ELECTRON DENSITY MEASUREMENTS
FOR PLASMA ADAPTIVE OPTICS

A Dissertation

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by

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Over the past 40 years, there has been growing interest in both laser communications and directed energy weapons that operate from moving aircraft. As a laser beam propagates from an aircraft in flight, it passes through boundary layers, turbulence, and shear layers in the near-region of the aircraft. These fluid instabilities cause strong density gradients which adversely affect the transmission of laser energy to a target. Adaptive optics provides corrective measures for this problem but current technology cannot respond quickly enough to be useful for high speed flight conditions. This research investigated the use of plasma as a medium for adaptive optics for aero-optics applications.

When a laser beam passes through plasma, its phase is shifted proportionally to the electron density and gas heating within the plasma. As a result, plasma can be utilized as a dynamically controllable optical medium. Experiments were carried out using a cylindrical dielectric barrier discharge plasma chamber which generated a sub-atmospheric pressure, low-temperature plasma. An electrostatic model of this design was developed and revealed an important design constraint relating to the geometry of the chamber.

Optical diagnostic techniques were used to characterize the plasma discharge. Single-wavelength interferometric experiments were performed and demonstrated
up to 1.5 microns of optical path difference ($OPD$) in a 633 nm laser beam. Dual-wavelength interferometry was used to obtain time-resolved profiles of the plasma electron density and gas heating inside the plasma chamber. Furthermore, a new multi-wavelength infrared diagnostic technique was developed and proof-of-concept simulations were conducted to demonstrate the system’s capabilities.
For my grandfather, Robert S. Neiswander
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CHAPTER 1

INTRODUCTION

Wavefront correction for airborne laser applications such as directed energy weaponry and laser communications rely on adaptive optics to provide optical path corrections that account for the local compressible flow field. The status quo technology for optical wavefront correction uses electro-mechanical deformable mirrors. These are expensive, fragile, subject to fatigue, and most importantly, have a limited bandwidth that is far below the time scale required to correct for energetic turbulent flow structures generated by an aircraft in transonic or supersonic flight. This research investigates the implementation of plasma as a dynamic medium to perform wavefront control. The result is “Plasma Adaptive Optics” (PAO) that utilizes a relation between the plasma electron density and its index of refraction. The advantage of the PAO is that there are no moving parts so that it is not fragile or subject to fatigue, and its temporal response can be two orders of magnitude higher than the fastest deformable mirror.

The prototype PAO lens uses a non-thermal plasma that is spatially confined within a low-pressure chamber. The plasma generator is an AC dielectric-barrier discharge (DBD). The DBD plasma has a number of advantages: it can be driven at high frequencies, the glow-to-arc transition is delayed which provides a significantly larger operating range compared to DC devices, it is easily adapted to nearly any geometry, and it can generate stable plasmas over a large range of
pressures. In addition, if the probing wave frequency is large compared to both the effective collision frequency and the plasma frequency, the attenuation of the probing beam is negligible.

The maximum frequency response of the PAO is limited by the electron-ion recombination time scale. This should give the PAO a frequency bandwidth that is several orders of magnitude higher than the fastest deformable mirrors.

The goal of the PAO lens design is to produce a stable plasma of controllable refractive index to provide for phase control of a laser beam. The induced phase shift becomes a function of the voltage applied to the PAO lens. Once a PAO lens is shown to possess sufficient refractive index properties and frequency response for aero-optics applications, the design will be used in a large array of PAO lenses. Each PAO lens in the array will be individually addressable to allow for localized phase control. A wavefront passing through the array will be corrected by prescribing the correct voltages to each PAO lens.

Subsequent chapters detail the background information and completed research of this project. Chapter 2 introduces fundamental optics concepts, the aero-optics problem, and a brief description of adaptive optics. In Chapter 3, derivations for the interactions between electromagnetic waves (light) and plasma are presented. Furthermore, a short summary is provided of experimental plasma diagnostic techniques relevant to this research. Chapter 4 details the design of the PAO lens with supporting experimental and theoretical data. In Chapter 5, data is presented from a proof-of-concept experiment demonstrating the ability of the PAO lens to produce changes in a laser wavefront. Chapter 6 presents dual-wavelength interferometric experiments to determine the time resolved electron density and gas temperature changes inside the plasma chamber as plasma forms.
Chapter 7 introduces a new plasma measurement system which probes the plasma with multiple infrared wavelengths to determine the plasma electron density and change in temperature.
CHAPTER 2

GENERAL OPTICS

In this chapter, the foundational theories for the interaction of fluids and optical electromagnetic waves are derived and presented. From geometric optics, it will be seen that amount of distortion, or aberration, in a laser beam can be described by a quantity known as the optical path difference. From Fourier optics, it will be seen that when a beam is propagated over long distances, diffraction effects can cause significant reduction in the amount of light energy delivered. Finally, the topics of aero-optics and adaptive optics are introduced with brief discussions.

2.1 Geometric Optics

The theory of geometric optics gives very simple approximations to complicated, real world phenomena. In this treatment, light is considered to propagate along straight lines, or rays, through space. The speed at which light travels is dependent upon the medium through which it is traveling. The refractive index or index of refraction is the ratio of the speed at which light travels in a medium relative to the speed of light in a vacuum,

\[ N = \frac{c}{v} \]  \hspace{1cm} (2.1)

where \( N \) is the refractive index, \( c \) is the speed of light in a vacuum, and \( v \) is the
speed of light propagation through a medium. Denser media like glass typically have high refractive indices around 1.5 whereas low density media like air have refractive indices very close to unity. Consequently, light travels very quickly through air (similar to in vacuum) and much slower through glass.

Now, consider a collection of identical light rays arranged in parallel all propagating together in the same direction at some initial time $t_0$. As a group, these rays represent a beam of light much like a collimated laser beam. As long as the medium they are traveling through is uniform, these rays will always remain completely in line with one another. The locus of points at the forefront of the rays is called the *wavefront*. This terminology will become better justified in the following section, once the concept of *phase* is introduced.

Up to this point, the wavefront has remained *planar*; all of the rays have traveled the exact same distance in the same amount of time. Now consider if the beam enters an area of non-uniform index of refraction. Some of the rays pass through lower refractive index areas causing them to speed up. Other rays pass through higher refractive index areas causing them to slow down. At some later time $t_0 + \Delta t$, the wavefront of the beam is no longer planar. Some of the rays have traveled ahead and others are far behind. Figure 2.1 depicts this propagation scenario. After passing through a non-uniform refractive index field, the wavefront of light becomes *aberrated*.

In this situation, some rays traveled faster and some rays traveled slower. This difference in each ray’s speed has caused an aberration of the wavefront. This is the fundamental concept of the *optical path length* (OPL), defined as the integral
Figure 2.1. Geometric propagation of a wave through a medium of non-uniform refractive index.

of the refractive index over a specified path between arbitrary points \( s_1 \) and \( s_2 \),

\[
OPL(x, y, t) = \int_{s_1}^{s_2} N(x, y, t) ds. \tag{2.2}
\]

Here the \( OPL \) and \( N \) are left as scalar fields of positions \( x \) and \( y \), and allowed to vary in time \( t \). It is important to note that the \( OPL \) is the apparent length a light ray traveled between \( s_1 \) and \( s_2 \). A ray passing through glass will generate a larger \( OPL \) relative to a ray traveling through air.

Consider the example corollary to Figure 2.1. At time \( t_0 + \Delta t \), each ray has a unique \( OPL \) value depending upon its path through the non-uniform medium. The distribution of \( OPL \) at \( t_0 + \Delta t \) is related to the shape of the light wavefront. The magnitude of the wavefront distortion is called the \textit{optical path difference}
A convenient definition for $OPD$ is the mean removed $OPL$

$$OPD(x, y, t) = OPL(x, y, t) - \bar{OPL}(x, y) \quad (2.3)$$

where the bar indicates the spatial average over the aperture. The $OPD$ is actually a conjugate reference of the wavefront \[49\]. More specifically, the wavefront lags at locations where the $OPD$ is positive, and it leads where the $OPD$ is negative.

It will be shown in the next section that a spatially varying $OPD$ will significantly diminish the propagation of laser energy. This phenomenon is the basis of entire fields of research across multiple disciplines. In particular, the implications for aero-optics will be discussed in the fourth section of this chapter.

2.2 Fourier Optics

Diffraction is an optical phenomenon that is not accounted for by geometric optics. This section will show that by considering the wave nature of light—thereby allowing the light to constructively and destructively interfere with itself—the diffraction phenomenon can be described. The first published observation of diffraction was made by Grimaldi in 1665 \[29\]. In his experiment, he passed light through an aperture and observed that the resulting image did not have crisp boundaries between light and dark areas as would be predicted by geometric optics. Instead, there was a gradual change from light to darkness. Although the theory was not developed at the time, it is now known that diffraction was responsible for this observation.

In 1678, Huygen postulated that light could be described as a series of wavefronts, or lines of points where the beam has the same phase. This laid the foundational groundwork for diffraction theory. Huygen’s principle states that
Figure 2.2. Huygen’s principle describes the propagation of waves as the superposition of in-phase spherical point sources along a phase front.

the propagation of light may be described by the superposition of new spherical point sources, or wavelets, along a wavefront. These new point sources radiate light away from the initial wavefront and form a new wavefront. The concept is shown in Figure 2.2. Huygen’s principle incorporates two important concepts: (1) light could be described as a wave and (2) a wave propagates in directions normal to its wavefronts.

Mathematical representations of diffraction did not develop until the late 1800s when Maxwell’s equations were used to describe light as an electromagnetic wave. The Huygens-Fresnel equation is one such representation. It is valid for a wave traveling through a dielectric medium assuming that the medium is linear, isotropic, homogeneous, non-dispersive, and non-magnetic [29]. Furthermore,
it is assumed that the components of the electric and magnetic fields can all be represented using a scalar wave equation. Consider the complex, scalar aperture distribution

\[ U(\xi, \eta) = A(\xi, \eta)e^{i\phi(\xi, \eta)} \]  

(2.4)

where \(A(\xi, \eta)\) is an amplitude function and \(\phi(\xi, \eta)\) is the wavefront. Both are defined as spatial functions in the \(\xi - \eta\) plane which is orthogonal to the propagation axis. After the beam propagates a distance \(z\), the new wave \(U(x, y)\) located in the \(x - y\) plane (parallel to the \(\xi - \eta\) plane) is described by the Huygens-Fresnel principle

\[ U(x, y) = \frac{z}{i\lambda} \iint_A U(\xi, \eta) \frac{e^{ikr}}{r^2} d\xi d\eta \]  

(2.5)

where \(\lambda\) is the wavelength, \(A\) is the area in the \(\xi - \eta\) plane, \(k\) is the wavenumber, and \(r\) is the distance between two points in each plane, given by

\[ r = \sqrt{(x - \xi)^2 + (y - \eta)^2 + z^2}. \]  

(2.6)

For brevity, the derivation of the Huygens-Fresnel equation is not presented here, but the reader is encouraged to see Fowles [25] and Born and Wolf [12] for a more complete discussion. It must be noted that in the derivation of Equation 2.5, it has already been assumed that \(r \gg \lambda\).
2.2.1 Near Field (Fresnel Approximation)

The computation of the double integrals in Equation 2.5 is non-trivial so a Taylor expansion is substituted for $r$ as

$$r = z \sqrt{1 + \left( \frac{x - \xi}{z} \right)^2 + \left( \frac{y - \eta}{z} \right)^2} \approx z \left[ 1 + \frac{1}{2} \left( \frac{x - \xi}{z} \right)^2 + \frac{1}{2} \left( \frac{y - \eta}{z} \right)^2 \right]$$  \hspace{1cm} (2.7)

The quantity $r$ appears twice in Equation 2.5: once in the exponential function in the numerator and once as $r^2$ in the denominator. With acceptable error, the one-term expansion is substituted for $r$ in the denominator and the two-term expansion is substituted for $r$ in the exponential term [29]. Equation 2.5 becomes

$$U(x, y) = e^{ikz} \frac{e^{i \frac{k}{\lambda z} (x^2 + y^2)}}{i \lambda z} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U(\xi, \eta) \exp \left[ \frac{ik}{2z} \left( (x - \xi)^2 + (y - \eta)^2 \right) \right] d\xi d\eta. \hspace{1cm} (2.8)$$

Note the limits of integration now span from $-\infty$ to $\infty$ because problematic singularities caused by the $r^2$ term have been removed. The term $e^{i \frac{k}{\lambda z} (x^2 + y^2)}$ is factored out of the integrals to become

$$U(x, y) = \frac{e^{ikz} e^{i \frac{k}{\lambda z} (x^2 + y^2)}}{i \lambda z} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ U(\xi, \eta) e^{i \frac{k}{\lambda z} (\xi^2 + \eta^2)} \right] e^{-i \frac{k}{\lambda z} (x \xi + y \eta)} d\xi d\eta. \hspace{1cm} (2.9)$$

To simplify this equation, the $x$ and $y$ coordinates are normalized [56] as

$$f_x = \frac{kx}{2\pi z} \hspace{1cm} f_y = \frac{ky}{2\pi z}. \hspace{1cm} (2.10)$$
and the Fresnel diffraction equation becomes

\[ U(x, y) = e^{ikz} e^{i\frac{2\pi}{\lambda}(f_x^2 + f_y^2)} \int \int_{-\infty}^{\infty} \left[ U(\xi, \eta) e^{i\frac{k}{\lambda} (\xi^2 + \eta^2)} \right] e^{-i2\pi(f_x \xi + f_y \eta)} d\xi d\eta. \]  

(2.11)

Finally, the wavenumber \( k \) may be replaced with \( \frac{2\pi}{\lambda} \) to give,

\[ U(x, y) = e^{iz} e^{i\frac{\pi \lambda z}{\lambda}} \int \int_{-\infty}^{\infty} \left[ U(\xi, \eta) e^{i\frac{\pi \lambda z}{\lambda} (\xi^2 + \eta^2)} \right] e^{-i2\pi(f_x \xi + f_y \eta)} d\xi d\eta. \]  

(2.12)

Equations 2.9 and 2.12 are two equivalent forms of the Fresnel diffraction integral. The Fresnel diffraction integral describes the propagation of light from an aperture within the near field where the light wave is proportional to the Fourier transform of the product of the initial aperture distribution and a quadratic phase function.

### 2.2.2 Far Field (Fraunhofer Approximation)

The Fresnel approximation reduced the complexity of the Huygens-Fresnel principle (Equation 2.5) by approximating the distances between points in the aperture plane and the propagated plane. The Fraunhofer approximation makes a further assumption that

\[ z \gg \frac{1}{2} k (\xi^2 + \eta^2)_{\text{max}}. \]  

(2.13)

This causes the quadratic phase term in Equation 2.9 to become unity,

\[ e^{i\frac{k}{\lambda} (\xi^2 + \eta^2)} \approx 1, \]  

(2.14)
and the propagated wave after a distance \( z \) is then

\[
U(x, y) = e^{ikz} e^{\frac{i\lambda}{2} (x^2 + y^2)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U(\xi, \eta) e^{-i\frac{2\pi}{\lambda} (x\xi + y\eta)} d\xi d\eta. \tag{2.15}
\]

The \( x \) and \( y \) coordinates are normalized using Equation 2.10 and the wavelength is substituted for the wavenumber using \( k = \frac{2\pi}{\lambda} \) to give

\[
U(f_x, f_y) = e^{i\frac{\pi}{2} \frac{2\pi}{\lambda}} e^{i\pi f_x^2 + f_y^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U(\xi, \eta) e^{-i2\pi (f_x\xi + f_y\eta)} d\xi d\eta. \tag{2.16}
\]

Equation 2.16 describes the propagation of light from an aperture into the far field. It should be noted that the integral term is simply the Fourier transform of the aperture function \( U(\xi, \eta) \).

Equation 2.16 is valid as long as the Fraunhofer assumption in Equation 2.13 is true. The far field is defined as \( z \gg \frac{1}{2} k (\xi^2 + \eta^2)_{max} \). Figure 2.3 shows the evaluation of the critical line at \( z = \frac{1}{2} k (\xi^2 + \eta^2)_{max} \) as a function of circular aperture diameter \( D = \sqrt{\xi^2 + \eta^2} \). Lines are shown for a helium neon laser and a CO\(_2\) laser with wavelengths of 0.633 \( \mu m \) and 10.6 \( \mu m \), respectively. This treatment assumes that the propagating beam is perfectly collimated. It should be noted that the focal plane of the beam is also the far field.

2.3 Far Field Irradiance

The irradiance, or radiant flux density, distribution \[40\] is

\[
I(f_x, f_y) = |U(f_x, f_y)|^2. \tag{2.17}
\]
Figure 2.3. Distance to the far field as a function of aperture diameter $D$.

From Equations 2.16 and 2.17, the far field irradiance is then

$$I(f_x, f_y) = \frac{1}{\lambda^2 z^2} \left| \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A(\xi, \eta) e^{i\phi(\xi, \eta)} e^{-i2\pi(f_x \xi + f_y \eta)} d\xi d\eta \right|^2. \quad (2.18)$$

The wavefront $\phi(\xi, \eta)$ relates to the $OPD(\xi, \eta)$ as

$$\phi(\xi, \eta) = \frac{2\pi}{\lambda} OPD(\xi, \eta) \quad (2.19)$$

and can be used in Equation 2.18 to give the far field irradiance as a function of $OPD(\xi, \eta)$,

$$I(f_x, f_y) = \frac{1}{\lambda^2 z^2} \left| \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A(\xi, \eta) e^{i\frac{2\pi}{\lambda}OPD(\xi, \eta)} e^{-i2\pi(f_x \xi + f_y \eta)} d\xi d\eta \right|^2. \quad (2.20)$$
The Fourier transform into the far field implies that any spatial variations in
the aperture function $U(\xi,\eta)$ will result in irradiant energy leaving the \textit{on-axis},
or zero frequency, to become side-lobe energy content. These variations may exist
in the amplitude $A(\xi,\eta)$ and the wavefront $OPD(\xi,\eta)$ of the aperture function.
An ideal aperture with planar amplitude and no $OPD$ will result in a maximum
on-axis irradiance. Any aperture function with variations in $A$ or $OPD$ over $(\xi,\eta)$
will unequivocally have reduced on-axis intensity.

Figures 2.4 and 2.5 show the normalized far field irradiance for near field
wavefronts that are planar and aberrated. For the planar OPD, the on-axis far field
irradiance takes its maximum possible value. For the aberrated OPD, Fraunhofer
diffraction causes irradiant energy to spatially dissipate across the axis, causing
the on-axis energy to be significantly reduced.

2.3.1 Strehl Ratio

The Strehl ratio is the ratio of the maximum on-axis irradiance of an aberrated
beam to the maximum on-axis irradiance of an unaberrated beam \cite{60}. The
term \textit{aberrated} refers to a wavefront with a non-planar phase, or $OPD(\xi,\eta) \neq 0$.
Similarly, an \textit{unaberrated} beam would have zero $OPD$ across the entire aperture.
The term \textit{on-axis} denotes the center of the aperture in the far field where $f_x = f_y = 0$. Using Equation 2.20 the Strehl ratio is

$$S = \frac{I(f_x, f_y)}{I_{OPD=0}(f_x, f_y)} \bigg|_{f_x=f_y=0} = \left( \frac{\iint_{-\infty}^{\infty} A(\xi,\eta) e^{i\frac{2\pi}{\lambda} OPD(\xi,\eta)} e^{-i2\pi(f_x\xi+f_y\eta)} d\xi d\eta}{\iint_{-\infty}^{\infty} A(\xi,\eta) e^{-i2\pi(f_x\xi+f_y\eta)} d\xi d\eta} \right)^2$$  \hspace{1cm} (2.21)

For the remainder of this section, assumptions will be applied to the Strehl ratio
to further simplify it. First, it is assumed that the amplitude $A(\xi,\eta)$ remains
Figure 2.4. A planar OPD in the near field (left) gives maximum on-axis irradiance in the far field (right).

Figure 2.5. An aberrated OPD in the near field (left) has significantly reduced on-axis irradiance in the far field (right). The wavefront shown was measured using the Notre Dame Compressible Shear Layer Facility.
constant across the aperture. This is accomplished by assuming that the aperture is circular and very far away from the far field image plane. The aperture may then effectively be treated as a point source with planar waves in the far field. This allows the $A(\xi, \eta)$ terms to move outside of the integrals,

$$S = \frac{I(f_x, f_y)|_{f_x=f_y=0}}{I_{OPD=0}(f_x, f_y)} = \left| \int_{-\infty}^{\infty} e^{i\frac{2\pi}{\lambda}OPD(\xi, \eta)} d\xi d\eta \right|^2,$$

or

$$S = \left| \frac{1}{A} \int_{-\infty}^{\infty} e^{i\frac{2\pi}{\lambda}OPD(\xi, \eta)} d\xi d\eta \right|^2,$$

(2.22)

where $A$ is the aperture area. Figure 2.6 shows slices taken from the irradiance patterns of the data in Figures 2.4 and 2.5. Here, the on-axis irradiance is located at zero. Note that the irradiance values have been normalized by the theoretical maximum found by propagating a planar wavefront. Therefore, the Strehl ratio has a maximum value of unity when the wavefront is planar and is less than unity when the wavefront is aberrated.

A two-term Taylor expansion is used to approximate the exponential term in Equation 2.23. The expansion is performed about $\phi = 0$ which, from Equation 2.19, implies that the $OPD$ must be very small relative to the wavelength $\lambda$. Ross [52] shows that by using the first and second moments of $OPD$ and the rms wavefront error, the Strehl ratio can be approximated as

$$S(\sigma) = 1 - 4\pi^2 \sigma^2$$

(2.24)
where $\sigma$ is the rms wavefront error given by

$$
\sigma^2 = \frac{1}{A} \int_A (\phi(\xi, \eta) - \overline{\phi(\xi, \eta)})^2 d\xi d\eta,
$$

$$
= \frac{2\pi}{\lambda A} \int_A (OPD(\xi, \eta) - \overline{OPD(\xi, \eta)})^2 d\xi d\eta.
$$

Here, the bar indicates the mean and $A$ is the aperture area. Equation 2.24 is known as the Marèchal approximation to the Strehl ratio. A more complete derivation of the Marèchal approximation is available in Ross 2009 [52]. From a statistical approach, one can arrive at a more general equation for the Strehl ratio. This approach assumes a probability density function for the phase error and applies it to Equation 2.22. Assuming a Gaussian noise distribution for the

Figure 2.6. Far field on-axis irradiance profiles shown for planar wavefronts and aberrated wavefronts.
phase error [39, 52], the exponential form of the Strehl ratio is
\[
\overline{S(\sigma)} = e^{-\sigma^2} = e^{-\left(\frac{2\pi}{\lambda} \text{OPD}_{\text{rms}}\right)^2}
\] (2.26)

A more thorough discussion of the Marèchal approximation is available in Porter 2013 [50]. For cases when the phase error does not have a Gaussian distribution, he presents an alternative approximation using basis functions.

2.3.2 Effect of Wavelength

The magnitude of an induced phase shift is proportional to the ratio of the \( \text{OPD} \) to the wavelength. So for a given \( \text{OPD} \), the phase distortion is more severe for shorter wavelengths. For example, a 0.5 \( \mu \text{m} \) \( \text{OPD} \) produces 0.3 radian phase shift in a 10.6 \( \mu \text{m} \) laser and a 2.4 radian phase shift in a 1.315 \( \mu \text{m} \) laser. Figure 2.7 shows the effect of the wavelength dependency on the Strehl ratio. As wavelength increases, the Strehl ratio grows rapidly, starting at about 1 \( \mu \text{m} \). In the visible light region, the Strehl ratio is very close to zero. From the mid-IR to far-IR region, the Strehl ratio asymptotically approaches unity.

Equation 2.20 shows that due to the wave nature of light, diffraction effects will always appear in the far field. Every aperture must be of finite size, and because of the Fourier transform term, the far field will never be identical to the near field. The black, dashed line in Figure 2.6 shows the far field irradiance for an ideal near field wavefront with uniform amplitude and zero \( \text{OPD} \). The irradiance pattern is known as an Airy pattern or diffraction limited spot, and is described by

\[
I(r) = \frac{4J_1(r)^2}{r^2}
\] (2.27)
where $r$ is the radius from the center of the aperture and $J_1$ is the first order Bessel function. The central lobe encompasses 84% of the total irradiant energy and is referred to the Airy disk. Its maximum on-axis value is

$$I_0 = \frac{P}{\pi z^2 \lambda^2} \sim \frac{P d^2}{z^2 \lambda^2} \quad (2.28)$$

where $P$ is the laser output power and $d$ is the aperture diameter [36]. Figure 2.8 shows the diffraction limited irradiance for an Airy disk normalized by the diffraction limited irradiance at 1 $\mu$m. The irradiance quickly falls as $1/\lambda^2$. Consequently, there is a diminished on-axis irradiance delivery for longer wavelengths.

2.4 Aero-Optics Problem

In the previous sections, it was shown that as a beam of light propagates into the far field, irradiant energy is dispersed across the beam’s aperture. This is directly due to the wave nature of light, and the phenomenon of diffraction in particular. Any aberration of the light’s phase in the near field results in further reduction in on-axis energy. This loss of irradiance delivered to the target introduces problems in a wide range of applications such as astronomy, remote sensing, laser microscopy, laser surgery, and aero-optics. The latter of these will be discussed in detail.

