WIND TUNNEL EXPERIMENTS ON THE EFFECT OF COMPRESSIBILITY ON
THE ATTRIBUTES OF DYNAMIC STALL

A Dissertation

Submitted to the Graduate School
of the University of Notre Dame
in Partial Fulfillment of the Requirements
for the Degree of

Doctor of Philosophy

by

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______________________________
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March 2012
Experimental results and analysis of an airfoil oscillated in pitch are presented at Mach numbers from 0.2 to 0.6 and Reynolds numbers up to \(3.5 \times 10^6\). The design of a novel pitch-mechanism and dynamic stall rig is discussed. High-speed surface pressures and the airfoil’s instantaneous incidence angle are simultaneously sampled and used to characterize the temporal pressure field, static and dynamic loading, and cycle and time-resolved aerodynamic stability under harmonic prescribed incidences.

This work focuses on three distinct problems in regard to dynamic stall and the influence of Mach number, as follows:

1. **The stall vortex convection properties:** The low pressure signature of the stall vortex’s travel aft over the chord is tracked through deep dynamic stall conditions and moderate to high reduced frequencies. The gestation or growth period of the dynamic stall vortex and its ultimate convection rate are quantified. Increased Mach number decreased the gestation period of the stall vortex and the duration the stall vortex resided over the airfoil chord. The surface pressures and surface pressure gradients found at low-speed as the stall vortex migrates over the chord are shown to be substantially altered following shock-induced dynamic stall.
2. **Shock induced dynamic stall and surface flow features:** A focused schlieren system is designed and constructed to glean information regarding the temporal evolution of shock structures on the airfoil’s upper surface. The flow visualization is used to interpret the corresponding surface pressures measured through the on-board pressure instrumentation. A $\lambda$-shock is shown to develop near the leading-edge. The foot of the developing shock remains near the leading-edge whereas the shock front travels aft over the chord. At a critical incidence, the flow separates, and an off-surface shock structure propagates upstream toward the leading-edge. The effect of roughness on the shock’s formation is examined.

3. **Compressibility effects on aerodynamic damping:** The ensemble averaged pitch-moment is used to determine the net cycle damping. The time-resolved damping, discussed here for the first time, is shown to be calculable through a Hilbert transform of the pitch-moment. This time-based damping analysis offers new incite into the mechanism of stall flutter in helicopter rotor operations. The test includes a comprehensive investigation of forcing parameters (mean angle of attack, amplitude, and reduced frequency), including the effect of free (untripped) and forced (tripped) laminar to turbulent transition. It is demonstrated that the cycle-integrated aerodynamic damping coefficient masks the physics underlying the destabilizing effect of the dynamic stall process. In particular, conditions that exhibit positive cycle-integrated aerodynamic damping, often include time intervals of large negative aerodynamic damping during the pitching cycle. These negatively-damped portions of the pitch cycle could account for high-frequency stall flutter divergence from the rotor control input in helicopter applications.
This work is dedicated to my mother, father, and family.
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LIST OF SYMBOLS

English

A  analytic series angle-of-attack, $A = \alpha + i\bar{\alpha}$
$\mathcal{H}[]$  Hilbert transform operator
$\mathcal{H}^{-1}[]$  inverse Hilbert transform operator
$M$  analytic series pitch-moment, $M = M + i\dot{M}$
$\mathcal{Y}$  dummy analytic series, $\mathcal{Y} = y + i\dot{y}$
$A(t)$  time-resolved damping magnitude or envelope, $A = \sqrt{M_R^2 + M_I^2}$ [Nm]
$A_1$  dummy forcing amplitude
$A_\alpha(t)$  envelope function of angle-of-attack data-series, $A_\alpha(t) = \sqrt{\alpha^2 + \dot{\alpha}^2}$
$A_{C_m}(t)$  dimensionless, time-resolved damping amplitude or envelope, $A_{C_m} = \sqrt{C_{M_R}^2 + C_{M_I}^2}$
$b$  semi-chord, $b = 2c$ [in]
$B_a$  influence coefficient resulting from changes in angle-of-attack
$C$  viscous damping constant [lbf-fts/rad or Nms/rad]
$c$  chord length [in]
$c_a$  speed of sound
$C_a$  dimensionless, chord or axial force coefficient
$C_l$  dimensionless, coefficient of lift
$C_m$  dimensionless, coefficient of moment
$C_n$  dimensionless, normal force coefficient
$C_p$  dimensionless, coefficient of pressure
$C_w$  dimensionless, work-per-cycle coefficient. $C_w \equiv \int C_{m_{real}} d\alpha$
$C_{d_p}$  dimensionless, coefficient of drag (viscous term excluded)
$C_{m_{real}}$  real component of Theodorsen’s solution for a pitching airfoil
$D_p$  drag force (viscous term excluded) [lbf or N]
\( \frac{dy}{dx} \) airfoil surface slope, \( \frac{dy}{dx} = \tan^{-1} \theta \)

\( f \) prescribed oscillation frequency [Hz]

\( f_1 \) 1/rev or cyclic input frequency, [Hz]

\( f_2 \) torsion input frequency, [Hz]

\( f_s \) sampling frequency, \( f_s = 5050.50 \) [Hz] for all data

\( h(t) \) time-dependent viscous aerodynamic damping, [lbfs/rad or Ns/rad]

\( h_0(t) \) time-dependent viscous aerodynamic damping coefficient

\( I \) Structural inertia, [slug-ft\(^2\) or kgm\(^2\)]

\( K \) spring-stiffness constant, [lbf-ft/rad or Nm/rad]

\( k \) reduced frequency, \( k = \pi f c/U_{\infty} \)

\( L \) lift force [lbf or N]

\( M \) pitch-moment, [lbf-ft or Nm]

\( m \) exponent used in vortex trajectory fitting

\( M_1 \) inertia term in aerodynamic equation-of-motion, [kgm\(^2\)]

\( M_2 \) aerodynamic viscous damping term in aerodynamic equation-of-motion, [Nms/rad]

\( M_3 \) aerodynamic stiffness used in aerodynamic equation-of-motion, [Nm/rad]

\( N \) number of complete pitching cycles in a data-series

\( n \) dimensionless, normal direction to the airfoil surface; or generic integer, \( n = 1, 2...N \)

\( P \) pressure [psi or Pa]

\( P_0 \) total pressure [psi or Pa]

\( P_s \) static pressure [psi or Pa]

\( q \) dynamic pressure, \( q = \frac{1}{2} \gamma P_s M_{\infty}^2 \) [psi or Pa]

\( R \) rotor radius [ft]

\( r \) radial rotor coordinate

\( R^2 \) coefficient of determination in regression analysis of convection properties

\( R_d \) average roughness height, [\( \mu m \)]

\( r_i \) geometry (length) used in kinematic study of pitch-mechanism

\( R_{rms} \) root-mean-square roughness height, [\( \mu m \)]

\( s \) dimensionless, tangent direction along the airfoil surface, \( s = x \cos(\theta) \)

\( S^+ \) dimensionless, surface vorticity flux
$S_1$ dimensionless, peak surface vorticity flux where $dP/dx < 0$, source of strength of the dynamic stall vortex

$S_2$ dimensionless, peak surface vorticity flux where $dP/dx > 0$, vorticity sink

$T$ temperature [$^\circ$F]; oscillation period, $T = 1/f [s]$

$t$ time [s]

$U$ airspeed, [length/time]

$U_\infty$ freestream airspeed, [m/s]

$U_L$ blade section normal velocity

$U_s$ air velocity tangential to airfoil surface, [ft/s or m/s]

$u_{rotor/U_\infty}$ relative velocity of the rotor-blade with respect to the flight speed

$V$ transpiration velocity, [ft/s or m/s]

$V/U_\infty$ convection speed of the stall vortex

$W_{cycle}$ Work-per-cycle, [lbf-ft or Nm]

$X$ horizontal coordinate used in kinematic analysis of mechanism

$x$ dimensionless, chordwise position, $x = x^*/c$; or dummy variable

$x^*$ chordwise position [in]

$x_{ac}$ dimensionless, aerodynamic center

$x_{C_{min}}$ dimensionless chord location of minimum $C_p$

$x_{cp}$ dimensionless, center of pressure

$x_{DSV}$ dimensionless position of the dynamic stall vortex

$x_{sonic}$ dimensionless, chordwise extent of supersonic flow

$Y$ vertical coordinate used in kinematic analysis of mechanism

$y$ dimensionless, chord normal position, $y = y^*/c$; or dummy variable

$y^*$ chord normal position [in]

$r$ position vector used in kinematic analysis of pitch-mechanism

$M_\infty$ Mach number

$Re_c$ dimensionless, Reynolds number based on the airfoil chord length

Conventions

$1/rev$ 1 revolution of a single rotor-blade

$1P$ single frequency, harmonic forcing

$B$ baseline or unmodified airfoil

$MT$ masking-tape trip placed on airfoil leading-edge
P320 P320 grit tape trip placed on airfoil leading-edge
P400 P400 grit tape trip placed on airfoil leading-edge

Greek

$\alpha$ angle of attack [°]
$\alpha_0$ mean angle of attack [°]
$\alpha_1$ oscillation amplitude for 1/rev forcing [°]
$\alpha_2$ oscillation amplitude for torsional forcing [°]
$\alpha_{\text{max}}$ maximum incidence [°]
$\alpha_{\text{min}}$ minimum incidence [°]
$\beta$ angle used in pitch mechanism kinematic analysis [rad]; Prandtl-Glauert compressibility correction, $\beta = \sqrt{1 - M_{\infty}^2}$; dummy constant used in explanation of the properties of the HT
$\delta\tau$ dimensionless, stall vortex residence time
$\delta\tau_g$ dimensionless, stall vortex gestation period
$\Delta C_p$ dimensionless difference in trailing-edge pressure coefficient between when stall vortex resides over the last chordwise pressure port and when the stall vortex begins to convection downstream
$\delta$ difference operator; stiffness ratio
$\gamma$ ratio of specific heats of air
$\gamma_1, \gamma_2$ generic phase angles, [rad]
$\kappa$ aerodynamic spring or elastic stiffness, [N/rad]
$\mu$ dimensionless, advance ratio, $\mu = U_{\infty}/\Omega R$; dynamic viscosity [Pa·s]
$\nu$ kinematic viscosity, [ft²/s or m²/s]
$\Omega$ rotor frequency [RPM or cps]; or angle with respect to the horizontal used in the pitch-mechanism analysis, [rad]; or arbitrary motion phase angle as described in the damping analysis; or vorticity as described in the surface vorticity flux equation [1/s];
$\omega(t)$ instantaneous circular frequency, [rad/s]
$\omega_0$ natural frequency, [rad/s]
\( \omega_1 \) 1/rev or cyclic input circular frequency (often denoted as simply \( \omega \)), \( \omega_1 = 2\pi f_1 \) [rad/s]

\( \omega_2 \) torsion input circular frequency \( \omega_2 = 2\pi f_2 \) [rad/s]

\( \phi \) phase angle [radians or degrees]

\( \psi \) azimuthal rotor coordinate ['\( \psi \)']; time-resolved phase different between \( \alpha \) and aerodynamic pitch-moment [rad]

\( \rho \) density [kg/m\(^3\)]

\( \sigma \) solidity, \( \sigma = \text{area}_{\text{rotor-blades}}/\pi R^2 \)

\( \tau \) non-dimensional time, \( \tau = \pi f_1 t/\mathcal{K} \)

\( \tau_{x=x_{dsy}} \) dimensionless instant in time when the stall vortex is first measured

\( \theta \) input angle, \( \theta = \omega t \) used in pitch mechanism analysis, [rad]; angle of airfoil surface with respect to the horizontal, [rad]

\( \theta_i \) \( i = 1, 2, ... 11 \), angle with respect to the horizontal used in kinematic study of pitch mechanism

\( \xi \) cycle-integrated, 2D total aerodynamic damping [lbfs/rad or Ns/rad]

\( \Xi_{\text{cycle}} \) cycle-integrated, dimensionless 2D aerodynamic damping factor

\( \xi \) total aerodynamic damping, [Ns/rad]

Subscripts

\( \alpha \) pitch contribution

\( \infty \) freestream value

\( i \) imaginary part, component out-of-phase with \( \alpha \)

\( r \) real part, component in-phase \( \alpha \)

\( \max \) maximum value

\( \min \) minimum value

airfoil value defined at airfoil surface

\( c/4 \) airfoil quarter-chord, \( x=0.25 \)

cycle net value found through a single oscillation period

ds dynamic (lift) stall value

\( f \) ultimate convection property

\( ff \) forward flight value

inlet windtunnel inlet value
Local valued (used to define a local Mach number on the airfoil surface, $M_l$)

- $ms$: Dynamic (moment) stall value
- $ss$: Static stall value
- $w$: Wall-bounded or initial convection property

**Superscripts**

- $'$: Fluctuating quantity
- $*$: Complex-valued; for $x^*$, $x^*_{DSV}$, and $y^*$, the asterisk denotes dimensioned values
- $\bar{}$: Amplitude
- $\dot{}$: Time derivative, $\frac{d}{dt}$
- $\tilde{}$: Hilbert transformed data-series
- $D$: Pitch-down half-cycle, $\dot{\alpha} < 0$
- $U$: Pitch-up half-cycle, $\dot{\alpha} > 0$
- $P$: Pressure side of the airfoil
- $S$: Suction side of the airfoil
ACKNOWLEDGEMENTS

First and foremost, I would like to thank Gabrielle Young for her utmost patience, unyielding support, and constant encouragement. At times when I didn’t, she instinctively knew when I had to get to work, when I had to take a break, and when I had to move on to the next thing. Without her, I would still be writing the introduction.

I would also like to address special thanks to my advisor, Dr. Thomas C. Corke who provided guidance, wisdom, and, what at times seemed like clairvoyance into the challenging and exciting problems centered on dynamic stall and plasma flow control. He provided an opportunity to research and study a problem that I have come to greatly enjoy. He is a true mentor to me.

I would also be amiss not to thank (many times) Dr. Flint O. Thomas who not only serves on my committee but acted as more of a co-advisor throughout this research work. His office door was always open, his advice was always brilliant, and his depth of knowledge was bottomless.

I am forever indebted to Dr. Mark Wasikowski of Bell Helicopter. Mark and I spent many nights in the lab together working on the dynamic stall rig, writing code, and trying to figure out something that would make all of this research come together. Mark was a constant resource and phenomenal partner in this work. I doubt many other graduate students have had the fortune of working with such an invested industry partner.

Although not directly involved with my research, I would like to thank Dr. Patrick
Dunn, for whom I worked as a teaching assistant most of my semesters at Notre Dame. Dr. Dunn taught me how to think outside of the box, and is a continual reminder of how fun engineering can be.

Although this list is long, and this acknowledgment is further down on the list, it by no means indicates that I consider the help, never-ending enthusiasm, and constant drive of Dusty Coleman and Katie Thorne, two graduate students I have worked on this project with, as anything less than amazing. Katie and I started this work together, and, while it saddened me to see her depart onto astronomy, I am forever grateful for her help. We built the dynamic stall rig together, and without her, the pitch-mechanism would still be completely misaligned. DC, as I call him, was and still is an invaluable aspect to this work. He had a hand in almost everything contained in this document. Not aware of what he’d be getting himself into, Dusty stepped up to the challenge of this project and, somehow, along the way, I convinced him that I knew what I was doing. I look forward to reading his contributions to the field in the near future.

I also owe a lunch, dinner, and well-deserved vacation to Robert Chlebek, Leon Hluchota, Rodney McClain, Joel Preston, Mike Swadener, and Marilyn Walker. These people were the gold cogs behind this work. Rob and Rod spent many sleepless nights at Whitefield Laboratory as I slowly varied the collective angle of the pitch mechanism, again and again. Simply put, Mike solved our problems – when something went awry, Mike dropped what he was working on, and offered his full support. His advice and craftsmanship were vital to this research. Joel spent countless hours designing, building, and debugging our electronics. And I spent countless hours bugging Joel, in my attempts to gain some of his vast wisdom. His door was always open. Leon built several of the components of our test rig and was another, critical member of this team. His handiwork saved our tunnel entries on more than one occasion. Last, but not least, Mar-
ilyn helped with so many aspects of the project, and, simply, could always be counted on to make my life easier.

A special thank-you is owed to the University of Notre Dame who granted me a Presidential fellowship to pursue my research ambitions and thirst for education.

Thank you to all my peers and colleagues whom have made my experience at Notre Dame a most enjoyable one.

Many bones go out to my canine companions, Callie and Olive, who stayed by my side dutifully through this (long) writing process. They were there when long nights turned into even longer mornings, sacrificing many walks in the name of academia.

University of Notre Dame, Patrick O. Bowles
March 2012
CHAPTER 1

INTRODUCTION

*The helicopter approaches closer than any other [vehicle] to fulfillment of mankind’s ancient dreams of the flying horse and the magic carpet.*

-Igor Sikorsky

Dynamic stall manifests itself on an airfoil subject to rapid, transient motion where its incidence surpasses the static stall limit[106]. The fluid interactions and aerodynamic attributes that accompany the unsteady motion stand in stark contrast to the features exhibited in steady flow. These salient characteristics include:

1. a delay in separation to angles of attack exceeding the static stall limit;
2. an eruption of vorticity from the leading-edge that organizes into a well-defined structure, the dynamic stall vortex (DSV), and then convects aft over the chord;
3. a vortex induced fluctuating, non-linear pressure field;
4. massive excursions in aerodynamic loads;
5. substantial hysteresis when the forces are viewed as functions of angle of attack;
6. and the possibility for a positive transfer of energy from the freestream to the lifting-body – this leads to the stall flutter phenomenon.

More than sixty years of experimental, computational, and empirical work has focused on dynamic stall, motivated by dynamic stall’s impact on rotorcraft, “supermaneuverable” fixed-wing aircraft, compressors, wind turbines, as well as birds, marine
life, and insects. Despite this large body of literature – reviewed in Chapter 2 – dynamic stall continues to promote investigations into the physics inherent to the problem as well as strategies to eliminate or control its detrimental aspects.

The current research investigates dynamic stall with an eye toward rotorcraft operation; high-load conditions, i.e., rapid maneuvers, high-altitude flight, or high-forward velocity, can result in dynamic stall events. Ultimately, the problem stands as the principal barrier restricting the helicopter’s flight envelope. Exceeding the stall boundary results in an abrupt growth in blade torsion, excessive load transmission to the control system, and increased airframe and cabin vibration - see Johnson[76], Tarzanin[79], McCroskey and Fisher[107], Benson[6], and Bousman[8]. Deep dynamic stall (discussed in Chapter 3) induced airloads consume rotor and control system fatigue life[6, 100] and result in costly maintenance and undue risk. Usable drivetrain power is neglected, ignoring decades of engine and transmission advances[100, 126].

Dynamic stall is traditionally considered a retreating-blade problem, where the diminished dynamic pressure exhibited in forward flight demands blade operation at large angles of attack, sometimes exceeding the static stall angle. Bousman[8], through work on the UH-60A Airloads Program, found that dynamic stall occurred at multiple rotor locations – on the retreating-blade, as well as in the fourth and first quadrants of the rotor disk.

On the opposite side of the rotor disk, high-speed forward flight promotes shock induced stall on the advancing blade. High altitude flight leads to a slight increase in Mach number of the blade’s oncoming flow due to a decrease in temperature and drop in the speed of sound, $c_T$. Additionally, blade loading increases due to the drop in density. Although rotorcraft operate at altitudes generally less than 20,000 ft, small variations in the Mach number cause abrupt changes in dynamic stall onset characteristics.
Dynamic stall induces large, blade-root torques. The torque has two important contributors – the blade pitch moment, which grows to prohibitive magnitudes through the dynamic stall process itself, and, additional, rapidly fluctuating moments caused by the blade’s aeroelastic response. Extensive prior research, most notably the work of McCroskey and colleagues [10, 15, 99, 101, 103–106, 108, 109], has focused on the pilot control input to the rotor-blade, studying the loads and aerodynamic damping that result from variation in the collective and cyclic of the controlled motion. Fewer studies have focused on shock-induced dynamic stall, particularly the aerodynamic damping behavior witnessed under compressible flow conditions. Thus, the first objective of this research is to examine, in detail, dynamic stall under high Mach and Reynolds numbers.

A corollary to this objective, while considerably complicating matters, is the fact that the boundary-layer near the leading edge of rotor-blades is not well understood; Chandrasekhar et al. [27] noted that it is unlikely that a laminar separation bubble resides on an in-flight rotor. Numerical and experimental results indicate that the separation bubble plays a critical role in determining the stall characteristics – see, again, § 2.2.2. The airfoil tested here exhibits a well-defined separation bubble in steady flow; in order to remove potential dissimilarity between the wind tunnel tests and flight dynamic stall behavior, several trip elements were applied to the leading-edge of the airfoil.

The second objective of the design of the Notre Dame Dynamic Stall Facility concerns the aerodynamic loads due to elastic motion. Certain combinations of blade motion and dynamic stall result in stall flutter – a single-degree-of-freedom, self-triggered and self-limiting pitch motion of an elastic body [51]. Carta and Lorber [23, 94] describe propeller stall flutter as a series of events initiated by a small amplitude oscillation which quickly grows into a larger, constant amplitude oscillation at the blade’s natural
frequency. The end-state, termed a limit-cycle, occurs when the elastic body’s structural damping inhibits further growth in pitch amplitude, balancing the aerodynamic work gained from the airstream. Unlike propeller stall flutter, the rotor-blade, under most conditions, fails to demonstrate limit-cycle behavior due to a stabilizing reduction in angle of attack and reduced pitch moment as the blade reaches the advancing side of the rotor disk. Still, the unsteady aerodynamic work and transfer of energy from the airstream to the rotor-blade creates significant blade-twist and can rapidly alter the blade incidence, generating additional loads, high frequency blade motion, and dangerous overall rotor dynamic response.

The rotor-blade angle of attack is a function of the pilot control input (collective and cyclic), the blade elastic deformation, and an induced angle resulting from various downwash velocities within the rotor flight environment. Traditional wind tunnel simulations of dynamic stall incorporate rigid, two-dimensional (2D) airfoils that inhibit the elastic twist of the model. As discussed by Liiva and Davenport[91], a rotor-blade rarely executes simple sinusoidal pitch oscillation – its incidence combines the pilot’s 1/rev input with varying amount of higher-frequency elastic motion. Previous stall flutter research relies on flight tests[6], full or sub-scale rotor models[9, 107, 144] or torsion springs built into the pitch drive[91] to investigate stall flutter loads. Here, considerable effort in the design of the facility was directed at superimposing a second oscillation amplitude onto the motion of the airfoil in order to mimic the elastic response of a rotor-blade. A unique pitch mechanism was designed and constructed to superimpose the blade torsional deformation angle. The mechanism can prescribe a time-dependent, biharmonic incidence of the form,

$$\alpha(t) = \alpha_0 + \alpha_1 \sin(2\pi f_1 t) + \alpha_2 \sin(2\pi f_2 t),$$  

(1.1)
where I describes the cyclic or ‘1P’ behavior, and II represents the torsion mode, which often occurs in non-integer multiples of the 1P, e.g., 6.7P.

A significant amount of data has been acquired under biharmonic forcing, and has revealed that small amplitude ($\alpha_2 < 2^\circ$), high frequency perturbations drastically alter the aerodynamic loads and spectral load behavior. The underlying motivation of the mechanism design, however, deals with the effect the biharmonic motion has on the aerodynamic damping. These tools are still being developed as the underlying assumptions used to define aerodynamic damping (single frequency harmonic motion) are not adequate to analyze these datasets. As such, this dissertation postpones the discussion of these very interesting measurements until the future.

Two major tools were used to discern the flow behavior:

1. simultaneous measurements of the airfoil’s surface pressures, instantaneous incidence angle, and the freestream pressures;
2. high speed flow visualization via a focused schlieren system of the leading edge flow field ($0 \leq x \leq 0.08$).

The pressure measurements are used to determine instantaneous, ensemble, and higher-order statistical flow properties, as well as first and second integral quantities such as the load coefficients and aerodynamic work, respectively. The schlieren still images are used to qualify the flowfield, providing a clear visual aid when interpreting the surface pressure measurements in dynamic, transonic flow.

Appendix A provides a brief introduction to helicopter flight, readers unfamiliar with dynamic stall are directed there first. Chapter 2 presents a review of related literature, focused on windtunnel dynamic stall experiments, although some mention is made of flight tests, computational results, and progress towards controlling dynamic stall. Chapter 3 provides the design and goals of the dynamic stall rig at Notre Dame’s Whitefield Laboratory, as well as a detailed discussion of the measurement devices and
data acquisition. A brief chapter on the unique ensemble averaging routines developed and employed here is provided in Chapter 4. The results are broken into three distinct chapters, as each chapter is nearly self-contained and focused on a niche dynamic stall problem. Chapter 5 addresses the convection properties of the stall vortex in a compressible flow field. Chapter 6 discusses the design and implementation of the focusing schlieren system used to identified shock behavior at \( M_x \geq 0.47 \). Torsional damping is defined, and, through utilization of a Hilbert transform of the measured aerodynamic pitch-moment, is analyzed in time for the first time (to this author’s knowledge). Finally, Chapter 8 offers conclusions drawn from this analysis. Suggestions for future investigations into compressible dynamic stall attributes and the requirements for successful flow control are also discussed.
CHAPTER 2

SUMMARY OF DYNAMIC STALL LITERATURE

Comprehensive reviews of dynamic stall are presented by McCroskey[104, 106], Carr[11, 12], and in the book, Principles of Helicopter Aerodynamics, by Leishman[89]. Aside from a short history lesson, this review is directed toward important developments and their pertinence to the current study.

2.1 Introduction: A brief history of unsteady airfoil research

The foundation of unsteady aerodynamics dates back to the separate work of Katzmayr (1922), Wagner (1925), Glauert (1929), and Theodorsen (1935). Excluding the experimental work of Katzmayr[83], these investigations were theoretical, providing analytic solutions to incompressible flow around thin airfoils (flat plates) of infinite span subject to time-dependent motion and small disturbances. Kussner (1935) and von Karman and Sears (1938) and Sears (1941), Lomax (1953), and Loewy (1957) solved the unsteady problems associated with freestream gusts, compressible flow, and the returning rotor wake, respectively. See Johnson[75], Fung[51], or Leishman[89] for a review of unsteady airfoil theory.

Kramer (1932) was one of the first to experimentally document the augmented lift associated with dynamic stall through experiments with freestream gusts and the subsequent change in the effective incidence. Bailey and Gustafson (1939)[5] and later
Gustafson and Myers (1946)[60] observed and photographed the stalled portion of the rotor disk on an autogyro using tufts and a blade-mounted camera; the tuft visualization showed that the retreating blade was prone to stall during forward flight. Halfman, Johnson, and Haley[61] measured the loads on a 2D airfoil oscillated in both pitch incidence and vertical translation.

Early rotor stall investigations concentrated on aeroelastic problems, such as stall flutter, e.g., see References. [9, 66, 144]. Ham and Garelick[63] were the first to realize the vortical nature of the rotor-blade flow field. Pressure measurements on a small, model rotor captured the signature of a non-linear, low-pressure disturbance that initiated near the upper surface leading-edge and moved aft over the chord as the blade incidence increased. Integration of the pressures over the chord revealed severe pitch-moment hysteresis, including an in-phase, nose-down pitch moment and pitch motion (unstable damping). Ham realized that the motion of the blade showed periodic shedding of “very concentrated positive vorticity,” and, in a subsequent paper[65] linked the occurrence of stall flutter with the “dynamic stall phenomenon.”

Following Ham, research shifted to the parameter space defining dynamic stall attributes. For an airfoil oscillated in pitch, these parameters are primarily (in no particular order) $\alpha_0$, $\alpha_1$, $k$, $Re_c$, and $M_{\infty}$; other tests examined airfoil geometry[109] and forcing conditions, including, transient pitch motions[81, 95, 124], heaving or plunging oscillations[20, 133], the effect of pitch axis location[82], and varying freestream velocity[111]. Investigators sought to identify the cause of the separation delay associated with the unsteady motion, the physical mechanism for stall onset, as well as derive analytic and empirical models to determine the loads for rotor analysis and design. In parallel, the theory of unsteady separation developed, highlighting fundamental differences in steady and unsteady boundary-layers and the sequence of events leading
to separation – see, e.g., Sears and Telionis[121], Telionis[128], van Dommelen and Shen[134], and, more recently, Haller[62].

Early dynamic stall research rarely concerned freestream speeds above $M_\infty = 0.3$, despite the rotor environment extending to the transonic regime, reaching tip Mach numbers of 0.9[36, 126]. McCroskey[105, 109] showed that the flow around the leading-edge of rotor airfoil sections is accelerated to speeds 3-5 times $U_\infty$. Supercritical flow developed despite $M_\infty$ as small as 0.18. Clearly, it is necessary to investigate dynamic stall under compressible flow conditions. Carr and Chandrasekhara are the largest contributors to the current understanding of the role of compressibility plays on dynamic stall. The tandem constructed a facility at the NASA Ames Research Center (ARC) dedicated to dynamic stall research – see Carr and Chandrasekhara[13].

Current dynamic stall investigations seek to expand the rotorcraft’s operational envelope and are largely aimed at rotor design and passive or active dynamic stall control – see the survey by Lorber[97]. Candidate control schemes include variable airfoil geometry[32–35, 84, 87, 119], leading-edge blowing[127, 139], leading-edge plasma actuation[37, 93, 112], vortex generators[100, 132], synthetic jets[40, 50, 132], and fixed-wing devices such as slots[18], leading-edge droop[78, 100], pressure-side tabs (gurney flaps)[35, 78], or trailing-edge flaps[49, 52–54]. Greenblatt et al.[57–59] showed that periodic excitation of the leading-edge flow further delays dynamic stall onset. Despite success in achieving a further delay in separation or in marginalizing the detrimental effects of DSV, most of the above control strategies have yet to be implemented at the combination of $Re_c$ and $M_\infty$ applicable to the rotor-environment.
2.2 Dynamic stall morphology

For a steady airfoil, separation occurs when the flow fails to follow the curvature of the surface, demarcated by the incidence, $\alpha_{ss}$, in which large-scale ($O(c)$) separation takes place, and attended by an abrupt loss in lift, escalated drag, and a nose-down pitch moment. An oscillating airfoil also reaches an angle at which the flow separates and aerodynamic performance degrades; however, for rapid motions, this angle can be in considerable excess of $\alpha_{ss}$. Once dynamic stall occurs, the instantaneous forces are substantially larger than their static counterparts and are accompanied by hysteresis with respect to the time dependent incidence, $\alpha(t)$.

Figure 2.1 summarizes the development of dynamic stall; 2.1a illustrates the different stages of deep dynamic stall through the aerodynamic loads; 2.1b is an oil particle flow visualization of a single stall cycle. The following discussion outlines each of the states in detail; complimentary descriptions of the stages of dynamic stall are provided in References [11, 86, 89, 104].

2.2.1 Stage 1: Delayed separation

Stage 1 in Figure 2.1 describes the delay in boundary-layer separation associated with the up-stroke, ($\dot{\alpha} > 0$); this separation/stall lag is attributed to three sources:

1. Quasi-steady thin airfoil theory shows that $\dot{\alpha} > 0$ results in a positive apparent camber[89].

2. The downwash induced through vortex shedding in the wake weakens the suction side adverse pressure gradient[129, 137].

3. Unsteadiness energizes the boundary-layer and slows separation.

For attached flow, the induced camber manifests itself as a change in lift, $C_l(\alpha) < C_l^{A}(\alpha)$ for $\dot{\alpha} > 0$ and $C_l(\alpha) > C_l^{A}(\alpha)$ for $\dot{\alpha} < 0$. Ericsson and Reding[41] showed
Stage 3: Shear layer rolls up, creating an energetic, leading edge vortex that convects downstream over the chord. Lift and drag reach a maximum, as does the nosedown pitching moment.

Stage 2: Airfoil exceeds static stall angle. Lift slope increases and separation bubble forms at leading edge.

Stage 1: Lift and moment resemble quasi-static loading, flow remains well attached to airfoil.

Stage 5: Flow reattaches from leading edge to trailing edge. Aerodynamic loads return to original values.

Stage 4: Vortex passes into airfoil wake and flow reaches a state of full separation. Lift stall accompanied by drag and nose-down moment reduction.

(a) Airloads and flowfield schematic associated with deep dynamic stall cycle for a single degree of freedom airfoil oscillating in pitch

(b) Oil particle flow visualization of a NACA 0015. Particles illuminated using a Argon Ion laser sheet and captured with a Photron high-speed camera. \( \alpha = 15^\circ + 10^\circ \sin \omega t, k = 0.16 \), \( U_\infty = 10 \text{ m/s} \).[37]

Figure 2.1. Illustration of dynamic stall events, adapted from McCroskey et al.[105] and Corke et al.[37]

that \( \dot{\alpha} > 0 \) induces a leading-edge “jet.” The Magnus effect accelerates the near-wall boundary-layer, increasing its ability to negotiate the adverse pressure gradient[43, 44].

Carta[21] analytically showed that the ratio of the unsteady pressure coefficient to the steady pressure coefficient was less than unity for positive pitch rates. Increased reduced frequency further attenuates the adverse pressure gradient. Walker et al.[138] added that increases to the pitch rate promote a more energetic suction peak and DSV.
Separation in unsteady boundary layers was first described by Telionis[128]. Scruggs[120] showed that time-dependence delays the formation of boundary layer flow reversals. Telionis and Sears subsequently showed that separation always occurs some time after the unsteady boundary layer first exhibits reversed flow. The lack of separation despite flow reversal implies that the shear stress, $\mu \frac{du}{dy}|_{wall}$, fails as an indicator of unsteady boundary layer separation[121, 128].

2.2.2 Stage 2: Dynamic stall onset mechanisms

Stage 2 in Figure 2.1 marks the first appearance and subsequent growth of the dynamic stall vortex; the ejection of vorticity from the airfoil surface and into the inviscid flow is a sequence of events regarded as the van Dommelen and Shen process[134]. Onset is attributed to several, often conflicting, mechanisms for the range of $Re_c$ and $M_{\infty}$ experienced on a rotor. These mechanisms include:

1. gradual or classic trailing edge separation,

2. the abrupt breakdown of boundary-layer flow reversals,

3. the bursting of a separation bubble, or

4. shock induced separation.

Certain stall situations, particularly those that encourage thin-airfoil stall – see Abbott and von Doenhoff[1] – coincide with the sudden breakdown of the laminar-to-turbulent separation bubble[108], a mechanism originally proposed by Johnson and Ham[77] in regard to dynamic stall onset. For rotor applicable $Re_c$ ($O(1 \times 10^6)$), but where $M_{\infty} < 0.3$, Mcalister et al.[103] and McCroskey et al.[109] found that the separation bubble persisted through the leading edge suction collapse, imposing no direct effect on the flow’s breakdown. Tripping the laminar boundary layer forward of the
bubble caused negligible changes in the separation behavior, pressure field, or aerodynamic loads. McCroskey determined that two alternative mechanisms precipitated stall. The first, trailing-edge stall, consists of the gradual, trailing-edge to leading-edge separation of the turbulent boundary layer. The dynamic stall vortex originated further aft on the chord under trailing-edge stall characteristics. McCroskey et al.[108] labeled the second stall mechanism “abrupt trailing-edge stall.” Stall paralleled the abrupt forward motion or breakdown of a flow reversal within the turbulent boundary layer, as first documented by Mcalister and Carr[102] in a water tunnel using hydrogen bubble flow visualization. Hot-wire and hot film measurements by McCroskey et al.[105] showed that the reversal originated near the trailing edge and moved forward over the suction side of the airfoil. The reversal had little effect on the turbulent boundary layer or aerodynamic behavior, that is, until the forward motion of the reversal ended abruptly near the quarter-chord. Immediately thereafter, if not concurrently, the turbulent boundary-layer separated, briskly moving up and downstream. This initiated the dynamic stall vortex within the first few percent of the chord. Lee and Gerontakos[86] found similar results using a Tao Systems Senflex© high-spatial resolution hot-film sensor array, albeit at $Re_c \approx 10^5$.

A penchant toward leading-edge stall accompanies the development of super critical flow at the leading-edge. McCroskey et al.[109] tested eight different rotor airfoil geometries – all but two of the airfoils displayed trailing-edge or abrupt trailing edge stall characteristics at low Mach number. At $M_{x_e} = 0.3$, seven of the eight geometries exhibited leading-edge stall traits, developing local regions of supersonic flow on the suction-surface[10, 101, 105, 109]. McCroskey and Fisher[107] found similar results for a model rotor operating in high speed forward flight conditions. Relying primarily on flow visualization tools, Chandrasekhara et al.[27, 29, 31] and Dyken et
al. [39] found that stall onset commenced with the breakdown of the separation bubble under moderate Mach numbers, $M_{\infty} \in [0.2, 0.35]$. Chandrasekhara and Ahmed [26] measured the local velocity and vorticity near the leading-edge of an oscillating airfoil at $M_{\infty} = 0.3$ using a two component laser velocimeter. A velocity deficit above the leading-edge suction surface revealed that a separation bubble formed as the airfoil increased incidence; simultaneously the vorticity increased up to a critical angle, at which point it remained constant and the vortex separated from the airfoil’s surface. A sudden increase in streamwise velocity through the measurement volume indicated laminar bubble breakdown.

At higher Mach number, $(M_{\infty} \sim 0.3-0.4)$, Chandrasekhara et al. [31] found that complex interactions between the separation bubble and weak shock waves in the supersonic region initiated dynamic stall. At yet higher Mach numbers $(M > 0.45)$, the supersonic region’s extent increased considerably, and strong, normal shock waves formed as the flow negotiated the adverse pressure gradient. The shocks led to an abrupt thickening and subsequent separation of the boundary-layer. The supersonic region enveloped the laminar separation bubble still present on the airfoil surface, and the bubble played no role in the initial formation of the DSV. An oblong vortex formed after the shock, its extent normal to the airfoil surface constrained by the supersonic flow nearby. Heat flux gauge studies showed that large, positive wall shear-stress gradients preceded each compressible dynamic stall onset mechanism [27].

Following onset, vorticity erupts from the airfoil surface into the inviscid, outer flow field. Flow visualizations indicate that a energetic, rotating structure grows near the leading-edge over a finite but short time period, e.g. see Mcalister and Carr [102] and Walker et al. [138]. Discrete vortex simulations by Shih et al. [124] show that the vorticity originates from the leading-edge boundary layer (indeed, the airfoil surface is
the only source of vorticity in the flowfield) and coalesces into the DSV. Particle image velocimetry (PIV) measurements of a pitching airfoil by Raffel et al.[115] and Wernert et al.[143] corroborate such numerical studies.

2.2.3 Stage 2-3: Moment and then lift stall

The low-pressure vortex’s travel aft over the suction-side of the airfoil drags the center of pressure toward the trailing-edge, creating an acute, nose-down pitch moment. Moment stall predates lift stall – originally discussed by Harris and Pruyn[70] – and occurs shortly after vortex formation. The lag between moment stall and lift stall correlates to the time it takes for the vortex to traverse the airfoil chord, as discussed by Bousman[8].

The low-pressure core of the vortex augments the peak suction, increasing the effective lift-curve-slope – however, increasing compressibility negates this effect. The vortex acts as natural mechanism for momentum exchange between the would-be separated shear layer and the freestream flow, sustaining lift until Stage 3 in Figure 2.1. At this point, the vortex passes into the airfoil’s wake. Total flow separation ensues, causing a drastic drop in lift, peak nose-down moment, and a rapid increase in drag due to the wake growth.

2.2.4 Stage 4-5: Reattachment

On the down-stroke, Stage 4 in Figure 2.1, the airfoil increased pressure of the separated flow over the suction side of the airfoil cultivates less lift. The narrowing wake decreases the drag and a less severe pitching moment is incurred as the center of pressure of the separated flow shifts toward the mid-chord. The aforementioned hysteresis evolves from the disparate flow states that evolve on the pitch-up and pitch-down stroke.
forces – a function of both the overshoot in attached flow through the up-stroke and the lag in flow reattachment on the down-stroke. Increased reduced frequency, although beneficiary on the up-stroke, further delays reattachment on the down-stroke. As the airfoil returns to its minimum angle of attack. assuming that this angle is less than the stall angle, the flow reattaches from the leading-edge to the trailing edge and the aerodynamic coefficients return to their attached flow values.

Recovery from dynamic stall is a non-repeatable process and significant differences in loading can occur from one cycle to the next - e.g., see Young[80], Wernert et al.[142], or Green and Galbraith[55]. Liiva and Davenport[92] inferred that the scatter common to the stalled-flow data is not a function of windtunnel turbulence or model imperfections as the attached flow behavior marginal cycle-to-cycle differences. Liiva and Davenport proposed that large-scale turbulent eddies form above the airfoil following separation; constrained to surface pressure measurements, their research had no incite into the flowfield topology. PIV measurements by Shih et al.[124] and by Wernert et al.[143] do, however, indicate that the separated shear layer contains non-reproduced vortex structure from one cycle to the next. Wernert added that the extent of the non-reproducibility is contingent on the size of the separated wake and the reduced frequency of the oscillation. To account for the cycle-to-cycle differences, the loading variations require statistical description; accordingly, ensemble averaging is employed to smooth the data in a tractable manner. McAlister et al[101] showed that fifty pitch cycles yielded converged statistics. The number of cycles required for a “smooth” force variation depends on the test conditions, ensemble discretization, and the accuracy of the transducers. Chapter 3 contains a detailed description of the ensemble routine used for the present study.
2.3 Dynamic stall regimes

McCroskey[109] divided dynamic stall into four separate categories - no stall, stall onset, light stall, and deep stall. The majority of dynamic stall research focuses on deep stall, e.g., see[64, 65, 79, 92, 108]; however, deep stall promotes severe forces and vibrations and is avoided under normal rotorcraft operations[12]. McCroskey et al.[104] delineated the stall regimes through the maximum angle of attack, $\alpha_{\text{max}} = \alpha_0 + \alpha_1$. Each dynamic stall regime is summarized as follows:

- **No Stall** - airfoil trajectory remains below the the static stall angle. Loads are well predicted by unsteady aerodynamic theory.

