DARK MATTER BEYOND “WIMP” ERA

Abstract

by

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One of the key ingredients of the standard model of Cosmology is dark matter. The existence of dark matter has been established from numerous astrophysical and cosmological observations, but its interaction nature still remains enigmatic. Assuming dark matter is a single particle thermal relic, a canonical paradigm of dark matter with a prediction of its interaction cross section with the standard model particles is developed. Miraculously, the size of the estimated cross section is just within the range of current experiments, encouraging various searches to look for the direct interaction of dark matter with the visible matter. However, after more than a generation of experiments, no unambiguous signature of dark matter has been detected, and most of the targeted parameter space proposed by the single particle thermal relic is excluded. As a result, we are led to move away from the simple single particle thermal relic paradigm and explore alternative scenarios.

Here, four alternative dark matter frameworks are discussed and their phenomenological signature are explored. 1) There is another particle in the dark sector near degenerate with the dark matter that has efficient annihilation rate to the standard model; whereas the dark matter itself has inefficient annihilation rate to the standard model states. 2) Dark matter annihilates to a pair of meta-stable mediators, which subsequently decay to standard model particles. 3) Dark matter is asymmetric and its relic abundance is set by annihilating away the symmetric component. 4)
Dark matter is decoupled from the photon thermal sector and has negligible initial abundance; but then gets populated through its feeble interaction with the standard model.

All of these scenarios, can successfully produce the right relic density of dark matter with ‘natural’ size couplings, while being safe from the current experimental bounds. Furthermore, some of these models have clear phenomenological signatures within the reach of the next generation experiments.
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CHAPTER 1

INTRODUCTION

The long-sought Higgs boson, first observed in 2012, was the last piece of the Standard Model (SM) of particle physics to be directly discovered. The SM provides a robust and self-consistent framework that can explain nearly all presently observed microscopic phenomena, providing high-energy physicists and cosmologists with an accurate model for understanding the visible universe.

Yet physicists are convinced that the SM is not the final theory of nature, but merely the low energy limit of a more fundamental model. One of the most compelling reasons to believe that there is physics “beyond the SM” (BSM) is the existence of dark matter (DM). While little is known with certainty about this new particle (or collection of particles), it is believed that DM is massive and that it interacts gravitationally, though it is not part of the SM.

The existence of DM has been manifested through its gravitational interactions in various astrophysical observations. By studying these observations (of which a more detailed discussion is given in 1.1), we have some clues about the properties of DM; namely, we suspect the DM to be mostly cold (non-relativistic), to be stable on cosmological time scales, and to constitute most of the matter content of the universe.

Given the abundance of DM in the universe today, one of the main questions that arises is how the DM abundance came to its current value. The most commonly considered scenario proposes that the DM is a thermal relic; that is, the DM was in thermal equilibrium with SM particles when the universe was hot, but fell out of equilibrium once the universe’s temperature fell below the mass scale of DM. More
specifically, as the universe’s temperature drops, there is eventually not enough energy to produce the DM particles, but DM still continues to annihilate. Eventually, due to the expansion of the universe, the DM particles become so sparse they can no longer find each other to annihilate. As a result, the abundance of DM “freezes out” to current level. Within this framework, even without choosing a specific model for the DM, rough estimates can be obtained for the cross section of interaction between the DM and the SM sector, which will be laid out in section 1.2.

Fortunately, the estimated cross section $\sim 10^{-26}\text{cm}^3/\text{s}$ is within the reach of current experiments. Having thus far found no signals, it appears that this generic estimate is inconsistent with observation. The main objective of my research has been to find well-motivated DM models that can saturate the relic abundance while being safe from the current experimental bounds. I also have worked on improving techniques to enhance our sensitivity to DM production in experiments such as those at the Large Hadron Collider (LHC).

1.1 Astrophysical Evidence of Dark Matter

The first person to conjecture the existence of dark matter was Zwicky in 1933, to account for the radial velocity dispersion of galaxies in the Coma cluster [1]. Zwicky noticed that the outer objects in the cluster were moving much faster than expected, given the gravitational potential of the visible matter. To reconcile the observed velocities with the Virial theorem, he suggested that there exists a “dark” matter that is much more abundant than visible matter. Later, similar observations were made in the Virgo cluster and then in our local cluster [1].

In the late 1970s, another set of observations by Rubin and collaborators on the rotational velocity of spiral galaxies also indicated that the luminous matter alone cannot be responsible for the motion of stars in the outer reaches of the galaxy [1]. The rotational velocity $v$ of an object orbiting the center of a galaxy at a radius
$r_r$, enclosing a mass of $M(r)$ is $v(r) \sim \sqrt{M(r)/r}$. Thus, for radii greater than the radius of the visible galaxy, the rotational velocity was expected to decline. Instead, $v(r)$ measurements show a flat rotation curve with respect to the radius (Fig. 1.1), leading to a dark matter mass density goes like $\rho(r) \sim 1/r^2$.

![Figure 1.1](image)

**Figure 1.1.** Measured rotation curve of NGC6503 [2]. The best fit of the data, and modeled contributions from the halo, disk and circum-galactic gas are also shown. As one sees, the contribution from the halo extends far beyond the visible disk of the galaxy, and dominates the rotation curve at large radius.

Another piece of evidence for DM comes from gravitational lensing. According to the principle of General Relativity, the spacetime near a massive object curves,
resulting in curvature of geodesic in the vicinity of the massive object. Thereby, light coming from background objects bends as it passes through the gravitational field of a massive object in the foreground. This “gravitational lensing” causes the background objects to appear brighter and more distorted than they would otherwise be \cite{3} (as shown in Fig. 1.2). The observed gravitational lensing by galactic clusters and individual galaxies is far greater than can be explained by visible matter, reaffirming the existence of DM.

Figure 1.2. The bending of light due to the gravitational potential of a cluster can be clearly observed in this image. Credits: NASA, Andrew Fruchter, and the ERO team.

One of the best known examples of gravitational lensing is the so-called Bullet Cluster, which is really two clusters of galaxies in collision. In this system, the mass distribution deduced from gravitational lensing is contrasted against the X-ray emitting baryonic distribution. Once again, a non-luminous matter is needed to explain the discrepancy between the two observations. Furthermore, as shown in Fig. 1.3 we can see the visible matter (shown in red) slowing down because of their
interaction in this collision, whereas DM (shown in blue) pass through, oblivious to the collision. This particular observation also rules out some of the rival explanations for DM, including those that modify gravity at galactic distance.

Figure 1.3. A composition image of the Bullet Cluster, showing the collision of two clusters. The red represents the visible matter based on the emitted X-rays. The blue represents the distribution of DM inferred by measurements of gravitational lensing. In the collision, the visible matter interacts, but DM seems to be collision-less, passing through with little or no interaction.

All of the aforementioned observations lead us to believe there is more matter than just the visible matter. But to quantify the relic density of dark matter $\rho_{DM}$ compared with the total energy density of the universe $\rho_{total}$, it would be useful to review our knowledge of relic density of baryonic matter $\rho_b$. The value of $\rho_b$ is deduced from multiple measurements, namely: Big Bang nucleosynthesis, counting the number of stars, and acoustic baryonic oscillation derived from the anisotropies in the Cosmic Microwave Background.
According to hot Big Bang cosmology, protons and neutrons bind to create the light elements (e.g., Hydrogen, Helium, and Lithium), around 3 minutes after the Big Bang. This epoch is known as the big bang nucleosynthesis (BBN). The abundance of these light elements depends on the $\rho_b$ [4]. Therefore, if we knew the primordial abundances of light elements, we could extract $\rho_b$. Abundances are usually observed at later times, and may not necessarily represent the primordial abundances. To faithfully reconstruct the primordial abundances, one must study extremely un-evolved systems, such as dwarf galaxies (because of little stellar nucleosynthesis) and distant quasars (showing the early stage of the universe). The current measurements of relative abundances appear to be very precise, leading to a very accurate $\rho_b/\rho_\gamma$, where $\rho_\gamma$ is the density of photons [4].

The footprint of DM can also be found in the angular power spectrum of thermal anisotropies in the Cosmic Microwave Background (CMB). The CMB is a picture of the universe with photon radiation left over from the time neutral atoms were formed and could no longer interact with photons. According to the CMB, the universe is very homogenous with rare instances of fluctuations that are the galaxies and stars. From studying these anisotropies in the uniform background, we can infer the ratio of matter to photon: Recall that photons move fast and radiate uniformly; while matter moves much slower and collapses due to its gravitational potential to form structures. In particular, we can study the angular power spectrum of these thermal anisotropies (as shown in Fig. 1.4), where the location of the peaks and their relative amplitudes can provide us with information about the total energy of the universe, the ratio of baryonic matter to total energy density $\Omega_b = \rho_b/\rho_{\text{total}}$, and the relative size of baryonic matter to non-baryonic matter [5]. CMB measurements indicate that the universe is flat, and that non-baryonic matter is about 5 times more abundant than the baryonic matter. The relative abundances that can be derived from CMB measurement is an orthogonal technique to BBN, and they are consistent with each
Figure 1.4. The power spectrum of thermal anisotropies for various multipole moments in the CMB is shown. Different curves correspond to different values of baryonic density compared with the total energy density: $\Omega_b = \rho_b/\rho_{\text{total}}$. Changing the baryonic density changes the amplitude of the peaks as well as their location. We see that the data is most consistent with $\Omega_b = 0.05$. Changing the DM density will also affect the shape in a similar fashion, and thus by fitting the data, we are able to attain the relative abundance of DM to $\rho_{\text{total}}$ as well.

Within the SM, the only stable electrically neutral matter in the SM, are the neutrinos. Neutrinos are in fact very abundant in the universe; however, due to their lightness ($\ll ev$), neutrinos remain relativistic after the universe cools and cannot
contribute much to the formation of large-scale structure, as relativistic matter tend to erase structures in the universe rather than forming them. Therefore, we are convinced that neutrinos are not DM, and we suspect DM to be non-relativistic.

In this section, we have shown that DM is about 5 times more abundant than the baryonic matter. Also, from various cosmological studies, we have concluded that it is non-relativistic at present day, electrically neutral, and stable in cosmological scales. Therefore, models of DM are not entirely unconstrained. In the following, we will discuss some of the canonical understanding of the DM.

1.2 A Thermal History of Dark Matter

According to the hot Big Bang paradigm, the universe started out hot and then cooled over time. In the early universe, when the temperature was significantly larger than particle’ mass \(T \gg m\), the particles were in thermal equilibrium– that is their production rate is the same as its annihilation rate. As the temperature decreases, the particles eventually fell out of equilibrium and became non-relativistic \(T < m\).

Let us take \(X\) to be a particular particle. Once \(T < m_X\), there is not enough energy in the universe to produce \(X\), but \(X\) continues to annihilate. Therefore, the \(X\) number density \(n_X\) falls rapidly. If \(X\) decays, the number density will eventually go to zero. If \(X\) does not decay, and for example has a \(2 \rightarrow 2\) annihilation process, its the annihilation rate is given by \(\Gamma \sim n_X \sigma_{2 \rightarrow 2} v\), where \(\sigma\) is the cross section, and \(v\) is the velocity of \(X\). In this scenario, \(X\) will continue to annihilate, until the universe has expanded enough that \(X\) particles can no longer find each other to annihilate. Thus, eventually the abundance of \(X\) freezes out due to the expansion of the universe.

For simplicity, let us assume that the most relevant annihilation of \(X\) is \(XX \rightarrow AB\), where \(A\) and \(B\) are any particles with \(m_A + m_B < 2m_X\). The evolution of the
\( n_X \) is described by the Boltzmann equation

\[
\dot{n}_X + 3Hn_X = - \int d\Pi_X d\Pi_X d\Pi_A d\Pi_B (2\pi)^4 \delta^4(p_X + p_X - p_A - p_B) \\
\times |M_{XX\rightarrow AB}|^2 (f_X f_X - f_A f_B),
\]

(1.1)

where \( H \) is the Hubble parameter, \( d\Pi_i \equiv \frac{dp_i}{(2\pi)^3 \frac{1}{2}E_i} \) is the phase space density, and \( f_i \) is the distribution function for a given state (assuming zero chemical potential \( f_i = e^{E_i/T} \)).

Using the conservation of energy and momentum, we can rewrite the expression above in terms of only \( X \) dependences:

\[
\dot{n}_X + 3Hn_X = -\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle (n_X^2 - n_{\text{eq}}^2).
\]

(1.2)

Here \( v_{\text{rel}} \) is the relative velocity between the \( X \) particles, \( n_{\text{eq}} \) is the equilibrium number density, and \( \sigma_{\text{ann}} \) is the annihilation rate of \( XX \rightarrow AB \). When \( X \) is relativistic \((T \gg m_X)\), one finds \( n_{\text{eq}} \sim T^3 \). For non-relativistic \( X \), the abundance of \( X \) exponentially decreases \((n_{\text{eq}} \sim (m_X T)^{3/2} e^{-m_X/T})\), reflecting the fact that creation of \( X \) is kinematically forbidden and so \( X \) will only annihilate.

Therefore, in this view (which we call “canonical”) the evolution of a massive particle \( X \) is as following:

- At \( T \gg m_X \), the rate of creation of \( X \) matches its annihilation. \( X \) is in thermal equilibrium with other relativistic particles (in particular photons).

- Once the temperature falls below \( m_X \), the creation of \( X \) requires being on the tail of the thermal distribution. Hence the number density of \( X \) drops exponentially, but \( X \) is still in thermal equilibrium.

- If \( X \) were to remain in thermal equilibrium, the abundance of \( X \) today would be infinitesimal. But due to the expansion of the universe, the \( X \) particles will
eventually become so sparse that the probability of $X$ finding another with which to annihilate is negligible. From this point on, the comoving number density of $X$ stays constant (i.e., freezes out).

![Figure 1.5. The evolution of a thermal relic DM as a function of time (or inverse temperature). The red region (roughly) represents the thermal equilibrium, the green region shows when DM becomes non-relativistic and only annihilates, followed by the blue region when the comoving number density freezes out due to the expansion of the universe. [6].](image)

Although we have to actually solve the Boltzmann equation to get the point of freeze out, we can make the following simplification to get an approximate understanding:

1. The Hubble parameter during the radiation-dominated era scales as $H \sim T^2/M_{\text{Pl}}$, where $M_{\text{Pl}}$ is the Planck mass. (Recall that the Hubble parameter is the rate at which the universe expands.)
2. Freeze out occurs when the annihilation rate of $X$ is the same as the Hubble parameter

$$n_X \langle \sigma v \rangle \sim H \sim T_f^2 / M_{Pl}. \quad (1.3)$$

Assuming $X$ is in non-relativistic regime, we estimate $n_{eq} \sim (m_X T)^{3/2} e^{-m_X / T}$. Let us define a dimensionless parameter $x_f \equiv m_X / T_f$, where $T_f$ refers to the freeze-out temperature. Rewriting $n_{eq}$ in terms of $x_f$, and plugging it into Eq (1.3), we get

$$\left( \frac{m_X^2}{x_f} \right)^{3/2} e^{-x_f \langle \sigma v \rangle} \sim \frac{m_X^2}{x_f^2 M_{Pl}^2} \Rightarrow x_f \sim \log \left( \frac{m_X \langle \sigma v \rangle}{M_{Pl} \sqrt{x_f}} \right). \quad (1.4)$$

Because $x_f$ only depends logarithmically on the parameters, its value does not vary much over a large range of parameters. Using the typical weak-scale values for $m_X \sim 100$ GeV and $\langle \sigma v \rangle \sim 10^{-26}$ cm$^3$/s, we get $x_f \sim 25$. The relatively large value of $x_f$ indicates that the particle $X$ is very non-relativistic (cold) by the time of freezes out, a result that persists over almost all possible masses for $X$.

3. So far we have been talking about a generic $X$ particle. If we now assume $X$ is the DM, then from astrophysical observations we know the relative abundance of $X$ compared to the total energy density of the universe. Observations tell us that the total energy density is the critical density $\rho_{\text{crit}} = 3H^2 / (8\pi G)$, with $G$ being the Newtonian gravitational constant. The critical density is the density required to have a flat universe \footnote{In other words, one in which the locally parallel lines stay parallel globally.} The relative density of DM compared to the
critical density is defined as

$$\Omega_{X,0} = \frac{\rho_{X,0}}{\rho_{\text{crit}}} = \frac{n_{X}(T_0)}{\rho_{\text{crit}}(T_0)} m_X,$$

where $T_0$ is the present time. Due to the expansion of the universe, the number density changes as a function of time (or equivalently temperature), even though the number of $X$ particles is assumed to remain constant. In order to track the density of $X$, we introduce a dimensionless parameter, yield, which is defined as $Y_X \equiv n_X/S$, where $S$ is the entropy density. Because the universe’s entropy is almost entirely due to the background cosmic photons, the entropy density is proportional to $T^3$. The advantage of using dimensionless parameter $Y$ is that they are invariant under the expansion of the universe, and so the yield at the time of freeze out is the same as the yield today, $n_{X}(T_f)/T_f^3 = n_{X}(T_0)/T_0^3$. As a result, we have

$$\Omega_X = \frac{T_0^3 n_{X}(T_f)}{T_f^3 \rho_{\text{crit}}(T_0)} m_X.$$

On the other hand, from Eq. (1.3), we get $n_{X}(T_f) \sim T_f^2/\langle \sigma v \rangle M_{\text{Pl}}$. Hence,

$$\Omega_X = \frac{T_0^3}{\rho_{\text{crit}}(T_0)} \frac{m_X 1}{T_f \langle \sigma v \rangle M_{\text{Pl}}} \simeq 0.25 \times \left( \frac{2.2 \times 10^{-26} \text{cm}^2/\text{s}}{\langle \sigma v \rangle} \right) \quad (1.5)$$

$$\simeq 0.25 \times \left( \frac{10^{-9} \text{GeV}^{-2}}{\langle \sigma v \rangle} \right) \quad (1.6)$$

The cross section roughly scales as $\sigma \sim \alpha_X^2 / \Lambda^2$ by dimensional analysis. Putting a weak scale coupling $\alpha_X \simeq 1/137$ and matching the correct relic abundance of DM, leads to $\Lambda \sim 100 \text{ GeV}$. Miraculously, particles with these scale mass and cross sections are also within the reach of current experiments, and so this coincidence is known as
the “WIMP miracle”, where WIMP refers to a weakly interacting massive particle.

Additionally, there are some open questions about the weak scale of the SM (addressed in greater details in Section 1.4.1), and most extensions of the SM attempting to solve these questions predict new state of particles near the weak scale. If any of these particles are stable on the cosmological time-scale, they can be well-motivated dark matter candidates. In the next section, different means of DM detection will be discussed.

1.3 Dark Matter Searches

There are three broad fronts on which DM searches are carried out: direct and indirect, and at colliders. Direct Detection means looking for the interaction of terrestrial apparatuses with dark matter in the vicinity of the earth. From rotational velocity of stars in galaxies, we have an estimation for the distribution of DM in the cosmos and near the earth. Using simulations of DM in the MilkyWay, we expect there to be streams of non-relativistic DM constantly passing through the earth. Since this distribution is a key input for predicting the rate of DM scattering with SM particles near earth, it is important to get an accurate description of DM density in galaxies. Large numerical simulations of structure formations can be used to find a density profile of DM in our galaxy. The most commonly used density profile is the Navarro-Frenk-White profile (NFW profile), which considers only the DM gravitational potential and not the baryonic matter:

\[ \rho(r) = \frac{\rho_0}{\left( \frac{r}{r_s} \right) \left[ 1 + \frac{r}{r_s} \right]^2}, \]

(1.7)

\(^2\)Although including the baryonic matter can improve the density distribution, the NFW profile appears to be in a descend agreement with the observation
where $r$ is the distance from the center of the halo and $r_s$ and $\rho_0$ are some reference radius and densities—usually $r_s$ is taken as the radius in which $r^2 \rho(r)$ reaches its local maximum, leading to $\rho_0 = 4\rho(r_s)$.

Figure 1.6. (a) In the direct detection, DM around the earth scatters off a SM particle and recoils. In indirect detection (b), DM in the cosmos annihilates to SM particles, and the final products of this interaction may be detectable through observations. At a Collider, DM particles can be pair produced and studied (c). If any of these diagrams are allowed, then all of them are allowed due to “crossing symmetry”.

Knowing the local density of DM is crucial to find the DM-SM scattering rate. The principle of the direct detection experiment is to let DM in the vicinity of the earth scatter off a particle in a ground-based detector, while then recoil with some energy $E_R$. From the perspective of the DM particle, the nucleus of the target atom is the scattering object. If a DM scatters with a nucleus of mass $m_N$, the nuclear recoil energy is

$$E_R = \frac{q^2}{2m_N},$$

(1.8)

where $q$ is the momentum transfer, roughly equal to $q \sim m_x v$, with $v \sim 10^{-3}$ being the estimated speed of the incoming DM particle. One of the strongest bounds on
DM interaction is from the LUX experiment. LUX uses a xenon target with mass $m_N \sim 120$ GeV. The energy threshold of LUX is about a few keV, which means their sensitivity declines for $m_X \lesssim 10$ GeV; and the optimal sensitivity is achieved for a DM mass of $m_X \sim 100$ GeV. Therefore, direct detection is an ideal place to test the WIMP miracle. The current bound on the interaction of DM with SM particles is $\sigma < 10^{-45}$ cm$^2$, which is much smaller than the predicted WIMP cross section ($\sim 10^{-36}$ cm$^2$). It is important to mention that in the direct detection experiments, DM can only interact with the nuclei (and thus gluons and light quarks). Therefore, the small cross section of DM with the nuclei cannot be taken as indicative of small cross sections of DM with all SM particles.

![Figure 1.7](image_url)  

Figure 1.7. The current bounds on DM-nucleon interaction according to the LUX experiment.

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$^3$PandaX experiment has bounds compatible with LUX [7].
Another means for DM discovery is by looking for evidence of its interaction with itself, and with SM particles in astronomical environment, where we might expect those interaction rates to be enhanced (e.g., where the density of the DM is greatest). If the DM interacts with SM states, the same processes that can result in DM direct detection can also lead to DM annihilation to SM states, as shown in Fig. 1.6. Through careful observations of photons, cosmic rays, and neutrinos from the cosmos, we may be able to extract some clues about the particle nature of this elusive component of the universe.

The most straightforward place to look for evidence of DM annihilation is simply in the galaxy itself, but in the halo (which has smaller density but larger volume) or in the galaxy core (higher density but smaller volume). Using a DM density profile (such as the NFW profile) the expected rate and energy spectra of DM annihilation end products such as gamma rays, neutrinos, and positrons can be derived. For instance, the flux of gamma rays produced at energy $E_\gamma$ that comes from annihilation of DM with mass $m_{DM}$ as a function of angle of observation $\psi$ (shown in Fig. 1.8) is given by

$$\frac{d^2\Phi}{d\Omega dE_\gamma} = \frac{\langle \sigma v \rangle}{8\pi \eta m_{DM}^2} \left( \sum_f \frac{dN}{dE_\gamma} Br_f \right) J(\psi), \quad (1.9)$$

where $\eta$ depends on whether DM is complex ($\eta = 2$) or self-conjugate ($\eta = 1$), $\langle \sigma v \rangle$ is the DM annihilation cross section, and $Br_f$ is the branching ratio to various final states $f$. The expected number of photons is deduced based on the final state particles. For simplicity, indirect detection experiments usually assume 100% branching ratio to one particular SM particles, as shown in Fig. 1.9.

Depending on the final state, photons maybe radiated. Finally, $J$ is the line-of-sight (LOS) integral that depends on the distribution of the DM $\rho(r)$:

$$J(\psi) = \int_{\text{LOS}} \rho^2(r) d\ell.$$
In Eq. (1.9), the $J(\psi)$ depends on the DM density profile, while the rest of the factors are dependent of the specific model the DM. From the observation of the flux $\Phi$, we can potentially exclude (or discover) various classes of models. However, due to the large background, the bounds from indirect detection are not as severe as the direct detection.

Of course, there may be other environments in which the density of the DM is enhanced providing for a stronger (and potentially different) set of observables. A prime example is the interior of stars, especially neutron stars which will be the subject of Chapter 4.

Finally, DM can also be produced at colliders. Currently, the LHC provides our best laboratory for studying models of DM. At the LHC, two high energy beams of protons collide at $E_{\text{cm}} = 14\text{ TeV}$. Depending on the DM mass and its interaction strength with SM particles, DM could be produced. Because DM is by definition weakly interacting at best, DM cannot be directly observed at the experiment; its presence is only manifested as missing transverse momentum. More specifically, if the momenta of detected particles in the plane transverse to the beam do not sum to zero, conservation of momentum tells us that there were particles missed by the detector. The only particles in the SM that are completely invisible to the LHC are neutrinos, and thus are a background to DM searches.

If we can produce DM at the LHC, we can find its cross section interaction with the SM, mass, and spin. This is in contrast with the direct detection that tells us
Figure 1.9. The current bounds on DM annihilation cross section to visible matter. The top left plot assumes 100% branching ratio of DM particles to a pair of muons, whereas the top right one assumes 100% branching ratio to a pair of b-quarks. Similarly the bottom left (right) panel assume DM particles annihilate to pair of tau ($W^\pm$). The solid line is the limit, while the dashed line is median expected. The green and yellow regions are respectively 1σ and 2σ deviations [8].

only the cross section rate, and DM mass and indirect detection, which only gives us the rate. However, one disadvantage of the LHC, is that we cannot be sure the inferred particle is really a DM candidate or a BSM particle that happens to be stable at collider time-scale. Whereas, a signal detected in the direct and indirect detection would be from a particle that is stable on cosmological time-scale.

There is one other advantage of the collider DM search compared to other mentioned methods. In direct and indirect detection, we rely only on detecting the DM that is already present in the universe. However, many particle physics models of Dm imply the existence of other new but unstable states that accompany the DM state, a set of particles and interactions that we call the “dark sector”. At the LHC, we can
produce these other dark sector particles and more fully understand the spectrum. This is easier to see in the context of specific models. We will review some of the most popular, and most studied, DM models in the next section and see how LHC searches can be a much more powerful probe of DM.

DM detection experiments, motivated at least in part by the WIMP miracle, have targeted the Weak scale DM with weak-scale mass and cross sections. To date, no unambiguous signal has been detected yet. In the following, we will see a few example of models of DM that are well-motivated and evade the current experimental bounds.

1.4 Models of Dark Matter

My research is focused on exploring various methods for being safe from the current experimental bounds while still providing a compelling model of the dark matter. These ideas fall into two broad classes: we can either use the parameter freedom found in explicit models of DM to help it evade current constraints, or we can relax one or more of the conditions assumed in the thermal relic paradigm. For example, in the first case, we might allow there to be more than one particle in the dark sector such that the dynamics of these various particles allow the model to evade the current bounds. In the second case, the DM may be produced out of thermal equilibrium. But in the either scenario we give up the WIMP miracle as mere coincidence.

1.4.1 Supersymmetry

Supersymmetry is the unique non-trivial extension of the Poincare group and links fermions to bosons. More specifically, supersymmetry requires that for every boson, there is a fermion with exactly the same quantum numbers and mass, and vice versa. Because in the SM, there are no such fermion/boson pairs satisfying this description, supersymmetric theories propose doubling the degrees of freedom of the
SM. Supersymmetric models, first introduced by Wess and Zumino in [10, 11], are among the most studied new physics models. There are also several important phenomenological merits of supersymmetric theories, two of which are 1) providing a DM candidate (in R-parity preserving models) and 2) solving the “hierarchy problem”. In the following, I will briefly explain the hierarchy problem and how supersymmetric theories solve the problem. I will also discuss the DM candidates in supersymmetric models.

SM fermions and gauge bosons are forbidden to acquire a mass in the unbroken phase of the electroweak symmetry due to the chiral and gauge symmetries, respectively. After electroweak symmetry breaking via the Higgs field acquiring a non-zero vacuum expectation value (VEV), fermions and gauge boson get a mass proportional to the VEV. Consequently, the quantum corrections to the fermions and gauge boson masses are also proportional to the VEV, because in the limit VEV → 0 their masses should vanish.

The mass parameter of the Higgs field ($\mu_H$ in the Lagrangian term $\mathcal{L} \supset \mu_H^2 |H|^2$), on the other hand, is not protected by any symmetry and thus is vulnerable to quantum corrections. The measured value of $\mu_H$ is around 100 GeV, but this parameter is quadratically sensitive to the cutoff scale of new physics ($\Lambda$) through quantum corrections:

$$\Delta\mu_H^2 \sim \Lambda^2 + \cdots \quad (1.10)$$

Above the scale $\Lambda$, the SM as an effective field theory (EFT) breaks down. So far, the only known cutoff scale is $M_{Pl} \sim 10^{19}$ GeV, which is when gravity (not part of the SM) becomes strong and relevant. If there are no scales of new physics up to $M_{Pl} \gg \mu_H$, fixing $\mu_H$ at its measured value requires highly tuned input parameters. This is known as the electroweak hierarchy problem. In other words, the low value of $\mu_H$ compared with the $M_{Pl}$ is not “natural”, unless there is a symmetry that protects the value of $\mu_H$ or there is a new physics scale in the vicinity of 100 GeV.
In the supersymmetry, due of the connection between fermions and bosons, the number of fermionic degrees of freedom matches the bosonic ones. Thus, fermions and bosons contribute equally to the quantum (or loop) corrections of $\mu_H$, but with opposite signs:

$$
\Delta \mu_H^2 = -\frac{|\lambda_f|^2}{8\pi^2} [\Lambda^2 + \cdots] \quad \text{fermions contributions}
+ \frac{\lambda_s}{8\pi^2} [\Lambda^2 + \cdots] \quad \text{bosons contributions,}
$$

(1.11)

where $\lambda_f$ and $\lambda_s$ are the interaction strength with the Higgs field. If both interact with the Higgs field equally $|\lambda_f|^2 = \lambda_s$, the two contributions cancel each other out at all orders. This is what happens in the exact supersymmetry scenario. However, experimentally we have not found the super-partners of the SM and so supersymmetry cannot be an exact symmetry of nature. The hope is that it is softly broken at a scale near $m_{\text{soft}} \sim 100\,\text{GeV}$ and thus the quantum corrections are sensitive to $m_{\text{soft}}$ rather than the planck scale.

Theories with supersymmetry also suffer from another experimental observation and that is the proton decay: vanilla supersymmetric theories include terms that result in proton decay, but observations tell us that protons do not decay at the cosmological scale. Therefore, supersymmetric models usually include an R-parity ($\mathbb{Z}_2$ symmetry) with even charges for SM particles and odd charges for super-partners to prevent proton decay. Another consequence of this parity is a stable super-partner particle that can be a dark matter candidate.

In the Minimal Supersymmetric Standard Model (MSSM), which is a model that supersymmetrises only the SM, there are a few particles that satisfy the requirement to be a DM. The most favorite DM candidates among them are the neutralinos – a linear combination of the super-partner of the neutral electroweak gauge bosons (Bino $\tilde{B}$, two Higgsinos $\tilde{H}_1$ and $\tilde{H}_2$, and Wino $\tilde{W}_3$). There are in total four neutralinos in the
MSSM with masses

\[
\mathcal{L}_{\text{neutralino mass}} = -\frac{1}{2} \psi^T M_\tilde{\chi} \psi + \text{c.c},
\]

where

\[
M_\tilde{\chi} = \begin{pmatrix}
M_1 & 0 & -c_\beta s_W m_Z & s_\beta s_W m_Z \\
0 & M_2 & c_\beta c_W m_Z & -s_\beta c_W m_Z \\
-c_\beta s_W m_Z & c_\beta c_W m_Z & 0 & -\mu \\
s_\beta s_W m_Z & -s_\beta c_W m_Z & -\mu & 0
\end{pmatrix}.
\] (1.12)

The entries \(M_1, M_2,\) and \(\mu\) are the mass parameters for Bino, Wino, and Higgsino respectively. Moreover, \(s_\beta = \sin \beta, \ c_\beta = \cos \beta,\) where \(\beta\) is the ratio of the two higgs vacuum expectation values (VEVs). Similarly, \(s_W = \sin \theta_W\) and \(c_W = \cos \theta_W,\) where \(\theta_W\) is the weak angle \(\equiv \cos^{-1} \frac{m_W}{m_Z}.\) The DM, the lightest supersymmetric particle (LSP), is commonly the lightest neutralinos. If DM is a pure Bino \((M_2, \mu \gg M_1)\) it does not annihilate efficiently enough in the thermal relic paradigm to produce the need abundance. Pure Higgsino \((M_2, M_1 \gg \mu)\) or Wino \((M_1, \mu \gg M_2)\) LSPs, on the other hand, annihilate too efficiently and so have a relatively small abundance. However, if the LSP is a mixture of Bino with either of Higgsino or Wino, the correct relic abundance may be obtained. This scenario is known as the “Well-Tempered Neutralino” and was first introduced in [12].

In Chapter 2 a Well-tempered scheme where the LSP is a mixture of Bino and Higgsino is explored. In particular, we assumed the mass parameter for the Bino \(M_1\) and Higgsino \(\mu\) are almost equal size, but the rest of the supersymmetric mass particles are much larger. As a result, there are three almost degenerate light neutralinos, and two light charginos (the super-partner of charged electroweak gauge bosons), and the rest of the particles are heavy. If \(M_1 \sim \mu \sim O(100\ GeV),\) this scenario can explain the hierarchy problem while providing a viable DM candidate, and so it is very close

\(^4\)In supersymmetric models, the higgs sector has to be expanded to two higgs doublets to make sure the theory is anomaly free.
to the WIMP miracle paradigm. The main difference is that DM is near degenerate with some other MSSM particles, which results in a much richer phenomenology. Furthermore, this DM appears to be in the blind spot of direct detection [13], and so safe from the current bounds. The main objective of Chapter 2 is to explore the signature of the DM in this scenario at the run II of LHC.

1.4.2 Froggatt-Nielson

In the thermal relic WIMP paradigm, we assumed DM with weak-scale mass interacts with $O$(weak) couplings with the SM particles. An alternative hypothesis could be that DM is decoupled from the Standard Model and annihilates to a metastable BSM mediators that subsequently decay to Standard Model particles. In this case, we could still have $O$(weak) - scale DM mass and coupling to the mediator but avoid the current bounds. Such scenarios can be realized in many BSM models, and the one particular example addressed in Chapter 3 is the Froggatt-Nielson (FN) mechanism.

FN mechanism attempts to explain the hierarchy of fermion masses and their patterns of mixing in the SM. Fermion masses as explained in 1.4.1 are protected from quantum corrections by the approximate chiral symmetry. However, the puzzling issue is the large separation between the fermion masses; namely, the top quark with 174 GeV mass is many orders of magnitude heavier than neutrinos being with less than an eV mass. FN mechanism proposes a symmetry (usually $U(1)_{FN}$) under which the fermions are distinguished. In this approach, there is a scalar field $S$ (the “flavon”) a SM singlet field, that acquires a VEV and breaks the symmetry. This breaking is communicated to the fermions at different powers of $\epsilon \equiv \langle S \rangle / \Lambda_{FN}$, where $\Lambda_{FN}$ is the scale at which FN mechanism as an EFT breaks down. The mass scale $\Lambda_{FN}$ is usually associated to the mass of heavy vector-like fermions $F$ that are integrated out, in the low energy approximation. The $F$ fermions are charged under the SM
and $U(1)_{\text{FN}}$ symmetry such that they have the following interactions

$$\mathcal{L} \supset Sf + \Lambda_{\text{FN}} F \bar{F},$$

(1.13)

where $f$ is a SM fermion. Some of the examples of such interactions are shown in Fig. 1.10. Integrating out the heavy fermions, results in higher dimensional terms (non-renormalizable terms $\equiv$ interactions suppressed by the cutoff scale):

$$\mathcal{L} \supset y^{ij}_{q} u \left( \frac{S}{\Lambda_{\text{FN}}} \right)^{n_{ij}} HQ_{i} u^{c}_{j} + y^{ij}_{d} \left( \frac{S}{\Lambda_{\text{FN}}} \right)^{m_{ij}} \bar{H}Q_{i} d^{c}_{j}. $$

(1.14)

Figure 1.10. Some examples of the interaction diagrams including the heavy vector-like fermions $F_{i}$ with mass $\sim \lambda_{\text{FN}}$. Left-handed quarks are shown by $Q$, whereas the right-handed up (down) quarks are shown by $u(d)$. The indices indicate the flavor (generation) [14].