Aero-optics is a field of study primarily involving the transfer of laser energy from a moving aircraft to a target. Some modern aero-optic applications include airborne laser communication and directed energy weapons. In either case, a laser beam exits an aircraft and passes through the unsteady, turbulent air in the nearby regions of the aircraft. These fluctuations are transient and can be on the order of many kilohertz.
Figure 2.7. Marèchal approximation shows that the Strehl ratio increases with wavelength.

Figure 2.8. Diffraction theory shows that shorter wavelengths deliver higher peak irradiance.
The index of refraction of air is related to its density by the Gladstone-Dale relationship,

\[ N = 1 + K_{GD} \rho \]  \hspace{1cm} (2.29)

where \( N \) is the refractive index, \( \rho \) is the air density, and \( K_{GD} \) is the Gladstone-Dale constant is defined by the Cauchy’s dispersion formula \([12, 36]\) as

\[ K_{GD} = 0.223 \times 10^{-3} \left( 1 + \frac{7.52 \times 10^{-15}}{\lambda^2} \right) \left[ \frac{m^3}{kg} \right]. \]  \hspace{1cm} (2.30)

Here, \( \lambda \) is the wavelength of the electromagnetic wave passing through the fluid medium.

It is evident from Equation 2.29 that regions of air with spatial gradients of density will have corresponding spatial gradients of index of refraction. A beam propagating through such a medium will become aberrated and the on-axis irradiant energy in the far field will be reduced. This is a major concern for aero-optics where transient density gradients exist near the aircraft as it moves through the air. A laser wavefront leaving the aircraft passing through these density gradients will suffer a significant loss in the on-axis energy in the far field. In the case of laser communication, this loss of energy can inhibits data transfer. In directed energy weaponry, this decreases the effectiveness of the weapon and might compromise a targets destruction.

From 1972 until 1983, the Air Force Weapons Laboratory developed the Airborne Laser Lab (ALL), a Boeing KC-135A Stratotanker with a high-energy 10.6 \( \mu \)m chemical laser capable of destroying targets while in flight \([34]\). The data in Figure 2.7 shows that at this relatively long wavelength the aero-optical effects are small. However, Figure 2.8 shows that the maximum on-axis intensity is very low.
in comparison to shorter wavelengths. More recent experiments were conducted under the Airborne Laser (ABL) and Advanced Tactical Laser (ATL) programs. These programs utilized a Boeing 747-400F and a C-130 Hercules respectively, which were outfitted chemical oxygen iodine lasers (COIL) of wavelength 1.315 \( \mu \text{m} \). The shorter wavelength allows for much higher on-axis energy delivery but is more susceptible to aero-optical effects.

2.5 Adaptive Optics

The term *adaptive optics* refers to optical systems which can actively correct for aberrations and maximize irradiance delivery to the target. A typical adaptive optics system is shown in Figure 2.9. The closed-loop controlled system works in the following fashion: (1) a reference beam from a reflection or spark at the target is aberrated through the atmosphere and local aero-optic disturbances and arrives at the adaptive optics system; (2) the aberrated reference beam reflects off an adaptive optic device and the wavefront is measured with the wavefront sensor; (3) based upon the \( \text{OPD} \) (error) of the reference beam wavefront, the computer prescribes a new correction for the adaptive optic device; (4) an outgoing beam is pre-aberrated as it reflects off the adaptive optic device; (5) the aberrated outgoing beam is restored to planar wavefronts after passing through the aero-optic and atmospheric disturbances; (6) maximum irradiance is delivered to the target.

To quantify aberrations, a wavefront can be decomposed into different modes using the Zernike polynomials. Figure 2.10 shows some common low order modes including piston, tip/tilt, defocus, astigmatism, coma, and spherical aberrations. Simple aberrations may be easily described using these polynomials but more
complex aero-optics wavefronts require too many modes for this decomposition to be efficient. Instead, wavefronts are reconstructed from wavefront slope measurements using a least-squares fitting.

There are a number of different devices that may be used to perform corrections in an adaptive optics system. The most common is the deformable mirror (DM), a reflective surface which can spatially distort itself to induce a spatially variant OPD onto incoming light. DM designs use many different methods of control technology including: discrete segmented mirrors, deformable piezoelectric driven ceramic sheets, deformable electrostatic driven membranes, discrete piezoelectric actuated membranes, edge actuating mirrors [60], and deformable magnetic driven membranes [23]. Due to inherent inertial limitations, there is a trade-off in DMs between frequency response and spatial resolution. For low order corrections, fast steering mirrors be used for tip/tilt corrections at rates of many kilohertz [58],
and fast steering micro-mirrors up to 68 kilohertz [46]. DMs for higher order corrections typically operate in the hundreds of hertz to low kilohertz [26, 60], and micro-DMs operate up to 42 kHz [33].

The speed of the wavefront corrections is important in aero-optics. Cicchiello and Jumper showed that the deformable mirror should perform corrections at least ten times per cycle for a Strehl ratio of 0.8 [17, 47]. From hot-wire measurements, Gordeyev and Cress showed that for a Mach 0.4 flow, the structures behind a laser turret fluctuate in the low kilohertz range [30]. Therefore, for real-time wavefront corrections, the DM would need to operate at a minimum frequency in the 10s of kilohertz. Therefore, there is a clear need for wavefront correction technology capable of higher frequency response.
CHAPTER 3
OPTICS OF PLASMA

As an electromagnetic wave passes through plasma, its propagation velocity changes because the plasma has a different index of refraction than the surrounding gas. This index of refraction is dependent on the electron density of the plasma. Consequently, the phase of a laser passing through plasma will be altered depending on the electron density of the plasma. For adaptive optic purposes, if the electron density of the plasma can be controlled precisely and quickly then it will provide means for high speed optical phase corrections and wavefront phase control.

The sections of this chapter derive important relations to describe the optical properties of a plasma. The derivations are all based on first principles and provide meaningful insight into the interaction between plasma and light.

3.1 Plasma Frequency

The plasma frequency is the frequency with which a cloud of free electrons naturally oscillates at. This quantity will be important later in determining a plasma’s index of refraction. This derivation of the plasma frequency expression follows one presented by Feynman in his Lectures on Physics [24].

Consider an electrically neutral plasma where the number of electrons, \( n_e \), is equal to the number of ions, \( n_i \). The charge of the ions perfectly balance out the
charge of the electrons, \( n_i = n_e \). If a small perturbation is introduced, the charged particles will move away from their equilibrium positions. With many orders of magnitude difference in mass, the movement of the ions may be neglected in comparison with the motion of the much lighter electrons. Therefore, the ion number density unchanged but the electron number density becomes

\[
n_e^* = n_e \left( 1 - \frac{ds}{dx} \right)
\]  

(3.1)

where \( x \) is the direction of the perturbation and \( s \) is the distance which the electrons were perturbed. The charge density within the plasma is the sum of the charges from electrons and ions,

\[
\rho = -qn_e \left( 1 - \frac{ds}{dx} \right) + qn_e = qn_e \frac{ds}{dx}
\]  

(3.2)

where \( q \) is the electron charge. Being non-zero, the charge density creates an electric field described with Maxwell’s equation

\[
\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}
\]  

(3.3)

where \( \epsilon_0 \) is the vacuum permittivity constant. With Equation 3.2 this becomes

\[
\frac{dE_x}{dx} = \frac{qn_e}{\epsilon_0} \cdot \frac{ds}{dx}.
\]  

(3.4)

The induced electric field is then

\[
E_x = \frac{n_e q}{\epsilon_0} s
\]  

(3.5)
and the force acting on a single electron displaced by a distance $s$ is then

$$F_x = -qE_x = -\frac{n_e q^2}{\epsilon_0}s. \quad (3.6)$$

From Newton’s second law, the equation of motion for the displaced electron is

$$m\frac{d^2s}{dt^2} = -\frac{n_e q^2}{\epsilon_0}s \quad (3.7)$$

where $m$ is the electron mass. The solution to the differential equation shows that the electron harmonically oscillates in time

$$s(t) = C_1 \cos \left(\sqrt{\frac{n_e q^2}{m \epsilon_0}}t\right) + C_2 \sin \left(\sqrt{\frac{n_e q^2}{m \epsilon_0}}t\right). \quad (3.8)$$

with the frequency

$$\omega_p = \sqrt{\frac{n_e q^2}{m \epsilon_0}}. \quad (3.9)$$

This frequency is defined as the plasma frequency and is entirely dependent on the electron density of the plasma.

### 3.2 Refractive Index

To form an expression for the index of refraction of plasma, the contributions from the free electrons and the surrounding neutral gas are each considered individually. Later, these expressions will be combined for a complete expression describing the index of refraction of plasma.
3.2.1 Wave Equation

Maxwell’s equations govern the electrical and optical properties of the plasma

\[ \nabla \cdot \mathbf{E} = -\frac{1}{\epsilon_0} \nabla \cdot \mathbf{P} \quad \text{(Gauss’s law)} \quad (3.10) \]

\[ \nabla \cdot \mathbf{H} = 0 \quad \text{(Gauss’s law for magnetism)} \quad (3.11) \]

\[ \nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t} \quad \text{(Maxwell-Faraday equation)} \quad (3.12) \]

\[ \nabla \times \mathbf{H} = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \frac{\partial \mathbf{P}}{\partial t} + \mathbf{J} \quad \text{(Ampere’s circuital law).} \quad (3.13) \]

Here \( \mathbf{E} \) is the electric field vector, \( \epsilon_0 \) is the vacuum permittivity constant, \( \mathbf{P} \) is the polarization vector for electric dipoles, \( \mathbf{H} \) is the magnetic field vector, \( \mu_0 \) is the free space permeability constant, and \( \mathbf{J} \) is the current density vector. Note that it has been assumed that the magnetization and volume density of electric charge are identically zero.

Taking the curl of the Maxwell-Faraday equation (Equation 3.12) gives

\[ \nabla \times \nabla \times \mathbf{E} = -\mu_0 \nabla \times \frac{\partial \mathbf{H}}{\partial t} \quad (3.14) \]

The time-derivative of Ampere’s circuital law (Equation 3.13) gives

\[ \nabla \times \frac{\partial \mathbf{H}}{\partial t} = \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} + \frac{\partial^2 \mathbf{P}}{\partial t^2} + \frac{\partial \mathbf{J}}{\partial t}. \quad (3.15) \]
Now, both equations are combined to eliminate the magnetic field \( \mathbf{H} \)

\[
\nabla \times \nabla \times \mathbf{E} = -\mu_0 \left( \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} + \frac{\partial^2 \mathbf{P}}{\partial t^2} + \frac{\partial \mathbf{J}}{\partial t} \right).
\]

(3.16)

Substituting in \( \mu_0 \varepsilon_0 = 1/c^2 \) this becomes

\[
\nabla \times \nabla \times \mathbf{E} + \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = -\mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2} - \mu_0 \frac{\partial \mathbf{J}}{\partial t}.
\]

(3.17)

Equation (3.17) is the general equation for an electromagnetic wave within a polarizable and conducting medium. It will be used in the following sections to derive the refractive index of the free electrons and of the neutral gas inside a plasma.

When determining the electron contribution to the index of refraction, all electrons are considered to be free charge carriers. This implies a finite current density \( \mathbf{J} \neq 0 \) and no polarization \( \mathbf{P} = 0 \). Conversely, for the neutral gas contribution the electrons are all considered to be bound to nuclei. This implies zero current density \( \mathbf{J} = 0 \) and a finite polarization \( \mathbf{P} \neq 0 \).

The understanding of the electron and heavy particle contributions to the refractive index are critical to the goals of this research. For this reason the complete derivations for both are presented in this section.

3.2.2 Electron Contribution

The electron contribution to the refractive index is determined by considering the plasma as a purely conducting medium. The formulation presented follows derivations given by Folwes and Duschin and Pawlitschenko [21, 25]. In this case, there is no polarization \( \mathbf{P} = 0 \) but the free electrons in the plasma allow current to flow \( \mathbf{J} \neq 0 \).
Consider an electron placed inside an external electric field $E$. The field induces a force on the electron with a damping force,

$$m \frac{dv}{dt} + m \nu v = -qE$$  \hspace{1cm} (3.18)

where $v$ is the electron velocity and $\nu$ is the electron collision frequency. The current density $J = -n_e q v$ is substituted in for the velocity to give

$$\frac{dJ}{dt} + \nu J = \frac{n_e q^2}{m} E.$$  \hspace{1cm} (3.19)

The electric field and current density are assumed to vary harmonically in time such that $J \rightarrow J e^{-i\omega t}$ and $E \rightarrow E e^{-i\omega t}$. This gives

$$\frac{d}{dt} (J e^{-i\omega t}) + \nu J e^{-i\omega t} = \frac{n_e q^2}{m} E e^{-i\omega t}.$$  \hspace{1cm} (3.20)

which simplifies to become

$$J (-i\omega + \nu) = \frac{n_e q^2}{m} E.$$  \hspace{1cm} (3.21)

Rearranging terms gives an expression for the current density as a function of the electric field,

$$J = \frac{n_e q^2}{m \nu} \cdot \frac{1}{1 - i\frac{i\omega}{\nu}} E.$$  \hspace{1cm} (3.22)

It has already been stated that there is no polarization. From Equation (3.10) this implies that $\nabla \times \nabla \times E = \nabla^2 E$. The general wave equation in (3.17) then becomes

$$\nabla^2 E = \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} + \mu_0 \frac{\partial J}{\partial t}.$$  \hspace{1cm} (3.23)
and substituting in Equation 3.22 gives

\[ \nabla^2 E = \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} + \mu_0 \frac{n_e q^2}{m \nu} \cdot \frac{1}{1 - i \frac{\omega}{\nu}} \cdot \frac{\partial E}{\partial t}. \] (3.24)

A solution to this differential equation is a wave that is harmonic in direction \( z \) and time \( t \),

\[ E = E_0 e^{i(kz - \omega t)} \] (3.25)

where \( k \) is the wavenumber. Substituting Equation 3.25 into 3.24 reveals that \( k \) must satisfy the relation,

\[ k^2 = \frac{\omega^2}{c^2} + \mu_0 \frac{n_e q^2}{m \nu} \cdot \frac{i \omega}{1 - i \frac{\omega}{\nu}}. \] (3.26)

Using the plasma frequency of Equation 3.9 and \( \mu_0 \epsilon_0 = 1/c^2 \) this may be rearranged as

\[ k^2 = \frac{\omega^2}{c^2} - \frac{\omega_p^2}{c^2 (1 + i \frac{\nu}{\omega})}. \] (3.27)

Note that the wavenumber can have imaginary components, \( k = k_r + i k_i \). The electromagnetic wave is then

\[ E = E_0 e^{i(kz - \omega t)} = E_0 e^{i k_r z} e^{-k_i z} e^{-i \omega t}. \] (3.28)

The real part of Equation 3.28 produces a spatial oscillation in the wave and the imaginary part produces a spatial decay. This implies energy from the electromagnetic wave will be absorbed into the surrounding plasma, which can be quantified using the optical thickness \( \tau \),

\[ \frac{I}{I_0} = e^\tau \] (3.29)
where \( I = |\mathbf{E}|^2 \) is the irradiance of a wave inside plasma and \( I_0 \) is the initial irradiance. An optically thin or lossless plasma is one with negligible absorption so that \( \frac{I}{I_0} = 1 \) and \( \tau = 0 \). A plasma is considered optically thin if the wavenumber has no imaginary components. This occurs when the wave frequency is much greater than the electron collision frequency, \( \omega \gg \nu \). With this condition met, the wavenumber becomes

\[
k^2 = \frac{\omega^2}{c^2} - \frac{\omega_p^2}{c^2}.
\]  

(3.30)

The validity of this assumption is dependent upon the pressure of the gas. At low pressure the mean free path of particles is very long and the collision frequency may be considered low. However, at higher pressures the density of particles and the collision frequency increases and \( \omega \gg \nu \) may not still be valid. Raizer gives an estimation for the effective collision frequency as a function of pressure for air as

\[
\nu = (3.9 \times 10^9 \text{ s}^{-1}\text{Torr}^{-1}) p
\]

(3.31)

where \( p \) is the pressure of the background gas of the plasma in Torr \([51]\). Figure 3.1 shows the region where a plasma may be considered optically thin based on pressure and wavelength assuming that \( \omega \gg \nu \) is valid when \( \omega \) is greater than \( \nu \) by an order of magnitude (\( \omega > 10\nu \)).

The wavenumber in Equation 3.30 is still complex if \( \omega < \omega_p \) and, thus, the plasma will attenuate any electromagnetic frequencies below the plasma frequency. As \( \omega \) approaches \( \omega_p \), the wavenumber asymptotically approaches zero; the plasma is considered transparent when \( \omega \gg \omega_p \). Figure 3.2 shows the critical electron density as a function of wavelength with the shaded region indicating where the
plasma can be considered transparent. For a wave to propagate through the plasma, its wavelength must be less than the critical wavelength,

$$\lambda_{\text{crit}} < \frac{2\pi c}{\sqrt{n}_e q^2 \sqrt{m \epsilon_0}}.$$  \hfill (3.32)

The expression can be equivalently written in terms of the critical electron density,

$$n_{e,\text{crit}} < \frac{4\pi^2 \epsilon_0 c^2 m}{q^2 \lambda^2}.$$  \hfill (3.33)

which is the maximum electron density that a wave of wavelength $\lambda$ can pass through without being attenuated.

From Equation (3.30), the index of refraction can be expressed as

$$N^2 = \frac{c^2}{\omega^2} k^2 = 1 - \frac{\omega_p^2}{\omega^2}. \hfill (3.34)$$

This equation represents the contribution of the free electrons within a plasma to the overall refractive index change. It has already been assumed that $\omega \gg \omega_p$. Therefore, a Taylor expansion is used for $\sqrt{1 - \left(\frac{\omega_p}{\omega}\right)^2}$ for $\frac{\omega_p}{\omega}$ about zero to give

$$N = 1 - \frac{\omega_p^2}{2\omega^2}. \hfill (3.35)$$

Substituting in the expressions for $\omega$ and $\omega_p$, it is evident that the refractive index is then linearly related to the electron density

$$N = 1 - \frac{q^2 \lambda^2 n_e}{8\pi^2 \epsilon_0 c^2 m}. \hfill (3.36)$$

Equations (3.35) and (3.36) represent the free electron contribution to the refractive index assuming that the plasma is lossless and transparent. Equation (3.36)
Figure 3.1. Pressure and wavelength relationship showing where plasma may be considered optically thin or lossless ($\omega \gg \nu$).

Figure 3.2. Electron density and wavelength relationship showing where plasma may be considered transparent ($\omega \gg \omega_p$).
indicates that free electrons in a perfect vacuum will produce an index of refraction less than unity. To completely model the plasma index of refraction, the effect of the background gas must also be considered.

### 3.2.3 Neutral Gas Contribution

The neutral gas contribution to the refractive index is determined by considering the plasma as a dielectric medium. The formulation presented follows derivations given by Folwes [25] and Duschin and Pawlitschenko [21]. In this case, the neutral gas is polarizable ($\mathbf{P} \neq 0$) and there are no free electrons to conduct charge ($\mathbf{J} = 0$).

An electron bound to a nucleus is modeled as a damped spring oscillator. When placed inside an electric field, this system is described with the differential equation,

$$ m \frac{d^2 \mathbf{r}}{dt^2} = -m \gamma \frac{d \mathbf{r}}{dt} - K \mathbf{r} - q \mathbf{E}. \tag{3.37} $$

Here, $\mathbf{r}$ is the electron displacement vector, $\gamma$ is a frictional damping constant, and $K$ is a restoring force constant. It is assumed that the electron displacement and electric field vary harmonically in time such that $\mathbf{r} \rightarrow r e^{-i\omega t}$ and $\mathbf{E} \rightarrow E e^{-i\omega t}$,

$$ m \frac{d^2}{dt^2} (re^{-i\omega t}) = -m \gamma \frac{d}{dt} (re^{-i\omega t}) - K re^{-i\omega t} - qE e^{-i\omega t}. \tag{3.38} $$

A solution for the electron displacement is found to be

$$ \mathbf{r} = \frac{-q \mathbf{E}}{-m\omega^2 - i\omega m\gamma + K}. \tag{3.39} $$

The polarization of an isotropic medium due to a displaced electron can be de-
scribed as

\[ P = -n_b q \mathbf{r}. \]  
(3.40)

where \( n_b \) is the number density of bound electrons. Combination of the latter two expressions gives an equation for the polarization of a bound electron as a function of the electric field,

\[ P = \frac{n_b q^2}{-m\omega^2 - i\omega m\gamma + K} \mathbf{E}. \]  
(3.41)

This is substituted into the general wave equation of Equation 3.17 to give

\[ \nabla (\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} + \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = -\mu_0 n_b q^2 \left( \frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega} \right) \frac{\partial^2 \mathbf{E}}{\partial t^2} \]  
(3.42)

where \( \omega_0 \) is oscillating, bound electron’s effective resonant frequency given by

\[ \omega_0 = \sqrt{\frac{K}{m}}. \]  
(3.43)

It can be shown that \( \nabla \cdot \mathbf{E} = 0 \) must be true for Equations 3.41 and 3.10 to be mutually satisfied. Substituting \( \mu_0 \epsilon_0 = 1/c^2 \), the wave equation becomes

\[ \nabla^2 \mathbf{E} = \frac{1}{c^2} \left( 1 + \frac{n_b q^2}{m \epsilon_0} \cdot \frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega} \right) \frac{\partial^2 \mathbf{E}}{\partial t^2}. \]  
(3.44)

A solution to this differential equation is a wave that is harmonic in direction \( z \) and time \( t \),

\[ \mathbf{E} = E_0 e^{i(kz - \omega t)}. \]  
(3.45)

Substitution of Equation 3.45 into 3.44 reveals that the wavenumber \( k \) must satisfy
the condition
\[ k^2 = \frac{\omega^2}{c^2} \left( 1 + \frac{n_b q^2}{m \epsilon_0} \cdot \frac{1}{\omega_0^2 - \omega^2 - i \gamma \omega} \right). \]  
(3.46)

This is rewritten in terms of refractive index using the expression \( N^2 = \frac{\omega^2}{c^2} k^2 \).

\[ N^2 = 1 + \frac{n_b q^2}{m \epsilon_0} \cdot \frac{1}{\omega_0^2 - \omega^2 - i \gamma \omega}. \]  
(3.47)

Equation (3.47) describes the neutral gas contribution to the refractive index for specific electrons bound with restoring force \( K \) and damping \( \gamma \). In reality, there are many electrons that may be bound with different bond strengths. To account for this, the refractive index equation is traditionally written as

\[ N^2 = 1 + \frac{n_b q^2}{m \epsilon_0} \sum_j \frac{f_j}{\omega_{0,j}^2 - \omega^2 - i \gamma_j \omega} \]  
(3.48)

where \( f_j \) is the population fraction for a specific electron bonding (\( K \) and \( \gamma \)).

The drawback to this approach is the neutral gas contribution is a non-trivial calculation requiring extensive knowledge of the electron bonding in the neutral gas particles. Duschin and Pawlitschenko propose using the Cauchy dispersion equation for modeling the neutral gas influence [21] which is derived from a Taylor expansion [12] of Equation (3.48). The refractive index then becomes

\[ N = A + \frac{B}{\lambda^2}. \]  
(3.49)

The terms \( A \) and \( B \) are empirically determined constants particular for a single gas at standard temperature and pressure. Ideally, one would use separate \( A \) and \( B \) values for ions and neutrals respectively but it is acceptable to use only the values from neutrals [10].
The preceding full derivation for heavy particles was presented for completeness and insight into the physical mechanism of the heavy particle influence. All further discussion on the plasma refractive index will use the expression of Equation 3.49.

3.2.4 Total Index of Refraction

The total refractive index change due to plasma is found from combining the free electron contribution in Equation 3.35 with the neutral gas (bound electron) contribution in Equation 3.49 to produce

\[ N = 1 - \frac{\omega_p^2}{2\omega^2} + A + B \frac{\lambda}{\lambda^2} \]

(3.50)

Note that values for \( A \) and \( B \) correspond to a specific gas at standard temperature and pressure. At low pressures the refractive index is linearly related to pressure and density [21] and the heavy particle term can be rewritten in terms of the density ratio \( \frac{n_h}{n_{h0}} \).

\[ N = 1 - \frac{\omega_p^2}{2\omega^2} + \left( A + B \frac{\lambda}{\lambda^2} \right) \frac{n_h}{n_{h0}} \]

(3.51)

Here, \( n_h \) is the number density of the surrounding gas at pressure and \( n_{h0} \) is the number density of the surrounding gas at standard temperature and pressure. The heavy particle term is now mathematically equivalent to the Gladstone-Dale relationship from Equation 2.29. The complete description for the index of refraction of a plasma is,

\[ N = 1 - \frac{q^2 \lambda^2 n_e}{8\pi^2 \epsilon_0 c^2 m} + \left( A + B \frac{\lambda}{\lambda^2} \right) \frac{n_h}{n_{h0}} \]

(3.52)
3.3 Plasma Optical Path Difference

Using the definition of optical path length in Equation 2.2, the $OPL$ through a region of plasma is

$$OPL = \int_0^L \left[ 1 - \frac{q^2 \lambda^2 n_e(s)}{8\pi^2 \epsilon_0 c^2 m} + \left( A + \frac{B}{\lambda^2} \right) \frac{n_h(s)}{n_{h0}} \right] ds$$  \hspace{1cm} (3.53)

where $L$ is the length of plasma. Assuming that the plasma is spatially uniform across $L$, this becomes

$$OPL = L \left( 1 - \frac{q^2 \lambda^2 n_e}{8\pi^2 \epsilon_0 c^2 m} + \left( A + \frac{B}{\lambda^2} \right) \frac{n_h}{n_{h0}} \right).$$  \hspace{1cm} (3.54)

The plasma induced $OPD$ is then

$$OPD_p = OPL_{on} - OPL_{off} = L \left( \frac{-q^2 \lambda^2 \Delta n_e}{8\pi^2 \epsilon_0 c^2 m} + \left( A + \frac{B}{\lambda^2} \right) \frac{\Delta n_h}{n_{h0}} \right)$$  \hspace{1cm} (3.55)

where $\Delta n_e$ and $\Delta n_h$ are the respective changes in electron density and heavy particle density between the on and off states. When plasma forms, the heavy particle density $n_h$ decreases due to gas heating, making $\Delta n_h$ a negative quantity.

