressor (station 6), operating as a vacuum pump. A portion of the compressor discharge air, at the maximum temperature in the cycle (∼ 450 K), is combined with fluid drawn from atmosphere (station 9) and recirculated back to the turbine through a lobbed mixer. The recycling process is managed by a pair of throttling valves and allows for a highly controllable turbine inlet temperature.

![Figure 4.1. Schematic of ND-TRT facility](image)

A variable speed AC electric motor, marked by station 6, is capable of generating 500 HP at 5,000 RPM, with a maximum speed of 5,250 RPM. The motor is mechanically linked to the turbine and compressor by gearboxes at station 3 and 5. This configuration is a unique feature of the facility as the turbine torque supplements the motor torque to power the compressor. The compressor is equipped with variable inlet guide vanes and diffuser guide vanes to modulate pressure ratio at fixed speed.
A total of 890 HP is required to drive the compressor at full operation. The facility allows for a typical HPT pressure ratio to be reached with reasonable blade size for this power requirement. A phase-shift torquemeter at station 2 is used to measure the turbine power output.

A distinguishing feature of the facility is the use of active magnetic bearings, as opposed to more conventional roller bearings. The high speed shaft is levitated during rig operation resulting in minimal power dissipation and precise torque measurements. The shaft can be offset statically and/or dynamically for simulating eccentric or off-centered axis of rotation [70]. Auxiliary ball bearings act as a fail-safe in the event of a magnetic bearing failure.

4.1.2 ND-TRT Stage

The stage chosen for this study is similar to that of a high-pressure turbine found in air-breathing propulsion and power generation applications. A cross-section of the stage flow path is shown in Figure 4.2.

Three distinct planes are identified in the schematic (dashed red lines). The first (US-1) gives the position of upstream measurements which are located 1.75 rotor...
axial tip chords away from the nozzle leading edge. There are two downstream measurement planes (DS-1/DS-2) at 1 and 2.5 chord lengths from the rotor trailing edge. A discussion of the instrumentation is found in Section 4.1.3. The nozzle and rotor consist of 47 and 74 blades, respectively. A description of the casing component is available in Section 3.2.2. The casing hardware was installed over the turbine rotor as shown schematically above.

The parameters used to define the design operating point are summarized in Table 4.1. The inlet temperature of the rig (i.e. 356 K) is significantly lower than that of a gas turbine engine (i.e. a turbine inlet temperature ∼ 2,000 K). There is, however, an adequate temperature difference between the gas and casing over much of the domain which corresponds to an accurately measurable amount of wall normal heat transfer. The active cooling system (i.e. within the casing) was used to augment the radial temperature gradient in the case such that heat transfer coefficient was obtained from the extrapolation method described in Section 1.2. For this experiment, the ratio of $T_w/T_{aw} > 0.8$ and thus $q''$ vs. $T_w$ data were described by a first order fit with the slope interpreted as $h_{aw}$ [51]. The heat transfer coefficients determined here may be equivalent to those in an actual engine depending on the ratio of $T_w/T_{aw}$. If cooling, for example, drives the $T_w/T_{aw}$ ratio below 0.8 in a gas turbine engine, the experimental $h_{aw}$ data remains applicable if properly corrected, e.g. by correlations in [46]. Therefore, heat transfer coefficient is an important calculation in this study considering the physical wall temperature and heat flux profiles are of considerably lower magnitudes than those in engine hardware.

A heat soaking procedure was employed to ensure that the rig was at thermal equilibrium prior to data acquisition. This is critical process, particularly when thermal measurements are the focus of a test program. The compressor exit temperature requires the greatest time in the facility to reach equilibrium and was monitored during startup. A typical startup involved raising turbine inlet temperature ∼ 10
TABLE 4.1

ND-TRT DESIGN POINT

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheel Speed</td>
<td>10,144</td>
<td>RPM</td>
</tr>
<tr>
<td>$T_{T,inlet}$</td>
<td>356</td>
<td>K</td>
</tr>
<tr>
<td>$\pi_{T/T}$</td>
<td>2.3</td>
<td>-</td>
</tr>
</tbody>
</table>

K above the design condition and remaining at that operating point for about 30 minutes; inlet/recycling valves were then adjusted gradually until the design $T_{T,inlet}$ was reached. The external temperature history of the rotor casing (as well as of the embedded measurements) was tracked at multiple circumferential positions during the heat soak. The test was started when these temperature measurements agreed within a prescribed tolerance of less than 0.5 K. The average time of a heat soak was between 1.5 and 2 hours.