The effective yukawas and mixing between quarks are therefore generated as powers of $\langle S \rangle / \Lambda_{\text{FN}}$. The ratio $\epsilon \equiv \langle S \rangle / \Lambda_{\text{FN}}$ is estimated to be around the Cabbibo angle which is 0.23.

In this paradigm, we can add a DM particle that is also charged under the $U(1)_{\text{FN}}$ and therefore interacts with the flavon. If we assume DM is a fermion, odd under a $\mathbb{Z}_{2}$ symmetry and neutral under SM gauge symmetries, the annihilation of DM to
SM particles is small (usually suppressed by $\Lambda_{FN}$) and inefficient to match the relic abundance. However, depending on the relative masses of the DM and the flavon (2 degrees of freedom: CP even eigenstate, or CP odd eigenstate), DM could annihilate to a pair of flavon components. Arranging the couplings of DM to the flavon to be $O(\epsilon = 0.23)$, a weak scale DM could saturate the relic abundance. Therefore, in this scenario, the WIMP particle annihilates to a pair of meta-stable mediators, and the mediators subsequently decay to the SM particles. This approach can easily evade the current direct detection bounds, because the effective coupling of DM to SM particles is small. In Chapter 3, the details of the model and the constraints are discussed in great details.

1.4.3 Non-thermal dark matter relic

As we are challenging the assumptions put in the thermal relic WIMP paradigm, we could also consider the cases where DM is not a thermal relic. Two of such cases discussed here are 1) DM is an asymmetric fermion and its relic abundance is set by annihilating away the symmetric component 2) DM starts with negligible abundance and gets produced through a very small interaction with the SM (this paradigm is known as “freeze in”).

A coincidence that does not get addressed in the WIMP paradigm is the curious comparability of DM abundance with that of the visible matter:

$$\Omega_{DM} = 5\Omega_{\text{visible matter}}, \quad (1.15)$$

while their abundance could have been orders of magnitude apart. This coincidence may suggest that DM abundance is related to the visible matter abundance. In the visible sector, due to an asymmetry between matter and antimatter, the abundance is set by annihilating away the symmetric component. If there were no asymmetries,
the matter and anti matter would keep annihilating and we would not exist. A similar story can occur in the dark sector where the DM candidate is asymmetric. There are three main conditions for the generation of particle asymmetry in a sector, known as the Sakharov conditions which demand the violation of charge (C), charge-parity (CP), and being out of thermal equilibrium. The source of C and CP violation responsible for the asymmetry in the visible sector is still unknown and a plausible scenario is that DM plays a role in it, and thus the dark sector is also asymmetric.

Asymmetric DM, not only needs to be stable but also requires the DM particle to be distinguished from its anti-matter. Therefore, DM must be charged under a different group than a simple $Z_2$ parity. Furthermore, assuming the asymmetries in the dark sector and SM are linked, as the relic abundance of DM is about 5 times greater than the visible matter, the natural mass for asymmetric DM is $5 m_p \sim 5$ GeV. This scheme is quite different from the WIMP paradigm, but still well-motivated. The direct detection experiment constrains the parameter space of such models for DM mass $> 10$ GeV, but has loose bounds for lighter DM (because the recoil energy is smaller than the experimental threshold). The conventional indirect detection – detecting the SM particles from DM annihilation– does not apply, as the remaining DM in the universe are only DM particles and no anti-particles. However, asymmetric DM that has attractive interaction with the SM particles can accumulate in heavy neutrons stars and affect their behavior. A particular model of an asymmetric DM, with a more precise description of the mechanism, outlining the conditions for the formation of the blackhole inside the neutron star and eventually collapsing it are provided in Chapter 4. Means of testing this model in direct detection detection and indirect detection are also discussed.

Yet, another way to move away from the thermal relic is to challenge the assumption that DM was as abundant as the photons in the early universe. A viable conjecture is that DM was produced out of thermal equilibrium, and had a negligi-
ble abundance in the early universe. The current relic of DM comes from its feeble interaction with the SM. The evolution of DM, in this scenario, is very different from thermal relic and strongly depends on the type of DM-SM interaction. This class of model fall under the rubric “freeze-in”, and they were introduced as a general mechanism for DM production only recently [15]. In this paradigm, DM never attains thermal equilibrium with the SM particles, and through its feeble coupling only gets produced. However, the rate of production is so low that DM particles cannot find each other to annihilate. Therefore, in this paradigm the comoving number density of DM only increases as opposed to freeze-out paradigm. One important concern with the general freeze-in mechanism is how are DM-SM small couplings generated. The question to this answer can also drastically change the evolution of DM in early universe. There are two main solutions:

- DM interacts with SM through renormalizable, but very small \((O(10^{-7}))\) couplings: This framework is known as the “IR freeze-in” or infrared freeze-in. That is because the yield (proportional to number density) of DM is dominated at low temperatures, near the mass of DM particle. Hence, this scenario is independent of high temperature physics, but the small couplings in this scenario are un-natural [15].

- DM-SM interaction occurs via non-renormalizable (suppressed by powers of the cutoff scale \(\Lambda\)), but \((O(1))\) couplings: This framework gets the name “UV freeze-in”, because it is highly sensitive to high temperature physics as the yield of DM is dominated at high temperatures.

In Chapter 5 UV-freeze-in is discussed in great details for various powers of \(\Lambda\) suppression, and it is contrasted against IR-freeze-in as well as thermal relic DM.
Supersymmetry has been a scenario strenuously sought at the LHC. While the lack of signals from standard searches has put several constraints on the spectrum of superpartners, and especially on the colored superpartners, these standard searches and channels have problems dealing with compressed supersymmetric mass spectra [16–19]. One circumstance in which several superpartners with similar mass are expected is neutralino dark matter (DM). It is well known that the MSSM with R-parity has candidates that explain the relic density of DM, the lightest neutralino being perhaps the most natural candidate. Taking into consideration all available experimental data, one finds that for pure states, Bino, wino or Higgsino, there are difficulties accommodating the measured dark matter relic density because, either the Bino does not interact sufficiently and overcloses the universe or Higgsinos and winos annihilate too efficiently and have to be over a TeV and thus outside LHC detection range, to explain the DM density. On the other hand, a non-trivial mixture (i.e. well-tempered) of the Bino and the Higgsino, or the Bino and wino, or all three, can reproduce the measured DM abundance with masses in the hundreds of GeV [12, 13, 20–26].

Naturalness is also a powerful guide to the mass spectrum of beyond the standard model (BSM) scenarios [27–37]. As the Higgsino mass parameter \( \mu \) enters at tree level into the expression for the Higgs mass, Higgsinos must be near the weak scale, \( O(200 \text{ GeV}) \), to remain natural\(^1\). Other superpartner masses, such as the Bino mass

\(^1\)Unless the relationship between \( \mu \) and other Higgs soft mass parameters is fixed by some UV
$M_1$ and the wino mass $M_2$ also contribute to the Higgs mass, though at loop level. Therefore the Bino and wino may be significantly heavier than the weak scale while remaining natural.

Viewed in the light of naturalness, the Bino-Higgsino admixture stands out among other well-tempered scenarios and is a prime target for LHC searches. This admixture will be the focus of this paper, with the study of well-tempered possibilities involving the wino deferred to later work. Well tempered Bino-Higgsino scenarios are characterized by small inter-electroweakino splittings; in terms of Lagrangian parameters, well tempered Bino-Higgsinos with $|\mu| < 200$ GeV have

$$M_1 \simeq |\mu| - 25 \text{ GeV}, \quad (2.1)$$

where $M_1$ is the Bino soft mass parameter and $\mu$ is the Higgsino mass. Translated into mass eigenvalues, the above relation implies the splitting between the lightest neutralino $\tilde{\chi}_1^0$ (the lightest supersymmetric particle (LSP)) and the next two neutralinos $\tilde{\chi}_2^0, \tilde{\chi}_3^0$, as well as the splitting between the lightest chargino $\tilde{\chi}_1^\pm$ and the LSP are all $\lesssim m_Z$.

The combination of the light Bino-Higgsino neutralino sector masses, preferred by naturalness arguments, and the small inter-state splitting puts the electroweakino sector of these models in a confounding place; the states are light enough to be produced abundantly at the LHC, but the small splitting among states makes conventional analyses difficult. Conventional analyses, assuming all sleptons are heavier than the electroweakinos, are based on the trilepton plus missing energy signal, $pp \to 3\ell + E_T$.

This final state is generated by the production of heavier electroweakinos $pp \to \tilde{\chi}_1^\pm \tilde{\chi}_2^0$, followed by the decays $\tilde{\chi}_1^\pm \to W^{\pm}(\ell^\pm\nu) + \tilde{\chi}_1^0, \tilde{\chi}_2^0 \to Z(\ell^+\ell^-) + \tilde{\chi}_1^0$. As $m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_1^0}$ fall below $m_Z$, the sensitivity of this approach degrades; the intermediate dynamics [38].
$W^\pm$ and $Z$ bosons become off-shell and their subsequent lepton decays are too soft to trigger upon efficiently.

One way to combat the loss of sensitivity in the trilepton plus $E_T$ channel is to look for electroweakinos produced in association with a hard photon or jet, $pp \rightarrow \widetilde{\chi}_1 \widetilde{\chi}_1^0 + j/\gamma$. Since the initial state radiation (ISR) can be used as a triggerable object, rather than the electroweakino decay products, the subsequent cuts can be loosened, opening up sensitivity to smaller electroweakino mass splittings. The price one pays for this approach is a significant loss in rate. The decrease depends on the jet and photon trigger thresholds, but is roughly $1/50$ for electroweakinos produced with a 100 GeV jet at a 14 TeV LHC \cite{37, 39, 40}.

In this paper we present an alternative analysis strategy for nearly degenerate electroweakinos that does not rely on large missing tranverse momentum concomitant with hard initial state radiation. Instead, we look to a different final state, $\ell^+ \ell^- + \gamma + E_T$. Electroweakino final states containing photons can certainly arise from initial or final-state radiation, such as $pp \rightarrow \widetilde{\chi}_1^+ \widetilde{\chi}_1^- + \gamma \rightarrow \ell^+ \ell^- + \gamma + E_T$, however photons can also come from neutralino decay \cite{41–43}, $\widetilde{\chi}^0_{2,3} \rightarrow \widetilde{\chi}^0_1 + \gamma$. Because decays to photons are a loop-level effect and are therefore sometimes neglected in electroweakino phenomenology. Indeed, Tevatron studies have placed bounds on neutralino masses in gauge-mediated scenarios by searching for their decays to photons, $Z$ bosons, and gravitinos \cite{44–46}. Photon decays are two body processes and can easily compete with three-body decays through an off-shell electroweak gauge boson, such as $\widetilde{\chi}^0_{2,3} \rightarrow \ell^+ \ell^- \widetilde{\chi}^0_1$. As we will show, photons from neutralino decays can yield a more easily distinguished signal in future high luminosity collider data.

Assuming $\widetilde{\chi}^0_{2,3} \rightarrow \gamma + \widetilde{\chi}^0_1$ makes up a portion of our electroweakino signal, the next question is what else is present. One possibility is that $\widetilde{\chi}^0_{2,3}$ is produced in association with a chargino, $pp \rightarrow \widetilde{\chi}^\pm \widetilde{\chi}^0_{2,3}$, which leads to a final state of $\ell + \gamma + E_T$\footnote{Throughout this work we will focus on electroweakino decays that yield leptons. The back-}. This process
has the benefits of a large production rate relative to other electroweakino processes and the $O(100\%)$ branching fraction of the chargino to $W^*$. However, this final state has a large SM background from $W(\ell\nu) + \gamma$, produced via $\sigma(pp \rightarrow W^{\pm}(\ell\nu)\gamma) \sim 30\text{ pb}$ at the LHC ($\sqrt{s} = 14\text{ TeV}$). While there are certainly kinematic handles that can distinguish $W + \gamma$ from $\tilde{\chi}^{\pm,0}_{2,3}$ production, the starting diboson cross section is so enormous ($> 100$ times the signal) that an electroweakino search using only a single-lepton final states looks extremely challenging.

We therefore focus on the final state $\ell^+\ell^- + \gamma + E_T$. While there are many electroweakino production and decay paths that arrive at this final state, we find $pp \rightarrow \tilde{\chi}_3^0\tilde{\chi}_2^0, \tilde{\chi}_{2,3}^0 \rightarrow \gamma E_T, \tilde{\chi}_3^0 \rightarrow \ell^+\ell^- E_T$ – one neutralino decays to a photon plus LSP and the other decays to a same-flavor lepton pair via an off-shell $Z$ – has the best combination of rate and kinematic discernibility from SM processes. One immediate benefit of the $\ell^+\ell^- + \gamma + E_T$ final state is that there is no diboson background. There are formidable backgrounds coming from $pp \rightarrow VV\gamma$, where $V$ are any combination of $W^\pm/Z/\gamma^*$, and from $pp \rightarrow \gamma^*/Z(\tau^+\tau^-) + \gamma$ where both of the taus decay leptonically. However, we find the signal can be separated from the SM using a combination of $m_{\ell\ell}$ and angular cuts.

The layout of the remainder of this paper is as follows: In Sec. 2.1 we review the existing limits on Higgsino-Bino admixtures, then explore how the inter-electroweakino splitting, the overall electroweakino mass scale, and the relic density are interrelated. Next, In Sec. 2.2 we introduce the $\ell^+\ell^-\gamma + E_T$ final state and study its rate and kinematic properties. Our main results are presented in Sec. 2.3 where we motivate and implement an analysis that isolates the Higgsino-Bino signal from the SM background. We first test our analysis on four benchmark points, then, discuss how our strategy fares in a wider region of parameter space. In Sec. 2.4 we grounds for hadronic final states are orders of magnitude larger, especially considering the low energies (i.e small splittings) we are interested in.
comment on how our fairly idealized setup holds up under more realistic experimental conditions. Finally, Sec. 2.5 contains our conclusions. Some technical details can be found in the appendices.

2.1 The Mass Splitting and Relic Abundance of Bino-Higgsinos

In this section we determine Bino-Higgsino mass splittings and relic abundances for the mass parameter ranges $M_1 = 100 - 250$ GeV, $|\mu| = 100 - 250$ GeV, and for $\tan \beta = 2$ and 10. Here and throughout this paper, it is assumed that the wino (mass parameter $M_2$), and all other supersymmetric particles are decoupled; their masses are set to $\sim 3$ TeV in numerical computations. We do not explicitly determine how a large Higgs mass is generated, but the model building details which yield $m_h = 125$ GeV should not affect the results of this paper\footnote{While we do not specify how the mass of the Higgs is generated, heavy stops and F or D term contributions could be responsible for raising the Higgs mass. In any case, in this study the stop and gluino are assumed to completely decouple from electroweakinos and we ignore them in our treatment of the electroweakino mass spectrum.}. The treatment of well-tempered neutralinos given below – masses, collider and dark matter properties – can be easily adapted to mixed Bino-wino scenarios, which we leave to future work.

2.1.1 Status of Bino and Higgsino collider searches

Many of the most stringent bounds on the Bino, Higgsino, and Bino-Higgsino admixture were set over a decade ago by LEP and LEPII. The exclusion of charginos produced via $e^+e^- \to \tilde{\chi}_1^\pm \tilde{\chi}_1^-$ at LEPII bounds the lightest chargino mass, $m_{\tilde{\chi}_1^\pm} > 103$ GeV; since we have decoupled the wino, in our setup this limit is essentially a limit on $|\mu|$. Recent multilepton plus $E_T$ studies at the LHC have restricted some mixed neutralino parameter space\footnote{While we do not specify how the mass of the Higgs is generated, heavy stops and F or D term contributions could be responsible for raising the Higgs mass. In any case, in this study the stop and gluino are assumed to completely decouple from electroweakinos and we ignore them in our treatment of the electroweakino mass spectrum.}. As we have decoupled all sleptons, the limits that apply are for electroweakinos that decay via $W^{(*)}\tilde{\chi}_2^0/Z^{(*)}$ and generate a $3\ell + E_T$ final state. When $m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0}$ is greater than $m_Z$, $\tilde{\chi}_2^0$ (assumed degenerate
with $\tilde{\chi}_1^\pm$) are excluded up to 400 GeV for massless $\tilde{\chi}_1^0$ and up to 350 GeV for $\tilde{\chi}_1^0$ lighter than $\sim 150$ GeV. For more degenerate spectra, $m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_1^+} - m_{\tilde{\chi}_1^0} < m_Z$, the bounds are even weaker, with no limits for $m_{\tilde{\chi}_1^1} > 100$ GeV.

Looking forward to the 14 TeV LHC run\(^4\), the sensitivity of $3\ell + E_T$ searches to scenarios with $m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_1^+} - m_{\tilde{\chi}_1^0} > m_Z$ will extend greatly\([49]\), but the sensitivity to nearly degenerate spectra will not. New collider techniques to search this area of neutralino parameter space are necessary. This region where the electroweakino spectra is quasi-degenerate also has a compelling connection with dark matter – as we will see, well-tempered Bino-Higgsino scenarios with minimal fine-tuning ($|\mu| \lesssim 200$ GeV) typically have inter-electroweakino mass splittings of $O(25$ GeV).}

2.1.2 Bino-Higgsino mass splitting

Given the insensitivity of current LHC searches to sub-$m_Z$ inter-electroweakino splitting, our next step is to analyze what regions of Bino-Higgsino parameter space lead to quasi-degenerate spectra. Under our assumption of a decoupled wino, the mass eigenstates of the Bino-Higgsino depend on the masses $M_1$, $\mu$, $m_Z$ and the angles $\theta_W$ and $\beta$. In a basis with a column vector, which from top to bottom has first the Bino $\tilde{B}$ and then the Higgsino mass states defined as $\tilde{H}_1 \equiv (\tilde{H}_u - \tilde{H}_d)/\sqrt{2}$ and $\tilde{H}_2 \equiv (\tilde{H}_u + \tilde{H}_d)/\sqrt{2}$, the mass mixing matrix is given by

$$
\mathcal{M} = \begin{pmatrix}
M_1 & -s_\beta + c_\beta s_W m_Z & s_\beta - c_\beta s_W m_Z \\
-s_\beta - c_\beta s_W m_Z & \mu & 0 \\
s_\beta + c_\beta s_W m_Z & 0 & -\mu
\end{pmatrix},
$$

where $s_\beta$ and $c_\beta$ represent $\sin \beta$ and $\cos \beta$ respectively, and $s_W$ is $\sin \theta_W$.

The masses and composition of the neutralinos determine how they are produced and decay. To see how the masses and mass splittings $m_{\tilde{\chi}_{1,2,3}^0} - m_{\tilde{\chi}_1^0}$ shift as we vary

---

\(^4\)The current energy of the LHC is 13 TeV.
the relationship between $|\mu|$, $M_1$ and $\tan \beta$, it is useful to first explore some limiting cases. If $|\mu| \gg M_1$, the heaviest two neutralinos $\tilde{\chi}_2^0$ and $\tilde{\chi}_3^0$ will be Higgsino like, and the mass splittings between each of these heavy neutralinos and the Bino-like LSP $\tilde{\chi}_1^0$ will be sizable. This spectrum is shown in the left-hand panel of Fig. 2.1. If, on the other hand, $|\mu| \ll M_1$ (sample spectrum shown in the right panel of Figure 2.1), the lightest two neutralinos $\tilde{\chi}_1^0$ and $\tilde{\chi}_2^0$ will be Higgsino like. This means that while the mass splitting between $\tilde{\chi}_3^0$ and $\tilde{\chi}_1^0$ will remain sizable as before, the mass splitting between $\tilde{\chi}_2^0$ and $\tilde{\chi}_1^0$ will be tiny.

To interpolate between these limits, we proceed numerically. The mass splittings for $|\mu|, M_1 \in [100 \text{ GeV}, 250 \text{ GeV}]$ are shown below in Fig. 2.2. In the upper right (left) of the $\mu < 0 (\mu > 0)$ plots, we see the limit $M_1 \gg |\mu|$, while we see the $|\mu| \gg M_1$ limit in the lower left (right) corner. In the middle region, where $|\mu| \sim M_1$, the neutralino spectrum becomes compressed and there are broad regions of parameter space where either $m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0}$, $m_{\tilde{\chi}_3^0} - m_{\tilde{\chi}_1^0}$, or both are less than the $Z$ mass; these slices are the regions of greatest interest for our study. While the splittings are less than $m_Z$ in these slices, they do not become arbitrarily small. In the region $|\mu| \sim M_1$, the splitting between $\tilde{\chi}_3^0$ and $\tilde{\chi}_1^0$ is always greater than $\sim 30 \text{ GeV}$. The $\tilde{\chi}_2^0 - \tilde{\chi}_1^0$ splitting can be smaller, i.e. in the $|\mu| \ll M_1$ limit, but for $M_1 \sim |\mu|$ the splitting rarely dips below $\sim 20 \text{ GeV}$. As we explain in greater detail in Sec. 2.3, splittings of this size are interesting from a collider perspective; the splittings are large enough that particles emitted as one neutralino decays to another can be efficiently detected at the LHC, yet the splittings are too small for neutralinos to decay via on-shell $W^\pm/Z$.

Comparing the four plots in Fig. 2.2 we see that the degree of degeneracy depends on the sign of $\mu$ and $\tan \beta$. The off-diagonal matrix entries in Eq. (2.2) grow in magnitude with $\tan \beta$, thus the inter-state splitting also increases with $\tan \beta$. To understand the effect of the sign of $\mu$, consider the limit that $M_1 \sim |\mu|$, and $\tan \beta = 1$. In this case, $m_{\tilde{\chi}_3^0} - m_{\tilde{\chi}_2^0} \sim m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0} = \frac{1}{2} \left( m_{\tilde{\chi}_3^0} - m_{\tilde{\chi}_1^0} \right) \sim m_W \tan \theta_W$. However, for
Figure 2.1. The mass splitting for different ranges of $|\mu|$ and $M_1$. On the left side, $M_1 < |\mu|$ and so the two Higgsino-like states are heavier than the Bino-like state. This corresponds to the lower, outside edge of the plots shown in Fig. 2.2. Both the splitting $m_{\tilde{\chi}_3^0} - m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0}$ are large, making this spectrum amenable to studies with a photon and dilepton pair in the final state. The right side shows the opposite regime, where $|\mu| < M_1$. This results in the Higgsinos having smaller masses than the Bino, and is shown in the upper, inside edge of the plots in Fig. 2.2. In this case the splitting $m_{\tilde{\chi}_3^0} - m_{\tilde{\chi}_1^0}$ is large and the splitting $m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0}$ is small.

When $M_1 \sim -\mu$, we find $m_{\tilde{\chi}_3^0} \sim m_{\tilde{\chi}_2^0} > m_{\tilde{\chi}_1^0}$. The splitting between $m_{\tilde{\chi}_3^0} - m_{\tilde{\chi}_1^0}$ for $\mu$ positive is greater than $\mu$ negative. Hence, as reflected in the left and right halves of Fig. 2.2, the mass splittings $m_{\tilde{\chi}_{2,3}^0} - m_{\tilde{\chi}_1^0}$ for a positive $\mu$ are greater than that of a negative $\mu$. Combining these trends, the smallest inter-neutralino splittings occur when $\tan \beta$ is small and $\mu < 0$ while the splittings are largest for large $\tan \beta, \mu > 0$.

Finally, we note that the well-tempered/forged Bino-Higgsino chargino mass, when $|\mu| > M_1$, will be very nearly the mass of $\tilde{\chi}_2^0$. 

\[ M_1 \gtrsim |\mu| \]
2.1.3 Bino-Higgsino relic abundance

The inter-neutralino mass splittings also have ramifications for neutralino dark matter relic abundance, since the lightest neutralino is assumed to be stable. Before describing how the $M_1 - \mu$ split affects mixed Bino-Higgsino relic abundance, we note that a lightest neutralino that is purely Bino or Higgsino does not make a viable electroweak-scale (100 GeV – 1 TeV) dark matter candidate. In order for a pure Bino LSP to be viable it must coannihilate with another sparticle$^5$, while a purely Higgsino LSP can be viable only if its mass is fine-tuned to be greater than 1 TeV, Refs. [12, 13, 25, 50, 51].

Although low (sub-TeV) mass Higgsino and Bino dark matter are separately disfavored, if the Bino mixes appreciably with the Higgsino, a viable relic dark matter candidate – the ‘well-tempered’ neutralino scenario – can emerge. Consider the smooth transition from the case where $M_1 \ll |\mu|$ to that of $M_1 \gg |\mu|$ by simultaneously lowering $|\mu|$ and raising $M_1$ (moving along a diagonal line from the lower-outer edge to the upper-inner edge of the plots in Fig. 2.2). As the mass of $|\mu|$ and $M_1$ get closer, more annihilation channels open to the Bino LSP via an off-shell Higgs or chargino coannihilation, and the relic abundance decreases. Thus we expect that for a well-tempered Bino-Higgsino, the correct relic abundance is achieved in parameter space where $M_1 \lesssim |\mu|$.

The required Bino-Higgsino mass relation is made more explicit in Fig. 2.2 where we overlay the relic abundance contours on top of the contours of inter-Higgsino splitting. We find that the Higgsino and Bino mass parameters producing the correct

$^5$Coannihilating Bino DM is limited by LEP sfermion constraints, e.g. $m_{\tilde{\ell}^\pm} > 100$ GeV. Assuming Bino coannihilation with a right handed slepton, in the limit $m_{\tilde{B}} < m_{\tilde{\ell}^\pm}$, the Bino relic, 

$$\Omega_{\tilde{B}} h^2 \approx 10^{-2} \left( m_{\tilde{\ell}^\pm}/(100 \text{ GeV}) \right)^2 / M_1^2,$$

indicates that the Bino must be lighter than $\sim 50$ GeV, a mass range constrained by the Z width.
Figure 2.2. Mass splitting and dark matter relic abundances are shown for Bino-Higgsino admixtures. The mass splitting between the next-to-lightest neutralino ($\tilde{\chi}_2^0$) and the lightest neutralino ($\tilde{\chi}_1^0$) measured in GeV are indicated with dashed blue lines. The orange bands display the mass splitting between $\tilde{\chi}_3^0$ and $\tilde{\chi}_1^0$. The black lines show dark matter relic abundances.

dark matter abundance for a well-tempered Bino-Higgsino can be approximated by

$$M_1 \simeq |\mu| - 25 \text{ GeV}$$

(2.3)

in the limit that all sparticles besides the Bino and Higgsino are heavy and with only mild dependence on $\tan \beta$ and the sign of $\mu$. The neutralino relic abundance is indicated in Fig. 2.2 by black lines in each of the panels. The line marked with
\( \Omega h^2 = 0.12 \) in an oval corresponds to the well-tempered parameter space yielding the observed dark matter abundance\(^6\).

Dark matter direct-detection experiments, such as CDMS, XENON, and LUX have placed some constraints on well-tempered parameter space Refs. \(^{54-57}\). However, recent work in Ref. \(^{13}\) has emphasized that near certain pieces of the well-tempered region, and especially for \( \tan \beta \leq 2 \) and \( \text{sign}(M_1) \neq \text{sign}(\mu) \), the LSP of the Bino-Higgsino will have a vanishing coupling to the Higgs. Thus for these well-tempered regions of \( M_1 - \mu \) parameter space, spin-independent direct detection will be less-sensitive to a relic Bino-Higgsino, making concomitant collider studies of the Bino-Higgsino especially important for probing the entirety of MSSM dark matter possibilities. For example, the two points in green in the top left panel of Fig. \(^{2.2}\) have \( \sigma_{SI} \lesssim 10^{-45} \text{ cm}^2 \), where \( \sigma_{SI} \) is the spin-independent dark matter-nucleon scattering cross-section\(^7\). The recent LUX result, which constrains a \( m_\chi \sim 100 \text{ GeV} \) LSP to have \( \sigma_{SI} < 2 \times 10^{-45} \text{ cm}^2 \) at 90\% confidence, Ref. \(^{55}\), does not exclude these points. In addition, if one allows the CP-odd neutral Higgs mass \( m_A \) to be light, Ref. \(^{25}\), yet heavy enough to avoid \( A \to \tau\tau \) searches at the LHC, the plausible nucleon-scattering blind regions extend to more parameter space. Particularly, for small \( \tan \beta \) and \( -\mu \sim M_1 \), as studied in this paper, the Bino-Higgsino would be entirely unconstrained by any planned direct detection study, so long as \( m_A \sim 300 \text{ GeV} \).

It is important to note that nucleon-scattering blind regions exhibit some electroweak fine-tuning. Indeed, it has long been appreciated that because well-tempered neutralino relic abundance is sensitive to small shifts in electroweakino and Higgsino mass parameters, there is fine-tuning associated with the simple requirement that

\(^6\)In this study we calculate Bino-Higgsino relic abundance with micrOMEGAs\(^3\) \(^52\), and use a mass spectrum derived with Suspect\(^2\) \(^53\) for values set at the low scale to avoid complications with large logs coming from the decoupling of the other states, we do not include radiative corrections to the masses of the neutralinos or charginos.

\(^7\)We use output from micrOMEGAs\(^3\) \(^52\) to determine DM-nucleon scattering.
mixed neutralinos freeze out with the correct dark matter relic abundance, Ref. \[12\].

For more lengthy discussions see Refs. \[13 50 51\].

Pushing past the well-tempered region, as $\mu$ is lowered closer to $M_1$, the lightest neutralino will annihilate more efficiently, and the total neutralino relic abundance will continue to drop, because the LSP will be more Higgsino-like. A line in parameter space that fits this description is $\Omega h^2 = 0.02$, shown on all of the plots in Fig. 2.2. In most situations, this still lies in the shaded orange regions of parameter space, where all of the mass splittings of the lightest three neutralinos is less than $m_Z$, meaning this ‘well-forged’ spectrum can also be found at colliders via decays of neutralinos to photons and dileptons, a topic explored at more length in Section 2.2.

2.2 Branching Ratios and Cross-Sections for Bino-Higgsinos

In Section 2.1 we showed that a wide swathe of parameter space for which the Bino-Higgsino splittings are $O(20 - 70 \text{ GeV})$ produces a fraction or the entire dark matter relic abundance. Past searches for light electroweakinos have focused on $pp \rightarrow \tilde{\chi}^\pm \tilde{\chi}_2^0$ in the context of a three lepton signal, where the Bino-Higgsino splittings are greater than mass of $Z$ or $W^\pm$ boson, see Refs. \[37 39 40 58 62\].

To gain sensitivity to quasi-degenerate electroweakino spectra, another option is needed. One possibility is to search for electroweakinos produced in association with hard initial state radiation, looking in the final state $\slashed{E}_T + j + X$. As the additional radiation in the event can be used as a trigger, subsequent cuts can be relaxed and soft decay products from the decays among the electroweakinos can be picked out. In the extreme limit of electroweakino splittings $\ll \text{ GeV}$, this technique becomes a mono-jet search, a standard dark matter collider signature \[63 69\]. Though this technique is sensitive to smaller mass splittings, the need to produce hard initial state radiation in addition to the electroweakino pair reduces the cross section substantially, order $50 (p_{T,j} > 100 \text{ GeV}, 14 \text{ TeV})$ \[40\] and signal rate becomes the limiting factor.
2.2.1 Bino-Higgsinos in photon decays

Rather than rely on associated radiation to access compressed electroweakino spectra, we propose looking for pair production of Bino-Higgsino electroweakinos in the final state $\ell^+ \ell^- \gamma + \not{E}_T$.

While there are several possible avenues for electroweakino pairs to arrive at this state, the strategy we advocate is best suited to pair production of heavy neutralinos which decay, one to $\ell^+ \ell^- \tilde{\chi}^0_1$ and the other to $\gamma + \tilde{\chi}^0_1$. Neutralino decays to photons are often neglected, since the decay is a loop-level process, proceeding via a $W^\pm$-chargino loop. However, when the neutralino spectrum gets squeezed, the photon decay mode becomes competitive. Specifically, as the splitting among neutralinos shrinks below $m_Z$, neutralino decays through the $Z$ become three-body decays and are phase-space suppressed. Combined with the small branching fraction of the $Z$ to leptons – the most clearly identifiable decay products – it is certainly feasible that $BR(\tilde{\chi}^0_2,3 \rightarrow \gamma \tilde{\chi}^0_1) \simeq BR(\tilde{\chi}^0_2,3 \rightarrow Z^*(\ell^+ \ell^-)\tilde{\chi}^0_1)$. We will make this relation among decay modes more concrete shortly. One set of Feynman diagrams showing the $\tilde{\chi}^0_2,3 \rightarrow \ell^+ \ell^- \tilde{\chi}^0_1$ and $\tilde{\chi}^0_2,3 \rightarrow \gamma \tilde{\chi}^0_1$ decays are given in Fig. 2.3.

![Feynman diagrams](image)

Figure 2.3. Decays of $\tilde{\chi}^0_3$ through a dilepton pair and $\tilde{\chi}^0_2$ through a photon.
Having specified the final state we intend to study, the viability and sensitivity of our search depends on i.) the rate of electroweakino (specifically neutralino) production, ii.) the branching fraction of the neutralino pairs into the $\ell^+\ell^-\gamma + E_T$ final state, and iii.) the size and kinematic characteristics of the SM backgrounds. The production cross section and branching fractions of neutralinos vary as we move in Bino-Higgsino parameter space $(\mu, M_1, \tan \beta)$ and will be addressed in turn in this section. We will study the SM backgrounds in more detail in Sec. 2.3.

2.2.2 Production of Bino-Higgsinos

Turning first to the production, one element of the signal rate is how many electroweakino subprocesses contribute to our final state. Several different electroweakino pair-production modes are possible, i.e. $\tilde{\chi}_1^\pm \tilde{\chi}_1^\mp, \tilde{\chi}_2^0 \tilde{\chi}_2^0, \tilde{\chi}_3^0 \tilde{\chi}_1^0$, etc., however as we will show later on, the mode driving the $\ell^+\ell^-\gamma + E_T$ signal is $pp \rightarrow \tilde{\chi}_2^0\tilde{\chi}_3^0$. In Fig. 2.4, we plot the production cross-section of these heavier neutralinos, $pp \rightarrow \tilde{\chi}_2^0\tilde{\chi}_3^0$ as a function of $\mu$ and $M_1$ for $\tan \beta = 2, 10$. The cross sections are largest when the neutralinos are lightest and decrease more slowly as $|\mu|$ is increased compared to increasing $M_1$.

Mixed Bino-Higgsinos are produced through an s-channel $Z$ or $W^\pm$ boson. However, as the Bino is inert under $W^\pm/Z$ interactions, the neutralino mass eigenstates are produced in proportion to their Higgsino fraction. In the mass range pertinent to LHC studies, the well-tempered line that quenches the observed relic abundance of dark matter has $M_1$ about 25 GeV less than $|\mu|$. In this case, the production cross section will be larger for the heavier neutralinos than the lightest neutralino, because $\tilde{\chi}_2^0$ and $\tilde{\chi}_3^0$ have larger Higgsino components than $\tilde{\chi}_1^0$. This is illustrated in Fig. 2.4 where we see a sharp drop in the cross section when $|\mu| < M_1$ indicating a large Bino component in $\tilde{\chi}_2^0, \tilde{\chi}_3^0$. One might expect $pp \rightarrow \tilde{\chi}_2^0\tilde{\chi}_2^0, \tilde{\chi}_3^0\tilde{\chi}_3^0$ to have a similar size cross section as $pp \rightarrow \tilde{\chi}_2^0\tilde{\chi}_3^0$, however due to the fact that the two Higgsinos have oppo-
Figure 2.4. Lines for the Bino-Higgsino relic abundance and the cross section \( pp \rightarrow \tilde{\chi}^0_2\tilde{\chi}^0_3 \) at the 14 TeV LHC are indicated with oval bubbles and rectangular bubbles, respectively. The green points in parameter space are studied in this paper for the signal \( pp \rightarrow \tilde{\chi}^0_2\tilde{\chi}^0_3 \rightarrow \tilde{\chi}^0_1\tilde{\chi}^0_1\ell^+\ell^-\gamma \).

site hypercharge, the \( Z \) couplings to same-flavor neutralinos (i.e. \( \tilde{\chi}^0_i\tilde{\chi}^0_i \)) are highly suppressed compared to mixed flavor.