Considering the plasma as an ideal gas, the heavy particle density is

$$n_h = \frac{P N_A}{RT}$$  \hspace{1cm} (3.56)

where $N_A$ is the Avogadro constant ($6.022 \times 10^{23}$ mol$^{-1}$) and $P$, $R$, and $T$ are respectively the pressure, gas constant, and temperature of the background gas.

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The change in heavy particle density after plasma formation is then

\[ \Delta n_h = - \left( \frac{N_A P}{R} \right) \left( \frac{T_{on} - T_{off}}{T_{on} T_{off}} \right). \]  

(3.57)

where \( T_{on} \) and \( T_{off} \) are the temperatures of the background gas with plasma on and off respectively. The formation of Equation 3.57 requires the assumption that the generation of plasma is an isobaric process. This asserts that changes in heavy particle density are solely due to changes in the gas temperatures. Substituting this into Equation 3.55 gives

\[ OPD_P = L \left( \frac{-q^2 \lambda^2 \Delta n_e}{8\pi^2 \epsilon_0 c^2 m} \right) + L \left( \frac{A}{n_{h0}} + \frac{B}{n_{h0} \lambda^2} \right) \left( \frac{N_A P}{R} \right) \left( \frac{T_{on} - T_{off}}{T_{on} T_{off}} \right). \]  

(3.58)

3.4 Effects of Wavelength, Electron Density, and Temperature Change

The plasma induced \( OPD \) can be decomposed into its electron contribution and heavy particle contribution as

\[ OPD_e = \left( \frac{-q^2 \lambda^2 \Delta n_e}{8\pi^2 \epsilon_0 c^2 m} \right) \]  

(3.59)

and

\[ OPD_h = L \left( \frac{A}{n_{h0}} + \frac{B}{n_{h0} \lambda^2} \right) \left( \frac{N_A P}{R} \right) \left( \frac{T_{on} - T_{off}}{T_{on} T_{off}} \right). \]  

(3.60)

For plasma adaptive optics, the electron contribution is of particular importance because it responds much faster than the heavy particle contribution \[43\]. It is favorable to have very high electron densities and minimal heavy particle contribution in order to maximize the frequency response of the plasma adaptive
optic device. In other words, it is desirable to maximize the ratio

\[
\frac{OPD_e}{OPD_h} = \left( \frac{-q^2 \Delta n_e}{8\pi^2 \epsilon_0 c^2 m} \right) \left( \frac{\lambda^4 n_{h0}}{A \lambda^2 + B} \right) \left( \frac{R}{N_A P} \right) \left( \frac{T_{on} T_{off}}{T_{on} - T_{off}} \right).
\] (3.61)

Equation 3.61 provides a great deal of information about the factors influencing the plasma induced \( OPD \). An electron-dominated \( OPD \) \( \frac{OPD_e}{OPD_h} \gg 1 \) may be achieved by:

1. Selecting a gas with low \( A \) and \( B \) values;
2. Probing with a longer wavelength;
3. Reducing the pressure of the background gas;
4. Generating a high plasma electron density;
5. Minimizing the temperature increase when plasma forms.

Option (1) is the easiest to accomplish by using a low-dispersion gas such as helium, which features \( A \) and \( B \) values about an order of magnitude lower than air. Option (2) may be accomplished in the laboratory by using an infrared or microwave measurement system. For the intended aero-optic application for plasma adaptive optics, however, longer wavelengths are highly unfavorable due to their lower peak irradiance (Figure 2.8).

The remaining options (3) through (5) are strongly coupled and cannot be changed independently of one another. For example, reducing the background pressure will directly increase \( \frac{OPD_e}{OPD_h} \) and reduce the gas heating during plasma formation, but it will also significantly reduce the plasma electron density. Furthermore, applying higher power to sustain the plasma will generate a higher electron density but will significantly increase the gas temperature. This coupling
of terms poses a challenge in the design and applications for plasma adaptive optics.

Another useful metric is the ratio of the electron $OPD_e$ to the total $OPD$. When equal to one, it indicates the $OPD$ is generated purely by free electrons, and at zero, it indicates the $OPD$ is due to only to heavy particles. Figures 3.3, 3.4, and 3.5 show plots of $\frac{OPD_e}{OPD}$ as functions of wavelength, gas pressure, and gas temperature increase for both air and helium plasmas. Data are shown for an electron density of $10^{14}$ cm$^{-3}$.

In Figure 3.3 it is evident that the $OPD$ measured by visible and near-infrared wavelengths would be dominated by heavy particles for air and for helium. Notably, it is clear that helium is a significantly better gas to use because it can achieve equivalent $OPD_e$ values at much shorter wavelengths. Air would require wavelengths greater than about 10 microns for the $OPD$ to become electron dominated.

Figures 3.3 and 3.5 show that using a low-pressure, cold plasma would be ideal for plasma adaptive optics. As the pressure and temperature of the background gas increases, the $OPD_e$ becomes less significant. The experiments discussed here use gas pressures between 20 to 40 Torr. In this range, the helium produces a much higher $OPD_e$ than air. If the background gas is higher than 100 Torr or if the temperature increases by more than 100 K, both gases produce a heavy particle dominated $OPD$ at a wavelength of $\lambda = 1\mu$m.

3.5 Plasma Diagnostics using Interferometry

Plasma diagnostics is an expansive field spanning many decades of research, numerous disciplines of study, and involving an assortment of different experimen-
Figure 3.3. Fraction of electron $OPD$ as function of wavelength for air (solid) and helium (dashed).

Figure 3.4. Fraction of electron $OPD$ as function of background gas pressure for air (solid) and helium (dashed).
Figure 3.5. Fraction of electron $OPD$ as function of temperature increase by plasma for air (solid) and helium (dashed).

tal techniques. One common plasma diagnostic technique is laser interferometry for plasma electron density measurements, which is the focus of this present work. This section will present a historical background of interferometric plasma diagnostics, the setup of a Michelson interferometer, the theory of optical interference, and the theory for multiple-wavelength interferometry.

3.5.1 Historical Background

Plasma diagnostics was prominently introduced to the world in the late 19th century by the scientists Crookes, Townsend, Thomson, and Langmuir. In this period, early concepts of charged particles, plasma, and electron and ion collision processes were discovered. Further significant advancements in plasma diagnostics did not occur until after World War II with the advent of computer and laser
technology [35].

In 1953, Dolgov and Mandelstam made the first experimental observation of the effect of free electrons on an interferogram [35]. In 1958, Alpher and White measured interferograms for thermally ionized argon plasma and suggested that a multi-wavelength experiment could provide an exact determination of the electron density [2]. In 1959, they presented interferometric measurements for oxygen and nitrogen plasmas measured at three distinct visible wavelengths. They proposed that longer wavelengths would be better suited to increase electron sensitivity [3, 4]. Ascoli-Bartoli and Rasetti also commented on the dependence of wavelength in plasma diagnostic interferometry in 1959 and 1960 [5, 6].

In 1962, Dyson et al. used an optical Michelson interferometer to measure the electron density of a high temperature plasma, noting that the heavy particle contribution to the index of refraction was negligible due to a low background pressure of $2 \times 10^{-2}$ Torr. The electron density of the plasma was found to be on the order of $10^{15}$ cm$^{-3}$.

Up to this point in time, interferometric experiments had been conducted with high intensity discharge lamps with narrow-band filters to achieve coherence. In 1963, Ashby and Jephcott performed what appears to be the first laser interferometric plasma electron density measurement [7]. The experiment used a helium-neon laser which simultaneously emitted 633 nm and 3.39 µm wavelengths. Instead of using a traditional Michelson interferometer setup, they used the laser cavity itself as the interferometer. The beam exiting the laser passed through a six foot plasma chamber and reflected off a mirror directly back through the plasma chamber and into the laser cavity. This beam passed through the laser cavity and exited through other side onto a visible detector. Changes in the plasma index
of refraction produced interference at the detector. Using optical glass and germanium filters they were able to isolate the visible and infrared phase shifts to determine the electron density, which was found to be on the order of $10^{15}$ cm$^{-3}$.

In 1972, Woolsely and Illingsworth performed electron density measurements of a Z pinch argon plasma using a Jamin interferometer with a helium-neon laser. In this case, the researchers justified the use of the short optical wavelength because the background gas pressure was only $15 \times 10^{-3}$ Torr. They reported electron density values on the order of $10^{15}$ cm$^{-3}$.

In 1998, Schittenhelm et al. used two-wavelength interferometry to measure the plasma electron density of a laser induced plasma in air, helium, and argon at atmospheric pressure [54]. The experimental setup used a Michelson interferometer with wavelengths of 532 nm and 633 nm. They asserted that in the visible region, the refractive index of the heavy particles was independent of wavelength, thus simplifying the determination of the electron density. They measured electron densities on the order of $10^{19}$ cm$^{-3}$ and estimated gas temperatures of 22,000 K in air, 12,000 K in argon, and only 2,500 K in helium.

In 2000, Liepold et al. performed electron density measurements on a micro-hollow cathode discharge in air at atmospheric pressure using heterodyne interferometry [43]. They probed the plasma with single wavelength from a 10.6 µm CO$_2$ laser. At atmospheric pressure, the contribution from the electrons was estimated to be only 0.5% in comparison to the heavy particle contribution. Using a single wavelength laser prevented them from truly disambiguating the electron and heavy particle components. Instead, they used the time scales of the plasma induced phase shift to distinguish between the electron contribution and the slower heavy particle contribution. They reported electron densities for the plasma on
the order of $10^{13}$ cm$^{-3}$

In 2010, Urabe et al. used an experimental setup very similar to Leipold to measure the electron density of an atmospheric pressure glow discharge (APGD) [61, 62]. The background gas was pure helium at atmospheric pressure and the plasma was generated with a 30 kHz ac signal with a 3 kV peak voltage. They found electron densities on the order of $10^{12}$ cm$^{-3}$. As the name implies, the AGPD is specifically designed to operate in the glow regime and, thus, features much lower electron densities than normal dielectric barrier discharges which operate in the streamer discharge.

In 2008, Dong et al. measured the spectral line shapes of argon to determine the temperature and electron density of a dielectric barrier discharge in argon at sub-atmospheric pressures ranging from 75 to 450 Torr [20]. The plasma device featured cylindrical liquid (water) electrodes separated by a gap distance of 1.5 mm. The plasma was driven with a 50 kHz sinusoid. The voltage is unreported but in an earlier paper with a similar plasma setup, they reported voltages on the order of 2 kV [19]. The electron densities are reported to be on the order of $10^{14}$ to $10^{15}$ cm$^{-3}$ depending on the pressure. They also reported gas temperature values between about 420 K to 480 K depending on the pressure. Although this experiment did not use interferometry, the plasma conditions in this paper are the closest match found in literature to the work presented here.

3.5.2 Michelson Interferometer

In general, interferometric measurement systems use a coherent light source (typically a laser) which is split into two paths and then later recombined. Upon recombination, any changes in refractive index or physical length between the
two paths produce constructive and destructive interference due to the coherent nature of the light.

The Michelson interferometer was first developed by Albert A. Michelson in 1881 [45]. It is still among the most commonly used interferometers today due, in part, to its simplicity. A schematic of a Michelson interferometer is show in Figure 3.6. In this figure, light enters the interferometer from the left and passes through a beamsplitter which splits the light into two paths (arms), where 50% is reflected and 50% is transmitted. The reflected light passes through a compensator, reflects off a mirror, passes back through the compensator, and returns to the beamsplitter. The transmitted light passes through a sample, reflects off a mirror, passes back through the sample and returns to the beamsplitter. As both beams return to the beamsplitter, they split again and 50% of the energy returns to the source and 50% exits the interferometer. If the optical path difference between the two arms is shorter than the coherence length of the light, then interference is produced when the beams recombine. Measurement of the interfer-
ference pattern (fringes) allows for very precise determination of refractive index of the sample in the interferometer. For this work, the Michelson interferometer is used to determine the electron density of plasma by way of the index of refraction, shown in Equation 3.52, and discussed further in the next sections.

Note that the transmitted arm travels through the beamsplitter three times whereas the reflected arm travels through the beamsplitter only once. Traditionally, compensators were necessary to compensate for this difference due to very short coherence lengths. With the introduction of lasers, which can feature coherence lengths of meters or greater, the compensator is not always necessary.

3.5.3 Theory of Interference

In this section, a short derivation for the interference of two electromagnetic waves is presented. In general, the electric field of light radiation may be described with

\[ E(z, t) = A \exp \left[ i(kz - \omega t + \phi) \right], \quad (3.62) \]

where \( A \) is some arbitrary amplitude, \( k \) is the angular wavenumber, \( z \) is a propagation distance, \( \omega \) is the wave frequency, \( t \) is time, and \( \phi \) is the phase shift. Inside the interferometer (Figure 3.6), the electric fields of the light in each arm are

\[ E_1(z, t) = A_1 \exp \left[ i(kz_1 - \omega t + \phi_1) \right], \quad (3.63) \]

and

\[ E_2(z, t) = A_2 \exp \left[ i(kz_2 - \omega t + \phi_2) \right]. \quad (3.64) \]
If the amplitude of the beam entering the interferometer is $2A$ then in each arm the amplitude is $A_1 = A_1 = A$. The total electric field at the exit of the interferometer is the superposition of Equations 3.63 and 3.64

$$E(z,t) = E_1(z,t) + E_2(z,t). \quad (3.65)$$

and the irradiance is $[25]$,

$$I(z,t) = |E|^2 = (E_1 + E_2) \cdot (E_1^* + E_2^*) = |E_1|^2 + |E_2|^2 + 2A^2 \cos [k(z_2 + \phi_2 - kz_1 - \phi_1)] = 2A^2 (1 + \cos [k\Delta z + \Delta \phi]) \quad (3.66)$$

where $\Delta z = z_2 - z_1$ and $\Delta \phi = \phi_2 - \phi_1$. The initial irradiance of the beam is $I_0 = (2A)^2 = 4A^2$ making the irradiance at the exit of the interferometer

$$I(z,t) = \frac{I_0}{2} (1 + \cos [k\Delta z + \Delta \phi]). \quad (3.67)$$

Equation 3.67 is the fundamental equation for interference. The two beams are in total constructive interference when

$$|k\Delta z + \Delta \phi| = 2m\pi \quad \text{for } m = 0, 1, 2, 3, ... \quad (3.68)$$

and all the initial irradiant energy is output from the interferometer ($I = I_0$). Conversely, when

$$|k\Delta z + \Delta \phi| = (2m - 1)\pi \quad \text{for } m = 0, 1, 2, 3, ... \quad (3.69)$$
the two beams are in total destructive interference and no irradiant energy is output \((I = 0)\). During destructive interference, the irradiant energy is returned to the source.

Note that Equation 3.67 shows that the interference is a function of phase. Assuming that \(\Delta z = 0\), if both beams are perfectly in phase then \(\Delta \phi = 0\) and the beams add constructively. Any lead or lag in one beam will cause \(\Delta \phi \neq 0\) and produce interference. It will be seen in the following section that by using interferometry, it is possible to quantify this phase shift \(\Delta \phi\) and \(\text{OPD}\) for diagnostic purposes.

3.5.4 Multiple-Wavelength Interferometry

Multiple-wavelength interferometry is a plasma diagnostic technique used to determine plasma electron density that does not require any assumptions about the plasma electron density or heavy particle density. Recall from Equation 3.55 that the plasma induced \(\text{OPD}\) has two unknowns: the change in electron density \(\Delta n_e\) and the change in heavy particle density \(\Delta n_h\). Both parameters may be determined by acquiring measurements of the plasma-induced \(\text{OPD}\) at different wavelengths in order to construct a linear system of equations based off of Equation 3.55. This system may then be solved to give the two unknowns. Note that at least two wavelengths are required although more can be used to produce a better estimation of the unknowns.

An interferometer, such as the one pictured in Figure 3.6, has the ability to measure very precise differences in the optical path difference between the two arms of the interferometer. When a plasma sample is placed in one arm, the interferometer can effectively measure the plasma induced \(\text{OPD}\), as will be seen
in Chapters 5, 6, and 7.

Given a set of $M$ plasma induced OPD measurements at $M$ distinct wavelengths, it is the goal of this section to explain the procedure for determining the plasma electron and heavy particle density changes. First the OPD measurements are converted to phase shifts using

$$\Delta \phi = \frac{2\pi}{\lambda} \text{OPD}. \quad (3.70)$$

Equation 3.55 is then rearranged to give

$$\left( \frac{1}{2\pi 2L} \right) \Delta \phi(\nu) = \left( \frac{-q^2 \lambda^2}{8\pi^2 c^2 \varepsilon_0 m} \right) \Delta n_e + \left( \frac{A}{n_{h0}} + \frac{B}{n_{h0} \lambda^2} \right) \Delta n_h. \quad (3.71)$$

Here, the plasma length $L$ is multiplied by two to account for the double-pass of the Michelson interferometer. Now, a linear system of equation is formed with the experimental measurements in the left hand side,

$$\frac{1}{4\pi L} \begin{bmatrix} \Delta \phi(\lambda_1) \\ \Delta \phi(\lambda_2) \\ \vdots \\ \Delta \phi(\lambda_M) \end{bmatrix} = \begin{bmatrix} -\frac{q^2 \lambda_1^2}{8\pi^2 c^2 \varepsilon_0 m} & \frac{A}{n_{h0}} + \frac{B}{n_{h0} \lambda_1^2} \\ -\frac{q^2 \lambda_2^2}{8\pi^2 c^2 \varepsilon_0 m} & \frac{A}{n_{h0}} + \frac{B}{n_{h0} \lambda_2^2} \\ \vdots & \vdots \\ -\frac{q^2 \lambda_M^2}{8\pi^2 c^2 \varepsilon_0 m} & \frac{A}{n_{h0}} + \frac{B}{n_{h0} \lambda_M^2} \end{bmatrix} \begin{bmatrix} \Delta n_e \\ \Delta n_h \end{bmatrix}. \quad (3.72)$$

Let $\Phi = Cx$ denote the linear system such that $\Phi$ is the $m \times 1$ phase vector, $C$ is the $m \times 2$ coefficients matrix, and $x$ is the $m \times 1$ solution vector. This solution vector is determined via the pseudoinverse,

$$x = (C^T C)^{-1} C^T \Phi, \quad (3.73)$$
The first element in \( x \) is the change in plasma electron density and the second element is the change in heavy particle density. The latter may be used with Equation 3.57 to solve for the temperature of the background gas when plasma is on,

\[
T_{on} = \frac{T_{off} N_A P}{N_A P + \Delta n_h R T_{off}}. \tag{3.74}
\]

Finally, with \( \Delta n_e \) and \( T_{on} \) known, Equations 3.59 and 3.60 may be evaluated to determine the electron and heavy particle contributions to the total OPD.
CHAPTER 4

PLASMA ADAPTIVE OPTIC LENS DESIGN

4.1 Introduction

“Plasma adaptive optic (PAO) lens” is a term used to describe a plasma device used for optical corrections. The sole purpose of the PAO lens is to apply a phase shift, or equivalently induce an optical path difference (OPD), on a laser beam. The magnitude of the phase shift is a function of the applied voltage, pressure, and frequency.

4.2 Background

This research uses a device known as a dielectric barrier discharge (DBD) actuator. The DBD actuator consists of two electrodes separated by a dielectric medium. When an ac potential is applied across the electrodes, a strong electric field develops at the dielectric surface. Within the electric field, free electrons are accelerated and gain large amounts of kinetic energy. These electrons collide with much larger neutral gas particles and kinetic energy is transferred. When sufficient energy is transferred, the neutral gas particle ionizes and sheds an electron. The collision of a single and a neutral particle produces two electrons and an ion. This process repeats producing an electron avalanche, or very rapid breakdown of the
neutral particles into ions. With sufficient ac electric field, the plasma will sustain itself.

The DBD is be considered a non-equilibrium plasma because the kinetic energy of the ions is much less than the kinetic energy of the electrons, where the kinetic energy is related to temperature by the Boltzmann constant, $E \propto k_B T$. This implies that the ion temperature (gas temperature) is many orders of magnitude less than the electron temperature. For this reason, the DBD is generally referred to a non-thermal plasma.

The DBD may be either completely homogeneous or filamentary. In a homogeneous discharge, the gas ionization is a spatially uniform and indiscernible. When higher voltages are applied, the DBD become filamentary where the discharge is comprised of many small ionization channels, or filaments. This is also known as streamer breakdown. The filamentary/streamer breakdowns generally feature much higher electron densities. In this work, the plasma devices used all operate in this regime.

Four different PAO lens geometries were investigated. For various reasons, they were eliminated in favor of other designs. All past designs are included in the Appendix for the reader’s interest. The current design of the PAO lens uses a cylindrical dielectric barrier discharge capable of producing and sustaining plasma while maintaining optical access for a laser to pass through the plasma. A schematic of the basic PAO lens is shown in Figure 4.1. The device consists of a borosilicate tube of diameter $D$ with optical windows at each end and sealed with silicone. Copper tape electrodes are wrapped around the outside of the borosilicate tube at each end. The pressure inside the tube is lowered using a vacuum pump. Upon applying a sufficient ac voltage to the copper tape electrodes, a plasma
forms inside the chamber. A laser passing through the optical windows is phase shifted by a specified amount due to the presence of plasma inside the chamber.

4.3 Objective

Early experiments to miniaturize the PAO lens revealed an interesting plasma behavior: for a specified PAO lens diameter $D$ and small electrode gap distances $d$ the plasma formed along the inner wall of the lens. However, at larger gap distances the plasma always formed in the center of the lens. The two plasma regimes are depicted in Figure 4.2. The physical quantity responsible for producing the wall-plasma or center-plasma was identified as the electrode-gap-distance-to-diameter ratio, $d/D$.

In this chapter, the role of the $d/D$ ratio is investigated. Using experimental and theoretical approaches, it will be shown that a critical $d/D$ exists which corresponds to the transition point between wall and center plasmas. Future PAO lens designs must feature gap distances large enough to satisfy the critical $d/D$ value to ensure a center plasma.
4.4 Experiment

Experiments to investigate the $d/D$ dependency used a cylindrical PAO lens with an adjustable electrode gap distance as shown in Figure 4.3. It featured a 9.0 mm outer-diameter borosilicate tube with a 1.0 mm thick wall. The tube was 100 mm long and passed through three acrylic blocks. The two end blocks had optical windows to allow for optical access through the length of the PAO lens. The center block was threaded with a lead screw passing through it. Turning the lead screw caused the middle block to translate along the tube.

Two brass electrode collars were positioned on the outside of the borosilicate tube. They were 6.35 mm long and featured a 10 mm outer-diameter and a 0.9 mm thick wall. The right electrode was fixed to the right-most acrylic block. The left electrode was fixed to the sliding acrylic block. Rotating the lead screw allowed for accurate adjustment of the gap distance between the two electrodes. The gap distance was measured with Cen-tech digital calipers to an accuracy of 0.01 mm.
4.4.1 Plasma Generation

Pressure inside the adjustable PAO lens was lowered with a JB Platinum DV-285N vacuum pump that was connected to a port in one of the acrylic end blocks. The pressure was measured with a Heise H51011 absolute pressure gauge. To generate plasma inside the adjustable PAO lens, a large AC voltage potential was applied across the two electrodes. The AC signal was a 14 kilohertz sine-wave generated with an Agilent 33220A function generator. Two Crown XTi4000 amplifiers and two Corona Magnetics transformers were used to boost the voltage of the function generator output. The high-voltage signal was measured with a LeCroy PHV4-1903 high-voltage probe. Current was measured with a Pearson 2100 current monitor. The voltage and current measurements were acquired using a LeCroy WaveRunner LT264 oscilloscope and downloaded to a computer using a Transmission Control Protocol (TCP) connection.
4.4.2 Image Acquisitions

Images of the plasma inside the adjustable PAO lens were taken with a Spot Idea digital camera with a five-megapixel resolution and 12-bit color depth. The lens used was a Micro-Nikkor 55mm lens at an aperture of $f/8$. The image exposure time was 2.3 seconds. The camera was positioned to image through the optical windows and into the PAO lens.

4.4.3 Experimental Results

Using the adjustable PAO lens, twelve electrode-gap distances were tested. They ranged between 2.7 mm to 12.6 mm. Each gap distance represents a unique $d/D$ value. The pressure inside the PAO lens was held constant at 25.8 Torr. The peak-to-peak voltage potential used to sustain plasma inside the adjustable PAO lens was measured for each gap distance. The gap distances, $d/D$ ratios,
and voltage potentials used in the experiment are listed in Table 4.1.

For each gap distance, an image of the plasma luminescence inside the PAO lens was acquired, as shown in Figure 4.5. The images were post-processed to generate an azimuthally-averaged radial profile of the plasma for each gap distance. These radial profiles are shown in Figure 4.6. The color scales in the images vary from image to image to better emphasize the change in spatial structure of the plasma luminescence.

