NUMERICAL AND THEORETICAL ANALYSIS OF AERO-OPTICS WITH APPLICATION TO AN OPTICAL TURRET

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Abstract
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When an optical beam is transmitted through a compressible turbulent flow, it is distorted due to the non-uniform speed of light resulting from density fluctuations. The distortions caused by turbulent airflow surrounding the exit pupil of a beam director, known as aero-optical phenomena, are a major impediment to applications of airborne laser systems. The objectives of this research are to gain a deep understanding of the fluid dynamical causes of aero-optical phenomena and to improve the ability to predict them in practically important flow configurations.

To better understand the spectral properties of aero-optical distortions, a general expression for the phase-distortion spectrum is derived from the ideal-gas law and Gladstone-Dale relation. Using this expression, the relationship among the optical distortion magnitude, turbulence length scales, and aperture size is examined, and resolution requirements for wavefront sensor measurements and numerical simulations are analyzed. Large-eddy simulations (LES) of weakly compressible, temporally evolving shear layers are used to verify theoretical results and to investigate turbulent density fluctuations in relation to pressure and temperature fluctuations. Computational results support theoretical findings and confirm that if the log slope of the one-dimensional density spectrum in the inertial range is \(-m_\rho\), the optical phase distortion spectral slope is given by \(-(m_\rho + 1)\). The value of \(m_\rho\) is shown to
be dependent on the ratio of shear-layer free-stream densities and bounded by the spectral slopes of temperature and pressure fluctuations.

One of the most widely used optical transmission platforms is the hemisphere-on-cylinder turret. When employed on an aircraft, the bluff-body shape of the turret creates a complex turbulent flow that can cause severe optical aberrations at even low subsonic Mach numbers. To investigate the turret aero-optics, wall-modeled LES is used to simulate flow over a turret at Mach 0.4 and Reynolds number of $2.3 \times 10^6$, and the optical distortions are computed using ray tracing. Basic flow and optical statistics show agreement with available experimental data. It is found that the optical distortion over a majority of the turret field-of-regard is solely dependent on the lookback angle. Regions of the field-of-regard where the lookback angle alone is not sufficient to relate viewing angle to optical distortion are those strongly affected by the horn vortices in the turret wake. Wavefront statistics as functions of viewing angle are presented along with the effects of pressure and temperature fluctuations on optical distortions. Topics important to the design of aero-optical mitigation strategies, such as steady lensing, beam jitter and the connection between optical distortions and turret pressure fluctuations, are discussed and quantified.
For my parents, my wife, and my daughter.
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**Roman symbols**

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<td>$a$</td>
<td>Speed of sound</td>
</tr>
<tr>
<td>$A$</td>
<td>Tip coefficient (rad/s)</td>
</tr>
<tr>
<td>$A_f$</td>
<td>Control volume face area</td>
</tr>
<tr>
<td>$B$</td>
<td>Tilt coefficient (rad/s)</td>
</tr>
<tr>
<td>$B_p$</td>
<td>Turbulent pressure constant</td>
</tr>
<tr>
<td>$c_0$</td>
<td>Speed of light in a vacuum</td>
</tr>
<tr>
<td>$C$</td>
<td>Piston component of a wavefront</td>
</tr>
<tr>
<td>$C_D$</td>
<td>Drag coefficient</td>
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<tr>
<td>$C_f$</td>
<td>Skin-friction coefficient</td>
</tr>
<tr>
<td>$C_F$</td>
<td>Force coefficient</td>
</tr>
<tr>
<td>$C_M$</td>
<td>Moment coefficient</td>
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<td>$C_n$</td>
<td>Refractive index structure coefficient for atmospheric turbulence</td>
</tr>
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<td>$C_p$</td>
<td>Pressure coefficient</td>
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<tr>
<td>$\hat{C}$</td>
<td>Coherence function</td>
</tr>
<tr>
<td>$D$</td>
<td>Turret diameter</td>
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<tr>
<td>$D_{Ap}$</td>
<td>Aperture diameter</td>
</tr>
<tr>
<td>$D_m$</td>
<td>Coefficient of molecular diffusion</td>
</tr>
<tr>
<td>$D$</td>
<td>Differencing operator</td>
</tr>
<tr>
<td>$E$</td>
<td>Total energy; one-dimensional power spectrum</td>
</tr>
<tr>
<td>$E^{2D}$</td>
<td>Two-dimensional power spectrum</td>
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\( f_{\phi}, H_i \) Electromagnetic wave propagation spectral filter functions

\( F \) Flux; aperture filter function

\( g, h \) Generic functions for operation definitions

\( I \) Irradiance

\( I_0 \) Diffraction-limited irradiance

\( k \) Optical wavenumber

\( \mathbf{K} \) Spatial wavenumber vector

\( K_{GD} \) Gladstone-Dale coefficient

\( \ell \) Characteristic turbulence length scale

\( \ell_c \) Cutoff length scale

\( L \) Length of the optically active turbulence field

\( L_0 \) Peak turbulence length scale

\( L_{AO} \) Nondimensional optical-turbulence length scale

\( L_{ref} \) Reference length scale

\( m \) Spectral slope

\( M \) Mach number

\( M_c \) Convective Mach number

\( n \) Index of refraction

OPD Optical path difference

OPL Optical path length

\( p \) Pressure

\( Pr \) Prandtl number

\( Pr_t \) Turbulent Prandtl number

\( q \) Heat flux vector

\( \text{rms} \) Root-mean-square

\( \mathbf{r} \) Correlation separation vector

\( R \) Specific gas constant
<table>
<thead>
<tr>
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<tr>
<td>$Re$</td>
<td>Reynolds number</td>
</tr>
<tr>
<td>$\mathcal{R}$</td>
<td>Correlation function</td>
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<tr>
<td>$\hat{\mathcal{R}}$</td>
<td>Normalized correlation function</td>
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<tr>
<td>$s$</td>
<td>Ratio of two freestream densities in a shear layer; optical beam propagation path</td>
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<tr>
<td>SGS</td>
<td>Sub-grid scale</td>
</tr>
<tr>
<td>SR</td>
<td>Strehl Ratio</td>
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<tr>
<td>$t$</td>
<td>Time</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature</td>
</tr>
<tr>
<td>$T_0$</td>
<td>Total temperature</td>
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<tr>
<td>$u_i$</td>
<td>$i^{th}$ component of velocity; $i^{th}$ spatial POD mode</td>
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<td>$U$</td>
<td>Velocity</td>
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<td>$\mathbf{U}$</td>
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<td>$v_i$</td>
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</tr>
<tr>
<td>$V_{cv}$</td>
<td>Grid cell volume</td>
</tr>
<tr>
<td>$x, y, z$</td>
<td>Flow field coordinates</td>
</tr>
<tr>
<td>$x', y', z'$</td>
<td>Beam local coordinates</td>
</tr>
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**Greek symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
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<tbody>
<tr>
<td>$\alpha$</td>
<td>Flux blending parameter; Lookback angle</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Turbulent temperature constant; modified elevation angle</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Specific heat ratio</td>
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<tr>
<td>$\Gamma$</td>
<td>Gamma function</td>
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<tr>
<td>$\delta_{ij}$</td>
<td>Kronecker delta</td>
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<tr>
<td>$\delta_\theta$</td>
<td>Momentum thickness</td>
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<tr>
<td>Symbol</td>
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</tr>
<tr>
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<td>Vorticity thickness</td>
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<tr>
<td>$\Delta$</td>
<td>Filter width; change in value</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Turbulence dissipation rate</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Kolmogorov turbulence length scale</td>
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<td>$\kappa$</td>
<td>Scalar wavenumber</td>
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<tr>
<td>$\kappa_i$</td>
<td>$i^{th}$ component of the spatial wavenumber vector</td>
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<td>$\kappa_c$</td>
<td>Cutoff wavenumber</td>
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<tr>
<td>$\kappa_{vk}$</td>
<td>Von Karman constant</td>
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<tr>
<td>$\lambda$</td>
<td>Taylor microscale</td>
</tr>
<tr>
<td>$\lambda_i$</td>
<td>$i^{th}$ POD eigenvalue</td>
</tr>
<tr>
<td>$\lambda_{opt}$</td>
<td>Wavelength of the optical beam</td>
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<tr>
<td>$\mu$</td>
<td>Dynamic viscosity</td>
</tr>
<tr>
<td>$\mu_t$</td>
<td>Turbulent eddy viscosity</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Kinematic viscosity</td>
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<tr>
<td>$\xi$</td>
<td>Relative Error of squared OPL from a simulation to actual OPL value</td>
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<tr>
<td>$\rho$</td>
<td>Density</td>
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<tr>
<td>$\rho_{SL}$</td>
<td>Air density at sea-level</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Viscous stress; nondimensional time</td>
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<tr>
<td>$\tau_w$</td>
<td>Wall shear-stress vector</td>
</tr>
<tr>
<td>$\phi'$</td>
<td>Optical phase distortion relative to the mean phase</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>Three-dimensional power spectrum</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Turbulence temperature dissipation rate</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Optical beam frequency (rad/s); Vorticity</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Filtering domain</td>
</tr>
<tr>
<td>Notation</td>
<td>Description</td>
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</tr>
<tr>
<td>( )</td>
<td>Time average of a quantity</td>
</tr>
<tr>
<td>( )′</td>
<td>Complex conjugate of a quantity</td>
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<td>( )′′</td>
<td>Fluctuating component of a quantity</td>
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<tr>
<td>&lt;( )&gt;</td>
<td>Spatial average of a quantity</td>
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<td>( )̂</td>
<td>Spatially filtered quantity</td>
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<tr>
<td>( ) ̃</td>
<td>Farve filtered quantity</td>
</tr>
<tr>
<td>( )⊥</td>
<td>Coordinates perpendicular to the propagation direction of a quantity</td>
</tr>
<tr>
<td>( )*</td>
<td>Nondimensional wavenumber quantity</td>
</tr>
<tr>
<td>( )^N</td>
<td>Normalized optical quantity</td>
</tr>
<tr>
<td>( )^+</td>
<td>Value of a quantity in wall-units</td>
</tr>
<tr>
<td>( )∞</td>
<td>Freestream value of a quantity</td>
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CHAPTER 1

INTRODUCTION

1.1 Overview

As airborne optical systems have become more desirable for directed energy purposes and point-to-point communication with aircraft, understanding the physical mechanisms of optical beam distortions and finding an effective platform for laser transmission have become requisite areas of research. While airborne beam transmission systems have been sought after and under various stages of development since the 1970’s, in modern systems the wavelength of the optical beam has decreased in an effort to increase the intensity of the beam on a target in the farfield. An undesirable consequence of smaller wavelength beams is an increased sensitivity to aero-optic distortions, or optical distortions due to density fluctuations induced by turbulent flow near the beam aperture. Aero-optic effects can result in a severe degradation of the signal fidelity and on-target intensity of an optical beam as it propagates away from an aircraft [25].

While atmospheric turbulence between the aircraft and its target is also an issue in airborne beam transmission, its effects on lasers have been extensively studied [93, 94, 39] and adaptive-optics systems have been used to correct for its distortion for decades [48]. In comparison, aero-optic distortions are more difficult to correct using adaptive-optics methods because of much smaller characteristic time and length scales; systems must be able to operate at frequencies on the order of kilohertz to mitigate aero-optic distortions compared to the hertz range required by atmospheric
optical distortions. Additionally, the study of aero-optic distortions lacks some fundamental theoretical descriptions of distortion dynamics that can help establish corrective system requirements and assist in the development of potential correction methodologies.

As mentioned, an effective laser transmission platform is required for airborne beam transmission systems. A key problem arises when platforms that are optimal for optical transmission in terms of field-of-regard and geometry tend to be sub-optimal in terms of aero-optical effects. A prime example is the hemisphere-on-cylinder turret, which is widely used in optical systems since it provides a 360° view around the turret. In airborne systems, however, the bluff-body shape of the hemisphere-on-cylinder creates a complex, turbulent flow that causes severe aero-optical aberrations at even low subsonic Mach numbers. The combination of boundary layers, horseshoe vortices, separated shear layers, and disturbances caused by features on the turret surface form a complex aerodynamic environment around the turret that is highly unsteady. This presents a considerable challenge in predicting the flow field and in designing flow control techniques to mitigate aero-optical effects [34, 110].

The first goal of the research contained in this dissertation is to improve the fundamental understanding of aero-optic distortion dynamics. This is accomplished by extending classic results for the spectral behavior of electromagnetic wave propagation through the turbulent atmosphere to accommodate the unique qualities of aero-optic distortions. Secondly, using high-fidelity computational fluid dynamics (CFD), a free shear layer analogous to a separated shear layer from an optical transmission platform in flight will be used to validate theoretical results and characterize a fundamental aero-optic flow. The third goal of this research is to use what we learned from the first two parts to improve the understanding of the aerodynamic and aero-optic behavior of the hemisphere-on-cylinder turret in subsonic flow. Using wall-modeled large-eddy simulation (LES), the compressible flow field around the
turret is computed to study the unique aero-optical and flow features of this canonical optical platform in flight and to explore topics important to the design of aero-optic mitigation strategies.

1.2 Aero-Optics Fundamentals

Aero-optics, the distortion of light due to the presence of turbulent flow, can be directly attributed to the spatially and temporally evolving density field in compressible flows. Connected through the Gladstone-Dale relation [27], these density fluctuations cause local changes in the index of refraction, \( n \), as

\[
n(x,t) = 1 + K_{GD} \rho(x,t),
\]

where \( x \) is the position vector of flow field, \( \rho \) is the density of air, and \( K_{GD} \) is the Gladstone-Dale coefficient, which can be taken as a constant across the visible and near-visible optical wavelengths of interest. An initially collimated beam passing through a region of varying refractive index will become distorted due to difference in the local speed of light. These optical distortions are typically measured in terms of the optical path length (OPL) which can be found by integrating the index of refraction through the beam propagation path, \( s \),

\[
\text{OPL}(x',y',t) = \int_{s_0}^{s_1} n(x,t)ds,
\]

where \( x' \) and \( y' \) are coordinates in the plane perpendicular to the optical beam.

As light is composed of electromagnetic waves, the propagation of an optical beam is governed by Maxwell’s equations. By making reasonable assumptions that time scales of the flow are much larger than those of the optical transmission and that the smallest spatial scale of the turbulence is much larger than the optical wavelength,
Maxwell’s equations for a continuous beam at a single frequency can be reduced to the Helmholtz equation [110],

\[ \nabla^2 U + \frac{\omega^2 n^2}{c_0^2} U = 0. \]  

(1.3)

In Eqn. (1.3), \( U \) is a scalar component of the electric field at a frequency \( \omega \) and \( c_0 \) is the speed of light in a vacuum. Assuming that beam local coordinates are aligned with the flow coordinate system for simplicity, a solution of the Helmholtz equation of the form \( U(x, y, z) = U_0(x, y, z) \exp(-ikz) \) can be sought, where \( k = \omega/c_0 \) is the free-space optical wavenumber. Because refractive index fluctuations in aero-optical applications are weak and caused by turbulent flow scales much larger than the optical wavelength, the optical beam can be assumed to propagate entirely in the axial (z) direction with a slowly changing amplitude [110]. In addition, assuming that the region where aero-optical distortions occur is small, it can be shown that diffraction effects are very weak across the aero-optic turbulence region. As such, terms corresponding to diffraction effects can be ignored [112]. The resulting equation after these paraxial and weak-diffraction assumptions are made can be integrated through the turbulence region (given as \( z = 0 \) to \( z = L \)) to obtain

\[ U(x, y, L) = U_0(x, y, 0) \exp \left( -ik \int_0^L n(x, y, z)dz \right). \]  

(1.4)

Eqn. (1.4) shows that the aero-optic region of the flow causes changes to the local beam phase while the amplitude term, \( U_0 \), remains unchanged. Upon inspection, the integrand in Eqn. (1.4) is the OPL as described in Eqn. (1.2) where \( ds = dz \) since diffraction is weak.

The relative difference in optical path length over a beam cross-section is called the optical path difference (OPD) and is found by removing the spatial average (denoted...
here by angle brackets) of OPL at each time instant,

\[ \text{OPD}(x, y, t) = \text{OPL}(x, y, t) - \langle \text{OPL}(x, y, t) \rangle. \]  

(1.5)

The term given by \( k\text{OPD} \), or \( 2\pi\text{OPD}/\lambda_{\text{opt}} \) where \( \lambda_{\text{opt}} \) is the optical wavelength, is called the optical phase distortion, \( \phi' \), and it describes the relative phase lag or lead of a beam compared to the mean phase. Furthermore, OPD can be decomposed into three components: steady lensing, tip/tilt, and a high-order distortion. Steady lensing is the time-average of the OPD and represents the effect of the mean density nonuniformity. Tip/tilt is the linear, planar component of the OPD that produces beam-jitter effects, deflecting the entirety of the beam off the pointing direction. The tip/tilt components are usually determined together with the spatial average of the OPL or piston, \( \langle \text{OPL}(x, y, t) \rangle \), using a least-square surface fitting method. By evaluating the values of \( A, B, \) and \( C \) in a least-square minimization of the function

\[ G = \int \int_{A_p} [\text{OPL}(x, y, t) - (A(t)x + B(t)y + C(t))]^2 dx dy \]  

(1.6)

at each time instant, beam tip, tilt, and piston, \( A(t)x + B(t)y + C(t) \), respectively, can be determined. The high-order distortions that remain after the steady lensing and tip/tilt have been removed from OPD characterize the effect of sub-aperture scale turbulence on the shape and intensity of the beam phase distortion.

When the size of the aperture is much larger than the length scales of the turbulence, the so called large-aperture approximation can be used. The large aperture approximation provides a relationship between the ratio of on-target irradiance of an aberrated beam to the irradiance of a diffraction limited beam, called the Strehl ratio, SR, and \( \text{OPD}_{\text{rms}} \), the spatial root-mean-square (rms) of the higher-order OPD.
The large-aperture approximation is given as:

\[ \frac{\overline{SR}}{I_0} \approx \exp \left[ -\left( \frac{2\pi \text{OPD}_{\text{rms}}}{\lambda_{\text{opt}}} \right)^2 \right] = \exp \left[ -\overline{\phi}_{\text{rms}}^2 \right], \quad (1.7) \]

where the overbar denotes the time mean of a quantity. Eqn. (1.7) is in the time-averaged form as suggested by Steinmetz [89]. The approximation of Strehl ratio from phase distortion was originally proposed by Marechal [59] and modified by Mahajan [53, 54] for instantaneous quantities. As the expression indicates, for a given \text{OPD}_{\text{rms}}, a beam’s sensitivity to aero-optical effects is strongly dependent on the optical wavelength and is more severe for shorter wavelengths. While the applicability of the large-aperture approximation may be questionable for beams with non-Gaussian OPD distributions or large phase distortions, it remains a powerful tool to relate properties of the optical wavefront to the far-field beam distortion [57, 75].

Additional fundamental information about the physics of aero-optic distortions will be presented in Chapter 2 as a part of the development of theoretical descriptions of aero-optic distortions.

1.3 Previous Aero-Optics Research of Interest

In this section, previous research pertaining to the goals defined for the present research will be review. First, prior theoretical research into aero-optics, particularly the spectral behavior of optical distortions, will be presented and will be followed by the body of research into the aero-optics of shear layers. Then, previous experimental and numerical studies of the aero-optics and fluid dynamics of subsonic hemisphere-on-cylinder turrets will be discussed.
1.3.1 Theoretical Work in Aero-Optics

As mentioned in the opening of this chapter, the study of electromagnetic wave propagation through a turbulent media has an extensive history but the focus of previous studies is dominated by atmospheric turbulence. In studies of optical distortions caused by the atmosphere, a key and seemingly ubiquitous assumption is that local changes in the index of refraction are dominated by fluctuations in temperature (and humidity in some applications) and that the effects of pressure changes are negligible, which appears to be valid for the atmosphere [38]. In aero-optic flows, however, the effects of aerodynamics and compressibility enhance turbulent pressure fluctuations and it would be inappropriate to neglect these effects [83].

Theoretical research into aero-optic distortions has historically focused on time-mean statistical descriptions of the optical phase distortion. One of the most notable contributions in theoretical aero-optics is Sutton’s linking equation which relates average phase distortions to the statistical properties of the turbulent density field [91]. The linking equation provides a simple relationship between the local aerodynamic field and the optical distortions but it does not provide information about the dynamics of the distortion itself. A more extensive presentation of early aero-optics theory and experiments up to 1985 can be found in the review article by Sutton [92]. However, in this review little is stated about the dynamics of optical distortions, potentially stemming from sensor limitations at the time of the article.

The high sampling rate needed to probe aero-optic distortions constrained researchers until the late 1980’s when the first instrument described by Malley et al. [55] was developed. While these first devices only resolved a single, small-aperture beam, further study of aero-optics was enabled by the development of high speed, two-dimensional Shack-Hartmann sensors for aero-optics [20, 32]. With these tools, the modern field of aero-optics has emerged where high fidelity, time-resolved measurements of flows allows researchers to non-intrusively probe compressible flow
fields, study large-scale turbulence structures, and develop potential adaptive-optics schemes for airborne optical transmission systems. Reviews of modern aero-optic applications and theory can be found by Jumper & Fitzgerald [41], Wang et al. [110], and Jumper & Gordeyev [32]. The latter also provides some historical context to the study of aero-optics over the past several decades.

While time-resolved measurement techniques have been developed and extensively used, little has been done to extend the classic results for the spectral behavior of electromagnetic wave propagation through the turbulent atmosphere to aero-optic distortions. While the power spectral density of optical distortions has been presented previously for subsonic boundary layers [35, 84, 108], supersonic boundary layers [31], shear layers and jets [21, 17], and bluff body flows [109], not much has been done to explain the spectral behavior within a well defined theoretical framework. An exception is the research performed by Gao et al. [24] where they attempted to identify the universal behavior of the phase distortion structure functions in optical transmissions through the supersonic boundary layer. The lack of a theoretical description for the spectral behavior of aero-optic distortion dynamics and the inapplicability of theory for atmospheric optical distortions to describe them drives the need to extend well known theories of optical wave propagation to the study of beam propagation through an aero-optic field.

1.3.2 Investigations of Shear Layer Aero-Optics

The optical distortion caused by a turbulent free or separated shear layer has been an important topic in the study of aero-optics. The shear layer flow structure can be found in a number of aero-optical devices including laser cavities and optical turrets of various shapes. While the basic fluid dynamics of a compressible shear layer has been thoroughly investigated and reasonably well established for a number of years both experimentally [7, 73] and numerically [72], the aero-optic distortion
mechanism in weakly compressible shear layers was not correctly identified until the work by Fitzgerald & Jumper [18, 19] in the early 2000s. Prior to their research, it was presumed that the effects of pressure fluctuations were negligible compared to temperature fluctuations as is the case in atmospheric turbulence [79]. The opposite has shown to be true in several cases. The notion that pressure fluctuations are primarily responsible for aero-optical aberrations in compressible shear layers has been verified in several numerical investigations of spatially developing separated shear layers using computational fluid dynamics [81, 101, 109] and numerous experimental investigations since the work of Fitzgerald and Jumper. A very thorough review of past and present studies of shear layer aero-optics and several other aero-optic flows can be found in the review paper by Jumper & Gordeyev [32].

Most experimental aero-optic shear layer investigations have been conducted using a single gas as would be expected in nearly all airborne applications. However, a series of investigations into subsonic shear layers with two different free-stream gases (and/or two streams with different freestream indexes of refraction) have been made by Dimotakis et al. [17] and others [13, 119]. In these experiments, Rayleigh scattering is used to capture instantaneous index-of-refraction fields which are then integrated along the beam-propagation path to calculate OPD. Even with flow streams with two different refractive index coefficients, results from Dimotakis et al. agree with Fitzgerald & Jumper that large scale shear layer structures are the driver of dominant optical distortions. Additionally, they reported a region of streamwise wavenumber phase-distortion spectrum in the turbulent shear layer with a power law slope of $-2$ but the presence of a clear power-law region appears unclear in the figures. In the same paper, the optical distortion of a two-gas jet showed a spectral power law slope of $-2.5$ which is close to the value of $-8/3$ predicted by Tatarski for the turbulent atmosphere [93].

Measurements of aero-optic distortions by transitional supersonic mixing layers
using a similar method were also made by Gao et al. [23] by measuring the density field using nano-based planar laser scattering (NPLS). In their work, several statistics pertaining to OPD were presented including the second-order structure function of OPD. They found a log linear region in their structure function calculations with a slope near 1.1, translating to a spectral slope of $-2.1$, close to the results presented by Dimotakis et al. While this may seem to be inconsistent with the classic $5/3$ slope of second-order OPD structure function by Tatarski, upon inspection of instantaneous NPLS images presented in their paper it is evident that the shear layer is still undergoing transition in large portions of the window in which the structure function is computed, and thus the validity of the comparison to a fully developed turbulent field is questionable.

Complete understanding of the aero-optical distortion in turbulent shear layers is an ongoing endeavor but a few important conclusions can be drawn. The dominant distortion mechanism in all known cases appears to be linked to the large scale roller structures in the mixing layer. Closest to most airborne aero-optic applications, experiments and simulations with a single gas demonstrate that these structures create large pressure wells that alter the density field and create large, coherent optical distortions coincident with the roller vortex structure. In mixed gas experiments where the freestream index of refraction of the gases are different, the large scale mixing of the structures creates interfaces of different index of refraction as the gases actively mix between the two streams.

The spectral behavior of optical distortions in a compressible shear layer still remains to be well defined in terms of the inertial range behavior and to be linked to results from theoretical research. Additionally, the combined effects of two different freestream values of index of refraction and turbulent pressure fluctuations have not been thoroughly studied to understand how they affect the aero-optic distortion at large.
1.3.3 Experimental Research in Hemisphere-on-Cylinder Turret Aero-Optics

The body of literature on the aerodynamics of hemisphere-on-cylinder turrets and the resulting aero-optics contains a large volume of experimental work. A summary of early research compiled by Gilbert and Otten [26] describes work conducted before the early 1980’s when larger wavelength gas-dynamic lasers were the primary high powered lasers used in aero-optics applications. The large wavelength combined with a lack of high frequency wavefront sensors forced research in this era to focus on time-averaged optical distortions. As expected with larger wavelength lasers, phase distortions were much smaller than those seen with modern smaller wavelength beams.

More recent work up to 2010 was discussed in a review paper by Gordyev and Jumper [30] and showcased findings for hemisphere-on-cylinder and hemispherical turrets in multiple configurations and at several Mach numbers. These more recent works were able to utilize 2-D wavefront sensors that can measure instantaneous wavefronts up to 100 kHz with very good accuracy. This has enabled aero-optics researchers to directly relate beam distortions to fluid phenomena and even gain insight into important aerodynamic properties [2, 36]. Wind tunnel surface visualization of a turret performed by Gordeyev et al. [33] helped establish the flow topology that is known today with horseshoe vortices rolling up in front of the turret base, horn vortices forming in the separated wake, and the unsteady separation of the boundary layer over the top of the turret. An illustration of the flow structures is shown in Figure 1.1.

Static pressure measurements in the centerline of the turret were also collected at several Mach numbers and at Reynolds numbers near 2.3 million [33]. The data showed turbulent boundary layer separation at approximately 115° along the centerline of the turret. Later, DeLucca et al. [16] used unsteady pressure measurements on the surface of the turret collected from pressure taps and pressure-sensitive paint
to conclude that global pressure fluctuations due to large flow structures can strongly affect the unsteady jitter of the beam through flow induced vibration of the turret beamtrain. The bulk of the aero-optical data on hemisphere-on-cylinder turrets in the last decade can be directly attributed to the Airborne Aero-Optics Laboratory (AAOL) at the University of Notre Dame and its associated projects [42]. From the in-flight data collected, the viewing angle dependence of OPD for turrets in subsonic flight has been established [49, 50] and new methods to analyze optical wavefronts have been developed [1].

There has been limited work investigating the role of realistic turret features on the flow field beyond the effects of flat versus conformal windows and the effects of different cylinder height to diameter ratios. Research into the effects of the smaller turret features like those on the AAOL turret, shown in Figure 1.2, is restricted to recent work by Gordeyev et al. [35] and DeLucca et al. [16] but neither explored the aero-optical impact beyond beam jitter effects. Both Gordeyev et al. and DeLucca et al. showed that the presence of small geometric features can induce small vortical
structures that cause local pressure fluctuations. These features can be seen as analogous to flow control devices, which have been investigated extensively by Vukasinovic et al. [103, 104, 105, 106, 34]. Using passive, active, and hybrid passive/active flow control, these works have shown improvement in the aero-optical environment around turrets.

In all, previous experimental research has provided extensive information about the characteristics of the flow around hemisphere-on-cylinder turrets and the trends of the aero-optical distortion at different flow conditions and viewing angles. There is also some evidence that suction, blowing, or passive control devices on the turret surface can have considerable impact on the flow and the resulting aero-optics.
1.3.4 Computational Research in Aero-Optics

Numerical studies of aero-optics began in the late 1980’s and early 1990’s but due to a lack of computational power, early research into flows at realistic engineering Mach and Reynolds numbers used simulation methods with limited suitability for aero-optical predictions such as RANS and Euler methods [12, 98, 85]. Studies using higher fidelity methods like direct numerical simulation (DNS) and large-eddy simulation (LES) began shortly after to study simple flow topologies like free shear layers and channels flows [97, 96, 14]. Because of their ability to resolve optically relevant flow structures in typical aero-optical flows [58] and low computational cost relative to DNS, LES and hybrid RANS/LES methods have emerged as the most popular methods for simulating aero-optical flows. In the last ten years, LES based methods have been used to study turbulent shear layers, open-cavity flows, turbulent boundary layers, flow over a cylinder, flow over cylindrical turrets, flow over realistic three dimensional turrets, and the optical distortions by a helicopter tip vortex [44]. A full review of computational aero-optics as of 2012 is provided by Wang et al. [110].

The first use of computational methods to investigate the optics of shear flows was performed by Truman & Lee [97] who used DNS data of a incompressible shear flow and modeled the index of refraction as a passive scalar. While treating $n$ as a passive scalar does not capture the true behavior of the refractive index, they were able to identify the importance of turbulence anisotropy in the rms of phase distortions in relation to the direction of beam propagation. In the previously mentioned work of Fitzgerald & Jumper [19], a physics model called the weakly-compressible model (WCM) was coupled with a discrete-vortex method simulation of a spatially developing shear layer to analyze optical distortions. The WCM allows for the thermodynamic quantities of a weakly-compressible flow to be deduced from the velocity field using the Euler equations and a simplified version of the energy equation. They showed that their model predicted the experimentally observed aero-optical aberrations.
tions for a subsonic shear layer when other models failed to do so accurately.

Using implicit LES (ILES) with high-order numerical methods, Visbal investigated the optical distortion by a number of spatially developing shear layers including 2D matched and mismatched temperature shear layers, 3D laminar and transitional shear layers, and 3D forced shear layers [101]. In all shear layer configurations, Visbal found that the optical distortion is dominated by effects of the pressure wells in the shear layer structures but did not scrutinize the statistical behavior of optical distortions in the flows. In his simulation of 2D shear layers with matched total temperature, Visbal noted a sharp density gradient across the braid region of the shear layer that were also seen in investigations by Rennie et al. [76] when a 2D shear layer was simulated using the COBALT flow solver. This feature is not seen when the WCM is used to compute the density field and has not been observed in experiments. It was argued by Visbal that this feature is a result of the transient pressure source term in the total temperature equation that was assumed to be small in the WCM. This difference between the two methods and the behavior of the total temperature in the shear layer is still an active area of discussion.

Previous numerical investigations into the fluid dynamics and aero-optical environment around a smooth, canonical hemisphere-on-cylinder turret have generated mixed results. Nahrstedt et al. [71] used a Partially-Averaged Navier-Stokes (PANS) method with a $k$-$\varepsilon$ turbulence model to simulate the flow over an optical turret matching the experimental set-up in Gordeyev et al. [33] However, the optical distortions based on the PANS solutions were consistently several times smaller than experimental results. Ladd et al. [47] studied the same turret configuration using an unsteady Reynolds-Averaged Navier-Stokes (uRANS) approach and Detached Eddy Simulation (DES), and observed improvement in both flow field and optical results but showed a strong dependency on the type of RANS model and the mesh resolution used.
Morgan and Visbal [63, 64] used hybrid RANS and ILES to simulate the flow in the same experimental configuration and another hemisphere-on-cylinder turret. While the flow structures were much more adequately resolved than previous simulations, the optical distortions calculated were still quite different from experimental results. Using the same computational approach and geometry, White et al. [112] found steady lensing and tip/tilt removed OPD_{rms} to be much smaller than experimental values.

There have also been computational investigations into the use of flow control to improve the aero-optical environment around a turret. Morgan and Visbal [65, 66, 67] computationally studied the role of oscillatory blowing/suction and steady suction through both a slot and a porous surface for conformal and flat windows using the previously mentioned hybrid RANS/ILES method. They observed a significant delay in separation and reduction in wake size when using suction, but optical results were not discussed. Overall, previous hemisphere-on-cylinder turret aero-optical predictions using hybrid RANS/LES methods have not provided consistent and satisfactory results. The reasons for this are likely due to their inability to resolve all of the optically important flow scales or accurately predict the boundary-layer separation and wake dynamics.

1.4 Computational Methods

To successfully study turbulent flow and aero-optics using computational approaches, a simulation must be able to accurately resolve the unsteady flow field and account for momentum and energy interactions across all relevant scales of the turbulence. While there are several methods of solving the fluid equations at varying levels of fidelity, the unsteady nature of aero-optics rules out traditional Reynolds averaged Navier-Stokes methods (RANS) as these formulations are designed to provide mean flow quantities only. Similarly, unsteady RANS (uRANS) approaches are
unfit to resolve a range of realistic turbulence structures central to aero-optics since the method involves ensemble averaging and can only capture the largest-scale motions. At the the finest level of resolution, DNS resolves the flow field down to the Kolmogorov scales and uses no additional modeling to account for flow physics, but is often far too computationally expensive for flow conditions commonly found in aero-optical systems because of the large scale-separation at high Reynolds numbers.

Mani et al. [58] demonstrated theoretically that the level of resolution required to accurately capture aero-optical effects is less than that for direct numerical simulation. This places the most effective simulation methods for aero-optics in the realm of LES and combinations of LES with RANS elements: wall-modeled LES and DES. Large-eddy simulation solves a spatially filtered version of the full set of flow equations and provides modeling to account for the effect of scales smaller than those resolved by the computational grid. LES has shown to be an effective technique to compute turbulent flows out of the reach of DNS where the separation of the largest to smallest scales of a flow becomes too great.