2.2.3 Branching fraction of Bino-Higgsinos

The next ingredient is the branching fraction of \( \tilde{\chi}^0_2\tilde{\chi}^0_3 \) into \( \ell^+\ell^-\gamma + E_T \). Of the two decays we are envisioning, \( \tilde{\chi}^0_{2,3} \rightarrow \gamma + \tilde{\chi}^0_1 \) is the more exotic \cite{42,70,74} and worth further scrutiny. The branching ratios \( BR(\tilde{\chi}^0_2 \rightarrow \gamma\tilde{\chi}^0_1) \) and \( BR(\tilde{\chi}^0_3 \rightarrow \gamma\tilde{\chi}^0_1) \) are shown in Fig. 2.5 as a function of \( \mu \) and \( M_1 \) for \( \tan \beta = 2 \). We have overlaid the
mass splittings $m_{\tilde{\chi}_3^0} - m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0}$ on the branching ratio contours, as the splitting controls how suppressed the competing off-shell $Z$ decay modes are. The size of $BR(\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 \gamma)$ roughly follows the size of the mass splitting and peaks where $|\mu| \sim M_1$, though the transition is sharper. The sharpness of the transition is due to a level crossing of the $\tilde{\chi}_2^0, \tilde{\chi}_3^0$ eigenvalues. Specifically, as the diagonal elements of Eq. (2.2) become degenerate, the mixing angles get large, suddenly altering the composition of the neutralinos. If a neutralino (either $\tilde{\chi}_2^0$ or $\tilde{\chi}_3^0$) inherits a large Bino component, its $Z$ couplings all drop. Since the dominant mechanism of $\tilde{\chi}_2^0, \tilde{\chi}_3^0$ decay is via $Z$, when these couplings drop, the total width drops, and the branching ratio to photons – which involves a different set of mixing parameters than the $Z$ modes – jumps.

Combining the production and decay rates, we see that the $\ell^+\ell^-\gamma + E_T$ final state explored in this paper is well suited for, but not limited to, well-tempered neutralino parameter space. We now move on to the third factor in this mode’s viability, the SM backgrounds, and suggest a set of collider analysis cuts to separate this background from the electroweakino signal.

2.3 Compressed Electroweakinos from Photon + Dilepton at the LHC

The collider final state we are interested in extracting from compressed electroweakinos is $\ell^+\ell^- + \gamma + E_T$. In the standard model, there are a number of processes which give rise to this final state. The dominant backgrounds for the electroweakino $\gamma + \ell^+\ell^- + E_T$ signal are

\begin{align}
pp &\to t\bar{t} \gamma |_{\text{dilepton decay}} \\
pp &\to \gamma^* / Z (\tau^+ \tau^-) \gamma |_{\text{dilepton decay}} \\
pp &\to VV \gamma |_{\text{dilepton decay}}
\end{align}

(2.4)
Figure 2.5. The branching fraction for Bino-Higgsinos decays to photons or dileptons and the LSP are shown for $\tan \beta = 2$ and $\mu < 0$. The black line is the well-tempered region indicating where Bino-Higgsinos produce the observed relic abundance in our universe. The green points mark the benchmarks studied in section 2.3.

where the photon is radiated from a charged particle in the initial or final state. In the $VV\gamma$ background, $V$ corresponds to all combinations of $W^\pm/Z/\gamma^*$, though in practice the dominant contribution comes from $W^+W^-\gamma$. The presence of missing energy, multiple electromagnetic objects, and little to no hadronic activity strongly limits what backgrounds can arise. There are other processes which can contribute to

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the $\ell^+\ell^-\gamma + E_T$ final state through object mis-reconstruction (fakes) or other realities of pileup and hadronic chaos in the LHC environment. We believe that, for the final state we are interested in, these environmental backgrounds are manageable. We will therefore ignore them for now, deferring more detailed comments until Sec. 2.4.

To show that the $\ell^+\ell^-\gamma + E_T$ electroweakino final state can be effectively discriminated from these backgrounds, we pick four benchmark points from the well-forged and well-tempered parameter space. These points are marked as green dots in Figs. 2.2, 2.4, and 2.5. The points A and B both have a negative value for $\mu$ and $\tan\beta = 2$, which leads to small mass splittings between the neutralinos. Points C and D have $\tan\beta = 10$, which creates larger mass splittings. A summary of these benchmark points is given in Table 2.1. It will be shown that the smaller mass splitting in points A and B not only leads to a higher branching ratio to photons, but also leads to more distinct kinematics than the larger splitting of points C and D.

There are also electroweakino processes other than $pp \rightarrow \tilde{\chi}_2^0\tilde{\chi}_3^0$ which generate a $\ell^+\ell^-\gamma + E_T$ final state. For example:

\begin{align}
pp &\rightarrow \gamma \left( \tilde{\chi}^+ \rightarrow \tilde{\chi}_1^0\ell^+\nu_\ell \right) \left( \tilde{\chi}^- \rightarrow \tilde{\chi}_1^0\ell^0\bar{\nu}_\ell \right), \\
pp &\rightarrow \left( \tilde{\chi}_2^0 \rightarrow jj\tilde{\chi}_3^0 \right) \left( \tilde{\chi}_3^0 \rightarrow \gamma\tilde{\chi}_2^0 \rightarrow \gamma\ell^+\ell^-\tilde{\chi}_1^0 \right), \\
pp &\rightarrow \left( \tilde{\chi}^+ \rightarrow \tilde{\chi}_1^0jj' \right) \left( \tilde{\chi}_3^0 \rightarrow \gamma\tilde{\chi}_2^0 \rightarrow \gamma\ell^+\ell^-\tilde{\chi}_1^0 \right), \\
pp &\rightarrow \gamma \left( \tilde{\chi}^+ \rightarrow \tilde{\chi}_1^0jj' \right) \left( \tilde{\chi}_2^0 \rightarrow \ell^+\ell^-\tilde{\chi}_1^0 \right).
\end{align}

We refer to the processes in Eq. \((2.5)-(2.8)\), which are explained in more detail in Appendix A, as ‘alternative signals’ because they have a different final state photon kinematic distribution than the dominant signal $pp \rightarrow \tilde{\chi}_2^0\tilde{\chi}_3^0 \rightarrow \gamma\ell^+\ell^-\tilde{\chi}_1^0\tilde{\chi}_1^0$, and are harder to distinguish from the SM background. For instance, the two chargino production in \((2.5)\) has nearly the same collider morphology as the $WW\gamma$ background.
TABLE 2.1

VALUES OF INTEREST FOR THE FOUR BENCHMARK POINTS

<table>
<thead>
<tr>
<th>Benchmark points</th>
<th>Point A</th>
<th>Point B</th>
<th>Point C</th>
<th>Point D</th>
</tr>
</thead>
<tbody>
<tr>
<td>µ</td>
<td>-150 GeV</td>
<td>-180 GeV</td>
<td>-145 GeV</td>
<td>150 GeV</td>
</tr>
<tr>
<td>$M_1$</td>
<td>125 GeV</td>
<td>160 GeV</td>
<td>120 GeV</td>
<td>125 GeV</td>
</tr>
<tr>
<td>tan β</td>
<td>2</td>
<td>2</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>$m_{\tilde{\chi}_0^1}$</td>
<td>124.0 GeV</td>
<td>157 GeV</td>
<td>105 GeV</td>
<td>103 GeV</td>
</tr>
<tr>
<td>$m_{\tilde{\chi}_0^2}$</td>
<td>156.9 GeV</td>
<td>186 GeV</td>
<td>150 GeV</td>
<td>153 GeV</td>
</tr>
<tr>
<td>$m_{\tilde{\chi}_0^3}$</td>
<td>157.4 GeV</td>
<td>188 GeV</td>
<td>163 GeV</td>
<td>173 GeV</td>
</tr>
<tr>
<td>$\sigma(pp \to \tilde{\chi}_2^0 \tilde{\chi}_3^0)$</td>
<td>394 fb</td>
<td>200 fb</td>
<td>345 fb</td>
<td>287 fb</td>
</tr>
<tr>
<td>$BR(\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 \gamma)$</td>
<td>0.0441</td>
<td>0.0028</td>
<td>0.0017</td>
<td>0.0014</td>
</tr>
<tr>
<td>$BR(\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 \ell^+ \ell^-)$</td>
<td>0.0671</td>
<td>0.0712</td>
<td>0.0702</td>
<td>0.0700</td>
</tr>
<tr>
<td>$BR(\tilde{\chi}_3^0 \to \tilde{\chi}_1^0 \gamma)$</td>
<td>0.0024</td>
<td>0.0767</td>
<td>0.0115</td>
<td>0.0102</td>
</tr>
<tr>
<td>$BR(\tilde{\chi}_3^0 \to \tilde{\chi}_1^0 \ell^+ \ell^-)$</td>
<td>0.0714</td>
<td>0.0613</td>
<td>0.0447</td>
<td>0.0304</td>
</tr>
<tr>
<td>$\sigma(pp \to \tilde{\chi}_2^0 \tilde{\chi}_3^0 \to \gamma \ell^+ \ell^- \tilde{\chi}_1^0 \tilde{\chi}_1^0)$</td>
<td>1.297 fb</td>
<td>1.125 fb</td>
<td>0.279 fb</td>
<td>0.205 fb</td>
</tr>
</tbody>
</table>

These alternative signals are lumped together with the primary process, $pp \to \tilde{\chi}_2^0 \tilde{\chi}_3^0$, to form the electroweakino signal in all of our simulations.

We conducted an analysis of these benchmark points and their backgrounds using Monte Carlo event generators to simulate LHC proton-proton collisions with a center of mass energy $\sqrt{s} = 14$ TeV. To generate supersymmetric mass parameters for the signal events, we used the spectrum generated by SuSpect 2.43 [53] with the decays calculated by SUSY-HIT [75]. The resulting parameter card was used with PYTHIA 6.4 [76] to generate the events and perform subsequent showering, hadronization and decays. The background processes were generated with MG5@NCLO [77], again using PYTHIA 6.4 for showering and hadronization. To simulate collider acceptance
in this analysis, we implemented jet clustering and required that partons pass angular
cuts and $p_T$ thresholds as detailed below. For a more extended discussion of collider
triggers and efficiencies, see Section 2.4.

The final state involves exactly two leptons and one photon. We identify lepton
candidates by requiring they have $|\eta| < 2.5$ and $p_T > 8$ GeV. Photon candidates also
must have $|\eta| < 2.5$ and $p_T > 20$ GeV. We require the leptons to be isolated from
each other, the photon, and jet candidates. For each lepton (photon) candidate we
check the hadronic energy within a radius of $\Delta R < 0.4$. If the hadronic energy is
greater than 5% of the lepton (photon) energy, the lepton (photon) is added to the jet
seeds. The jets were combined using Fastjet3 \cite{78} with the anti-$k_T$ jet algorithm and
jet radius of 0.5; subsequently, we impose a minimum jet $p_T$ of 25 GeV and a rapidity
of $|\eta| < 2.5$ on all jets. A final check removes any lepton or photon within $\Delta R < 0.4$
of a jet. The leptons are then sorted by their transverse momentum, defining the
lepton with largest $p_T$ as $\ell_1$ and sub-leading $p_T$ as $\ell_2$. We then use the 8 TeV dilepton
trigger defined as

$$p_{T,\ell_1} > 20 \text{ GeV} \& \ p_{T,\ell_2} > 8 \text{ GeV}.$$ \hspace{1cm} (2.9)

For our analysis, we require that there are only two leptons, and that these two
leptons need to be a same flavor opposite sign pair (SFOS). In addition, we require
that all events have a single photon. The photon $p_T$ criteria is quite high

$$p_{T,\gamma} > 20 \text{ GeV},$$ \hspace{1cm} (2.10)

which helps to reduce soft photon backgrounds. For the rest of the analysis, we refer
to equations \Eq{2.9} and \Eq{2.10} as the basic selection.

To further separate the signal from the background, we can make use of several
kinematic features peculiar to the signal. Fig. 2.6 shows an illustration of a possible
event to help visualize the kinematic features.
The two leptons should be minimally separated, while the angle between the photon and the dilepton system should be large. The two $\chi_0^3$s are in nearly opposite directions leading to small amounts of missing energy.

- No jets $p_T > 25$ GeV, $|\eta| < 2.5$ in the event. The signal comes completely from electroweak production and therefore contains little hadronic activity. Meanwhile, backgrounds such as $t\bar{t} + \gamma$ are characterized by at least two jets and are strongly suppressed by this condition.

- $|\Delta \phi_{\ell_1, \ell_2}| < \pi/2$, where $\Delta \phi_{\ell_1, \ell_2}$ is the azimuthal angle between the two leptons. In the signal, both leptons come from the decay of either $\chi_2^0$ or $\chi_3^0$ and tend to be close together. This is in contrast to the $VV\gamma$ and $t\bar{t}\gamma$ backgrounds where the leptons come from two separate $W$ bosons, or the $\gamma^*/Z(\tau^+\tau^-)+\gamma$ where the leptons come from two taus. Thus, by placing a cut on the maximum $|\Delta \phi_{\ell_1, \ell_2}|$, we can remove a large fraction of the background without affecting the signal. The area normalized distributions for $|\Delta \phi_{\ell_1, \ell_2}|$ are shown in the first panel of Fig. 2.7.

- $10$ GeV $< m_T(\ell_i) \lesssim m_W$, where $m_T(\ell_i)$ is the transverse mass formed from
Figure 2.7. Area normalized distributions of $|\Delta \phi_{\ell,\ell}|$, $m_T(\ell_1)$, and $|\Delta \phi_{\ell\ell,\gamma}|$ for events that have passed the trigger and the 0 jet constraint. Point A has mass splitting of the neutralinos $\sim 25$ GeV while point C has splittings on the order of 50 GeV. The larger splitting causes all cuts to be less effective than the lower mass splitting case.

either of the two leptons and the missing energy. A minimum threshold of $m_T(\ell_i, \slashed{E}_T) > 10$ GeV removes a large fraction of the $\gamma^*/Z(\tau^+\tau^-) + \gamma$ background without throwing away much of the signal. An upper limit on $m_T(\ell_i, \slashed{E}_T) < m_W$ removes large portions of the $t\bar{t} + \gamma$ and $VV + \gamma$ backgrounds. The area-normalized distributions for $m_T(\ell_i, \slashed{E}_T)$ for the backgrounds of our benchmark signal points are shown below in the second panel of Fig. 2.7. The $m_{T2}$ vari-
able was also examined and found to provide good separation between signal and the $\gamma^*/Z(\tau^+\tau^-) + \gamma$. However, we found that using $m_T$ for both leptons individually provided better background discrimination than $m_{T2}$ for the other backgrounds. Note that, while in the preceding we have quoted a cut of $m_T(\ell_i, \vec{E}_T) < m_W$, the actual value of this cut will be listed below, and will depend on the particulars of the parameter space point analyzed (i.e. do other necessary cuts already exclude $W$ boson containing backgrounds).

- $|\Delta\phi_{\ell\ell-\gamma}| > 1.0$, where $\Delta\phi_{\ell\ell-\gamma}$ is the azimuthal angle between the dilepton pair and the photon. In the signal the dilepton pair and the photon come from separate neutralino decays, $\chi_{2,3}^0 \rightarrow \ell^+\ell^-\chi_1^0$, $\chi_{3,2}^0 \rightarrow \gamma\chi_1^0$ and therefore tend to be well separated in the detector. Photons that come from soft final state radiation, such as in the dominant $\gamma^*/Z(\tau^+\tau^-) + \gamma$ background, do not have this separation and are dominated by configurations where the photon is as close to one of the leptons as the isolation cuts allow.

- $m_{\ell\ell} \ll m_Z$. For the signal the maximum of this distribution is set by the inter-electroweakino splitting, while the background distributions is broad and peaked at $\sim 50$ GeV ( $\sim 40$ GeV for $\gamma^*/Z(\tau^+\tau^-) + \gamma$). Therefore, by imposing a cut on the maximum allowed value of $m_{\ell\ell}$, we retain the signal while suppressing all backgrounds. The optimal $m_{\ell\ell}$ window depends on the signal point under consideration. For the sake of thoroughness, we note that we did not enforce a minimum invariant mass on $m_{\ell\ell}$.

In addition to these primary kinematic handles, we find several other variables that show small separation between the signal and the background. These include the photon $p_T$, the amount of missing energy, and the angles between the missing energy and the photon or dilepton system. Details of these cuts can be found in Appendix A.1. The two $\tilde{\chi}^0$s are nearly back-to-back which yields a small amount of
missing energy, and there is preferred orientation of the photon or dilepton relative to the $E_T$. This is in stark contrast to ISR-based searches \cite{37,39,40}, where the signal is characterized by large amounts of missing energy.

The actual numerical values that optimize the analysis vary from benchmark to benchmark. To determine the optimal set of cuts we scan over the possible lower and upper bound of the kinematic variables. At each step a simple significance, defined by $S/\sqrt{B}$, is calculated, where the signal cross section does not use the ‘alternative’ signals. We keep the cut which maximizes this value as it leads to the smallest necessary integrated luminosity to achieve a significance of 5. After the optimal cut for each variable is found, the resulting significances are compared and the largest one is chosen. After each cut is chosen, the process starts over again keeping the previous cuts fixed. While it is likely that other optimization procedures would yield slightly different numbers, we believe our qualitative conclusions are robust.

The benchmark points $A$ and $B$ have comparable splittings, which leads to very similar cuts. We therefore take the average of these cut values and define the ‘small mass splitting cuts’. The cut values and resulting significances are summarized below in Table 2.2, where the signal cross sections now include the ‘alternative signals’ of equations (2.5)-(2.8). From these cuts we estimate that Point $A$ could be discovered with an integrated luminosity of 430 fb$^{-1}$ and Point $B$ could be discovered with 620 fb$^{-1}$ of data.

Similarly, benchmark points $C$ and $D$ have comparable mass splittings so their cuts are averaged for the ‘large mass splitting cuts’, which are shown in Table 2.3. The benchmark points $C$ and $D$ have smaller initial cross sections, but the kinematics are also more similar to the backgrounds which makes the cuts less effective. We estimate that point $C$ will be take 4300 fb$^{-1}$ of integrated luminosity to discover, while point $D$ will take 1900 fb$^{-1}$. The required luminosities are large, but within the scope of a
### TABLE 2.2

CUTS USED TO ISOLATE THE SIGNAL FOR THE BENCHMARK

POINTS A AND B

<table>
<thead>
<tr>
<th>'small mass splitting' cuts</th>
<th>Cross section ([\text{ab}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cut</td>
<td>Signal A</td>
</tr>
<tr>
<td>0) Basic Selection</td>
<td>281</td>
</tr>
<tr>
<td>1) (N_{jets} = 0)</td>
<td>181</td>
</tr>
<tr>
<td>2) (</td>
<td>\Delta\phi_{\ell_1,\ell_2}</td>
</tr>
<tr>
<td>15 GeV &lt; (m_T(\ell_2)) &lt; 50 GeV &lt; (m_T(\ell_1)) &lt; 60 GeV</td>
<td>52.4</td>
</tr>
<tr>
<td>4) (</td>
<td>\Delta\phi_{\ell\ell-\gamma}</td>
</tr>
<tr>
<td>5) 30 GeV &lt; (p_{T,\gamma}) &lt; 100 GeV</td>
<td>36.9</td>
</tr>
<tr>
<td>6) (\not{E}_T) cuts</td>
<td>26.8</td>
</tr>
<tr>
<td>7) (m_{\ell\ell} &lt; 24) GeV</td>
<td>23.3</td>
</tr>
</tbody>
</table>

high-luminosity LHC run.

We have shown that the \(\ell^+\ell^-\gamma + \not{E}_T\) signal is more effective at the lower mass splittings of points \(A\) and \(B\) than it is for points \(C\) and \(D\). A large reason for this is the value of \(m_{\ell\ell}\) which is determined by \(m_{\tilde{\chi}^0_{3,2}} - m_{\tilde{\chi}^0_1}\). In Fig. 2.8, we plot the \(m_{\ell\ell}\) distributions for points \(A\) and \(C\). The red hashed regions are the signals examined in this paper and the blue region are the 'alternative signals'. The small mass differences in point \(A\) leads to an \(m_{\ell\ell}\) peak which is at lower values, which significantly helps reduce the \(\gamma^*/Z(\tau^+\tau^-) + \gamma\) background. One then expects that the efficiency of this signal should get even better for lower mass splittings. However, as the splitting is decreased much more than the \(\sim 30\) GeV observed in points \(A\) and \(B\), the leptons
### TABLE 2.3

CUTS USED TO ISOLATE THE SIGNAL FOR THE BENCHMARK POINTS C AND D

<table>
<thead>
<tr>
<th>'large mass splitting' cuts</th>
<th>Cross section [ab]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cut</td>
<td>Signal C</td>
</tr>
<tr>
<td>0) Basic Selection</td>
<td>256</td>
</tr>
<tr>
<td>1) (N_{jets} = 0)</td>
<td>157</td>
</tr>
<tr>
<td>2) (</td>
<td>Δφ_{ℓ_1,ℓ_2}</td>
</tr>
<tr>
<td>3) (10 \text{ GeV} &lt; m_T(ℓ_1) &lt; 100 \text{ GeV}) (\bigcup) (10 \text{ GeV} &lt; m_T(ℓ_2) &lt; 95 \text{ GeV})</td>
<td>47.9</td>
</tr>
<tr>
<td>4) (8 \text{ GeV} &lt; E_T &lt; 95 \text{ GeV})</td>
<td>45.8</td>
</tr>
<tr>
<td>5) (m_{ℓℓ} &lt; 39 \text{ GeV})</td>
<td>42.8</td>
</tr>
</tbody>
</table>

become too soft to trigger on efficiently. We therefore expect that the smallest mass splitting, \(\min(m_{\tilde{χ}^0_2} - m_{\tilde{χ}^0_1}, m_{\tilde{χ}^0_3} - m_{\tilde{χ}^0_1})\), that this signal can be used for is \(\sim 25 \text{ GeV}\). The regions of parameter space for this can be found in Fig. 2.2.

The benchmark points C and D are harder to find with this signal, especially using only the process \(pp \rightarrow \tilde{χ}^0_3\tilde{χ}^0_2 \rightarrow ℓ^+ℓ^- + γ + E_T\). These points, which have a larger mass splitting (though \(m_{\tilde{χ}^0_3} - m_{\tilde{χ}^0_1}\) are still below \(m_Z\)), suffer from a drop in the branching ratio to the photon as well as the effectiveness of the \(m_{ℓℓ}\) cut. However, there is hope. When the difference in mass increases between \(m_{\tilde{χ}^0_3} - m_{\tilde{χ}^0_1}\), the mass difference between \(m_{\tilde{χ}^0_3} - m_{\tilde{χ}^0_2}\) also increases. This opens up the possibility of cascade decays such as those shown in Eqs. (2.6), (2.7). For example, the main difference between points C and D is the difference in mass between \(\tilde{χ}^0_3\) and \(\tilde{χ}^0_2\): 13 GeV for point C and 23 GeV for point D. For point D, the \(\tilde{χ}^0_3 - \tilde{χ}^0_2\) splitting is large enough.
that the photon and leptons from the decay

$$\tilde{\chi}_3^0 \to \tilde{\chi}_2^0 + (\gamma, \ell^+ \ell^-)$$  \hspace{1cm} (2.11)

to be triggered. However, the geometry of the decays changes when the process goes through a cascade, and the photon and dilepton system are no longer back-to-back. The change in topology renders the $|\Delta \phi_{\ell \ell - \gamma}|$ ineffective, so we do not use it in the ‘large mass splitting cuts’ of Table 2.3. The lepton separation $|\Delta \phi_{\ell_1, \ell_2}|$, the $m_T$ of the leptons, and $m_{\ell \ell}$ are still useful cuts, since the lepton properties are still constrained by the inter-electroweakino splitting. Using the ‘large mass splitting’ set of cuts, we can extend the region where this search method is effective to regions where all of the splittings are less than $m_Z$ and $\tilde{\chi}_2^0$ and $\tilde{\chi}_3^0$ are split by around $20 - 40$ GeV. Applying some of the ‘small mass splitting’ cuts, such as $|\Delta \phi_{\ell \ell - \gamma}|$ or $E_T$, to scenarios
C and D does result in better $S/B$ than achieved in Table 2.3, but the signal cross section drops so low that a much higher luminosity is needed to achieve a significance of 5.

2.4 Other Backgrounds

The analysis we have presented above neglects several details, which we discuss in more detail here. First, only physics backgrounds have been included; environmental backgrounds such as object misidentification (fakes) and overlapping partonic collisions have been neglected. Our analysis relies on the multiple electromagnetic objects, two leptons and a photon, which reduces the likelihood that our signal can be faked by multi jet processes. However, our analysis also relies on fairly soft leptons – the subleading lepton $p_T$ cut is 8 GeV – and softer leptons are more easily faked by jets.

Being more quantitative, one fake background comes from $W^\pm (\ell \nu) + \gamma + \text{jets}$, where one of the jets is mistaken for a lepton. After basic cuts, the lowest order cross section at 14 TeV for $pp \to W(\ell \nu) + \gamma + \text{jet}$ is 12.5 pb. Randomly selecting one of the jets in the event to be treated as a second lepton then passing the ‘fake’ $\ell^+ \ell^- + \gamma + j$ events through our ‘small-splitting scenario’ analysis cuts, we find an efficiency of 0.12%. The net contribution of this fake process to the background is then the product of the signal rate, the analysis cut efficiency, and the rate for a jet to fake a lepton $\epsilon_{j \rightarrow \ell}$, which we take to be $p_T$ independent and fixed at 0.01%. This value is the most conservative rate quoted (for $p_T,j = 10$ GeV) in the study in Ref. [79] (based on 7 TeV) multiplied by 1.3 to account for the fact that we are simulating events at 14 TeV. The result is $\sigma(pp \to W(\ell \nu) + \gamma + j)_{fake} = 1.5$ ab, which is small compared to both the other backgrounds and our benchmark signals.

In most supersymmetry searches the missing energy is large, so pure QCD backgrounds are not an issue. Our signal does not have large missing energy, so we have
to consider a wider set of fakes. One example is $pp \rightarrow \gamma + \text{jets}$ with two jets faking leptons. Generating $\gamma + jj$ events with MG5@NCLO (including $\gamma \bar{b}b$), treating the two parton-level jets as leptons, and imposing all non-$E_T$ cuts, we find the rate to be $\sim 5 \epsilon_{j \rightarrow \ell}^2 \text{nb}$. To pass our signal, these events still need to acquire some $E_T$. Small amounts of $E_T$ are easy to acquire in the busy LHC environment from pileup or other soft interactions/decays. We estimate the fraction of events with $E_T > 10 \text{GeV}$ by the fraction of minimum bias events passing this threshold [80], $\sim 10\%$.[8] Including a factor of 0.5 to crudely incorporate an efficiency to pass the ‘small-splitting’ angle-related $E_T$ cuts and plugging in the value for $\epsilon_{j \rightarrow \ell}$, the result is 2.5 ab. As with $\sigma(W(\ell\nu) + \gamma + j)_{\text{fake}}$, this rate is subdominant to the irreducible background. A more accurate estimate would require overlaying minimum bias events on top of fake $\gamma + \text{jets}$ events and treating the combination as single events. Such detailed treatment is beyond the scope of this paper.

A second environmental background worth mentioning is double parton scattering (DPS), two independent partonic collisions within the same initial proton pair. This background was brought up in Ref. [40] in the context of electroweakino searches and found to be small. However, Ref. [40] studied electroweakinos produced in association with a hard ISR jet, a qualitatively different kinematic region than we are studying here. Nevertheless, we believe the DPS background to be safely negligible because of the odd assortment of final state particles that our analysis employs. Specifically, while a $3\ell + \nu$ final state can be faked by the combination of $pp \rightarrow W(\ell\nu)$ and a low-mass Drell-Yan event, there is no simple secondary process that can be combined with $W$ production to make $\ell^+\ell^- + \gamma + E_T$. Similarly, $pp \rightarrow Z(\nu\bar{\nu})$ is a useless ingredient because it provides no net $E_T$. One possible DPS candidate is $pp \rightarrow \ell^+\ell^-\gamma$ (Drell-Yan plus a photon emission) combined with $pp \rightarrow Z(\nu\bar{\nu}) + j$. The cross section

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8These estimates are based off of 7 TeV data. At 14 TeV, higher pileup could make this fraction higher.
for $pp \rightarrow \ell^+\ell^−\gamma$ with basic cuts is $\sim 20\,\text{pb}$, however after imposing all lepton and photon-based analysis cuts (but neglecting and $E_T$-based cuts) the rate drops to 72 fb. The cut most responsible for suppressing $pp \rightarrow \ell^+\ell^-\gamma$ is $|\Delta\phi_{\ell_1,\ell_2}| < 1.05$, since the leptons from $pp \rightarrow \ell^+\ell^-\gamma$ are preferentially produced back-to-back. Combining the $pp \rightarrow \ell^+\ell^-\gamma$ rate with the cross section for $pp \rightarrow Z(\nu\bar{\nu}) + j \sim 10\,\text{nb}$, and using the DPS estimation reviewed in Ref. [81], we find:

$$\sigma_{DPS}(pp \rightarrow (\ell^+\ell^-\gamma) + (Z(\nu\bar{\nu})j)) = (72\,\text{fb}) \times \frac{10\,\text{nb}}{(\sigma_{\text{eff}} = 12\,\text{mb})} \ll 1\,\text{ab}. \quad (2.12)$$

This source of DPS background is orders of magnitude too small to impact our signal, even allowing for $O(1)$ variation in $\sigma_{\text{eff}}$ or the individual cross sections.

While our study of environmental backgrounds has not been exhaustive, the low rates exhibited here give us confidence that our estimates based on physics backgrounds alone in Sec. 2.3 are reasonable.

Another place where our analysis has been optimistic is our use of 8 TeV LHC lepton trigger thresholds. Once the LHC ramps up to 14 TeV, the increasingly chaotic environment may necessitate raising these thresholds. Higher thresholds hurt our analyses since our signal tends to have a softer lepton spectrum than the background. To quantify how increased thresholds affect the sensitivity, we have redone the previously presented analyses with lepton thresholds pushed to 30 GeV for the leading lepton and 10 GeV for the subleading lepton. With the higher thresholds, benchmark $A$ ($B$) requires $1400\,\text{fb}^{-1}$ ($2100\,\text{fb}^{-1}$), roughly three times the value at lower threshold. The drop in significance is motivation for the 14 TeV LHC experiments to keep the lepton trigger thresholds as low as possible. The loss in significance may be offset somewhat by diversifying the search to include ISR, i.e. $pp \rightarrow \chi_2^0\chi_3^0 + j$, as the recoil of the electroweakinos off the initial jet is inherited by their decay products and can lead to higher trigger efficiency. This signal diversification is not free, however, since
the background for $\ell^+ \ell^- \gamma + \not{E}_T + j$ is large. A devoted study is needed to determine the ideal mixture of zero and one (or more) jet channels.

Another place where our study could be improved is the modeling of the significance; we used a simple cut-based $S/\sqrt{B}$ measure to quantify the sensitivity. More sophisticated, multi-variate approaches can likely take additional advantage of the shape differences between the electroweakino signals and the SM backgrounds. Finally, all signal and background numbers have been computed using leading order cross sections. The $K$ factors for the signal and dominant backgrounds are similar and somewhat larger than 1 [82–85]. Simply slapping on these factors, $S/\sqrt{B}$ will shift slightly. However, our study focuses on a peculiar corner of phase space and it is possible that higher-order effects in this region are different than in the overall cross section.

2.5 Conclusions

In this work we have presented an alternate search channel for electroweakinos based on the final state $\ell^+ \ell^- \gamma + \not{E}_T$. This final state comes about from a variety of electroweakino sources, but the signal we find most easily captured is $pp \rightarrow \widetilde{\chi}_2^0 \widetilde{\chi}_3^0$, where one of the heavier neutralinos decays to a lepton pair and the other to a photon and LSP. The radiative decay mode $\widetilde{\chi}_2^0 \rightarrow \gamma \widetilde{\chi}_1^0$ is usually ignored since it typically has a small branching fraction. However, when the electroweakino spectrum is compressed, more conventional electroweakino decay modes become suppressed and the $\gamma + \widetilde{\chi}_1^0$ mode can be competitive and even dominant. The parameter space where the electroweakino spectrum is compressed overlaps significantly with the so-called ‘well-tempered’ region, i.e. where admixtures of Bino and Higgsino or Bino and wino can act as dark matter. The lack of strong LHC bounds on compressed electroweakino spectrum, combined with the potential connection to dark matter makes seeking out new electroweakino search strategies a must.
Focusing on Bino-Higgsino admixtures, we mapped out how quantities like the mass splitting, branching ratios, and relic abundance depend on the supersymmetry inputs. After identifying and studying the parameter space of interest, we presented our search strategy. By exploiting kinematic features of the signal such as low dilepton invariant mass, low hadronic activity, and small azimuthal separation between the leptons we were able to reduce the SM backgrounds \((VV\gamma, \gamma^*/Z(\tau^+\tau^-) + \gamma \text{ and } t\bar{t}\gamma)\) enormously. This strategy is viable in any Bino-Higgsino scenarios where the heavier neutralinos \(\tilde{\chi}^0_{2,3}\) are heavier than the LSP by \(O(25 - 70 \text{ GeV})\); if the splitting is smaller than 25 GeV, the final state particles are too soft to trigger on efficiently, while if the splitting is large enough that \(\tilde{\chi}^0_{2,3}\) can decay to an on-shell \(Z\), the photon branching fraction plummets. Signal events with smaller \(m_{\ell\ell}\) are easier to distinguish from the background, so our search performs best when \(m_{\tilde{\chi}^0_{2,2}} - m_{\tilde{\chi}^0_1}\) is close to the lower threshold. Translated into supersymmetry parameters, the \(\ell^+\ell^-\gamma + E_T\) search is best suited to \(M_1 \lesssim |\mu|\), with \(\mu < 0\) and small \(\tan \beta\). As an example, we find neutralinos with spectrum set by \(M_1 = 125 \text{ GeV}, \mu = -150 \text{ GeV}, \tan \beta = 2\) can be discovered with our technique with 430 fb\(^{-1}\). The amount of required luminosity increases as the overall mass scale of the electroweakinos is raised or as the splitting between \(\tilde{\chi}^0_{2,3}\) and the LSP grows. Once we increase the value of \(\tan \beta\) the splitting increases and our signal becomes more difficult to differentiate from the background and we need luminosities at the ab\(^{-1}\) level.

The search strategy we have demonstrated for the well-tempered Bino-Higgsino could be applied to other dark matter frameworks. One simple application is to other neutralino mixtures, such as Bino-wino, however it can also be applied to any mixtures of a light fermion singlet and fermion \(SU(2)\) doublets with hypercharge 1/2 or to \(SU(2)\) charged scalar or vector dark matter with electroweak scale masses. Indeed, for any dark matter state with couplings so small it would overclose the universe for an electroweak scale mass (i.e. the Bino), if the annihilation rate for
this state is increased via mixing with other heavy states that transform non-trivially under $SU(2)$ (like a pair of Higgsinos), the mass splittings between the singlet and heavier states can be detected via decays to photons and off-shell Z bosons. More generally and for the same reasons, the collider final state of $MET + \gamma + \ell^+ + \ell^-$ proposed in this article can be applied to any $O(200\text{ GeV})$ relic dark matter whose freeze-out is dictated by couplings to electroweak gauge bosons.

For Bino-Higgsino mixtures, the region where our search works best is exactly where direct detection searches struggle, since for low $\tan \beta$ and $\mu < 0$ the couplings of the LSP to the Higgs boson are vanishingly small and the prospects for direct detection experiments are not great. It is important to stress again that searches for electroweakinos are not limited by the energy of the collision but by the luminosity, therefore a possible upgrade in the luminosity of the LHC could be key to be able to discover these blind spots.
CHAPTER 3

THERMAL DARK MATTER VIA THE FLAVOR PORTAL

The nature and origin of dark matter (DM) remain elusive. Since the Standard Model (SM) does not account for a DM candidate, it is natural to seek one in extensions of it devised to confront its other problems. This approach enjoys an obvious merit: a single theory can account for (at least) two problems. Thus, solutions to the electroweak (EW) hierarchy problem (e.g., weak scale supersymmetry and little Higgs) provide DM when a “new physics” parity is imposed, right-handed neutrinos introduced to explain small neutrino masses, or axions introduced by the Peccei-Quinn resolution to the strong CP problem, may serve as DM – and so on. Can DM be addressed in the problem of fermion flavors?