The profiles in Figure 4.6 clearly show the wall-plasma and center-plasma regimes. At the smallest gap distance of $d/D = 0.3$ the plasma layer is very thin and directly on the inner wall of the PAO lens. As the gap distance increases up to $d/D = 0.5$ the plasma layer thickens but remains attached to the inner wall. As the gap distance increases above $d/D = 0.5$ the plasma remains attached to the wall but the location of the maximum intensity begins to move inwards toward the center of the PAO lens. At $d/D = 1.25$ the plasma coalesces at in the center of the PAO lens. At $d/D = 1.4$ the plasma remains in the center. The data suggests that the center-plasma regime occurs for values of $d/D$ above 1.25.

4.5 Theory

To better understand the wall-plasma and center-plasma regimes observed in the experiment, an electrostatic model for the cylindrical PAO lens is derived. The assumptions for the analytical model are: (1) steady-state; (2) axisymmetric; (3) no dielectric present; (4) no plasma present; (5) infinitely thin electrodes.
### TABLE 4.1

**EXPERIMENTAL PARAMETERS**

<table>
<thead>
<tr>
<th>$d/D$ Ratio</th>
<th>$d$ [mm]</th>
<th>Voltage [kV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>2.7</td>
<td>1.8</td>
</tr>
<tr>
<td>0.4</td>
<td>3.6</td>
<td>2.3</td>
</tr>
<tr>
<td>0.5</td>
<td>4.5</td>
<td>2.2</td>
</tr>
<tr>
<td>0.6</td>
<td>5.4</td>
<td>3.0</td>
</tr>
<tr>
<td>0.7</td>
<td>6.3</td>
<td>3.2</td>
</tr>
<tr>
<td>0.8</td>
<td>7.2</td>
<td>3.5</td>
</tr>
<tr>
<td>0.9</td>
<td>8.1</td>
<td>3.5</td>
</tr>
<tr>
<td>1.0</td>
<td>9.0</td>
<td>3.5</td>
</tr>
<tr>
<td>1.1</td>
<td>9.9</td>
<td>3.5</td>
</tr>
<tr>
<td>1.2</td>
<td>10.8</td>
<td>3.5</td>
</tr>
<tr>
<td>1.25</td>
<td>11.25</td>
<td>3.5</td>
</tr>
<tr>
<td>1.4</td>
<td>12.6</td>
<td>3.5</td>
</tr>
</tbody>
</table>
Figure 4.5. Experimental images of the plasma luminescence for different $d/D$ ratios.

Figure 4.6. Azimuthally-averaged profiles of the plasma luminescence for different $d/D$ ratios.
4.5.1 Voltage Potential of a Charged Ring

We consider the vector locations of a point charge positioned on a circle of radius $R$ in the $xy$ plane,

$$\mathbf{P}_0 = R \cos \theta \hat{i} + R \sin \theta \hat{j} + 0\hat{k} \quad (4.1)$$

and an arbitrary point $\mathbf{P}$ in the $xz$ plane,

$$\mathbf{P} = \rho \hat{i} + 0\hat{j} + z\hat{k}. \quad (4.2)$$

The distance $a$ between $\mathbf{P}_0$ and $\mathbf{P}$ is

$$a = (\rho^2 + R^2 + z^2 - 2\rho R \cos \theta)^{1/2}. \quad (4.3)$$

The differential voltage potential at $\mathbf{P}$ due to the charge at $\mathbf{P}_0$ is

$$dV = \frac{1}{4\pi \epsilon_0} \frac{dq}{a} \quad (4.4)$$

where $\epsilon_0 = 8.85418 \times 10^{-12} \text{ Fm}^{-1}$ is the vacuum permittivity and the differential charge is

$$dq = \frac{q}{2\pi R} Rd\theta. \quad (4.5)$$

An expression for the voltage potential at $\mathbf{P}$ induced by a ring of charge lying in the $xy$ plane is found from integrating the differential voltage potential in Equation $4.4$ over the interval $\theta = [0, 2\pi]$. Observing the symmetry in the integral, this
becomes

\[ V = \frac{q}{4\pi^2\epsilon_0\rho} \int_0^\pi \frac{d\theta}{(\rho^2 + z^2 + R^2 - 2\rho R \cos \theta)^{1/2}}. \]  \hfill (4.6)

This is rewritten as

\[ V = \frac{q}{4\pi^2\epsilon_0\rho} \left( \frac{1}{2\rho R} \right)^{1/2} \int_0^\pi \frac{d\theta}{(b - \cos \theta)^{1/2}} \]  \hfill (4.7)

where

\[ b = \frac{\rho^2 + R^2 + z^2}{2\rho R}. \]  \hfill (4.8)

Equation (4.7) is an elliptic integral with the solution

\[ V_{\text{ring}} = \frac{q}{2\pi^2\epsilon_0} \frac{1}{\alpha} K \left( \frac{4R\rho}{\alpha} \right) \]  \hfill (4.9)

where K is the complete elliptic integral of the first kind \[15, 28\]. The parameter \( \alpha \) is

\[ \alpha = (\rho + R)^2 + z^2. \]  \hfill (4.10)

Equation (4.9) gives the voltage potential at any point in the \( xz \) plane due to a charged ring of radius \( R \) in the \( xy \) plane centered at the origin.
Figure 4.7. A charged ring in the xy plane.

Figure 4.8. Superposition of two charged rings of diameter $D$ and separated by a distance $d$. 
4.5.2 Electric Field of a Charged Ring

The electric field is the negative gradient of the potential. For a single charged ring the electric field is

\[ \mathbf{E}_{\text{ring}} = -\nabla V_{\text{ring}} = -\frac{\partial V_{\text{ring}}}{\partial \rho} \hat{e}_\rho - \frac{1}{\rho} \frac{\partial V_{\text{ring}}}{\partial \theta} \hat{e}_\theta - \frac{\partial V_{\text{ring}}}{\partial z} \hat{e}_z. \]  

(4.11)

The vector components are

\[ E_{\rho,\text{ring}} = \frac{q}{4\pi^2 \epsilon_0} \frac{1}{\sqrt{\alpha \beta \rho}} \left( \beta K \left[ \frac{4R\rho}{\alpha} \right] - \gamma E \left[ \frac{4R\rho}{\alpha} \right] \right), \]  

(4.12)

\[ E_{\theta,\text{ring}} = 0, \]  

(4.13)

\[ E_{z,\text{ring}} = \frac{q}{2\pi^2 \epsilon_0} \frac{z}{\sqrt{\alpha \beta}} E \left[ \frac{4R\rho}{\alpha} \right]. \]  

(4.14)

Here E is the complete elliptic integral of the second kind and

\[ \beta = (R - \rho)^2 + z^2, \]  

(4.15)

\[ \gamma = R^2 - \rho^2 + z^2. \]  

(4.16)

Taking the radius \( \rho \to 0 \) recovers the form for the \( z \)-component of the electric field magnitude along the \( z \)-axis given in Brockhaus et. al \[13\],

\[ E_{z,\text{ring}}(0, z) = \frac{1}{4\pi \epsilon_0} \frac{qz}{(R^2 + z^2)^{3/2}}. \]  

(4.17)
4.5.3 Cylindrical PAO Lens Model

To model the PAO lens, two oppositely charged rings are superimposed. Each have a diameter $D$ and are separated by a gap distance $d$ as shown in Figure 4.8. The voltage potential and electric field components between the two rings are then

$$V = V_{\text{ring}}(\rho, z - d/2) - V_{\text{ring}}(\rho, z + d/2).$$  \hspace{1cm} (4.18)

$$E_\rho = E_{\rho,\text{ring}}(\rho, z - d/2) - E_{\rho,\text{ring}}(\rho, z + d/2),$$  \hspace{1cm} (4.19)

$$E_\theta = 0,$$  \hspace{1cm} (4.20)

$$E_z = E_{z,\text{ring}}(\rho, z - d/2) - E_{z,\text{ring}}(\rho, z + d/2).$$  \hspace{1cm} (4.21)

The magnitude of the electric field is a spatial function of $\rho$ and $z$,

$$E(\rho, z) = \sqrt{E_\rho(\rho, z)^2 + E_z(\rho, z)^2}. $$  \hspace{1cm} (4.22)

All solutions presented here are in the $z = 0$ plane. The model for the electric field magnitude of the PAO lens becomes

$$E(\rho) = \sqrt{E_\rho(\rho)^2 + E_z(\rho)^2}. $$  \hspace{1cm} (4.23)

4.5.4 Theory Results

Solutions for the electric field magnitude inside the PAO lens were generated using Equation 4.23. Variables were chosen to mimic the experimental setup. All
the variables used in the solutions are listed in Table 4.2. The solutions for the
twelve gap distances tested in the experiment are shown in Figure 4.9. The color
scales in Figure 4.9 vary from image to image to better emphasize the electric
field’s spatial structure.

The theoretical model’s profiles appear very similar to the experimental pro-
files. At $d/D = 0.3$ the concentration of highest electric field magnitude is thin
and along the inner wall of the PAO lens. As the the gap distance increases,
the concentration of highest electric field magnitude thickens and pulls away from
the walls. Above $d/D = 1.2$ the concentration of highest electric field magnitude
coalesces into the center of the chamber.

4.6 Comparison of Experiment and Theory

The experimental data gave average profiles of the plasma luminescence, a
quantity related to the level of plasma ionization inside the PAO lens. The the-
Figure 4.9. Theoretical profiles of the electric field magnitude for different $d/D$ ratios.

theoretical data gave profiles of electric field magnitude inside the PAO lens. It is known that regions of high electric field magnitude are generally indicative of high plasma ionization regions. Therefore, it is valid to qualitatively compare the experimental data to the theoretical data.

It was observed in both the experiment and theory that as the $d/D$ value increases, the concentration of plasma inside the PAO lens moves from the wall and eventually coalesces in the center. Tracking this movement at different $d/D$ provided a convenient metric for comparing the experiment and theory. Figures 4.10 and 4.11 show plots of the experimental and theoretical data respectively. Local maximums clearly move from the wall towards the center as the $d/D$ ratio increases. The vertical dashed line in each plot shows the radial location of the local maximum, $\rho_{max}$, that was detected for each case.
Figure 4.10. Experimental data showing the radial distribution of plasma luminescence for different $d/D$ ratios (solid). Vertical lines showing maximum location (dashed) and inflection location (dash-dot).

Figure 4.11. Theoretical data showing the radial distribution of electric field magnitude for different $d/D$ ratios (solid). Vertical lines showing maximum location (dashed).
The values for $\rho_{max}$ were normalized by the full radius $R$ as

$$\rho_{max}^* = \frac{\rho_{max}}{R}. \quad (4.24)$$

Note that the value $\rho_{max}^* = 1$ indicates a wall-plasma and $\rho_{max}^* = 0$ indicates a center-plasma. Figure 4.12 shows a plot of the the $\rho_{max}^*$ values calculated from the experimental data and the theory. The theory shows good agreement for $d/D$ values below 0.5 when the plasma is very close to the wall and above 1.1 when the plasma is very close to the center of the PAO lens. The values of $d/D$ between 0.6 and 1.1 show the poorest agreement. In this range are the transitional states between the wall-plasma and center-plasma.

There must exist a critical $d/D$ value that distinguishes between a wall-plasma and a center-plasma. The experimental data bracket this critical value in a notably narrow margin between $d/D = 1.2$ and $d/D = 1.25$. Solutions from the theory identify this critical value to be $d/D = 1.22$. The agreement between the experiment and theory here is extremely good.

As mentioned earlier, the range of $d/D$ between 0.6 and 1.1 showed the poorest agreement between the experiment and theory. Comparing the plots in Figures 4.10 and 4.11 reveal an interesting feature: the experiment has an extra inflection point which the theory does not predict. The inflection point locations are shown in Figure 4.10 with vertical dash-dot lines. Reasons for the inflection points have not yet been determined but could be caused by space charge accumulation at the dielectric surface.

Considering the theory lacks this extra inflection point, a new normalization method for the experimental data was introduced. This adjusted normalization
used the radius of the inflection point $\rho_{\text{inflection}}$ rather than the electrode radius,

$$\rho_{\text{max}}^{**} = \frac{\rho_{\text{max}}}{\rho_{\text{inflection}}}.$$  \hspace{1cm} (4.25)

Figure 4.13 shows the comparison of the experimental and theoretical data using this adjusted normalization. The adjusted normalization altered the experimental data in the region from $d/D = 0.6$ to $d/D = 1.1$ to show extremely good agreement against the theory. This suggests that there is a virtual radius inside the cylindrical PAO lens and within this radius the plasma behaves exactly as the electrostatic model predicts.

Figure 4.12. Comparison of the experiment and theory for the radial distribution of plasma normalized by the total radius.
4.7 Conclusions

The cylindrical PAO lens has promising features in the field of adaptive optics. The work presented has parametrized the electrode gap distance and diameter of the PAO lens using the ratio $d/D$. Depending on the $d/D$, two distinct plasma regimes were identified: surface-plasma and center-plasma. Experimental and theoretical approaches were applied to better understand the role of $d/D$ on the PAO lens’ plasma regime.

Experimental data was presented that showed the radial profiles of the plasma inside the PAO lens for different $d/D$ ratios. Both wall-plasma and center-plasma were observed. The theoretical model determined that the critical $d/D$ ratio dividing the wall-plasma and center-plasma regimes is 1.22. Any $d/D$ ratio below 1.22 generates concentrations of plasma between the wall and the center. Good agreement was found between the experiment and theory. Furthermore, a virtual
radius was discovered to exist inside the PAO lens where the plasma within behaves exactly as the electrostatic model predicts.

The geometrical constraints learned in this study will facilitate the design of new PAO lenses. With a sufficiently small PAO lens, it will be possible to create arrays of multiple PAO lenses to provide for spatial wavefront correction for aero-optics applications.
5.1 Background

A proof of concept experiment was conducted to measure the optical path difference produced with a PAO lens. The experiment used a 15.2 cm PAO lens that generated an air plasma at three different sub-atmospheric pressures. Measurements included $OPD$, and current. The interferometric measurements were done using a 633 nm helium neon laser and interferogram images were taken using a digital camera. A non-dimensional relationship for the plasma electron density and applied power was created and found to have good agreement with the data. The PAO lens exhibited values of $OPD$ up to 1.5 $\mu m$.

5.2 Experiment

5.2.1 Experimental Setup

A non-thermal, DBD plasma was generated inside a hollow Pyrex cylinder. A schematic and photograph of the Pyrex cylinder are shown in Figure 5.1. The Pyrex cylinder was 15.2 cm in length, 1.9 cm in diameter, and had a wall thickness of 0.28 cm. Both ends of the cylinder were sealed with 2.54 cm diameter, 5 mm thick optical glass disk. The laser beam in the interferometer setup passed through the optical glass ends of the cylinder. A vacuum port with a valve was located
halfway along the length of the cylinder. This was connected to a vacuum pump that was used to evacuated the air in the cylinder to a low pressure. Once the pressure was set, the valve closed the port and the vacuum pump was disconnected. Copper tape electrode strips were placed at both ends of the Pyrex tube and wrapped completely around the outer circumference. A high-voltage AC input was applied to the electrodes. The AC waveform was a sine-wave at 14kHz.

The Pyrex cylinder was located in one arm of a Michelson interferometer that was assembled for the experiments. A schematic of the optical setup is shown in Figure 5.2. The Pyrex cylinder is labeled PAO lens in the schematic. The Michelson interferometer was used to measure the $OPD$ produced by the plasma in the Pyrex cylinder. The $OPD$ is related to the index of refraction of the plasma.
Figure 5.2. Schematic of the interferometer.
The index of refraction is a function of the plasma electron density\cite{10}.

The light source used in the optical setup was a 25 mW, 633 nm helium-neon laser. A 8 \(\mu\)m pinhole spatial filter was used to collimate and produce nearly planar wavefronts of the incoming laser. The laser beam split along two paths. The reference path, labeled “b”, reflected off the beam-splitter and then was reflected directly back. The measurement path, labeled “a”, passed through the beam-splitter and the PAO lens and was reflected back. The reflecting mirror in path “a” was placed on a mechanical stage to allow adjustment of the path length. The beams were then recombined and imaged onto a digital camera where the interferograms were stored. Analysis of the interferograms was performed in post-processing.

Experiments were conducted with different gas static pressures inside the Pyrex cylinder. At each pressure, a range of AC voltage amplitudes were applied to cause the air inside Pyrex cylinder to ionize. At each AC voltage, voltage and current time-series data and interferogram images were acquired. A schematic of the automated experiment control and data acquisition system is shown in Figure 5.3.

A TCP-controlled Agilent 33220A function generator was used to generate the input carrier waveform for the plasma. The function generator output was transformed into a high voltage, low current signal using two Crown XTi4000 amplifiers and two Corona Magnetics transformers. The gas pressure inside the chamber was measured with a Heise H51011 absolute pressure gauge. The voltage potential of the output from one of the transformers was measured with a LeCroy PHV4-1903 high voltage probe, and the current is measured with a Pearson 2100 current monitor. Both signals were acquired with a LeCroy WaveRunner LT264
Figure 5.3. Schematic of the automated data acquisition and control system used for the PAO lens analysis.
oscilloscope. The oscilloscope’s A/D converter has 8-bit resolution and a 1 GHz sampling frequency. Both were suitable for capturing some of the plasma discharge spikes that appeared in the current measurements. Data from the oscilloscope was passed to the computer using a TCP socket connection. Interferograms images were taken with a Spot Image 12-bit, 5-megapixel digital camera. The camera’s active sensor size was 5.7mm horizontal by 4.28 mm vertical. A 100 mm lens was placed in front of the camera and adjusted so that the interferogram completely filled the sensor. The camera was triggered with a TTL pulse from the computer.

During data acquisition, the following actions are performed: (1) a TTL trigger was sent to the camera to acquire a reference (voltage-off) interferogram; (2) the computer set the appropriate voltage on the function generator and enabled the output to generate a plasma; (3) a TTL trigger was sent to the camera to capture the active (voltage-on) interferogram; (4) the computer set the appropriate time/voltage scales on the oscilloscope and acquired 12 periods of the current and voltage waveforms; (5) the output to the function generator was turned off; (6) twenty second timeout was performed. This process repeated for a range of function generator voltages.

5.2.2 Data

Two interferograms were recorded (reference and active) for every voltage-pressure combination examined in the experiment. A sample pair of interferograms are shown in Figure 5.4. The axes are shown in pixels where 1 pixel corresponds to 3.88 nm and the distance between two fringes is equal to the laser’s wavelength (633 nm). Image processing techniques were applied to each interferogram image to determine the fringe locations. The shift that occurred in the fringe locations
between the cases with the plasma present, and corresponding reference, were used to determine the spatial values of $OPD$.

The image processing first consisted of cropping the interferograms to a fixed region of interest selected to include the largest rectangular area within the circular interferogram image. A two-dimensional digital filter was then convolved with each image to remove high-frequency noise. The digital filter did not distort the locations of the fringes. The corresponding fringes were then carefully registered between each reference and active image pairs.

Each digitized interferogram consisted of a 2-D matrix of values that were proportional to light intensity. A single matrix row consisted of a sinusoidal light intensity data series with peaks corresponding to bright areas and valleys corresponding to dark areas. The boundary between light and dark areas represented an interferometric fringe. A fringe-tracking algorithm was used to locate the center of each fringe, and keep a count of the fringe numbers. This process was repeated for every row within the interrogation region.

A shift in the position of a specific fringe between the active and reference images corresponds to the local $OPD$, in pixels. From the pixel-valued $OPD$, the observed phase shift, $\phi$, is found using

$$\phi = OPD_{px} \frac{2\pi}{\lambda_{px}},$$  \hspace{1cm} (5.1)

where $OPD_{px}$ is the $OPD$ in pixels and $\lambda_{px}$ is the spatial wavelength of the interferograms in pixels.

The $OPD$ in physical dimensions is

$$OPD = OPD_{px} \frac{\lambda}{\lambda_{px}},$$  \hspace{1cm} (5.2)
Figure 5.4. A sample pair of interferograms with the plasma off (top) and plasma on (bottom).
where \( \lambda \) is the laser wavelength. For the Michelson interferometer\[12, 44\], the average refractive index of the PAO lens with electrode spacing \( d \) is then,

\[
N = 1 + \phi \frac{1}{2d} \frac{\lambda}{2\pi}.
\]  

(5.3)

We were seeking a relation between the plasma electron density and the index of refraction found in the experiments. Recall from Equation 3.52 in Chapter 3, the refractive index of plasma is

\[
N = 1 - \frac{q^2}{8\pi^2\epsilon_0 c^2 m} \lambda^2 n_e + \left( A + \frac{B}{\lambda^2} \right) \frac{n_h}{n_{h0}}.
\]  

(5.4)

For these experiments, it is assumed that the heavy particle influence is negligible, and therefore

\[
N = 1 - \frac{q^2}{8\pi^2\epsilon_0 c^2 m} \lambda^2 n_e.
\]  

(5.5)

This assumption may not be valid considering that the plasma is probed with a very short wavelength. In the next chapter, experimental data is presented without requiring this assumption. Equation 5.5 is rewritten for the electron density,

\[
n_e = 2(1 - N) \left( m_e \epsilon_0 \right) \left( \frac{2\pi}{\lambda c} \right)^2.
\]  

(5.6)

The instantaneous power of the plasma is the product of current and voltage. For the discrete voltage and current time series collected during the experiments, the power is

\[
P = \frac{1}{J} \sum_{j=1}^{J} V_j I_j
\]  

(5.7)

where \( J \) encloses an integer number of cycles. In the experiment, discrete time
series data for the plasma applied voltage and current were recorded for post-
calculation of the power. Another quantity of interest is the power level at which
stable plasma initiates inside the chamber. This was found from inspection of the
phase-angle between voltage and current,\[\phi_{VI} = \cos^{-1}\left(\frac{P}{P_{RMS}}\right)\]

where $P_{RMS}$ is the root-mean-square of power. Without plasma, the phase shift
between the voltage and current remained at about 90°. At the initiation of
plasma there was a sudden decrease in phase. From experimental observations it
was noted that “stable” plasma did not form until the phase shift was less than
about 85°. This was the metric we adopted to ensure that a stable plasma was
present during the measurements.

5.3 Dimensional Modeling

A model for the plasma electron density was derived from a Buckingham-
Pi analysis[14]. The included variables were power $P$, electron density $n_e$, gas
pressure $p$, gas temperature $T$, and the gas constant $R$. The five properties are
expressed in basic dimensions (mass $M$, length $L$, time $t$, temperature $G$) given
by the following equations.

\[
P \triangleq ML^2t^{-3} \quad \text{(5.9)}
\]

\[
n_e \triangleq L^{-3} \quad \text{(5.10)}
\]

\[
p \triangleq ML^{-1}t^{-2} \quad \text{(5.11)}
\]

\[
T \triangleq G \quad \text{(5.12)}
\]

\[
R \triangleq L^2t^{-2}G^{-1} \quad \text{(5.13)}
\]

The dimensional matrix was constructed and shown below in reduced echelon form.

\[
D = \begin{bmatrix}
1 & 0 & 0 & 0 & 2 \\
0 & 1 & 0 & 0 & -2 \\
0 & 0 & 1 & 0 & 4/3 \\
0 & 0 & 0 & 1 & -1 \\
\end{bmatrix}
\]

With five variables and four dimensions, we expected one pi-group. The right null-space vector of the over-determined system \( D \) gave the pi-group,

\[
\Pi = \frac{n_e^{4/3}P^2}{p^2RT} \quad \text{(5.14)}
\]

We sought some function \( f \) such that

\[
f(\Pi) = 0. \quad \text{(5.15)}
\]

A solution was found and written in terms of electron density,

\[
n_e = n_{e0} + K \left( p^2RT \right)^{3/4} \left( \frac{1}{P^{3/2}} - \frac{1}{P_0^{3/2}} \right) \quad \text{(5.16)}
\]
where \( n_{e0} \) and \( P_0 \) are the electron density and power at the initiation point of stable plasma. \( K \) is an experimentally determined constant that absorbs information pertaining to the plasma actuator geometry, dielectric properties, and gas ionization properties.

The model predicts the electron density for any subsequent power as a function of the gas pressure or temperature. In particular it shows the weighting (dominance) of each parameter. The model requires knowing the power and electron density at plasma initiation at any pressure of interest, and an experimentally determined constant.

Figure 5.5. Current-voltage data for plasma off and plasma on.
Figure 5.6. Power-voltage characteristics for the plasma as a function of pressure.

5.4 Results

Experiments were conducted to validate the electron density model given in Equation 5.16. This model is the key element in relating the $OPD$ of the plasma lens with the input voltage or plasma power.

Interferograms were collected for three air pressures inside the plasma chamber: 25.86 Torr, 33.61 Torr, and 41.73 Torr. At each pressure, data was taken at 16 different AC voltage levels ranging from 14 to 44 kV$_{p-p}$. Atmospheric temperature
and pressure were recorded before each test. Analysis of the interferogram image pairs provided the \( \text{OPD} \), and the index of refraction. Post-analysis of the current and voltage data series was used to determine the power in the analytic model.

Figure 5.5 shows an example phase plot of voltage-current data series. Each AC cycle forms an approximate ellipse. The phase plot is made up of 12 AC periods. The inner, red ellipse corresponds with a lower AC voltage that was below the level needed to ionize the air. The outer, black ellipse corresponds to an AC voltage that was large enough to ionize the air. This is evident by the high frequency discharge spikes that when averaged, cause an asymmetry in the voltage-current phase plot that when integrated, results in a net power dissipation. The net power was calculated based on Equation 5.7.

