TIME-DEPENDENT DARK ENERGY
AND THE FLUX POWER SPECTRUM OF THE LYMAN $\alpha$ FOREST

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Abstract

by

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Dark energy is the as yet unidentified mechanism responsible for the accelerated expansion of the universe. Understanding this mechanism is one of the most important problems in modern cosmology for two reasons. First, dark energy is estimated to be responsible for $\approx 70\%$ of the energy content in the Universe, which means that we cannot have a complete model for the evolution of the structure in the Universe without understanding dark energy. Second, the current Standard Model of physics does not provide an explanation for dark energy. This means that understanding dark energy could serve as a gateway to physics beyond the current Standard Model.

The onus of constraining dark energy falls on observations. Since dark energy affects the expansion of the Universe, studying the behavior of dark energy currently requires the use of observables sensitive to the expansion. These include Type Ia supernovae, baryon acoustic oscillations, the anisotropies in the cosmic microwave background, and weak gravitational lensing. However, at present, the constraints provided by these probes are still rather weak. This leaves the pool of viable dark energy models quite large. There are two ways to alleviate this problem: the first is to acquire more data from those observational probes that are already employed, and the second is to develop complimentary observational probes.

In this thesis I take the latter approach by exploring the use of the Lyman $\alpha$ Forest
as a probe of dark energy. In particular, I have investigated whether time-dependent dark energy leaves an observationally detectable signature in the flux power spectrum of the Lyman α Forest.

To this end, I have run five high-resolution, large-scale cosmological simulations using a modified version of the publicly available smoothed-particle hydrodynamics code GADGET-2. Each simulation employed a different dark energy model. Four of these dark energy models are dynamical models while the fifth is the standard cosmological constant, or vacuum energy. I then developed efficient massively-parallel codes in order to extract both synthetic Lyman α Forest spectra and synthetic flux power spectra from my simulations.

These power spectra were then compared against one another using the k-sample Anderson-Darling test. The results of these tests indicate that there is insufficient statistical distinction between the power spectra calculated from my dynamical dark energy simulations and the power spectra calculated from a cosmological constant simulation ($\alpha = 0.05$ significance level). The effects of my chosen dark energy models on the power spectrum are so small as to suggest that there is likely no prospect of future observations being able to distinguish between power spectra from different dark energy models. This implies that other approaches to the Lyman α Forest must be explored to find a significant signature of the time-dependent dark energy.
Don’t panic.

Douglas Adams, *The Hitchhiker’s Guide to the Galaxy*

This thesis is dedicated to my grandfather, Ralph F. Coughlin, who was the best person I have ever known. You were always there for me, always patient, always kind, and always ready with a smile and a thumbs up. I miss you, I love you, and I hope I gave them hell.
Figure 1.1  This figure, obtained from Hubble (1929), demonstrates Hubble’s Law whereby the recessional velocity of a galaxy that is not gravitationally bound to us is proportional to its distance. The solid line and filled in circles refer to the individual galaxies, whereas the open circles and dashed line refer to when the galaxies are collected together into groups. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 17

Figure 1.2  This figure, obtained from Leavitt and Pickering (1912), shows the period in days on the x-axis and the magnitude on the y-axis. The two curves refer to the magnitudes of the Cepheids at their maximum and minimum magnitude. This marked the discovery of the period-luminosity relation for Cepheids, which was vital to being able to determine distances on cosmological scales. The units on the y-axis should be km s$^{-1}$. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 18

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Figure 1.4 This figure (Ellison, 2000) shows an example of the Lyα Forest.  
The Lyα emission line is prominently featured, and establishes the redshift of the quasar via $\lambda = \lambda_0 (1 + z)$, where $\lambda$ is the observed wavelength and $\lambda_0$ is the rest wavelength of the Lyα transition $\approx 1216$ Å. Blueward of this emission line is the multitude of Lyα absorption lines that comprise the Lyα Forest.

Figure 1.5 This figure (Planck Collaboration et al., 2015) shows the marginalized posterior distributions for $w_0$ and $w_a$ for various combinations of data sets. The Planck + BSH is the combination of the CMB, BAO, SNIa, and $H_0$, Planck + WL is the CMB and weak lensing data, Planck + BAO/RSD is the CMB and BAO/redshift-space distortions, and Planck + WL + BAO/RSD is the CMB, weak lensing, and BAO/redshift-space distortions. These distributions show the weak observational constraints on dark energy. The dashed lines correspond to $w_0 = -1$ and $w_a = 0.0$, which are the parameters for the cosmological constant model of dark energy, discussed in Section 2.2.1.

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Figure 2.1 This figure is Fig. 4 in Riess et al. (1998). The top panel shows the Hubble diagram from their SNIa dataset along with three cosmological fits. The first, and best, fit describes a flat universe dominated by dark energy. The other two are describe a flat universe without dark energy and an open universe without dark energy, respectively. The bottom panel shows the differences between the data and the models with respect to the $\Omega_{m,0} = 0.2, \Omega_{DE,0} = 0.0$ model.

Figure 2.2 This figure demonstrates the effect that different amounts of dark energy have on the luminosity distance as a function of redshift. Including more dark energy increases the acceleration of the expansion, which leads to larger distance measures. The blue curve, which is for a model with $\Omega_{m,0} = 0.3, \Omega_{DE,0} = 0.7$, and $h = 0.6731$ in a flat universe, is closest to the best-fit model preferred by the current observational data. This is reproduction of Figure 5.2 from Amendola and Tsujikawa (2010).
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Figure 2.5  This figure, recreated from Amendola and Tsujikawa (2010), shows the dependence of $\mathcal{R}$ on the amount of dark energy. The horizontal green band represents the allowed bounds on $\mathcal{R}$ as determined by WMAP (Komatsu et al., 2009).

Figure 2.6  This figure (Eisenstein et al., 2005) shows the detection of the BAO signal in the two-point correlation function as determined from the SDSS survey. The bump occurs at $\approx 150\text{ Mpc}$ comoving, which allows it to act as a standard ruler since it corresponds to a known size.

Figure 2.7  This figure, recreated from Amendola and Tsujikawa (2010), shows the BAO distance as a function of redshift for three different cosmological models: ($\Omega_{m,0} = 0.3, \Omega_{DE,0} = 0.7$), ($\Omega_{m,0} = 1.0, \Omega_{DE,0} = 0.0$), and ($\Omega_{m,0} = 0.1, \Omega_{DE,0} = 0.9$). The measured BAO distances are shown as the black points. The data, while sparse, is still clearly best-fit by the ($\Omega_{m,0} = 0.3, \Omega_{DE,0} = 0.7$) model, which is in accordance with the values determined from age measurements, the CMB, and SNIa.

Figure 2.8  This figure, reproduced from Amendola and Tsujikawa (2010), shows $d\Omega_{DE}/dN$ where $N = \ln(a)$ as a function of the scale factor. Trying to understand why $d\Omega_{DE}/dN$ peaks only near the present day is the coincidence problem.
Figure 3.1  This figure represents the core concept behind SPH. Here, several tracer particles that are used to discretize and follow the fluid flow are shown. In particular, the smoothing circles (spheres in three dimensions) for particles A and B are indicated by the blue and red circles, respectively. The smoothing lengths for particles A and B (the radii of the blue and red circles, respectively) are given by $h_A$ and $h_B$. Furthermore, the neighbor particles for particle A (those that fall within particle A’s smoothing circle) are shown in light blue, and those particles that do not contribute to particle A’s density as given by Eq. (3.7) because they lie outside the smoothing circle are shown in red. The greyscale gradient filling particle A’s smoothing circle denotes the role of the smoothing kernel $W(\mathbf{r}, h)$ in distributing particle A’s mass within the smoothing circle, and serves as a reminder of the fact that this distribution is not uniform, but given by Eq. (3.6). Lastly, the $x$ denotes an arbitrary point $\mathbf{r}$ in space where we might want to calculate the density of the fluid. In order to do this, Eq. (3.7) must be evaluated, in which case the sum is over all of the particles whose smoothing circles contain point $x$, and the distance from the particle to point $x$ is given by $r_A$ and $r_B$ for particles A and B, respectively (which is $r_A = |\mathbf{r} - \mathbf{r}_A|$ and likewise for particle B).

Figure 3.2  This is Figure 1 from Springel et al. (2001) and it shows a schematic diagram of the BH tree. Starting with the root node, which represents the simulation volume as a whole, each node is broken up into four (eight in three dimensions) equally sized daughter nodes until each node contains either zero or one particles. The nodes containing zero particles are discarded.

Figure 3.3  This figure shows a schematic of how each LOS is set up (albeit in two dimensions instead of three). A random point on the $x = 0$ cube face is chosen to be point A (the location of the quasar and the start of the LOS). The LOS then runs parallel to the $x$ axis to the opposite cube face; this is point B (the location of the observer and the end of the LOS). It is broken up into bins called pixels, the edges of which are denoted by the slash lines on the LOS. For practical purposes, the location of the pixel is taken to be the center of the bin, denoted by the blue x. Each pixel has a width $\Delta_p$ associated with it denoted with the bracket.
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Figure 3.5 This figure, from Faucher-Giguère et al. (2008), shows the effects of continuum bias using mock spectra. The thick black line represents the true, known, continuum and the thin black line represents the estimated continuum. The figure highlights how the discrepancy increases at higher redshift owing to the increase in absorption. The two lines in the $z = 4.5$ panel correspond to a spectrum generated with a hydrogen photoionization rate of half (blue) and twice (red) their fiducial value.

Figure 3.6 This figure, from Bolton et al. (2017), shows their simulated power spectrum as compared with observational data. Those models with the “cc” appended to their name have had the continuum correction applied. The figure clearly shows a deficiency in power across all scales for those models without the continuum correction. Every model has baryonic physics included.

Figure 4.1 This figure shows the density projected along the $z$ axis for my L-40-1024 (cosmological constant) simulation at four different redshifts. The figure clearly shows the presence and evolution of the web-like large-scale structure.

Figure 4.2 (a): The evolution of the dark energy EOS $w(z)$ for each of the dark energy models that I consider in this thesis. The blue line corresponds to the cosmological constant ($w_0 = -1, w_a = 0$). The other models were chosen to be close the edges of the 95% confidence $w_0 - w_a$ parameter space as determined by Planck Collaboration et al. (2015), save for model DE2-40-1024 ($w_0 = -1.1, w_a = 1.3$), which was deliberately chosen to be outside of the allowed range. See text for details. We see that the EOS for different dark energy models can vary considerably in their behavior, thereby affecting the expansion history of the universe in unique ways via Eq. 1.49. (b): The time derivative of the cosmic scale factor as a function of redshift for each of the dark energy models considered in this paper. This panel highlights the differences in the expansion history due to dark energy. The color coding and legend are the same for both panels.
Figure 4.3 This figure shows the temperature-density relation from my L-40-1024 simulation at \( z = 3.0 \). There are two distinct aspects of the relation: the straight line and the block-like region. The straight line arises from the temperatures of the field particles and the block-like region arises from the temperatures of the halo particles. The horizontal bands in the block-like region are because every particle in a given halo will have the same temperature, owing to the assumption that halos can be modeled as isothermal spheres. Only 5% of the particles are shown for visual clarity.

Figure 4.4 This figure shows an example synthetic Ly\( \alpha \) Forest spectrum that was extracted from my L-40-1024 simulation at \( z = 3.00 \). The LOS used passes through the center of the simulation volume parallel to the x-axis. Panel (a) shows the flux along the LOS, panel (b) shows the number density of neutral hydrogen along the LOS, and panel (c) shows the temperature of neutral hydrogen along the LOS.

Figure 4.5 This figure shows the same LOS for each of my dark energy models, though vertically offset from one another for visual clarity. The LOS used is the same as that in Figure 4.4. Each dark energy model, save for DE2-40-1024, gives rise to a nearly identical spectrum.

Figure 4.6 This figure shows the power spectrum for my L-40-1024 simulation as compared with the observed power spectrum from Iršič et al. (2017); McDonald et al. (2000, 2006) at \( z = 4.2, 3.8, 3.0, 2.7 \) and 2.2. The only caveat here is that data from McDonald et al. (2000) at \( z = 2.2 \) was actually measured at \( z = 2.41 \), and is being compared to my simulated power spectrum at \( z = 2.2 \). The synthetic power spectrum shown here was generated using synthetic Ly\( \alpha \) Forest spectra that had been run through the degradation process described in Section 3.6.1.

Figure 4.7 This figure shows the power spectrum from each dark energy model at \( z = 4.2, 3.8, 3.0, 2.7 \) and 2.2.

Figure 4.8 This figure shows the effects of using a smaller box size on the power spectrum. We have plotted the power spectrum for both the L-40-1024 (solid) and L-15-428 (dashed) simulations at \( z = 3.00 \). We see that at larger scales, the two power spectra are nearly identical, but as we move to smaller scales we see that there is an excess of power in the L-15-428 simulation.
Figure 4.9  This figure shows the effects of varying spatial resolutions on the power spectrum. Here we plot the L-100-1024 (dotted), L-25-428 (dashed), and L-40-428 (dash dot) simulations alongside the fiducial L-40-1024 (solid) simulation. Here we see that at large scales each of the power spectra are nearly indistinguishable from one another. At smaller scales, however, there is a fairly substantial difference. The L-100-1024 simulation has substantially under-predicted power whereas the L-25-428 simulation has over-predicted power as compared to the others. The L-40-428 and L-40-1024 simulations are nearly identical at all scales.

Figure 5.1  This figure, from Snedden et al. (2016), shows the temperature-density relation from a cosmological simulation that included UV heating, radiative cooling, star formation, and supernova feedback at $z = 3$. This can be compared with my temperature-density relation given in Figure 4.3.
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Throughout my time in graduate school, I was confused and stuck far more often than not. As such, I sent more than my fair share of emails to various authors whose papers I was following. While all of these people, from Dr. Tom Theuns, Dr. Luca
Amendola, Dr. Shinji Tsujikawa, Dr. Cameron Hummels, Dr. Matthew Turk, Dr. Nathan Goldbaum, Dr. James Bolton, and Dr. Romeel Davé provided me with help and encouragement, none were more supportive than Dr. Serena Bertone. You stayed in contact and continued helping me long after most other people would have given up. I am eternally grateful that you were kind enough to answer my initial email asking for help and for your continued support. I’m not sure I would have been able to finish my programs without your help.

To that end, most of my time in graduate school was spent developing and, let’s be honest, debugging various programs. Due to this, I would also like to sincerely thank Dr. In-Saeng Suh and the rest of the CRC support team for their countless hours of help while I tried to get my simulations off the ground and running. You have easily saved me months of time. Additionally, I would like to thank Richard Stallman for the development of gdb, which is hands down the greatest program ever written. Without this excellent debugger I never, ever would have finished the programs developed for this thesis.

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CHAPTER 1

INTRODUCTION AND BACKGROUND

There is a theory which states that if ever anyone ever discovers what the universe is for and why it is here, it will instantly disappear and be replaced by something even more bizarre and inexplicable. There is another theory which states that this has already happened.

—Douglas Adams, The Restaurant at the End of the Universe

1.1 Overview

There is now ample evidence that the universe began in a so-called hot big bang. Such evidence includes the expansion of the universe (Hubble 1929), the existence of the cosmic microwave background (CMB) (Penzias and Wilson 1965), and the success of big bang nucleosynthesis (BBN) (Alpher et al. 1948) in predicting the cosmic elemental abundances. In this model, spacetime started out as a quantum fluctuation that began expanding. After undergoing a theorized period of extremely rapid expansion known as inflation (Guth 1981; Starobinski 1979), the universe continued to expand and cool adiabatically. This led to the creation of baryons and, eventually, the distribution of stars, galaxies, and intergalactic material that comprises the large-scale structure in the universe known as the cosmic web (Bond et al. 1996). The cosmic web is comprised of three primary types of structures: clusters of galaxies, filaments of gas, dust, stars, and galaxies, and strongly under-dense regions between the filaments known as voids (e.g., Snedden 2015).

In this model, the big bang provided the initial impetus that set the expansion into motion. But then, without any further driving mechanism, and under the attractive
pull and infinite range of gravity, the expansion should slow down over time, as shown in Section 1.2.3. This change in the expansion rate governs how both the density and temperature in the universe change, thereby affecting the formation and growth of structure in the universe. Understanding the expansion history of the universe is therefore integral to understanding how the universe has evolved. This understanding is also important to model the future fate of the universe, as well. Such questions as will the expansion continue on unabated, asymptotically approach zero, or will it eventually stop and then begin to collapse back in on itself?

In order to answer the above questions, one must know: the current expansion rate $H_0$ and the amount of gravitating matter in the universe, the latter of which determines the deceleration parameter of the expansion $q_0$. Due to this, cosmology was widely thought of as the “quest for two numbers” (Sandage, 1970) up until the late nineties.

Then, in the late nineties, in an effort to determine $q_0$, two independent teams studying Type Ia supernovae (SNIa) announced an astounding result: the expansion of the universe was not decelerating as had been previously thought, but was, in fact, accelerating (Perlmutter et al., 1999; Riess et al., 1998)!

Since this initial discovery, cosmic acceleration has been confirmed by several other, independent, observational probes: e.g., baryon acoustic oscillations (BAOs) (e.g., Blake et al., 2011; Cole et al., 2005; Eisenstein et al., 2005), the cosmic microwave background (CMB) anisotropies (e.g., Komatsu et al., 2011; Planck Collaboration et al., 2014, 2016b; Spergel et al., 2003), and weak gravitational lensing studies (e.g., Abbott et al., 2016; Garcia-Fernandez et al., 2016; Schrabback et al., 2010). The evidence supporting the existence of cosmic acceleration is discussed in more detail in 2.1.

There are three possible explanations for cosmic acceleration: first, our understanding of gravity as described by Einstein’s general theory of relativity (GR) is
incomplete (modified gravity) (e.g., Carroll et al., 2005; Lue et al., 2004); second, there is some exotic and as-yet unknown new mechanism called dark energy responsible for continuing to drive cosmic expansion, and third, we are living inside of a very under-dense region and there really is no cosmic acceleration at all, it simply appears as if there is (inhomogeneous models) (e.g., Célerier, 2000; Enqvist, 2008; Iguchi et al., 2002; Tomita, 2000, 2001). However, no convincing inhomogeneity model has been put forward to date (e.g., Amendola and Tsujikawa, 2010) and modified gravity models are strongly restricted by local gravity constraints (e.g., Amendola and Tsujikawa, 2010). As such, this thesis is primarily concerned with the second option: dark energy. Understanding what, exactly, comprises dark energy is one of the most prominent open questions in physics (e.g., Amendola and Tsujikawa, 2010). There are two primary reasons for this importance: first, since dark energy is estimated to comprise roughly 70% of the universe’s energy budget (e.g., Planck Collaboration et al., 2015), we cannot have a complete model for how the universe has evolved without understanding it, and second, dark energy represents a gateway to understanding the physics beyond the current Standard Model.

As discussed further in Section 1.7 while there is a plethora of data and observational probes used to confirm the existence of cosmic acceleration, the observational constraints on what, exactly, dark energy is are still quite weak. Due to this, the primary goal of this thesis is to explore whether or not the flux power spectrum of the Lyman $\alpha$ Forest (Ly$\alpha$ Forest) can be used as a new observational probe for constraining the pool of viable dark energy models.

The remainder of this chapter is laid out in the following manner: I first discuss the prerequisite cosmological physics in Section 1.2, the evolution of matter species in the universe in Section 1.3, the expansion of the universe in Section 1.4, cosmological distance measures in Section 1.4.2 and the formation of large-scale structure in Section 1.5. The chapter concludes with a brief introduction to the Ly$\alpha$ Forest, which
is the key observable explored in this dissertation, in Section 1.6 and the project’s motivation in Section 1.7.

1.1.1 Notation

Before continuing, a brief discussion of the notation used in this thesis is in order. I use a system of natural units whereby $c = \hbar = k_B = 1$ unless otherwise specified, where $c$ is the speed of light in vacuum, $\hbar$ is the reduced Planck’s constant, and $k_B$ is the Boltzmann constant. This system of units has the effect of putting all quantities in terms of energy. In this chapter and in Chapter 2, Greek indices run from zero to three, where zero indicates the temporal component of a tensor. Latin indices run from one to three and the values one to three indicate the three spatial components. The Einstein summation convention is also employed. This says that repeated indices on the same term are summed over. When discussing quantities in a cosmological context instead of a tensorial one a subscript 0 indicates the value of that quantity at the present time (e.g., $H_0$ is the present-day value of the Hubble parameter $H$, which is a function of time). Also, throughout this thesis I work under the normalization that the present-day value of the scale factor is one (i.e., $a_0 = 1$). Lastly, a boldface font indicates a vector quantity (e.g., $\mathbf{r} \equiv \vec{r}$).

1.2 Cosmological Background

1.2.1 The Cosmological Principle

In order to build a mathematical description of the universe and its evolution, we first need a foundation to build off of. One part of this foundation is the so-called cosmological principle. Simply put, the cosmological principle states that on sufficiently large scales the universe is both homogeneous and isotropic. Homogeneity means that the universe possesses translational symmetry; i.e., no matter how much an observer translates along a given direction, the universe will always look the same.
Isotropy means that the universe possesses rotational symmetry; i.e., no matter how an observer rotates about a given axis, the universe will always look the same. The central caveat here, however, is that the cosmological principle holds only on “the largest of scales.” It is clear that the universe is neither homogeneous nor isotropic on the smallest scales. The natural question, then, is what scales qualify as large enough?

Obtaining observational evidence of homogeneity and isotropy is difficult, but observations suggest that scales on the order of tens of megaparsecs should be sufficient for homogeneity (e.g., Mo et al., 2010). Evidence of isotropy is far easier to come by than evidence of homogeneity. Probably the single best piece of evidence that we have in favor of isotropy on large scales is the CMB. While anisotropies have been detected (e.g., Smoot et al., 1977, 1992), the amplitudes of these fluctuations are of the order $10^{-5}$ (Planck Collaboration et al., 2016b).

The cosmological principle can be thought of as an extension of the Copernican Principle. That is, there is nothing special about our particular location in the universe. This idea has several important consequences. The first is that the same laws of physics in all locations in space and time. Secondly, it implies that our observable sample of the universe is a fair sample. This justifies any inferences and statistical measurements made in the local universe. See Mo et al. (2010) for a more thorough discussion of the cosmological principle.

1.2.2 The Friedmann-Robertson-Walker Metric

For the purposes of this thesis, the most practical implication of the cosmological principle is that it uniquely specifies a metric tensor that can be used to model the dynamical evolution of the universe.

A metric tensor is simply a mathematical object that allows for coordinate distances to be converted into physical distances (e.g., Dodelson, 2003). Without a
metric, there could not be an adequate description of spacetime or its curvature, which are both vitally important to understanding gravity as described by GR. By having this metric uniquely specified, one can derive equations that describe how the universe evolves. The metric of a homogeneous and isotropic universe is known as the Friedmann-Robertson-Walker (FRW) metric, and it was first studied independently by Friedmann (Friedmann, 1922, 1924), Lemaître (Lemaître, 1931, 1933), Robertson (Robertson, 1935, 1936a,b), and Walker (Walker, 1937) in the early twentieth century.