4.1.3 Primary Instrumentation

Custom rakes, manufactured by Aerodynamic Engineering Inc., were used to set the design operating point. Three inlet rakes (US-1) contained 5 Kiel-type total pressure probes at prescribed spanwise and circumferential positions. Each rake port was paired with a Type-K thermocouple for total temperature measurement. Three boundary layer (DS-1) and 10 exit (DS-2) rakes, designed with 6 Kiel-type probes per instrument, were used to measure conditions downstream of the rotor.

Data from arrays of casing embedded thermocouples (as described in Section 3.2.2) were input to an IHCP algorithm (detailed in Chapter 2). Wall temperature
and heat flux distributions were the outputs of this analysis, used to estimate heat transfer coefficient and other related quantities. A rectangular domain was defined with the 4 boundary temperatures profiles constrained by natural cubic spline functions. Here, a $\Delta x/\ell$ or $\Delta y/\ell = 1$ was specified, depending on the boundary. Temperature continuity was enforced at the corners of the domain. The internal sensor arrays were located at 4 circumferential positions. The locations of the measurement planes were constant relative to the nozzle vanes, i.e. same nozzle “clocking”. At a given condition, the IHCP was solved 4 separate times. The solutions, i.e. one for each location, were obtained using data from the corresponding thermocouple array. The plane-to-plane variations in the inverse solutions were less than 0.5 K. These variations are consistent with the small circumferential non-uniformities of the inlet temperature profile as well as the low level of random measurement and positional errors (that were deduced from Chapter 3 validation experiments) associated with the embedded sensors. In Section 4.3, a single inverse solution is presented at each test point corresponding to the average of those obtained from the 4 available arrays.

The data acquisition (DAQ) system was comprised of a custom LabVIEW™ interface with integrated devices from Nation Instruments™ (NI) and Esterline Corp. An Esterline NetScanner™ 98RK-1 was used to measure differential pressure. Individual measurements were referenced to a static port upstream or downstream of the rotor as appropriate. Reference pressures were measured by Setra® Model 270 transducers with an operating range of 0-137.9 kPA. Temperatures were acquired via 2 NI cFP-1804 chassis, with 4 NI cFP-TC-120 modules per chassis. A NI SCXI-1000 chassis was configured with 2 NI SCXI-1102 and 2 NI SCXI-1303 modules for an additional 128 temperature measurements. Analog voltage outputs were obtained from both NI cDAQ-9205 and NI cDAQ-9239 modules using NI cDAQ-9178 chassis. The 4 NI cDAQ-9239 modules (16 total measurements) were reserved for measurements requiring high precision such as torque and tip clearance. These modules are true
differential DAQ systems.

4.2 Computational Setup

The following is an overview of the numerical model and solver used in steady-state RANS simulations. These simulations have been validated extensively by Takakura [77] in a previous work using aerodynamic measurements. Here, the purpose of the CFD was to offer a point of comparison for the experimental casing heat transfer results obtained via the IHCP. The inverse measurement technique (validated in Chapter 3) provides a means of thermal validation for a commercial CFD code.

4.2.1 Numerical Model

To implement the steady solution, the flow domain was split into two sub-domains: the nozzle in a stationary (inertial) reference frame and the rotor in a rotating (non-inertial) reference frame. Information was communicated between the nozzle and rotor domains using a mixing plane model. The Spalart-Allmaras closure model was selected based on the work of Perez [70] and Takakura [77].

A schematic of the simplified flow geometry is shown in Figure 4.3. The inlet boundary (marked in red) was set 4.75 axial nozzle chords upstream of the nozzle leading edge, where a pressure inlet boundary condition was specified as uniform in the axial direction in accordance with experimental data. The inlet turbulence intensity was set to 5.5% based on measurements by Schmitz [73]. The exit boundary was extended 5 axial rotor chords downstream of the rotor trailing edge. A pressure outlet boundary condition was specified with an imposed constraint of radial equilibrium. This exit condition was established by setting the hub static pressure.

All other surfaces in the domain were modeled as walls. The momentum conditions were specified as in [77]. All walls other than the casing were set to adiabatic. For the thermal boundary condition at the casing, simulations were run for several
different constant temperatures, in addition to specified temperature profiles (based on experiment) and the adiabatic case.

The ideal gas law was specified to compute the air density, and the specific heat of air was set to a constant 1,006.43 J/kg-K. The thermal conductivity of air was input as a constant 0.0242 W/m-K, and Sutherland’s viscosity law was enforced. A constant molecular weight of 28.966 kg/kgmol was used for air.

