Abstract

by

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The purpose of this research was to control crossflow-induced boundary-layer transition on a cone at angle of attack in hypersonic quiet flow. A 7° half-angle cone model with interchangeable nosetips was designed and fabricated from stainless steel, polyether ether ketone (PEEK), and Macor. Transition was characterized using infrared thermography and Kulite pressure transducers in the Boeing/AFOSR Mach-6 Quiet Tunnel at Purdue University. A plasma-based active flow-control system was used to control the transition location of the stationary crossflow waves, which manifested themselves as hot streaks on the cone. The transition location was accelerated by critical forcing (where the actuator wavenumber equals the wavenumber of naturally largest amplitude waves) and delayed by subcritical forcing (where the actuator wavenumber is larger than the natural waves). The disturbance wavenumber input of the plasma actuators was observed downstream on the model for many of the plasma-on runs, demonstrating that the plasma actuators introduced discrete forcing into the flow. The precise locations of the hot streaks varied for different nosetips, presumably due to differences in their microscale roughness.

The experimental data were used to inform an improved stability analysis. Stationary crossflow vortex N-factors were calculated over the surface of a yawed circular
cone using computationally predicted and experimentally observed wavenumber distributions. A wavelet analysis was conducted on the experimental surface heat-flux data to construct a spatial mapping of the local largest amplitude wavenumbers of the stationary crossflow waves, which were between 40 and 80 per circumference. Significant spatial variation was observed. The results from the wavelet analysis informed the stability analysis. The computed integration marching directions demonstrated good agreement with the experimentally observed paths. N-factors were calculated by integrating the local amplification rate corresponding to the most amplified experimental wavenumbers. The calculations were repeated based on non-dimensional computationally varying wavenumber ratios, which were dimensionalized by the experimental data. The computed N-factors showed good agreement between the two techniques. N-factors were also computed using the computationally predicted most unstable wavenumbers. The results showed decreased agreement with the other two cases, suggesting that this assumption does not properly model the crossflow transition process.
DEDICATION

To my wife, Kate, and to my family, whose unending love and support made this work possible.
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<td>$c_p$</td>
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CHAPTER 1
INTRODUCTION AND MOTIVATION

Hypersonic flight, which occurs at speeds above Mach 5, is relevant to missions such as atmospheric re-entry, prompt global strike, high altitude flight, and easier space access. At these speeds, a flight vehicle will be subjected to high heating loads, which will have a large influence on the vehicle’s design and performance characteristics. These effects are largely influenced by the state of the boundary layer, which can be laminar, transitional, or turbulent. The transition process is largely influenced by disturbances in the freestream or on the surface of a body introducing disturbances into the boundary layer through a process known as receptivity. These disturbances can either decay or grow. If the disturbance amplitude becomes large enough, rapid non-linear growth can occur, leading to a transition to turbulence and high heating loads on the body.

It is imperative to accurately understand, predict, and control the transition process in order to achieve optimal and effective vehicle designs. If the boundary layer transition location can be delayed, a larger portion of the vehicle can be subjected to lower, laminar heating rates. Ensuring that the vehicle’s thermal protection system can withstand these thermal loads while also optimizing its weight to maximize performance attributes such as payload, range, and maneuverability is a major design driver of future hypersonic vehicles.

The crossflow instability is one mechanism that can lead to transition to turbulence in a three-dimensional hypersonic boundary layer. This instability can manifest itself on different geometries that may be implemented on flight vehicle designs and
is therefore highly relevant to the research community. Therefore, ground-based tests that address the control of this instability are critical to the understanding of hypersonic boundary layer transition and flight vehicle design.

1.1 Crossflow Instability

There are several instabilities that can cause transition in a three-dimensional boundary layer including the centrifugal, streamwise, and crossflow instabilities. The crossflow instability can occur on different configurations such as swept wings, cylinders, rotating disks, and cones at angle of attack [80]. On a cone at angle of attack in hypersonic flow, the bow shock is not axisymmetric, creating a circumferential pressure gradient. This pressure gradient generates curved streamlines within the boundary layer by providing a centripetal force, thinning the windward boundary layer and thickening the leeward boundary layer [55, 84].

Within the boundary layer, pressure is approximately constant in the wall-normal direction. Velocity is zero at the wall due to the no-slip condition and the spanwise velocity (perpendicular to the inviscid streamline) is also zero at the boundary layer edge. These two velocity boundary conditions result in a spanwise velocity profile with an inflection point, as seen in Figure 1.1 [1]. This inflection point provides a source of an inviscid instability, which leads to the development of stationary and/or traveling crossflow vortices that can lead to transition to turbulence [80]. Traveling crossflow modes are caused by freestream noise and/or turbulence, and cause vortical disturbances that convect with the bulk flow. Stationary crossflow modes are caused by surface roughness or any other steady forcing and produce disturbances that are stationary on the geometry.
1.2 Plasma-Based Active Flow Controllers

Recently, there has been considerable interest in using active and passive flow controllers to manipulate crossflow-induced boundary layer transition location on test models, motivating the current work. Saric and Reed first introduced the concept of distributed roughness as a means of passive laminar flow control for the crossflow instability in supersonic flow [78, 79]. They used distributed roughness elements to control crossflow-induced transition on a 73° swept wing in a Mach-2.5 flow. They proposed that this laminar flow control could be accomplished with roughness, holes, or a glow discharge applied in a discrete pattern. The experiments used roughness elements located near the wing leading edge to control transition location. For a 1.7 mm roughness element spacing, transition was moved on the swept wing from roughly 30% to 55% of the chord length [78]. Saric and Reed showed that a proper spacing for the roughness elements should be less than the spacing of the most amplified stationary waves to delay transition [78].

Schuele et al. ran experiments at Mach 3.5 in quiet flow using a 7° half-angle cone at 4.2° angle of attack [84]. Schuele used passive patterned roughness elements
and Dielectric Barrier Discharge plasma actuators located at the neutral point of the most amplified stationary waves to influence transition. Data were collected using two plasma actuator designs: one at the naturally amplified wavenumber \( m = 45 \) and one at a modenumber \( 1.5 \times \) the naturally amplified wavenumber \( m = 68 \) to suppress the crossflow instability. Schuele showed that the \( 1.5 \times \) subcritical forcing was effective in delaying transition for this geometry.

Corke et al. used passive discrete roughness elements to control stationary crossflow waves at Mach 6 under noisy-flow conditions \[19\]. Oil flow visualization and a traversing total pressure Pitot probe system were used to make measurements regarding the state of the boundary layer. Transition location was delayed by 25% using the subcritically spaced configuration. A non-linear interaction between the stationary and traveling waves was hypothesized, which could explain discrepancies between crossflow-induced transition location in quiet and noisy tunnels.

Ward et al. looked at the crossflow instability on a circular cone at angle of attack in a Mach-6 quiet tunnel using temperature sensitive paint and fast acting pressure transducers (PCBs and Kulites). The experiments found that transition was induced by the crossflow instability under fully quiet flow. Ward used roughness elements, as developed by Saric and Reed, in a circumferential pattern located 2 inches from the nose tip in a torlon insert to influence stationary crossflow waves. A pattern of 50 elements was found to promote the generation of stationary crossflow vortices, thereby advancing the transition location forward. The experiments studied the effects of varying the diameter, height, and spacing of the roughness elements. It was found that the location of crossflow-induced transition moved upstream for an increased roughness element diameter or height \[95\].

This work builds upon the results of Ward and Schuele by implementing a plasma-based active flow control system on a circular cone at an angle of attack at Mach 6 under quiet-flow conditions. While it is known that crossflow-induced boundary
layer transition can be successfully delayed at Mach 6 in the BAM6QT with passive roughness elements and in the NASA Langley Mach-3.5 Quiet Tunnel with a plasma flow controller, this project will seek to demonstrate the effectiveness of a plasma-based active flow controller at Mach 6. The system was designed with the goal of both accelerating and delaying crossflow-induced boundary layer transition on the test model.