While typically much less costly than DNS, even LES can become exceedingly expensive computationally for high Reynolds number, wall-bounded flows, where the smallest energetic scale in the near-wall region shrinks with increasing Reynolds number. In these cases, hybrid RANS-LES methods like wall-modeled LES and DES have become very useful. Compared to the fully-resolved LES where no additional modeling is provided for the effect of walls on the flow, DES and wall-modeled LES incorporate RANS elements in varying degrees to reduce the computational cost. There are several styles of DES in the literature, including delayed detached eddy simulation (DDES) and improved delayed detached eddy simulation (IDDES), but the general philosophy is to use RANS in near-wall regions of the flow, which generally include the entire attached boundary layer, and LES in separated flow regions where the turbulence needs to be resolved accurately [86].
Wall-modeled LES solves the filtered fluid dynamics equations everywhere on a computational mesh that does not adequately resolve the near wall region, where a RANS type model is used to provide approximate wall boundary conditions to account for the effects of the unresolved near-wall turbulence dynamics. This approach relaxes the resolution restriction in the inner boundary layer while still resolving turbulence structures in the outer boundary layer. DES tends to be less expensive than wall-modeled LES, but turbulent fluctuations in the boundary layer, which can affect flow separation and the subsequent development of the separated shear layer, are lost due to the RANS treatment in DES. This restriction can cause significant deviations from the actual flow and large errors in optical predictions.

As mentioned previously, White et al. [112] used DES to simulate the flow around a hemisphere-on-cylinder turret but noted that there could be “potential limitations of hybrid [RANS-LES] approaches due to the lack of large scale structures in the area where RANS is active.” Wall-modeled LES can overcome these difficulties since RANS is only applied in the near-wall region (typically less that 5% of the boundary layer) which does not contribute significantly to optical distortions [108]. The bulk of the boundary layer is computed with LES and contains realistic turbulence structures. In the present work, LES and wall-modeled LES are used to simulate the flow field and aero-optics of temporally evolving shear layers and hemisphere-on-cylinder turrets, respectively.

1.4.1 Flow Solver

To compute the flow, a compressible, unstructured mesh solver developed by Cascade Technologies Inc. named CharLES is used [46]. As mentioned previously about LES methods, CharLES solves the spatially filtered, compressible, Navier-Stokes equations:

\[
\frac{\partial \hat{\rho}}{\partial t} + \frac{\partial \hat{\rho} \hat{u}_i}{\partial x_i} = 0, \tag{1.8}
\]
\[
\frac{\partial \hat{\rho} \tilde{u}_i}{\partial t} + \frac{\partial}{\partial x_j} (\hat{\rho} \tilde{u}_i \tilde{u}_j + \hat{p} \delta_{ij}) = \frac{\partial \tilde{T}_{ij}}{\partial x_j} - \frac{\partial \tau_{ij}^{SGS}}{\partial x_j},
\]
(1.9)

\[
\frac{\partial \hat{\rho} \tilde{E}}{\partial t} + \frac{\partial}{\partial x_i} \left[ (\hat{\rho} \tilde{E} + \hat{p}) \tilde{u}_i \right] = \frac{\partial \tilde{q}_i}{\partial x_i} + \frac{\partial \tilde{u}_i \tilde{T}_{ij}}{\partial x_j} - \frac{\partial \tilde{q}_i^{SGS}}{\partial x_i},
\]
(1.10)

where, \( \rho \) is the density, \( u \) is the velocity, \( p \) is the pressure, and \( E \) is the total energy. The equations are non-dimensionalized using \( \rho_{ref}, a_{ref}, T_{ref}, \mu_{ref}, \) and \( L_{ref} \), reference values of density, sound speed, temperature, dynamic viscosity, and length scale. The hat represents spatial filtering over a domain \( \Omega \) by a convolution kernel \( G \) with a filter width \( \Delta \),

\[
\hat{f}(x) = \int_{\Omega} G_{\Delta}(x - x'') f(x'') dx'',
\]
(1.11)

and the tilde represents Favre filtering, given by

\[
\tilde{f} = \frac{\hat{f}}{\hat{\rho}}.
\]
(1.12)

The filtered total energy in Eqn. (1.10), \( \hat{\rho} \tilde{E} \), is the combination of internal and kinetic energy of the fluid,

\[
\hat{\rho} \tilde{E} = \frac{\hat{p}}{\gamma - 1} + \frac{1}{2} \hat{\rho} \tilde{u}_k \tilde{u}_k.
\]
(1.13)

The filtered heat flux term, \( \tilde{q}_i \) is given by

\[
\tilde{q}_i = \frac{\mu}{(\gamma - 1) Re Pr} \frac{\partial \tilde{T}}{\partial x_i},
\]
(1.14)

where \( \gamma \) is the specific heat ratio, \( Re = \rho_{ref} c_{ref} L_{ref}/\mu_{ref} \) is the reference Reynolds number, \( Pr = \mu_{ref} c_p / k \) is the Prandtl number, and \( \tilde{T} \) is the filtered temperature.

The filtered viscous stress tensor, \( \tilde{\tau}_{ij} \), is given by

\[
\tilde{\tau}_{ij} = \frac{\mu}{Re} \left( \frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} - \frac{2}{3} \frac{\partial \tilde{u}_k}{\partial x_k} \delta_{ij} \right).
\]
(1.15)

To close the system of equations, the ideal gas law, \( \hat{p} = \hat{\rho} \tilde{T} / \gamma \) is used for the pressure.
and the dynamic viscosity, \( \mu \), is found as a function of \( \tilde{T} \) via the expression
\[
\mu = \mu_{\text{ref}} \tilde{T}^{0.76}.
\]
The two subgrid-scale (SGS) quantities, the subgrid stress tensor, \( \tau_{ij}^{SGS} \), and the subgrid heat flux vector, \( q_i^{SGS} \), are a result of spatial filtering nonlinear terms and are defined as
\[
\tau_{ij}^{SGS} = \hat{\rho} \left( \tilde{u}_i \tilde{u}_j - \tilde{\bar{u}}_i \tilde{\bar{u}}_j \right),
\]
\[
q_i^{SGS} = \hat{\rho} \left( \tilde{u}_i \tilde{T} - \tilde{\bar{u}}_i \tilde{\bar{T}} \right).
\]
These terms account for the effects of small-scale flow physics that occur below the filter width \( \Delta \) and need to be modeled. In CharLES, the Vreman [102, 118] model is used to model the effects of sub-grid scale motions.

CharLES is a fully explicit solver that utilizes a third order Runge-Kutta method in time and a low-dissipative finite-volume scheme in space. To enable simulations of turbulent flows with complex geometries, a novel blending method is used to combine non-dissipative central flux with a dissipative upwind flux to provide computational stability when the computational mesh quality is not ideal. The percent of upwind flux used in the solver varies spatially depending on the local quality of the mesh so that the numerical dissipation due to the use of an upwind scheme is minimized.

This flux blending method is constructed by first writing Eqns. (1.8)-(1.10) in the finite-volume formulation:
\[
\frac{\partial \mathbf{U}}{\partial t} V_{cv} + \sum_f \left( F_e + F^d \right) A_f = SV_{cv},
\]
where \( \mathbf{U} = \left[ \hat{\rho}, \hat{\rho} \tilde{\bar{u}}_i, \hat{\rho} \hat{E} \right] \) is the vector of variables representing the average state in the control volume, \( f \) denotes the face of a control volume, \( V_{cv} \) is the cell volume, \( F^e \) is the face-normal inviscid (Euler) flux, \( F^d \) is the face-normal diffusive flux, \( A_f \) is the face area, and \( S \) is a source vector. To begin calculating the Euler flux across each face of a control volume, left and right biased states at the face are calculated using
a 3\textsuperscript{rd} order polynomial reconstruction. The reconstruction of the data in this way allows for the solver to be nominally 2\textsuperscript{nd}-order central difference in space when the left and right states are averaged and sufficiently biased to allow for an upwinding scheme to introduce dissipation. In addition, CharLES’ flux construction method recovers to a 4\textsuperscript{th}-order central difference scheme in areas where the mesh is uniform and Cartesian.

The Euler flux is then found by blending the previous non-dissipative central flux and a dissipative upwind flux via

\[ F^e = (1 - \alpha) F_{central} + \alpha F_{upwind}, \]

where \( \alpha \) is a blending parameter that ranges from 0 to 1 and the upwind flux is found using an HLLC approximate Riemann solver [95]. The value of \( \alpha \) depends on the skew-symmetry of the differencing operator, \( D \), that is found from the flux reconstruction. Since a stable and non-dissipative differencing operator is skew-symmetric, the ability of the reconstructed states to form a stable non-dissipative differencing operator can be assessed by analyzing \( D \). If \( D \) is perfectly skew-symmetric, \( D + D^T = 0 \), denoting a stable central-difference scheme. CharLES uses the row-norm of \( D + D^T \) to determine a value for \( \alpha \),

\[ \alpha = c \| D + D^T \|, \]

where \( c = 2 \) is a constant found via numerical tests. The advantage of this scheme is that dissipation is only added in regions where the mesh quality is bad and stability of the simulation is improved for complex flows. However, this requires a high quality mesh to achieve a simulation with little added artificial dissipation. A more thorough description of the CharLES numerical foundation can be found in Khalighi et al. [46].

CharLES is parallelized using MPI and has been shown to exhibit excellent parallel efficiency on large computer clusters. Its scalability was utilized in the first CFD
simulation to use more than one million computing cores [70].

1.4.2 Aero-Optics Calculation

To calculate aero-optic distortions, separate beam grids are embedded in the LES mesh for each propagation direction. At each time step when the OPL is calculated, the density is interpolated from the LES mesh onto the beam mesh using a second-order gradient based method,

$$\rho(x_i) = \rho(x_{cv}) + \frac{\partial \rho}{\partial x} \bigg|_{x_{cv}} \cdot (x_i - x_{cv}),$$  \hspace{1cm} (1.21)

where $x_{cv}$ is the location of the control volume centroid within which the interpolation point $x_i$ lays. The refractive index is then computed and integrated along the beam path according to Eqns. (1.1) and (1.2).

By having each processor only interpolate and integrate points that are local to its region of the domain, using the same beam grid for each viewing angle, and using separate processors to write optical results, hundreds of viewing angles can be computed efficiently even with large beam grids. The aero-optics calculation has been parallelized so that all angles require only a single collective communication to compute the OPL, leading to good parallel scaling. In Figure 1.3, the scaling for the calculation of 268 viewing angles with increasing number of processors is shown. Each beam grid contains about 5.4 million points for a total of over 1.6 billion points for all angles. The optics calculation currently scales very well up to 4096 cores where the effect of scaling begins to saturate.
Figure 1.3. Mean time to compute 268 optical beams, each with 5.4 million points on the beam grid, with increasing number of processors: □, actual scaling; ○, ideal scaling.
CHAPTER 2

THEORETICAL DESCRIPTION OF THE SPECTRAL BEHAVIOR OF AERO-OPTIC DISTORTIONS

In this chapter a general expression for turbulent index-of-refraction fluctuations in aero-optic flows is derived in terms of pressure and temperature fluctuations, and then combined with wave propagation theory to describe the spectral behavior of aero-optic distortions. In the first section, an approximation of the index-of-refraction spectrum will be constructed starting from the ideal gas law and Gladstone-Dale relation. It will then be applied to find an approximation for the spectrum of phase distortions for a beam propagating through an aero-optic field. Using the theoretical results, the remainder of the chapter will include discussions pertaining to OPD_{rms} with and without simple adaptive optic corrections, wavefront sensor resolution requirements, and numerical simulation resolution requirements.

2.1 Derivation of the Refractive Index Spectrum

As mentioned in Section 1.2, the index of refraction for air can be related to its density via the Gladstone-Dale relation, Eqn. (1.1). Density can be related to the temperature and pressure of air via the ideal gas law, \( \rho = \frac{p}{RT} \), where \( R \) is the specific gas constant for air. Combining the two equations, an expression for the index of refraction of air can be formulated based on the temperature and pressure,

\[
n = 1 + K_{GD} \frac{p}{RT}.
\]  

(2.1)
Any time-varying quantity \( g(t) \) can be decomposed into the sum of its time mean, \( \overline{g} \), and fluctuating component, \( g'(t) \), as \( g(t) = \overline{g} + g'(t) \). Applying this decomposition to time variable quantities in Eqn. (2.1) leads to

\[
\overline{n} + n' = 1 + K_{GD} (\overline{\rho} + \rho') = 1 + K_{GD} \overline{\rho} \left( \frac{1 + \rho'/\overline{p}}{1 + T'/\overline{T}} \right).
\] (2.2)

Assuming that \( T'/\overline{T} \) is small, to first order

\[
\frac{1 + \rho'/\overline{p}}{1 + T'/\overline{T}} \approx 1 + \frac{\rho'}{\overline{p}} - \frac{T'}{\overline{T}}.
\] (2.3)

As a result, the Gladstone-Dale relation can be approximated as

\[
\overline{n} + n' = 1 + K_{GD}(\overline{\rho} + \rho') \approx 1 + K_{GD} \overline{\rho} \left( 1 + \frac{\rho'}{\overline{p}} - \frac{T'}{\overline{T}} \right).
\] (2.4)

Removing the time mean of Eqn. (2.4) from itself, the fluctuating component of index of refraction is

\[
n' = K_{GD}\rho' \approx K_{GD} \overline{\rho} \left( \frac{\rho'}{\overline{p}} - \frac{T'}{\overline{T}} \right).
\] (2.5)

The three-dimensional autocorrelation function of the index of refraction is defined as

\[
\mathcal{R}_{nn}(\mathbf{x}_0, \mathbf{r}) = n'(\mathbf{x}_0, t) n'(\mathbf{x}_0 + \mathbf{r}, t),
\] (2.6)

where \( \mathbf{x}_0 = x_0 \hat{i} + y_0 \hat{j} + z_0 \hat{k} \) is the point about which the correlation function is defined and \( \mathbf{r} = r_x \hat{i} + r_y \hat{j} + r_z \hat{k} \) is the displacement vector relative to \( \mathbf{x}_0 \). Assuming that the flow field is statistically homogenous and isotropic, the location of \( \mathbf{x}_0 \) is arbitrary and using Eqn. (2.5), \( \mathcal{R}_{nn} \) can be expanded as,

\[
\mathcal{R}_{nn}(\mathbf{r}) = K_{GD}^2 \mathcal{R}_{\rho\rho}(\mathbf{r})
\approx (K_{GD}\overline{\rho})^2 \left( \frac{\mathcal{R}_{\rho\rho}(\mathbf{r})}{\overline{\rho}^2} + \frac{\mathcal{R}_{TT}(\mathbf{r})}{\overline{T}^2} - \frac{\mathcal{R}_{TP}(\mathbf{r})}{\overline{\rho}\overline{T}} - \frac{\mathcal{R}_{pT}(\mathbf{r})}{\overline{p}\overline{T}} \right).
\] (2.7)
Here $R_{T_p}$ and $R_{pT}$ are the temperature-pressure and pressure-temperature cross-correlation functions, respectively, defined as

$$R_{gh}(x_0, r) = g'(x_0)h'(x_0 + r), \quad (2.8)$$

where $g'$ and $h'$ are two fluctuating quantities. When the flow is statistically homogeneous and isotropic, $R_{gh}(r) = R_{hg}(-r)$. To find the three-dimensional wavenumber spectrum of the turbulent refractive index, $\Phi_{nn}$, Fourier transform is taken in all three spatial dimensions to obtain

$$\Phi_{nn}(K) = \left(\frac{1}{2\pi}\right)^3 \int\int\int_{-\infty}^{\infty} R_{nn}(r)e^{i(K\cdot r)}dr, \quad (2.9)$$

where $K = \kappa_x \hat{i} + \kappa_y \hat{j} + \kappa_z \hat{k}$ is the spatial wavenumber vector. Applying the Fourier transform to Eqn. (2.7), one obtains

$$\Phi_{nn}(K) = K^2 G_D \Phi_{pp}(K)$$

$$\approx (K G_D \bar{p})^2 \left(\frac{\Phi_{pp}(K)}{\bar{p}^2} + \frac{\Phi_{TT}(K)}{\bar{T}^2} - \frac{2 \text{Re} [\Phi_{pT}(K)]}{\bar{p} \bar{T}}\right). \quad (2.10)$$

It should be noted that the real parts of the temperature-pressure and pressure-temperature cross-spectra, also termed cospectra, are equal ($\text{Re} [\Phi_{pT}(K)] = \text{Re} [\Phi_{Tp}(K)]$) and the sum of their imaginary parts is zero ($\text{Im} [\Phi_{pT}(K)] + \text{Im} [\Phi_{Tp}(K)] = 0$) due to the symmetry of the sum of their correlation functions.

This result for the refractive index spectrum is very similar in form to expressions found by Friehe et al. [22], Hill et al. [38], and McBean & Elliott [61] in their investigations of index-of-refraction fluctuations in the atmosphere. In the first two of these investigations, however, the pressure was appropriately taken to be negligible in comparison to the optical effects of temperature and humidity. As a result, expressions were found for the index-of-refraction spectra as a function of fluctu-
ating temperature spectra, fluctuating humidity spectra, and temperature-humidity cospectra. McBean & Elliott wrote an expression for the refractive index spectrum using all three components (pressure, temperature, and humidity) but found that contributions from pressure fluctuations were small. Due to the conditions of typical aero-optic applications, the effects of humidity are expected to be negligible but the form of the current results are similar in composition to expressions found in these works.

Using the relationship between the three-dimensional spectra and the one-dimensional energy spectra, \( E \), for homogeneous and isotropic turbulence [4]

\[
\Phi(K) = \frac{E(\kappa)}{4\pi\kappa^2}, \tag{2.11}
\]

where \( \kappa \) is the scalar wavenumber, \( ||K|| \), we can rewrite Eqn. (2.10) in terms of the one-dimensional energy spectra,

\[
\Phi_{nn}(\kappa) \approx \left( \frac{K_{GD\rho}}{4\pi}\right)^2 \left( \frac{E_{pp}(\kappa)}{p^2} + \frac{E_{TT}(\kappa)}{T^2} - \frac{2 \text{Re} [E_{pT}(\kappa)]}{pT} \right). \tag{2.12}
\]

If it is assumed that the Reynolds number is large and the flow is only weakly compressible, the velocity statistics can be approximated by incompressible representations. To examine the inertial subrange behavior of index-of-refraction fluctuations, the pressure spectrum predicted for the inertial subrange based on Kolmogorov’s scaling will be used,

\[
\frac{E_{pp}(\kappa)}{p^2} = B_p \bar{\rho} \varepsilon^{4/3} \kappa^{-7/3}, \tag{2.13}
\]

(taken from Monin & Yaglom [62]) where \( \bar{\rho} \) is the average sound speed, \( \varepsilon \) is the turbulent dissipation rate, and \( B_p \) is a constant. Similarly, an inertial subrange spectrum of temperature can be assumed from the Kolmogorov-Obukhov-Corrsin
prediction of the 1-D temperature spectrum [111],

\[
\frac{E_{TT}(\kappa)}{T^2} = \frac{\beta \chi \varepsilon^{-1/3} \kappa^{-5/3}}{T^2} = \frac{\gamma^2}{4} R^2 \beta \chi \varepsilon^{-1/3} \kappa^{-5/3}, \tag{2.14}
\]

where \( \beta \) is a constant, \( \chi \) is the average temperature dissipation rate given as

\[
\chi = 2D_m \left( (\partial T/\partial x)^2 + (\partial T/\partial y)^2 + (\partial T/\partial z)^2 \right),
\]

and \( D_m \) is the coefficient of molecular diffusion. Combining these expressions for the pressure and temperature spectra and Eqn. (2.12), we obtain

\[
\Phi_{nn}(\kappa) \approx \left( \frac{\gamma K_{GD} D}{4\pi a^4} \right)^2 \left( B_p \varepsilon^{4/3} \kappa^{-13/3} + R^2 \beta \chi \varepsilon^{-1/3} \kappa^{-11/3} - \frac{2R}{\rho \kappa^2} \text{Re} [E_{wT}(\kappa)] \right). \tag{2.15}
\]

Clearly, Eqn. (2.15) implies that the spectral behavior of the index of refraction will depend on contributions from the pressure, temperature, and the pressure-temperature cospectra. This is a more general expression for the refractive index spectrum in aero-optic turbulence than the classic expression used in most atmospheric turbulence studies given as

\[
\Phi_{nn}(\kappa) = 0.033 C_n^2 \kappa^{-11/3}, \tag{2.16}
\]

where \( C_n^2 \) is the refractive index structure coefficient that accounts for the strength of the turbulence and its value is proportional to the variance of the index of refraction.

Comparing the dependence on wavenumber between Eqn. (2.15) and Eqn. (2.16), it is noted that the spectrum for refractive index commonly used for atmospheric turbulence only accounts for the effect of temperature fluctuations. It can be expected that the expressions are equivalent when the pressure fluctuations are negligible.

By using the more general spectral representation derived here, we can proceed to develop theoretical expressions for the phase distortion for flows where the pressure fluctuations are appreciable.
An implication of this result is that for some flow conditions, density and refractive index spectra may be dominated by either temperature or pressure fluctuations and as such, will have different spectral behaviors. This phenomenon was observed by Winarto & Davis in their studies of density fluctuations in turbulent jets [115]. When the static temperature of the jet differed from the ambient air temperature, the one-dimensional density spectrum had an inertial subrange with a slope $-m_\rho = -5/3$ indicating temperature dominant density fluctuations. When the static temperature of the jet was matched to the ambient, a spectral slope near $-7/3$ was observed, indicating that the turbulent density fluctuations were dominated by pressure fluctuations.

It is important to note that the one-dimensional spectral slopes of $-m_T = -5/3$ and $-m_p = -7/3$ mentioned above for the temperature and pressure spectra, respectively, are formulated for homogenous, isotropic turbulence at very large Reynolds numbers. Both experimentally and computationally, the spectral roll-off for temperature and pressure have been observed to be smaller when the flow conditions are different and/or the Reynolds number is moderate.

A brief literature search for the spectral behavior of thermodynamic quantities for flows in free space reveals a range of values seen for a variety of flow conditions. For the spectral slope of temperature fluctuations, grid turbulence experiments did not observe the $-5/3$ spectral slope predicted by Kolmogorov scaling until $Re_\lambda$ reached values above 500 [69]. Here $Re_\lambda$ is the Reynolds number based on the Taylor microscale defined as $Re_\lambda = u'_{\text{rms}}\lambda/\nu$, where $u'_{\text{rms}}$ is the root mean square of the turbulent velocity, $\lambda$ is the Taylor microscale, and $\nu$ is the kinematic viscosity. In shear flow experiments, a $m_T$ of 4/3 was measured at $Re_\lambda$ values near 200 and did not reach the $-5/3$ slope until $Re_\lambda$ reached values upwards of 2000 [88]. The spectral slope of pressure was founded to be $-5/3$ in DNS of homogenous and isotropic turbulence at $Re_\lambda = 235$ [100] and between $-6/3$ to $-7/3$ for $Re_\lambda = 200$ to 1200 [99], respectively,
in experimental measurements of a turbulent jet. Using DNS, Gohto & Fukayama [28] noticed very small regions with a slope of $-7/3$ starting at $Re_\lambda = 284$ but also observed that the spectrum could appear to have longer power law regions if fitted to smaller slopes.

In all, literature on free turbulent flows indicates that the spectral slope appears to be in the range of $-4/3$ to $-5/3$ for temperature and in the range of $-5/3$ to $-7/3$ for pressure depending on Reynolds number and flow conditions. The reason for the appearance of different spectral slopes at lower Reynolds numbers for temperature and pressure compared to the Kolmogorov scaling is likely multifaceted and related to the bottleneck phenomena in the Fourier transformation [111] and turbulent anisotropy [88].

2.1.1 Alternative Derivation of Temperature Spectra via the Strong Reynolds Analogy

A popular mechanism for modeling and explaining temperature effects on density fluctuations in aero-optics research has been adiabatic heating and cooling of flow structures [117, 35, 37]. This has been especially effective in the study of turbulent boundary layers where turbulent pressure fluctuations are small and temperature effects are the dominant source of optical disturbances. Morkovin [68] developed the original Strong Reynolds Analogy (SRA) assuming that $p'$ is small and the total temperature, $T_0 = T + u^2/2c_p$, of the flow is constant. If the static temperature and the velocity are decomposed into their mean and fluctuating components, an expression for the fluctuating temperature can be found in terms of the fluctuating velocity given as,

$$\frac{T'}{T} = -(\gamma - 1)M^2 \left( \frac{u'}{u} + \frac{u'^2}{u^2} \right), \quad (2.17)$$
where \( \overline{M} \) is the time-averaged Mach number, \( \overline{u/c} \), and the \( u^2/\overline{u}^2 \) term is usually considered to be negligible. Eqn. (2.17) is what is typically referred to as the Strong Reynolds Analogy and in the proper conditions provides a useful expression for relating temperature and velocity fluctuations.

However, this expression fails to accurately predict temperature fluctuations in the study of non-adiabatic wall flows which are of particular interest in aero-optics. A more general expression for temperature when total temperature changes are not ignored was developed by Walz [107], given by

\[
\frac{T}{T_\infty} = \frac{\bar{T}_w}{T_\infty} + \frac{\bar{T}_r - \bar{T}_w}{T_\infty} \left( \frac{\bar{U}}{U_\infty} \right) - r\frac{(\gamma - 1)}{2} \left( \frac{\bar{U}}{U_\infty} \right)^2,
\]

(2.18)

where \( \bar{T}_w \) is the Farve-averaged wall temperature, \( T_\infty \) is the free-stream temperature, \( \bar{T}_r \) is the Farve-averaged recovery temperature, \( U_\infty \) is the free-stream velocity, \( \bar{U} \) is the Farve-averaged mean velocity, and \( r \) is the recovery factor. This equation is commonly referred to as the Extended Strong Reynolds Analogy (ESRA). If it is linearized for small fluctuations in temperature and velocity, it becomes

\[
\frac{T'}{T_\infty} = \left( \frac{u'}{U_\infty} \right) \left[ \frac{\bar{T}_r - \bar{T}_w}{T_\infty} - r(\gamma - 1) \left( \frac{\bar{U}}{U_\infty} \right) \right].
\]

(2.19)

As can be seen from both Eqns. (2.19) and (2.17), \( T'/T_\infty \propto u'/U_\infty \) and \( \propto u'/\overline{u} \), respectively, and an expression for the temperature spectrum can be constructed from this information.

Using the standard SRA expression from Eqn. (2.17), we find that

\[
\frac{\mathcal{R}_{TT}(x_0, r)}{T^2} = (\gamma - 1) \frac{\overline{M}^4 \mathcal{R}_{uu}(x_0, r)}{\overline{u}^2}.
\]

(2.20)

Assuming that the flow field is homogenous and isotropic and integrating according to Eqn. (2.9), the three-dimensional turbulent temperature spectra is found to be
equal to
\[ \frac{\Phi_{TT}(K)}{T^2} = (\gamma - 1)^2 M^4 \frac{\Phi_{uu}(K)}{u^2}. \]  \tag{2.21}

Eqn. (2.11) can be used to relate \( \Phi_{uu} \) to the well known one-dimensional Kolmogorov velocity spectrum \[4\],
\[ E_{uu}(\kappa) = C_u \varepsilon^{2/3} \kappa^{-5/3}, \]  \tag{2.22}
where \( C_u \) is a constant, to develop an expression for \( \Phi_{TT} \),
\[ \frac{\Phi_{TT}(\kappa)}{T^2} = (\gamma - 1)^2 M^4 C_u \varepsilon^{2/3} \frac{1}{4\pi u^2} \kappa^{-11/3}. \]  \tag{2.23}

Comparing with the three-dimensional temperature spectrum term in Eqn. (2.15), the two expressions for the temperature spectrum contain the same dependence on wavenumber, \( \kappa^{-11/3} \). Through this comparison, the characterization of some aero-optic phenomena using adiabatic heating/cooling to describe density fluctuations and theoretical predictions of turbulent temperature fluctuations are in agreement spectrally. It would be expected that the current analysis of aero-optic phase distortions could be extended to aero-optic research that utilizes SRA and ESRA without making extensive changes to the previously derived temperature spectrum.

2.2 Phase Correlation Function of a Plane Wave Propagating Through an Aero-Optic Field

In this section, the theoretical basis for the phase distortions for a plane electromagnetic wave propagating through a medium with weak index-of-refraction fluctuations is given. As given in the introduction, from Maxwell’s equations, the equations that describe the propagation of a continuous laser with a single wavelength can be reduced to the scalar Helmholtz equation given by Eqn. (1.3). With approximations that the diffraction effects are weak across the aero-optic region and the beam
propagates primarily in the axial direction, a solution was found for the Helmholtz equation showing that the primary distortion mechanism are changes in the local beam phase, $\phi'$.  

To develop an expression for the spectral behavior of $\phi'$, we look to construct the autocorrelation function for the relative phase distortion in the 2-D plane perpendicular to the propagation direction,

$$R_{\phi\phi}(\mathbf{x}_0, \mathbf{r}_\perp, L) = \overline{\phi'(\mathbf{x}_0, L)\phi'(\mathbf{x}_0 + \mathbf{r}_\perp, L)},$$  \hspace{1cm} (2.24)

where $\mathbf{r}_\perp = r_x \hat{i} + r_y \hat{j}$, $\phi'$ is the complex conjugate of $\phi'$, the optically active region of the flow is in the region from $z = 0$ to $L$, and the simplification is made that the coordinate system local to the beam is aligned with the flow coordinate system. An expression for the phase fluctuations of a plane wave propagating through a random medium can be derived directly from the Helmholtz equation using the first Rytov solution [94, 39] and is given as

$$\phi'(\mathbf{x}_0, L) = \int_0^L \left[ \int_{-\infty}^\infty e^{i \mathbf{K}_\perp \cdot \mathbf{x}_0} H_i(L - z, \mathbf{K}_\perp)dv(\mathbf{x}_0, z, \mathbf{K}_\perp) \right]dz,$$  \hspace{1cm} (2.25)

where $\mathbf{K}_\perp$ is the transverse wavenumber vector, $\kappa_x \hat{x} + \kappa_y \hat{y}$, $\mathbf{x}_0 \perp = x_0 \hat{i} + y_0 \hat{j}$, and a pseudo-spectral representation of the fluctuating index-of-refraction field is used, where

$$n'(\mathbf{x}_0, \mathbf{r}) = \int \int e^{i \mathbf{K}_\perp \cdot \mathbf{x}_0} dv(\mathbf{x}_0, z, \mathbf{K}_\perp).$$

In this representation, $n'$ is in the physical space in the propagation direction and in wavenumber space for the transverse directions. This is done for analytic convenience and more information about the construction of the pseudo-spectral representation and the complete definition of the differential, $dv$, can be found in Appendix A of [39]. $H_i$ is the phase component of the Fourier transform of the approximate Green’s function solution to the Helmholtz equation for a plane wave and represents the effects of wave propagation from 0 to
The expression for $H_i$ is given by

$$H_i(L - z, \mathbf{K}_\perp) = k \cos \left[ \frac{(L - z)^2}{2k \kappa^2} \right], \quad (2.26)$$

where $\kappa^2 = \kappa_x^2 + \kappa_y^2$. If wave propagation (diffraction) effects are assumed to be weak ($H_i \to k$), it can be seen that Eqn. (2.25) reduces to $k \int n'dz$, or the phase distortion as traditionally defined within aero-optics.

Using this expression for $\phi'$, the 2-D phase autocorrelation, Eqn. (2.24), can be written as

$$R_{\phi\phi}(x_0, r_\perp, L) = \int \int e^{i\mathbf{K}_\perp \cdot r_\perp} H_i(L - z', \mathbf{K}_\perp) H_i(L - z'', \mathbf{K}_\perp)$$

$$F_n(x_0, |z' - z''|, \mathbf{K}_\perp) \, d\kappa_\perp \, dz'' \, dz' \quad (2.27)$$

where

$$F_n(x_0, |z' - z''|, \mathbf{K}_\perp) = \int_{-\infty}^{\infty} e^{i\kappa \cdot z} \Phi_{nn}(\mathbf{K}) \, d\kappa_z, \quad (2.28)$$

and $z'$ and $z''$ are propagation directions for the phase distortions, $\phi'(x_0)$ and $\phi'(x_0 + r_\perp)$, respectively, evaluated in the correlation function. If it is assumed that the index-of-refraction field is homogenous in the $z$-direction and the index-of-refraction spectrum is radially symmetric, Eqn. (2.27) can be reduced to

$$R_{\phi\phi}(r_\perp, L) = 2\pi^2 k^2 L \int_0^{\infty} \kappa_\perp J_0(\kappa_\perp r_\perp) f_{\phi'}(\kappa_\perp) \Phi_{nn}(\kappa_\perp) \, d\kappa_\perp, \quad (2.29)$$

where $J_0$ is the Bessel function of first kind and order zero, $r_\perp^2 = r_x^2 + r_y^2$, and $f_{\phi'}$ is a spectral filter function,

$$f_{\phi'}(\kappa_\perp) = 1 + \frac{\sin(\kappa^2_\perp L/k)}{\kappa^2_\perp L/k}, \quad (2.30)$$
In application, the flow typically cannot be assumed to be statistically homogeneous in the propagation direction. As such, a formulation for the phase autocorrelation function that is dependent on an index-of-refraction spectrum that changes in the propagation direction must be determined. If the spectral dependence on the scalar transverse wavenumber, $\kappa_\perp$, is the same at any point along the propagation direction, the index-of-refraction spectrum is dependent on separable components of the scalar transverse wavenumber and the propagation direction, $z$. In this particular case, $\Phi_{nn}$ can be decomposed into a product of its $\kappa_\perp$ and the propagation distance dependent components as $\Phi_{nn} = \Phi_{1n}(x_0, z)\Phi_{2n}(\kappa_\perp)$. In this decomposition, $\Phi_{1n}(x_0, z)$ is proportional to the variance of the index of refraction at location $z$ and the expression for $\Phi_{2n}(\kappa_\perp)$ does not change along the beam path. To simplify the expression for the phase correlation, a variable $\eta$ is defined as $\eta = z' - z''$. Then, by showing that $H_i(L - z', K_\perp) \approx H_i(L - \eta, K_\perp)$ [39], and again assuming radial symmetry of the refractive-index spectrum, Equation (2.27) can be rewritten as

$$R_{\phi\phi}(x_0, r_\perp, L) = (2\pi)^2 \int_0^L \Phi_{1n}(x_0, \eta) \left[ \int_0^\infty \kappa_\perp J_0(\kappa_\perp r_\perp) H_i(L - \eta, \kappa_\perp)^2 \Phi_{2n}(\kappa_\perp) d\kappa_\perp \right] d\eta. \quad (2.31)$$

It is noted that the component of autocorrelation function within the brackets is similar to Eqn. (2.29) with the only difference being the filtering functions $f_{\phi'}$ and $H_i$. Notably, the outer integral has no effect on the transverse wavenumber dependence of the correlation function, only serving to emphasize and de-emphasize regions of the flow field along the propagation.