The grid was generated by Takakura [77] using Pointwise® 17.1R4. Takakura conducted an accompanying grid refinement study. A structured mesh was chosen and the nozzle and rotor domains consisted of ∼2.3 and ∼7.6 million cells, respectively. A single rotor blade passage was modeled for use with a rotational periodic boundary condition. Grid points adjacent to the wall boundaries in the tip region were positioned such that $y^+ \sim 1$.

4.2.2 Solver

ANSYS® FLUENT® 17.0 was used to run the numerical simulations on the Center for Research Computing cluster. Details of the solver algorithm are provided by ANSYS® [8]. The pressure-based solver was applied to the 3-D compressible flow problem. Pressure-velocity was treated with the coupled scheme. Spatial dis-
cretization was set to second order upwind for all parameters (except pressure, set to standard) and gradients were computed using a least squares cell based approach. Additional information regarding the mixing plane formulation is available in [70] and [77].

4.3 Experimental Results and Comparisons to CFD

First, to give added confidence to the IHCP technique, an example that is representative of the typical temperature differences between internal measurements and an inverse solution (at the corresponding sensor locations) is provided in Figure 4.4. The inverse solution is that which minimizes an ordinary least squares norm defined by these differences, as in Equation 1.2. Here, axial sensor location as a percentage of rotor chord is plotted on the abscissa and the radial distance from the surface, scaled by a characteristic length, defines the ordinate. The squares mark the sensor locations and are colored by the difference between the measurement and IHCP output at that point in the domain.

![Figure 4.4. Typical difference between internal temperature measurements and an IHCP solution [K]](image)

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The differences are negligible upstream of the rotor where the internal temperature field is nearly constant and the heat flux into the casing is largest. The maximum differences (±0.35 K) are at the locations farthest away from the boundaries (i.e. 50% and 77% rotor chord and \( y/L \approx 0.13 \) and 0.23). The root mean square average difference over all measurements is less than 0.2 K. The result confirms that random errors in the embedded temperature measurements were small, i.e. an inverse solution governed by the heat equation can accurately describe the internal measurements. The combination of small random errors and the design of the thermocouple array such that Chapter 2 criteria were met, suggests that the inverse solver error is small (i.e. within approximately 3% of the temperature difference of the surface condition).

Contours of internal casing temperature were calculated by solving the IHCP and the results are presented in Figure 4.5. The embedded thermocouple data were acquired at the design aerodynamic operating condition. The internal measurement array was constructed such that \( \Delta y/\ell = 0.3 \). Temperatures are nondimensionalized by,

\[
\tau = \frac{T - T_{T,exit}}{T_{T,inlet} - T_{T,exit}}.
\] (4.1)

Here, the radial distance from the surface is shown as \( y/L \) and the axial domain is referenced to the rotor chord length. The label (a.) marks the condition where no cooling fluid was circulated in the OD cavity of the part. The casing temperature is observed to decrease with axial position as work is extracted from the flow by the turbine. The shape of the near wall contours at positions less than 60% rotor chord indicates that heat is being transferred from the gas to the casing. Downstream, there is a strong axial temperature gradient and the casing adds heat to the cooler flow.

The temperature solutions labeled (b.) – (d.) are for conditions in which the active cooling system was used. The cooling level was increased from (b.) \( \rightarrow \) (c.) \( \rightarrow \) (d.) in accordance with the decrease in the overall metal temperatures. One of the purposes
of the cooling system was to augment the radial temperature gradient into the casing. This system functioned according to design as is particularly evident based on a comparison between (a.) and (b.) at the upstream axial positions. The added cooling appears to have a smaller effect downstream where the heat transfer is mostly axial. Overall, the shape of the internal temperature fields remains relatively constant with cooling. The cooling acts to shift the temperature and heat flux profiles in a manner proportional to the heat transfer coefficient distribution.

Surface temperature and heat flux boundary conditions for each of the cases in Figure 4.5 are displayed in Figure 4.6. The nondimensionalized casing wall temperature is plotted as a function of axial position in (a.) and a nondimensionalized heat flux is shown in (b.).
Increased Cooling

Figure 4.6. Surface casing temperature (a.) and wall normal heat flux (b.) profiles for variable cooling conditions at design operation.