Fermion masses are hierarchical across many orders, and mix in peculiar patterns. That these may be accidents of nature is an explanation we find unsatisfactory. A simple alternative may be found in the mechanism of Froggatt and Nielsen (FN) \[86\], that extends the SM gauge group with a (global or local) symmetry. The lightness of fermions $f$ is then arranged by mixing with heavy fermions $F$ vector-like under the new and SM symmetries: $SFf + \Lambda_{\text{FN}} F\bar{F}$, where $S$ is the “flavon”, a scalar field acquiring a vev $v_s$ and breaking the symmetry. The SM Yukawa matrices are now nothing but powers of $\epsilon \equiv \langle S \rangle / \Lambda_{\text{FN}}$ in an effective field theory (EFT), with $\epsilon$ usually fixed to the Cabibbo angle $\simeq 0.23$. Both the mass and mixing hierarchies can be obtained now, but flavor changing neutral currents (FCNCs) are inevitable. To avoid constraints from FCNCs, it is found that $\Lambda_{\text{FN}} > 2 \text{ TeV}$ \[87\].

It is to this picture that we wish to add DM. A profitable pursuit, one that
gives experiments a well-motivated target, is to identify the class of parameters that results in the correct relic abundance through $2 \to 2$ annihilations, in the spirit of such multi-parameter DM frameworks as supersymmetric neutralinos [12, 88, 89], minimal DM [90, 91], secluded WIMPs [92], effective WIMPs [93, 94], and forbidden DM [95, 96]. In other words, our first goal is to locate the “relic surface”. Our other guiding principle is to add no more than a minimal set of mass scales to the FN mechanism. To begin with, we are not interested in the case of DM annihilations to the vector-like $F$’s, as this puts DM mass $> \Lambda_{\text{FN}}$ and generally out of current reach. Thus, through operators suppressed by suitable powers of $\Lambda_{\text{FN}}$, DM must annihilate to SM fields and the flavon quanta that are obtained by expanding $S$ around its vev,

$$S = \frac{1}{\sqrt{2}} (v_s + \sigma + i\rho).$$

$\Lambda_{\text{FN}}$ is now the “messenger scale” for DM interactions with SM and $S$, or in other words, the cutoff of our theory. Following the FN procedure, we will arrange our EFT interactions by populating this scale with additional vector-like fermions. One can broadly see where this leads if DM is a fermion singlet $\chi$. Assuming it to be odd under a $Z_2$ symmetry in order to avoid the operator $LH \chi$, one may find that interactions with all SM species must be suppressed by negative powers of $\Lambda_{\text{FN}}$, sometimes with extra suppression from factors of $v/\Lambda_{\text{FN}}$ (where $v$ is the Higgs vev) as well as from powers of $\epsilon$ (determined by the FN charges of SM and $\chi$). It can be verified – and we will explicitly show it – that these effects cause $\chi \overline{\chi} \to \text{SM SM}$ to be too feeble, with the cross-section $\langle \sigma v \rangle$ many orders smaller than $(\eta)^2 2.2 \times 10^{-26}$ cm$^3$ s$^{-1}$ (where $\eta = 1$ (2) for Majorana (Dirac) DM) that is required for the correct abundance. DM interactions with the flavon, on the other hand, need not be $\Lambda_{\text{FN}}$-suppressed and may be arranged with marginal operators, as we shall show in this work. Couplings of $\mathcal{O}(0.1 - 1)$ are easily arranged, rendering annihilations to the
flavon particles $\sigma$ and $\rho$ a viable avenue. Thus $\chi$ can be a “secluded WIMP” \cite{92,97}: it achieves the correct relic density by annihilating primarily to mediators (here $\sigma$ and $\rho$), while keeping direct couplings to SM small. In this first paper, we will focus on a scenario where $\chi$ is charged under the flavor symmetry and interacts with the flavon through a renormalizable Yukawa term $y_{DM}\chi\chi S$; the best constraints on this species of secluded WIMP come from indirect limits imposed by flavor experiments. In a follow-up paper \cite{98}, we will extend our findings to cases where DM-flavon interactions are non-renormalizable, identify parametric families that lead to correct freezeout (while including SM annihilation channels that may become important), and derive all relevant constraints.

A most remarkable feature here is the hand played by the non-zero flavor charge of DM. Due to this charge, DM mass must stem from symmetry-breaking as $\propto v_s \sim \epsilon \Lambda_{FN}$. And parametrically, the cross-section of $\chi$’s annihilation to the flavon mediators is given by $\langle \sigma v \rangle \sim y_{DM}^4/m_\chi^2$. Since perturbative unitarity limits $m_\chi$ from above \cite{99}, an upper limit on $\Lambda_{FN}$ is imposed. As a result, lower limits on $\Lambda_{FN}$ which may be placed by future flavor experiments or high-energy collider searches can potentially falsify our premise. Moreover, since perturbativity at the flavor scale restricts $y_{DM}$ to be $O(1)$, we know from the lore of the “WIMP miracle” that, to obtain the characteristic $\langle \sigma v \rangle_{th} = (\eta) \times 10^{-26}$ cm$^3$ s$^{-1}$, $m_\chi$ must be $\lesssim 10$ TeV. Thus, although all the masses introduced here ($\Lambda_{FN}$, $v_s$, $m_\chi$) were \textit{a priori} free to be arbitrarily heavy, requiring correct freezeout puts them all within current experimental reach. This attribute of a low-energy flavor-breaking scale emerging from a connection between DM and flavor was pointed out in \cite{100}. It is comparable to \cite{101-104}, where a low $\Lambda_{FN}$ is obtained by breaking the flavor symmetry with electroweak Higgs doublets. See also \cite{105} for model-independent constraints on low-scale flavor-breaking.

The above features will be spoiled if DM is a scalar, in which case it can have a renormalizable interaction with the Higgs doublet through a portal term: $|\chi|^2 |H|^2$. 

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Annihilations to the SM Higgs boson must dominate unless the coupling is tuned to be small, and there is no intimate relation between the DM abundance and the FN mechanism. For these reasons our study will only focus on fermionic DM.

A Froggatt-Nielsen portal to DM was explored in [100], but the presence of the CP-odd flavon was omitted and emphasis was not placed on obtaining the correct relic abundance. Here we will show that the CP-odd flavon plays a primary role in freeze-out. The status and prospects of the CP-odd flavon were explored in comprehensive detail in [87], whose results we will use extensively in this work. For other works that explore the interface between flavor and dark matter, see [106–116] and the references in [117].

This paper is laid out as follows. We first review the FN mechanism in Sec. 3.1. DM is carefully incorporated into this set-up in Sec. 3.2. We will find this a non-trivial task: we begin with a brief overview of simple models in Sec. 3.2.1 and show them to be ineffective or unsatisfactory, before moving on to a successful model that requires $\chi$ to be charged under $U(1)_{\text{FN}}$. Sec. 3.3 discusses constraints and future prospects, and Sec. 3.4 concludes the paper.

3.1 Froggatt-Nielsen mechanism

We begin with a brief review of the FN mechanism; for a more thorough review, see [118]. Ingredients relevant for embedding DM (performed in the next section) will be given emphasis. The FN mechanism introduces an array of heavy vector-like fermions of mass $\Lambda_{\text{FN}}$ transforming under the SM gauge group as well as under a new symmetry that is either global, local or discrete; we will use a global $U(1)_{\text{FN}}$ for illustration. The symmetry also transforms a new complex scalar $S$, the flavon, and all SM fermions excepting the top quark; the Higgs doublet is neutral under it. Conventionally, $S$ is assigned a $U(1)_{\text{FN}}$ charge -1, which we assume hereafter. The charge assignments ensure that in the theory below $\Lambda_{\text{FN}}$, fermions couple to the
Higgs doublet only via non-renormalizable terms containing several powers of $S$ (or no power in the case of the top quark):

$$\mathcal{L} \supset y_{ij}^{(u)} \left( \frac{S}{\Lambda_{\text{FN}}} \right)^{m_{ij}} \bar{Q}_i u_j \tilde{H} + y_{ij}^{(d)} \left( \frac{S}{\Lambda_{\text{FN}}} \right)^{n_{ij}} \bar{Q}_i d_j H ,$$  \hspace{1cm} (3.2)

where $\tilde{H} = i\sigma_2 H^*$ and the exponents $m_{ij}, n_{ij}$ are determined by the FN charges of the fermions. For simplicity, we have assumed only quarks to be charged under the FN symmetry, though the mechanism can be easily extended to leptons as well. The $U(1)_{\text{FN}}$ symmetry breaks if $S$ develops a vev $v_s$, giving rise to Yukawa couplings that are parameterically powers of $\epsilon = v_s/\sqrt{2}\Lambda_{\text{FN}}$. Thus, fermion masses and mixings originate in both electroweak symmetry breaking (EWSB) and flavor breaking, with their relative sizes set by the number of $\epsilon$ powers. The size of $\epsilon$ is traditionally fixed by matching with measurements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix: $\epsilon \approx |V_{us}| \approx |V_{cb}| \approx 0.23$. Once the hierarchies are fixed to the right order this way, dimensionless $O(1)$ coefficients $y_{ij}^{(u,d)}$ can bring the CKM entries and fermion masses to their measured values.

Thus far we have described the FN mechanism without explicit reference to the flavor group at work, which can be continuous, discrete, global, local, abelian, or non-abelian. In our work we choose a global $U(1)_{\text{FN}}$ for simplicity. Symmetry-breaking must introduce a potentially troublesome Goldstone boson $\rho$, disfavored by cosmological constraints if it couples to the SM $\gamma_{119,121}$. This problem is evaded if the pseudoscalar $\rho$ acquires a non-zero mass through explicit breaking$^2$

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$^1$The $y_{ij}$ must be complex to account for the CKM phase.

$^2$Alternatively, $U(1)_{\text{FN}}$ may be either (i) gauged, which may however introduce anomalies since the left- and right-handed fermions are charged differently, or (ii) discretized, in which case there is no Goldstone boson. Through the operator $S^N/\Lambda_{\text{FN}}^{N-4}$, where $N$ is the dimension of the $Z_N$ group, one has $m_\rho^2 \sim \epsilon^{N-4}v_s^2$. Successful FN models require $N \leq 16$ since the up quark demands an $\epsilon^8$ suppression, implying a very light $\rho$ that is already excluded by flavor constraints $^7$. 

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potential:

\[
V(H, S) = -\mu_s^2 |S|^2 + \lambda_s |S|^4 \\
+ \lambda_{sh} |S|^2 H \dagger H - b^2 (S^2 + \text{H.c.}) ,
\]

(3.3)

giving rise to the physical masses

\[
m_{\sigma}^2 = 2\lambda_s v_s^2, \quad m_{\rho}^2 = 4b^2 ,
\]

(3.4)

with \( b^2 > 0 \) and \( v \simeq 246 \text{ GeV} \). Though \( b^2 \) is a free parameter, as it is the only term explicitly breaking \( U(1)_{\text{FN}} \), it is multiplicatively renormalized and can be naturally smaller than the other scales here. Thus we require \( m_{\rho} \) to lie below \( \Lambda_{\text{FN}} \) and assume the mass hierarchy in Ref. [87]:

\[ m_{\rho} < m_{\sigma} \simeq v_s < \Lambda_{\text{FN}} . \]

Eq. 3.2 determines the Yukawa couplings \((g_s)_{ij}\) of \( \sigma \) and \( \rho \) with quark pairs of families \( i \) and \( j \), written out explicitly in Appendix B. These couplings generate tree-level FCNC processes, due to which the FN set-up confronts limits from measurements of meson mixing, meson decays, and top quark decays, with the strongest constraints imposed by the neutral kaon mixing CP-violation parameter \( \epsilon_K \) [87]. The latter constrains the masses of \( \sigma \) and \( \rho \), which can be translated to limits in \( \{ \lambda_s, v_s, m_{\rho} \} \) space\(^3\). When \( m_{\rho} \gg 200 \text{ GeV} \), the contribution of the flavon quanta to the Wilson coefficients of \( \Delta F = 2 \) operators goes as \((g_\sigma)^2_{sd}/m_\sigma^2 \propto (\lambda_s v_s^4)^{-1}\) (from Eqs. 3.4 and 3.11). Thus a lower limit on \( v_s \) and \( \Lambda_{\text{FN}} = v_s/(\sqrt{2} \epsilon) \) may be obtained if we require

\(^3\)Through the rest of the paper we take into account the fact that the right-hand sides of Eq. 3.4 are \( 2 \times \) those in Ref. [87].
the coupling $\lambda_s$ to be perturbative. From Ref. [87], we have

$$\lambda_s \leq 4\pi \quad \Rightarrow \quad v_s \geq 670 \text{ GeV}$$

$$\Rightarrow \quad \Lambda_{\text{FN}} \geq 2.07 \text{ TeV} , \quad (3.5)$$

which will serve for the purposes of this paper as absolute lower limits on $v_s$ and $\Lambda_{\text{FN}}$.

Once we embed DM into the FN picture, the above constraints may also restrict DM masses. Therefore, we will revisit these constraints in more detail in Sec. 3.3, while in the next section we proceed to our principal task of adding DM.

3.2 Incorporating DM

3.2.1 General model-building

Having described the interactions of the flavon quanta $s = \sigma, \rho$ with SM states, we turn to our central program of incorporating fermionic DM into the FN setup.

The following are general considerations to keep in mind before we delve into the details of model-building.

- As mentioned in the Introduction, we impose a $Z_2$ symmetry under which SM fields and $S$ are even and $\chi$ is odd. This prevents operators of the form $(S/\Lambda)^k H L \chi$ that could result in DM decay, where $k \geq 0$ is an integer.

- We also mentioned in the Introduction that annihilations to SM species are suppressed by inverse powers of the cutoff scale and $\epsilon$, and that successful freezeout is only obtained through annihilations to the flavons. Hence our emphasis in the following will be on DM interactions with flavons. All these low-energy interactions are assumed to arise from vector-like fermions integrated out at the scale $\Lambda_{\text{FN}}$. Some of these vector-like fermions, $F_\chi, \bar{F}_\chi$, must be charged odd under the $Z_2$ symmetry that stabilizes DM. In principle these vector-like
fermions could have arbitrary masses, but we have chosen for them a common mass $\Lambda_{FN}$ in line with our objective of keeping the number of new mass scales at a minimum. Thus the theory at high scales would appear as

$$\mathcal{L} \supset S \chi \bar{F} \chi + \Lambda_{FN} F \bar{F} \chi .$$

• Annihilations of a DM pair into $\sigma + \rho$ are $s$-wave, whereas annihilations into $\sigma\sigma$ and $\rho\rho$ are $p$-wave. We see this from parity considerations. The fermion-pair initial state has $P = (-1)^{L+1}$, thus the $L = 0$ transition is allowed for the parity-odd $\sigma\rho$ final state, and forbidden for the parity-even $\sigma\sigma$ and $\rho\rho$ final states.

• As $\chi$ can in principle be either neutral or carry an (arbitrary) $U(1)_{FN}$ charge, we must seek a successful model by sifting through the possibilities.

With these considerations, we now explore the freeze-out of DM neutral or charged under $U(1)_{FN}$.

3.2.1.1 $U(1)_{FN}$-neutral DM

Here DM has a bare Majorana mass $m_\chi$, and connects to the flavon via the lowest dimension operator $\chi\chi|S|^2/\Lambda_{FN}$. Assuming CP-violating phases vanish, the interactions

$$\chi\chi\rho^2/\Lambda_{FN} , \; \chi\chi\sigma^2/\Lambda_{FN} , \; \epsilon \chi\chi\sigma$$

are obtained from Eq. [3.1]. These give rise to the annihilation of $\chi$ to pairs of $\sigma$ and $\rho$, which are $p$-wave-suppressed. In addition, annihilation through the first two is $\Lambda_{FN}$-suppressed, and that through the third is non-trivial to arrange: $\chi$ must be heavier than $\sigma$ to kinematically allow it, and at least an order of magnitude lighter than $\Lambda_{FN}$ for the EFT to be valid. From Eq. [3.4], this means $v_s < m_\chi \ll \Lambda_{FN}$ for the
quartic coupling $\lambda_s \sim 1$. However, this is not possible since $v_s \simeq \epsilon \Lambda_{FN}$. Of course, the special hierarchy $m_\sigma < m_\chi \ll \Lambda_{FN}$ may be contrived if $\lambda_s \ll 1$, but we do not pursue this possibility since we expect the region of viability to be small for light $\sigma$ in the face of flavor constraints. Also, as we mentioned in the Introduction, we wish to keep the introduction of new mass scales minimal (violated in this case by the introduction of $m_\chi$).

To summarize, $U(1)_{FN}$-neutral DM can possibly lead to successful freeze-out through $p$-wave annihilations to $\sigma$ pairs, but only a small region would survive flavor constraints. A larger region of viability is possible if DM can annihilate to $\sigma + \rho$ through the $s$-wave instead. In the next sub-section we will show that DM charged under $U(1)_{FN}$ is more successful in this respect.

3.2.1.2 $U(1)_{FN}$-charged DM

DM charged under $U(1)_{FN}$ must acquire its mass, and interactions with the flavon quanta $s$, via symmetry-breaking. Specifically, DM acquires a Dirac mass and couplings to $S$ through the operator

$$y_\chi \left( \frac{S}{\Lambda_{FN}} \right)^n S \chi_a \chi_b ,$$

(3.6)
given by

$$m_\chi = \frac{y_\chi}{\sqrt{2}} v_s \epsilon^n, \quad g_{\chi \chi \chi} = (n + 1) \frac{y_\chi}{\sqrt{2}} \epsilon^n ,$$

(3.7)

where $n$ determines $Q_\chi \equiv$ the collective charge of $\chi_a$ and $\chi_b$ as

$$Q_\chi = (n + 1)/2 .$$
Without loss of generality, we take $y_\chi$ to be real. The only free parameters in this set-up are now

\begin{align}
\text{Scales} & : \Lambda_{\text{FN}}, b^2, \\
\text{Charge} & : Q_\chi, \\
\text{Couplings} & : y_\chi, \lambda_s, \lambda_{sh}.
\end{align}  \tag{3.8}

It is among these parameters that we must find successful freezeout conditions and identify the relic surface. For our phenomenological treatment in Sec. 3.3 we neglect $\lambda_{sh}$, for it plays little role in our freezeout: as we will show in Sec. 3.2.2 its influence by means of turning on a small Higgs-$\sigma$ mixing is negligible.

We now proceed to find our desired conditions. First, we notice that Eq. 3.7 allows for the $s$-wave process $\chi \bar{\chi} \to \sigma \rho$. Both the $s$- and $t$-channel diagrams in Fig. 3.1 contribute, and lead to the annihilation cross-section given in Appendix D.
For $m_\rho \ll m_\chi$, this is schematically

$$\langle \sigma v \rangle \sim \frac{1}{s} |\mathcal{M}|^2 \sim \frac{1}{4m_\chi^2} f'' \sim \frac{\epsilon^2}{4\Lambda_{FN}^2} f', \quad (3.9)$$

where $f''$ and $f'$ are functions of $y_\chi, \lambda_s, n$ and $\epsilon$. The above equation implies our set-up can give us a potential upper limit on the Froggatt-Nielsen scale $\Lambda_{FN}$ when we require the thermal cross-section $\langle \sigma v \rangle_{th} = 4.4 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}$. This must happen when we require that the coefficient $y_\chi$ be perturbative ($y_\chi \leq 4\pi$). In the following we will derive this upper limit on $\Lambda_{FN}$ for a few select cases.

Let us begin our investigation of DM annihilations with the case of $n = 0 (\Rightarrow Q_\chi = 1/2)$. Here

$$m_\chi = \frac{y_\chi}{\sqrt{2}} v_s, \quad g_{s\chi\chi} = \frac{y_\chi}{\sqrt{2}}$$

$$\Rightarrow \langle \sigma v \rangle \simeq \frac{3}{2048\pi} \frac{y_\chi^2}{v_s^2}, \quad (3.10)$$

where in the second line we have used Eq. D.2 and set $m_\chi = m_\sigma$ for simplicity. This can certainly lead to successful freezeout, provided $v_s$ is not so large as to make $y_\chi$ non-perturbative. This is not a cause for concern, since Eq. 3.5 implies $y_\chi \geq 1.9$ if we require the correct abundance at $m_\chi \gg m_\rho$. (For $m_\chi \sim m_\rho$, there is no lower bound on $y_\chi$.) Annihilations to $\sigma + \rho$ are kinematically allowed so long as $m_\sigma + m_\rho < 2m_\chi \Rightarrow 2b^2 < (y_\chi - \sqrt{\lambda_s})^2 v_s^2$. Requiring $y_\chi \leq 4\pi$ gives $\Lambda_{FN} \leq 13.65 \text{ TeV}$, which is allowed by the limit in Eq. 3.5.

To sum up, we have found our first successful freezeout scenario without contriving a compressed mass spectrum. Our work will chiefly concern this scenario, for reasons that will become apparent when we inspect the effect of increasing $n$.

As we increase $n$, Eq. 3.7 implies that $\chi$ gets lighter, reducing the phase space
available for annihilation to $\sigma + \rho$. (One may try to recover some phase space by tuning $\lambda_s$ small and making $\sigma$ light, but at the cost of tension with kaon mixing constraints.) Thus the $p$-wave flavon modes ($\sigma\sigma$ and $\rho\rho$) and SM modes gain in importance. Moreover, inserting Eq. 3.7 into Eq. 3.9 \( \langle \sigma v \rangle \propto y^2_n (n + 1)^4 \epsilon^2 / \Lambda_{FN}^2 \) in the $\lambda_s \to 0$ limit, implying that as we increase $n$, the upper bound on $\Lambda_{FN}$ from $y_\chi$ perturbativity gets stronger. Eventually this upper bound will fall below the lower bound in Eq. 3.5. For $m_\chi = m_\sigma$, this occurs at $n = 4$, which gives us an important condition for successful freezeout:

$$n \leq 3, \text{ or } Q_\chi \leq 2.$$  

Conditions on the parameters in Eq. 3.8 that render desired annihilation modes kinematically allowed may be derived in a straightforward manner from Eqs. 3.4 and 3.7.

Our next task is to show that, after imposing these conditions and locating our relic surface, our set-up is quite viable in the face of dark matter experiments. To this end, we pick a single scenario for phenomenological study, $Q_\chi = 1/2$ ($n = 0$). Our choice is motivated by the following reasons.

(1) As we just showed, the $n = 0$ case provides the maximum phase space for the channel $\chi \bar{\chi} \to \sigma\rho$, allowing it to dominate the annihilation over a large parametric region. This simplifies the phenomenological analysis.

(2) As Eq. 3.6 is a marginal operator for $n = 0$, we may relax the assumption that $Z_2$-odd vector-like fermions with a common mass $\Lambda_{FN}$ generate DM-flavon interactions at low energies, and assume no more than the presence of a pair of dark fermions with a combined $U(1)_{FN}$ charge of unity.\(^4\)

\(^4\)We also assume that neither individual charge $Q_a^{\chi}, Q_b^{\chi}$ is zero. If one of the $\chi_i$ is $U(1)_{FN}$-neutral, a Majorana mass $\frac{1}{2}M_{m_\chi} \chi_i \chi_i$ and the operator $|S|^2 |\chi_i \chi_i| / \Lambda_{FN}$ are allowed, confounding the freezeout analysis and potentially introducing physical complex phases.
\[m_\chi = \sqrt{2} y_\chi v_s\]
\[m_\sigma = \sqrt{2} \lambda_s v_s\]
\[m_\rho = 2 \, b\]

Figure 3.2. The spectrum studied in this work. The \(U(1)_{\text{FN}}\) symmetry breaks at \(\sqrt{2} \epsilon\) below the Froggatt-Nielsen scale \(\Lambda_{\text{FN}}\), giving masses to dark matter \(\chi\) and the CP-even flavon \(\sigma\) at the symmetry-breaking scale \(v_s\). Shown for illustration is a hierarchy in which \(m_\chi > m_\sigma\). The mass of the CP-odd flavon \(\rho\), acquired through a freely tunable explicit symmetry breaking parameter \(b^2\), is assumed \(< 2m_\chi - m_\sigma\) in order to allow for DM annihilations to \(\rho + \sigma\).

The spectrum of scales in our scenario is sketched in Fig. 3.2. In general \(m_\chi, v_s\) and \(m_\sigma\) reside at a common \(\mathcal{O}(\text{TeV})\) scale, while \(m_\rho\), a free parameter, can be much lower. The relation between these masses and scales will play a decisive role in our phenomenology. We note again that the case of \(Q_\chi = 2\) was studied in Ref. [100], but without taking the CP-odd \(\rho\) into account or matching the relic abundance.

3.2.2 Flavon mode domination: an illustration

In the Introduction we had estimated that DM annihilations to all SM final states will be suppressed. We had also surmised that freezeout will be dictated by s-wave annihilations to flavon quanta. In the previous sub-section, after identifying \(U(1)_{\text{FN}}\)-charged DM as a workable scenario, we derived freezeout conditions ignoring SM modes and including only flavon modes. We now demonstrate the accuracy of our assumptions by quantifying these estimates, which form the crux of our paper.

The left panel of Fig. 3.3 shows the \(\langle \sigma v \rangle\) of various annihilation modes against the ratio \((m_\rho + m_\sigma)/2m_\chi\), with the thermal cross-section \(\langle \sigma v \rangle_{\text{th}} = 4.4 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}\) shown for reference. We have chosen \(m_\chi = 950 \text{ GeV}, \ y_\chi = 1.4, \ \lambda_s = 0.25\) and
Figure 3.3. **Left:** Cross sections of various DM annihilation channels as a function of \((m_\sigma + m_\rho)/2m_\chi\), keeping \(m_\chi = 950\) GeV, \(y_\chi = 1.4\), \(\lambda_s = 0.25\), and \(\lambda_{sh} = 0.1\). Annihilations to the CP-even \(\sigma + \text{CP-odd } \rho\) are seen to dominate over all other modes. **Right:** Contours of \(y_\chi\) resulting in the correct relic abundance, fixing \(m_\sigma = m_\chi\). See text for more details.

\(\lambda_{sh} = 0.1\) for illustration; this puts \(m_\sigma\) at 678 GeV. The relevant SM modes, \(hh, t\bar{c}, b\bar{b}, c\bar{c}, gg, \) and \(\gamma\gamma\) are plotted in brown, red, green, magenta, dot-dashed black, and dot-dashed orange respectively, the flavon modes \(\sigma\rho\) and \(\rho\rho\) in solid blue and dashed blue. Our parametric range kinematically forbids the \(\sigma\sigma\) mode, but allowing it does not change our conclusions. We checked our calculations against MicrOmegas 4.3 \cite{micr} and found very good agreement.

Let us begin our task by first inspecting the SM final states. Annihilations to Higgs bosons \((\chi\bar{\chi} \rightarrow \sigma^* \rightarrow hh)\) proceed through the \(\lambda_{sh}\) vertex in Eq. 3.3, and suppressed by the twofold effect of its \(p\)-wave nature and the large mass of \(\sigma\) in the propagator. This is why the cross-section is three orders of magnitude below \(\langle \sigma v \rangle_{\text{th}}\). Even for \(\lambda_{sh}\) as large as 1, the above effects would keep the cross-section at a factor of 100 below the \(\langle \sigma v \rangle_{\text{th}}\). In general, turning on the coupling \(\lambda_{sh}\) would induce \(h-\sigma\) mixing, introducing potential constraints from LHC Higgs measurements. However, due to the hierarchy between the mass scales \(m_h \sim v\) and \(m_\sigma \sim v_s\), the mixing angle comes out to be \(\lesssim 0.1\), which is safe from these constraints. For this reason, and
because $\lambda_{sh}$ plays no role in the freezeout, we consistently neglect it throughout the rest of the paper. As a consequence, we will also not be concerned with ($p$-wave suppressed) annihilations to the electroweak bosons that would have been prompted by a non-zero $\lambda_{sh}$.

Annihilations to SM fermions are highly suppressed as well. These must proceed through flavon mediation in the $s$-channel; since both $\rho$ and $\sigma$ couple to fermion pairs through the Higgs doublet (as seen in Eq. [3.2]), a factor of $(v/\Lambda_{FN})^2 < 10^{-2}$ appears in the cross-section. The relative contributions of the fermion modes is determined by the number of $\epsilon$ powers in the DM-flavon coupling, which is shown in Appendix B. Re-writing Eq. B.4 (up to $O(1)$ coefficients) as

$$g_s^u = \frac{1}{v_s} \begin{pmatrix} 8m_u & \epsilon m_c & \epsilon^3 m_t \\ \epsilon^3 m_c & 4m_c & \epsilon^2 m_t \\ \epsilon^5 m_t & \epsilon^2 m_t & 0 \end{pmatrix},$$

$$g_s^d = \frac{1}{v_s} \begin{pmatrix} 7m_d & \epsilon m_s & \epsilon^3 m_b \\ \epsilon m_s & 5m_s & \epsilon^2 m_b \\ \epsilon m_b & \epsilon^2 m_b & 3m_b \end{pmatrix},$$

we see that except for $t\bar{c}$, $b\bar{b}$ and $c\bar{c}$, all other fermion modes are too feeble.

The presence of a global $U(1)_{FN}$ anomaly in our set-up gives rise to the annihilation modes $\chi\bar{\chi} \rightarrow \rho^* \rightarrow gg, \gamma\gamma$. Calculating the $\rho gg$ and $\rho \gamma\gamma$ couplings using the color and electromagnetic anomaly coefficients that originate from quark triangle diagrams [123], we find the $gg$ cross-section comparable to $t\bar{c}$, and the $\gamma\gamma$ cross-section 100 times smaller.

We are now left with annihilations to two flavons. The $\sigma\rho$ mode, contributing $> 95\%$ to the total cross-section, is dominantly $s$-wave (with the $p$-wave contribution so negligible as to vary the solid blue curve only minutely). As advertised in Sec. 3.2.1.2.
this annihilation can proceed through an $O(1)$-sized coupling to produce $\langle \sigma v \rangle_{\text{th}} = 4.4 \times 10^{-26}$ $\text{cm}^3$ $\text{s}^{-1}$. We see this clearly in the solid blue curve. The $\rho \rho$ (and $\sigma \sigma$) mode is $p$-wave. Consequently, its cross-section is suppressed by about an order of magnitude with respect to the $\sigma \rho$ mode. The cross-section also drops sharply as $m_\rho$ approaches $2m_\chi - m_\sigma$ and shrinks the phase space open for annihilation.

Finally, the right panel of Fig. 3.3 shows, in the $m_\rho$-$m_\chi$ plane, contours of $y_\chi$ that result in successful freezeout. We have set $m_\sigma = m_\chi$ in this plot, which from Eqs. 3.4 and 3.10 implies $\lambda_s = y_\chi^2 / 4$ along each contour. As $m_\rho$ is raised, the phase space available for $\chi \bar{\chi} \rightarrow \sigma \rho$ is reduced, requiring a slight increase in $m_\chi = m_\sigma$ to recover the thermal cross-section. One also observes that larger couplings are needed for heavier DM to overcome the $m_\chi^{-2}$ suppression of the annihilation cross-section.

In the next section we explore the various signals and constraints of our set-up, and show that our parameter space on the relic surface is by and large allowed by flavor and dark matter experiments. Searches best suited for finding our scenario are also identified and discussed.

### 3.3 Phenomenology

Since our DM gets its relic abundance effectively by annihilating to mediators, it is a “secluded WIMP” [92], generally hidden from the Standard Model and hence expected to be probed poorly by direct detection and colliders. And though our annihilation is $s$-wave, allowing our set-up to submit to indirect detection searches, our DM is generally too heavy to produce sufficient photonic flux to be seen. However, flavor-changing processes can competently probe the mediators $\rho$ and $\sigma$.

In this section we will discuss the constraints on our scenario from these various experiments, and predict our future prospects. We will begin with flavor experiments, recasting the findings of Ref. [87] in our parameters and finding bounds on $m_\chi$ and $m_\rho$. The most stringent limits here are from kaon mixing measurements. Next we
discuss constraints from direct detection, and show that future searches would reach
regions that are allowed by kaon mixing. Then we briefly discuss the poor (current
and future) sensitivity of indirect detection. Finally, we show that current LHC limits
too are weak, and explore promising DM signatures for Run 2.

3.3.1 Meson mixing

Both the CP-even $\sigma$ and CP-odd $\rho$ exhibit flavor violating couplings at tree level
(Eq. 3.11), generating FCNCs and incurring low-energy constraints from meson mix-
ing in neutral $K, B$ and $D$ systems \cite{87}. The strongest limits come from kaon mixing
since the SM contribution is rendered small by the GIM mechanism, i.e. it is both
loop- and CKM-suppressed.

Ref. \cite{87} recast results from the UTfit collaboration \cite{124} onto the $m_\rho - v_s$ plane,
showing that measurements of the CP-violation parameter $\epsilon_K$ outdo all other flavor
constraints\footnote{This result may seem to be an artifact of the $O(1)$ phases that appear in the Yukawa
textures derived in \cite{87}, but one must, for the reasons explained above, expect $\epsilon_K$ to outconstrain the CP-
preserving $\Delta m_K$ for most Yukawa textures. We thank F. Bishara (M. Bauer) for raising (clarifying)
this point.}. We use this result to our show our constraints in the $m_\rho - m_\chi$ plane in
the left panel of Fig. 3.5, taking advantage of the relation between $m_\chi, y_\chi$ and $v_s$
in Eq. 3.10. We fix $y_\chi = 2.2$ and show with a dashed curve a contour of $\Omega_\chi h^2 = 0.12$.
In the regions above this contour, DM is overabundant for this $y_\chi$. The effect of
varying this coupling is seen in Fig. 3.3, however, it must be remembered that raising
or lowering $y_\chi$ would correspondingly tighten or loosen the $\epsilon_K$ bound on $m_\chi$.

The dark shaded region is excluded at 95\% C.L. by the $\epsilon_K$ measurement, with an
illustrative $\lambda_s$ value of 0.25. As explained in Sec. 3.1, the bound comes from tree-level
contributions to $\Delta F = 2$ operator Wilson coefficients, which depend on $m_\rho$ and $m_\sigma$.
For $m_\rho > 200$ GeV, these contributions scale as $\lambda_s^{-1} v_s^{-4} \propto \lambda_s^{-1} (y_\chi/m_\chi)^4$, hence giving
a flat bound across $m_\rho$. It will prove useful to recast this as a scaling of the lower
bound on $m_\chi$ in terms of the couplings:

$$m_\chi |_{\epsilon_K} \propto \frac{y_\chi}{\lambda_s^{1/4}}.$$ (3.12)

For $m_\rho < 200 \text{ GeV} \ll m_\sigma$, the flavon contributions to the Wilson coefficients scale as $(v_s m_\rho)^{-2} \propto (m_\chi m_\rho / y_\chi)^{-2}$, giving a bound on $m_\rho$ that falls inversely with $m_\chi$. The dip feature seen between these two regions comes from an accidental cancellation in the Wilson coefficient at $m_\sigma = m_\rho$ due to destructive interference.

This plot illustrates clearly that regions favored by our freezeout scenario are quite viable vis-à-vis flavor constraints. The choice of 2.2 is the smallest $y_\chi$ that allows our scenario to escape the $\epsilon_K$ bound for all $m_\rho > 200 \text{ GeV}$. We find that for $1.2 \leq y_\chi \leq 2.2$, our set-up is viable when the relic contour is trapped in the dip feature. For $y_\chi < 1.2$, we are completely excluded.

Since regions where our relic contours are excluded mostly correspond to $m_\rho > 200 \text{ GeV}$, where the scaling in Eq. 3.12 roughly applies, for our discussions below we will use this equation for making comparisons with constraints from DM experiments. In the following sub-sections we will pay particular attention to whether these experiments have future sensitivities to parametric regions not excluded by flavor probes.
Figure 3.5. **Left:** 95% C.L. bounds from $\epsilon_K$ measurements (dark shaded region excluded) in the $m_\chi-m_\rho$ plane, at $\lambda_s = 0.25$ and $y_\chi = 2.2$. The blue dashed curve is a contour of $\Omega_\chi h^2 = 0.12$; the region above it leads to overabundant DM. The region to the right of the red line is kinematically forbidden. **Right:** Spin-independent DM-nucleon scattering cross-sections as a function of $m_\chi$ for $y_\chi = 1.0$, 1.4 and 2.2 (red, green, and blue curves), versus current constraints from Xenon1T (solid black) and future sensitivities at LZ and DARWIN (dotted and dot-dashed). The cloverleaf on each $y_\chi$ curve shows the $m_\chi$ that leads to $\Omega_\chi h^2 = 0.12$. **Both panels:** The pink star in the right-hand panel at $m_\chi = 960$ GeV is replicated at the same mass in the left-hand panel – this is as an example indication that regions excluded by direct detection are usually even more deeply excluded by kaon mixing constraints.