For a formal derivation of the FRW metric from the cosmological principle the reader is referred to Weinberg (1972). I simply present the final FRW metric as

\begin{equation}
\begin{aligned}
    ds^2 &= -dt^2 + a^2(t) d\sigma^2 \\
    &= -dt^2 + a^2(t) \left[ \frac{dr^2}{1-Kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right],
\end{aligned}
\end{equation}

where \(a(t)\) is a dimensionless scale factor that parameterizes the expansion of the spacetime in question, \(K\) is the curvature parameter (with units of one over area) that can be normalized by a coordinate transformation to be \(K \in \{1, 0, -1\}\) and describes the overall curvature of the spacetime. A value of \(K = 0\) corresponds to a flat spacetime, \(K = 1\) is a closed spacetime, and \(K = -1\) refers to an open spacetime. The \(dr^2\) describes the radial interval, and has units of distance. It is also a comoving coordinate (discussed in Section 1.4). The \(d\sigma^2\) term encapsulates the purely three-space aspect of the spacetime interval.

Strictly speaking, \(ds^2\) is the spacetime interval, and in relativity it is an invariant between inertial observers of particular events. However, the spacetime interval can be expressed in terms of the metric tensor via

\begin{equation}
    ds^2 = g_{\mu \nu} dx^\mu dx^\nu,
\end{equation}

where \(g_{\mu \nu}\) is the metric tensor, and \(dx^\mu\) and \(dx^\nu\) are coordinates. The relationship given in Eq. (1.2) is what allows the metric to convert coordinate distances into
physical distances, as described above. It is an unfortunate fact that in the literature it is extremely common to refer to $ds^2$ as the metric, which can create confusion about whether one is referring to $ds^2$ or $g_{\mu\nu}$. I have succumbed to peer pressure and will therefore be following this convention throughout the rest of this chapter. Just note, however, that the metric tensor for the spacetime interval Eq. (1.1) is actually

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{a^2}{1-Kr^2} & 0 & 0 \\ 0 & 0 & a^2 r^2 & 0 \\ 0 & 0 & 0 & a^2 r^2 \sin^2 \theta \end{pmatrix}. \quad (1.3)$$

The spatial part of the spacetime interval ($d\sigma^2$) given in Eq. (1.1) can be written in another, unified, form that will be particularly useful when discussing cosmological distance in Section 1.4.2 (e.g., Amendola and Tsujikawa 2010)

$$d\sigma^2 = d\chi^2 + f_K^2(\chi)(d\theta^2 + \sin^2 \theta d\phi^2), \quad (1.4)$$

where

$$f_K(\chi) = \begin{cases} 
\sin \chi & K = +1, \\
\chi & K = 0, \\
\sinh \chi & K = -1
\end{cases} = \frac{1}{\sqrt{-K}} \sinh \left( \sqrt{-K} \chi \right), \quad (1.5)$$

and comes from using $r = \sin \chi$ for $K = +1$, $r = \chi$ for $K = 0$, and $r = \sinh \chi$ for $K = -1$ in Eq. (1.1).

1.2.3 The Friedmann Equations

It is important to note that Eq. (1.1) is derived purely from the geometrical considerations given by the cosmological principle. The only aspect of the FRW metric that is left unaccounted for is the form of the scale factor. The equations
governing the evolution of the scale factor are called the Friedmann Equations, and they are derived from the incorporation of the FRW metric in the Einstein Field Equations (EFE). The EFE are given as

\[ R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi GT_{\mu\nu}, \]  

(1.6)

where \( R_{\mu\nu} \) is the Ricci Tensor, \( R \) is the Ricci Scalar, \( G \) is Newton’s universal gravitational constant, and \( T_{\mu\nu} \) is the energy-momentum tensor.

Eq. (1.6) forms the heart and soul of GR, which is the best theory of gravity that we have. In words, Eq. (1.6) says that “Space acts on matter, telling it how to move. In turn, matter reacts back on space, telling it how to curve.” (Misner et al., 1973)

The left-hand side of Eq. (1.6) describes the curvature of spacetime, as the Ricci Tensor and Scalar both describe curvature as deviations from flatness, and they are defined in terms of the metric. Their definitions are rather cumbersome, however, and so the interested reader is referred to Carroll (2004); Misner et al. (1973); Weinberg (1972) for a far more complete overview of GR and the mathematics behind it. The right-hand side, through the energy-momentum tensor, describes the distribution of matter and energy in spacetime.

It turns out that there are only four non-zero components of the Ricci Tensor when applying the FRW metric. These are:

\[ R_{00} = -3\frac{\ddot{a}}{a}, \]  

(1.7)

\[ R_{11} = \frac{a\ddot{a} + 2\dot{a}^2 + 2K}{1 - Kr^2}, \]  

(1.8)

\[ R_{22} = r^2(a\ddot{a} + 2\dot{a}^2 + 2K), \]  

(1.9)

\[ R_{33} = r^2(a\ddot{a} + 2\dot{a}^2 + 2K)\sin^2\theta, \]  

(1.10)

where a dot signifies a time derivative. These four components of the Ricci Tensor
give rise to the Ricci Scalar $R = R^{\mu\nu}R_{\mu\nu}$ as

$$R = 6 \left[ \frac{\ddot{a}}{a} + \left( \frac{\dot{a}}{a} \right)^2 + \frac{K}{a^2} \right]. \quad (1.11)$$

The last piece of the EFE that needs to be specified is the form of the energy-momentum tensor. For this tensor, one usually assumes a perfect fluid. A perfect fluid is completely described by its energy density $\rho$ and isotropic pressure $P$, which allows it to fit in nicely with the cosmological principle. The form of a perfect fluid is (e.g., Carroll 2004)

$$T_{\mu\nu} = (\rho + P)U_\mu U_\nu + Pg_{\mu\nu}, \quad (1.12)$$

where $U_\mu$ and $U_\nu$ represent the components of the four-velocity of the fluid. In order to remain isotropic in an expanding universe (and therefore in line with the cosmological principle), the fluid must be at rest in comoving coordinates (discussed in Section 1.4). This makes the comoving four-velocity take the form $U^\mu = (1, 0, 0, 0)$.

By now applying Eqs. (1.7) – (1.10), Eq. (1.11), and Eq. (1.12) to Eq. (1.6), we obtain two equations (from the 00 and $ii$ components of the EFE):

$$H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2}, \quad (1.13)$$

$$3H^2 + 2\dot{H} = -8\pi GP - \frac{K}{a^2}, \quad (1.14)$$

where

$$H \equiv \frac{\dot{a}}{a} \quad (1.15)$$

is the Hubble parameter. Eq. (1.13) is known as the Friedmann Equation, and it governs the evolution of the universe’s expansion in terms of its overall curvature and energy density.

We arrive at the second Friedmann Equation, also known as the acceleration equation, by eliminating the curvature terms $K/a^2$ from Eq. (1.13) and Eq. (1.14).
This gives:

\[
\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3P).
\] (1.16)

Since \( a(t) \) governs the expansion of the universe, \( \ddot{a} \) represents the acceleration of that expansion. As such, we see from Eq. (1.16) that the universe is only static in the very specific case of \( P = -\frac{1}{3} \rho \). For any other case (assuming that both \( \rho \) and \( P \) are positive), we see that the universe is decelerating due to the negative sign in Eq. (1.16). This is the mathematical justification for the view that the expansion was decelerating as described in Section 1.1. Both Eq. (1.13) and Eq. (1.16) are vitally important to the discussion of dark energy, as we shall see in Chapter 2.

The density given in Eq. (1.13) is a composite density, which is to say that it inherently includes the densities of all of the various components of the universe. Put another way,

\[
\rho \equiv \rho_m + \rho_r + \rho_{\text{DE}},
\] (1.17)

where \( \rho_m \) is the energy density of non-relativistic matter (both baryonic and dark), \( \rho_r \) is the energy density of relativistic matter, and \( \rho_{\text{DE}} \) is the energy density of dark energy. The matter species in the universe are discussed in Section 1.3.

Eq. (1.13) can be written in a form that is far more convenient to observations by introducing the concept of the critical density \( \rho_c \). The critical density is the density required in order to realize a flat (\( K = 0 \)) universe. By setting \( K = 0 \) in Eq. (1.13) and solving for \( \rho \), we obtain

\[
\rho_c = \frac{3H^2}{8\pi G}.
\] (1.18)

If we plug Eq. (1.17) into Eq. (1.13) and then divide both sides by \( H^2 \), we obtain

\[
1 = \frac{8\pi G}{3H^2} (\rho_m + \rho_r + \rho_{\text{DE}}) - \frac{K}{a^2 H^2},
\] (1.19)

and notice that the first term of the right-hand side of Eq. (1.19) is simply the density
of each matter species in the universe divided by the critical density. We now define the so-called density parameters for each matter species as the ratio of the density of that species to the critical density. That is:

\[
\Omega_m \equiv \frac{\rho_m}{\rho_c} = \frac{8\pi G \rho_m}{3H^2}, \quad (1.20)
\]

\[
\Omega_r \equiv \frac{\rho_r}{\rho_c} = \frac{8\pi G \rho_r}{3H^2}, \quad (1.21)
\]

\[
\Omega_{DE} \equiv \frac{\rho_{DE}}{\rho_c} = \frac{8\pi G \rho_{DE}}{3H^2}, \quad (1.22)
\]

are the matter, radiation, and dark energy density parameters. Additionally, it is convenient to define

\[
\Omega_K \equiv -\frac{K}{a^2 H^2}, \quad (1.23)
\]

as the curvature density parameter. The present-day values of these density parameters are given by using \( H_0 \) and \( \rho_{i,0} \), where \( \rho_i \) refers to the energy density of component \( i \) (either non-relativistic matter, relativistic matter, dark energy, or curvature). The density parameters allow us to write the Friedmann Equation (1.13) as

\[
1 = \Omega_m + \Omega_r + \Omega_{DE} + \Omega_K. \quad (1.24)
\]

In words, Eq. (1.24) simply states that adding up the contributions from each component of the universe must equal the whole at every moment in time.

1.3 Matter Species in the Universe

Despite being written in terms of observables, Eq. (1.24) is only useful if we know how the density parameters evolve in time. This behavior is described by the continuity equation. From the covariant derivative\(^1\) of the energy-momentum tensor

\(^1\)A covariant derivative, usually denoted with a ‘;’, is the generalization of the concept of a partial derivative such that the derivative operator transforms as a tensor. A tensor is a mathematical object
\( T^{\mu\nu} = 0 \), whose vanishing is the embodiment of energy-momentum conservation in relativity, we obtain

\[
\dot{\rho} + 3H(\rho + P) = 0,
\]

(1.25)

which is the continuity equation (e.g., Carroll 2004).

Eq. (1.25) can be solved by specifying an equation of state (EOS) that stipulates how the pressure depends upon the density. This EOS is usually written as

\[
w \equiv \frac{P}{\rho}.
\]

(1.26)

In general, \( w = w(a) \), which is to say that the EOS is a function of time. By using Eq. (1.26) in Eq. (1.25), we obtain an equation for the energy density as a function of the scale factor:

\[
\rho(a) = \rho_0 \exp \left[ 3 \int_a^{a_0} \frac{1 + w(a')}{a'} da' \right].
\]

(1.27)

In order to solve Eq. (1.27), the form of \( w \) for each of the various matter species in the universe must be known. This can be found by appealing to quantum statistical mechanics. The phase-space distribution of particles with momentum \( p \) in equilibrium and temperature \( T \) is given by (e.g., Amendola and Tsujikawa 2010)

\[
f(p) = \frac{1}{\exp \left[ \frac{E - \mu}{T} \right] \pm 1},
\]

(1.28)

where \( E = \sqrt{p^2 + m^2} \) is the energy of the particles, \( \mu \) is the chemical potential of the species, and the plus or minus signs refer to fermions (the Fermi-Dirac distribution) or bosons (Bose-Einstein distribution), respectively. Eq. (1.28) can then be used to get the energy density and pressure for a given species as (e.g., Amendola and Tsujikawa that transforms in a particular way (e.g., Carroll 2004) Tensors are useful because they allow for the laws of physics to be written in a coordinate-free manner (e.g., Carroll 2004).
\[
\rho = \frac{g^*}{2\pi^2} \int_m^{\infty} dE \frac{(E^2 - m^2)^{\frac{1}{2}}}{\exp\left[\frac{(E-\mu)}{T}\right] + 1} E^2, \quad (1.29)
\]

\[
P = \frac{g^*}{6\pi^2} \int_m^{\infty} dE \frac{(E^2 - m^2)^{\frac{3}{2}}}{\exp\left[\frac{(E-\mu)}{T}\right] + 1}, \quad (1.30)
\]

where \(g^*\) is the number of degrees of freedom.

### 1.3.1 Relativistic Matter

In order to evaluate Eq. (1.29) and Eq. (1.30) for relativistic particles, we use the fact that, for relativistic particles, the kinetic energy is much greater than the rest energy. This corresponds to \(T >> m\) or taking the limit as \(m \to 0\). This limit leads to the result (e.g., Amendola and Tsujikawa 2010)

\[
\rho_r = \begin{cases} 
\frac{\pi^2}{90} g^* T^4 & \text{(Bosons)}, \\
\frac{7\pi^2}{230} g^* T^4 & \text{(Fermions)},
\end{cases} \quad (1.31)
\]

\[
P_r = \rho_r / 3, \quad (1.32)
\]

where we see from Eq. (1.32) that the EOS for relativistic particles is

\[
w_r = \frac{1}{3}. \quad (1.33)
\]

Applying Eq. (1.33) to Eq. (1.27), we see that the evolution of the energy density for relativistic particles goes as

\[
\rho_r(a) = \rho_{r,0} a^{-4}. \quad (1.34)
\]
1.3.2 Non-Relativistic Matter

For non-relativistic matter, the condition is opposite. That is, the rest energy is much larger than the thermal kinetic energy, so $T << m$, which gives

$$\rho_m = g_s m \left( \frac{mT}{2\pi} \right)^{\frac{3}{2}} \exp \left[ -\frac{m - \mu}{T} \right], \quad (1.35)$$

$$P_m = \frac{T}{m} \rho_m. \quad (1.36)$$

Since $T << m$, we get from Eq. (1.36) that

$$w_m = 0. \quad (1.37)$$

Applying Eq. (1.37) to Eq. (1.27), the evolution of the energy density for non-relativistic matter goes as

$$\rho_m(a) = \rho_{m,0} a^{-3}. \quad (1.38)$$

1.3.3 Dark Energy

For dark energy, the case is somewhat more complicated by the fact that Eq. (1.28) does not apply. This means that the EOS for dark energy is unknown and must be derived based on the particular model for dark energy that is chosen. This is explored further in Chapter 2. For now, we note that there is no \textit{a priori} reason that $w_{\text{DE}}$ should be a constant, as it is for both relativistic and non-relativistic matter. As such, the integral in Eq. (1.27) must be left general. This term is the so-called dark factor and is given by

$$\zeta(a) = \exp \left[ 3 \int_a^1 \frac{1 + w'(a') \text{DE}}{a'} da' \right]. \quad (1.39)$$
This means that the energy density for dark energy is

\[ \rho_{\text{DE}}(a) = \rho_{\text{DE},0}\zeta(a). \]  

(1.40)

Since the exact form for \( w \) is currently unknown and we lack the large number of measurements of \( w \) necessary for non-parametric inference, we must use some parameterized form of \( w \) in order to compare with observations (e.g., Amendola and Tsujikawa, 2010; Corasaniti and Copeland, 2003). See Section 2.2.2 for more on parameterizations of \( w \). Here we employ the parameterization introduced in Linder (2003) and Chevallier and Polarski (2001):

\[ w(a) = w_0 + w_a(1 - a) = w_0 + w_a \frac{z}{1 + z}, \]

(1.41)

where \( w_0 \) is the present-day value of the equation of state \( w(a = 1) \) and \( w_a \) is its derivative \( \frac{dw}{da}\big|_{a=1} \).

1.3.4 The Hubble Equation

Armed with the evolution of the energy densities of each of the matter species in the universe, we can now proceed to write Eq. (1.24) in the form that we will use throughout the rest of this thesis. This is done by substituting in Eq. (1.38), Eq. (1.34), and Eq. (1.40) in for the energy density in each of Eq. (1.20), Eq. (1.21), and Eq. (1.22):

\[ \Omega_m = \frac{8\pi G}{3H^2}\rho_{m,0}a^{-3}, \]

(1.42)

\[ \Omega_r = \frac{8\pi G}{3H^2}\rho_{r,0}a^{-4}, \]

(1.43)

\[ \Omega_{\text{DE}} = \frac{8\pi G}{3H^2}\rho_{\text{DE},0}\zeta(a). \]

(1.44)

Plugging Eq. (1.42) – Eq. (1.44) into Eq. (1.24) and performing a little algebra
gives:

\[ 1 = \Omega_r + \Omega_m + \Omega_K + \Omega_{DE}, \quad (1.45) \]

\[ 1 = \frac{8\pi G}{3H^2}\rho_r a^{-4} + \frac{8\pi G}{3H^2}\rho_m a^{-3} - \frac{K}{a^2H^2} + \frac{8\pi G}{3H^2}\rho_{DE,0}\zeta(a), \quad (1.46) \]

\[ \frac{H^2}{H_0^2} = \frac{8\pi G}{3H_0^2}\rho_r a^{-4} + \frac{8\pi G}{3H_0^2}\rho_m a^{-3} - \frac{K}{a^2H_0^2} + \frac{8\pi G}{3H_0^2}\rho_{DE,0}\zeta(a), \quad (1.47) \]

\[ \frac{H^2}{H_0^2} = \Omega_r,0a^{-4} + \Omega_m,0a^{-3} + \Omega_K,0a^{-2} + \Omega_{DE,0}\zeta(a), \quad (1.48) \]

\[ H(a)^2 = H_0^2 [\Omega_r,0a^{-4} + \Omega_m,0a^{-3} + \Omega_K,0a^{-2} + \Omega_{DE,0}\zeta(a)], \quad (1.49) \]

where Eq. (1.49) is known as the Hubble Equation, and it gives the expansion history of the universe in terms of the present-day values of the density parameters for relativistic matter, non-relativistic matter, curvature, and dark energy, all of which can be determined from observations.

1.4 The Expanding Universe

The prevailing view in the early twentieth century was that the universe was static. Then, in 1929, Edwin Hubble published observational evidence (Hubble, 1929), now known as Hubble’s Law, that supported the claim of an expanding universe:

\[ v = H_0d, \quad (1.50) \]

where \( v \) is the magnitude of the recessional velocity of an object and \( d \) is the distance to that object.

Hubble’s Law states that those galaxies that are not gravitationally bound to each other are all receding away from one another at a speed that is proportional to their distance. Hubble’s Law is demonstrated in Fig. 1.1.

The discovery of Hubble’s Law traces was made possible by the period-luminosity
Figure 1.1: This figure, obtained from Hubble (1929), demonstrates Hubble’s Law whereby the recessional velocity of a galaxy that is not gravitationally bound to us is proportional to its distance. The solid line and filled in circles refer to the individual galaxies, whereas the open circles and dashed line refer to when the galaxies are collected together into groups.

relationship for Cepheid variable stars, which was discovered by Henrietta Leavitt (Leavitt and Pickering, 1912). This relationship allows one to measure the period of pulsation of a Cepheid and obtain the luminosity of the star, as shown in Fig. 1.2.

The luminosity of a star $L$ is an intrinsic property and is related to the distance $d$ and flux $f$ via (e.g., Maoz, 2007)

$$d = \sqrt{\frac{L}{4\pi f}}. \quad (1.51)$$

Leavitt’s period-luminosity relation, when calibrated using distances measured with stellar parallax (e.g., Turner et al., 2010), therefore allowed cosmological distances to be measured for the first time.

This procedure for calculating distances was employed by Hubble at the Mount Wilson observatory. Furthermore, spectroscopic measurements of these stars had been made by Vesto Slipher (Slipher, 1917), which allowed for a determination of
Figure 1.2: This figure, obtained from Leavitt and Pickering (1912), shows the period in days on the x-axis and the magnitude on the y-axis. The two curves refer to the magnitudes of the Cepheids at their maximum and minimum magnitude. This marked the discovery of the period-luminosity relation for Cepheids, which was vital to being able to determine distances on cosmological scales. The units on the y-axis should be km s\(^{-1}\).

the radial velocities of the stars. Using the distances and velocities of the Cepheid variables, Hubble was able to formulate his now famous law in 1929.

The crux of the concept of an expanding universe lies in the interpretation of Hubble’s Law. For example, the data show that non-gravitationally bound galaxies are receding from one another with a speed that is proportional to their distance. The simplest explanation is that the galaxies are simply moving through space. This interpretation of the data has several problems, however. Chief among them is that, if this were the case, then the expansion would have a center. This is in violation of the cosmological principle and the observational evidence that supports it, mainly the extremely high degree of isotropy of the CMB.

In light of this, we need another interpretation. The next simplest way to inter-
pret the data is to say that the galaxies are not moving *through* space, but rather *with* space. This leads to the conclusion that it is the distances themselves that are changing, as predicted by Friedmann.

We can see this by introducing the concept of comoving coordinates $x_c$. These are coordinates that change along with the supposed expansion that we are positing is occurring. That is, an object that is one meter away in comoving coordinates will always be one meter away, regardless of how the universe expands. This is contrasted with proper, or physical, coordinates $x_p$ that do not change with the expansion. That is, an object that is one meter away in proper coordinates at one moment in time will be farther away in proper coordinates at some later moment in time, dependent upon how the expansion is actually occurring. These two sets of coordinates are related via the scale factor as

$$x_p = a(t)x_c.$$  \hspace{1cm} (1.52)

We can get a handle on what the velocity of an object in an expanding universe looks like by taking the time derivative of Eq. (1.52) to obtain:

$$v \equiv \dot{x}_p = \dot{a}x_c + ax_c,$$ \hspace{1cm} (1.53)

where the first term in Eq. (1.53) is known as the Hubble velocity and the second term is known as the peculiar velocity. For distant galaxies (such as those not gravitationally bound to us) peculiar velocities are small (e.g., Mo et al. 2010), so making use of Eq. (1.52), we see that Eq. (1.53) can be written as

$$v \approx v_H = \dot{a}x_c = \frac{\dot{a}x_p}{a} = Hx_p,$$ \hspace{1cm} (1.54)

which is just Hubble’s Law. If the universe were static, then $\dot{a} = 0$, which means that, from the definition of $H$ in Eq. (1.15), $H = 0$. As can be seen from Fig. 1.1, Hubble
did not measure $H = 0$, which is therefore in keeping with what would be expected from an expanding universe. Additionally, if it is space itself that is expanding, then the problem of a center is circumvented as well, which keeps this idea in line with the observational evidence.

1.4.1 Cosmological Redshift

Cosmological redshift refers to the elongation of the wavelength of light due to the expansion of the universe. It is often simply referred to as the redshift, though this is dangerous without context as there are several other sources of redshift in astrophysics, including redshift due to peculiar motion and gravitational redshift. In this thesis, however, as we are explicitly concerned with the expansion of the universe, I will refer to cosmological redshift simply as redshift without fear of confusion, unless otherwise noted.