The resulting power as a function of the applied AC voltage level for the three gas pressures is shown in Figure 5.6. The voltage at which the power begins to increase marks the condition where the air first ionizes. The voltage where this occurs decreases with decreasing pressure. Once the air ionizes, there is an approximately linear relation between voltage and power.

Visual observations of the plasma indicated the possibility of unstable behavior that particularly occurred at lower voltages, just above the minimum needed to ionize the air. The term \textit{stable} is used to describe plasma with no visual transients in its structure and formation; an \textit{unstable} plasma flickers and can move around within the chamber. A method to identify the region of stable plasma generation involved monitoring the phase angle between the voltage and current. This is plotted as a function of power for the three gas pressures in Figure 5.7. The horizontal dashed lines indicate boundaries where visual observations indicated that there was no plasma, unstable plasma, or stable plasma. At lower power or volt-
Figure 5.7. Phase-angle between voltage and current as a function of plasma power for different pressures.
age levels that were below the minimum needed to ionize the air, the phase angle between the current and voltage was $90^\circ$. Once minimum voltage was exceeded, the plasma formed and the phase angle decreased. The phase change was a direct result of a change in the total capacitance of the plasma as it initiated and became stronger. However, observations indicated that the plasma was stable only when the phase angle was less than $85^\circ$. Therefore for each of the gas pressures, the plasma initiation power used in the model analysis was the minimum power value at which the voltage-current phase angle was less than $85^\circ$.

The $OPD$ values were found from analysis of interferogram image pairs. Spatial $OPD$ maps as a function of the AC voltage for the three gas pressures are shown in Figures 5.8 to 5.10. The cylindrical electrode configuration of the PAO lens created a spatial gradient of the plasma with highest electron density in the center. This was observable during the experiments and is quantified in the $OPD$ contours. As the voltage potential increased, the region where the air was ionized increased. This required higher voltages at the higher gas pressures.

The maximum $OPD$ values from each spatial map in Figures 5.8 to 5.10 were used to calculate respective electron density values. The results were used with Equations 5.1, 5.2, 5.3, and 5.6 to determine the electron density. The electron density and $OPD$ are plotted versus the plasma power in Figure 5.11 for the three gas pressures. The symbols correspond to the measured values. The curves are based on the analytic model given in Equation 5.16. The agreement is very good.

A maximum $OPD$ of approximately 1.5 $\mu$m was achieved at the highest pressure. The dynamic range of the $OPD$ of the PAO lens depends on the plasma power and gas pressure.

Experimental values were required for the electron density model, specifically,
Figure 5.8. Spatial contours of $-\text{OPD}$ in $\mu\text{m}$ as a function of voltage for a gas pressure of 25.86 Torr.

Figure 5.9. Spatial contours of $-\text{OPD}$ in $\mu\text{m}$ as a function of voltage for a gas pressure of 33.61 Torr.
the power and electron density values at stable plasma initiation, and the experimental constant $K$. The former was determined based on the criterion given by Figure 5.7. For the latter, a new variable, $n_e^*$, was defined and given as

$$n_e^* = \left( \frac{p^2 RT}{P^2} \right)^{3/4}.$$  \hspace{1cm} (5.17)

A plot of the electron density, $n_e$, versus $n_e^*$ should produce a straight line with a slope that corresponds to the experimental constant, $K$. This is shown in Figure 5.12. The data at the two lower pressures of 25.86 Torr and 33.61 Torr fall reasonably well on a straight line. At the higher pressure of 41.73 Torr, the fewer number of data points makes it more difficult to assess. Averaging the two

Figure 5.10. Spatial contours of $-OPD$ in $10 \mu m$ as a function of voltage for a gas pressure of 41.73 Torr.
Figure 5.11. Electron density and $OPD$ as a function of plasma power. Curves correspond to the analytic model given by Equation 5.16.
slopes at the lower pressures, the experimental constant was

\[ K = -7.187 \times 10^{13}. \]  

(5.18)

This was the value for \( K \) that was used in the model curve fits to the data in Figure 5.11.

The comparison between the electron density and \( OPD \) model, and experimental results was encouraging. At the lower two pressures, very good agreement was observed, particularly at higher voltages. At lower voltages, the model over-predicted the electron density. This is an indication that the power/electron density initiation values might be too high. Future work will attempt to pinpoint these locations with greater accuracy.

5.5 Conclusions

This experiment demonstrated that a PAO lens will induce an \( OPD \) on a laser beam. The \( OPD \) produced by the plasma was characterized and related to the plasma electron density. The effect of gas pressure and plasma power were investigated. Good agreement was observed between the experiment and analytic model. The maximum \( OPD \) increased with the gas pressure inside the PAO lens and a maximum \( OPD \) of approximately 1.5 \( \mu \)m was achieved in the experiment. Notably, even larger \( OPD \) values could be achieved by using multiple devices in series. Air was used as the gas in the PAO lens for these feasibility experiments. Note that the determination of electron density depended on the assumption that the optical effect of the heavy particles was negligible. The validity of this assumption will be discussed in the following chapter.
Figure 5.12. Electron density, $n_e$, as a function of $n_e^*$ defined in Equation 5.17. Curves correspond to the analytic model given by Equation 5.16.
CHAPTER 6
DUAL-WAVELENGTH INTERFEROMETRY

6.1 Introduction

In the previous chapter, values of \( OPD \) and electron density were measured by probing a plasma with a single, visible wavelength. It was assumed that the index of refraction (and \( OPD \)) was not affected by the heavy particles (neutrals, metastables, ions) within the plasma. This a valid assumption for very long wavelengths (THz, microwaves) but it is a poor assumption when probing the plasma with visible light. In order to correctly measure the plasma index of refraction in the visible and IR wavelength range requires probing the plasma with at least two wavelengths.

Dual-wavelength interferometry is a measurement technique that was first suggested for plasma diagnostic purposes by Alpher and White in the late 1950s [2][4]. This technique probes a plasma with two distinct wavelengths, typically one visible and one infrared, in order to determine the plasma electron density and heavy particle density. Although the this setup is more complicated than traditional interferometry, it will be seen later in this chapter that by using two wavelengths, a linear system of equations may be formed in order to solve for the plasma densities.

This chapter will present the experimental setup for the plasma generation and the dual-wavelength interferometer; the data processing procedure; and, data measured with the dual-wavelength interferometer for air and helium plasmas.
6.2 Experimental Setup

Descriptions of the experimental setup for this dual-wavelength interferometry experiment are divided into four sections: (1) the plasma chamber, (2) the electronics to generate plasma, (3) the optics and Michelson interferometer, and (4) the experimental procedure and data acquisition. Each of these components will be addressed individually in the following subsequent sections.

6.2.1 Plasma Chamber

The plasma chamber used in this experiment featured the same cylindrical, double dielectric barrier design introduced in Chapter 4 and tested in Chapter 5. Figure 6.1 shows a general schematic of the plasma chamber. In comparison to past designs, significant improvements were made to reduce gas leaks so that the chamber was able to hold a constant sub-atmospheric pressure without requiring a vacuum pump to run continuously.

The chamber was constructed using commercial-off-the-shelf ConFlat (CF) components. CF flanges use copper gaskets placed between flange faces that are permanently deformed by “knife-edge” azimuthal grooves when the flanges are bolted together. This approach forms a mechanical, leak-tight seal able to maintain pressures as low as $1 \times 10^{-13}$ Torr (ultra-high vacuum) \[41\].

The dielectric of the plasma chamber was a borosilicate glass tube (McMaster-Carr 8729K37) that featured a 0.5” outer diameter and a wall thickness of 60 mil. The tube was sealed at each end to CF 1.33” flanges using quick-disconnect adapters (A&N Corporation 133XQ50). Each CF 1.33” flange was then connected to a zero-length reducer which stepped up to CF 2.75” (A&N Corporation 5000287). Finally, the CF 2.75” flanges were sealed to CF 2.75” flanges featuring
1” calcium-fluoride (CaF$_2$) windows (Thor Labs VPCH512) which allowed the lasers to pass through the length of the chamber. Calcium fluoride was selected because it has good transmission properties for both of the laser wavelengths used in this experiment.

Two 1/16” NPT ports were machined into one of the zero-length reducers. In one of these ports was a 0-15 psi vacuum pressure transducer (Honeywell 19C015PV4K) which allowed real-time monitoring of the background gas pressure during plasma generation. In the second port was a plug valve (Swagelok SS-4P4T1) which allowed the plasma chamber to be closed off. A pressure line from the plug valve went to a pressure manifold where it connected to four lines: (1) deep vacuum pump (JB Platinum DV-285N) rated to 0.015 Torr, (2) gas cylinder with ultra-high purity helium (Airgas HE UHP35), (3) vacuum pressure gauge (Heise H51011), and (4) bleed valve. Manipulation of the pressure manifold allowed control of the plasma chamber pressure and gas. Once the pressure was set, the plug valve was closed to seal off the plasma chamber and the vacuum pump was turned off.

The plasma chamber was leak-tested using a Varian 959 Macrotorr Helium Leak Detector and shown to have a leak rate of less than 2×10$^{-8}$ cm$^3$/s. Estimating the total volume inside the plasma chamber to be about 15 cm$^3$, it would take nearly 24 years for the chamber to leak to atmospheric pressure at this leak rate.

At each end of the borosilicate tube, copper tape electrodes were wrapped around the outside of the tube. The spacing between the electrodes was 1.5” corresponding to a $d/D$ ratio of about 4. From the experiments of Chapter 4, it was known that this spacing would produce a center plasma. Spacers, made
of 1/4” thick acrylic, were used to insulate the electrodes to prevent arcing to the stainless-steel quick-disconnects. The glass-acrylic interface was sealed using a general purpose epoxy. The electronics used to generate the plasma will be discussed in the following section.

6.2.2 Plasma Generation

Plasma generation inside the plasma chamber required a high-voltage, low-current ac signal. A 2 kHz sine-wave carrier signal from a function generator (Agilent 33220A) was passed to the inputs of two audio amplifiers (Crown XTi4000). Four channels of output from the audio amplifiers were each passed through two 1 ohm high-power resistors to create a load similar to an audio speaker. From the resistors, the signals were joined and passed to two low-frequency transformers (Corona Magnetics CMI-5225) that were 180° out-of-phase. This produced two high-voltage, low-current ac signals that were 180° out-of-phase. The high-voltage signals were then connected to the copper tape electrodes of the plasma chamber. With the signals 180° out-of-phase, there was effectively twice the voltage potential between the electrodes. The maximum voltage potential used in these experiments was about 30 kV.

During the interferometric measurements, the plasma carrier signal was modulated with a second function generator (Agilent 33220A) causing the plasma to repeatedly ignite and extinguish in turn. The purpose of this modulation was to enable measurements of the plasma induced phase response over a range of applied voltage potentials during a single test. A schematic of the custom modulating waveform is shown in Figure 6.3. At the lowest modulation voltage, the applied voltage to the plasma chamber was less than 1 kV and there was no
Figure 6.1. Schematic of the plasma chamber.

Figure 6.2. Pictures of the plasma chamber with plasma off (left) and with plasma on (right).
plasma present. The modulation ramped up over a quarter of the modulation period, linearly increasing the plasma chamber voltage to about 30 kV. During this voltage increase, plasma ignited and became more intense inside the plasma chamber. The modulation held a constant voltage for a quarter of the modulation period, maintaining the plasma chamber at about 30 kV. Then the modulation ramped down over a quarter of the modulation period, linearly decreasing the plasma chamber voltage to below 1 kV. During this voltage decrease, the plasma extinguished. Using this plasma excitation scheme, it was expected that the time-resolved OPD profiles measured with the interferometer would display a similar modulation behavior. The modulation frequency used in the experiments was 25 Hz, which produced 80 carrier cycles per modulation period. The modulating function generator was set up to burst 100 cycles per test (corresponding to a 4 second test time).
The voltage potential and the current of one of the high-voltage signals were measured using a high-voltage probe (LeCroy PHV4-1903) and a wide-band current monitor (Pearson 2100). Both measurement signals were connected to a digital oscilloscope (LeCroy Waverunner 6100) sampling at 50 MHz.

6.2.3 Interferometer

The experiment used a dual-wavelength Michelson interferometer to measure the plasma induced $\text{OPD}$s as the plasma was modulated on and off. The experiment utilized a visible 633 nm helium neon laser (CVI Melles Griot 25-LHP-828-230) and an infrared 3.39 $\mu$m helium neon laser (Newport Corporation R-32172). The difference in the wavelengths allowed for the determination of the time-resolved changes in electron density and heavy particle density using the procedure described in Section 3.5.4.

A schematic of the interferometer is depicted in Figure 6.4. The two lasers were aligned and then joined onto the same path using an N-BK7 window with an anti-reflective coating (Thor Labs WG11050-B) which passes only visible light. The beams were passed through an optical chopper (Digirad C-980) set at a 1000 Hz chopping frequency.

Entering the Michelson interferometer, the beams passed through a 2” diameter potassium bromide (KBr) compensator and beamsplitter pair (FTIR.com). They were enclosed in a custom mount with a 10 mil air gap between the surfaces. Note that KBr is a hygroscopic material, so special care had to be taken to keep the lab under 40% humidity to prevent damaging the optics. This was accomplished by using two dehumidifiers (Hisense DH-70K1SJE5) which ran 24 hours a day. When the beamsplitter and compensator were not in use, they were
placed in a desiccator chamber (VWR 47751-772) for safe keeping.

The beamsplitter split the beams into two paths, one through the plasma chamber (plasma arm) and one through air (reference arm). In the reference arm, the beams passed through quiescent air, reflected off a mirror, and returned to the beamsplitter. In the plasma arm, the beams passed through the plasma chamber, reflected off a mirror, passed again through the plasma chamber, and returned to the beamsplitter. Both mirrors (Thor Labs PF05-03-P01) featured surface flatness values of $\lambda/10$ @ 633 nm. Each arm of the interferometer was 16” long, and the mirror in the plasma arm was mounted on a linear stage which allowed for adjustments between tests if necessary.

At the exit of the interferometer, the two beams recombined and passed through an aperture set to block stray reflections. A second AR-coated optical window then split the beams by passing the visible light and reflecting the infrared light. The visible interference signal was measured with a dc-coupled silicon photoconductive detector (Thor Labs PDA36A). The infrared interference signal was measured with an ac-coupled lead selenide photoconductive detector (Thor Labs PDA20H). The photodetector signals were recorded using a 16-bit data acquisition system (National Instruments PCI-6225) and a custom LabVIEW VI.

Special care was taken to isolate the system from external sources of noise. The entire system was built on an optical table (VERE Incorporated 48724E1FM) with air-spring vibration isolation legs (VERE Incorporated VIL30) to reduce noise from external vibrations. The interferometer was enclosed inside a acrylic box, shown in Figure 6.4 to minimize noise from air currents and acoustics in the room. The optical chopper, which vibrates due to its mechanical motion, was placed on a cantilever arm which held it in place over the optical table without
making contact with the floating table. Finally, all cables were aligned so that they left the table together at a specific location and then passed through a large apparatus which individually clamped each cable firmly in place to prevent cable vibrations from affecting the table.

6.2.4 Experimental Procedure and Data Acquisition

Before acquiring data, the alignment of the interferometer was checked to make sure there was acceptable contrast for both wavelengths. If not, adjustments were made to the reference arm mirror tip and tilt. If suitable alignment was not achieved, then the full interferometer was realigned.

Next, a “disturbance” test was conducted. The purpose of this test was to determine the voltage values corresponding to 0 and $\pi$ in the interferograms for each detector. This was accomplished by collecting eight seconds of photodetector data as the pump was turned on and the plasma chamber was evacuated. Reducing the pressure inside the chamber resulted in the interferograms cycling multiple times through their full 0 to $\pi$ range. As it will be seen in Section 6.3, post-processing this disturbance data is critical to calculating the phase of the interference data.

After acquiring the disturbance data, the plasma chamber was set to the appropriate pressure and closed off using the plug valve. When testing helium, the chamber was evacuated to below 1 Torr and then purged up to atmospheric pressure with helium gas. This process was repeated 10 times before filling to the desired testing pressure. Once the pressure was set, the interferometer enclosure was closed and the setup was left to sit for a few minutes to dampen out table vibrations.

As mentioned earlier, the two signals from the photodetectors were recorded in
Figure 6.4. Schematic of the dual-wavelength interferometer.
LabVIEW at 125 kHz. The voltage and current data of the plasma were recorded on the oscilloscope because its faster sampling time of 50 MHz could better resolve the short-duration current spikes that are characteristic of a dielectric barrier discharge. The oscilloscope’s timebase was configured to record a single modulation cycle. The timing of the entire acquisition system was triggered by the modulating function generator, as shown in Figure 6.5. Activating the modulating function generator would simultaneously trigger the oscilloscope and LabVIEW DAQ system as it produced 100 modulation cycles (Figure 6.3) at 25 Hz. This cycled the plasma on and off 25 times per second for four seconds with a peak voltage of about 30 kV. After the modulation cycles finished, the LabVIEW VI was set to record an additional four seconds of “plasma off” data for reference in the data processing.

6.3 Data Processing

Each experiment produced six measurements: (1) visible laser signal of the disturbance test, (2) infrared laser signal of the disturbance test, (3) visible signal of the plasma test, (4) infrared signal of the plasma test, (5) plasma voltage, and (6) plasma current. The visible and infrared signals were processed to determine the time-resolved change in plasma electron density and heavy particle density. The plasma voltage and plasma current data were processed to estimate the power draw of the plasma chamber. Each measurement will be discussed in detail in the following sub-sections.
Figure 6.5: Schematic between test acquisition and triggering system.
6.3.1 Photodetector Processing

The objective of the photodetector signal processing was to determine the plasma induced phase shifts recorded by both the visible and infrared detectors. From these phase shifts, a linear system of equations was set up to determine the time-resolved change in plasma electron density and change in heavy particle density. This section will detail the relevant steps performed to obtain these plasma parameters from the raw photodetector data.

Recall that both beams passed through an optical chopper before entering the interferometer (Figure 6.4). This produced a square-wave modulation in the signals of each detector which first needed to be removed before any analysis could be performed. To remove the chopping modulation, the data points in each chopper cycle were replaced with the corresponding peak-to-peak, as shown in Figure 6.6.
Next, it was necessary to convert the detector voltages into interference phase values. Figure 6.7 demonstrates the relationship between the interference irradiance and phase. The measured interference irradiance was modeled with a cosine wave where the maximum signal, $V_{\text{max}}$, corresponded to a phase of 0 radians. The minimum signal, $V_{\text{min}}$, corresponded to a phase of $\pi$ radians. The instantaneous voltage, $V(t)$, measured by a photodetector was related to the phase of the interference with \[ V(t) = V_{\text{min}} + \frac{V_{\text{max}} - V_{\text{min}}}{2} \left( 1 + \cos \phi(t) \right). \] \hfill (6.1)

The instantaneous phase of the interference could then be expressed as \[ \phi(t) = \cos^{-1} \left( 2 \left( \frac{V(t) - V_{\text{min}}}{V_{\text{max}} - V_{\text{min}}} \right) - 1 \right). \] \hfill (6.2)

Values of $V_{\text{max}}$ and $V_{\text{min}}$ were determined for each detector using the disturbance data (discussed in Section 6.2.4). During each disturbance test, the photodetector voltages were recorded as the vacuum pump was turned on thereby evacuating the plasma chamber. The reducing pressure inside the plasma chamber caused the interference to cycle through numerous 0 to $\pi$ intervals, as shown in Figure 6.8. It was observed that the frequency of the 0 to $\pi$ cycles decreased as the pressure dropped over time. As a result, the disturbance data contained a broad range of frequency content. The gains of the photodetectors were frequency dependent and so the values for $V_{\text{max}}$ and $V_{\text{min}}$ also depended on the modulating frequency. The disturbance data was bandpass filtered around the carrier signal modulation frequency so that the resulting $V_{\text{max}}$ and $V_{\text{min}}$ would be most applicable to the experimental data. There remains significant room for improvement in the determination of $V_{\text{max}}$ and $V_{\text{min}}$ which will be discussed further in 6.5.
Figure 6.7. Schematic showing the relation between the photodetector voltage and the phase of the measured interference.

Figure 6.8. Disturbance test data measured by the visible detector (top) and the infrared detector (bottom). Red lines show the data bandpass filtered around the carrier modulation frequency.
After finding the maximum and minimum interference voltages for each detector, Equation 6.2 was evaluated to determine the instantaneous phase measured at each photodetector. Figures 6.9 and 6.10 show the time-series of the phase shifts for each laser wavelength during plasma formation. From zero to four seconds, the plasma was ignited and extinguished 100 times according to the carrier signal amplitude modulation. After four seconds, the plasma remained off. This data demonstrated that the plasma was able to induce modulations in the interference signals for both the visible and infrared wavelengths.

To reduce the effect of noise, the time-series phase data were cycle-averaged across all 100 modulation cycles. Figures 6.11 and 6.12 show the cycle-averaged phase shifts for the visible and infrared detectors respectively. The data from each cycle are shown as points and the cycle-averages are shown as lines. It was observed that the direction of the visible and infrared phase shifts were opposite one another. It was also noted that the shift directions could change from test-to-test. This was explained by recognizing that the shift direction was dependent on the initial phase of the interference, which was arbitrarily by set by the interferometer alignment. Thus, any adjustment in the mirror positions or alignment would change the initial phase of the interference.

It was known from Equation 3.71 that the plasma induced phase shifts at both wavelengths should be negative considering that $\Delta n_e > 0$ and $\Delta n_h < 0$. Furthermore, to solve for the changes in the plasma densities, the relative phase difference was required, not the absolute phase shift. Therefore, each phase shift was converted into a relative phase difference and then forced to be negative. The resulting phase differences measured at each detector are shown in Figure 6.13.

Finally, the plasma induced phase differences, $\Delta \phi$, were used to determine the
Figure 6.9. Time series of phase measured during an experiment from the visible detector (top) and infrared detector (bottom).

Figure 6.10. Zoomed view of data from Figure 6.9 showing the visible detector signal (top) and infrared detector signal (bottom).
time-resolved changes in electron density and heavy particle density. For each point in time, the linear system of equations in Equation 3.72 was constructed using the phase shift data from each wavelength,

\[
\frac{1}{4\pi L} \begin{bmatrix}
\Delta \phi_{\lambda_1,j} \\
\Delta \phi_{\lambda_2,j}
\end{bmatrix} = \begin{bmatrix}
\frac{-q^2 \lambda_1^2}{8\pi^2 e^2 \epsilon_0 m} \frac{A}{n_{\text{h0}}} + \frac{B}{n_{\text{h0}} \lambda_1^2} \\
\frac{-q^2 \lambda_2^2}{8\pi^2 e^2 \epsilon_0 m} \frac{A}{n_{\text{h0}}} + \frac{B}{n_{\text{h0}} \lambda_2^2}
\end{bmatrix} \begin{bmatrix}
\Delta n_{e,j} \\
\Delta n_{h,j}
\end{bmatrix},
\]  

(6.3)

where the subscript \( j \) denotes the time index, \( \lambda_1 = 0.633 \ \mu m \), and \( \lambda_2 = 3.39 \ \mu m \). Each system was solved using a constrained least-squares solver (MATLAB \texttt{lsqlin}) to give values for the time-resolved changes in electron density and heavy particle density. From the change in heavy particle density, the time-resolved temperature change of the plasma was found using Equation 3.57. Experimental determinations for \( \Delta n_e, \Delta n_h \), and \( \Delta T \) are presented in Section 6.4.

6.3.2 Power Processing

During each experiment, the voltage and current of the plasma were measured. Due to the amplitude modulation of the carrier signal (Figure 6.3), it was necessary to process the electrical data over each cycle of the carrier waveform. For each carrier cycle, the peak-to-peak voltage and average power were calculated. Note that the actual voltage potential was twice the measured value because the ac signals driving the plasma were 180° out-of-phase. Average power was calculated using Equation 5.7.

Figure 6.14 shows typical current-voltage data acquired over one modulation cycle, clearly demonstrating the discharge current spikes that are characteristic of a dielectric barrier discharge. Furthermore, the circular profile of the current-voltage plot is indicative of the capacitive nature of the plasma circuit. Figures
Figure 6.11. Cycle-averaged phase shift measured by the visible detector for plasma on (blue) and plasma off (black).

Figure 6.12. Cycle-averaged phase shift measured by the infrared detector for plasma on (green) and plasma off (black).
6.15 and 6.16 show typical voltage and current characteristics of the plasma over one modulation cycle. Data such as these were processed after each experiment to determine the voltage and power characteristics of the plasma to supplement the interferometer measurements.

6.4 Results and Discussion

Data was collected for air and helium plasmas at a background pressure of 20 Torr and a maximum voltage potential of about 30 kV. The plasma carrier signal was a sine wave at 2 kHz and modulated at 25 Hz for 100 cycles using the custom modulation scheme shown in Figure 6.3. The length of the plasma was approximately 1.5”. For each gas, tests were performed following the procedure described in Section 6.2.4.
Voltage and current data was collected during each test and processed to obtain the peak-to-peak voltage and power, shown in Figures 6.17 and 6.18 for the air and helium tests. It was observed that the modulation shape and maximum voltage potential were identical for both gasses. In Figures 6.19 and 6.20, the computed power of the air and helium plasma tests are presented. For both gasses, the measured power was about 3 watts. The accuracy of the power measurement was dependent on how well the current spikes from plasma were resolved during data acquisition. However, because the measurement technique was consistent for all tests, it was considered valid to compare the power values relative to one another.