2.3 Wavenumber Spectrum of Aero-Optic Distortions

In this section, the result for the phase distortion correlation will be used to derive expressions for the one and two dimensional wavenumber spectrum of optical distortions in homogeneous turbulent flow fields focusing on their spectral behavior
in the inertial subrange.

2.3.1 One-Dimensional Wavenumber Spectrum of Phase Distortions

The one-dimensional spectrum of phase distortion along the $x$-direction, $E_{\phi\phi}(\kappa_x)$, can be calculated by taking the Fourier transform of the phase autocorrelation in the $\mathbf{r}_\perp = r_x \hat{i}$ direction. Assuming radial symmetry in the correlation function as before,

$$E_{\phi\phi}(\kappa_x) = \int_{-\infty}^{\infty} R_{\phi\phi}(r_x, L)e^{-ir_x\kappa_x}dr_x = 2 \int_{0}^{\infty} R_{\phi\phi}(r_x, L) \cos(r_x \kappa_x)dr_x. \quad (2.32)$$

In addition, a simplification to the correlation for a homogenous turbulent field given by Eqn. (2.29) can be made by assuming the spatial wavenumber in the inertial region that we are focused on is large and taking the limit as $\kappa_x^2 \rightarrow \infty$ of the spectral filter function,

$$\lim_{\kappa_x^2 \rightarrow \infty} \frac{\sin(\kappa_x^2 L/k)}{\kappa_x^2 L/k} = 0. \quad (2.33)$$

Taking this limit, the result can be expected to be accurate for a portion of the inertial subrange at high Reynolds numbers when the Kolmogorov length scale is very small. Using this simplification for $R_{\phi\phi}$, the expression for $E_{\phi\phi}$ becomes

$$E_{\phi\phi}(\kappa_x) = 4\pi^2k^2L \int_0^{\infty} \left[ \int_0^{\infty} \kappa_x J_0(\kappa_x r_x) \Phi_{nn}(\kappa_x) d\kappa_x \right] \cos(r_x \kappa_x)dr_x. \quad (2.34)$$

If a simple inertial subrange model for the index-of-refraction spectrum of the form $\Phi(\kappa) \propto \kappa^{-m^{3D}}$ like those used in Section 2.1, where $m^{3D}$ is the spectral slope of the three-dimensional spectrum, is inserted into Eqn (2.34), a singularity exists for values of $m^{3D}$ greater than 2 as the integral is evaluated at zero. To avoid this singularity, a spectral model of turbulence must be used that is bounded as $\kappa \rightarrow 0$. A popular model of choice is the von Karman model of the turbulent spectrum where
the spectrum of some zero-mean fluctuating quantity $g'$ is equal to,

$$
\Phi_{gg}(\kappa) = C_{gg} \overline{g'^2} \kappa^2 \ell^5 \left(1 + \kappa^2 \ell^2 \right)^{-m_{3D}/2-1},
$$

(2.35)

where $\ell$ is a characteristic large scale [114]. This spectrum separates the inertial subrange from low-wavenumber scales of turbulence at a peak value of $\kappa \ell = 1$, corresponding to a peak turbulence length scale, $L_0$, equal to $2\pi \ell$. The integral of the model spectrum across all wavenumbers will equal the variance of $g'$, $\overline{g'^2}$, and the value of $C_{gg}$ is a constant that satisfies this relation.

In the model given by Eqn (2.35), values of $\kappa \ell < 1$ approach zero at a rate of $\kappa^2$ representing the decaying behavior of turbulent structures larger than $L_0$. Values of $\kappa \ell > 1$ decrease at a rate of $\kappa^{-m_{3D}}$ and represent the inertial subrange of the turbulence. While the previous equation more accurately represents both the large-
scale and inertial wavenumber range, a simpler form of the von Karman spectral model,

$$\Phi_{gg}(\kappa) = C_{gg}g^2\ell^3 \left(1 + \kappa^2\ell^2\right)^{-m_3^{3D}/2}, \quad (2.36)$$
is frequently used by Tatarski [94] and Ishimaru [39] in their texts on wave propagation. This simpler form trends to a constant as $\kappa \to 0$ but maintains the same inertial subrange behavior of Eqn (2.35). In Figure 2.1, plots of both forms of the von Karman spectrum are given. For the calculations in this subsection and the next, the spectral model in Eqn (2.36) will be used since it eases complications in some integral evaluations and the focus in these subsections is on the inertial subrange behavior.

Using this simpler spectral model for the refractive-index spectrum with an arbitrary spectral slope of $m_3^{3D}$, $E_{\phi\phi}$ in Eqn. (2.34) can be shown to be

$$E_{\phi\phi}(\kappa_x) = 2\pi^{5/2}k^2C_{nn}n^2L \ell^2 \left(1 + \kappa_x^2\ell^2\right)^{1-m_3^{3D}/2} \frac{\Gamma\left((m_3^{3D} - 1)/2\right)}{\Gamma\left(m_3^{3D}/2\right)}, \quad (2.37)$$

where $\Gamma$ is the gamma function, $\Gamma(t) = \int_0^\infty x^{t-1} \exp(-x)dx$. It can be seen that within the inertial subrange, $E_{\phi\phi}(\kappa_x) \propto \kappa_x^{1-m_3^{3D}}$.

If the same model spectrum for density fluctuations is assumed as,

$$\Phi_{\rho\rho}(\kappa) = C_{\rho\rho}\rho^2\ell^3 \left(1 + \kappa^2\ell^2\right)^{-m_3^{3D}/2}, \quad (2.38)$$
a general expression for the wavenumber spectrum can be found without using the linearized ideal gas law approximation found in Section 2.1. The dependence of the one-dimensional phase spectrum on density spectrum can then be calculated to find a result similar to Eqn. (2.37),

$$E_{\phi\phi}(\kappa_x) = 2\pi^{5/2}k^2K_{GD}^2C_{\rho\rho}\rho^2L \ell^2 \left(1 + \kappa_x^2\ell^2\right)^{1-m_3^{3D}/2} \frac{\Gamma\left((m_3^{3D} - 1)/2\right)}{\Gamma\left(m_3^{3D}/2\right)}. \quad (2.39)$$

Importantly, Eqn. (2.39) expresses that if the three-dimensional density spectrum has
a spectral slope proportional to $-m_\rho^{3D}$, the one-dimensional phase fluctuation spectrum will have a spectral slope of $-(m_\rho^{3D} - 1)$. Equivalently, if the inertial subrange of the one-dimensional density spectrum has a spectral slope of $m_\rho$, the one-dimensional phase fluctuation spectrum will have a slope of $-(m_\rho + 1)$ because $m_\rho = m_\rho^{3D} - 2$.

A similar relation can be observed for the temperature, pressure, and pressure-temperature components in the linearized spectrum of index-of-refraction fluctuations, Eqn (2.15), derived earlier. The inertial subrange model presented for each of the terms in Eqn (2.15) are replaced with the von Karman model of the turbulent spectrum resulting in the following expression for the refractive index spectrum:

$$\Phi_{nn}(\kappa) \approx \frac{\gamma K_G D \rho}{4\pi^2} \left[ B_p \varepsilon^{4/3} \ell^{13/3} \left[ 1 + \kappa_\rho^2 \ell^2 \right]^{-13/6} \right. $$

$$+ R^2 \beta \chi \varepsilon^{-1/3} \ell^{-11/3} \left[ 1 + \kappa_\rho^2 \ell^2 \right]^{-11/6} \left. - \frac{2R}{\rho k^2} \text{Re} [E_{pT}(\kappa)] \right). \quad (2.40)$$

Using this expression in Eqn. (2.34) and integrating for $E_{\phi\phi}$ in terms of the temperature spectrum, pressure spectrum, and cospectrum gives

$$E_{\phi\phi}(\kappa_x) \approx \pi \left( \frac{\gamma K_G D \rho k}{\sigma^2} \right)^2 \left( \frac{\sqrt{\pi}}{2} B_p \varepsilon^{4/3} \ell^{10/3} \left[ 1 + \kappa_\rho^2 \ell^2 \right]^{-5/6} \frac{\Gamma(5/3)}{\Gamma(13/6)} \right. $$

$$+ \frac{\sqrt{\pi}}{2} R^2 \beta \chi \varepsilon^{-1/3} \ell^{8/3} \left[ 1 + \kappa_\rho^2 \ell^2 \right]^{-4/3} \frac{\Gamma(4/3)}{\Gamma(11/6)} $$

$$- \frac{8\pi R}{\rho} \int_0^\infty \left[ \int_0^\infty J_0(\kappa_x r_x) \text{Re} [\Phi_{pT}(\kappa_x)] d\kappa_x \right] \cos(\kappa_x r_x) dr_x \right). \quad (2.41)$$

Notably, the wavenumber spectra contains two different spectral slopes as $\kappa_x$ becomes large, $-10/3$ and $-8/3$, in addition to an unknown contribution from the pressure-temperature cospectra. The predominant slope is dependent on the conditions of the flow as seen in atmospheric flows where the $-8/3$ slope is almost exclusively observed due to the dominance of temperature fluctuations in the turbulent atmosphere.
The discussion in the Section 2.1 on the spectral slope and its dependence on Reynolds number and flow structure have direct consequences for the spectral slope of optical phase distortions. If the flow of interest is of low Reynolds number and/or in shear, it is possible that the observed spectral slope of the one-dimensional temperature spectra can appear to be $-4/3$ instead of the Kolmogorov predicted $-5/3$. Similarly, the pressure spectral slope for low Reynolds numbers can appear to have a value of $-5/3$ compared to the Kolmogorov predicted $-7/3$. In this scenario it would be expected that the temperature would contribute a component of the one-dimensional phase spectra that appears to have a spectral slope of $-7/3$ and the pressure, a component that appears to go as $-8/3$. The total phase spectral behavior would then depend on the strength of each of these contributions together minus the contribution from the cospectra.

To derive the expressions given in Eqns. (2.39) and (2.41), an assumption was made that the flow was homogenous in the propagation direction. As discussed at the end of the Section 2.1, an expression was derived for a flow where the wavenumber dependence of the index-of-refraction spectrum was separated from strength of the refractive index fluctuations along the integration path. For several flow configurations this may be an adequate assumption but may be insufficient for flows such as boundary layers where the spectral dependence on wavenumber can change with the distance from the wall. For these flows, a more complex coupling between the index of refraction strength and spectral dependence on wavenumber must be considered and integrated according to Eqn. (2.27). For the case where the decoupling of fluctuation strength and wavenumber dependence is appropriate and the refractive-index variance is smoothly varying in $z$, Woo & Ishimaru [116] showed that the phase spectral slope has the same dependence on the wavenumber as the homogenous case. Therefore, in the case where this decoupling is valid, relations for the spectral slope of phase distortion spectra are expected to be proportionally the same as those found
here for the homogenous turbulence case.

2.3.2 Two-Dimensional Wavenumber Spectrum of Phase Distortions

In addition to the one-dimensional spectrum, understanding the connection between the two-dimensional spectrum of the phase distortions and the index-of-refraction spectrum is of fundamental importance as projecting and receiving apertures are primarily two-dimensional. The two-dimensional phase spectrum, $E_{\phi \phi}^{2D}(\kappa_\perp)$, can be written as

$$E_{\phi \phi}^{2D}(\kappa_\perp) = \int_\infty^{-\infty} R_{\phi \phi}(r_\perp, L) e^{-i r_\perp \cdot \kappa_\perp} dr_\perp. \quad (2.42)$$

Assuming that the correlation is radially symmetric and utilizing the Hankel transform, Eqn. (2.42) can be rewritten as

$$E_{\phi \phi}^{2D}(\kappa_\perp) = 2\pi \int_0^\infty R_{\phi \phi}(r_\perp, L) J_0(\kappa_\perp r_\perp) r_\perp dr_\perp. \quad (2.43)$$

Using the same simplification as in the previous section in the limit of large spatial wavenumbers,

$$E_{\phi \phi}^{2D}(\kappa_\perp) = 4\pi^3 k^2 L \int_0^\infty \left[ \int_0^\infty \Phi_{mn}(\kappa_\perp) J_0(\kappa_\perp r_\perp) \kappa_\perp d\kappa_\perp \right] J_0(\kappa_\perp r_\perp) r_\perp dr_\perp. \quad (2.44)$$

In essence, Eqn. (2.44) is taking the inverse Fourier transform and then the Fourier transform of the three-dimensional index-of-refraction spectrum assuming radial symmetry. If the same spectrum for the three-dimensional index-of-refraction spectrum is used as Eqn. (2.37), then $E_{\phi \phi}^{2D}(\kappa_\perp)$ is equal to

$$E_{\phi \phi}^{2D}(\kappa_\perp) = 4\pi^3 k^2 C_m n^2 L \ell^3 \left(1 + \kappa_\perp^2 \ell^2\right)^{-m^3/2}. \quad (2.45)$$

From this solution, it can be seen that within the inertial subrange $E_{\phi \phi}^{2D}(\kappa_\perp) \propto \kappa_\perp^{-m^3}$. Again applying the same process as in the one-dimensional case and using the
general result from Eqn. (2.45) together with the linearized index-of-refraction spectrum derived in Eqn. (2.15), an expression for the 2-D spectrum of phase distortion in terms of temperature, pressure, and pressure-temperature cospectra can be approximated as

$$E_{\phi \phi}^{2D}(\kappa_\perp) \approx \left( \frac{\pi^2 K_G D \bar{p} k L^{1/2}}{\alpha^2} \right)^2 \left[ B_p \varepsilon^{4/3} \ell^{13/3} \left( 1 + \kappa_\perp^2 \ell^2 \right)^{-13/6} + R^2 \beta \varepsilon^{-1/3} \ell^{11/3} \left( 1 + \kappa_\perp^2 \ell^2 \right)^{-11/6} - \frac{8\pi R}{\rho} \text{Re} \left[ \Phi_{pT}(\kappa_\perp) \right] \right]. \quad (2.46)$$

Here, the 2-D phase spectra in the limit of large spatial wavenumbers has contributions from pressure proportional to $\kappa_\perp^{-13/3}$, from temperature proportional to $\kappa_\perp^{-11/3}$, and some unknown contribution from the pressure-temperature cospectra. It follows that like the one-dimensional case, an exact expression for the 2-D phase distortion spectra can be written from the 3-D density spectrum. Examining Eqn. (2.45), the spectral slope of the 3-D density spectra, $-m_\rho^{2D}$, will be the same as spectral slope of the 2-D phase distortion spectra.

### 2.4 Approximation and Behavior of OPD$_{\text{rms}}$ Based on Model Spectrum

#### 2.4.1 OPD$_{\text{rms}}$ of an Infinite Aperture Beam

An analytic expression for the two-dimensional phase distortion spectra makes it convenient to write an expression for the relationship between OPD$_{\text{rms}}$ and the spatial wavenumber, providing insight into the link between turbulent length scale and far-field optical behavior. First, the two-dimensional OPD spectrum can be related to the 2-D phase distortion spectrum by $E_{\text{OPD}}^{2D} = E_{\phi \phi}^{2D} / k^2$. The variance of the OPD can be found by integrating the 2-D OPD spectrum over all transverse wavenumbers and the OPD$_{\text{rms}}$ can be found by taking the square root of the variance. The fraction of total OPD$_{\text{rms}}$ of an infinite aperture captured up to some non-dimensional cutoff
wavenumber, \( \kappa^*_c = \kappa_c \ell \), can be written as

\[
\frac{\text{OPD}_{\text{rms}}(\kappa^*_c)}{\text{OPD}_{\text{rms}}} = \sqrt{\int_0^{\kappa^*_c^2} E_{\text{OPD}}^{2D}(\kappa_\perp \ell)(\kappa_\perp d(\kappa_\perp \ell))}
\]

(2.47)

Because of the importance of the low-wavenumber spectral range, the more complete von Karman spectral model given by Eqn. (2.35) will be used for index-of-refraction spectrum. As demonstrated by Eqn. (2.44), the 2-D phase distortion spectrum calculation is a Fourier transform followed by an inverse Fourier transform of the refractive-index spectrum. Without making any assumptions about the spectral filter, \( f_{\phi'} \), it is expected that the 2-D OPD spectrum will have the form

\[
E_{\text{OPD}}^{2D}(\kappa_\perp) = C_{\text{OPD}}(\kappa_\perp \ell)^2 \left[ 1 + (\kappa_\perp \ell)^2 \right]^{-m_{3D}/2 - 1} \left[ 1 + \frac{\sin((\kappa_\perp L/k_\ell)^2)}{\kappa_\perp^2 L/k_\ell} \right].
\]

(2.48)

where \( C_{\text{OPD}} \) is a constant equal to \( 4\pi^3 C_{nn} \sqrt{n^2 L \ell^3} \). If we define the non-dimensional wavenumber, \( \kappa^* = \kappa_\perp \ell \), and a non-dimensional optical-turbulence spatial scale as \( L_{AO} = \sqrt{L/k_\ell^2} \), the previous equation can be rewritten as

\[
E_{\text{OPD}}^{2D}(\kappa^*, L_{AO}) = C_{\text{OPD}}\kappa^*^2 \left[ 1 + \kappa^*^2 \right]^{-m_{3D}/2 - 1} \left[ 1 + \frac{\sin((\kappa^* L_{AO})^2)}{(\kappa^* L_{AO})^2} \right].
\]

(2.49)

Depending on the value of \( L_{AO} \), the spectral filter function affects different regions of the model spectrum as shown in Figure 2.2. Above approximately \( \kappa^* = 1/L_{AO} \), the spectrum is identical to the von Karman spectrum. Below that value, there is a weak oscillation and then the spectrum settles to the von Karman spectrum multiplied by a factor of two. For typical aero-optics applications where the optical wavenumber is on the order of millions and the turbulence integral length scale and optical integration distance is on the order of centimeters to meters, values of \( L_{AO} \) are small, typically on the order of \( 0.001 - 0.01 \). As can be seen in Figures 2.2(a) and 2.2(b), a change in the shape of model spectrum due to the spectral filter is not
Figure 2.2. Two-dimensional OPD spectrum given by Eqn. (2.49) (---) compared to the solution using the more complete von Karman spectrum, given by Eqn. (2.35), (---) both normalized by $C_{\text{OPD}} = 4\pi^3 C_{\text{mn}} m^2 L^3$ for values of $L_{AO} = (a) 0.001, (b) 0.01, (c) 1.0, and (d) 100.0$. For this example, $m^{3D} = 11/3$. 

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seen until high non-dimensional wavenumbers and for the following analysis will be ignored. As such, the value of the spectral filter term, \( \sin((\kappa^* L_{AO})^2)/(\kappa^* L_{AO})^2) \), in Eqn. (2.49) will be taken as equal to one.

Using this assumption and evaluating the denominator in Eqn. (2.47), the total value of \( \text{OPD}_{\text{rms}} \) for an infinite aperture is proportional to

\[
\sqrt{\frac{4}{m_n^{3D}(m_n^{3D} - 2)}}. \tag{2.50}
\]

Solving for the numerator of Eqn. (2.47) shows that it is proportional to

\[
\sqrt{\frac{4(1 + \kappa_c^*)^{-m_n^{3D}/2}}{m_n^{3D}(m_n^{3D} - 2)}} - \frac{2\kappa_c^2(1 + \kappa_c^*)^{-m_n^{3D}/2}}{m_n^{3D} - 2} \tag{2.51}
\]

Since Eqns. (2.50) and (2.51) are proportional to the same constant, Eqn. (2.47) can be written as equal to Eqn. (2.51) divided by Eqn. (2.50).

Plotting this expression in Figures 2.3 for two values of \( m_n^{3D} \), it can be seen that the large scale structures between \( 0.2 < \kappa^* < 2 \) contribute nearly 70% to 80% of the total \( \text{OPD}_{\text{rms}} \) for an infinite aperture. This supports the notion that large scales of turbulence are the most optically active. Also, it can be seen between the two line plots of \( m_n^{3D} \), larger values of \( m_n^{3D} \) contain more of their total \( \text{OPD}_{\text{rms}} \) in low wavenumber range since the energy decays faster in the inertial subrange.

### 2.4.2 \( \text{OPD}_{\text{rms}} \) of an Finite Aperture Beam and Effects of Tip/Tilt Removal

While useful in illustrating the importance of turbulent scales in terms of their contribution to \( \text{OPD}_{\text{rms}} \), integrating from a wavenumber of zero in Eqn. (2.47) implies that the size of the optical aperture is infinite. As the aperture size is typically on the same order of the size of energy containing turbulence structures, the effects of the aperture as a high-pass filter on the total \( \text{OPD}_{\text{rms}} \) can not be ignored. To account for
the finite size of the aperture or other aperture effects, a function working to filter part of the OPD spectrum, $F$, in the evaluation of the OPD variance can be defined as

$$\text{OPD}_{\text{rms}} = \sqrt{\int_{0}^{\infty} E_{\text{OPD}}^{2D}(\kappa^*) F(\kappa^*/\kappa^*_\text{Ap}) \kappa^* d\kappa^*}. \quad (2.52)$$

For a collimated plane wave, the filter function modifying the spectrum for the effects of finite aperture size is the same as a filter that accounts for the reduction in OPD variance if an adaptive optic correction of the first Zernike mode, piston, was applied to the wavefront[80, 90]. This filter is given by the function

$$F_P(\kappa^*/\kappa^*_\text{Ap}) = 1 - 16 \left[ \frac{J_1(\pi \kappa^*/\kappa^*_\text{Ap})}{2\pi \kappa^*/\kappa^*_\text{Ap}} \right]^2 \quad (2.53)$$
where \( \kappa_{Ap}^* = \frac{2\pi \ell}{D_{Ap}} = \frac{L_0}{D_{Ap}} \) is the wavenumber pertaining to an aperture of diameter \( D_{Ap} \), and the \( P \) subscript denotes piston removal. This filter equation is very similar to the one found by Siegenthaler [82] to account for the removal of piston in an aero-optical wavefront except in this work we are operating in the spatial wavenumber domain instead of temporal frequency domain via the frozen flow hypothesis. From here on, accounting for piston removal in the wavefront and accounting for a finite aperture will be used interchangeably since they are mathematically equivalent in terms of the filter function.

In adaptive-optic beam systems, an additional corrective component that affects the impact of near aperture size structures is tip/tilt correction. Calculated in a similar fashion to the piston filter function, by projecting the phase spectra onto a spectral representation of the Zernike polynomials for the tip and tilt modes, the filter functions for an aperture with tip/tilt removed is given by

\[
F_{TT}(\kappa^*/\kappa_{Ap}^*) = 1 - 64 \left[ \frac{J_2(\pi \kappa^*/\kappa_{Ap}^*)}{2\pi \kappa^*/\kappa_{Ap}^*} \right]^2,
\]

(2.54)

and with both piston and tip/tilt removed is given by

\[
F_{PTT}(\kappa^*/\kappa_{Ap}^*) = 1 - 16 \left[ \frac{J_1(\pi \kappa^*/\kappa_{Ap}^*)}{2\pi \kappa^*/\kappa_{Ap}^*} \right]^2 - 64 \left[ \frac{J_2(\pi \kappa^*/\kappa_{Ap}^*)}{2\pi \kappa^*/\kappa_{Ap}^*} \right]^2,
\]

(2.55)

where \( J_1 \) and \( J_2 \) are Bessel functions of the first kind of order one and two, respectively. Shown in Figure 2.4 are the filter functions given by the Eqns. (2.53)–(2.55). It is shown that with only piston removal, structures larger than the aperture size, \( \kappa^*/\kappa_{Ap}^* < 1 \), will still have an effect on the total \( \text{OPD}_{\text{rms}} \) but their influence decreases as they become much larger than the aperture size. Structures smaller than the aperture size are almost entirely captured as expected. From this figure, it is evident that the removal of tip/tilt acts to remove some influence of structures on \( \text{OPD}_{\text{rms}} \) over a
Figure 2.4. Filter functions, $F(\kappa^*/\kappa_{Ap}^*)$, for an aperture corresponding to $\kappa_{Ap}^*$: finite aperture/piston removal, \textcolor{blue}{\cdot\cdot\cdot}; tip/tilt removal, \textcolor{red}{\cdot\cdot\cdot}; piston and tip/tilt removal, \textcolor{black}{\cdot\cdot\cdot}.

band of wavenumbers between $0.1 < \kappa^*/\kappa_{Ap}^* < 2$, removing the most spectral content at $\kappa^*/\kappa_{Ap}^* \approx 0.73$. The simultaneous removal of piston and tip/tilt acts as a high-pass filter, removing energy from structures at sizes larger than half the aperture size.

The filter functions act by projecting the phase spectrum onto spectral representations of Zernike basis modes. The variance of the spectrum projected onto the Zernike mode is found by integrating across all wavenumbers and is then removed from the variance of the entire OPD spectrum to account for the ‘correction’ of that kind of aberration from the wavefront. This is different than the method of Siegenthaler[82], who assumed a form of the phase distortion in physical space, but interestingly, the results are virtually the same. This method of projecting onto the Zernike modes is equivalent to accounting for Z-tilt correction for tip/tilt on a wavefront.

The effect of the filter functions on the two-dimensional OPD spectrum is exam-
ined and shown in Figure 2.5 where the model spectrum has been multiplied by $F_P$ and $F_{PTT}$ filter functions for four different aperture sizes, $D_{Ap}/L_0 = 100, 10, 1, 0.1$.

In the piston/finite aperture filtered only case, for nearly all aperture sizes the largest contribution to the OPD$_{rms}$ comes from structures of size near $L_0$. When both the tip/tilt and piston filters are applied, aperture diameters smaller than $L_0$ see the peak contribution to OPD$_{rms}$ shift closer to $\kappa_{Ap}^*$. Since tip/tilt filters out structures larger than the aperture, aperture sizes larger than $L_0$ do not see much change in the influence of the spectral peak and thus do not see much of a decrease in OPD$_{rms}$ when piston and tip/tilt are removed.

Figure 2.5. The model spectrum (---) including the (a) piston (---) and (b) tip/tilt filter (---) function for multiple aperture sizes. Wavenumber values corresponding to the aperture size are denoted by ●●●.

As the aperture diameter decreases, it is expected that the total value of OPD$_{rms}$ also decreases. The value of OPD$_{rms}$ for a filtered aperture relative to the infinite
aperture OPD$_{\text{rms}}$ is given by

$$\frac{\text{OPD}_{\text{rms}}(\kappa^*_{\text{Ap}})}{\text{OPD}_{\text{rms}}(\infty)} = \sqrt{\frac{\int_{0}^{\infty} E_{\text{OPD}}^{2D}(\kappa^*) F(\kappa^*/\kappa^*_{\text{Ap}}) \kappa^* d\kappa^*}{\int_{0}^{\infty} E_{\text{OPD}}^{2D}(\kappa^*) \kappa^* d\kappa^*}}. \quad (2.56)$$

Numerically integrating the previous equation for different aperture sizes, the effect of aperture size on OPD$_{\text{rms}}$ with and without tip/tilt removed is shown in Figure 2.6. When tip/tilt has not been removed, the OPD$_{\text{rms}}$ is roughly the same as the infinite aperture case for aperture sizes at and larger than the peak turbulence scale. As aperture size decreases, the value of OPD$_{\text{rms}}$ when compensating for the finite aperture size shrinks relatively slowly; an aperture five times smaller than the outer scale of turbulence sees a decrease in OPD$_{\text{rms}}$ of only about 40-50% compared to the infinite aperture. As in Figure 2.3, there is a noticeable difference between the curves with different values of $m_{3D}^D$. The case with turbulent pressure dominated index-of-refraction fluctuations, $m_{3D}^D = 13/3$, sees a bigger decrease in OPD$_{\text{rms}}$ with a decrease in aperture size than the temperature dominated case, $m_{3D}^D = 11/3$.

When tip/tilt is removed from the aperture along with piston, there is an appreciable decrease in the OPD$_{\text{rms}}$ compared with the piston removed only case. At an aperture size of five times smaller than $L_0$, the tip/tilt removed aperture sees a total reduction of OPD$_{\text{rms}}$ from the infinite aperture case of about 60% for the temperature dominated optical distortions and about 75% for the pressure dominated optical distortions, which is an improvement of approximately 25% compared to the finite aperture only in both cases. For $\kappa^*_{\text{Ap}} = 1$, the improvement in optical distortion removal from tip/tilt is limited to only about 10-15% and the improvement in optical behavior goes to zero as the aperture diameter increases.

The improvement in optical behavior that is afforded from tip/tilt removal and piston removal compared to having only piston removal is visualized in Figure 2.7. As observed previously, the only appreciable decrease in OPD$_{\text{rms}}$ occurs after $\kappa^*_{\text{Ap}} > 1$
Figure 2.6. Ratio of OPD$_{\text{rms}}$ for finite aperture removed and finite aperture and tip/tilt removed to the OPD$_{\text{rms}}$ of an infinite aperture: — finite aperture only removed, $m_n^{3D} = 11/3$; —— finite aperture and tip/tilt removed, $m_n^{3D} = 11/3$; —— finite aperture only removed, $m_n^{3D} = 13/3$; — finite aperture and tip/tilt removed, $m_n^{3D} = 13/3$. 
and the largest improvement is gained within the range of $\kappa^*_A p = 1$ to 10. Beyond $\kappa^*_A p = 10$, additional effects of tip/tilt removal begins to slowly level off. Below $\kappa^*_A p = 1$, tip/tilt correction does little to remove any optical distortion. These results have direct implications in the design of adaptive-optic schemes for aero-optics and demonstrates the importance of the $L_0/D$ ratio. For aperture sizes near the peak turbulence length scale and larger, including tip/tilt removal will provide little to no improvement in far-field optical performance.

There are a few important items to note when interpreting these results. When using $\text{OPD}_{\text{rms}}$ to link wavefront behavior to the farfield intensity, the large-aperture approximation (LAA) assumes that the wavefront is piston and tip/tilt removed and for the LAA to be applied for any statistical distribution of OPD, the aperture must be larger than the characteristic turbulence length scale. As a result the piston
only removed $\text{OPD}_{\text{rms}}$ cannot be used to link the wavefront $\text{OPD}_{\text{rms}}$ to the farfield irradiance. However, these results are presented since examining the filtering behavior of piston removal provides insight into the effects of a finite aperture on $\text{OPD}_{\text{rms}}$.

In addition, information is presented in this section for when the aperture is smaller than the peak turbulence length scale. This conflicts with the applicability of the LAA when the probability distribution of the OPD is non-Gaussian. In their paper, Porter et al. [75] discusses the consequences of using the LAA on “not-so-large apertures” and demonstrates using data from the AAOL that for apertures smaller than the characteristic turbulence length scale, the LAA under-predicts the time-averaged Strehl ratio. It follows that using the LAA for results in this chapter when $\kappa_{Ap}^* > 1$ may also under-predict the value of the Strehl ratio.

2.4.3 Resolution Requirements on Wavefronts to Accurately Capture $\text{OPD}_{\text{rms}}$

With a filtered spectrum for different aperture sizes and adaptive optics correction, it is possible to study the effect of wavefront resolution based on the flow conditions. Figure 2.8(a) shows the non-dimensional wavenumber of resolution required to resolve 95% of the total $\text{OPD}_{\text{rms}}$, $\kappa_{95}^*$, versus different values of $\kappa_{Ap}^*$ for two different values of $m_{n}^{3D}$, with and without tip/tilt removed. Assuming the resolution of a wavefront sensor acts as a ‘perfect’ spectral filter where the filter fully resolves scales of the flow up to the sensor resolution, the Nyquist criterion requires that the wavefront must have $N = 2\kappa_{95}^*/\kappa_{Ap}^*$ points across the aperture in each direction to resolve the required length scales. Examining the plots, the difference between the resolution requirement for an optical distortion with $m_{n}^{3D} = 11/3$ compared to $m_{n}^{3D} = 13/3$ is about 1.5 times greater for both tip/tilt removed and not removed cases. For example, if an optical aperture is the same scale as the turbulence peak scale, $\kappa_{Ap}^* = 1$, and tip/tilt is removed, examining Figure 2.8(b) for $m_{n}^{3D} = 13/3$, $N \approx 8$ and for $m_{n}^{3D} = 11/3$, $N \approx 13$. The required resolution for the aperture (assuming a square wavefront
sensor) would be approximately $8 \times 8$ for the $m_{n}^{3D} = 13/3$ case compared to about $13 \times 13$ for $m_{n}^{3D} = 11/3$. Thus, at this value of $\kappa_{Ap}^{*}$, an optical distortion dominated by pressure fluctuations (corresponding to $m_{n}^{3D} = 13/3$) requires approximately 2.5 times fewer subapertures on a wavefront sensor to accurately capture the OPD$_{rms}$ compared to temperature dominated optical aberrations (corresponding to $m_{n}^{3D} = 11/3$).

The removal of tip/tilt also has a large effect on the resolution requirement at smaller apertures. Since a bulk of the variance in the optical distortion for the finite aperture only case is attributable to the large turbulent scales even when the aperture is small, it does not take much resolution to capture all of the variance. On the other side, since tip/tilt shifts the bulk of the optical distortion to scales closer to the aperture size, the number of points required to accurately capture the OPD$_{rms}$ levels off at a constant (Figure 2.8(b), dot and dash-dot lines). This means that for apertures below a certain size, the same number of grid points will be required on a wavefront sensor even as the aperture size is decreased. Below $\kappa_{Ap}^{*} = 1$ as the aperture size increases, there is almost no difference for the required length scaled need to be resolved whether or not tip/tilt is removed. In both cases, the wavefront sensor must be able to accurately capture the optically active large scales of the flow. Thus as the aperture size increases, so does the number of points across the aperture needed to accurately capture OPD$_{rms}$. These observations have obvious implications for the design of aero-optic systems, where the resolution of the wavefront sensor has to be sufficiently high to accurately capture the optical distortion but not too high to over-burden sensor and corrective systems.

2.5 Improvement to Computational Resolution Requirements for Aero-Optical Simulations

Using the results from the previous sections relating the importance of different length scales to optical distortions, the necessary resolution to capture optical dis-
Figure 2.8. (a) Non-dimensional wavenumber of wavefront grid resolution and (b) number of grid points across a wavefront diameter required to resolve 95% of the total OPD\textsubscript{rms} for a finite aperture/piston filtered and piston and tilt/tilt filtered wavefront. In both: \( m_n^{3D} = 11/3; \) piston only filtered, \( m_n^{3D} = 13/3; \) piston and tilt/tilt filtered, \( m_n^{3D} = 11/3; \) piston only filtered, \( m_n^{3D} = 13/3; \) piston and tilt/tilt filtered, \( m_n^{3D} = 13/3. \)
tortions using CFD can be evaluated. In the only previous attempt at establishing computational requirements for aero-optics, Mani et al. [58] used an approximation for the index-of-refraction spectrum to calculate the mean squared error of the OPL and found that for a simulation to maintain a solution less than an error ratio, ξ, the following inequality must be satisfied,

\[
\frac{12\pi^3 \gamma^2 B_p}{7\lambda_{opt}^2} \left[ \frac{(\overline{\pi} - 1)^2 \varepsilon^{4/3}}{c_{\infty}^4} \right] \left( \frac{l_c}{2\pi} \right)^{7/3} \Delta z < \xi, \tag{2.57}
\]

where \( l_c \) is the cutoff length of the computational grid used in the calculation and \( \Delta z \) is the length of the aero-optic region in Mani et al.’s notation, equivalent to \( L \) used in the present study. However, to derive this expression, Mani et al. assumed that refractive index fluctuations are only a function of turbulent pressure fluctuations. Since this assumption does not include the effects of temperature fluctuations, for some flows Eqn. (2.57) may underestimate the optical distortion and provide inaccurate results.