Here, heat flux is presented as,

\[ \phi_q = \left( \frac{q''}{k_{metal}} \right) \left( \frac{c}{T_{T,inlet}} \right), \]

where \( q'' \) is the physical heat flux (calculated from differentiating the temperature field), \( k_{metal} \) is the thermal conductivity of the casing, \( c \) is the rotor chord length, and \( T_{T,inlet} \) is the stage inlet temperature. The colors represent 4 different levels of active cooling which vary from no cooling (dark red) to maximum cooling (dark blue). The temperature profiles exhibit a consistent trend characterized by a \( \Delta x/\delta \approx 0.2 \). The profile shape changes minimally with the addition of cooling. The offset provided by the cooling system was limited by the specifications of the chiller and the diameter of the copper tubes.

Heat flux distributions are more complex than those of the wall temperature. For the no cooling case, the largest heat flux into the part is found upstream of the rotor. A negative \( \phi_q \) corresponds to heat flux into the casing by definition. The value remains relatively constant until approximately 35% rotor chord. There is a
slight decrease in heat flux before the rotor to a point near the leading edge with the addition of cooling. The incident heat flux decreases from 35% to nearly 90% chord and a change in sign is observed within this axial range, i.e., heat is transferred from the casing to the flow. The location of the adiabatic condition is dependent on the cooling level. As more cooling is added, the \( q'' = 0 \) point moves farther downstream, as expected. There is a decrease in the amount of heat transferred from the casing to the flow at \( x > 90\% \) chord, where the casing wall temperature approaches the gas temperature.

An uncertainty analysis based on the sensitivity coefficients was conducted using Equation 1.3. The variance of the internal temperature measurements was calculated according to,

\[
\sigma^2_{\dot{Y}} = \sigma^2_{TC} + \sigma^2_x \left( \frac{\partial T}{\partial x} \right)^2 + \sigma^2_y \left( \frac{\partial T}{\partial y} \right)^2,
\]

where \( \sigma_x \) and \( \sigma_y \) are the standard deviations of random positional errors. The effect of positional errors was recognized as proportional to the temperature gradients within the part and is additive to the measurement noise (\( \sigma_{TC} \)). Input variances values were set according to those presented in Section 2.5. The total uncertainty is a combination of the above contribution of random error, the Figure 2.9 estimation of inverse solver error using “exact” internal temperatures (i.e., assuming \( \sigma = 0 \) for \( \Delta x/\delta = 0.2 \)), and an estimation of the potential bias associated with the DAQ calibration. The error is effectively constant over the domain as the \( T_w \) distributions are of high \( \delta \) (i.e., regions of high wavenumber content are not present which would increase the overall uncertainty). In summary, an uncertainty in wall temperature of 1.2% of the stage \( \Delta T \) is expected. The estimated uncertainty in normalized heat flux is \( 1.3(10)^{-3} \), less than 5% of the inlet value for the no cooling case (using the same procedure as for the wall temperature).

The casing internal temperature field contains low wavenumber content in the re-
gion of the internal measurements and is thus subject to small gradients that would otherwise amplify positional errors. Inverse solutions were obtained from a relatively fine array of sensors and the combined effects of individual measurement errors (which are assumed small based on Chapter 3 and Figure 4.4) are reduced through regularization. This conclusion is in agreement with the Monte Carlo simulations in Chapter 2 when the $\Delta y/\ell$ criteria is met.

Inverse solutions from multiple cooling cases were used along with the extrapolation method (refer to Section 1.2) to quantify heat transfer coefficient. The result is given in terms of a Stanton number,

$$\text{St} = \frac{h_{aw}}{(\rho_{inlet})(u_{fluid})(c_{p,inlet})}. \quad (4.4)$$

The Stanton number is a measure of the heat transferred into a fluid to the thermal capacity of the fluid. Heat transfer coefficient is normalized by the product of the turbine inlet density, specific heat, and a representative flow speed.

Stanton number distribution is displayed in Figure 4.7 (b.) and the indirect method used to determine this parameter is illustrated in (a.). Here, nondimensionalized heat flux is plotted as a function of nondimensionalized wall temperature for 8 different levels of active cooling (solid black circles) at 3 different axial locations. The data at each location were fit with a first order polynomial, shown as dashed lines in (a.); line colors were used to distinguish between the different axial locations, marked by “X” symbols of the corresponding color in (b.). The slope of a polynomial generated using physical quantities was interpreted as the adiabatic heat transfer coefficient. The intersection of the best fit lines with $\phi_q = 0$ was taken as the adiabatic wall temperature. The process was conducted at all axial locations in the domain (only 3 are shown above for clarity) to produce the Stanton number curve. The small deviations of the heat flux vs. wall temperature from the linear fit at a given location provide confidence in the measurements.
The Stanton number is relatively constant over the domain. There is a decrease in 
St with axial position up to about 20% rotor chord. A slight increase in the parameter 
is observed aft of 20% rotor chord until roughly 60% chord, followed by a decrease 
moving further downstream. The trend is consistent with the rotor compressive 
heating phenomenon (discussed in Section 1.4).