Since the redshift arises from the expansion of the universe, it is intimately linked to the scale factor. We can see this relationship by considering the FRW metric Eq. (1.1). For light we have the case $ds^2 = 0$, which follows from the definition of velocity as distance over time:

\begin{align*}
  c &= \frac{a(t)d\sigma}{dt}, \\
  c^2 &= \frac{a^2(t)d\sigma^2}{dt^2}, \\
  c^2 dt^2 &= a^2(t)d\sigma^2, \\
  0 &= -c^2 dt^2 + a^2(t)d\sigma^2,
\end{align*}

where Eq. (1.58) is just Eq. (1.1) with $ds^2 = 0$ (temporarily reintroducing factors of $c$ for clarity and recall that $d\sigma$ is given in comoving coordinates). Since Eq. (1.58) holds for the full spatial aspect of the metric $d\sigma^2$, this means it also holds for the special case of purely radial motion, which we consider for ease of computation but
without costing us generality. The brief derivation below follows Maoz (2007).

If we consider two wavefronts being emitted from a galaxy located at a comoving coordinate \( r_e \) at time \( t_e \) and \( t_e + dt_e \) and then arriving at an observer at times \( t_0 \) and \( t_0 + dt_0 \), there is then one instance of Eq. (1.58) for each wavefront (reverting to \( c = 1 \)):

\[
\int_{t_e}^{t_0} \frac{dt}{a(t)} = \int_0^{r_e} \frac{dr}{\sqrt{1 - Kr^2}}. \tag{1.59}
\]

\[
\int_{t_e}^{t_0 + dt_0} \frac{dt}{a(t)} = \int_0^{r_e} \frac{dr}{\sqrt{1 - Kr^2}}. \tag{1.60}
\]

Since \( r \) is a comoving coordinate, the right-hand sides of Eq. (1.59) and Eq. (1.60) must be equal because \( r \) does not depend on time. This, in turn, implies that the left-hand sides must also be equal to one another. This gives us:

\[
\int_{t_e}^{t_0 + dt_0} \frac{dt}{a(t)} - \int_{t_e}^{t_0} \frac{dt}{a(t)} = 0, \tag{1.61}
\]

\[
\int_{t_e}^{t_0} \frac{dt}{a(t)} - \int_{t_e}^{t_e + dt_e} \frac{dt}{a(t)} + \int_{t_0}^{t_0 + dt_0} \frac{dt}{a(t)} - \int_{t_e}^{t_0} \frac{dt}{a(t)} = 0, \tag{1.62}
\]

where the second step broke up the \( \int_{t_e}^{t_0 + dt_0} \frac{dt}{a(t)} \) term into three separate integrals. Since the time between the emission of successive wavefronts is a great deal smaller than the timescale over which the universe’s expansion changes appreciably we can approximate the values of the scale factor between the emission events as a constant and then as another constant between the two reception events. This approximation yields:

\[
\frac{\Delta t_e}{a(t_e)} = \frac{\Delta t_0}{a(t_0)}. \tag{1.63}
\]

If we now use the fact that \( \nu \lambda = c \), where \( \lambda \) is the wavelength of the light and \( \nu \) is the frequency of the light along with the fact that the period of oscillation for light is \( T = 1/\nu \) and that the time between the emission of successive wavefronts must be
one period, we get that \( T = \Delta t \). Plugging this into Eq. (1.63) gives:

\[
\frac{\Delta t_0}{\Delta t_e} = \frac{\lambda_0}{\lambda_e} = \frac{\nu_e}{\nu_0} = \frac{a(t_0)}{a(t_e)} = \frac{1}{a(t_e)} \equiv 1 + z,
\]

where \( z \) is the cosmological redshift because, from Eq. (1.64)

\[
z = \frac{\lambda_0}{\lambda_e} - 1 = \frac{\lambda_0 - \lambda_e}{\lambda_e},
\]

which shows that \( z \) describes a shift in wavelength in keeping with the definition of the classical Doppler effect, though in this case the relative motion between the source and observer is due to the expansion of the universe.

1.4.2 Cosmological Distances

The expansion of the universe complicates the measurement of distance on cosmological scales. In practice, there are only two quantities that can be directly measured for any astronomical object: the flux \( f \) and angular size of the object \( \vartheta \) (this is the angle subtended by the object on the sky). These two quantities allow for the determination of two different distances, known as the luminosity distance \( d_L \) and the angular diameter distance \( d_A \). These two distances are given in terms of the luminosity \( L \) of the object and the actual size \( D \), which are both intrinsic properties. These distances are defined as (e.g., Amendola and Tsujikawa, 2010)

\[
d_A = \frac{D}{\vartheta},
\]

\[
d_L = \sqrt{\frac{L}{4\pi f}},
\]

\[\text{With the first detection of gravitational waves (Abbott et al., 2016), there is now a third observable quantity, albeit a third that is, presently, not readily measurable for most astronomical sources.}\]
where in Eq. (1.66) the small angle approximation \( \sin \vartheta \approx \vartheta \) has been used. If the universe were not expanding then \( d_A = d_L \) as we would expect. In general, however, they are different in an expanding universe.

1.4.2.1 Comoving Distance

The comoving distance to an object can be found from the FRW metric. As was the case when deriving the redshift, we here make use only of the radial component of \( d\sigma^2 \) for ease of calculation without any loss of generality. We use the form of the FRW metric as given by Eq. (1.4) because it more simply encodes information about the curvature of the universe. If a source located at \( \chi = \chi_1 \) emits a light signal at \( t = t_e \) and we observe it at \( \chi = 0 \) and \( t = t_0 \), then the comoving distance \( d_c \) obtained from the FRW metric is

\[
d_c \equiv \chi_1 = \int_0^{\chi_1} \! d\chi = \int_{t_e}^{t_0} \! \frac{dt}{a(t)},
\]

\[
d_c = H_0^{-1} \int_0^z \! \frac{dz'}{E(z')},
\]

where

\[
E(z) \equiv \frac{H(z)}{H_0},
\]

\( E(z) \) is known as the expansion factor and in obtaining Eq. (1.69) I used \( dt = -dz/[H(1 + z)] \) which comes from Eq. (1.64) and Eq. (1.15).

1.4.2.2 Luminosity Distance

Eq. (1.67) is really just the definition of flux, which is defined as energy per time per area. Since the source emits light in all directions, the wavefronts traveling outwards from the source define an expanding sphere that is centered on the object. When the boundary of this sphere reaches us, the radius of the sphere is just the luminosity distance to the object. However, due to the expansion of the universe,
the source luminosity $L$ is different than the luminosity we would calculate from Eq. (1.67) if we knew the distance to begin with. This is because the wavelength of the light is elongated as the universe expands, which changes its energy and, therefore, its luminosity. This means that the observed flux is a function of this observed luminosity $L_0$. The flux we observe is given by

$$f = \frac{L_0}{S}, \quad (1.71)$$

where $S$ is the area of the sphere centered on us that extends out to the source. The radius of this sphere is just $f_K(\chi)$ (given in Equation (1.5)), which makes $S = 4\pi f_K^2(\chi)$. This makes the luminosity distance

$$d_L^2 = f_K^2(\chi)\frac{L}{L_0}. \quad (1.72)$$

The final step is to therefore find $L/L_0$. To do this, we use the fact that $L = dE/dt$ and, for a photon, $E = 1/\lambda$. This yields:

$$\frac{L}{L_0} = \frac{dE}{dE_0} \frac{dt}{dt}, \quad (1.73)$$

$$= (1 + z)^2, \quad (1.74)$$

where in the last step we have used Eq. (1.64). If we now plug in Eq. (1.74), Eq. (1.5), and Eq. (1.69) into Eq. (1.72) we obtain

$$d_L = \frac{(1 + z)}{H_0\sqrt{\Omega_K,0}} \sinh \left( \sqrt{\Omega_K,0} \int_0^z \frac{dz'}{E(z')} \right). \quad (1.75)$$

1.4.2.3 Angular Diameter Distance

In the case of the angular diameter distance, the angle subtended by the object $\vartheta$ is measured, which means that the only part of Eq. (1.66) that is unknown is $D$. In
order to find $D$, we consider that the object is on the surface of a sphere centered on us with radius $a(t)f_K(\chi)$. Using the definition of arc length $l = R\vartheta$, where $l$ is the arc length on the surface of the sphere ($l = D$, in this case) and $R$ is the radius of the sphere. One then has

$$D = a(t)f_K(\chi)\vartheta. \quad (1.76)$$

Plugging Eq. (1.76) into Eq. (1.66) and using Eq. (1.5) and Eq. (1.64) gives

$$d_A = \frac{1}{H_0(1+z)\sqrt{\Omega_{K,0}}} \sinh \left( \sqrt{\Omega_{K,0}} \int_0^z \frac{dz'}{E(z')} \right). \quad (1.77)$$

This leads to the Etherington relation

$$d_A = \frac{d_L}{(1+z)^2}. \quad (1.78)$$

Eq. (1.78) shows that, in general, the luminosity distance and the angular diameter distance differ from one another in an expanding universe (though for very low redshifts the two are approximately the same).

1.5 The Formation of Large-Scale Structure in the Universe

As discussed below in Section 1.7, I am interested in studying the effects of time-dependent dark energy on the flux power spectrum of the Ly$\alpha$ Forest because it provides a way of probing the large-scale structure (LSS) in the universe. In order to understand how dark energy affects the formation and evolution of this structure, and the role of the power spectrum in probing this structure, I present a brief overview of the theory of structure formation in the universe. I refer the interested reader to Dodelson (2003); Mo et al. (2010) for more complete treatments of this topic.

The prevailing paradigm for the formation and evolution of LSS in the universe is that the initial energy density in the universe was not perfectly smooth, but rather had
small inhomogeneities. Over time, the slightly denser regions began to gravitationally
attract more matter than the surrounding regions owing to their slightly increased
density, whereas the under-dense regions were vacated by the same process. This
eventually led to the cosmic web that we observe today.

The most rigorous and complete way to study the formation and evolution of
LSS is to perturb GR. However, such a treatment is outside the scope of the modest
overview I am presenting here. As such, for simplicity, I here consider only Newtonian
perturbation theory.

There are four equations that govern the evolution of a fluid: the continuity
equation Eq. (3.1), the Euler equation Eq. (3.2), the Poisson equation Eq. (3.3), and
an equation of state relating the pressure and density. Since we are interested in
studying how density inhomogeneities give rise to LSS, we write the density as

\[ \rho(x, t) = \bar{\rho}(t)[1 + \delta(x, t)], \]

(1.79)

where \( x \) is the comoving position, \( \bar{\rho}(t) \) is the average background density, and \( \delta \)
the density perturbation contrast (i.e. the overdensity). Using Eq. (1.79) and writing
Eqs. (3.1),(3.2), and Eq. (3.3) in terms of comoving coordinates gives

\[
\frac{\partial \delta}{\partial t} + \frac{1}{a} \nabla \cdot [(1 + \delta) \mathbf{v}] = 0,
\]

(1.80)

\[
\frac{\partial \mathbf{v}}{\partial t} + \frac{\dot{a}}{a} \mathbf{v} + \frac{1}{a} (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{\nabla \Phi}{a} - \frac{\nabla P}{a \bar{\rho}(1 + \delta)},
\]

(1.81)

\[
\nabla^2 \Phi = 4\pi G \bar{\rho} a^2 \delta,
\]

(1.82)

\[
\Phi \equiv \phi + \frac{a \ddot{a} x^2}{2}.
\]

(1.83)

The equation of state is generally determined based on the thermodynamic pro-
cesses that the fluid is undergoing. For an ideal, non-relativistic, monotomic gas, the
first law of thermodynamics gives (e.g., Mo et al., 2010)

\[
\frac{\nabla P}{\rho} = c_s^2 \nabla \delta + \frac{2}{3} (1 + \delta) T \nabla S, \tag{1.84}
\]

where \(S\) is the specific entropy and \(c_s\) is the adiabatic sound speed given by

\[
c_s = \left( \frac{\partial P}{\partial \rho} \right)^{\frac{1}{2}}_S. \tag{1.85}
\]

Applying Eq. (1.84) to the Euler equation, keeping only the linear terms, and then combining the resulting equations gives the equation of motion for the perturbations (e.g., Mo et al., 2010)

\[
\frac{\partial^2 \delta}{\partial t^2} + 2 \frac{\dot{a}}{a} \frac{\partial \delta}{\partial t} = 4\pi G \bar{\rho} \delta + \frac{c_s^2}{a^2} \nabla^2 \delta + \frac{2}{3} \bar{T} \frac{a}{a^2} \nabla^2 S, \tag{1.86}
\]

where \(\bar{T}\) is the temperature of the background. We see from Eq. (1.86) that both the density and entropy contribute to the evolution of the density fluctuations. This gives rise to two different sets of initial conditions: isentropic, for which the initial perturbation is in the density, and isocurvature perturbations, for which the initial perturbation is in the entropy. This corresponds to spatial variations in pressure, which causes compression and expansion, thereby giving rise to density fluctuations.

In this linear regime that we are considering, we expand the perturbation fields using some basis. In this particular case, it turns out that it is convenient to choose these basis functions to be plane waves. This allows us to represent the perturbation fields by their Fourier transforms. Taking the Fourier transform of Eq. (1.86) gives

\[
\frac{d^2 \delta_k}{dt^2} + 2 \frac{\dot{a}}{a} \frac{d \delta_k}{dt} = \left[ 4\pi G \bar{\rho} - \frac{k^2 c_s^2}{a^2} \right] \delta_k - \frac{2}{3} \frac{\bar{T}}{a^2} k^2 S_k. \tag{1.87}
\]

Solving Eq. (1.87) provides the evolution for the various perturbation modes in the density field.
If we now consider a pressureless fluid (such as dark matter) with isentropic initial conditions, as are predicted by inflation, (e.g., [Mo et al., 2010], then Eq. (1.87) reduces to

\[ \frac{d^2 \delta_k}{dt^2} + 2 \frac{\dot{a}}{a} \frac{d\delta_k}{dt} = 4\pi G \bar{\rho} \delta_k. \]  

(1.88)

If \( \delta_+ \) and \( \delta_- \), referred to as the growing mode and decaying mode, respectively, are two known solutions to Eq. (1.88), then these two solutions obey (Mo et al., 2010)

\[ \delta_- \dot{\delta}_+ - \delta_+ \dot{\delta}_- = Ca^{-2}, \]  

(1.89)

where \( C \) is a constant. Eq. (1.16) can be written as

\[ \frac{dH}{dt} + H^2 = - \frac{4\pi G}{3} \bar{\rho}, \]  

(1.90)

which, when differentiated once with respect to time, gives

\[ \frac{d^2 H}{dt^2} + 2 \frac{\dot{a}}{a} \frac{dH}{dt} = 4\pi G \bar{\rho} H, \]  

(1.91)

where I used \( \bar{\rho} = \bar{\rho}_m \propto a^{-3} \). This shows that both \( \delta_k \) and \( H \) obey the same differential equation. Since \( H \) decreases with time, we can identify \( H \) as the decaying mode solution; i.e., \( \delta_- \propto H \). Using this in Eq. (1.89) lets us solve for the growing mode as

\[ \delta_+ \propto H(t) \int_0^t \frac{dt'}{a^2(t')H^2(t')} \propto D_1(t), \]  

(1.92)

where \( D_1(t) \) is the linear growth factor. Eq. (1.92) therefore shows that the growth factor is dependent upon \( H \), which, in turn, is dependent upon the dark energy model being used. The power spectrum is then related to the density perturbations as (Mo et al., 2010)
\[ P(k, t) = \langle |\delta(k, t)|^2 \rangle, \tag{1.93} \]

where the angled brackets indicate an average. Therefore, we see that the power spectrum is a function of the density perturbations that are, in turn, a function of the growth factor and, therefore, the Hubble parameter. However, as this is not a direct relation, it can be expected that the signature from dark energy on the power spectrum, if it exists, will be a small effect.

1.6 The Lyman \( \alpha \) Forest

The Ly\( \alpha \) Forest is the tight collection of absorption lines present in the spectra of quasars (also known as QSOs, or quasi-stellar objects). See Mo et al. (2010) for an excellent discussion of quasars. Light emitted by the quasar invariably encounters material in the intergalactic medium (IGM) as it travels towards the observer. This material consists of a small fraction of neutral hydrogen. When light at a wavelength of \( \approx 1216 \) Å encounters this neutral hydrogen, it is absorbed via the Ly\( \alpha \) transition\(^3\).

As the light continues to travel and the universe expands, the wavelength of the light is redshifted away from its rest wavelength. Additionally, light at shorter wavelengths is elongated towards the required wavelength, which allows for further Ly\( \alpha \) absorption to occur. Therefore, each cloud of absorbing material along the line of sight (LOS) between the observer and the quasar is imprinted onto the spectrum as an absorption feature that has been shifted from the rest wavelength of \( \approx 1216 \) Å. This is what forms the Ly\( \alpha \) Forest. A schematic of the Ly\( \alpha \) Forest is shown in Fig. 1.3 and an example of the Ly\( \alpha \) Forest is shown in Fig. 1.4.

The Ly\( \alpha \) Forest was first predicted by Gunn and Peterson (1965) and Scheuer (1965) and then observed by Lynds and Stockton (1966), Burbidge et al. (1966),

\(^3\)Ly\( \alpha \) absorption is when the electron in the hydrogen atom transitions from the ground \((n = 1)\) state to the first \((n = 2)\) excited state.
Figure 1.3: This figure (Wright, 2004) shows a schematic of the Lyα Forest. The upper portion of the figure highlights the physical situation. Light is emitted by the quasar and encounters material in the IGM that contains neutral hydrogen. This neutral hydrogen causes Lyα absorption. Then, as the light continues to travel the universe expands, which causes the wavelength of the light to become longer (redder). This elongation results in the position of the absorption line being shifted to a longer wavelength in accordance with the redshift (distance) of the absorber. Concurrently, light at shorter wavelengths that previously could not induce Lyα absorption is elongated to $\approx 1216$ Å and is subsequently able to be absorbed by neutral hydrogen. This tight collection of redshifted Lyα absorption lines is known as the Lyα Forest. The lower portion of this figure gives an example of the spectrum we would observe from the situation shown in the upper portion. Here the color of the line becomes redder to the right in order to indicate that it corresponds to longer wavelengths on the spectrum. That is, the light that is redshifted the most is the light shown in the quasar’s emission line (the strong peak at the right end of the spectrum) because the quasar is the most distant object observed in this situation. The intervening clouds are not as far away and so the light is not stretched (redshifted) as much.
Figure 1.4: This figure [Ellison, 2000] shows an example of the Lyα Forest. The Lyα emission line is prominently featured, and establishes the redshift of the quasar via $\lambda = \lambda_0(1 + z)$, where $\lambda$ is the observed wavelength and $\lambda_0$ is the rest wavelength of the Lyα transition $\approx 1216 \, \text{Å}$. Blueward of this emission line is the multitude of Lyα absorption lines that comprise the Lyα Forest.

Stockton and Lynds (1966), and Kinman (1966). However, it was not until later observations that the observed absorption lines were confirmed to be Lyα lines of H Romani (Baldwin et al., 1974). Even then, a coherent picture of the absorbers responsible for causing the Lyα Forest remained elusive until later when hydrodynamic simulations were able to show that the Lyα Forest was caused by fluctuations in the density field between the observer and the QSO, thereby making a map of the cosmic web (e.g., Bi, 1993; Cen et al., 1994; Hernquist et al., 1996; Theuns et al., 1998).

Of particular interest to my project is the fact that, on large scales, the distribution of baryonic matter largely follows that of the dark matter (e.g., Meiksin and White, 2001). As shown in Section 3.6, the absorption in the Lyα Forest is related to the matter density. This means that the power spectrum (see Section 1.7) of the flux transmission is related to the density fluctuation $\delta$ (e.g., Croft et al., 1998; McDonald et al., 2000; Mo et al., 2010). Since the Lyα Forest is readily observable across a large
redshift range, the flux power spectrum of the Lyα Forest provides a way of studying the underlying cosmic density field, described in Section 1.5, the evolution of which is sensitive to dark energy. See Rauch (1998) for an excellent review of the history and physics of the Lyα Forest.

1.7 Motivation

Currently, observational constraints on allowed dark energy models are quite weak (e.g., Planck Collaboration et al., 2015), as shown in Figure 1.5. This has given rise to a plethora of models in the literature, ranging from the vacuum energy (Zel’dovich, 1968) all the way to exotic fields known as ghosts (e.g., Piazza and Tsujikawa, 2004). Each model proposes both a different physical mechanism by which the expansion is accelerated and, as such, a different expansion history for the universe. There is currently no *a priori* reason to favor one model over any other.

From a purely empirical standpoint, then, determining the expansion history (which is equivalent to measuring the Hubble parameter $H$ at various times throughout the cosmic evolution) serves as a way of constraining dark energy. While straightforward in theory, this exercise is extremely difficult in practice, and has so far only been done for a small handful of very low redshifts (e.g., Freedman and Madore, 2010).

Adding to this observational challenge is the fact that dark energy did not begin to dominate the dynamics of the universe until relatively recently (e.g., Amendola and Tsujikawa, 2010). For much of the universe’s history, dark energy was sub-dominant to the other forms of matter. This tends to suppress any model-specific signatures that may have been present in the past. Furthermore, the present-day value of the dark energy EOS, $w_0$, has been measured to be very close to $-1$ (e.g., Garnavich et al., 1998; Planck Collaboration et al., 2015). This creates a strong degeneracy between all of the viable dark energy models because each must exhibit similar behavior at
Figure 1.5: This figure shows the marginalized posterior distributions for $w_0$ and $w_a$ for various combinations of data sets. The Planck + BSH is the combination of the CMB, BAO, SNIa, and $H_0$, Planck + WL is the CMB and weak lensing data, Planck + BAO/RSD is the CMB and BAO/redshift-space distortions, and Planck + WL + BAO/RSD is the CMB, weak lensing, and BAO/redshift-space distortions. These distributions show the weak observational constraints on dark energy. The dashed lines correspond to $w_0 = -1$ and $w_a = 0.0$, which are the parameters for the cosmological constant model of dark energy, discussed in Section 2.2.1.
recent times in order that $w(z = 0) \approx -1$. These two facts cause model differences to be extremely difficult to detect.

This degeneracy implies several things: first, the observable differences between viable dark energy models will most likely be small. Second, no one observational probe will be capable of breaking this degeneracy on its own (e.g., Gerke and Efstathiou, 2002). It is therefore only by accruing many observations of multiple probes that enough evidence can be gathered in order to put forward a convincing case for one model over any other (e.g., Kujat et al., 2002). There are, however, two primary problems with the current observational data with regards to dark energy: first, the contiguous redshift range that is probed is quite small, going out only to $z \approx 2$, and second, the number of measurements is relatively small.

Due to these issues, here we investigate the viability of using the Ly$\alpha$ Forest as a tool for constraining time-dependent dark energy. The Ly$\alpha$ Forest has already been shown to be a powerful tool (e.g., Weinberg et al., 2003) for constraining cosmological parameters and studying the physical properties of the IGM, but its use as a probe of dark energy is relatively unexplored (see, however Greig, 2013; Kujat et al., 2002; Lidz et al., 2003; Viel et al., 2003). The benefits of the Ly$\alpha$ Forest are that it, unlike other observables, probes a large, contiguous, redshift range (from $z \approx 2$ to $z \approx 6$)$^4$, it covers a redshift range not probed by other observables, and has a large number of excellent observations and readily available data. The redshift ranges of several dark energy probes are illustrated in Figure 1.6.