Using the method from Mani et al. together with the linearized spectrum for index-of-refraction fluctuations derived in Eqn. (2.15), the inequality is rewritten as

\[
\frac{12\pi^3 \gamma^2 (\overline{\pi} - 1)^2 \Delta z}{c_{\infty}^4 \lambda_{opt}^2} \left[ \frac{B_p \varepsilon^{4/3}}{7} \left( \frac{l_c}{2\pi} \right)^{7/3} + \frac{R^2 \beta \chi}{5\varepsilon^{1/3}} \left( \frac{l_c}{2\pi} \right)^{5/3} \right] \left[ \frac{2R}{\bar{\rho}} \int_{2\pi/l_c}^{2\pi/\eta} \text{Re} \left[ \frac{E_pT(\kappa)}{\kappa} \right] d\kappa \right] < \xi. \tag{2.58}
\]

where \( \eta \) is the Kolmogorov turbulence length scale. If the cospectrum term is smaller in magnitude than both of the other terms, the corrected expression will require a smaller grid length scale than predicted by Mani et al. However, if twice the cospectrum is larger in magnitude than the temperature spectrum, the resolution requirement may be less than predicted by Mani et al.

Compared to Eqn. (2.57), the more general expression derived here unfortunately lacks the ability to be used as a ‘back of the envelope’ equation for aero-optics resolu-
tion requirement. The terms in the equation by Mani et al. could each be related to parameters of a simulation. In Eqn. (2.58), the cospectrum and the average temperature dissipation rate, $\chi$, are difficult to approximate given the general parameters of the problem being simulated. Because of its convenience, the work by Mani et al. still stands as the best starting point for estimating the mesh requirements in aero-optic simulations. As stated however, a further increase in resolution may be required to actually attain the error ratio specified in Eqn. (2.57), depending on the flow conditions.

2.6 Conclusions

The work in this chapter provides an analysis of the spectral behavior of aero-optic distortions in the context of statistical turbulence theory and electromagnetic wave propagation. The results are found for the most basic cases, only offering a view into the behavior of a planar, small wavelength electromagnetic wave propagating through weakly-compressible, high-Reynolds number homogeneous turbulence. While these limitations exist, important conclusions can still be drawn about the role of different scales of turbulence in optical distortions, especially with regard to aperture size and spectral behavior of index-of-refraction fluctuations due to pressure and temperature.

Beginning from the Gladstone-Dale relation and the ideal gas law, general expressions for the three-dimensional spectra of index-of-refraction and density fluctuations are derived. The results indicate that if the three-dimensional spectrum of turbulent index-of-refraction or density fluctuations have an inertial region that follows a power law slope of $-m^{3D}$, then the measured 1-D phase distortion spectrum or OPD spectrum will have a slope of $-(m^{3D} - 1)$, and the 2-D phase distortion or OPD spectrum follows a $-m^{3D}$ power law in the inertial subrange. It can be written equivalently that if the one-dimensional density or index-of-refraction spectrum have an inertial subrange that follows a power law slope of $-m$, the 1-D phase distortion spectrum...
will have a slope of \(-(m+1)\) and the 2-D phase distortion spectrum will have a slope of \(-(m+2)\). For a high-Reynolds number, weakly-compressible turbulent flow, it is expected that the spectral behavior of phase distortions will depend on a combination of effects from temperature, pressure, and pressure-temperature cospectra. If the turbulence is homogenous, temperature will add a contribution in the inertial subrange of the 3-D spectrum with a power law spectral slope of \(-11/3\), pressure will have a contribution with a power law spectral slope of \(-13/3\), and pressure-temperature cospectra will have an unknown deleterious contribution. This means that \(m^{3D}\) could have a spectral slope of \(-11/3\), \(-13/3\), or a value in between. The effects of Reynolds number and flow configuration on these values are also discussed in this chapter.

The chapter continues to use these results along with a model spectrum with different values of \(m^{3D}\) to examine the role of different turbulence scales, aperture size, and tip/tilt removal on \(\text{OPD}_{\text{rms}}\). For cases where the optical distortion is dominated by either temperature or pressure fluctuations, it is clearly evident that the most optically active scales of the flow are the large scales of the turbulence and finite aperture filtering alone does not eliminate their influence even at small aperture sizes. Overall, when index-of-refraction fluctuations are dominated by turbulent pressure fluctuations, a better reduction in \(\text{OPD}_{\text{rms}}\) is achieved when adaptive-optics methods are employed or when the aperture size is reduced. Approximate resolutions for wavefronts at different aperture sizes, spectral slopes, and with and without tip/tilt were calculated using the model spectrum. For all cases, temperature dominated optical distortions require a higher wavefront resolution to resolve pertinent optical features.

Finally, the resolution requirement developed by Mani et al. is revisited using results from the chapter. The revised version of the resolution requirement is more restrictive in some cases compared to the original but is expected to more accurately capture the full extent of turbulent index-of-refraction fluctuations. Its complexity
prevents it from replacing the approximation for resolution length scale developed by Mani et al. however, and their results remain as a good starting point in determining the required grid spacing for aero-optical simulations.
CHAPTER 3

NUMERICAL SIMULATIONS OF SHEAR LAYER AERO-OPTICS

To verify theoretical results presented in the previous chapter and to investigate the optical behavior of a flow system fundamental to aero-optics, simulations of weakly compressible, temporally-evolving shear layers (also called mixing layers) are presented in this chapter. In the next section, the simulation setup and configuration are described followed by results from flow field quantities. The rest of this chapter contains an in-depth discussion into the statistical behavior of aero-optics in shear layers and the effects of temperature and pressure on optical distortions.

3.1 Introduction

To computationally evaluate results from the last chapter and to investigate a flow fundamental to aero-optics, simulations of temporally evolving shear layers are performed due to its simple configuration, the availability of previous high-fidelity simulations to be used for simulation validation, and similarity to separated shear layers commonly found in aero-optic applications. While other fundamental flows could have potentially served the same purpose, no other flow contains the simplicity of the shear layer while enabling the easy introduction of a density gradient that could be utilized to isolate the individual effects of temperature and pressure on turbulent density fluctuations.

LES is employed to simulate the shear layer flow at a sufficiently large Reynolds number to identify a clear inertial subrange. The domain size and simulation initialization are similar to those of the baseline case (termed A3) in the compressible shear
layer research of Pantano & Sarkar [72]. In Figure 3.1, the domain for the current numerical investigations is shown, where $U_1$ corresponds to the upper stream of the flow and $U_2$ is the lower stream. $\Delta U = U_2 - U_1 = 0.6a_0(0)$, where $a_0(0)$ is the speed of sound at the center of the shear layer at the simulation start, so that the freestream velocities at the top and bottom of the mixing layer are equal to $\pm 0.3a_0(0)$. A common measure used to quantify compressibility effects in shear flows is the convective Mach number, $M_c$, defined as

$$M_c = \frac{\Delta U}{a_1 + a_2},$$  \hspace{1cm} (3.1)$$

where $a_1$ is the sound speed in the upper stream of the flow and $a_2$ is the sound speed in the lower stream of the flow. In all simulation configurations in this chapter $M_c = 0.3$ so that the effect of fluid compressibility is weak and on the brink of the rule-of-thumb limit for the incompressible flow approximation typically taken as $M_c = 0.3$.

The domain dimensions are defined relative to the initial momentum thickness of the mixing layer, $\delta_\theta(0)$, and are equal to $L_x = 345\delta_\theta(0)$ and $L_y = 86\delta_\theta(0)$ in the streamwise and spanwise directions, respectively, to be the same as the A3 case by Pantano & Sarkar. It should be noted that in the current simulation, the streamwise coordinate is $x$, the spanwise coordinate is $y$, and the transverse coordinate is $z$. The coordinate system is chosen such that the aero-optic distortion is calculated along the $z$ coordinate in keeping with previous notation. The computational domain is periodic in both the streamwise and spanwise directions. At the top and bottom of the domain are numerical sponge layers used to damp and prevent the reflection of acoustic disturbances in the finite domain [56, 6]. To accommodate the additional length of the sponge layer, the size of the domain in the $z$ direction is approximately 15% larger than Pantano & Sarkar at $L_z = 200\delta_\theta(0)$.

The number of control volumes in each direction of the computational domain is given as $N_x \times N_y \times N_z = 1200 \times 300 \times 696$, 2.3 times the resolution relative
to $\delta_\theta(0)$ in each direction compared to Pantano & Sarkar, and the mesh spacing is approximately equal in all three directions with a uniform distribution. To improve the convergence of statistical results, all statistical quantities presented in this chapter have been ensemble averaged over three individual simulations for each of the three computational configurations (to be discussed in the following paragraphs) along with spanwise and streamwise averaging when possible. The three ensemble simulations are identical except for different initial random perturbation fields.

The flow is first initialized using a hyperbolic tangent profile for the streamwise velocity,

$$u(0) = 0.5\Delta U \tanh \left( -\frac{z}{2\delta_\theta(0)} \right), \quad (3.2)$$

and zero for the other two velocity components. To accelerate the transition of the shear layer to a fully turbulent state, a random field of divergence-free velocity perturbations with a prescribed isotropic spectrum,

$$\Phi_{u_i u_j}(\kappa) = \left( \frac{\kappa}{\kappa_0} \right)^4 e^{-2\left( \frac{\kappa}{\kappa_0} \right)^2}, \quad (3.3)$$
as used by Pantano & Sarkar, is applied to the initial velocity field where $\kappa_0$ is the peak wavenumber. This is accomplished using the method described by Rogallo [77] and the peak wavenumber is selected so that there are 24 peak wavelengths across the streamwise domain. The intensity of the fluctuations are scaled so that their root-mean-square value is 10% of $\Delta U$ and the fluctuations are confined to the region between $-50 < z/\delta_0(0) < 50$ of the domain by applying an exponential ramping function. While the application of the ramping function to the perturbations destroys the divergence-free property of the initial random field, the resulting acoustic disturbances are weak and quickly propagate out of the domain.

### TABLE 3.1

**INITIAL FREESTREAM VALUES FOR SHEAR LAYER SIMULATIONS**

<table>
<thead>
<tr>
<th>Simulation</th>
<th>$s$</th>
<th>$\rho_1/\rho_0$</th>
<th>$\rho_2/\rho_0$</th>
<th>$T_1/T_0$</th>
<th>$T_2/T_0$</th>
<th>$p_1/p_0$, $p_2/p_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>2</td>
<td>1.05</td>
<td>0.9756</td>
<td>1.0244</td>
<td>1.0249</td>
<td>0.9761</td>
<td>1.0</td>
</tr>
<tr>
<td>3</td>
<td>1.35</td>
<td>0.8511</td>
<td>1.1489</td>
<td>1.1750</td>
<td>0.8704</td>
<td>1.0</td>
</tr>
</tbody>
</table>

*Note:* Subscript 0 denotes centerline values.

The initial temperature also follows a hyperbolic tangent profile with the same form as Eqn. (3.2). Since the pressure is held constant across the mixing layer, the initial density profile also has a hyperbolic tangent profile as imposed by the ideal gas law. Three simulation configurations are used to investigate the effect of temperature and pressure on phase distortions. The three simulations differ in their ratio of lower
to upper stream density, \( s = \rho_2/\rho_1 \). To investigate the flow condition where the pressure is expected to play the largest role in index-of-refraction fluctuations, \( s \) is set to 1.0. In the second case, \( s \) is equal to 1.05 to represent a more realistic value of \( s \) in a subsonic separated shear layer. This value is found by examining the ratios of density across separated shear layers in previous computations; simulations of a hemisphere-on-cylinder turret found \( s \) across the separated shear layer to be approximately 1.03 [60] and in computations of cylindrical turrets [109], density ratios in the range of 1.06 – 1.075 were observed across the separated shear layer. To investigate conditions where temperature effects are expected to completely dominate index-of-refraction fluctuations, \( s = 1.35 \). The initial freestream values of thermodynamic quantities for the three simulations can be found in Table (3.1).

In each simulation, the Reynolds number based on the momentum thickness, \( Re_\theta = \Delta U \delta_\theta/\nu_0 \), where \( \nu_0 \) is the average kinematic viscosity at the centerline of the flow, is initially 1600. This is ten times the Reynolds number in the DNS of Pantano & Sarkar. The decision to increase the Reynolds number from the DNS value was made after initial calculations at the DNS Reynolds number did not exhibit a strong, identifiable inertial subrange. While conducting a DNS instead of an LES would have been desirable because of the absence of modeling error and the reduced resolution error in DNS, the clear identification of the inertial subrange behavior is essential to the goals of the simulations and a significant increase in Reynolds number is required.

3.2 Flow Field Results

Major qualitative differences between the three shear layers can be seen in Figure 3.2 where contours of density normalized by the initial centerline density, \( \rho_0 \), are shown for a section of the shear layer at a time step after the flow is fully turbulent. Three different mechanisms of density fluctuations can be observed from the contours. The first mechanism, most easily identified in Figures 3.2(a) & 3.2(b), is the drop in
Figure 3.2. Contours of $\rho/\rho_0$ for a portion of the flow field for the three simulations: (a) $s = 1.0$, (b) $s = 1.05$, and (c) $s = 1.35$. 
pressure at the center of large shear layer roller structures and in the smaller scale
turbulent vortex structures. The second mechanism is viscous heating and is only
clearly visible in the $s = 1.0$ shear layer. The viscous heating is only seen in the
turbulent band of flow near the centerline of the domain and appears as a decrease
in density. The third mechanism is turbulent mixing of different density fluids across
the two freestreams. This may be the only density fluctuation mechanism visible in
Figure 3.2(c) and is partially visible in Figure 3.2(b). While the presence of large scale
vortical structures are not as visible in the density field for the $s = 1.35$ simulation,
periodic ejections of high and low density fluid into the opposite freestream can be
seen and are evidence of large-scale vortex action.

3.2.1 Comparison with Previous Computations and Experiments

To validate the results from the mixing layer simulations, flow field results from
the $s = 1.0$ simulation are presented against comparable simulations; the previously
mentioned weakly compressible DNS by Pantano & Sarkar (case A3) and the DNS
of an incompressible mixing layer performed by Rogers & Moser [78] (presented in
their paper as the baseline simulation). Compared to the current simulation and the
simulation by Pantano & Sarkar, Rogers & Moser imposed initial conditions from
boundary layer computations to initialize their flow field. Near the end of their
simulations, Pantano & Sarkar observed an $Re_\theta$ of approximately 1530 while Rogers
& Moser reached $Re_\theta = 2420$. For the $s = 1.0$ mixing layer, at a non-dimensional
time $\tau = t \Delta U / \delta_\theta(0) = 517.5$ at the end of the simulation, $Re_\theta = 14670$ is reached on
average. For the $s = 1.05$ and $s = 1.35$ mixing layers, the average values of $Re_\theta$ are
14793 and 14352, at $\tau = 518.7$ and 517.8, respectively.

The flow field data presented in the following figures have been time-averaged
when the flow is in the self-similar range as well as spanwise, streamwise, and ensemble
averaged across the three simulations. In the $s = 1.0$ simulation, self-similarity is
reached at $\tau$ greater than 264 and until $\tau = 517.5$, which is comparable to the time averaging range used by Pantano & Sarkar, $\tau = 261 - 518$.

Figure 3.3. Self-similar mean streamwise velocity profile: \textbullet\, $s = 1$ simulation; \textcolor{blue}{- - - - - -}, A3 Pantano & Sarkar [72]; \textcolor{red}{- o - - - - -}, Spencer & Jones [87]; \textcolor{green}{- - - - - - - - - - - - -}, Bell & Mehta [5].

Figure 3.3 compares the average streamwise velocity against the transverse coordinate, $z$, normalized by the vorticity thickness of the shear layer, $\delta_\omega = \Delta U / (\partial U / \partial z)_{max}$. Experimental results from investigations of the self-similar region of incompressible shear layers performed by Bell & Mehta [5] and Spencer & Jones [87] are also presented. The velocity profile compares very well with Pantano & Sarkar’s DNS and compares reasonably well with experimental measurements. Figure 3.4 shows a com-
Figure 3.4. Root-mean-square values of (a) streamwise, (b) transverse, and (c) spanwise components of velocity, and (d) the Reynolds shear stress for the matched temperature shear layer simulation: \( s = 1 \) simulation; A3 Pantano & Sarkar; Rogers & Moser [78]; Spencer & Jones [87]; Bell & Mehta [5].
parison of root-mean-square of streamwise, spanwise, and vertical velocity fluctuations and the spanwise component of the Reynolds stress from the $s = 1.0$ simulation against computational and experimental results. Even though the current simulation is performed at a value of $Re_{\theta,0}$ ten times larger than the previous DNS and uses subgrid scale modeling, the shape of the rms and Reynolds stress profiles compare well with previous calculations. The peak values also compare well for all quantities except for the peak streamwise velocity rms which is roughly $5\% - 10\%$ larger than those in the previous simulations. In all, it appears that the current simulation captures the characteristics of the mixing layers with reasonable accuracy and can be used to analyze the optical properties of the flow. It follows that since the other two simulations for $s = 1.0$ and $s = 1.35$ use the same mesh and computational parameters except for the initial temperature and density profiles, all of the simulations are expected to be able to capture the flow and optical physics faithfully.

3.2.2 Evaluation of the Density Spectrum Composition

With the results of both temperature matched and mismatched shear layers, we seek to analyze the composition of the density spectra and evaluate the validity of the density linearization presented in Section 2.1. Figure 3.5 shows the streamwise wavenumber spectra for density, temperature, pressure, and twice the pressure-temperature cospectra for all simulations at the mixing layer mid-plane, $z/\delta_\theta(0) = 0$. The streamwise wavenumber spectra is averaged in the spanwise direction and over a time window from $\tau = 290.5$ to 334.9, approximately a third of a large-scale eddy-turnover time within that window, to improve statistical convergence in the inertial wavenumber subrange. There are noticeable differences between the spectra in Figure 3.5(a) and Figure 3.5(c), most obviously in the spectral levels of temperature fluctuations. In the matched density shear layer case, the temperature spectrum is nearly an order of magnitude smaller than the other components until higher wavenumbers
where it is comparable to the cospectra magnitude.

In Figure 3.5(c), the temperature spectrum is much larger than the pressure spectra and temperature-pressure cospectra. The temperature spectra and density spectra are very close in the $s = 1.35$ simulation, where the density has the same spectral slope as the temperature. In the matched temperature shear layer, the density spectrum seems to be more closely connected to the pressure and cospectrum and it appears to be a combination of all components. The $s = 1.05$ is an intermediate case where the pressure and density spectrum still appear to be highly connected at low to intermediate wavenumbers but at larger wavenumbers, the increased level of the temperature spectrum has an effect on the density spectrum.

While the spectral slope of the cospectrum in the inertial subrange was undefined in the previous chapter, in the $s = 1.0$ and $s = 1.05$ simulations these slopes are very similar to those observed in the pressure spectra. The inertial subrange behavior of the cospectrum in the $s = 1.35$ simulation is difficult to ascertain from the current results.

To verify the accuracy of the ideal gas linearization utilized in the previous chapter, the same process used to derive the three-dimensional index-of-refraction spectra is used to obtain an expression for the one-dimensional density spectra,

$$
\frac{E_{pp}(x_0, \kappa)}{\bar{p}^2} \approx \frac{E_{pp, \text{approx}}(x_0, \kappa)}{\bar{p}^2} = \frac{E_{pp}(x_0, \kappa)}{\bar{p}^2} + \frac{E_{TT}(x_0, \kappa)}{\bar{T}^2} - 2\text{Re}\left[\frac{E_{pT}(x_0, \kappa)}{\bar{p} \bar{T}}\right].
$$

By comparing $E_{pp, \text{approx}}$ to $E_{pp}$, the accuracy of the linearization can be examined. Figure 3.6 shows the relative percentage error of the density spectra for the three shear layers as a function of the streamwise wavenumber. The relative error is less than 1% for $s = 1.0$, less than 2.5% for $s = 1.05$, and less than 5% for a majority of the spectra for $s = 1.35$. These results demonstrate the accuracy of the linearization introduced in Chapter 2 for aero-optic flows and indicate that the error incurred by
Figure 3.5. Streamwise wavenumber spectra for the density, temperature, pressure and temperature-pressure co-spectra at $z/\delta_\theta(0) = 0$ for (a) $s = 1.0$, (b) $s = 1.05$, and (c) $s = 1.35$: $\rho$, $p$, $T$; $2 \times p-T$ co-spectra.
the linearization increases with increasing temperature fluctuation.

Figure 3.6. Relative percentage error, $100 \times |E_{\rho\rho} - E_{\rho\rho,\text{approx}}|/E_{\rho\rho}$, of the density spectra approximated by Eqn. (3.4) compared to the computed density spectra for (a) $s = 1.0$, (b) $s = 1.05$, and (c) $s = 1.35$.

Using the spectral levels of thermodynamic variables, a cursory evaluation of the approximation by Mani et al. [58], that the density spectrum is only a function of pressure, in their study of the resolution requirement for aero-optics can be made by comparing the spectral levels of pressure and density. In the $s = 1.0$ simulation, the pressure spectral levels are higher than density spectral levels. However, the
shape of the pressure and density spectra are similar and the resolution requirement formulated by Mani et al. can be expected to be accurate. Out of all conditions, in the $s = 1.05$ case the pressure and density spectra are nearly equivalent and the formulation by Mani et al. would give a very good estimate of the necessary resolution. For cases like $s = 1.35$ where the temperature spectrum dominates the density spectrum, Eqn. (2.57) poorly approximates the behavior of the OPD and accounting for the temperature in the resolution requirement as in Eqn. (2.58) would be necessary. As values of $s$ near 1.05 are expected for subsonic separated flows, Mani et al.’s criterion appears to be very good for estimating the resolution needed in these flows. However, if the flow has a larger temperature or density gradient across the area of interest, the resolution needs to be finer in these regions.

3.2.3 Statistics of Flow Variables

To examine thermodynamic differences among the three shear layers, rms values of density, pressure, temperature, and streamwise velocity are presented in Figure 3.7 versus the transverse coordinate normalized by vorticity thickness. The values in these figures are computed via the same averaging method as described in Sec. 3.2.1. The streamwise velocity rms is normalized by $\Delta U$ and the pressure, temperature, and density rms are normalized by the local average values along the $z$ direction. As shown in Figures 3.7(a) and 3.7(b), the dependence of the level and shape of streamwise velocity and pressure fluctuations on density ratio appears to be weak for this range of $s$ values. The only noticeable difference is a slight decrease in the peak value in pressure rms and a shift of the profiles to the low density side of the flow in pressure and streamwise velocity rms for the $s = 1.35$ shear layer.

Temperature fluctuations in the three shear layers are very different however. Most notably, the level of fluctuations in the $s = 1.35$ case is several times larger than the $s = 1.0$ and $s = 1.05$ cases and extends over a larger region of the domain.
Figure 3.7. Root-mean-square profiles of (a) streamwise component of velocity, (b) pressure, (c) temperature, and (d) density:  
- $s = 1.0$;  
- $s = 1.05$;  
- $s = 1.35$.  

For the $s = 1.35$ case, for values in the range $-0.4 < z/\delta_\omega(\tau) < 0.4$, the temperature rms varies little. This is in contrast to the $s = 1.05$ case where a peak in temperature fluctuations exists on the high density side of the flow near $z/\delta_\omega(\tau) = -0.4$ and the $s = 1.0$ case where the level of temperature rms is relatively smooth and evenly distributed between $-0.5 < z/\delta_\omega(\tau) < 0.5$. It is easy to deduce that the increase in temperature fluctuations between the three cases is directly related to turbulent mixing as $\Delta T$ across the shear layer increases with $s$.

The rms of density fluctuations follows the same trend as temperature rms, with peak values increasing with increasing $s$. However, the density rms also appears to show that there is a suppression of density fluctuations in the low density region of the flow for the $s = 1.05$ and $s = 1.35$, cases which is not present in the $s = 1.0$ simulation. This suppression is significant enough that for the $s = 1.05$ case the density rms drops below the baseline $s = 1.0$ case in the low density stream.

To further investigate this phenomenon, the variance of the density fluctuations can be written to first order using the ideal gas law linearization as,

$$\frac{\rho'\rho'}{\rho^2} \approx \frac{T'\rho'}{T^2} + \frac{T'^2}{T^2} - 2\frac{T'}{pT},$$

(3.5)

where $g'^2$ is the variance of a quantity $g'$, and $g'h'$ is the covariance of two quantities, $g'$ and $h'$. This equation was first given by Winarto & Davis in their studies of density, pressure, and temperature fluctuations in turbulent jets [115]. In Figure 3.8, the density variance approximated by Equation (3.5) for each of the three shear layers is shown along with the actual density variance and the components of the density variance given by the right-hand side of Eqn. (3.5). Figure 3.8 shows that the linear approximation very accurately represents the density variance for the $s = 1.0$ and $s = 1.05$ cases but begins to deviate from the actual density variance in the $s = 1.35$ case. For the $s = 1.05$ and $s = 1.35$ cases, the peak of the density variance coincides
Figure 3.8. Density variance budget for the (a) $s = 1.0$, (b) $s = 1.05$, and (c) $s = 1.35$ shear layers: $\frac{\overline{\rho'^2}}{\overline{\rho^2}}$, $\frac{\overline{T'^2}}{\overline{T^2}}$, $\frac{\overline{\rho'^2}}{\overline{\rho^2}}$, $\frac{-2\overline{\rho' T'}}{\overline{\rho T}}$, $\times$, Approximate $\frac{\overline{\rho'^2}}{\overline{\rho(z)^2}}$ given by Eqn (3.5).
with the peak of the temperature variance. It is especially interesting to note that for the \( s = 1.05 \) case the pressure variance peak is more than twice the temperature variance peak. The coincidence of the locations of the density and temperature variance peaks in the presence of a much larger pressure variance peak was also observed in the separated shear layer of Wang & Wang [109].

This phenomenon can be explained by examining the value of \(-2p'T'\), which is negative on the low density side of the shear layer and positive on the high density side for the two temperature and density mismatched shear layers. The peak magnitude of the covariance term is also larger on the low density side for each. This positive \( p'T' \) on low-density side/negative \( p'T' \) on high-density side behavior of temperature-pressure covariance along with the larger covariance magnitude on the low-density side was also observed by Winarto & Davis [115] across the mixing region of a turbulent jet. This behavior of the covariance is best explained by the contours in Figures 3.9 - 3.11.

In Figure 3.9(a), the fluctuating pressure and temperature are shown in a spanwise normal plane at a single time instant for the \( s = 1.0 \) shear layer. The contours of pressure and temperature of the same sign line up almost exactly, leading to an almost entirely positive pressure-temperature covariance as seen in Figure 3.9(b). As there is no mixing of different temperature streams in this flow, it is most likely that the heating and cooling of the shear layer here is of adiabatic nature and directly linked to the large-scale pressure wells.

Figure 3.10(a) shows contours of the fluctuating temperature and pressure for the \( s = 1.05 \) shear layer, and an entirely different mechanism for the covariance behavior is observed. The vortical action of the large shear layer structures centered in the negative pressure fluctuation regions pulls low-temperature fluid into the low-density stream of the flow and high-temperature fluid into the high-density stream of the flow. This in turn creates a region of positive \( p'T' \) in the low-density stream (low
Figure 3.9. Iso-contours of fluctuating temperature and pressure in a spanwise plane for $s = 1.0$. (a) Fluctuating temperature contours overlain with line contours of positive (solid) and negative (dashed) pressure fluctuations; (b) contours of pressure fluctuations multiplied by temperature fluctuations at a single time instant.
Figure 3.10. Iso-contours of fluctuating temperature and pressure in a spanwise plane for $s = 1.05$. (a) Fluctuating temperature contours overlain with line contours of positive (solid) and negative (dashed) pressure fluctuations; (b) contours of pressure fluctuations multiplied by temperature fluctuations at a single time instant.
Figure 3.11. Iso-contours of fluctuating temperature and pressure in a spanwise plane for \( s = 1.35 \). (a) Fluctuating temperature contours overlain with line contours of positive (solid) and negative (dashed) pressure fluctuations; (b) contours of pressure fluctuations multiplied by temperature fluctuations at a single time instant.
temperature fluid convected into a negative fluctuating pressure area) and a region of negative $p'T'$ in the high-density stream (high temperature fluid convected into a negative fluctuating pressure fluctuation area).

In the high pressure region between shear layer rollers, the opposite occurs. The ejection of high and low temperature fluid into the opposite stream drives the mean temperature of the shear region to values between the two freestream temperatures. When high and low temperature fluid about to be convected into the opposite stream sits in the shear region of the flow between vortices, relative to the local mean temperature the components of fluctuating temperature are positive and negative, respectively. When coupled with the positive fluctuating pressure area between vortices, it creates a region of positive $p'T'$ in the low-density stream (high temperature fluid into a positive fluctuating pressure area) and a region of negative $p'T'$ in the high-density stream (low temperature fluid into a positive fluctuating pressure area). Figure 3.12 is an illustration of the physical mechanism described above. This same action is seen in the $s = 1.35$ shear layer but the levels of the instantaneous product of the fluctuating temperature and pressure is larger overall than in the $s = 1.05$ shear layer. As the contribution from the $p'T'$ covariance to the density variance is $-2p'T'$, positive covariance results in a suppression of $\rho^2$ and vice versa.

This in effect increases the level of density fluctuations on high density side of the shear layer and decreases it on the low density side in the $s = 1.05$ and $s = 1.35$ cases. For the density matched shear layer, the covariance is positive and symmetric about the shear layer center and reduces the peak density fluctuations. In all three cases, the covariance has a significant effect on the shape and intensity of the density fluctuations but the behavior is starkly different when a density gradient is introduced across the shear layer because of the mixing by the shear layer.
Figure 3.12. Illustrations of the $p'T'$ generation mechanism in turbulent shear flows.
3.3 Analysis of Shear Layer Optical Path Difference

To calculate the aero-optical distortion, the density is integrated vertically (along the $z$-direction in Figure 3.1) through the mixing layer to calculate the OPL. The spatial mean of the OPL at each time step is then removed from the wavefront to calculate OPD. The OPD is normalized in this chapter using a normalization similar to the one used in Gordeyev et al. [30] but with the local density at the centerline, $\rho_0$, convective Mach number, and initial momentum thickness, $\delta_\theta(0)$,

$$\text{OPD}^N = \frac{\text{OPD}}{\rho_{SL} M_c^2 \delta_\theta(0)}$$ \hspace{1cm} (3.6)

Instantaneous snapshots of OPD$^N$ for each $s$ condition are plotted in Figure 3.13. The $s = 1.0$ and $s = 1.05$ shear layers are seen to have more spanwise coherent structures than the $s = 1.35$ one, which has more small scale structures.

Since the shear layer flow violates the assumption of homogenous turbulence in the derivation of the analytic equations for wavenumber spectrum in Sections 2.1 and 2.2, the wavenumber dependence of the density spectrum along the propagation direction must be examined. In Figure 3.14, the streamwise wavenumber spectrum for each shear layer is examined at five locations along the propagation direction. In all cases, the inertial subrange behavior appears to remain at the same spectral slope at different values of $z/\delta_\theta(0)$. At the outermost locations where the spectrum is the most different, their contribution to optical phase distortions is expected to be minimal since the magnitude of density fluctuations have decreased by more than an order of magnitude compared to the peak density variance.

From these results, it would appear that the decomposition of the turbulent index-of-refraction fluctuations into a fluctuation magnitude component dependent on the propagation distance, $\Phi_{1n}(x_0, \eta)$, and a wavenumber dependent component, $\Phi_{2n}(\kappa_\perp)$, is a valid assumption for this flow. As this is the case, Woo & Ishimaru [116] showed
Figure 3.13. Instantaneous contours of normalized OPD for the (a) $s = 1.0$, (b) $s = 1.05$, and (c) $s = 1.35$ cases at $\tau = 312.7$. All simulations started using the same initial perturbations.
Figure 3.14. Streamwise wavenumber spectra of density at various $z/\delta_\theta(0)$ locations for (a) $s = 1.0$, (b) $s = 1.05$, and (c) $s = 1.35$ mixing layers:

- $-$, $z/\delta_\theta(0) = 0$;
- $-$, $z/\delta_\theta(0) = 10$;
- $-$, $z/\delta_\theta(0) = 20$;
- $-$, $z/\delta_\theta(0) = -10$;
- $-$, $z/\delta_\theta(0) = -20$;

(a) $m_\rho = 5.5/3$, (b) $m_\rho = 4.75/3$, (c) $m_\rho = 4/3$. 
that the phase distortion frequency spectral slope has the same dependence on the
index-of-refraction spectral slope as in the homogenous turbulence case, as mentioned
at the end of Section 2.2.

3.3.1 One-dimensional Spectrum of Optical Path Difference

In Figure 3.15, the spectrum of density fluctuations along the centerline of each
shear layer is shown, along with the wavenumber spectra compensated for their iner-
tial subrange spectral slope. The spectrum is computed and averaged in the spanwise
direction, time-averaged over the interval $\tau = 290.5 - 334.9$, and ensemble-averaged
over three simulations each. By using the compensated spectra, $E_{\rho \rho}/[(\kappa_x \delta_\theta(0))^{m_\rho} \delta_\theta(0) \overline{\theta^2}]$, it is easier to identify a power-law region with a certain spectral slope, $-m_\rho$. The
temperature matched shear layer appears to have a spectral slope between that of the
turbulent pressure and temperature spectra, a value of $m_\rho = 5.5/3$. Figure 3.15(b)
shows that for $s = 1.05$, the value of $m_\rho$ decreases to $4.75/3$ and in Figure 3.15(c) a
value of $m_\rho = 4/3$ is observed in the $s = 1.35$ shear layer, the latter corresponding to
the expected spectral slope for temperature in a shear flow at low Reynolds numbers.