This project also sought to design an optimized plasma-based active flow control system to manipulate transition location on the test model. In order to design the most effective actuator, two designs were pursued: one with actuators spaced at the wavenumber of the vortices at the transition location and one with actuators spaced at the wavenumber of the disturbances at their neutral points (where the disturbances begin to amplify). In order to create the most efficient actuator designs, the naturally amplified wavenumber of the stationary crossflow waves needed to be determined. Tests were conducted on a cone test model at Mach 6 with a steel nosetip to experimentally observe the naturally most amplified wavenumbers of the stationary waves. These measured wavenumbers informed both of the actuator designs.

1.3 Improved Stability Analysis

In general, “stability analysis” refers to a method of decomposing a Navier-Stokes solution into two parts; an unperturbed “basic state” and a perturbation on this solution. One can then formulate a set of equations, which depend heavily on leveraging the assumptions made about the basic state quantities, which form a complex eigenvalue problem having complex exponential solutions. All the methods in this family are derived by inserting the decomposed flow variables into the full Navier-Stokes equations and simplifying by truncating small terms. A fallout of the simplifications made is that the disturbances typically take the form of sinusoidal waves in time and the non-resolved (homogeneous) spatial directions. In this context, linear stability
theory (LST) assumes only one non-homogeneous direction (wall normal). Both linear and nonlinear parabolized stability equations (PSE) assume one rapidly varying non-homogeneous direction, as in LST, but add the assumption of a second “slowly-varying” non-homogeneous direction to account for streamwise development of the disturbance. The PSE can be solved linearly (single disturbance) or non-linearly (accounting for multiple disturbances and their interactions).

Computational work pertaining to the crossflow instability has focused on modeling the stability and receptivity of three-dimensional boundary layers using a variety of computational tools including LST, PSE, and direct numerical simulation (DNS). Each of these techniques have different advantages. LST analyses offer the simplest and least computationally intensive approach, but they ignore nonlinear and non-parallel effects [58]. These assumptions can lead to discrepancies between computed predictions and experimental results. PSE can take into account nonlinear and non-parallel effects, thus more accurately predicting the transition process [15, 50, 51]. DNS simulates the fully nonlinear governing equations, thus providing the most detailed results with the least number of assumptions [26]. This technique, however, requires substantial computational power.

Regardless of the chosen technique, previous computational works modeling the stationary crossflow vortices have relied on seeding initial disturbances on the computational grid in order to excite disturbances at a characteristic wavelength [26]. While arbitrarily spaced disturbances can successfully excite disturbances of a given wavenumber, connecting the wavenumber of the controlled disturbance input to the wavenumber of instabilities at transition is not straightforward because the local wavenumber varies as the disturbances evolve downstream [64]. An open question in the research community, therefore, is how to accurately predict the wavenumber of the largest amplitude disturbances.

Previous research efforts using the PSE method on conical geometries have at-
tempted to more accurately model the spatial evolution of the wavenumber of the stationary crossflow waves by holding the total number of waves constant and adjusting the local wavenumber based on the local geometric circumference. In this approach, the wavenumber is assumed to be uniform azimuthally, but varies axially. While this assumption partially accounts for the spatial evolution of the wavenumber, it does not fully capture the spatial evolution of the largest amplitude stationary waves. Recently, more robust computational techniques have been implemented to address this challenge. Kuehl et al. proposed a technique to account for the local wavenumber variations in the marching direction of their stability analysis conducted on a yawed circular cone [51]. This method relied on the assumption that the evolution of wavenumber was a function of radius of curvature. Because the local streamlines are inclined relative to a constant azimuth, the local radius of curvature was taken to be an effective radius in the wall-normal direction. This technique required an assumption of the initial wavenumber where the disturbances were seeded. Moyes et al. refined the computational technique of Kuehl et al. to compute the local effective wavenumber and implemented this on the HiFIRE-5 elliptic cone geometry [64].

Moyes et al. proposed an adjustment to the basic state to model the evolution of wavenumber measured in the direction perpendicular to the marching path (in this case taken along a crossflow vortex path). This methodology assumed that the number of waves remained constant in the space between the arbitrarily seeded computational vortex paths. As the paths moved along the cone, the distance between paths, measured in azimuthal angle, could increase or decrease. This change in spacing, along with the local orientation of the disturbance path, was used to determine what the local effective wavenumber would be [64]. While these techniques more accurately modeled the changes in local wavenumber as the disturbances progressed downstream, they still cannot predict the actual wavenumber of the stationary crossflow waves that would be observed experimentally or in flight.
In this work, a new technique is presented to more accurately model the spatial wavenumber evolution of the largest amplitude stationary crossflow waves. The local wavenumbers are computed from experimental data, providing greater insight into the boundary-layer physics and resulting in a more robust computational approach. The local wavenumbers of the vortices are extracted from experimental observations of stationary waves, taken on a yawed circular cone under quiet-flow conditions at Mach 6 in the BAM6QT. The experimental wavenumbers are computed via a wavelet analysis of the surface heating distribution, which captures the axial and azimuthal wavenumber variation. The wavenumbers of the stationary waves with the local largest amplitude are thereby obtained. The stability analysis then computes N-factors by integrating the local amplification rate along computed vortex paths corresponding to the largest amplitude wavenumbers observed in the experiment. This method is advantageous compared to previous techniques because it directly measures the evolution of the disturbance wavenumbers. Additionally, a second stability analysis is conducted, implementing a computational technique based on vortex path spreading similar to the work of Moyes et al. This technique yields a nondimensional wavenumber ratio, seeking to more accurately predict the spatial wavenumber variation of the stationary crossflow waves. The results are dimensionalized by the experimental data for each respective path. Finally, the stability analysis is conducted using the computationally predicted most unstable wavenumbers.
CHAPTER 2

LITERATURE REVIEW

2.1 Subsonic Crossflow Studies

The crossflow instability was first observed by Gray during a survey of different flight and ground tests involving configurations with swept wings [33]. Using sublimation techniques, transition location was quantified and stationary streaks were observed in the local flow direction near the leading edge of swept wings. Subsequently, many experimental works were carried out in the subsonic flow regime on different geometries. Using flight test results from Gray, Owen and Randle defined a crossflow Reynolds number based on the maximum crossflow velocity and boundary-layer height where the crossflow velocity is within 10% of the maximum [66]. Leckoudis and Mack evaluated the effects of compressibility on the stability of flow over a swept wing [54, 57]. They found that compressibility significantly reduced the amplification rate and orientation of the most unstable wave. Malik and Poll studied the crossflow instability on a yawed cylinder in incompressible flow [59]. They showed curvature of the surface geometry and streamlines to have a stabilizing effect on the disturbances. Poll also carried out separate experiments on a yawed swept cylinder [71]. The crossflow instability was characterized in the form of both stationary and traveling waves. Stationary waves were visualized by surface evaporation or oil flow techniques. Regularly spaced streaks were observed and aligned approximately in the direction of the inviscid flow. Travelling waves were observed in the form of large-amplitude waves at a frequency near 1 kHz. This was the first time high frequency traveling waves were observed experimentally.
2.2 Super- and Hypersonic Crossflow Studies

In 2004, Schneider reviewed computational analyses and experimental ground and flight tests of super- and hypersonic transition on circular cones \cite{83}. An early synthesis of linear stability theory and experimental results was conducted by Malik and Balakumar, who studied 3-D flow on a 5° semi-vertex sharp cone at 2° angle of attack at Mach 3.5 \cite{58}. N-factor integration was performed along inviscid streamlines. N-factors were found to be much higher along the leeward ray than the windward ray. These N-factors were not related to the crossflow instability due to their azimuthal position. Boundary-layer instability between the windward and leeward rays was attributed to the crossflow instability. The corresponding experimental work was conducted by King at Mach 3.5 for a sharp 5° half-angle cone at 0.6°, 2°, and 4° angle of attack under noisy and quiet flow \cite{48}. Transition was detected using Preston tubes by slowly increasing the tunnel stagnation pressure until the normalized Preston tube pressure saw an abrupt increase. Reynolds number was varied until transition was achieved on different azimuthal angles of the cone. Using this technique, transition fronts were mapped onto the cone surface for both noisy and quiet data. An N-factor of 10 for the most amplified disturbances correlated with experimental transition fronts within 35 percent \cite{58}. The difference between computed and experimentally observed transition fronts was attributed to assumptions in the LST analysis. These included ignoring non-linear, surface curvature, and non-parallel effects.