The uncertainty in heat transfer coefficient is influenced by the number of mea-
surement points used to define the linear fit, the amount of heat flux, the temperature 
range achieved between the wall and adiabatic condition, and the uncertainty in wall 
temperature and heat flux measurements. It is estimated as 7% of the inlet value 
by statistical analysis. The 8 individual $q''$ vs. $T_w$ data points used to determine 
h_{aw} were perturbed by all 256 combinations of $\pm \sigma_{T_w}$ and $\pm \sigma_{q''}$. The 95% confidence 
interval of the vector of potential heat transfer coefficients was chosen as a measure 
of uncertainty. A conservative value is reported based on the largest confidence in-
terval calculated over the domain at the 16% rotor chord location. Similarly, the
uncertainty in adiabatic wall temperature is estimated as 3% of the stage \( \Delta T \). The slight upward bend in the Stanton number curve downstream of the rotor is within the measurement uncertainty. Physically, the St is probably constant or marginally decreasing in this region. The observation could be a consequence of the lack of internal temperature measurements beyond the 100% rotor chord location that, if available, would improve the inverse result.

Numerical simulations yield time-resolved information that is helpful in elucidating the physics of turbine casing heat transfer. Computationally predicted contours of casing adiabatic wall temperature are shown in Figure 4.8 (a.). Axial position relative to the rotor chord is plotted on the abscissa and circumferential position, \( \theta \), is on the ordinate. The locations of the rotor blade tips relative to the casing are identified using dashed black lines. Here, the direction of rotation is downward. The adiabatic wall temperature is high upstream of the blade row and close to that at the stage inlet. The small temperature variations in this region are due to the rotor potential field. There is a reduction in adiabatic wall temperature over the rotor passage as work is extracted from the fluid. The highest \( \tau_{aw} \) is observed over the blade tips as a consequence of rotor compressive heating. The adiabatic wall temperatures in this location are nearly 15% greater than the turbine inlet value.

The \( \tau_{aw} \) computations were circumferentially averaged to produce the dashed blue profile in Figure 4.8 (b.). Here, adiabatic wall temperature is given as a function of axial position. This process allows for the comparison of time-resolved CFD to time-averaged experimental results. The experimental adiabatic wall temperatures, obtained by applying the extrapolation method to inverse solutions, are shown on the same plot as a solid purple curve. Overall, the results are in excellent agreement, particularly between 0 and 80% rotor chord. Time-averaged adiabatic wall temperatures are relatively constant upstream of the rotor. The values then decay as a closely linear function of \( x \) starting at about 20% rotor chord until a nearly constant
Figure 4.8. Contours of $\tau_{aw}$ from CFD (a.) and comparison of the circumferentially averaged CFD profile to experiment (b.)

downstream condition is reached.

There is a greater rise in the experimentally determined adiabatic wall temperature upstream of the rotor to roughly 20% chord than observed in the CFD (this difference is outside of the measurement uncertainty). Aft of 80% rotor chord, the experimental curve is relatively constant with a small upward trend (similar to that observed in the experimental heat transfer coefficient). Whereas the CFD result continues to decay between approximately 80% and 100% chord to a lower downstream value than the experiment.

The downstream discrepancy between the experiment and CFD warrants further study at this time. The temperature fields are dominated by an axial gradient ($i.e. \partial T/\partial x$) aft of 80% rotor chord. The measurements here may therefore be less reliable due to the small amount of wall normal heat flux, $i.e.$ near 0. The extrapolation method used to calculate $T_{aw}$ would only amplify these potential errors. To test this theory, a casing could be designed with a cooling/heating system that allows for a greater $\partial T/\partial y$. Alternatively, the adiabatic wall temperatures could be measured directly on a casing of low thermal conductivity. This was attempted in
proof-of-concept experiments and is not recommended due to issues relating to the survivability of the epoxy in a turbine rig. Lastly, additional internal sensors could be placed at locations farther downstream (i.e. > 100 % rotor chord) to verify that the mismatch is not an artifact of extrapolating the inverse domain too far beyond the axial extent of the measurements.