Conceptually, the justification for using the Ly$\alpha$ Forest to probe dark energy is as follows. Since dark energy possesses a negative energy density, it is thought that the effects of dark energy, with respect to the cosmic web, should be most apparent on the morphology of voids (e.g., Biswas et al., 2010; Bos et al., 2012; Lavaux and

$^4$The Ly$\alpha$ Forest is actually visible at lower redshifts than this, but not from the ground, because the Ly$\alpha$ line is shifted into the UV. As such, these measurements of the low-redshift Ly$\alpha$ Forest must be made from space.
Figure 1.6: This figure shows the Hubble parameter $H(z)$ as a function of redshift. The highlighted regions show the redshift range presently spanned by the corresponding cosmological probe. We see that the Ly$\alpha$ Forest spans a much larger redshift range than either BAO or SNIa, as well as extends to redshifts that those probes do not.

The absorbers responsible for the Ly$\alpha$ Forest should reside primarily in the clusters and filaments (e.g., Bi et al., 1995; Cen et al., 1994). However, along a given line of sight (LOS), some of these absorbers will be separated by the voids. As such, the separation of these absorbers in redshift space should act as a tracer of the evolution of the voids (e.g., Viel et al., 2003). An example of this concept is shown in Figure 1.7.

Stated more rigorously, the linear growth factor $D_1$ is a function of $H(z)$, which means that it is dependent upon the dark energy model being used. For high redshifts ($z > 2$), the best way to measure $D_1$ is through the flux power spectrum of the Ly$\alpha$ Forest (Kujat et al., 2002; Seljak et al., 2002). This means that, by simulating the flux power spectrum of the Ly$\alpha$ Forest under various dark energy models, I am searching for the empirical effects of the linear growth factor’s dependence on $\zeta(a)$.

In particular, I look at a statistic of the Ly$\alpha$ Forest known as the flux power spectrum (hereafter referred to simply as the power spectrum). A power spectrum is a measure of an amplitude as a function of scale. As discussed in more detail in Section 4.1 on large scales the distribution of baryons that gives rise to the Ly$\alpha$
Figure 1.7: This figure, from Bolton et al. (2017), illustrates the concept of using the Lyα Forest as a probe of the cosmic web. The figure shows the projected gas density in the simulation at $z = 2$. The dashed line corresponds to the line-of-sight through the simulation volume, and the corresponding Lyα Forest spectrum arising from neutral hydrogen absorption along this line-of-sight is shown in the green curve below the dashed line. The blue curve above the dashed line shows the gas density along the line-of-sight.
Forest follows that of the underlying dark matter distribution that forms the frame of the cosmic web, meaning that the flux power spectrum is related to the underlying dark matter power spectrum of the cosmic web (e.g., [Croft et al., 1998, 1999]). This implies that the effects of dark energy on the voids should manifest themselves in the power spectrum at large scales, where the absorbers are separated by the voids. Additionally, studying the Lyα Forest is an independent and complimentary approach to searches based on the SNIa redshift-distance relation, the CMB, BAO, and gravitational lensing.

The primary goal of this thesis, then, is to answer the following question: does time-dependent dark energy leave an observationally significant signature in the flux power spectrum of the Lyα Forest? In order to answer this question, I make use of high-resolution, large-scale cosmological simulations. Each simulation is run with a different dark energy model. From my simulations I extract synthetic Lyα Forest spectra, which I use to predict the flux power spectrum for each dark energy model. The power spectra from the time-dependent dark energy models are compared to the power spectra from the cosmological constant in order to determine whether or not the effects of dark energy on the power spectrum are strong enough to be detected.

This thesis is laid out in the following manner: Dark energy is discussed in detail in Chapter 2. The numerical methods employed for this project are discussed in Chapter 3. My results are presented in Chapter 4. Conclusions and discussion are given in Chapter 5.
CHAPTER 2
DARK ENERGY

“One and one and one is three,” got to be good looking ’cause he’s so hard to see
—John Lennon/Paul McCartney, Come Together

This chapter provides an introduction and overview of dark energy. I begin with a review of the observational evidence supporting the existence of cosmic acceleration in Section 2.1 before moving on to various specific models of dark energy, including the cosmological constant in Section 2.2.1, quintessence in Section 2.2.2.1, k-essence in Section 2.2.2.2, and phantom models in 2.2.2.3.

2.1 Observational Evidence

2.1.1 Type Ia Supernovae

The first, and still the strongest, piece of evidence in favor of cosmic acceleration is the luminosity distance vs. redshift relation based upon SNIa. A Type I supernova is identified by the lack of hydrogen lines in its spectrum and is further classified as a Type Ia supernova if its spectrum contains a line of singly ionized silicon (e.g., Hillebrandt and Niemeyer, 2000). While the exact progenitor for SNIa remains ambiguous (e.g., Parrent et al., 2014), it is believed that they are caused when a white dwarf approaches the Chandrasekhar limit of $\approx 1.4M_\odot$ (e.g., Parrent et al., 2014). This is because as a white dwarf approaches the Chandrasekhar limit leads to gravitational collapse as electron degeneracy pressure is overcome.
The two most popular progenitor models for SNIa are the singly-degenerate and doubly-degenerate models. In the former case, a white dwarf exists in a binary system. As this companion star evolves, it expands and fills its Roche Lobe, which causes material to accrete onto the white dwarf, thereby increasing its mass. The doubly-degenerate case involves two white dwarfs in a binary system that spiral together and eventually merge.

Since the white dwarf going supernova will have approximately the same mass and chemical composition in each instance, each SNIa reaches nearly the same absolute magnitude $M$ at peak brightness of $M \approx -19$ (e.g., Hillebrandt and Niemeyer, 2000). This fact makes SNIa excellent standardizable candles.

The absolute magnitude, apparent magnitude $m$, and luminosity distance are related by the distance modulus

$$m - M = 5 \log \left( \frac{d_L}{10 \text{pc}} \right).$$

(2.1)

Since the absolute magnitude is known and the apparent magnitude is observed, the luminosity distance can be calculated (e.g., Perlmutter et al., 1999, Riess et al., 1998). By measuring the luminosity distances of many SNIa at various redshifts, the redshift dependence of the the luminosity distance can be established and compared with the theoretical expectation from Eq. (1.75).

The first observational evidence of cosmic acceleration came from Riess et al. (1998) and Perlmutter et al. (1999), though a non-zero $\Omega_{DE,0}$ had been previously suggested in the literature (e.g., Ostriker and Steinhardt, 1995a,b). Results from Riess et al. (1998) are presented in Fig. 2.1. This figure shows that the measured luminosity distances are larger than what can be accounted for in a universe without cosmic acceleration. The implication of these larger distances is that the expansion rate of the universe must be accelerating in order to allow for the SNIa host galaxies
to have receded as far as they have. Additionally, by having the distances at several
different redshifts, it is apparent that the expansion rate has continued to accelerate
with time.

Fig. 2.2 more clearly demonstrates the marked effect of dark energy on the lu-
minosity distance. As the amount of dark energy is increased (the acceleration is
increased), the distance measures become increasingly large.

At the time that Fig. 2.1 was generated, the standard cosmological model was of
a flat, matter dominated universe. However, it is also possible for an open universe to
generate luminosity distances larger than those possible in a flat matter dominated
universe. This creates a possible degeneracy between dark energy and curvature.
However, studies of the CMB anisotropies have constrained the curvature of the
universe to be very close to zero (Komatsu et al., 2011; Planck Collaboration et al.,
2014, 2016b), which implies that an open curvature is not the source of the larger-
than-expected luminosity distances.

2.1.2 The Age of the Universe

With the discovery of the expansion of the universe by Hubble in 1929, it became
apparent that the universe was not infinitely old. This is because if galaxies are
continuously receding from each other, then they must have been closer together in
the past. Running this scenario to its logical extreme results in the singularity from
which the universe was birthed and the idea of the Big Bang.

A lower limit for the age of the universe can be determined by measuring the ages
of the oldest stars. Such stars are generally found in globular clusters, and have ages
up to $\approx 13$ Gyr (e.g., Carretta et al., 2000).

This observational limit can be compared to the theoretical value for the age of
the universe. By using the relation between time and redshift $dt = -dz/[(1+z)H]$ we
can integrate from the birth of the universe (which occurs at $a = 0$ and corresponds
Figure 2.1: This figure is Fig. 4 in Riess et al. (1998). The top panel shows the Hubble diagram from their SNIa dataset along with three cosmological fits. The first, and best, fit describes a flat universe dominated by dark energy. The other two are describe a flat universe without dark energy and an open universe without dark energy, respectively. The bottom panel shows the differences between the data and the models with respect to the $\Omega_{m,0} = 0.2, \Omega_{DE,0} = 0.0$ model.
Figure 2.2: This figure demonstrates the effect that different amounts of dark energy have on the luminosity distance as a function of redshift. Including more dark energy increases the acceleration of the expansion, which leads to larger distance measures. The blue curve, which is for a model with $\Omega_{m,0} = 0.3, \Omega_{\Lambda,0} = 0.7$, and $h = 0.6731$ in a flat universe, is closest to the best-fit model preferred by the current observational data. This is reproduction of Figure 5.2 from Amendola and Tsujikawa (2010).

to $z = \infty$) to the present day of $a_0 = 1$ (corresponding to $z_0 = 0$) to get

$$t_0 = H_0^{-1} \int_0^\infty \frac{dz'}{E(z')(1 + z')} , \quad (2.2)$$

which yields an age of $\approx 10$ Gyr in the case of a flat universe without dark energy (e.g., Salaris et al. [1997] and references therein). This value is smaller than the lower limit for the age as determined from observations. There are two ways of resolving this issue: the first is to include dark energy in the evaluation of Eq. (2.2), and the second is to posit that the universe has negative curvature, as shown in Fig. 2.3.

Since both dark energy and negative curvature have the same effect on the calculated age, there is therefore a degeneracy between them, as noted in 2.1.1. Fortunately, measurements of CMB anisotropies provide strict constraints on the value of $\Omega_{K,0}$ that are consistent with zero and, therefore, a flat universe. This suggests
Figure 2.3: The age of the universe as a function of $\Omega_{m,0}$ for both a flat and open universe (with no dark energy in the open case). When the universe is completely matter dominated (i.e. $\Omega_{m,0} \rightarrow 1$), the age dips below the lower limit set by observations ($\approx 11\text{Gyr}$). This problem is alleviated in the flat case by including dark energy. From the dashed curve we see that a larger $\Omega_{K,0}$ in an open universe without dark energy is also capable of increasing the calculated age, thereby establishing a degeneracy between dark energy and open curvature. I use $H_0 = 67.31\text{km}\text{s}^{-1}\text{Mpc}^{-1}$.

that dark energy is consistent with the measured age of the universe. Indeed, it is encouraging that the values $\Omega_{m,0} \approx 0.3$ and $\Omega_{\text{DE},0} \approx 0.7$ that best-fit the SNIa data are also the values that give rise to an age of the universe that is consistent with globular cluster measurements, as the two probes are completely independent of one another.

2.1.3 The Cosmic Microwave Background

First discovered by accident in 1964 by Penzias and Wilson (Penzias and Wilson 1965), the CMB is the strongest piece of evidence supporting the Big Bang. It is the surface of last scattering from the era of recombination at $z \approx 1100$ (e.g., Amendola and Tsujikawa 2010), when the universe had expanded and cooled enough to allow electrons and protons to combine, thereby removing the source of scattering for photons and allowing them to free stream (e.g., Liddle and Lyth 2006).

The primordial density field was not uniform (due to quantum fluctuations during inflation) (e.g., Liddle and Lyth 2006), and the slight differences in density manifest
Figure 2.4: This figure ([Planck Collaboration et al., 2016a]) shows the power spectrum of the temperature anisotropies in CMB in the upper panel and the residuals are shown in the lower panel.

themselves as acoustic waves in the primordial plasma. Once recombination occurred, however, the source of the pressure driving these waves was removed\(^1\) and these anisotropies were frozen into the temperature distribution of the photons. They are observable today as the peaks in the power spectrum of the CMB\(^2\), which is shown in Fig. 2.4.

The angular location of the peaks in the power spectrum depends upon the expansion history of the universe through the angular diameter distance and the so-called

\(^1\)The photons and baryons were coupled together via Compton scattering. Once recombination occurred, the free electrons were removed from the plasma, which ended the Compton scattering and allowed the photons to free-stream. (e.g., Mo et al., 2010)

\(^2\)A power spectrum is simply the measure of a fluctuation strength as a function of scale.
Figure 2.5: This figure, recreated from Amendola and Tsujikawa (2010), shows the dependence of $R$ on the amount of dark energy. The horizontal green band represents the allowed bounds on $R$ as determined by WMAP (Komatsu et al., 2009).

The shift parameter $\mathcal{R}$ defined as

$$\mathcal{R} = \sqrt{\frac{\Omega_{m,0}}{\Omega_{K,0}}} \sinh\left(\sqrt{\Omega_{K,0}} \int_0^{z_r} \frac{dz}{E(z)} \right), \quad (2.3)$$

where $z_r$ is the redshift of recombination. The dependence of $\mathcal{R}$ on the amount of dark energy is shown in Fig. 2.5. The horizontal green band represents the allowed values of $\mathcal{R}$ as determined by WMAP (Komatsu et al., 2009). This is also consistent with $\Omega_{DE,0} \approx 0.7$ as determined by both age of the universe measurements and the SNIa data.
2.1.4 Baryon Acoustic Oscillations

Just as the CMB anisotropies are the result of primordial quantum fluctuations that become frozen into the photon distribution after recombination, baryon acoustic oscillations (BAOs) are due to the freeze-in of these fluctuations into the baryonic density field after recombination. BAOs were first detected as a peak in the galaxy correlation function in Eisenstein et al. (2005).

Fig. 2.6 shows the first detection of the BAO signal in observational data, as measured by Eisenstein et al. (2005). The signal occurs in the two-point correlation function at a comoving scale of \( \approx 150 \) Mpc. Analogous to how SNIa serve as standard
candles for the luminosity distance, the BAO signal acts as a standard ruler for the angular diameter distance. Since the actual size of the scale is known and the angular size can be measured on the sky, the BAO signal allows for an accurate determination of the angular diameter distance.

The distance as determined from the BAO signal depends upon the amount of dark energy in the universe, as illustrated in Fig. 2.7. Fig. 2.7 plots the BAO distance for three different cosmological models alongside the actual measurements, shown as black points. While there are only two data points, the data are clearly best-fit by the cosmological model with \((\Omega_{m,0} = 0.3, \Omega_{DE,0} = 0.7)\). This agrees with the values as determined by the other observational probes. Since these probes are independent from one another, the data presents a picture of a universe whereby most of the energy content is in the form of some unknown substance that gives rise to an accelerated expansion.

2.2 Models of Dark Energy

2.2.1 The Cosmological Constant

The cosmological constant is the simplest model of dark energy. It posits that the mechanism behind cosmic acceleration is the negative pressure generated by the vacuum state of spacetime itself. A vacuum energy has been shown to exist, for example, through experiments demonstrating the Casimir Effect (e.g., Casimir, 1948; Casimir and Polder, 1948; Lamoreaux, 1997, though see, e.g., Jaffe (2005) for a different opinion). Since there is a non-zero energy density associated with the vacuum, GR says that it must gravitate, entering the EFE as a new component of the energy-momentum tensor

\[
R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} = 8\pi G \left( T_{\mu\nu}^{(M)} + T_{\mu\nu}^{(\text{vac})} \right),
\]  

(2.4)
Figure 2.7: This figure, recreated from Amendola and Tsujikawa (2010), shows the BAO distance as a function of redshift for three different cosmological models: $(\Omega_m,0 = 0.3, \Omega_{DE,0} = 0.7)$, $(\Omega_m,0 = 1.0, \Omega_{DE,0} = 0.0)$, and $(\Omega_m,0 = 0.1, \Omega_{DE,0} = 0.9)$. The measured BAO distances are shown as the black points. The data, while sparse, is still clearly best-fit by the $(\Omega_m,0 = 0.3, \Omega_{DE,0} = 0.7)$ model, which is in accordance with the values determined from age measurements, the CMB, and SNIa.
where the net energy-momentum tensor has been broken up into a matter component \( T^{(M)}_{\mu\nu} \) and a vacuum component \( T^{(\text{vac})}_{\mu\nu} \) such that \( T_{\mu\nu} = T^{(M)}_{\mu\nu} + T^{(\text{vac})}_{\mu\nu} \).

In order to specify the form of \( T^{(\text{vac})}_{\mu\nu} \), we must impose that it be both Lorentz invariant such that it looks the same to all observers. These two restrictions imply that the energy density must be both constant and proportional to the metric (e.g., Carroll et al. 1992)

\[
T^{(\text{vac})}_{\mu\nu} = -\rho_{\text{vac}} g_{\mu\nu}.
\]

(2.5)

Comparing Eq. (2.5) with the form of the energy-momentum tensor of a perfect fluid Eq. (1.12) we see that Eq. (2.5) is a perfect fluid with \( P = -\rho_{\text{vac}} \), which gives an EOS of \( w = -1 \). Plugging Eq. (2.5) into Eq. (2.4) we obtain

\[
R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T^{(M)}_{\mu\nu},
\]

(2.6)

where

\[
\Lambda \equiv 8\pi G \rho_{\text{vac}}
\]

(2.7)

is the cosmological constant.

The introduction of the cosmological constant to the EFE also changes the form of the Friedmann and acceleration equations to

\[
H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2} + \frac{\Lambda}{3},
\]

(2.8)

\[
\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3P) + \frac{\Lambda}{3},
\]

(2.9)

3If this were not the case, then there would be a preferred direction, and this would violate the cosmological principle.
We see from Eq. (2.9) that a positive cosmological constant has a repulsive effect. Historically, this was the reason that Einstein originally introduced the cosmological constant. Einstein retracted the cosmological constant when Hubble’s observations helped to show that the universe was not static but was, in fact, expanding. While the possible existence of a cosmological constant remained a part of the literature, it gained credibility as a result of the SNIa observations in the late nineties (Perlmutter et al., 1999; Riess et al., 1998).

Despite its mathematical simplicity, association with a known physical mechanism, and consistency with all current observational data, the cosmological constant is plagued by two major problems: the fine-tuning problem and the coincidence problem.

2.2.1.1 The Fine-Tuning Problem

The fine-tuning problem refers to a discrepancy between the observed and theoretical values of $\rho_{\text{vac}}$. The observed value of $\rho_{\text{vac}}$ can be estimated from observations of $H_0$. This is because, presently, dark energy is the dominant component of the universe and so from Eq. (2.8) we see that $\Lambda \approx H_0^2$. Using this relation, along with a value of $H_0 \approx 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and Eq. (2.7) gives

$$\rho^{(\text{obs})}_{\text{vac}} \approx 10^{-47}\text{GeV}^4.$$

In order to calculate the theoretical value of $\rho_{\text{vac}}$, I follow Carroll et al. (1992). The process starts by calculating the energy of the ground state (the zero-point energy) of a simple harmonic oscillator in quantum mechanics, which works out to be (e.g., Sakurai 1985)

$$E_0 = \frac{1}{2} \hbar \omega,$$

where $\hbar$ has been temporarily reintroduced for clarity and $\omega$ is the frequency of
oscillation for the particle. This situation generalizes to quantum field theory by considering a quantum field to be a collection of simple harmonic oscillators, which gives (e.g., Carroll et al., 1992)

$$E_0 = \sum_j \frac{1}{2} \hbar \omega_j.$$  (2.12)

The sum in Eq. (2.12) can be evaluated by imposing periodic boundary conditions and then letting the size $L$ of the imposed volume go to infinity. Imposing periodic boundary conditions forces the wavelength in the $i$'th direction to be

$$\lambda_i = \frac{L}{n_i}.$$  (2.13)

where $n_i$ is an integer. Solving Eq. (2.13) for $n_i$ and converting the wavelength to wavenumber $k$ via $\lambda = (2\pi)/k$ gives

$$n_i = \frac{k_i L}{2\pi},$$  (2.14)

which is the number of values of $k$ (modes) that fit within the range $(k_i, k_i + dk_i)$. Eq. (2.14) allows us to write the sum in Eq. (2.12) as an integral

$$E_0 = \frac{\hbar L^3}{2(2\pi)^3} \int w_k d^3k.$$  (2.15)

Eq. (2.15) can be evaluated by using the relation $w_k^2 = k^2 + m^2/\hbar^2$ to obtain

$$\rho_{\text{vac}} \equiv \lim_{L \to \infty} \frac{E_0}{L^3} = \frac{\hbar}{2(2\pi)^3} \int_0^{k_{\text{max}}} 4\pi k^2 dk (k^2 + \frac{m^2}{\hbar^2})^{\frac{1}{2}},$$  (2.16)

where the integral over $d^3k \to 4\pi k^2 dk$ is just the integral over the spherical volume of radius $k$ and $k_{\text{max}}$ is some maximum cutoff scale.\(^\text{4}\) This maximum cutoff scale is

\(^4\)This is a diverging sum, known as the UV divergence.

\(^5\)This maximum cutoff scale is required because quantum field theory is believed to be a low-
usually taken to be the Planck scale, which is the value of $k$ corresponding to the Planck energy $E_P \approx 10^{19}\text{GeV}$. This value is $k_{\text{max}} = E_P/\hbar$, so if we revert back to natural units whereby $\hbar = 1$, then we have $k_{\text{max}} \approx 10^{19}\text{GeV}$.

The next step in evaluating Eq. (2.16) is to note that the integral is dominated by very large $k \gg m^2$, which means we have the approximation

$$\rho_{\text{vac}} \approx 4 \frac{1}{16\pi^2} \int_0^{k_{\text{max}}} k^3 dk = \frac{k_{\text{max}}^4 \hbar}{16\pi^2} \approx 10^{74}\text{GeV}^4, \tag{2.17}$$

which is $\approx 120$ orders of magnitude larger than the value inferred from observations.

If correct, this difference implies that something must cancel the theoretical value to about 120 decimal places in order to match the observed value, hence the fine-tuning problem. If $k_{\text{max}}$ is larger than the Planck scale, this problem only becomes worse as $\rho_{\text{vac}} \to \infty$. The $k_{\text{max}}$ required to match the theoretical and observed values is $k_{\text{max}} \approx 10^{-3}\text{cm}^{-1}$, which corresponds to an energy lower than the binding energy of hydrogen, and so cannot be correct (Carroll et al., 1992).

A part of what makes this problem particularly nefarious is that it must be solved regardless of whether or not the cosmological constant is the source of dark energy, as the vacuum energy is already known to be a real entity. A vanishing constant usually implies the existence of a symmetry (e.g., Amendola and Tsujikawa, 2010), but there is currently no known symmetry that can give rise to a vanishing vacuum energy while remaining consistent with the known laws of physics (Carroll, 2004), though see, e.g., Hawking (1984); Kachru et al. (2000); Mukohyama and Randall (2004); Yokoyama (2002) for attempts to solve this problem.

energy (“effective”) theory that is only valid up to some scale where, above such a scale, it is possible that new interactions and particles might become important.
2.2.1.2 The Coincidence Problem

The second problem facing the cosmological constant is known as the coincidence problem. The energy density associated with the vacuum remains constant throughout cosmic evolution. The energy density related to matter, however, falls off as $\rho_m = \rho_{m,0}(1 + z)^3$. This means that there is only one point in all of cosmic history when the energy densities of dark energy and matter are the same, and this point is very close to the present-day.