Progress since 2004 has benefited from improved computational techniques and instrumentation, new facilities, and successful flight tests. Casper et al. studied the effect of freestream noise on roughness-induced transition for a slender cone at zero and 6° angle of attack \cite{10}. Transition behind a roughness element was delayed with quiet flow compared to noisy flow. Kroonenberg et al. used IR thermography to visualize the stationary crossflow vortices on a 7° half-angle cone at 6.6° angle of attack \cite{92}. Power spectra were computed from surface pressure measurements to deter-
mine the frequency and spatial variation of disturbances within the boundary layer. Peaks in the spectra near 400 kHz indicated the presence of second-mode instabilities. Additionally, a technique was developed to compute the relative amplification rates of the experimentally observed crossflow vortices. This involved computing the amplitude of high-pass filtered heat-flux data fit to a sine function.

Wadhams et al. conducted ground tests in support and development of the HiFIRE-1 flight tests [94]. Schlieren images, heat-transfer measurements, and pressure data were obtained on the full-scale HIFiRE-1 flight geometry. Berger et al. conducted experiments in support of HiFIRE-1 in the NASA Langley 20 in. Mach-6 tunnel [4]. Global heat transfer, used to infer the state of the boundary layer, was obtained using phosphor thermography. Kimmel et al. studied data from the HiFIRE-1 test flight [45, 46]. The test flight obtained pressure, temperature, and heat-transfer measurements during ascent and reentry. Data obtained during the ascent phase of the flight were used to map a transition front which could calibrate N-factor transition correlations. During the ascent phase, the freestream Mach number was greater than 5 and the angle of attack was less than 1°. This meant second mode was predicted to dominate the transition process. Willems et al. conducted post-flight ground experiments in support of HIFiRE-1 to match the conditions experienced during flight including Mach number, Reynolds number, and angle of attack [96]. These tests provided a better source of comparison with the flight-test data because the flight vehicle flew at higher angles of attack than expected during the descent phase. This work used high-frequency PCB pressure transducers and IR thermography to collect pressure and temperature data on the model surface.

Edelman and Schnedier investigated the nonlinear breakdown of hypersonic stationary crossflow waves in Mach-6 quiet flow on a sharp 7° half-angle cone at 6° angle of attack [29]. They measured the growth and breakdown of the secondary instabilities of the stationary crossflow waves. These secondary instabilities, which
form in regions of high shear caused by the upwelling of large stationary vortices, are the proximate cause of transition. Edelman and Schneider used PCB piezoelectric pressure sensors to measure pressure fluctuations on the surface of the cone and temperature sensitive paint (TSP) to determine the location and amplitude of the stationary waves. A secondary instability was measured as it grew over a range of amplitudes until the onset of turbulence. The initial linear growth rate of the secondary instability was about 50 m$^{-1}$, with a peak growth before breakdown of about $e^{3.8}$. High-frequency instabilities grew under hot streaks observed in the TSP, which were attributed to the thin troughs between vortices. Low-frequency instabilities grew more slowly between the hot streaks, suggesting the instability frequencies scaled with the local boundary-layer thickness.

Craig and Saric investigated the crossflow instability in a hypersonic laminar boundary layer in the low-disturbance Mach-6 Quiet Tunnel at Texas A&M University [24]. Experiments were performed on a 7° right, circular cone with an adiabatic wall condition at 5.6° angle of attack. Measurements were made using a constant-temperature hot-wire anemometer system with a frequency response up to 180 kHz. They observed the growth and saturation of the stationary crossflow waves. A traveling wave coexisted with the stationary waves and occurred in a frequency band centered around 35 kHz. The behavior of the modes was largely consistent with their low-speed counterparts prior to saturation of the stationary wave. Craig and Saric also noted that the stationary crossflow vortices dominated the flow field in hypersonic quiet flow on the model [23].

Chang et al. developed the Langley Stability and Transition Analysis Code (LASTRAC), which implements LST and PSE to predict transition on two-dimensional, axisymmetric, and infinite swept-wing boundary layers [12]. They validated the code for several configurations including Mach-2 flow over a flat plate, Mach-6 flow over a compression cone, and Mach-2 flow over a swept wing with roughness control. They
concluded that transition N-factor correlation remains the most viable option for transition prediction. Li et al. used LST and PSE to study transition over a circular cone at 3° and 6° angle of attack at Mach 6 \[55\]. The computations were modeled to match the corresponding experimental conditions in the Boeing/AFOSR Mach-6 Quiet Tunnel (BAM6QT) at Purdue University. Two configurations were studied: flow over a yawed circular cone and flow over a compression cone. The maximum stationary crossflow N-factors were computed along streamline trajectories and found to be between 10 and 20. These N-factors provided assessments of predicted transition fronts and disturbance characteristics, including frequency content and wavenumbers.

Balakumar and Owens studied the stability characteristics on a cone at angle of attack in hypersonic flow using DNS to solve the three-dimensional Navier-Stokes equations in cylindrical coordinates \[2\]. Simulations were conducted at Mach 6 at nominally identical freestream conditions compared to experiments in the BAM6QT. Simulations containing between 30 and 70 stationary crossflow vortices were performed by modeling roughness elements on the simulated cone. N-factor computations predicted transition would occur more rapidly on the yaw side of the cone (between the windward and leeward rays), with the earliest transition detection occurring at \( \phi = 110^\circ \). The wavenumbers of the stationary crossflow vortices evolved as the vortices progressed downstream.

In summary, the focus of past research efforts involving circular cones in hypersonic flow has been to understand the complex mechanisms of crossflow-induced transition for this configuration. Research has focused on obtaining good agreement between transition prediction from computational data as well as experimental data from flight and ground tests. The effect of freestream noise level on stationary crossflow waves is not completely understood. Quiet tunnel tests have shown a large impact of tunnel noise on transition location due to stationary crossflow on various geometries \[41, 95\]. On the other hand, amplification and breakdown of station-
ary crossflow waves have been detected in conventional noisy facilities [19]. Limited flight test results appear to agree better with quiet-tunnel transition locations than conventional noisy results [43].

2.3 Plasma-Actuated Flow Control

The term “plasma” was first defined by Langmuir, who denoted it as a net electrically neutral region of gas discharge [53]. This definition was expanded, referring to a plasma as a system of particles whose collective behavior is characterized by long-range Coulomb interactions [52]. A dielectric-barrier-discharge (DBD) plasma actuator is classified as a low-temperature non-equilibrium discharge. Several experiments have been conducted to study the physics of DBD plasma actuators. Enloe et al. studied the space-time evolution of the ionized air light emission on a DBD actuator in a static environment, focusing on a 2D region of plasma [30]. The light emission was assumed to indicate the plasma density. They observed the air ionized only over part of the AC cycle, and the light emissions was characterized by narrow spikes indicating microdischarges. Orlov investigated the effects of voltage and AC frequency on the extent and velocity of a DBD discharge [65]. The extent of the discharge increased linearly with increasing voltage and was independent of frequency. The velocity of the discharge increased linearly with both voltage and AC frequency. Post showed that the maximum velocity induced by a DBD actuator was limited by the area of the dielectric barrier because this determines how much charge is stored at the dielectric surface [72].