3.3.2 Direct detection

DM can scatter with nucleons through flavon exchange, potentially introducing constraints from direct detection searches. It is well-known that fermion DM scattering with nucleons via pseudoscalar mediator exchange produces a spin-dependent cross section that is velocity-suppressed \[125\]. Thus, only the exchange of the CP-even $\sigma$ is relevant to our scenario. As sketched in Fig. 3.4, this leads to scattering via light quark operators at the tree level and via gluon operators through heavy quark loops. In general, we expect the rates to be small since they will be suppressed by a
large $m_\sigma \ (> v_s)$.

Setting aside considerations of relic density for the moment, we now inspect the constraints. We computed our spin-independent cross-section $\sigma_{\text{SI}}$ using the formulae in Appendix D. From Eqs. 3.7, 3.11, and D.3, this cross-section scales as

$$\sigma_{\text{SI}} \propto \mu_{\chi N}^2 m_p^2 \left( \frac{y_\chi^8}{\lambda_s m_\chi^6} \right).$$

(3.13)

The right panel of Fig. 3.5 plots $\sigma_{\text{SI}}$ against $m_\chi$ for three choices of $y_\chi$: 1.0 (red), 1.4 (blue) and 2.2 (green), and fixing $\lambda_s$ to 0.25. The $y_\chi^8$ scaling may be seen by comparing among these curves at some $m_\chi$. The 90% C.L exclusion cross-sections (with their 1 and 2 $\sigma$ bands) set by Xenon1T [126] are provided for reference. Due to the scaling in Eq. 3.13, our limit tightens with $y_\chi$. In general, we expect direct detection to constrain the DM mass poorer than kaon mixing. For instance, one may read off the plot that for $y_\chi = 2.2$, $m_\chi \gtrsim 960$ GeV. This is a weaker bound than the kaon mixing one, illustrated by the pink star at the corresponding $m_\chi$ in both the left and right panels of Fig. 3.5. Variations in $y_\chi$ will not change this behavior. Since the exclusion cross-section rises only gently across $m_\chi$ in this region, we expect from Eq. 3.13 that the limit on $m_\chi$ scales as $y_\chi^{4/3}$. On the other hand, from Eq. 3.12 we know that the kaon mixing $m_\chi$ bound scales as $y_\chi$, which is not much slower than the direct detection scaling.

Our future direct detection prospects are quite interesting. To check these, we compare our $\sigma_{\text{SI}}$ with the projected reaches of the LUX-ZEPLIN (LZ) [127] and DARWIN [128] experiments, provided in the figure. For $y_\chi = \{1.0, 1.4, 2.2\}$, LZ is sensitive to $m_\chi \lesssim \{0.7, 1.0, 1.6\}$ TeV and DARWIN to $m_\chi \lesssim \{0.8, 1.2, 2.1\}$ TeV. Amusingly, there emerge three distinctive future prospects of our relic surface for the three $y_\chi$ choices. We show this by placing a cloverleaf on our $\sigma_{\text{SI}}$ curves for each $y_\chi$ at the $m_\chi$ that gives $\Omega_\chi h^2 = 0.12$ (the part of the $\sigma_{\text{SI}}$ curve to the left/right of
the cloverleaf corresponds to DM freezing out under/over-abundantly. Also, as seen in the left panel of Fig. 3.5, the relic contour is near-insensitive to \(m_\rho\), and mostly picked by \(m_\chi\). By scanning the cloverleaves (at \(m_\chi \simeq \{0.55, 1.1, 2.7\} \text{ TeV}\)) and the points where our \(\sigma_{\text{SI}}\) curves intersect with LZ and DARWIN, we conclude that

(a) both LZ and DARWIN can reach \(y_\chi = 1.0\),

(b) LZ cannot, but DARWIN can, reach \(y_\chi = 1.4\),

(c) neither LZ nor DARWIN can reach \(y_\chi = 2.2\).

As we had mentioned in the previous sub-section, for \(y_\chi \in [1.2, 2.2]\) our relic contour evades the \(\epsilon_K\) bound in the dip feature. Thus we have a small range of \(y_\chi\) that gives the correct abundance, is currently viable with respect to all constraints, and is discoverable by future direct detection searches. (Note that although \(y_\chi > 2.2\) evades the \(\epsilon_K\) bound even without help from the dip feature, it is not discoverable at direct detection.) Though these results were obtained after fixing \(\lambda_s\), varying it would not change the broad conclusions.

We end this sub-section on a final note. Throughout the above, we have set \(\lambda_{sh} = 0\) following the motivation in Sec. 3.2.2, but had we turned it on and enabled mixing with the Higgs boson, some contribution to scattering cross-sections due to Higgs and gauge boson exchange may have arisen; however, these are completely negligible due to the small mixing angles quoted in Sec. 3.2.2.

3.3.3 Indirect detection

Fermi-LAT observations of gamma rays from stacked dwarf galaxies \[129\] have set 90\% C.L. limits on DM annihilation cross-sections that can reach \(\langle \sigma v \rangle_{\text{th}} = (\eta) 2.2 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}\), potentially affecting our set-up since our annihilation \(\chi \chi \rightarrow \sigma \rho\) is mainly \(s\)-wave. However, these limits are generally much weaker than the \(\epsilon_K\) bound discussed in Sec. 3.3.1. Consider the strongest Fermi-LAT bound, that from a 100\% \(b\bar{b}\) final state: for \(\langle \sigma v \rangle = \langle \sigma v \rangle_{\text{th}}\), DM mass \(\gtrsim 100 \text{ GeV}\). Although our annihilation sce-
nario is different – SM final states are products of $\sigma$ and $\rho$ decay, \textit{i.e.} we have cascaded, as opposed to direct, annihilations – the corresponding limit on $m_\chi$ must not be too far from 100 GeV. Indeed, we find that $m_\chi \gtrsim 175$ GeV from a naive recasting that assumes (a) the integrated photon flux from our DM cascaded annihilation equals that from direct annihilation to $b\bar{b}$, (b) equal masses, and hence decay branching ratios, for $\sigma$ and $\rho$. This limit falls well short of the one corresponding to the smallest $m_\chi$ spared by the $\epsilon_K$ bound: $m_\chi \gtrsim 760$ GeV, which occurs at $y_\chi = 1.2$ as explained in Secs. 3.3.1 and 3.3.2. Future indirect detection prospects too are dim. By recasting the sensitivities provided in [130], we find that only $m_\chi \lesssim 325$ GeV is within Fermi-LAT’s reach.

These conclusions, based on order-of-magnitude estimates, would hold against variations in $\lambda_s$. But they would change dramatically if we choose $n > 0$ ($Q_\chi > 1/2$) in Eq. 3.6, since Eq. 3.7 implies $m_\chi$ can now be lighter and hence in the range of Fermi-LAT. This warrants a thorough investigation of the indirect detection phenomenology of our $n > 0$ parametric families, which we undertake in [98].

3.3.4 LHC

The 13 TeV LHC can potentially probe our scenario. The collider prospects of $\rho$ have already been thoroughly explored in Ref. [87]. To summarize these briefly:

- the collider phenomenology primarily arises from the fact that $\rho$ couples strongest to bottom-bottom and top-charm; this can be seen in Eq. (3.11).

- the production rates are dominated by some combination of $b\bar{b} \rightarrow \rho$, $gb \rightarrow b\rho$ and $gc \rightarrow t\rho$, depending on $m_\rho$,

- the decay branching ratios of $\rho \rightarrow b\bar{b}$ and $\rho \rightarrow t\bar{c}$ dominate,

- at a 100 TeV collider, studies of top $\rightarrow \rho +$ charm can exclude the region $m_\rho \leq 175$ GeV, whereas $\sigma(gc \rightarrow t\rho) \times \text{BR}(\rho \rightarrow tc)$ can exclude $175 \text{ GeV} \leq$
\[ m_\rho \leq 1000 \text{ GeV for } v_s \lesssim 1 \text{ TeV}. \]

In the following we focus on the LHC and study additional signals – our smoking guns – generated by our introduction of DM. Since we use Ref. [87]'s Yukawa texture, and since generally \( \{m_\rho, m_\sigma\} < 2m_\chi \), the branching fractions of the flavon quanta remain the same, and we wish to clarify that adding our DM does not alter the phenomenology of [87], only augment it.

Using the fact that our DM must be detected as missing energy (in association with a visible particle) and that our mediator couples strongest to \( b\bar{b} \) and \( t\bar{c} \), we will show that searches for heavy quarks + \( E_T \) (mono-bottom and mono-top), as well as monojet searches, are our best strategy. In general, we expect these signals to be difficult to observe. We may see this from two considerations: (1) Being \( Z_2 \)-odd, \( \chi \) must be pair-produced, either through \( \sigma \) production followed by its invisible decay or through an off-shell \( \rho \). The former is kinematically suppressed since \( m_\sigma \simeq v_s \) is heavy, the latter phase-space suppressed, (2) In most of our signals, the initial state involves the sea quarks \( b \) and \( c \) that have low PDFs.

It is for the above reasons that we expect to be quite safe from current bounds, as we will show explicitly at the end of this sub-section. Due to this lack of constraints, we will restrict ourselves in the following to only a qualitative discussion of our best-case signals. A more thorough treatment involving careful background estimates and signal-enriching techniques will be dealt in forthcoming work [98].

While below we present our LHC signals with DM production through off-shell \( \rho \), it must be kept in mind that contributions from processes mediated by a possibly light \( \sigma \) may also be relevant.
3.3.4.1 Monojets

Missing transverse energy accompanied by a single jet is a popular channel for LHC DM searches \[131, 132\]. This signal arises in our set-up from the diagrams in Fig. 3.6. Contributions of the form \(q\bar{q} \rightarrow \rho^* j\) and those involving lighter quarks in the loop will be negligible because of the weaker couplings. The main backgrounds are \(Z(\nu\bar{\nu}) + j, W(\nu\ell) (\ell\text{ fakes a jet}), W(\nu\ell) + j (\ell\text{ is missed})\). Another significant but poorly understood background comes from QCD jet mismeasurement, usually minimized by a tight \(E_T\) cut.

3.3.4.2 Mono-b

\(bg \rightarrow \rho^* b\) (Fig. 3.6) can contribute to a mono-bottom signal, with subdominant contributions from the flavor-changing processes \(qg \rightarrow \rho^* b\), where \(q = d, s\). \(b\bar{b}\rho^*\) production may also contribute when one of the \(b\)’s is missed.

The dominant SM backgrounds are still \(Z(\nu\bar{\nu})/W(\nu\ell) + j/c\), where the ordinary or \(c\)-tagged jet is misidentified as a \(b\) and the \(\ell\) is missed. These contributions, though appearing to be suppressed by mistag rates, outdo direct \(b\) production \[133\]: \(gb \rightarrow bZ(\nu\bar{\nu})\) is the only irreducible background, and just like the signal process \(bg \rightarrow \rho^* b\), is PDF suppressed. Future improvements in \(b\)-tagging algorithms may reduce the \(Z/W + j/c\) background, and cutting high on \(E_T\) would help suppress all the backgrounds.

3.3.4.3 Mono-top

The flavor off-diagonal coupling \(gtc\) is comparable in size to \(gbb\) (see Eq. (B.3)) and helps in obtaining a large signal of mono-tops \[134\]. See Fig. 3.6 for the attendant diagrams. With the charm PDF higher than the bottom, this channel may be more

\[\text{\footnotesize 6 Although light quarks in the initial state will enhance the cross section due to their PDFs, the coupling of } \rho \text{ to light quarks is so small that these contributions are sub-dominant to } bg \rightarrow \rho^* b.\]
Figure 3.6. Our main signal processes producing mono-jet, mono-\( b \) and mono-top signatures. See text for details on backgrounds and signal-enrichment.

relevant than mono-\( b \).

Hadronically decaying tops may be particularly advantageous, since the branching ratio is high (\( =68\% \)) and since the top mass may be reconstructed from visible final states, aiding in background reduction. The main background is QCD multijets with mismeasured jets, which can be controlled with a high \( E_T \) requirement. This may boost the top quark and give near-collinear final products. Hence, instead of three distinct jets, the top may be detected as a single fat jet. All this makes our mono-top
similar to our mono-jet (in both signal and backgrounds), except here we further demand $m_{\text{fat jet}} \simeq m_t$ and b-tagging.

Having detailed our signals, we now demonstrate the safety of our scenario from constraints at the 8 TeV LHC. Ref. [135] places these bounds for monojet signals on an effective cutoff $\Lambda$ that mediates $\chi$-q interactions ($q = u, d, s, c, b$), which can be recast to our scenario by mapping our parameters to the definition of $\Lambda$. First, we choose the smallest DM mass spared by kaon mixing: 760 GeV (see the previous two sub-sections). If $m_\sigma \sim m_\chi$ and $m_\rho < m_\chi$ (as is true in most regions on our relic surface), the propagator is dominated by the momentum required to pair-produce DM, $2m_\chi$, which is then our cutoff. Thus $\Lambda = 1520$ GeV for both $\sigma$-mediated and $\rho$-mediated DM production. However, Ref. [135] finds the tightest bound to only be $\Lambda > 50$ GeV. Similarly, we can recast the mono-\(b\) bound in Ref. [133], which is even weaker: $\Lambda > 90$ GeV. Finally, mono-top signals studied in Ref. [136] set bounds on a model analogous to ours. While our $(g_\sigma)_{tc}$ at the parametric point considered above is 0.01, Ref. [136]'s upper limit on the coupling of a pseudoscalar mediator to the up and top quark $\sim O(1)$. Our actual limit is much weaker, since our $\rho$ and $\sigma$ production must proceed through the sea charm quark in the initial state as opposed to the valence up quark in Ref. [136], and also because our production rates are hurt by a 3-body final state when $\rho$ is in the propagator. Analogous to our indirect detection limits, our collider limits are generally weaker than flavor limits because $m_\chi \sim v_s$. Again similar to indirect detection, we expect the LHC to probe our scenario better at $n > 0$ since $m_\chi \propto \epsilon^n v_s$ (Eq. 3.7).

Though we have only considered mono-X searches, other signals involving $\rho^* \rightarrow \chi\bar{\chi}$ may be explored. E.g. a $b\bar{b}+E_T$ signal can arise via QCD $b$-pair production with one of the $b$'s radiating a $\rho^* \rightarrow \chi\bar{\chi}$. The lack of PDF suppression may bolster the signal, but the background also becomes more significant. Moreover, the signal rate falls away for heavy DM. These various tensions make this avenue a potentially
interesting study.

3.4 Conclusions

In this paper, we investigated the conditions under which fermionic dark matter embedded in the Froggatt-Nielsen mechanism, with a cutoff scale of $\Lambda_{\text{FN}}$, can freeze out to give the observed abundance. Annihilations to SM species are suppressed by the cutoff scale, while those to the flavon quanta can proceed with $\mathcal{O}(1)$ couplings. If neutral under $U(1)_{\text{FN}}$, DM can undergo $p$-wave-suppressed annihilations to pairs of the CP-even flavon $\sigma$, provided a compressed spectrum is contrived. If charged under $U(1)_{\text{FN}}$, DM can annihilate freely to a CP-odd flavon $\rho + \text{CP-even flavon } \sigma$ in the $s$-wave, as the hierarchy $m_\rho \ll m_\chi$ can be naturally arranged. Flavor constraints on the FN vev $v_s$ and perturbativity of the DM-flavon coupling $y_\chi$ together restrict the collective $U(1)_{\text{FN}}$ charge of DM to $Q_\chi \leq 2$. Perturbativity also sets upper limits on the FN cutoff $\Lambda_{\text{FN}}$, implying that future experiments sensitive to this scale (such as precision low-energy measurements or a 100 TeV collider) may be able to falsify our scenarios.

We focused on the case of $Q_\chi = 1/2$, which is viable over a larger parametric region than the cases of higher $Q_\chi$. We found that while direct detection, indirect detection and LHC searches provide weak constraints, measurements of the CP-violation parameter $\epsilon_K$ in kaon mixing probe this scenario well. Future direct detection searches can unearth regions allowed by kaon mixing, but future indirect detection searches cannot. We also discussed possible DM signatures at LHC Run-2 and strategies for enriching the signal over background. The clearest signal of our hypothesis would be a triple discovery of the pseudoscalar flavon, CP-even flavon and DM on the “relic surface” of our parameters, such as along the contours shown in the right panel of Fig. 3.3. These possibilities will be explored in greater detail in a forthcoming paper [98]. In it we will more closely examine the implications of indirect detection on the

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entire parameter space in Eq. 3.7, studying each $Q_\chi$ scenario in detail. We will also undertake a fuller collider study of our LHC signals.

Our set-up can be trivially extended to include the leptonic FN mechanism, in which case future AMS-02 measurements could become relevant. Gauging the $U(1)_{FN}$ symmetry is another possibility, potentially introducing $Z'$ bosons as a DM annihilation channel, and as an avenue for a new set of constraints. Spin-0 DM takes our analysis into non-trivial directions, since by virtue of the Higgs portal term $|\chi|^2|H|^2$, annihilations could now be shared amongst flavon- and Higgs-pair channels. We leave all these possibilities for future study.

In summary, it is intriguing that thermal freezeout provides a target for exploring not only the identity of dark matter but also the apparatus behind flavor.
CHAPTER 4
HIGGS PORTALS TO PULSAR COLLAPSE

Although the WIMP coincidence is a motivating framework to look for DM around weak scale, the WIMP paradigm does not explain why the relic abundance of DM is comparable to that of baryonic matter: $\Omega_{DM} \sim 5\Omega_B$ [137, 138]. The baryonic relic abundance itself was determined by a baryon asymmetry arising at some temperature hotter than the baryon-anti-baryon freeze-out temperature in the early universe, $T \gtrsim 10$ MeV. While the exact source of the baryon asymmetry is unknown, the Sakharov conditions stipulate that particle charge (C) and charge-parity (CP) must be violated out of thermal equilibrium to generate a particle asymmetry. There are numerous models that meet these conditions for baryons, including out of equilibrium decays [139, 140] and the Affleck-Dine mechanism [141–145].

Because the source of C and CP violation responsible for the baryon asymmetry is unknown, it is plausible, and indeed the coincidence $\Omega_{DM} \sim 5\Omega_B$ suggests, that the dark sector participated in the generation of the baryon asymmetry. In the simplest dark asymmetry models (see e.g. [146–148]), DM is charged under a quantum number that is composed of or identical to baryon number, and the DM abundance is set by the same process that sets the baryon abundance. Such models usually imply a DM mass range $m_X = 1 – 15$ GeV and collectively fall under the rubrik of asymmetric dark matter (ADM) (see Refs. [149–151] for recent reviews).

After the dark asymmetry is generated, the final ADM relic abundance will depend on having a large annihilation cross-section at freeze-out. ADM freeze-out cross-sections must be larger than that of WIMP dark matter, because if the dark
asymmetry provides for most of the dark matter relic abundance, ADM annihilation cross-sections must exceed the standard picobarn-size DM freeze-out cross-section. Otherwise, ADM would freeze-out to an overabundance and collapse the universe. Altogether, these relic abundance requirements motivate ADM coupled to light mediators, because a picobarn cross-section is difficult to attain using heavy ($\gtrsim 100$ GeV) mediators, while remaining consistent with collider bounds [152, 153] on low mass dark matter (i.e. $m_X = 1 - 15$ GeV). However, collider bounds on light DM can be evaded if DM annihilates to SM particles through a light mediator mixed with a SM boson [155].

In addition to evading collider bounds and matching cosmological requirements, light mediator dark matter also fits a number of astrophysical anomalies [149, 156–162], including the core-cusp problem – some dwarf galaxies exhibit unexpectedly cored DM density profiles [163, 164] – and the too big to fail problem – there is a dearth of massive Milky Way satellite galaxies [165, 167].

More recently, radio surveys of the galactic center have discovered a missing pulsar problem [168, 169]. Based on the number of progenitor stars which collapse into pulsars, around 10 young pulsars were expected to have already been found within the Milky Way’s central parsec. However, so far no young pulsars have been observed.

In this paper, we demonstrate that the missing pulsar problem specifically motivates asymmetric fermionic dark matter coupled to the SM through a light Higgs portal mediator, which would collect in and collapse pulsars at the galactic center, yet remain consistent with older pulsars seen in less DM-dense regions. The structure of this paper is as follows: in the remainder of the introduction, we provide a review of the missing pulsar problem. In section 4.2, we introduce the Higgs portal model, and catalogue constraints on it from relic abundance, primordial nucleosynthesis, direct

\[^1\text{DM-nucleon couplings for } m_X = 0.01 - 1 \text{ GeV and sub-TeV mediators are most constrained by quarkonium}\rightarrow\text{invisible decays at Belle and BES [154].}\]
detection, and Higgs invisible width measurements. In section 4.3 we determine what Higgs portal parameter space could simultaneously explain the absence of pulsars at the galactic center, while remaining consistent with old pulsars observed near the solar position. Section 4.4 gives the expected maximum age of pulsars as a function of distance from the galactic center. We present concluding remarks in section 4.5. Appendix E details estimates of the expected pulsar population at the galactic center. Appendix E.3 addresses the velocity dependence of the DM-nucleon cross-section.

4.1 The Missing Pulsar Problem

Radio surveys of the galactic center have not found as many pulsars as expected [168]. Pulsars are rapidly rotating, strongly magnetized neutron stars with spin period $P$ and approximate spin lifetime $\tau_p \lesssim P/2\dot{P}$, formed in the supernova collapse of (8-20 solar mass) heavy progenitor stars (see [170] for a review). There are about 300 heavy progenitor stars within a parsec of the galactic center [171], which signify the presence of at least 500 young (0.01-100 Myr) pulsars in the same region [168, 169, 172].

In addition to these young pulsars, one thousand old ($\sim$ Gyr) millisecond pulsars (MSPs) are expected to reside in the central parsec. Millisecond pulsars form when a neutron star in a binary system accretes material and angular momentum from its binary companion, thereby spinning up to millisecond pulse periods. The dense stellar environment of the galactic center should host many binary systems; extrapolating from the population of millisecond pulsars observed in star-dense globular clusters (e.g. Terzan5 has $\sim$ 100 MSPs [173]), an estimated $500 \sim 10^4$ millisecond pulsars should abide in the central few parsecs [174].

In addition to young and millisecond pulsars, there are also magnetars, which have a $\gtrsim$ 100 times stronger magnetic field and slower pulses than most pulsars. To date, a single magnetar 0.1 parsecs from the galactic center, is the only pulsar discovered
inside the central 50 parsecs [173,176]. Prior to the measurement of this magnetar’s radio pulse dispersion [177], it was reasonable to assume that pulsars had not been seen in the galactic center, because their radio pulses were smeared out by compton scattering off a dense intervening column of electrons.

However, using radio pulses from the magnetar, the density of electrons between the GC magnetar and earth has been measured. This measurement implies that, with the most conservative assumptions, prior radio surveys of the galactic center [178,179] should have found at least 10 young pulsars and 4 millisecond pulsars [168]. (Note that only about a tenth of existing pulsars will have pulses that beam towards earth.) A Bayesian analysis of the missing young pulsar problem found that the 6.6 GhZ galactic center survey [180] puts a 99% upper limit of 200 young pulsars in the central parsec, comparable to 500 expected using high mass progenitor data [169].

In Appendix E we provide more detail on methodologies for estimating the galactic center pulsar population. There are a number of possible explanations for the missing pulsar problem – the 300 heavy progenitor stars observed inside the central parsec could be a recent anomaly, and not indicative of heavy progenitor populations 100 Myr ago [168]. Some estimates of the galactic center millisecond pulsar and young pulsar populations assume a typical stellar initial mass function in the galactic center (see Appendix E). The actual initial mass function of the galactic center may be “top heavy” [181], resulting in more black holes and fewer pulsars produced through core collapse of GC high mass progenitor stars. In this paper, we focus on a different, intriguing possibility: asymmetric fermionic dark matter coupled to SM particles through a Higgs portal may collapse pulsars in the galactic center.

4.2 Higgs Portal Mediators For Pulsar Collapse

While a number of astrophysical anomalies motivate fermionic dark matter coupled to an MeV-scale vector or scalar mediator [155,158,182], in the case of pulsar-
collapsing dark matter (PCDM), scalar mediators and asymmetric DM are required. The dark matter must be asymmetric, so that it can collect in pulsars without annihilating to SM particles \[183, 184\]. If it is coupled to a light scalar mediator, it will only have attractive self-interactions\(^2\) permitting DM self-attractive forces to overcome Fermi degeneracy pressure and initiate black hole collapse at the center of pulsars.

Assuming the dark mediator \(\phi\) is a real scalar, and \(X\) is a dirac fermion, we add the following terms to the SM Lagrangian,

\[
\mathcal{L} = \mathcal{L}_{\text{SM}} + i \bar{X} \gamma^{\mu} X + \frac{1}{2} (\partial \phi)^2 - a \phi(|H|^2 - v^2/2) - b \phi^2 (|H|^2 - v^2/2) - g_D \phi \bar{X} X - m_\phi^2 \phi^2 - m_X \bar{X} X
\]  

(4.1)

Thus \(\phi\) and \(X\) interact with the visible sector through \(H - \phi\) mixing \[186–188\]. Note that we have taken \(H \rightarrow h + v/\sqrt{2}\) as the Higgs vacuum expectation value (vev) after electroweak symmetry breaking and have shifted the \(H - \phi\) coupling so that \(\phi\) does not get a vev. In the low momentum limit, and assuming \(a, m_\phi \ll v, m_h\), the effective coupling of \(\phi\) to standard model particles is

\[\epsilon_h \equiv \sqrt{2}av/m_h^2.\]  

(4.2)

The coupling of the light mediator to DM, \(g_D\) (or equivalently \(\alpha_D = g_D^2/4\pi\)), is bounded from below by relic abundance and CMB constraints. For thermal dark matter, relic abundance requirements put a lower bound on the annihilation cross-section of dark matter \(<\sigma v> \gtrsim 10^{-25}\)cm\(^3\)/s, so that the universe does not collapse. If \(^2\)Note that DM-mediator couplings of the form \(g_D \bar{X} \gamma^\mu X \phi_\mu\) will be repulsive for \(XX \rightarrow XX\) and \(\bar{X} X \rightarrow \bar{X} X\) scattering, whereas scalar couplings \(g_D \bar{X} X \phi\) produce purely attractive self-interactions. In both these cases the potential is Yukawa \(|V| = g_D^2 \exp(-m_\phi r)/4\pi r\). See \[185\] for a treatment of pseudoscalar and axial couplings, which we do not consider here.
the scalar mediator is lighter than dark matter, this implies

\[ \alpha_D \gtrsim 11 \times 10^{-5} \left( \frac{\langle \sigma v \rangle}{10^{-25} \text{cm}^3/\text{s}} \right)^{1/2} \left( \frac{m_X}{\text{GeV}} \right)^{1/2} \left( \frac{x_f}{20} \right)^{1/2} \]  

(4.3)

where \( m_X \) is the mass of DM, \( x_f \equiv m_x / T_{\text{FO}} \), and \( T_{\text{FO}} \) is the freeze-out temperature of DM [155]. For asymmetric dark matter, requiring that \( \Omega_x h^2 > 0.11 \) (since the relic abundance is assumed to be generated by a particle asymmetry), the annihilation cross-section should be \( \langle \sigma v \rangle > 4.5 \times 10^{-26} \text{cm}^3/\text{s} \) for Dirac fermions [189].

Because the mediator \( \phi \) must decay before big bang nucleosynthesis, there is a lower bound on its coupling to SM particles. Big bang nucleosynthesis (BBN) takes place in the CMB radiation era after about a second. In this era, any extra particles that can decay and inject energy lead to an enhancement in the nuclear interaction rate. In particular, a light mediator with a mass of \( 1 - 100 \text{ MeV} \) is below the \( ^4\text{He} \) binding energy; therefore it would disassociate deuterium and lithium isotopes, while leaving \( ^4\text{He} \) relatively undisturbed. Thus there is a constraint on any light degree of freedom besides those of the SM [190]. To avoid these constraints, we require that \( \phi \) decay before BBN. For \( m_\phi < 2m_\mu \), the total width of \( \phi \) is

\[
\Gamma_\phi = \frac{\epsilon^2 m^2_{\phi} m_\phi}{16\pi v^2} \left( 1 - \frac{4m^2_\mu}{m^2_\phi} \right)^{3/2} \Theta(m_\phi - 2m_e) \\
+ \frac{\epsilon^2 \alpha^2 D m^3_\phi}{512\pi v^2} \left[ A_1 + \left( \frac{8}{3} \right) A_{1/2} + \left( \frac{1}{3} \right) A_{1/2} \right]^2, \quad (4.4)
\]

where the second contribution is for \( \phi \to \gamma\gamma \), with \( A_1 = -7 \) from the W loop, and \( A_{1/2} = 4/3 \) is the contribution from heavy quark loops [191]. In Figure 4.1, the dark cyan dotted line indicates parameter space where the lifetime of \( \phi \) is less than a second. Note that even this constraint can be lifted entirely by coupling \( \phi \) to additional light dark states (e.g. sterile neutrinos [192]).

\( \phi - H \) mixing also contributes to the invisible Higgs decay width. The additional
contributions to the Higgs invisible decay width from (4.1) are

$$\Gamma_{h\text{inv}} = \alpha_D \epsilon_h^2 m_h \left(1 - \frac{4 m_X^2}{m_h^2}\right)^{3/2} + \frac{b^2 v^2}{8 \pi m_h} \left(1 - \frac{4 m_\phi^2}{m_h^2}\right)^{1/2}. \quad (4.5)$$

Requiring the Higgs→invisible branching ratio be less than 40% (from e.g. [193]) of the total SM Higgs width ($\Gamma_H \sim 4.1$ MeV) puts an upper bound on $\epsilon_h$. Since we only consider $m_\phi \ll m_h$, the constraint on the higgs invisible decay requires $b \ll 0.013$. Assuming the contribution from $b$ is negligible, in Figure 4.1 we indicate the constraint on $\epsilon_h$ from restricting the Higgs invisible width to be less than 40% of the SM expectation [194–196].

DM-nucleon scattering experiments such as LUX [55], XENON100 [54], and SuperCDMS [197] also constrain Higgs portal parameter space. A calculation of DM-nucleon scattering is given in Appendix E.3 in which we include the full dependence of the scattering amplitude on $m_\phi$ and the velocity of DM in the nucleon’s rest frame. Figure 4.1 shows the current bounds [55, 197] (solid magenta line) and future reach [198] (dashed lines) of DM direct detection experiments.

4.3 Collection and Collapse of Higgs Portal Dark Matter in Pulsars

Another possible method for detecting asymmetric Higgs portal dark matter, is to find pulsars imploding (or having imploded) in regions where ambient dark matter is dense and can rapidly collect in pulsars. After enough asymmetric dark matter has been captured and thermalized in a pulsar, it may form a black hole that swallows the pulsar [183, 184]. Usually, fermions will not form a black hole until $N_{ch} \sim (m_{Pl}/m_X)^3$ particles have agglomerated, where $N_{ch}$ is the Chandrasekhar limit for fermionic matter. The asymmetric Higgs portal model we consider has attractive self-interactions (from $g_D \phi \bar{X} X$) that counteract Fermi degeneracy pressure, effectively decreasing $N_{ch}$. Indeed, attractive self-interactions are required for fermionic DM-
induced pulsar collapse, because the maximum number of DM particles captured by a neutron star in a gigayear, $\sim 10^{41} \text{(GeV}/\text{m}_X)$, is much smaller than the Chandrasekhar number, $\sim 10^{57} \text{(GeV}/\text{m}_X)^3$.

$\mu = 80 \text{ or } 0.2 \text{ GeV/cm}^2$, $\tau_{SS} = 0.01 \text{ or } 1 \text{ Gyr}$, $v = 150 \text{ or } 200 \text{ km/s}$, $T_{SS} = 10^4 \text{ K}$, $M_{SS} = 1.4 \text{ M}_\odot$, $R_{SS} = 10 \text{ km}$

Figure 4.1. Direct detection, cosmological, and collider bounds on Higgs portal dark matter are shown as a function of $\epsilon_h \equiv \sqrt{2}a v/m_h^2$, along with viable galactic center pulsar-collapsing parameter space, shaded grey. Space above the solid black, blue and pink lines are excluded by Gyr old pulsars outside the GC, constraints on the Higgs invisible decay width, and direct detection, respectively. Constraints from BBN exclude space below the dotted cyan line. The DM density ($\rho_X$), pulsar lifetime ($\tau_{NS}$), velocity dispersion ($v$), and pulsar temperature ($T_{NS}$), mass ($M_{NS}$), and radius ($R_{NS}$) used to plot the solid and dotted-dashed black lines are given at the top of the plot. The parameter space along the dotted-dashed (solid) black line would collapse pulsars older than $\sim 10^7$ years inside the Milky Way’s central 500 parsecs, while permitting up to $\sim 10^{10}$ year old pulsars outside of the galactic center.
4.3.1 DM collection

Dark matter particles are captured in neutron stars at a rate given by [184, 199, 201, 216],

\[
C_X = \sqrt{\frac{6}{\pi}} \left( \frac{\rho_X}{\bar{v}_X} \right) \frac{N_B \xi v^2_{esc}}{m_X} \left[ 1 - \frac{1 - \exp(-B^2)}{B^2} \right] f(\sigma_{nX}), \tag{4.6}
\]

where \( \rho_X \) is the DM density around the neutron star, \( \bar{v}_X \sim 200 \text{ km/s} \) is the velocity dispersion of the dark matter-neutron star system, \( N_B = 1.2 \times 10^{57} \) is the number of nucleons in a 1.4\(M_\odot\) neutron star, \( v_{esc} \simeq 0.7 \) is the escape velocity from the surface of a neutron star, and \( \xi = \text{Min}[m_X/(0.2 \text{ GeV}), 1] \) is a factor accounting for Pauli blocking of DM-nucleon scattering. Because the dark matter will have a semi-relativistic momentum when falling through the neutron star’s rest frame, the size of its momentum \( p_X \sim m_X \) indicates that heavy \((m_X \gg m_B)\) dark matter may not transfer enough momentum to be captured by the neutron star. The term in square brackets, with the quantity \( B^2 = 6v^2_{esc} m_X m_B / \bar{v}_X^2 (m_X + m_B)^2 \), accounts for this diminution in capture of heavier dark matter. Finally, the term dependent on the dark matter-nucleon scattering cross-section, \( \sigma_{nX} \), is given by \( f(\sigma_{nX}) = \sigma_{sat}(1 - \exp(-\sigma_{nX}/\sigma_{sat})) \).

When the dark matter-nucleon scattering cross-section is small, \( \sigma_{nX} \lesssim 10^{-45} \text{ cm}^2 \), this function returns \( \sigma_{nX} \), but as \( \sigma_{nX} \) gets larger, the geometric cross-section per nucleon in the neutron star saturates. Note that for lighter dark matter, this saturation cross-section will depend on Pauli blocking [184, 216], \( \sigma_{sat} = R_{NS}^2/0.45N_B\xi \), where \( R_{NS} \sim 10 \text{ km} \) is the neutron star radius. We detail our calculation of the dark matter-nucleon scattering cross-section \( \sigma_{nX} \) for a given set of Higgs portal parameters \((\epsilon_h, m_X, m_\phi, \alpha_D)\) in Appendix E.3.
4.3.2 DM state at collapse

To determine the critical number of dark fermions that will initiate black hole collapse in the neutron star interior, we use the Virial equation for a test particle at the edge of a sphere of fermions thermalized at the center of the neutron star,

\[-2E_k + \left(\frac{4\pi}{3}\right)^{1/3}G\rho_B N_X^{2/3}m_Xy^2 + V_{Yuk} = 0, \quad (4.7)\]

where the first term is the virialized kinetic energy, the second term is the virialized gravitational potential from baryons in the neutron star, and the third term is the virialized form of the Yukawa potential. It is reasonable to assume all DM particles are thermalized, because results from [215, 220] indicate that for the parameter space under consideration, dark matter thermalizes inside the neutron star on timescales \(\lesssim 10^5 \text{ yrs.} \) \(N_X\) is the number of dark matter particles collected, and \(V_{Yuk} = \sum r_j \alpha_D e^{-m_\phi r_j} (1/r_j + m_\phi), \) with \(r_j\) being the inter-particle distance. Once \(V_{Yuk} \gg 2E_k,\) the dark matter will collapse into a black hole. In these calculations, we can neglect the dark matter self-gravity as being small (in what follows, see [216] for more details). In Eq. (4.7), we have defined \(y = 1.6m_\phi r/N_X^{1/3},\) which is the exponent of the Yukawa potential if only the nearest fermions to the test particle contribute to its potential; note that \(x = 1.6r/N_X^{1/3}\) is the nearest-neighbor inter-fermion distance for \(N_X\) fermions evenly distributed in a sphere of radius \(r.\)

4.3.2.1 Strongly-screened

If \(y \gg 2,\) only the nearest-neighbor particles will matter in determining the Yukawa potential. We will call this the “strongly-screened” limit, the limit in which all but the nearest-neighbor fermions can be neglected. The strongly-screened virial-
ized Yukawa potential is

\[ V_{Yuk}^{strong} = 8\alpha_D \left( m_\phi e^{-y/y} + m_\phi e^{-y} \right), \] (4.8)

where we assume eight nearest neighbor particles.