The redshift associated with the transition to a dark energy dominated universe can be calculated by equating $\rho_{\text{vac}}$ and $\rho_m$, which gives

$$z_{\text{coinc}} = \left( \frac{\Omega_{\text{DE},0}}{1 - \Omega_{\text{DE},0}} \right)^{\frac{1}{3}} - 1 \approx 0.3. \tag{2.18}$$

Fig. 2.8 illustrates the coincidence problem by showing how $\Omega_{\text{DE}}$ changes as a function of the scale factor (or time). The only peak occurs now. Understanding why this peak occurs, and why it occurs so close to the present is the coincidence problem.
There are several proposed solutions to the coincidence problem. The first, and least satisfying, is that this really is just a coincidence. A popular explanation, however, is one that invokes the anthropic principle. For instance, if cosmic acceleration started too early, then it would disrupt the formation of large-scale structure, which, in turn, affects the formation of galaxies and, therefore, life. The argument is that we see the energy density values that we do because those are the ones most conducive to life. See Amendola and Tsujikawa (2010) and references therein for more on the coincidence problem and its proposed solutions.

It is worth noting that the coincidence problem is not unique to the cosmological constant. Even if dark energy is given by some mechanism other than the cosmological constant, we must still understand why the two energy densities coincide at the present epoch. Another way of stating this problem is that we need to understand why cosmic acceleration starts so late in the history of the universe, i.e. after large-scale structure has had time to form. For an excellent review of the cosmological constant see Carroll et al. (1992).

2.2.2 Dynamical Dark Energy

If dark energy is not given by the cosmological constant, then the possibility for dynamical dark energy opens up. It should be noted that there is no known solution to the fine-tuning problem. Dynamical dark energy models help, but do not solve it. Rather, it is hoped a proper theory of quantum gravity will provide a solution. As such, most dynamical dark energy models attempt to resolve the coincidence problem, with varying degrees of success, though none fully solve it. Most importantly, however, dynamical dark energy models are both allowed by current observations and are not discriminated against by theory. As such, dynamical dark energy models must be considered viable until they are either confirmed or ruled out by observations. Whereas the equation of state of the cosmological constant is given
by $w = -1$, the equation of state for dynamical dark energy is allowed to change with time, $w = w(a)$. In the next subsection I present a brief overview of several of the most popular dynamical dark energy models.

It should be noted that the models presented below are not the dark energy models used in my simulations. These models are presented solely to illustrate the variety of physical mechanisms responsible for cosmic acceleration that exist. In theory, it is possible to reconstruct the EOS by differentiating Equation (1.49) with respect to $z$ (Amendola and Tsujikawa, 2010). In practice, however, this process suffers from the problem that observational data are known only at discrete redshifts and are affected by errors (Amendola and Tsujikawa, 2010). This means that any distance measures obtained from observations must undergo some smoothing process before differentiation is possible (Amendola and Tsujikawa, 2010). This means that either the distance or the EOS must be parameterized in some way in order to be compared with observations (Amendola and Tsujikawa, 2010). While these parameterizations do not arise from any particular physical model, they are nevertheless useful for being able to directly compare with observations that employ them.

2.2.2.1 Quintessence

Quintessence was first introduced by Caldwell et al. (1998). It is a scalar field $\phi$ whose dynamics are dominated by an effective potential $V(\phi)$. It is the cosmic evolution of the field along this potential that gives rise to cosmic acceleration. The equation of state of a quintessence field is given by (e.g., Amendola and Tsujikawa, 2010)

$$w \equiv \frac{P_\phi}{\rho_\phi} = \frac{\ddot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)},$$

and the evolution of the scalar field is given by (e.g., Amendola and Tsujikawa, 2010)

$$\ddot{\phi} + 3H \dot{\phi} + V_\phi = 0,$$
where $V_{\phi} \equiv dV/d\phi$. The Hubble parameter in Eq. (2.20) is obtained by plugging in the energy density for quintessence $\rho_{\phi} \equiv \frac{1}{2} \dot{\phi}^2 + V(\phi)$ in for $\rho_{\text{DE}}$ in Eq. (1.13) (and assuming a flat universe). This gives

$$H^2 = \frac{8\pi G}{3} \left[ \frac{1}{2} \dot{\phi}^2 + V(\phi) + \rho_M \right], \quad (2.21)$$

where $\rho_M$ is the background energy density combining both non-relativistic and relativistic matter and $w_{\phi}$ is obtained by solving Eq. (2.20) for $\phi(t)$.

If the kinetic energy of the field dominated then $w_{\phi} \approx 1$, which, when plugged into Eq. (1.25), gives $\rho_{\phi} \propto a^{-6}$. This falls off faster than any other matter component in the universe and so cannot give rise to late-time cosmic acceleration. The condition for late-time cosmic acceleration translates into $w < -1/3$ from Eq. (1.16). Applying this condition to Eq. (2.19) implies that $\dot{\phi} < V(\phi)$, so that the potential must dominate the dynamics.

The most promising aspect of quintessence models is a property exhibited by some potentials known as tracking. Tracking behavior makes quintessence models extremely insensitive to initial conditions. The quintessence field “tracks” (follows) whatever the dominant component of the universe is at the time. In other words, during the radiation dominated epoch of the universe, the quintessence field follows the radiation energy-density and during the matter dominated epoch it follows the matter energy-density. The existence of tracking in a given quintessence model depends upon the slope of the potential.

The tracking condition is given by (Steinhardt et al., 1999)

$$\frac{VV_{,\phi\phi}}{V_{,\phi}^2} > 1, \quad (2.22)$$

where a comma is indicative of a derivative (meaning $V_{,\phi} = dV/d\phi$).

With the imposition of the condition in Eq. (2.22) the field solutions can be shown
to run from unstable points towards stable ones (e.g., Amendola and Tsujikawa 2010), giving rise to accelerated expansion.

Tracking behavior also provides a more natural explanation to the coincidence problem. Since the quintessence field naturally follows the evolution of the dominant component of the universe, there is no need to fine-tune the dark energy starting values in order to give rise to a cosmic acceleration that begins near the present-day.

However, this does not mean that the coincidence problem is solved. The problem still remains of explaining why the cosmic acceleration occurs so late in cosmic history and therefore why $\Omega_{DE,0} \approx \Omega_{m,0}$ only now. Although it slightly alleviates the fine tuning problem since it can be near the Planck scale at the Planck epoch and reduced to the current scale at the present epoch, a smallness problem still remains (e.g., Weinberg 2000).

2.2.2.2 K-Essence

A canonical scalar field is one with a Lagrangian of the form

$$L_\phi = -\frac{1}{2} g_{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi).$$  \hspace{1cm} (2.23)

That is, the field has a standard kinetic energy term analogous to the form $(1/2) mv^2$, which is the kinetic energy from classical mechanics. However, it is possible to have a non-canonical scalar field, the Lagrangian of which is given by (e.g., Amendola and Tsujikawa 2010)

$$L_X = P(\phi, X),$$  \hspace{1cm} (2.24)

where $X \equiv -\frac{1}{2} g_{\mu\nu} \partial_\mu \phi \partial_\nu \phi$ is the kinetic energy of the field and $P$ is some arbitrary function of the field and its kinetic energy. Such a model was originally developed for inflation (Armendáriz-Picón et al. 1999), and then later applied as a model of dark energy (Armendáriz-Picón et al. 2000, 2001; Chiba et al. 2000). K-essence therefore
refers to kinetic quintessence and, unlike quintessence, it is the kinetic energy of
the field that gives rise to cosmic acceleration. Whereas quintessence models are
differentiated by their form of $V(\phi)$, k-essence models are differentiated by their form
of $P(\phi, X)$.

The equation of state of k-essence is given by (e.g., Amendola and Tsujikawa
2010)

$$w_\phi = \frac{P_\phi}{\rho_\phi} = \frac{P}{2X P_X - P}. \quad (2.25)$$

Like quintessence, k-essence also possesses tracking behavior, which helps to alleviate
the coincidence and fine-tuning problems, but does not solve them, as k-essence cannot
answer why it is only today that the dark energy and matter density parameters
are roughly the same and so small.

2.2.2.3 Phantoms

It may be possible for $X$ to have a negative sign in some forms of $P$ (e.g., Caldwell
2002). Such models are called phantom or ghost fields because they roll up their
potentials instead of down them. They are characterized by $w < -1$. Phantoms
generally lead to a scenario known as the Big Rip, which occurs when the potential is
unbounded from above, therefore causing the energy density $\rho_\phi$ to increase towards
infinity. This results in a doomsday runaway cosmic acceleration that eventually tears
the universe apart.

Despite being allowed by current observations, phantoms have several theoretical
problems, the first of which is that they possess unstable vacuum states (e.g., Carroll
et al. 2003; Cline et al. 2004). The second issue plaguing phantom models is that
they have been shown to give rise to a universe in which the cosmological principle is
violated; that is, they generate a universe that is neither homogeneous nor isotropic
(e.g., Gannouji et al. 2006). The third issue facing phantom models is that the
field has been strongly constrained to originate from very low-energy physics (e.g.,
Amendola and Tsujikawa (2010). However, this physics must be completely hidden from the standard model except for gravity so as not to affect any of the theoretical results already confirmed by experiment, and hence the ghost moniker.

The dark energy models presented here are but a small sample of some of the more popular models in the literature. The interested reader is referred to Amendola and Tsujikawa (2010) and references therein for a much more complete review of dark energy models.
CHAPTER 3

NUMERICAL METHODS

Twenty-four hour banking? I don’t have time for that.

—Steven Wright

This chapter presents the numerical and algorithmic details of the codes used in this thesis. I begin with a description of the simulation code, GADGET-2 \cite{Springel2005}, including a discussion of the core gravitational calculation and the smoothed-particle hydrodynamics (SPH) prescription. As described in Section 4.1, all of my simulations are dark matter only simulations for reasons of speed. However, calculating synthetic Lyα Forest spectra requires knowledge of the densities and temperatures of baryonic matter. Both of these quantities are hydrodynamic variables. As such, these are determined in post-processing, as described in Sections 3.1 and 3.4. This is followed by a description of my flux power spectrum code in Section 3.7.

Unless otherwise noted, all numerical integrations and interpolations were done using the \texttt{gsl} \footnote{https://www.gnu.org/software/gsl/} software package in C. The Fourier transforms were performed with \texttt{FFTW2} \footnote{http://www.fftw.org/} and \texttt{FFTW3} \footnote{http://wwwmpa.mpa-garching.mpg.de/gadget/} libraries in C. Furthermore, GADGET-2 \footnote{http://wwwmpa.mpa-garching.mpg.de/gadget/}, the Amiga Halo Finder (AHF) \footnote{http://popia.ft.uam.es/AHF/Download.html} \cite{Gill2004,Knollmann2009}, and the data visual-
ization package yt\(^1\) (Turk et al., 2011) are all also publicly available. A number of analysis codes were written by me\(^2\) including a temperature calculation code (t\texttt{spec}), a spectral extraction code (e\texttt{xspec}), and flux power spectrum code (p\texttt{spec}). These are all publicly available.

### 3.1 Smoothed Particle Hydrodynamics

The dynamics of a fluid are described by five equations: the continuity equation, the Euler equation, the Poisson equation, the internal energy equation, and an equation of state. The continuity equation describes conservation of mass, the Euler equation is the equation of motion, the Poisson equation describes the effect of gravity on the fluid, and the equation of state relates the thermodynamic state variables (such as pressure, temperature, and density). The first three of these are given by (e.g., Mo et al., 2010)

\[
\begin{align*}
\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{u} &= 0, \\
\frac{D\mathbf{u}}{Dt} &= -\frac{\nabla P}{\rho} - \nabla \phi, \\
\nabla^2 \phi &= 4\pi G \rho, \\
\frac{De}{dt} &= -\frac{P}{\rho} \nabla \cdot \mathbf{u},
\end{align*}
\]

where \(\mathbf{r}\) is the proper coordinate, \(\mathbf{u}\) is the velocity of the fluid, \(\rho\) is the density of the fluid, \(e\) is the specific internal energy, and \(D/Dt \equiv \partial/\partial t + \mathbf{u} \cdot \nabla\) is the convective derivative, which is the time derivative in the frame that is moving with the fluid. There are therefore two core challenges facing any astrophysical simulation: modeling the hydrodynamic interactions and modeling the gravitational interactions.

\(^1\)http://yt-project.org/

\(^2\)https://bitbucket.org/polaris42/
For the hydrodynamic interactions, there are two primary methods used for modeling a fluid: grid-based and particle-based methods. Here, we are concerned specifically with the SPH method (Gingold and Monaghan, 1977; Lucy, 1977). See O’Shea et al. (2005) for an excellent comparison of SPH and grid-based codes. In SPH, the central idea is that the fluid is sampled by discrete tracer particles that move along with the fluid. This makes the evolution Lagrangian in nature.

The first problem SPH needs to address is how to calculate the density of a fluid that has been discretized into particles; that is, to define the density at a point. The solution to this problem is to take the mass that is represented by each tracer particle and disperse it such that each particle defines not just a point in space but also a region around itself. GADGET-2 uses a spherical region known as the smoothing sphere. This means that the density at a given point \( r \) inside this sphere due to a given particle \( j \) is given by (e.g., Bodenheimer et al., 2007)

\[
\rho_j(r) = m_j W(|r - r_j|, h_j),
\]

where \( W(|r - r_j|, h) \) is known as the smoothing kernel, \( m_j \) is the mass of the particle, and \( h \) is the smoothing length of the particle. The smoothing kernel has units of inverse volume and describes how the mass of the particle \( j \) is distributed within the smoothing sphere. The smoothing length is the radius of the smoothing sphere.

The choice of smoothing kernel is vitally important (e.g., Dehnen and Aly, 2012) as it affects the mass distribution for each particle. This has strong implications for the hydrodynamic interactions. GADGET-2 makes use of a cubic polynomial smoothing
kernel of the form (Springel, 2005)

\[
W(r, h) = \frac{8}{\pi h^3} \begin{cases} 
1 - 6 \left( \frac{r}{h} \right)^2 + 6 \left( \frac{r}{h} \right)^3 & 0 \leq \frac{r}{h} \leq \frac{1}{2}, \\
2 \left( 1 - \frac{r}{h} \right)^3 & \frac{1}{2} < \frac{r}{h} \leq 1, \\
0 & \frac{r}{h} > 1,
\end{cases}
\] (3.6)

where \( r \equiv |\mathbf{r} - \mathbf{r}_j| \). Eq. (3.6) is a spline kernel with compact support, which means that the distribution has a finite range of influence after which no mass from particle \( j \) is distributed. This is important numerically in order to reduce the computation time for the hydrodynamic forces since only those particles with non-negligible contributions to the quantity being calculated are considered (as opposed to summing over all particles, which is slow).

Since each tracer particle has its own smoothing sphere, in order to calculate the full, physical, density at the point \( \mathbf{r} \), SPH needs to include the effects of every smoothing sphere that contains said point. As such, the physical density at a particular point is the sum of the densities of all of the smoothing spheres that contain the desired point (e.g., Bodenheimer et al., 2007)

\[
\rho(\mathbf{r}) = \sum_j^N m_j W(|\mathbf{r} - \mathbf{r}_j|, h_j). 
\] (3.7)

SPH next needs to determine the size of the smoothing lengths for each particle. There are two ways that this is generally done. The first is to require that a nearly constant number of particles are contained within the smoothing sphere. The second method, and the way that GADGET-2 determines the smoothing lengths, is to require that there is a nearly constant mass contained within the smoothing sphere

\[
N\bar{m} = \frac{4\pi h^3 \rho^3}{3}. 
\] (3.8)
In Eq. (3.8) $\bar{m}$ is the average particle mass and $N$ is the desired number of so-called neighbor particles that one wishes to be contained within the smoothing sphere. This is a free parameter that can affect the numerical convergence of the calculation (e.g., Dehnen and Aly 2012).

Eq. (3.7) and Eq. (3.8) have to be solved iteratively in order to correctly determine the smoothing length, since each time the smoothing length is updated the density changes as well. The reason that each particle needs its own smoothing length as opposed to using a uniform smoothing length is because a uniform smoothing length does not perform well when there is a large dynamic range in the densities involved in the simulation, as is the case in cosmological simulations (e.g., Bodenheimer et al. 2007). These core concepts of SPH are illustrated in Figure 3.1.

In addition to being able to calculate densities, SPH must also be able to calculate other fluid quantities such as pressure and temperature in order to solve Eqs. (3.1) – (3.2). These are obtained at a given point in space by treating the tracer particles as interpolation points on an irregular grid and performing a density-weighted interpolation between the tracer particles (e.g., Bodenheimer et al. 2007)

$$A(r) = \sum_{j=1}^{N} m_j \frac{A_j}{\rho_j} W(|r - r_j|, h_j), \quad (3.9)$$

where $A$ can be any physical quantity associated with the fluid. For a far more complete treatment of SPH see Bodenheimer et al. (2007); Price (2004, 2012).

3.2 Gravitational Algorithms

In addition to being able to calculate the hydrodynamic forces, GADGET-2 also needs a way to calculate the gravitational force between the tracer particles. This is done using N-body methods, which naturally complement the SPH paradigm. However, since the gravitational force on a given particle depends upon its gravitational
Figure 3.1: This figure represents the core concept behind SPH. Here, several tracer particles that are used to discretize and follow the fluid flow are shown. In particular, the smoothing circles (spheres in three dimensions) for particles A and B are indicated by the blue and red circles, respectively. The smoothing lengths for particles A and B (the radii of the blue and red circles, respectively) are given by $h_A$ and $h_B$. Furthermore, the neighbor particles for particle A (those that fall within particle A’s smoothing circle) are shown in light blue, and those particles that do not contribute to particle A’s density as given by Eq. (3.7) because they lie outside the smoothing circle are shown in red. The greyscale gradient filling particle A’s smoothing circle denotes the role of the smoothing kernel $W(r, h)$ in distributing particle A’s mass within the smoothing circle, and serves as a reminder of the fact that this distribution is not uniform, but given by Eq. (3.6). Lastly, the $x$ denotes an arbitrary point $r$ in space where we might want to calculate the density of the fluid. In order to do this, Eq. (3.7) must be evaluated, in which case the sum is over all of the particles whose smoothing circles contain point $x$, and the distance from the particle to point $x$ is given by $r_A$ and $r_B$ for particles A and B, respectively (which is $r_A = |r - r_A|$ and likewise for particle B).
interaction with every other particle, performing a direct summation of these forces is quite slow for large numbers of particles. As such, GADGET-2 employs a Barnes-Hut (BH) tree algorithm (e.g., Appel 1985, Barnes and Hut 1986, Dehnen 2000) in order to reduce the computation time from $O(N^2)$ to $O(N \log(N))$.

The concept of a BH tree is to break the simulation volume up into cubes known as nodes, beginning with the simulation volume as a whole, which is known as the root node. The root node is then broken up into eight equal sized nodes known as daughter nodes, and each of these daughter nodes is further broken up into eight additional daughter nodes until each node contains either zero or one particles. These end nodes that only contain zero or one particle are called leaf nodes. An example of a BH tree is shown in Fig. 3.2.

For each node, the center of mass and total mass are calculated. Once built, the BH tree is then used to perform the neighbor search for each particle so that the smoothing lengths and densities can be calculated as described in Section 3.1. In order to calculate the gravitational force from this information, the tree is walked...
recursively for each particle. If the so-called opening criterion

\[
\frac{GM}{r^2} \left( \frac{l}{r} \right)^2 \leq \alpha |\mathbf{a}|
\]  

(3.10)
is satisfied, (where \( M \) is the node mass, \( l \) is the side-length of the node, \( r \) is the distance between the particle in question and the center of mass of the node, \( \alpha \) is a manually set tolerance parameter, and \( |\mathbf{a}| \) is the size of the acceleration found in the previous time-step\(^7\)) then the contribution of that node to the gravitational acceleration of the particle in question is obtained using the monopole moment of that node. If Eq. (3.10) is not satisfied then this means that the node is too close to the particle in question and the node is said to be opened. This means that the tree walk descends into the node’s daughter nodes until either Eq. (3.10) is satisfied or a node with only a single particle is encountered. In other words, if a node is far enough away from the particle in question, all of the particles contained within that node are treated as one aggregate particle with mass equal to the total mass of the node and a position equal to the center of mass of the node. Those nodes that are close enough to the particle in question are opened, which means that the walk considers the daughter nodes. This treatment of distant nodes as aggregate particles greatly reduces the number of calculations that need to be performed when compared to direct summation methods. See Springel (2005) for more details.

3.3 Halo Finding

After running a simulation, the next step in the pipeline is to identify halos within the data. The physical conditions in halos are different than those of the low density IGM gas that gives rise to the Ly\(\alpha\) Forest (e.g., Bertone 2003). Therefore, the

\(^7\)For the first force calculation this information does not exist and so GADGET-2 uses the standard BH opening criterion of \( l/r < \alpha \).
temperatures for halo particles need to be calculated differently than those that do not belong to halos.

To this end, I employ the Amiga Halo Finder (AHF) \cite{gill2004, knollmann2009}, a parallel and publicly available halo finding code. AHF identifies halos through a hierarchical grid generated through adaptive mesh refinement. This procedure has several advantages, chief of which is the ability to naturally identify sub-structure within each halo via the more refined grid levels. The ability to use substructure is important as it allows for the temperature calculation to be refined through the use of the properties of the subhalo as opposed to being restricted to the properties of the host. Additionally, AHF has the ability to incorporate non-standard dark energy models into its halo calculation out of the box, which makes it ideally suited for my purposes.

The size of halos is determined by the virial radius, which is defined through

\[ \rho(R_{\text{vir}}) = \Delta_{\text{vir}} \rho_b, \] (3.11)

where \( \rho(R_{\text{vir}}) \) is the density contained within the virial radius \( R_{\text{vir}} \), defined below, \( \rho_b \) is the background density, which I choose to be the critical density, and \( \Delta_{\text{vir}} \) is the virial overdensity, which is taken to be 200, by convention in the literature. The parameters I used for my halo finding runs are given in Table 3.1.\footnote{These parameters are all defined in the AHF user manual, available at \url{http://popia.ft.uam.es/AHF Documentation.html}.}

The \texttt{LgridDomain} parameter defines the number of cells for the top-level grid and \texttt{LgridMax} defines the highest level of refinement. \texttt{NperDomCell} is the number of particles required to trigger a refinement of the top-level grid and \texttt{NperRefCell} is the number of particles required to trigger a refinement of a refinement grid. \texttt{VescTune} is used to determine the speed at which particles are considered unbound to a halo and \texttt{NminPerHalo} is the minimum number of particles required for a halo. \texttt{RhoVir} is a flag used to determine
<table>
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</tr>
</tbody>
</table>

whether the critical density or the background density is used to find the virial radius of the halo, \( \text{DVir} \) is the overdensity required when finding the virial radius of a halo, and \( \text{MaxGatherRad} \) limits the maximum size of the halos identified. The interested reader is referred to Gill et al. (2004); Knollmann and Knebe (2009) and references therein for more detailed information about the inner workings of AHF and the numerics of halo finding in general.