Plasma actuators are effective in a wide variety of applications, including the control of the boundary-layer transition, flow separation, lift and drag properties of airfoils, noise and vibrations, and shock wave patterns [88]. One of the first experiments to use DBD actuators as a flow-control device was performed by Mhitaryan et al. to investigate flow separation control on an airfoil at angle of attack in sub-
sonic flow [63]. The operated an AC driven DBD actuators with a frequency range of 50 - 570 Hz and demonstrated up to a 30% drag force decrease and 40% lift force increase. They also concluded that the DBD actuator affected the flow through the ionic wind (momentum transfer) mechanism. Roth et al. studied the boundary layer on a flat plate at low subsonic speeds using DBD actuators [74]. They observed that the ambient air was drawn toward the covered electrode at the surface of the plate. Measurements indicated a wall-normal mean velocity profile similar to a tangential wall jet in the presence of DBD actuation.

Although plasma actuators were initially only considered effective at low subsonic speeds, experiments have demonstrated their effectiveness in high subsonic, transonic, supersonic, and hypersonic speed regimes. Klimov et al. conducted experiments to study the effects of plasma on shock wave propagation in supersonic flow [49]. They observed an increased shock wave velocity in the presence of a gas discharge. Matlis et al. developed an AC driven plasma anemometer to measure mean and dynamic mass-flux variations in super- and hypersonic flows and at frequencies of up to 200 kHz [61]. This sensor was applicable to measurements in shock tubes, high-enthalpy flows, and shock-wave boundary-layer interaction experiments. Meyer et al. conducted experiments to study shock modification in a Mach 2.5 flow over a quasi 2D wedge at 0° angle of attack [62]. A non-equilibrium discharge was generated and was capable of weakening the oblique shock waves over the wedge.
3.1 Model

A 7° half-angle circular cone was designed and used in the experiments (Figure 3.1). The model has three main components, which were manufactured from 304 stainless steel, polyether ether ketone (PEEK), and Macor. The model base was machined from stainless steel. This component features an adaptor to connect to the sting in the wind tunnel and attachment points to accommodate interchangeable nosetips and PEEK shells. A channel also runs down the axis of this piece in order to allow access for a high voltage wire that is required to run the plasma-actuated nosetips. Two shells were constructed from PEEK in order to obtain global temperature maps on this section of the model. PEEK, which has a high emissivity and low thermal conductivity, is well-suited for IR thermography [35, 39, 42, 96]. One PEEK shell has a nominally smooth surface finish and was designed to obtain unobstructed temperature readings on the model surface. The second PEEK shell, shown in Figure 3.1, featured eight Kulite pressure transducers. These sensors measured pressure simultaneously with IR temperature measurements, allowing for a more complete characterization of the boundary layer to be conducted. The model also allowed for interchangeable nosetips to be used, all with a 1.0 mm tip diameter. A steel nosetip was used to investigate the natural evolution of the stationary vortices and several different Macor nosetips featuring the plasma actuators were used to investigate their flow control capabilities (Figure 3.1). The plasma nosetips used a dielectric-barrier-discharge plasma actuator as a means of flow control. This consists
of two electrodes, one with a high-voltage input and one at ground, with a dielectric layer between them. In this approach, the surface electrode consisted of a copper layer on the model surface, where the plasma was generated. This was grounded through the model and subsequently the wind tunnel. A buried electrode consisted of a high-voltage wire running internally through the model. The dielectric barrier consisted of Macor nosetips. Macor was chosen for its high dielectric strength. Its material properties are given in Table 3.1. Finally, the model was 41.35 cm (16.3 in.) long with a 10.16 cm (4 in.) base diameter. This is the maximum size that can be reliably started in the BAM6QT at 6° angle of attack.

(a) Steel nosetip configuration

(b) Plasma-actuated nosetip configuration

Figure 3.1. Model schematic.
Figure 3.2. Model with interchangeable nosetips.

<table>
<thead>
<tr>
<th>$\delta$ (kV/mm)</th>
<th>$c_p$ (J/kg·K)</th>
<th>$k$ (W/m·K)</th>
<th>$\rho$ (kg/m$^3$)</th>
<th>$h$ (mm$^2$/s)</th>
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<td>790</td>
<td>1.46</td>
<td>2520</td>
<td>0.73</td>
</tr>
</tbody>
</table>

TABLE 3.1

MATERIAL PROPERTIES OF MACOR

An extension to the original model has been designed and is currently in fabrication (Fig. 3.3). This component will extend the overall model length from 0.4 m to 1.2 m for testing in the AFOSR–Notre Dame Large Mach-6 Quiet Tunnel (AN-DLM6QT). The model was designed to take full advantage of the large open-jet test
section in this facility, thus being capable of achieving higher $Re_x$ values. The extension is made of aluminum. This will improve the machinability, cost, and weight of the component. One disadvantage of using aluminum in the design is that it has a low emissivity, ill-suited for IR thermography. Running et al. developed a technique to wrap an aluminum test model in a matte black film to increase the emissivity for IR measurements [76]. This model will replicate the same technique.

![Figure 3.3. Model with extension (dimensions in m).](image)

3.2 Boeing/AFOSR Mach-6 Quiet Tunnel

The experimental data presented in this work were collected in the Boeing/AFOSR Mach 6 Quiet Tunnel (BAM6QT) at Purdue University. The tunnel is a Ludwieg tube design, which was chosen to minimize cost while achieving fairly high freestream Reynolds numbers [81]. This design utilizes a long driver tube, followed by a converging-diverging nozzle that accelerates the flow to the designed Mach number, and finally a vacuum tank at the downstream end of the tunnel.

For this work, the BAM6QT ran under quiet-flow conditions. The tunnel is
capable of achieving quiet-flow freestream pressure fluctuations of less than 0.02% of the mean pressure through several design features. The tunnel nozzle is polished to a mirror finish to inhibit roughness-induced boundary layer transition. In the contraction section of the nozzle, a suction slot removes the boundary layer growing at this axial location of the tunnel. A valve allows the bleed air to bypass the diverging nozzle and test section to feed directly to the vacuum tank. A new, laminar boundary layer then begins to form downstream of this location [16]. This feature removes tunnel-wall boundary layer fluctuations that could lead to a turbulent wall boundary layer in the test section [38]. By leaving the bleed line closed, the tunnel can be run in noisy-flow conditions. When the bleed line is open, quiet flow is achieved up to a maximum quiet stagnation pressure.

3.3 Instrumentation

3.3.1 Infrared (IR) Camera

The surface temperature of the PEEK model was measured with an InfraTec 8300 hp IR camera. This is a 14-bit camera with a mid-wave spectral range of 2.0–5.5 µm. The detector format is 640 × 512 pixels. Images were acquired at 355 Hz with a wide-angle lens having a focal length of 13 mm and field of view of 43.0 × 32.9 degrees. The camera captures the images using a global shutter method, in which all pixels in a given temporal frame are captured simultaneously [36]. This is important because all pixels are captured in the same moment in time. This is in contrast to a rolling-frame operation, in which the image is captured row by row and each pixel corresponds to a different value of time. The camera was mounted on the right (north) side of the BAM6QT, with flow moving from right to left. The viewing angle of the camera was approximately 10 degrees from perpendicular to the model axis. A calcium fluoride (CaF₂) window was used because of its transmission range of 0.15–9.0 µm.