4.3.2.2 Coulombic

If \( y \ll 2 \), on the other hand, the exponential piece of the Yukawa potential approaches unity, and the Yukawa potential becomes Coulombic. In other words, the Yukawa potential \( (\propto e^{-m_\phi r}) \) will not be suppressed inside radius \( 1/m_\phi \). The number density of dark matter fermions at the center of the star is given by \( 1/x^3 \). Hence, the number of fermions contributing to the Coulomb-like potential inside radius \( 1/m_\phi \), is \( N_{co} = 4\pi/3m_\phi^3x^3 \), which gives a virialized potential term,

\[ V_{Yuk}^{Coul} = 3\alpha_D N_{co}m_\phi = 4\pi\alpha_D m_\phi/y^3. \] (4.9)

4.3.2.3 Degenerate

Before the onset of collapse, the dark matter fermions collected inside radius \( r \) will be degenerate if more than

\[ N_{deg} = 5 \times 10^{27}(r/cm)^3(m_X/\text{GeV})^{3/2} \] (4.10)

have collected, assuming an ambient temperature of \( T = 10^4 \text{ K} \). In this case the kinetic energy is given by

\[ E_{k}^{deg} = (9\pi N_X/4)^{2/3}/2m_Xr^2 = (3\pi^2)^{2/3}m_\phi^2/2m_Xy^2 \] (4.11)
and the pre-collapse radius of the dark matter, determined by solving Eq. (4.7) with the last term omitted is

\[ r_{th, deg} = 2.4 \times 10^{-4} N_X^{1/6} (\text{GeV}/m_X)^{1/2} \text{ cm}. \] (4.12)

4.3.2.4 Non-degenerate

If less than \( N_{deg} \) particles have collected before collapse, the kinetic energy is simply \( E_{non-deg}^k \sim 3k_B T/2 \) and the pre-collapse thermal radius is

\[ r_{th, nondeg} = 80(\text{GeV}/m_X)^{1/2} \text{ cm}, \] (4.13)

again assuming a neutron star temperature of \( 10^4 \) K.

4.3.3 Critical DM number for collapse

There are four states from which the collapse of DM can begin: the dark matter may be either degenerate or non-degenerate and either strongly-screened or Coulombic. We can determine which state precedes collapse as a function of model parameters \((\alpha_D, m_X, m_\phi)\) using Eq. (4.7).

4.3.3.1 Degenerate

If more than \( N_{deg} \) particles have collected, the dark matter will be degenerate. Substituting \( E_{deg}^k \) into Eq. (4.7), and dropping the baryonic gravity term (which should be negligible after collapse begins), we can determine what parameter space initiates Coulombic collapse by requiring \( y \leq 2 \) in

\[ (3\pi^2)^{2/3} m_\phi^2 / 2 m_X y^2 = 4\pi \alpha_D m_\phi / y^3, \] (4.14)
from which we find that collapse will begin from a Coulombic (strongly-screened) state when $\alpha \gtrsim m_\phi / m_X$ ($\alpha \lesssim m_\phi / m_X$). If collapse begins from a Coulombic, degenerate state, the number of particles required for collapse can be found by solving Eq. (4.7) for $N_{\text{coll}}$,

$$N_{\text{coll}}^{\text{degCoul}} > 10^{25} \alpha^{-6} (m_\phi / \text{MeV})^{12} (\text{GeV} / m_X)^9. \quad (4.15)$$

If on the other hand, collapse begins from a strongly-screened degenerate state, Eq. (4.7) must be solved numerically with $V_Y^{\text{strong}}$. Specifically, we solve Eq. (4.7) for $y$, using successively larger values of $N_{\text{coll}}$, until no solution can be obtained where $y > 2$, indicating collapse has begun. More details of this numerical solution can be found in [216].

4.3.3.2 Non-degenerate

If less than $N_{\text{deg}}$ particles have collected, dark matter will not be degenerate, and the kinetic energy will be $3k_B T/2$. For all parameter space of interest ($m_X \gtrsim 0.1$ GeV), the number of particles required to begin collapse in a Coulombic, non-degenerate state exceeds $N_{\text{deg}}$ [216]. Hence, we only need to consider collapse from a strongly-screened non-degenerate state. To determine the critical number of particles required, we insert $E_k^{\text{non-deg}}$ in Eq. (4.7) and solve numerically for $N_{\text{coll}}$. As collapse proceeds, and $y$ (along with $r$) shrinks, the fermions will eventually become degenerate. The value of $y$ at which the fermions become degenerate can be found by substituting $r_{\text{th,deg}}$ into $y$,

$$y_{\text{deg}} = 20 (m_\phi / \text{MeV})(\text{GeV} / m_X)^{1/2}. \quad (4.16)$$
Figure 4.2. Constraints from SuperCDMS and LUX (pink) along with bounds from old pulsars (black) on asymmetric Higgs portal dark matter are shown as a function of $\epsilon_h \equiv \sqrt{2} av/m_h^2$, for a number of values of $\alpha_D$ and $m_\phi$. The DM density ($\rho_X$), pulsar lifetime ($\tau_{NS}$), velocity dispersion ($v$), and pulsar temperature ($T_{NS}$), mass ($M_{NS}$), and radius ($R_{NS}$) used to plot the black lines are given at the top of the plot. Note that parameter space along each black line would collapse pulsars older than $\sim 10^6$ years inside the Milky Way’s central 500 parsecs, while permitting $\sim 10^9$ year old pulsars outside of the galactic center.

If $y_{deg} < 2$, we can substitute Eq. (4.16) into Eq. (4.14), and find that collapse continues through a Coulombic, degenerate state so long as

$$\alpha > 0.02(m_\phi/\text{MeV})^2(\text{GeV}/m_X)^{3/2}. \quad (4.17)$$

If $y_{deg} > 2$, then substituting Eq. (4.16) into Eq. 4.7 (dropping the baryonic term), we find that collapse proceeds so long as

$$\alpha > 10^{-6} e^{y_{deg}}(\text{MeV}/m_\phi)(1 + 1/y_{deg})^{-1}. \quad (4.18)$$

After dark matter collapses into a black hole, in order to destroy the neutron
star, it must accrete surrounding baryons faster than it radiates via the Hawking mechanism [221]. This condition is fulfilled if,

\[-1/(15360\pi G^2 N_{\text{coll}}^2 m_X^2) + 4\pi \rho_B G^2 N_{\text{coll}}^2 m_X^2 / c_s^3 > 0, \tag{4.19}\]

where the first term is the Hawking radiation rate, the second term is the Bondi accretion rate, and $c_s \sim 0.3$ is the sound speed of baryons in the neutron star. If the black hole grows, it will destroy the neutron star in $t_{\text{des}} \sim 2000(\text{GeV}/m_X)/(10^{40}/N_{\text{coll}})$ yrs.

In Figure 4.1 and 4.2 we display parameter space which is excluded by the existence of $10^9$ year old pulsars outside the galactic center (where the DM density is $\rho_X \sim 0.2 \text{ GeV/cm}^3$). Note that the left side of these bounds terminate where $N_{\text{coll.}} > C_X \tau_{NS}$, while the right side terminates when the condition Eq. (4.19) fails, meaning the black holes formed by collapsing dark matter are too small too grow and instead evaporate. In Figure 4.1 we show model space which would collapse $10^6 - 10^7$ year old pulsars in the galactic center, while allowing for old pulsars outside the galactic center. Note that we assume a galactic center DM density ($\rho_X \sim 80 \text{ GeV/cm}^3$) determined by DM self-interactions, as we discuss in the next section.

4.4 Dark Matter Self-Interactions and Pulsar Age Curves

To determine which Higgs portal PCDM models precipitate pulsar destruction in the galactic center, we need to know the mass distribution of DM in the Milky Way. It is commonly assumed that DM is cold and collisionless (CCDM), and simulations of CCDM structure formation yield halo densities that follow the Navarro-Frenk-White (NFW) profile: $\rho(r) = \rho_0/(r/R_s) (1 + r/R_s)^{-2}$, where $\rho_0$ and $R_s$ are characteristic density and scale radius respectively, and $r$ is the distance from the galactic center. However, the CCDM paradigm has been challenged recently by some observations of small scale structure. These include dwarf halo profiles which appear underdense at
their centers [163, 164], too few small satellite galaxies near the Milky Way, and no large satellite galaxies [165][167].

These small scale structure anomalies can be addressed with the addition of sizable DM self-interactions $\sigma_{SI} \sim 10^{-24}$ cm$^2$/GeV. However, this seems disfavored by self-interaction bounds from “bullet” collisions in galactic clusters [222, 223] and the observed ellipticity of galactic cluster cores [224]. Typically these require DM self-interactions smaller than $\sigma_{SI}/m_X \sim 10^{-24}$ cm$^2$/GeV. If, on the other hand, DM self-interactions are velocity dependent through coupling to a light mediator ($m_\phi = 1 – 100$ MeV), bounds from large scale observations can be evaded [149, 156–162]. This is because the cross-section can be large in, for example dwarf galaxies, where the characteristic velocity is 50 km/s, while the mediator is more off-shell and interactions are weaker for DM velocities $\sim 1000$ km/s, in galactic clusters. Much of the parameter space shaded grey in Figure 4.1 will have velocity-dependent self-interactions suitable for resolving small scale anomalies, while remaining consistent with large scale bounds [162, 182].

Some simulations indicate that DM with significant self-interactions $\sigma_{SI} \gtrsim 10^{-24}$ cm$^2$/GeV, will cause the Milky Way’s DM density profile to be flat ($\sim$ GeV/cm$^3$) within the central 2 kiloparsecs [225, 227]. However, a recent study which included the gravitational potential of baryons showed that SIDM in Milky Way size halos will create a 500 parsec DM core, with a larger constant central density ($\sim$ 80 GeV/cm$^3)$ [228]. For SIDM, the density of the central core will depend mostly on central the baryonic density, $\rho(r) = \rho_0 \exp[h(r)]$, where $h(r)$ satisfies Jean’s equation and is given by

$$h(r) = \frac{-2\pi G \rho_0 r_0^2}{\sigma_0^2} y \left( \sigma_0 - \frac{2y^3}{3} \right).$$

(4.20)

In the Milky Way, $\rho_0 = 80$ GeV/cm$^3$ is the central DM density, $y = r/(r + r_0)$
parameterizes distance from the galactic center, \( r_0 = 2.7 \text{ kpc} \), \( \sigma_0 = 160 \text{ km/s} \) is a constant velocity dispersion. We show this DM density profile in the upper left inset of Figure 4.3, along with the DM velocity dispersion we assumed, which was derived by fitting observed star velocities in the Milky Way \([218, 229]\).
In Figure 4.3 we use the DM density profile in Eq. (4.20) to predict the maximum pulsar age as a function of distance from the galactic center. The solid (dotted-dashed) line shows the maximum age curve for asymmetric Higgs portal models that predict $\gtrsim 10^6$ ($\gtrsim 10^7$) year old pulsars will collapse in the galactic center. The radial distance from the galactic center and the characteristic radii of pulsars in the Milky Way, shown with blue and pink data points, are taken from the ATNF pulsar catalogue [230]. It should be pointed out that the characteristic age of pulsars $\tau_p \lesssim P/2\dot{P}$, often vastly overestimates the age of the pulsar. Indeed, it has become clear that this overestimate may be systematically large for all millisecond pulsars with $\tau_p \gtrsim 10^9$ years [170, 231]. This effect results from an initial pulse period, $P_0$, that is not much smaller than the current pulse period $P$. In this case the full expression for the pulsar age, $\tau_p^{\text{full}} \simeq P/2\dot{P} - P_0/2\dot{P}$, is required, but $P_0$ is unknown. Note in Figure 4.3 that many of the pulsar “characteristic” ages exceed the age of the universe.

However, millisecond pulsars that formed in a binary system with a white dwarf (WD) provide an alternative way to check the age of the pulsar. The binary white dwarf’s temperature and mass can be fit to WD cooling curves to independently determine the age of the binary system. One example is pulsar J1738+0333, which has a characteristic age of 4 Gyr, and whose white dwarf has an apparent age of 0.5– 5 Gyr [232]. Therefore, for the purpose of bounding DM parameters with pulsar ages, it is most conservative to assume pulsars outside the galactic center have reached ages of about a gigayear.

4.5 Conclusions

Higgs portal DM-induced pulsar collapse could explain the apparent paucity of galactic center pulsars. Fermionic asymmetric dark matter coupled to the SM through a Higgs portal can collapse pulsars older than $\sim 10^6$ years inside the galactic center,
while not collapsing $10^9$ year old pulsars near the solar position. This is because the expected DM density in the galactic center ($\rho_X = 80 \text{ GeV/cm}^3$) is larger than in the rest of the halo ($\rho_X = 0.2 \text{ GeV/cm}^3$), leading to increased DM capture in GC pulsars.

Based on the number of GC pulsar progenitor stars, GC radio surveys should have already found $O(10)$ pulsars in the central parsec. However, none have been observed, and it is unlikely that this is solely the result of measurement limitations [168], although it could result from an overestimated GC pulsar population. We have showed that for DM masses $m_X = 0.1 - 100\text{GeV}$, with Higgs portal mediator masses $5 - 20\text{MeV}$, DM-mediator couplings $\alpha_D = 0.001 - 0.1$, and mediator-Higgs mixings $\epsilon_h = 10^{-8} - 10^{-2}$ asymmetric Higgs portal dark matter provides an explanation for the absence of GC pulsars: older GC pulsars collapse into black holes after enough DM is captured to form a black hole that will grow in pulsar interiors. As pulsars are discovered in the galactic center by future radio surveys and radio telescopes, (e.g. FAST [233] and the Square Kilometer Array [234]) PCDM would manifest as a maximum pulsar age that increases with distance from the GC.

Applied to old pulsars seen outside the GC, our results set bounds on fermionic, asymmetric Higgs portal dark matter that are often more stringent than those set by direct detection experiments. However, while much of the Higgs portal PCDM parameter space is inaccessible to present terrestrial direct detection, we have shown that next-generation direct detection experiments will either find or exclude it. In addition, through its coupling to a light mediator, Higgs portal PCDM naturally fits one explanation for the core-cusp phenomenon: light mediator DM self-scattering is resonantly enhanced at smaller momenta in dwarf galaxies, and is diminished at larger momenta in spiral galaxies and galactic clusters.

Finally, we note that a recent proposal of Fuller and Ott [235] (advanced with all reserve) has linked PCDM to fast radio bursts [236–240]. We leave this and other
complementary probes of PCDM to future work.
CHAPTER 5

ULTRAVIOLET FREEZE-IN

In the freeze-out paradigm of dark matter (DM), such as the much studied WIMP scenario, the DM is initially in thermal equilibrium and its abundance evolves with its equilibrium distribution until it decouples from the thermal bath. After decoupling the DM comoving number density is constant and (for appropriate parameter values) can give the observed relic density. Models of freeze-in DM [15, 241–245] provide a very different picture of the evolution of the DM abundance. In this setting it is supposed that the DM is not in thermal equilibrium with SM particles and has a negligible initial number density. However, DM interacts very feebly with the SM particles, and over time an abundance suitable to match the relic density is produced. DM in freeze-in mechanism never attains thermal equilibrium with the SM. Consequently, it is only produced due to the interaction with SM, but its annihilation is negligible. In other words, the number density of DM also stay low enough that they never find each other to annihilate.

This is illustrated in Fig. 5.1. For the DM abundance to be initially negligible, and subsequently set by the freeze-in mechanism (rather than freeze-out), the hidden sector must be thermally decoupled from the visible sector bath at all times, which implies that the portal operators must be extremely small. For instance, for TeV scale DM produced via $2 \rightarrow 2$ scattering of bath states involving a renormalisable portal interaction the coupling dressing this operator should be typically $\langle 10^{-7}$ in order to avoid equilibration of the DM with the visible sector. Such DM states are sometimes referred to as feebly interacting massive particles, or FIMPs.
Freeze-in, as a general mechanism for DM production, was proposed only recently and thus many important aspects remain to be studied. In particular, a huge class of models has been largely neglected and the purpose of this paper is to rectify this. Freeze-in using renormalisable interactions has been considered in some detail; here instead we examine the alternative possibility, that freeze-in production proceeds via non-renormalisable operators. A suitable DM abundance can potentially be generated by freeze-in via such effective operators, which we refer to as UltraViolet (UV) freeze-in, and in this case the DM abundance depends sensitively on the reheat temperature. Conversely, we use InfraRed (IR) freeze-in to refer to the class of models in which the sectors are connected via renormalisable operators, in which case the DM abundance is set by IR physics and is independent of the reheat temperature. The different thermal histories associated to these DM frameworks are illustrated in Fig. 5.1.

The two basic premises of the general freeze-in picture are that

\[ Y \equiv \frac{n_{DM}}{S} \]

with respect to inverse scaled temperature \( x \propto T^{-1} \) for the freeze-out \( Y_{FO} \), IR freeze-in \( Y_{IR} \) and UV-freeze-in \( Y_{UV} \) scenarios.

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\footnote{This framework builds upon earlier specific realisations, most notably the production of right-handed neutrinos, axinos, and gravitinos, and see also [243][244].}
• The hidden and visible sectors are thermally disconnected,

• The inflaton decays preferentially to the visible sector, not reheating the hidden sector.

Consequently, it is a model independent statement that, due to the out-of-equilibrium dynamics, DM production will proceed through freeze-in via any non-renormalisable operator which is not forbidden by symmetries. Further, the expectation from UV completions of the SM is that distinct sectors of the low energy theory are generically connected by UV physics.

On the other hand, IR freeze-in relies on a rather special construction in which the (renormalisable) portal operators have diminutive couplings, however the naïve expectation is that dimensionless parameters should be near unity. Whilst such feeble couplings are not inconceivable (the electron Yukawa $Y_e \sim 10^{-6}$ is one example), such decoupling is readily achieved if the visible sector and hidden sector are only connected via high dimension operators.

The UV freeze-in scenario bears some resemblance to models of non-thermal DM [246]. The two frameworks both require the DM to be thermally decoupled from the visible sector, they also rely on particular realisations of inflation, and both lead to a DM abundance which is dependent on the reheat temperature. However, there are also important distinctions between these frameworks. In non-thermal DM, the DM has sufficiently small couplings with the visible sector such that energy exchange between the sectors is negligible and the DM relic density is set primarily by inflaton decay. In contrast, in UV freeze-in the DM is dominantly populated by energy transfer from the visible sector to the hidden sector.

In this paper we examine a range of motivated operators for UV freeze-in and discuss potential connections with other aspects of high scale physics. The paper is structured as follows: In Sect. 5.1 we develop the physics behind UV freeze-in using a number of toy models which exemplify several interesting features. In particular,
we discuss high dimension operators with many body final states. Further, we ex-
amine examples in which a field involved in the portal operator develops a vacuum
expectation value (VEV). We consider the constraints which arise from avoiding sec-
tor equilibration in Sect. 5.2 and find that this leads to bounds on the parameter
space, but that large classes of viable models can be constructed. Subsequently, in
Sect. 5.3 we propose a number of simple models, motivated by beyond the Standard
Model (BSM) physics, which realise the UV freeze-in picture. Specifically, we con-
sider possible connections with axion models and $Z'$ portals. We also comment on
the prospect of identifying the scale of UV physics given the DM mass, portal oper-
ator and the magnitude of the reheat temperature. In Sect. 5.4 we provide a brief
summary, alongside our closing remarks.

5.1 General Possibilities For UV Freeze-In

The possibility of freeze-in via non-renomalisable operators has been briefly dis-
cussed in [15, 241, 242, 245]. One of the distinguishing features of UV freeze-in is
that DM production is dominated by high temperatures, and so the abundance is
sensitive to the reheat temperature $T_{RH}$. Whilst this possibility has been previously
remarked upon as less aesthetic due to the dependence on the unknown value of $T_{RH}$,
it is nevertheless very well motivated as it is a generic expectation that sectors which
are decoupled at low energy may communicate via high dimension operators. In this
section we examine some general classes of toy models in which the hidden and visible
sector are connected only by effective contact operators.

5.1.1 Dimension-five operators with two and three-body final states

We shall start by discussing the simple toy model of UV freeze-in outlined by
Hall, March-Russell & West [15] (see also [244] for a similar model in the context
of supersymmetry); this will provide a basis from which to examine more realistic
scenarios in subsequent sections. In this toy model a scalar DM state $\varphi$ freezes-in due to a dimension five operator of the form\footnote{As $\varphi$ appears linearly in the operator it can not be stabilised by a simple $Z_2$. We examine the issue of stability in Sect. \ref{s:5.3} for specific models, but note here that an enlarged symmetry could accommodate this operator and stabilise $\varphi$. Alternatively, $\varphi$ could be an unstable hidden sector state which decays to the DM. At present we use this example as a simple toy model to illustrate freeze-in via non-renormalisable operators.}

$$\mathcal{L} \supset \frac{1}{\Lambda} \varphi \bar{\psi}_1 \psi_2 \phi ,$$

(5.1)

where $\phi$ is a boson in the thermal bath, $\psi_i$ are bath fermions, and $\Lambda$ is the mass scale at which the effective operator is generated. Throughout this section we shall use $\varphi$ and $\chi$ to denote scalar and fermion hidden sector states, respectively, and use $\phi$ and $\psi$ to indicate scalars and fermions in the thermal bath. Let us suppose, for the time being, that $\phi$ does not develop a VEV. An abundance of $\varphi$ can freeze-in via $2 \rightarrow 2$ scattering processes involving the bath states: $\phi \psi_1 \rightarrow \varphi \psi_2$.

The change in number density $n$ can be described by the Boltzmann equation (see e.g. \cite{247})

$$\dot{n}_\varphi + 3H n_\varphi = \int d\Pi_\varphi d\Pi_\psi_1 d\Pi_\psi_2 d\Pi_\phi (2\pi)^4 \delta^{(4)} (p_\psi_1 + p_\phi - p_\psi_2 - p_\varphi) \times \left[ |\mathcal{M}|^2_{\varphi \psi_1 \rightarrow \varphi \psi_2} f_\varphi f_\psi_1 - |\mathcal{M}|^2_{\varphi \psi_2 \rightarrow \varphi \psi_1} f_\varphi f_\psi_2 \right] ,$$

(5.2)

where $d\Pi_i \equiv \frac{d^3 p_i}{(2\pi)^3} \frac{1}{2E_i}$ and $f_i$ is the distribution function for a given state. We shall assume that the various states are in thermal equilibrium and thus Maxwell-Boltzmann distributed, $f_i \sim e^{-E_i/T_i}$, with the visible sector states $\phi, \psi_i$ distributed with respect to the temperature of the thermal bath $T$, whereas the DM $\varphi$ is part of a cold hidden sector at initial temperature $T_\varphi \simeq 0$. Correspondingly, this implies that $f_\varphi \simeq 0$, and
the initial number density of $\varphi$ is negligible

$$n_\varphi \equiv \frac{g_\varphi}{2\pi^3} \int d^3 p f_\varphi \simeq 0,$$

(5.3)

where $g_\varphi$ is the number of internal degrees of freedom of $\varphi$. Therefore, the latter term in the Boltzmann equation proportional to $f_\varphi$ (the back-reaction) can be neglected. This is the standard picture of the freeze-in scenario, which we shall adopt throughout. Further, we assume here that the portal operator is always sufficiently feeble that it does not bring the hidden sector into thermal equilibrium with the visible sector. We shall examine the specific requirement of this condition in Sect. 5.2.

It follows that the Boltzmann equation can be rewritten as an integral with respect to centre of mass energy as follows [15]

$$\dot{n}_\varphi + 3Hn_\varphi \simeq \frac{3T}{512\pi^6} \int_{m_\varphi^2}^{\infty} ds \ d\Omega \ P_{\varphi \psi_1 \psi_2} \ |\mathcal{M}|^2_{\varphi \psi_1 \rightarrow \varphi \psi_2} \frac{1}{\sqrt{s}} K_1 \left( \frac{\sqrt{s}}{T} \right),$$

(5.4)

where $K_1$ denotes a Bessel function of the second kind and

$$P_{ij} = \frac{1}{2\sqrt{s}} \sqrt{s - (m_i + m_j)^2} \sqrt{s - (m_i - m_j)^2}.$$

(5.5)

In the limit that the particle masses involved in the scattering are negligible compared to the temperature, scattering via the dimension five operator $\frac{1}{\Lambda} \varphi \bar{\psi}_1 \psi_2 \varphi$ is described by a matrix element the form

$$|\mathcal{M}|^2_{\varphi \psi_1 \rightarrow \varphi \psi_2} \sim \frac{s}{\Lambda^2},$$

(5.6)

where $\sqrt{s}$ is the centre of mass energy of the scattering at temperature $T$. Unless otherwise stated, throughout this paper we assume that the masses of the various states are substantially smaller than both $\Lambda$ and the reheat temperature $T_{RH}$. 

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It follows from Eq. (5.4) & (5.6) that the Boltzmann equation reduces to the form \(^{15}\)

\[
\dot{n}_\varphi + 3Hn_\varphi \simeq \frac{T}{512\pi^5\Lambda^2} \int_0^\infty ds \frac{s^{3/2}K_1(\sqrt{s}/T)}{16\pi^5\Lambda^2} \simeq \frac{T^6}{16\pi^5\Lambda^2} .
\]  

(5.7)

Using the relation \(\dot{T} = -HT\) \(^{247}\), this can be re-expressing in terms of the yield \(Y \equiv \frac{n}{S}\) (where \(S\) is the entropy density) to obtain

\[
\frac{dY_{UV}}{dT} \simeq -\frac{1}{SHT} \frac{T^6}{16\pi^5\Lambda^2} \simeq -\frac{45M_{Pl}}{1.66 \times 32\pi^7g_{*}^{s} \sqrt{g_{*}^{s}} \Lambda^2} ,
\]  

(5.8)

in terms of the effective number of degrees of freedom in the bath \(g_{*}^{S,\rho}\) \(^{247}\). Using

the definitions \(S = \frac{2\pi^2g_{*}^{S}T^3}{45}\) and \(H = \frac{1.66\sqrt{g_{*}^{s}T^2}}{M_{Pl}}\), for \(M_{Pl}\) the (non-reduced) Planck mass, then integrating with respect to temperature (between \(T = 0\) and \(T = T_{RH}\)) gives \(^{15}\)

\[
Y_{UV} \simeq \frac{180}{1.66 \times (2\pi)^7g_{*}^{s} \sqrt{g_{*}^{s}} \Lambda^2} \left(\frac{T_{RH}M_{Pl}}{\Lambda^2}\right) .
\]  

(5.9)

The important thing to note is that the yield depends on the reheat temperature of

the visible sector. This is in contrast to the case of freeze-in via renormalisable inter-

actions, in which the yield only depends on the coupling \(\lambda\) and particle masses \(^{15}\).

As we reproduce in Appendix \(F\) the DM yield due to IR freeze-in is parametrically

\[
Y_{IR} \sim \lambda^2 M_{Pl}/m_{DM} .
\]  

(5.10)

With the above example in mind, we extend this analysis to consider a range of
effective operators of varying mass dimension and involving different combinations of
fields. It is important to recognise that operators of large mass dimension typically
lead to many-body final states. Indeed, as we examine below, even the simplest

\(^{3}\text{Neglecting the mass in the lower limit of the integral leads to only percent-level deviations in the result.}\)
extension of the previous example to dimension five operators $\frac{1}{\Lambda} \phi_1 \phi_2 \phi_3 \phi_4 \phi$ involving four bath scalars $\phi_i$ and scalar DM $\varphi$, which allows freeze-in production via scattering $\phi_1 \phi_2 \to \phi_3 \phi_4 \varphi$, leads to a 3-body phase space.

The Boltzmann equation describing DM production via $2 \to 3$ scattering is given by

$$\dot{n}_\varphi + 3Hn_\varphi = 6 \int d\Pi_1 d\Pi_2 f_1 f_2 |M|_{2\to3}^2 \text{DLIPS}_3, \quad (5.11)$$

where DLIPS$_3$ denotes the differential Lorentz invariant phase space for 3-body final states and the numerical prefactor accounts for permutations of initial and final states. An evaluation of the 3-body phase space (see Appendix G) allows the Boltzmann equation to be rewritten in a form reminiscent of Eq. (5.7)

$$\dot{n}_\varphi + 3Hn_\varphi = \frac{6T}{(4\pi)^7} \int_0^\infty ds \frac{s^{3/2}}{2} |M|_{2\to3}^2 K_1 \left( \frac{\sqrt{s}}{T} \right). \quad (5.12)$$

We have assumed here that the final state masses can be neglected. By dimensional analysis the associated matrix element is parametrically

$$|M|_{2\to3}^2 \sim \frac{1}{\Lambda^2}. \quad (5.13)$$

Substituting this into the Boltzmann equation and expressing our result in terms of the yield we obtain

$$\frac{dY_\varphi}{dT} \simeq -\frac{1}{SHT} \frac{6T}{(4\pi)^7} \frac{1}{\Lambda^2} \int_0^\infty ds s^{3/2} K_1 \left( \frac{\sqrt{s}}{T} \right). \quad (5.14)$$

Performing the integrals over $s$ and, subsequently, temperature we find the form of the DM yield

$$Y_\varphi \simeq \frac{135}{1.66 \times (2\pi)^9 g_s^3 \sqrt{g_s}} \left( \frac{T_{RH} M_{Pl}}{\Lambda^2} \right). \quad (5.15)$$

Up to a numerical suppression of $\sim 10^{-2}$, this is similar in form to Eq. (5.9) and,
notably, also depends linearly on the reheat temperature.

The DM yield may be related to the relic density as follows

$$\Omega_\phi = \frac{m_\phi Y_\phi S_0}{\rho_c} \simeq 0.2 \times \left( \frac{m_\phi}{1 \text{ TeV}} \right) \left( \frac{Y_\phi}{10^{-13}} \right), \quad (5.16)$$

where \( \rho_c \) denotes the critical density and \( S_0 \) is the present day entropy density, evaluated at \( T_0 = 2.75 \text{ K} \sim 10^{-4} \text{ eV} \). In the latter expression we have approximated \( \sqrt{g_s^* g^{*}} \sim 10^3 \). We can choose judicious parameter values such that the observed relic density \( (\Omega_\phi h^2 \approx 0.1) \) is obtained for a given value of the DM mass. For example, choosing a canonical DM mass of 1 TeV, Eq. (5.15) can be rewritten

$$Y_\phi \simeq 10^{-13} \times \left( \frac{T_{\text{RH}}}{3 \times 10^8 \text{ GeV}} \right) \left( \frac{10^{16} \text{ GeV}}{\Lambda} \right)^2. \quad (5.17)$$

It should be noted that in non-minimal models the DM produced via freeze-in might be able to subsequently annihilate. This would introduce further terms in the Boltzmann equation. If there are additional hidden sector interactions, and light hidden sector states into which the DM can annihilate, then this can give rise to a period of annihilation and freeze-out in the hidden sector, which may dilute the DM relic density. To maintain a degree of predictability, throughout we shall assume that there are no such additional hidden sector interactions which can lead to DM pair annihilation. On the other hand, if the DM only interacts via the suppressed portal operator then, as the DM never enters thermal equilibrium, the rate of annihilation back to the visible sector is always negligible compared to the rate of production. In some sense the DM is immediately frozen-out on production.

5.1.2 High dimension operators with many-body final states

We would like to understand how this generalises to operators of increasing mass dimension. For UV freeze-in involving an operator of mass dimension \( n + 4 \), the
cut-off enters in the denominator of the yield as \( \Lambda^{2n} \). Thus, still assuming that none of the fields involved acquire non-zero VEVs, the generic expectation is that the DM yield \( Y_{(n)} \) due to this operator should scale as follows

\[
Y_{(n)} \sim \frac{M_{Pl} T_{RH}^{2n-1}}{\Lambda^{2n}},
\]

the factor of \( M_{Pl} \) coming from the Hubble parameter. One important issue which arises, however, is that the phase space becomes increasingly large and complicated. Here we shall make certain assumptions and approximations to obtain an order-of-magnitude estimate of the yield.

Consider the dimension-\((n + 4)\) operator \( \frac{1}{\Lambda^n} \phi_1 \phi_2 \cdots \phi_{n+3} \phi \) which corresponds to \((n+2)\)-body final state phase space for scattering events \( \phi_1 \phi_2 \rightarrow \phi_3 \cdots \phi_{n+3} \phi \). Variant operators might be considered but this toy example should be illustrative of a more general issue regarding the interplay between mass dimension and phase space. For simplicity here we assume that there is only a single relevant high dimension operator; the converse scenario would imply a sum over the various operators in the Boltzmann equation.

The Boltzmann equation describing DM production via \( 2 \rightarrow n + 2 \) scattering is given by

\[
\dot{n}_\phi + 3Hn_\phi = \int d\Pi_1 d\Pi_2 f_1 f_2 |M|_{(n)}^2 \text{DLIPS}_{(n+2)}.
\]

The differential phase space grows like \( \text{DLIPS}_{(n+2)} \propto s^n \) and we make the approximation

\[
\text{DLIPS}_{(n+2)} \sim \left[ \frac{s}{4\pi^2} \right]^n \text{DLIPS}_{(2)}.
\]

The square bracket provides a parametric estimate of the additional phase space suppression due to the \((n + 2)\)-body final state. This is a somewhat crude approx-

\[\text{We neglect here permutations of initial states. If included this leads to a combinatorial enhancement, but as the DM abundance is highly sensitive to } \Lambda, \text{ for } n > 1 \text{ this is of lesser importance.}\]
imation, but for low $n$ should give an order-of-magnitude estimate. For increasing $n$ the suppression to the cross section coming from the phase space should become more severe and strongly suppress operators of high ($n \gg 1$) mass dimension. From dimensional analysis

$$|\mathcal{M}|_{(n)}^2 \sim \left( \frac{1}{\Lambda^2} \right)^n,$$

and thus the Boltzmann equation can be expressed

$$\dot{n}_\phi + 3H n_\phi \simeq \frac{2T}{(4\pi)^5 \Lambda^{2n}} \left[ \frac{1}{4\pi^2} \right]^n \int_0^\infty ds \ s^{(2n+1)/2} K_1(\sqrt{s}/T).$$

(5.22)

We will check the resulting estimate against the 3-body result calculated in Sect. 5.1.1.

The integral over $s$ has a closed form expression which for $n \in \mathbb{N}$ is given by

$$\int_0^\infty ds \ s^{(2n+1)/2} K_1(\sqrt{s}/T) = 4^{n+1} T^{2n+3} n!(n+1)!.$$

(5.23)

Using this result the Boltzmann equation can be rewritten

$$\dot{n}_\phi + 3H n_\phi \simeq \frac{1}{(2\pi)^7} \left( \frac{n!(n+1)!}{\pi^{2n-2}} \right) \frac{T^{2n+4}}{\Lambda^{2n}}.$$

(5.24)

From the above we obtain an expression for the DM yield

$$Y_{(n)} \simeq \frac{90}{1.66 \times (2\pi)^9 \sqrt{g_* g_\gamma^5}} \left( \frac{n!(n+1)!}{\pi^{2n-1}} \right) \int_0^{T_{\text{RH}}} dT \ \frac{M_{\text{Pl}} T^{2n-2}}{\Lambda^{2n}} \frac{1}{2n-1} \left( \frac{n!(n+1)!}{\pi^{2n-1}} \right) \left( \frac{M_{\text{Pl}} T_{\text{RH}}^{2n-1}}{\Lambda^{2n}} \right).$$

(5.25)

The form of Eq. (5.25) conforms with our expectations for the parametric scaling discussed in Eq. (5.18). Moreover, this shows that a range of operators of varying mass dimension should be able to reproduce the observed relic density. We consider some examples below.
Firstly, we can check this result against our 3-body calculation by setting $n = 1$

$$Y_{(1)} \approx \frac{360}{1.66 \times (2\pi)^{10}} \frac{1}{\sqrt{g_s^2 g_b^2}} \frac{M_{Pl} T_{RH}}{\Lambda^2}.$$  \hspace{1cm} (5.26)

Comparing with Eq. (5.15), we find that these two expressions agree up to an $\mathcal{O}(1)$ factor. For suitable parameter values the observed relic density can be reproduced for a large range of DM masses, comparing with Eq. (5.16). As a concrete example, consider a model with 1 TeV DM, a yield of appropriate magnitude is found for

$$Y_{(1)} \approx 10^{-13} \times \left( \frac{T_{RH}}{3 \times 10^8 \text{ GeV}} \right) \left( \frac{10^{16} \text{ GeV}}{\Lambda} \right)^2.$$  \hspace{1cm} (5.27)

Observe, unlike typical models of freeze-out and IR freeze-in, the yield is independent of $m_\phi$, the DM mass, provided $m_\phi \ll T \ll \Lambda$. Thus in UV freeze-in one can find the observed DM relic density for different values of the DM mass by simply rescaling the yield.