### 3.4 Calculating Temperatures

Once the particles that belong to halos have been identified and the properties of those halos have been calculated using AHF, the temperatures of each particle can be determined. I call the particles that do not belong to halos “field” particles,
and they represent the relatively diffuse IGM that is responsible for most of the Lyα Forest. The temperature calculation is different for field and halo particles owing to the different physical conditions present in the two environments.

3.4.1 Halo Particles

For particles flagged as belonging to halos, temperatures are given by the virial temperature of the halo (this assumes that the halo behaves as an isothermal sphere), which is given by (e.g., Mo and White 2002)

$$T_{\text{vir}} = \frac{\mu m_p V_c^2}{2k_B},$$

(3.12)

where $\mu$ is the mean molecular weight, $m_p$ is the proton mass, $k_B$ is the Boltzmann constant, and $V_c$ is the circular velocity. I take $\mu = 0.588$, which corresponds to a primordial composition with a hydrogen mass fraction $X = 0.76$ and a helium mass fraction of $Y = 0.24$ (e.g., Mo and White 2002). The circular velocity is given by (e.g., Mo and White 2002)

$$V_c = \left(\frac{GM_{\text{vir}}}{R_{\text{vir}}}\right)^{\frac{1}{2}},$$

(3.13)

where $M_{\text{vir}}$ is the virial mass (the mass contained within the virial radius $R_{\text{vir}}$). The virial radius is given by (e.g., Mo and White 2002)

$$R_{\text{vir}} = \left[\frac{GM_{\text{vir}}}{100H^2(z)}\right]^{\frac{1}{2}}.$$

(3.14)

In my case, both the virial radius and virial mass are provided by AHF, which means that they can be immediately used to calculate $V_c$ and $T_{\text{vir}}$. 

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3.4.2 Field Particles

Field particles represent the gas in the diffuse IGM. The physical conditions of this gas are largely governed by the interplay between photoionization heating due to the UV background, Compton cooling, and adiabatic cooling due to the expansion of the universe (e.g., Bertone 2003, McQuinn 2016).

Under these assumptions, Hui and Gnedin (1997) found that the IGM can be described by a power law equation of state

\[ T = T_0 (1 + \delta)^{\gamma - 1}, \]  

(3.15)

where \( T_0 \) is the characteristic temperature of the IGM at a given redshift, \( \delta = \rho / \bar{\rho} - 1 \) is the overdensity (also sometimes referred to as the density contrast), and \( \gamma \) is the adiabatic index. The form of \( T_0 \) is given by (Theuns et al. 1998)

\[ T_0(z) = \left\{ \left[ \Omega_\text{b} h t'_{HH} L'_e (1 + z)^{\frac{3}{2}} \right] / \left[ 1 - \frac{L'_e t'_{HH} (1 + z)^{\frac{3}{2}}}{h t'_{\text{heat}}} \right] \right\}^{\gamma - 1}, \]  

(3.16)

where \( L'_e = 1.7 \times 10^{-20} \text{ergs}^{-1} \text{cm}^3 \text{K}^{0.7}, \) \( L'_c = -7.31 \times 10^{-30} \text{ergs}^{-1} \text{cm}^3 \text{K}^{-1}, \) \( t'_{\text{heat}} = 5.41 \times 10^{-11} \text{ergs} \text{cm}^3 \text{K}^{-1}, \) and \( t'_{HH} = 2.06 \times 10^{17} \text{s}. \) These parameters are from Theuns et al. (1998) and have been shown to accurately reproduce the thermal history of the IGM (e.g., Schaye et al. 1999). In practice, I use a preconstructed table kindly provided by S. Bertone in order to interpolate to the proper value of \( T_0 \) at each redshift.

The overdensity is calculated by taking \( \rho \) to be the density of the particle, computed using GADGET-2’s density calculation and \( \bar{\rho} = \Omega_m(z) \rho_c, \) where the matter density parameter as a function of redshift is given by (Mo and White 2002)

\[ \Omega_m(z) = \frac{\Omega_{m,0} (1 + z)^3}{E^2(z)}. \]  

(3.17)
For the adiabatic index in Eq. (3.16), I use $\gamma = 1.588$, which has been shown (e.g., Bertone, 2003) to match the thermal history of the IGM at intermediate redshifts ($2 \lesssim z \lesssim 5$). $E(z)$ is the expansion factor defined in Eq. (1.70).

3.5 Calculation of Neutral Hydrogen Fractions

The Ly$\alpha$ Forest is due to Ly$\alpha$ absorption that occurs in neutral hydrogen. This means that in order to extract a synthetic spectrum from a GADGET-2 simulation the mass fraction of neutral hydrogen $X_{\text{HI}}$ for each particle needs to be calculated. The evolution of neutral hydrogen is determined by the interplay between ionization and recombination events, given as

$$\frac{dX_{\text{HI}}}{dt} = \alpha_{\text{HII}} n_e X_{\text{HII}} - X_{\text{HII}}(\Gamma_{\gamma\text{HI}} + \Gamma_{e\text{HI}} n_e),$$

(3.18)

where $\alpha_{\text{HII}}$ is the recombination coefficient for ionized hydrogen, $n_e$ is the electron number density, $X_{\text{HII}}$ is the mass fraction of ionized hydrogen, $\Gamma_{\gamma\text{HI}}$ is the photoionization rate for neutral hydrogen, and $\Gamma_{e\text{HI}}$ is the collisional ionization rate for neutral hydrogen.

The recombination rate of ionized hydrogen is given by (e.g., Bertone, 2003)

$$\alpha_{\text{HII}} = 6.3 \times 10^{-11} \frac{T_n^{-\frac{2}{3}} T_{10}^{-\frac{1}{2}}}{1 + T_{10}^{\frac{3}{2}}} \text{s}^{-1},$$

(3.19)

where $T_n \equiv T/(10^n \text{K})$. The electron number density is given by (e.g., Bertone, 2003)

$$n_e = e n_{\text{H}} = \frac{2 - Y}{2(1 - Y)} n_{\text{H}},$$

(3.20)

where $e$ is the electron fraction, $Y$ is the helium abundance, and $n_{\text{H}}$ is the total
hydrogen number density. For a given particle, the hydrogen number density is

\[ n_H = \frac{X_H \rho}{m_H}, \]

where \( m_H \) is the mass of hydrogen. Photoionization affects mostly the cold, low-density gas in the IGM and is due primarily to the UV background \( J \). As such, the photoionization rate depends upon \( J \) as (e.g., Osterbrock 1989)

\[ \Gamma_{\gamma HI}(z) = \int_{\nu_H}^{\infty} 4\pi J(\nu, z)\sigma_H(\nu) \frac{d\nu}{h\nu}, \]

where \( \nu_H = 3.2 \times 10^{15} \text{s}^{-1} \) is the threshold frequency for hydrogen, \( \sigma_H(\nu) \) is the photoionization cross-section given by (Osterbrock 1989)

\[ \sigma_H(\nu) = A_0 \left( \frac{\nu}{\nu_H} \right)^4 e^{4 - 4 \arctan(\epsilon)/\epsilon} \frac{1 - e^{-2\pi/\epsilon}}{1 - e^{-2\pi/\epsilon}}, \]

where

\[ A_0 = \frac{2^8 \pi}{3e^4} \left( \frac{1}{137} \right) \pi a_0^2 = 6.3 \times 10^{-18} \text{cm}^2, \]

\[ \epsilon = \sqrt{\frac{\nu}{\nu_H}} - 1. \]

In practice, Eq. (3.22) is solved numerically by making use of the UV flux spectrum as provided in the Haardt and Madau 2005 table (Haardt and Madau 1996).

The collisional ionization rate is given by (e.g., Bertone and White 2006)

\[ \Gamma_{e HI} = 1.17 \times 10^{-10} T^4 e^{-157809.1 \frac{1}{T}} \text{cm}^3 \text{s}^{-1}. \]

Eq. (3.18) can be solved by assuming photoionization equilibrium, which implies \( dX_{HI}/dt = 0 \) (e.g., Schaye and Dalla Vecchia 2008). Furthermore, the IGM is mostly
ionized over my redshift range of interest \(4 \lesssim z \lesssim 2\), which means we can approximate \(X_{\text{HI}} \approx 1\) (e.g., McQuinn [2016]), allowing for the neutral fraction to be solved for as

\[
X_{\text{HI}} = \frac{\alpha_{\text{HII}}n_e}{\Gamma_{\gamma\text{HI}} + \Gamma_{e\text{HI}}n_e}
\]

for each particle.

3.6 Spectral Extraction

Once the densities, temperatures, and neutral fractions have been calculated for each particle, a synthetic spectrum can be extracted from the simulation data. The first step is to cast a line of sight (LOS) through the simulation volume. Each LOS starts at a randomly chosen position on the \(x = 0\) face of the simulation cube. The starting point is denoted as point A. From there, I extend the LOS to the opposite cube face while keeping it parallel to the \(x\)-axis of the simulation volume. This point is denoted as point B. Following the conceptual framework laid out in Hummels et al. (2016), point A corresponds to the location of the quasar whose spectrum is being simulated. Point B corresponds to the location of the observer.

With the LOS defined, the next step is to break it up into bins (referred to as pixels). I chose a pixel width \(\Delta_p\) of 0.01 comoving Mpc h\(^{-1}\). This value was chosen because it was found to balance the spectral resolution with the overall runtime required to extract the desired number of spectra for our box size of \(L = 40\) Mpc h\(^{-1}\), as discussed in Section 4.2. For LOS parallel to the \(x\)-axis, this spectral resolution corresponds to \(N = 3000\) pixels. This gives a width in velocity space of \(\Delta_{\text{vel}} = 1.3\) km s\(^{-1}\). A schematic LOS is shown in Figure 3.3. In order to calculate a spectrum along the LOS, the optical depth \(\tau\) for each pixel must first be calculated. This is done by following the method presented in Bertone and White (2006); Theuns et al. (1998), discussed below.
Figure 3.3: This figure shows a schematic of how each LOS is set up (albeit in two dimensions instead of three). A random point on the \( x = 0 \) cube face is chosen to be point A (the location of the quasar and the start of the LOS). The LOS then runs parallel to the \( x \) axis to the opposite cube face; this is point B (the location of the observer and the end of the LOS). It is broken up into bins called pixels, the edges of which are denoted by the slash lines on the LOS. For practical purposes, the location of the pixel is taken to be the center of the bin, denoted by the blue \( x \). Each pixel has a width \( \Delta_p \) associated with it denoted with the bracket.

The first step is to determine which particles’ smoothing spheres intersect the LOS. This is done by simply calculating the perpendicular distance \( d \) between the particle and the LOS and then comparing this value to the particle’s smoothing length. If \( h > d \), then the particle intersects the LOS. The perpendicular distance is given by

\[
d^2 = \left[ (r_{A,x} - r_{P,x}) + (r_{B,x} - r_{A,x})t \right]^2 + \left[ (r_{A,y} - r_{P,y}) + (r_{B,y} - r_{A,y})t \right]^2 + \left[ (r_{A,z} - r_{P,z}) + (r_{B,z} - r_{A,z})t \right]^2,
\]

\[
t = -\frac{(r_A - r_P) \cdot (r_B - r_A)}{|r_B - r_A|^2}, \tag{3.29}
\]

where \( r_A \) is the vector from the origin to point A, \( r_B \) is the vector from the origin to point B, and \( r_P \) is the vector from the origin to the particle being considered.

For those particles whose smoothing spheres do intersect with the LOS, the next step is to determine which pixels are contained within the smoothing sphere. This is done by using the perpendicular distance in order to figure out which pixel the particle projects down onto (which I refer to as the intersection pixel) and then putting the smoothing length in units of the pixel width (which gives the number of pixels that can fit within the smoothing length). The maximum possible range of pixels that
could be contained within the smoothing sphere is then given by the number of pixels that fit within the smoothing length adjacent to the intersection pixel. This procedure is illustrated in Figure 3.4.

With the pixels intersected by the particle determined, the next step is to calculate the particle’s contribution to the density, neutral weighted temperature, and neutral weighted velocity to each of these pixels $j$, as these quantities are needed in order to determine $\tau$ (Bertone and White, 2006; Theuns et al., 1998):

$$\rho_{X_{HI},j} = \sum_i X_{HI,i} m_i W(r_{ij,h_i}), \quad (3.30)$$

$$(\rho T)_{X_{HI},j} = \sum_i X_{HI,i} m_i T_i W(r_{ij,h_i}), \quad (3.31)$$

$$(\rho v)_{X_{HI},j} = \sum_i X_{HI,i} m_i W(r_{ij,h_i}) v_{tot,i}, \quad (3.32)$$
where $X_{HI,i}$ is the neutral fraction of particle $i$, $m_i$ is the mass of particle $i$, $W(r_{ij,h_i})$ is the SPH smoothing kernel between the particle $i$ and the pixel $j$, $h_i$ is the smoothing length of particle $i$, $T_i$ is the temperature of particle $i$, and $v_{tot,i}$ is the total velocity $v_p + v_H$ (where $v_p$ is the peculiar velocity and $v_H$ is the Hubble velocity) projected along the direction of the LOS for particle $i$.

Once Eq. (3.30)-Eq. (3.32) have been evaluated for every particle, they can be used to calculate the optical depth for each pixel as:

$$
\tau_k = \sum_j \sigma_{\alpha} c n_{HI,j} \delta_{\text{pix}} \sqrt{\pi V_{HI,j}} \exp \left[ -\left( \frac{v_{H,k} - v_{HI,j}}{V_{HI,j}} \right)^2 \right],
$$

where the speed of light $c$ has been reintroduced for clarity, $\sigma_{\alpha} = 4.45 \times 10^{-18}\text{cm}^2$ is the cross section for the Lyman $\alpha$ transition, $n_{HI,j}$ is the number density for neutral hydrogen of pixel $j$ given by dividing Eq. (3.30) by the mass of hydrogen, $\delta_{\text{pix}}$ is the pixel width, and $V_{HI,j}$ is the neutral hydrogen Doppler parameter that governs the thermal broadening of the spectral lines for pixel $j$, given by

$$
V_{HI,j} = \sqrt{\frac{2k_B T_j}{m_H}}.
$$

In Eq. (3.33), $v_{H,k}$ is the Hubble velocity of pixel $k$ and $v_{HI,j}$ is the velocity of neutral hydrogen in pixel $j$ obtained by dividing Eq. (3.32) by Eq. (3.30). This exponential term is needed because, if the gas were not moving in the pixel then the absorption would be a Gaussian centered at the location of the pixel. However, if the gas is moving with some peculiar velocity then the Gaussian is shifted away from its original pixel and the peak of the Gaussian will fall in a different pixel. This means that it is possible for every pixel to make a contribution to the optical depth of every other pixel due to the motion of the neutral gas $v_{HI,j}$. It is for this reason that there the sum in Eq. (3.33) is over all pixels.

The quantity $v_{H,k}$ is the Hubble velocity of pixel $k$, given by
\[ v_{H,k} = H(z_k)d_k, \]  

where \( d_k \) is the distance between pixel \( k \) and point B, and \( H(z_k) \) is the Hubble parameter at the redshift of pixel \( k \). The redshift of each pixel is found by following the method presented in Hummels et al. (2016). Briefly, this method requires that I first find the redshift extent of the simulation volume by solving

\[
\frac{c}{H_0} \int_{z_B}^{z_A} \frac{dz'}{E(z')} - L = 0 \quad (3.36)
\]

for \( z_B \), which is the redshift of the observer. In Eq. 3.36, \( c \) has been reintroduced for clarity and \( z_A \) is the redshift of the quasar, which I take to be the redshift of the snapshot. I solve Eq. 3.36 using Newton’s method. Once the velocity extent of the box is known, this allows us to assign a redshift to each pixel along the LOS according to

\[
z_k = z_A - \sum_{i=0}^{k-1} dz_i \quad (3.37)
\]

where

\[
dz_i = -\frac{\Delta_i}{l}(z_B - z_A) \quad (3.38)
\]

and \( \Delta_i = \frac{l}{N} \) is the width of pixel \( i \) (in practice, this is a constant for each pixel), \( l \) is the length of the LOS, and \( N \) is the number of pixels along the LOS.

Once the optical depth has been calculated for each pixel, the spectrum is normalized to match the mean optical depth as determined from observations. To this end, I use the relation found by Kim et al. (2002)

\[
\bar{\tau}(z)_{\text{obs}} = 0.0032(1 + z)^{3.37}. \quad (3.39)
\]
The observational mean as determined from Eq. (3.39) is compared to the theoretical mean given by

$$\bar{\tau}_{\text{theor}} = -\ln \left( \frac{\sum_{i=1}^{N} \exp \left( -\tau_i \right)}{N} \right)$$

(3.40)

in an iterative process whereby the absolute value of the difference between the theoretical and observed means are compared to a user defined tolerance parameter. If $|\bar{\tau}_{\text{obs}} - \bar{\tau}_{\text{theor}}| > \epsilon$, where $\epsilon$ is the tolerance parameter, I calculate

$$f_{\text{norm}} = \frac{\bar{\tau}_{\text{obs}}}{\bar{\tau}_{\text{theor}}}$$

(3.41)

I then multiply the optical depth of every pixel by $f_{\text{norm}}$ and evaluate Eq. (3.40). This process is repeated until the mean observed and theoretical values are within a tolerance $\epsilon = 0.01$. The flux for each pixel $i$ can then be calculated as

$$F_i = e^{-\tau_i}$$

(3.42)

in order to complete the calculation of the spectrum.

3.6.1 Adding Realism to Synthetic Spectra

In order to compare my synthetic Ly$\alpha$ Forest spectra with observational data, I first post-process the spectra in order to match the properties of real, observed spectra, such as those obtained from the Cosmic Origins Spectrograph (COS) and other high resolution ($R \approx 40000$) echelle spectrographs (Bolton et al., 2017; Hummels et al., 2016). This process follows the methods presented in Bolton et al. (2017); Greig (2013); Hummels et al. (2016). The first step is to convolve the synthetic spectra with a line-spread function (LSF) to match the properties of a real instrument. In practice, I follow Bolton et al. (2017) and convolve my spectra with a Gaussian kernel with full-width-half-max (FWHM) of 7 km s$^{-1}$. The convolution is done using
the `convolve` routine and the `Gaussian1dKernel` from the Python package `astropy` (Astropy Collaboration et al., 2018). The FWHM is related to the standard deviation of the Gaussian via the relation $\text{FWHM} = 2\sqrt{2 \ln(2)} \sigma_{\text{lsf}}$.

After this convolution, the spectra are re-binned onto pixels of a more realistic width, 3 km s$^{-1}$ (Bolton et al., 2017) using a cubic spline interpolation (Greig, 2013). In practice, this interpolation is done using the `interp1d` routine from the Python package `scipy` (Jones et al., 2001). After re-binning, each pixel has noise with a fixed signal-to-noise (SNR) added to it, which is meant to mimic the read-out noise of a detector. Following Greig (2013), I assume that the noise is thermally limited, which allows the noise to be modeled by a Gaussian. I use a SNR of 50. The SNR is related to the standard deviation of the Gaussian via the relation $\sigma_{\text{noise}} = 1/\text{SNR}$ (Greig, 2013). The values for the FWHM, re-binned pixel size, and noise were all chosen to be comparable to what is obtained from high resolution spectra (Bolton et al., 2017).

The final step in this post-processing pipeline is to correct the spectra for continuum bias. The effective optical depth $\tau_{\text{obs}}$, the fitting formula for which is given in Equation (3.39), used to normalize the spectra depends upon the mean flux $\langle F \rangle$ (Faucher-Giguère et al., 2008) as

$$\tau_{\text{obs}} = -\ln \left[ \langle F \rangle(z) \right].$$

(3.43)

Observationally determining $\langle F \rangle$ requires accurately estimating the quasar continuum. This, however, is notoriously challenging to do (e.g., Faucher-Giguère et al., 2008). In particular, estimating the continuum generally becomes more difficult at higher redshifts owing to the increase in HI absorption (e.g., Bolton et al., 2017). This increased absorption means that transmission peaks reaching unity become rarer at high redshift, which makes estimating the continuum more difficult (Faucher-Giguère et al., 2008). In particular, this lack of transmission peaks allows for the possibil-
Figure 3.5: This figure, from Faucher-Giguère et al. (2008), shows the effects of continuum bias using mock spectra. The thick black line represents the true, known, continuum and the thin black line represents the estimated continuum. The figure highlights how the discrepancy increases at higher redshift owing to the increase in absorption. The two lines in the $z = 4.5$ panel correspond to a spectrum generated with a hydrogen photoionization rate of half (blue) and twice (red) their fiducial value.

Faucher-Giguère et al. (2008) found that at low redshift ($z = 2$) there was very little difference between the estimated continuum and the true continuum, owing to the presence of many transmission peaks. However, at higher redshift ($z = 4$), they found that the continuum fits underestimated the true continuum by $\approx 12\%$, as shown in Figure 3.5.

The redshift trend of this continuum correction $C_{\text{corr}}$ is given by (Faucher-Giguère et al., 2008).
Following Bolton et al. (2017), I apply the continuum correction given by Equation (3.44) to each pixel using

\[ F_{i,\text{corr}} = \frac{F_{i,\text{deg}}}{C_{\text{corr}}}, \]  

where \( F_{i,\text{deg}} \) is the flux of the pixel after the LSF, re-binning, and noise have all been added. Without including this correction, Bolton et al. (2017) showed that the power was under-predicted at all scales, even when including baryonic physics. This is demonstrated in Figure 3.6.

I note that this degradation procedure is only applied to my spectra when comparing them to observational data. In the cases when I am comparing my synthetic
spectra to each other, I use the “ideal” spectra that emerge from the procedure described in Section 3.6.

3.7 The Flux Power Spectrum

The primary statistic of the Lyα Forest used in this thesis is the power spectrum calculated along the LOS. A power spectrum is a measure of the strength of an amplitude as a function of scale. This makes it a very powerful statistic for measuring the clustering of structure in the universe. In the case of the flux power spectrum, it acts as a proxy for the matter power spectrum on large scales because the LOS pierces various structures in the cosmic web, thereby associating the absorption with cosmological structure. The amplitude being considered is therefore the amplitude of the density contrast in these structures.

The flux power spectrum is given by

\[ P_{F_p}(k) = N |F_{p,k}|^2, \tag{3.46} \]

where \( F_{p,k} \) is the Fourier transform of

\[ F_p = \frac{e^{-\tau}}{\langle e^{-\tau} \rangle} - 1. \tag{3.47} \]

I use the quantity \( F_p \) in my Fourier transforms for the power spectrum instead of the flux given in Eq. (3.42) because \( F \) is very sensitive to changes in the mean flux \( \langle e^{-\tau} \rangle \) (Hui et al. 2001), which, given the uncertainties in the UV background, could serve to wash out any desired signal. The normalization of the power spectrum \( N \) is found by dividing out the total number of counts that occur in each \( k \) bin and then multiplying by the length of the Lyα Forest spectra in velocity space (e.g., Bertone).

\[^9\text{In fact, the power spectrum is the Fourier transform of the correlation function, which is a direct measure of clustering in real-space (e.g., Amendola and Tsujikawa 2010).}\]
The Fourier transforms are taken of $F_p$ in velocity space (that is, using the Hubble velocity of each pixel as the abscissa). As such, the corresponding frequencies $k$ have units of s km$^{-1}$. Since each pixel along the LOS exists at a different point in velocity space, it will have its own frequency $k_i$ associated with it. The frequencies are found via:

$$k_i = \frac{2\pi i}{T},$$  \hspace{1cm} (3.48)

where $i$ indicates the bin index and $T$ is the period. For discrete Fourier Transforms (DFTs), the signal is assumed to be periodic over the range in which there is data, so this is simply the length of the spectrum in velocity space. My power spectra are then binned according to the $k$ values used in McDonald et al. (2000). This binning is chosen because the data presented in McDonald et al. (2000) spans eighteen values of $k$ from $k = 0.00284$ to $k = 0.142$. So, while there is only data from McDonald et al. (2000) at three of my five snapshot redshifts, the dataset contains more data points at each redshift and covers a larger $k$ range than the other two datasets I use, Iršič et al. (2017); McDonald et al. (2006).
CHAPTER 4

RESULTS

There’s a fine line between fishing and just standing on the shore like an idiot.