- **Stall Onset** - for the helicopter, stall onset is the analogue to the operation of a fixed wing aircraft at maximum lift. It represents the useful lift of a particular airfoil and rotor if excess drag and moment are to be avoided. Airloads maintain a resemblance to their steady counterparts.

- **Light Stall** - a subset of dynamic stall where vortex shedding, although present, is poorly defined. The onset, growth, and convection of the vortex is sensitive to all unsteady parameters, including $M_\infty$, $Re_c$, $k$, $\alpha_0$, and $\alpha_1$, as well as boundary-layer separation characteristics. Lift stall and peak moment are less severe than deep stall. The boundary between stall onset and light stall was found to be abrupt and was identified by the appearance of moment stall. The separation region is on the order of the airfoil thickness[104].

- **Deep Stall** - deep stall represents the “fully-developed” vortex shedding phenomenon[104]. Airloads fluctuate drastically and exhibit large peak forces and severe hysteresis. Loads show little sensitivity to $Re_c$, airfoil geometry, or type of motion. The viscous separation region extends to the order of the airfoil chord[104]. Figure 2 in Reference[109] illustrates the load attributes associated with the four dynamic stall regimes.

2.4 Compressibility effects on dynamic stall and damping

Compressible flow affects dynamic stall attributes in complex ways; the following items highlight the important distinctions associated with compressible dynamic stall behavior:
1. at $M_x > 0.3$ (and as low as $M_x = 0.18$), supersonic flow limits the acceleration of the fluid around the leading edge, shortening the aforementioned separation delay, suppressing the suction peak and peak force coefficients;

2. compressibility alters the onset mechanism for dynamic stall - first, it promotes leading edge stall through bubble bursting which, subsequently, transitions to shock-induced dynamic stall with increasing $M_x$;

3. compressibility weakens the strength of the dynamic stall vortex, discussed by Chandrasekhara and Carr[27] and separately by Lorber and Carta[95].

These effects are discussed by Chandrasekhara, Carr, Ahmed, Dyken, and Wilder in research stemming from NASA ARC, e.g., see [12–14, 17, 25–29, 31, 39, 123]. Here, the review focuses on the limited number of studies attempting to discern the effects of compressibility on the aerodynamic damping behavior.

Liiva[91] found that increasing the Mach number led to more stable aerodynamic loads due to transonic flow effects. For an amplitude of $\alpha_1 = 5^\circ$, Liiva found only a reduction in damping as the maximum angle of attack passed through the static stall angle; whereas, at $M_x = 0.4$, the airfoil exhibited a large region of unstable, negative damping. Liiva reasoned that a stable expansion fan and shock system replaced the sudden, leading-edge flow separation exhibited at $M_x = 0.4$. Logically, Liiva believed that transonic flow would eliminate the large nose-down pitching moment as the expansion/shock system adjusts to changes in $\alpha$ through slight readjustments of the leading-edge pressures and shock location.

Lorber and Carta also remarked that, although a shock formed in their $M_x = 0.4$ test case, and a stall vortex clearly initiated after the shock at $x/c = 0.060$, the pressure signature of the vortex was not observed past $x/c = 0.57$. At low $M_x$, the low-pressure vortex cores carries the airfoil center of pressure aft toward the trailing edge. If the vortex is weakened or diffused due to the compressible flow field, then the peak nose-down moment will be greatly reduced, and, further, the blade motion may remain stable
throughout the cycle.

2.5 Trip effects on compressible dynamic stall

Trips are applied in order to produce large Reynolds number similarity via the removal of laminar to turbulent transition. Martin et al.[99] and McCroskey et al.[105, 109] experimented with trips in order to remove the separation bubble, as discussed previously, and to ascertain the dynamic stall onset mechanism. These tests focused on $M_{\infty} < 0.3$.

Chandrasekhar et al.[30] found that application of the traditional tripping guidelines proved inappropriate for compressible dynamic stall. One out of five tested trips proved successful in tripping the laminar boundary layer and removing the separation bubble. The trip promoted an increase in the peak suction and further delay in dynamic stall onset. This suggests that the ideal trip should

1. alleviate the adverse pressure gradient, and

2. fail to introduce any disturbance that would promote shock separation.

The masking tape trip used in this study met these two conditions, as it caused an increase in surface flow velocity and showed no indication of shock alteration at the higher Mach number test points. The grit trips, on the other hand, reduced the adverse pressure gradient at the cost of a significant reduction in $C_{p_{\text{min}}}$. The backward facing edge of the grit tape caused shocks to form and remain fixed at $M_{\infty} = 0.6$. This is a result of the large height of the trip compared to the boundary layer height $- O(10\delta)$. 

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CHAPTER 3

EXPERIMENTAL SETUP AND PILOT TESTING

3.1 Background: An overview of windtunnel dynamic stall simulation

The airflow witnessed by helicopter rotor-blade consists of three components, the blade section normal velocity, $U_\perp$, stemming from the flap-degree-of-freedom; the inflow velocity, $U_\parallel$, affected by the flight speed, rotor rpm, radial position, and lead-lag degree of freedom; and the angle of attack, $\alpha(t)$, prescribed through the pilot control stick but strongly affected by the blade elastic properties and local, fluid interactions with the rotor flow field. Figure 3.1 depicts the relevant terms.

The velocities vary periodically during operation. Liiva[91] estimated the cumulative spectral behavior as follows:

- $U_\perp$
  - at 1/rev because of flapping motion,
  - at a freq of 2/rev and higher because of flap bending at appreciable amplitude, and
  - across a wide spectrum due to rotor wake interactions;

- $U_\parallel$
  - sinusoidally at 1/rev due to forward flight speed,
  - higher frequencies due to chordwise bending (marginal), lead/lagging blade and horizontal gusts,
  - across a wide spectrum due to rotor wake interactions.
Figure 3.1. Rotor-blade section velocities and angles with respect to air and rotor disk.

- $\alpha$ varies
  - at 1/rev because of the cyclic pitch control input, and
  - at higher frequencies because of blade torsional response.

The current experiment examines only the $\alpha$ component of the blade aerodynamics. Windtunnel investigations of the pressure and load dependence resulting from a forced $\alpha(t)$ are historically carried out via 2D pitching experiments, e.g., see References[12, 31, 64, 92, 99, 108, 109, 115]. A smaller body of literature tackles the aerodynamics associated with $U_\perp$ (see [20, 91, 133]) and $U_\parallel$ (see Pierce et al.[111]). These studies and the differences resulting from the three forced motions indicate that the primary source of aerodynamic loading results from the blade prescribed incidence, $\alpha(t)$.

Constant angular pitch rate tests have been studied to eliminate angular acceleration from the problem, e.g., see Ham and Garelick[67], Lorber and Carta[95], or Jumper et al.[81]. Pitch and plunge tests can be accomplished via classic mechanisms such as the crank rocker, crank slider, or cam-follower[22, 91, 126] and are easily incorporated in standard windtunnel test sections. Servo motors offer the ease of direct drive and have been utilized by Post[113] and Lombardi[93]. The servo motor’s large, inherent moment of inertia constrains the attainable reduced frequency, limiting tests to small $Re_c$ and $Ma$ in order to match rotor reduced frequencies.
In order to verify that windtunnel simulation of dynamic stall captures the fundamental rotor aerodynamic physics, McCroskey[107] examined stall on a model rotor-blade instrumented with miniature pressure transducers and skin friction gages. Forces determined from the integrated pressures showed that the three-dimensional nature of the flow field failed to cause significant divergence from the forces and moments found in two-dimensional windtunnel studies. Investigation of the individual pressure traces showed the presence of the DSV. The study by Carta and Carlson[22] afforded similar results. In-flight stall characteristics of a Sikorsky Blackhawk (tested through the UH-60A Airloads Program) showed striking similarity with 2D windtunnel dynamic stall simulation. The only dissimilarity occurred near the outboard rotor, $r/R > 0.865$, where the tip vortex dynamics dominated the flow.

3.2 Experimental Setup

The experiment was conducted at the University of Notre Dame’s $M_x = 0.6$, 3 by 3 ft closed-return windtunnel at the Hessert Research Center for Aerospace at Whitefield. The test objectives were to obtain detailed information regarding the dynamic stall behavior of the Bell airfoil. The windtunnel and test section are depicted in Figures 3.2 and 3.3, respectively. The tunnel has three, interchangeable 3 ft square by 9 ft test sections in which one has been reserved and retrofitted for compressible dynamic stall studies. The windtunnel is powered by a 1750 hp, variable rpm AC motor that drives an 8 ft diameter, 2-stage high-solidity axial fan. The air in the wind tunnel is cooled by 40 °F water provided by a 125 ton chiller coupled with a 1000 ton-hr ice-storage system. Chilled-water circulates through cooling vanes downstream of the fan. The cooled air allows the tunnel to reach an equilibrium temperature at each operating Mach number with minimal pressure loss or temperature gradients across the vanes.
Figure 3.2. Top down view of the $M_x=0.6$ windtunnel at Whitefield Laboratory. Scale =0.005.

The tunnel has a contraction ratio of 6:1 and conditions the flow prior to the contraction with a honeycomb wall of 5 in depth and nominal dimension of 0.25 in. Five # 28 wire screens follow the honeycomb. The wind tunnel is designed to have a very low turbulence intensity level of $u'/U_\infty = 0.05\%$, that is comparable to free-flight conditions.

3.2.1 Bell airfoil

The airfoil under consideration is a proprietary design of Bell Helicopter but similar in design to modern rotor-blade sections. The Bell airfoil has an thickness-to-chord ratio, $t/c$, of approximately 10% and very modest camber line. The last 10% of the airfoil is a thin tab and typical of rotor-blade designs. The wing was constructed from four aluminum (7075-T6) 0.33 in thick ribs. Two cutouts in each rib facilitated the installation and wiring of the airfoil’s pressure sensors. A 1 in wide rib with flanges housed the airfoil instrumentation at the half-span. A wire EDM machine cut the leading-
and trailing-edges (7075-T6). 30 mil aluminum sheet covered the suction and pressure surface in the range 0.1 ≤ x ≤ 0.7 where it smoothly interfaced with the leading and trailing edges. #4 screws threaded into tapped holes in the ribs to secure the aluminum skins; and bolts fastened the ribs to the leading and trailing edges. Figure 3.4 shows the model assembly. Figure 3.5 shows the fabricated model with and without pressure instrumentation.

3.2.2 Airfoil pressure instrumentation

Thirty-one, single-ended, absolute pressure transducers – manufactured by Kulite and Endevco – were located on the suction and pressure surfaces to measure the model static pressure. A cosine distribution was used to position the sensors,

\[ x_n = 1 - \cos \left( \frac{n \pi}{2N} \right), \quad (3.1) \]
where \( N = 15, \ n = 0, 1, 2, \ldots N-1, \) and \( x_n = x_n^* / c \) is the non-dimensional chord position of the \( n^{th} \) transducer. The last station was placed at \( x = 0.995 \). A transducer resides on both the suction and pressure side of the airfoil at each station. A single transducer sits at the leading edge. The single ended instruments allow for the determination of the (pressure) drag on the model. The transducers were staggered about the airfoil’s mid-span to facilitate installation and access to the wires. Table 3.1 lists each station’s chordwise position. Section 3.4 discusses the transducers in more detail.

3.2.3 Reynolds and Mach relationship

Typical helicopter rotor-blades have a chord that is approximately 10 - 24 in, operating under \( Re_c \) of \( 1 \times 10^6 \) to \( 1 \times 10^7 \). The relationship between \( Re_c \) and \( M_{c} \) for the current model is shown in Figure 3.6. No attempt was made at varying the Reynolds number independently of the Mach number.
3.2.4 Test section interior assembly

An exploded view of the compressible dynamic stall test insert is shown in Figure 3.7. Table 3.2 provides the part list. The insert was designed so that the quarter chord of the model aligns with the center of the test section. The airfoil was mounted horizontally between two, 0.75 in aluminum splitter plates, which, in turn, were positioned approximately 10.75 in from the test section walls. End plates were formed from 0.5 in thick Lexan and fixed to the airfoil sides. The end plates were lubricated with synthetic grease and held in place by a teflon backed aluminum enclosure bolted to each
TABLE 3.1

PRESSURE TRANSDUCER INSTALLATION

<table>
<thead>
<tr>
<th>Station</th>
<th>$x$ [%]</th>
<th>Model</th>
<th>Station</th>
<th>$x$ [%]</th>
<th>Model</th>
</tr>
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<td>0.00</td>
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<td>8</td>
<td>33.3</td>
<td>Kulite XCL-152</td>
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<td>Kulite XCL-062</td>
<td>9</td>
<td>41.2</td>
<td>Kulite XCL-152</td>
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<td>Kulite XCL-062</td>
<td>10</td>
<td>50.0</td>
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<td>59.3</td>
<td>Kulite XCL-152</td>
</tr>
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<td>69.1</td>
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<td>79.2</td>
<td>Endevco 8515C</td>
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<td>19.1</td>
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<td>14</td>
<td>89.5</td>
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<td>25.7</td>
<td>Kulite XCL-152</td>
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<td>99.5</td>
<td>Endevco 8515C</td>
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</table>

Figure 3.6. Linear relationship between $Re_x$ and $M_{\infty}$. 

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Figure 3.7. Exploded view of the compressible dynamic stall insert

splitter plate and designed to provide a pseudo-labyrinth seal. The seal is necessary to minimize pressure leakage between the static freestream pressure and the airfoil flow field. The aft 25% of the chord oscillates near the fixed splitter plate walls. 1/8 in clearance on each side of the trailing edge was incorporated into the model design to avoid damage if the model sheared against the splitter plates. Surface oil flow-visualization indicates that the gaps do not compromise the centerline pressure measurements at the rear port locations.

Hardened steel pins press-fit into both sides of the leading and trailing edge and through the Lexan plates. One set of pins is used to attach an aluminum (7075-t6) tube so that the instrumentation wires can exit the test section. A hardened steel, six-
TABLE 3.2

COMPRESSIBLE DYNAMIC STALL INSERT PART LIST

<table>
<thead>
<tr>
<th>Part #</th>
<th>Name</th>
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<th>Material</th>
<th>Man./Dist.</th>
<th>PN</th>
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</thead>
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<td>torque tube (TT)</td>
<td>1</td>
<td>AISI 4140</td>
<td>VT</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>TT retaining ring (RR)</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>TT bushing</td>
<td>1</td>
<td>PEEK</td>
<td>-</td>
<td>-</td>
</tr>
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<td>-</td>
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<td>Al 2024</td>
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<td>Al/frelon backing</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

sided parallel key spline attaches to the other pair of pins. The spline aligns with the quarter-chord of the airfoil.

Two bearings press-fit into custom pillow blocks outside the splitter plates. The
Figure 3.8. Final configuration of the compressible dynamic stall test section

aluminum tube used to relay the wires is push-fit through one of the bearings. The bearings and pillow blocks transmit the model generated lift and drag to the test section floor where the pillow blocks are bolted. A hardened steel rod, deemed the “torque tube,” passes through a PEEK bushing mounted into the test section’s center window. The interior end of the tube is press fit into the remaining bearing. The end of the torque tube fits the spline mounted to the airfoil pins and allows the torque tube to transmit the control torque from the mechanism, located outside the test section, to the airfoil. The torque tube connects to the pitching mechanism via 100 tooth involute spline. Figure 3.8 shows the assembled compressible dynamic stall rig.
3.2.5 Pitch mechanism

The Bell Helicopter designed mechanism has the capability to pitch the airfoil at single or dual frequencies and amplitudes, i.e.,

\[ \alpha = \alpha_0 + \alpha_1 \sin(2\pi f_1 t + \phi_1) + \alpha_2 \sin(2\pi f_2 t + \phi_2). \]  \hspace{1cm} (3.2)

where \( \alpha \) is the instantaneous angle of attack of the airfoil, \( \alpha_0 \) is the mean angle of attack, \( \alpha_1 \) is the 1P amplitude, \( \alpha_2 \) is the second amplitude, \( \omega_1 \) and \( \phi_1 \) are the 1P circular frequency and phase, respectively, and, \( \omega_2 \) and \( \phi_2 \) are the circular frequency and phase of the second input (where, of course, \( \omega_1 t = \theta_1 \) and \( \omega_2 t = \theta_2 \)). The third term on the \emph{rhs} of Eq. 3.2, as previously discussed, was included to mimic the high-frequency, small amplitude elastic twist induced by dynamic stall/stall flutter. The design coupled a four and five-bar linkage, resulting in a 9 bar mechanism with 11, single-degree-of-freedom joints. Subsequently, the mechanism has two degrees of freedom (the motor inputs). See Figure 3.9 and Table 3.3 for the mechanism configuration and parts list, respectively.

A large, freestanding steel stand supports the pitch mechanism components. Two, Marathon 10 HP, Black Magic 420 VAC electric motors bolt to a 0.25 in steel sheet on the stand and drive the mechanism at two user defined frequencies. The motors are equipped with Dynapar internal encoders monitored by Yaskawa F-7 drives to maintain constant RPM. Motor 1 provides 1P input of the pitch trajectory, typically 350 RPM - 400 RPM on a rotor, whereas Motor 2 provides the elastic response, 2/rev-7/rev. The mechanical components and motors were sized for 1P and torsional frequencies of approximately 20 Hz (as to include large reduced frequencies at high Mach number) and 50 Hz, respectively, whereas the motors have maximum frequency of 60 Hz each.
A DuraFlex flexible coupling transmits the motor torque to each driveshaft. The drive shaft is supported by two Dodge Imperial pillow blocks. The pillow blocks are bolted down to a 2-in thick steel plate. Admittedly, the plate is oversized, but it provides the necessary level between the motor shaft and drive shafts, eliminating additional parts. Pneumatic calipers mount to a C-channel above the pillow blocks and braking discs secured to the driveshafts provide an emergency mechanical stop. A 12 in diameter flywheel mounts to the front face of the 1P drive shaft and a 10 in diameter flywheel mounts to the “torsion” drive shaft. The flywheel assembly includes four major components, listed as follows:

- Rear flywheel - solid 3 in thick steel cylinder
- Front flywheel - 1.5 in thick steel cylinder with hog-out for spindle travel

Figure 3.9. Compressible dynamic stall pitch mechanism, side and front CAD.
### TABLE 3.3

MECHANISM PARTS LIST

<table>
<thead>
<tr>
<th>Part#</th>
<th>Part Name</th>
<th>Part#</th>
<th>Part Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Stand</td>
<td>2</td>
<td>Marathon Black Magic 10 HP motor (2)</td>
</tr>
<tr>
<td>3</td>
<td>Duraflex coupling (2)</td>
<td>4</td>
<td>TB Woods bushing and key (4)</td>
</tr>
<tr>
<td>5</td>
<td>1P torsion driveshafts</td>
<td>6</td>
<td>Baldor-Dodge Imperial pillow blocks (4)</td>
</tr>
<tr>
<td>7</td>
<td>Brake caliper (3)</td>
<td>8</td>
<td>Lg/sm flywheel</td>
</tr>
<tr>
<td>9</td>
<td>Vespel bushings (2/FW)</td>
<td>10</td>
<td>Idler arm</td>
</tr>
<tr>
<td>11</td>
<td>Idler ground</td>
<td>12</td>
<td>Idler</td>
</tr>
<tr>
<td>13</td>
<td>Revolute joints (pins)</td>
<td>14</td>
<td>1P pushrod</td>
</tr>
<tr>
<td>15</td>
<td>Pitch-link</td>
<td>16</td>
<td>Torque tube/pitch horn involute spline</td>
</tr>
<tr>
<td>17</td>
<td>Pitch horn</td>
<td>18</td>
<td>1P flywheel assembly</td>
</tr>
<tr>
<td>19</td>
<td>Disc brake (2)</td>
<td>20</td>
<td>Torsion flywheel assembly</td>
</tr>
<tr>
<td>21</td>
<td>Pneumatic calipers</td>
<td>22</td>
<td>Torsion pushrod</td>
</tr>
<tr>
<td>23</td>
<td>Walking beam</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Acme thread and bearing assembly
- Spindle - steel block with tap to allow travel on the acme thread. The block has a 1.5 diameter solid protrusion that extends out from the front face of the flywheel and attaches to a pushrod

The acme thread and spindle allow for a continuous variation of the spindle’s offset distance from the center of the flywheel. The offset controls the pitch amplitude of the
airfoil, allowing a continuous range of amplitudes for both the 1P and torsion inputs. The amplitude ranges are shown in Figure 3.11, as derived in Section 3.3. Figure 3.10 shows the skeleton diagram for the mechanism. The offset distance are \( r_1 \) for the 1P input and \( r_5 \) for the torsion input, corresponding to Motor 1 and Motor 2 frequency inputs, respectively.

Push-rods, \( r_2 \) and \( r_6 \), attach to the spindles’ ends and to a beam below via revolute joints. The walking beam, a solid beam with three revolute joints and divided into \( r_7 \) and \( r_8 \) for analysis, mechanically adds the continuous, circular motion of the spindles and outputs a vertical translation to the pitch link, \( r_{10} \), through a spherical bearing. The pitch link connects to the pitch horn, \( r_{11} \), through a second spherical bearing. The involute spline, mentioned previously, is used to connect the pitch horn to the torque tube, yielding the airfoil’s incidence, \( \alpha(t) \). An additional link, \( r_3 \) constrains the motion of the revolute joint where \( r_2 \) and \( r_7 \) meet. Restraining the torsion input, \( r_5 \), and taking care to mitigate movement of the torsion pushrod, \( r_6 \), reverts the mechanism to a crank-rocker and single frequency, 1P motion can be obtained.

Rotation of the torque tube within the pitch horn allows for a variation in mean angle of attack in increments of 3.6°. Altering the length of the pitch link through its rod end bearings provides ten degrees of continuous variation in the mean angle of attack without effecting the sinusoidal motion of the output. When used in conjunction, the pitch horn and pitch link allow for any mean angle of attack, i.e., \( \alpha_0 \in [0^\circ, 360^\circ] \). Table 3.4 presents a comparison between flight operation and the experimental parameter space obtainable through the mechanism and windtunnel.
Figure 3.10: Skeleton diagram of the biharmonic pitching mechanism. Solid vectors: physical links. Dashed vectors: geometry to fixed joints. Dash-dot circles: spindle orbits.
TABLE 3.4

TEST SPACE OF THE NOTRE DAME DYNAMIC STALL FACILITY
(NDDSF)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Formula</th>
<th>Operational range</th>
<th>Experimental range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{\infty}$</td>
<td>$U_{\infty}/\sqrt{\gamma RT}$</td>
<td>0 - 0.9</td>
<td>0 - 0.6</td>
</tr>
<tr>
<td>$Re$</td>
<td>$\rho U_{\infty}L/\mu$</td>
<td>$1.0 \times 10^6 - 1.0 \times 10^7$</td>
<td>$1.0 \times 10^6 - 3.5 \times 10^6$</td>
</tr>
<tr>
<td>$f_1$</td>
<td>$6$ Hz - $7$ Hz</td>
<td>0 Hz - 20 Hz</td>
<td></td>
</tr>
<tr>
<td>$k$</td>
<td>$\pi f c/U_{\infty}$</td>
<td>0 - 0.1</td>
<td>0 - 0.1</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>-</td>
<td>$0^\circ$ - $10^\circ$</td>
<td>$0^\circ$ - $27.5^\circ$</td>
</tr>
<tr>
<td>$f_2$</td>
<td>$f_2/f_1 = \omega_2/\omega_1$</td>
<td>$4/f_1$ - $7/f_1$</td>
<td>$0/f_1$ - $7/f_1$</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>-</td>
<td>$0^\circ$ - $5^\circ$</td>
<td>$0^\circ$ - $5^\circ$</td>
</tr>
</tbody>
</table>

3.3 Pitch-mechanism kinematic analysis

Each linkage in Figure 3.10 is defined by a vector, e.g., $r_3$, where,

$$r_3 = r_3 e^{i \theta_3}. \quad (3.3)$$

Here $\theta_3$ is the orientation of the vector with respect to the horizontal, defined at the vector’s tail. Definition of the mechanism in this manner leads to the two independent vector loop equations,

$$r_1 + r_2 - r_3 - r_4 = 0 \quad (3.4)$$
$$r_5 + r_6 + r_7 - r_3 - r_9 = 0. \quad (3.5)$$
The first vector loop equation is identical to the classic four bar mechanism, and has a closed form solution. However, the second equation, representative of a five bar mechanism, does not have a closed form solution. Similarly, the mechanism output, $\alpha$, cannot be easily represented in terms of $r_1$, $r_5$, $\theta_1$, and $\theta_5$, and, instead, requires a numerical solution.

Decomposition of each vector loop equation into its $X$ and $Y$ components yields a system of four simultaneous equations, listed as follows,

$$f_1 = r_1 \cos(\theta_1) + r_2 \cos(\theta_2) - r_3 \cos(\theta_3) - r_4 \cos(\Omega_1) = 0,$$  \hspace{1cm} (3.6)

$$f_2 = r_1 \sin(\theta_1) + r_2 \sin(\theta_2) - r_3 \sin(\theta_3) - r_4 \sin(\Omega_1) = 0,$$  \hspace{1cm} (3.7)

$$f_3 = r_5 \cos(\theta_5) + r_6 \cos(\theta_6) - r_7 \cos(\theta_7) - r_3 \cos(\theta_3) - r_9 \cos(\Omega_2) = 0,$$  \hspace{1cm} (3.8)

$$f_4 = r_5 \sin(\theta_5) + r_6 \sin(\theta_6) - r_7 \sin(\theta_7) - r_3 \sin(\theta_3) - r_9 \sin(\Omega_2) = 0,$$  \hspace{1cm} (3.9)

where $f_i, i = 1, 2, 3, 4$, represents the above equations. The nonlinear equations can be solved via Newton’s Method, an iterative numerical method used to solve systems of $n$ homogeneous equations in $n$ unknowns. $\theta_1$ and $\theta_5$ are the inputs for the above system of equations, provided a predetermined length for $r_1$ and $r_5$. The angles $\Omega_1$ and $\Omega_2$ are constants (grounds). All lengths are known, leaving $\theta_2$, $\theta_3$, $\theta_6$, and $\theta_7$ unknown. A Taylor series of $f = [f_1...f_4]$ is then, in matrix form,

$$f(x)|_{x=x^0} + A(x - x^0) = 0$$  \hspace{1cm} (3.10)

where $x$ is the array of unknowns, $x^0$ is an initial guess of the solution, and $A$ is the Jacobian of the system of equations. For the four homogenous equations in Equation 3.6, Equation 3.10 takes the form
\[
\begin{bmatrix}
  f_1 \\
  f_2 \\
  f_3 \\
  f_4
\end{bmatrix}
= x^0 + \begin{bmatrix}
  \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_4} \\
  \vdots & \ddots & \vdots \\
  \frac{\partial f_4}{\partial x_1} & \cdots & \frac{\partial f_4}{\partial x_4}
\end{bmatrix}
\begin{bmatrix}
  \theta_2 \\
  \theta_3 \\
  \theta_6 \\
  \theta_7
\end{bmatrix}
- \begin{bmatrix}
  \theta_2^0 \\
  \theta_3^0 \\
  \theta_6^0 \\
  \theta_7^0
\end{bmatrix}
= \begin{bmatrix}
  0 \\
  0 \\
  0 \\
  0
\end{bmatrix},
\] (3.11)

where

\[
\begin{bmatrix}
  \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_4} \\
  \vdots & \ddots & \vdots \\
  \frac{\partial f_4}{\partial x_1} & \cdots & \frac{\partial f_4}{\partial x_4}
\end{bmatrix}
= \begin{bmatrix}
  -r_2 \sin(\theta_2) & r_3 \sin(\theta_3) & 0 & 0 \\
  r_2 \cos(\theta_2) & -r_3 \cos(\theta_3) & 0 & 0 \\
  0 & r_3 \sin(\theta_3) & -r_6 \sin(\theta_6) & -r_7 \sin(\theta_7) \\
  0 & -r_3 \cos(\theta_3) & r_6 \cos(\theta_6) & r_7 \cos(\theta_7)
\end{bmatrix}.
\] (3.12)

Solving for \(x\), yields

\[
x = -A^{-1}f(x)|_{x=x^0} + x^0
\] (3.13)

Since Equation 3.13 is based on a Taylor series approximation, the solution to \(x\) is not exact, hence the solution is used as a new guess and the process iterates until convergence. Choosing an initial guess is often difficult when applying Newton’s method; however, the initial configuration of the mechanism is known, and the unknowns vary only slightly from the initial state. Therefore,

\[
\begin{bmatrix}
  \theta_2^0 \\
  \theta_3^0 \\
  \theta_6^0 \\
  \theta_7^0
\end{bmatrix}
= \begin{bmatrix}
  \frac{3\pi}{2} \\
  0 \\
  \frac{3\pi}{2} \\
  \pi
\end{bmatrix}.
\] (3.14)

Once the unknowns are found, each position vector in Figure 3.10 is known relative to the origin of \((X,Y)\). The output angle, \(\alpha\), is the angle \(r_{11}\) makes with the horizontal.
Taking advantage of the law of cosines, $\alpha$ is found as follows

$$
\begin{align*}
r_{10}^2 &= r_{11}^2 + r_{12}^2 + 2r_{10}r_{11} \cos(\alpha + \beta) \\
\alpha &= \cos^{-1} \left( \frac{r_{10}^2 - r_{11}^2 - r_{12}^2}{2r_{10}r_{12}} \right) - \beta,
\end{align*}
$$

where $r_{12}$ and $\beta$ are defined in Figure 3.10. Strictly speaking, $r_{12}$ is also an unknown, however, it is easily determined via $r_8$. Newton’s method was employed to determine the functional relationship between the spindle offsets, $r_1$ and $r_5$, and the resulting pitch amplitudes, $\alpha_1$ and $\alpha_2$, respectively. The results are shown in Figure 3.11. The half-stroke of $r_1$ is 2.2 in whereas the half-stroke of $r_5$ is 1.2 in.

![Figure 3.11. Mode amplitudes as a function of the flywheels’ half-strokes. $\alpha_1$, $r_5 = 0$ [in] ⊙ $\alpha_2$, $r_1 = 0$ [in] □.](image)

$\alpha_1 = 12.5r_1 - .2$

$\alpha_2 = 4.85r_5$
3.3.1 Mechanism output

Newton’s method was also used to compute a theoretical mechanism output, provided $\theta_1 = \theta_1(t)$ and $\theta_5 = \theta_5(t)$. Neglecting torsional input, $r_5 = 0$, the airfoil 1P output is shown in Figure 3.12. Incorporating the torsional input gives, for this particular case, the non-harmonic output shown in Figure 3.13.

3.4 Instrumentation

The following section provides a brief overview of the instruments used for this investigation. Calibration procedures and results are also discussed.
Figure 3.13. Two-mode mechanical output as determined from Newton’s method, \( r_1 = 0.8 \) in, \( \phi_1 = 0^\circ \), \( r_5 = 0.25 \) in, \( \phi_5 = 3\pi/2 \), \( f_1 = 4 \) Hz, \( f_2 = 22.8 \) Hz.

3.4.1 Kulite

Kulite pressure transducers use an isolated silicon on silicon diaphragm. Pressure disturbances cause the diaphragm to deflect, causing a change in resistance of a wheatstone bridge. A passive RLC circuit tailored to each individual Kulite provides temperature compensation in the range \( T \in [80 \, ^\circ\text{F}, 180 \, ^\circ\text{F}] \). Acceleration sensitivity was near \( 5.0 \times 10^{-4} \%\text{FS/g} \). During the course of the experiment, the equilibrium temperature of the windtunnel was found to increase substantially with \( M_\infty \), often from 70 \( ^\circ\text{F} \) to 140 \( ^\circ\text{F} \) for \( M_\infty \) from 0.2 to 0.6, respectively. Despite the lower temperature falling outside the compensated range, the factory issued calibrations showed excellent repeatability across the range of temperatures indicated. Both Martin et al[100] and Lorber and Carta[95] calibrated the onboard sensors for both pressure and temperature. Here, no effort was made to calibrate the sensors at the different operational temperatures. The
absolute transducers have a pressure range up to 25 psi. The large range allowed for simple diagnostics of the transducers with a small air compressor. Typical factory sensitivities are 4 mv/psi with a natural frequency of 240 kHz.

The model numbers for the Kulites are shown in Table 3.1. XCL series transducers are cylindrical probes, with diameters of 66 mil, 95 mil, and 152 mil for the -062, -093, and -152 models, respectively. Each sensor was installed as close as possible to the model surface to minimize cavities.

3.4.2 Endevco

The Endevco 8515C-15 measures pressure via a piezoresistive four arm wheatstone bridge. The sensor has compensated temperature range of -65 °F to 250 °F and similar acceleration attributes as the Kulite models. The natural frequency of the Endevco model is 180 kHz. The model’s 30 mil thin profile was ideally suited for installation onto the airfoil TE tab. The trailing edge of the airfoil was milled and the Endevcos were secured with epoxy. Channels milled into the outer surface of the trailing edge allowed for the transducers’ wires. After installation, the channels were filled with epoxy so that the airfoil surface remained smooth. The transducer diameter measures 0.25 in. The absolute sensors had a range up to 15 psi and approximate sensitivity of 13 mv/psi.

3.4.3 Freestream pressure measurement

Two Model 270 Setra barometric pressure transducers were used to measure the freestream total and static pressures. The Model 270 has a range of 8.70 psi to 15.95 psi (600 mb to 1100 mb), whereas freestream pressures are expected to range from 11.29 psi to approximately 14.7 psi. The high accuracy (0.05% FS), absolute sensors
are necessary in order to zero the Kulites and Endevcos. The factory calibrations are shown in Figure 3.14 (note, the calibrations are nearly identical).

3.4.4 On-board pressure calibration

For a significant portion of the experiment (runs numbers below RUN2548), the factory issued calibrations for the both the Kulite and Endevco pressure transducers were used to determine each individual sensor’s output. These calibrations produced reasonable pressure distributions. Section 3.4.7 compares the pressure distribution on the airfoil measured with the onboard-transducers and an XFOIL panel-method prediction. The accuracy of the factory calibrations were called into questioned due to the numerous loading cycles the transducers were forced to endure. To calibrate, the sensor was placed under vacuum. Tygon tubing and an open three-way valve connected a Setra Model 270, the Kulite/Endevco, and a handheld, pneumatic vacuum gun. A static
Figure 3.15. Calibration results for onboard pressure sensors, including instrumental amplifier gain and offset. Typical calibration output, Incorrect calibration, steady calibration.

calibration was not possible for the majority of the sensors due to their installation in the airfoil at the time of calibration. Instead, a dynamic calibration was performed. Care was taken to release the vacuum pressure slowly so that both the response times of the Kulite/Endevcos and the Setra did not effect the procedure. Figure 3.15 highlights the calibration for Port 10S. The ‘incorrect calibration’ resulted from intentionally releasing the vacuum in an abrupt fashion. The induced non-linearity indicated a poor calibration. Non-linear calibrations were discarded and the procedure repeated. 10S was well sealed within the center rib flange on the model, and allowed for a static calibration. The typical dynamic calibration and the static calibration align, and both methods yield a sensitivity of -185 mV/psi. Additionally, the results of the calibration showed that the factory sensitivities drifted only marginally in the time between receiving the transducers and performing the above procedure.
3.4.5 Tunnel static pressure calibration

A formal test section calibration was conducted with the airfoil model removed and a pitot static tube placed in the center of the insert apparatus, at the location where the model would normally be located. The inlet static pressure was measured with a Setra Model 270 transducer from a tap at the top of the tunnel. Two pitot-static probes were placed in the test section. The first probe was placed approximately 2 ft from the tunnel inlet to test the equivalence between the inlet and upstream test section pressures. The second probe, referred to as the ‘cal’ probe, was placed so that the static ports aligned with the pitch axis of the mechanism (the model quarter-chord). Refer to Figure 3.3 for the location of the pressures used for the calibration. Model 270 transducers recorded $P_1$ and $P_2$.

The three static pressures were recorded as the tunnel’s fan speed was varied from 0 to 60 Hz. The inlet pressure, $P_{inlet}$, and the upstream probe, $P_1$, saw nearly identical values. A fourth-order polynomial was used to fit the relationship between $P_{inlet}$ and the static pressure at the cal probe, $P_2$. The calibrations are shown below in Figure 3.16; Equations 3.17 and 3.18 list the derived fits. During static and dynamic tests, $P_{inlet}$ was recorded as the freestream static pressure and Equation 3.18 was used as a correction to determine $P_{s,xx}$. This was necessary for two reasons: first, the calibration accounts for the decrease in static pressure resulting from the interior apparatus blockage; and, secondly, any static pressure measured downstream of the inlet witnesses significant fluctuations at a frequency equal to $f_1$ due the changing incidence of the model.

$$P_1 = 0.994 P_{inlet} + 0.082 \quad (3.17)$$

$$P_2 = 0.001491 P_{inlet}^4 - 0.06557 P_{inlet}^3 + 0.9744 P_{inlet}^2 - 3.757 P_{inlet} - 1.835 \quad (3.18)$$
The same procedure deduced the relationship between the fan speed and the tunnel Mach number. Figure 3.17 presents the tunnel calibration. Note that the difference between the Mach number measured at station 1, corresponding to the upstream static pressure, and the Mach number at the cal probe grows significantly with increasing fan speed. Both relationships proved fairly linear; however, the last data point diverges from the exhibited trend. More than likely, the tunnel motor and fan could no longer obtain the required pressure differential. Regardless, the point falls outside the desired parameter space.

3.4.6 Angle of attack

A Positek RIPS P500 inductive rotary sensor measured the airfoil instantaneous incidence. The sensor output 0 to 5 Volts over a 0° to 60° range (or a -5 to 0 V output
if the sensor was rotated 180°. A rigid torque coupling fastened the wire shaft to the rotary sensor. The coupling allowed slight misalignment and absorbed axial thrust. The sensor required calibration each tunnel entry, consistently yielding a sensitivity near ± 12.5 °/V (the offset is set close to 30 °).

3.4.7 Chord pressure verification

XFOIL, a free, 2D panel method code was used as a diagnostic tool to determine a reference for the airfoil static pressure measurements. XFOIL uses and $e^a$ scheme to determine boundary-layer transition and separation characteristics. Comparisons between the XFOIL and measured data are illustrated in Figure 3.18. Overall, both the factory calibrations and the in-situ calibrations show good qualitative and quantitative agreement with the XFOIL prediction.
3.5 Data acquisition

The Kulite and Endevco signals pass through an instrumental amplifier, where gain and offset can be adjusted. Approximate gain settings of 50 and 20 were used for the Kulites and Endevcos, respectively. The decreased sensitivity of the Kulites required the additional gain. The exact gain factors produced by the instrumental amplifiers and associated circuitry were determined by applying a known voltage input (battery) across the channel input and measuring the board output. A low-pass, anti-aliasing filter conditions the signal prior to reaching the sample-and-hold op-amps. All signals were
acquired by a Microstar Laboratories DAP5380a and three MSXB 028 simultaneous sampling boards at a sample frequency, $f_s$, of 5.050 kHz. The DAP provides 14-bit analog-to-digital conversion; however, resolution is reduced through the 12-bit MSXB 028 boards. The Setra pressure transducers and the Positek angle sensor condition their respective outputs and are directed straight to the sample-and-hold cards. The data acquisition code was written in Mircostar Laboratories’ DAPVIEW software. All raw data, in the form of quantization levels, are stored on disk for reduction and future reference. A flow chart representing the data conditioning and acquisition process is shown in Figure 3.19.

![Figure 3.19. Signal conditioning and acquisition](image_url)
3.6 Data reduction

The Fortran program STALL_FLUTTER.exe reduced the experimental data. The raw signals acquired by the data acquisition processor are converted from quantized levels to a voltage. Instrument calibrations are applied and the airfoil’s pressure distribution is determined. All quantities of interest are derived and stored. System calls to gnuplot are used to display the data. Figure 3.20 highlights the general steps required to reduce the test data.

3.6.1 Coefficient of pressure and local Mach number

The $C_p$ was found as follows,

$$C_p = \frac{2}{\gamma M_\infty^2} \left[ \frac{P_{\text{airfoil}}}{P_{\infty}} \right] - 1,$$

(3.19)
where $\gamma$ is the ratio of specific heats for air, $P_{s,\text{airfoil}}$ and $P_{s,\infty}$ are the static pressures on the airfoil surface and in the freestream, respectively. Recall, $P_{s,\infty}$ is a fourth-order polynomial function of $P_{\text{inlet}}$.

The local Mach number, $M_l$, on the airfoil surface can be found through rearrangement of the isentropic relationship,

$$\frac{P_{s,\text{airfoil}}}{P_0} = \left[ 1 + \frac{\gamma - 1}{2} M_l^2 \right]^{\frac{\gamma}{\gamma - 1}}.$$  \hspace{1cm} (3.20)

Solving for $M_l$, yields

$$M_l = \sqrt{\frac{2}{\gamma - 1} \left[ \left( \frac{P_0}{P_{s,\text{airfoil}}} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right]},$$  \hspace{1cm} (3.21)

where $P_s$ has been replaced by the static pressures on the airfoil surface. $M_l$ can be used to gauge the presence and extent of supersonic flow over the airfoil.

### 3.6.2 Integrated forces

The aerodynamic force coefficients were determined from integrating the pressure distribution on the airfoil. Figure 3.21 illustrates the airfoil coordinate system used in the force calculations. See [89] for derivation.