Taking a few further examples, consider dimension-six ($n = 2$) and dimension-seven ($n = 3$) operators (with 4 and 5 body final states, respectively), the estimates for the freeze-in yield of $\phi$ for these cases are

$$Y_{(2)} \approx 10^{-13} \left( \frac{T_{RH}}{5 \times 10^{13} \text{ GeV}} \right)^3 \left( \frac{10^{16} \text{ GeV}}{\Lambda} \right)^4,$$

$$Y_{(3)} \approx 10^{-13} \left( \frac{T_{RH}}{4 \times 10^{14} \text{ GeV}} \right)^5 \left( \frac{10^{16} \text{ GeV}}{\Lambda} \right)^6.$$  \hspace{1cm} (5.28)

Thus for a given operator and fixed UV scale, the observed DM abundance can typically be obtained by adjusting the reheat temperature. We shall comment in Sect. 5.3.3 on motivated values of $T_{RH}$. Moreover, we learn that in the presence of multiple high dimension portal operators of varying mass dimension (but common $\Lambda$), generally the physics will be determined by the operator(s) with smallest mass dimension. This conforms with the standard intuition regarding effective field theory.
5.1.3 VEV expansions of high dimension operators

If any of the fields involved in the high dimension operator acquire VEVs then, by expanding the operator around the vacuum, one can construct a sequence of terms dressed by couplings of different mass dimension \[15\]. Let us again examine the simple example given in \[15\], involving the dimension-five operator in Eq. (5.1). Suppose that the bath scalar field has a non-zero VEV \(\langle \phi \rangle \neq 0\), expanding around this VEV gives

\[
L \supset \lambda \bar{\psi}_1 \psi_2 \phi + \frac{1}{\Lambda} \bar{\phi} \psi_1 \psi_2 \phi ,
\]

(5.29)

where we have identified \(\lambda \equiv \frac{\langle \phi \rangle}{\Lambda}\). To ensure validity of the effective field theory we will assume that \(\langle \phi \rangle \ll \Lambda\) and therefore \(\lambda \ll 1\). In this example the yield will receive both an IR contribution from the first term (which appears as an operator with a dimensionless coupling after symmetry breaking) and a UV contribution from the latter term. The IR contribution is assumed to be generate by decays \(\psi_1 \rightarrow \psi_2 \phi\), this is reproduced in Eq. (F.6) of the Appendix. By calculating the two contributions, it can be shown \[15\] that the yield is dominated by the IR contribution if

\[
\frac{3\pi^3 \langle \phi \rangle^2}{T_{\text{RH}} m_{\psi_1}} > 1 .
\]

(5.30)

It is notable that this condition does not depend on \(\Lambda\).

Let \(T_*\) be the critical temperature associated to the spontaneous breaking of some symmetry, due to a scalar field involved in the UV freeze-in operator developing a VEV. It should be expected that \(T_* \sim \langle \phi \rangle\). The VEV expansion of Eq. (5.29) is only valid for \(T < T_*\), but as the yield due to IR operators is temperature independent, it will not depend on the temperature \(T_*\) at which the IR operator is generated. Thus it is not required that \(T_{\text{RH}} < T_*\) in this case.

The situation is more complicated if the VEV expansion leads to additional UV freeze-in operators. For \(T_* < T_{\text{RH}}\), one can find UV contributions which depend on
rather than $T_{\text{RH}}$. This is because for temperatures $T > T_\ast$ there is no VEV expansion until thermal evolution (due to expansion) causes $T$ to drop below $T_\ast$. We shall illustrate this with an example below. Importantly, if the phase transition happens after the point at which DM freeze-in terminates, i.e. below the mass of the bath states involved in the freeze-in process ($T_\ast < m_{\text{bath}}$), then no further (UV or IR) contributions will be generated.

Let us consider an example in which the VEV insertion does not lead to operators with dimensionless coefficients, but results in several UV contributions. One manner of realising this scenario is by dressing the SM Yukawa operators with a pair of DM states

$$L \supset \frac{1}{\Lambda^3} H ar{Q} u^c \chi \chi \rightarrow \frac{v}{\Lambda^3} \bar{u} u \chi \chi + \frac{1}{\Lambda^3} h \bar{u} u \chi \chi.$$  

Note that the DM can be stabilised by the assumption of $\chi$-parity, such that the Lagrangian is invariant under $\chi \rightarrow - \chi$. After electroweak symmetry breaking (EWSB) one makes the appropriate expansion around the vacuum to obtain the four fermion operator. Henceforth we shall call $O_1$ the operator dressed by $v/\Lambda^3$ and the latter term $O_2$.

As one might anticipate the VEV expansion leads to two operators which provide UV contributions to the yield. The operator $O_2$ leads to freeze-in via $2 \rightarrow 3$ scattering processes such as $hq \rightarrow q\bar{\chi} \chi$. From dimensional analysis the matrix element is of the form

$$|\mathcal{M}|^2_{O_2} \sim \frac{s^2}{\Lambda^6}.$$  

As previously, we can describe the production of DM through the Boltzmann equation given in Eq. (5.12). It follows that the DM yield due to $O_2$ is of the form

$$Y_{O_2} = \frac{9}{8\pi^5} \frac{45}{1.66 \sqrt{g_s^2 g_s^2}} \left( \frac{T_{\text{RH}}^5 M_{\text{Pl}}}{\Lambda^6} \right).$$  

(5.33)
Similarly, the operator $\mathcal{O}_1$ results in DM production via $2 \rightarrow 2$ scattering $\bar{q}q \rightarrow \bar{\chi}\chi$

$$|\mathcal{M}|^2_{\mathcal{O}_1} \sim \langle H \rangle^2 \left( \frac{s^2}{\Lambda^6} \right). \quad (5.34)$$

The relevant Boltzmann equation is analogous to Eq. (5.7)

$$\dot{n}_\phi + 3Hn_\phi \simeq \frac{(H)^2 T}{512 \pi^5 \Lambda^6} \int_0^\infty ds \ s^{5/2} K_1(\sqrt{s}/T) \simeq \frac{3(H)^2 T^8}{2\pi^5 \Lambda^6}. \quad (5.35)$$

Hence the yield is given by

$$Y_{\mathcal{O}_1} \simeq \frac{3 \times 45}{1.66 \times 4\pi^7} \frac{1}{\sqrt{g_e g_\nu}} \left( \frac{M_{Pl}}{\Lambda^6} \right) \int_0^{T_{\text{max}}} \langle H \rangle^2 T^2 \ dT. \quad (5.36)$$

The maximum temperature $T_{\text{max}}$, which is the upper limit of the integral, depends on whether the reheating temperature is above or below the critical temperature $T_*$ at which the Higgs develops a VEV and can be expressed as $T_{\text{max}} = \min(T_*, T_{RH})$. Thus there are two possible cases, which we examine below, depending on whether the reheat temperature is above or below the phase transition. As the operator in Eq. (5.31) involves the Higgs VEV, the critical temperature is around $T_* \sim 100$ GeV, associated to the electroweak phase transition (EWPT).

Provided $T_* > T_{RH}$ the VEV expansion is valid, and thus the portal operator $\mathcal{O}_1$ is active, for all physically relevant temperatures. Therefore $T_{\text{max}} = T_{RH}$, and identifying $\langle H \rangle = v$, the ratio of the two contributions is

$$\frac{Y_{\mathcal{O}_2}}{Y_{\mathcal{O}_1}} = \frac{9}{2\pi^2} \left( \frac{T_{RH}^5}{T_{\text{max}}^3 v^2} \right) = \frac{9}{2\pi^2} \left( \frac{T_{RH}}{v} \right)^2. \quad (5.37)$$

For VEV expansions which do not lead to dimensionless couplings, because $T_* \sim v$ one expects $Y_{\mathcal{O}_2}/Y_{\mathcal{O}_1} < 1$ for $T_* > T_{RH}$. More generally, if $T_* > T_{RH}$ the expectation is that the contribution coming from the operator in the VEV expansion dressed by
the coefficient with smallest (negative) mass dimension will dominate the yield.

For $T_* < T_{RH}$ the phase transition takes place during the cooling of the thermal bath, in this case production via the VEV expanded operator only occurs for $T < T_*$ and the dominant contribution will be generated at $T \sim T_*$. Further, thermal fluctuations may be important in determining the field expectation value and it is expected\footnote{A more careful study of these thermal effects would be of interest, but it is beyond the scope of this work.} that $\langle H \rangle \sim T$. Evaluating Eq. (5.36), the ratio of the contributions coming from Eq. (5.31) in this case is instead

$$\frac{Y_{O_2}}{Y_{O_1}} \simeq \frac{15}{2\pi^2} \left( \frac{T_{RH}}{T_*} \right)^5.$$  \hfill (5.38)

Since by assumption $v < T_{RH}$, the operator $O_2$ is typically dominant. More generally, we expect that for alternative operators typically the term with no explicit VEVs in the expansion around the vacuum will provide the most significant contribution to the yield. In the case that the VEV expansion generates a dimensionless coupling, the associated yield is independent of $T_{RH}$ and $T_*$, and the criteria under which this operator dominates will be described by an equation analogous to Eq. (5.30).

For scattering processes $hq \rightarrow q\bar{\chi}\chi$ to occur in the thermal bath it is required that the SM states can be thermally produced. This implies that $T_{RH} \gtrsim 100$ GeV if $H$ is the SM Higgs. In the case that the Higgs is thermally produced, this pushes the model into the regime in which the operator $O_2$ always gives the dominate contribution. In the converse scenario that the Higgs is not thermally produced then $O_2$ will be exponentially suppressed, but $O_1$ can still potentially lead to DM production.

Further complications can arise if the VEV expansion gives several terms, and thus multiple contributions to the DM yield, or if there are multiple scalar fields, especially if the scalars develop VEVs due to spontaneous symmetry breaking taking place at different scales. We shall leave these more complicated possibilities until
they arise in motivated examples.

5.2 Sector Equilibration Constraints

For the DM relic density to be established by freeze-in production (rather than freeze-out) it is imperative that the DM is not brought into thermal equilibrium with the visible sector due to its interactions through the portal operator. For the case of IR freeze-in via renormalisable interactions, for instance due to $\lambda \bar{\psi}_1 \psi_2 \phi$, it has been argued that the hidden sector should not thermalise with the visible sector, provided $\lambda \lesssim 10^{-6} \sqrt{m_{\text{bath}}/100 \text{ GeV}}$.

In this section we derive an analogous condition for the case that the portal is due to a non-renormalisable operator, leading to UV freeze-in.

It will be useful to introduce the freeze-out temperature $T_{\text{FO}}$, the temperature at which a state in thermal equilibrium decouples from the thermal bath, defined such that at $T = T_{\text{FO}}$

$$n\langle \sigma v \rangle = H.$$  \hspace{1cm} (5.39)

The requirement that the DM is always out of thermal equilibrium is equivalent to $Y < Y^{eq}$. This implies two conditions:

- For $T_{\text{RH}} > T > m_{\text{DM}}$, the bath number density is $n^{eq} \simeq T^3/\pi^2$ and thus the DM number density is non-equilibrium provided $n_{\text{DM}} \ll T^3/\pi^2$.

- For $m_{\text{DM}} > T$, the equilibrium number density is Boltzmann suppressed and thus to avoid $n_{\text{DM}}$ coinciding with $n^{eq}$ it is required that DM freeze-out occurs for $T_{\text{FO}} \gg m_{\text{DM}}$.

The UV operator freezes-in a DM abundance at $T_{\text{RH}}$, which is effectively frozen-out. Clearly, an upper bound is given by the scenario in which a near thermal abundance

\footnote{In the case that a VEV expansion gives a renormalisable operator, then identifying $\lambda = \langle \phi_1 \rangle \cdots \langle \phi_n \rangle / \Lambda^n$ one should apply this IR constraint, in addition to requirements on high dimension operators in this section.}
is generated in the early universe, subsequently decouples, and then evolves to the present modified only by entropy conservation. Including the entropy factor, this gives (for a real scalar DM particle)

\[ n_{DM}(T_0) = \left( \frac{s_0}{s(T_{RH})} \right) n_{DM}(T_{RH}) = \left( \frac{g^S_\star(T_0)}{g^S_\star(T_{RH})} \right) \left( \frac{1.2 T_0^3}{\pi^2} \right). \]

Further, we use that the DM relic density is given by

\[ \Omega_{DM} = \frac{n_{DM}(T_0) m_{DM}}{\rho_c}. \]

Comparing with the observed value \( \Omega_{DM} \approx 0.2 \), this gives a bound on the DM mass

\[ m_{DM} \gtrsim \frac{g(T_{RH}) \pi^2 \rho_c \Omega_{DM}}{g(T_0)} \frac{T_0^3}{1.2 T_0^3} \approx 0.4 \text{ keV}, \quad (5.40) \]

where we have used that \( g(T_0)/g(T_{RH}) \approx 3.36/100 \). Thus this places a model independent bound on the DM mass.

Next we examine the second requirement: \( T_{FO} \gg m_{DM} \). The freeze-out temperature can be found by solving Eq. (5.39), which we expand below

\[ \langle \sigma v \rangle \left( \frac{T_{FO}^3}{\pi^2} \right) = H(T_{FO}) \approx 1.66 \sqrt{g_\star} \frac{T_{FO}^3}{M_{Pl}}. \quad (5.41) \]

Thus the requirement that freeze-out occurs before the mass threshold \( m_{DM} \) is given by

\[ T_{FO} \approx \frac{1.66 \sqrt{g_\star} \pi^2}{M_{Pl} \langle \sigma v \rangle} \gg m_{DM}. \quad (5.42) \]

For a specific portal operator this can be re-expressed in terms of the UV scale \( \Lambda \) at which the operator is generated. Let us consider an example. Recall the Lagrangian term studied previously for UV freeze-in by \( 2 \rightarrow 2 \) scattering: \( \mathcal{L} \supset \frac{1}{\Lambda} \varphi \bar{\psi}_1 \psi_2 \phi \). In this
case the matrix element is given by Eq. (5.6) and the associated cross section is

\[ \langle \sigma v \rangle \sim \left( \frac{1}{4\pi} \right)^3 \frac{1}{\Lambda^2}. \]  

(5.43)

It follows that the requirement Eq. (5.42) can be expressed for this operator as below

\[ \Lambda \gg \left( \frac{m_{DM}M_{Pl}}{1.66 \times 2^{\alpha_5} \sqrt{g_\star^a}} \right)^{1/2} \simeq 10^7 \text{ GeV} \left( \frac{m_{DM}}{0.4 \text{ keV}} \right)^{1/2}. \]  

(5.44)

Moreover, when combined with the constraint of Eq. (5.40), this implies an extreme lower bound on the scale of new physics of 10^7 GeV, as indicated above.

In Fig. 5.2 we display the constraints on the parameter space of various models. In particular we look at the UV freeze-in portals considered in Sect. 5.1.2 & 5.1.3. Using the derived forms of the yields, we present contours of \( \Lambda \) and \( T_{RH} \) which give the observed relic density \( \Omega_{DM}h^2 \approx 0.1 \) for a range of DM masses. We overlay these contour plots with the relevant constraints. Towards the lower right of the parameter space in each plot the yields are low and require increasingly larger DM masses in order to reproduce the observed relic density. In certain regions of parameter space the DM mass which would be required is larger than the \( T_{RH} \) and thus such models can not give the correct DM abundance; this is indicated by the grey shaded regions in Fig. 5.2. The purple shaded regions indicate parameter values which lead to sector equilibration, as given in Eq. (5.40), the DM enters thermal equilibrium \( Y = Y_{\text{eq}} \), thus the abundance will not be set via the freeze-in mechanism. In the red highlighted region \( T_{RH} > \Lambda \), the effective field theory breaks down, and the dark matter abundance is not set by UV freeze-in. Note also that because the DM is never in thermal equilibrium the unitarity bound of freeze-out DM [99] does not apply to freeze-in DM [15] and thus there is no upper bound on the DM mass.
Figure 5.2. We consider operators $\frac{1}{\Lambda^n} \phi_1 \phi_2 \cdots \phi_{n+3} \varphi$, as in Sect. 5.1.2 (with $n = 1, 2, 3$), for DM with mass 1 PeV (solid), 1 TeV (dashed), 1 GeV (dotted), and 1 MeV (dot-dashed). Contours indicating values of $T_{RH}$ and $\Lambda$ appropriate to match the observed DM relic density via UV freeze-in are shown, following Eq. (5.15), (5.16) & (5.25). The grey shaded areas indicate regions of parameter space in which the observed relic density can not be obtained. The parameter space highlighted in red shows where the effective theory breaks down and thus the UV freeze-in picture is not valid. The requirement $Y_{DM} < Y^{eq}$, that the DM does not come in to thermal equilibrium, equivalent to $m_{DM} > 0.4$ keV, is shown in orange.

5.3 UV Freeze-In and BSM Physics

Having discussed a range of possibilities in the context of toy models, we now turn to constructing explicit models based on extensions of the SM, motivated by outstanding problems. These BSM scenarios generally require additional states to be introduced at new physical scales above the weak scale. One interesting possibility is that the operator(s) responsible for UV freeze-in are generate at this scale of new
physics. We shall also discuss motivated BSM models which lead to high dimension operators with VEV expansions at temperatures above the weak scale. It was remarked in [15] that the high dimension operator responsible for UV freeze-in might arise from GUT scale physics, here we examine some alternative scenarios.

5.3.1 $Z'$ portal

First we consider the scenario in which the SM gauge group is extended by an additional U(1) gauge symmetry, which is broken at some high scale $\Lambda$ (for a general review see e.g. [248])

$$SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)'. \quad (5.45)$$

If some visible sector states and the DM are both charged under this new gauge symmetry, then the associated massive gauge boson can provide a portal that links these two sectors.

Additional U(1) gauge groups are a generic expectation of string theory compactifications, see e.g. [249, 250], as supported by scans of vacua of Heterotic string theory. Further, GUTs based on $E_6$ or SO(10) can introduce extra U(1) factors from the breaking of these larger groups [251]

$$E_6 \rightarrow SO(10) \times U(1) \rightarrow SU(5) \times U(1)'. \quad (5.46)$$

In type IIB theories extra U(1)'s can arise from isolated branes; moreover, brane stacks are associated to symmetry groups $U(N) \sim SU(N) \times U(1)$, where the U(1) factor is (pseudo)-anomalous. This U(1) anomaly is cancelled via the Green-Schwarz mechanism, and as a result the $Z'$ acquires a mass near the string scale. It is interesting to note that in type IIB theories the string scale can be lowered substantially compared to the Planck mass if the moduli are stabilised at LARGE volume [252].
Alternatively, from an IR perspective, it is conceivable that a global quantum number of the SM model might be gauged. In the SM baryon number $B$ and lepton number $L$ appear as accidental symmetries and are typically broken in extensions of the SM, for instance GUTs. However, it is possible that some global quantum number of the SM may arise from an exact gauged symmetry. An appealing possibility is that the combination $B - L$ is gauged as this is anomaly free provided the spectrum includes right-handed neutrinos. If one assumes that DM is charged under $B - L$, then the $U(1)_{B-L}$ gauge boson can provide a portal operator which connects the DM and the SM fermions. If $B - L$ is gauged, typically it must be broken at a high scale to give masses to the neutrinos via the seesaw mechanism. Once the $Z'$ is integrated out this generates effective operators which connect the SM fermions and the DM, suppressed by the (intermediate) scale at which $U(1)_{B-L}$ is broken. This can potentially lead to UV freeze-in for appropriate parameter choices.

More generally, suppose that the DM $\chi$, $\bar{\chi}$ and the SM fermions are charged under some new group $U(1)’$. It follows that the SM states can pair-annihilate and produce DM states via $\bar{q}q \rightarrow Z’ \rightarrow \bar{\chi}\chi$. In the UV, as usual, interactions mediated by the (heavy) $Z’$ appear in the Lagrangian through the covariant derivative in the gauge invariant kinetic terms

$$ L \supset i\bar{Q}\gamma^\mu Q + i\bar{u}\gamma^\mu u + i\bar{\chi}\gamma^\mu \chi + \cdots, \quad (5.47) $$

where $\gamma = \phi + iy’Z’ + \cdots$, the ellipsis denote the gauge fields of the SM and $y’$ is the $U(1)’$ charge. Once the $Z’$ is integrated out this leads to four-fermion interactions, suppressed by $\Lambda = y’\chi’/m_{Z}$, in the effective Lagrangian of the form

$$ \mathcal{L}_{\text{eff}} \supset \frac{1}{\Lambda^2} Q\gamma^\mu Q\bar{\chi}\gamma^\mu \chi + \frac{1}{\Lambda^2} \bar{u}c\gamma^\mu u^c\bar{\chi}\gamma^\mu \chi + \cdots. \quad (5.48) $$

This is similar to $O_1$ studied in Eq. (5.31), however the prefactor is different, as is the
Lorentz structure, as here we have integrated out a Lorentz vector. DM production will proceed fairly analogously and the DM yield is given by (cf. Eq. (5.25))

\[ Y_2 \approx \frac{45}{1.66 \times 2^6 \pi^{12}} \sqrt{g_s g_*} \frac{M_{Pl} T_{RH}^3}{\Lambda^4}. \]  

(5.49)

It should be noted that the presence of an additional U(1) gauge group will generically lead to kinetic mixing via operators of the form \( F_{\mu\nu} F'_{\mu\nu} \). Such interactions can provide a renormalisable portal operator between the visible and hidden sectors. As we are primarily concerned here with UV freeze-in, we shall assume that such operators are negligible. This kinetic-mixing portal has been previously studied in the context of IR freeze-in [241]. Discussion on UV freeze-in via \( Z' \) also appear in [1].

5.3.2 The axion portal

We shall next consider a realisation of the simple toy model of UV freeze-in originally considered in [15], and discussed here in Eq. (5.29), based on the axion solution to the strong CP problem. This will provide an example in which a VEV expansion introduces additional freeze-in portals. It is widely thought that the most viable solution to the strong CP problem is the Peccei-Quinn (PQ) mechanism, which dynamically sets the \( \bar{\theta} \)-parameter to zero [253]. Such ‘axion portals’ have been contemplated previously within the context of DM freeze-out, e.g. [254]. We shall take a DFSZ-type model, where a type II two Higgs doublet is supplemented with an additional SM singlet scalar \( S \) transforming under the PQ symmetry

\[ S \to e^{2i\beta} S, \quad H_u \to e^{2i\beta} H_u, \quad H_d \to e^{-2i\beta} H_d. \]  

(5.50)

The SM fermions transform as \( q_L \to e^{i\beta} q_L \) and \( q_R \to e^{-i\beta} q_R \). We further supplement this model with SM singlet Weyl fermions \( \chi, \bar{\chi} \) which transform with equal charges
under the PQ symmetry\footnote{It is useful for our purposes that the states are Dirac as a Majorana fermion $\chi'$ would necessarily be a singlet under the PQ symmetry and thus, in the absence of additional symmetries (e.g. lepton number), this would allow the renormalisable operator $L H_u \chi'$.}. The state $\chi$ is the DM candidate, it can be stabilised by a $Z_2$ $\chi$-parity. Potentially, a stabilising discrete symmetry might arise as a subgroup of the PQ symmetry. This might be considered as a ‘toy’ setting, as we shall not confront the various naturalness problems \cite{255,256} which arise in such axion models, but it will illustrate the general principle.

In addition to the Yukawa couplings we can build the following Lorentz, gauge, and PQ invariant combinations of these fields:

\[ H_u^\dagger H_u, \quad H_d^\dagger H_d, \quad S^\dagger S, \quad H_u^\dagger H_d S^2, \quad \chi \bar{\chi}. \quad (5.51) \]

These field combinations allow us to construct the following Lagrangian terms involving the DM bilinear

\[ \mathcal{L} \supset \frac{1}{\Lambda} S^\dagger S \chi \bar{\chi} + M_\chi \chi \bar{\chi} + \text{h.c.} + \cdots. \quad (5.52) \]

The scalar field $S$ develops a non-vanishing VEV $v_a$ at scale $f_a = 2v_a \gg m_Z$, which preserves electroweak symmetry, but spontaneously breaks the PQ symmetry.

Let us assume the following mass hierarch:

\[ m_Z \ll m_\sigma \lesssim f_a < T_{RH} < \Lambda. \quad (5.53) \]

For $T_{RH} > f_a$, UV freeze-in can proceed via scattering $SS^\dagger \rightarrow \chi \bar{\chi}$. The matrix element for this process is $|\mathcal{M}|^2 \sim s/\Lambda^2$, which is similar in form to Eq. (5.6). Thus the UV contribution to the yield is (up to $\mathcal{O}(1)$ factors) as Eq. (5.9)

\[ Y_{UV} \sim \frac{360}{1.66 \times (2\pi)^7 g_s^2 \sqrt{g_s^*}} \left( \frac{T_{RH} M_{Pl}}{\Lambda^2} \right). \quad (5.54) \]
There is a similar $T_{RH}$-dependent contributions to the yield from the operators $H_i^\dagger H_i \bar{\chi}^c$ for ($i = u,d$) which has the same form to that given above. The correct relic density is found for $\Lambda \simeq 10^{9} \sqrt{m_\chi T_{RH}}$. A further model-dependent limit comes from the assumed mass hierarchy Eq. (5.53)

$$m_\chi \simeq \frac{\Lambda^2}{10^{18} T_{RH}} < \frac{\Lambda^2}{10^{18} m_\sigma}.$$  \hspace{1cm} (5.55)

As we expect from the Lagrangian that $m_\chi \sim f_a^2 / \Lambda$ and $m_\sigma \sim f_a$, this implies the following consistency constraint on the hierarchy of scales: $f_a \lesssim \Lambda / 10^6$. The above requirements can be simultaneously satisfied with reasonable parameter values.

At energies above the EWPT, but after PQ breaking one can re-examine the operators appearing in Eq. (5.52) following a VEV expansion around the vacuum of $S$. The singlet field can be decomposed into radial and axial components

$$S = \left( f_a + \frac{\sigma}{\sqrt{2}} \right) e^{ia / \sqrt{2} f_a}. \hspace{1cm} (5.56)$$

The axial field $a$ is identified with the axion.

Expanding around the VEV of $S$, the radial component $\sigma$ provides the following operators

$$\frac{1}{\Lambda} \chi \bar{\chi}^c S^\dagger S \rightarrow \frac{f_a^2}{\Lambda} \chi \bar{\chi}^c + \frac{f_a}{\Lambda} \sqrt{2} \sigma \chi \bar{\chi}^c + \frac{1}{\Lambda} \frac{\sigma^2}{2} \chi \bar{\chi}^c. \hspace{1cm} (5.57)$$

For $T \gtrsim m_\sigma$ the state $\sigma$ is part of the thermal bath, as it is kept in thermal contact via interactions involving products of the bilinear operators in Eq. (5.51) involving $H_u, H_d$ and $S$ if these have $O(1)$ coefficients.

Once the temperature drops below the PQ breaking scale (but whilst still above $m_\sigma$), freeze-in can proceed via direct decay of heavy $\sigma$ states to DM pairs, leading to an IR contribution to the yield. Comparing with the form of the IR freeze-in yield
given in [15], and reproduced in Eq. (F.6) of Appendix [F] one finds

\[ Y_{\text{IR}} \sim \frac{135}{4\pi^3 g_*^S \sqrt{g_*^s}} \frac{M_{\text{Pl}} \Gamma_{\sigma}}{m_\sigma^2} \sim \frac{135}{4\pi^3 g_*^S \sqrt{g_*^s}} \left( \frac{M_{\text{Pl}} f_a^2}{m_\sigma \Lambda^2} \right), \]

where \( \Gamma_{\sigma} \sim \frac{m_\sigma f_a^2}{\Lambda^2} \) is the partial width of \( \sigma \to \chi \bar{\chi} \). The condition under which the UV contribution will be dominant is

\[ \frac{Y_{\text{UV}}}{Y_{\text{IR}}} \sim \frac{m_\sigma T_{\text{RH}}}{f_a^2} \gtrsim 1. \]

Note also that after EWSB there is a further VEV expansion involving the Higgs fields which leads to the portal operators of the form

\[ \frac{1}{\Lambda} H_i^\dagger H_i \chi \chi^c \to \frac{v_i^2}{\Lambda} \chi \chi^c + \frac{v_i}{\Lambda} \sqrt{2} h_i \chi \chi^c + \frac{1}{2} \frac{h_i^2}{\Lambda} \chi \chi^c. \]

This leads to IR and UV freeze-in contributions to the yield, similar to Eq. (5.54) & (5.58), in the case that \( m_\chi < m_Z \).

It would be of interest to embed the above model into a supersymmetric extension of the SM, similar to e.g. [254]. The advantages of a supersymmetric implementation is that the type II structure of the Higgs sector (required to employ the DFSZ model and to avoid constrained flavour changing processes) is an automatic consequence of holomorphy of the superpotential. In addition this might also alleviate the naturalness problem [255] associated with destabilising the weak scale, and the DM can be stabilised by \( Z_2 \) R-parity if it is the lightest supersymmetric particle.

5.3.3 The reheat temperature

In Sect. 5.3.1 & 5.3.2 we have discussed motivated scales of new physics which might generate the high dimension operators. An interesting alternative to this approach is to consider special values for the reheat temperature \( T_{\text{RH}} \) and use this,
in conjunction with the DM relic density, to identify the unknown UV scale. One drawback of this scenario is that very little is known about the reheat temperature. Precision measurements of primordial elements due to Big Bang nucleosynthesis are thought to imply that \( T_{\text{RH}} \gtrsim \text{few MeV} \). Models of inflation typically suggest an upper bound around \( T_{\text{RH}} \lesssim 10^{16} \text{ GeV} \), see e.g. [257]. Moreover, if \( T_{\text{RH}} \) is high then in principle this can lead to problems with long-lived exotic relics which can over-close the Universe, the classic example being the cosmological gravitino problem of supergravity [258]. This implies the upper bound on the reheat temperature in models of supergravity is typically \( T_{\text{RH}} \lesssim 10^{10} \text{ GeV} \), with substantially stronger bounds if the gravitino is light.\(^8\)

The following extreme cases may be of particular interest:

- The maximum expected from simple models of inflation: \( T_{\text{RH}} \sim 10^{16} \text{ GeV} \).
- The upper bound from the cosmological gravitino problem: \( T_{\text{RH}} \sim 10^{10} \text{ GeV} \).
- The lower bound from precision measurement of primordial elements: \( T_{\text{RH}} \sim 10 \text{ MeV} \).

The first two scenarios might be motivated through considerations of environmental selection if there is some anthropic pressure which favours high \( T_{\text{RH}} \), with the cosmological gravitino problem imposing a catastrophic boundary at \( T_{\text{RH}} \sim 10^{10} \text{ GeV} \) in supersymmetric models.

Let us consider a specific example involving dimension-(\( n+4 \)) operator \( \frac{1}{\Lambda^n} \phi_1 \phi_2 \cdots \phi_{n+3} \varphi \) of the scalar toy model studied in Sect. 5.1.2. For DM with mass around 100 GeV the yield required to obtain the observed relic density is \( Y \sim 4 \times 10^{-12} \), as discussed in Eq. (5.16). To obtain the correct relic abundance via freeze-in through the dimension-five operator with a reheat temperature of \( T_{\text{RH}} \sim 10^{16} \text{ GeV} \), requires a UV scale as

\(^{8}\)It has been further argued [259] that, with some assumptions, once the contribution from axinos is also included this can lead to an even more stringent upper bound: \( T_{\text{RH}} \lesssim 10^5 \text{ GeV} \).
indicated below
\[ Y_{(1)} \simeq 4 \times 10^{-12} \left( \frac{T_{RH}}{10^{16} \text{ GeV}} \right) \left( \frac{M_{Pl}}{\Lambda} \right)^2. \] (5.61)

The magnitude of the UV scale \( \Lambda \) in this example may be suggestive of a connection with Planck Scale physics.

It is not implausible that future observations might indicate the reheat temperature (given some assumptions regarding the model of inflation). Ultimately to test UV freeze-in DM, and disambiguate it from other frameworks, it will be necessary to determine the DM mass, the UV scale \( \Lambda \) and \( T_{RH} \). Recently the BICEP2 collaboration claimed they had observed primordial tensor modes [260], and thus could infer \( T_{RH} \), however this result is currently disputed [261]. If the BICEP2 signal survives further scrutiny we shall comment on this in a dedicated paper.

5.4 Conclusion

This work has provided an exploratory study of the model building opportunities which arise for the UV freeze-in mechanism. We considered general aspects of this scenario in the context of toy models and demonstrated that interesting and phenomenologically viable models can be constructed in motivated settings of BSM physics. In Sect. [5.1] we examined various toy models which encapsulate the fundamental features of UV freeze-in. Typically high dimension operators lead to many body final states, and the case of DM production via \( 2 \rightarrow 3 \) UV freeze-in was carefully studied. Subsequently, we attempted to quantify DM production associated to more complicated phase spaces. Further, we discussed the potential impact of spontaneous symmetry breaking on UV freeze-in, in particular, we identified a new case of interest in which the VEV expansion leads only to additional UV contributions, and does not generate an IR freeze-in portal. In this scenario the DM yield depends on both the reheat temperature and the critical temperature of symmetry breaking.
Sect. 5.2 examined the constraints on UV freeze-in from the requirement that the hidden sector and visible sector do not equilibrate and we argued that this can lead to bounds on the DM mass.

In Sect. 5.3 we presented realistic models of UV freeze-in and related these to interesting BSM scenarios. We suggested that UV freeze-in might be connected to motivated solutions of prominent puzzles of the SM, specifically we consider an example involving the Peccei-Quinn mechanism. A further example was presented in which the UV freeze-in portal is generated by integrating out a heavy $Z'$. This is appealing as $Z'$ arise in many extensions of the SM and additional U(1) gauge groups are common in realistic string compactifications. It should be evident from our discussions that UV freeze-in offers a large range of possibilities for DM model building and that there are many interesting aspects yet to be explored.

UV freeze-in presents a new manner of obtaining non-thermal DM, with a relic abundance directly related to the reheat temperature, and provides an interesting alternative to the conventional ideas regarding the DM thermal history. For the DM relic density to be set through UV freeze-in it is required that reheating of the hidden sector is negligible and that the DM is connected to the visible sector via non-renormalisable operators. Once one assumes that the abundance of DM is initially depleted, one might argue that freeze-in via high dimension contact operators presents a more generic mechanism than IR freeze-in portals, which require very small renormalisable couplings or complicated effective operators involving several scalar fields with non-vanishing VEVs. Moreover, from a UV perspective, it is a fairly general expectation that distinct sectors in the low energy theory may become coupled through the high scale physics, and we have presented some examples of this principle in the above.
CHAPTER 6

CONCLUSION

The problem of dark matter (DM) is indeed one of the most exciting open questions in particle physics, astrophysics, and cosmology. The existence of DM is deduced from various astrophysical and cosmological observations. In fact, the cosmological observations of the cosmic-microwave-background constrain the DM relic density to $\Omega \sim 0.25$. Furthermore, it is believed that DM is mostly non-relativistic, and stable on the cosmological scale. The canonical paradigm for DM relic abundance is that it is a single particle thermal relic: starting with abundance comparable to photons in the early universe, but then dropping in number density once the temperature falls below its mass, and then freezing out due to the expansion of the universe. In this naive picture, we expected DM mass and interaction to close to be around weak scale. In other words, DM was suspected to be a weakly interacting massive particle (WIMP). Although current DM experiments are sensitive to the weak scale, no signal of DM has been detected yet, which lead us to move away from the WIMP paradigm. Here, we studied four alternative frameworks where one or more of single particle thermal relic are challenged.