—Steven Wright

This chapter presents the results of my cosmological simulations, spectral extraction, and power spectrum calculations.

4.1 Simulations

The simulations performed for this thesis were run using the publicly available SPH code GADGET-2 [Springel 2005], which is described in Section 3.1 and Section 3.2. The vanilla version of GADGET-2 assumes that dark energy is due to a cosmological constant. This means that the code makes use of Eq. (2.8) when determining how the simulation volume expands in time. However, the goal of this thesis was to investigate the effects of time-dependent dark energy. This means that the default calculation of the Hubble parameter needed to be changed in order to work with dynamical dark energy by implementing Eq. (1.49) [Dolag et al. 2004].

Since Eq. (1.49) involves the dark factor $\zeta(a)$, the form of which is unknown, there are two approaches to solving this problem. The first approach involves choosing a particular, explicit, dark energy model, solving for that model’s form of $w(a)$ and then implementing that form in GADGET-2. The second approach involves using a parameterized version of $w(a)$, as described in Section 3.3, and implementing that
into GADGET-2. While there are pros and cons to each method, I elected to go with the later option in this thesis.

The reasons for this choice are as follows. First, currently, the number of dark energy models is quite large, including, but not limited to, quintessence, k-essence, phantom fields, coupled dark energy models, chameleon fields, and unified models of dark energy and dark matter. Each of these models have their own substantial parameter spaces that need to be explored (Amendola and Tsujikawa 2010). This makes selecting an explicit model difficult. Second, using an explicit model of dark energy makes comparison with observations challenging because, currently, observations have constrained the behavior of dark energy using a parameterized model (e.g., Planck Collaboration et al. 2015). Due to these reasons, I chose to use the parameterized form of $w(a)$ given in Eq. (1.41).

Implementing Eq. (1.41) into GADGET-2 was relatively straightforward. The use of Eq. (1.41) in Eq. (1.49) gives rise to an analytic expression

$$H^2(a) = H_0^2 \left[ \Omega_{r,0} a^{-4} + \Omega_{m,0} a^{-3} + \Omega_{K,0} a^{-2} + \Omega_{DE,0} \left\{ a^{-3(1+w_0+w_a)} e^{-3w_a(1-a)} \right\} \right].$$  \hfill (4.1)

The values of $w_0$ and $w_a$ are read in from the GADGET-2 parameter file, which provides a simple method for running multiple simulations with varying dark energy models. For reasons of speed, Eq. (4.1) is tabulated at the beginning of the run and this look-up table is then used to perform interpolation to the value of $H$ at the desired redshift.

I ran five simulations\footnote{Any and all of my simulation data will be made available upon request.} for this thesis. One of these simulations, which I take to be my fiducial simulation, was run using the cosmological constant as its dark energy
TABLE 4.1

SIMULATION DARK ENERGY PARAMETERS

<table>
<thead>
<tr>
<th>Simulation Name</th>
<th>(w_0)</th>
<th>(w_a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L-40-1024</td>
<td>−1.0</td>
<td>0.0</td>
</tr>
<tr>
<td>DE1-40-1024</td>
<td>0.0</td>
<td>−3.0</td>
</tr>
<tr>
<td>DE2-40-1024</td>
<td>−1.1</td>
<td>1.3</td>
</tr>
<tr>
<td>DE3-40-1024</td>
<td>−2.0</td>
<td>0.0</td>
</tr>
<tr>
<td>DE4-40-1024</td>
<td>−2.0</td>
<td>2.0</td>
</tr>
</tbody>
</table>

model \((w_0 = -1, \ w_a = 0)\). This simulation is shown in Figure [4.1]. The other four simulations were dynamical models. The values of \(w_0\) and \(w_a\) that I used for these four simulations were selected from the posterior distribution given in [Planck Collaboration et al. (2015)], which is shown in Figure [1.5] and hereafter referred to as the Planck posterior. In particular, three of my four dynamical dark energy models had their parameters chosen such that they were close to the fringes of the allowed parameter space of the Planck posterior. The remaining dark energy model was deliberately chosen to lie outside of the bounds provided by the Planck posterior. This was done for two reasons: first, I wanted an example of an extreme dark energy model which might show that the power spectrum is sensitive to the time-dependence in the dark energy model. The second reason was to use this model as a check on the observational bounds provided by the Planck posterior. There are several caveats to this model selection, however, and these are discussed in Chapter [5]. The values of \(w_0\) and \(w_a\), along with the names of each of my simulations, are given in Table [4.1].
Figure 4.1: This figure shows the density projected along the $z$ axis for my L-40-1024 (cosmological constant) simulation at four different redshifts. The figure clearly shows the presence and evolution of the web-like large-scale structure.
The EOS for each of these five dark energy models, along with their effects on the expansion history of the universe, are shown in Figure 4.2. Figure 4.2 shows that the expansion histories predicted by four of our dark energy models (all save DE2-40-1024) are extremely similar to one another, particularly at high redshift when dark energy was sub-dominant to matter and radiation. However, at lower redshifts \((z \lesssim 1)\), each dark energy model starts to differentiate itself from the others due to dark energy becoming the dominant component in the universe. As expected, the expansion history for the DE2-40-1024 model is quite different from the other four at nearly all redshifts, including at higher redshifts when dark energy is sub-dominant, indicating the extreme effects that dark energy can have on the expansion history.

Figure 4.2 highlights the dark energy model degeneracies discussed in Chapter 1; namely that different viable dark energy models are difficult to discriminate between because at higher redshifts they are sub-dominant and give rise to similar expansion histories and at lower redshifts each model must converge to give \(w_0 \approx -1\). However, by restricting myself to the fringes of the Planck posterior, the models I have selected manage to provide a fairly large spread in both \(w_0\) and \(H_0\). It would seem, then, that the ideal redshift range for discriminating between dark energy models is \(0.5 \lesssim z \lesssim 1.5\), which is a redshift range where the Ly\(\alpha\) Forest is in the UV, and therefore must be observed from space. However, there is a lack of observational data for the Ly\(\alpha\) Forest at these redshifts owing to the increased difficulty in performing such observations. This is discussed further in Chapter 5.

Simulating the Ly\(\alpha\) Forest requires a very high spatial resolution in order to properly resolve the small-scale structure that gives rise to the Ly\(\alpha\) Forest. In particular, it has been suggested (McDonald, 2003) that a spatial resolution of \(\approx 40 \ Mpc \ h^{-1}\) comoving in a \(40 \ Mpc\) comoving box is required in order to achieve the proper convergence for the power spectrum. To this end, each of my simulations evolved a distribution of \(1024^3\) dark matter particles in a box of side length \(L = 40 \ Mpc \ h^{-1}\) comoving. Due to this high resolution requirement, each simulation contains only
Figure 4.2: (a): The evolution of the dark energy EOS $w(z)$ for each of the dark energy models that I consider in this thesis. The blue line corresponds to the cosmological constant ($w_0 = -1, w_a = 0$). The other models were chosen to be close the edges of the 95% confidence $w_0 - w_a$ parameter space as determined by Planck Collaboration et al. (2015), save for model DE2-40-1024 ($w_0 = -1.1, w_a = 1.3$), which was deliberately chosen to be outside of the allowed range. See text for details. We see that the EOS for different dark energy models can vary considerably in their behavior, thereby affecting the expansion history of the universe in unique ways via Eq. 1.49. (b): The time derivative of the cosmic scale factor as a function of redshift for each of the dark energy models considered in this paper. This panel highlights the differences in the expansion history due to dark energy. The color coding and legend are the same for both panels.
dark matter particles out of consideration for the total run-time of the simulation. Including baryons and, in particular, baryonic physics such as radiative cooling, UV heating, star formation, and Type Ia and Type II supernova feedback (Snedden et al., 2016), drastically increases the run-time of the simulation because the BH tree must be walked several additional times at each time-step. A simulation at my spatial resolution including the aforementioned baryonic physics and the same number processors would require several months to finish. Each of my dark matter only simulations took around ten days with 72 processors. The parameters for my simulations are given in Table 4.2.

### TABLE 4.2

**SIMULATION PARAMETERS**

<table>
<thead>
<tr>
<th>Simulation Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of DM Particles</td>
<td>$1024^3$</td>
</tr>
<tr>
<td>Number of SPH Neighbors (tolerance)</td>
<td>$48(\pm 3)$</td>
</tr>
<tr>
<td>Softening Length (kpc $h^{-1}$ comoving)</td>
<td>1.56</td>
</tr>
<tr>
<td>Box Size (Mpc $h^{-1}$ comoving)</td>
<td>40.0</td>
</tr>
<tr>
<td>Particle Mass</td>
<td>$5.21 \times 10^6 M_\odot h^{-1}$</td>
</tr>
<tr>
<td>$z_{\text{start}}$</td>
<td>49.0</td>
</tr>
<tr>
<td>$z_{\text{end}}$</td>
<td>2.2</td>
</tr>
<tr>
<td>$\Omega_{m,0}$</td>
<td>0.315</td>
</tr>
<tr>
<td>$\Omega_{b,0}$</td>
<td>0.0456</td>
</tr>
<tr>
<td>$\Omega_{\text{DE},0}$</td>
<td>0.685</td>
</tr>
<tr>
<td>$H_0$ (km $s^{-1}$Mpc$^{-1}$)</td>
<td>67.31</td>
</tr>
</tbody>
</table>
Using only dark matter particles in my simulations is justified because in the low-density, mildly non-linear environments typically responsible for the Lyα Forest, the baryon distribution largely follows that of the underlying dark matter on large scales (e.g., Meiksin and White [2001]). Further, the effects of baryonic physics, such as galactic winds, have been shown to be small at large scales (Bertone and White [2006]). Since dark energy is a large-scale phenomenon, the region of the power spectrum that I am most interested in is at large-scales, which makes the dark matter only approximation a suitable one for this case.

In GADGET-2, the number of SPH neighbors is not a property of dark matter particles. However, as discussed in Section 3.6, extracting synthetic spectra requires knowledge of the densities of each particle. These densities were found by adapting GADGET-2’s density calculation to work on dark matter particles in post-processing. This is where the neighbor parameter was employed.

In addition to the density calculation, the next most important aspect of my post-processing pipeline was the temperature calculation. As described in Section 3.4, my temperature calculation scheme was bimodal. That is, those particles that were identified as belonging to a halo had their temperatures calculated according to the virial properties of their host halo whilst the remainder of the particles were treated as field particles that comprised the IGM. The temperature-density relation is shown in Figure 4.3. Figure 4.3 shows two regions: the straight line and the block-like region. The straight line arises from the temperatures of the field particles, which follow a power law, and the block-like region arises from the temperatures of the halo particles. The horizontal bands comprising the block-like region stem from the fact that every particle in a given halo will have the same temperature, but a different density, owing to the assumption that halos can be modeled as isothermal spheres.

Each of my simulations was started from the same set of initial conditions. These initial conditions were generated using the publicly available second-order Lagrangian
Figure 4.3: This figure shows the temperature-density relation from my L-40-1024 simulation at $z = 3.0$. There are two distinct aspects of the relation: the straight line and the block-like region. The straight line arises from the temperatures of the field particles and the block-like region arises from the temperatures of the halo particles. The horizontal bands in the block-like region are because every particle in a given halo will have the same temperature, owing to the assumption that halos can be modeled as isothermal spheres. Only 5% of the particles are shown for visual clarity.
perturbation theory code \texttt{2LPTic} (Scoccimarro et al., 2012). The simulations were evolved from $z = 49$ down to $z = 2.2$, with snapshots written at $z = 4.2, 3.8, 3.0, 2.7$, and 2.2. These five redshifts were chosen because they align with those of the observed power spectrum data in Iršič et al. (2017); McDonald et al. (2000, 2006), which were the observational datasets I used in this work. Further, writing the simulation data at these five redshifts covers a large redshift range over which there is power spectrum data while at the same time being conscious of available disk space. Each snapshot requires $\approx 40$ GB of storage space. This means that five snapshots per simulation and five simulations gives a total space requirement of $\approx 1$ TB. Further, because I engage in an intensive post-processing pipeline in order to calculate the densities, temperatures, and neutral hydrogen fractions, this intermediate data gets saved until the results are finalized. This increases the required storage space by several times, resulting in each simulation requiring $\approx 1$ TB, including restart files, or $\approx 5$ TB total. This does not include the data from the simulations run as a part of my resolution study, discussed in Section 4.4.

4.2 Synthetic Spectra

Once my simulations were run and the post-processing pipeline for calculating densities, temperatures, and neutral hydrogen mass fractions, as described in Chapter 3 was completed, I extracted 1152 synthetic Ly$\alpha$ Forest spectra from each snapshot using the method described in Section 3.6. An example spectrum is shown in Figure 4.4. Figure 4.4 has three panels: panel (a) plots the flux $F = \exp^{-\tau}$, panel (b) plots the number density of neutral hydrogen, and panel (c) plots the temperature of neutral hydrogen along the LOS. This figure serves as a consistency check on my spectral extraction method, as it shows that whenever there is an absorption feature in the spectrum, there is an increase in the density of neutral hydrogen, as expected. Further, the temperature range shown ($\approx 10^4 - 10^5$ K) is in keeping with the observed
temperature for the neutral hydrogen responsible for the Ly$\alpha$ Forest in the IGM (e.g., Becker et al., 2011), owing to the UV background.

In Figure 4.5 I plot the same LOS for each dark energy model, though vertically offset from one another for visual clarity. This figure highlights several points about the effects of dark energy on the Ly$\alpha$ Forest. The first point is that, for each dark energy model save for DE2-40-1024, the features in the spectra are nearly identical. The only slight differences between these four models is that there is a slight offset along the abscissa for the spectral features owing to the way that dark energy affects $v_H$, but this shift is small. As expected, the DE2-40-1024 model gives rise to a synthetic spectrum that is, visibly, quite distinctive from the other four. Most noticeably, perhaps, is the fact that DE2-40-1024 reaches Hubble velocities that are several hundred kilometers per second larger than those reached by the other four
Figure 4.5: This figure shows the same LOS for each of my dark energy models, though vertically offset from one another for visual clarity. The LOS used is the same as that in Figure 4.4. Each dark energy model, save for DE2-40-1024, gives rise to a nearly identical spectrum.

dark energy models.

4.3 Power Spectra

In order to generate a power spectrum for a given snapshot, I run my pool of synthetic Lyα Forest spectra through the procedure presented in Section 3.7.

Figure 4.6 shows the power spectrum from my L-40-1024 simulation as compared to the observational data of Iršić et al. (2017); McDonald et al. (2000, 2006). Figure 4.6 shows that, at each redshift, the simulated power spectrum matches the shape of the observed power spectra quite well. However, for nearly every $k$, and at every redshift save for $z = 3.8$ and $z = 3.0$, the simulated power spectra under-predict the amount of power. This is especially true at larger values of $k$ (corresponding to smaller spatial scales). See Chapter 5.

Figure 4.7 shows the power spectrum arising from each dark energy model at five different redshifts, $z = 4.2, 3.8, 3.0, 2.7$ and 2.2. The figure highlights that each dark energy model, save for DE2-40-1024, gives rise to an extremely similar power spectrum at all redshifts. The DE2-40-1024 predicts slightly less power at all scales.
Figure 4.6: This figure shows the power spectrum for my L-40-1024 simulation as compared with the observed power spectrum from Iršič et al. (2017); McDonald et al. (2000, 2006) at $z = 4.2, 3.8, 3.0, 2.7$ and 2.2. The only caveat here is that data from McDonald et al. (2000) at $z = 2.2$ was actually measured at $z = 2.41$, and is being compared to my simulated power spectrum at $z = 2.2$. The synthetic power spectrum shown here was generated using synthetic Lyα Forest spectra that had been run through the degradation process described in Section 3.6.1.
than my other four dark energy models.

4.3.1 Comparing Power Spectra

The goal of this project was to investigate the question: does dark energy leave an observationally detectable signature in the flux power spectrum of the Lyman α Forest? To answer this question, I started by comparing the simulated power spectra arising from each dark energy model to the power spectra arising from my cosmological constant simulation. The idea behind this is that we cannot distinguish between power spectra at the simulation level, where the situation is “ideal,” meaning that we possess full knowledge of the three-dimensional matter distribution and all of the prescriptions describing the physical processes involved, then they certainly cannot be determined in observations where the situation is far messier, with the presence of noise, systematic errors, statistical errors, and biases.

To perform this comparison, I chose to use the k-sample Anderson-Darling (AD) test (Anderson and Darling, 1952). I decided to adopt the AD test for several reasons. First, it is distribution free, and second, when compared with the Kolmogorov-Smirnov (KS) test, the AD test puts more emphasis on the tails of the distribution, whereas the KS test emphasizes differences between distributions near the center. Since dark energy is a large-scale phenomenon, I expect most of the differences between power spectra, if there are any at all, to occur on the largest scales (smallest $k$), rather than in the central parts of the power spectrum. Third, due to its increased sensitivity and ability to always be applied, the AD test has recently begun to be recommended over the KS test in astronomy (e.g., Babu and Feigelson, 2006).

When comparing power spectra, I chose the L-40-1024 simulation as my fiducial simulation to compare my other simulations to, since the cosmological constant is the de facto dark energy model in modern cosmology. The results of my comparison are given in Table 4.3. I used scipy in order to conduct this test. Table 4.3 shows
Figure 4.7: This figure shows the power spectrum from each dark energy model at $z = 4.2, 3.8, 3.0, 2.7$ and $2.2$. 
TABLE 4.3

ANDERSON-DARLING TEST RESULTS

<table>
<thead>
<tr>
<th>z</th>
<th>AD(^{1.5})</th>
<th>C.V.(^{1.6})</th>
<th>AD(^{2})</th>
<th>C.V.(^{2})</th>
<th>AD(^{3})</th>
<th>C.V.(^{3})</th>
<th>AD(^{4})</th>
<th>C.V.(^{4})</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.20</td>
<td>-1.23</td>
<td>1.96</td>
<td>-0.97</td>
<td>1.96</td>
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<td>1.96</td>
<td>-1.23</td>
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<td>-0.95</td>
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<td>1.96</td>
<td>-1.23</td>
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<td>3.00</td>
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<td>-1.23</td>
<td>1.96</td>
<td>-1.23</td>
<td>1.96</td>
</tr>
</tbody>
</table>

\(^{1}\) \(w_0 = 0.0, w_a = -3\)

\(^{2}\) \(w_0 = -1.1, w_a = 1.3\)

\(^{3}\) \(w_0 = -2.0, w_a = 0.0\)

\(^{4}\) \(w_0 = -2.0, w_a = 2.0\)

\(^{5}\) AD indicates the value of the Anderson-Darling test statistic

\(^{6}\) C.V. indicates the critical value for the chosen significance level of \(\alpha = 0.05\)

that my AD statistics are lower than the critical values for \(\alpha = 0.05\) in every case, indicating that I cannot reject the null hypothesis that the power spectra were drawn from the same distribution. This indicates that there is no statistically significant evidence of a signature from dark energy being imprinted on the flux power spectrum of the Ly\(\alpha\) Forest at the \(\alpha = 0.05\) significance level.

4.4 Resolution Study Simulations

In addition to my five fiducial simulations, I also ran four additional simulations for use in a resolution study. The names and parameters of these four resolution study
TABLE 4.4
RESOLUTION STUDY SIMULATION PARAMETERS

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Particles</th>
<th>Softening Length (kpc h(^{-1}) comoving)</th>
<th>(L) (Mpc h(^{-1}) comoving)</th>
<th>(M)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L-100-1024</td>
<td>(1024^3)</td>
<td>5.80</td>
<td>100</td>
<td>(8.14 \times 10^7 M_\odot h^{-1})</td>
</tr>
<tr>
<td>L-15-428</td>
<td>(428^3)</td>
<td>1.40</td>
<td>15</td>
<td>(3.76 \times 10^6 M_\odot h^{-1})</td>
</tr>
<tr>
<td>L-25-428</td>
<td>(428^3)</td>
<td>2.33</td>
<td>25</td>
<td>(1.74 \times 10^7 M_\odot h^{-1})</td>
</tr>
<tr>
<td>L-40-428</td>
<td>(428^3)</td>
<td>3.73</td>
<td>40</td>
<td>(7.13 \times 10^7 M_\odot h^{-1})</td>
</tr>
</tbody>
</table>

These simulations cover several different bases. The first is strictly the effect of box size. That is, the L-15-428 simulation has the same spatial resolution as my fiducial simulations, but is evolved in a box of length \(L = 15 \text{ Mpc} \ h^{-1}\). The purpose of this simulation was to explore the effects of box size on both the amount of small and large-scale power. The L-40-428 simulation was run in a box of the same size as my fiducial simulations, but with a different spatial resolution courtesy of a greatly reduced particle number. This was done to explore the effects of resolution due to particle number. The remaining two simulations, L-25-428 and L-100-1024, were run at lower spatial resolution than my fiducial simulations and with a lower and larger box size, respectively. The purpose of these two simulations was to explore the general effects of resolution on the power spectrum. Each of these four resolution study simulations was run using the cosmological constant as their dark energy model.

The results of my resolution study are shown in Figure 4.8 and Figure 4.9. The effects of smaller box size are shown in Table 4.8, which shows the power spectrum for the L-40-1024 and L-15-428 simulations at \(z = 3.0\). At larger scales the two power...
Figure 4.8: This figure shows the effects of using a smaller box size on the power spectrum. We have plotted the power spectrum for both the L-40-1024 (solid) and L-15-428 (dashed) simulations at $z = 3.00$. We see that at larger scales, the two power spectra are nearly identical, but as we move to smaller scales we see that there is an excess of power in the L-15-428 simulation.

spectra are nearly identical, but at smaller scales, the L-15-428 simulation has an excess of power as compared to the L-40-1024 simulation.