The expressions used to determine the non-dimensional forces are

$$C_n = \int_0^1 \left( C_p^p - C_p^s \right) dx,$$  \hspace{1cm} (3.22)

$$C_a = \int_0^1 \left( C_p^p \frac{dy^p}{dx} - C_p^s \frac{dy^s}{dx} \right) dx,$$  \hspace{1cm} (3.23)

where the subscript $n$ refers to the force in the normal direction and the subscript $a$
Figure 3.21. Airfoil coordinate system illustrating normal, axial, lift, and drag forces.

refers to the axial, or chord force direction, shown in Figure 3.21. The normal and axial force coefficients are mapped to determine the lift and drag coefficients, shown in Equations 3.24 and 3.25.

\[ C_l = C_n \cos(\alpha) + C_a \sin(\alpha) \]  \hspace{1cm} (3.24)

\[ C_d = C_n \sin(\alpha) - C_a \cos(\alpha) \]  \hspace{1cm} (3.25)

The moment about the leading edge is shown in Equation 3.26,

\[ C_{mLE} = - \int_0^l \left( C_{p}^P - C_{p}^S \right) x \, dx, \]  \hspace{1cm} (3.26)

and is mapped to the quarter chord through the addition of the normal force and its lever-arm,

\[ C_{m/Q} = C_{mLE} + C_n/4. \]  \hspace{1cm} (3.27)

The above integrals are computed using a trapezoidal integration scheme, reported in Section 4.1. The equations hold true for steady or unsteady motion. In the steady
case, data was taken at discrete angles of attack, whereat all pressures and forces were determined from approximately 10 s of data. In the unsteady case, data was taken for approximately 100 cycles. As noted by Ahmed and Chandrasekhara[3] and Green and Galbraith[55], significant cycle-to-cycle differences in the pressures and load are exhibited during dynamic stall, particularly during stall recovery and flow reattachment. Ensemble averages were employed to show the mean dynamic stall behavior for all single-frequency oscillations, removing inherent randomness common to the dynamic stall problem.

3.7 Leading-edge modifications

Fluorescent dyed oil and ultra-violet (UV) light showed the existence of a leading-edge separation bubble on the airfoil under steady flow conditions – see Appendix B. The role the bubble plays in the dynamic separation characteristics of the airfoil were unknown prior to this investigation. Several standard roughness elements were used to ‘trip’ the flow at the airfoil leading-edge with the intention to remove the leading-edge separation bubble. The oil and UV light were also used to visualize the effect of the trip on the steady flow. The roughness elements or grit size were chosen to match the trip characteristics discussed by Chandrasekhara et al.[30]. Overall, the effect of the trip should be minimal and produce the following attributes:

- a marginal increase in peak suction pressure (the turbulent boundary layer should be more energetic than the laminar boundary layer),
- a delay in dynamic stall angle,
- trailing-edge separation and stall behavior,
- the trip should not introduce flow disturbances that would promote the formation of shocks at high freestream Mach number.
TABLE 3.5

LEADING-EDGE TRIP CHARACTERISTICS

<table>
<thead>
<tr>
<th>Material</th>
<th>Thickness [in]</th>
<th>$R_a$ [$\mu$m]</th>
<th>$R_{rms}$ [$\mu$m]</th>
<th>$\frac{R_a}{\delta_{M_8 = 0.4}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MT</td>
<td>0.00055</td>
<td>3.64</td>
<td>4.74</td>
<td>0.72</td>
</tr>
<tr>
<td>P400</td>
<td>0.007</td>
<td>4.57</td>
<td>6.03</td>
<td>0.91</td>
</tr>
<tr>
<td>P320</td>
<td>0.008</td>
<td>4.39</td>
<td>5.64</td>
<td>0.87</td>
</tr>
</tbody>
</table>

Four versions of the airfoil were tested throughout the duration of the experiment. A baseline, un-tripped airfoil, and an airfoil with three different roughness elements - masking tape, P400 grit, and P320 grit. Each trip was placed near the leading edge, $x/c \in [0.005, 0.03]$. The adhesive backing on the masking tape was strong enough to retain the trip throughout the Mach test points. 3M Super Weatherstrip Adhesive (black) secured the grit to the leading-edge. Table 3.5 lists the thickness, average roughness, $R_a$, and rms roughness, $R_{rms}$, of the roughness strips as measured with a Zygo white light interferometer. The table also displays the average roughness normalized by the Blasius boundary layer height at $M_{\infty} = 0.4$ and $x = 0.03$. The measurements do not reflect the added thickness due to the weatherstripping adhesive.
CHAPTER 4

ENSEMBLE AVERAGING METHODS

Several ensemble methods were examined in order to derive the most accurate representation of the single-frequency airloads. Individual sampled angles of attack were sorted into the closest collocation point or “bin.” The sample index of that incidence was recorded so that the instantaneous pressures and loads could be sorted accordingly. The ensemble discretization is examined in this section. RUN 1331 is used throughout the following analysis.

4.1 Initial processing

For a time signal,

\[ \alpha = \alpha(t) \quad (4.1) \]
\[ C_{p_i} = C_{p_i}(t) \quad (4.2) \]

where \( \alpha \) is the instantaneous angle of attack of the airfoil and \( C_{p_i} \) is the instantaneous coefficient of pressure on the airfoil at the \( i \)th port. These pressures later appear as \( C_{p_i}^P \) and \( C_{p_i}^S \) for pressure and suction, respectively, where \( i \in [0 : 15] \). After acquiring the
time-signal and digitizing we have

$$\alpha = \alpha(n)$$  \hspace{1cm} (4.3)

$$C_{p_i} = C_{p_i}(n)$$  \hspace{1cm} (4.4)

where \( n \in [1, 2, \ldots N] \) and \( N \) is the number of sample points. \( \alpha(n) \) is the discrete counterpart of \( \alpha(t) \). A discrete Fourier transform of \( \alpha(n) \) is employed to determine its spectral content, allowing the cyclic frequency, \( f_1 \), to be deduced. With \( f_1 \) known, a 4-pole Butterworth low-pass filter is applied to \( \alpha(n) \) with a cutoff frequency of \( 4f_1 \). Here, the multiple of four has been empirically determined- the factor consistently produced a “clean” indication of the airfoil’s pitch direction over the study’s wide variety of test conditions. Figure 4.1 displays the magnitude response of the filter. The sampled incidence was filtered in forward and reverse, correcting the phase response of the filter and accounting for the -160dB/dec slope in the transform’s response. The filtered position signal will be referred to as \( \hat{\alpha}(n) \). Care was taken not to further process the initial and final segments of \( \hat{\alpha}(n) \) (and the corresponding pressures) due to lag in the filter response. \( \alpha(n) \) and \( \hat{\alpha}(n) \) are compared in Figure 4.2.

The filter removes the high-frequency, random signal fluctuations in order to create a monotonically increasing/decreasing incidence. A simple forward difference method applied to the filtered signal, i.e., \( \delta \hat{\alpha} = \hat{\alpha}_{n+1} - \hat{\alpha}_n \), is then computed. If \( \delta \hat{\alpha} > 0 \), then the airfoil has increased its angle of attack, etc., creating a high (1) or low (0) direction signal. The direction is used in sorting the pressure and load for the ensemble calculations. Figure 4.3 shows the airfoil’s direction for the time snapshot shown in Fig 4.2 (\( \hat{\alpha} \) is scaled and offset for visual comparison).

\( C_{p_i}^p \) and \( C_{p_i}^S \) are computed using the \( P_{p_i}^p \), \( P_{p_i}^S \), \( P_s \), and \( P_0 \) – measured with absolute pressure transducers. The forces on the airfoil are then computed numerically for \( n \in \)
Figure 4.1. 4-Pole Butterworth filter used in data processing. Magnitude response, Cutoff frequency.

\[ [1, N] \text{ as follows:} \]

\[
\Delta C_{pj} = (C^p_{pj} - C^s_{pj}) \tag{4.5}
\]

\[
\Delta C_p \frac{dy}{dx_j} = C^p_{pj} \cdot \frac{dy}{dx_j} - C^s_{pj} \cdot \frac{dy}{dx_j} \tag{4.6}
\]

\[
C_n(n) = \frac{1}{2} \sum_{i=0}^{N_j-1} (x_{i+1} - x_i)(\Delta C_{pj+1} + \Delta C_{pj}) \tag{4.7}
\]

\[
C_a(n) = \frac{1}{2} \sum_{i=0}^{N_j-1} (x_{i+1} - x_i) \left( \Delta C_p \frac{dy}{dx_{i+1}} + \Delta C_p \frac{dy}{dx_i} \right) \tag{4.8}
\]

\[
C_{mE}(n) = \frac{1}{2} \sum_{i=0}^{N_j-1} (x_{i+1} - x_i) \left( \Delta C_{pj+1} \cdot x_{i+1} + \Delta C_{pj} \cdot x_i \right) \tag{4.9}
\]

\[
C_l(n) = C_n(n) \cdot \cos(\hat{a}(n)) + C_a(n) \cdot \sin(\hat{a}(n)) \tag{4.10}
\]

\[
C_d(n) = C_n(n) \cdot \sin(\hat{a}(n)) - C_a(n) \cdot \cos(\hat{a}(n)) \tag{4.11}
\]

\[
C_{mE}(n) = C_mE(n) + C_n(n)/4 \tag{4.12}
\]
where $N_x$ is the number of pressure ports. Other numerical integration routines, e.g. Simpson’s method, were not considered.

4.2 Method 1: Linear discretization

The range of the airfoil, $\alpha \in [\alpha_{\text{min}}, \alpha_{\text{max}}]$ is broken into 201 points (200 intervals) and linearly spaced from $\alpha_{\text{min}}$ to $\alpha_{\text{max}}$. The collocation points for averaging are the midpoint of each bin, $(\alpha_{i+1} - \alpha_i)/2$. Subsequently, the average of the quantities in Equations (5)-(10) are computed, e.g.,

$$\left\langle C_{l_i} \right\rangle = \frac{1}{N} \sum_{n=1}^{N} c_{l_i}^{(n)} \quad \alpha_i \in [\alpha_i - \Delta, \alpha_i + \Delta]$$

(4.13)

where $\alpha_i = (\alpha_1, \alpha_2, ..., \alpha_N)$ are the collocation points and $\Delta = (\alpha_{\text{max}} - \alpha_{\text{min}})/(2N)$. Decreasing the size of $\Delta$ leads to a less “smoothed” version of the ensemble, if desired.
Figure 4.3. Direction signal and comparison to instantaneous angle of attack.

Figure 4.5a shows the results of the ensemble in terms of the lift and quarter-chord moment. Due to the cycle-to-cycle deviations post separation, the average forces show odd trends near the maximum angle of attack. Comparing the averaged forces to their instantaneous counterparts, shown in Figure 4.4, yields several unsettling conclusions. The ensemble fails to capture the change in $C_l$ near $\alpha_{max}$ and the break in lift between the pitch-up stroke and pitch-down stroke poorly represents the data. Overall, cycle-to-cycle forces appear qualitatively similar, with slight fluctuations in peak values. Figure 4.5b presents a histogram of the linear discretization. Clearly, the discretization favors $\alpha_{min}$ and $\alpha_{max}$ due to the decreased pitch velocity and constant sampling rate.
4.3 Method 2: Linear discretization using the filtered incidence

An alternative method is to use $\hat{\alpha}(n)$ as opposed to $\alpha(n)$ when determining the ensemble. Figure 4.6a highlights the results. It is clear that the average more accurately captures the cycle to cycle behavior of the loads, without producing inaccurate artifacts. Method 2 captures the increase $C_l$ but fails to fill the void in points near lift stall and the peak nose-down moment. The histogram for Method 2 is displayed in Figure 4.6b. Again, more points are captured at the maximum and minimum angles of attack. Also worth noting, the histograms appear smoother and more symmetric compared to Fig-
Figure 4.5. Ensemble forces and histogram for a linearly discretized ensemble algorithm. Up-stroke ○. Down-stroke ●.

\[
\langle C_{l_i} \rangle = \frac{1}{N} \sum_{n=1}^{N} C_{l_{i,n}}^{(n)}, \quad \hat{\alpha}_j \in [\hat{\alpha}_j - \Delta, \hat{\alpha}_j + \Delta]
\]  

(4.14)

4.4 Method 3: Cosine angular discretization

Method 1 and Method 2 produce histograms that favor \( \alpha_{\text{min}} \) and \( \alpha_{\text{max}} \), reducing the resolution of the rapidly changing forces near \( \alpha_{\text{max}} \). Clearly, smaller bin size at \( \alpha_{\text{max}} \) are necessary. To achieve the desired distribution, the range of motion was discretized
using a cosine spacing, where the smallest $\Delta \alpha$'s were at $\alpha_{\text{min}}$ and $\alpha_{\text{max}}$. Figure 4.7a and 4.7b show the resulting force coefficients and histograms. Unlike the previous results, the histogram resulting from the cosine distribution yields a more consistent dispersal of the data points. However, instead of too many points at the endpoints, the distribution yields to few. Resolution of the stall behavior is, on the other hand, quite good in comparison to the linear discretization techniques.
Figure 4.7. Ensemble forces and histogram for a cosine discretized ensemble algorithm based on the filtered incidence. Up-stroke ○. Down-stroke ●.

4.5 Method 4: Cosine angular discretization with endpoint logic

The lack of points found at the min/max angle of attack using the cosine intervals of Method 3 may produce erroneous statistical quantities. This is a result of too points captured in the end point bins as the bin size approaches zero. In order to avoid potential problems, method 4 combines bins near the endpoints if the number of points are less than 3 standard deviations removed from the mean number of points per bin. Ultimately, this technique was used to average all 1P data.
4.6 Comparison of ensemble average and maximum and minimum forces

Figure 4.9 demonstrates the difference between the ensemble averaged maximum and minimum forces for deep dynamic stall, i.e., the worse case scenario in terms of cycle-to-cycle differences. For the attached loads at small $\alpha$, the ensemble average and maximum and minimum forces closely agree. Significant divergence occurs shortly after moment-stall, near $\alpha = 14^\circ$ on the up-stroke, affecting both the lift, moment, and drag (not shown) similarly. The largest deviations are observed following lift stall as the flow fully separates. Significant contrast continues as the airfoil pitches down, slowly decreasing as the flow reattaches to the airfoil. Once the flow does reattach,
near $\alpha = 7^\circ$, there exists little distinction between the ensemble and extrema forces. Stall onset and light stall exhibit similar patterns where the magnitudes of the extrema and difference from the ensemble profile correlate primarily to vortex presence and the degree of flow separation.

Figure 4.9. Maximum and minimum forces (solid fill) compared to the ensemble average for a deep dynamic stall case (RUN685). $M_x=0.27$, $k = 0.056$, $\alpha = 10.0^\circ + 10.0^\circ \sin \omega_1 t$. Ensemble average $\text{---}$; $2\pi \alpha/\beta$ $\text{---}$; $\pi \alpha/\beta$ $\text{---}$; $\pi \alpha$ $\text{---}$.
CHAPTER 5

CONVECTION PROPERTIES OF THE DYNAMIC STALL VORTEX

The dynamic stall vortex resides over the chord of the airfoil for only a short duration. During this lifespan, the vortex exhibits three distinct phases – the accumulation of surface vorticity, the accretion or ‘roll-up’ of vorticity into a concentrated structure, and the convection of the vortex over the chord and into the airfoil’s wake. Although the ideal study of the vortex’s evolution requires time-resolved flow field measurements, e.g. PIV, surface pressures can be used to study the 1D vortex kinematics. This chapter focuses on the prescribed forcing parameters and Mach related effects on the 1D stall vortex kinematics. This is determined from a close, but objective examination of the light and deep dynamic stall test data.

5.1 Background

The vortex kinematics can be deduced by tracking the maximum velocities, or equivalently, the minimum pressures on the airfoil upper-surface via the static pressure transducers distributed along the airfoil chord. Past studies – see [56, 95] – have followed this approach to compute the stall vortex convection speed, \( V \), defined as

\[
V = \frac{\Delta x_{pav}}{\Delta t},
\]

(5.1)
where $\Delta x^{*}_{DSV}$ and $\Delta t$ are the change in position in of the stall vortex during the time period $\Delta t$. See the detailed review and experiment by Green et al.[56] for a discussion of related works as well as the convection speed of the stall vortex for several airfoil geometries ($Re_c = 1.5 \times 10^6$, $M_{\infty} = 0.11$).

Graphical methods tracing the minimum pressure can be employed to determine $V$ – see [22, 71]. Lorber and Carta[95] revised this calculation using linear regression analysis, therein removing subjective errors. The chord locations of the surface pressure instrumentation and the ‘time of minimum pressure’ are fit to determine the convection speed. Data in [95] from ramp forcing histories showed that the vortex speed was constant for $x \in (0.2, 0.8 - 0.9)$. Figure 5.1a and Figure 5.1b (see Figure 12 and 18, respectively, in [95]) highlight results for the convection speed at $M_{\infty} = 0.2$ for different reduced pitch-rates ($A = \frac{\dot{\alpha}}{2U_{\infty}}$) and motion ranges.

The stall vortex subroutine of the Leishman-Beddoes dynamic stall model[88] de-

![Image](image1.png)

(a) Location of the stall vortex for 0-30° ramps at $M_{\infty} = 0.2$ and several reduced pitch-rates.

![Image](image2.png)

(b) Effect of pitch-angle range on the stall vortex propagation at $A = 0.01$ and $M_{\infty} = 0.2$

Figure 5.1. Vortex trajectories for the SSC-A09, as presented by Lorber and Carta[95]
scribes the stall vortex path as

\[ x = 1 - \cos \left( \frac{\pi \tau}{\delta \tau} \right), \quad (5.2) \]

where \( \tau = 0 \) at the onset of separation conditions (defined as the instant in time when the normal force exceeds a threshold value) and \( \tau = \delta \tau \) when the vortex reaches the trailing edge. \( \delta \tau \) is the time the vortex resides over the airfoil chord. The model is compared to a measured vortex trajectory in Figure 5.3.

Significant ambiguities exist in the literature regarding the speed the vortex convects downstream\[56, 82, 89\]. This uncertainty stems from airfoil design, separation mechanisms, and motion history considerations; in general, the stall vortex convection speed falls into the range \( V / U_\infty \in (0.3, 0.5) [56, 89] \), with a weak dependence on Mach number\[88\]. As correctly captured in Eq. 5.2, the stall vortex does not move at a constant speed as it propagates over the chord. The effects of Reynolds number and compressibility on the vortex motion, including shock-induced dynamic stall, are not adequately discussed in the literature. This chapter provides further insight into the convection problem, and addresses the change in vortex motion associated with shock-induced separation.

5.2 Calculation of the vortex convection properties

Linear, least-squares regression analysis of the time of minimum pressure and the fixed position of the pressure transducers was used to determine the convection speed, \( V \). To mitigate cycle-to-cycle differences, \( \Delta t \) in Equation 5.1 was found from the ensemble incidence, \( \langle \alpha \rangle \). The time, \( t \), in which the stall vortex passes over a port is shown
in Equation 5.3.

\[ t = \frac{\arcsin\left(\frac{\alpha - \alpha_1}{\alpha_1}\right)}{2\pi f_1} \]  

(5.3)

Using the non-dimensional values, \( x = x^* c \) and a non-dimensional time, \( \tau \), defined as

\[ \tau = \frac{\pi f_1 t}{k} = \frac{U_{\infty} t}{c}, \]  

(5.5)

Equation 5.1 becomes

\[ \frac{V}{U_{\infty}} = \frac{\Delta x}{\Delta \tau}. \]  

(5.6)

Equation 5.6 is equivalent to finding the slope (in a least-squares sense) of the vortex position as a function of time, i.e., \( x(\tau) \). The parameter \( \tau \) offers the opportunity to glean information on the time-scales of the stall related events; \( \tau \) measures time in terms of chord lengths, i.e., for \( \tau = 1 \), a freestream fluid particle would traverse a distance equal to one chord.

Two of the three stages involved in the evolution of the stall vortex can be directly captured from the static pressure measurements. Changes in the airfoil pressure distribution are not manifested until the vortex begins to detach from the airfoil surface. Accordingly, this analysis does not delve into the first vortex stage – the accumulation and eruption of surface vorticity. During the subsequent gestation process, the accretion of surface vorticity continues, sourced from the favorable pressure gradient, as discussed by Reynolds and Carr[118]. The DSV velocity is small, inhibited by the large, adverse pressure gradient[71] immediately following it. The stall vortex accelerates away from the wall as the accrual of vorticity slows and eventually halts. At this point, the vortex has reached its ‘fully-developed’ stage and convects aft over the chord. The disparate
convection rates of the stall vortex found between the gestation period and its penultimate speed are referred to here in the context of a ‘wall-bounded’ and ‘fully-developed’ vortex, with respective non-dimensional rates $V/U_\infty|_w$ and $V/U_\infty|_f$.

$V/U_\infty|_w$ was calculated over a small distance immediately following the onset of separation. The onset point defined as the location of the pressure transducer to first detect a rearward moving pressure wave. The chord used to determine $V/U_\infty|_w$ was, unfortunately, subjective – too large a length over-predicts $V/U_\infty|_w$, whereas too small a length reduces the number of points available for the regression analysis. Generally speaking, the different ranges used to find $V/U_\infty|_w$ biased the data but had little affect on the observed trends in regard to the prescribed motion parameters or Mach number. Ultimately, $V/U_\infty|_w$ was found from $x \in [x_{DSV}, x_{DSV} + 0.13]$. The convection speed of the fully developed DSV was determined by fitting $\tau$ in the range $x \in (0.3, 1.0]$. This approach was found acceptable for all light and deep dynamic stall data.

Three additional kinematic properties were determined through this analysis - the inception point of the stall vortex, $x_{DSV}$, the gestation period of the stall vortex, $\delta \tau_g$, and the residence time of the stall vortex, $\delta \tau$.

The vortex inception point was computed from the $x$ location of the minimum $\tau$, subject to the condition that all subsequent $\tau$ were greater, i.e., $\tau_{x_{DSV}} < \tau_{x > x_{DSV}}$. In other words, this requires that the stall vortex act as a downstream convecting wave. Further note, $x_{DSV}$ is discrete due to the location of the pressure instrumentation. In the following discussions it is implied that the inception point is always located at a point $x \in (x_{DSV} - 1, x_{DSV}]$.

The gestation period of the stall vortex was estimated from the curves found for the wall bounded and fully developed vortex stages, i.e.,

$$x = V/U_\infty|_w \tau + b_w \quad x \in [x_{DSV}, x_{DSV} + 0.13],$$

(5.7)
\[ x = \frac{V}{U_x|_f} \tau + b_f \quad x \in [0.3, 1], \tag{5.8} \]

where \( b_w \) and \( b_f \) are the y-intercepts found from the regression analysis. The intercept of the two curves is used to estimate the end of the gestation period and the start of the fully developed vortex stage. Setting Equations 5.7 and 5.8 equal yields the gestation time, \( \tau_g \),

\[ \tau_g = \frac{b_w - b_f}{V/|U_x|_f - V/|U_x|_w}. \tag{5.9} \]

The gestation period is then, simply, the difference between the detachment of the stall vortex and \( \tau_g \),

\[ \delta \tau_g = \tau_g - \tau_{x=x_{DSV}}. \tag{5.10} \]

Similarly, the residence time was calculated as the difference in time between the vortex reaching the trailing-edge and the time in which the vortex first appeared, i.e.,

\[ \delta \tau = \tau_{x=1} - \tau_{x=x_{DSV}}. \tag{5.11} \]

5.2.1 Filtering the experimental data

For attached flow, stall-onset, or cases where the stall vortex’s presence was not discernible over the chord, the regression analysis failed, and resulted in large standard errors and poor coefficients of determination, \( R^2 \). For \( k \geq 0.05 \) and \( \alpha_{\text{max}} - \alpha_{\text{ss}} \geq 2.0^\circ \), the vortex is fairly well defined. The latter statement can be interpreted as a light stall requirement. Increases in the reduced frequency tend to increase the signature of the stall vortex, as indicated by the more pronounced \( C_p \)’s over the upper-surface pressure ports. The regression analysis was more amenable to the targeted test points \( k = 0.05, 0.075, 0.10 \). The lower-bound on \( k \) is typically considered the lowest reduced frequency that incurs significant unsteady effects[89]. Shock induced stall at \( M_{\infty} \) >
0.48 produced a vortex signature at lower incidences, \( \alpha_{\text{max}} - \alpha_{ss} \geq -1.0^\circ \), and, as such, was included in the following sections’ analysis. The survey included in this chapter considers only a subset of the available data, and, unless otherwise noted, adheres to the following restrictions:

1. trip: masking tape;

2. \( \alpha_{\text{max}} - \alpha_{ss} > 2.0^\circ \) when \( M_\infty < 0.50 \);

3. \( \alpha_{\text{max}} - \alpha_{ss} > -1.0^\circ \) when \( M_\infty \geq 0.50 \);

4. \( k > 0.040 \);

5. \( R^2_f \geq 80\% \);

where \( R^2_f \) is the coefficient of determination of the fit for \( \frac{V}{U_\infty} \bigg|_f \). The masking tape trip forced the separation mechanism to be constant throughout the Mach ranged examined (turbulent boundary layer separation as opposed to the breakdown of a transition bubble). The restriction on the ‘goodness-of-fit,’ \( R^2_f \), had a similar effect as that of the stall penetration angles. These restrictions allowed for data sets of size \( N = [46, 53, 60, 57, 7] \) points for \( M_\infty \approx [0.2, 0.3, 0.4, 0.5, 0.6] \).

Overall, the data presented in the ensuing sections required a discerning eye. The variety of forcing conditions and consequent fluid behavior did not lend itself to sweeping generalizations that make developing computer codes an easy task. That said, the majority of the data points have been checked on an individual basis to ensure accuracy.

5.3 Example analysis of the vortex trajectory

Figure 5.2 highlights the ensemble pressures and loads for a low-speed (\( M_\infty = 0.2 \)), deep dynamic stall case. Figure 5.2a illustrates the ensemble behavior of each
suction-side pressure transducer. 95 cycles were used to determine the averages. The pressure coefficients are offset by \(-C_p^S = 4.2\). Each curve is referenced to its own origin (the tic mark before the \(x\) locations of the transducers). The unwrapped normal force, pitch moment, and incidence are shown at the top of the figure. A strong suction peak in the pressure distribution occurs at \(x = 0.006\) and \(\omega t = 0.35\), and is followed by an abrupt increase in pressure. This breakdown is the signature of the onset of dynamic stall vortex and is communicated aft to the port at \(x = 0.022\). The low-pressure disturbance continues to convect downstream over the chord (down on Figure 5.2a) as time progresses. The vortex passes into the wake as the airfoil nears its peak incidence. The aerodynamic loads are presented in Figure 5.2b as a function of \(\alpha\). The vortex had a strong effect on the loads – best exhibited by the abrupt break in pitch-moment and increase in \(C_{n\alpha}\). Reattachment fails to occur until the airfoil reaches its minimum angle of attack.

The times of each local pressure minimum illustrated in Figure 5.2a are plotted against the position of the transducers and depicted in Figure 5.3. The convection speeds for \(x \in (0.001, 0.10)\) and \(x \in (0.30, 1.0]\) are indicated. The \(x = 0.006\) trace exhibits a minimum pressure (\(\tau = 1.76\)) corresponding to the separation of the stall vortex. The leading-edge transducer (\(x = 0.00\)) fails to exhibit a sudden increase in pressure, but does reach a minimum shortly after the \(x = 0.006\) signal at \(\tau = 1.92\). This minimum is not associated with the passage of a stall vortex, and is instead marked by a filled circle in Figure 5.3. This convention is adopted throughout this chapter. The stall vortex travels aft over the chord with an initial rate of \(V/\infty \big|_w = 0.048\). The convection speed during the formation of the stall vortex is an order-of-magnitude less than its terminal rate, \(V/\infty \big|_f\). During the gestation period, \(\delta \tau_g\), the vortex speed changes considerably. Aft of \(x = 0.30\), the vortex moves at a nearly constant rate of \(V/\infty = 0.40\).
Figure 5.2. Example result for ensemble pressures and aerodynamic loads, RUN 2321. $M_\infty = 0.2$, $\alpha = 10^\circ + 8.5^\circ \sin \omega t$, $k_1 = 0.10$.

- pitch-up, --- pitch-down, - - - steady.

for a duration $\tau = 1.68$. Overall, the behavior exhibited in Figure 5.3 is qualitatively similar to that found by Lorber and Carta[95] for ramp pitch motions, as shown previously in Figures 5.1a and 5.1b (note $\tau$ in [95] follows a separate definition, $\tau = t/T$, where $T$ is the sample period – this definition does not have physical significance in the associated figures unless the sample period is known). Figure 5.3 graphically illustrates both the gestation period, $\delta \tau_g$, and the vortex residence time, $\delta \tau$. 
Figure 5.3. Stall vortex position and time-of-minimum pressure, RUN2321.
- Fit for fully-developed stage.
- Fit for ‘wall’ stage.
- Pressure minimum not associated with vortex passage, DSV present.

5.4 The effect of $\alpha_0$, $\alpha_1$ and $k$ on the convection properties of the stall vortex at fixed Mach numbers

This section segregates the data set into the Mach test points, $M_{\infty} \approx 0.2, 0.3, 0.4, 0.5,$ and 0.6. The primary amplitudes of the sinusoidal pitch motions used throughout this study were 1P flywheel offsets corresponding to $\alpha_1 = 5^\circ$ and $8^\circ$. At high 1P frequencies, mechanical imbalance in the pitch-mechanism caused substantial shaking of the motor stand and increased the prescribed amplitude of the oscillation. As such, the following sections often refer to the input amplitudes of $5^\circ$ and $8^\circ$, whereas the measured amplitudes are plotted or tabulated.

Throughout this section, each Mach number is presented in a similar fashion, highlighting the loads, pressure distributions, and vortex paths for several cases. The data set is then examined in order to qualitatively extract the convection properties depen-
dence on the prescribed motion parameters. The attributes of the light and deep stall cases examined in this section are tabulated in Table 5.1.

5.4.1 Convection properties at $M_\infty \approx 0.2$

The ensemble averaged loads, pressures and vortex paths for the pitch cases $\alpha \approx \alpha_0 + 5.1^\circ \sin \omega t, k = 0.08$ are shown in Figures 5.4 and 5.6. The pressures in Figure 5.4 are plotted on a single vertical axis. The mean incidence, amplitude, and reduced frequency for the three runs are tabulated in Figure 5.6 with the vortex 1D paths.

RUN1954 is best characterized as a light dynamic stall case; RUN1870 and 1968 are both examples of deep stall. Each record produced a strong suction peak ($C_{p_{min}} < -10.0$) at $x = 0.006$. The initial footprint of the stall vortex occurred just aft of this port, captured by the pressure transducer at $x = 0.022$. The low-pressure, minima found in the $C_p$ plots of Figure 5.4 signify the passage of the stall vortex. This ‘wave’ sweeps aft over the chord as time increases. For each case, the influence of the stall vortex is clear, even as the vortex reaches the TE (---). The vortex induced pressure, $\Delta C_p$, at the TE for RUN1954 is $\Delta C_p = -0.33$, whereas it measured $-0.55$ for RUN1870 and $-0.65$ for RUN1968. From a Biot-Savart standpoint, the smaller $\Delta C_p \Delta$ (increased suction) indicate the presence of a stronger vortex filament, provided the distance between the stall vortex and TE has not decreased (flow visualization shows the opposite is true for increasing $\alpha_0$, i.e., a deep stall vortex, shed at larger $\alpha$, remains as distance significantly further above the airfoil surface than a light stall vortex shed at small $\alpha$). The time-histories of the upper-surface pressures, normal force, and pitch moment for RUN1870 are shown in Figure 5.5 for comparison to the ensemble average. The spurious peaks captured in the pressure time-series were from an undetermined source. The noise is not phased-locked with the 1P frequency, and washed out of the ensemble average.
<table>
<thead>
<tr>
<th></th>
<th>( M_{\infty} \approx 0.20 )</th>
<th>( M_{\infty} \approx 0.30 )</th>
<th>( M_{\infty} \approx 0.4 )</th>
<th>( M_{\infty} \approx 0.5 )</th>
<th>( M_{\infty} \approx 0.6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>RUN#</td>
<td>1954 1870 1968</td>
<td>1997 2003 1900</td>
<td>2119 2134 2140</td>
<td>2054 2239 2060</td>
<td>2072 2078 2083</td>
</tr>
<tr>
<td>( M_{\infty} )</td>
<td>0.20 0.19 0.19</td>
<td>0.30 0.30 0.29</td>
<td>0.41 0.41 0.41</td>
<td>0.53 0.54 0.53</td>
<td>0.60 0.59 0.58</td>
</tr>
<tr>
<td>( \alpha_{DS} - \alpha_{ss} \ [^\circ] )</td>
<td>2.15 3.23 3.96</td>
<td>1.70 2.09 2.69</td>
<td>1.78 3.09 3.39</td>
<td>2.79 3.26 3.13</td>
<td>0.35 1.53 1.88</td>
</tr>
<tr>
<td>( C_{N_{\text{max}}} )</td>
<td>1.52 1.60 1.65</td>
<td>1.38 1.44 1.55</td>
<td>1.21 1.30 1.32</td>
<td>1.39 1.37 1.42</td>
<td>1.26 1.37 1.38</td>
</tr>
<tr>
<td>( C_{\text{min}} )</td>
<td>-0.18 -0.24 -0.27</td>
<td>-0.18 -0.21 -0.24</td>
<td>-0.10 -0.16 -0.16</td>
<td>-0.15 -0.14 -0.16</td>
<td>-0.09 -0.15 -0.15</td>
</tr>
<tr>
<td>( C_{p_{\text{min}}} )</td>
<td>-9.81 -10.0 -10.1</td>
<td>-7.30 -7.31 -7.58</td>
<td>-4.59 -4.64 -4.59</td>
<td>-3.53 -3.46 -3.56</td>
<td>-2.95 -3.06 -3.09</td>
</tr>
<tr>
<td>( M_{\text{t_{\text{max}}}} )</td>
<td>0.72 0.71 0.72</td>
<td>1.05 1.04 1.03</td>
<td>1.24 1.24 1.24</td>
<td>1.57 1.56 1.56</td>
<td>1.72 1.71 1.70</td>
</tr>
<tr>
<td>( x_{\text{c_{P_{\text{min}}}}} )</td>
<td>0.006 0.006 0.006</td>
<td>0.006 0.006 0.006</td>
<td>0.006 0.006 0.006</td>
<td>0.049 0.022 0.022</td>
<td>0.087 0.087 0.086</td>
</tr>
<tr>
<td>( S_{1} )</td>
<td>-662 -748 -693</td>
<td>-471 -467 -492</td>
<td>-386 -392 -392</td>
<td>-300 -334 -311</td>
<td>-224 -284 -301</td>
</tr>
<tr>
<td>( S_{2} )</td>
<td>605 652 636</td>
<td>464 462 482</td>
<td>292 279 284</td>
<td>182 157 205</td>
<td>80.0 148 148</td>
</tr>
<tr>
<td>( \Delta C_{p} )</td>
<td>-0.33 -0.55 -0.65</td>
<td>-0.60 -0.63 -0.70</td>
<td>-0.44 -0.58 -0.60</td>
<td>-0.49 -0.47 -0.54</td>
<td>-0.32 -0.52 -0.52</td>
</tr>
<tr>
<td>( \frac{\Delta C_{p}}{%} )</td>
<td>3.4 5.3 10.2</td>
<td>8.2 8.6 9.2</td>
<td>9.7 12.4 13.0</td>
<td>13.9 13.5 15.3</td>
<td>10.9 16.7 16.9</td>
</tr>
<tr>
<td>( V/</td>
<td>U_{\infty}</td>
<td>_{f} )</td>
<td>0.19 0.31 0.29</td>
<td>0.45 0.39 0.42</td>
<td>0.44 0.49 0.39</td>
</tr>
<tr>
<td>( V/</td>
<td>U_{\infty}</td>
<td>_{w} )</td>
<td>0.038 0.053 0.046</td>
<td>0.39 0.044 0.048</td>
<td>0.031 0.033 0.029</td>
</tr>
<tr>
<td>( V/</td>
<td>U_{\infty}</td>
<td>_{s} )</td>
<td>-0.034 -0.018 -0.031</td>
<td>-0.034 -0.018 -0.031</td>
<td>-0.053 -0.040 -0.047</td>
</tr>
<tr>
<td>( x_{DSV} )</td>
<td>0.006 0.022 0.022</td>
<td>0.006 0.006 0.006</td>
<td>0.022 0.022 0.022</td>
<td>0.086 0.048 0.086</td>
<td>0.134 0.134 0.134</td>
</tr>
<tr>
<td>( \delta \tau_{g} )</td>
<td>4.37 3.70 3.60</td>
<td>3.79 3.45 3.23</td>
<td>3.97 4.05 4.13</td>
<td>1.78 1.56 1.91</td>
<td>1.64 2.70 1.97</td>
</tr>
<tr>
<td>( \delta \tau )</td>
<td>7.50 5.97 6.18</td>
<td>5.58 5.76 5.08</td>
<td>5.89 5.85 6.27</td>
<td>4.67 4.02 4.84</td>
<td>5.25 4.71 4.52</td>
</tr>
<tr>
<td>( m )</td>
<td>1.39 1.70 1.63</td>
<td>2.10 1.50 1.85</td>
<td>2.18 2.10 2.06</td>
<td>1.66 1.26 1.15</td>
<td>1.13 1.98 1.31</td>
</tr>
<tr>
<td>( 2\Xi_{\text{cycle}}/\pi k )</td>
<td>-1.17 -0.59 0.10</td>
<td>1.08 1.46 1.77</td>
<td>1.32 1.81 2.04</td>
<td>1.24 1.34 1.87</td>
<td>0.84 1.20 1.68</td>
</tr>
</tbody>
</table>
The stall vortex motion for the three cases examined here is shown in Figure 5.6. An increased mean incidence, \( \alpha_0 \), promoted the stall vortex to form at a reduced \( \tau \). This is a result of \( \alpha_0 \) approaching \( \alpha_{ss} \). Subsequently, the stall vortex forms ‘earlier’ during the pitch cycle. For a fixed \( k \) and \( \alpha_1 \), the starting signature of the vortex appears at roughly the same incidence, \( \alpha \approx 15^\circ \). The two convection speeds, \( \frac{V}{U_\infty} \bigg|_w \) and \( \frac{V}{U_\infty} \bigg|_f \), for RUN1954 are significantly smaller than the convection speeds experienced during the deep stall runs. The duration of the stall vortex, measured by \( \delta \tau \), lasted more than a chord in time longer through light stall than during deep stall. RUN1954 promoted the largest value of negative damping, \( \Xi \_\text{cycle} = -0.14 \), compared to \( \Xi \_\text{cycle} = 0.07 \) and 0.01 for RUNS 1870 and 1968, respectively.

Figure 5.7 highlights each of the three convection properties for the filtered data set at \( M_{\infty} = 0.20 \). The stall vortex initial speed, \( \frac{V}{U_\infty} \bigg|_w \), shows little dependence on \( \alpha_0 \). A slight difference exists between the 5\(^\circ\) and 8\(^\circ\) input amplitudes, with average ratios 0.044 and 0.041, respectively. Increasing \( k \) caused an increase in the initial convection rate.

For \( \alpha_1 \approx 5^\circ \), \( \frac{V}{U_\infty} \bigg|_f \) varies from 0.20 to 0.40 for \( \alpha_0 \in (7^\circ, 10^\circ) \). For \( \alpha_1 \approx 8^\circ \), \( \frac{V}{U_\infty} \bigg|_f \) varies from 0.20 to 0.32 for \( \alpha_0 \in (10^\circ, 13^\circ) \). The trends indicate that the convection rate was dependent on the maximum amplitude, \( \alpha_0 + \alpha_1 \); this implies that the severity of the stall regime played an important role in setting the stall vortex speed. This dependency is exhibited by the individual cases described previously (Figures 5.4 and 5.6). Figures 5.7d and 5.7e are combined in Figure 5.8 to show the dependence on maximum incidence. Additionally, Figure 5.7f indicates a linear dependence of \( \frac{V}{U_\infty} \bigg|_w \) and \( \frac{V}{U_\infty} \bigg|_w \) on \( k \). The correlation between \( \frac{V}{U_\infty} \bigg|_f \) and \( k \) is supported by the observations in [95].

The duration the vortex remains over the airfoil is weakly correlated to \( \alpha_0 \) and showed no dependency on \( \alpha_1 \). \( \delta \tau \) was, however, affected by the reduced frequency. The
Figure 5.4. Ensemble average aerodynamic loads and upper-surface pressures, $M_\infty = 0.20$. Loads: pitch-up, pitch-down, steady reduced pitch-rate was also considered and a similar trend was exhibited, i.e., increasing $A$ caused $\delta \tau$ to decrease. However, $\delta \tau$ was more dependent on $k$, in a least-squares sense, for the sinusoidal tests involved in this experiment.
Figure 5.5. Time-histories of loads and upper surface pressures for RUN1870, \( M_\infty = 0.20 \)

<table>
<thead>
<tr>
<th>( M_f \geq 1.0 )</th>
<th>( M_f \geq 1.31 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.0005</td>
<td>-0.0005</td>
</tr>
<tr>
<td>-0.0022</td>
<td>-0.0022</td>
</tr>
<tr>
<td>-0.0049</td>
<td>-0.0049</td>
</tr>
<tr>
<td>-0.0086</td>
<td>-0.0086</td>
</tr>
<tr>
<td>-0.0134</td>
<td>-0.0134</td>
</tr>
<tr>
<td>-0.0191</td>
<td>-0.0191</td>
</tr>
<tr>
<td>-0.0257</td>
<td>-0.0257</td>
</tr>
<tr>
<td>-0.0331</td>
<td>-0.0331</td>
</tr>
<tr>
<td>-0.0412</td>
<td>-0.0412</td>
</tr>
<tr>
<td>-0.0493</td>
<td>-0.0493</td>
</tr>
<tr>
<td>-0.0691</td>
<td>-0.0691</td>
</tr>
<tr>
<td>-0.0792</td>
<td>-0.0792</td>
</tr>
<tr>
<td>-0.0895</td>
<td>-0.0895</td>
</tr>
</tbody>
</table>

5.4.2 Convection properties at \( M_\infty \approx 0.3 \)

Supersonic flow first appears at a freestream speed near \( M_\infty = 0.30 \) for the Bell airfoil. From inspection of the measured pressure distribution at a variety of prescribed
Figure 5.6. Vortex position for $M_\infty = 0.20$ filled symbols: no DSV, ..... Fits from convection rate analysis.

motions, the breakdown of the LE flowfield and ensuing vortex motion is very similar to the dynamics witnessed at $M_\infty \approx 0.2$. At first glance, the small region of supersonic flow does not alter the vortex behavior.