A similar procedure as used by Lavagnoli et al. \[51\] was executed to output the adiabatic heat transfer coefficient from CFD. The fluid side heat transfer coefficient is numerically determined based on local flow field properties (e.g. turbulence level, temperature, and velocity profiles) input to a temperature wall function as described in \[8\]. A set of 4 different constant wall temperature boundary conditions, \(\tau = [0.35, 0.76, 0.83, 1.15]\), were assigned and the \(q''(x, \theta)\) field was computed for each case. The extrapolation method was applied in 2-D by fitting a line to \(q''(x, \theta)\) vs. \(T_w(x, \theta)\) at all \((x, \theta)\) locations. Information from the adiabatic wall temperature calculation was included in the fit for a total of 5 points per location. The slopes of the lines were taken as \(h_{aw}(x, \theta)\). The results were then used to calculate \(St(x, \theta)\), contours of which are plotted in Figure 4.9 (a.).

![Figure 4.9. Contours of St from CFD (a.) and comparison of the circumferentially averaged CFD profile to experiment (b.)](image-url)
The highest Stanton numbers occur far upstream and over the pressure-side edge of the rotor blades between approximately 40% and 100% rotor chord. Large St values over the rotor correspond to regions of supersonic absolute Mach numbers in the tip gap. The casing experiences a significantly lower heat transfer rate over the blade passage. Stanton numbers are approximately 1/4 of the domain maximum in this region.

The St computations were circumferentially averaged, as was done for the adiabatic wall temperature results, to yield the dashed profile in Figure 4.9 (b.). The experimental distribution is shown on the same plot as a solid orange curve. The experiment and CFD are in excellent agreement over the rotor domain. Here, the results are effectively equivalent in both magnitude and trend. Upstream of the rotor, the computed St is lower than in the experiment. The experimental and computational Stanton numbers experience similar rates of decay in the upstream locations that are characterized by a spatial offset. This is perhaps explained by variations in the experimental and computed thermal boundary layers due to geometric differences between the physical hardware and the model. The rig was assembled from individual components and small clearance gaps may have been present upstream of the casing that were modeled as solid surfaces in the CFD.

The casing surface does not respond to temperature changes at the blade rate, i.e. the thermal diffusion timescale is much slower than that of the blade-passing. Although time-resolved information is useful in understanding the physics of the turbine endwall aerothermal field, the casing is subject to a time-averaged temperature field as shown in Figure 4.10 (a.). Here, the temperature on the casing wall is constant in \( \theta \) and varies with axial position. This profile was input as a casing wall temperature boundary condition in CFD. The time-averaged contours reduce to the Figure 4.10 (b.) curve (\( \tau \) vs. % rotor chord) which was chosen to match the experimentally obtained wall temperature distribution for the no cooling situation. The accompanying
simulations provide a means of comparison between the time-averaged heat transfer profile computed from gas-side quantities and that estimated from the IHCP (i.e. using embedded thermocouple measurements).

Contours of wall normal heat flux are an output of the computations described above. Figure 4.10 (c.) shows the $\phi_q$ results plotted as a function of axial and circumferential position. The time-resolved heat flux is similar to the Stanton number computations shown in Figure 4.9 (a.). The regions of highest heat flux (blue) occur over the pressure-side of the blade as a consequence of the leakage flow. The maximum heat flux at these locations is roughly 6 times higher than the inlet value.

The $q''$ computations were circumferentially averaged (dashed pink curve) and compared to the experimentally determined time-averaged heat flux profile for the no cooling case (solid red curve) in Figure 4.10 (d.). The curves are of similar trend and show an excellent overall agreement, particularly between approximately 40% and
80% rotor chord. There is an offset between the experimental and CFD distributions at positions less than 20%. Here, the experimentally predicted heat flux into the casing is greater than the CFD value. The observation is consistent with the higher experimental Stanton numbers in this region as compared to the computations (refer to 4.9(b.)). Potential variations in the actual and computed thermal boundary layers could be used as an argument for shifting the computational curve downward such that the differences in \( \phi_q \) aft of 80% rotor chord would be approximately halved.

4.4 Surface Roughness Effects

A major utility of the inverse measurement technique is its ease of application in geometries with complex surfaces. The turbine designer may be motivated to add a surface treatment to the over-rotor casing for purposes of tip leakage desensitization. The stage efficiency and rub dynamics may be improved with such treatments; however, the added surface roughness on the casing wall will augment heat transfer. A casing was machined with a surface roughness of approximately 1% of the characteristic length, \( L \). The casing contained the same internal temperature sensor and active cooling package as the smooth wall (SW) part detailed above and will be termed the “PCT” for the remainder of the section. Internal temperature data were acquired for the PCT at the Table 4.1 operating point and used in an inverse analysis to quantify casing heat transfer. A summary of the results for the PCT will be presented in this section followed by a comparison to those for the SW hardware.