The IR camera was used to obtain non-intrusive, global surface temperature mea-
TABLE 3.2

MATERIAL PROPERTIES OF PEEK

<table>
<thead>
<tr>
<th>ε</th>
<th>$c_p$ (J/kg·K)</th>
<th>$k$ (W/m·K)</th>
<th>$\rho$ (kg/m³)</th>
<th>$h$ (mm²/s)</th>
</tr>
</thead>
<tbody>
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<td>0.91</td>
<td>1026</td>
<td>0.29</td>
<td>1300</td>
<td>0.217</td>
</tr>
</tbody>
</table>

Measurements on the model through a technique known as IR thermography. Thermal radiation is emitted by matter at a nonzero temperature. This radiated energy is transported by electromagnetic waves to the IR camera. A detector in the IR camera was sensitive to the thermal radiation emitted by the model in the desired infrared range. The detector generates a raw electrical signal that is converted to surface temperature data using the factory calibration taking into account the material properties of PEEK, the transmissivity of the IR window, and the directional emissivity of the model (see section 4.1 for more information).

3.3.2 Kulite Pressure Transducers

To confirm transition and its onset location, the PEEK shell incorporates eight access holes for the use of Kulite XCQ-062-15A sensors, allowing for pressure and IR data to be collected simultaneously. These sensors have a frequency response up to 30–40% of their resonant frequency, which is 270–285 kHz. The sensors are mechanically stopped at 15 psia in order to withstand the high pressures encountered in the BAM6QT when pressurizing the driver tube.

Kulite XCQ-062-15A sensors were used in the experiment for several reasons. First, Borg and Kimmel successfully demonstrated the sensors’ ability to determine the state of the boundary layer on an elliptical cone (HiFIRE 5), where the crossflow
instability was the dominant transition mechanism [6]. These tests took place in the same facility (the BAM6QT) and for similar freestream test conditions as the current work. Borg and Kimmel demonstrated that the Kulites could measure the onset of transition using two different techniques. First, they analyzed transition behavior at one location using one sensor only, in which the freestream unit Reynolds number was varied over the course of several runs to achieve varying $Re_x$ values at this location. Second, they analyzed the transition behavior during multiple runs using sensors at different locations for one freestream unit Reynolds number, achieve varying $Re_x$ values at different locations. The Kulite XCQ-062-15A sensors also have a small diameter (0.168 cm) compared to other pressure transducers such as PCBs. A small sensor diameter is desirable as it will interfere less with IR measurements. Finally, the chosen Kulites have a flat frequency response at low frequencies. Although the focus of this work is to measure and influence transition location using flow controllers, a secondary objective that can be accomplished with the Kulites is measuring low frequency disturbances within the boundary layer. The traveling crossflow instability has been shown to have a frequency of about 30–55 kHz [6, 95]. Future tests can examine the effect, if any, of the plasma-based system on the traveling waves. Additionally, the plasma is driven at an AC frequency of 3–5 kHz. The Kulites can be used to determine if the plasma itself is introducing any low frequency modulation to the flow in the boundary layer.

The Kulite XCQ-062-15A sensors were placed in a 2 by 4 grid on the model, as seen in Figure 3.1. Axially, the sensors were spaced 25 mm apart, with the farthest downstream pair of sensors located 10 mm from the back edge of the model. Azimuthally, the sensors are placed 15° apart, with the upper row of sensors located 10° from the interface between the PEEK and stainless steel frustums (located at the top of the image). The placement of the sensors (both axially and azimuthally) was motivated by past experimental results (see Ref. [28, 95, 99]). The crossflow
instability occurs between the windward and leeward rays of the cone and its growth rate is a maximum when the crossflow Reynolds number is a maximum, which occurs at $\phi = 130^\circ$. It was desirable to place the Kulites near this azimuth. Because the geometry was axisymmetric, the model could also be rotated to acquire Kulite data at various azimuths relative to the windward ray. Axially, the sensors in the new PEEK shell were spaced to cover most of the area visible in the IR images. The downstream sensors were placed as far downstream as possible, limited by the counter sink on the interior of the model.
4.1 Heat-Flux Calculations

Surface heat-flux maps were calculated for the different test cases by inputting calibrated infrared temperature traces into the QCALC subroutine written by Boyd and Howell [7] (and later translated into MATLAB [39]). These traces were corrected for directional emissivity effects arising from the viewing angle of the camera. The code calculates the surface heat flux as a function of time from the temperature history at a point by numerically solving the transient 1-D heat equation using a second order Euler-explicit finite difference approximation in Cartesian, cylindrical, or spherical coordinates. An adiabatic, isothermal, or time-varying back-face boundary condition can be chosen. QCALC was run independently for each pixel, treating its temperature history, as measured by the infrared camera, as the front-face boundary condition. Each frame was registered to within 1/100 pixel accuracy of a reference image. The images were also mapped to a \(x-\phi\) coordinate system based on an image processing code before input into QCALC [75]. For these tests, each point was solved in cylindrical coordinates, and the local wall thickness and radius of curvature was taken into account. An isotermal back-face boundary condition was assumed, based on an initial pre-run temperature distribution measured by the IR camera. The facility experiences about 200 ms of steady conditions in between reflections of the expansion wave within the driver tube and an overall run time of about 5 s. The combination of a short run time and thick model wall (\(\approx 20-25\) mm) meant the calculated surface heat flux was rather insensitive to the back-face boundary
condition. The material properties of PEEK, used in the testing and data analysis, are given in Table 3.2. In this one-dimensional analysis, spanwise and streamwise conduction were neglected. Constant and homogeneous material properties were assumed.

There are several sources of uncertainty in the surface temperature and corresponding heat flux. The IR camera measures raw intensity values, which are converted to surface temperature using the factory calibration curve for the wide-angle lens. Running observed that the surface temperature computed by the camera software was within 0.20% of that measured with a surface thermocouple when the model’s emissivity and the transmissivity of the IR window were taken into account. In this work, the factory calibration was used to convert from raw intensity to surface temperature data, taking into account the transmissivity, $\tau$, of the CaF$_2$ IR window and the directional emissivity, $\epsilon_n$ of the model. The transmissivity of CaF$_2$ was taken to be 0.94. In the spectral range of the camera between 2.0–5.5 $\mu$m, this value will vary by less than ±0.01. This results in a temperature uncertainty of ±0.04%.

The emissivity of a material is a measure of how much thermal radiation it emits into its environment relative to an ideal black body of the same size and shape. The emissivity of many materials is sensitive to their surface temperatures, which would introduce error into the calculated heat flux. Gülhan and Braun conducted experiments at Mach 6 on a PEEK model in the DLR Hypersonic Wind Tunnel (H2K). They noted that the variance of the emissivity of PEEK was negligible in the calibrated temperature measurement range of their IR camera (283 to 423 K). In the current work, the calibrated range of the camera was similar. Moreover, the range of surface temperatures actually encountered was smaller yet, roughly 300 to 315 K. The temperature variation of the emissivity of PEEK was considered negligible in the current work.
Emissivity also varies with viewing angle, the angle between the camera and the local normal at each point on the model’s surface. Cerasuolo found that PEEK has a nearly constant directional emissivity of 0.91 for viewing angles less than 30° and that emissivity rapidly decreases for angles larger than 80° \cite{11}, as seen in Figure 4.1. The post-processing in the current work calculates the viewing angle at each pixel and calculates heat flux using the corresponding directional emissivity \cite{77}. Figure 4.2 shows an example of the computed viewing angle at each pixel on the model during a run. The viewing angles of the pixels on the PEEK shell used in the data analysis ranged from 0° to 75°, with approximately 90% of the data having viewing angles of less than 60°. Hot streaks associated with the stationary crossflow instability were only observed in the area with viewing angles under 60°. The uncertainty of the computed viewing angles on the model ranged from 0.5° to 2.75°, giving a maximum viewing angle uncertainty of 7.7% at a viewing angle of 18°. The corresponding error in emissivity at this low viewing angle was negligible. The highest error in directional emissivity was 4% and occurred near the highest viewing angle of 75°.