The ensemble averaged loads and pressures for the pitch cases $\alpha \approx \alpha_0 + 5.2 \sin \omega t$, $k = 0.08$ are shown in Figures 5.9. The cases were chosen to represent increasing stall severity, from left to right. Qualitatively, the loads and pitch-up/pitch-down hysteresis are similar to $M_\infty = 0.2$. The influence of the stall vortex on the lift and pitch-moment is clear. Each case produced a suction peak ($C_{p_{\text{min}}} < -7.0$) at $x=0.006$. The red line in each of the $C_p$ plots denotes the sonic pressure at that sample’s measured Mach number. The maximum local Mach number for each case was found to be approximately 1.04. The induced pressures at the TE are $\Delta C_p = -0.60$, $-0.63$, and $-0.70$ for $\alpha_0 = 10.3^\circ$, $11.6^\circ$, and $13.2^\circ$, respectively. As before, a single time-history case, Figure 5.10, is presented for comparison to the ensemble pressures and loads of RUN2003 as well as the $M_\infty = 0.2$ behavior. Aside from the decreased amplitude and slight
Figure 5.7: Effect of 1P parameters on stall vortex convection properties, $M_{x_0} \approx 0.2$.

- $\alpha_1 \approx 5^\circ$ (input), $\triangle \alpha_1 \approx 8^\circ$ (input)
phase-shifts, the time-histories at $M_{\alpha} = 0.2$ and $M_{\alpha} = 0.3$ indicate that the stall processes are governed by the same physical mechanism - an abrupt breakdown of the leading-edge flow.

The measured motion parameters for the three cases are tabulated in Figure 5.11 along with the resulting vortex trajectories. From the time of minimum pressure measurements, the stall vortex originated at $x = 0.006$ for each case, further forward on the chord than the three runs examined at $M_{\alpha} = 0.20$. Depending on the motion parameters, both the $M_{\alpha} = 0.2$ and $0.3$ test points resulted in a nearly equal likelihood that the initial vortex signature was captured by either the first or second suction-side pressure transducers, positioned at $x = 0.006$ or 0.022, respectively. Despite the different ensemble pressure behavior and aerodynamic loads seen in Figure 5.9 and logged

![Graph](image)

**Figure 5.8.** Fully developed vortex convection speed dependence on $\alpha_{max}$, $M_{\alpha} \approx 0.2$
Figure 5.9. Ensemble average aerodynamic loads and upper-surface pressures, $M_\infty = 0.30$. Loads: \textbullet\textbullet pitch-up, \textbullet\textbullet pitch-down, \textsolidarrow steady

in Table 5.1, the vortex paths are nearly identical in Figure 5.11 (aside from the time-shift), and produce similar convection properties. Although the effect of increasing $\alpha_0$ appears to increase the strength of the stall vortex (inferred by the increased TE suction.
pressures), the outcome on the convection properties is not as deterministic as found at $M_\infty = 0.2$ with regard to $\alpha_0$ or $\alpha_{max}$. This is examined more thoroughly through the entire $M_\infty = 0.3$ data set.
Figure 5.11. Vortex position for $M_\infty = 0.30$ filled symbols: no DSV, 
--- Fits from convection rate analysis.

Although the appearance of supersonic flow did not seem to affect the load or chord
pressure’s in a qualitative sense, the overall stall vortex convection properties lack the
trends found at $M_\infty \approx 0.2$. The initial convection speed of the stall vortex, $V/U_\infty|_w$,
is less dependent on mean or amplitude of the forcing; the linear increase in $V/U_\infty|_w$
found due to an increase in $k$ at $M_\infty \approx 0.2$ remains at $M_\infty \approx 0.3$.

Similar to the vortex gestation behavior, the ultimate convection speed of the stall
vortex shows no functional relationship to either $\alpha_0$ or $\alpha_1$. The dependency on stall
regime found at $M_\infty \approx 0.2$ is eliminated. There remains a dependency of $V/U_\infty|_f$ on
$k$.

The duration the stall vortex is present over the airfoil falls into four constant bands
or groupings depending on the figure inspected. In Figure 5.12g, the bands are primarily
a consequence of the prescribed reduced frequency, and, to a much lesser degree, the
oscillation amplitude. The combination of $\alpha_1 = 5^\circ$ and $k \approx 0.05$ results in $\delta\tau \approx 8$,
wheres a $k > 0.05$ results in $\delta\tau \approx 5.5$ An amplitude of $8^\circ$ caused slight variations -
\( \delta \tau \approx 7.2 \) for \( k \approx 0.05 \) and \( \delta \tau \approx 5.4 \) for \( k > 0.05 \). Similar to \( M_{\infty} \approx 0.2 \), the vortex’s residence time over the chord decreases with increasing \( k \).

The fact that the qualitative behavior of the separation and convection properties of the stall vortex fail to be effected by \( \alpha_{\text{max}} \), i.e., the stall regime, suggests that the small region of supersonic flow developed at \( M_{\infty} \approx 0.3 \) inhibits further growth of the stall vortex. This observation was similarly purported by Lorber and Carta[95] on the SCC-A09 airfoil at \( M_{\infty} = 0.4 \) – the first Freestream Mach number where the upper surface pressures exceeded the sonic limit for that geometry.

### 5.4.3 Convection properties at \( M_{\infty} \approx 0.4 \)

Proceeding in the same fashion as the previous sections, the ensemble loads and pressures for the family of sinusoids described by \( \alpha \approx \alpha_0 + 5.2 \sin \omega t, k = 0.07 \) are shown in Figure 5.13. A single light stall case is presented in Figure 5.13a. Deep stall cycles, RUNS 2134 and 2140, are illustrated in Figure 5.13b and c. For each case, the strongest suction peak occurred at \( x=0.006 \) \( (C_p_{\text{min}} < -4.5) \), corresponding to a peak local Mach number near 1.24. The chordwise extent of supersonic flow increased with increasing \( \alpha \) on the up-stroke, moving forward toward the leading-edge; the chordwise extent of the supersonic flow was only a small portion of the chord \( (x < 0.5\%) \) and reemerged during the downstroke. The stall vortex’s presence on the ensemble loads is not as evident as it was at \( M_{\infty} \approx 0.2 \) or 0.3. The stall delay is qualitatively similar, but the increment to \( C_{l_{\text{r}}} \) and the pitch-moment break are not as apparent, especially between the two deep dynamic stall cases. The vortex induced minimum pressures at the trailing-edge measured \( \Delta C_p = -0.44, -0.58, \) and \( -0.60 \) for \( \alpha_0 = 7.5^\circ, 9.7^\circ, \) and \( 10.9^\circ \), respectively. Two cycles of instantaneous pressures and loads are shown in Figure 5.14 for RUN2134. At \( M_{\infty} \approx 0.4 \) the mechanism for dynamic stall is somewhat
Figure 5.12: Effect of 1P parameters on stall vortex convection properties, $M_{\infty} = 0.3$

- $\alpha_0 \approx 5^\circ$ (input), $\Delta \alpha_1 \approx 8^\circ$ (input)
Figure 5.13. Ensemble average aerodynamic loads and upper-surface pressures, $M_\infty = 0.41$. Loads: \(\cdot\) pitch-up, \(\cdot\cdot\) pitch-down, \(\cdot\cdot\cdot\) steady
‘gray’. Both the surface pressure measurements and the focusing schlieren images revealed that no strong, normal shocks develop until $M_\infty > 0.47 \pm 0.02$. Instead, separation involves the interaction of weak shock waves with the boundary layer – see [31].

The vortex trajectories and measured motion parameters are shown in Figure 5.15. $x_{DSV}$ was 0.022 for each case but shifted between $x = 0.006$ and $x = 0.022$ for the $M_\infty = 0.4$ dataset. Although the loads and pressures differ between the three cases, the vortex trajectories are similar and result in similar convection properties. Up to and including $M_\infty \approx 0.4$, the overall shape and characteristics of the vortex trajectories are qualitatively similar and well approximated by Equation 5.2. The outcome of the interaction of the boundary layer and weak shocks on the separation behavior and the convection properties makes for a challenging interpretation of the effect of the prescribed motion parameters. Figure 5.16 highlights the results, but, unlike the lower freestream speeds, obvious trends are difficult to infer. The reduced-frequency stands out as the only parameter correlated to the time-dependent convection properties. However, this dependency appears multi-branched, showing opposite trends below and above $k = 0.075$ for $V/U_\infty |_f$ and $\delta \tau$, and possibly for $V/U_\infty |_w$ as well.

5.4.4 Convection properties at $M_\infty \approx 0.5$

At $M_\infty \approx 0.5$, separation is governed by the formation of a shock near the leading-edge. The local Mach number at the airfoil surface exceeds 1.31 and a normal shock induces separation in the range $x \in [0.022, 0.086]$, subject to the prescribed $\alpha(t)$. The ensemble loads and upper-surface pressures are shown in Figure 5.17 for three separate cases of increasing stall severity at $M_\infty = 0.53$. The shock phenomenon produces several ramifications on the resulting light and deep dynamic stall behavior. Compared
to $M_\infty \approx 0.2, 0.3,$ and 0.4, the effect of the stall vortex, although prominent in the associated $C_p$ distributions, is not witnessed through the ensemble $C_n$. I.e, there is no distinct change in $C_{n_\alpha}$ as the airfoil pitches through $\alpha_{ss}$ and the stall vortex forms. The
vortex itself is produced by the separation caused by the normal shock. Meanwhile, the
LE flow remains attached, maintaining a pressure distribution that varies linearly with
$\alpha$. Separation follows the stall vortex downstream, whereas the shock moves forward
toward the LE – also inducing separation (this separation is limited by $\alpha_{max}$, and, for
the cases considered here does not reach $x = 0$). Thus, there exists two, opposing
separation patterns in shock-induced dynamic stall. The rapid break in $C_{m_{c/4}}$ is still
apparent as the vortex induced pressure at the TE is significant. The $\Delta C_p$ values at
$M_{\infty} \approx 0.53$ are $-0.49$, $-0.47$, and -0.54 for RUNS 2054, 2239, and 2060, respectively.

The simultaneous forward motion of the shock and aft propagation of the stall vortex
is further visualized in the time-histories of the upper-surface pressures for RUN2239,
found in Figure 5.18, and, perhaps better, by the vortex trajectories in Figure 5.19. The
filled symbols in Figure 5.19, although still representative of the minimum pressures
at that chord location, are immediately followed by a rapid increase in pressure. This
increase in pressure corresponds to a falloff in local Mach number from $\approx 1.56$ to

![Figure 5.15. Vortex position for $M_{\infty} = 0.41$ filled symbols: no DSV,
---- Fits from convection rate analysis.](image)
Figure 5.16: Effect of 1P parameters on stall vortex convection properties, $M_\infty = 0.4$

\( \diamond \alpha_1 \approx 5^\circ \) (input), \( \triangle \alpha_1 \approx 8^\circ \) (input)
subsonic velocities, indicative of the passage of a shock wave. Also worth noting, as compared to non-shock induced separation cases, the convection speed, \( V/|U_{\infty}|_f \), is constant over a much larger section of the chord, \( x \in (0.175, 1] \). In Figures 5.6, 5.11, and 5.15, the gestation period occurs over a time \( \tau_g \approx 3 - 4 \) chords. At \( M_\infty = 0.53 \), the vortex forms and reaches its terminal speed in \( \delta \tau_g < 1.8 \) chords over a range of \( x \) less than 0.05c. This is a consequence of the sudden breakdown of the potential flowfield due to the normal shock.

With the competing separation mechanisms, particularly the shock motion, another convection property can be introduced as it might be of some interest. Fitting the forward motion of the minimum pressures of the shock in a similar fashion as the pressures associated with the movement of the stall vortex, yields the convection speed of the shock wave. Keeping with the general notation, this convection rate is referred to as \( V/|U_{\infty}|_s \) and is tabulated in Table 5.1 along with the DSV convection properties. Overall, the shock motion is slow, as it moves upstream against a favorable pressure gradient, and is comparable to the convection rate of the stall vortex as it moves downstream, opposite the adverse pressure gradient.

Figure 5.20 provides the entire forcing parameter, \( M_\infty \approx 0.50 \) data subset. Correlations to either \( \alpha_0 \) or \( \alpha_1 \) are again very weak for \( V/|U_{\infty}|_w \), \( V/|U_{\infty}|_f \), and \( \delta \tau \). The multi-branched behavior with respect to \( k \) witnessed at \( M_\infty \approx 0.4 \) is replicated at \( M_\infty \approx 0.5 \).

5.4.5 Convection properties at \( M_\infty \approx 0.6 \)

Only 5° input amplitudes were considered at \( M_\infty \approx 0.6 \), although, as previously discussed, the measured amplitudes were larger due to mechanical shaking. The ensemble loads and chord pressures for three select runs are shown in Figure 5.21. Similar to the observations at \( M_\infty = 0.53 \), the normal force shows no increase in \( C_{n_x} \); \( C_{m_{x/4}} \) in-
Figure 5.17. Ensemble average aerodynamic loads and upper-surface pressures, $M_{\infty} = 0.53$. Loads: ••• pitch-up, •• pitch-down, • steady

creases marginally and then abruptly breaks. These traits are once again attributed to the same physical pressure variation over the chord – simultaneous movement of the DSV downstream, initiated by a strong, normal-shock, and the upstream progressing shock wave. For each illustrated case, the shock fails to produce notable separation at $x = 0.006$, a function of $\alpha_{\text{max}}$. These features are best viewed through a closer examina-
Figure 5.18. Time-histories of loads and upper surface pressures for RUN2239, $M_\infty = 0.53$.

At $M_\infty \approx 0.6$, the shock forms in the range $x \in [0.086, 0.134)$. A stall vortex is produced – indicated by the induced suction pressure of the vortex at the TE (seen in the $C_p$ figures at the bottom of
Figure 5.19. Vortex position for $M_x = 0.53$ filled symbols: shock,

.... Fits from convection rate analysis.
Figure 5.20: Effect of 1P parameters on stall vortex convection properties, $M_\infty \approx 0.5$

- $\alpha_1 \approx 5^\circ$ (input), $\triangle \alpha_1 \approx 8^\circ$ (input)
Figure 5.21. Ensemble average aerodynamic loads and upper-surface pressures, $M_\infty = 0.60$. Loads: 
- pitch-up, 
- pitch-down, 
- steady
Figure 5.21 as well as for RUN2078 in Figure 5.22). At $M_{\infty} = 0.6$, these induced pressures measure $\Delta C_p = -0.31, -0.53, \text{ and } -0.56$. The drop off in $\Delta C_p$ that has occurred with increasing $M_{\infty}$ is not observed in moving from $M_{\infty} = 0.53$ to $M_{\infty} = 0.60$. The peak suction pressures have retreated to the chord position $x = 0.0865$ and measure $C_{p\text{min}} = -2.95, -3.06, -3.09$ for RUNS 2072, 2078, and 2083. The respective local Mach numbers are 1.72, 1.71, 1.70 (the local Mach numbers decrease due to the slight decrease in the measured $M_{\infty}$, tabulated in Figure 5.23).

The time of minimum pressures are shown in Figure 5.23. The aft propagation of the DSV and simultaneous forward motion of the shock wave are exhibited. The downstream moving, DSV is first detected at $x = 0.134$ for each case – significantly further aft on the chord than at $M_{\infty} \approx 0.2, 0.3, \text{ or } 0.4$. The gestation period increases slightly over that observed at $M_{\infty} = 0.53$. However, this ‘gestation’ period is radically different than that previously observed, and, rather, the stall vortex appears to form and convect with a much more subtle change between $V/U_{\infty}|_w$ and $V/U_{\infty}|_f$. The convection properties of the stall vortex and normal shock are compiled in Table 5.1. The number of points taken at this Mach number that met the desired restrictions are insufficient for statistical interpretation but, nonetheless, are shown for completeness in Figure 5.24.

5.5 The effect of increasing Mach number on the stall vortex convection properties

A deep dynamic stall cycle is examined at each Mach test point to highlight the stall vortex kinematics with respect to increasing compressibility. Similar to the prior approach, the entire light and deep dynamic stall dataset (conforming to the restrictions outlined in §5.2.1) is then examined.
5.5.1 Deep Stall: $\alpha_1 \approx 8^\circ$, $k \approx 0.075$

Several runs were selected to highlight the effect of compressibility on the stall vortex 1D kinematics for $\alpha_1 \approx 8^\circ$ and $k \approx 0.073 \pm 0.03$. The prescribed motions for the
Figure 5.23. Vortex position for $M_\infty = 0.60$ filled symbols: shock, 
Figure 5.23. Vortex position for $M_\infty = 0.60$ filled symbols: shock, 
Figure 5.23. Vortex position for $M_\infty = 0.60$ filled symbols: shock, 
--- Fits from convection rate analysis.

<table>
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<th>$M_\infty$</th>
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<th>$\alpha_1$</th>
<th>$k$</th>
</tr>
</thead>
<tbody>
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<td>0.60</td>
<td>3.1</td>
<td>7.0</td>
<td>0.069</td>
</tr>
<tr>
<td>2078</td>
<td>0.59</td>
<td>5.6</td>
<td>6.8</td>
<td>0.070</td>
</tr>
<tr>
<td>2083</td>
<td>0.59</td>
<td>7.3</td>
<td>6.2</td>
<td>0.071</td>
</tr>
</tbody>
</table>
Figure 5.24: Effect of 1P parameters on stall vortex convection properties, $M_{\infty} = 0.6$

- $\circ \alpha_1 \approx 5^\circ$ (input), $\triangle \alpha_1 \approx 8^\circ$ (input)
selected cases were chosen to represent similar stall penetration results: $\alpha_{ds} - \alpha_{ss} \geq 3.0$. Due to the limited test points at $M_\infty = 0.6$, this stall penetration behavior did not occur; instead, the largest amplitude/stall penetration case at $M_\infty = 0.6$ was selected.

Figure 5.25 illustrates the normal force, pitch-moment, and vortex paths for the Mach numbers 0.20, 0.30, 0.41, 0.54, and 0.59. Table 5.2 documents key information describing the runs’ attributes, including the stall vortex convection properties. The runs showcase the expected trends associated with compressibility with respect to $C_n$, $C_{m_{c/4}}$, $C_{p_{min}}$, etc. The rapid break in $C_{m_{c/4}}$ for each run indicates the presence of a stall vortex. The increase of $C_{p_{u}}$ found at $M_\infty = 0.2$ and 0.3 is not as distinguished at higher Mach due to the simultaneous increase in $Re_c$, which encourages flow attachment at the LE.

The vortex paths, at first glance, are qualitatively similar; however, further study reveals notable contrast. Foremost, the characteristic path of the stall vortex changes sharply, shifting ‘up’ (aft on the chord), as the freestream speed increases from $M_\infty = 0.41$ to 0.54. This abrupt change coincides with the appearance of a normal shock at $M_\infty = 0.47$ and the subsequent, radical change in separation mechanism.

For $M_\infty \leq 0.41$, the stall vortex is born out of the abrupt breakdown of the LE turbulent boundary-layer. Despite the similar onset mechanism for the three cases, the vortex paths at $M_\infty = 0.20, 0.30, 0.41$ possess several distinctions. The vortex path at $M_\infty = 0.20$ demonstrates a gradual acceleration from inception to $V/U_\infty |_f$. This suggests that the breakdown of the favorable pressure gradient ‘feeds’ the stall vortex, increasing its strength and size. The dependence on $\alpha_{max}$ found in Figure 5.8 also indicates that the size/inertia of the stall vortex can be altered at this speed. Flow visualization and/or flow field measurements are required to support this hypothesis; similar features have been established at incompressible freestream velocities (20 ft/s)[4].
Figure 5.25: Mach effects on stall vortex trajectory. filled symbols: no DSV, ---- Fits from convection rate analysis.
### TABLE 5.2
MACH EFFECTS ON THE CONVECTION PROPERTIES OF THE STALL VORTEX

| RUN# | M_x | α_0[°] | α_1[°] | k | αDs - α-sc | \(C_{N_{max}}\) | \(C_{P_{max}}\) | \(\mu_{C_{P_{max}}}\) | \(S_1\) | \(S_2\) | \(\Delta C_p\) | \(\frac{\Delta C_p}{C_{P_{max}}}\%\) | \(\frac{1}{U_8}u_{\infty}\) | \(x_{DSV}\) | \(\delta\tau\) | \(\delta\tau\) | \(\mu\) | \(2\pi/\mu k\) |
|------|------|--------|--------|---|------------|----------------|----------------|----------------|--------|--------|------------|----------------|----------------|----------------|----------------|--------------|--------------|----|--------------|
| 2314 | 0.20 | 8.14   | 8.34   | 0.074 | 3.45       | 1.59           | -0.25          | -7.25          | 0.63   | 0.006  | -206       | 325             | -0.70          | 9.7            | 0.27           | 0.040        | 0.006       | 4.33 | 7.42         | 1.40        | 0.07         |
| 2440 | 0.30 | 9.35   | 8.51   | 0.075 | 3.18       | 1.77           | -0.30          | -6.56          | 0.97   | 0.006  | -367       | 402             | -0.96          | 14.7           | 0.44           | 0.040        | 0.006       | 3.25 | 5.21         | 1.84        | 0.87         |
| 2374 | 0.41 | 7.48   | 8.84   | 0.073 | 3.74       | 1.54           | -0.23          | -4.83          | 1.27   | 0.006  | -420       | 289             | -0.71          | 14.7           | 0.50           | 0.052        | 0.022       | 2.56 | 4.07         | 1.93        | 1.24         |
| 2524 | 0.54 | 4.46   | 8.83   | 0.070 | 3.71       | 1.51           | -0.20          | -3.51          | 1.60   | 0.049  | -344       | 192             | -0.53          | 15.2           | 0.37           | 0.050        | 0.086       | 2.59 | 4.84         | 1.39        | 1.15         |
| 2078 | 0.59 | 5.63   | 6.77   | 0.070 | 1.53       | 1.37           | -0.15          | -3.05          | 1.71   | 0.089  | -283       | 148             | -0.52          | 16.9           | 0.29           | 0.074        | 0.134       | 2.70 | 4.71         | 1.98        | 1.20         |
At $M_\infty = 0.30$ and 0.41, the stall vortex forms and quickly reaches its terminal velocity. This demonstrates that, unlike at $M_\infty = 0.20$, the stall vortex ‘growth’ is curtailed soon after formation. A similar conclusion is reached when reconsidering the absence of correlation between stall regime and the stall vortex convection properties found in § 5.4.2 and 5.4.3. This phenomenon coincides with the onset of supersonic flow near $x_{DSV}$ such that it appears supersonic flow limits the stall vortex growth. One would logically believe that this also produces a ‘weaker’ DSV. This claim, purported by Lorber and Carta[95] and Chandrasekhara and Carr[27], is the subject of Appendix D. The increase in extent and local Mach number of the upper-surface supersonic flow from $M_\infty = 0.30$ to 0.41 further mitigates the gestation or ‘roll-up’ period, $\delta \tau_g$. These three cases are compared in Figure 5.26, where each curve has been displaced to account for the different onset times, $\tau_x = x_{DSV}$. From these three curves, the gestation period, $\delta \tau_g$ was estimated from Equation 5.9. Figure 5.26b highlights the behavior of $\delta \tau_g$ for each of the five Mach numbers discussed. The three cases examined prior to shock-induced dynamic stall show a clear decrease in $\delta \tau_g$ as the Mach number is increased. Examining the other convection properties in Table 5.2, a dependence of $V/U_\infty |_w$ on $M_\infty$ is not clear through this Mach range; drastic differences should not be expected as the adverse pressure gradient opposing the vortex convection downstream is qualitatively similar (see Figure 5.27). $V/U_\infty |_f$ increased with $M_\infty$ up to $M_\infty = 0.41$. Also, the time in which the vortex resides over the airfoil decreases as $M_\infty$ increases, from $\delta \tau = 7.42$ at $M_\infty = 0.20$ to $\tau = 5.21$ and 4.07 at $M_\infty = 0.30$ and 0.41, respectively. The reduction in $\delta \tau$ stems from two sources – the increase in $V/U_\infty |_f$, as well as the decrease in $\delta \tau_g$.

At $M_\infty = 0.54$ and 0.59, a normal shock is present on the airfoil surface. The retreating position of the minimum pressure forces $x_{DSV}$ aft on the chord. RUN2524
Figure 5.26. Vortex trajectories and gestation period.

exhibits a longer gestation period than the cases observed in § 5.4.4 at $M_{\infty} \approx 0.53$. The exact cause is uncertain, but seems to stem from the different amplitudes of the prescribed pitch motion. Unlike the trend up to $M_{\infty} = 0.41$, $V/U_\infty|_f$ decreases as $M_{\infty}$ further increases. The gestation period and residence time of the stall vortex remain approximately the same.

Figure 5.27 examines the propagation of the stall vortex in more detail by considering the upper-surface pressures and pressure gradient at the time-of-minimum-pressures (note, the pressure gradient is only shown for $x > 0.03$). In other words, the chord pressures are examined as the stall vortex convects over each instrumentation port. The time ‘snapshots’ are offset by $x = 1.05$, with individual ordinates marked. The first (leftmost) snapshot for each Mach number corresponds to 1 chord prior to the onset of separation. The figure immediately to its right highlights the pressure field at $\tau_{x=\text{DSV}}$, and then, moving left to right, at $\tau_{x=\text{DSV}+1}$, $\tau_{x=\text{DSV}+2}$, etc. The last four images capture
the pressure field at \( \tau = 0.5, 1.0, 1.5, \) and 2.0 chords following the stall vortex’s entrance into the airfoil wake. Due to \( x_{DSV} \) increasing with \( M_\infty \), the number of pressure distributions shown for the higher Mach cases are less than at \( M_\infty = 0.20 \) or 0.30.

At \( M_\infty = 0.2 \), the stall vortex is distinguishable at each snapshot in time. A strong, adverse pressure opposes the vortex’s motion downstream. This adverse gradient dissipates slowly compared to the higher Mach cases. Further, it maintains its position downstream of the DSV as the vortex moves toward the TE. At \( M_\infty = 0.30 \) and 0.41, the adverse pressure gradient slows the vortex’s motion following inception but, as the vortex proceeds over the chord, the pressure gradient downstream of the vortex is weaker than the gradient at \( M_\infty = 0.20 \). The dissipation of the adverse pressure gradient following the vortex’s inception results in a smaller \( \delta r_g \) and larger \( V/U_\infty|f\).

For the shock induced stall cases at \( M_\infty = 0.54 \) and 0.59, the adverse gradient maintains its position following the stall vortex’s location a longer duration than that exhibited at \( M_\infty = 0.41 \). This slows the convection of the stall vortex, slightly increasing \( \delta r_g \). Eventually, the adverse gradient (abruptly) collapses and the stall vortex propagates into a nearly constant pressure field. Interferometric photographs of dynamic stall on a NACA 0012 by Chandrasekhara et al.[29] revealed that the vortex structure was significantly different following shock induced separation when compared to the stall vortex structure incurred through the bursting of a separation bubble at \( M_\infty = 0.41 \). Similar to the pressure gradient measurements here, the interferograms indicate that the stall vortex convects into a constant pressure field. Further, the circular fringes associated with the stall vortex at \( M_\infty = 0.2 \) are not found at \( M_\infty = 0.45 \) for the NACA 0012, replaced by ‘half-fringes’ (see Figures 3&4 in [29]). The interferograms, pressure gradient, and \( \delta r_g \) differences suggest that the stall vortex might be mislabeled following shock development – in fact, each of these measurements indicate more of a dynamic
stall gust than a coherent, vortical structure. The vortical nature of the flow following shock induced separation requires further measurement and study, most likely, through PIV.

5.6 Vortex trajectory fitting

Returning to the Leishman-Beddoes vortex trajectory, Equation 5.2, the only provision available to adjust for the discussed vortex behavior is captured by the time-constant $\delta \tau$. However, the performance of the model can be improved via the following two adjustments,

$$x = 1 + x_{DSV} - \cos \left[ \frac{\pi}{2} \left( \frac{\tau}{\delta \tau} \right)^{(m)} \right], \quad (5.12)$$

where, typically, $m \in [1, 2]$ and can depend on $M_{\infty}$. The addition of $x_{DSV}$ accounts for the vortex formation a finite distance from the leading-edge. $m$, however, was found to be more of a ‘tuning’ parameter and showed little correlation to the prescribed motion parameters or Mach number. Equation 5.12 is compared to the experimental data and for the original LB model ($m = 1$) in Figure 5.28.

5.6.1 Stall vortex convection properties and Mach number trends

Figures 5.29 through 5.32 illustrate the behavior of the convection properties as the freestream Mach number increases up to $M_{\infty} \approx 0.6$. Significant vertical scatter exists at each Mach test point and is related to the prescribed motion parameters discussed previously, primarily reduced frequency. The convection properties, excluding $\delta \tau$, show a change in character as the Mach number increases from $M_{\infty} = 0.4$ to $M_{\infty} = 0.5$. This behavior, is, of course, linked to the different separation mechanisms involved in the production of the dynamic stall vortex. Whether that stall vortex develops due to the abrupt, viscous LE breakdown of the boundary layer or shock induced separation, its
Figure 5.27. Upper-surface pressures and pressure gradient at the time of minimum pressures at several Mach numbers. $C_p^S$, $\frac{dC_p^S}{dx}$
presence over the chord of the airfoil was discernible at each Mach number investigated.

As the freestream Mach number is increased, the peak suction pressure moves aft on the chord. At a critical incidence, the pressure field can no longer be maintained. The peak suction pressure falls off quickly, reducing the favorable pressure gradient and coalescing into DSV[118] (see Appendix D). As a result, the first sign of the stall vortex occurs just after the chord location of the peak suction pressure. For $M_\infty < 0.45$, the stall vortex inception point is near the LE, $x < 0.05$. As shocks develop at higher Mach, the peak suction and vortex inception point move aft on the chord. This behavior is shown in Figure 5.29.

The initial speed of the stall vortex is governed by the adverse pressure gradient near its inception point. At small $M_\infty$, the adverse pressure gradient is large and the stall vortex moves slowly after inception. The adverse pressure gradient tends to collapse more rapidly as compressibility effects increase, specifically as weak or strong shocks
The vortex inception point, $x_{i,v}$, and Mach number, $M_\infty$, are shown in Figure 5.29. Mach effect on the stall vortex inception point.

$\diamond$ data, $\bullet$ average value, $\cdots$ bezier curve following average values

develop. Once a normal shock forms, the stall vortex moves even faster following inception, as the adverse pressure gradient is negligible. These trends are found in Figure 5.30.

The fully developed stall vortex’s speed with respect to $M_\infty$ is shown in Figure 5.31. $V/U_\infty|_f$ increases with increasing Mach number for similar prescribed motions up to $M_\infty \approx 0.4$. The increase in convection rate coincides with apparent limitations on the size and growth of the stall vortex due to the emergence of supersonic flow. Once the shock forms, the low-pressure signature of the stall vortex remains essentially the same. However, the pressure field is significantly different. The drop in $V/U_\infty|_f$ following shock induced stall is concurrent with the constant pressure behavior of the downstream flow, such that the unsteady separation characteristics are more similar to a jet or gust than a DSV.

The residence time, $\delta \tau$, decreased with increasing $M_\infty$. This was primarily driven
Figure 5.30. Mach effect on the stall vortex initial convection speed. 

Figure 5.31. Mach effect on the fully developed stall vortex convection speed.
by the reduction in stall vortex’s gestation period.

5.7 Conclusions

The stall vortex convection properties for light and deep dynamic stall of the Bell airfoil were computed from the surface pressure instrumentation. The vortex trajectories were determined from the low-pressure propagation of the stall vortex aft over the chord. This analysis successfully determined the effect of the prescribed motion parameters of sinusoidal pitch motion-histories as well as the effect of Mach number. The study showed the following:

1. At $M_\infty = 0.2$, the stall vortex convection properties correlated to $\alpha_{max}$. This dependence was not found at higher Mach number.

2. The gestation period of the stall vortex decreased with increased $M_\infty$, implicating that the growth of the stall vortex is cutoff due to the appearance of supersonic flow.
3. The low-pressure signature of the “stall vortex” following shock-induced stall remains similar to the signature of the stall vortex following viscous breakdown of the boundary layer; however, the development of the surface pressures downstream of the ‘vortex’ is drastically different than the surface pressures observed at $M_x \approx 0.20, 0.30$ and even 0.4. It is proposed that the stall mechanism behaves more inline with a jet or gust of air than a concentrated, vortical structure. The lack of circular fringes in the interferograms of [29] support this conclusion.
CHAPTER 6

FOCUSED SCHLIEREN FLOW VISUALIZATION OF COMPRESSIBLE AND
SHOCK-INDUCED DYNAMIC STALL

Anticipated difficulties associated with other flow visualization techniques and their
ability to resolve compressible flow fields motivated the design and construction of
a schlieren system. A focusing system was selected over a more traditional, z-type
arrangement due to the poor optical quality of the test section windows and airfoil
endplates. The system was used to study the influence of Mach number on the onset of
steady and unsteady separation of the Bell airfoil. This chapter documents the design
and performance of the schlieren system. The results demonstrate the effect of Mach
number on the separation characteristics, including the development of shocks on the
airfoil upper-surface. Comparisons of the schlieren images to the onboard pressure
sensors allow for the determination of the effect of the shock-waves on the surface
static pressure.

6.1 Background

Flow visualization of low-speed dynamic stall includes hydrogen bubbles[102],
smoke-wire[4, 138], and oil-particle[93] techniques, to name a few. Imaging of the
compressible dynamic stall flow field has relied on optical methods capable of sensing
the changes in refractive index\(^1\). These techniques include stroboscopic schlieren[25,

\(^1\)For gases, \(n - 1 = \kappa \rho\), where \(\kappa = 0.23 \text{ cm}^3/\text{g}\) and \(n\) is the refractive index, \(n\).[122]
and point-diffraction-interferometry (PDI)[16, 17, 29, 123]. The sensitivity of a schlieren system is proportional to the gradient of the refractive index, \( \partial n / \partial x \), whereas interferometry is proportional to the 0\(^{th}\) derivative, i.e., \( n \). For flows involving small spatial refractive disturbances, such as shock waves, the schlieren system is more applicable as a flow visualization tool. Unfortunately, it is limited in its ability to quantify the density/pressure field; the point-diffraction-interferometry (PDI) technique developed by Carr et al.[17] allowed the time-resolved computation of the unsteady compressible flow field but provides poor resolution of shock features[29].

As the developing pressure field of the Bell airfoil is determined by the onboard, high-frequency pressures transducers, a schlieren system was selected over that of an interferometer. A z-type schlieren system, schematically depicted in Figure 6.1, integrates all distortions along its optical centerline due to the collimated (parallel) light. For the NDDSF, this would include the unwanted effects of refraction/density variations fostered by fluid interactions with the tunnel walls, splitter plates, bearing supports, and optical imperfections in the test section windows and Lexan endplates. Fear that these undesired effects would mask the flow field, a focusing schlieren (FS) system was preferred. A schematic of the FS optical layout is shown in Figure 6.2. The parabolic mirrors used in the z-type schlieren, that act to collimate the light source, are replaced by a fresnel lens in the FS. The placement of the light source beyond the focal length of the fresnel causes the light to converge past the lens. Since the light is nonparallel, each plane along the optic centerline (moving from the source-side to the assembly-side of FS) has an optical conjugate, i.e., a ‘re-imaging’ plane. Selection of the optical components – the fresnel lens, schlieren lens, relay lens, etc. – and variations in their placement, alters the size and location of the conjugate planes. Here, the fluid ‘disturbance’ is of central interest, and is usually the result of a wing, boundary-layer, etc. The lenses
Figure 6.1. Schematic of a Z-type schlieren system.

and placement of each component are selected in order to best capture the schlieren effect caused by this disturbance. The source grid discretizes the light source, providing a finite number of light rays that pass through the inhomogeneous fluid disturbance. Each ray is refracted to some extent. As the ray passes through the cutoff-grid, the portion of that ray that was refracted is blocked. This yields the schlieren effect and is captured on the image-plane (typically a camera or sheet of frosted glass). The selection of the system’s lenses and spacial layout requires tradeoffs in desired sensitivity, depth-of-focus, and field-of-view. These tradeoffs are best discussed by Settles[122]. The FS was implemented to serve as a visualization tool; its initial design and construction follows the systematic approach submitted by Weinstein[141].

A comprehensive review of lens-and-grid schlieren techniques is provided by Weinstein[140] and/or Settles[122]. In addition to pure visualization, novel applications of the FS technique have been demonstrated by Vander creek[135] and Hargather[69], for deflectometry and seedless PIV measurements, respectively.
6.2 Components and design

The optical components of the FS system include a 0.65 by 0.55 m \( f = 800 \) mm Fresnel lens, an \( A = 101.6 \) mm aperture \( f = 500 \) mm plano-convex ‘schlieren’ lens, a 127 mm aperture \( f = 150 \) mm relay lens recovered from a bench-top magnifying lamp, a high-speed camera (Photron FASTCAM SA1.1) the source and cutoff-grids, and the LED array. The lens was removed from the FASTCAM and the schlieren image was projected straight onto the camera’s CMOS array. Each raw schlieren image was capture with \( 1024 \times 1024 \) lines of resolution and 8-bit (256 levels) grayscale. Light Inc. (Silver Spring, MD), an advertisement and display company, produced the source-grid for the experiment on a clear sheet of 24 x 24 x 0.25 in scratch-resistant polycarbonate (McMaster-Carr PN: 8707K134). The source-grid was fashioned from equally spaced (0.125 in) opaque and translucent horizontal lines that effectively divide the single light source into \( n \), discrete sources. To fabricate the opaque lines on the source-grid, black vinyl bands were transferred to the polycarbonate sheet. The placement of the light source beyond the focal point of the Fresnel lens forces the discretized light to converge.
through the test section at various angles with respect to the optical centerline. The converging light allows the schlieren to focus on a ‘plane’ within the test section; the source components and flow disturbances are re-imaged on the assembly side of the system. The schlieren lens is placed near the re-imaged light source. The cutoff-grid was a photographic negative of the source-grid, creating a knife-edge for each of the \( n \) discrete sources. Its construction is discussed in more detail in § 6.2.2. Vertical translation of the cutoff-grid blocks light from each discrete source, limiting the light that reaches the relay lens/camera and providing the schlieren effect. The relay lens and FASTCAM are used to capture the desired field within the test section. Translating either changes the focal plane location.

The light source, discussed in § 6.2.1, was mounted to a camera tripod. The source-grid and Fresnel lens were supported by Thor filter holders (PN:FP02) and Newport PRC-3 rail guides. The guides secured the grid and lens to two Newport PRL-12 rails, mounted to Thor 95 mm optic rails, which, in turn, were secured to a Vere 24 x 24 in optic breadboard. The source side sub-assembly could be adjusted along or normal to the optic centerline.

The assembly side opto-mechanical components were mounted to a Vere 2 x 6 ft optics breadboard. Newport PRL rails (-36 and -24) and Newport PRC-3 rail carriers allowed relocation of the components without the need for realignment along the optic centerline. The schlieren lens was mounted directly to an optics post and post holder, and shrouded with a wood box and black-cloth bellows that extended to the cutoff-grid. Foam inserts between the lens and wood box, as well as the bellows, blocked stray light. A 5 x 4 in Toyo Cut Film holder locked the film used for the cutoff-grid into a 5 x 4 in, large-format Graflok camera back purchased through Glennview. The Graflok was bolted to a Tech. Ops right-angle camera bracket intended for large-format rail cam-
Figure 6.3: Final focusing schlieren configuration for compressible dynamic stall flow visualization.
eras (and also purchased through Glennview). It was modified and secured to a XYZ translation stage (Thor Labs PT3A and Newport 481-A). Fine adjustment of the cutoff is required for proper alignment between it and the source-grid, as well as control of the schlieren sensitivity. A Melles-Griot universal lens holder (07 LHA 703) and Newport damped support-rod (model 75 equipped with 370-RC clamp) positioned the relay lens behind the cutoff-grid. The FASTCAM translates along the optical centerline using a Melles-Griot optical rail (PN: 07 ORP 003) and rail carriers (PN: 07 OCP 441). A Newport jack (M-EL 120) varied the camera’s elevation. Both the light and assembly sides of the schlieren setup rested on stands constructed from 80/20 extruded aluminum. All optics were centered vertically along the optical centerline, 76 in from the laboratory floor. The axis passed through the test section parallel to the airfoil span, located approximately 0.25 in aft of the leading-edge with the airfoil near a 10° angle-of-attack. The position of the source and assembly sides of the schlieren system was cumbersome but manageable with the aid of a laser level. The level was mounted in place of the light source and all system components were aligned to mitigate reflections. The schlieren components were incorporated into a CAD model of the experiment to ensure proper alignment before purchase; the final configuration is depicted in Figure 6.3. Note, the table supports for the both the source and assembly sides of the FS system, as well as the tri-pod used to mount the light source, are removed for clarity. Table 6.1 lists the distances between components, as depicted previously in Figure 6.2.