One of the simplest ways we can challenge the WIMP paradigm is to assume that DM particle is not alone in setting the relic abundance. Rather, there are other particles nearby (almost degenerate), that can co-annihilate with DM and play a significant role in setting the relic abundance. Namely, we could have a set up where the annihilation of DM to the SM particles is small, but DM’s co-annihilation with another particle to the SM is significant. One of the most studied frameworks for
such DM model, with multiple particles in the dark sector, is the Minimal Super-
symmetric Standard Model. A spectrum satisfying the aforementioned description
is discussed in Chapter 2 and its phenomenological signature is explored in details
at the Large Hadron Collider (LHC). Such spectrum is in the blind spot of the
Direct Detection (DD) and Indirect Detection (indirect detection) is insensitive to it.
A new search technique for this spectrum based on final states with missing trans-
verse energy, a photon, and a dilepton pair, $\ell^+ \ell^- + \gamma + E_T$ is discussed. Unlike
traditional searches, which perform best for mass splitting $\gtrsim m_Z$ between the DM
and the other-almost-degenerate particle, our search favors smaller mass splitting;
degenerate electroweakinos typically have a larger branching ratio to photons, and
the cut $m_{\ell\ell} \ll m_Z$ effectively removes on-shell Z boson backgrounds while retaining
the signal. This feature makes our technique optimal for ‘well-tempered’ scenarios,
where the dark matter relic abundance is achieved with inter-electroweakino splittings
of $\sim 20 - 70$ GeV. Additionally, our strategy applies to a wider range of scenarios
where the lightest neutralinos are almost degenerate, but only make up a subdomi-
nant component of the dark matter – a spectrum we dub ‘well-forged’. Focusing on
Bino-Higgsino admixtures, an optimal cuts and expected efficiencies for several bench-
mark scenarios were presented, and it was found that Bino-Higgsino mixtures with
$m_{\tilde{\chi}_2^0} \lesssim 190$ GeV and $m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0} \cong 30$ GeV can be uncovered after roughly $600 \text{ fb}^{-1}$
of luminosity at the 14 TeV LHC. Scenarios with lighter states require less data for
discovery, while scenarios with heavier states or larger mass splittings are harder to
discriminate from the background and require more data. Unlike many searches for
supersymmetry, electroweakino searches are one area where the high luminosity of
the next LHC run, rather than the increased energy, is crucial for discovery.

Moving away one more step from the WIMP paradigm, we can have DM anni-
hilating to metastable mediators, which then decay to SM particles. The generic
terms used for this class of models is called “secluded WIMPs”. Such mediators
are a common prediction of many extensions of SM. The particular model presented in this work, is the Froggatt-Nielsen (FN) mechanism which proposes a symmetry under which fermions are charged. As a consequence of the explicit breaking of this symmetry and electroweak symmetry breaking, the fermions acquire mass. Thus the various masses of fermions are due to their various charge under the FN symmetry. We added DM to this picture, allowing it to get mass from the breaking of the FN mechanism, and discussed its annihilation rate to SM particles and the new particles introduced in the FN mechanism. In Chapter 3 we showed that a weak scale DM can have $O(\text{weak interaction coupling})$ with a meta stable mediator introduced in FN mechanism, and thus have the required annihilation cross section in the thermal relic paradigm to produce the right amount of relic. Means to testify such scenario is also discussed, proposing the next generation of indirect detection might exclude or discover some of the parameter space.

Not having found any signals might also indicate that the thermal relic framework is not the correct picture of DM evolution throughout the history of the universe. So non-thermal DM was also investigated in this paper. One of the scenarios was the asymmetric DM, motivated by the comparable relics of DM and the visible matter. In this setup, DM at the present time does not annihilate but can accumulate inside neutron stars and change their phenomenology than otherwise expected. Moreover, there are pulsars apparently missing from the galactic center. We presented a model where this disappearance could be because asymmetric fermionic dark matter ($m_X = 1 - 100$ GeV) coupled to a light scalar ($m_\phi = 5 - 20$ MeV), which mixes with the Higgs boson destroyed the pulsar. We point out that this pulsar-collapsing dark sector can be discovered by upcoming direct detection experiments. Another implication is a maximum pulsar age curve that increases with distance from the galactic center, with a normalization that depends on the couplings and masses of dark sector particles. Finally, we use old pulsars outside the galactic center to place bounds on asymmetric
Higgs portal models.

Yet, another avenue to deviate from the thermal relic is to assume DM is thermally decoupled from the visible sector. The observed relic density can potentially be obtained via freeze-in production of dark matter. Originally in such models it is assumed that the dark matter is connected to the thermal bath through feeble renormalisable interactions. Here, rather, we considered the case in which the hidden and visible sectors are coupled only via non-renormalisable operators. This is arguably a more generic realisation of the dark matter freeze-in scenario, as it does not require the introduction of diminutive renormalisable couplings. We examine general aspects of freeze-in via non-renormalisable operators in a number of toy models and present several motivated implementations in the context of beyond the standard model physics, and studied some examples in greater details.

All of the discussed models have viable parameter space that can produce the right relic abundance and still be safe from the current experimental bounds. Moreover, the input parameters in the models discussed are around the natural size which is consistent with theorist’s prejudices. Most of the parameter space of the models discussed are within the reach of the next generation of experiments and so they can be testified in experiments. There are still many interesting and well-motivated scenarios beyond the WIMP paradigm that deserve further exploration, which is my interest in pursuing in the future.
APPENDIX A

ADDITIONAL PHOTON+DILEPTON PRODUCTION PROCESSES

In this appendix we detail alternate electroweakino dilepton + photon production modes.

The process
\[ pp \to \gamma \left( \tilde{\chi}^+ \to \tilde{\chi}_1^0 \ell^+ \nu_\ell \right) \left( \tilde{\chi}^- \to \tilde{\chi}_1^0 \ell^- \bar{\nu}_\ell \right) \]  \hspace{1cm} (A.1)
involves the 2 to 3 body production of a photon and two charginos, each decaying leptonically. This process contains the same final state particles as the neutralino production considered. There are also collider processes which fall into the photon + dilepton category once we allow for extra final state products that are either soft or missed by the detector. The processes
\[ pp \to \left( \tilde{\chi}_2^0 \to jj \tilde{\chi}_1^0 \right) \left( \tilde{\chi}_3^0 \to \gamma \tilde{\chi}_2^0 \to \gamma \ell^+ \ell^- \tilde{\chi}_1^0 \right) \]  \hspace{1cm} (A.2)
\[ pp \to \left( \tilde{\chi}^+ \to \tilde{\chi}_1^0 jj' \right) \left( \tilde{\chi}_3^0 \to \gamma \tilde{\chi}_2^0 \to \gamma \ell^+ \ell^- \tilde{\chi}_1^0 \right) \]
can have sizeable cross sections for large values of \( \tan \beta \) or \( \mu > 0 \). These two processes involve a cascade decay \( \tilde{\chi}_3^0 \to \tilde{\chi}_2^0 \to \tilde{\chi}_1^0 \). This does not happen for smaller values of \( \tan \beta \) and \( \mu < 0 \) because the Higgsino splittings are smaller, keeping \( m_{\tilde{\chi}_2^0} \) nearly degenerate with \( m_{\tilde{\chi}_3^0} \). Finally, the process
\[ pp \to \gamma \left( \tilde{\chi}^+ \to \tilde{\chi}_1^0 jj' \right) \left( \tilde{\chi}_{2,3}^0 \to \ell^+ \ell^- \tilde{\chi}_1^0 \right) \]  \hspace{1cm} (A.3)
is another 2-3 production. Here the photon is again directly produced instead of
being a decay product. The other two particles produced are a chargino which decays hadronically and a neutralino which goes through a leptonic decay.

A.1 Minor Cuts

In this section we motivate and explain some of the minor cut used in our analysis. Following the rest of the paper, the cuts are broken up into ‘small mass splitting’ and ‘large mass splitting’ scenarios.

A.1.1 Photon transverse momentum:

‘small mass’ splitting (A and B): $30 \text{ GeV} < p_{T,\gamma} < 100 \text{ GeV}$

‘large mass’ splitting (C and D): $45 \text{ GeV} < p_{T,\gamma} < 135 \text{ GeV}$

The transverse momenta of the signal photon is determined by the mass splitting of the neutralinos as well as the boost of the parent particle. The lower bound removes the background from soft final-state radiation. After the leptonic angular cuts $|\Delta \phi_{\ell_1, \ell_2}|$ and $|\Delta \phi_{\ell\ell-\gamma}|$ have been established, the surviving background events have the leptonic system recoiling off a hard initial-state radiation photon. The signal photon is not as hard as these, placing an upper bound.

A.1.2 Missing energy magnitude and orientation:

‘small mass’ splitting (A and B):

$E_T < 65 \text{ GeV}, 0.2 < |\Delta \phi(E_T, \gamma)| < 2.7,$ and $1.0 < |\Delta \phi(\ell\ell, E_T)| < 2.9$

‘large mass’ splitting (C and D): no cut
In many searches for supersymmetry or Dark Matter, the expectation is for large amounts of missing transverse energy. However, in much of the well-forged parameter space, the dominant production is to nearly degenerate Higgsinos, $\tilde{\chi}^0_{2,3}$, produced back-to-back. When they decay to the LSP, the net result is the two unobserved particles are approximately back-to-back. The vector sum of the LSP momenta cancels to some degree, implying that the overall amount of $E_T$ will be small (at least compared to traditional supersymmetry searches).

The direction of the photon relative to the missing transverse energy can also help separate signal from background. In the $\gamma^*/Z(\tau^+\tau^-)+\gamma$ background, after demanding high $|\Delta\phi_{\ell\ell-\gamma}|$ and low $|\Delta\phi_{\ell\ell}|$, the surviving configurations have the neutrinos moving in the same direction as the leptons and in the opposite direction of the photon; thus, $|\Delta\phi(\ell\ell, E_T)|$ is nearly 0. The signal typically does not have this topology. Figure A.1 shows example event configurations for the signal and background.
Figure A.1. The kinematics (after major cuts) of \( \tau\tau\gamma (\tilde{\chi}_2^0\tilde{\chi}_3^0) \) on left (right).

In the \( \tau\tau\gamma \) picture, the azimuthal angle between the \( \slashed{E}_T \) and the \( \gamma \) is expected to be near \( \pi \). In the signal the \( \slashed{E}_T \) vector will point neither towards the photon nor the dilepton system.
APPENDIX B

YUKAWA COUPLINGS

The quark masses arising from Eq. 3.2 are

\[(m_u)_{ij} = y_{ij}^{(u)} \epsilon Q_{qi} - Q_{uj} \frac{\nu}{\sqrt{2}}, (m_d)_{ij} = y_{ij}^{(d)} \epsilon Q_{qi} - Q_{dj} \frac{\nu}{\sqrt{2}}, \]  

(B.1)

with the possibility

\[
\begin{pmatrix}
Q_{q1} & Q_{q2} & Q_{q3} \\
Q_u & Q_c & Q_t \\
Q_d & Q_s & Q_b
\end{pmatrix} =
\begin{pmatrix}
-3 & -2 & 0 \\
5 & 2 & 0 \\
4 & 3 & 3
\end{pmatrix}
\]

yielding the correct masses [87].

The corresponding Yukawa interactions with the physical Higgs are identified as

\((Y_{ij}^{u}/\sqrt{2})h \bar{u}_{Li} u_{Rj}\) and \((Y_{ij}^{d}/\sqrt{2})h \bar{d}_{Li} d_{Rj}\) with

\[Y_{ij}^{u} \equiv y_{ij}^{(u)} \epsilon Q_{qi} - Q_{uj}, \quad Y_{ij}^{d} \equiv y_{ij}^{(d)} \epsilon Q_{qi} - Q_{dj}. \]

(B.2)

From Eqs. 3.1 and 3.2, one also obtains the Yukawa interactions

\[g_{ij}^q \sigma \bar{f}_{Li} f_{Rj} + g_{ij}^q \gamma_5 \rho \bar{f}_{Li} f_{Rj}, \]

(B.3)
with

\[
(g^u_{\sigma})_{ij} = i (g^u_{\rho})_{ij} = y^u_{ij} (Q_{qi} - Q_{qj}) e^{(Q_{qi} - Q_{qj})} \frac{v}{\sqrt{2} v_s},
\]

\[
(g^d_{\sigma})_{ij} = i (g^d_{\rho})_{ij} = y^d_{ij} (Q_{qi} - Q_{qj}) e^{(Q_{qi} - Q_{qj})} \frac{v}{\sqrt{2} v_s}.
\]

(B.4)

More explicitly,

\[
g^u_s = \frac{1}{v_s} \begin{pmatrix}
8m_u & \epsilon m_c & \epsilon^3 m_t \\
\epsilon^3 m_c & 4m_c & \epsilon^2 m_t \\
\epsilon^5 m_t & \epsilon^2 m_t & 0
\end{pmatrix},
\]

\[
g^d_s = \frac{1}{v_s} \begin{pmatrix}
7m_d & \epsilon m_s & \epsilon^3 m_b \\
\epsilon m_s & 5m_s & \epsilon^2 m_b \\
\epsilon m_b & \epsilon^2 m_b & 3m_b
\end{pmatrix}.
\]

(B.5)

The $g^{(u,d)}_s$ matrices are brought to the mass basis via the biunitary transformations that diagonalize the Higgs Yukawas $Y^{(u,d)}_{ij} \equiv y^{(u,d)}_{ij} e^{n_{ij}(m_{ij})}$. Due to the misalignment between the Higgs and flavon Yukawa bases, the $\sigma$ and $\rho$ can mediate flavor-violating interactions that may be subject to meson-mixing constraints.
APPENDIX C

ORIGINS OF PSUEDOSCALAR MASS AND DM OPERATORS

In this appendix we provide toy models for the origins of \( m_\rho \) in Eq. 3.4 and the non-renormalizable operator in Sec. 3.2.1.2 that gives DM its mass and interactions with \( \sigma \) and \( \rho \).

We begin with possible spurionic origins of the \( b^2 \) term that we have discarded as too contrived or fine-tuned, before settling on the choice we have used in this paper.

C.1 Dimension-0

If \( b^2 \) originated in a spurion that gave a dimensionless co-efficient, viz.,

\[
V(H,S) \supset -b^2(S^2 + S^\dagger 2) \xrightarrow{\text{UV}} -(Y/\Lambda_{\text{new}})b^2 S^2 + \text{H.c.},
\]

then the explicit breaking is “hard”: a quartic term \( \propto S^4 \) involving the same spurion is mandatory, allowing for loop corrections to \( b^2 \) and making it quadratically sensitive to \( \Lambda_{\text{new}} \).

C.2 Dimension-1

Suppose

\[
V(H,S) \supset -b^2(S^2 + S^\dagger 2) \xrightarrow{\text{UV}} -\mu_Y (YS^2 + Y^\dagger S^\dagger 2),
\]

where \( \text{dim}[\mu_Y] = 1 \). This possibility, considered in Ref. 262, raises questions about the size of the scale \( \mu_Y \). Including all the relevant terms, minimizing this potential
and finding $\langle S \rangle$, $\langle Y \rangle$ in terms of its parameters, it may be found that to obtain $b^2 < v_s$ (as befitting an explicit symmetry-breaking parameter), the $|S|^2|Y|^2$ quartic coupling must be fine-tuned to be unnaturally small.

### C.3 Dimension-2

This is the model we have chosen for our treatment of DM. The $b^2$ term in Eq. 3.3 can arise when the $U(1)_{FN}$ sector is “weakly coupled” to a hidden $U(1)'$ sector. Consider the potential

$$V(S,Y) = \mu^2_S |S|^2 + \lambda_s |S|^4$$
$$+ \mu^2_Y |Y|^2 + \lambda_y |Y|^4$$
$$+ \lambda_{sy} |S|^2|Y|^2 + \lambda_{sy}' S^2 Y^2 + \text{H.c.},$$

(C.1)

with $S$ and $Y$ carrying $U(1)_{FN} \otimes U(1)'$ charges $(1,0)$ and $(0,1)$ respectively. $V$ is invariant under $U(1)_{FN} \otimes U(1)'$ in the limit $\lambda_{sy}' \to 0$, so it is technically natural for $\lambda_{sy}'$ to be small. Therefore, if $Y$ gets a vev, a small $b^2$ may be generated.

Now we discuss the origins of the operator $(S/\Lambda_{FN})^n S \chi \chi$. In order to generate DM interactions suppressed by $\Lambda_{FN}$, we must introduce new particles at this cutoff scale. As $\chi$ is $Z_2$-odd, so must these particles. A simple choice, taking a leaf out of the original FN mechanism, is vector-like fermions $F_\chi, \bar{F}_\chi$ of mass $\Lambda_{FN}$ with the following UV Lagrangian:

$$\mathcal{L} \ni S \chi \bar{F}_\chi + \Lambda_{FN} F_\chi \bar{F}_\chi \chi.$$
This appendix collects formulae used in the calculation of the relic density and direct detection cross sections.

D.1 Relic Abundance

The relic abundance is given by

$$\Omega \chi h^2 = 0.12 \times \frac{2.2 \times 10^{-26} \text{ cm}^3/\text{s}}{\langle \sigma v \rangle} \times \left( \frac{x_f}{25} \right) \left( \frac{106}{g_*} \right)^{1/2}, \quad (D.1)$$

where \( x_f \equiv m_\chi/T_{\text{freezeout}} \) and \( g_* \) counts the entropy degrees of freedom at freezeout.

The annihilation cross-section \( \langle \sigma v \rangle \) is given by

$$\langle \sigma v \rangle = \frac{g_{\chi \chi}^4}{256\pi m_\chi^4(m_\rho^2 - 4m_\chi^2)^2(m_\rho^2 + m_\sigma^2 - 4m_\chi^2)^2} \sqrt{m_\rho^4 - 2m_\rho^2(m_\rho^2 + 4m_\chi^2) + (m_\sigma^2 - 4m_\chi^2)^2} \times$$

$$\left( (m_\rho^4 + m_\rho^2(m_\sigma^2 + 2 + 4m_\chi^2)(2\lambda_s m_\chi v_s) + 2m_\chi(m_\sigma^2 - 4m_\chi^2)(\lambda_s v_s - 2m_\chi)^2 \right)^2$$

(D.2)

where the coupling \( g_{\chi \chi} = \sqrt{2}y_\chi \epsilon. \)

D.2 Direct Detection

The spin-independent DM-nucleon scattering cross section is given by

$$\sigma_{\chi N}^{\text{SI}} = \frac{4\mu^2_{\chi N}m_N^2}{\pi} \left( \frac{g_{\sigma \chi \chi}g_{\sigma bb}}{m_b m_\sigma^2} \right)^2 f_N^2, \quad (D.3)$$
with \( \mu_{\chi N} \) the DM-nucleon reduced mass, \( g_{\sigma \chi \chi} = \sqrt{2} y_{\chi} \epsilon \) and \( g_{\sigma bb} \) is given in Eq. B.4.

The effective nucleon-DM coupling \( f_N \) arises from the \( \chi-\chi \)-gluon-gluon operator via heavy quark loops and is given by \[ f_N = \frac{2}{27} f_{T\Gamma}^N c_b, \] (D.4)

with \( f_{T\Gamma}^N \) the gluon mass fractions in nucleons,

\[ f_{T\Gamma}^{\text{proton}} = 0.925, \quad f_{T\Gamma}^{\text{neutron}} = 0.922, \]

and \( c_b \) a QCD correction factor = \( 1 + 11\alpha_s(m_b)/4\pi = 1.19 \). We have neglected light quark contributions to \( f_N \) due to the tiny sizes of their \( g_{\sigma qq} \) couplings.
APPENDIX E

GALACTIC CENTER PULSAR POPULATION

This appendix details methods for estimating the number of pulsars, both young and millisecond, at the galactic center.

E.1 Young Pulsars in the Central Parsec

A simple way to estimate the expected number of young pulsars in a region is to count the number of 8-20 solar mass, high mass stars, \( N_{\text{hms}} \). It is expected that the majority of these will expire in core collapse supernovae and form pulsars [168, 172]. More massive stars have larger hydrogen cores which burn faster, and so as a general rule the luminosity of stars scales as \( L \sim M^{3.5} \). Normalizing to a sol-type star, the lifetime of a high mass star \( (t \sim M/L) \) is given by \( t_{\text{hms}} = 10^{10}(M/M_\odot)^{-2.5} \) yrs. Assuming that the abundance of each star type stays constant in the region of interest, this implies there are \( \sim 5N_{\text{hms}}/3 \) young pulsars, because the lifetime of the high mass stars is 6 Myr, and a typical lifetime for a young pulsar is 10 Myr.

It follows that the inner parsec of the galactic center which hosts \( \sim 300 \) high mass stars should also host \( \sim 500 \) young pulsars, \( \sim 50 \) of which would beam towards earth, \( \sim 25 \) of which should have already been detected given the recently measured radio scattering dispersion (courtesy of a bright magnetar) [168]. However, this calculation is only valid if high mass stars in the galactic center were as abundant historically as they are now; high mass stars may have been less abundant 20 million years ago [171]. The next generation of surveys should permit imaging of 1-2 solar mass stars [266],
which as the byproducts of high mass stars, will provide a better indication of the
historical abundance of high mass progenitor stars.

E.2 Older, Millisecond Pulsars

Millisecond or recycled pulsars form when a neutron star spins up in a binary
system by accreting gas from its companion \[170\]. Most millisecond pulsars have
been found inside globular clusters, which have stellar densities as large as \(1 - 10^3\)
solar masses per cubic parsec, comparable to a stellar density of \(10^3 M_\odot/pc^3\) and
\(10^6 M_\odot/pc^3\) in the central 100 parsecs and central parsec of the milky way, respec-
tively. One way to estimate the central parsec millisecond pulsar population is to
scale up the millisecond population in globular clusters (about 10-50 millisecond pul-
sars per cluster) \[267\]. Millisecond pulsar formation is expected to be greater in the
central parsec, because of its higher stellar density and higher escape velocity (400
vs. 50 km/s), allowing it to retain a larger fraction of young neutron stars (with an
average birth velocity of 400 km/s). Given these factors, \(\sim1000\) millisecond pulsars
are expected in the central parsec \[174, 267, 268\].

We can also consider a simpler, more conservative method for estimating the mil-
losecond pulsar population of the central parsec. Before a neutron star has finished
accreting gas from its companion, it will necessarily be in a low mass x-ray binary.
As gas falls into the compact neutron star, it often emits x-rays. Hence, we should
expect the observation of low mass x-ray binaries to correlate with the number of
millisecond pulsars. Indeed, studies have shown that both of these correlate with
the stellar encounter rate in globular clusters, \(\Gamma_c \sim \rho_c r_c^3 v_c^{-1}\), where these variables
are respectively the density, radius, and linear velocity dispersion of a globular clus-
ter \[269, 270\]. Roughly 10 - 20 times more millisecond pulsars than low mass x-ray
binaries have been found in globular clusters \[269\]. Extrapolating from the four x-ray
binaries found in the central parsec \[271, 272\], one would expect \(\sim40-80\) detectable
millisecond pulsars in the central parsec, of which (accounting for increased radio pulse dispersion at the galactic center) \( \sim 5-10 \) would have been detected by the 14 Ghz survey \([168, 178]\).

### E.3 \( \sigma_{nX} \) Velocity Dependence for Higgs Portal DM

The matrix element for dark matter t-channel scattering off nuclei is

\[
i\mathcal{M} = g_D \epsilon_N \frac{[\bar{u}(p_4)u(p_1)][\bar{u}(p_3)u(p_2)]}{t - m_\phi^2 + i\epsilon}, \tag{E.1}
\]

where \( p_{1,3} \) and \( p_{2,4} \) are the dark matter and nucleon initial and final 4-momenta, respectively. The prefactor \( \epsilon_N \simeq 3 \times 10^{-3} \times \epsilon_h \) parameterizes the coupling of \( \phi \) to nuclei, largely determined by the gluon hadronic matrix element \([273]\).

In the center momentum frame of the interaction, the four momenta are given by

\[
\begin{align*}
p_1 &= (E_1, 0, 0, p_{cm}) \\
p_2 &= (E_2, 0, 0, -p_{cm}) \\
p_3 &= (E_3, -p_{cm}\sin \theta, 0, -p_{cm}\cos \theta) \\
p_4 &= (E_4, p_{cm}\sin \theta, 0, p_{cm}\cos \theta). \tag{E.2}
\end{align*}
\]

To approximate the cross-section at a direct detection experiment or in a neutron star, we integrate the square of the amplitude,

\[
\frac{1}{4} |\mathcal{M}|^2 = g_D^2 \epsilon_N^2 \frac{4(2m_n^2 + p_{cm}^2(1 - \cos \theta))(2m_n^2 + p_{cm}^2(1 - \cos \theta))}{(-2p_{cm}^2(1 - \cos \theta) - m_\phi^2)^2}, \tag{E.3}
\]

over a range of nuclear recoil energies \( E_{\text{recoil}} \), specifically ranging over incoming DM momenta \( 0.001\gamma v m_X \) to \( \gamma v m_X \), where \( \gamma = 1/\sqrt{1 - v^2} \), and \( v \) is a typical DM velocity.
in the nucleon’s rest frame ($v \approx 10^{-3} \, c$ and $0.7 \, c$ for direct detection experiments and pulsars, respectively). Here we define the momentum transfer $Q \equiv \sqrt{p^2_{cm}(1 - \cos \theta)}$, which can be related to the recoil energy with $p^2_{cm}(1 - \cos \theta) \sim 2m_nE_{\text{recoil}}$.

Some recent work has shown that Dirac dark matter with a strong ($\alpha_D \gtrsim 0.1$) coupling to a light scalar mediator, may form bound states at freeze-out \cite{191, 274}. This bound state dark matter can have a cross-section enhanced by $N_D^2$, the square of the number of bound fermions, as long as the momentum transfer of the interaction is less than the binding energy $Q \ll BE_0$. For the purposes of this study, we assume no bound state enhancement to the cross-section – this assumption is certainly valid for the capture of DM in neutron stars where $Q \sim m_X \gg BE_0$. However, there may be some enhancement to direct detection capture when $\alpha_D^2 \gg 4m_\phi/m_X$, which would shift direct detection bounds on $\epsilon_h$ in Figures 4.1 and 4.2.
APPENDIX F

IR FREEZE-IN OF DARK MATTER

The IR yield due to $2 \to 2$ scattering via a four-point scalar interaction with the matrix element $|\mathcal{M}|^2 = \lambda^2$ was calculated in [15]. This result is referenced in the text, so we give it here for completeness. The abundance of the DM $\phi$ is initially zero, and it is produced via the operator $\lambda \phi_1 \phi_2 \phi_3$, where $\lambda$ is a feeble dimensionless coupling. Consider $2 \to 2$ scattering where the momenta of the incoming bath particles are labelled $p_1, p_2$ and outgoing state momenta labelled $p_3, p_\phi$.

The matrix element associated to scattering via this four-point interaction is $|\mathcal{M}|^2 = \lambda^2$. The Boltzmann equation which describes DM production in this set-up is

$$\dot{n}_\phi + 3Hn_\phi \simeq 3 \int d\Pi_1 d\Pi_2 d\Pi_3 d\Pi_\phi f_1 f_2 |\mathcal{M}|^2 (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_\phi) . \quad (F.1)$$

As previously, we re-express this as an integral with respect to centre of mass energy

$$\dot{n}_\phi + 3Hn_\phi \simeq \frac{3T}{512\pi^6} \int_{m_\phi^2}^{\infty} ds \ d\Omega \ P_{12}P_{3\phi} |\mathcal{M}|^2 \frac{1}{\sqrt{s}} K_1 \left( \frac{\sqrt{s}}{T} \right) , \quad (F.2)$$

where $P_{ij}$ is defined in eq. (5.5). If the bath state masses can be neglected, this reduces to

$$\dot{n}_\phi + 3Hn_\phi \simeq \frac{3T\lambda^2}{512\pi^5} \int_{m_\phi^2}^{\infty} ds \left( \frac{s - m_\phi^2}{\sqrt{s}} \right) K_1 \left( \frac{\sqrt{s}}{T} \right) \simeq \frac{3m_\phi T^3 \lambda^2}{128\pi^5} K_1 \left( \frac{m_\phi}{T} \right) . \quad (F.3)$$
Converting this to a yield one obtains the result \[ Y_\phi \simeq \frac{135}{512\pi^6(1.66)g_*^2\sqrt{g_*^2}} \frac{M_{Pl}\lambda^2}{m_\phi}. \] (F.4)

For the case of direct freeze-in due to decays of heavy bath states \( \psi_H \) to a lighter state \( \psi_L \) in the thermal bath and DM \( \phi \) via the interaction \( \lambda \bar{\psi}_H \psi_L \phi \) this is instead

\[
\dot{n}_\phi + 3Hn_\phi \simeq 2\int d\Pi_H \Gamma_H m_H f_H \\
\simeq 2\int_{m_H}^{\infty} \frac{\Gamma_H m_H}{2\pi^2} \sqrt{E_H - m_H} \ e^{-\frac{E_H}{T}} \ dE_H \\
\simeq \left( \frac{\Gamma_H m_H^2 T}{2\pi^2} \right) K_1 \left( \frac{E_H}{T} \right).
\] (F.5)

It follows that the yield is of the form \[ Y_\phi \simeq \frac{135}{8\pi^3(1.66)g_*^2\sqrt{g_*^2}} \left( \frac{M_{Pl}\Gamma_H}{m_H^2} \right). \] (F.6)
APPENDIX G
UV FREEZE-IN OF DARK MATTER VIA 2 → 3 SCATTERING

Consider 2 → 3 scattering where the momenta of the incoming particles are labelled $p_1, p_2$ and outgoing state momenta labelled $p_3, p_4, p_\varphi$, where $\varphi$ indicates that it associated to the DM and the other states are part of the thermal bath. The Boltzmann equation for the production of $\varphi$ via these scatterings are given by

$$\dot{n}_\varphi + 3Hn_\varphi = \int d\Pi_1d\Pi_2f_1f_2|\mathcal{M}|^2\text{DLIPS}_3, \quad (G.1)$$

where $\text{DLIPS}_3$ denotes the Differential Lorentz Invariant Phase Space for 3-body final states

$$\text{DLIPS}_3 = d\Pi_3d\Pi_4d\Pi_\varphi(2\pi)^4\delta^{(4)}(p_1 + p_2 - p_3 - p_4 - p_\varphi). \quad (G.2)$$

To evaluate the Boltzmann equation we shall first look at simplifying the form of the RHS; we start by observing that

$$d^3p_1d^3p_2 = (4\pi|p_1|E_1\,dE_1)(4\pi|p_2|E_2\,dE_2)^{1/2}\cos \theta. \quad (G.3)$$

It is convenient to make the following change of variables (following broadly)

$$E_+ \equiv E_1 + E_2, \quad E_- \equiv E_1 - E_2, \quad s = 2E_1E_2 - 2|p_1|p_2|\cos \theta. \quad (G.4)$$

It follows that the volume element can be rewritten in these new variables as follows

$$\int d\Pi_1d\Pi_2 = \int \frac{1}{(2\pi)^4} \frac{dE_+dE_-ds}{8} = \int \frac{1}{(2\pi)^4} \frac{\sqrt{E_+^2 - s}}{4} dE_+ds, \quad (G.5)$$
since \( |E_-| \leq \sqrt{E_+^2 - s} \) the integral over \( E_- \) can be evaluated \( \int dE_- = 2\sqrt{E_+^2 - s} \).

The Boltzmann equation reduces to the form

\[
\dot{n}_\varphi + 3Hn_\varphi = \int_0^\infty ds \int_{\sqrt{s}}^\infty dE_+ e^{-E_+/T} \frac{1}{(2\pi)^4} \frac{\sqrt{E_+^2 - s}}{4} |\mathcal{M}|^2 \text{DLIPS}_3
\]  \tag{G.6}

\[
= \frac{T}{(2\pi)^4} \int_0^\infty ds \frac{\sqrt{s}}{4} |\mathcal{M}|^2 K_1 \left( \frac{\sqrt{s}}{T} \right) \text{DLIPS}_3.
\]  \tag{G.7}

Now turning to the DLIPS factor; in the centre of mass frame, we have \( \vec{p}_1 + \vec{p}_2 = 0 \) and hence by conservation of momentum \( \vec{p}_3 = -(\vec{p}_4 + \vec{\varphi}) \). From which it follows that

\[
E_3^2 = |p_4|^2 + |p_\varphi|^2 + 2|p_4||p_\varphi| \cos \theta_4 \varphi,
\]  \tag{G.8}

using \( \vec{p}_4 \cdot \vec{p}_\varphi = |p_4||p_\varphi| \cos \theta_4 \varphi \). The phase space differential, given in eq. (G.2), reduces to

\[
\text{DLIPS}_3 = \frac{1}{(2\pi)^5} \frac{d^3p_4}{|p_\varphi||p_4| E_3} \frac{d^3p_\varphi}{\delta(\sqrt{s} - \sqrt{|p_4|^2 + |p_\varphi|^2 + 2|p_4||p_\varphi| \cos \theta_4 \varphi} - |p_4| - |p_\varphi|)}
\]  \tag{G.9}

where we have assumed that the masses of the particles are negligible compared to \( \sqrt{s} \). Further, we define

\[
\overline{\cos \theta} \equiv \frac{s - 2\sqrt{s}(|p_4| + |p_\varphi|) + 2|p_4||p_\varphi|}{2|p_4||p_\varphi|},
\]  \tag{G.10}

such that \( \overline{\cos \theta} \) is a solution to the delta function. Therefore, we can write

\[
\delta(\sqrt{s} - \sqrt{|p_4|^2 + |p_\varphi|^2 + 2|p_4||p_\varphi| \cos \theta_4 \varphi} - |p_4| - |p_\varphi|) = \frac{\delta(\cos \theta_4 \varphi - \overline{\cos \theta})}{|p_4||p_\varphi|/E_3}.
\]  \tag{G.11}

It follows, after some simplifications, that

\[
\text{DLIPS}_3 = \frac{1}{(2\pi)^5} \frac{d \cos \theta_4 \varphi}{d \phi_4 \varphi} \frac{dp_4^0}{d \Omega} \frac{dp_\varphi^0}{d \varphi} \frac{1}{8} \delta(\cos \theta_4 \varphi - \overline{\cos \theta}),
\]  \tag{G.12}
where we have used that $d^3 p_4 = |p_4|^2 d(\cos \theta_4 \phi) d\phi_4 d\rho_4^0$ and $d^3 p_\varphi = |p_\varphi|^2 d\Omega d\rho_\varphi^0$. As the amplitude does not depend on any angle, the angular integrals are trivial

\[
\int d\Omega = 4\pi, \quad \int d\phi_4 = 2\pi, \quad \int d\cos \theta_4 \phi_4 \delta(\cos \theta_4 \phi_4 - \cos \theta) = 1. \tag{G.13}
\]

Consequently, the differential phase space factor simplifies substantially

\[
\text{DLIPS}_3 = \frac{1}{(2\pi)^3} \frac{d\rho_4^0 d\rho_\varphi^0}{4} = \frac{1}{(2\pi)^3} \frac{s}{16} dx_1 dx_2, \tag{G.14}
\]

where we have made the following change of variables (following e.g. [275]) in the latter equality

\[
p_3^0 = (1 - x_1 + x_2) \frac{\sqrt{s}}{2}, \quad p_4^0 = x_1 \frac{\sqrt{s}}{2}, \quad p_\varphi^0 = (1 - x_2) \frac{\sqrt{s}}{2}. \tag{G.15}
\]

Substituting eq. (G.14) into eq. (G.7), and integrating over the $x_i$, we obtain the result

\[
\dot{n}_\varphi + 3H n_\varphi = \frac{T}{2^{13} \pi^7} \int_0^\infty ds \int_0^1 dx_2 \int_{x_2}^1 dx_1 \ s^{3/2} |\mathcal{M}|^2 K_1 \left( \frac{\sqrt{s}}{T} \right)
\]

\[
= \frac{T}{(4\pi)^7} \int_0^\infty ds \ s^{3/2} |\mathcal{M}|^2 K_1 \left( \frac{\sqrt{s}}{T} \right). \tag{G.16}
\]
BIBLIOGRAPHY


98. *** Non-standard form, no INSPIRE lookup performed ***


122. *** Non-standard form, no INSPIRE lookup performed ***