Photodetector data from the dual-wavelength interferometer was processed to determine the cycle-averaged plasma induced phase difference for each wavelength. Figures 6.21 and 6.22 display the phase differences for the air and helium tests. It was noticed that data for the visible laser had a considerably larger spread in...
Figure 6.15. Voltage versus time over one modulation cycle with peak-to-peak values (solid) and root-mean-square values (dashed).

Figure 6.16. Current versus time over one modulation cycle with root-mean-square values (dashed).
Figure 6.17. Peak-to-peak voltage measurements from the air plasma tests.

Figure 6.18. Peak-to-peak voltage measurements from the helium plasma tests.
Figure 6.19. Power measurements from the air plasma tests.

Figure 6.20. Power measurements from the helium plasma tests.
comparison with the infrared laser. This was attributed to the fact that the visible wavelength was more susceptible to temperature differences in the background gas. When pumping down the plasma chamber and also when forming plasma, the temperature of the background gas was inadvertently changed. If sufficient time was not given before acquiring the next test, then the drift in gas temperature could have caused fluctuations that would have been most noticeable in the visible laser measurements.

The time-resolved changes in plasma electron density and heavy particle density were found using the plasma induced phase difference from each test. Figures 6.23 and 6.24 present the changes in electron density for both air and helium, showing the individual data from each test and the total average from all tests. Error bars are shown indicating the standard deviation. The large variance in the helium electron density profiles was due to the aforementioned variations in the phase differences in Figure 6.22. As was expected, the electron density profiles resembled the shape of the modulating waveform. However, in the regions where the voltage was ramping up or ramping down, the least-squares solver seemed to produce less consistent results. At maximum voltage, the air and helium data both featured maximum electron densities slightly above $10^{14}$ cm$^{-3}$, which were within the expected range. Dong et al. measured electron densities near $10^{15}$ cm$^{-3}$ for a dielectric barrier discharge at pressures from 70 Torr to 375 Torr [20]. Considering that the experiments presented here were performed at much lower pressure and with a different plasma geometry, the values for electron densities around $10^{14}$ cm$^{-3}$ was deemed reasonable. It was surprising, however, that almost no change was observed in the electron density profiles between the air and helium tests. This is something that will be discussed further in this chapter.
Figure 6.21. Cycle-averaged phase differences from the air plasma tests.

Figure 6.22. Cycle-averaged phase differences from the helium plasma tests.
Figure 6.23. Time-resolved change in electron density from the air plasma tests (points) shown with the total average of all tests (line). Error bars indicate the standard deviation between tests.

Figure 6.24. Time-resolved change in electron density from the helium plasma tests (points) shown with the total average of all tests (line). Error bars indicate the standard deviation between tests.
In Figures 6.25 and 6.26, the time-resolved changes in heavy particle density (and temperature) for both the air and helium tests are presented. It was noticed that the least-squares solver performed well in determining the change in heavy particle density because the profiles appeared more continuous than the profiles in Figures 6.23 and 6.24. Furthermore, good agreement was observed between the individual tests for each gas. In comparing the air to helium, however, it was noticed that the calculated temperature change for the helium plasma was about three times larger than that for air. This was completely against the expectations for the experiments. One possible explanation for this occurrence is that the helium inside the plasma chamber was contaminated with air. If enough air was in the system, then solving Equation 6.3 using dispersion constants (A and B) for helium would not be valid and the resulting change in temperature would be biased much higher than it should be. This diagnosis explains the observations in the helium temperature profile. In addition, it also explains why the power, phase, and electron density results for air and helium were all almost identical. Recommendations for correcting this problem will be presented in Section 6.5.

In comparing the temporal profiles for electron density and heavy particle density, it was noted that there was a distinct difference in their shapes. In the electron density profiles, the plateaus corresponding to constant maximum voltage potential were much wider with sharper corners, indicating that the electrons had responded much faster to the carrier modulation in comparison to the heavy particles. This result was expected due to the fact that the heavy particle time constant is much larger in comparison to the electron time constant 43.
Figure 6.25. Time-resolved change in heavy particle density (left axis) and temperature (right axis) from the air plasma tests (points) shown with the total average of all tests (line). Error bars indicate the standard deviation between tests.

Figure 6.26. Time-resolved change in heavy particle density (left axis) and temperature (right axis) from the helium plasma tests (points) shown with the total average of all tests (line). Error bars indicate the standard deviation between tests.
6.5 Conclusions

In this chapter, dual-wavelength interferometry was used to determine time-resolved changes in electron density and gas temperature of the cylindrical double dielectric barrier discharge geometry presented in Chapter 4. The dual-wavelength technique is advantageous to single-wavelength interferometry because it does not require any assumptions in the determination of the plasma densities. Measured values for plasma electron density were shown to be on the order of $10^{14}$ cm$^{-3}$, which was within the expected range [20]. Temperature data showed that the gas temperature increased by about 35 K. Although the data presented were preliminary tests, the measurement system has demonstrated the ability to measure time-resolved changes in electron density and gas temperature during plasma generation.

In working with this interferometer, potential sources of error were identified which could improved upon to better the measurement system. A major source of error was in the determination of the minimum and maximum phase voltages ($V_{\text{max}}$ and $V_{\text{min}}$) of the interference pattern. In the work presented, these values were determined from processing disturbance data generated by decreasing the pressure inside the plasma chamber. It should be noted that other disturbance sources had previously been tested, including using speakers and vibrating motors, but were all found to produce inconsistent results. It remains necessary to find a reliable disturbance technique capable of producing cyclical 0 to $\pi$ phase shifts for both wavelengths. This could potentially be solved by using a high-precision motorized stage to jog the reference mirror back and forth at constant velocity.

As noted earlier, it was suspected that the helium tests presented in Section 6.4 were contaminated with air. The pressure system must be improved so that
gasses may be confidently changed without concern of air contamination. This most likely would be solved by replacing the current pressure manifold with a glassware vacuum mixing manifold and by replacing all of the Tygon tubing with vacuum tubing rated for helium.

Finally, it was observed during experiments that the phase of the interferometer tended to drift between experiments. The interferometer required constant adjustments between tests to ensure that the starting phase was not near 0 or $\pi$ where the sensitivity was the lowest. In these starting locations, there was also a higher probability of interference phase wrapping, which was undesirable because it complicated the data processing. More investigations are necessary to determine the source of the phase drifts in order to prescribe a solution.

In summary, preliminary tests show that the dual-wavelength interferometer was effective in determining time-resolved changes in electron density and gas temperature of a dielectric barrier discharge. More work is necessary to improve repeatability and reduce certainty in the measurements. Ultimately, this system needs to be validated against a second plasma diagnostics technique, such as terahertz time-domain spectroscopy [38], to provide confidence in the measurements.
7.1 Introduction

Fourier transform infrared (FT-IR) spectrometry is an experimental measurement technique which was first developed in the 1950s \cite{11} and is widely active today. These systems use a continuous spectrum of infrared light to probe a sample material to provide information about the chemical makeup of the material. FT-IR features several advantages in comparison with dispersive or grating spectroscopy techniques \cite{18, 22, 37} and have evolved from traditional transmission spectroscopy into more advanced techniques such as attenuated total reflectance (ATR) \cite{53, 55}, diffuse reflectance \cite{27}, spectral photoconductivity \cite{48}, and Raman spectroscopy \cite{16}.

In this chapter, the design of an FT-IR system to measure the electron and heavy particle densities of a plasma is introduced and demonstrated with simulated data. Conventional FT-IR systems are not suitable for these measurements because they cannot make direct measurements of dispersion (phase) which is required for plasma diagnostics. For this reason, a custom phase-sensitive FT-IR system will be presented.

The experimental setup for the phase-sensitive FT-IR interferometer is depicted in Figure 7.1. The system uses a heating element to emit an infrared
spectrum of light which is collimated using an off-axis parabolic mirror. The collimated beam passes into a Michelson interferometer where the two beams are split into two paths denoted the plasma arm and the reference arm.

In the plasma arm, the light travels through a plasma chamber similar to the ones used in Chapters 4 and 5. The light then reflects off a stationary mirror, passes through the plasma chamber a second time, and returns back to the beamsplitter. In the reference arm, the light travels through air and reflects off a moving mirror and travels back to the beamsplitter.

At the exit of the interferometer, the two beams recombine and generate interference corresponding to the phase difference between the two paths. The interference is focused onto an infrared detector which is connected to a data acquisition system which records the signal.

The moving mirror is the key element of an FT-IR system. It traverses at constant velocity in a path parallel to the beam path and the motion effectively changes the optical path length of the reference interferometer arm. To perform this motion, FT-IR systems typically use a voice-coil or a piezoelectric motorized stage. When the mirror is positioned such that the optical path lengths of both arms of the interferometer are equal, there is zero phase difference and no interference. At this location, all wavenumbers feature perfect constructive interference and the irradiance of the light exiting the interferometer is at its maximum value. This location is referred to as the zero-path difference (ZPD).

As the mirror displaces from the ZPD, the optical path lengths of the interferometer become imbalanced causing interference, which reduces the irradiance at the exit. Recall from Equation 2.2 that optical path length depends on the wavenumber (wavelength) of light. This means for at any position outside of the
each wavenumber will travel a different optical path length, thereby producing different amounts of interference. It follows that each wavenumber will produce perfect constructive interference at specific locations of the mirror and perfect destructive interference at a different locations of the mirror. As the mirror displaces with constant velocity, certain wavenumbers become amplified and others attenuated which effectively produces a continuous spectrum of amplified infrared wavenumbers exiting the interferometer. It is this characteristic of FT-IR systems that makes them especially appealing for plasma diagnostics.

In Section 3.5.4, it was explained that the determination of plasma electron density and heavy particle density requires measuring the phase of the plasma at multiple wavelengths. Figure 3.3 shows that infrared light is optimal for measuring both the electron and heavy particle effects. Thus, an FT-IR measurement system would be ideal for probing the plasma across such a range of infrared wavelengths.

The system is referred to as being phase-sensitive because the placement of the plasma chamber inside the interferometer allows for direct phase measurements of the plasma. This key difference is evident in comparing the plasma location of Figure 7.1 to the sample position in Figure 7.2.

The remainder of this chapter will present the design and simulation of a phase-sensitive FT-IR system suitable for plasma diagnostic measurements. Section 7.2 presents the background theory necessary for FT-IR analysis and Section 7.2 discusses the pertinent design considerations and requirements for the FT-IR system. Section 7.4 details the setup of a MATLAB simulation of the phase-sensitive FT-IR interferometer and Section 7.5 presents the results of the simulations.
Figure 7.1. Schematic of the phase-sensitive FT-IR interferometer for plasma measurements.

Figure 7.2. Schematic of a traditional FT-IR interferometer.
7.2 Theory

The electric fields of the infrared radiation inside the interferometer are modeled as one-dimensional waves. Accounting for interference, the irradiance of the two beams at the exit of the interferometer is described with Equation 3.67, which is restated below

\[ I(z, t) = \frac{I_0}{2} (1 + \cos [k\Delta z + \Delta \phi]). \] (7.1)

Recall that \( \Delta z = z_2 - z_1 \) and \( \Delta \phi = \phi_2 - \phi_1 \) where “1” denotes the arm with the plasma and “2” denotes the arm with the moving mirror. If \( \phi_1 \) and \( \phi_2 \) represent the total phase shifts of each beam due to the optical path lengths through the interferometer, then it is valid to assert that \( z = 0 \) at the exit of the interferometer. It follows that the extra path encountered in “2” due to the mirror displacement may then be accounted for by any non-zero \( z_2 \) values.

Let \( k = 2\pi\nu \) where \( \nu \) is the spectroscopic wavenumber and let \( z_2 \rightarrow \delta \) where \( \delta \) represents the change in optical path length due to the mirror displacement. If \( x \) is the physical mirror displacement then \( \delta = 2Nx \) where \( N \) is the refractive index of the background gas of the interferometer (air) and the factor of two accounts for the double-pass of the Michelson interferometer. The irradiance at the exit of the interferometer at a single wavenumber is then

\[ I(\delta, \nu) = \frac{I_0}{2} (1 + \cos [2\pi\nu\delta + \Delta \phi]). \] (7.2)

The infrared source is emits a continuous spectrum of infrared radiation with a
spectral radiance of \( U(\nu) \) and an irradiance of

\[
I_0(\nu) = U(\nu)\epsilon \Theta \Delta
\]

(7.3)

where \( \epsilon \) is the emissivity of the infrared source, \( \Theta \) is the throughput of the optical system, and \( \Delta \nu \) is the wavenumber resolution of the FT-IR system. Without interference, \( I_0(\nu) \) would ideally be the same at the entrance and exit of the interferometer. In reality, they are not due to optical losses, transmittance properties of the optical system, and the spectral response of the detector. Without interference, the effective signal received at the detector would then be

\[
B(\nu) = \frac{I_0}{2} = \frac{1}{2} U(\nu)\epsilon \Theta \Delta \nu T(\nu) R(\nu) \xi
\]

(7.4)

where \( T(\nu) \) is the total transmittance spectrum of the optical system, \( R(\nu) \) is the responsivity of the detector, and \( \xi \) is the overall optical efficiency of the optical system. Considering the respective units for each term in Equation 7.4

\[
V = (W \text{ sr}^{-1} \text{ cm}^{-2} (\text{cm}^{-1})^{-1}) (\text{sr cm}^2) (\text{cm}^{-1}) (V \text{ W}^{-1}),
\]

(7.5)

shows that \( B(\nu) \) has units of volts. Note that transmittance and efficiency are unitless parameters valued from zero to one. The interference signal at the detector is then

\[
S_\nu(\delta, \nu) = B(\nu) (1 + \cos [2\pi \nu \delta + \Delta \phi]).
\]

(7.6)

The subscript \( \nu \) in 7.6 is used to remind the reader that this the interference signal for an individual wavenumber of the light spectrum emitted from the infrared source. The total interference signal (interferogram) produced by the light exiting
the interferometer is found by superimposing the signals from all wavenumbers together,

\[
S(\delta) = \int_{0}^{\infty} S_\nu(\delta, \nu) d\nu \\
= \int_{0}^{\infty} B(\nu) (1 + \cos [2\pi \nu \delta + \Delta \phi]) d\nu, \quad (7.7)
\]

\[
= \int_{0}^{\infty} B(\nu) d\nu + \int_{0}^{\infty} B(\nu) \cos [2\pi \nu \delta + \Delta \phi] d\nu.
\]

This is rewritten as

\[
S(\delta) = \frac{1}{2} S(0) + \int_{0}^{\infty} B(\nu) \cos [2\pi \nu \delta + \Delta \phi] d\nu. \quad (7.8)
\]

by considering that at the zero path difference, or \( \delta = 0 \), the interference is

\[
S(0) = 2 \int_{0}^{\infty} B(\nu) d\nu. \quad (7.9)
\]

Typically, infrared detectors used in FT-IR systems are ac-coupled which implies that the measured signal contains only the modulating component and no dc-component (mean-removed). Therefore, the dc term in Equation (7.8) is removed and the interferogram signal is redefined as

\[
S(\delta) = \int_{0}^{\infty} B(\nu) \cos [2\pi \nu \delta + \Delta \phi] d\nu. \quad (7.10)
\]

Noticing that the interferogram \( S(\delta) \) is a \textit{real} quantity and asserting that the signal spectrum \( B(\nu) \) is Hermitian, Equation (7.10) is rewritten as

\[
S(\delta) = \frac{1}{2} \int_{-\infty}^{\infty} B(\nu) \exp [i (2\pi \nu \delta + \Delta \phi)] d\nu. \quad (7.11)
\]
Note that this treatment is non-trivial and the reader is encouraged to see Bell 1972 for a full proof [11]. The constant and exponential phase term are absorbed into $B(\nu)$ to give

$$S(\delta) = \int_{-\infty}^{\infty} B'(\nu) \exp[i(2\pi\nu\delta)] d\nu,$$

where the complex interferogram signal spectrum is

$$B'(\nu) = \frac{1}{2} B(\nu) \exp[i\Delta\phi] = \frac{1}{4} U(\nu) e^{\Theta \Delta\nu T(\nu)} R(\nu) \xi \exp[i\Delta\phi].$$

Equation (7.12) shows that the interferogram signal $S(x)$ is the inverse Fourier transform of the complex signal spectrum $B'(\nu)$. Thus, the complex signal spectrum, which contains information about the transmittance and the phase difference through the interferometer, can be found from the Fourier transform of the measured interferogram,

$$B'(\nu) = \int_{-\infty}^{\infty} S(\delta) \exp[-i2\pi\nu\delta] d\delta.$$

The phase shift between the two arms of the interferometer are found using

$$\Delta\phi(\nu) = \tan^{-1} \left( \frac{\text{Im} B'(\nu)}{\text{Re} B'(\nu)} \right).$$

This measurement system is referred to as a phase-sensitive FT-IR interferometer because by placing the sample (plasma) inside the interferometer, it becomes possible to measure determine $\Delta\phi$ between across a spectrum of wavenumbers, as evident in Equation (7.15). In traditional FT-IR systems, the sample is placed
outside of the interferometer so $\Delta \phi = 0$. In this case, analysis of a measured interferogram would return only the transmittance information, or $B'(\nu) = B(\nu)$, for absorption spectroscopy purposes.

7.3 Design

The FT-IR interferometer is designed around a list of experimental parameters shown in Table 7.2. The remainder of this section will present subsequent calculations necessary to design the FT-IR system which satisfies these design parameters.

7.3.1 Mirror Travel Distance

As the mirror moves from $x = 0$ to $x = l$, the data acquisition system records $N$ points equally spaced $\Delta x$ apart,

$$N = \frac{l}{\Delta x}. \quad (7.16)$$

The total number of points may also be derived in the spectral-domain. By way of the Fourier transform, the spectra of the time-domain signal ranges from $-\nu_{\text{max}}$ to $\nu_{\text{max}}$ at a resolution of $\Delta \nu$. Therefore, the total number of points acquired during a single mirror scan from $x = 0$ to $x = l$ must be

$$N = \frac{2\nu_{\text{max}}}{\Delta \nu}. \quad (7.17)$$

Using (7.16) and (7.17) an expression is found for the maximum mirror displacement,

$$l = \frac{2\Delta x \nu_{\text{max}}}{\Delta \nu}. \quad (7.18)$$
### TABLE 7.1

**FT-IR INTERFEROMETER DESIGN PARAMETERS**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum wavenumber</td>
<td>( \nu_{\text{min}} )</td>
<td>833 (12)</td>
<td>cm(^{-1}) ((\mu)m)</td>
</tr>
<tr>
<td>Maximum wavenumber</td>
<td>( \nu_{\text{max}} )</td>
<td>1250 (8)</td>
<td>cm(^{-1}) ((\mu)m)</td>
</tr>
<tr>
<td>Wavenumber resolution</td>
<td>( \Delta \nu )</td>
<td>4</td>
<td>cm(^{-1})</td>
</tr>
<tr>
<td>Mirror velocity</td>
<td>( V' )</td>
<td>1.25</td>
<td>cm/s</td>
</tr>
<tr>
<td>Interferogram resolution</td>
<td>( \Delta x )</td>
<td>0.5</td>
<td>(\mu)m</td>
</tr>
<tr>
<td>Beam diameter</td>
<td>( D )</td>
<td>6.35</td>
<td>mm</td>
</tr>
<tr>
<td>Infrared source area</td>
<td>( A_S )</td>
<td>1.8</td>
<td>mm(^2)</td>
</tr>
<tr>
<td>Infrared source temperature</td>
<td>( T )</td>
<td>1200</td>
<td>°C</td>
</tr>
<tr>
<td>Infrared source emissivity</td>
<td>( \epsilon )</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>Detector specific detectivity</td>
<td>( D^* )</td>
<td>( 2 \times 10^{10} )</td>
<td>cm Hz(^{1/2})W(^{-1})</td>
</tr>
<tr>
<td>Detector area</td>
<td>( A_D )</td>
<td>1.0</td>
<td>mm(^2)</td>
</tr>
<tr>
<td>Detector half-angle</td>
<td>( \alpha_M )</td>
<td>30</td>
<td>degrees</td>
</tr>
<tr>
<td>Detector responsivity</td>
<td>( R )</td>
<td>1000</td>
<td>volts/watt</td>
</tr>
<tr>
<td>Rms mirror position error</td>
<td>( \Delta x_{\text{rms}} )</td>
<td>20</td>
<td>nm</td>
</tr>
<tr>
<td>Overall optical efficiency</td>
<td>( \xi )</td>
<td>0.1</td>
<td></td>
</tr>
</tbody>
</table>
The spatial resolution is rewritten in terms of the mirror velocity and sampling frequency, $F_s$,

$$\Delta x = \frac{V'}{F_s}. \quad (7.19)$$

By the Shannon-Nyquist sampling theorem, the maximum measured frequency is equal to one-half the sampling frequency, $f_{\text{max}} = F_s/2$, where the maximum frequency present is $f_{\text{max}} = 2V'\nu_{\text{max}}$. Therefore, the maximum wavenumber in terms of sampling frequency is

$$\nu_{\text{max}} = \frac{F_s}{4V'}. \quad (7.20)$$

Combining Equations 7.18, 7.19, 7.20 gives the simplified expression for the mirror scan distance

$$l = 2\frac{F_s}{4V'} \cdot \frac{V'}{F_s} \cdot \frac{1}{\Delta \nu} = \frac{1}{2\Delta \nu}. \quad (7.21)$$

Thus, the mirror scan length is purely a function of the desired FT-IR interferometer wavenumber resolution. Greater mirror scan distances yield higher resolution spectra, but requires more time per scan. The desired resolution of $4 \, \text{cm}^{-1}$ requires a scan length of $l = 0.125 \, \text{cm}$.

### 7.3.2 Single-Sided vs. Double Sided Scan

When acquiring data with an FT-IR system, there are two options: a *single-sided* scan or a *double-sided* scan. A single-sided scan acquires data as the mirror moves from $x = 0$ to $x = l$, whereas a double sided scan acquires data from $x = -l$ to $x = l$. The double-sided scan takes twice as much time and generates twice as
much data, but it allows for phase determination and corrections.

With no dispersion between the interferometer arms, the resulting interferogram is perfectly symmetric about $x = 0$. When dispersion is present, the interferogram is no longer symmetric due to the exponential phase shift in Equation 7.11. To capture this asymmetry it is necessary to acquire data on both sides of $x = 0$. Note that a double-sided scan acquires twice as much data,

$$N_{DS} = 2N,$$

but does not change the spectral resolution, $\Delta \nu$, of the interferometer. The double-sided scan may be equivalently thought of as acquiring twice as many points over a scan length $l$. Thus, for a double-sided scan the effective sampling interval is

$$\Delta x_{DS} = \frac{\Delta x}{2},$$

and the effective scan interval is $x = -\frac{l}{2}$ to $\frac{l}{2}$. The change in $\Delta x$ increases the maximum wavenumber in the spectrum to be

$$\nu_{max,DS} = 2\nu_{max}.$$  

### 7.3.3 Oversampling Ratio

From Equation 7.20, the minimum sampling frequency required to resolve the maximum wavenumber $\nu_{max}$ is

$$F_{S,min} = \frac{4V'}{\nu_{max}}.$$  

Data may be oversampled for better resolution of the interferogram, and the
oversampling ratio is

$$\beta = \frac{F_S}{F_{S,\text{min}}} = \frac{1}{4\nu_{\text{max}} \Delta x_{DS}}.$$

(7.26)

Note that a double-sided scan inherently doubles the oversampling ratio. It should be expected from Fourier theory that faster sampling in the time-domain increases the maximum wavenumber in the frequency-domain, so the new maximum wavenumber is

$$\nu_{\text{max},OS} = \frac{1}{4\Delta x_{DS}}.$$

(7.27)

For the desired parameters listed in Table [7.2], the minimum sampling frequency is 6250 Hz. The oversampling ratio is 8 making the actual sampling frequency 50 kHz. The maximum wavenumber in the spectra would be 10,000 cm\(^{-1}\).

### 7.3.4 Collimating and Focusing Optics

The degree of collimation of the infrared source has a significant effect on the achievable wavenumber resolution $\Delta \tilde{\nu}$ of the system. As the mirror travels away from the ZPD location, the off-axis rays have different optical path lengths than the on-axis rays. When the path difference between the on and off-axis rays equals $1/2\lambda$, they become out of phase with each other and the fringe contrast at the detector disappears completely. To a first order approximation, the largest allowable divergence half-angle for a given resolution is [31]

$$\alpha_{\text{max}} = \left( \frac{\Delta \tilde{\nu}}{\nu_{\text{max}}} \right)^{1/2}.$$

(7.28)

Using the design values given in Table [7.2], it is found that the maximum allowable
half-angle for divergence is about 57 milliradians. This value is a vital constraint considered when selecting the focal lengths of the collimating and focusing optics, as shown in Figure 7.1. The minimum focal length of the collimating off-axis parabolic mirror is

\[ f_c = \frac{2}{2 \sin \left( \frac{\Delta \nu}{\nu_{\text{max}}} \right)^{1/2}}, \quad (7.29) \]

where \( D \) is the beam diameter. The infrared source is placed exactly at the focal point of this collimating off-axis parabolic mirror.