The third and fourth rows in Table 6.1 characterize the schlieren system’s performance attributes - see Weinstein[141] for design metrics. The system was structured to magnify the flow near the model’s leading-edge (m=1.55) with minimum depth-of-sharp-focus\(^2\) (DS=2Aw/l). The pitch-mechanism and splitter plates blocked a signifi-

\(^2\)The depth-of-sharp-focus describes the thickness of the 'plane' in which the schlieren effect is in-
TABLE 6.1

FOCUSED SCHLIEREN FINAL CONFIGURATION PARAMETERS

<table>
<thead>
<tr>
<th>A [mm]</th>
<th>d [m]</th>
<th>l [mm]</th>
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<td>m</td>
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<td>DU [mm]</td>
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<td>0.48</td>
<td>21.69</td>
<td>106</td>
</tr>
</tbody>
</table>

cant portion of the converging light as it progressed from the source to the assembly side of the system. Further, the FASTCAM’s placement at a close distance to the relay lens reduced the field of view (FOV). The narrowed FOV is not reflected in Table 6.1, but is approximately 65 mm. The translucent line width of the cutoff-grid, b, was measured from the exposed and developed cutoff-grid.

6.2.1 Light source

LEDs manufactured by LEDEngin (PN: LZ4-00WW15) were assembled in a 3 x 3 array to serve as the light source for the schlieren. Hargather[69] reported that the LEDs were capable of producing a 0.5 μs light pulse, with rise and fall times less than 0.05 μs[69]. Similar results were found when benchmarking the LEDs used here. The light grid provided diffuse, warm-white light for both the exposure of the cutoff-grid and the schlieren illumination. The light array was powered by a Mastech DC power supply (PN: HY3010D). The exposure time was controlled by adjusting the pulse frequency from an Agilent Function Generator (PN: 33120A). The circuit could be triggered by an
Figure 6.4. Schlieren light source

external TTL signal and could also adjust the duty cycle in order to vary the illumination flux. This mode of operation was used for the high-speed schlieren video. Since the lights were limited to a 15 W output, a Pearson current monitor (Model 2100) measured the current through the voltage line to ensure that the output power remained below 15 W/LED. The LEDs were mounted to 3 x 4 x 0.25 in aluminum plate with thermally conductive adhesive. Two 9 in heat pipes extended from the rear of the aluminum plate for additional heat transfer. Two photos of the constructed light source is shown in Figure 6.4.

A 45 mm polyamide tab extends from the LED bulb to its solder points. This tab prevented the top-center LED from being properly adhered to the aluminum plate. Eventually the heat generated by the LED caused it to burnout. All schlieren images included within this document utilized the eight remaining LEDs, pictured in Figure 6.4b.
6.2.2 Cutoff-grid development

The FS system requires an optically exact reproduction of the source-grid, as seen from the assembly side of the system, i.e., the cutoff-grid is the optical conjugate of the source-grid. In order to meet this design criteria, an un-exposed sheet of film is placed at the re-imaging plane of the source-grid. The film is then exposed, removed from the system and developed, and realigned with the source-grid.

Creation of the cutoff-grid is the primary obstacle in implementing an FS system, especially considering that the best tools for creating the cutoff-grid are disappearing, if not obsolete already, due to the maturation of the digital photography age. The technique requires hard-to-find, high-contrast film and developing chemicals. The development process also requires a dark room.

To accommodate this last demand the model shop at Whitefield Laboratory was modified to block incident light. The cutoff-grid fabrication consisted of the following steps:

1. The film was loaded into the Toyo film holder under safelight in the darkroom.
2. The Toyo film holder was placed in the Graflok camera back in the setup.
3. The airfoil was removed (pitched to $\alpha = 90^\circ$) from the FOV.
4. The light source was strobed at a set frequency and duty cycle (effectively setting the exposure time, $t_s$) and at a set voltage, $V_{exp}$ (controlling the light flux).
5. The dark-slide of the film holder was opened, the film emulsion was exposed to the strobe, and dark-slide closed, all over the course of a single flash.
6. The Toyo film holder was removed from the Graflok back.
7. The film was removed from the film holder and developed in the darkroom under safelight. The chemistry consisted of the following steps: developer $\rightarrow$ stop bath $\rightarrow$ fixer $\rightarrow$ rinse (running water) $\rightarrow$ wetting agent. All chemicals and running water were maintained near a temperature of 70 °F.
8. The film was allowed to dry before it was returned to the film holder.

After drying, the developed cutoff-grid was placed back into the film holder and Graflock back. The $XYZ\Theta$ translation stage was used to adjust the film’s orientation and position to align with the source-grid. Despite the high-resolution from the 4 degree-of-freedom stage, small adjustments of the film’s position within the Toyo holder are still required for best alignment. The film attributes, the chemicals used to develop the film, and the exposure settings are listed in Table 6.2. Originally, only the Ilford film and Arista II developer were purchased; however, the A/B developer destroyed the emulsion on the Ilford film, prompting the purchase of Kodak T-Max developer, and later Ultraline lith film. Heightened contrast of the gray-toned Ilford film required long development times, $t_d$, and high illumination flux (related to $V_{exp}$). Overexposure, caused by the combination of a poorly matched exposure time, $t_e$, and flash setting – controlled through the applied voltage, $V_{exp}$ – resulted in the opaque lines leaking into the translucent bands. Overdevelopment yielded similar effects. Overall, several dozen film sheets were exposed and developed before a satisfactory cutoff-grid was created. The Ultraline lith film was purchased to obtain higher contrast and was compatible with the Arista II Part A/B developer. Lith film, as opposed to gray-tone film, is binary, i.e., black or translucent. Additional benefits of the lith film include a drastic reduction in development time. However, the non-standard 5 x 4 in size of the film demanded that each sheet be cut to fit the Toyo film holder. Additionally, the 4 mil thick film fit loosely into the film holder, increasing the difficulty associated with realigning the source and cutoff-grid after the cutoff-grid was developed.
### Table 6.2

<table>
<thead>
<tr>
<th>Film</th>
<th>Ilford Ortho Plus</th>
<th>Ultrafine Ortho Litho</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>5 x 4 in (standard)</td>
<td>5 x 4 in (non-standard)</td>
</tr>
<tr>
<td>ISO</td>
<td>80</td>
<td>10</td>
</tr>
<tr>
<td>Safelight</td>
<td>red</td>
<td>red</td>
</tr>
<tr>
<td>Contrast</td>
<td>continuous graytone</td>
<td>lithographic (black/white)</td>
</tr>
<tr>
<td>Developer</td>
<td>Kodak T-Max</td>
<td>Arista II Part A/B</td>
</tr>
<tr>
<td>Stop Bath</td>
<td>Arista Indicator Stop Bath</td>
<td>Arista Indicator Stop Bath</td>
</tr>
<tr>
<td>Fixer</td>
<td>Ilford Rapid Fixer</td>
<td>Ilford Rapid Fixer</td>
</tr>
<tr>
<td>Wetting agent</td>
<td>Arista Flo Wetting Agent</td>
<td>Arista Flo Wetting Agent</td>
</tr>
<tr>
<td>( t_e )</td>
<td>( \approx 0.2 \text{ s} )</td>
<td>( \approx 1.4 \text{ s} )</td>
</tr>
<tr>
<td>( V_{exp} )</td>
<td>14.7 V</td>
<td>11.5 V</td>
</tr>
<tr>
<td>( t_d )</td>
<td>10 min (minimum)</td>
<td>1 min (maximum)</td>
</tr>
<tr>
<td>( t_f )</td>
<td>6 min (minimum)</td>
<td>1 min (minimum)</td>
</tr>
</tbody>
</table>

6.3 An explanation of the schlieren images

The focusing schlieren system had to contend with the pitch-mechanism and test section apparatus. Both collaborated to block the light (rays) entering the test section, transmitting their shadows to the image-plane. Figures 6.5a and 6.5b show a raw schlieren image and that image’s relation to the bearing support and pitch horn. Due to the shadows cast by these objects, only 8.15% of the LE flow field can be viewed.

The oscillation of the airfoil normal to the optic centerline resulted in varying misalignment. This misalignment produced a 3D view of the airfoil. This is also illustrated in Figure 6.5b. Unfortunately, the top-down view of the upper surface and the resulting shadow from the span of the airfoil often conceals features of the separating boundary-layer that could be of interest. Careful study of the images, however, allows one to
dissociate the fluid behavior from that of the moving surface.

![Schlieren image](image1)

(a) Raw schlieren image  
(b) Schlieren image with airfoil and apparatus masks

Figure 6.5. Raw focused schlieren image and relationship to test apparatus,  
\[ \alpha = 11^\circ \text{ (steady) at } M_{\infty} = 0.4. \]

6.4 Schlieren image post-processing

Although the high speed flow, weak shocks, normals shocks, and separation features are discernible in the raw schlieren images, post-processing enhances their detail. The images throughout this document have been processed in a variety of ways. These techniques include linear contrast stretching (with and without clipping), histogram equalization, and unsharp masking – see [122]. Often, multiple methods were used in conjunction in order to accentuate the flow details under discussion.

Figure 6.6 illustrates the raw schlieren images of shock-induced dynamic stall, contrasting the results obtained with the two different films used for the cutoff grid.
Figure 6.6. Raw focusing schlieren image of shock-induced dynamic stall, 
\[ \alpha = 6^\circ + 5^\circ \sin \omega t \] at \( M_\infty = 0.5 \) and \( k = 0.030 \)

The most useful method to improve schlieren detail during post-processing is through the removal of the background image and subsequent mathematical operations, e.g. scaling, on the native gray levels. This is accomplished by taking a tare – an image without the model present – and subtracting it from the schlieren image. For unsteady motion, the tare or ‘background’ constantly changes. As such, this method requires phase-locked flow-on and flow-off photos. Flow-off images were not recorded in this study. Instead, where applicable, the schlieren photos captured at \( M_\infty = 0.3 \) were used as tares (Mach 0.3 exhibited the weakest density gradients). After subtracting the tare, the image was cropped to focus on the shock structure. This allowed the contrast of the image to be stretched over a larger range due to the elimination of the darkest blacks. For the Ilford cutoff, however, there remained a very bright (white) region corresponding to the fastest moving flow upstream of the shock that also limited
the contrast stretching range. Figure 6.7a highlights several versions of Figure 6.6a in which the described operations were performed to improve visualization of the normal shock details. A similar procedure was applied to Figure 6.6b, as shown in Figure 6.7b. In the end, the so-called ‘best image’ is often a matter of opinion.

6.5 Results

The FS test matrix is shown in Table 6.3. The test points, or Schlieren runs (S.RUN#), were recorded twice, once with the Ilford cutoff-grid, and a second time with the Ultrafine cutoff-grid. The LED light source was set to a stroboscopic mode, triggered by the camera at 1kFPS and 4kFPS for the Ilford and Ultrafine grids, respectively. The supply voltage and flash duration also varied between the two grids in order to provide sufficient light to the high-speed camera; the light settings for the Ilford film were $V_{\text{strobe}} = 15.9$ V and $t_{\text{strobe}} = 150$ µs, whereas the settings used with the Ultrafine cutoff-grid in place were $V_{\text{strobe}} = 27.0$ V and $t_{\text{strobe}} = 20$ µs. The higher intensity, shorter duration flash corresponded to the increased camera framerate.

6.5.1 Steady schlieren images

Only the Ilford cutoff-grid was used to capture the steady separation behavior. Further, these flow visualizations only served to establish the presence and basic structure of shocks at each of the test Mach numbers. The steady flow schlieren images are shown in Figure 6.8. The stills were taken at pre-stall angles of attack to establish shock presence. Due to compressibility effects, this pre-stall angle decreased with increasing $M_{\infty}$.

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3Missing RUN numbers correspond to cases in which the light source or cutoff-grid settings were varied. This excludes RUN38 and RUN39 which were dual-mode pitch inputs. RUN38: $\alpha = 4^\circ + 5^\circ \sin(\omega_1 t) + 1^\circ \sin(\omega_2 t)$, $M_{\infty} = 0.6$, $k_1 = 0.025$, $k_2 = 0.1$; RUN39: $\alpha = 6^\circ + 5^\circ \sin(\omega_1 t) + 1^\circ \sin(\omega_2 t)$, $M_{\infty} = 0.6$, $k_1 = 0.03$, $k_2 = 0.1$. It also excludes RUN4 where the freestream velocity was increased from $M_{\infty} = 0.4$ to $M_{\infty} = 0.5$ over the course of the run.
Figure 6.7: Comparison of shock detail following schlieren post-processing.
### TABLE 6.3

FS TEST MATRIX

<table>
<thead>
<tr>
<th>$M_\infty$</th>
<th>( \alpha ) [ \theta ]</th>
<th>( \alpha_0, \alpha_1 ) [ \theta ]</th>
<th>( k )</th>
<th>S.RUN# (Ilford) [ steady ]</th>
<th>S.RUN# (Ultrafine) [ unsteady ]</th>
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</thead>
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<tr>
<td>baseline</td>
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<td>14</td>
<td>9, 5</td>
<td>0.05</td>
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<tr>
<td></td>
<td>0.4</td>
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<td>6, 5</td>
<td>0.038, 0.05</td>
<td>2</td>
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<td></td>
<td>0.5</td>
<td>11</td>
<td>6, 5</td>
<td>0.03, 0.05</td>
<td>3</td>
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<tr>
<td></td>
<td>0.6</td>
<td>9</td>
<td>3, 5</td>
<td>0.025, 0.05</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td>11</td>
<td>6, 5</td>
<td>0.025, 0.05</td>
<td>5</td>
</tr>
<tr>
<td>P320</td>
<td>0.3</td>
<td>13</td>
<td>8, 5</td>
<td>0.05</td>
<td>13</td>
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<tr>
<td></td>
<td>0.4</td>
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<td>6, 5</td>
<td>0.038, 0.05</td>
<td>15</td>
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<tr>
<td></td>
<td>0.5</td>
<td>11</td>
<td>6, 5</td>
<td>0.03, 0.05</td>
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<td></td>
<td>0.6</td>
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<td>4, 5</td>
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</tr>
<tr>
<td></td>
<td>0.6</td>
<td>11</td>
<td>6, 5</td>
<td>0.025, 0.05</td>
<td>21</td>
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<tr>
<td>P400</td>
<td>0.3</td>
<td>14</td>
<td>9, 5</td>
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<td>11</td>
<td>6, 5</td>
<td>0.025, 0.05</td>
<td>32</td>
</tr>
</tbody>
</table>

Each image in Figure 6.8 was post-processed using an identical routine, such that the local flow speeds can be qualitatively compared. Unfortunately, the routine washes out some of the shock structure exhibited in Figure 6.8h and i.

The first row of figures corresponds to the baseline airfoil without LE modification. The following two rows are for the airfoil with grit trips placed at $x \in [0.005, 0.030]$. The trips are not discernible in the photographs. These images show the extent and, in a qualitative sense, the speed of the flow near the Bell airfoil LE. Light regions correspond to the highest flow speeds. The spurious, bright region on each image near the pressure-side of the LE (partially covered by the airfoil masks) was a consequence of the misalignment between the assembly side optics and the optical centerline. For
the baseline, a shock forms at $M_\infty = 0.47$ and quickly moves aft on the chord. The shock was not captured in either Figure 6.8c or Figure 6.8d as it had moved out of the field-of-view, but its presence was confirmed by a pseudo-steady run in which the freestream Mach number was slowly varied from $M_\infty = 0.4$ to 0.5. Shocks formed for both trips at $M_\infty = 0.6$ but not at $M_\infty = 0.5$. Both the P320 and P400 trips resulted in an oblique shock at $M_\infty = 0.6$ that originated at the rear of the trip tape. This is most likely an effect of the 2D aspect of the sandpaper tape and not an outcome of the distributed, 3D roughness elements.

6.6 Pressure and flow visualization correlations

This section compares the surface static pressure measurements and the flow visualization images captured by the focused schlieren system. The pressure measurements and the flow visualization were not simultaneously captured. This, unfortunately, was considered too late in the test schedule. Instead, equivalent trajectories are considered and the correlations based on careful examination of the schlieren images and pressure measurements.

6.6.1 $M_\infty = 0.3$, $\alpha = 9^\circ + 5^\circ \sin \omega t$, $k = 0.05$

Figure 6.9 compares the schlieren images with the measured upper-surface pressures from RUN2737. The instantaneous loads at the top of Figure 6.9e indicate deep stall characteristics. The upper-surface pressures, shown in the stacked $C_p^s$ plot, are labeled according to the selected schlieren images. Similar to other chapters, the pressures are offset but evaluated by the same vertical scale.

---

The stroboscopic, LED light source accepts any TTL input. The data acquisition system for the onboard and freestream sensors has a TTL output that could be used to trigger both the light source and Photron camera.
Figure 6.8: Steady state schlieren images of Bell airfoil LE at $M_\infty = 0.3, 0.4, 0.5, 0.6$. 
Observing the schlieren image in Figure 6.9a, the flow is attached, indicated by the lack of inhomogeneity in the image, as well as the corresponding upper-surface pressures. Boundary layer details are difficult to detect in this image due to the small boundary layer height associated with the large \( Re_x \) (\( \delta_{x=0.05} \in (150, 250) \) \( \mu m \) for \( M_\infty \in (0.2, 0.6) \)), as well as the reduced contrast due to the airfoil span. However, as shown in Figure 6.9b, a dark-region indicating high-pressure flow rises from the surface of the airfoil. This behavior is suggestive of a transition bubble. Careful examination of the high-speed video reveals that, despite the bubble’s presence, breakdown of the flow originates with the forward movement of the turbulent boundary-layer and not the bursting of the separation-bubble. This behavior agrees with that observed by Lorber and Carta at similar \( Re_c [95] \). The LE pressures respond non-linearly. At \( x = 0.0 \) and 0.006, forward of the bubble, the \( C_p \)’s increase sharply. The pressure traces at \( x = 0.022 \) through 0.134 show a cusp (pressure increase) that moves forward toward the LE. This is most pronounced by the signals at \( x = 0.134 \) and \( x = 0.086 \). Soon thereafter (\( \Delta t = 2.5 \) ms), the flow separates and the footprint of the stall vortex emerges at \( x = 0.006 \). The DSV convects downstream, inducing a drop in pressure as it moves aft. The breakdown of the LE flow is captured in Figure 6.9c. A high-pressure, separated flow follows in the stall vortex’s path. The separated shear-layer is seen in Figure 6.9d.

6.6.2 \( M_\infty = 0.4, \alpha = 6^\circ + 5^\circ \sin \omega t, k = 0.05 \)

The unmodified LE schlieren stills and upper-surface pressure traces (RUN2766) are shown together in Figure 6.10 for the \( M_\infty \) study. As before, the four points step through the separation process for the Bell airfoil. Unlike at \( M_\infty = 0.3 \), this particular run is categorized as stall onset. The developing pressure field does not produce the same degree of contrast as at \( M_\infty = 0.3 \) due to the less severe and thus more homo-
Figure 6.9. Stacked $C_p$ plot of upper-surface pressures and correspondence to selected schlieren images. Unmodified LE, $M_{\infty} = 0.3$.

$M_l < 1.0$
geneous flow field. Still, the schlieren images reveal a similar separation pattern as the $M_\infty = 0.3$ study. Image 1 is taken during the pitch-up stroke where the flow is fully attached. Point 2 marks the onset of separation, where the flow is again influenced by the forward motion of the turbulent boundary-layer. Weak shocks that are nearly invisible in the schlieren images ride atop the separating turbulent boundary layer. The rest of the fluid behavior can be expounded identically to the $M_\infty = 0.3$ survey.

6.6.3 $M_\infty = 0.5, \alpha = 6^\circ + 5^\circ \sin \omega t, k = 0.05$

The prescribed motion at $M_\infty = 0.5$ caused light dynamic stall behavior. Figure 6.11 highlights the upper-surface pressures from RUN2794 and selected frames from the FS high-speed video. Differences between the flow features at $M_\infty = 0.5$ and the previous Mach number comparisons are immediately apparent in both the pressure development and the FS stills. Weak shocks form on the upstroke of the airfoil motion, as highlighted in Figure 6.11a. A short time later (3.7 ms), the weak shocks coalesce to form a series of stronger, normal shock waves. During this shock development process, the shock has little impact on the developing pressure field, serving to turn the flow so that it follows the airfoil curvature and force the peak suction pressure aft on the chord. It is suspected that these shocks further combine to produce a larger, normal shock downstream on the chord but out of the view of the schlieren system. The shock structure is unstable and eventually breaks down. This causes separation following the shock, as demonstrated through the increasing rms of the pressure signals. From there, the shock propagates upstream, seemingly increasing its strength. The shock eventually breakdowns, and is immediately followed by a forward moving, off-surface or free shock structure that is apparent in Figure 6.11c. This is also visible through examination of the pressure traces at $x = 0.022$ and at a small time step later, $x = 0.006$. Full separation commences once
Figure 6.10. Stacked $C_p$ plot of upper-surface pressures and correspondence to selected schlieren images. Unmodified LE, $M_x = 0.4$.

- $M_I < 1.0$, $M_I \geq 1.0$
the shock reaches the LE. The turbulent boundary-layer does not play a significant (if any) role in the stall process at $M_\infty = 0.5$. Upstream propagated shock waves are briefly discussed in the review by Tijdeman and Seebass[130]. In their review, they describe the behavior of the shock similarly to the observations included here.

6.6.4 $M_\infty = 0.6, \alpha = 4^\circ + 5^\circ \sin \omega t, k = 0.05$

At $M_\infty = 0.6$, a series of weak shocks form at $\alpha = 6^\circ$ and propagate out of view without coalescing into a normal shock. This is shown in Figure 6.12. Instead, the foot of a $\lambda$-shock emerges from the LE – as seen in the second schlieren image. From the surface-pressure traces, the shock front emerges ahead of $x = 0.086$ and then moves to a location $x \in (0.134, 0.191]$. This is detected by examining the pressure trace at $x = 0.086$. The ‘kink’ in the signal just prior to the sudden pressure decrease (increased $-C_p^S$) indicates the passage of the normal shock front. A similar kink is seen in the trace at $x = 0.134$ a moment later. The lambda-shock causes a pressure plateau in the LE pressures; this flatline response abruptly rises as the shock front proceeds aft. For this trajectory, the flow forward of $x \leq 0.049$ fails to separate. The jump and falloff in the flatline response corresponds to points in time when the $\lambda$-shock first envelopes that chord point on the up-stroke and when it retreats on the down-stroke, respectively. During this time, the flow remains essentially stagnant. As the footprint of the $\lambda$-shock thickens at the LE – shown in schlieren image 3 – the pressure-distribution remained unaltered. The flow eventually breaks down behind the shock front, and the shock propagates upstream albeit at distance well above the surface as shown in the fourth schlieren still. For this prescribed motion case, the shock does not move upstream past $x = 0.049$.

A second set of flow visualization was taken at $M_\infty = 0.6$. However, the trajectory
Figure 6.11. Stacked $C_p$ plot of upper-surface pressures and correspondence to selected schlieren images. Unmodified LE, $M_{x0} = 0.5$.

- $M_t < 1.0$, $M_t \geq 1.0$, $M_t \geq 1.31$
was $\alpha = 4^\circ + 5^\circ \sin \omega t$. Consequently, it included a maximum angle of attack of only $9^\circ$. Separation did not occur at either reduced frequency investigated ($k = 0.025$ and 0.050).

6.7 Morphology of shock-induced stall

The visualization of the flow through the FS system allowed a substantial amount of information to be gleaned from the surface pressures. From the correlations considered in this chapter, the morphology of the shock-induced dynamic stall process is better understood. A cartoon of a $\lambda$-shock is depicted in Figure 6.13 for reference.

Figure 6.14 graphs the upper-surface pressures during two cycles of shock-induced dynamic stall. The salient flow features are indicated through the use of different arrows, patterns, and dashed lines, as indicated on the figure. The prescribed motion for the depicted case, RUN2820, is $\alpha = 5.08^\circ + 4.85^\circ \sin \omega t$, $M_\infty = 0.6$, $k = 0.05$. The flow features several distinct patterns associated with the formation, travel, and breakdown of the $\lambda$-shock. This discussion, for the most part, limits its focus to the first stall cycle in Figure 6.14 (most features are replicated in the second stall cycle, but, as dynamic stall is a pseudo-random process, dissimilarities should be expected). Weak shocks initiate the formation of the $\lambda$ pattern. The shock front is the first feature of the flow-field detected, originating at $x \in [0.049, 0.086]$ and $t/T = 0.26$ ($\alpha = 5.6^\circ$). The shock front proceeds aft, halting at a chord location $x \in (0.134, 0.191)$ where its motion slows due to a balance of the favorable and adverse pressure gradient. The stagnant flow within the $\lambda$-shock is very low-pressure ($C_{p_{\text{min}}} = -2.9$, $M_l = 1.70$), abruptly rising as the shock front passes, and then leveling – a feature more pronounced the closer the port location is to the shock front. The shock eventually becomes unstable and a ‘stall vortex’ emanates from the chord location immediately following the shock.
Figure 6.12. Stacked $C_p$ plot of upper-surface pressures and correspondence to selected schlieren images. Unmodified LE, $M_x = 0.6$.

- $M_l < 1.0$, red
- $M_l \geq 1.0$, blue
- $M_l \geq 1.31$
Figure 6.13. Schematic of a $\lambda$-shock formed during dynamic stall

front. The low-pressure wave sweeps aft causing the first instance of moment stall ($t/T = 0.37$) and subsequently lift stall ($t/T = 0.40$). Simultaneous to the aft motion of the vortex-like structure, the upstream flow separates due to the forward movement of a secondary shock-wave. This shock, shown previously to reside above the airfoil surface in Figures 6.11c and 6.12d, induces separation as far forward as $x = 0.022$. The exact source of the secondary shock was out of the FS field-of-view. It is suspected that it is caused by an acoustic reflection due to the sudden breakdown of the shock front. Flow visualizations of this interaction region would indicate its exact source. In the immediate aftermath of the secondary-shock reaching the LE ($t/T = 0.41$), a wave of low-pressure flow sweeps aft over the chord. This induces a nose-up moment (as the rest of the flow over the chord remains separated), and then a second moment break (it also provides a rise in lift). It is not clear if this pattern, also witnessed in other motion-histories, is a side-effect of the slowed-pitch variation near $\alpha_{max}^5$ or, possibly, the result of a second stall vortex. The non-sinusoidal motion does not alter the prior physics of

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$^5$As the physical pitch-frequency (12.98 Hz) is large at $M_x = 0.6$, this is not believed to be caused by a lack of inertia in the 1P flywheel. Instead, it is suspected that the large aerodynamic moment causes live-twist of the torque-tube.
the shock-induced flow but does add intriguing stall behavior (especially when one considers the elastic rotor-blade). On the downstroke, the $\lambda$-shock reemerges, first detected at $x = 0.191$, $t/T = 0.71$). The shock front starts downstream and moves forward toward the LE as the airfoil’s incidence decreases. The presence of the $\lambda$-shock through the down-stroke of the second stall cycle in Figure 6.14 is more prominent than the first cycle.

Several general dynamic stall trends hold through the shock-induced stall process. This includes a moment-stall that precedes lift stall, occurring at the point in time when the influence of the stall vortex is first realized at $c/4$. Lift stall occurs as the stall vortex reaches the TE. Flow reattachment starts at the LE and moves downstream over the chord. Comparison of Figure 6.14 to the suction pressures in Figures 6.10 and 6.9 at $M_\infty = 0.3$ and 0.4, respectively, reveal the very different nature of shock-induced and LE, compressible dynamic stall.

6.8 Trip effect on the shock structure

The spanwise trip placed at the near LE of the Bell airfoil was to remove laminar-to-turbulent transition. This certainly occurred, but at the cost of a reduced suction peak over the entire Mach test range. At $M_\infty \leq 0.4$ the schlieren images indicate that separation occurred abruptly without the presence of a bubble.

Figure 6.15 compares the unmodified airfoil shock and separation development at $M_\infty = 0.5$ to the altered behavior caused by the two trips. Each image in the first column was chosen to represent an identical incidence, subsequent images (moving left-to-right) occur at a time 3 ms after the preceding schlieren. The P320 grit eliminated the formation of a normal shock. Although weak shock waves formed, the separation at the LE was governed by an abrupt, forward movement of the trailing-edge separation point.
This was inferred by the lack of a separation-bubble as opposed to direct visualization. The P400 grit delayed the onset of both weak and stronger shock formation. The rear of the trip tape and the associated pressure drop caused the formation of a small $\lambda$-shock that grows in strength as $\alpha$ increases. The foot of the $\lambda$-shock remained positioned at...
the rear of the trip-tape. The breakdown of the shock is shown in Figure 6.17. This breakdown was much different than that observed on the baseline airfoil and did not indicate the forward motion of a shock. Instead, the shock appeared to weaken rapidly and scatter into weaker shocks. Figure 6.16 compares the upper-surface pressures for similar trajectories with and without the applied trips. A reduction in the magnitude and the extent of the supersonic LE flow is clear.

Figure 6.17 shows the FS visualization images at $M_\infty = 0.6$ for the baseline and modified Bell airfoil. The images were taken from the high-speed video at time intervals of 3.7 ms, moving left-to-right. At small incidences, the attached flow behaves similarly for the three cases. Both trips, however, promote the formation of shocks prior to the unmodified airfoil. The P320 and P400 trips replaced the thin foot structure of the $\lambda$-shock, a weak oblique shock wave, on the unmodified airfoil with a much stronger oblique shock originating at the back-facing step of the trip tape ($x = 0.03$). The increased strength of the oblique shock mitigated the need for the strong, normal shock downstream. This also greatly reduced the LE suction pressure that develop due to the $\lambda$-shock formation. Accordingly, the shock front does not travel as far downstream on the chord, stopping near $x = 0.086$ for the P320 trip and in the range $x \in [0.086, 0.134]$ with the P400 trip applied (as determined from the upper-surface pressures for similar prescribed motion cases, shown in Figure 6.18). As discussed previously, following the breakdown of the $\lambda$-shock front, the baseline airfoil’s LE flow remains attached, at least for the case captured by the FS. Figure 6.12 best highlights the associated pressure field. Here, however, the upper-surface pressures shown in Figure 6.18 for the unmodified airfoil case represent a more drastic stall than that captured by the high-speed video. This also applies to the $C_p^S$’s for the P320 and P400 grit in Figure 6.18.

When the normal-front of the $\lambda$-shock collapses on the baseline model, it retreats
Figure 6.15: Schlieren flow visualization of trip effect on shock development process, $M_\infty = 0.5$, $\alpha \approx 6^\circ + 5^\circ \sin \omega t$, $k = 0.05$. 
Figure 6.16: Upper-surface pressure for baseline and tripped airfoils, $M_{\infty} = 0.50$, $k = 0.05$. 
toward the LE. Considering the rapid pressure fluctuations that ensue (best shown in Figures 6.14 and 6.18a) the process can only be described as chaotic. For the tripped airfoil, as the flow separates following the normal shock, the shock front is seen to retreat toward the LE where it impacts the strengthened oblique shock. For the P320 grit, this impact resulted in the weakening and ultimate dispersion of the oblique shock wave. Multiple, weak normal shocks emerge and interact in a very chaotic way with the separated boundary-layer. For the P400 grit, the collision between the oblique shock and the shock front is dramatic, and, initially, the oblique shock is swept away (toward the LE). However, it quickly reemerges and interacts in a very unsteady manner with weaker shocks as well as the separated boundary layer. The ‘thick’ shock structure seen in Figure 6.17l is an image of the initial impact of the shock front and oblique wave. The trips also promoted earlier shock-induced stall onset than the baseline model. The stall process has initiated in both Figures 6.17h and 6.17l but not yet in Figure 6.17d.

6.9 Summary

The FS system allowed flow visualization of the separation mechanics of the baseline Bell airfoil, particularly the near surface features of shock induced dynamic stall. At low-speeds, \( M_\infty \leq 0.4 \), separation of the unmodified leading-edge was governed by an abrupt breakdown of the LE turbulent boundary-layer. At higher speed, \( M_\infty \geq 0.5 \), the flow breaks down after the formation of \( \lambda \)-shock over the airfoil chord. Important findings stemming from this experiment include:

1. Multiple normal shocks form and coalesce into a stronger shock wave over a small spatial extent during the unsteady stall process. This shock pattern takes a \( \lambda \) shape at \( M_\infty = 0.6 \).

2. The normal shock travels downstream with increasing angle of attack as it grows in strength but is not immediately followed by separation. A secondary, off-
Figure 6.17: Schlieren flow visualization of trip effect on shock development, $M_\infty = 0.6, \alpha \approx 5^\circ + 5^\circ \sin \omega t, k = 0.05$. 

(a) Baseline, $\alpha = 4.2^\circ$ (b) Baseline, $\alpha = 6.2^\circ$ (c) Baseline, $\alpha = 7.4^\circ$ (d) Baseline, $\alpha = 8.9^\circ$

(e) P320, $\alpha = 4.2^\circ$ (f) P320, $\alpha = 6.2^\circ$ (g) P320, $\alpha = 7.4^\circ$ (h) P320, $\alpha = 8.9^\circ$

(i) P400, $\alpha = 4.2^\circ$ (j) P400, $\alpha = 6.2^\circ$ (k) P400, $\alpha = 7.4^\circ$ (l) P400, $\alpha = 8.9^\circ$
Figure 6.18: Upper-surface pressure for baseline and tripped airfoils, $M_x = 0.60, k = 0.05$. 

(a) $\alpha = 5.5^\circ + 4.8^\circ \sin \omega t$

(b) $\alpha = 5.3^\circ + 4.8^\circ \sin \omega t$

(c) $\alpha = 5.6^\circ + 4.8^\circ \sin \omega t$
surface shock wave moves upstream following the first sign of separation, coinciding with the rearward convection of a stall-vortex like structure. This shock weakens as it reaches the LE. Further investigation into the initiation of the secondary shock wave is required.

3. The extreme sensitivity of the LE flowfield to the presence of the trips used to modify the separation behavior indicates that any LE flow control must have a physical height much less than $\delta$. Otherwise, the control device risks promoting premature, shock-induced dynamic stall (not to mention losing all control authority).

4. Although not discussed, increasing the reduced frequency served to delay shock-induced stall onset, as expected. This was directly observed in the FS high-speed videos.
CHAPTER 7

COMPRESSIBILITY EFFECTS ON AERODYNAMIC DAMPING

An experimental investigation into the relationship between Mach number and torsional aerodynamic damping is presented. The investigation centers on the Bell airfoil oscillated at pitch amplitudes near 5° or 8° with several roughness elements applied to the leading-edge (masking-tape, P400 grit, and P320 grit). The mean angle of attack was incremented at approximately 0.5° intervals to characterize attached flow through deep dynamic stall regimes. Unsteady surface pressures were recorded and integrated to determine the net cycle damping coefficient, \( \Xi_{\text{cycle}} \), and, through the Hilbert Transform, the time-resolved aerodynamic damping coefficient, \( \Xi(t) \). The derivation of both are discussed in detail. Results indicate that the effect of increased Mach number caused an increase in \( \Xi_{\text{cycle}} \), however, this average statistic masked properties of the temporal flow field that led to unstable damping during portions of the pitch cycle. The time-resolved aerodynamic damping analysis offers new insight into the mechanism of transient stall flutter in rotorcraft operations that were otherwise masked by the cycle-averaged statistics.

7.1 Background: Stall flutter

Increase in the forward speed or maneuverability of the traditional helicopter is limited by the load excursions incurred in deep dynamic stall. A second prohibitive
factor preventing performance gains exists due to the aeroelastic interaction between
the dynamic, separated flow field and the blade structural attributes. This instability,
called stall flutter, leads to rapid variations in rotor torsion, excessive vehicle vibration,
and fatigue life degradation.

Stall flutter is a single-degree-of-freedom, aeroelastic problem that results from neg-
ative system (aerodynamic + structural) damping, defined in §7.2. It is not a result of
interaction between aerodynamic disturbances, e.g., the Von Karman street in the wake
or Kelvin-Helmholtz structures in the separated shear layer, and the rotor-blade nat-
ural torsion frequency[42]. Stall flutter occurs as the system’s damping approaches
zero or becomes negative, causing the blade motion to become unstable, i.e., the pitch
amplitude grows at the blade’s natural torsion frequency[51]. A constant oscillation
amplitude emerges as the energy transfer between the aerodynamics and blade struc-
ture balances. Thus, stall flutter is often described as ‘self-excited but self-limiting,’
as it is induced by the airloads and bounded by structural dissipation[42, 68, 94]. The
limit-cycle end state starts with small amplitude, forced oscillations centered near $\alpha_{ss}$,
commensurate with propeller operations[94] but not that of a rotor in forward flight.
For a helicopter, a limit-cycle oscillation is not often realized as the blade’s reduced
incidence on the advancing side of the rotor disk encourages attached flow and positive
damping[68].

As discussed by Carta and Niebanck[24], certain combinations of forward speed
and high disc-loading produce negatively damped regions of the rotor disk. Figure 7.1
shows multiple stall events recorded in flight tests of an UH-60A (Sikorsky Black Hawk
helicopter)[8]. The stall sequence initiates on the retreating-blade near $\psi = 270^\circ$. Two
additional stall events occur as the blade rotates through $\psi = 45^\circ$. The cyclic control
input to the blade over this azimuthal range prescribes a decreasing incidence; the pres-
ence of three stall cycles suggests high-frequency, stall flutter related divergence from the control input. Figure 7.2 depicts the succession of events that cause stall flutter and its ramifications.

7.2 The work per cycle and the damping coefficient

This section outlines the derivation of the work per cycle and net aerodynamic damping coefficient as formulated by Carta and Neiback[24]. The interested reader is referred to this reference for supplementary details. The analysis forms the starting point to the quasi-nonlinear examination considered in §7.4.

The differential work, $dW$, incurred by a moment through a change in incidence,
Figure 7.2. Sequence of dynamic stall related events

d\alpha, is the product of the real components of each term[24],

\[ dW = M_R d\alpha_R. \]  

The net energy transfer between the airstream and airfoil for a single-degree-of-freedom torsional motion about an arbitrary axis is found through the work per cycle, \( W[24] \),

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defined as

\[ W \equiv \oint dW = \int M_R d\alpha_R. \]  

(7.2)

The cyclic integral indicates that the integration is performed over a complete cycle of motion, \( \alpha \in [\alpha(t), \alpha(t + T)] \). Normalization yields

\[ C_w = \oint C_{m_R} d\alpha_R. \]  

(7.3)

To calculate the integral in Eq. 7.3, Carta and Neibanck[24] assumed a harmonic incidence and pitch-moment,

\[ \alpha^* (t) = \alpha_1 (\cos \omega t + i \sin \omega t) = \alpha_1 e^{i\omega t}, \]  

(7.4)

\[ C^*_m(t) = \bar{C}^*_m e^{i\omega t} = (\bar{C}_{m_R} + i \bar{C}_{m_I}) e^{i\omega t} = \bar{C}_m e^{i(\omega t + \psi)}. \]  

(7.5)

It is important to note that, unlike \( \alpha_1 \), the moment amplitude, \( \bar{C}^*_m \), is complex; the pitch moment is not guaranteed to be in phase with the prescribed incidence. Instead, \( C^*_m \) is composed of both an in-phase amplitude, \( \bar{C}_{m_R} \), and a quadrature (out-of-phase) amplitude, found through the imaginary amplitude in \( C^*_m \), \( \bar{C}_{m_I} \). The last relation in Eq. 7.5 is the so-called polar or phasor representation[89], where the real-valued amplitude is

\[ \bar{C}_m = \sqrt{\bar{C}^2_{m_R} + \bar{C}^2_{m_I}} \]  

(7.6)

and where \( \psi \), the phase angle between the pitch-moment components, is shown in
Eq. 7.7. Figure 7.3 depicts the relevant terms.

\[ \psi = \arctan \left( \frac{\bar{C}_{mI}}{\bar{C}_{mR}} \right). \]  

(7.7)

Returning to Eq. 7.5, \( \bar{C}_{mR} \) is the real part of the moment amplitude, not the real part of the total pitch moment. The total, real pitch-moment, \( C_{mR} \), is determined by expanding the exponential in Eq. 7.5 and collecting terms, such that

\[ C_{mR} = \bar{C}_{mR} \cos \omega t - \bar{C}_{mI} \sin \omega t. \]  

(7.8)

For completeness, the imaginary part is

\[ C_{mI} = \bar{C}_{mI} \cos \omega t + \bar{C}_{mR} \sin \omega t. \]  

(7.9)
Returning to the work-per-cycle, the differential, real incidence from Eq. 7.4 is

\[ d\alpha_R = -\alpha_1 \sin \omega t \cdot d(\omega t). \] (7.10)

Substitution of Eq. 7.8 and Eq. 7.10 into Eq. 7.3 and integrating over a single cycle, \( \omega t \in [0, 2\pi] \), yields

\[ C_w = \pi \alpha_1 \tilde{C}_m, \] (7.11)

Eq. 7.11 shows that the aerodynamic work-per-cycle is directly proportional to the quadrature component of the pitch-moment amplitude. A positive value of \( C_w \) indicates work done by the air on the airfoil and is unstable. Returning to Eq. 7.5, \( C_w \) can also be expressed as

\[ C_w = \pi \alpha_1 \tilde{C}_m \sin \psi, \] (7.12)

where the work is negative if \( \psi \) is a lag and positive if \( \psi \) is a lead, as depicted in Figure 7.3. To relate the work to the aerodynamic damping, Carta and Neibank[24] considered a single-degree-of-freedom system capable of torsional motion only. For a linear, damped, stiff structure, the differential equation of motion for free vibration is

\[ I\ddot{\alpha} + C\dot{\alpha} + K\alpha = 0, \] (7.13)

where \( I \), the structure’s inertia, \( C \), the structural damping, and \( K \), the structure’s stiffness, are time-invariant. Eq. 7.13 considers the structure in the absence of aerodynamic forces. The homogeneous solution is \( \alpha = \alpha_1 e^{i\omega t} \). Inserting \( \alpha \) into Eq. 7.13 yields the characteristic equation

\[ \left[ -\omega^2 I + i\omega C + K \right] \alpha_1 = 0, \] (7.14)

where, as noted by Carta[110], the structural damping is the imaginary part of Eq. 7.14.
By analogy, the damping of the aerodynamic system must also be imaginary.