The expectation from Eqn. (2.39) is that the optical phase distortion from the
simulations will contain a spectral slope of $m_{\text{OPD}} = m_\rho + 1$. Figure 3.16 confirms
that the OPD for each case contains a region where the spectral roll-off is indeed
equal to $m_\rho + 1$ (equivalently $m^{3D}_\rho - 1$ for homogenous flow) in the same inertial
 subrange of wavenumbers as the density spectra. This result also confirms that
for this flow, the separation of the index-of-refraction spectrum into independent
transverse wavenumber and a variance related components is valid. It is important
to mention that this may not be the case for all situations, notably, if the beam is
tilted with respect to the orientation of the shear layer.
Figure 3.15. Streamwise wavenumber spectra of density at $z/\delta_\theta(0) = 0$ for the (a) $s = 1.0$, (b) $s = 1.05$ and (c) $s = 1.35$ mixing layers. Insert plots are of the compensated wavenumber spectra, $E_{\rho\rho}/[(\kappa_x\delta_\theta(0))^{m_\rho}\delta_\theta(0)/\rho_0^2]$ with values of (a) $m_\rho = 5.5/3$, (b) $m_\rho = 4.75/3$, and (c) $m_\rho = 4/3$. Dashed lines in each figure reflect lines with these power law slopes.
Figure 3.16. Streamwise wavenumber spectra of the normalized optical path difference for the (a) $s = 1.0$, (b) $s = 1.05$ and (c) $s = 1.35$ mixing layers. Insert plots are of the compensated wavenumber spectra, $E_{\text{OPD}}^N/[(\kappa_x \delta_\theta(0))^{m_{\text{OPD}} \delta_\theta(0)}]$ with values of (a) $m_{\text{OPD}} = 8.5/3$, (b) $m_{\text{OPD}} = 7.75/3$, (c) $m_{\text{OPD}} = 7/3$. Dashed lines in each figure reflect lines with these power law slopes.
3.3.2 Two-dimensional Spectrum of Optical Path Difference

The two-dimensional spectrum of OPD$^N$, $E_{\text{OPD}}^{2D}$, is calculated and shown in Figure 3.17. The spectrum is time-averaged over the interval $\tau = 290.5 - 334.9$, and ensemble-averaged over the three ensemble simulations for each shear layer. The largest qualitative difference in the spectrum of the three cases is the prominence of peaks at $\kappa_x \delta_\theta(0) = \pm 0.07285$, $\kappa_y \delta_\theta(0) = 0.0$ corresponding to the large scale vortical structures. The peaks are easy to identify in the $s = 1.0$ and $s = 1.05$ simulations, but are harder to locate in the $s = 1.35$ shear layer due to the increased overall intensity of fluctuations.

Black contour lines have been placed at wavenumbers within the inertial subrange of the spectrum in each of the images in Figure 3.17. They appear to have a “noisy” appearance due to a lack of statistical convergence. These contours show radial symmetry, demonstrating the isotropy of the turbulence at the small scales of the flow for all values of $s$. Taking slices of the two-dimensional spectrum along the $\kappa_y$ and $\kappa_x$ directions, Figure 3.18 shows that beyond the large scales of the flow, the spectral levels of OPD$^N$ are independent of direction. Further examining the spectra shows that the spectral slopes for the $s = 1.0$, $s = 1.05$, and $s = 1.35$ cases are approximately equal to $m_{\text{OPD}}^{2D} = 11.5/3$, $10.75/3$, and $10/3$, respectively. For each case, $m_{\text{OPD}}^{2D}$ is equal to $m_{\text{OPD}} + 1$ and $m_\rho + 2$ as predicted by the calculations for the inertial subrange slope in the previous chapter. This seems to support that in homogenous turbulence, $m_{\text{OPD}}^{2D} = m_\rho^{3D}$. This result should be interpreted carefully for non-homogenous flows like the shear layer however, where the flow contains obvious inhomogeneity but the relation $m_{\text{OPD}}^{2D} = m_\rho + 2$ still holds for a beam propagating perpendicular to the shear layer.

Using the two-dimensional OPD spectrum, the dependence of the OPD$_{\text{rms}}$ on different scales of the flow can be examined similar to the method in Section 2.4.1. By calculating the square-root of the integral of the spectrum in Figure 3.17 up to
Figure 3.17. Contour plot of $E_{ODN}^{2D}/\delta_{\theta}(0)^2$ for the (a) $s = 1.0$, (b) $s = 1.05$, and (c) $s = 1.35$ mixing layers colored in log scale. Level contours for three levels within the inertial subrange are plotted to showcase the radial symmetry of the spectrum at higher wavenumbers due to small-scale turbulence isotropy.
Figure 3.18. $E^2_{\text{OPD}}/\delta_0(0)^2$ for the (a) $s = 1.0$, (b) $s = 1.05$, and (c) $s = 1.35$ mixing layers along the $\kappa_\perp = \kappa_y \hat{j}$ direction, $\kappa_\perp = \kappa_x \hat{i}$ direction, Dashed lines denote power law slopes of (a) $m^2_{\text{OPD}} = 11.5/3$, (b) $m^2_{\text{OPD}} = 10.75/3$ and (c) $m^2_{\text{OPD}} = 10/3$. Inset plots are of the compensated wavenumber spectra, $E^2_{\text{OPD}}/[(\kappa_{x,y} \delta_0(0))m^2_{\text{OPD}}\delta_0^2(0)]$.
the cut-off wavenumber, \( \kappa^*_c = \kappa_{\perp c} \ell \), the contribution of wavenumbers up to \( \kappa^*_c \) over the total \( \text{OPD}_{\text{rms}} \) can be found and compared to theoretical predictions. The results are compared to theoretical curves computed to match the \( m^{2D}_{\text{OPD}} \) of each simulation and are shown in Figure 3.19 where \( L_0 \) was taken as the length-scale corresponding to the peak wavenumber identified in the two-dimensional spectrum, \( L_0 = 86.25 \delta_\theta(0) \) and \( \ell = L_0/2\pi \).

![Figure 3.19](image-url)

**Figure 3.19.** Fraction of \( \text{OPD}_{\text{rms}} \) captured up to the cutoff wavenumber \( \kappa^*_c \) to the total \( \text{OPD}_{\text{rms}} \) for the \( s = 1.0 \) (--), \( s = 1.05 \) (---), and \( s = 1.35 \) (-----) shear layers. Lines without symbols show the curves predicted by the theory presented in Chapter 2 by Eqn. (2.47) for values of \( m^{3D}_n = 11.5/3, \; m^{3D}_n = 10.75/3, \) and \( m^{3D}_n = 10/3 \) for the \( s = 1.0, \; s = 1.05, \) and \( s = 1.35 \) simulations, respectively. Lines with symbols show the values found from large-eddy simulations.

While the theoretical and computational curves do not match very well at lower values of \( \kappa^*_c \), the curves are almost exactly the same for the final 10%-15% of the \( \text{OPD}_{\text{rms}} \) fraction. The mismatch at lower wavenumbers could be due to a number
of reasons but the effect of the finite and non-square shape of the aperture in the computation and the inability of the model spectrum to capture the anisotropy of the spectrum at large-scales are probably the most important. Since the model spectrum has been defined to match the slope of the inertial subrange of the two-dimensional OPD spectrum in the computation, once $\kappa^*_{c}$ reaches the wavenumbers of the inertial subrange both theoretical and computational curves agree well. This may indicate that the model spectrum is too simplistic to model the large-scale effects of the optical distortions but is very effective for identifying the effects of the intermediate scales of the flow on the overall optical disturbance. This is especially important in the evaluation of resolution requirements as described in the previous chapter and the results in Figure 3.19 show good agreement in the value of $\kappa^*_{c}$ needed to capture 95% of the OPD$_{rms}$.

3.3.3 Autocorrelation of Optical Path Difference

In addition to spectrum, the normalized autocorrelation of the OPD, $\hat{R}_{\text{OPD}}$, is an important statistical quantity that can be used to investigate the spatial structure and behavior of optical distortions. Defined as

$$\hat{R}_{\text{OPD}}(x_0, r_\perp) = \frac{\text{OPD}(x_0)\text{OPD}(x_0 + r_\perp)}{\text{OPD}(x_0)^2}, \quad (3.7)$$

the autocorrelation presented here is equivalent to the correlation function defined in Section 2.1 normalized by the variance at $r_\perp = 0$. $\hat{R}_{\text{OPD}}$ is calculated by ensemble averaging Equation (3.7) for different values of $x_0$ in the $x$-$y$ plane, over the time period $\tau = 290 - 335$, and then over the three independent simulations for each value of $s$. The results for the three shear layers are shown in Figure 3.20 and the contrast between them shows the effect of the freestream density gradient on the overall structure of the OPD.
Figure 3.20. Contour plots of the autocorrelation function of OPD, $\hat{R}_{OPD}$, for the (a) $s = 1.0$, (b) $s = 1.05$, and (c) $s = 1.35$ shear layers.

In all cases, there is a clear spanwise coherent structure to the OPD and regularly spaced bands of correlated and anti-correlated structures. These are linked to the large scale shear layer vortical structures. The most notable difference is that in the spanwise direction, the magnitude of the correlation function in the $s = 1.35$ shear layer decreases much faster than in the $s = 1.0$ and $s = 1.05$ simulations. This is clear in Figure 3.21, where slices of $\hat{R}_{OPD}$ have been taken along the center of the spanwise and streamwise direction. In the streamwise direction, shown in Figure 3.21(a), each shear layer shows maximums and minimums in $\hat{R}_{OPD}$ at the same values of $r_x$, but the magnitude of the autocorrelation function is decreased in the $s = 1.35$ case compared to the other two. The same decrease is seen in the spanwise slice, Figure 3.21(b), where the $s = 1.35$ shear layer reaches a lower correlation value than the other two which are still highly correlated within the computational domain. The difference between the $s = 1.0$ and $s = 1.05$ shear layers are much smaller. The magnitude of the correlation function away from $r = 0$ is slightly decreased in the $s = 1.05$ shear
layer compared to $s = 1.0$ but overall, two correlation functions are very similar in terms of their shape and magnitude.

\[ \hat{R}_{OPD} \]

Figure 3.21. OPD autocorrelation function, $\hat{R}_{OPD}$, in the (a) streamwise and (b) spanwise direction: \( s = 1.0 \); \( s = 1.05 \); \( s = 1.35 \).

3.4 Analysis of Temperature and Pressure Decomposed Optical Path Difference

To further investigate the individual contributions of pressure and temperature to aero-optic distortions, a decomposition of the optical path difference can be made by utilizing the small temperature fluctuation linearization of density from Chapter 2, $\rho' \approx \bar{\rho}(\rho'/\bar{\rho} - T'/\bar{T})$. Using this relationship, the OPL can be approximated as

\[
\text{OPL} \approx \int_0^L 1 + K_{GD} \bar{p} + K_{GD} \bar{p} \left( \frac{\rho'}{\bar{\rho}} - \frac{T'}{\bar{T}} \right) \, dz. \tag{3.8}
\]
It can then be shown that the first two terms in the integrand are just the mean component of the OPL. Solving for the OPD, the relation can be written as

\[
\text{OPD} \approx \int_0^L K_{GDp} \frac{\rho'}{p} \, dz - \int_0^L K_{GDT} \frac{T'}{T} \, dz = \text{OPD}_p + \text{OPD}_T,
\]

(3.9)

where the pressure OPD is defined as \( \text{OPD}_p = \int_0^L K_{GDp} \frac{\rho'}{p} \, dz \) and the temperature OPD is defined as \( \text{OPD}_T = -\int_0^L K_{GDT} \frac{T'}{T} \, dz \).

Calculating the \( \text{OPD}_p \) and \( \text{OPD}_T \) from the flow field database, the ability of the linearization of the OPD to accurately represent the actual OPD is shown in Figure 3.22 for all three shear layers at an instantaneous sample along the streamwise direction. There is little difference between the actual \( \text{OPD}^N \) and the total \( \text{OPD}^N \) reconstructed from \( \text{OPD}^N_p \) and \( \text{OPD}^N_T \) in all cases. In addition to displaying the accuracy of the linearization, Figure 3.22 also shows the difference in instantaneous \( \text{OPD}^N \) signals between the three cases. While the \( \text{OPD}^N_T \) appears to have much more high frequency content and overall larger amplitude in the \( s = 1.35 \) simulation compared to the \( s = 1.0 \) and \( s = 1.05 \) cases, \( \text{OPD}^N_p \) seems to remain very similar in all three. Similarly, the \( \text{OPD}^N_T \) in the \( s = 1.05 \) shear layer has an increase in small scale fluctuations when compared to the \( s = 1.0 \) shear layer.

### 3.4.1 One-dimensional Spectrum of \( \text{OPD}_p \) and \( \text{OPD}_T \)

In Figure 3.23, the one-dimensional wavenumber spectra of \( \text{OPD}^N_p \) and \( \text{OPD}^N_T \) along the streamwise direction are depicted. For all three, the spectral level of \( \text{OPD}^N_p \) remains at similar levels. Spectral levels of \( \text{OPD}^N_T \), on the other hand, increase by nearly two orders of magnitude across all wavenumbers in the \( s = 1.35 \) case and by a notable amount in the \( s = 1.05 \) compared to the \( s = 1.0 \) simulation. For each case, the \( \text{OPD}^N_p \) appears to have a region of spectral slope equal to \(-8.5/3\). \( \text{OPD}^N_T \) appears to have a range of wavenumbers with a slope of \(-7/3\) for the density mismatched
Figure 3.22. Instantaneous values of normalized optical quantities for the (a) $s = 1.0$, (b) $s = 1.05$, and (c) $s = 1.35$ shear layers: $\text{OPD}^N$, $\text{OPD}_p^N$, $\text{OPD}_T^N$, $\text{OPD}^N$, $\text{OPD}_p^N + \text{OPD}_T^N$. 

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shear layers but a slightly higher value of \( m_{OPD_T} \) for the \( s = 1.0 \) shear layer.

These results further support the idea that the OPD is composed of components with different spectral behaviors in the inertial subrange and the spectral behavior of the OPD will depend on the strengths of these components added together. Compared with the spectrum of OPD in Figure 3.16 where the total OPD of each case have three different spectral slopes in the inertial subrange Figure 3.23 directly demonstrates the relationship given by Eqn. (3.4) — OPD is a combination of temperature and pressure signals that have two intrinsically different spectral behaviors in the inertial subrange.

3.4.2 Autocorrelation of \( OPD_p \) and \( OPD_T \)

The normalized autocorrelation function of \( OPD_p \) and \( OPD_T \) can be calculated to look at the spatial structure of the OPD components. In Figure 3.24 the autocorrelation functions of \( OPD_T \), \( \hat{R}_{OPD_T} \), are presented. For the density matched shear layer, the temperature OPD autocorrelation looks similar to the OPD autocorrelation and likely indicates that most of the temperature fluctuations are due to adiabatic heating and cooling in shear layer vortical structures. The \( OPD_T \) autocorrelation of the density mismatched shear layers shows an indication of shear layer vortex structure but the correlation outside of the local field decreases quicker with larger values of \( s \). Slices of the \( \hat{R}_{OPD_T} \) field in the streamwise and spanwise directions are shown in Figure 3.25 and indicates the same information. In both the streamwise and spanwise directions, the \( \hat{R}_{OPD_T} \) of the shear layers with mismatched freestream density decays to smaller values than the \( s = 1.0 \) case.

Contrary to what is observed for \( \hat{R}_{OPD_T} \), the pressure OPD autocorrelation appears almost exactly alike for each of the three cases. For the two-dimensional contour of autocorrelation shown in Figure 3.26 and the streamwise and spanwise slices shown in Figure 3.27, the location of maximums and minimums in the correlation, the
Figure 3.23. One-dimensional spectrum of OPD$_p^N$ (solid) and OPD$_T^N$ (dashed) for the (a) $s = 1.0$, (b) $s = 1.05$, and (c) $s = 1.35$ shear layers: $m_{OPD_p} = 8.5/3$; $m_{OPD_T} = 7/3$. 
Figure 3.24. Contour plots of the autocorrelation function of OPD$_T$, $\hat{R}_{\text{OPD}_T}$, for the (a) $s = 1.0$, (b) $s = 1.05$, and (c) $s = 1.35$ shear layers.

Figure 3.25. OPD$_T$ autocorrelation function, $\hat{R}_{\text{OPD}_T}$, in the (a) streamwise and (b) spanwise directions: $\,$, $s = 1.0$; $\,$, $s = 1.05$; $\,$, $s = 1.35$. 
magnitude of the correlation, and the spatial characteristics of the autocorrelations match very well. This implies that even with the density mismatch, the spatial characteristics of optical distortions due to pressure fluctuations are mostly unchanged and the differences in optical behavior between the three cases are primarily due to index-of-refraction changes caused by temperature fluctuations.

Figure 3.26. Contour plots of the autocorrelation function of $\text{OPD}_p$, $R_{\text{OPD}_p}$, for the (a) $s = 1$, (b) $s = 1.05$, and (c) $s = 1.35$ shear layers.

### 3.4.3 Decomposition and Magnitude of $\text{OPD}_{\text{rms}}$

The effect of pressure and temperature on optical distortions can also be investigated by examining the variance of the OPD in the shear layers. By decomposing
the OPD into OPD\(_p\) and OPD\(_T\) as previously, the OPD\(_{rms}\) is approximately equal to

\[
\text{OPD}_{rms} \approx \sqrt{\text{Var}[\text{OPD}_p] + \text{Var}[\text{OPD}_T] - 2\text{Cov}[\text{OPD}_p, \text{OPD}_T]}, \quad (3.10)
\]

where Var[\(g'\)] and Cov[\(g', h'\)] are the variance and covariance operators, \(\overline{g'^2}\) and \(\overline{g'h'}\), respectively.

In Table (3.4.3) each of the components of this equation and the actual OPD\(_{rms}\) is computed and tabularized. As can be observed, for both density matched and mismatched shear layers, the variance of OPD\(_p\) is nearly the same. This is compared to the variance of OPD\(_T\) which differs by two orders of magnitude between the \(s = 1.0\) and \(s = 1.35\) simulations and a factor of about three between the \(s = 1.0\) and \(s = 1.05\) simulations. The covariances in all cases are close. The approximate OPD\(_{rms}\) from the linearization is close to the values computed directly from the density in the \(s = 1.0\) and \(s = 1.05\) cases but is over-predicted by nearly 4% in the \(s = 1.35\) shear layer.

These results support the notion that the optical distortions due to pressure fluc-
tations are not affected much by reasonable differences in the density of the two shear layer streams. The optical distortion due to temperature fluctuations, however, are strongly affected by the difference in temperature across the shear layer and can become the dominant source of optical distortions when the difference in the two streams is sufficiently large.

**TABLE 3.2**

APPROXIMATE VALUES OF VARIANCE, COVARIANCE, AND RMS FOR THE THREE SHEAR LAYER SIMULATIONS FOR DIFFERENT OPD QUANTITIES

<table>
<thead>
<tr>
<th>Quantity</th>
<th>s = 1</th>
<th>s = 1.05</th>
<th>s = 1.35</th>
</tr>
</thead>
<tbody>
<tr>
<td>Var[OPD\textsubscript{N_p}]</td>
<td>3.8836×10\textsuperscript{-6}</td>
<td>4.1834×10\textsuperscript{-6}</td>
<td>4.0864×10\textsuperscript{-6}</td>
</tr>
<tr>
<td>Var[OPD\textsubscript{N_T}]</td>
<td>2.0414×10\textsuperscript{-7}</td>
<td>5.9057×10\textsuperscript{-7}</td>
<td>1.3558×10\textsuperscript{-5}</td>
</tr>
<tr>
<td>Cov[OPD\textsubscript{N_p},OPD\textsubscript{N_T}]</td>
<td>7.9916×10\textsuperscript{-7}</td>
<td>8.4743×10\textsuperscript{-7}</td>
<td>5.9468×10\textsuperscript{-7}</td>
</tr>
<tr>
<td>Approx. OPD\textsubscript{rms} [Eqn. (3.10)]</td>
<td>1.5777×10\textsuperscript{-3}</td>
<td>1.7547×10\textsuperscript{-3}</td>
<td>4.0564×10\textsuperscript{-3}</td>
</tr>
<tr>
<td>OPD\textsubscript{rms}</td>
<td>1.5770×10\textsuperscript{-3}</td>
<td>1.7541×10\textsuperscript{-3}</td>
<td>3.8995×10\textsuperscript{-3}</td>
</tr>
</tbody>
</table>

*Note:* Presented data is averaged over the same time span as spectral quantities.

### 3.5 Conclusions

Overall, the research in this chapter has shed new light on not only the optical behavior of turbulent shear layers but also in the broader area of the fluid mechanics
of turbulent density fluctuations in shear flows. To validate previously presented theoretical results and examine the aero-optics of a fundamental flow of interest, high-fidelity simulations of three weakly-compressible mixing layers are performed; one condition where the pressure fluctuations are expected to dominate the optical distortion, another where the temperature fluctuations are expected to dominate, and a final condition close to observed computational studies of separated shear layers. This is accomplished by matching the density in the first simulation and having the ratio of the bottom stream and top stream density equal to 1.35 and 1.05 in the second and third conditions, respectively, by adjusting the bottom and top stream temperatures.

Analysis of the simulation data shows good results when compared to previous direct numerical simulations of weakly compressible and incompressible shear layers. The data from the simulation is also used to show that the error of using a linearized expression of the ideal gas law to approximate density is small for all cases. In addition, once the flow developed and became fully turbulent, the simulation data demonstrated that the spectral dependence of density on spatial wavenumber does not have any significant change along the propagation direction. As such, it is expected that the relation between the spectral slope of density and optical phase distortion in the inertial subrange for the shear layer flow will be the same as the relation found for a beam propagating through homogenous turbulence.

After computing the OPD of the flow field, it was shown that the one-dimensional spectral slope magnitude of OPD for all mixing layers obeys a relation that was predicted by the theoretical results, \( m_{\text{OPD}} = m_\rho + 1 \). The value of \( m_\rho \) was observed to be affected by the flow conditions, where the spectral slope of the mismatched density shear layers were noticeably less than that of the matched density shear layer. Additionally, it was observed that the two-dimensional spectral slope magnitude of OPD for each mixing layer follows \( m_{\text{OPD}}^{2D} = m_\rho + 2 \), also as predicted by the theoretical
results. Then, by using the computational results for the two-dimensional spectrum, the fraction of the total OPD$_{rms}$ captured up different wavenumbers was examined. These curves were compared with theoretical results for the dependence of OPD$_{rms}$ on cutoff wavenumber and while there was disagreement at lower wavenumbers, at high wavenumbers agreement between computational and theoretical results was very good.

Examining the normalized autocorrelation functions of the OPD for the shear layers demonstrated that while evidence of the classic shear layer spanwise correlated structures are present in all cases, they are much more prominent in the density matched and $s = 1.05$ shear layer. To further study the individual effects of the pressure and temperature on optical distortions in the shear layers, OPD was decomposed into a temperature OPD, OPD$_T$, and a pressure OPD, OPD$_p$, using the ideal gas linearization. Computing these components of OPD and taking the one-dimensional spectrum further demonstrated that OPD is composed of components with different spectral behaviors in the inertial subrange and the spectral behavior of the total OPD depends on the strength of each component added together. This supports the theoretical results in the previous chapter where the phase distortion spectrum is found to be equal to the sum of temperature, pressure, and pressure-temperature spectral components. When examining the autocorrelation of these components, large differences are observed in $\hat{R}_{OPD_T}$ for different values of $s$ but nearly no difference is seen in $\hat{R}_{OPD_p}$. This indicates that with different values of $s$ the structure of optical distortions due to temperature vary greatly but the optical distortions due to pressure are unchanged and maintain a structure closely tied to the roll-up of shear layer vortices.

Finally, OPD$_{rms}$ is decomposed into contributions from the OPD$_p$ variance, OPD$_T$ variance, and OPD$_p$-OPD$_T$ covariance using the same linearization. It is found that in each of the shear layer configurations, there is little difference between the values
of $\text{OPD}_p$ variance, indicating that the optical distortion due to pressure fluctuations are the same even though the free-stream densities are different. This is compared to the $\text{OPD}_T$ variance, which increases by nearly two order of magnitude from the $s = 1.0$ to the $s = 1.35$ case and is the main source of the increase in total $\text{OPD}_{\text{rms}}$ in all cases.
A canonical geometry in aero-optics, the hemisphere-on-cylinder turret offers great advantages in terms of the field-of-regard but when employed on an airborne platform it suffers from flow conditions detrimental to far-field beam fidelity. In this and the next chapter, a numerical study of the flow field and aero-optics of the hemisphere-on-cylinder will be presented, building on analysis methods developed in the previous two chapters.

In this chapter, the numerical simulation set-up is discussed and the results are validated against experimental measurements. Additionally, the sensitivity of computed flow to mesh resolution are presented along with flow field results from the simulations. Chapter 5 will present more results from the simulations focusing on the aero-optics of the turret flow system.

4.1 Flow Simulation Set-Up and Methods

In simulating the flow over a hemisphere-on-cylinder turret at realistic conditions the high Reynolds number imposes a significant challenge. The strong near-wall mesh resolution requirement [15] restricts the ability of traditional LES methods to simulate this turret flow where the Reynolds number based on the turret diameter, $D$, is equal to $2.3 \times 10^6$ based on experimental conditions [30]. To reduce the number of grid points required for the LES of wall-bounded flows at high Reynolds numbers, a wall model is used to treat the near-wall region and provide approximate wall
boundary conditions to the LES. The wall model is of the basic stress-balance type solving the following equations [43],

\[
\frac{d}{dy} \left[ (\mu + \mu_t) \frac{du}{dy} \right] = 0, \tag{4.1}
\]

\[
\frac{d}{dy} \left[ (\mu + \mu_t)u \frac{du}{dy} + c_p \left( \frac{\mu}{Pr} + \frac{\mu_t}{Pr_t} \right) dT \right] = 0, \tag{4.2}
\]

where \(\mu\) is the viscosity, \(c_p\) is the specific heat at constant pressure, \(Pr = 0.7\) is the Prandtl number, and \(Pr_t\) is the turbulent Prandtl number which is set to be 0.9 in the present simulations. In the model, the turbulent eddy viscosity is defined using a mixing length model with near wall damping as described by Cabot and Moin [11], where \(\mu_t = \kappa_{vk} \mu y^+ [1 - \exp(-y^+/A)]^2\) with \(\kappa_{vk} = 0.41\), \(A = 17\), and \(y^+\) is the distance from the wall in wall units. The equations are solved on a near wall grid embedded in the LES mesh. The outer boundary conditions for temperature and tangential velocity on this embedded mesh are Dirichlet and interpolated from the LES solution, and the wall boundary conditions are no-slip and adiabatic. From the solution of the wall model equations, the shear stress and heat flux at the wall are calculated and applied as approximate boundary conditions for LES.

In the simulation, the domain size is \(15D\) in the streamwise \((x)\) direction, \(5D\)
in the wall-normal ($y$) direction, and $10D$ in the spanwise ($z$) direction, where $D$ is the diameter of the turret, and the center of the turret is located at $4D$ from the inlet. To match the AAOL flight test turret, the ratio of the cylinder height to hemisphere diameter, $H/D$, is 0.375. Three computational meshes are used in the turret computation; the finest mesh contains 401.9 million cells, the medium mesh contains 200.5 million cells, and the coarsest mesh contains 83.3 million cells. With each refinement, the emphasis is on the region immediately surrounding the turret. The entire mesh is composed of hexahedral cells and generated using a multi-block structure. On the turret and bottom wall, the wall model is applied using a stretched grid containing 125 points in the wall-normal direction.

### TABLE 4.1

<table>
<thead>
<tr>
<th>Mesh</th>
<th>No. cells ($\times 10^6$)</th>
<th>$\Delta y_1$ on turret</th>
<th>$\Delta t$</th>
<th>Statistical averaging time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse</td>
<td>83.3</td>
<td>$1.05 \times 10^{-3}D$</td>
<td>$8.0 \times 10^{-4}D/a_\infty$</td>
<td>$300D/a_\infty$</td>
</tr>
<tr>
<td>Medium</td>
<td>200.5</td>
<td>$9.0 \times 10^{-4}D$</td>
<td>$5.5 \times 10^{-4}D/a_\infty$</td>
<td>$600D/a_\infty$</td>
</tr>
<tr>
<td>Fine</td>
<td>401.9</td>
<td>$8.25 \times 10^{-4}D$</td>
<td>$4.65 \times 10^{-4}D/a_\infty$</td>
<td>$300D/a_\infty$</td>
</tr>
</tbody>
</table>

The first off-wall node for LES is located approximately $8.25 \times 10^{-4}D$ from the turret surface for the fine mesh, $9.0 \times 10^{-4}D$ for the medium mesh, and $1.05 \times 10^{-3}D$ for the coarse mesh. The grid spacings of the first off-wall control volume in the wall-normal and streamwise directions along the turret centerline in wall units for
all grids are shown in Figure 4.2. At the bottom wall, the first off-wall node is located $1.0 \times 10^{-3}D$ from the wall for the fine and medium resolution meshes and $1.25 \times 10^{-3}D$ from the wall for the coarse mesh. The wall layer interpolation height is located between the second and third control volume center for all three simulations.

![Image](image1)

Figure 4.2. Grid spacings in terms of (a) first wall-normal cell height ($\Delta x_1^+$) and (b) streamwise cell length at the turret surface ($\Delta y_1^+$) in wall units along the centerline:  
- Fine mesh;  
- medium mesh;  
- coarse mesh.

The inlet condition is provided by a mean turbulent boundary layer profile taken from a 2-D RANS simulation using FLUENT of a flat-plate boundary layer to match the boundary layer thickness of Gordeyev et al. [34] which is 10% of the turret diameter. Because the boundary layer is thin relative to the turret height, the turbulent fluctuations in the boundary layer are not expected to have a major effect on the vortical structures around the turret and are therefore not provided at the inlet. The
outlet boundary employs a sponge layer of thickness $5D$ prior to the boundary that damps out vortical structures and acoustic waves before the flow approaches the outlet. Similarly, a sponge region is placed at the top of the domain with a thickness of $1.5D$. A running-time average of the flow variables is employed as the outlet boundary condition. In the spanwise direction, periodic boundary conditions are used at $-5D$ and $5D$ from the turret center. Fully upwind layers with a thickness of $D$ at the inlet and $5D$ at the outlet are employed to prevent the reflection of outgoing waves back into the domain.

The simulation is performed at a Reynolds number of $Re_D = 2.3 \times 10^6$ to match the experimental condition [30]. The free-stream Mach number is 0.4. The flow fields for each simulation are initialized with a solution obtained from coarser-mesh simulations and run for at least $75D/a_\infty$ until transients are eliminated. A constant time step is then employed for time marching with values of $\Delta t = 4.65 \times 10^{-4}D/a_\infty$ for the fine mesh simulation, $\Delta t = 5.5 \times 10^{-4}D/a_\infty$ for the medium mesh, and $8 \times 10^{-4}D/a_\infty$ for the coarse mesh. Aero-optical and flow field statistics are calculated over a time period of 16 flow-through times, or $600D/a_\infty$ in the medium mesh simulation and 8 flow-through times, or $300D/a_\infty$, in the coarse and fine mesh simulation. In the medium mesh simulation, instantaneous flow fields of velocity, pressure, and density in the entire domain are collected every 90 time steps, or $0.0495D/a_\infty$, over this time-span, generating a database of approximately 100 TB for in-depth analysis of the flow field and optics.

4.2 Mesh Sensitivity and Comparison with Experimental Measurements

In Figure 4.3, to briefly examine the canonical flow topology of a hemisphere-on-cylinder turret before making comparisons of flow quantities from different resolution simulations, the streamlines of the time-averaged velocity field are shown. Flow coming toward the turret near the wall is pulled into the necklace vortex that then
Figure 4.3. Streamlines of the time-averaged velocity field colored by the dimensionless velocity magnitude.
wraps around the turret base and is convected downstream. At the hemisphere surface, the flow separates and forms a recirculation region behind the center of the turret. The core of this low-speed recirculating flow is pushed downstream by the separated shear layer forming near the cylindrical base and develops into two counter-rotating horn vortices.

4.2.1 Comparisons of Flow Quantities from Different Resolution Simulations

To verify that the computational solution is insensitive to the computational mesh, a convergence study is performed by comparing results from different grid resolutions. Figure 4.4 shows the time-averaged pressure coefficient along the turret centerline. For all three simulations, the pressure coefficient compare very well with the experimental measurements of Gordyev et al. [30, 33] and show convergence to the fine mesh solution as resolution increases. For all simulations, the peak value reaches approximately $-1.2$ at $85^\circ$ and after separation, the fine and medium resolution solutions return to a back pressure coefficient of nearly $-0.4$. Compared to the other two meshes, the back pressure in the coarsest mesh appears slightly higher. All three simulations show flow separation at $110^\circ - 115^\circ$ as observed in the experimental data.

In Figures 4.5-4.7, several comparisons of flow statistics among the fine, medium, and coarse simulations are presented. In each figure, the first profile is located at the turret center ($x/D = 4.0$) and each subsequent profile is displaced by $0.25D$ downstream to show the evolution of the flow as it moves through the turret wake.

Figure 4.5 shows the time-averaged streamwise velocity in the same center-plane of the turret, cutting through the separated shear layer and the wake. The differences among the profile are small, showing good agreement with respect to the magnitude of the freestream velocity and the mean structure of the shear layer. The difference
Figure 4.4. Pressure coefficient along the turret centerline; 
- Fine mesh;  - medium mesh;  - coarse mesh;  - Experimental measurements of Gordeyev et al. [30, 33].

Figure 4.5. Mean streamwise velocity along the turret centerline; 
- Fine mesh;  - medium mesh;  - coarse mesh;  - zero value reference line for each profile. The first profile for each is located at the center of the turret with subsequent profiles placed 0.25D downstream of the previous profile.
Figure 4.6. Root-mean-square of streamwise velocity fluctuations along the turret centerline; ——, Fine mesh; —, medium mesh; ——, coarse mesh; ——, zero value reference line for each profile. The first profile for each is located at the center of the turret with subsequent profiles placed 0.25D downstream of the previous profile.

between the fine and medium resolution profiles also appears to be smaller than the difference between the medium and coarse simulations indicating convergence toward the fine mesh solution. The same is true for the rms of the streamwise velocity fluctuations along the turret centerline in Figure 4.6, where the shape and the peaks appear to be most similar in the medium and fine resolution solutions. There does appear to be a difference in the peak rms value 0.5D and 0.75D downstream from the turret center, but the coarser resolution solutions approach the fine mesh solution elsewhere. There are noticeable oscillations in the solution of the $u'_{rms}$ below the shear layer due to the relatively long time required for statistical convergence in this flow region. As such, it is hard to draw conclusions about the sensitivity of the $u'_{rms}$ solution to the mesh in this area of the flow.

Of inherent importance to the study of aero-optics are density fluctuations. In Figure 4.7, the rms of density fluctuations are shown in the wake along the turret centerline, $z/D = 0.0$. Good agreement among all three resolutions is achieved up until approximately $D$ downstream of the turret. In the far wake, there are noticeable discrepancies between the coarse simulation and the fine and medium simulations but
Figure 4.7. Root-mean-square of density fluctuations along the turret centerline; —, Fine mesh; —— medium mesh; —--, coarse mesh; -- zero value reference line for each profile. The first profile for each is located at the center of the turret with subsequent profiles placed 0.25D downstream of the previous profile.