The surface temperature on the model was converted to heat flux using QCALC. Additional sources of uncertainty were introduced in this process. QCALC solves the 1-D heat equation, neglecting lateral conduction. The thermal conductivity, \( k \),
density, $\rho$, and specific heat capacity, $c_p$, of PEEK were taken to be 0.29 W/m·K, 1300 kg/m³, and 1026 J/kg·K, respectively [75]. Reported values give a maximum range for these parameters of ±10%, ±1.5%, ±3.1%, respectively [93]. Taking into account this variation gives a maximum heat-flux uncertainty of ±28 W/m². The uncertainty of the transmissivity of the IR window gives a maximum heat-flux uncertainty of ±12 W/m². The IR camera has an accuracy and resolution of ±1.0 K and 0.02 K, respectively [36]. This results in a maximum surface temperature uncertainty of ±0.33% and a corresponding maximum heat-flux uncertainty of ±130 W/m². This was the primary source of uncertainty, which was approximately 15% of the lowest laminar heating rates observed along the hot streaks at the lowest Reynolds number test case ($Re = 5.4 \times 10^6$ /m).

4.2 Transition Location Assessment

The surface heat-flux data indicated evidence of stationary crossflow vortices manifested in hot streaks. 2-D heat-flux profiles were extracted from the data along...
hot streaks and compared with mean-flow calculations conducted by Dr. Matthew Tufts, AFRL/RQHF (for more information on the mean-flow computations, see Appendix A). The data were collected at the same conditions as the computations ($Re = 8.03 \times 10^6 /m$). A heat-flux distribution of the experimental data is given in Figure 4.3. Two paths are given, one along a vortex that exhibited a large heat-flux increase, and one along a vortex that remained near constant values. In Figure 4.4, the corresponding heat-flux profiles from these paths are compared to the fully laminar and fully turbulent heating rates obtained computationally. Figure 4.4a shows the heat-flux along vortex path two, which appears to be fully laminar. No significant rise in heat flux was observed for this path. Figure 4.4b shows the heat-flux along vortex path one, which appears to be transitional. The laminar region of the data shows good agreement with the computations. A sharp increase in the heating rate on the model was observed at $x \approx 340$ mm. This sharp rise deviated from the expected laminar heating rate, suggesting that transition onset had occurred. The vortex did not appear to become fully turbulent; the heat-flux was still increasing at the back edge of the model. The magnitude of the heat flux along this path never reached the expected computational turbulent heating rate.

For the experimental data, a transition location was found by computing linear fits for two portions of the data: one where heat flux was decreasing moving downstream and one where heat flux was increasing downstream, if observed. The intersection point of these fits was concluded to be an estimate of the transition location. Uncertainty estimates were computed by varying the domains of these two regions [44]. Based on this technique, the transitional vortex given in Figure 4.4b saw a transition location near $x = 340$ mm with an uncertainty of $\pm 10$ mm. This location corresponded to where the heat-flux departed from the laminar heating rate in the mean-flow calculations.

The laminar mean-flow results show good agreement with the low heating rates of
Figure 4.3. Heat-flux distribution. $\alpha = 6^\circ$, $Re = 8.71 \times 10^6 /m$.

the experimentally observed hot streaks. The lower heating rates observed between the hot streaks were over-predicted by the laminar mean-flow results. The stationary crossflow vortices distort the mean flow, making the boundary layer alternately thinner (higher heating rates) and thicker (lower heating rates). The low heating rates between the hot streaks were attributed to this distortion.

4.2.1 Comparison Between Experimental Data and Heating Correlation

To provide greater confidence in the computed experimental heat-flux data, a correlation developed by Kimmel et al. was compared with the experimental results. This correlation gives the laminar and turbulent Stanton numbers on a cone at $0^\circ$ angle of attack. Figure 4.5 shows the comparison between the predicted laminar and turbulent Stanton number with our experimental data at $Re = 8.70 \times 10^6 /m$. The experimental data was extracted along a line of constant azimuth $10^\circ$ from the windward ray, which was the lowest azimuth where data were available. A $13^\circ$ turning angle was input to the correlation, corresponding to the $7^\circ$ cone half angle plus $6^\circ$.
Figure 4.4. Computational and experimental heat-flux along two vortex paths.

angle of attack. The results show very good agreement. This gives confidence that neither the experimental data nor mean-flow computations contain significant error.
4.3 Wavelet Analysis

Stationary crossflow amplification rates are a function of the wavenumber of the vortices. Integrating these amplification rates into an amplitude ratio requires an assumption of which wavenumber to use. Directly extracting these wavenumbers from experimental heat-flux data can improve the stability analysis. Previously, the dominant wavelengths of the heating distribution were calculated with a Discrete Fourier Transform (DFT) analysis of a spanwise heat-flux profile [85, 99]. In this application, the usefulness of the FFT was limited because it contained no information regarding the azimuthal variation of the wavelengths.

The wavelet analysis is a mathematical technique well-suited for this application. It will be used to determine the wavenumber with the local largest amplitude over the region of interest. By decomposing a time series into time–frequency space, one is able to determine both the dominant modes of variability and how those modes vary in time [90]. The wavelet transform can be used to analyze time series that contain non-
stationary power at many different frequencies [25]. In this case, azimuthally varying surface heat-flux data were analyzed, meaning the analysis was done spatially rather than temporally.

In this work, the continuous wavelet transform (CWT) was used to obtain spatial wavenumber distributions from experimental data. An analytic Morse Wavelet was chosen as the mother wavelet because it is effective in analyzing modulated signals [56]. The analytic wavelet form was affected by the time-bandwidth product ($P^2$) and the symmetry parameter ($\gamma$), which are non-dimensional parameters defined in the frequency domain. Here, $\gamma = 3$, which resulted in a wavelet function with zero skewness. Using a $P^2$ value of 60 in combination with $\gamma$ resulted in the desired frequency response. In this technique, a wavelet basis function that depends on a non-dimensional time parameter is chosen [90]. This function must have zero mean and be localized in both time and frequency space [31]. The discrete wavelet transform utilizes an orthogonal basis function while the continuous wavelet transform utilizes a non-orthogonal basis function [90]. The non-orthogonal transform is useful for temporal series analysis, where smooth, continuous variations in wavelet amplitude are expected [90].

The CWT is defined as:

$$W_\psi(t, s) = \int_{-\infty}^{\infty} \frac{1}{s^n} \psi^*\left(\frac{\tau - t}{s}\right)x(\tau)d\tau,$$

where $s$ is the wavelet scale, $\tau$ is the localized time index, $\psi$ is the wavelet function and (*) denotes complex conjugate [56, 90]. Here, the non-orthogonal Morse wavelet is used and defined as [56]:

$$\psi(\omega) = U(\omega)a_{\beta, \gamma}\omega^{\frac{P^2}{2}} e^{-\omega^2}.$$ (4.2)

where $U(\omega)$ is the unit step, $a_{\beta, \gamma}$ is the normalizing constant, $P^2$ is the time-
bandwidth product, and $\gamma$ characterizes the symmetry of the Morse Wavelet.

By varying the wavelet scales and translating along the localized index, a picture showing both the amplitude of any features versus the scale and how this amplitude varied spatially was constructed \[90\]. Figure 4.6 shows an example surface heat-flux contour mapped to a $x-\phi$ coordinate system. The streaks of high surface heating due to stationary crossflow vortices are clearly visible in these data. Most of the vortices appear to be fully laminar because they do not see an increase in heating rate expected for a transitional vortex. A vortex at $\phi = 160^\circ$ exhibits higher heating rates far downstream on the model, suggesting that it is transitional.