The linear equation of motion for the system, i.e., the structure moving in an airstream with ‘aerodynamic inertia, damping, and stiffness’ is

\[ I \ddot{\alpha} + C \dot{\alpha} + K \alpha = M_1 \ddot{\alpha} + M_2^* \dot{\alpha} + M_3^* \alpha \quad (7.15) \]

where the real and imaginary components of \( M_1, M_2^*, \) and \( M_3^* \) are given by Theodorsen[129] or Bisplinghoff et al.[7]; for this application, the amplitudes are not of significance. The assumption that \( M_1 \) only contains real components is inline with References [7] and [24]. The removal of this assumption does not effect the derivation of the aerodynamic damping. The \( rhs \) of Eq. 7.15 is also a second-order, ordinary differential equation with \( constant \) coefficients. Carta and Neibanck include two sources of damping; \( M_2^r \) is a viscous damping force, whereas \( M_3^s \) is considered structural or hysteretic damping due to ‘internal friction,’ as described in [51]. A discussion of these components is included in §7.4. Eq. 7.15 has the solution \( \alpha = \alpha_1 e^{i \omega t} \) for small damping (such that the motion is essentially sinusoidal), substitution gives the characteristic equation

\[ \{ \left[ -\omega^2 (I - M_1) - \omega M_2^r + K - M_3^s \right] + i \left[ \omega (C - M_2^r) - M_3^s \right] \} \alpha_1 = 0. \quad (7.16) \]

The total aerodynamic damping, \( \xi \), is then the imaginary part of Eq. 7.16 minus the structural damping \( (\omega C) \),

\[ \xi \equiv -\omega M_2^r - M_3^s \quad (7.17) \]

Equating the \( rhs \) of Eq. 7.15 and the \( rhs \) of Eq. 7.5, both of which represent the unsteady
pitch moment, and scaling Eq. 7.15 by $qc^2 = \frac{1}{2}\rho U_x^2 c^2$, yields

$$(\bar{C}_{m_r} + i\bar{C}_{m_3}) = \{[-\omega^2 C_{m_1} - \omega C_{m_3} + C_{m_3}] + i[\omega C_{m_2} + C_{m_3}]\} \alpha_1. \tag{7.18}$$

From inspection,

$$\bar{C}_{m_3} = [\omega C_{m_2} + C_{m_3}] \alpha_1, \tag{7.19}$$

or, comparing to equation Eq. 7.17,

$$\xi = -qc^2 \frac{d\bar{C}_{m_3}}{d\alpha_1} = -qc^2 \frac{\bar{C}_{m_3}}{\alpha_1}. \tag{7.20}$$

Eq. 7.20 may be rewritten in a dimensionless form to define the cycle aerodynamic damping coefficient, $\Xi_{cycle}$, by dividing through by the moment per unit span $qc^2$,

$$\Xi_{cycle} = \frac{\xi}{qc^2} = -\frac{d\bar{C}_{m_3}}{d\alpha_1} = -\frac{\bar{C}_{m_3}}{\alpha_1}. \tag{7.21}$$

In polar representation[61, 116], the damping is

$$\Xi_{cycle} = \frac{d\bar{C}_m}{d\alpha_1} \sin \psi = -\frac{\bar{C}_m}{\alpha_1} \sin \psi. \tag{7.22}$$

From Eq. 7.22, phase leads indicate negative damping, or positive work, and unstable airloads.

The point of this exercise is to relate the aerodynamic damping, $\Xi_{cycle}$, to the work per cycle, $C_w$, as the latter is easily determined from experimental measurements – see §7.2.2. To avoid a costly error, recall that $\bar{C}_{m_3}$ and $C_w$ are functions of $\alpha_1$ and that simple substitution of Eq. 7.11 into Eq. 7.21 results in an incorrect relation. Instead,
return to Eq. 7.5 and consider
\[ \tilde{C}_m e^{i\omega t} = \frac{\pi k^2}{2} B^* \alpha_1 e^{i\omega t}, \] (7.23)
where \( \frac{\pi k^2}{2} B^* \alpha_1 \) is the dimensionless aerodynamic moment given by Theodorsen’s method (\( B^* \) is the so-called influence coefficient, shorthand for several contributions to the total moment due to changes in \( \alpha \)). Equating the imaginary parts of Eq. 7.23 and canceling exponential terms gives
\[ \tilde{C}_{m_I} = \frac{\pi k^2}{2} B_0 \alpha_1. \] (7.24)
Substitution of Eq. 7.24 into Eq. 7.11 and Eq. 7.21, and noting \( B_0 \) is independent of \( \alpha_1 \), provides a simple functional relationship between \( \Xi \) and \( C_w \),
\[ \Xi_{\text{cycle}} = -\frac{C_w}{\pi \alpha_1^2} = -\frac{1}{\pi \alpha_1^2} \int C_m d\alpha. \] (7.25)
Clockwise oriented loops in a \( C_{m_{1/4}}(\alpha) \) plot signify negatively damped fractions of the pitch cycle. Figure 7.4 depicts both a light and deep dynamic stall cycle with negative damping present.

7.2.1 Thin-airfoil damping and comparison
For harmonic pitch oscillations in potential flow with aerodynamic center and pivot axis at \( x = 1/4 \), the influence coefficient \( \beta^*_\alpha \) is simply \( M^*_\alpha[7] \), where
\[ M^*_\alpha = M_{\alpha R} + iM_{\alpha I} = \frac{3}{8} - \frac{i}{k}, \] (7.26)
Expanding the pitch-moment in Eq. 7.5 gives

\[ C_{m_{c/4}}^* = \frac{\pi k^2}{2} \alpha_1 (M_{\alpha_R} + iM_{\alpha_I}) (\cos \omega t + i \sin \omega t), \] (7.27)

with real and imaginary parts

\[ C_{m_{c/4R}} = \frac{\pi k^2}{2} \alpha_1 (M_{\alpha_R} \cos \omega t - M_{\alpha_I} \sin \omega t), \] (7.28)
\[ C_{m_{c/4I}} = \frac{\pi k^2}{2} \alpha_1 (M_{\alpha_I} \cos \omega t + M_{\alpha_R} \sin \omega t). \] (7.29)

Substitution of Eq. 7.26 into Eq. 7.28 gives the real component of the quarter chord pitch moment as

\[ C_{m_{c/4R}} = \frac{\pi k \alpha_1}{2} \left[ \sin \omega t + \frac{3}{8} k \cos \omega t \right]. \] (7.30)

From inspection, the pitch-moment from the model is predominately out-of-phase with the incidence \((\alpha = \alpha_0 + \alpha_1 \cos \omega t)\). To determine the phase angle, \(\psi\), Eq. 7.7 and Eq. 7.26 are combined,

\[ \psi = \tan^{-1} \left( -\frac{8}{3k} \right). \] (7.31)
Figure 7.5. Phase and amplitude of the damping coefficient pitch-moment response from Theodorsen’s method

As observed, the theoretical pitching-moment always lags \( \alpha \) indicating stable airloads.

The corresponding amplitude is \( \tilde{C}_{m/4} \)

\[
\tilde{C}_{m/4} = \frac{\pi k}{2} \frac{\alpha_1}{\alpha} \sqrt{1 + \frac{9}{64} k^2}.
\] (7.32)

Figure 7.5 illustrates the effect of reduced frequency on the Phase and amplitude of the damping coefficient response of the pitch-moment for typical rotor values of \( k \) and \( \alpha_1 \). Note, \( \psi \approx -\pi/2 \) for all \( k \) of interest; the \( k^2 \) term in Eq. 7.32 has little impact on the moment amplitude, such that \( \tilde{C}_{m/4} \approx \frac{\pi k}{2} \alpha_1 \).

Use of the prescribed incidence, \( \alpha = \alpha_0 + \alpha_1 \cos \omega t \) [24], and well-known trigonometric identities transforms Eq. 7.30 into

\[
C_{m/4r} = \frac{\pi k}{2} \left[ \frac{3}{8} k (\alpha - \alpha_0) \pm \sqrt{\alpha_1^2 - (\alpha - \alpha_0)^2} \right],
\] (7.33)

replacing the explicit dependence on \( \omega t \) with \( \alpha \).

Substitution of Eq. 7.30 into Eq. 7.25 and integrating, or, the more simple alterna-
Theodorsen’s method concludes that the damping coefficient is strictly positive, linearly proportional to the reduced frequency [94, 129], and independent of $\alpha_1$. This result is in good agreement with experimental measurements up to stall onset – see [94].

Figure 7.6 contrasts Theodorsen’s prediction to the ensemble pitch-moment for attached flow at $M_\infty = 0.19$. Here, the time dependence has been replaced by the airfoil incidence. The arrows indicate the direction of increasing time.
7.2.2 Experimental determination of $\Xi_{\text{cycle}}$

The integral is experimentally computed from the area enclosed in the ensemble $C_{m/4}(\alpha)$ hysteretic curve[19],

$$
\Xi_{\text{cycle}} = \frac{1}{\pi \alpha_1^2} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} C_{m/4}^D - C_{m/4}^U \, d\alpha,
$$

(7.36)

where the superscripts $U$ and $D$ denote the up- and down-strokes of the oscillation, respectively, and $C_{m/4}$ is determined from integration of the chord-pressures. Unless otherwise noted, the damping coefficient was found from the 1P ensemble averaged quarter-chord moment, $\langle C_{m/4} \rangle$. The numerical determination of the integral was carried out via a trapezoidal scheme in STALLFLUTTER.exe. The damping of the experimental case shown in Figure 7.6 is $\Xi_{\text{cycle}} = 0.066$, whereas the value predicted by Eq. 7.34 is $\Xi_{\text{cycle}} = 0.081$. The corresponding ratio of measured damping to theoretical damping is $\Xi_{\text{cycle}} / \pi k = 0.81$, and is similar to that found by Lorber and Carta[96] for attached flow (albeit different prescribed motion).

To resolve the effect of the pseudo-random nature of dynamic stall, albeit in a very discrete fashion, $\Xi_{\text{cycle}}$ can be determined as

$$
\Xi_{\text{cycle}}(n) = -\frac{1}{\pi \alpha_1^2} \int_{t_1}^{t_1+T} C_{m/4} \frac{d\alpha}{dt} \, dt,
$$

(7.37)

where $n = 1, 2, ..., N$, and $N$ is the number of complete cycles in the time history. Substitution of $d\alpha = \frac{d\alpha}{dt} dt$ is not necessary at this point, as $\Xi(n)$ could be determined through its original formulation. Eq. 7.37 can also be used to determine the average damping over the time-series,

$$
\Xi_{\text{cycle}} = -\frac{1}{N} \frac{1}{\pi \alpha_1^2} \int_{0}^{N*T} C_{m/4} \frac{d\alpha}{dt} \, dt
$$

(7.38)
Computation of $\Xi_{cycle}$ found through Eq. 7.36 to values found through Eq. 7.38 yield excellent agreement ($error = O(\Xi \times 10^{-2})$).

Mcalister et al.[103] inspected the integrand of Eq. 7.37 in the form

$$-\frac{1}{\pi \alpha^2} C_{m_{/4}} \frac{d\alpha}{d(\omega_1 t)}, \quad (7.39)$$

which normalizes the integrand by the prescribed circular frequency. The integrand, however, can not be interpreted as the time-derivative of the damping, as $C_{m_{/4}}$ is obviously a function of time. STALLFLUTTER.exe computes this quantity, however, its meaning should not be confused with $\dot{\Xi}$.

7.3 Net damping and dynamic stall regimes

Depending on motion history, boundary layer separation characteristics, and compressibility effects, light dynamic stall trajectories may promote two closed loops in the $C_{m_{/4}}$ trace, such that the second loop is negatively damped[89, 104, 109]. Deep stall trajectories promote stall early in the pitch-cycle such that the stall vortex convects into the wake during the oscillation’s up-stroke[89]. The stall-vortex-free, separated flow allows for a third, stable sub-loop in the $C_{m_{/4}}$ curve, increasing $\Xi$. Prior investigations[106, 109] have shown that light dynamic stall was most prone to large, negative values of $\Xi$ for rotor-blade geometries[94–96].

7.4 Derivation of $\Xi(t)$

The cycle integrated, net damping values from Eq. 7.25 and its experimental counterpart, Eq. 7.36, fail to account for the time-resolved damping characteristics experienced through the dynamic stall process on 2D airfoils or during in-flight rotor opera-
tions. As such, little information is gleaned from calculation of $\Xi$, aside from whether the net airloads are stable or unstable. This section addresses this problem, undertaking a different approach to the calculation of the aerodynamic damping based on the (discrete) Hilbert transform.

7.4.1 The Hilbert Transform (HT)

The HT takes a data-series, whether that series is analytical (e.g, $\cos \omega t$) or measured (e.g. $C_n(n)$), and shifts its phase by $-\pi/2[48]$. The HT is frequently used in signal processing, e.g., amplitude and phase modulation and demodulation, single-sideband modulation and demodulation, and bandpass filtering[114]. Interesting applications of the HT and its extensions, particularly the Hilbert-Huang method for empirical-mode-decomposition (EMD), include feature detection in images[85], determination of wave spectra[136], and linear and non-linear vibration analysis[45, 46, 48, 125]. The latter includes flutter on fixed-wing air vehicles[72].

For the signal, vibration, and flutter problems, the HT allows computation of the time dependent, nonlinear or quasi-nonlinear system characteristics. These ‘modal parameter’[46] calculations primarily focus on the instantaneous amplitude, often referred to as the envelope function, $A(t)$, and instantaneous frequency, $\omega(t)$, of the data-series. In turn, $A(t)$ and $\omega(t)$ can be used to study the system damping, stiffness, mass, etc[48]. In a similar light, the HT provides a means to determine the aerodynamic damping in the time-domain. Before approaching the unsteady aerodynamic problem, the fundamentals and useful properties of the Hilbert transform are described.
7.4.1.1 Definition and properties of the Hilbert transform

The review and tutorial paper by Feldman[47] summarizes the HT and provides simple examples of its utility in signal processing and vibration analysis.

Let \( y(t) \) be a real-valued data-series. The HT of \( y(t) \), denoted here as \( \tilde{y}(t) \), is defined

\[
\tilde{y}(t) \equiv H[y(t)] = -\frac{1}{\pi} \mathcal{P} \int_{-\infty}^{\infty} \frac{y(\tau)}{\tau - t} d\tau,
\]

with inverse

\[
y(t) \equiv H^{-1}[\tilde{y}(t)] = -\frac{1}{\pi} \mathcal{P} \int_{-\infty}^{\infty} \frac{\tilde{y}(\tau)}{\tau - t} d\tau,
\]

where \( H \) is the Hilbert operator and \( \mathcal{P} \) denotes the Cauchy principal value of the improper integral. From Eq. 7.40 and Eq. 7.41, it is apparent

\[
y \leftrightarrow H \tilde{y}
\]

such that \( y \) and \( \tilde{y} \) form a HT pair. The analytic data-series is a complex series whose imaginary part is the HT of its real part such that the Cauchy-Riemann conditions are satisfied. Thus, the analytic data-series, \( \mathcal{Y}(t) \), is defined

\[
\mathcal{Y}(t) \equiv y(t) + i \cdot \tilde{y}(t) = A(t)e^{i\phi(t)} = A(t) \left( \cos \phi(t) + i \sin \phi(t) \right)
\]

where \( A(t) \) and \( \phi(t) \) are

\[
A(t) = \sqrt{y^2 + \tilde{y}^2},
\]

\[
\phi(t) = \arg(\mathcal{Y}),
\]

and represent the series’ time-dependent amplitude and phase, respectively. Differentiating Eq. 7.45 leads to the so-called instantaneous frequency, \( \omega(t) \). Physically, \( \mathcal{H}[-] \) can
be better understood in frequency space through the Fourier transform[47] of Eq. 7.40.

In frequency space, \( \mathcal{H}[\cdot] \) shifts the phase of the spectrum by \(-\pi/2\) without altering its amplitude. Thus, in convolution notation, Eq. 7.40 is \( \tilde{y} = -\frac{1}{j\pi} * y \). For a real-valued time-series \( y(t) \), \( \tilde{y}(t) \) is also real-valued; the analytic data-series, \( \mathcal{Y}(t) \), is, by definition, complex-valued – analytic series in this chapter are not denoted by * in order to separate there meaning from other complex terms. For example, \( \mathcal{H}[y = \cos \omega t] = \sin \omega t \), and the analytic series is \( \mathcal{Y}(t) = \cos \omega t + i \sin \omega t = e^{i\omega t} \).

Useful properties of the Hilbert transform – see [74] or [114] for rigorous proof – and its application to unsteady aerodynamics problem or general engineering problems include:

1. Linearity: \( \mathcal{H}[\beta_1 y_1 + \beta_2 y_2] = \beta_1 \ast \tilde{y}_1 + \beta_2 \ast \tilde{y}_2 \), where \( \beta_i \) is a constant

2. Multiple transforms: \( \mathcal{H} [\mathcal{H}[y]] = -y \)

3. Derivative: \( \frac{d}{dx} \mathcal{H}[\cdot] = \mathcal{H} \left[ \frac{d}{dx} \cdot \right] \)

4. Orthogonality: \( \int_{-\infty}^{\infty} y \cdot \tilde{y} dt = 0 \)

5. Transform of analytic signals: \( \mathcal{H}[\mathcal{Y}] = -i \mathcal{Y} \)

6. Transform of a product of analytic signals: \( \mathcal{H}[\mathcal{Y}_1 \cdot \mathcal{Y}_2] = \mathcal{Y}_1 \mathcal{H}[\mathcal{Y}_2] = \mathcal{H}[\mathcal{Y}_1] \mathcal{Y}_2 = -i \mathcal{Y}_1 \cdot \mathcal{Y}_2 \)

7. Operation on a constant: \( \mathcal{H}[constant] = 0 \)

8. Multiplication of signals with non-overlapping spectra: \( \mathcal{H} [y_1(t) \cdot y_2(t)] = y_1(t) \mathcal{H}[y_2(t)] \)

Additional relations between the Hilbert transform and complex signals are analyzed by Reddy and Swamy[117].
The discrete Hilbert transform (DHT) is carried out using FFT and IFFT routines, for example, via the MATLAB® command Hilbert.m which accepts the single\textsuperscript{1} argument $y[n]$ and returns the $n$ valued analytic data-series $\mathcal{Y}[n]$, where $\text{Im} \{Y[n]\} = \tilde{y}[n]$. Li et al.[90] discuss the DHT in depth and offer suggestions for efficient computation. Luo et al. delve into common errors associated with the DHT including substantial inaccuracies in the computation of $\omega[n]$ (the discrete, instantaneous phase) from noisy data. Luo et al. also offer a simple algorithm for the Hilbert transform based on the FFT. In the work later in this chapter, the output of STALLFLUTTER.exe was processed by an auxiliary MATLAB® script to determine the analytic series of the ensemble averaged pitching-moment.

7.4.1.2 Limitations of the Hilbert transform

The Hilbert transform is a global or integral operator[47]; the entire data-series is required to determine the mapping. Consequently, adaptations are required in order to perform Hilbert based operations in realtime applications.

7.4.2 The equations of motion (EOM) for a single-degree-of-freedom aerodynamic pitch-moment with time-varying damping and stiffness

By consideration of the aerodynamic loads due to prescribed torsional oscillations in a uniform airstream, this section describes the EOM for the airfoil’s incidence. The EOM for the system is similar to that of Carta and Neibach, where $\alpha(t)$ is the output to the input pitch-moment, $M(t)$. Written in this form, the quasi-nonlinear EOM is nearly identical to the linear one, Eq. 7.15, the exception being that nonlinearities are introduced by assuming time-dependent aerodynamic damping and stiffness.

\textsuperscript{1}Hilbert.m also accepts two inputs, $y[m]$ and $n$ such that $m < n$ and $y$ is zero padded to reduce spurious results in the discrete Fourier operations, e.g. Gibbs phenomenon.
The SDOF EOM for the pitch-moment with time-dependent aerodynamic damping, 
$h^*(t)$, and aerodynamic stiffness, $\kappa^*(t)$, is

$$\pi \rho b^4 \dddot{\alpha}(t) + h^*(t) \ddot{\alpha}(t) + \kappa^*(t) \alpha(t) = M(t) \tag{7.46}$$

where $h^* = h_R + ih_I = \bar{h}e^{i\gamma_1(t)}$ is the viscous damping moment and $\kappa^* = \kappa_R + i\kappa_I = \bar{\kappa}e^{i\gamma_2(t)}$ is the aerodynamic stiffness. Further, this EOM can account for non-linearities inherent to the dynamic stall problem, unlike formulations that rely on constant amplitude, constant phase damping or stiffness[110]. The complex, viscous damping, $h^*$, and aerodynamic stiffness, $\kappa^*$, contribute to the total aerodynamic damping (imaginary terms) and stiffness (real terms), as will be shown in §7.4.3; this assumption is examined more critically in §7.5.1.2 through examination of the attached flow dynamic stall data set. The damping and stiffness terms used here are identical to $M_2^*$ and $M_3^*$ in Eq. 7.15. Substitution of the polar representations of $h^*$ and $\kappa^*$ into Eq. 7.46 yield

$$\pi \rho b^4 \dddot{\alpha} + \bar{h}(t)e^{i\gamma_1(t)} \ddot{\alpha} + \bar{\kappa}(t)e^{i\gamma_2(t)} \alpha = M. \tag{7.47}$$

Eq. 7.47 can also be written

$$\dddot{\alpha} + 2h_0(t)e^{i\gamma_1(t)} \ddot{\alpha} + \omega_0^2(t)e^{i\gamma_2(t)} \alpha = \frac{1}{\pi \rho b^4}M, \tag{7.48}$$

where the viscous damping coefficient is defined

$$h_0 \equiv \bar{h}/2\pi \rho b^4, \tag{7.49}$$
and the undamped natural frequency is

$$\omega_0 \equiv \sqrt{\kappa/\pi \rho b^4}. \quad (7.50)$$

In Eqs. 7.47 to 7.48, although not indicated, $\alpha$, its derivatives, and $M$ remain time dependent.

### 7.4.3 The Hilbert transform of the EOM

In order to employ the Hilbert transform analysis, each term in Eq. 7.48 is replaced by its analytic signal counterpart found through the Hilbert transform; this generates the new EOM

$$\ddot{A} + 2h_0(t)e^{i\gamma_1(t)} \dot{A} + \omega_0^2(t)e^{i\gamma_2(t)} A = \frac{1}{\pi \rho b^4} M, \quad (7.51)$$

where $A = \alpha + i\dot{\alpha} = \alpha_1 e^{i\omega t}$ (harmonic motion) and $M = M + i\ddot{M} = A(t)e^{i\phi(t)}$.

In applying the HT, it is assumed that the spectra of $h_0(t)e^{i\gamma_1(t)}$ and $\omega_0^2(t)e^{i\gamma_2(t)}$ do not overlap with $\dot{\alpha}$ or $\alpha$, respectively. This assumption is valid as both the damping and natural frequency terms result from the aerodynamic loads. The loads themselves will never be completely in-phase and phase-locked with the prescribed angle-of-attack.

Substitution of $A$, $M$, and the first and second derivatives of $A$ (for harmonic motion these terms are zero) into Eq. 7.51 gives

$$-\omega^2 - 2h_0 \omega \sin\gamma_1 + \omega_0^2 \cos\gamma_2 + i \left(2h_0 \omega \cos\gamma_1 + \omega_0^2 \sin\gamma_2 \right) = \frac{1}{\alpha_1 \pi \rho b^4} (M + i\ddot{M}) e^{-i\omega t} \quad (7.52)$$

Equating real and imaginary parts in Eq. 7.52 yields a system of equations; for this work, only the imaginary part of Eq. 7.52 is required, however, the real part is shown
for completeness,

\[ -\omega^2 - 2h_0\omega \sin \gamma_1 + \omega_0^2 \cos \gamma_2 = \frac{1}{\alpha_1 \pi \rho b^4} (M \cos \omega t + \dot{M} \sin \omega t) \quad (7.53) \]

\[ = \frac{A(t)}{\alpha_1 \pi \rho b^4} \cos(\phi(t) - \omega t) \quad (7.54) \]

\[ 2h_0\omega \cos \gamma_1 + \omega_0^2 \sin \gamma_2 = \frac{1}{\alpha_1 \pi \rho b^4} (\dot{M} \cos \omega t - M \sin \omega t) \quad (7.55) \]

\[ = \frac{A(t)}{\alpha_1 \pi \rho b^4} \sin(\phi(t) - \omega t) \quad (7.56) \]

By comparison to Eq. 7.17, the \( \text{lhs} \) of Eq. 7.55 is the total (viscous and internal/hysteretic sources) aerodynamic damping, \(-\xi/\pi \rho b^4\). Therefore,

\[ \xi(t) = -\frac{1}{\alpha_1} \left( \frac{M \cos \omega t - \dot{M} \sin \omega t}{M_I} \right) = -\frac{A(t)}{\alpha_1} \sin(\phi(t) - \omega t) \quad (7.57) \]

where \( M_I \) denotes the quadrature (out-of-phase) pitch-moment component, and \( \psi(t) \) is the phase lead or lag between the pitch moment and the angle of attack. Eq. 7.57 is equivalent in form to Eq. 7.20. Non-dimensionalizing Eq. 7.57, the total aerodynamic damping coefficient becomes

\[ \Xi(t) = \xi/qc^2 = -\frac{1}{\alpha_1} \left( \bar{C}_m \cos \omega t - \bar{C}_m \sin \omega t \right) \quad (7.58) \]

\[ = -\frac{A_{\bar{C}_m}(t)}{\alpha_1} \sin \left( \tan^{-1} \left( \frac{\bar{C}_m}{\bar{C}_m} \right) - \omega t \right) \quad (7.59) \]

where \( A_{\bar{C}_m}(t) = \sqrt{\bar{C}_m^2 + \bar{C}_m^2} \). Similarly, the phase difference between the pitch moment and \( \alpha \) can be determined directly from \( \alpha, \bar{C}_m \), and their respective Hilbert
transforms[48], namely

\[
\psi(t) = \phi(t) - \omega t = \tan^{-1}\left(\frac{C_{m_r}}{C_{m_q}}\right)
\]

(7.60)

\[
= \tan^{-1}\left(\frac{\alpha \cdot \tilde{C}_m - \tilde{\alpha} \cdot C_m}{\alpha \cdot C_m + \tilde{\alpha} \cdot \tilde{C}_m}\right),
\]

(7.61)

where \(C_{m_r}\) and \(C_{m_q}\) are the in-phase and quadrature pitching moment coefficients. In terms of the amplitude and phase difference, the total aerodynamic damping coefficient is

\[
\Xi(t) = -\frac{A_{C_m}(t)}{\alpha_1} \sin \psi(t).
\]

(7.62)

Eq. 7.62 illustrates how the phase lead or lag between the pitch moment and the prescribed angle of attack determines the sign of the aerodynamic damping; its form is identical to previous derivations of the aerodynamic damping[7, 110, 116]. Further, it remains that a phase lag, \(-\pi < \psi(t) < 0\) produces positive damping; whereas, \(0 < \psi(t) < \pi\) indicates negative damping or a local exchange of energy from the airstream to the airfoil. \(\Xi\) reflects the stability of the pitch moment, which, in turn, is manifestation of the developing pressure field. An unstable pitch-moment, \(\Xi < 0\), implies unstable pressure loading, and can be described as a necessary condition to excite stall flutter of an elastic body.

The novelty of the approach, much unlike previous investigations of aerodynamic damping that rely on \(\Xi_{cycle}\), is that Eqs. 7.59 or 7.62 allow the time-resolved aerodynamic damping to be calculated. The average damping, \(\Xi_{avg}\), is then \(\Xi_{avg} = \frac{1}{T} \int_0^T \Xi(t) dt\), where \(T\) is the period of a single oscillation. Although \(\Xi_{avg}\) is not examined explicitly in this paper, the difference between \(\Xi_{avg}\) and \(\Xi_{cycle}\) (Eq. 7.36) was typically \(\Xi_{cycle} \times 10^{-3}\).

For further utility and possible application to arbitrary unsteady motion, the analytic-series \(\mathcal{A}\), which is based on the airfoil’s incidence, can be stated as
Here, the amplitude of the motion and even instantaneous phase can vary in time. This, however, introduces several additional terms to the lhs of Eq. 7.52 involving $\dot{A}_a(t)$, $\dot{\Omega}(t)$, $\ddot{A}_a(t)$, and $\ddot{\Omega}(t)$. These terms are zero for pure harmonic motion and were left out of the above derivation. For arbitrary motion, e.g., heaving oscillations, the meaning of these additional terms is not quite clear in terms of the aerodynamic loads. However, for the total aerodynamic damping, it follows

$$\Xi(t) = \frac{A_{C_m}(t)}{A_a} \sin \psi(t).$$

(7.63)

where, $\psi = \phi - \Omega$ and is still determinable through Eq. 7.61. Retaining this formulation lumps the additional terms stemming from the derivatives of $A_a$ and $\Omega$ into the lhs of Eq. 7.52; this assumption casts these terms as additional damping sources, however, it may be desired to subtract these terms from Eq. 7.63. Prescribed motion and analysis of this type is left for future work but will be necessary for the biharmonic data.

The data processed in this chapter was based on the ensemble averaged pitching-moment, $\langle C_{m_{\alpha}} \rangle$, and the collocation points (angle-of-attack) used to find the average. As such, $\alpha$ was a perfect harmonic signal. The true angle-of-attack differed most from the collocation points when the pitching-moment reached its largest magnitude. For attached flow cases this difference was negligible. For light and deep dynamic stall, this difference was largest as the DSV reached the trailing-edge and $C_{m_{\alpha}}$ grew to its peak value. The effect of this behavior, however, is not investigated in the analysis of this chapter. More importantly, it is not expected that this affects the most critical aspect of this work, as the minimum time-resolved damping occurs prior to the peak pitch moment, well before the difference in the true incidence and the ensemble incidence reaches a maximum.

Lastly, errors arise if the mean moment and incidence (the DC components of the
data) are not removed from $C_m(t)$ and $\alpha(t)$ before the time-series are processed – see [48]. The DC biases the amplitude, $A_{C_m}(t)$, and changes the instantaneous frequency, $\dot{\psi}$. This does alter the analysis as the cyclic integral used to determine $\Xi_{cycle}$ eliminates the mean value of the pitch-moment as well.

### 7.4.4 The relationship to the damping-per-cycle, $\Xi_{cycle}$

Through Eq. 7.58 or Eq. 7.59, the damping can be examined in the time domain. The average damping, $\Xi_{avg}$, is then $\Xi_{avg} = \frac{1}{T} \int_0^T \Xi(t) dt$. Although the latter quantity is not examined explicitly in this chapter, it is obvious from the later plots of $\Xi(t)$ that the ‘time-average,’ and the area integrals used to determine $\Xi_{cycle}$ (Eq. 7.36) are nearly identical. The difference between $\Xi_{avg}$ and $\Xi_{cycle}$ (Eq. 7.36) was typically $\Xi_{cycle} \times 10^{-3}$.

### 7.4.5 Example: Theodorsen pitch moment

Theodorsen’s method is used to generate a model problem to demonstrate the utility of the Hilbert transform analysis. The quarter chord pitch-moment of an airfoil undergoing harmonic oscillation is[7] given in Eq. 7.27 where, recall, $M_{\alpha \dot{\alpha}} + i M_{\alpha \dot{\alpha}} = 3/8 - i/k$. Also recall, the amplitude and phase difference of the aerodynamic moment are (from [89]) $A = \frac{\pi k}{2} \alpha_1 \sqrt{1 + \frac{9}{8} k^2}$ and $\psi = \arctan\left(-\frac{8}{3k}\right)$, respectively. It is the goal of the Hilbert analysis to correctly determine the amplitude, phase, and subsequent aerodynamic damping (Eq. 7.34) from only the real part of Eq. 7.27, as shown in Eq. 7.30 is

$$C_{m_{\text{real}}} = \Re\{C_{m_{\text{real}}}^*\} = \frac{\pi k \alpha_1}{2} (3/8 k \cos \omega t + \sin \omega t). \quad (7.64)$$

$C_{m_{\text{real}}}$ was generated for discrete time steps over the course of a single cycle and plotted as function of $\alpha$ for $k = 0.1$ and a prescribed motion, $\alpha = 6^\circ + 8^\circ \cos \omega t$. This is shown as the oval labeled $C_{m_{\text{real}}}$ in Fig. 7.7a. The discrete Hilbert transform of
\( C_{m\text{real}} \) corresponds to the red long-dashed line that is labeled \( \tilde{C}_m \). It is identical to the imaginary part of Eq. 7.30.

As previously noted, the in-phase moment coefficient, \( C_{mg} \), and quadrature moment coefficient, \( C_{mi} \) have theoretical values \( C_{mg} = 3\pi k^2 \alpha_1/16 = 4.1 \times 10^{-4} \) and \( C_{mi} = -\pi k \alpha_1/2 = -2.193 \times 10^{-2} \). These values, as determined through the Hilbert transform, are simply the denominator and numerator of Eq. 7.61, and are also plotted in Fig. 7.7a. The Hilbert based analysis of the discrete data-series \( (C_{m\text{real}}) \) accurately captures both \( C_{mg} \) and \( C_{mi} \). The same information is plotted as a function of \( \omega t \) in Fig. 7.7b.

Fig. 7.7c represents the aerodynamic damping in polar form, as found in Eq. 7.63. The radial component indicates the magnitude of the damping, \( A_{C_m}/\alpha_1 \), and the angle is \( \psi \). The small arrow in quadrant four is directed at a single point within the red circle, which is \( (A_{C_m}/\alpha_1, \psi) = (0.157, -1.53) \). This solution found through the DHT agrees with Theodorsen’s solution.

Finally, Fig. 7.7d highlights the time-resolved damping, which, for the model problem is constant. Both the numerical solution, \( \Xi(t) \), and the theoretical solution given in Eq. 7.34 are shown but they are indistinguishable from one-another, such that \( 2\Xi(t)/\pi k = 1.0 \). Numerical errors associated with the HT are focused at the endpoints but are relatively small (< 1.4%).

Figure 7.8 considers Theodorsen’s model with additive, random noise. 0.5% noise caused a maximum error in the temporal damping, \( \Xi(t) \), of 14.6%. Since the accuracy of the HT method is susceptible to noise, a feature of HT analysis discussed by Luo et al.[98], the ensemble averaged pitch-moment was utilized to calculate the damping properties.

With the understanding gained from the model problem, the influence of increasing Mach number on both the damping-per-cycle and the time-resolved aerodynamic
(a) The real moment \( C_{m_{\text{real}}} \) (used in this example problem as the measured data-series), its DHT, \( C_m \), and the in-phase and quadrature pitching moments, \( C_{m_R} \) and \( C_{m_I} \), respectively. The arrows on \( C_{m_{\text{real}}} \) indicate the direction of increasing time.

(b) An illustration of (a) as a function of \( \omega t \) instead of \( \alpha \).

(c) The polar representation of the aerodynamic damping, \((A_{C_{m_1}}/\alpha_1, \psi)\) for Theodorsen’s method is found within the red circle.

(d) The time-resolved damping coefficient.

Figure 7.7. Hilbert transform analysis of Theodorsen’s unsteady pitching moment, \( k = 0.1, \alpha = 6^\circ + 8^\circ \cos \omega t \).

damping is next investigated. Note, for the Bell airfoil, increased angle of attack, \( \alpha \), creates a nose-down moment. Consequently, and unlike Eq. 7.64, the phase lies in the third quadrant, \(-\pi < \psi < -\pi/2\), for cases in which the flow remains attached to the airfoil upper-surface and \( \alpha_{\text{max}} < \alpha_{ss} \) and \( \dot{\alpha} > 0 \).
7.5 Compressibility effects on $\Xi_{cycle}$ and $\Xi(t)$

The influence of increasing Mach number on the damping-in-pitch behavior of the Bell airfoil was investigated from $M_\infty = 0.2 - 0.6$. Each of the following sections studies both the average and temporal behavior of the damping coefficient. Note, to facilitate the HT analysis, as well as avoid errors, the mean pitch-moment and $\alpha_0$ were
7.5.1 Attached flow damping

Attached flow pitching oscillations are generally stable and adhere reasonably well to incompressible, thin-airfoil theory[24, 65, 94] for low to moderate $M_\infty$. The damping coefficient for attached flow for each of the LE modified data subsets is shown in Figure 7.9 to highlight the variation in damping due to $M_\infty$.

The data in Figure 7.9 were selected to maintain similar maximum incidence with respect to the trials’ Mach dependent stall behavior. To allow additional points, a small range of attached flow cases were considered such that $\alpha_{max} - \alpha_{ss} \in (-2.6^\circ, -2.0^\circ)$. The parameter $2\Xi_{cycle}/\pi k$ is studied over that of $\Xi_{cycle}$ to account for experimental variations removed from the data.
in the reduced frequency\(^2\). The low-speed data at \( M_\infty = 0.2 \) is similar to the findings of Lorber and Carta\cite{94} for \( \alpha_1 = 6^\circ \) and \( k = 0.15 \) – see Figure 8 in \cite{94}. Both experiments resulted in measured damping values less than the thin-airfoil prediction at low-speed (despite different airfoil geometries). Figure 7.9 indicates a clear increase in damping as \( M_\infty \) rises for each LE modification. The trips had a mild and somewhat random effect on the attached flow damping compared to the baseline, with the largest variation witnessed at \( M_\infty = 0.6 \). Since the damping is a second integral of the pressure field, the attached flow damping behavior can be expected to adhere to the Prandtl-Glauert rule\(^3\). Application of the correction to the aerodynamic damping coefficient yields

\[
\frac{2 \Xi_{\text{cycle}}}{\pi k} = \frac{2 \Xi/\pi k|_0}{\beta},
\]

(7.65)

where \( \beta = \sqrt{1 - M^2_\infty} \) and \( 2 \Xi/\pi k|_0 \) refers to incompressible behavior. The red, dashed line in Figure 7.9 transforms \( 2 \Xi/\pi k|_0 = 0.85 \), picked to represent the aggregate trip low-speed damping behavior, to higher freestream speeds. The correction agrees arguably well up to \( M_\infty = 0.4 \) for all cases, but diverges quickly from Eq. 7.65 for \( M_\infty > 0.45 \). This can be expected as the airfoil’s critical Mach number is near 0.3; larger \( M_\infty \) produce significant regions of \( M_l > 1 \) and forced a rearward travel of the center of pressure. To provide a better fit of the measured data, the Prandtl-Glauert transformation is replaced by \( 2 \Xi/\pi k|_0 / \sqrt{1 - BM^2_\infty} \) where the numerator and \( B \) are found from least-squares analysis. The values of \( B \) for each LE modification are significantly greater than unity, as shown in Figure 7.9. Subsequently, the Prandtl-Glauert singularity decreases (\( M_\infty = 0.75 \) for the masking-tape trip) from \( M_\infty = 1 \). This suggests that the damping is

\(^2k \text{ varied from 0.051 to 0.045 for } M_\infty \in [0.2, 0.6] \text{ for the masking-tape data. This discrepancy resulted from the static pressure calibration discussed in } \S 3.4.5.\)

\(^3\text{The Prandtl-Glauert transform is derived from linearization of the inviscid, steady potential flow equation. Its application is usually restricted below transonic flight regimes, } M_\infty < 0.7.\)
Figure 7.10. Ensemble lift and moment for select attached flow cases (masking-tape trip). $k \approx 0.05$, —— pitch-up, —— pitch-down, ——— steady

substantially more sensitive to $M_\infty$ than, for example, $C_{l_v}$, as the steady lift-curve-slope agreed fairly well with the transform over the entire Mach range of the experiment – see Figure B.9. It is possible that other mechanisms, aside from the non-linearities induced through the formation of shocks, play a significant role in the aerodynamic damping behavior at $M_\infty > 0.45$.

To begin the search for these mechanisms, Figure 7.10 examines the individual lift and moment coefficient plots for three attached flow cases at $M_\infty \approx 0.2, 0.4$, and 0.6. The lift is presented to indicate the severity of the stall, which, for attached flow, is negligible. However, close scrutiny reveals that there exists a lift defect (hysteresis) that increases with $M_\infty$. The area inclosed by the moment traces also increases with $M_\infty$, leading to the previously observed increase in $\Xi$. The three cases and their time-dependent damping characteristics found through the HT of the measured $C_m$ and $\alpha$ are shown in Figures 7.11 through 7.13.