The focal length, \( f_f \), of the focusing off-axis parabolic mirror must be selected such that the light completely fills the detector area. This is determined by considering the magnification of the imaging system, which is equal to the ratio of the focusing focal length to the collimating focal length,

\[ M = \frac{f_f}{f_c}. \quad (7.30) \]

The magnification may also be expressed as the ratio of the height of the detector, \( h_D \) to the height of the infrared source \( h_S \), which is equivalent to

\[ M = \left( \frac{A_D}{A_S} \right)^{1/2}, \quad (7.31) \]

where \( A_D \) is the area of the detector and \( A_S \) is the area of the infrared source. Combining Equations 7.30 and 7.31 gives an expression for the length of the focusing mirror as a function of the collimating focal length and the areas of the detector and infrared source,

\[ f_f = f_c \left( \frac{A_D}{A_S} \right)^{1/2}. \quad (7.32) \]
Using focal lengths of $f_c$ and $f_f$ will perfectly image the entire source area onto the detector area. Considering the design parameters in Table 7.2, the minimum collimating focal length is 56 mm. To appropriately image the infrared source onto the detector, the focusing off-axis mirror should have a focal length of 42 mm.

7.3.5 Infrared Source

In FT-IR systems, the infrared source is typically a small heating element which emits a blackbody spectrum of light in the visible and infrared ranges. The spectral range and energy output from the infrared source is a function of temperature and emissivity. The spectral radiance, or the power emitted per unit area per steradian per wavenumber, of a perfect blackbody source is given by the Planck equation

$$U(\nu) = \frac{2hc^2\nu^3}{\exp\left(\frac{hc}{kT}\right) - 1} \text{ W cm}^{-2} \text{ sr}^{-1} \text{ (cm}^{-1})^{-1}$$  \hspace{1cm} (7.33)

where $h$ is Planck’s constant ($6.62606957 \times 10^{-34}$ W s$^2$), $c$ is the speed of light ($2.9972458 \times 10^{10}$ cm/s), and $k$ is Boltzmann’s constant ($1.3806488 \times 10^{-23}$ J/K). It’s obvious that an infrared source operating at a higher temperature will emit more radiant energy which would result a larger signal at the detector. It is desirable to use the hottest infrared source possible in an FT-IR system. This sometimes requires liquid cooling of the optics to prevent misalignments as they heat up.

Figure 7.3 shows the blackbody spectral brightness for a range of different infrared source temperatures. It is evident that higher temperatures cause higher spectral energy to be emitted from the source. However, as the temperature
increases the location of the peak irradiance moves to higher wavenumbers (shorter wavelengths) making the source less efficient for the infrared region.

The dashed black lines in Figure 7.3 indicate the wavenumber range listed in Table 7.2. This range between 833-1250 cm\(^{-1}\) is not ideal for a source operating at 1200\(^\circ\) C but is practical for plasma measurements. Lower wavenumbers would have an even weaker signal at the detector and higher wavenumbers would require more expensive optical components.

![Figure 7.3. Blackbody spectral radiance versus wavenumber shown for four different temperatures.](image-url)
7.3.6 Coherence Length

The coherence length, $l_c$, of the infrared radiation in the FT-IR interferometer is estimated with

$$l_c = \frac{\lambda_{pk}^2}{\Delta \lambda} \quad (7.34)$$

where $\lambda_{pk}$ is the wavelength of peak irradiance and $\Delta \lambda$ is the spectral linewidth of the source spectrum (full width half maximum). Recall that the wavelength is simply the inverse of the spectroscopic wavenumber $\lambda = \nu^{-1}$. For a perfect blackbody source operating at 1200°C, the coherence length of the full, unfiltered spectrum would be just 1.8 µm.

From Equation 7.34, the coherence length may be increased either by increasing the peak wavelength $\lambda_{pk}$ or by filtering the spectrum to reduce the spectral linewidth $\Delta \lambda$. Filtering the full blackbody spectrum to the spectral range listed in Table 7.2 (833-1250 cm$^{-1}$) increases the coherence length to 14 µm. It should be emphasized that this is still an extremely short coherence length which implies that to achieve this an experimental setup would need to be aligned with micron-precision to achieve this.

7.3.7 Signal-to-Noise Estimation

The signal-to-noise is the ratio of the signal power, $S'$, to the noise power, $N'$, in the optical system,

$$SNR = \frac{S'}{N'} \quad (7.35)$$

Accounting for transmittance losses and also the fact that the infrared source is
not a perfect blackbody emitter, the effective spectral radiance through the FT-IR system is

\[
U'(\nu) = U(\nu)\epsilon T_{\text{sys}}(\nu),
\]

(7.36)

where \( T_{\text{sys}}(\nu) \) is the overall system transmittance and \( \epsilon \) is the emissivity of the infrared source. Recall that \( U'(\nu) \) has units of power per unit area per steradian per wavenumber. Therefore, the total radiant power, or signal power, of the system is the product of the effective spectral radiance, the throughput of the system \( \Theta \), and the wavenumber \( \Delta\nu \),

\[
S' = U(\nu)T_{\text{sys}}(\nu)\epsilon\Theta\Delta\nu\xi.
\]

(7.37)

The latter term \( \xi \) is the overall optical efficiency of the system which is used to account for other losses in the optical system. It usually has a value of around 0.1 [31]. The value used for throughput is the minimum throughput of the system, limited by either the interferometer or the detector. The interferometer throughput is

\[
\Theta_I = \frac{\pi^2 D^2 \Delta\nu}{2\nu_{\text{max}}}
\]

(7.38)

where \( D \) is the diameter of the beams inside the interferometer. The detector throughput can be approximated with [31, 32]

\[
\Theta_D = 2\pi(1 - \cos\alpha_M)A_D
\]

(7.39)

where \( \alpha_M \) is the field-of-view half-angle of the detector. The throughput used in
Equation 7.37 is

\[ \Theta = \min \{ \Theta_I, \Theta_D \} . \]  

(7.40)

The noise equivalent power (NEP) of the detector is

\[ NEP = \frac{\sqrt{A_D}}{D^*} \]  

(7.41)

where \( A_D \) is the detector area and \( D^* \) is the specific detectivity of the detector.

Over a measurement of \( t \) seconds, the noise power is

\[ N^t = \frac{NEP}{\sqrt{t}} = \frac{\sqrt{A_D}}{D^* \sqrt{t}} \]  

(7.42)

The signal-to-noise estimation for the system becomes

\[ SNR = \frac{S^t}{N^t} = U(\nu)T_{sys}(\nu) \epsilon \Theta \Delta \nu \xi D^* \sqrt{\frac{t}{A_D}} . \]  

(7.43)

This is rewritten in terms of sampling frequency, \( F_S = t^{-1} \),

\[ SNR = \frac{U(\nu)T_{sys}(\nu) \epsilon \Theta \Delta \nu \xi D^*}{\sqrt{F_S A_D}} . \]  

(7.44)

which is then converted to decibels,

\[ SNR_{dB} = 10 \log \left( \frac{U(\nu)T_{sys}(\nu) \epsilon \Theta \Delta \nu \xi D^*}{\sqrt{F_S A_D}} \right) . \]  

(7.45)

The signal to noise ratio can be increased by increasing either the spectral radiance of the infrared source, the system throughput, the wavenumber resolution, or the detector specific detectivity. It may also be increased by decreasing the sampling frequency or using a smaller detector. Figure 7.4 shows the estimated SNR as a
function of wavenumber. For the parameters listed in 7.2, the maximum SNR is about 31 dB. Note that without changing the experimental setup, this SNR may be further increased by averaging the data from multiple mirror scans together. This concept will be discussed further in Section 7.4.3.

7.3.8 Moving Mirror Specifications

The key element in the FT-IR interferometer is the mechanism which scans the moving mirror back and forth. The precision of this movement is critical in determining the performance of the optical system. Ideally, the motor moves the mirror with perfectly constant velocity, but in reality it will always have some fluctuations. Any error in velocity causes the interferogram to be sampled at unequal intervals, which is equivalent to adding noise to the signal \[31\]. This
error can be represented as the root mean square of the mirror position error, $\Delta x_{rms}$ and the maximum obtainable signal-to-noise ratio is
\[
SNR_{max} = \frac{4}{\Delta x_{rms} \nu_{max}}.
\] (7.46)

Using the parameters in Table 7.2 ($\Delta x_{rms} = 20 \text{ nm}$), the maximum obtainable signal-to-noise ratio is about 32 dB. This is comparable to the SNR value found in the previous section.

In addition to the position error, the linearity of the mirror motion is also critical to the FT-IR interferometer performance. Linearity is a measure of the “straightness” of travel. Consider that if the mirror’s travel path is not perfectly parallel with the infrared beam, then the two extreme off-axis rays of the beam will have different optical path lengths during the motion. Any difference in the optical path length of the rays within the beam would produce a loss of fringe modulation. This becomes evident when the optical path difference is about 0.1$\lambda$ [31]. Considering $\gamma$ to be the maximum allowable motor tilt angle (the angle between the mirror movement and the beam path) it can be shown that [31]
\[
\gamma < \frac{1}{20D \nu_{max}}.
\] (7.47)

For small angles, $\gamma \times 100\%$ is equivalently the motor linearity. Over the travel distance $l$, the maximum transverse displacement of the mirror is
\[
l_t < l \gamma = \frac{1}{40D \Delta \nu \nu_{max}}.
\] (7.48)

The maximum allowable mirror tilt angle can be increased by decreasing either the beam diameter or the maximum wavenumber. For the design parameters
listed in Table 7.2, the maximum allowable tilt angle is 63 $\mu$rad and the maximum transverse mirror displacement is 80 nm. This high sensitivity to position error and movement linearity during the mirror travel puts extremely exacting requirements on the motor and demands very specialized equipment.

7.3.9 Summary of Design

Given the design parameters in Table 7.2, the previous sections of this chapter worked out pertinent requirements and characteristics for the phase-sensitive FT-IR interferometer. A summary of these results is listed in Table 7.3.9.

7.4 Simulation Setup

To evaluate the effectiveness of the FT-IR design presented in the previous section, the optical system was simulated in MATLAB. Figure 7.5 depicts a visual flowchart of the fundamental elements of the simulation. First, the blackbody spectrum is generated and windowed according to the design parameters in Table 7.2. Geometric optics is used to calculate the optical path lengths for each arm of the interferometer and Fourier optics is used to then generate the detector signal representing the interferogram. The system’s signal-to-noise ratio is estimated and noise is added to the interferogram accordingly. Finally, the simulated interferogram is analyzed to determine the plasma electron density and heavy particle density. The following sections will provide more details for each of these steps in the simulation. Results from the simulation are presented in Section 7.5.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of points per side</td>
<td>$N$</td>
<td>2500</td>
<td></td>
</tr>
<tr>
<td>Mirror scan distance</td>
<td>$l$</td>
<td>±0.125</td>
<td>cm</td>
</tr>
<tr>
<td>Oversampling ratio</td>
<td>$\beta$</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>Minimum sampling frequency</td>
<td>$F_{S,\text{min}}$</td>
<td>6250</td>
<td>Hz</td>
</tr>
<tr>
<td>Actual sampling frequency</td>
<td>$F_S$</td>
<td>50000</td>
<td>Hz</td>
</tr>
<tr>
<td>Maximum oversampled wavenumber</td>
<td>$\nu_{\text{max,OS}}$</td>
<td>10000</td>
<td>cm$^{-1}$</td>
</tr>
<tr>
<td>Maximum divergence</td>
<td>$\alpha_{\text{max}}$</td>
<td>57</td>
<td>mrad</td>
</tr>
<tr>
<td>Collimating focal length</td>
<td>$f_c$</td>
<td>56</td>
<td>mm</td>
</tr>
<tr>
<td>Focusing focal length</td>
<td>$f_f$</td>
<td>42</td>
<td>mm</td>
</tr>
<tr>
<td>Coherence length (unfiltered)</td>
<td>$l_c$</td>
<td>1.8</td>
<td>$\mu$m</td>
</tr>
<tr>
<td>Coherence length (filtered)</td>
<td>$l_{c,f}$</td>
<td>14</td>
<td>$\mu$m</td>
</tr>
<tr>
<td>Interferometer throughput</td>
<td>$\Theta_I$</td>
<td>0.006</td>
<td>sr cm$^2$</td>
</tr>
<tr>
<td>Detector throughput</td>
<td>$\Theta_I$</td>
<td>0.008</td>
<td>sr cm$^2$</td>
</tr>
<tr>
<td>Maximum SNR (optics)</td>
<td>$SNR_{\text{max,o}}$</td>
<td>31</td>
<td>dB</td>
</tr>
<tr>
<td>Maximum SNR (mirror)</td>
<td>$SNR_{\text{max,m}}$</td>
<td>32</td>
<td>dB</td>
</tr>
<tr>
<td>Moving motor linearity</td>
<td>$\gamma$</td>
<td>0.0063</td>
<td>%</td>
</tr>
<tr>
<td>Moving motor max displacement</td>
<td>$l_t$</td>
<td>80</td>
<td>nm</td>
</tr>
</tbody>
</table>
Figure 7.5. Flowchart for the phase-sensitive FT-IR interferometer simulation.
7.4.1 Modeling the System Transmittance

Transmittance is a material property valued over $[0, 1]$ which describes the fraction of incident radiation that passes through a material. A material with zero transmittance will not pass any radiation while a material with unity transmittance will pass all incident radiation. As the light propagates through the interferometer, its spectrum is filtered due to the transmittance properties of the beamsplitter, compensator, plasma chamber windows, and detector window. Considering the transmittance for each optical component, the total transmission spectrum for the system is

$$T_{\text{sys}}(\nu) = T_{\text{bs}}(\nu)^\alpha T_{\text{comp}}(\nu)^\beta T_{\text{window}}(\nu)^\gamma T_{\text{det}}(\nu)^\eta$$  \hspace{1cm} (7.49)$$

where the subscripts $bs$, $comp$, $window$, and $det$ denote the beamsplitter, compensator, plasma chamber windows, and detector window respectively. The superscripts $\alpha$, $\beta$, $\gamma$, and $\eta$ denote how many times the beam passes through each component. In the plasma arm of the interferometer, the beam travels through the beamsplitter three times, once through the compensator, four times through plasma chamber windows, and once through the detector window. For reference, Figure 7.6 shows the propagation of the beams through the beamsplitter and compensator. In the reference arm, the beam passes once through the beamsplitter, three times through the compensator, and once through the detector window.

The transmittance spectrum of the plasma arm will be lower than the reference arm due to the beam passing through the plasma chamber windows. Designing for the worst case (lowest transmittance), the superscript values of Equation 7.50...
are selected to be,

\[ T_{\text{sys}}(\nu) = T_{bs}(\nu)^3 T_{\text{comp}}(\nu) T_{\text{window}}(\nu)^4 T_{\text{det}}(\nu). \]  
(7.50)

The beamsplitter and compensator are germanium coated potassium bromide (KBr), the plasma chamber windows are zinc selenide (ZnSe) with an 8-12 micron anti-reflective coating, and the detector window is an uncoated zinc selenide. In terms of material, the overall system transmittance is

\[ T_{\text{sys}}(\nu) = T_{KBr}(\nu)^4 T_{ZnSe,AR}(\nu)^4 T_{ZnSe}(\nu). \]  
(7.51)

Table 7.3 summarizes the materials and transmittance properties and lists references for spectral transmittance data. Figure 7.7 shows the spectral transmitt-
tance data for each material with the overall system transmittance. After propagating through the interferometer, the beam is significantly attenuated across all wavenumbers. Maximum transmittance occurs near $\nu = 1000 \text{ cm}^{-1}$ which is roughly the center of the wavenumber range of the designed phase-sensitive FT-IR interferometer.

Figure 7.7. Spectral transmittance profiles each optical medium in the FT-IR interferometer.
<table>
<thead>
<tr>
<th>Component</th>
<th>Material</th>
<th>Lower Cutoff [cm$^{-1}$ ($\mu$m)]</th>
<th>Upper Cutoff [cm$^{-1}$ ($\mu$m)]</th>
<th>Data Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beamsplitter</td>
<td>KBr †</td>
<td>450 (22)</td>
<td>7000 (1.4)</td>
<td>[1]</td>
</tr>
<tr>
<td>Compensator</td>
<td>KBr †</td>
<td>450 (22)</td>
<td>7000 (1.4)</td>
<td>[1]</td>
</tr>
<tr>
<td>Windows</td>
<td>ZnSe ‡</td>
<td>833 (12)</td>
<td>1250 (8)</td>
<td>[42]</td>
</tr>
<tr>
<td>Detector Window</td>
<td>ZnSe</td>
<td>500 (20)</td>
<td>20000 (0.5)</td>
<td>[59]</td>
</tr>
</tbody>
</table>

† Features a multilayer Germanium coating.
‡ Features an anti-reflective coating with an 8-12 micron passband.
7.4.2 Determining the Optical Path Difference

The imbalance of optical path length (OPL) between the two arms of the interferometer is the mechanism responsible for generating interference at the exit of the interferometer. Using geometric optics, the optical path difference, OPD, between the two arms of the interferometer is determined, which is necessary in the next section to produce the interference pattern.

As the two beams travel through their respective paths inside the interferometer, each accumulates different optical path lengths as a result of passing through different materials. Figure 7.8 shows the all of the materials that the simulation accounts for in each arm of the interferometer. Index of refraction values for each material are listed in Table 7.4 and plotted versus wavenumber in Figure 7.9.

The OPL of each interferometer arm is

\[ OPL(\nu) = \sum_{\text{All materials}} N_i(\nu) l_i \]  

(7.52)

where the subscript \( i \) denotes a specific optical medium in the optical path, \( N_i(\nu) \) is the wavenumber dependent index of refraction, and \( l_i \) is the length of the particular medium.

In the plasma arm, the beam passes through the plasma chamber, reflects off the stationary mirror, passes through the plasma chamber again, and returns to the beamsplitter. The optical path length of this beam is

\[ OPL_1(\nu) = 2 [2N_{\text{air}}(\nu)l_{\text{air,1}} + 2N_{\text{window}}(\nu)l_{\text{window}} + 2N_{\text{gas}}(\nu)l_{\text{gas}} + \ldots \]  

\[ N_{\text{plasma}}(\nu)l_{\text{plasma}}] + 3N_{\text{bs}}(\nu)l_{\text{bs}} + N_{\text{comp}}(\nu)l_{\text{comp}} + N_{\text{air}}(\nu)l_{\text{gap}} \]  

(7.53)

In the reference arm, the beam passes through quiescent air, reflects off the moving
mirror, and returns to the beamsplitter. The total optical path length of this beam is

\[
OPL_2(\nu) = 2N_{\text{air}}(\nu)l_{\text{air},2} + N_{\text{bs}}(\nu)l_{\text{bs}} + 3N_{\text{comp}}(\nu)l_{\text{comp}} + 3N_{\text{air}}(\nu)l_{\text{gap}}.
\]  

(7.54)

Assuming that the beamsplitter and compensator are of identical material and thickness, the optical path difference between the two arms is

\[
OPD(\nu) = OPL_2(\nu) - OPL_1(\nu)
\]

\[
= 2[N_{\text{air}}(l_{\text{air},2} - 2l_{\text{air},2} + l_{\text{gap}}) - N_{\text{plasma}}l_{\text{plasma}} - \ldots
\]

\[
2(N_{\text{window}}l_{\text{window}} + N_{\text{gas}}l_{\text{gas}})].
\]

(7.55)

7.4.3 Simulating the Interferogram

To simulate the interferogram, Equation 7.12 is discretized such that

\[
S(\delta) = \sum_{\nu} B'(\nu) \exp[i(2\pi\nu\delta)] \Delta\nu
\]  

(7.56)

where the wavenumber \(\nu\) is defined over \([-\nu_{\text{max,OS}}, \nu_{\text{max,OS}}]\) with a spacing of \(\Delta\nu\). The retardation \(\delta = 2N_{\text{air}}(\nu)x\) is defined for the mirror displacement, \(x\), over \([-\frac{l}{2}, \frac{l}{2}]\) with a spacing of \(\frac{\Delta x}{2}\), where the factor of \(\frac{1}{2}\) accounts for the double-sided scan. The complex signal spectrum \(B'(\nu)\) is given with

\[
B(\nu) = \frac{1}{4} U(\nu)\epsilon\Theta T(\nu)R(\nu)\xi \exp[i\Delta\phi]
\]

(7.57)
Figure 7.8. Diagram of optical mediums in the paths of each arm of the interferometer.
Figure 7.9. Index of refraction values of different materials modeled in simulations.
## Expressions for Material Index of Refraction

<table>
<thead>
<tr>
<th>Material</th>
<th>Index of Refraction, ( N(\nu) = )</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air †</td>
<td>( 1 + \left( A + \frac{B}{\nu^2} \right) \frac{n_h}{n_{h0}} )</td>
<td>[21]</td>
</tr>
<tr>
<td>Gas †</td>
<td>( 1 + \left( A + \frac{B}{\nu^2} \right) \frac{n_h}{n_{h0}} )</td>
<td>[21]</td>
</tr>
<tr>
<td>Plasma †</td>
<td>( 1 - \frac{q^2}{8\pi^2 c^2 \epsilon_0 \omega^2} n_e + \left( A + \frac{B}{\nu^2} \right) \frac{n_h}{n_{h0}} )</td>
<td>[21]</td>
</tr>
<tr>
<td>ZnSe</td>
<td>( \left( 1 + \frac{4.2980149\nu^{-2}}{\nu^{-2} - 0.1920630}\right) + \frac{0.62776557\nu^{-2}}{\nu^{-2} - 0.37878260^2} + \frac{2.8955633\nu^{-2}}{\nu^{-2} - 46.994595^2} \right)^{1/2} )</td>
<td>[9]</td>
</tr>
<tr>
<td>KBr</td>
<td>( \left( 1.39408 + \frac{0.79221\nu^{-2}}{\nu^{-2} - 0.146^2} + \frac{0.01981\nu^{-2}}{\nu^{-2} - 0.173^2} \right) + \frac{0.15587\nu^{-2}}{\nu^{-2} - 0.187^2} + \frac{0.17673\nu^{-2}}{\nu^{-2} - 0.6061^2} + \frac{2.06217\nu^{-2}}{\nu^{-2} - 87.72^2} \right)^{1/2} )</td>
<td>[9]</td>
</tr>
</tbody>
</table>

† Values for \( A \) and \( B \) listed in Table A.1

which is equivalent to Equation 7.13 except the \( \Delta \nu \) term has been removed because it is already accounted for in Equation 7.56. The phase component of \( B'(\nu) \) is calculated with the expression for \( OPD \) in Equation 7.55

\[
\Delta \phi(\nu) = 2\pi \nu \text{OPD}(\nu).
\] (7.58)

Therefore, the total complex irradiance spectrum is

\[
B'(\nu) = \frac{1}{4} U(\nu) \epsilon T_{sys}(\nu) \Theta \xi R(\nu) \exp\left[i \left(2\pi \nu \text{OPD}(\nu)\right)\right].
\] (7.59)

Simulated interferograms for the FT-IR system may be generated using Equations 7.56 and 7.59. Note that these interferograms are mathematically perfect aside from computational round-off error, implying that the Fourier analysis would reconstruct the phase difference perfectly. Thus, these interferograms would not
yield any useful information about how the physical FT-IR system would actually perform. To remedy this, noise is added such that the resulting signal-to-noise ratio corresponds to the FT-IR system designed in Section 7.3. The noise power of the system is calculated with Equation 7.42. Noise data with maximum amplitudes equal to the square-root of the noise power value are superimposed onto the interferogram signal. The expression for the interferogram in Equation 7.56 becomes

\[ S(\delta) = \chi(\delta) + \frac{\Theta \xi \Delta \nu}{4} \sum_{\nu} U(\nu) T_{sys}(\nu) R(\nu) \exp[i(2\pi \nu OPD(\nu))] \exp[i(2\pi \nu \delta)] \]

(7.60)

where \( \chi(\delta) \) is a set of random numbers (noise) generated at each mirror position. The values of \( \chi(\delta) \) feature a uniform distribution and amplitudes ranging between \( \pm \sqrt{N} \).

To increase the SNR of the measurement system, multiple mirror scans may be added together in a process called co-adding. During an acquisition, the sign of the interferogram signal is positive and the sign of the noise is random [57]. By averaging multiple scans together, the magnitude of the signal is retained while magnitude of the noise is reduced. Over an infinite number of scans, the magnitude of the noise would be zero. The effect of co-adding improves the signal to noise ratio by [57]

\[ SNR \propto K^{1/2} \]

(7.61)

where \( K \) is the number of scans co-added together. Incorporating this into Equa-
tion 7.60 produces

$$S(\delta) = \chi(\delta) + \frac{\Theta \xi \Delta \nu K^{1/2}}{4} \sum_{\nu} U(\nu) T_{\text{sys}}(\nu) R(\nu) \exp\left[i (2\pi \nu \text{OPD}(\nu))\right] \exp\left[i (2\pi \nu \delta)\right].$$

(7.62)

To simulate FT-IR interferograms, Equations 7.33, 7.40, 7.51, and 7.55 are evaluated for the physical FT-IR system. These are substituted into Equation 7.62 which is evaluated over the full domains of $\delta$ and $\nu$.

7.4.4 Reconstructing the Phase

This section makes use of a Fourier phase analysis to obtain the plasma induced phase shift, $\Delta \phi_P(\nu)$. It will be shown in Section 7.4.5 that $\Delta \phi_P(\nu)$ can be used to solve for the changes in electron density and heavy particle density induced by the plasma.