This difference appears to be only in the flow region underneath the shear layer. Most importantly, the medium-mesh and fine-mesh results agree well.

To examine the mesh sensitivity of the flow solution in a region where the flow field is highly three-dimensional due to a horn vortex, profiles of mean z-component of velocity, $W$, and $\rho'_{\text{rms}}$ are shown in a spanwise slice one-quarter diameter from the turret diameter ($z/D = 0.25$) in Figures 4.8 and 4.9, respectively. In Figure 4.8, profiles of the mean spanwise velocity show the impact of the horn vortex circulation can be noticed where there is jet-like area of negative $W$ pulling flow inward to the centerline of the turret for the first 1.25D after the turret center. This strong inward fluid motion in the near wake is countered later in the wake by flow moving in the positive $z$-direction near the wall. This bulk rotation of the flow continues far downstream as the horn vortices convect away from the turret. In the far wake, the profiles for all three simulations are similar but in the near wake, there are some differences in the location of the jet-like structure. As the mesh is refined, the peak negative $W$ in the near wake is located higher and the medium and fine mesh solutions agree very well after 0.75D downstream from the turret center.
Figure 4.8. Mean spanwise velocity at different streamwise locations at an off-centerline plane of $z/D = 0.25$; — — —, Fine mesh; — — — —, medium mesh; — — , coarse mesh; — — — —, zero value reference line for each profile. The first profile for each is located at the center of the turret with subsequent profiles placed $0.25D$ downstream of the previous profile.

Figure 4.9. Root-mean-square of density fluctuations at different values of $x/D$ at an off-centerline plane of $z/D = 0.25$; — — — —, Fine mesh; — — — —, medium mesh; — — — — —, coarse mesh; — — — — —, zero value reference line for each profile. The first profile for each is located at the center of the turret with subsequent profiles placed $0.25D$ downstream of the previous profile.
Figure 4.10. Mean streamwise velocity at different streamwise locations in a plane through \( y/D = 0.1875 \); — — —, Fine mesh; — —, medium mesh; — — —, coarse mesh; — — — —, zero value reference line for each profile. The first profile for each is located at the center of the turret with subsequent profiles placed 0.25\( D \) downstream of the previous profile.

While the peak values of \( \rho'_{\text{rms}} \) in each profile of Figure 4.9 are similar for the fine and medium simulations, there is some variability in the profile shape between all three simulations. At the profiles \( D \) and 1.25\( D \) from the turret centerline, the coarse mesh simulation predicts a lower shear layer position and peak density rms than the other two simulations. At all locations, the fine and medium simulations agree well. In all simulations, the peak \( \rho'_{\text{rms}} \) location moves closer to the bottom wall and the profile broadens as \( x/D \) increases.

In Figure 4.10, profiles of the mean streamwise velocity in the turret wake in a wall-normal slice through the center of the cylindrical turret base (\( y/D = 0.1875 \)) show good convergence towards the fine-mesh solution. While the fine and medium mesh simulations show very similar results, the coarse simulation under-predicts the thickness of the wake near the cylinder base. This can also be noticed in Figure 4.11, where line plots of the \( \rho'_{\text{rms}} \) are shown in the same plane through the cylindrical turret base. Again, the coarse mesh under-predicts the wake thickness compared to
Figure 4.11. Root-mean-square of density fluctuations at different streamwise locations in a plane through $y/D = 0.1875$; ——, Fine mesh; ——, medium mesh; ——, coarse mesh; ——, zero value reference line for each profile. The first profile for each is located at the center of the turret with subsequent profiles placed $0.25D$ downstream of the previous profile.

the medium and fine mesh simulations.

4.2.2 Aerodynamic Coefficients

The unsteady forcing on the turret exerted by the surrounding flow is of fundamental aerodynamic importance. In Figure 4.12(a) and Figure 4.12(b), the time-histories of the drag coefficient,

$$C_D = \frac{F_z}{\frac{1}{2} \rho_{\infty} U_{\infty}^2 D^2},$$

(4.3)

and spanwise force coefficient,

$$C_{F_z} = \frac{F_z}{\frac{1}{2} \rho_{\infty} U_{\infty}^2 D^2},$$

(4.4)

of the turret from the medium mesh simulation are shown. Note that $D^2$ is used here as the reference area for convenience. While the drag coefficient is dominated by the difference in the pressure from the front to the back of the turret and contains higher
Figure 4.12. Time histories of the instantaneous (a) drag coefficient, (b) spanwise force coefficient, and (c) moment coefficient about an axis aligned vertically through the turret center from the medium-resolution simulation.
frequency oscillations, the spanwise force coefficient fluctuations are mainly caused by alternating, lower frequency, coherent vortical structures on either side of the turret.

In addition to unsteady forcing, the torquing of the turret along its vertical axis due to shear stress on the turret surface is of concern as it could lead to mechanical jitter in turrets without adequate damping. Figure 4.12(c) shows the time history of the moment coefficient aligned with the vertical axis through the turret center,

\[ C_{M_y} = \frac{M_y}{\frac{1}{2} \rho_\infty U_\infty^2 D^3}. \]  

(4.5)

A visual comparison between \( C_{M_y} \) and \( C_{F_z} \) shows that many of the same signal structures are contained in each and that there may be a physical connection between the spanwise forcing on the turret and the yaw moment. In Table 4.2, the mean and rms of the drag coefficient and the rms of spanwise force coefficient are provided with the contributions of the shear stress and pressure to the coefficients, by subscripts \( f \) and \( p \) respectively. Contributions from the shear stress are only collected in the medium and fine mesh simulations. However, in each, the shear stress contributes very little to the overall forcing (< 3% in the drag coefficient) and is largely insignificant to the total value of the coefficients. The rms of the moment coefficient about the vertical axis is also included in Table 4.2.

In terms of the convergence of the aerodynamic coefficients across all three simulations, the difference in the value of \( C_{D,p} \) between the medium and coarse mesh simulations is 14% of the medium simulation value and between the fine and medium mesh simulations, the difference is 2.3% of the fine simulation value. The rms values of \( C_{D,p} \) and \( C_{F_z,p} \) are close for all simulations, and within 8% of the fine mesh value. In all, the aerodynamic coefficients approach the fine-mesh solution or are very close in value for all simulations.
### Table 4.2

Turret Aerodynamic Coefficients Computed From the Three Simulations

<table>
<thead>
<tr>
<th>Simulation</th>
<th>$C_{D,p}$ mean</th>
<th>$C_{D,p}$ rms</th>
<th>$C_{D,f}$ mean</th>
<th>$C_{D,f}$ rms</th>
<th>$C_D$ mean</th>
<th>$C_D$ rms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse</td>
<td>0.2427</td>
<td>0.0094</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Medium</td>
<td>0.2822</td>
<td>0.0109</td>
<td>0.0076</td>
<td>7.672 ×10$^{-5}$</td>
<td>0.2898</td>
<td>0.0109</td>
</tr>
<tr>
<td>Fine</td>
<td>0.2889</td>
<td>0.0102</td>
<td>0.0075</td>
<td>7.231 ×10$^{-5}$</td>
<td>0.2964</td>
<td>0.0103</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Simulation</th>
<th>rms $C_{M_y}$</th>
<th>rms $C_{F_x,p}$</th>
<th>rms $C_{F_x,f}$</th>
<th>rms $C_{F_x}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse</td>
<td>–</td>
<td>0.0363</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Medium</td>
<td>5.244 ×10$^{-5}$</td>
<td>0.0364</td>
<td>8.485 ×10$^{-5}$</td>
<td>0.0365</td>
</tr>
<tr>
<td>Fine</td>
<td>4.803 ×10$^{-5}$</td>
<td>0.0366</td>
<td>8.595 ×10$^{-5}$</td>
<td>0.0367</td>
</tr>
</tbody>
</table>
4.3 Flow Field Results

In Figure 4.13(a)-4.13(c), instantaneous visualizations of the unsteady flow field are shown illustrating the highly unsteady and turbulent nature of the flow over the turret. Figure 4.13(a) shows two contour levels of $\lambda_2$ (whose definition is given by Jeong & Hussain[40]) where blue contours are lower values of $\lambda_2$ (indicating stronger vortical structures) and red contours are higher values of $\lambda_2$. Most noticeable is the high level of turbulence in the flow – vortical structures are highly dispersed throughout and are intermittent in the far wake. The majority of strong vortical structures are present in the separated shear region and in the region where the horn vortices are formed.

The large vortical structures in the wake are better visualized in this flow by removing the time-average of the pressure field from an instantaneous field to calculate the pressure fluctuations. Iso-surfaces of the pressure fluctuations at a value 0.7% lower than the mean and colored by density fluctuations are depicted in Figure 4.13(b). While there is a continual development of vortical structures generated near the separation, the strongest vortex cores in the flow are formed at the separation point on the cylinder base. These vertically aligned vortices form at irregular periods and are partially convected into the horn vortices as they develop. The bottom of the vortices continue to convect downstream from the turret while the near hemisphere portions wrap around the horn vortices and tend to settle into slower recirculation regions and the horn vortex cores. As a result, these vortices can form long, snaking structures in the wake like those seen at the bottom right of Figure 4.13(b).

These pressure fluctuations are strongly correlated to density fluctuations in the turret wake and several of the structures in Figure 4.13(b) can also be observed in the low density regions of Figure 4.13(c). The alternating high and low density regions possess a band-like quality and these banded structures are commonly observed convecting in the downstream direction in optical wavefronts. While only the near
Figure 4.13. Instantaneous fields in the turret wake from the medium mesh simulation: (a) Iso-surfaces of $\lambda_2$: blue structures denote stronger, coherent vortices while red structures represent weaker vortices; (b) iso-surfaces of pressure fluctuations 0.7% lower than the local mean value colored by density fluctuations, ranging from -2.5% (blue) below the local average to 0.5% (red) above the local average; (c) iso-surfaces of density fluctuations: Red and blue regions are 1.25% larger and smaller than the local mean density value, respectively.
turret structures will affect the aero-optics, these large scale structures in the wake can effect the dynamics of near turret density fluctuations.

In Figure 4.14, contours of the instantaneous density fluctuations and $y$-component vorticity in a wall-normal slice midway through the turret cylinder, $y/D = 0.1875$, are shown. In both contours, shear layers are formed from the separation of the flow on both sides of the cylinder and develops into vortical structures convecting downstream in the near wake as shown in Figure 4.14(a). The strong vortical structures generated by the shear layer are the cause of the regions of low fluctuating density present downstream on either side of the turret in Figure 4.14(b). As mentioned, periodically a large vortical structure is generated in the wake of the turret and an incidence of this phenomenon is seen in the positive $z/D$ region of the flow between $D$ and $3D$ downstream of the turret center in both Figure 4.14(b) and Figure 4.14(a). While portions of the large vortex core can be easily identified by the large negative density fluctuations in Figure 4.14(b), in the instantaneous vorticity contour, the location of the vortex is coincident with an increase in local, small-scale positive and negative vorticity and the large scale flow structure is difficult to identify.

In Figure 4.15 time-averaged streamwise velocity, pressure, density, and temperature are shown in the $x$-$y$ plane cutting through the turret center. Several important flow features can be observed. In Figure 4.15(a), deceleration and then acceleration of the flow over the turret are present followed by boundary layer separation and formation of a separated shear layer. The highest speed attained by the flow is nearly 50% larger than the freestream velocity, approaching Mach 0.6. The separated flow reattaches to the wall at approximately $1.25D$ behind the turret, and an increase in temperature in Figure 4.15(c) near this point is coincident with the reattachment. Also notable in the temperature contours is the presence of a mean temperature gradient across the shear layer, similar to what is simulated in two of the temporal shear layers in Chapter 3. This implies that along the turret centerline there is
Figure 4.14. Contours of (a) vorticity and (b) density fluctuations in an $x$-$z$ plane cutting midway through the turret cylinder ($y/D = 0.1875$) taken from the fine mesh simulation.
some increase in $\overline{\rho'^2}$ in the shear region due to the $\overline{p'T'}$ mixing mechanism as was seen in the $s = 1.05$ and $s = 1.35$ temporal shear layer simulations. The formation of the necklace vortex $0.25D$ in front of the turret is evident in Figure 4.15(b) and Figure 4.15(d) by the areas of low density and pressure. The recirculation region of the flow can also be noticed in these two figures where there is a large region of low pressure and density immediately behind the turret.

To examine the mean movement of fluid in the wall-normal direction, the mean wall-normal velocity in a slice midway through the cylinder is shown in Figure 4.16. Primarily the effects of the necklace and horn vortices are visible. Upstream, the necklace vortex pulls the flow away from the wall as the flow approaches the turret and toward the wall near the turret front. The necklace vortex wraps around the turret but weakens as it convects downstream. The effect of the horn vortex is confined to the wake of the turret, pulling fluid on either side of the wake upward and convecting flow downward in the centerline of the wake. The region of the largest
Figure 4.16. Contours of time-averaged wall-normal velocity, $V/U_\infty$, in an $x$-$z$ plane cutting midway through the turret cylinder ($y/D = 0.1875$) from the fine mesh simulation.

downward velocity in the centerline behind the turret is coincident with the boundary layer reattachment in the wake.

In Figure 4.17, the time-average pressure and the skin friction coefficient from the wall model,

$$C_f = \frac{||\tau_w||}{\frac{1}{2} \rho U_\infty^2},$$

(4.6)

where $||\tau_w||$ is the magnitude of the mean wall shear stress, on the turret surface is shown. The pressure on the surface of the turret shows the largest value of pressure is at the very front of the turret at the stagnation point and the flow accelerates in all directions around the turret. The location of the lowest pressure on the cylinder is closer to the front of the turret compared to the hemispherical section of the turret. Because of this, the flow appears to detach earlier on the cylindrical portion of the turret. In Figure 4.17(b) the skin friction based on the shear stress magnitude shows that the flow remains attached to the turret surface longer on the hemispherical
portion of the flow compared to the cylinder, and very close to the wall, effects from the necklace vortex on the shear stress can be seen.

The impact of the flow system on the wall on which the turret is mounted was studied by Gordeyev et al. [33] using fluorescent oil to visualize the surface-flow topology as shown in Figure 4.18(b). A visualization of the surface flow topology from the simulation is shown in Figure 4.18(a) and is found by integrating streamlines from the time-averaged velocity on the computational cells nearest to the wall. Colored contours in Figure 4.18(a) are of the time-averaged pressure on the wall. Evidence of several key flow features are present in each figure. In both the experiment and computation, streamlines are attracted to the locations of vortex structures in the flow. The necklace vortex wraps around the turret and aligns with the flow direction as they move downstream. Signs of the horn vortices are present in the turret wake, moving downstream and away from the turret centerline. The reattachment location can also be seen in both visualizations where the streamlines are diverging behind
Figure 4.18. (a) Streamlines integrated close to the bottom surface in the medium resolution simulation, contour colored by $\bar{p}/p_\infty$. (b) Surface-flow topology visualized using fluorescent oil (taken from Gordeyev et al. [33]) over a $M = 0.35$ hemisphere-on-cylinder turret.
the turret in the wake of the turret. The reattachment point is coincident with an increase in pressure as shown in the Figure 4.18(a).

4.3.1 Density Variance Budget Analysis

To examine the role of pressure and temperature in causing density fluctuations, which are at the heart of aero-optical distortions, the decomposition of the density variance presented in Chapter 3,

\[
\frac{\rho'\rho'}{\rho^2} \approx \frac{\rho'^2}{\rho^2} + \frac{T'^2}{T^2} - 2\frac{\rho'T'}{\rho T},
\]

is used to investigate the turret flow field. The results in this section are based on the medium resolution simulation. In Figure 4.19, contours of the components of the density variance budget given by Eqn. (4.7) at a streamwise-normal plane immediately behind the turret \((x/D = 4.5)\) are shown. The density fluctuations in the separated shear layer generated from the turret’s cylindrical base are both stronger and affect a wider region than the shear layer generated from the turret hemisphere. This is due to the much stronger pressure fluctuations in the turret cylinder shear layers compared to the hemisphere shear layer as shown in Figure 4.19(b). The effect of the temperature variance on density variance is much weaker than the pressure and the difference in peak temperature variance between the cylinder and hemisphere shear layers is much smaller compared to density and pressure.

In Figure 4.19(d), the temperature-pressure covariance is shown. The shear layers emanating from the cylindrical base show largely negative values of the covariance component, indicating that the pressure and temperature fluctuations in these shear layers are strongly correlated. In the shear layer generated by the hemisphere, there is a positive contribution from the covariance component indicating the shear layer mixing phenomenon described in Chapter 3 is present. Examining the temperature
Figure 4.19. Components of the density variance budget in the $x/D = 4.5$ plane at the back of the turret.
field around the turret, the physical explanation for why a positive contribution to the covariance term in the density variance budget is only seen over the top of the hemisphere and not the turret sides is because the acceleration of the flow is greater over the hemispherical portion of the turret. As such, the temperature gradient across the shear layer emanating from the top of the hemisphere is much greater than the temperature gradient across the cylinder shear layers.

Figure 4.20. Components of the density variance budget at a streamwise normal slice in the turret wake, $x/D = 5.5$.

Further downstream at $x/D = 5.5$, in Figure 4.20 the influence of the horn vortices has rotated the top of the shear layers generated by the turret cylinder inward
toward the centerline, pulled the shear layer from the turret hemisphere toward the bottom wall, and thickened the region affected by large density, pressure, and temperature fluctuations. It is noted that the pressure variance plays a much larger role in density variance compared to the temperature variance, which is several times smaller. The only major difference between the pressure and density variance is that the areas of larger density variance are confined to the outer region of the wake and the pressure variance shows more widespread intensity throughout the wake region. The explanation for the difference is that the inner wake region has a large negative covariance component offsetting much of the inner wake density variance contributed by the pressure variance as shown in Figure 4.20(d). The outer wake region at the top of the wake shows a positive covariance component, showing that as the cylindrical shear layers convect with the horn vortices, mixing occurs over an extensive region resulting in a mild density variance increase.

Taking a more quantitative look at the density variance budget, Figure 4.21 shows the components of the density variance budget and the approximate density variance found by evaluating Eqn. (4.7) in the shear layer region in the turret centerplane, $z/D = 0.0$. At all locations there is no observable difference between the actual density variance and the approximate value supporting the validity of the linearization for this case. Also in all locations the peak value of temperature and pressure variance are similar but pressure variance tends to be an equal or largest component of density variance at most wall-normal locations. Still, at its peak value the density variance is approximately twice the peak variance of temperature or pressure. Additionally, the peak density variance is coincident with the peak temperature variance location and the change in sign of the covariance component is present in the shear layer as observed in Chapter 3.

Figure (4.22) shows the same quantities in a wall-normal plane where the turret hemisphere meets the cylinder ($z/D = 0.375$). Here pressure fluctuations are the
Figure 4.21. Components of the density variance budget in the turret center-plane, $z/D = 0.0$, in the shear layer region; $\overline{\rho'^2/\overline{\rho}^2}$; $\overline{T'^2/T^2}$; $\overline{\rho'^2/\overline{\rho}^2}$; $-2\overline{\rho'T'/\overline{\rho}T}$; $\times$, Approximate $\rho'^2/\overline{\rho}^2$ given by Eqn (4.7); ——, zero value reference line for each profile. The first profile is located at the center of the turret with subsequent profiles placed $0.25D$ downstream of the previous profile. All values have been multiplied by a factor of $10^4$. 
Figure 4.22. Components of the density variance budget in a slice at the top of the turret cylinder ($y/D = 0.375$): $\overline{\rho'}/\rho^2$; $\overline{T'}/T^2$; $\overline{\rho'^2}/\rho^2$; $-2\overline{p'}/T$; $\times$, Approximate $\rho'^2/\rho^2$ given by Eqn (4.7); $-$ $-$ $-$, zero value reference line for each profile. The first profile is located at the center of the turret with subsequent profiles placed $0.25D$ downstream of the previous profile. All values have been multiplied by a factor of $10^4$. 
dominant source of density variance. The peak density and pressure variance values are nearly the same at almost all locations and the peak temperature variance is several times smaller. However, the peak density fluctuations still occur at the same location as the peak temperature fluctuations even though the temperature effects are small. At all locations, the covariance component of the density variance budget is negative, implying that here the temperature fluctuations are largely correlated with pressure and due to adiabatic heating and cooling in shear layer structures. Again the approximate density variance found by evaluating Eqn. (4.7) is nearly identical to the density variance directly from the numerical simulation.

4.3.2 Turret Surface Pressure Proper Orthogonal Decomposition

Proper Orthogonal Decomposition (POD) has recently been used by Gordyev et al. [35] to investigate the behavior of global pressure fluctuations on the surface of optical turrets measured using pressure sensitive paint. Using POD allows for the decomposition of flow dynamics into its most important modes in a least-squares sense and provides structure in an otherwise largely ‘noisy’ signal. POD is typically used to decompose a signal into spatial modes that contain the largest possible variance of the signal and provides a corresponding energy that quantifies the contribution of that mode to the total energy of a signal.

If spatial POD modes are used to recreate a function, $g(x, t)$, the temporal evolution of each spatial mode must also be known. $g(x, t)$ can be decomposed as a summation of its POD spatial modes and their temporal coefficients as,

$$g(x, t) = \sum_{i=1}^{N} \sqrt{\lambda_i} u_i(t)v_i(x), \quad (4.8)$$

where $N$ is the number of modes (and spatial locations), $v_i$ is the $i^{th}$ spatial mode, $u_i$ is the set of temporal coefficients for the $i^{th}$ mode, and $\lambda_i$ is the eigenvalue corresponding
Figure 4.23. Build up of modal energy of the first 100 POD modes for the turret surface pressure.

to the $i^{th}$ mode. Note that $\sqrt{\lambda_i}$ has the same dimension as $g$ and $v_i$ and $u_i$ are both dimensionless functions. Further information about the mathematical basis of POD can be found in Appendix A.

To compute the POD of the pressure on the turret surface, pressure data from the simulation is collected at 7150 locations on the surface over a time period of $600D/a_\infty$ using the medium mesh simulation. Figure 4.23 shows the pressure surface variance build-up of the first 100 POD modes. The first 20 modes account for almost 50% of the total POD mode energy and the first two modes contain about 25% of the total energy.

Figure 4.24 shows the first five modes of the unsteady surface pressure. The first mode shows the dominant pressure fluctuation mode of the turret flow associated with large-scale shedding from the turret, similar to vortex shedding on a cylinder. This mode was also observed as the dominant surface pressure mode in all of the results from Gordeyev et al. [35] including cases where turret surface features (gaps,
Figure 4.24. First 5 POD modes of the turret surface pressure.
smiles) had effects on the pressure field. The second mode shows negative correlation between both sides of the turret hemisphere and cylinder and the turret top and rear, indicating oscillation between the two locations linked to the ‘breathing’ of the wake. There is also an interesting positive correlation in the second mode between pressure fluctuations at the separation line and the location where the hemisphere and the cylinder meet in the wake of the turret. This correlation in a ‘breathing’ type mode also appears in some experimental results in Gordeyev et al. [35] but no physical explanation is presently known. Higher order modes have finer spatial structures and are generally associated with pressure fluctuations in the recirculation region and vortex shedding.

Figure 4.25 shows the power spectra of the temporal coefficients of the first five surface pressure POD modes. For all the modes, the spectra contain a large amount of very low frequency content at frequencies lower than $St_D = 0.2$. As the mode number increases there is an increase in high frequency spectral levels. In Gordeyev et al.’s [35] results, a spectral peak is observed for the first five modes at $St_D = 0.2$. While the simulation spectra appear to have some phenomena at that frequency, the duration of the simulation does not offer a sufficiently long time span to accurately identify the existence of a clear peak in the computational data.

The pressure modes corroborate what is observed in the time history of $C_{F,z}$ in Section 4.2.2. While the flow field is highly three dimensional, alternating vortical structures at low frequencies dominate the forcing on the turret and they represent the predominant behavior of the turret wake.

4.4 Conclusions

The basic flow statistics presented in this chapter demonstrate that the turret simulations have achieved mesh insensitivity, and the results can adequately represent the flow and optical results as presented in experiments. By examining profiles of
Figure 4.25. Temporal mode spectrum of the first 5 POD modes of the turret surface pressure.
flow quantities in the turret wake, the pressure coefficient along the turret centerline, and aerodynamic coefficients, the simulations show a level of convergence toward the finest simulation results.

Further examination of the flow quantities shows that the turret flow field is highly three-dimensional and periodically exhibits very large-scale vortical structures in its wake. A major flow feature that strongly impacts the aero-optics is the horn vortices generated in the wake of the turret which are captured in the simulations.

Examining the density variance budget in the separated shear layer and in the near wake of the turret reveals that the source of density fluctuations are different across different regions of the wake. In the shear layers generated by the turret cylinder, density fluctuations are dominated by pressure fluctuations. Closer to the center-plane of the turret, temperature fluctuations and pressure-temperature interactions play a more significant role in the density variance. Further downstream in the wake, the convection of the cylinder shear layers by the horn vortices increases the role of pressure-temperature covariance in density variance but overall, pressure fluctuations are the largest source of density fluctuations.
CHAPTER 5

NUMERICAL SIMULATIONS OF A HEMISPHERE-ON-CYLINDER TURRET:
AERO-OPTICAL RESULTS

While the previous chapter presented the simulation flow field, the sensitivity of the flow results to mesh resolution, and flow field statistics, the current chapter focuses on the analysis of optical results from the simulations. Both a survey of the magnitude, structure, and spectral behavior of optical distortions and an analysis of optical results with potential importance to the mitigation of aero-optical distortions are presented. Several analyses in this chapter utilize methods from Chapter 2 and Chapter 3 to examine the optical distortions caused by the flow over the turret.

5.1 Optical Results

To calculate the OPL from the turret flow field, separate beam grids are embedded in the LES mesh for each angle. The beam grids are generated to match the LES mesh resolution and extended approximately $2D$ outward from the turret surface in the beam propagation direction to ensure that it covers the entire optically active region of the flow. Each beam grid has a $D/2 \times D/2$ cross-section to allow for the investigation of aperture size effects on aero-optics. At each time step when the OPL is calculated, the density is interpolated from the LES mesh using a second-order gradient based method and integrated along the beam path using the method described in Chapter 1. In the present simulations, 390 viewing-angles are calculated and each beam grid in the coarse, medium, and fine simulations contains approximately 3.8, 5.5, and 11.3 million points, respectively. 275 of the viewing-angles are distributed equally
over the surface of the turret hemisphere by area and the additional remaining 115 viewing-angles are distributed over the separated region of the turret to increase the resolution of results in that area. Results presented for OPD have had steady-lensing, piston, and tip/tilt removed and are normalized using the normalization of Gordeyev et al. [30],

\[ \text{OPD}^N = \frac{\text{OPD}}{\rho_{\text{PSL}} M_2^2 D}. \]  

(5.1)

The data is presented with an aperture size \( D/3 \times D/3 \) unless noted otherwise, to match the aperture size of the AAOL turret. For all sections, the data presented is from the medium-mesh simulation to maintain consistency of results across all of the analysis. In addition, data is collected for twice the time-span for the medium simulation compared to the coarse and fine simulations, leading to smaller statistical errors.

5.1.1 Comparison of OPD\(_{\text{rms}}\) for Different Mesh Resolutions

In Figure 5.1, the tip/tilt removed, normalized OPD\(_{\text{rms}}\) along the centerline is shown for several experimental measurements [34, 33, 105] and the current simulations. The fine, medium, and coarse mesh simulations show some discrepancies consistently across most of the elevation angles, but the difference between the normalized OPD\(_{\text{rms}}\) results between each simulation decreases with mesh refinement. Overall, the comparison with experimental results are reasonable as the simulation results show a similar magnitude and trend with elevation angle compared to experimental measurements but are overall lower in value. It should be noted that the optical distortions calculated from the simulations are analogous to those measured using 2-D wavefront sensors and the better comparison with the 1-D Malley probe data is therefore likely fortuitous. In Gordeyev et al. [33], OPD\(_{\text{rms}}\) was measured with a 2-D Shack Hartmann wavefront sensor for a 12-inch hemisphere-on-cylinder
turret with $H/D = 0.375$ at a Mach number of 0.4 matching our current computation, whereas in Vukasinovik et al. [105] and Gordeyev et al. [34], tests were performed with a 24-inch conformal window turret with $H/D = 0.31$ at a Mach number of 0.3.

To examine the sensitivity of the optical results to the mesh resolution at different viewing angles, the normalized OPD$_{rms}$ for multiple elevation angles are calculated at two azimuthal angles, 145.2° and 124.5°, and are presented in Figures 5.3 and 5.4. The definition of beam viewing angles in terms of azimuthal and elevation angles and lookback and modified elevation angles is given in Figure 5.2. These azimuthal angles are chosen to allow for comparison with wavefront measurements from the AAOL that will continue in the next section of this chapter. In both figures, the coarse mesh simulation predicts higher values of OPD$_{rms}$ at higher elevation angles and lower values at lower elevation angles when compared to both the fine and medium mesh. With mesh refinement, the solutions approach the fine-mesh result. The different
Figure 5.2. The definition of beam viewing angles with respect to the flow direction, where $AZ$ is the azimuthal angle, $EL$ is the elevation angle, $\alpha$ is the lookback angle, and $\beta$ is the modified elevation angle.

Figure 5.3. The time-averaged normalized $\text{OPD}_{\text{rms}}$ as a function of elevation angle at off-center azimuthal angle of $145.2^\circ$:  \text{fine mesh};  \text{medium mesh};  \text{coarse mesh}.
behavior in the coarse mesh can be attributed to the lack of ability to accurately resolve density fluctuations in the horn vortex regions as shown in Figure 4.9.

While the results for the three mesh resolutions appears to demonstrate convergence of optical quantities, it should be noted that the normalized OPD$_{rms}$ at off-center viewing angles calculated from the simulation are significantly smaller than experimental data. At $AZ = 124.6^\circ$, $EL = 30.6^\circ$ a normalized OPD$_{rms}$ of $1.070 \times 10^{-6}$ is predicted in the medium resolution LES compared to $2.287 \times 10^{-6}$ found from AAOL. Similarly, at $AZ = 145.2^\circ$, $EL = 46.4^\circ$ the medium resolution LES predicts a normalized OPD$_{rms}$ of $1.004 \times 10^{-6}$ while the experimental value is $4.801 \times 10^{-6}$. The reasons for these larger discrepancies at off-center locations are at least due in part to the exclusion of some aerodynamically active geometric features on the computational turret like the trunnion gaps or “smiles” present on the AAOL flight turret [49, 50, 35].

Figure 5.4. The time-averaged normalized OPD$_{rms}$ as a function of elevation angle at off-center azimuthal angle of 124.5°: ---, fine mesh; -. -. -. medium mesh; ---, coarse mesh.
5.1.2 Comparison of Experimental and Computational Wavefronts and POD Mode Cross-Projection

To further examine the capability of the wall-modeled LES to capture the optical disturbance generated by the flow over a hemisphere-on-cylinder turret, POD is used to compare the spatial and temporal qualities of both the simulation and experimental data. Recently, POD has been used to identify the most important spatial components of aero-optical distortions [1, 29]. In the study of aero-optical wavefronts, POD is typically used to decompose a 2-D, time-dependent OPD signal into its most important spatial modes in terms of OPD$_{rms}$, modal energies, and temporal coefficients that describe how the modes evolve in time.

The motivation behind using POD with the CFD generated wavefronts is to evaluate the current capabilities to replicate the dynamic spatial and temporal behavior observed in experimental data. POD provides a means to decouple the overlying spatial structure of the optical distortion from the time evolving signal. It also provides a method to validate computationally predicted wavefront distortions against experimental data in terms of their spatial modes and corresponding proportion in the total energy.

Using off-center viewing angles where the aero-optic structures tend to be more viewing angle dependent than centerline angles, two comparisons between AAOL data and computational spatial POD modes from the medium resolution simulation are presented. As mentioned, the two viewing angles are at AZ = 124.2°, EL = 30.6° and AZ = 145.2°, EL = 46.4°. To ensure that the comparison is equivalent between computational and experimental counterparts, a series of post-processing steps are taken. First, the vectors of convective velocity of the aero-optical structures in the AAOL data and computational data are determined by the method described in Abado et al. [1]. Using the convective velocity vectors, the computational wavefronts are rotated to match the experimental orientation. This is necessary because while
Figure 5.5. First six POD modes of optical wavefronts at a viewing angle of $AZ = 124.5$, $EL = 30.6$ calculated from (a) simulation data and (b) AAOL measurements.

The computational window orientation is known, due to the complex system employed on the AAOL the exact orientation of the window with respect to the flow direction is uncertain. The computational aperture was next interpolated to match the lower resolution of the experimental wavefront aperture (a reduction of a 2-D aperture with $90 \times 90$ points to approximately $30 \times 30$ points). After that, the AAOL mask is applied to eliminate apertures not used in experiments to obtain a direct one-to-one spatial correspondence. With regard to matching the wavefronts temporally, it is observed that with and without down-sampling the computational wavefronts to match experimental sampling frequency, the spatial modes typically converge with approximately 6000 frames. The POD modes of both sets of data are then computed. The results for the LES and AAOL spatial modes for both datasets are shown in Figures 5.5 and 5.6.

A few observations can be made from comparing the spatial POD modes. In both datasets the first few modes contain the largest spatial structures which decrease in
size with the mode number. Also, modes tend to come in complimentary pairs that have similar energy values and similar structure sizes but are phase shifted. The difference in composition of the modes between the two datasets illustrate the sensitivity of wavefronts to the viewing angle. While the flow has only strongly separated near the bottom-right of the aperture in Figure 5.5, in Figure 5.6 most of the beam propagates through a separated shear layer. Comparing the results between the LES and AAOL, there is reasonable agreement with both the shape and position of the modal structures especially within the first few modes. This strongly implies that the wall modeled LES is capable of capturing the location and size of large scale, three-dimensional turbulence structures caused by flow separation on the turret. It should be noted that the blue areas of modes are negative values and red areas are positive, but they are interchangeable: since spatial modes are decoupled from time using POD, their sign is arbitrary and only indicates how the peaks and valleys of the mode are correlated to each other.
Figure 5.7. Spectra of temporal coefficients for the first four POD modes from computation (solid) and experiment (dashed) modes.