All three cases remain stable for the entire pitch-cycle as exhibited in (d) of each
Figure 7.11. Time-resolved damping analysis using the DHT, RUN1857 (MT)

\[ \text{M}_\infty = 0.2, k = 0.052, \alpha = 5.2^\circ - 5.1^\circ \cos \omega t \]

Mach number had little effect on the amplitude or phase lag of the pitch-moment, \( A_{C_m}(t) \) or \( \psi(t) \), between the example cases at \( \text{M}_\infty = 0.2 \) and 0.4, as indicated by the polar diagram in (c) of each figure. \( A_{C_m}(t) \) and \( \psi(t) \) remain nearly constant over the pitch-cycle, such that the polar traces out only a small area. Similarly, \( \Xi(t) \), normalized by the theoretical damping, is flat and near \( 2\Xi_{\text{cycle}}/\pi k \) for all \( \omega t \). The damping increased substantially at \( \text{M}_\infty = 0.6 \), primarily due to a large increase in lag (\( \psi(t) \rightarrow -\pi/2 \)), and, to a lesser degree, an increase in \( A_{C_m}(t) \). Figure 7.14 explicitly highlights the
behavior of the average phase and average amplitude for increasing $M_{\infty}$ for $k \approx 0.025, 0.025, 0.075, \text{ and } 0.10$. The phase lag increased as $M_{\infty}$ increased for each targeted $k$ at a roughly consistent rate. This linear behavior between reduced frequency and phase lag for oscillating airfoils in transonic flow is discussed in the review by Tijdeman and Seebass[130]. Also, it is obvious that increasing reduced frequency has a stabilizing effect on the airloads. The amplitude of the pitch moment remained fairly constant as $M_{\infty}$ increases but increased rapidly with increased $k$. 

Figure 7.12. Time-resolved damping analysis using the DHT, RUN1905 (MT) 
$M_{\infty} = 0.4, k = 0.048, \alpha = 3.2^\circ - 5.3^\circ \cos \omega t$
The general behavior of the damping coefficient at $M_{\infty} = 0.6$ is also shown to be significantly more unsteady than the lower $M_{\infty}$ cases – compare Figure 7.13d to Figures 7.12d and 7.11d. The maximum variation in damping about the cycle integrated value, $\Delta \Xi = \| \Xi(t) - \Xi_{\text{cycle}} \|$ is shown in Figure 7.15. Trends in the damping variation are difficult to distinguish in the attached flow regime. Several cases exhibited large variation at $M_{\infty} = 0.6$.

Figure 7.16 highlights the behavior of the Bell airfoil without LE edge trip at $M_{\infty} =$
0.6. The removal of the trip increased the damping where the primary mechanism was an increase in the phase lag between the prescribed incidence and the pitch-moment. The placement or removal of the trip also caused significant differences in the temporal behavior of $\Xi(t)$, suggesting how sensitive the airloads are to LE modifications at large
Figure 7.15. Absolute magnitude of the difference in time-resolved damping from the cyclic damping coefficient for attached flow (MT), $\alpha_1 \approx 5^\circ$, $\alpha_{\text{max}} - \alpha_{\text{ss}} \in (-3^\circ, -1^\circ)$

- $\diamondsuit k \approx 0.025$, $\bigcirc k \approx 0.05$, $\square k \approx 0.075$, $\triangle k \approx 0.10$

$M_{\infty}$, even in attached flow regimes. Figures 7.17 and 7.18 show $M_{\infty} = 0.6$ trials with the P320 and P400 grit LE trips.

7.5.1.1 Negative damping prior to stall onset

Figure 7.19 illustrates the minimum damping found from $\Xi(t)$. Through $M_{\infty} = 0.55$, the damping remains positive throughout the cycle. However, at $M_{\infty} = 0.6$, the nonlinearities introduced by the appearance of shocks on the airfoil upper surface caused local portions of the pitch cycle to experience negative damping.

These temporally unstable variations were masked by the overall, positive net cycle damping. These cases were confined to low-reduced frequency, $k \approx 0.025$ or 0.05 at $M_{\infty} = 0.6$. At $M_{\infty} \approx 0.55$, the damping approached zero, but remained positive throughout the cycle. Figure 7.20 highlights several $\alpha_0$ cases that exhibited slight nega-
Figure 7.16. Time-resolved damping analysis using the DHT, RUN2802 (B)

\[ M_{\infty} = 0.6, \, k = 0.048, \, \alpha = 1.4^\circ - 4.9^\circ \cos \omega t \]

tive damping through a portion of the pitch-cycle. These drops in damping consistently appeared near \( \omega t \approx 4.2 \) as \( \alpha \) decreased through \( \alpha_0 \). The earlier decrease in damping near \( \omega t \approx 2.1 \) grew more unstable and longer in duration as \( \alpha_0 \) increased. The source of instability is examined in Figure 7.21, which shows the damping in relation to \( \alpha \), \( C_{m_{v/4}} \) and the upper-surface pressure coefficients. The drop in damping begins as the shock’s motion aft over the chord halts. This induces a thick boundary layer following the shock that increases the unsteadiness in the pressure time-series downstream of the...
shock. The flow forward of the shock remains attached, causing a nose-up and unstable pitch-moment.

7.5.1.2 On the mechanism of damping in dynamic stall

The attached flow data presents an opportunity to examine the source of damping, which, from Eq. 7.46, is assumed to stem from a combination of viscous and hysteretic mechanisms. First, it is important to highlight some general observations of damped,
linear systems. A viscously damped system undergoing harmonic excitation can be described by the equation\[38, 47, 48, 131],

\[
\dot{x} + C \dot{x} + Kx = A_1 e^{j\omega t},
\]

Figure 7.18. Time-resolved damping analysis using the DHT, RUN2977 (P400) $M_x = 0.6, k = 0.049, \alpha = 1.5^\circ - 4.8^\circ \cos \omega t$
Figure 7.19. Minimum damping (MT), \( \alpha_1 \approx 5^\circ, \alpha_{\text{max}} - \alpha_{\text{ss}} \in (-3^\circ, -1^\circ) \)
\[ \circ k \approx 0.025 \quad \odot k \approx 0.05 \quad \square k \approx 0.075 \quad \triangle k \approx 0.10 \]

where \( x \) is a displacement and \( m, C, \) and \( K \) are the the mass, viscous damping force, and spring force of the system. It can be shown\(^{131}\) that

\[
W_{\text{cycle}} \propto C \| x \|^2 \cdot \omega, \tag{7.67}
\]

such that the work-per-cycle is proportional to the square of the output amplitude and linearly proportional to the oscillation frequency. This is the dependence found for the aerodynamic work-per-cycle, \( C_w \), in §7.2 (Eq. 7.35) based on Theodorsen’s model of \( C_{m_{\alpha/4}} \).

A system that exhibits hysteretic or structural damping fails to display a dependence on motion rate, \( \dot{x} \). The EOM for such a system is\(^{51}\)

\[
m\ddot{x} + K(1 + i\delta)x = A_1 e^{i\omega t}, \tag{7.68}
\]
Figure 7.20. Local regions of negative damping due to shock development on the airfoil upper surface. RUN2251, 2253, 2255, 2257 (top to bottom).

$M_\infty = 0.60, \alpha_0 = 2.7^\circ, 3.1^\circ, 3.6^\circ, 4.0^\circ, \alpha_1 \approx 4.5^\circ, k = 0.023$. -- $2\Xi_{cycle}/\pi k$
Figure 7.21. Upper-surface pressures, RUN2257, $\alpha = 4.0^\circ + 4.4^\circ \cos \omega t$.

where $\delta$ is a stiffness ratio. As shown in Eq. 7.68, structural damping stems from an elastic or spring force out of phase with the oscillation – identical to the contribution of
$M_{3a}$ in § 7.2 or $Im(\kappa^*)$ in Eq. 7.46. The work-per-cycle is then:[131]

$$W_{\text{cycle}} \propto \delta K \|x\|^2,$$

(7.69)

and is independent of forcing frequency. Examination of Figure 7.22 shows a clear dependence of the net cycle damping on $k$; this should not come as a surprise to those familiar with dynamic stall. What is interesting, is that this frequency dependence is consistent for each Mach test point and suggests that viscous damping is the predominant mechanism for the observed hysteresis, independent of compressibility or Reynolds number effects. One could make an engineering assumption and eliminate the out-of-phase component of $\kappa^*$ and relate $\Xi(t)$ directly to the viscous damping coefficient, $h_0(t)$, in Eq. 7.55. However, for low reduced frequencies near the stall angle, it is well known that the aerodynamic forces exhibit hysteresis. This hysteresis exists in the absence of motion and can therefore only be characterized as hysteretic in nature.

7.6 Mach effects on light and deep dynamic stall damping

The pitch-moment and other aerodynamic forces (or pressures) diverge from linear behavior once the oscillation nears or encompasses the stall angle. The mechanism for this is a combination of stall delay corresponding to the unsteady motion and the formation and eruption of the DSV. This section focuses on the behavior of the damping coefficient, both its average value, $\Xi_{\text{cycle}}$, and its time-dependent evolution, $\Xi(t)$. $\Xi$ reflects the stability of the pitch moment, which, in turn, is manifestation of the developing pressure field. An unstable pitch-moment, $\Xi < 0$, implies unstable pressure loading, and can be described as a necessary, aerodynamic condition to excite stall flutter of an elastic body.

Figure 7.23 highlights the typical behavior of the aerodynamic damping coefficient
as the airfoil trajectory is swept through stall (this sweep was accomplished by incremental increases to \( \alpha_0 \) holding \( \alpha_1 \) fixed) at six freestream Mach numbers. The stall angles for each Mach number are indicated by the colored arrows on Figure 7.23. The curves through the data points are set for visual reference only. Static data was not gathered for \( M_\infty \approx 0.55 \). Fig. 7.24 singles out five pitching-moments from Fig. 7.23 and is color coded in accordance with the Fig. 7.23. The moments are offset by \( C_{m_{1/4}} = +0.1 \). These pitching-moments were selected to maintain similar dynamic stall penetration at their respective Mach numbers.

From Fig. 7.23, only the low-speed, \( M_\infty = 0.2 \) sweep resulted in negative cycle damping. The minimum damping occurred as the maximum angle, \( \alpha_{\text{max}} = \alpha_0 + \alpha_1 \), was approximately 2\(^\circ\)-3\(^\circ\) past \( \alpha_{ss} \). This level of stall penetration classically falls into the light dynamic stall regime described by McCroskey[104]. As the maximum inci-

\[ \Xi_{\text{cycle}} \times 10 \]

\[ \begin{array}{cccc}
0.0 & 0.5 & 1.0 & 1.5 \\
0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 \\
\end{array} \]

\( k \) increasing

Figure 7.22. Mach number and reduced frequency effects on cycle damping in attached flow (MT), \( \alpha_1 \approx 5^\circ, \alpha_{\text{max}} - \alpha_{ss} \in (-3^\circ, -1^\circ) \)

\( \odot k \approx 0.025 \odot k \approx 0.05, \square k \approx 0.075, \triangle k \approx 0.10 \)
Figure 7.23. Effect of maximum incidence on the cycle integrated aerodynamic damping coefficient (MT), $k \approx 0.05, \alpha_1 \approx 5^\circ$.

Figure 7.24. Ensemble averaged pitching-moment for light stall, $\alpha_{ds} - \alpha_{ss} \approx 2^\circ$, at $M_\infty = 0.2, 0.3, 0.4, 0.5, 0.6, k \approx 0.05, \alpha_1 \approx 5^\circ$. Each trace in offset by $+0.1 C_{m_{c/4}}$. Solid lines: pitch-up, dashed lines, pitch-down.
dence increased, the cycle damping became more positive, which is a ramification of the separated flow field[24, 51].

The $M_\infty = 0.3 - 0.55$ data lacked minima in the damping. Instead, the cycle-integrated damping remained nearly constant with increasing Mach number until $\alpha_{max}$ exceeded $\alpha_{ss}$, where $\Xi_{cycle}$ then increased. The cycle-integrated damping for the $M_\infty = 0.6$ case remained positive throughout the angle of attack sweep, but it showed a small region of decreased damping near $\alpha_{ss}$, at $\approx 10.5^\circ$. Similar damping and Mach trends are reported by Liiva[91].

It is of interest to determine the cause of the increased damping encountered in moving from $M_\infty = 0.2$ to 0.6. To examine this in more detail, we return to Fig. 7.24 and, at first, consider the cycle-averaged perspective. At $M_\infty = 0.2$ a clockwise-oriented region within the pitching-moment exists for $\pm 2^\circ$ from $\alpha_{ss}$. This region of hysteresis in the pitch moment is indicative of the unstable damping coefficient observed in Fig. 7.23. Inspection of the pitch-moment cycle at $M_\infty = 0.3$, colored in red, shows no such crossover. The same can be said of the pitching-moment cycles at $M_\infty = 0.4$ and 0.5. A small, clockwise-oriented region within the pitch moment cycle at $M_\infty = 0.6$ is observed, but it is much smaller than the one exhibited at $M_\infty = 0.2$. From this point of view, one would conclude that only a stabilizing transfer of energy from the airfoil to the airstream occurred in these cases for $M_\infty = 0.3, 0.4,$ and 0.5. However at $M_\infty = 0.2$, the reverse occurred with a large destabilizing transfer of energy from the airstream to the airfoil. A similar, but smaller destabilizing energy transfer occurred at $M_\infty = 0.6$.

As in the attached flow study, the cycle-averaged perspective conceals important physics that lead to localized, unstable damping during dynamic stall. Furthermore, the role of the dynamic stall vortex in the production of positive and negative damping is
greatly misinterpreted without a time-resolved damping analysis. To better understand
how the aerodynamic damping evolves through dynamic stall, the five pitch-moment
cycles in Fig. 7.24 were analyzed using the discrete Hilbert transform. Fig. 7.25 il-
lustrates the time-resolved damping results for the $M_\infty = 0.2, 0.3$ and $0.6$ pitching-
moments. The left column of Fig. 7.25 shows the polar representation of the aerody-
namic damping for the pitch cycle. The figures in the right column of Fig. 7.25 simulta-
neously indicate the time-resolved damping, the ensemble pitching-moment, $C_{m_{\alpha/4}}$, and
the position of the dynamic stall vortex. The position of the dynamic stall vortex was
determined using the approach of Lorber and Carta[95] whereby its position is corre-
lated with the location of the minimum pressure on the suction side of the airfoil, as
in Chapter 5. Unlike Chapter D, $\tau$ is transformed to $\omega t$ to align with the damping and
pitch moment. Through this figure, the effect of the stall vortex on the damping and
pitch-moment can be described. Figure 7.26 shows the decomposition of $C_{m_{\alpha/4}}$ for the
three cases as a function of incidence (left column) and $\omega t$ (right column).

At $M_\infty = 0.2$, in Fig. 7.25a, the phase, $\psi(t)$, at $\alpha_{min}$ indicates that the aerodynamic
damping is neutrally stable. As $\alpha$ increases, $\psi(t)$ moves into the second quadrant. This
phase reaches a maximum value of $\psi = 1.92$ at $\omega t = 2.72$. As the phase decreases,
the pitch-moment amplitude, $A_{C_{m}}(t)$, increases such that the polar representation of the
damping sweeps out a large area. This causes the damping, $\Xi(t)$, whose sign is the
same as $\psi(t)$, to be negative for nearly the entire pitch-up portion of the cycle.

The phase lead in this instance is a consequence of the positive pitch-rate induced
delay of stall, and accompanying alleviation of the upper-surface adverse pressure gra-
dient. This unsteady motion effect allows the pitching-moment to remain almost con-
stant as $\alpha$ increases. Subsequently, from Fig. 7.25b, a near linear decrease in aerody-
namic damping occurs as $\alpha$ increases through the cycle, resulting in energy exchange
from the airstream to the airfoil.

The damping diverges from this linear decline, increasing the production of negative damping, as the initial signature of the stall vortex appears at \( x = 0.006 \) at a time \( \omega t = 2.36 \). As the favorable pressure gradient feeds the dynamic stall vortex, it slowly migrates over the chord. Shortly thereafter, the stall vortex reaches its terminal velocity as discussed in Chapter 5. The damping, \( \Xi(t) \), continues to decrease until the dynamic stall vortex arrives at \( x = 0.25 \), at a time \( \omega t = 2.49 \). At this instant, the maximum negative damping of \( \frac{\Xi(t)}{\sigma} = -13.89 \) occurs. The following rise in damping coincides with a pitch moment “stall” at which point the position of the dynamic vortex is aft of \( x = 0.25 \).

As the dynamic stall vortex convects downstream, the damping increases as the phase between the pitch-moment and instantaneous angle of attack recover, namely \( \psi \to -\pi \). For \( M_{x} = 0.2 \), the negative damping persists into the pitch-down portion of the cycle as the dynamic stall vortex is still inducing a nose-down moment that combines with the nose-down motion of the airfoil. The production of positive damping does not start until \( \omega t = 3.26 \), where the upper-surface flowfield becomes completely separated.

From these observations, the source of negative aerodynamic damping stems from the delay of stall due to the unsteady pitching motion. Negative damping is augmented through the growth and eruption of the dynamic stall vortex. It reduces as the dynamic stall vortex convects past the quarter-chord location, and the pitch-moment momentarily asymptotes (stalls). Thus the convection of the dynamic stall vortex past the quarter-chord location is stabilizing. Once the dynamic stall vortex passes into the airfoil wake, the fully separated flow field that follows initiates positive aerodynamic damping. This damping reaches a maximum as the flow fully reattaches.
Figure 7.25. Time-resolved damping analysis of light dynamic stall. The left column shows the polar representation of the time-resolved aerodynamic damping. The arrows indicate increasing time. The right column shows the time-resolved damping, pitch-moment, and position of the DSV. (a)&(b) \( M_\infty = 0.2, k = 0.051, \alpha = 9.8^\circ - 4.9^\circ \cos \omega t \). (c)&(d) \( M_\infty = 0.3, k = 0.051, \alpha = 10.7^\circ - 4.9^\circ \cos \omega t \). (e)&(f) \( M_\infty = 0.6, k = 0.046, \alpha = 5.4^\circ - 5.1^\circ \cos \omega t \).
Figure 7.26. Decomposition of the ensemble pitch-moment using the DHT for light dynamic stall cases. Left column: The ensemble pitch-moment, DHT, and in-phase and quadrature representations of $C_m(\alpha)$. Right column: The ensemble pitch-moment, DHT, and in-phase and quadrature representations of $C_m(\omega t)$. 
(a) & (b) RUN1953, $M_\infty = 0.2$, $k = 0.051$, $\alpha = 9.8^\circ - 4.9^\circ \cos \omega t$. 
(c) & (d) RUN2000, $M_\infty = 0.3$, $k = 0.051$, $\alpha = 10.7^\circ - 4.9^\circ \cos \omega t$. 
(e) & (f) RUN2264, $M_\infty = 0.6$, $k = 0.046$, $\alpha = 5.4^\circ - 5.1^\circ \cos \omega t$. 

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The large, negative damping at $M_{\infty} = 0.2$ does not come as a surprise, as the cycle-integrated damping, $\Xi_{\text{cycle}}$, clearly indicated unstable airloads in Fig. 7.23. However, the ratio of peak, negative damping, $\Xi_{\text{min}}$, to $\Xi_{\text{cycle}}$ is 7.73!

However, examining the time-resolved damping shown in Figs. 7.25c and 7.25d for $M_{\infty} = 0.3$, tells a much different story than the cycle-integrated statistics, which indicate that the aerodynamic damping is always positive. In fact, during the pitch-up stroke, we observe the same type of negative damping that is supplemented by the dynamic stall vortex, as was observed at $M_{\infty} = 0.2$ (albeit small in magnitude). In contrast to the $M_{\infty} = 0.2$ case, the time that the dynamic stall vortex resides over the airfoil is greatly reduced due to the appearance of local, supersonic flow. This was discussed in terms of a gestation period, $\delta \tau_{g}$, in Chapter 5. The downstream convection of the dynamic stall vortex ultimately results in positive damping of the same magnitude that was observed at $M_{\infty} = 0.2$.

As in the attached flow study, it remains that increased $M_{\infty}$ promotes a phase lag in the aerodynamic response of the pitching motion, and reduced aerodynamic load coefficients. This decreases the phase difference, $\psi(t)$, and the amplitude of the time-resolved damping, $A_{C_m}(t)$. However, at $M_{\infty} = 0.4$ and $M_{\infty} = 0.5$, large, negative damping prevails on the airfoil’s pitch-up portion of the cycle.

The appearance of strong shocks drastically alters the aerodynamic damping behavior, in a manner that is similar to the attached flow observations discussed previously. Figs. 7.28c and 7.28e show the Hilbert transform analysis for the pitching airfoil at $M_{\infty} = 0.6$. The peak phase difference, $\psi(t)$ is larger than that observed at $M_{\infty} = 0.3$ which then correlates with a larger magnitude of $\Xi_{\text{min}}$. As a result of the retreat in the peak suction, the first signature of the dynamic stall vortex is detected $x/c = 0.134$, which is immediately downstream of the shock front found from examination of the
surface pressures.

The surface pressure signature of the dynamic stall vortex changes radically in the aftermath of the shock induced dynamic stall. This can be seen by comparing the position of the stall vortex in Fig. 7.28e to that in Fig. 7.25d, or Fig. 7.25b. The gradual convection of the dynamic stall vortex following its inception is supplanted by a near constant speed, low-pressure ‘gust’ that moves downstream, as discussed in Chapter 5. This gust failed to induce the same positive aerodynamic damping that the dynamic stall vortex produced in the lower Mach number cases. As before, the peak positive damping contributions occur as the flow reattaches on the upper surface.

The observations above lead to the following conclusions regarding the influence of compressibility on $\Xi(t)$:

1. As seen in Eq. 7.59, $\Xi(t)$ depends on the instantaneous amplitude of $C_{m_{\alpha}}$. Increased $M_\infty$ decreases the non-dimensional aerodynamic loads – see Figure C.19 – and thereby decreases the magnitude of $\Xi(t)$.

2. The large phase leads at $M_\infty = 0.2$ diminish as the freestream speed increases. This is a consequence of a more gradual moment stall. This observation couples with the attached flow results that indicate $M_\infty$ increases the phase lag, $\psi \rightarrow -\pi/2$. The increased lag agrees with the work included in the review by Tijdeman and Seebass[130].

3. The effect of the stall vortex on the pitch-moment is less severe as $M_\infty$ increases.

In regard to the last item, the weakening of the stall vortex with increasing $M_\infty$ is briefly discussed by Lorber and Carta[95] through measurement of the vortex induced pressure. Similar observations regarding the stall vortex strength are noted here and expounded in Appendix D in terms of the surface vorticity flux. This observation is also supported by Figure 5.8 in which the stall vortex’s growth correlates to changes in $\alpha_{max}$ at $M_\infty = 0.2$ but fails to show the same correlation at $M_\infty \geq 0.3$.

It is important to consider that the stall vortex has an unstable impact on the pitch-moment only as it first develops and resides near the leading-edge. This coincides
with the rapid growth of the suction peak, which, in turn, favorably impacts the lift and the pitch-moment. Conversely, the rearward movement of the stall vortex and its propagation into the airfoil wake is favorable in terms of aerodynamic damping but causes moment stall and then lift stall. Obviously, these competing mechanisms make for a challenging control problem.

7.6.1 $\Xi_{\text{min}}$ vs. $\Xi_{\text{cycle}}$

For rotor operations, $\Xi_{\text{min}}$ is a more critical quantity than $\Xi_{\text{cycle}}$. Local, negative aerodynamic damping, especially of the magnitude presented in this chapter, may account for the stall flutter related divergence found during high-speed forward flight or high g-force maneuvers[8]. Figure 7.27 compares the two aerodynamic damping quantities for the different dynamic stall regimes at the five primary Mach number test points. At $\alpha_{\text{max}} < \alpha_{\text{ss}}$ the difference between $\Xi_{\text{min}}$ and $\Xi_{\text{cycle}}$ is negligible. However, $\Xi_{\text{min}}$ is significantly less than $\Xi_{\text{cycle}}$ past $\alpha_{\text{ss}}$. For $M_{\infty} > 0.2$, the minimum damping remained nearly equal. In addition, although the cycle damping at $M_{\infty} = 0.6$ is larger than the cycle-damping observed at the lower Mach numbers, the minimum damping is second only to the damping observed at $M_{\infty} = 0.2$. It is also important to note that the onset of instability occurs at smaller angles of attack as $M_{\infty}$ increases, which is a steady state compressibility effect.

7.6.2 Shock induced light dynamic stall damping

At $M_{\infty} = 0.6$, strong shocks develop as the airfoil increases $\alpha$. These shocks were found to destabilize the pitch-moment in attached flow oscillations. The reduced damping found at $\alpha_{\text{max}} = 10.5^\circ$ at $M_{\infty} = 0.60$ in Figure 7.23 is investigated in Figure 7.28 to determine the outcome of shock induced, light dynamic stall on the time-resolved
damping characteristics. Figure 7.28a shows a cross-over of the pitch moment, a feature seldom found in the test data at $M_\infty \approx 0.3, 0.4, \text{ or } 0.5$. From the phasor, Figure 7.28c, the maximum phase lead is $\psi_{\text{max}} \approx 2\pi/3$, significantly more unstable than the cases studied here at $M_\infty = 0.42 \text{ or } 0.49$. Figure 7.29 compares the damping evolution to the surface pressure behavior. The damping decreases due to the unsteady delay in stall; it becomes negative at $\omega t = 1.26$ but diverges at $\omega t = 1.85$ as the rearward propagation of the normal shock ends and the shock becomes stationary ($x \in (0.134, 0.191)$). Unlike

Figure 7.27. Minimum time-resolved damping compared to the damping-per-cycle for $\alpha_1 \approx 5^\circ \text{ and } M_\infty = (a) 0.2, (b) 0.3, (c) 0.4, (d) 0.5, (e) 0.6$. Open symbols $\Xi_{\text{cycle}}$, filled symbols $\Xi_{\text{min}}$. Arrows point to $\alpha_{ss}$.
at $M_\infty = 0.2$, the change in slope of the damping is not associated with the appearance of the stall vortex. This destabilizing mechanism associated with the shock was also observed under attached flow conditions. The decrease in aerodynamic damping continues until the stall induced forward moving shock reaches the LE and dissipates at $\omega t = 2.48$. At this point the damping increases and remains neutrally stable until $\omega t = 3.34$ when it becomes positive for the first time in the cycle.

7.6.3 Effect of amplitude on $\Xi_{cycle}$ and $\Xi(t)$

Amplitudes of $\alpha_1 \approx 5^\circ$ and $8^\circ$ were the primary oscillation amplitudes of the test matrix. Figure 7.30 highlights the damping at various $M_\infty$ with a prescribed amplitude of $\alpha_1 \approx 8^\circ$. As in Figure 7.23 for $\alpha_1 \approx 5^\circ$, only the $M_\infty = 0.2$ test data exhibits $\Xi_{cycle} < 0$. The minimum damping at $M_\infty = 0.2$, $\alpha_1 \approx 8^\circ$ occurs at $\alpha_{max} = 16^\circ$. This also is similar to the damping for $\alpha_1 \approx 5^\circ$. The increased forcing amplitude, however, generates a significantly more positive net damping coefficient. Figure 7.31 shows the time-resolved damping behavior for RUN2313, $M_\infty = 0.2$, $\alpha_1 = 8.0^\circ$ and should be compared to the plots for RUN1953 in Figures 7.25 and 7.26 for $M_\infty = 0.19$, $\alpha_1 = 5^\circ$. Each of the moments, Figure 7.26a and Figure 7.31a, shows a large region of negative damping. The amplitudes and temporal behavior of the quadrature component and in-phase component of the pitch moment are similar. As such, the phase of the two motions also varies through approximately the same angle, $\psi \in [2\pi/3, 7\pi/6]$. The difference in amplitude, $A_{C_\alpha}(t)/\alpha_1$, is the primary source for the variation in damping. Small $\alpha_1$ magnifies $\Xi_{cycle}$, as determined by Carta and Lorber[23, 94], and also found here for time-resolved damping, $\Xi(t)$. 

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Figure 7.28. Time-resolved damping analysis using the DHT, RUN2264 (MT)

\[ M_x = 0.60, \ k = 0.046, \ \alpha = 5.4^\circ - 5.1^\circ \cos \omega t \]
Figure 7.29. Upper-surface pressures, RUN2264, $M_\infty = 0.60$, $k = 0.046$

\[
\alpha = 5.4^\circ - 5.1^\circ \cos \omega t
\]

- $M_\infty < 1.0$
- $M_\infty > 1.0$
- $M_\infty > 1.31$

7.6.4 Effect of reduced frequency on $\Xi_{cycle}$ and $\Xi(t)$

Figure 7.32 displays $\Xi_{cycle}$ for each of the $M_\infty$ test points of the test matrix ($\alpha_1 = 5^\circ$) as a function of maximum incidence. The primary reduced frequencies investigated for the MT trip were $k \approx 0.05$ and 0.075. Several test points, although more sparsely
Figure 7.30. Cycle integrated aerodynamic damping coefficient (MT),

\[ \frac{2\Xi_{\text{cycle}}}{\pi k} \]

spent, were acquired at \( k \approx 0.025 \) and 0.10. The lines on the figure are for reference only.

At \( M_\infty = 0.2 \), increases to the reduced frequency caused increased cycle damping. The opposite trend is observed for \( M_\infty \in [0.3, 0.5] \). No observations are made in regard to \( M_\infty = 0.6 \) due to the limited range of \( k \) investigated.

Figures 7.33 and 7.34 show the time-resolved behavior of the damping coefficient for \( k = 0.026 \) and \( k = 0.11 \) at \( M_\infty = 0.19 \) to investigate the influence of \( k \) on local damping. At \( k = 0.026 \), the stall vortex signature is weak, as seen in Figure 7.33e; consequently, local minima in the upper-surface pressure were only detected up to \( x = 0.6 \).

It is obvious from Figure 7.33d and Figure 7.34d that, again, the net damping conceals important characteristics of the local damping. The minimum in damping for \( k = 0.026 \) was \( \Xi(t) = -1.0 \) at \( \omega t = 2.37 \) and \( \Xi(t) = -1.28 \) at \( \omega t = 2.95 \).
Figure 7.31. Time-resolved damping analysis using the DHT, RUN2313 (MT)

$M_{\infty} = 0.20, k = 0.049, \alpha = 8.15^\circ - 8.03^\circ \cos \omega t$
$k = 0.11$. Thus, the larger $k$ promotes larger unstable local damping; it also promotes a more rapid return to positive damping, reaching its maximum value ($\Xi(t) = 0.57$) at $\omega t = 3.6$. $k = 0.026$ remains neutrally damped for a period of $\omega t \in (2.7, 4.1)$ before the pitch-moment begins to lag the forcing incidence. The quick return to positive damping for $k = 0.11$ is the primary reason $\Xi_{\text{cycle}}$ is more positive for $k = 0.11$ than $k = 0.026$ despite larger phase-leads and $A(t)/\alpha_1$.

Figure 7.35 and Figure 7.36 show the HT of the pitch-moments to identify the cause of the different behavior with respect to $k$ at $M_x = 0.4$. At $k = .048$ (RUN2121, Figure 7.35), the phase lead incurred on the pitch-up stroke is small. Mach number related reduction in the pitch-moment causes a similar reduction in $A(t)$, and, consequently, the magnitude of $\Xi(t)$. From Figure 7.35e, the damping diverges only slightly as the stall vortex is released from the surface. As the moment fails to break abruptly, the phase between $\alpha$ and $C_{m_{\alpha/4}}$ remains small (but unstable). At $k = 0.096$ (RUN1913), a cross-over of the pitch moment is seen in Figure 7.36a. The phase lead on the up-stroke is greater than that at $k = 0.049$, whereas $A(t)$ remains small – see Figure 7.36c. The delay in separation due to the larger reduced frequency provides a longer duration of negative damping. For $k = 0.048$, the damping becomes positive at $\omega t = 2.5$; for $k = 0.096$, the damping remains negative until the down-stroke at $\omega t = 3.4$. The different time scales of positive and negative damping account for the more negative damping found at larger $k$.

7.6.5 LE trip effects on torsional damping coefficient

In terms of the cycle damping, interesting features in the damping behavior occurred due to the trips placed at the leading-edge. In general, the trips slow the LE pressure development, causing more stable airloads at the cost of a reduced $C_{p_{\text{mean}}}$ and,
Figure 7.32. Reduced frequency effects on cycle damping coefficient, $\alpha \approx 5^\circ$

A corresponding reduction in aerodynamic loading. Figure 7.37 and 7.38 show $\Xi_{cycle}$ as computed from incremental sweeps of $\alpha_0$ ($\alpha_1 \approx 5^\circ$).

At low-speed, $M_{\infty} \leq 0.3$, the trips significantly increase the damping through the stall cycle, as shown in Figure 7.37a. Figure 7.39 highlights the temporal evolution
Figure 7.33. Time-resolved damping analysis using the DHT, RUN1868 (MT)

\[ M_x = 0.19, \ k = 0.026, \ \alpha = 11.2^\circ - 4.9^\circ \cos \omega t \]
Figure 7.34. Time-resolved damping analysis using the DHT, RUN1871 (MT)

\[ M_x = 0.19, \ k = 0.11, \ alpha = 11.3^\circ - 5.3^\circ \cos \omega t \]
(a) The ensemble pitch-moment, DHT, and in-phase and quadrature representations of $C_m(\alpha)$

(b) The ensemble pitch-moment, DHT, and in-phase and quadrature representations as of $C_m(\omega t)$.

(c) Phase and amplitude of the damping coefficient

(d) Time-resolved damping, $2\Xi_c/\pi k$

(e) Damping and pitch moment relation to stall vortex

Figure 7.35. Time-resolved damping analysis using the DHT, RUN2121 (MT)

$M_c = 0.42, k = 0.048, \alpha = 8.2^\circ - 5.0^\circ \cos \omega t$
Figure 7.36. Time-resolved damping analysis using the DHT, RUN1913 (MT)

\( M_x = 0.42, k = 0.096, \alpha = 6.0^\circ - 6.7^\circ \cos \omega t \)
Figure 7.37. Net cycle damping and minimum pressure comparison for LE trips, $M_\infty = 0.2 - 0.4$. $C_p^{\text{sonic}}$
of the damping for similar $\alpha_{\text{max}}$, comparing the four different LE modifications. From Figure 7.39a and 7.39b, the MT is the most unstable, possessing large phase leads and damping amplitude. The baseline, unmodified LE showcases similar instability; however, the cyclic damping is considerably larger due to a slightly diminished level of negative damping on the upstroke and larger positive damping following stall. The differences between the MT and B are attributed to the presence of the laminar separation bubble. The P320 grit and P400 grit produced significantly more stable airloads, mitigating the negative damping contribution on the airfoil’s upstroke while maintaining
the positive damping contribution following stall. The grits also promoted stall earlier in the cycle, limiting the duration in which the airfoil was negatively damped.

As $M_\infty$ increases, the cyclic damping found between the four LE geometries con-
verges, as specifically highlighted by the $M_{\infty} = 0.4$ data shown in Figure 7.37e. Figure 7.40 highlights the time-resolved behavior of the damping at $M_{\infty} = 0.4$, $\alpha_{\text{max}} \approx 11^\circ$. Note, the range of each subfigure in Figure 7.40 was adjusted as the magnitude of the damping is significantly smaller than that exhibited at $M_{\infty} = 0.2$ (Figure 7.39). The MT case, RUN2106, exhibits positive damping throughout the cycle. The baseline, P320 trip, and P400 trip each exhibit similar peak negative damping and phase leads/lags. A larger, local positive damping region on the downstroke was exhibited by the baseline airfoil. This leads to the increased $\Xi_{\text{cycle}}$ compared to the grit trips. Similar features are found at $M_{\infty} = 0.5$ (not shown). At $M_{\infty} = 0.6$, the cycle damping of the two trip cases is significantly reduced from that of the B or MT near $\alpha_{\text{max}} = 9^\circ$. Figure 7.41 compares the time-resolved damping characteristics of the trips and baseline airfoil. The baseline airfoil exhibits stable airloads through the pitch-cycle. The MT, however, shows a small, negatively damped region. This minimum occurs at $\omega t = 2.67$ with a damping value of $\Xi = -0.084$. As before, the negative damping stems from the stabilization of a normal shock on the airfoil chord. This prompts an abrupt rise in the pitch-moment and consequent phase lead. The P320 trip caused significant leads in the pitch-moment behavior, incurring an earlier minimum damping, with peak $\Xi(\omega t = 2.60) = -0.69$. The P320 grit also produced large, positive damping on the downstroke. The mechanism for the observed damping behavior is similar to that observed by the MT modification. The 2D spanwise element of the trip tapes, each of which extends out of boundary layer at $M_{\infty} = 0.6$, causes a large enough pressure drop at the rear of the trip to force the developing $\lambda$-shock to remain at this location. The fixed shock, captured by the focusing schlieren system discussed in Chapter 6, results in significant negative damping.
<table>
<thead>
<tr>
<th>RUN#</th>
<th>LE Mod.</th>
<th>$k$</th>
<th>$\alpha_1$</th>
<th>$\alpha_0$</th>
<th>$\Xi_{cycle}$</th>
</tr>
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<tr>
<td>2106</td>
<td>MT</td>
<td>0.047</td>
<td>5.09°</td>
<td>5.93°</td>
<td>0.083</td>
</tr>
<tr>
<td>2764</td>
<td>B</td>
<td>0.050</td>
<td>5.08°</td>
<td>5.79°</td>
<td>0.084</td>
</tr>
<tr>
<td>2582</td>
<td>P320</td>
<td>0.050</td>
<td>5.30°</td>
<td>6.00°</td>
<td>0.062</td>
</tr>
<tr>
<td>2899</td>
<td>P400</td>
<td>0.050</td>
<td>5.02°</td>
<td>5.94°</td>
<td>0.064</td>
</tr>
</tbody>
</table>

(a) Phase and amplitude of the damping coefficient

(b) Temporal damping

Figure 7.40. Comparison of phase, amplitude, and damping response due to Mach number, $M_\infty = 0.4$
<table>
<thead>
<tr>
<th>RUN#</th>
<th>LE Mod.</th>
<th>$k$</th>
<th>$\alpha_1$</th>
<th>$\alpha_0$</th>
<th>$\Xi_{cycle}$</th>
</tr>
</thead>
<tbody>
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<td>3.72°</td>
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<tr>
<td>2814</td>
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<td>0.049</td>
<td>5.00°</td>
<td>3.94°</td>
<td>0.126</td>
</tr>
<tr>
<td>2949</td>
<td>P320</td>
<td>0.050</td>
<td>5.21°</td>
<td>3.85°</td>
<td>0.058</td>
</tr>
<tr>
<td>2679</td>
<td>P400</td>
<td>0.049</td>
<td>4.92°</td>
<td>4.00°</td>
<td>0.074</td>
</tr>
</tbody>
</table>

(a) Phase and amplitude of the damping coefficient

(b) Temporal damping

Figure 7.41. Comparison of phase, amplitude, and damping response due to Mach number, $M_\infty = 0.6$

7.7 Conclusions

This chapter represents an extensive effort to study the fundamentals of incipient stall flutter through the aerodynamic damping coefficient. Carta and Neibanck’s[24]
original formulation of $\Xi_{\text{cycle}}$ is improved upon and pushed into the time-domain via the (discrete) Hilbert transform. The time-resolved damping analysis provides significantly more information as to the physics of unstable loading on oscillating airfoils. Important results and observations emanating from this research include the following:

1. In attached flow where shocks failed to form on the airfoil upper surface, the time-resolved damping remained stable throughout the pitch cycle. $M_\infty$ increases contributed to stability in pitch. The primary mechanism of this improved stability owes itself to Mach induced phase lags.

2. Attached flow aerodynamic damping can be modeled by the Prandtl-Glauert rule up to the development of shocks on the airfoil upper surface.

3. The mechanism of aerodynamic damping was found to correlate to viscous damping. This observation was made through a study of the attached flow damping behavior and its dependence on reduced frequency, $k$. This behavior showed no dependence on Mach number.

4. Low reduced frequency, $k \approx 0.025$, and $M_\infty = 0.6$ conditions were shown to cause negative transients in damping under attached flow conditions. The mechanism for this instability was found to correlate to the duration of time when the $\lambda -$ shock’s front stopped proceeding aft over the airfoil’s upper surface. This same phenomenon was present in both the attached flow cases and through dynamic stall. For a rotor-blade, the unstable energy transfer dictated by these results may be a concern at high freestream Mach number due to the large values of $q$.

5. The dynamic stall vortex serves to augment negative damping so long as it remains forward of $x = c/4$. Its aft travel enforces moment stall and a decrease in the phase difference of $\alpha$ and $C_{m_{c/4}}$. This causes the damping to increase (more stable).

6. The lag in flow reattachment following separation in light and deep dynamic stall is stabilizing.

7. Increases to the oscillation amplitude promote stable airloads but at the expense of large, peak loads if stall occurs.

8. In the absence of a strong, stall vortex and rapid moment stall break, reduced frequency tends to increase negative damping by delaying stall and inducing a phase lead between the pitch-moment and $\alpha$. At lower Mach, increased $k$ causes large, negative damping over a short period but also promotes a large, positive
source of damping soon after the airfoil initiates the down-stroke. Consequently, time-resolved damping is very unstable but the cycle damping is more balanced.

9. LE grit trips promoted increased, positive aerodynamic damping at $M_\infty \leq 0.3$ compared to the baseline and MT models. This mechanism stems from a larger lag in the developing LE pressures (the trip slows the flow as it navigates the LE). This also reduces the aerodynamic loads, including the normal or lift forces.

10. The alteration of the shock growth and subsequent fixation of the shock at the trailing-edge of LE grit trips at $M_\infty = 0.6$ caused large, transient unstable loading (this unstable loading was large enough to cause noticeable twist on the model and was captured by the focusing schlieren system). This suggests a severe sensitivity to LE control mechanisms – devices $O(\delta)$ may promote shock induced unstable aerodynamic loads.

11. Applications of this study include on-blade measurement of the static pressures, integration to determine $C_{m_{\alpha\alpha}}$, and Hilbert transform to determine incipient stall flutter conditions. This method, as currently developed, would require determination of the blade section incidence – a nontrivial task.