The spanwise heat-flux profiles used in the wavelet analysis are finite in length. Because the CWT assumes the data to be cyclic, errors will occur at the beginning and end of the wavelet power spectrum \[90\]. The region in which edge effects are important is known as the cone of influence (COI). For this CWT analysis, wavenumber data obtained closest to the windward and leeward rays will fall under the COI. This is not problematic because the transition behavior on a yawed cone is affected by
multiple transition mechanisms and the COI does not cover azimuths where cross-
flow is the dominant transition mechanism. Near the leeward symmetry plane under
quiet-flow conditions, first and second mode disturbances amplify. This behavior is
similar to supersonic flow in a 2-d boundary layer [55]. Hot streaks associated with
the stationary crossflow vortices are not expected in this region. Close to the lee-
ward side (at $\phi \approx 165^\circ$), a separation line forms. In this region, the second mode is
expected to be the dominant transition mechanism.

The experimental data were analyzed to determine the spatial development of
the wavenumber of the naturally occurring largest amplitude stationary waves. To
conduct the wavelet analysis, spanwise heat-flux profiles were extracted as a function
of azimuth at a constant axial station. Four representative spanwise heat-flux profiles
are shown in Figure 4.7. They correspond to the $x = 310, 340, 370,$ and $400$ mm
axial stations of the heat-flux contour shown in Figure 4.6.

Each individual spanwise signal was analyzed using the CWT technique. The
wavelet results corresponding to the representative profiles in Figure 4.7 are given in
Figure 4.8. These plots show the local frequency content in terms of the wavenumber
at a given spatial position. The contour levels display the amplitude of the heat-
flux signal as a function of the wavenumber and azimuth. The resulting wavelength
resolution in these data was $1.80^\circ$. The black region on these plots marks the COI
from the wavelet analysis. These results show that the peak disturbance amplitudes
increase at farther downstream stations. This corresponds to an increase in heat-flux
along many of the vortices.

It was important to determine the Nyquist wavenumbers of the spanwise heat-flux
signals because this represents the highest spatial frequency (i.e. the lowest wave-
length and highest wavenumber) that could be measured. The sampling rate was
2.2 pixels per degree at an axial position of $x = 330$ mm. According to the Nyquist
condition, the Nyquist frequency is one half of this value, or 1.1 pixels per degree.
This translated to a Nyquist wavelength of 0.91° and a resulting Nyquist wavenumber of 400. The lowest wavenumber that could be reliably measured in the wavelet analysis was limited by the azimuthal range of the signal. This was computed to be 2.67 at an axial position of $x = 330$ mm. While it is theoretically possible to resolve wavenumbers up to 400 in the wavelet analysis, resolving high wavenumbers in the data is not possible. High wavenumber disturbances on the model’s surface would not result in a corresponding temperature distribution with high wavenumber because of the high spanwise conduction associated with high temperature gradients.
The wavelet analysis was conducted for spanwise heat-flux profiles taken in 1-mm axial increments. This series of CWT amplitudes was used to construct a map of the dominant wavenumber as a function of axial position and azimuth. For the wavelet analysis at a given $x$, the wavenumber corresponding to the peak amplitude was identified as a function of $\phi$. The red lines in Figure 4.8 trace these peak amplitudes. No peak wavenumber was selected if the maximum amplitude at a given azimuth was below 50. This threshold was set to exclude regions where no hot streaks were observed. Figure 4.9 shows the results of this analysis for the $Re = 8.03 \times 10^6$ /m
case (i.e., corresponding to Figures 4.6–4.8). Figure 4.9 shows the wavenumber $k$ (number of waves per circumference), where $k = 360^\circ/(\lambda/R)$ and $\lambda/R$ is the dominant wavelength normalized by the local radius of curvature of the cone. The contour indicates the spatial distribution of the wavenumber of the stationary crossflow waves.

The wavelet analysis revealed a dominant wavenumber between $k = 40$ and $k = 65$ in the crossflow region of the cone (between $\phi = 120^\circ$ and $160^\circ$) for this baseline case. This result agrees well with previous works at similar test conditions. Ward used roughness elements with a critical wavenumber of $k = 50$ to accelerate transition on a 7° half-angle cone at 6° angle of attack at Mach 6 in the BAM6QT [95]. Li et al. computed a theoretical wavenumber between $k = 40$ and $k = 50$ for this configuration corresponding to the most amplified stationary modes under similar freestream conditions, as seen in the BAM6QT [55].
4.4 Direct Measurement of Distance Between Hot Streaks

The most straightforward implementation of the wavelet analysis uses spanwise heat-flux profiles as the input, and thus the output wavelengths are normalized by the local radius of curvature of the cone’s circular cross-section at a given span. However, it would be more proper to determine the crossflow vortex wavelengths normal to the vortex paths and normalize them by the local radius of curvature of the elliptical cross-section resulting from the conic section at an oblique angle to the axis. A second approach to evaluating the experimentally observed wavenumber distribution, which accounts for the curving vortex paths, was executed. The first step was to identify every hot streak path in a heat-flux contour. The gradient of the surface heat flux along these paths was calculated to produce the vectors perpendicular to the hot streaks. Tangent lines connecting these perpendicular vectors were calculated in 1-mm increments. This technique resulted in the generation of lines perpendicular to the hot streaks resulting from the stationary crossflow vortices. The length of each segment of the perpendicular lines was computed, giving a direct measurement of the local wavelength normal to the hot streaks. Figure 4.10 shows the IR heat-flux data from Figure 4.6 with the vortex paths (black lines) and vortex-normal lines (white lines) superimposed on the heat-flux contours. All vortex paths are shown, but only 20 vortex-normal paths are included for clarity.

A contour plot of the length of the perpendicular line segments functions as a map of the normalized wavelength or wavenumber for this technique. The wavelength was normalized by the local radius of curvature of the elliptical conic section created when slicing the cone along a vortex-normal path. Figure 4.11 shows the results using this technique. Note that in analyzing the data from Figure 4.10 the vortex path closest to the leeward ray has been ignored because the azimuthal position of this hot streak lies outside the region where the crossflow instability is the dominant transition mechanism. The direct measurement technique computes the
distance between two hot streaks. Thus, the results of using this technique on \( n \) hot streaks contain \( n - 1 \) paths between them. Regions of high wavenumber are seen near the leeward ray of the model where the vortices converge due to the spanwise pressure gradient. The wavenumber generally increases moving downstream, except at the lowest azimuth. Figure 4.12 shows the computed angle of the hot streaks with respect to the cone axis. The angle of the hot streaks was seen to increase upstream and near the windward ray of the model. Values of 10° or less were seen in the regions where the crossflow instability is the dominant transition mechanism. This also agrees qualitatively with the behavior of computationally generated inviscid streamline paths [99] and vortex paths [55]. The error introduced in the normalized wavelength or wavenumber by failing to account for an elliptic, not circular, radius of curvature, is small (approximately 0.5–2%) in the region where stationary crossflow vortices are most amplified. Therefore, the simplification of using spanwise heat-flux profiles normalized by a circular radius of curvature introduces minimal error.
These results are similar to those from the wavelet analysis based on spanwise heat-flux profiles. In both cases, a region of wavenumber greater than 50 is seen downstream near the leeward ray. A region of low wavenumber is seen closer to the windward ray, where stationary vortices were not observed in the experiment. Figure 4.14 shows the difference in wavenumber as indicated by these two techniques, where the data from Figure 4.11 have been interpolated onto the mapping from Figure 4.9. Due to the paths of the vortex-normal profiles and the cone of influence in the wavelet analysis, the regions where the wavenumber could be resolved differ slightly between the techniques. Figure 4.14 indicates that generally the results between the two techniques differ by a wavenumber of five or less, compared to a wavelength resolution of 1.80° for the wavelet results. The two techniques yield similar results over much of the region of interest. For example, a band of $k \approx 45$ at $\phi = 70^\circ$, with minima above and below was observed using both techniques. The largest differences were observed near the windward and leeward rays, where the wavelet analysis with spanwise profiles yielded higher wavenumbers.