$AZ = 124.6^\circ, EL = 30.6^\circ$. 
Figure 5.8. Spectra of temporal coefficients for the first four POD modes from computation (---) and experiment (--.--) modes. 
\[AZ = 145.2^\circ, EL = 46.4^\circ.\]
The temporal coefficients of the spatial modes can also be investigated to gain insight into the dynamic behavior of each mode and to evaluate the simulation’s ability to reproduce the time-dependent behavior of wavefront structures of varying sizes. In Figures 5.7 and 5.8, the frequency spectra of the temporal coefficients for the first four modes are compared between the LES and AAOL data at both viewing angles. At the first viewing angle, the spectra of the first two modes appear to have similar peak values at a Strouhal number, \( St_D = fD/U_\infty \), of about 2 – 3 and similar roll-off slopes for both the computational and experimental data. The third and fourth modes have some discrepancies at lower frequencies, which may reflect a change in behavior due to differences in spatial mode shape. In Figure 5.8, the LES predicts higher spectral levels than the AAOL data at higher frequencies and the spectral peak is broader in the computational spectra. All have similar roll-off at high frequency and similar low frequency behavior. It should be noted that in Figure 5.8(b), the spectra comparison is between AAOL mode 2 and LES mode 3 and in Figure 5.8(c), the comparison is between AAOL mode 3 and LES mode 2 since these modes have the most similar spatial structure. A non-physical peak exists in all experimental spectra at \( fD/U_\infty \approx 10 \) and as such, it does not appear in the simulation data.

Since the POD modes represent a set of orthogonal basis functions, how well the computational modes represent the AAOL data can be evaluated by projecting the computational spatial modes onto the AAOL OPD data set and then finding the variance or “modal energy”. Shown in Figure 5.9 is a comparison of the energy of the AAOL spatial modes projected onto the AAOL data and the energy of the computational spatial modes projected onto the AAOL data. Both sets have been normalized by the total cumulative energy, so the figure shows the percentage of energy contained up to that mode. By definition of POD, the experimental curve represents the fastest rise in energy among any possible modal decompositions for
the AAOL data. In comparison, the computational modes track the experimental data fairly well. By the first 10 modes, the computational data only contain nearly 5%-10% less total energy than that of the experimental data and this percentage decreases with the number of modes used. This indicates that modal structures contained in the LES based optical calculations are also contained in the actual data. More importantly, it shows that the first few modal structures that account for the largest variance in OPD are similar in both data sets.

5.1.3 Turret Field-of-Regard OPD_{rms} Behavior

As mentioned, of the 390 viewing angles, 275 are evenly distributed over the turret by area and the other 115 viewing angles are concentrated in the horn vortex region. The viewing angles were distributed symmetrically with respect to the centerplane of the turret and the data is averaged across the plane of symmetry. To visualize all of the optical data collected and their optical distortion magnitude, Figure 5.1.3 shows a contour map of the normalized OPD_{rms}, for a D/3 aperture, for each viewing angle.
Figure 5.10. Iso-contour of normalized OPD_{rms} for an aperture size D/3 from the medium mesh simulation. Each black dot corresponds to the viewing angle of an optical beam calculated during the simulation.

Each black dot in the figure represents the viewing angle of an optical beam that is computed. The most prominent structures in Figure 5.1.3 are the horn vortices present at low elevation, off-center azimuthal angles that have the highest OPD_{rms}.

An alternative to describing a viewing angle from the azimuthal-elevation space is the lookback-modified elevation angle space as shown in Figure 5.2. The lookback angle is a more natural way to describe the viewing angle in terms of flow physics, since the growth of the boundary layer and separation point are strongly dependent on the lookback angle. In Figure 5.11(a), the OPD_{rms} of all viewing angles is plotted as a function of lookback angle. As can be noticed, a large number of the data points fall along a region of the scatter plot with the lowest values of OPD_{rms} for a given lookback angle. To further investigate, these points that collapse onto this common line are marked with a red circle in the figure and the points that fall outside of this region are marked by a blue diamond. In this outer region, the amount that the data
Figure 5.11. (a) Normalized OPD$_{\text{rms}}$ plotted as a function of lookback angle for an aperture size $D/3$. Red circles represent points that collapse onto a common line while blue diamonds do not collapse. (b) Normalized OPD$_{\text{rms}}$ plotted as a function of lookback angle with markers colored by their modified elevation angle, $\beta$: black circles, $0^\circ < \beta < 25^\circ$; red circles, $25^\circ < \beta < 45^\circ$; blue circles, $45^\circ < \beta < 65^\circ$; green circles, $65^\circ < \beta < 90^\circ$.

point deviates from that collapsed line is dependent on the modified elevation angle, $\beta$, as shown in Figure 5.11(b). As the modified elevation angle decreases, the OPD$_{\text{rms}}$ tends to increase for a given lookback angle. This compares well with the findings of DeLucca et al. [52], who showed similar results after flow separation.

Using this separation of viewing angles into points that collapse onto a common line and those that do not, Figure 5.1.3 can be recast as Figure 5.12(a) where the black dots representing beam viewing angles are replaced with the red circle and blue diamond markers. A pattern emerges, showing that all of the non-collapsed viewing angles (blue diamonds) fall into the region where the horn-vortices play a dominant role. To further illustrate the separation, a support vector machine (SVM) data classification method is used to define the boundary between the two regions as shown in Figure 5.12(b). Here, the red region corresponds to the area where points collapse and the blue region corresponds to the area where they do not collapse.
Figure 5.12. (a) Contours from Figure 5.1.3 but with markers corresponding to those in Figure 5.11 denoting points that fall onto a common line. (b) Contours showing the two regions identified from the map of points using a SVM classifying method. The red region denotes the region where points collapse onto a common region, and the blue region denotes the region where points do not collapse.

It is expected that the range of viewing angles where the horn vortex has such an effect would be dependent on the size of the beam aperture. Using the same method used to classify viewing angles based on the collapse of number of data points in the OPD$_{rms}$ versus lookback angle, the regions of viewing angles that are affected by the horn vortices are shown in Figure 5.13(a) for beam apertures of $D/2$, $D/3$, $D/4$, and $D/8$. As the aperture size increases, so does the region of viewing angles that are affected by the horn vortex. The region consistently becomes larger, except at the corners of the contours where the lack of data in the region prevent the classification method from drawing accurate boundaries. Notable is the common presence of a quiet zone in the centerline of the turret where the horn vortex plays a less important role compared to other shallow elevation angles.

A fit of the points in the collapsed region is shown in Figure 5.13(b). After the separation, the data fits all have a similar slope except for the $D/8$ case where the
aperture size is on the order of the large-scale optical distortions and may behave differently due to the elimination of their effects when tip/tilt is removed.

5.1.4 Autocorrelation and Structure Size of Optical Distortions

While the OPD$_{\text{rms}}$ provides an overall measure of the magnitude of OPD, it does not provide any information about the spatial structure or temporal behavior of the optical distortion. To investigate the spatial structure of the OPD, the two-point correlation of steady lensing, tip/tilt removed OPD is presented in this section. Similar to the definition of the autocorrelation given in Chapter 3, the normalized autocorrelation is equal to

\[
\hat{R}_{\text{OPD}}(x_0, r_\perp) = \frac{\text{OPD}(x_0) \text{OPD}(x_0 + r_\perp)}{\sqrt{\text{OPD}^2(x_0)} \sqrt{\text{OPD}^2(x_0 + r_\perp)}},
\]

(5.2)

where $x_0$ is the location about which the autocorrelation is defined and $r_\perp$ is the displacement vector relative to $x_0$ in a beam cross-section. For the calculations of
Figure 5.14. Two-point correlations of piston and tip-tilt removed OPD for viewing angles of (a) $AZ = 120^\circ$, $EL = 30^\circ$, (b) $AZ = 150^\circ$, $EL = 30^\circ$, (c) $AZ = 0^\circ$, $EL = 150^\circ$, and (d) $AZ = 0^\circ$, $EL = 170^\circ$. The aperture size is $D/3$.

the autocorrelation here, $x_0$ is at the center of the optical aperture.

In Figure 5.14, examples of two-point autocorrelations are given for four different viewing angles and the apertures are locally aligned with the azimuth and elevation angle coordinates. In Figure 5.14(a), the center of the aperture is located just down stream from the separation line at $AZ = 120^\circ$, $EL = 30^\circ$. While the autocorrelation shows a banded structure with strong correlation perpendicular to the flow direction as expected in the separation region, its value decreases from the peak quickly, likely
due to the size of vortical structures. Looking further backward into the separated wake at an off-center location, Figure 5.14(b) shows the autocorrelation at a viewing angle of $AZ = 150^\circ$, $EL = 30^\circ$. Here the correlation perpendicular to the flow direction is stronger, the autocorrelation decreases from its peak more gradually compared to the near separation location, and there are strong anti-correlated regions on both sides of the peak value along the convection direction due to coherent vortex structures in the separated shear layer.

At a viewing angle along the centerline at $AZ = 0^\circ$, $EL = 150^\circ$ a similar structure is observed in Figure 5.14(c) compared to the previous off-center location. There is strong spanwise correlation perpendicular to the flow direction and negative values of correlation along the flow direction. Looking almost directly through the turret wake at a viewing angle of $AZ = 0^\circ$, $EL = 170^\circ$, the autocorrelation takes a different appearance. Here the shape of the autocorrelation is more circular and lacks the band-like quality seen previously. This is caused by the horn vortices which at this angle convect flow inward from the sides of the aperture in addition to the primary flow convecting optically active structures from the top to the bottom of the aperture.

While examining the autocorrelation of individual apertures can provide a glimpse into the behavior of optical structures contained in the wavefront, a comprehensive description of the autocorrelation at each viewing angle in this way is difficult. Instead, an analysis of the size of wavefront structures approximately along the convection direction is presented, providing insight into the size of optical structures as a function of viewing angle. While the lookback angle does not precisely match the direction of the convection velocity for each aperture, it serves as a reasonable approximation for the convection velocity direction which may be difficult to compute at some angles.

An integral length scale, $\Lambda_{OPD}$, of the optical structure can be obtained by integrating
Figure 5.15. Autocorrelation length scale of OPD, $\Lambda_{\text{OPD}}/D$, integrated along the lookback angle direction: (a) contour on turret and (b) plotted versus lookback angle. Black circles, $0^\circ < \beta < 25^\circ$; red circles, $25^\circ < \beta < 45^\circ$; blue circles, $45^\circ < \beta < 65^\circ$; green circles, $65^\circ < \beta < 90^\circ$.

The autocorrelation between the zero crossings of the autocorrelation,

$$\Lambda_{\text{OPD}} = \int_{r_\alpha(0^-)}^{r_\alpha(0^+)} \hat{R}_{\text{OPD}}(x_0, r_\alpha)dr_\alpha,$$  \hspace{1cm} (5.3)

where $r_\alpha$ is the relative displacement along the $\alpha$ direction, $r_\alpha(0^-)$ is the zero crossing of the autocorrelation along the negative $r_\alpha$ direction, and $r_\alpha(0^+)$ is the zero crossing of the autocorrelation along the positive $r_\alpha$ direction.

Contours of this autocorrelation length $\Lambda_{\text{OPD}}/D$ on the turret surface are shown in Figure 5.15. The correlation length is very small in forward viewing angles and gradually increases as the lookback angle increases. There is a clear decrease in correlation lengths over all of the turret associated with unsteady separation. This decrease can also be seen in Figure 5.15(b) where $\Lambda_{\text{OPD}}/D$ has been plotted as a function of lookback angle. Upon reaching a local minimum near the separation angle $\alpha = 115^\circ$, the correlation increases nearly linearly with $\alpha$ due to the growth of shear layer structures until $\alpha = 155^\circ$. After this lookback angle, $\Lambda_{\text{OPD}}/D$ grows faster.
with increased lookback angle possibly due to the a change in the primary large-scale wavefront optical distortion transitioning from separated shear layer vortical structures to larger wake structures associated with periodic wake movement and the recirculation region. The correlation length also shows a trend in the modified elevation angle, $\beta$, where the largest correlation lengths are at smaller $\beta$ angles and the smallest correlation lengths are along the centerline of the turret.

5.1.5 Spectral Content of Aero-Optical Distortions

The temporal behavior of aero-optical distortions is of fundamental importance as the high frequency content of the distortions make them difficult to mitigate. To examine how the variance or ‘power’ of a signal is distributed by frequency, the power spectral density of the OPD is investigated. In addition to examining the spectrum of OPD, in Chapter 3 the decomposition of the OPD into its pressure component, $OPD_p$, and its temperature component, $OPD_T$, was examined to investigate the role of pressure and temperature in the optical distortion spectrum. While this decomposition was previously written for the condition where only steady lensing is removed, it can be shown to be valid when tip/tilt is removed from OPD, $OPD_p$, and $OPD_T$. That is, it can be mathematically proven that the individual components of tip and tilt of the total OPD are equal to the sum of the tip and tilt components from the $OPD_p$ and $OPD_T$.

In Figure 5.16, the $OPD^N$, $OPD^N_p$, and $OPD^N_T$ spectra are shown for four different viewing angles: two at off-center viewing angles and two viewing angles along the centerline of the turret. At the two off-center viewing angles, the pressure fluctuations dominate the low to mid frequency range and at higher Strouhal number, the $OPD^N_T$
Figure 5.16. Normalized, piston and tip-tilt removed OPD$^N$ spectrum (---), OPD$^N_p$ spectrum (--), and OPD$^N_T$ spectrum (---), at the center of the aperture for viewing angles of (a) AZ = 120°, EL = 30°, (b) AZ = 150°, EL = 30°, (c) AZ = 0°, EL = 150°, and (d) AZ = 0°, EL = 170°. Aperture size is $D/3$. 
spectral levels start to become comparable to \( \text{OPD}_p^N \). Close to the separation line in Figure 5.16(a), the \( \text{OPD}_p^N \) and \( \text{OPD}_T^N \) spectra have a peak near a \( St_D = 1.5 \) and there does not appear to be a well-developed inertial subrange in the midrange of frequencies. Looking further back into the turret wake at \( AZ = 150^\circ, EL = 30^\circ \) in Figure 5.16(b), the peaks appear to be at the same location and the spectral slope of the OPD in the inertial subrange now is smaller in magnitude than the \( \text{OPD}_p \) spectral slope as the \( \text{OPD}_T \) spectrum has a larger effect than before in the inertial subrange.

The spectra for two viewing angles looking along the turret centerline, \( AZ = 0^\circ, EL = 150^\circ \) and \( AZ = 0^\circ, EL = 170^\circ \), are shown in Figures 5.16(c) and 5.16(d), respectively. Compared to the off-center viewing angles, the contribution of \( \text{OPD}_T^N \) to the optical distortions along the centerline is significant for all frequencies. While at low frequencies the pressure fluctuations are still larger, at mid to high frequencies the spectral levels of \( \text{OPD}_T \) are comparable to or larger than \( \text{OPD}_p \). As a result, the OPD is expected to have a smaller spectral slope at these viewing angles since the spectral slope of optical distortions due to temperature fluctuations is smaller than optical distortions due to pressure fluctuations as shown in Chapter 3.

To examine the spectral slope and its dependence on viewing angle, the spectrum of OPD at the center of the aperture is shown for several azimuthal angles at a constant elevation angle of \( EL = 10^\circ \). A shallow elevation angle is chosen so that for most of the viewing angles investigated, the beam propagates through turbulence which has had more time to develop in comparison to elevation angles closer to the separation line and incipient shear layer. In Figure 5.17, spectra plotted for values of the azimuthal angle from 110° to 180° with increment of 10°. At the bottom curve in Figure 5.17, corresponding to the viewing angle at \( AZ = 110^\circ, EL = 10^\circ \), the flow has not yet separated and the spectral levels are much smaller than the other viewing angles. Starting with viewing angles at \( AZ = 120^\circ, EL = 10^\circ \), the beam
Figure 5.17. OPD spectral behavior: (a) Normalized OPD frequency spectra for several azimuthal angles at $EL = 10^\circ$ at the center of the aperture. The bottom curve is for $AZ = 110^\circ$ and the top curve is for $AZ = 180^\circ$, with increment of $10^\circ$. Each spectrum is displaced by a factor of $10^{0.5}$ above the previous data for clarity. (b) Curve fit for the spectral slope magnitude of all the OPD spectra calculated using the region between the vertical dashed lines in (a).
is propagating through the separated shear layer and an inertial subrange can be identified approximately in the frequency range bounded by the vertical dashed lines, \( 10^{0.5} < S_{tD} < 10^1 \). Also starting at \( AZ = 120^\circ \), a broad peak appears associated with the large scale shear layer structures in the spectra near \( S_{tD} = 2 \). As \( AZ \) increases, the peak shifts to lower frequencies as the size of the shear layer structures become larger. In addition, as \( AZ \) increases the strength of the spectral peak decreases and by \( AZ = 170^\circ \) and \( 180^\circ \), there is no identifiable low frequency peak.

In Chapter 3, it was shown that the inertial subrange slope of the OPD spectrum is dependent on the influence of pressure and temperature fluctuations on the optical distortion. It is expected that in regions where pressure fluctuations dominate optical distortions the spectral slope is larger in magnitude than in regions where temperature fluctuations are more important. As seen in Figure 5.16, for the examples of the OPD, \( \text{OPD}_p \), and \( \text{OPD}_T \) spectra shown, pressure fluctuations appear to have a larger role in viewing angles off the turret center, and along the turret centerline, \( \text{OPD}_T \) approaches the levels of \( \text{OPD}_p \). The slopes of the spectra in Figure 5.17(a), extracted by fitting a line to the spectra in the frequency band between the dashed lines, are shown in Figure 5.17(b). After separation, viewing angles where the horn vortex may have a larger effect (\( 120^\circ < AZ < 150^\circ \)) have a larger value of spectral slope than at locations near the centerline. After \( AZ = 150^\circ \), a gradual decrease in spectral slope from \( 8/3 \) to less than \( 7/3 \) is seen.

Even though these results confirm that the spectral slope of OPD depends on the viewing angle and the dominant source of optical distortions at a given viewing angle, the values of spectral slope are smaller than predicted from the theoretical results for a high Reynolds number flow. From the theoretical results presented in Chapter 2, for a high Reynolds number flow, it is expected that pressure dominated optical distortions would have a spectral slope near \( -10/3 \) and temperature dominated optical distortions would have a slope near \( -8/3 \). Current results agree more closely
with what was seen for a lower Reynolds number shear flow, where a slope of $-8/3$ and $-7/3$ are expected for pressure and temperature dominated OPD, respectively. A potential explanation for this behavior may be linked to the use of a wall-model to near the surface of the turret. While wall-modeling appears to capture the global effect of the near-wall turbulence dynamics when examining the flow field, the current boundary layer resolution may not be sufficiently high to resolve all of the turbulence scales relevant to the transition of the boundary layer to a turbulent separated shear layer. If so, the shear layer may still be undergoing transition at locations immediately following the separation. While fine scales of optical distortions do not appear to be completely faithful to the high-Reynolds number predictions, the large scale optical distortions responsible for the majority of the optical distortions are captured by the simulation as shown previously.

5.1.6 Investigation of OPD$_T$ and OPD$_p$ in Turret Flow Optics

As shown in Chapter 3, the decomposition of the optical distortion into its temperature and pressure terms leads to the following expression for the variance of the OPD in terms of the OPD$_p$ and OPD$_T$,

$$\text{Var}[\text{OPD}] \approx \text{Var}[\text{OPD}_p] + \text{Var}[\text{OPD}_T] - 2\text{Cov}[\text{OPD}_p, \text{OPD}_T]. \quad (5.5)$$

The error incurred in this approximation for all viewing angles as a function of lookback angle is shown in Figure 5.18. Across all viewing angles the percentage error of the OPD as calculated using the approximation given in Eqn. (5.4) relative to the actual OPD calculated directly from the density field is less than seven percent. This demonstrates that the linearization of the OPD is adequate in this flow and that the decomposition is valid even with the removal of tip/tilt from the components in Eqn. (5.4).
Figure 5.18. Relative error, $|\text{Var}[\text{OPD}] - \text{Var}[\text{OPD}_{\text{approx}}]|/\text{Var}[\text{OPD}]$, of the OPD variance approximation computed using Eqn. (5.5).

In Figure 5.19, contours of the variance of normalized OPD, OPD$_p$, OPD$_T$, and the OPD$_p$-OPD$_T$ covariance are shown as a function of viewing angle in elevation angle-azimuthal angle space. Comparing the contours of the OPD$_p$ and OPD$_T$ variance, Figure 5.19(b) and 5.19(c), respectively, it can be noticed that the contribution from the pressure OPD is larger than that of the temperature, and the pressure OPD affects a larger region of the field-of-regard. In addition, while the effect of the horn vortices on the shape of the contours is shown in both OPD$_p$ and OPD$_T$ variance contours, the locations of peak variance for the pressure OPD are farther away from the centerline of the turret compared to the temperature OPD. This indicates that the horn vortex increases fluctuations in pressure primarily in off-center regions and the centerline of the flow is left with a noticeable ‘gap’ where pressure OPD is much weaker. In the temperature OPD, some effect of the horn vortices is seen but the OPD$_T$ contributions are much more concentrated toward the centerline of the turret.

The contours of the OPD$_p$-OPD$_T$ covariance in Figure 5.19 agree with what was
Figure 5.19. Contours of (a) OPD variance, (b) OPD\textsubscript{p} variance, (c) OPD\textsubscript{T} variance and (d) −2 Cov[OPD\textsubscript{T}, OPD\textsubscript{p}].
presented in Section 4.3.1 where temperature and pressure are well correlated in the horn vortex region of the flow having a deleterious effect on overall OPD variance. The contours also show less correlation in the centerline of the turret. Interestingly, the covariance component of the OPD variance budget is negative for all viewing angles indicating that in the mean, the OPD_p and OPD_T are always correlated for this flow.

In Figure 5.20 the normalized rms for the OPD_p and OPD_T, and the normalized OPD_p-OPD_T covariance are plotted as a function of lookback angle to further examine the similarities and difference in their behavior. For all three quantities, their values increase monotonically as lookback angle increases and for a given lookback angle, the value of each quantity increases as modified elevation angle decreases.

A noticeable difference in the behavior of OPD_p and OPD_T as α increases is the wider distribution of values in the OPD_p after flow separation compared to OPD_T. For values of OPD_p near the centerline of the turret (β > 65°), there appears to be a ‘kink’ in the distribution of data around α = 135° where the data does not follow a more linear trend like the other groups of β angles. This is likely due to the ‘quiet’ zone for the OPD_p seen in the contour of Figure 5.19(b) along the turret where the variance is lower than other viewing angles with comparable elevation angles. This is in contrast to the rms of the normalized temperature OPD which follows a much more predictable trend with both α and β. It may be that the OPD_T is more closely related to the shear layer thickness and finer scale optical structures as the beam propagates through the wake of the turret and the OPD_p is closely connected to large shear layer structures whose intensity and location are heavily affected by the horn vortices and can be more susceptible to being filtered by tip/tilt removal.
Figure 5.20. Normalized (a) OPD\(_{p,\text{rms}}\), (b) OPD\(_{T,\text{rms}}\), and (c) OPD\(_p\)-OPD\(_T\) covariance plotted as a function of lookback angle: black circles, 0° < \(\beta\) < 25°; red circles, 25° < \(\beta\) < 45°; blue circles, 45° < \(\beta\) < 65°; green circles, 65° < \(\beta\) < 90°.
5.1.7 Joint Proper Orthogonal Decomposition of Optical Wavefronts

In the development of adaptive-optics systems to correct aero-optical distortions, the conjugate of the optical wavefront must be reproduced by a deformable mirror or other similar device prior to propagating a beam through the flow field. To efficiently represent the corrective shape of a deformable mirror to provide to an adaptive-optic control device, a reduced-order model of the wavefront that provides a compact basis for the correction reduces the large number of degrees-of-freedom that arises from propagating a beam through a turbulent flow. An obvious choice for this reduced-order basis is POD which provides an orthogonal basis ordered by the mode’s ability to capture the fluctuations of a signal. While recent methods proposed by Burns et al. [10, 8] have successfully utilized POD as a part of the adaptive-optics control methodology, the POD modes used in the method are only representative of the local viewing angle being corrected. For the design of adaptive systems it may be of use to investigate the most important mode shapes shared by all viewing angles of interest. For example, the most important mode shapes of all viewing angles would give a representation of the most common deformations needed in an adaptive-optics mirror for aero-optics corrections and a control system could be designed with this in mind.

To accomplish this task, the Joint Proper Orthogonal Decomposition (JPOD) described by Gordeyev et al. [35] is used. JPOD uses the same construction as regular POD but datasets with a common spatial structure are strung together as one continuous time series so that the single set of spatial basis modes computed from the POD takes all data into account. The result of the JPOD is a set of spatial modes that are the most efficient (in a least-squares sense) at representing the whole span of the input data. An important point for JPOD when using multiple datasets with some element of difference among them is that they must not only be made in a common form compatible with computing a POD but also be represented in a
common orientation. Since the description of the viewing angle of the wavefronts to the local wavefront coordinate system is a mapping from a three-dimensional spherical coordinate system native to the turret hemisphere to the cartesian coordinates of the planar wavefront, the two common descriptions of viewing angle are used in this analysis to orient the wavefronts. In one description, the $y'$ coordinate of the aperture is aligned with the elevation angle of the turret and the $x'$ coordinate the azimuthal angle and in the other description, the $y'$ coordinate of the aperture is aligned with the lookback angle of the aperture.

Before taking the JPOD of the wavefronts, each dataset is standardized (the wavefronts are normalized such that the time-mean spatial standard deviation of the wavefronts is unity) to prevent viewing angles with large values of OPD variance from dominating mode shapes and only viewing angles where the azimuthal angle is greater than 90° are used since wavefronts before separation contain little spatial content. The aperture size for all wavefronts is $D/3$ and the initial aperture resolution of $90 \times 90$ is reduced to $45 \times 45$ to relax the computational requirements of the JPOD calculation. When the wavefronts are initially calculated by the simulation, the aperture local $y'$ and $x'$ coordinates are aligned to the turret local elevation angle and azimuthal angle coordinates of the turret, respectively. To align the wavefronts into the turret lookback angle coordinates, the wavefront must be rotated by an angle $\theta_{rot}$ which can be found via the expression,

$$
\theta_{rot} = \text{sgn}(AZ) \frac{\arccos(\sin(EL) \cos(AZ))}{\sin^2(EL) \cos^2(AZ) + \sin^2(AZ) \cos^2(EL)}. 
$$

(5.6)

After rotating the wavefront by $\theta_{rot}$, each frame is interpolated onto a common grid to be used in the JPOD calculation.

While using the same initial data, the two sets of mode shapes resulting from the JPOD calculations in Figure 5.21 look very different. Mode shapes for the apertures
Figure 5.21. Joint Proper Orthogonal Decomposition of standardized, rear-facing optical wavefronts (azimuthal angle greater than 90°) when the optical aperture is locally aligned with (a) the elevation (EL) and azimuthal (AZ) angles, (b) the lookback (α) and modified elevation angle (β).

aligned in $AZ-EL$ do not appear to have shapes similar to those seen in other POD calculations where spanwise correlated structures are apparent. This is because the $AZ-EL$ orientation does not align well with the convection velocity of the aero-optical structures and the mode shapes try to provide a basis for wavefronts where the flow can be convecting in several different directions. In comparison, the basis found by orienting the wavefronts in the local $α-β$ space shown in Figure 5.21(b) look much more like the structures seen in typical aero-optical wavefronts. They show a strong spanwise correlation or symmetry about the midline and a banded structure due to a closer alignment of the wavefront orientation to the aero-optic convection velocity.

To examine the ability of the two sets of JPOD modes to represent their respective datasets (which contain the same amount of total variance plus error due to interpolation), Figure 5.22 shows the percentage of the buildup of mode variance with increasing number of JPOD modes. Throughout all of the modes, the data oriented in $α-β$ space does a better job representing the variability in the database than the
Figure 5.22. Comparison of the cumulative energy contained up to a mode in the joint Proper Orthogonal Decomposition for the aperture locally aligned with the elevation (EL) and azimuthal (AZ) angles, –×–, and the lookback (α) and modified elevation angle (β), –○–.

data oriented in AZ-EL space. For a majority of the range of modes shown here, the α-β space modes contain about 10% more total variance when compared to the other orientation.

This analysis also provides important clues to the effective design of adaptive-optics methods. If a given reduced-order basis is used to represent wavefronts such as Zernike modes or computed JPOD modes, there may be an optimal orientation of the aperture to reduce the number of modes needed to gain a level of correction. As seen in this analysis, if a common basis computed using POD or some other data-driven reduced order model is used to represent the optical wavefronts on a turret, it is more efficient to design the system such that the aperture is aligned with the α-β coordinate system or the convection velocity direction.
5.1.8 Steady-Lensing of Turret Aero-Optical Distortions

While the focus of the research contained in previous sections in this chapter has been on the higher-order wavefront that remains after steady-lensing and tip/tilt is removed, the steady lensing and tip/tilt components must still be identified and corrected to have a well functioning airborne optical system. The aero-optic steady-lensing component of the wavefront represents the time mean optical distortion caused by the aerodynamic effects of the flow around the turret. The mean component of the wavefront, $\overline{OPL}$, of a stationary beam is given by the expression

$$\overline{OPL} = \frac{1}{T} \int_{0}^{T} \int_{0}^{L} (1 + K_{GD} \rho) dz' dt = L + K_{GD} \int_{0}^{L} \bar{\rho} dz',$$

(5.7)

where $T$ is the time over which the average is computed. The steady lensing component of the wavefront is presented here with the piston, or spatial mean, removed since the spatial mean of an optical beam causes no distortion to the beam as it propagates to the farfield. As such, the steady lensing, or $\text{OPD}_{\text{lensing}}$, presented in this section has the form

$$\text{OPD}_{\text{lensing}} = \overline{OPL} - <\overline{OPL}>, \tag{5.8}$$

where angle brackets denote spatial averaging over the aperture. The steady lensing has also been normalized similar to the normalization of OPD,

$$\text{OPD}_{\text{lensing}}^{N} = \frac{\text{OPD}_{\text{lensing}}}{\rho_{\infty} M^2 D}. \tag{5.9}$$

Examples of the steady lensing for four different viewing angles are shown in Figure 5.23. In Figure 5.23(a) at a viewing angle of $AZ = 0^\circ$, $EL = 60^\circ$, the shape of the steady lensing is typical for forward looking viewing angles where steady tip and tilt dominate the wavefront. The steady lensing in this portion of the flow field
Figure 5.23. Normalized, piston removed steady-lensing component of optical wavefronts for viewing angles of (a) $AZ = 0^\circ$, $EL = 60^\circ$, (b) $AZ = 0^\circ$, $EL = 90^\circ$, (c) $AZ = 0^\circ$, $EL = 120^\circ$, and (d) $AZ = 124.5^\circ$, $EL = 30.6^\circ$. The aperture size is $D/3$. 
is primarily affected by the steady potential flow field caused by the acceleration of the flow over the front of the turret. As the elevation angle increases to the point where the beam propagates directly through the top of the turret at \( \text{EL} = 90^\circ \), the potential flow field begins to change and its effect on the steady lensing causes a very different effect as observed in Figure 5.23(b). Propagating just after the location of lowest pressure and density, the effect on \( \text{OPD}_{\text{lensing}}^N \) is a saddle like shape to the mean wavefront. Increasing the elevation angle even further to the point where the beam projects through the beginning of the separated shear layer at \( \text{AZ} = 0^\circ \), \( \text{EL} = 120^\circ \), the steady lensing has a band of large positive values across the center of the aperture, with negative values on either side of the band in the elevation angle directions. This band of positive \( \text{OPD}_{\text{lensing}}^N \) stretching across the aperture near the separation line can also be observed at off-center locations at \( \text{AZ} = 124.5^\circ \), \( \text{EL} = 30.6^\circ \) as shown in Figure 5.23(d). Here however, due to the orientation of the aperture the band stretches from the bottom-left of the aperture to the top-right and the thickness of the band changes along its length.

To better quantify the shape and intensity of the aero-optic steady lensing and how they change with viewing angle, the steady lensing component of each aperture is projected onto Zernike modes to find the Zernike mode amplitude. Zernike modes are of particular interest in the study of optical wavefronts as they represent a mode basis on a unit circle and are commonly used in the study and development of adaptive optics systems. Values for the \( n^{th} \) Zernike mode amplitude, \( a_n \), where \( n \) corresponds to Noll index of the Zernike modes, are found by computing,

\[
a_n = \frac{\int_0^{2\pi} \int_0^{D_{\text{Ap}}/2} Z_n \text{OPD}_{\text{lensing}}^N rdrd\theta}{\int_0^{2\pi} \int_0^{D_{\text{Ap}}/2} Z_n^2 rdrd\theta}, \quad (5.10)
\]

where the \( Z_n \) is the \( n^{th} \) Zernike polynomial mode that has been scaled from the unit disk to the aperture size. Several important Zernike modes used in this analysis are
shown in Figure 5.24.

In Figure 5.25 contours of mode amplitude on the hemispherical surface of the turret are visualized for the tip, tilt, and defocus components where the coordinates of the mode axis are locally aligned with the $\alpha$ and $\beta$ directions. The largest amplitude of any mode or viewing angles are the tip and tilt modes for forward and extreme rear viewing angles of the turret. There is a spanwise low value in the tip amplitude coincident with the saddle point seen in Figure 5.23(b) where the peak velocity is located across the turret. While tip amplitude has a very coherent structure to its distribution over the turret with a strong dependence on lookback angle and weak dependence on modified elevation angle, tilt has a more complicated distribution. In all, distributions tend to be simpler on the leading side of the turret where flow is attached and the distortion is primarily due to potential flow effects. In the wake, the separated shear layer combined with the convective effects of the horn vortices
Figure 5.25. Amplitude of Zernike mode representation of steady-lensing for apertures aligned locally with the $\alpha$-$\beta$ directions: (a) tip, (b) tilt, and (c) defocus. Contour locations denote the location of the center of the aperture for a given viewing angle.
complicates the distribution pattern. Defocus in Figure 5.25(c), however, has little dependence on \( \beta \) over nearly the entire turret.

When the values of the Zernike mode amplitude norm are plotted as a function of lookback angle in Figure 5.26, for all values of modified elevation angle there is a complete collapse for lookback angles less than 100° where the potential flow field dominates the steady lensing. Above 100° the effects of flow separation begin to make the steady-lensing much more dependent on \( \beta \). Figures 5.26(b), 5.26(c), and 5.26(d) show that the dependence of the norm on \( \beta \) in the separation region is more complex. The largest values of the norm are found leading the turret and looking back through the wake at shallow angles. There is also a faint band across the centerline of the turret, before the separation line that elevates values of the Zernike mode norm. By removing the steady tip and tilt Zernike mode amplitudes from the calculation, the peak value of the norm decreases by nearly an order of magnitude, leaving a bands along the front of the turret before the peak of the turret and a band near the separation line. Values in the region at the very front of the turret with the highest values of the total norm have been dramatically decreased with the removal of only steady tip and tilt. Additionally, removing both astigmatisms has the most effect in decreasing the intensity of the steady-lensing in the band of forward-looking viewing angles and in the horn vortex regions. By accounting for the defocus in the steady lensing in addition to the previous Zernike modes, the values of Zernike amplitude norm is decreased in forward viewing-angles, the band of values near the separation line, and in the horn vortex region.