Contours of internal casing temperature obtained via the IHCP are presented for the PCT in Figure 4.11. The axial distance as a percentage of rotor chord is on the abscissa and radial distance from the surface, \( y/L \), is on the ordinate. Results from the no cooling configuration are denoted by (a.) and the temperature field for the maximum cooling configuration is shown in (b.). Here, the heat flux is into the part over the entire domain for both situations based on the shape of the contours. The
addition of cooling not only reduces the overall temperature in the part, but appears to have a much greater effect on the radial temperature gradient than observed for the SW casing. The PCT will thus result in higher values of heat transfer coefficient than the SW casing.

Surface temperature and heat flux distributions are shown for tests with no cooling (dark red), medium cooling (light red), and maximum cooling (dark blue) in Figure 4.13. Normalized wall temperature is plotted against axial position in (a.) and $\phi_q$ is displayed in (b.) in the same manner.

The wall temperatures remain relatively constant at locations less than 20\% rotor chord. The subsequent decay of $\tau$ is closely related to a linear function of $x$. Profile trends are not changed significantly (for both $\tau$ and $\phi_q$) with the addition of cooling, i.e. cooling preserves the shape of a distribution and shifts the magnitude to a lower value. This observation is consistent with the SW results. Incident heat flux increases from an upstream value to a maximum point at approximately 30\% rotor chord for the no cooling case. The location of maximum heat flux is displaced slightly upstream as the cooling level is increased. Aft of 30\% rotor chord, $\phi_q$ decreases with a trend that is again characterized by a nearly linear function of $x$. 

Figure 4.11. Rough wall casing internal temperature solutions via the IHCP for (a.) no cooling, and (b.) max. cooling tests
Figure 4.12. B.C. for casing with elevated surface roughness: temperature (a.) and wall normal heat flux (b.) profiles for variable cooling

Heat transfer parameters for the PCT are compared to the SW results in Figure 4.13. The SW data are plotted as blue curves and the PCT results are shown as red. Figure 4.13 (a.) is a comparison of $\tau$ vs. axial position for the no cooling tests. The PCT is at a higher surface temperature than the smooth wall casing over the entire domain. The SW and PCT surfaces are defined by the same datum. The roughness height of the PCT is considered as beyond this datum, i.e. $y/L = -0.01$, and has a similar effect to adding fins into the flow. The PCT wall temperature profile is effectively constant between approximately -20% and 20% rotor chord. There is a more substantial decay in $\tau$ over this region for the SW casing. The PCT geometry has a limiting effect on the axial heat transfer within the part and results in wall temperatures that remain closer to the inlet value for a longer axial extent.

Wall heat flux profiles are assessed in (b.). Far upstream, both sets of hardware experience the same incident flux. The slope of $\phi_q$ remains relatively constant until roughly 30% chord for the SW, as opposed to for the PCT, where $\phi_q$ increases to a maximum value over the same range. The wall normal heat flux then experiences a
Figure 4.13. Comparison of smooth wall (blue) and rough wall (red) casings: (a.) surface temp. for no cooling, (b.) heat flux, (c.) St, and (d.) $\tau_{aw}$

greater rate of positional decay for the SW than the PCT. Downstream, the casing heats the gas for the SW, while heat transfer remains into the part for the PCT.

Stanton number distributions are compared in (c.). The PCT St is on average 80% greater than that of the SW over the entire domain. The increase in Stanton number between approximately 20% and 60% rotor chord for the SW casing is not observed for the PCT, where St remains constant in the same region. This increase in the SW part was associated with the rotor compressive heating phenomenon. The corresponding lack of an increase for the PCT indicates that the treatment is acting to suppress an adverse effect of the leakage flow.

Adiabatic wall temperatures are examined in (d.). The PCT $\tau_{aw}$ is on average 5% greater than that of the SW over the entire domain. Larger differences in these values occur aft of 40% chord. Comparatively, the PCT value remains much more constant.
over the domain than the SW result. The small upward trend in adiabatic wall temperature (and St) at downstream locations is not reflected in the PCT results. This is perhaps an artifact of the increased heat transfer rate associated with the PCT (where the near 0 heat transfer to the SW part may lead to inaccurate calculations via the extrapolation method).