The effects of resolution due solely to particle number as well as the resolution effects due to varying both the particle number and the box size are shown in Figure 4.9. Figure 4.9 shows the power spectrum for the L-40-1024, L-40-428, L-25-428, and L-100-1024 simulations at $z = 3.0$. This figure highlights several points: first, the largest box size reaches scales beyond what can be reached in those simulations with smaller box sizes. Second, just like as in Figure 4.8, the power spectra in Figure 4.9 are nearly identical at large scales. At small scales, however, the L-25-428 and L-
100-1024 simulations differ from the L-40-1024 and L-40-428, albeit in different ways. The L-100-1024 simulation under-predicts the amount of power at small scales as compared with L-40-1024. This is to be expected, however, as the L-100-1024 simulation has a lower spatial resolution, which means that the small-scale structure in the cosmic web simply isn’t resolved and so cannot contribute. The L-25-428 simulation over-predicts the amount of power at small scales as compared with L-40-1024. While L-25-428 has a lower spatial resolution than L-40-1024, the effect is similar to that which occurred in Figure 4.8 and is discussed more in Chapter 5. Interestingly, the L-40-1024 and L-40-428 simulations appear to have virtually identical power spectra at all scales. This indicates that, as is to be expected, a higher spatial resolution affects small scales the most and, secondly, that the resolution of my simulations does not play a role in either washing out any potential dark energy signals or introducing spurious effects at large scales. This means that my conclusions about not being able to detect signatures of dark energy in the power spectrum appear to be unaffected by simulation resolution.

This conclusion is reinforced by the results of the AD test I performed, comparing each resolution study power spectrum to my fiducial simulation. The results of this test are shown in Table 4.5. The test shows that there is not evidence to reject the null hypothesis at the $\alpha = 0.05$ significance level for the resolution study simulations.
Figure 4.9: This figure shows the effects of varying spatial resolutions on the power spectrum. Here we plot the L-100-1024 (dotted), L-25-428 (dashed), and L-40-428 (dash dot) simulations alongside the fiducial L-40-1024 (solid) simulation. Here we see that at large scales each of the power spectra are nearly indistinguishable from one another. At smaller scales, however, there is a fairly substantial difference. The L-100-1024 simulation has substantially under-predicted power whereas the L-25-428 simulation has over-predicted power as compared to the others. The L-40-428 and L-40-1024 simulations are nearly identical at all scales.
<table>
<thead>
<tr>
<th>AD</th>
<th>C.V.</th>
<th>AD</th>
<th>C.V.</th>
<th>AD</th>
<th>C.V.</th>
<th>AD</th>
<th>C.V.</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.21</td>
<td>1.96</td>
<td>-0.95</td>
<td>1.96</td>
<td>-1.17</td>
<td>1.96</td>
<td>-1.23</td>
<td>1.96</td>
</tr>
</tbody>
</table>

1. L-100-1024
2. L-15-428
3. L-25-428
4. L-40-428

AD indicates the value of the Anderson-Darling test statistic

C.V. indicates the critical value for the chosen significance level of $\alpha = 0.05$
CHAPTER 5

DISCUSSION AND FUTURE WORK

The other night I was lying in bed, looking up at the stars, and I wondered, “Where the hell is my roof”?

—Steven Wright

This chapter presents a summary of my project, my conclusions about the results that were given in Chapter 4 and a discussion of the limitations of my simulations, analysis, and dark energy model selection procedure. Finally, I conclude with a discussion of future work to improve and build upon what has been presented in this thesis.

5.1 Summary

This work has sought to answer the question: does time-dependent dark energy leave an observationally significant signature in the flux power spectrum of the Lyman α Forest? The implication of this question is: can the power spectrum be used to constrain the pool of viable dark energy models? The motivation behind this question is that the mechanism responsible for dark energy, while comprising a majority of the universe’s present energy density, is still unknown despite observational data from SNIa, BAOs, the CMB, and weak gravitational lensing. Due to this gap in our knowledge, we remain incapable of building a complete model of how the universe has evolved and will continue to evolve. Furthermore, dark energy represents a gateway to understanding physics beyond the current standard model.
Since dark energy affects the expansion history of the universe (that is, the form of the Hubble parameter), in order to test various models of dark energy one must use an observational probe that is sensitive to $H(a)$ in some way. In theory, the only quantities that fit this bill are measurements of the age of the universe $t(a)$, the growth factor $D(a)$, the luminosity distance $d_L(a)$, and the angular diameter distance $d_A(a)$ (Kujat et al., 2002).

Measurements of the age of the universe, as discussed in Section 2.1.2, provide evidence for the existence of dark energy, but do little to allow for discrimination between dark energy models. This is because such measurements only provide a lower limit for the age as opposed to a definite prediction. Observations of SNIa and BAOs provide determinations of the luminosity and angular diameter distances as discussed in Section 2.1.1 and Section 2.1.4 respectively. These observations provide definitive values that viable dark energy models must match. However, performing these observations is difficult and, as such, they have only been made for a handful of low redshifts. Similarly, a determination of the angular diameter distance is also provided by measurements of CMB anisotropies, as discussed in Section 2.1.3 but this is only for the redshift of the surface of last scattering $z \approx 1100$. The growth factor, in particular the linear growth factor $D_1(a)$, however, is best determined from the Ly$_\alpha$ Forest at higher redshifts (e.g., Kujat et al., 2002; Seljak et al., 2002).

The Ly$_\alpha$ Forest is a well-known and well-studied observable that has already proven to be a useful cosmological tool (e.g., Weinberg et al., 2003). Its application to dark energy, however, has remained relatively unexplored (though see Greig, 2013; Kujat et al., 2002; Seljak et al., 2002; Viel et al., 2003). The physical motivation behind this thesis was, therefore, to explore the potential effects of time-dependent dark energy on the flux power spectrum of the Ly$_\alpha$ Forest because of the sensitivity of the linear growth factor to dark energy.

In order to answer this question, I ran five high-resolution N-body simulations
using a version of GADGET-2 that was modified for this thesis. Each simulation employed a different dark energy model. One of my five models was the concordance cosmological constant model of $w_0 = -1$, $w_a = 0$ and the other four were dynamical models. Three of these dynamical models were chosen such that they were close to the $2\sigma$ (95% confidence level) edge of the $(w_0, w_a)$ posterior as determined by Planck. This was done in the expectation that the power spectra arising from these models would have the greatest disparity between them while at the same time still remaining observationally viable. The large disparity between the models was desired because the models with the greatest difference in $w(a)$ would have the most noticeable differences in their simulated power spectra. The fourth dynamical model was deliberately chosen to lie outside of the allowed bounds as determined by Planck in the expectation that, should the other three dynamical models not be discernible, a more extreme dark energy model would be. This would serve to establish that time-dependent dark energy leaves a detectable signature on the power spectrum. For each of my five simulations, I saved snapshots at $z = 4.2, 3.8, 3.0, 2.7$ and $2.2$.

From each snapshot I extracted 1152 synthetic Ly$\alpha$ Forest spectra, as described in Section 3.6 and used this pool of spectra to calculate an instance of the power spectrum, as described in Section 3.7.

I took the cosmological constant simulation, L-40-1024, to be my fiducial simulation. At each redshift I compared each of my other four power spectra to my fiducial power spectrum via the k-sample AD test, which tests the null hypothesis that each of the k samples (two, in this case) are drawn from the same distribution. The results of this test indicate that there is no significant evidence to reject this null hypothesis at the $\alpha = 0.05$ significance level. This indicates that, statistically, the power spectrum cannot be used to discriminate between different dark energy models. The conclusion of this thesis, then, is that time-dependent dark energy does not leave a statistically significant signature on the flux power spectrum within the $2\sigma$ bounds of the Planck
analysis. However, other approaches may show promise, as described below.

5.2 Discussion

5.2.1 Temperatures

Due to the high resolution requirements for simulating the Lyα Forest, my simulations were strictly N-body out of consideration for the total run time. So, each of the hydrodynamic quantities needed to extract synthetic spectra (such as density and temperature) had to be calculated in post-processing. Of these, the temperature calculation in particular is worth discussing in greater detail. This is because the thermal state of the IGM affects the Lyα Forest via smoothing the absorption along the LOS with thermal motion of atoms and by smoothing the physical distribution of matter via pressure support ([Peeples et al., 2010]). Therefore, correctly modeling the thermal state of the IGM is imperative when studying the Lyα Forest.

My temperature calculation scheme was bimodal based on whether or not the particle in question was contained within a halo. For particles that were contained within a halo, temperatures were calculated based upon the virial properties of their host halo. This, of course, is sensitive to the parameters that I used in order to perform the halo finding in the first place. While the values of the halo-finding parameters that I chose resulted in halos in keeping with observational values, the parameter space is wide enough to allow for variation. However, I do not expect this to affect my results in a significant manner for the following reasons. First, only a small fraction of particles belong to halos (on the order of 10%), and on the other hand, halos take up very little physical space when compared with the size of my simulation volume (from $R_{\text{vir}} \approx 420 \text{ kpc h}^{-1}$ at $z = 4.2$ to $R_{\text{vir}} \approx 720 \text{ kpc h}^{-1}$ at $z = 2.2$). Second, the Lyα Forest is primarily due to the diffuse IGM, which is modeled by field particles. The gas closer in to galaxies gives rise to types of absorption systems other than the Lyα Forest, namely Lyman limit systems (LLSs) and damped Lyα absorbers (DLAs).
The temperatures of the field particles in my simulations are obtained by using Equation (3.15). Equation (3.15) is derived from the first law of thermodynamics as applied to an unshocked, ionized fluid element at late-times post-reionization (McQuinn, 2016). In the case that the IGM is highly ionized, as is the case for later times after reionization ($z \lesssim 6$), the thermal state of the IGM is governed largely by photoheating from the UV background and Compton cooling (e.g., McQuinn, 2016). This results in all unshocked gas with $\delta \lesssim 10$ evolving towards a power law with $\gamma - 1 = 0.6$ and very little dispersion (e.g., Hui and Gnedin, 1997; McQuinn, 2016), thereby justifying the use of Equation (3.15) across the redshift range $2 < z \lesssim 5$.

The temperature-density relation from my simulations is given in Figure 4.3. This can be compared to the temperature-density relation from a cosmological simulation that includes UV heating, radiative cooling, star formation, and feedback from supernovae (Snedden et al., 2016) in Figure 5.1. This figure shows a great deal more higher density gas than my simulations, owing to the presence of radiative cooling and metal feedback. This lack of cooling affects the small-scale power of the power spectrum, contributing to the discrepancy between my simulated power spectra and the observations. However, my temperature-density relation captures most the lower density part of the relation that gives rise to the Ly$\alpha$ Forest, particularly the structure responsible for the large-scale power of interest to this project.

5.2.2 The UV Background

The UV background is characterized by a flux spectrum $J(\nu, z)$, where $\nu$ is frequency and $z$ is the redshift. The exact form of $J$ is believed to be due to contributions from both quasars and star forming galaxies (e.g., Haardt and Madau, 2012). While there are constraints on the quasar contribution, the contribution from star forming galaxies is far more uncertain, due mostly to the fact that the fraction of UV...
Figure 5.1: This figure, from Snedden et al. (2016), shows the temperature-density relation from a cosmological simulation that included UV heating, radiative cooling, star formation, and supernova feedback at $z = 3$. This can be compared with my temperature-density relation given in Figure 4.3.
photons that can escape from such a galaxy is unknown (e.g., Rivera-Thorsen et al., 2017; Vanzella et al., 2018 and references therein). Additionally, Madau et al. (1999) have shown that the known population of quasars and large starburst galaxies cannot provide enough UV photons to produce the ionization state of the present-day IGM. However, it is believed that small dwarf galaxies may contribute, as well.

This large uncertainty surrounding the UV background affects the numerical procedure for spectral extraction via the normalization of each spectrum, as discussed in Section 3.6. This is because scaling the pixel optical depths to match the mean observed optical depth is akin to re-scaling the UV background (e.g., McQuinn, 2016). This process is justified because the photoheating rate does not depend upon the amplitude of \( J \) (e.g., McDonald et al., 2000; McQuinn, 2016; Theuns et al., 1998). While collisional ionization is important for high temperature gas, this gas is always at high density, which means that it produces saturated absorption in Ly\( \alpha \) and therefore does not contribute to the Ly\( \alpha \) Forest (e.g., McDonald et al., 2000). The Ly\( \alpha \) Forest is caused by lower density, mildly non-linear material in the IGM.

Further, the UV background that I have assumed is spatially uniform, depending only on the redshift and the frequency. As stated in McQuinn (2016), it is difficult to determine how well such uniform models describe the real UV background. While observations constraining the UV background have been made, particularly at low redshift (e.g., Fechner, 2011), it is also known that the assumption of a perfectly uniform UV background cannot be completely valid, such as near very dense systems and during reionization (e.g., Becker et al., 2015; Rahmati et al., 2013). However, as my simulations take place well after reionization, and the variance in the UV background near very dense systems has been estimated to be \( O(1) \) (e.g., Bolton et al., 2006), this should not affect my conclusions.
5.2.3 Metal Lines

My simulations assumed that the IGM was solely comprised of hydrogen and helium. This, however, is not true. As stars form, evolve, and die, elements heavier than helium are synthesized\(^1\). These heavier elements are collectively known in astrophysics as metals. The winds from stars and supernovae drive the majority of these metals out of the host galaxy and into the IGM (e.g., Davé et al., 2011; Peeples et al., 2014). Indeed, it has been found that approximately half of the gas in the IGM by mass has been metal enriched to observationally detectable levels (e.g., Pieri and Haehnelt, 2004; Simcoe et al., 2004).

It has been found that metals account for \(\approx 10\%\) of the absorption in the Ly\(\alpha\) Forest at \(z = 2\) (e.g., Kirkman et al., 2005; Schaye et al., 2003). This fraction decreases as redshift increases. Simulations of the Ly\(\alpha\) Forest implicitly assume that all of these metal lines can be detected and removed from observational data. This, however, is not the case. As the metal lines are typically narrower than the Ly\(\alpha\) lines, limited resolution causes the features to blend together (e.g., McDonald et al., 2000). This blending strongly affects the amount of power in the power spectrum at small scales \((k > 0.16 \text{ km s}^{-1})\) (McDonald et al., 2000).

While my Ly\(\alpha\) Forest spectra do not include metal lines, they are compared to observational data that necessarily do. While attempts were made to remove metal lines from the observational data sets that I compared my power spectra to, these techniques are not perfect, and could add power to the observations at small scales, helping to account for the discrepancy between my power spectra and the observed power spectra.

Generally, the way that metals are removed from the power spectrum is by using the metal absorption lines redward of the quasar emission line (e.g., Iršić et al., 2017).

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\(^1\)In addition to hydrogen and helium, the Big Bang synthesized trace amounts of deuterium, tritium, the lithium-7 isotope, and the beryllium-7 isotope. The tritium and beryllium-7 were unstable, however, and decayed to helium-3 and lithium-7.
This is because metal absorption is the only type of absorption that occurs in this region of the spectrum, which means a metal line power spectrum can be determined. However, as these metal lines are redward of the quasar’s Lyα emission line, they occur at a higher redshift than the Lyα absorption whose power spectrum is being determined. As such, in order to remove the metal line power from the Lyα power at the redshift of interest, a subsample of quasar spectra at a lower redshift than the desired redshift is required (e.g., Iršić et al., 2017).

From this lower redshift subsample, the metal line power spectrum can be calculated for the higher, desired, redshift and then subtracted off, leaving just the power from Lyα absorption. This process is only a rough approximation, but Iršić et al. (2017) found that the value of the metal power \( P_m \) was generally smaller or of the same order as the rest of their statistical errors, implying that a better determination of the metal power would not significantly change their results.

As such, while contamination from metals can certainly play a role in creating the observed discrepancy between my simulated power spectra and observational data, particularly at small scales, it does not appear to be the dominant effect.

5.2.4 Resolution

Perhaps the largest effect on my results as far as discrepancy with observations is concerned is resolution. Resolution has been found to have a significant effect on simulated Lyα Forest spectra and their statistics (e.g., Bolton and Becker, 2009; Bolton et al., 2017; Lidz et al., 2010; Lukić et al., 2015; McDonald, 2003; McDonald et al., 2005; Meiksin and White, 2004; Tytler et al., 2009).

Bolton et al. (2017) found that a particle mass of \( m \approx 7.97 \times 10^5 \, h^{-1} M_\odot \) was sufficient for resolving the power spectrum to within \( 10 - 20\% \) on scales \( \log(k) > -1 \).

My simulations were run using the resolution recommended in McDonald (2003). This resolution results in particles with masses nearly an order of magnitude more
massive than those of Bolton et al. (2017). As my resolution study suggests, the effect of simulation resolution on small-scale power is substantial. Therefore, I conclude that resolution, along with the lack of baryons, account for the discrepancies between my simulated power spectra and the observations at small scales. However, as these factors primarily affect small scales, and will systematically affect each of my simulations, they do not affect my conclusions.

5.2.5 Model Selection

My simulations were run using four dynamical models of dark energy. These models were chosen such that they were near the fringes of the allowed observational values as determined by Planck. However, as a caveat to this model selection, it must be noted that the distribution I was sampling from is a posterior distribution where all other cosmological parameters (such as $\Omega_m,0$ and $\Omega_{DE,0}$) have been marginalized out. Since I pick explicit values for these cosmological parameters, direct comparison with such a posterior is not strictly viable.

It was the aim of Planck’s analysis to determine the best-fit values for $w_0$, $w_a$, and the other cosmological parameters. This was not the aim of my project. Such a Bayesian approach was neither viable nor applicable to my problem. In order to perform a comparable marginalization, I would need a very large grid of simulations with a dimensionality equal to the number of cosmological parameters that I wished to explore. Each axis of this higher dimensional grid would then vary the value of its corresponding cosmological parameter. However, from both a computational and storage-space perspective, creating such a grid was simply not feasible. Further, the purpose of such a grid is to allow for the implementation of Markov-Chain-Monte-Carlo (MCMC) methods that can be used to determine best-fit values for each cosmological parameter.

The aim of my project was to explore whether or not dynamical models of dark
energy left an observationally detectable signature in the power spectrum of the Lyα Forest. This meant that I was less interested in determining best-fit values for $w_0$ and $w_a$, but instead interested in the effects of varying $w_0$ and $w_a$ whilst holding the values of the other cosmological parameters constant. While the fact that I drew from Planck’s posterior distribution is complicated by my lack of marginalization, thereby making it impossible to tell if my particular combination of cosmological parameters (besides the fiducial combination) for a given simulation were observationally viable, this point is largely secondary. The main takeaway is that I was able to use Planck’s posterior as a guide for choosing reasonable values from the $w_0 - w_a$ parameter space and then explore the effects of $w_0$ and $w_a$ on the power spectrum.

5.2.6 Results

My synthetic Lyα Forest spectra appear visually reasonable. In addition to this, they produce physically reasonable values for the neutral hydrogen number density and temperatures, as shown in Figure 4.4.

The power spectra extracted from each of my simulation snapshots match the overall shape of the observed power spectra quite well, especially at the large scales of interest for this study. Indeed, at higher redshifts ($z = 4.2, 3.8$, and $3.0$) there is good overall agreement between my simulations and the observations at large scales. There is an under-prediction in the power at small scales for every redshift, however. This discrepancy in small scale power is, as discussed above, due to a combination of resolution and the absence of baryons in my simulations. The most important aspect of baryonic physics that I do not model is that of cooling. Cooling allows for the baryon distribution to reach much higher overdensities than the dark matter distribution (e.g., Peeples et al. 2010), therefore increasing the amount of power present on small scales. Increased resolution, of course, allows for these smaller, very over-dense regions to be resolved by the simulation. Additionally, at lower redshifts
(z = 2.7 and 2.2), there is an under-prediction of the power at all scales. As my resolution study showed very little effect on large-scale power, it is unlikely that this is the culprit. Further, while the absence of metals and baryons in my simulations certainly affects the overall power, these mostly affect small-scale power, as described above.

The next point of discussion is the normalization of my Lyα Forest spectra. It has been shown (e.g., [Bolton et al. 2017]) that simulations will under predict the power at all scales unless they account for a correction to the effective optical depth. This correction arises from systematic bias in the continuum level estimated in observational data (see Section 3.6.1) ([Faucher-Giguère et al. 2008]). Without including this correction, Bolton et al. (2017) showed that the power was under-predicted at all scales, even when including baryonic physics. However, Faucher-Giguere et al. (2008) showed that the strength of this correction ranged from ≈ 12% at z = 4 down to being negligible at z = 2. As such, this effect does not explain the under-prediction of power at low redshifts.

Of particular importance to me is less that the simulated power spectra perfectly match the observations, but rather knowing that the reasons for the discrepancy are systematic. That is, they affect each of my simulations in the same manner. As discussed above, there appear to be three primary reasons that my simulated power spectra under-predict the power as compared to observations: the absence of baryons, resolution effects, and my spectral normalization. Since baryons affect primarily small scales and dark energy is primarily a large-scale phenomenon, I do not expect the presence of baryons to affect each model in substantially different ways. Further, the effects of resolution should be systematic in nature, as should the effects of normalization since this process serves to simply scale everything up. This means that these effects, while helping my simulated power spectra better match observations, should not affect my result about being unable to use the power spectrum as a probe of dark
energy.

My result of being unable to detect the effects of dark energy in the power spectrum differs from what was found by Viel et al. (2003). Those authors determined that the dark energy models they employed could, in principle, be discriminated against using the optical depth power spectrum. It should be noted, however, that Viel et al. (2003) used dark energy models that all had a constant EOS, whereas I have used dynamical models. Furthermore, the simulations employed by Viel et al. (2003) were semi-analytic in nature whereas mine were N-body. These two factors would seem to be consequential enough to account for our differences in conclusions. This in no way invalidates the findings of Viel et al. (2003), as constant EOS dark energy models other than \( w(a) = -1 \) are still very much permitted by observations (Planck Collaboration et al., 2015). I simply point out that the effects from dynamical dark energy do not appear to be as prominent as those models considered in Viel et al. (2003).

5.3 Future Work

Over the last several years, the Ly\( \alpha \) Forest has shown itself to be a powerful cosmological tool. This thesis has investigated the flux auto-power spectrum, which is just one of many statistics that can be obtained from the Ly\( \alpha \) Forest. A related statistic is the cross-power spectrum. This is obtained from sight lines of close quasar pairs, and provides an alternative method to measuring the two-point correlation function. Interestingly, this has been suggested as a possible probe of dark energy (Seljak et al., 2002). This may work because it uses the scale at which the spectral index \( n \) of the matter power spectrum takes a particular value as a standard ruler by which \( H(z) \) can be determined. Since this statistic uses the angular diameter distance as opposed to peaks in the density field along the line of sight, it is likely that predictions for various dark energy models made using the cross-spectrum will
be more robust than those presented here. This makes the cross-spectrum worth investigating.

Additionally, there has been recent work regarding the use of the Lyα Forest in order to detect the BAO signature (e.g., Norman et al. 2009; Slosar et al. 2009). While the simulations employed in these works were significantly larger than those analyzed in this thesis. Slosar et al. (2009), in particular, employed a suite of dark matter only simulations with $3000^3$ particles in a $1.5 \text{Gpc } h^{-1}$ box. Their simulations were performed for a cosmological constant only. It would be illuminating to carry out the exercise using dynamical dark energy in order to make robust predictions about the BAO signal under dynamical dark energy models. This project is particularly relevant now that both the BAO signature has been detected in observational data of the Lyα Forest (Slosar et al. 2013) and the amount of observational data regarding BAOs is soon to be substantially increased, owing to the Euclid (e.g., Joachimi 2016) project, LSST (e.g., LSST Science Collaboration et al. 2009), the WEAVE-QSO survey (e.g., Pieri et al. 2016), and DESI (e.g., Levi et al. 2013).


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