The percent of the total Zernike amplitude norm after mode removal for each viewing angle is shown in Figure 5.27. In Figure 5.27(a), where the percent of the norm remaining after steady tip and tilt are removed is shown, nearly 70% of the mode norm is removed from the front and rear of the turret. The only regions with a sizable percentage of the total Zernike mode norm remaining are the two bands near
Figure 5.26. Total Zernike mode amplitude norm, using (a) all Zernike modes, (b) excluding tip and tilt, (c) excluding tip, tilt, and astigmatism, and (d) excluding tip, tilt, astigmatism, coma, and defocus: black circles, $0^\circ < \beta < 25^\circ$; red circles, $25^\circ < \beta < 45^\circ$; blue circles, $45^\circ < \beta < 65^\circ$; green circles, $65^\circ < \beta < 90^\circ$. 
Figure 5.27. Surface of the turret colored by the percent of total Zernike mode amplitude norm with (a) tip and tilt excluded, (b) tip, tilt, and astigmatism excluded, (c) tip, tilt, astigmatism, coma, and defocus excluded. Each point on the surface denotes the location of the center of the aperture for a given viewing angle.
Figure 5.28. Percent of total Zernike mode amplitude norm remaining at each viewing angle after removing (a) tip and tilt, (b) tip, tilt, and astigmatism, and (c) tip, tilt, astigmatism, coma, and defocus: black circles, $0^\circ < \beta < 25^\circ$; red circles, $25^\circ < \beta < 45^\circ$; blue circles, $45^\circ < \beta < 65^\circ$; green circles, $65^\circ < \beta < 90^\circ$.

the separation line and in the forward viewing direction. Removing astigmatism, shown in Figure 5.27(b) reduces the percentage of the total norm in both bands but has the most effect in the forward viewing band. By removing steady tip, tilt, astigmatism, coma, and defocus, the percentage of total Zernike mode norm over the entire turret is reduced to lower than 25% as shown in Figure 5.27.

Taking this same data and plotting it as a function of $\alpha$ in Figure 5.28, the two peaks at nearly $\alpha = 85^\circ$ and $115^\circ$ are clearly visible. When only tip and tilt are removed in Figure 5.28(a), the peak values are 100% of their total Zernike norm
indicating that their removal is completely ineffective at these locations. As more modes are removed in Figure 5.28(b) and Figure 5.28(c) these peaks significantly decrease. When steady tip, tilt, astigmatism, coma, and defocus are removed, viewing angles at $\alpha < 90^\circ$ have nearly all of the steady lensing removed from the wavefront and as noted previously, and the total percentage of total Zernike mode norm over the entire turret is reduced to lower than 25%.

While using the Zernike modes is convenient as basis to project the wavefront steady lensing onto because of their familiarity within the adaptive-optics community, they do not represent a completely orthogonal basis. Because of this, there is some correlation among mode shapes and portions of the mode amplitude can be represented in more than one mode. This may incur some error in the total norm of the Zernike modes compared to the total variance, but typically the error is small.

5.1.9 Analysis of Unsteady Aero-Optical Jitter

In addition to the steady-lensing and high-order components of the wavefront, the unsteady tip and tilt, or jitter, are of importance to the design of adaptive-optic systems. If the tip and tilt of the wavefront are not mitigated prior to propagating a beam through an aero-optical flow field, the beam is deflected near the aperture by some angle and stray from the intended target in the farfield.

In practice there are two types of jitter typically found in experimental measurements, aero-optic jitter and aero-mechanical jitter. While aero-optic jitter is a result of the unsteady density field around or near the aperture, aero-mechanical jitter is caused by the vibration of a optical device when exposed to unsteady pressure of a turbulent flow field. Because the two types of jitter are indistinguishable in optical measurements made in experiments, extracting the jitter caused purely by aero-optics is difficult. Previous experimental efforts by DeLucca et al. [51] used simultaneous accelerometer and wavefront measurements to separate mechanical from aero-optic
jitter via a linear stochastic estimator and more recently, Kemnetz et al. [45] developed a wavefront stitching technique that enables the extraction of tip and tilt from times series of wavefronts. Because of the difficulty in measuring the aero-optic jitter, it is of great interest to examine the jitter computed from the current simulations since it is purely aero-optic in nature.

As mentioned in the introduction, at each time instant the unsteady tip and tilt are computed from a wavefront by evaluating the values of $A$, $B$, and $C$ in a least-square minimization of the function

$$G = \int \int_{Ap} [OPL(x', y', t) - (A(t)x' + B(t)y' + C(t))]^2 dx'dy'$$

(5.11)

where $A(t)x'$, $B(t)y'$, and $C(t)$ are the beam tip, tilt, and piston, respectively. To remain consistent with the results presented, the $A$ and $B$ jitter components are normalized as

$$A^N = \frac{A}{\rho_{\infty} M^2}, B^N = \frac{B}{\rho_{\infty} M^2}.$$  

(5.12)

In Figure 5.29, contours of the root-mean-square of the $A$ and $B$ jitter coefficients and the norm of the two root-mean-squares for apertures that are locally aligned with the AZ - EL coordinate system are shown. In Figure 5.29(a) it can be seen that in this coordinate system the intensity of $A$ fluctuations are larger at shallow viewing angles looking through the turret wake. This is compared to the $B$ jitter component rms in Figure 5.29(b) whose contour shows larger values in two regions with larger elevation angles. When the norm of the $A$ and $B$ rms values is found, the overall jitter rms appears similar to the contours of OPD$_{rms}$, where the effect of the horn vortices are noticeable on the shape of the contours and the intensity of jitter fluctuations increase as the viewing angle looks further into the turret wake.

The same data from Figure 5.29(c) of the norm of the tip coefficient rms and tilt coefficient rms is plotted as a function of lookback angle in Figure 5.30. The norm
Figure 5.29. Contours of the normalized (a) $A$ jitter coefficient rms, (b) $B$ jitter coefficient rms, (c) norm of the $A$ and $B$ jitter coefficient rms for beams locally aligned in $AZ$ - $EL$ coordinates.
of the coefficient rms collapses for $\alpha < 100^\circ$. Above this angle, the norm increases as $\alpha$ increases and viewing angles closer to the horizon of the turret field-of-regard have increasingly greater norms.

While looking at the rms of the jitter coefficients gives an idea about the intensity of the jitter fluctuations, a practical concern in the design of adaptive optic systems is the necessary bandwidth of a potential corrective system. Typically, to mitigate tip/tilt effects, a fast steering mirror (FSM) is employed to correctively deflect the beam before transmission out of the aperture. The FSM system, in its simplest form, consists a flat mirror mounted on a control device capable of rapidly deflecting the mirror with precise control capabilities. To quantify the bandwidth needed to drive a FSM, the Strouhal number up to which a half and three-quarters of the total variance is contained in the $A$ and $B$ coefficient spectra, $St_{\frac{1}{2}}$ and $St_{\frac{3}{4}}$, respectively, is used. The half-power point of the variance, $St_{\frac{1}{2}}$, was previously used by Whiteley et al. [113] to define a bandwidth measure for the jitter in the study of aero-optic jitter. To find these values, the spectra of each coefficient is numerically integrated to solve
for $St_{\frac{1}{2}}$ and $St_{\frac{3}{4}}$,

$$\frac{1}{2} = \frac{1}{Var[A, B]} \int_{0}^{St_{\frac{1}{2}}} (E_{A,B}U_{\infty}/D) dSt_{D},$$  

(5.13)

$$\frac{3}{4} = \frac{1}{Var[A, B]} \int_{0}^{St_{\frac{3}{4}}} (E_{A,B}U_{\infty}/D) dSt_{D}. $$  

(5.14)

These quantities provide an idea of where in the frequency spectrum the most energetic jitter fluctuations lie and what frequency a FSM needs to be capable of resolving to correct for an amount of unsteady tip/tilt. Viewing angles where the $St_{\frac{1}{2}}$ or $St_{\frac{3}{4}}$ are small indicates that most of the jitter fluctuations are at lower frequencies and conversely, higher values indicate that more jitter fluctuation energy is contained at high frequencies.

In Figure 5.31 contours of $St_{\frac{1}{2}}$ for the $A$ and $B$ coefficients are presented. For forward looking angles of the turret, $St_{\frac{1}{2}}$ is small compared to that for the turret wake. In the rear of the turret, the $A$ coefficient has large values of $St_{\frac{1}{2}}$ in the horn vortex regions and the $B$ coefficient sees large values along a region after the separation line of the turret with stronger peaks on the side of the turret and the turret centerline.
Figure 5.32. $St_{\frac{1}{2}}$ for the (a) $A$ coefficient jitter and (b) $B$ coefficient jitter: black circles, $0^\circ < \beta < 25^\circ$; red circles, $25^\circ < \beta < 45^\circ$; blue circles, $45^\circ < \beta < 65^\circ$; green circles, $65^\circ < \beta < 90^\circ$.

Plotting the same data as a function of the lookback angle in Figure 5.32, the $A$ coefficient $St_{\frac{1}{2}}$ has a weak peak around $\alpha = 100^\circ$ and a stronger peak around $\alpha = 140^\circ$. In the stronger peak near $140^\circ$, values for viewing angles along the centerline have a lower $St_{\frac{1}{2}}$ compared to angles at other values of $\beta$. The $B$ coefficient $St_{\frac{1}{2}}$ has a small increase in value as the turret looks forward, possibly due to potential flow field interaction with the necklace vortex at the front of the turret. The peak location of $St_{\frac{1}{2}}$ is near $\alpha = 120^\circ$ but the value varies greatly depending on $\beta$.

In Figure 5.33, contours of $St_{\frac{3}{4}}$ are shown on the surface of the turret hemisphere for the $A$ and $B$ jitter coefficients. Qualitatively the contours appear similar to those for $St_{\frac{1}{2}}$ but clearly, the values of the peak Strouhal number required to capture the variance increases. Examining this data as a function of $\alpha$ in Figure 5.34, the same collapse for forward looking angles as previously shown for $St_{\frac{1}{2}}$ is noticed as well. For the $A$ jitter coefficient $St_{\frac{3}{4}}$, the peak around $\alpha = 100^\circ$ is more difficult to identify and the peak near $140^\circ$ has broadened. Again, viewing angles near the centerline of the turret have lower $St_{\frac{3}{4}}$ values compared to shallower viewing angles where the horn
Figure 5.33. Surface contours plots of $St_{\frac{\pi}{4}}$ for the (a) $A$ coefficient jitter and (b) $B$ coefficient jitter.

Figure 5.34. $St_{\frac{\pi}{4}}$ of (a) $A$ coefficient jitter and (b) $B$ coefficient jitter: black circles, $0^\circ < \beta < 25^\circ$; red circles, $25^\circ < \beta < 45^\circ$; blue circles, $45^\circ < \beta < 65^\circ$; green circles, $65^\circ < \beta < 90^\circ$. 
vortices have larger effects. The $B$ jitter spectrum now has larger values of Strouhal number to capture the variance of the jitter coefficient at extreme forward looking angles, indicating that there may be some high frequency fluctuations in jitter. There also exists a peak in $St_{34}$ near the separation line of the turret that gradually decreases as lookback angle increases. In the separated wake region, the $B$ coefficient $St_{34}$ is the largest along the centerline, contrary to what is noted for the $A$ jitter coefficient.

### 5.1.10 Statistical Connections Between Turret Pressure and OPD

In the development of aero-optical mitigation strategies, different methods of predicting wavefronts from optical and non-optical methods have been proposed. A class of possible methods utilizes non-optical measurements to predict the current or future optical state and use the prediction to apply an adaptive-optic solution as previously investigated by Burns et al. [9] and Arndino et al. [3]. Due to the high temporal resolution of unsteady pressure sensors, pressure measurements in particular are of interest to this class of aero-optic mitigation strategies since they can sufficiently resolve high-frequency aero-optic distortions. Studying the link between pressure and optical distortion experimentally requires simultaneous pressure and optical measurements which can be difficult when multiple viewing angles need to be considered and a high spatial resolution of pressure measurements is needed. Fortunately, using the simulation results the relation between pressure fluctuations and optical distortion can be studied for different viewing directions with high spatial resolution.

In Figure 5.35, contours of the cross-correlation of pressure on the surface of the turret and the optical signal at the aperture center at four different locations are shown. Here, the OPD-pressure cross-correlation is defined as,

$$\mathcal{R}_{\text{OPD},p}(x_0, r) = \frac{\text{OPD}(x_0)p'(x_0 + r)}{\sqrt{\text{OPD}^2(x_0)p'^2(x_0 + r)}}.$$  (5.15)
Figure 5.35. Contours of cross-correlation between the turret surface pressure and the OPD at the center of the aperture for four different viewing angles. The black dot denotes the location of the center of the aperture and the black line is in the lookback angle direction for the given viewing angle along which the cross-correlation is plotted in Figure 5.36.
Lookback Angle, $\alpha$

Figure 5.36. Correlation between the OPD at the aperture center and turret surface pressure as a function of lookback angle at different AZ angles: $\bullet$, $AZ = 120^\circ$, $EL = 30^\circ$; $\bigcirc$, $AZ = 130^\circ$, $EL = 30^\circ$; $\bigotimes$, $AZ = 150^\circ$, $EL = 30^\circ$; $\bigcirclearrowright$, $AZ = 170^\circ$, $EL = 30^\circ$. Vertical lines denote the location of the aperture center colored and patterned for their respective viewing angles.

where $x_0$ is the location at the center of the aperture. The first noticeable feature of the cross-correlation function on the turret surface is that the peak value does not coincide with the location of the aperture center for any of the viewing angles. In the first three viewing angles shown in Figure 5.35, the cross-correlation function is qualitatively similar where there are positive and negative bands of cross-correlation perpendicular to the lookback angle direction. In each, the peak cross-correlation location is downstream of the aperture center. The final contour, Figure 5.35(d), has much smaller values than the other three contours and while there appears to be bands of positive and negative cross-correlation perpendicular to $\alpha$ they are much weaker compared to the other examples.

In Figure 5.36, the data presented for the four previous viewing angles along the lookback angle direction from $90^\circ < \alpha < 180^\circ$ is shown. As noted in the previous figure, at each viewing angle there is a displacement between the peak cross-correlation...
Figure 5.37. Coherence between the OPD and turret surface pressure at the aperture center at different viewing angles:

- $AZ = 120^\circ, EL = 30^\circ$;
- $AZ = 130^\circ, EL = 30^\circ$;
- $AZ = 150^\circ, EL = 30^\circ$;
- $AZ = 170^\circ, EL = 30^\circ$.

and the location of the aperture center. For $AZ = 170^\circ, EL = 30^\circ$, the cross-correlation is too weak to define a true peak cross-correlation location. The physical explanation for this correlation peak ‘lag’ is attributed to the detachment of the shear layer after flow separation. The flow structure seen by the optical beam may not be the structure that has the greatest effect on the local pressure at the aperture. Analysis of the space-time correlation may show that there is a time-lag in the pressure and optical signal proportional to the displacement between the aperture center and peak correlation location divided by the convection speed of the shear layer structure.

To study the connection between the optical signal and pressure fluctuations at the aperture center, the spectral coherence between the two signals is computed at that location. The coherence, or the absolute value of two signal’s cross-spectrum
squared relative to the two signal’s power spectral density,

\[ \hat{C}_{\text{OPD},p'} = \frac{|E_{\text{OPD},p}(x_0)|^2}{E_{\text{OPD}}(x_0)E_{pp}(x_0)}, \]  

(5.16)

provides information about similarities between two signals at different frequencies akin to a correlation in the frequency domain. It is phase independent however, and any difference in the phase between the two signals is not reflected in the coherence. In Figure 5.37, the coherence between the pressure and OPD signals at the aperture center is shown for the four viewing angles. At the angle closest to the separation line, \( AZ = 120°, EL = 30° \), the \( p' \) and OPD signals show a non-trivial level of coherence across a large band of frequencies. Looking further back into the wake, a peak at lower frequencies emerges due to the large scale separated shear layer structures near \( St_D = 1 \sim 2 \). Viewing angles closer to the separation line show coherence across a larger range of frequencies and as \( AZ \) increases, all levels of coherence decrease indicating a decoupling of the local pressure signal from the optical signal.

It can be expected that, since the tip/tilt and piston are removed from the OPD, the correlation between pressure and OPD is a function of aperture size. In Figure 5.38, the correlation coefficient of the pressure fluctuations and OPD at the aperture center is shown for five different aperture sizes at \( AZ = 150°, EL = 30° \). The overall shape and characteristics of the correlation function does not change much with different aperture sizes but the magnitude of the correlation decreases for all lookback angles. With an aperture size of \( D/16 \), the peak in the correlation is nearly indistinguishable from the rest of the function. The removal of tip/tilt and piston acts as a high-pass filter by removing large scale optical distortions linked to shear layer structures and large pressure fluctuations. As a result, decreasing the aperture size increases the cutoff frequency of the high-pass filter on OPD and decreases \( \hat{R}_{\text{OPD},p'} \).

In Figure 5.39, the coherence between the OPD and pressure fluctuations is shown
Figure 5.38. Correlation between the OPD at the aperture center and turret surface pressure as a function of lookback angle for different aperture sizes at $AZ = 150^\circ, EL = 30^\circ$: $D/2$; $D/3$; $D/4$; $D/8$; $D/16$.

Figure 5.39. Coherence between the OPD at the aperture center and turret surface pressure for different aperture sizes at $AZ = 150^\circ, EL = 30^\circ$: $D/2$; $D/3$; $D/4$; $D/8$; $D/16$. 

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for the five aperture sizes. While all five aperture sizes show the largest value of coherence between $St_D = 1 \sim 2$, the coherence level is a strong function of the aperture size.

For adaptive-optic methods for high-order wavefronts using feed-forward information from pressure sensors on the turret surface, the results presented show that they have to account for the lag between the pressure signal and OPD. In addition, for viewing angles looking far through the turret wake, there is little connection between the surface pressure signal and the optical distortion. For different aperture sizes, the optical signal in larger apertures have more in common, both in correlation and in spectral behavior, with pressure fluctuations, but as the aperture decreases, the link between $p'$ and OPD signal diminishes.

5.2 Conclusions

In this chapter, optical quantities are analyzed to gain insight into the aero-optical properties of the hemisphere-on-cylinder turret in flight. While comparisons with experimental measurements of the pressure coefficient along the centerline of the turret show very good agreement and the simulation results demonstrate mesh convergence for flow and optical quantities, overall, the simulations under-predict optical distortions, especially at off-center angles. However, when more detailed comparisons are made using POD, the spatial and temporal qualities of large-scale structures in computational results match experimental measurements very well. The reason for the discrepancies may stem from the lack of realistic turret features on the computational turret and elements like vibrational disturbances in experimental measurements. The differences between computational and experimental results should and can not be ignored as they point to a lack of some physical element in the computation. Even with these differences, important insights into the aero-optic and fluid dynamic behavior of the hemisphere-on-cylinder turret can still be gathered from the results of
the simulations

As discussed in Chapter 4, the horn vortex regions are a major flow structure in the turret wake and serve to increase optical distortions due to their convection of strong vortices shedding from the cylindrical base into the turret line of sight. Viewing angles near the horn vortices show trends that differ from viewing angles outside of this region. These vortical structures exhibit increased pressure fluctuations that affect optical distortions compared to the turret centerline, where optical distortions due to temperature fluctuations play a larger role. These large vortices also serve to increase the unsteady jitter at viewing angles near the horn vortices and, if unaccounted for, may deflect the optical beam off of its intended target.

An investigation of the spatial structures of the high-order wavefronts after propagating through the aero-optic field, shows that across all viewing angles the autocorrelation length of the wavefronts along the lookback direction increases nearly linearly after the flow separates. By examining the frequency spectra of optical distortions at multiple azimuthal angles for a single elevation angle, it is found that the spectral slope of OPD is larger at off-center viewing angles compared to viewing angles along the centerline. This is a result of the dominant role of OPD$_p$ at off-center viewing angles and comparable spectral levels in the inertial subrange of OPD$_p$ and OPD$_T$ along the centerline. There remains questions about the ability of the wall-modeled LES to fully capture the behavior of the transition from boundary layer to turbulent separated shear layer since the magnitude of spectral slope of optical distortions is less than expected for a high Reynolds number flow.

In airborne optical systems the beam jitter must also be mitigated to maintain accurate propagation to the intended far-field target. From the simulation, the largest variance in tip and tilt is found near the horn-vortex region of the turret wake. The region with the most demanding bandwidth requirement for a fast-steering mirror to correct jitter is observed to be at viewing angles right after the separation line
and in the horn-vortex region of the wake. Additionally, if corrective measures are
designed to use pressure sensors at the aperture center in a feed-forward manner to
predict wavefront distortions, a ‘lag’ between the optical and pressure signals must
be taken into account. The spectral coherence between the aperture local pressure
signal and the optical signal decreases as the viewing angle is farther away from the
separation line and as the optical aperture size decreases. Additional analysis using
Joint POD shows that for corrective wavefront methods, it is more efficient for a
basis of wavefront distortions common to all backward looking viewing angles to be
locally oriented in $\alpha - \beta$ coordinates compared to $AZ - EL$ coordinates.
CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

While three individual research goals were identified for this dissertation in the introduction, together they attempt to accomplish one overarch goal: to improve our understanding of and ability to predict aero-optic distortions. Through theoretical and computational means, the presented research provides new insights into fundamental and applied aero-optic systems and how the conditions of the flow field can affect aero-optic behavior.

First, starting from the Gladstone-Dale relation and a linearized ideal-gas law, expressions for the turbulent index-of-refraction spectrum for a weakly compressible, homogenous flow is derived. It is found that the refractive index spectrum is a function of contributions from the pressure spectrum, temperature spectrum, and pressure-temperature cospectrum. The inertial range behavior of the refractive index spectrum depends on the strengths of each of these components, where the value of the spectral slope magnitude is bounded by the turbulent temperature and pressure spectral slope magnitude. By applying wave propagation theory developed for electromagnetic wave propagation through the turbulent atmosphere, relations between the spectral behavior of density or refractive index and the 1-D and 2-D optical phase distortion are found. The remainder of Chapter 2 uses these results to model the OPD, with and without simple adaptive optic corrections, to examine wavefront sensor resolution requirements, and to discuss numerical simulation resolution requirements.

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Then, to verify the results presented in Chapter 2 and examine a canonical aero-optical flow, Chapter 3 presents large-eddy simulations (LES) of temporally-evolving shear layers. To vary the effects of turbulent temperature mixing on density fluctuations and the resulting aero-optics, three different ratios of top to bottom freestream density, $s$, are presented. The ratios are chosen so that there is one case where turbulent pressure fluctuations are expected to dominate density fluctuations, one case where turbulent temperature fluctuations are expected to dominate density fluctuations, and one case where the ratio is selected to be close to values seen in previous computations.

Examinations of the density variance budget show that as the ratio of bottom to top freestream density increases, the role of temperature in density fluctuations increases. In addition, the introduction of a temperature gradient across the shear layer also introduces a large-scale mixing phenomenon that increases density fluctuations on the high-density side of the shear layer and suppresses density fluctuations on the low-density side. Results from the shear layer simulations confirm results from the theory in Chapter 2, that a relationship between density spectral slope, $m_\rho$, and 1-D and 2-D OPD spectral slopes, $m_{\text{OPD}}$ and $m_{\text{OPD}}^{2D}$, exists: $m_{\text{OPD}} = m_\rho + 1$ and $m_{\text{OPD}}^{2D} = m_\rho + 2$.

Using the linearized ideal gas law to decompose OPD into its temperature and pressure components, $\text{OPD}_T$ and $\text{OPD}_p$, respectively, the individual effects of each are examined. It is seen that even as $s$ increases, the autocorrelation, spectrum, and variance of $\text{OPD}_p$ are largely unchanged. However with increased $s$, $\text{OPD}_T$ greatly changes in terms of autocorrelation structure and variance and the change in optical distortion with $s$ is almost entirely due to temperature fluctuations.

Finally, results from wall-modeled large-eddy simulations of the subsonic flow over a hemisphere-on-cylinder turret are presented to examine the aerodynamic and aero-optical effects of a realistic airborne optical platform. The turret flow field is
highly three-dimensional and periodically exhibits very large-scale vortex formation in its wake. A primary feature of the wake is the two horn vortices generated after the flow separates and the aero-optic distortions in the turret wake is greatly affected by their presence. Viewing angles near the horn vortex region show increased levels of optical distortion and demonstrate different trends from the rest of the field-of-regard. At these horn-vortex viewing angles, optical distortions also have a larger contribution from $\text{OPD}_p$ than other viewing angles and the spectral slope of the OPD in the power-law region tend to be larger compared to viewing angles along the turret centerline.

Several results from simulation important to the development of adaptive-optics methods are also presented in Chapter 5. In addition to presenting the autocorrelation length of optical structures, steady-lensing behavior, and unsteady jitter of optical distortions, the joint Proper Orthogonal Decomposition (JPOD) of all backward facing viewing angles is presented. It was found that the common basis provided by the JPOD is more efficient when the aperture is locally aligned with the lookback and modified elevation angles compared to azimuthal and elevation angles. Additionally, the correlation and coherence of the turret surface pressure and the optical signal at the center of the aperture is presented for several viewing angles and different aperture sizes. It is seen that a lag between the optical and pressure signals exists after separation and as the viewing angle looked further through the wake, the coherence decreased across all frequencies. When the aperture size decreases, both the correlation and coherence between the optical distortion and surface pressure decrease.

6.2 Future Work

In each of the three main topics discussed in this research, there is the possibility for further investigations and the opportunity to gain a greater understanding
of aero-optics. For the theoretical modeling of aero-optic distortions, the simulation of shear layers, and the flow over realistic aero-optical turrets, future work and recommendations are presented in this section.

6.2.1 Spectral Description of Aero-Optic Distortions

There are many components in the spectral descriptions of aero-optic phase distortions that need to be extended upon. This list is not limited to: the combined effect of aero-optic and atmospheric distortions, the effects of compressibility on index-of-refraction spectrum, modeling the pressure-temperature cospectra, more accurate expressions for refractive index in large temperature fluctuations, effects of variation in refractive-index spectrum along the propagation length, the spectrum filter of actual adaptive-optic systems, the effects of large-scale anisotropy in the spectrum, aero-optic effects on non-planar waves, and so on.

Potentially, the current spectral description of aero-optic distortions could be used to find an optimal aperture size based on maximum farfield intensity. While, the large-aperture approximation does not hold for apertures $\kappa_\ast Ap > 1.0$, the Marechal approximation for the instantaneous Strehl ratio is accurate if the probability density function (PDF) of the OPD is Gaussian and phase distortion is small. If we assume that the PDF of OPD is Gaussian, (though there is evidence to the contrary [75]), the farfield irradiance relation can be used to find that,

$$I \propto \frac{PD^2}{L_{prop}^2 \lambda^2} \approx \frac{PD^2_{Ap}}{L_{prop}^2 \lambda_{opt}^2} \exp \left[ - \left( \frac{2\pi OPD_{rms,\infty} f_F(\kappa_\ast Ap)}{\lambda_{opt}} \right)^2 \right]. \tag{6.1}$$

where $f_F$ is the function given by Fig. (2.6), $P$ is the beam output power, and $L_{prop}$ is the propagation distance of the beam to the target. For certain values of $OPD_{rms,\infty}$ there is an optimal aperture size that is less than $L_0$. When $OPD_{rms,\infty}$ is small enough, $I$ scales linearly with aperture size. There are several parameters that
can be tuned in this equation and to fully explore its impact, some work would be required to link the dependencies of different parameters of the previous equation to the farfield irradiance.

6.2.2 Aero-Optic and Density Behavior of Temporally Evolving Shear Layers

Immediately, using the data collected for the presented simulations the behavior of the optical distortion and the spectral dependence of the OPD when the beam propagates at some angle with the shear layer can be explored. These results would be of fundamental importance to actual optical systems like the turret, where the beam propagates at some angle through the shear layer. Additionally, the compressible Mach number in these simulations are equal to 0.3 for all cases and effects of compressibility are very weak. This weakly compressible condition is convenient for examining of the density spectrum since for the incompressible case theoretical expressions for temperature and pressure spectrum are known. In reality, aero-optics is an inherently compressible phenomena and the strength of distortions is known to increase with Mach number.

The effects of compressibility on the density and thus the OPD would be a very interesting next step for this research, where $s$ and the convective Mach number could be adjusted to isolate pressure and/or temperature effects and to account for compressibility, respectively. It would be unsurprising to find that the strength of pressure fluctuations and $\text{OPD}_p$ have a strong relation to the convective Mach number but how much and how it compares to adiabatic heating and cooling and changes in the total temperature is of fundamental interest not only to aero-optics, but also for shear turbulence research.
6.2.3 Simulations of Aero-Optic Turrets

The current turret simulations represent some of the largest Reynolds number and largest unstructured, compressible CFD simulations to-date, however, there is still some evidence that higher resolution may be required in the boundary layer region of the turret. To achieve the boundary layer resolutions typically recommended in wall-model simulations \((y^+ \approx 30 - 50, \ x^+ \approx 150 - 200, \ z^+ \approx 100 - 120)\), a back-of-the-envelope estimation for the number of cells required for such a simulation is around 10 billion. In addition, because of the strong pressure gradient over the surface of the turret and the presence of boundary layer transition, a non-equilibrium wall model would capture more physics in the turret simulation than the equilibrium wall-model used in Chapter 4 and 5. However, a non-equilibrium wall-model for an unstructured flow solver comes at a significant computational expense. A wall-model of this type has been implemented in CharLES by Park et al [74] and it was found that the increase in time for solving the non-equilibrium wall model with the LES equations was \(2 \times -2.5 \times\) that of solving just the LES equations for the same mesh. That is compared to about \(1.3 \times\) for solving the non-equilibrium wall model with the LES equations. Fully realizing the recommended modeling and resolution would require significantly larger computational resources even with current state-of-the-art CFD.

Beyond a simulation with far increased resolution and modeling capability that may be several years out, recently a large amount of research in the aero-optic community has focused on transonic aero-optic flows. Using CFD to simulating the transonic flow over a three-dimensional turret would include several challenges in addition to the difficulties of large Reynolds number flows, namely resolving shockwaves and shockwave dynamics that are key in the transonic flow over an aero-optic turret. In this flow, the domain must be large enough to prevent shockwave reflection into the domain in addition to having sufficient resolution in the boundary layer region...
of the mesh to capture the boundary layer shockwave interactions.

In the study of three-dimensional turrets, computational aero-optics lacks a benchmark problem to compare optical and flow results from CFD to experimental data. While experimental methods in aero-optics have made considerable improvements in their ability to faithfully measure optical distortions, results typically published in the literature are not of conditions that are easily reproducible (e.g. the AAOL) or do not contain enough flow and optical data to validate a simulation. A recommendation for the future would be for the generation of experimental flow and optical data for a simple three-dimensional turret geometry, such as a hemisphere, that can be used for simulation validation. Using this data, solvers and modeling methodologies can be validated against the benchmark and better ideas about resolution and fidelity requirements for computational aero-optics can be established.
APPENDIX A

MATHEMATICAL BASIS OF PROPER ORTHOGONAL DECOMPOSITION

The mathematical basis of POD can be described via a constrained maximization problem using Lagrange multipliers. A unit vector, $v$, is desired that maximizes the variance of its projection on some data set $X$, where $v$ is a column vector of dimension $n$ and $X$ is a matrix of column vectors, $X = [X_1, X_2, ..., X_m]$ with each $X_j$ containing data at $n$ spatial locations at a time instant $j$. The goal in maximizing the variance of the projection is to capture the most energetic “structure” contained in the data set $X$ and encapsulate it in the vector $v$. The Lagrange function to maximize $\text{Var}[v^T X]$ constrained by $v^T v = 1$ can be written as

$$L(v, \lambda) = \text{Var}[v^T X] - \lambda(v^T v - 1), \quad (A.1)$$

and after expanding the variance term as

$$L(v, \lambda) = v^T \Sigma v - \lambda(v^T v - 1), \quad (A.2)$$

where $\Sigma = XX^T$ is the covariance matrix of $X$. Taking the derivative with respect to $v$ and $\lambda$ we find that,

$$\frac{\partial L}{\partial v} = 2\Sigma v - 2\lambda v = 0, \quad (A.3)$$

$$\frac{\partial L}{\partial \lambda} = -(v^T v - 1) = 0. \quad (A.4)$$

Equation (10) can be rewritten as an eigenvalue problem, $\Sigma v = \lambda v$, whose solutions for $v$ and $\lambda$ are the eigenvectors and eigenvalues of the covariance matrix $\Sigma$. Equation
(11) satisfies the constraint condition $v^T v = 1$. From the variance term in Equation (9) and using the results from Equation (10), it can then be proven that the variance of an eigenvector $v_i$ projected onto $X$ is given by its corresponding eigenvalue, $\lambda_i$:

$$\text{Var}[v_i^T X] = v_i^T \Sigma v_i = v_i^T \lambda_i v_i = \lambda_i v_i^T v_i = \lambda_i. \quad (A.5)$$

Therefore, the unit vector, $v_i$, that maximizes the variance of its projection on the data set, $X$, is the eigenvector corresponding to the eigenvalue $\lambda_i$ of the covariance matrix of $X$. Further, since the covariance matrix is positive-semidefinite, all eigenvectors of $\Sigma$ are orthogonal and can be ordered by the magnitude of its eigenvalues to create an orthonormal basis that, term-for-term, provides the best possible representation of the data in a least squares sense. This orthonormal basis formed from the eigenvectors of the covariance matrix and sorted by eigenvalue magnitude are referred to as POD “modes” where the largest eigenvalue mode is the first mode, and successive modes are numbered in order of descending eigenvalues.

If the POD modes are used to recreate the time-dependent wavefronts, $\text{OPD}(x', y', t)$, the temporal evolution of each spatial mode must also be known. $\text{OPD}(x', y', t)$ can be decomposed as a summation of its POD spatial modes and their temporal coefficients as,

$$\text{OPD}(x', y', t) = \sum_{i=1}^{N} \sqrt{\lambda_i} u_i(t) v_i(x', y'), \quad (A.6)$$

where $N$ is the number of modes (and spatial locations), $v_i$ is the $i^{th}$ spatial mode, $u_i$ is the set of temporal coefficients for the $i^{th}$ mode, and $\lambda_i$ is the eigenvalue corresponding to the $i^{th}$ mode. Note that $\sqrt{\lambda_i}$ has the same dimension as OPD and $v_i$ and $u_i$ are both dimensionless. Using notation where $\text{OPD}$ is the matrix form of $\text{OPD}(x', y', t)$ similar in construction to $X$, $U$ is a matrix of temporal coefficients for all modes, $D$ is a diagonal matrix containing the square root of the eigenvalues, and $V^T$ is a
matrix containing all spatial modes, Equation (A.6) can be rewritten as

\[ \text{OPD} = \text{UDV}^T. \]  \hfill (A.7)

Often, the most convenient method to determine \( \text{U} \), \( \text{D} \), and \( \text{V}^T \) is to perform the singular value decomposition of \( \text{OPD} \).
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