4.5 Conclusions

The inverse measurement technique was successfully demonstrated in a turbine rig, specifically in two separate over-rotor casing components. The work in Chapter 2 was used to aid in uncertainty quantification for a set of heat transfer parameters. Here, the quoted uncertainties are consistent with or better than those given in the literature, e.g. [80], for the same parameters (that are typically either measured or calculated using heat flux gauges). Experimental results yielded similar conclusions to other smooth wall casing studies in the literature, e.g. [28]. The inverse solutions from the smooth wall casing were used to validate CFD results, while the experimental and computational comparisons provided confidence in the measurement technique. Additionally, the rough wall casing to which the inverse analysis was applied represents a situation where HFGs are generally not applicable and computations are difficult.
CHAPTER 5

CONCLUSIONS

The development, validation, and application of an inverse heat transfer measurement technique were the major elements of this study. The method provides an alternative to contact sensors. The inverse technique should be considered in high heat flux situations and/or when complex geometries prohibit the installation of surface instrumentation. This chapter will summarize the major findings of each of the three components of the work and identify opportunities for additional studies.

The solution of the inverse conduction problem required an array of internal temperature measurements. A theoretical model was developed to identify and evaluate an internal sensor spacing criteria for accurate inverse solutions. The analysis was performed for both coarse and fine sensor grids as it was recognized that cost restrictions may limit the amount of available measurements in an experiment. The inverse problem was posed as that of a function estimation involving cubic splines characterized by \( \ell \). A set of test surface boundary conditions parameterized by \( \delta \) were considered. A sensor spacing criteria of \( \Delta x/\delta < 0.4 \) and \( \Delta y/\ell < 0.5 \) was suggested. Random measurement noise and positional errors were considered in defining these criteria.

To implement the technique, thermocouples were embedded in machined holes within metal hardware using an epoxy. The epoxy and metal were described by a thermal conductivity ratio of \( k_2/k_1 \approx 1 \). A series of two and three dimensional validations were performed which showed that the presence of the epoxy did not significantly bias the thermal measurement for a range of \( 0.75 \leq k_2/k_1 \leq 1.25 \). The
inverse conduction problem was thus solved on a homogeneous domain in which the material property mismatch between the epoxy and metal was neglected. A set of experimental validations were completed for three different boundary conditions: $\partial T/\partial x$ dominant, $\partial T/\partial y$ dominant, and case where gradients were present in both $x$ and $y$. Independent surface measurements via contact thermocouples and infrared thermography were used to validate the inverse technique within measurement uncertainty.

The inverse method was applied in a smooth wall transonic turbine casing and a casing with enhanced surface roughness. An active cooling system was installed in the component to control the radial temperature gradient in the part. The system allowed for the determination of heat transfer coefficient. Experimental findings were consistent with previous studies in the literature. The experimental data were used to validate a RANS solution. Overall, the results provided confidence in the measurement technique. The addition of surface roughness was shown to increase Stanton number by an average of 80% over the casing domain.

There are opportunities for future extensions of the work presented here. For instance, the accuracy of the inverse technique could be improved for situations in which the sensor spacing criteria is not met. The data processing algorithm would be modified for sparse grids of internal measurements, for example, by considering functional forms with broader support such as lower order terms in Fourier-Legendre series. For this class of problems, further regularization would be necessary to obtain physical solutions. A Bayesian approach, specifically the use of the Markov random field to model the prior distribution, would provide a means of regularization for coarse grids subject to high levels of random error. Test functions associated with higher wavenumber content over multiple axial locations could be considered and constrained by functional forms of more compact support such as wavelets. A demonstration of accurate predictions with fewer internal measurements would con-
tribute to wider use of the inverse technique.

The integration of a nonlinear direct solver would extend the applicability of the technique to experiments with higher temperature and heat flux ranges. This addition would be particularly important for acquiring inverse measurements in full-scale turbine tests or in geometries with complex internal cooling systems. A further extension to a two-material domain would be applicable to turbine casings with abradable coatings. Here, there would be a significant thermal conductivity variation between the two materials as abradable coatings are insulators. The technique could be applied in other components such as nozzle vanes or turbine rotor blades where a three-dimensional analysis would be required and more complex boundary conditions are present.

Additional steady and transient data for both the smooth and rough casings are available for future study. Examples of this include data at off-design conditions with purge flows and variable tip clearances. Time-resolved aerodynamic information from the validated CFD code could be studied for different tip leakage desensitization geometries in an attempt to reduce casing heat transfer while considering aerodynamic performance.