Through these examples, it was shown that conditions that exhibit positive cycle-integrated aerodynamic damping, often include large negative aerodynamic damping during the pitching cycle. These negatively-damped portions of the pitch cycle could account for high-frequency stall flutter divergence from the rotor control input that has been observed in helicopter applications.
CHAPTER 8

SUMMARY AND CONCLUSIONS

Compressible dynamic stall offers a number of challenging problems in unsteady aerodynamics, separation, and control. The phenomenon is full of rich fluid physics that affect the daily operation of flight vehicles. The dynamic stall facility at Notre Dame is unique, capable of operating at rotor flight Reynolds and Mach numbers, and provides a biharmonic prescribed incidence at rotor frequencies. The rig offers unlimited potential for investigations into the aerodynamics of both 1P and 1P+torsional oscillations.

Of the vast assortment of problems concerning compressible dynamic stall that can be investigated through the acquired data set of the Bell airfoil, this paper focuses three distinct problems. Those problems are summarized as follows:

1. The stall vortex convection rate: The behavior of the stall vortex’s convection over the airfoil was successfully studied experimentally through tracking of the minimum pressures over the chord. This study focused only on the Bell airfoil with the masking-tape trip. The analysis of deep dynamic stall showed that, prior to the appearance of supersonic flow, the properties of the stall vortex were correlated to both reduced frequency and the airfoil’s maximum incidence. This was not the case at higher freestream Mach numbers. Both the residence time and the gestation period of the stall vortex decreased with increased Mach number under similar forcing parameters. As shown in Chapter 7, the decrease in gestation period and subsequent weakening of the DSV) reduces both the duration of negative, time-resolved aerodynamic damping as well as the peak, minimum damping $\Xi_{min}$. The downstream pressure field of the stall vortex at $M_c \leq 0.4$ indicates an adverse pressure gradient. At $M_c \geq 0.5$, the stall vortex propagates into a constant pressure field. This observation, in conjunction with the experimental
results of [29], suggests that shock-induced dynamic stall produces a ‘stall vortex’ more inline with a jet or sudden release of air than a concentrated, vortical structure.

2. **Shock induced dynamic stall and surface flow features:** A focused schlieren system was successfully designed and built to experimentally visualize the leading-edge flowfield of the Bell airfoil. The system aided in the characterization of the separation events at different freestream Mach test points. The schlieren images showed the initial formation and coalescence of multiple shocks at $M_x > 0.47$ in both steady and unsteady flow. At $M_x = 0.6$, the shock pattern takes on a $\lambda$ shape. The focusing schlieren, in conjunction with the on-board pressure instrumentation, revealed the complex breakdown and series of events following shock-induced dynamic stall, including the simultaneous motion of a forward moving, off-surface shock and rearward propagating ‘stall-vortex’ at $M_x = 0.6$.

3. **Compressibility effects on torsion damping:** A return to the work-per-cycle and torsional damping due to harmonic pitch incidence revealed the inadequacy of the assumption of constant coefficients in the aerodynamic system equation of motion. This assumption was replaced by a quasi-nonlinear equation of motion and time variant aerodynamic viscous damping and stiffness. Through the Hilbert transform of the equation of motion, the time-resolved aerodynamic damping proved to take on an equivalent formulation to the cycle damping, but in terms of an analytic signal. The time-resolved damping was successfully determined from the experimentally integrated surface pressures and resulting ensemble averaged pitch-moment. This analysis provided a wealth of new information regarding the stability of the airloads through the pitch cycle. Foremost, the analysis revealed that the cycle-integrated damping, $\Xi_{cycle}$, conceals large durations of the pitch-cycle when the time-resolved damping was unstable (negative). This negative aerodynamic damping could be sufficient to excite stall flutter on a helicopter’s rotor-blade.

General observations and suggestions from the experimental database that were not expounded in the text include the following:

1. **Mach effects on stall delay:** Stall delay is augmented through increasing the reduced frequency of the 1P sinusoidal pitch inputs. For the same trajectory, increasing Mach number accelerates the onset of dynamic stall; however, this is a steady effect and not related to the unsteady motion of the airfoil. For the same reduced frequency and increasing Mach number, stall penetration, $\alpha_{ds} - \alpha_{ss}$, remains nearly constant.
2. **Mach and reduced frequency**: The effect of reduced frequency increases with increasing Mach number, specifically in terms of the induced lag. At high Mach number, the attached flow significantly lags the unsteady motion, such that the hysteresis between pitch-up and pitch-down strokes increases with \( M_8 \) and directly impacts \( \Xi_{\text{cycle}} \). In this way, compressibility acts in a similar fashion to reduced frequency. Further investigation is required.

3. **Strength and structure of the dynamic stall vortex**: The strength of the stall vortex, through consideration of the surface vorticity flux (Appendix D) as well as the induced pressures over the airfoil chord, decreases with increasing Mach number. This is closely related to the decrease in the gestation period of the stall vortex that results from the appearance of supersonic flow. Although the magnitude of the induced pressures decreased, it was found that the ratio \( \Delta C_p/C_{p_{\text{min}}} \) increases with \( M_8 \). This most likely has a marginal impact on lift and drag; but, as the pitch-moment is a measure of pressure balance, the effect of the reduced \( \Delta C_p \) at high \( M_8 \) is substantial. Again, further investigation is required into the study of compressibility on the development of the stall vortex and the vortex’s properties. Such investigations require measurements of the flow-field in order to compute the spanwise vorticity of the stall vortex. Flow field measurements are required to distinguish the differing properties of a low-speed, dynamic stall vortex, and the ‘stall-vortex’ generated in the aftermath of shock-induced dynamic stall.

4. **Biharmonic pitch inputs**: The biharmonic data set has been examined and shows that the prescribed forcing generates rapid fluctuations in lift, drag, and pitch-moment. Ongoing work is directed at studying the time-resolved damping of the biharmonic motion through the Hilbert transform. Time-frequency analysis (e.g., wavelet or empirical-mode-decomposition) is required to understand the effect of the instantaneous frequency of the prescribed motion.

5. **On the structural side**: The experimentally measured, time-resolved damping from 1P motions can be employed as an empirical model in simple, single-degree-of-freedom structural analysis of the rotor-blade to determine the impact of the variable damping on the blade dynamics – see [23]. The question then posed is: Can variable aerodynamic damping be used to predict localized, divergent blade operation, such as that illustrated by Bousman[8]?

6. **Flow control**: Control of dynamic stall is a complicated task. Maximum lift is a function of the prescribed motion, and is augmented by the formation of the stall vortex at lower \( M_8 \). Control mechanisms that reduce the vorticity flux of the developing potential pressure field will weaken the stall vortex and degrade lift. The same vortex and its propagation aft over the chord causes large excursions in the pitch-moment. To avoid moment-stall, the stall vortex must be
destroyed prior to its convection past $x = c/4$. Alternatively, a control mechanism that can entrain and redirect the vortex energy to the boundary layer and into the streamwise direction offers the key compromise between maintaining lift and eliminating moment-stall. The control mechanism itself, if placed near the leading-edge of the airfoil, must not extend above the surface of the airfoil. As the trip study showed, small discontinuities in airfoil geometry near the leading-edge lead to unstable shock production and augmented pitch loads at $M_{\infty} \geq 0.6$. The final wrench to the control problem stems from the time-resolved damping analysis. The delay in separation due to the unsteady motion of an oscillating airfoil comes at the cost of significant work imparted from the airstream to the airfoil. If this negative aerodynamic damping is to be avoided, then the control mechanism must diffuse the developing leading-edge pressure field and associated vorticity in such a way as not to promote separation. This directly conflicts with the desire for high-lift. In the end, a compromise must be reached, and this most likely requires control input at several positions on the airfoil upper surface.
APPENDIX A

HELICOPTER FLIGHT BASICS

A.1 The helicopter environment

The helicopter flight environment is shown schematically in Figure A.1. The rotor flow field features complex, time-varying blade angles of attack (a function of both radial and azimuthal position), out-of-plane mechanical and elastic blade motion, unsteady boundary layers, massive separation, vortex blade interaction, and varying inflow velocity - to name a few. Each unsteady aerodynamic problem effects vehicle performance and operation and may contribute to vibration, noise, excessive loading, or reduced control authority[36]. Additionally, the rotor blade may witness local, instantaneous Mach numbers, indicative of low speed, \( M_l < 0.2 \), transonic \( 0.7 < M_l < 1 \), and supersonic flow \( M_l > 1 \) within a single rotation[89, 92, 108].

A.1.1 Flight conditions

A typical helicopter flies at altitudes from sea-level to 20,000 ft. At high altitude, a slight increase in Mach number accompanies the reduction in temperature and consequent drop in the speed of sound, \( c_x \). Cruise speeds extend from 120 kn to 160 kn for commercial and military vehicles, respectively. Maximum speeds near 200 knots. Figure A.2 illustrates the forward flight Mach number, \( M_{ff} \), as a function of altitude (standard atmosphere, lapse rate -3.57°F/1000 ft) for several aircraft speeds. The Mach
increase is no greater that 8\% for typical rotorcraft operations. Additionally, loading on the rotor increases with increasing altitude, a byproduct of the decreased air density, i.e., $C_T/\sigma \propto 1/\rho$[8].

A.1.2 The rotor disk

The rotor disk is shown in Figure A.3 to illustrate conventional nomenclature. Here, $r$ is the position along the blade, $R$ the length of the rotor-blade, $\psi$ the azimuth of the blade, $\Omega$ the angular velocity of the rotor-blade, referred to as the “1/rev,” and $U_{ff}$, the forward flight speed. For constant rotational velocity, $\psi = \Omega t$. The rotor is most highly loaded in lift in the forward and aft quadrants of the disk, yet both positions
are void of serious aerodynamic problems[6]. Instead, blade stall commonly occurs on
the retreating side of the rotor disk[70, 79, 80], although other azimuthal locations may
also induce stall[8].

The advancing blade moves opposite the direction of the relative airflow caused
by forward flight. Consequently, the oncoming flow is faster for the advancing blade
than it is for the retreating blade, where the blade motion is in the same direction as
the airstream. The lift generated on the advancing blade must be balanced by the lift
generated on the retreating blade to avoid an unstable rolling moment. Therefore, the
retreating blade operates at larger angles of attack; and, as a result, is more susceptible
to stall. The increased retreating blade force compensates for the vehicle rolling mo-
ment that would otherwise occur[36] and eliminates the damaging blade-root stresses
that occur in the presence of an alternating blade force.

This qualitative description can be further understood through the following discus-
Consider the blade onflow velocity,

\[ u_{\text{rotor/}U_{ff}}(r, \psi) = \Omega r + U_{ff}\sin(\psi), \quad (A.1) \]

where \( u_{\text{rotor/}U_{ff}} \) is the relative velocity of the rotor-blade with respect to the flight speed.

For the advancing blade, \( \psi \in (0^\circ, 180^\circ) \), forward flight yields an increase in relative blade velocity, whereas a decrease is found for retreating blade, \( \psi \in (180^\circ, 360^\circ) \).

Obviously, \( u_{\text{rotor/}U_{ff}} \) experiences a 1/rev (once-per-revolution) sinusoidal variation. In most wind-tunnel tests \( u_{\text{rotor/}U_{ff}} \) is held constant (and relabeled \( U_\infty \)), although experiments varying the inflow velocity have been conducted[111]. Between the two positions \( \psi = 90^\circ \) and \( \psi = 270^\circ \), the difference in speed is exactly \( 2U_{ff} \); whereas, for \( \psi = 0^\circ \) and \( 180^\circ \), the onflow is unaffected by the forward flight velocity.

The blade Mach number is then \( M_\infty = \frac{u_{\text{rotor/}U_{ff}}}{c_\infty} \). Clearly, \( M_\infty \) is directly effected by the rotor frequency, radial position, and azimuthal angle. Figure A.4 illustrates the
typical Mach variation on the Sikorsky UH-60 Blackhawk rotor for the azimuthal locations $\psi = 90^\circ$ and $\psi = 270^\circ$. Note, the rotor rpm used in the Figure A.4 is larger than the nominal Blackhawk shaft rate (258 rpm). Higher forward flight speed requires an increase in the rotor shaft rate or diameter; increases in forward flight speed decrease the aeroelastic stability of the blade motion. Additionally, the blade and control loads increase because of the lopsided required incidence between the advancing and retreating blades. Both the aerodynamic efficiency and thrust capability of the rotor decrease, primarily due to the onset of retreating blade stall. The maximum tip speed is limited by compressibility and should remain below a Mach number 0.9. The maximum tip Mach number, $M_{1,90}$ is then,

$$M_{1,90} = \frac{(U_{ff} + \Omega R)}{c_\infty}, \quad (A.2)$$

as shown by Johnson[75].

Equation A.1 can be non-dimensionalized via the advance ratio, $\mu$. The advance ratio is defined

$$\mu = \frac{U_{ff}}{\Omega R} \quad (A.3)$$

and is the ratio of forward vehicle velocity to the rotor-blade tip speed. Substituting into Equation A.1 yields

$$\frac{u_{rotor}}{U_{ff}}(\Omega R) = \frac{r}{R} + \mu \sin(\psi). \quad (A.4)$$

Figure A.5 displays Equation A.4 for two radial positions and several advanced ratios. Note the large, inboard retreating-blade region where the inflow velocity is negative – the area of negative velocity is called the reversed flow region, that is, the flow propagates from the trailing- to the leading-edge.
Reversed flow further limits the retreating blade’s ability to produce lift, existing where \( \frac{u_{\text{rotor}}}{U_{ff}} < 0 \). Provided this restriction, Equation A.4 can be used to determine the extent of the reverse flow region, as follows

\[
\frac{u_{\text{rotor}}}{U_{ff}/(\Omega R)} < 0 \tag{A.5}
\]
\[
r/R + \mu \sin(\psi) < 0 \tag{A.6}
\]
\[
r < -\mu R \sin(\psi). \tag{A.7}
\]

For \( \psi \in (180^\circ, 360^\circ) \), the right hand side of Equation A.7 is positive. The result describes a circle of radius \( \mu R/2 \) centered at \( (r, \psi) = (\mu R/2, 270^\circ) \), as shown in Figure A.6. The reversed flow region extends radially outboard from the rotor hub with increasing advance ratio. For \( \mu = 0 \) (hover), no reverse flow region exists. For \( \mu = 1 \), the reversed flow stretches to the blade tip, but, for most operations, it has limited extent.
Figure A.5. Non-dimensional relative blade velocity dependence on azimuthal location. Amplitudes are $\mu = 0, 0.2, 0.4, 0.6, 0.8$, and 1.0.

and covers only the hub portion of the rotor disk - the rotor cutout (where there is no rotor-blade). However, at high forward flight speed, or high-loading, the deficit in inboard lift demands that the outboard sections of the rotor-blade produce even more lift than required by the dynamic pressure imbalance, eliciting larger angles of attack than already required. This can lead to tip stall on the retreating blade, spreading further inboard as $\mu$ increases.

A typical rotor disk angle of attack distribution, as found by Ham and Garelick[67], is shown in Figure A.7. The distribution of angle of attack is clearly asymmetric, demanding large angles of attack on the retreating side but marginal incidences immediately following $\psi = 0^\circ$. Despite the reduced angle of attack, the advancing blade can also operate near its stall boundary, however, this is typically dictated by the onset of
locally transonic or super sonic flow and accompanying shock separation. Twist is incorporated into the design of the rotor-blade in order to help achieve the required radial angle of attack distribution.

A.1.3 Operation

Increasing or decreasing a helicopter’s throttle does not directly vary the lift of the vehicle or, alternatively, the speed of the vehicle. Instead, the blade angle of attack is manipulated to obtain the vehicle’s desired flight path. A helicopter utilizes four basic control inputs to maneuver. Two of the control inputs, the torque pedals and throttle, do not directly effect the rotor-blade’s transient incidence. The torque pedals adjust the side-force produced by the tail rotor, allowing the helicopter to alter its forward orientation. The throttle varies the RPM of the rotor, and, under normal operating conditions, remains fixed.
Figure A.7. Typical angle of attack distribution for a helicopter in forward flight (140 knots, $\mu=0.33$), replotted from Ham[67].

The collective and cyclic control the rotor-blades’ azimuthal dependent angle of attack. The collective changes the mean angle of attack, $\alpha_0$. By adjusting the collective, the pilot varies the magnitude of the thrust, causing the helicopter to climb or descend. Rapid variation in the collective leads to high g-force maneuvers and large $C_T/\sigma$, where $C_T$ is the thrust coefficient and $\sigma$ is the rotor solidity ($\sigma = A_{\text{rotor-blades}}/\pi R^2$). The cyclic alters the local angle of attack of the rotor-blade and forces the airfoil to pitch or feather about the quarter chord with a prescribed amplitude, $\alpha_1$. The azimuthal variation in lift alters the direction of the thrust vector and compensates for the lift deficient retreating blade by pitching to higher angles of attack. The control over the tilt of the rotor disk plane allows the pilot to maneuver and increase the forward flight speed or advance ratio, $\mu$, of the helicopter. E.g., increased cyclic pitches each blade to
a higher angle of attack (larger lift) at the rear of the rotor disk, $\psi = 0^\circ$. The resulting forward tilt of the rotor disk promotes a forward component of thrust. Typically, the cyclic and collective controls are varied mechanically via a swashplate and pitch-links. More recently, individual blade control (IBC) has achieved the same effect using servo motors and hydraulics[73].

A.1.4 Reduced frequency

The main parameter governing the unsteady nature of the rotor-blade is the reduced frequency, $k$, defined by Liiva[91], in terms of physical pitch frequency, $f$, and, equivalently, by McCroskey[104], in terms of circular frequency, $\omega$, as follows:

$$k = \pi fc/U_{ff}, \quad (A.8)$$

$$k = \omega c/(2U_{ff}), \quad (A.9)$$

where $c$ is the chord. Leishman[89] indicates that $k$ characterizes the degree of unsteadiness associated with the problem and represents the ratio of times scales associated with the unsteady fluid properties to the time scales of the freestream. For $k = 0$, the flow is steady. For $0 < k < 0.05$, unsteady effects are generally small and routinely neglected. For $k \geq 0.05$, the flow is considered unsteady. Lastly, for $k \geq 0.2$, the problem is considered highly unsteady. Additionally, it is often necessary to think of the reduced frequency as a “lag” parameter, where the fluid reaction lags the motion input. Larger reduced frequencies correspond to increased disparities between the fluid time-scales and the time-scales of the aerodynamic body’s prescribed motion.

At $\psi = 270^\circ$, Equation A.9 can be manipulated to represent the retreating rotor-
blade,

\[ k = \pi \Omega c / (\Omega r - U_{ff}) \]  \hspace{1cm} (A.10)

For a fixed \( \Omega \), \( k \) decreases with decreasing forward flight speed, reaching a minimum value during hover. Thus, the largest reduced frequencies exist at the helicopter’s maximum speed at radial locations just outside the reversed flow region. For \( \psi = 90^\circ \text{irc} \), the largest reduced frequencies occur during hover and grow progressively smaller as the rotorcraft increases speed. In general, the reduced frequency is constantly changing due to the \( 1/\text{rev} \) inflow dependence on \( U_{ff} \), and, as a result, is ambiguous in regard to the unsteadiness associated with the helicopter problem\[89\]. For windtunnel tests, the reduced frequency is constant provided a fixed pitching frequency and freestream velocity.

### A.2 Dynamic stall in the rotor environment

At moderate to high disk loading, high speed forward flight, or high g-force maneuvers\[88\], the rotor-blade enters regimes where it can dynamically stall. Bousman\[8\] found that stall events occur at multiple sites during flight tests of a UH-60A, as shown Figure A.8. Stall increases the torsion transmitted to the rotor hub and control system, as shown in Figure A.9.

\( C_T/\sigma \) is a ratio of the helicopter thrust coefficient to the blade solidity (blade area/rotor disk area) and represents the average blade loading. Increased blade loading leads to stall; while faster forward flight promotes stall at reduced blade loading. This relationship between \( C_T/\sigma \) and \( \mu \) is often used to schematically represent the helicopter performance envelope, shown in Figure A.10.
Figure A.8. Rotor disk of a UH-60A (Sikorsky “Black Hawk”) performing several in-flight maneuvers. The symbols indicate the location of dynamic stall events. Replotted from Bousman[8].
Figure A.9. Blade root torsional loads as a function of blade loading, adapted from Leishman[89].

Figure A.10. Rotor performance limits. Adapted from Martin et al.[100].
APPENDIX B

STEADY DATA

Static data was recorded for the unmodified model and the three cases with the leading-edge boundary layer trips. This appendix briefly discusses the laminar separation bubble characteristics that was found near the Bell airfoil leading-edge. The static data is then presented in a series of figures and tables. Overall, the airfoil indicated no unusual static characteristics.

B.1 Laminar Separation Bubble Characteristics

A limited study of the laminar separation bubble characteristics was conducted to determine the position and extent of the bubble at relevant test points. Fluorescent yellow dyed 10,000 cSt silicone oil was painted on the suction surface of the model. The airfoil was positioned at incidences of 8°, 9°, 10°, and 11°, and the tunnel velocity was varied from Mach 0.2 to 0.6. The viscous oil slowly sheared in response to the local velocity on the airfoil surface over a test period of 3-5 minutes (dependent on the freestream speed). Regions of reversed flow, i.e., high shear stress ($\gamma \frac{du}{dy}$), cause the fluid to conglomorate and allows the bubble position and length to be measured. The measurements were not meant to be exact, but, instead, were intended to provide the necessary information to trip the airfoil successfully over the Mach and incidence range required for both the static and dynamic studies. The oil was applied to the forward 10%
the airfoil, as the separation was known to be located upstream. Figure B.1 indicates the center of the separation bubble, as well as its extent, denoted by the error bars. An XFOIL calculation for \( M_{\infty} = 0.4 \) of the transition location, i.e., that laminar separation point, agreed fairly well with the measured bubble properties. For all angles of attack considered, the separation bubble at \( M_{\infty} = 0.6 \) was not found; only a thin line of oil remained, indicative of an early stall angle. Figure B.2 shows the results of the oil flow visualization. A clear separation bubble exists near the leading-edge, visualized as the thin, spanwise band of accumulated oil.

Five trips were investigated throughout the study – masking-tape and P500, P400, P320, P180 grits. The effect of the P320 and P400 grit trips are shown in Figures B.3 and B.4, respectively. The trip was placed over the bubble extent in order to remove the reverse flow region and ensure that bubble bursting could not act as the dynamic
Figure B.2. Oil flow visualization of the baseline airfoil laminar separation bubble. \( M_{\infty} \approx 0.2, \alpha = 8^\circ \). No Trip.

Stall mechanism. The P180 grit proved too large and produced a large area of reverse flow following the tape, whereas the P500 grit did not seem to eliminate the separation bubble. Both the P400 and P320 trips introduced a small region of reversed flow immediately following the downstream end of the trip (backwards facing step). The thickness of the trip tape was only 4 mil, however, the pressure jump across the step will be shown to have significant impact on both the steady and unsteady airloads.

Surface flow visualization of the masking-tape trip showed that the separation bubble was not completely eliminated. However, the masking-tape did produce trailing-edge stall characteristics for the static cases where the baseline oil flow visualization and airloads indicated leading-edge stall behavior.
Figure B.3. Oil flow visualization of the baseline airfoil laminar separation bubble. \( M_\infty \approx 0.2, \alpha = 8^\circ \). P320 Grit applied to leading-edge, 
\( x \in [0.005, .030] \).

Figure B.4. Oil flow visualization of the baseline airfoil laminar separation bubble. \( M_\infty \approx 0.2, \alpha = 8^\circ \). P400 Grit applied to leading-edge, 
\( x \in [0.005, .030] \).
B.2 Static pressures and loads

The run numbers associated with the presented static data are shown in Table B.1. Static data was recorded for each Mach number from 0.2 to 0.6 in increments of 0.1. Approximately 40-60 points were used to define the static characteristics, dependent on $M_\infty$. Points were separated by 1° angle of attack, except over the range $\alpha \in (\alpha_{ss} - 3^\circ, \alpha_{ss} + 3^\circ)$, where the points were spaced by approximately 0.2° to improve stall resolution.

**TABLE B.1**

STATIC TEST MATRIX AND ASSOCIATED RUN NUMBERS

<table>
<thead>
<tr>
<th>LE modifications</th>
<th>$M_\infty \approx 0.2$</th>
<th>$M_\infty \approx 0.3$</th>
<th>$M_\infty \approx 0.4$</th>
<th>$M_\infty \approx 0.5$</th>
<th>$M_\infty \approx 0.6$</th>
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</thead>
<tbody>
<tr>
<td>NONE</td>
<td>2548</td>
<td>2549</td>
<td>2553</td>
<td>2554</td>
<td>2555</td>
</tr>
<tr>
<td>MASKING TAPE</td>
<td>1847</td>
<td>1849</td>
<td>1851</td>
<td>1853</td>
<td>1854</td>
</tr>
<tr>
<td>P400 GRIT</td>
<td>2827</td>
<td>2826</td>
<td>2825</td>
<td>2824</td>
<td>2823</td>
</tr>
<tr>
<td>P320 GRIT</td>
<td>2556</td>
<td>2557</td>
<td>2558</td>
<td>2559</td>
<td>2560</td>
</tr>
</tbody>
</table>
Figure B.5. Static loads. Baseline □, masking-tape ○, P400 grit △, P320 grit ◊.
Figure B.6. Static loads. Baseline □, masking-tape ○, P400 grit △, P320 grit ◊.
Figure B.7. Static loads. $M_{\infty}=0.6$. Baseline □, masking-tape ○, P400 grit △, P320 grit ◇.
Figure B.8. Static load attributes for the baseline and LE modified airfoil. Baseline □, masking-tape ○, P400 grit △, P320 grit ◊.

Figure B.9. Normal curve slope comparison for the baseline and LE modified airfoil. Baseline □, masking-tape ○, P400 grit △, P320 grit ◊, $2\pi/\beta$. 
Figure B.10. Pressure coefficient development with increasing angle of attack, $M_\infty = 0.2$
Figure B.11. Pressure coefficient development with increasing angle of attack, $M_{x_e} = 0.4$. Green regions highlight the extent of supersonic flow.
Figure B.12. Pressure coefficient development with increasing angle of attack, $M_\infty = 0.6$. Green regions highlight the extent of supersonic flow.
Figure B.13. Static chord pressure comparison for select angles of attack, $M_x=0.2$. Baseline □, masking-tape ⊙, P400 grit △, P320 grit ◊, $C_p^{\text{sonic}}$ (———).
Figure B.14. Static chord pressure comparison for select angles of attack, \(M_x=0.3\). Baseline \(\square\), masking-tape \(\bigcirc\), P400 grit \(\Delta\), P320 grit \(\diamond\), \(C_p^{\text{sonic}}\) (---).

\(C_p\) vs. \(x\) for:

(a) \(M_x=0.3, \alpha = 8.5^\circ\)
(b) \(M_x=0.3, \alpha = 10.0^\circ\)
(c) \(M_x=0.3, \alpha = 11.4^\circ\)
(d) \(M_x=0.3, \alpha = 14.5^\circ\)
Figure B.15. Static chord pressure comparison for select angles of attack, $M_x=0.4$. Baseline $\square$, masking-tape $\bigcirc$, P400 grit $\triangle$, P320 grit $\diamond$, $C^s_{p}$ (---).
Figure B.16. Static chord pressure comparison for select angles of attack, $M_\infty=0.5$. Baseline $\blacksquare$, masking-tape $\bigcirc$, P400 grit $\triangle$, P320 grit $\diamond$, $C_p^{sonic}$ (dashed line).
Figure B.17. Static chord pressure comparison for select angles of attack, $M_x=0.6$. Baseline $\Box$, masking-tape $\bigcirc$, P400 grit $\triangle$, P320 grit $\diamond$, $C_{p,\text{sonic}}$ (---).

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Figure B.18. Minimum pressure comparison for the baseline and modified airfoils. Baseline □, masking-tape ⊙, P400 grit △, P320 grit ◊.

Figure B.19. Chordwise extent of supersonic flow for steady data

(a) $M_\infty = 0.4$  
(b) $M_\infty = 0.5$  
(c) $M_\infty = 0.6$
### TABLE B.2

**STATIC STALL ANGLES**

<table>
<thead>
<tr>
<th></th>
<th>static stall angles, $\alpha_{ss}$ [°]</th>
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<tbody>
<tr>
<td></td>
<td>$M_{\infty}$ ≈ 0.2 0.3 0.4 0.5 0.6</td>
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<tr>
<td>Baseline</td>
<td>12.41 10.54 9.64 9.50 8.71</td>
</tr>
<tr>
<td>Masking tape</td>
<td>12.89 13.10 10.52 8.95 9.73</td>
</tr>
<tr>
<td>P400 grit</td>
<td>12.63 12.31 11.73 11.52 7.91</td>
</tr>
<tr>
<td>P320 grit</td>
<td>12.49 11.54 10.54 10.01 7.77</td>
</tr>
</tbody>
</table>

### TABLE B.3

**STATIC STALL-TYPE**

LE: leading-edge, TE: trailing-edge, MIXED: leading-/trailing-edge,
LE/S: shock induced leading-edge

<table>
<thead>
<tr>
<th></th>
<th>Stall-types</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M_{\infty}$ ≈ 0.2 0.3 0.4 0.5 0.6</td>
</tr>
<tr>
<td>Baseline</td>
<td>LE  TE  TE  LE/S  LE/S</td>
</tr>
<tr>
<td>Masking tape</td>
<td>MIXED  TE  TE  LE/S  LE/S</td>
</tr>
<tr>
<td>P400 grit</td>
<td>TE  TE  TE  TE  LE/S</td>
</tr>
<tr>
<td>P320 grit</td>
<td>TE  TE  TE  TE  LE/S</td>
</tr>
</tbody>
</table>
APPENDIX C

MACH EFFECTS ON PRESSURE DISTRIBUTION AND LOADS

Prior investigations, discussed in the review by Carr[12], have focused on the behavior of the aerodynamic loads and pressure distribution as the freestream Mach number is increased. This section utilizes a subset of the much larger test matrix to summarize several features supporting previous studies. Table C.1 lists the specific cases considered.

Figure C.1 shows the force coefficients and corresponding steady state data for $M_\infty = 0.2$ and several reduced frequencies. Increased $k$ results in several important characteristics, including:

- increased delay of stall angle,

- increased peak loading, and

- a delay in reattachment angle on the downstroke.

Figure C.2 highlights roughly the same motion history for the case of $M_\infty = 0.3$. Qualitatively, the behavior of the four cases considered are very similar to the cases shown in Figure C.1. However, the increased Mach number reduced the peak load coefficients, e.g., $C_{n_{\text{max}}} = 1.72$ for $M_\infty = 0.2$ and $k = 0.1$, while $C_{n_{\text{max}}} = 1.67$ at $M_\infty = 0.3$ and $k = 0.1$. The aerodynamic damping, significant at $M_\infty = 0.2$ is greatly reduced at
TABLE C.1

TEST CONDITIONS STUDIED

<table>
<thead>
<tr>
<th>M&lt;sub&gt;x&lt;/sub&gt;</th>
<th>α&lt;sub&gt;0&lt;/sub&gt;, α&lt;sub&gt;1&lt;/sub&gt; [°]</th>
<th>reduced frequency, k</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.025</td>
<td>0.050</td>
</tr>
<tr>
<td>0.2</td>
<td>11, 5</td>
<td>1868</td>
</tr>
<tr>
<td></td>
<td>8, 8</td>
<td>2312</td>
</tr>
<tr>
<td>0.3</td>
<td>11, 5</td>
<td>1892</td>
</tr>
<tr>
<td></td>
<td>8, 8</td>
<td>2342</td>
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<tr>
<td>0.4</td>
<td>11, 5</td>
<td>1922</td>
</tr>
<tr>
<td></td>
<td>8, 8</td>
<td>2372</td>
</tr>
<tr>
<td></td>
<td>6, 8</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>9, 5</td>
<td>2058</td>
</tr>
<tr>
<td></td>
<td>9, 8</td>
<td>2544</td>
</tr>
<tr>
<td>0.6</td>
<td>8, 5</td>
<td>2081</td>
</tr>
</tbody>
</table>

M<sub>x</sub> = 0.3. The large, negative damping areas are replaced by nearly identical pitch-moment values incurred on both the pitch-up and pitch-down strokes. This suggests that substantial lag experienced at M<sub>x</sub> = 0.2 in return of leading-edge suction and the associated nose-up moment, is greatly reduced at M<sub>x</sub>=0.3. Due to the dependency of Re<sub>c</sub> and M<sub>x</sub>, the exact cause of the diminished hysteresis/lag cannot be isolated as a Reynolds number or Mach number effect. Not nearly as effected is the delay in stall angle, i.e., for equal reduced frequencies, dynamic stall occurs after a similar increment in incidence above the Mach dependent steady stall angle. Examination of Figures C.3,
C.4, and C.5 illustrates that both these trends continue with increasing $M_x$. A reduced frequency of $k = 0.1$ was not obtained at $M_x = 0.6$ due to excessive vibration of the pitch mechanism and stand.

Figure C.1. 1P loads, $M_x = 0.2$, $\alpha_0 \approx 11^\circ$, $\alpha_1 \approx 5^\circ$. 
Figure C.2. 1P loads, $M_x=0.3$, $\alpha_0 \approx 11^\circ$, $\alpha_1 \approx 5^\circ$.

Figure C.3. 1P loads, $M_x=0.4$, $\alpha_0 \approx 11^\circ$, $\alpha_1 \approx 5^\circ$.
Figure C.4. 1P loads, $M_x = 0.5$, $\alpha_0 \approx 9^\circ$, $\alpha_1 \approx 5^\circ$.

Figure C.5. 1P loads, $M_x = 0.6$, $\alpha_0 \approx 8^\circ$, $\alpha_1 \approx 5^\circ$. 267
Figure C.6. 1P loads, \( M_x = 0.2, \alpha_0 \approx 8^\circ, \alpha_1 \approx 8^\circ \).

Figure C.7. 1P loads, \( M_x = 0.3, \alpha_0 \approx 8^\circ, \alpha_1 \approx 8^\circ \).
Figure C.8. 1P loads, $M_x=0.4$, $\alpha_0 \approx 8^\circ$, $\alpha_1 \approx 8^\circ$. 

(a) $k = 0.025$  
(b) $k = 0.05$  
(c) $k = 0.075$  
(d) $k = 0.10$
Figure C.9. 1P loads, $M_x = 0.5$, $\alpha_0 \approx 9^\circ$, $\alpha_1 \approx 8^\circ$. 
Figure C.10: Chord pressure development for (a)-(d): $k = 0.05$ and (e)-(h): $k = 0.075$. 
Figure C.11: Effect of reduced frequency on the local Mach number for fixed angle of attack before stall
Figure C.12: Effect of reduced frequency on the local Mach number for fixed angle of attack before stall
Figure C.13. Stacked $C_p$, $k = 0.05$, illustrating moment stall (M), lift stall (L), and peak nose-down moment sequence (PM).
Figure C.14. Stacked $C_p$, $M_{\infty} = 0.6$, RUN2082
Figure C.15. Minimum pressure for all test cases as a function of freestream Mach number

Figure C.16. Local Mach number for all test cases as a function of freestream Mach number, \( M_l = 2.60M_\infty + 0.29 \)
Figure C.17. Maximum unsteady normal force variation with Mach number, all test cases.

Figure C.18. Maximum unsteady axial force variation with Mach number, all test cases.
Figure C.19. Minimum unsteady pitch-moment variation with Mach number, all test cases.

Figure C.20. Maximum unsteady drag variation with Mach number, all test cases.
Figure C.21. Compressibility effects on dynamic stall angle of attack.
APPENDIX D

CONSIDERATION OF MACH EFFECTS ON THE STRENGTH OF THE DYNAMIC STALL VORTEX

D.1 Introduction: Surface vorticity flux

In the absence of quantitative flow field diagnostics away from the airfoil surface, determination of the vorticity produced through the compressible dynamic stall process is not possible. Consequently, correlating the effect of compressibility on the strength of the DSV is difficult. However, some inference can be made into the strength of the DSV through the vorticity flux generated by the airfoil surface. Due to the irrotational oncoming flow, the surface of the airfoil is the only source for vorticity in the dynamic stall process[2]. Reynolds and Carr[118] showed that the rate of vorticity generation, or flux, in the spanwise direction at the surface of a body depends on three quantities, shown in Equation D.1.

\[ \nu \frac{\partial \Omega}{\partial n} = \frac{\partial U_s}{\partial t} + \frac{1}{\rho} \frac{\partial P}{\partial s} + \frac{V \Omega}{\nu} \]  \hspace{1cm} (D.1)

The normal vorticity flux (LHS) is produced through three mechanisms, the surface acceleration (I), the potential flow pressure gradient (II), and surface transpiration (III). In the absence of mass transport through the airfoil surface, i.e. suction or blowing, III can be disregarded. A magnitude study by Chandrasekhara et al.[35] shows that I,
the surface acceleration term, is an order of magnitude smaller than flux generated by
the adverse pressure gradient. As discussed by Reynolds and Carr[118] and observed
in many experiments, e.g. [92, 96, 103, 109], an oscillating airfoil experiences an ex-
tremely high $dP/dx$ near the leading-edge. Further, Reynolds and Carr’s analysis of
driven, separated flows as an image system consisting of vortex pairs above and be-
low the a wall (or surface), indicated that the favorable pressure gradient, $dP/dx < 0$,
was the only source of vorticity. This is supported by the water-tunnel flow visualiza-
tions of Mcalister and Carr[102]. See Reynolds and Carr[118] (p.21-25) for a detailed
discussion and enlightening description of the dynamic stall process.

An approximation to the vorticity introduced by the airfoil surface into the inviscid
flow can be made through the relation

$$S = \frac{\rho}{\rho} \frac{\partial P}{\partial s}, \quad (D.2)$$

where, $S = \nu \frac{\partial \Omega}{\partial n}$ and the surface acceleration and vorticity transpiration t
erns are ne-
glected in Eq. D.1. Non-dimensionalizing Equation D.2, the surface vorticity flux, $S^+$
is

$$S^+ = \frac{2c}{U_\infty^2 \rho} \frac{\partial P}{\partial s}. \quad (D.3)$$

Substitution of the Mach number ($U_\infty = M_\infty \sqrt{\gamma RT}$), the ideal gas law ($\rho = P/RT$),
and $dx = \cos \theta ds$, yields

$$S^+ = \frac{2c}{\gamma M_\infty^2} \frac{\partial}{\partial x} (\ln(P)) \cos(\theta) \quad (D.4)$$

The discrete location of the pressure ports do not provide enough data points to accu-
rately determine derivative quantities. In order to improve the accuracy of the vorticity
flux calculation, shape-preserving, Akima splines were applied to the pressure distribution. The resulting polynomial coefficients were used to calculate $S^+$ at the pressure port stations as well as their midpoints for all time-samples. $S^+$ was also incorporated into the ensemble averaging routine used for 1/rev input.

Acharya and Metwally[2], in a study of low-speed dynamic stall, found that the peak minimum, $S_1$, and the peak maximum, $S_2$, of the vorticity flux had a significant influence on the stall vortex. Further, Acharya and Metwally identified $S_1$ as a source of vorticity, whereas $S_2$ was identified as a sink of vorticity. This description is consistent with Reynolds and Carr[118], as $S_1$ is a measure of the favorable pressure gradient and $S_2$ is a measure of the adverse pressure gradient.

Eq. D.4 is derived from the 2D momentum equation and irrotational, incompressible flow assumptions. As such, its meaning is questionable for dynamic stall studies in which compressibility effects are experience even at small $M_x$. However, it is included here as the trends associated with its behavior strongly agree with experimental observations.

D.2 Vorticity flux results

$$S^+(x) = \frac{2\gamma}{M_x^2} \frac{\partial}{\partial x} \ln(P) \cos(\theta) = \begin{cases} 
+ & x \in [0, x_{stag}] : \text{Pressure side} \\
- & x \in [x_{stag}, 1] : \text{Pressure side} \\
- & x \in [0, x_{cp_{min}}] : \text{Suction side} \\
+ & x \in [x_{cp_{min}}, 1] : \text{Suction side}
\end{cases}$$
Figure D.1. Surface vorticity flux sign behavior. $\ln(P)$ determined for $M_\infty = 0.2, \alpha = 11.53^\circ$ (RUN1847).
angle of attack, $\alpha$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>M$_\infty$ = 0.2</th>
<th>M$_\infty$ = 0.4</th>
<th>M$_\infty$ = 0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.05</td>
<td>8.00</td>
<td>10.36</td>
<td>11.10</td>
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<td>3.16</td>
<td>7.93</td>
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<tr>
<td>2.84</td>
<td>6.09</td>
<td>6.85</td>
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</table>

Figure D.2. Static vorticity flux for masking-tape trip. (a) M$_\infty$ = 0.2, (b) M$_\infty$ = 0.4, (c) M$_\infty$ = 0.6.
Figure D.3. Static vorticity comparison for different leading-edge modifications, $M_\infty = 0.2$, baseline, masking-tape, P400, P320.

Figure D.4. $S_1$ (open symbols) and $S_2$ (filled symbols) variation with Mach number. ○ MT, □ baseline, △ P320 Grit, ◆ P400 Grit. Test Conditions listed in Table C.1.
Figure D.5. Influence of Mach number on unsteady vorticity production term (MT). Vertical variation due to significant differences in dynamic stall attributes associated with $\alpha_{\text{max}}$ and $k$. 
Figure D.6. Influence of Mach number on unsteady vorticity sink term (MT). Vertical variation due to significant differences in dynamic stall attributes associated with $\alpha_{max}$ and $k$. 
Figure D.7. Unsteady surface vorticity evolution. Test Conditions listed in Table C.1.


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