Obtaining the plasma induced phase shift from the FT-IR system requires the acquisition of two interferograms: (1) one with plasma off and (2) one with plasma on. For the plasma on acquisition, it is assumed that the plasma state is constant for the duration of the measurement. The Fourier transform of each interferogram produces the complex spectra of the interferograms

$$B'_{\text{off}}(\nu) = B(\nu) \exp\left[i \phi_{\text{off}}(\nu)\right] = \mathcal{F}\mathcal{T}\{I_{\text{off}}(\delta)\},$$

$$B'_{\text{on}}(\nu) = B(\nu) \exp\left[i \phi_{\text{on}}(\nu)\right] = \mathcal{F}\mathcal{T}\{I_{\text{on}}(\delta)\}.$$  

(7.63)

The complex spectra is then decomposed to obtain the phase shifts between the
arms of the interferometer using Equation 7.15

\[ \Delta \phi_{\text{off}}(\nu) = \tan^{-1} \left( \frac{\text{Re} B'_{\text{off}}(\nu)}{\text{Im} B'_{\text{off}}(\nu)} \right), \]

\[ \Delta \phi_{\text{on}}(\nu) = \tan^{-1} \left( \frac{\text{Re} B'_{\text{on}}(\nu)}{\text{Im} B'_{\text{on}}(\nu)} \right), \]  

where

\[ \Delta \phi_{\text{off}}(\nu) = \phi_{2,\text{off}} - \phi_{1,\text{off}} \]

\[ \Delta \phi_{\text{on}}(\nu) = \phi_{2,\text{on}} - \phi_{1,\text{on}}. \]  

(7.64)

Recall that “2” denotes the arm with the plasma and “1” denotes the reference arm. In the plasma arm, \( \phi_{2,\text{on}}(\nu) \neq \phi_{2,\text{off}}(\nu) \) due to the formation of plasma which changes the optical path length. In the reference arm, however, \( \phi_{1,\text{on}}(\nu) = \phi_{1,\text{off}}(\nu) \) because there are no optical path length changes between the on and off cases. The difference of the on and off phase shifts effectively removes the influence of the reference arm and isolates the plasma induced phase shift

\[ \Delta \phi_P(\nu) = \Delta \phi_{\text{on}}(\nu) - \Delta \phi_{\text{off}}(\nu) \]

\[ = (\phi_{2,\text{on}}(\nu) - \phi_{1,\text{on}}(\nu)) - (\phi_{2,\text{off}}(\nu) - \phi_{1,\text{off}}(\nu)) \]

\[ = \phi_{2,\text{on}}(\nu) - \phi_{2,\text{off}}(\nu). \]  

(7.65)

(7.66)

The quantity \( \Delta \phi_P(\nu) \) contains a wealth of information about the plasma. Recall that plasma is a dispersive medium, so the plasma induced phase shift is dependent on the wavenumber of the radiation passing through it. \( \Delta \phi_P(\nu) \) contains
individual measurements of the plasma induced phase shift for all wavenumbers between the designated spectral range from $\nu_{min}$ to $\nu_{max}$.

7.4.5 Least-Squares Solution of Plasma Parameters

In the last section, the plasma induced phase shift was found from a Fourier phase analysis of interferogram data. Now the plasma induced phase shift will be processed to determine the electron density change and the heavy particle density change due to the formation of plasma.

The linear system of equations in Equation 3.72 is rewritten in terms of wavenumber,

$$
\begin{bmatrix}
\frac{1}{4\pi l_{plasma}} & \Delta \phi(\nu_1) \\
\Delta \phi(\nu_2) & \frac{-q^2}{8\pi^2 c^2 \varepsilon_0 \nu_1^2} \\
\vdots & \vdots \\
\Delta \phi(\nu_{2N}) & \frac{-q^2}{8\pi^2 c^2 \varepsilon_0 \nu_{2N}^2}
\end{bmatrix}
\begin{bmatrix}
\Delta n_e \\
\Delta n_h
\end{bmatrix}
= 
\begin{bmatrix}
\frac{A}{n_{ho}} + \frac{B}{n_{ho} \nu_1^2} \\
\frac{A}{n_{ho}} + \frac{B}{n_{ho} \nu_2^2} \\
\vdots \\
\frac{A}{n_{ho}} + \frac{B}{n_{ho} \nu_{2N}^2}
\end{bmatrix}
\begin{bmatrix}
\Delta n_e \\
\Delta n_h
\end{bmatrix}.
$$

(7.67)

where $\Delta n_e = n_{e,on} - n_{e,off}$ and $\Delta n_h = n_{h,on} - n_{h,off}$ and the subscripts on $\nu$ denote wavenumbers from $\nu_{min}$ to $\nu_{max}$. The system of equations is solved using the pseudoinverse in Equation 3.73 to give the change in electron density $\Delta n_e$ and the change in heavy particle density $\Delta n_h$. Equation 3.74 may then be used to determine the change in gas temperature due to the formation of plasma.

7.4.6 Summary of Simulation Setup

This section has presented the procedure to simulate interferograms from a phase-sensitive FT-IR interferometer. It has been shown from the analysis of
interferograms with and without plasma, it is possible to determine the plasma induced phase shift, optical path difference, electron density, change in heavy particle density, and change in gas temperature. This is an impressive amount of information acquired simultaneously using a non-invasive measurement technique. The following section will present simulation results and error estimates for this measurement system. Note that for the plasma cases presented, all simulations use a plasma length of 38 cm.

7.5 Simulation Data

FT-IR interferograms are generated using the procedure detailed in the previous section. The major steps in the simulation are summarized below.

1. Generate the blackbody spectral radiance with Equation 7.33.
2. Calculate the optical throughput of the system with Equation 7.40.
3. Generate the system’s total transmittance spectrum using Equation 7.51.
4. Calculate the noise power of the detector using Equation 7.42.
5. Calculate the optical path difference of the system using Equation 7.55 for two cases:
   (a) Plasma on using prescribed values for $\Delta n_e$ and $\Delta n_h$,
   (b) Plasma off using $\Delta n_e = 0$ and $\Delta n_{h,off}$.
6. Generate uniform noise and scale to $\sqrt{N'}$.
7. Generate the FT-IR interferograms using Equation 7.62 for two cases:
   (a) Plasma on using $OPD_{on}$,
   (b) Plasma off using $OPD_{off}$.
8. Calculate the complex spectra of both interferograms using Equation 7.63.
9. Calculate plasma induced phase shift with Equations 7.64 and 7.66.

10. Form the system of equations in Equation 7.67 and solve using Equation 3.73 for the “experimental” values of $\Delta n_{e,exp}$ and $\Delta n_{h,exp}$.

To evaluate the effectiveness of the phase-sensitive FT-IR interferometer for plasma diagnostics, a number of cases are investigated in this chapter including: the effect of dispersion on the shape of the interferogram, the isolated effects of changes in electron density and heavy particle density (temperature), the effect of co-adding mirror scans, and the error of the measured values for $\Delta n_{e,exp}$ and $\Delta n_{h,exp}$.

7.5.1 Effect of Dispersion

In optics, dispersion refers to the behavior of radiation as it passes through a material with a wavenumber dependent index of refraction. All of the materials inside the FT-IR system are dispersive, as evidenced in the plots for refractive index of Figure 7.9. Considering the relative scales of the ordinate in each plot, it is clear that zinc-selenide is the most dispersive material inside the interferometer, followed by potassium bromide. Air, helium, and plasma are very weakly dispersive in comparison. In this section, FT-IR simulations are run to demonstrate that the model can produce the expected results due to dispersion inside the interferometer. In these simulations, there is no plasma generation and no noise has been added to the interferograms.

Without dispersion, the interferometer OPD is not dependent on wavenumber and there exists a single mirror location (ZPD) such that $OPD = 0$ for all wavenumbers. At this location, the detector will return its maximum value, known as the centerburst, because all wavenumbers are interfering constructively. Fig-
Figure 7.10 shows a simulated interferogram of the FT-IR system with no dispersion present. In this simulation, both arms of the interferometer are empty, as in Figure 7.2. As expected, the detector reaches its maximum value at the ZPD and the interferogram is symmetric.

When a dispersive material is placed inside one arm of the FT-IR interferometer, the interferometer OPD becomes dependent on wavenumber. Likewise, the ZPD (where the mirror displacement balances the interferometer OPD) must also be dependent on wavenumber. There is no longer a single location where all the wavenumbers interfere constructively because they each have a unique ZPD. Instead, dispersion inside the interferometer spreads the energy contained in the centerburst across a wider range of mirror displacement. This is advantageous in some situations because it reduces the necessary dynamic range of the data acquisition system [31].

Figure 7.11 shows a simulated interferogram with weak dispersion. In this simulation, a 0.6 mm thick zinc selenide window was placed in one arm of the interferometer. The interferogram no longer features a centerburst and there is a slight asymmetry across it. Figure 7.12 shows a simulated interferogram with the plasma chamber inside the interferometer, as in Figure 7.1. The plasma chamber has two 3 mm zinc selenide windows so the dispersion effect much stronger. The maximum signal amplitude has decreased significantly and the energy is spread across a much wider range than the weak dispersion case. Furthermore, note that the frequency content of the interferogram is more clearly separated, with low frequencies on the left side and higher frequencies on the right side. For this reason, using dispersive media inside the interferometer produces what is commonly referred to as a “chirped” interferogram.
From these tests, the FT-IR simulations produce the expected results for dispersion. In the next section, the effect of plasma (which is weakly dispersive) will be investigated.

7.5.2 Effect of Electron Density and Heavy Particles

When plasma forms inside the plasma chamber, it changes the $\text{OPD}$ of the interferometer which thereby alters the interference pattern. In this section, FT-IR simulations are run to investigate the effect that changes in electron density and changes in heavy particle density have on the interferogram and reconstructed phase. In these simulations, no noise is added to the interferograms in order to clearly observe the effect of the plasma.

To investigate only the effect of electron density on the interferogram, the temperature change (heavy particle density change) of the plasma is set to zero.
Figure 7.11. Simulated interferogram with weak dispersion due to 0.6 mm of zinc selenide.

Figure 7.12. Simulated interferogram with strong dispersion due to 6 mm zinc selenide.
Three different electron densities are investigated: $10^{13}$, $10^{14}$, and $10^{15}$ cm$^{-3}$. The resulting interferograms for each electron density are shown in Figure 7.13. The top image shows the full interferogram and the bottom shows a zoomed view of the interferogram. The shift in the interferogram is very slight at $10^{13}$ cm$^{-3}$, but clearly increases with higher electron densities. The corresponding reconstructed shifts are shown in Figure 7.14. This demonstrates that a greater change in electron density produces a greater phase shift, and thus larger $OPD$.

To investigate only the effect of a gas temperature increase on the interferogram, the change in electron density is set to zero. Recall that the temperature change of the background gas in the plasma is related to the heavy particle density change with Equation 3.57. It will be seen that the gas composition also has significant effect on the interferogram and phase changes due to temperature increases. For this reason, both air and helium are considered. Figure 7.15 shows the resulting interferograms for air at 30 Torr with temperature changes of 50 K, 100 K, and 200 K, which are equivalent to heavy particle changes of $-1.4 \times 10^{17}$, $-2.4 \times 10^{17}$, and $-4.0 \times 10^{17}$ cm$^{-3}$ respectively. The interferogram shows distinct shifts with increasing temperature. In the corresponding reconstructed phase shifts in Figure 7.16, it is observed that the phase shifts span a much narrower range than the electron density changes in Figure 7.14. This is directly due to the fact that the range of $\Delta n_h$ values are much smaller than the $\Delta n_e$ values considered.

In helium, it is noticed that the interferogram and phase shifts are even less dramatic than in air, as shown in Figures 7.15 and 7.16. The shift in the interferogram is almost unnoticeable and the corresponding phase shifts are an order of magnitude less than those of air. This can be attributed to the gas dispersion constants of helium (Table A.1) which are much smaller than those for air. For
Figure 7.13. Simulated data showing shifts in the interferogram induced by changes in electron density. Top figure shows the full interferograms and the lower figure shows a zoomed view.
Figure 7.14. Calculated phase shifts after processing the interferograms of Figure 7.13. Data shows negative phase shift increasing with electron density.
this reason, using helium as a background gas for the plasma will minimize overall contribution from heating to the measured \( OPD \), which is favorable for plasma adaptive optics.

From the data presented here, it is evident that the plasma induced interferogram shifts measured in an FT-IR experiment would greatly depend on the magnitude of the changes in electron density and heavy particle density. However, it should be noted that because the FT-IR model incorporates the spatial resolution of the interferogram acquisition, these simulations show that all interferogram shifts would be measurable in the absence of noise. In the next section, the effect of noise on the interferogram and reconstructed phase will be investigated.

### 7.5.3 Effect of Noise and Mirror Scans

It was explained in Section 7.4.3 that the overall SNR of the measurement system may be increased by co-adding multiple mirror scans together. In this section, FT-IR simulations are performed with the effects of noise included. The effect of the number of mirror scans on the interferogram and reconstructed phase is investigated. These simulations are conducted with helium plasma with an electron density change of \( 10^{13} \text{ cm}^{-3} \) and a temperature change of 50 K, which corresponds to a heavy particle change of \(-1.4 \times 10^{17} \text{ cm}^{-1}\).

Figure 7.19 shows the simulated FT-IR data for \( K=1, 50, \) and 500 mirror scans. The left hand column shows the simulated interferogram and the right column shows the resulting reconstructed phase. The black lines in the phase plots show the plasma induced phase shift prescribed in the simulation. With only one scan, the interferogram is visibly noisy and the reconstructed phase does not follow the reconstructed phase at all. With fifty mirror scans, some noise is
Figure 7.15. Simulated data showing the shifts in the interferogram induced by changes in air temperature. Top figure shows the full interferograms and the lower figure shows a zoomed view.
Figure 7.16. Calculated phase shifts after from processing the interferograms of Figure 7.15. Data shows negative phase shift increasing with the air temperature change.
Figure 7.17. Simulated data showing the shifts in the interferogram induced by changes in helium temperature. Top figure shows the full interferograms and the lower figure shows a zoomed view.
Figure 7.18. Calculated phase shifts after from processing the interferograms of Figure 7.17. Data shows negative phase shift increasing with the helium temperature change.
still evident in the interferogram but the reconstructed phase begins to match the prescribed phase. With five hundred mirror scans, almost no noise is visible in the interferogram and the reconstructed phase matches very well with the prescribed phase.

The accuracy in the determination of the change in electron density and change in heavy particle density will significantly depend on how well the reconstructed phase matches the prescribed phase. From Figure 7.19 it is clear that the phase-sensitive FT-IR measurement system would certainly require many mirror scans to reproduce prescribed shift. In the next section, the error of the measurement system will be investigated as a function of mirror scans.

7.5.4 Measurement System Error

In the FT-IR simulations, the change in electron density $\Delta n_e$ and change in heavy particle density $\Delta n_h$ due to plasma are prescribed. The FT-IR model generates the resulting interferograms for plasma on and plasma off, which are then processed to obtain experimental values of $\Delta n_{e,exp}$ and $\Delta n_{h,exp}$. The experimental error of the measurement system for each term is

$$\Delta n_e \ Error = \left| \frac{\Delta n_e - \Delta n_{e,err}}{\Delta n_e} \right| \times 100\%,$$

$$\Delta n_h \ Error = \left| \frac{\Delta n_h - \Delta n_{h,err}}{\Delta n_h} \right| \times 100\%.$$  (7.68)

FT-IR simulations are run to determine how the error of the measurement system correlates to the number of mirror scans co-added together. The number of mirror scans ranges from 1 to 1000, and each error value is averaged over 100 individual simulated experiments. Simulations are conducted over a range of
Figure 7.19. Simulated interferograms (left) and reconstructed phase shifts (right) for 1, 50, and 500 mirror scans.
prescribed $\Delta n_e$ values and temperature changes ($\Delta n_h$ values).

Figure 7.20 shows the FT-IR measurement error for determining the change in electron density. The simulations used to construct this plot featured a helium plasma at 30 Torr with no change in temperature. Data are presented for three different electron densities: $10^{13}$, $10^{14}$, and $10^{15}$ cm$^{-3}$. The data shows that as the electron density increases, the measurement error decreases. This should be expected because higher electron densities yield a more measurable shift in the interferogram (Figure 7.13). At the $10^{13}$ cm$^{-3}$, the measurement error using 1000 mirror scans is about 50%, and at $10^{15}$ cm$^{-1}$, the error is under 1%.

Figures 7.21 and 7.22 shows the FT-IR measurement error for determining the change in temperature (heavy particle density) for air and helium, respectively. The simulations used to construct this plot featured a helium plasma at 30 Torr with no change in electron density. Data are presented for three different temperature changes: 50 K, 100 K, and 200 K. As the temperature change increases, the measurement error decreases. Again, this should be expected because larger temperature changes will produce a more measurable shift in the interferogram (Figures 7.15 and 7.17). It is interesting to note that the measurement error of air is much less than that for helium. Again, this may be attributed to the fact that the helium produces a much smaller shift in the interferogram in comparison with air.

7.6 Conclusions

The phase-sensitive FT-IR interferometer is an attractive measurement tool for plasma diagnostics for its ability to obtain phase measurements for a plasma across a range of infrared wavelengths. Simulations have shown that the accu-
Figure 7.20. Measurement system error for $\Delta n_e$ versus number of mirror scans for electron densities of $10^{13}$, $5 \times 10^{13}$, and $10^{14}$ cm$^{-3}$. 
Figure 7.21. Measurement system error for $\Delta n_h$ versus number of mirror scans for air temperature changes of 25, 50 and 100 K.

Figure 7.22. Measurement system error for $\Delta n_h$ versus number of mirror scans for helium gas temperature changes of 25, 50 and 100 K.
racy of the measurement system depend highly on the conditions of the plasma, namely electron density and temperature. Simulations show that stronger plasmas (ie. high electron density and high temperature) produce a larger effect on the measured interferogram and become easier to measure. The measurement error for stronger plasmas is under 10%, which is reasonable considering the magnitude of the values for electron density and heavy particle density. Weaker plasmas that produce a smaller effect on the measured interferogram are shown to be able to be resolved to within an order of magnitude.

Several improvements can be made to the system to improve the sensitivity and reduce the experimental error. The noise of the system can be reduced by acquiring more scans, decreasing the sampling frequency, or by using a detector with a smaller sensor area and higher specific detectivity. Some ways to improve the signal of the system are listed below.

- Use an infrared source with a higher emissivity or by operating at a higher temperature to generate more spectral radiance.
- Optimize the materials used in the interferometer to maximize the total system transmittance.
- Increase the interferometer throughput by increasing the beam diameter, increasing the spectral resolution $\Delta \nu$, or by decreasing the maximum wavenumber $\nu_{max}$
- Use a detector with higher responsivity.

Alternatively, the plasma induced phase shift may be increased increasing the length of the plasma. For the same measurement system and plasma conditions, this would effectively increase the shift in the interferogram.

In summary, the phase-sensitive FT-IR interferometer is an innovative mea-
urement system for plasma diagnostics for the fact that it can provide simultaneous measurements of electron density and temperature inside the plasma. FT-IR simulations have demonstrated that it is possible to determine these parameters, although special considerations must be made for the magnitudes of the electron density and temperature change expected in an experiment to ensure that the error of the measurements falls within acceptable bounds.
The focus of the presented work was to experimentally measure the electron density of a dielectric barrier discharge plasma in order to evaluate it as a medium for adaptive optics. This was motivated by the field of aero-optics, where plasma-based devices for wavefront corrections could potentially advance the state-of-the-art of the industry. In this work, a plasma geometry was designed and optimized, single and dual wavelength interferometric experiments were performed, and a new multi-wavelength measurement technique was proposed to better characterize the optics of plasma. This chapter will summarize the results found in this work and present conclusions and recommendations for the continuation of this study.

8.1 Summary of Work

A cylindrical double dielectric barrier discharge plasma chamber was designed specifically for plasma optics applications. Two distinct plasma formations were observed in experiments, denoted the wall-plasma and center-plasma regimes. It was found that the ratio of the electrode gap distance, $d$, to the cylinder diameter, $D$, determined how the plasma would form. A low-order electrostatic model was developed for the electric field magnitude inside the plasma chamber. The model was compared against experimental measurements of plasma luminescence
for a range of $d/D$ ratios. Good agreement was found between the model and experiment, suggesting that the formation of the weakly ionized gas inside the plasma chamber was dominated by the electric field. Furthermore, it was found that a $d/D$ ratio greater than 1.22 would guarantee a center-plasma regime, which was favorable for laser interferometric experiments because the laser beam passes through the center of the chamber.

Single-wavelength interferometry was performed to measure the $OPD$ induced by the formation of plasma inside the plasma chamber over a range of pressures and applied voltages. A 6" long plasma was shown to produce greater than a 1 $\mu$m $OPD$ at a 633 nm wavelength. It was further shown that the $OPD$ increased with pressure and applied voltage potential. Electron density was determined from the $OPD$ values by assuming that the gas heating was negligible. A non-dimensional model was developed to relate the electron density to the gas pressure and applied power. Later dual-wavelength experiments showed that this gas heating assumption was invalid and that the 1 $\mu$m shift was likely due almost completely to gas heating.

A dual-wavelength interferometer was constructed to simultaneously probe the plasma with both 633 nm visible and 3.39 $\mu$m infrared wavelengths. This system allowed for time-resolved determination of the electron density and the background gas heating. Results for air at 20 Torr and 30 kV showed electron densities on the order of $10^{14}$ cm$^{-3}$ and gas temperatures increases of around 35 K. Results for helium were inconclusive as it was suspected that there was air contamination inside the chamber. These experiments demonstrated the proof-of-concept for this measurement system, which will continue to be a vital tool in assessing the optics of the plasma.
Finally, a new plasma diagnostic technique was proposed using a phase-sensitive FTIR interferometer. This technique probed the plasma with a broad spectrum of infrared wavelengths in order to determine the plasma electron density and gas heating. The complete design of the measurement was presented and simulations were conducted to assess its effectiveness and accuracy. It was shown that the accuracy of the system was dependent on the gas composition and intensity of plasma. Recommendations were given to improve the signal-to-noise of the system for improved measurements.

8.2 Conclusions and Future Recommendations

This work has demonstrated techniques for the experimental determination of plasma electron density. The dual-wavelength interferometer has shown plasma electron values on the order of $10^{14}$ cm$^{-3}$. In relation to aero-optic wavefront control, it would require over 20 meters of plasma at this density to produce a 1 μm electron-induced OPD at a 1 μm wavelength, which is impractical for this application. A shorter length of plasma may be used if the beam is passed through it multiple times, mimicking the effects of a much longer plasma.

Improvements should be made to the system to increase confidence in the measurements. The pressure system should be revised so that other gasses may be used without the possibility of air contamination inside the chamber. Most importantly, a new method of generating the reference phase disturbances must be implemented to assure accuracy of the measured phase shifts. With these improvements made, the measurements from the system should be compared against a second plasma diagnostic technique to further validate the system.

The advantages of using the new phase-sensitive FTIR measurement system
over the dual-wavelength interferometer are: (1) it uses longer infrared wavelengths so that the measurements are more sensitive to the electron density of the plasma and (2) it probes the plasma with a full spectrum of infrared wavelengths which improves the least-squares determination for electron density. It is noted, however, that the experimental setup is much more complicated and special notice should be given to the very short coherence length of the light inside the interferometer.

Foundational work to characterize dielectric barrier discharges for this purpose have been presented in this work. Further improvements should be made to the measurement techniques to gain a more accurate representation of the plasma characteristics. Additionally, experiments should be conducted to more precisely characterize the frequency response of the electron density generation. Given the electron density measurements reported, these plasmas may not be suitable for piston-based aero-optic corrections. However, utilizing plasma for slope-based wavefront corrections may be a viable alternative and warrants further investigation.
A.1 Gas Dispersion Constants

TABLE A.1

GAS DISPERSION CONSTANTS

<table>
<thead>
<tr>
<th>Gas</th>
<th>A ($\times 10^{-5}$)</th>
<th>B ($\times 10^{-14}$ cm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H$_2$ †</td>
<td>13.58</td>
<td>1.02</td>
</tr>
<tr>
<td>He †</td>
<td>3.48</td>
<td>0.08</td>
</tr>
<tr>
<td>N$_2$ †</td>
<td>29.06</td>
<td>2.24</td>
</tr>
<tr>
<td>Ne †</td>
<td>6.66</td>
<td>0.16</td>
</tr>
<tr>
<td>Ar †</td>
<td>27.97</td>
<td>1.56</td>
</tr>
<tr>
<td>Kr †</td>
<td>41.89</td>
<td>2.92</td>
</tr>
<tr>
<td>Xe †</td>
<td>68.23</td>
<td>6.92</td>
</tr>
<tr>
<td>Air ‡</td>
<td>28.71</td>
<td>1.63</td>
</tr>
</tbody>
</table>

† Values from Duschin & Pawlitschenko [21].
‡ Value from Leipold et al. [43]
A.2 Plasma Chamber Designs

Figure A.1. Schematic of the plasma adaptive optic lens with dimensions shown in inches.

Figure A.2. Front view showing the plasma generated with design 1. The laser would pass directly through the plasma (in and out of the page).
Figure A.3. Schematic of the plasma adaptive optic lens design 2 with dimensions shown in inches.

Figure A.4. Top view showing the plasma generated with design 2. The laser would pass through the plasma from left to right.
Figure A.5. Schematic of the plasma adaptive optic lens design 3 with dimensions shown in inches.

Figure A.6. Front view showing the plasma generated with design 3. The laser would pass through the plasma (in and out of the page).
Figure A.7. Schematic of the plasma adaptive optic lens design 4 with dimensions shown in inches.

Figure A.8. Side view showing the plasma generated with design 4. The laser through the plasma from the left to right.