The wavelet analysis is much less time consuming than the one tracking hot streaks because very little work is needed to prepare the images for input to the wavelet analysis. The wavelet analysis is more robust for the small amount of noise present in the data and the amplitude threshold automatically determines the range over which the results are valid. For these reasons, and because the difference between techniques was not large, the wavelet analysis technique is preferred.

A third, hybrid technique was considered: manually identifying the hot streak paths, extracting the heat-flux paths perpendicular to the hot streaks, and calculating the wavelet analysis for those profiles. This method was found to combine the worst features of both methods — it was time-consuming, but didn’t result in precise wavelengths — so it was not pursued.
Figure 4.11. Spatial mapping of stationary crossflow vortex distribution using vortex-normal heat-flux profiles.

Figure 4.12. Angle of the stationary hot streaks relative to the cone axis.
Figure 4.13. Comparison of spatial mapping of stationary crossflow vortex distribution using wavelet analysis and vortex-normal heat-flux profiles.

Figure 4.14. Change in wavenumber between the wavelet and direct measurement techniques.
CHAPTER 5

RESULTS WITHOUT ACTUATORS

Using the CWT analysis, the spatial distribution of the hot streak wavenumbers was calculated for different runs to observe the effect of Reynolds number and angle-of-attack variation on the hot streaks. The experimental results discussed in this work were all taken on a 7° circular cone with a 1.0 mm nosetip diameter and a 101.6 mm base diameter. All wind tunnel runs were conducted at a freestream Mach number of 6. The freestream unit Reynolds number and angle of attack were varied to observe their effect on the wavenumber of the stationary waves. The freestream unit Reynolds number was varied by conducting individual runs at different total pressures.

5.1 Effect of Reynolds Number on Surface Heat Flux and Wavenumber

Figure 5.1 shows IR heat-flux data at six different freestream unit Reynolds numbers at 6° angle of attack. Hot streaks are clearly visible in each image. Heat flux generally increased with increasing Reynolds number. Note that for these data the colorbar scale changes to maximize the visibility of the features. Figures 5.1a–5.1d feature eight hot spots between approximately 145° and 165°. These correspond to the location of Kulite XCQ-062-15A pressure transducers, which were not used in the current work. Some vortices appear transitional based on the steep increase in heat-flux observed along their paths. For the three lowest Reynolds number cases, the streaks maintained their structure, spacing, and low levels of heating. This implied
that all of the vortices were laminar. At these lower Reynolds numbers, the disturbances are still growing linearly as they progress downstream, but the disturbance amplitudes have not grown large enough to cause transition to occur.

Transitional vortices first appeared near $\phi = 160^\circ$ at $Re = 8.03 \times 10^6$/m (Figure 5.1d), but this is in the separated zone close to the leeward plane. Between the separation line and the leeward ray, crossflow is not the dominant transition mechanism, so these streaks were not necessarily indicative of crossflow-induced transition. As the Reynolds number was further increased, transitional vortices were observed between $\phi = 120^\circ$ and $145^\circ$ (Figure 5.1e). At the highest Reynolds number case (Figure 5.1f), ample evidence of crossflow-induced transition was observed. Streaks between $\phi = 110^\circ$ and $150^\circ$ exhibited large increases in heat flux and they began to spread azimuthally. At higher Reynolds numbers, the higher heating rates along the hot streaks indicated that the disturbance amplitudes had grown large enough to cause transition to occur.

Above $Re = 8.5 \times 10^6$ /m, some hot streaks appeared to weaken, but give rise to adjacent streaks that became turbulent. Edelman observed similar structures on a cone at angle of attack at conditions similar to those of the current work [27]. He referred to them as “wishbone structures”, and suggested they may be due to a pair of co-rotating vortices. We suspect that the upstream streaks are due to the mean-flow distortion of the stationary crossflow waves, whereas the nascent streaks arise from the secondary instability that are the proximate cause of transition. The surface heating footprints of the secondary instabilities are anchored to the primary instabilities, but not directly in line with them.

The wavelet analysis was applied to these six test cases using spanwise heat-flux profiles as the input. The results, showing the local wavenumber distribution of the experimentally observed hot streaks, are given in Figure 5.2. The observed hot streaks, which correspond to the stationary crossflow waves, became more distinct
Figure 5.1. Effect of freestream unit Reynolds number on surface heat flux.

\[ \alpha = 6^\circ. \]
with increasing Reynolds number. This is reflected in the wavelet analysis, where amplitudes above the cut-off threshold were observed over a larger portion of the surface as the Reynolds number increased. For most cases, the wavenumber increased towards the leeward ray. The boundary layer is known to thicken near the leeward ray for this configuration, resulting in a distinct, mushroom-like structure \[55\]. The observed increase in the wavenumber near the leeward ray is related to the separation line that forms due to this phenomenon.

Between \[\phi = 100^\circ\] and \[140^\circ\], local wavenumbers between \[k = 40\] and \[55\] were observed. In this azimuthal range of the model, crossflow is the dominant transition mechanism. In this region, the wavenumbers were fairly uniform spatially and did not vary strongly as a function of Reynolds number. Closer to windward \(\phi \lesssim 100^\circ\), the wavenumbers are higher than in the zone of peak crossflow, and no distinct streaks were observed for \(\phi \lesssim 75^\circ\). Li computed amplification factors for stationary crossflow vortex instabilities for the same geometry and similar test conditions \[55\]. N-factors of 10 or higher were concluded to correspond to crossflow-induced transition. This was seen down to an angle of \(\phi = 80^\circ\) at \(x = 300\) mm and \(\phi = 60^\circ\) at \(x = 400\) mm in the computations \[55\]. Therefore, the observed increase in wavenumber in Figure 5.2 was still in the azimuthal range where crossflow-induced transition could be expected.

To demonstrate the repeatability of the data, multiple runs were conducted at nominally-identical unit Reynolds numbers. The model configuration, including the nosetip orientation, was held constant for these tests. Resulting heat-flux distributions showed that the hot streaks’ behavior was similar between runs. The streaks formed at the same azimuths and exhibited similar heating rates. The resulting wavenumber distributions were repeatable and showed good agreement between runs. This consistency suggests that the stationary crossflow vortices were anchored to model roughness, rather than being a stochastic function of the tunnel’s startup process and flow field establishment.
Figure 5.2. Effect of freestream unit Reynolds number on wavenumber. $\alpha = 6^\circ$. 

(a) $Re = 5.4 \times 10^6 / \text{m}$

(b) $Re = 6.6 \times 10^6 / \text{m}$

(c) $Re = 7.5 \times 10^6 / \text{m}$

(d) $Re = 8.5 \times 10^6 / \text{m}$

(e) $Re = 9.4 \times 10^6 / \text{m}$

(f) $Re = 10.5 \times 10^6 / \text{m}$
5.2 Effect of Angle of Attack on Surface Heat Flux and Wavenumber

Runs were conducted to investigate the effect of angle of attack on the local wavenumber of the vortices. Note that the colorbar scale varies among the subfigures to improve the visibility of the flow features. Data were collected for two unit Reynolds numbers at 4° and 6° angle of attack. At a lower angle of attack, the circumferential pressure gradient is weakened. This in turn reduces the magnitude of the crossflow velocity component in the boundary layer.

Figure 5.3 shows angle-of-attack variation at two unit Reynolds numbers. Figures 5.3a and 5.3b show two runs at \( Re = 7.9 \times 10^6 \) /m at 4° and 6° angle of attack. Clear, distinct hot streaks were observed for each run, but they did not appear to transition at the lower Reynolds number. At a 4° angle of attack, the hot streaks were most distinct between \( \phi = 100° \) and 160°. In contrast to the 6° angle of attack case, vortices were generally not observed between the windward ray and \( \phi = 100° \). This indicated the azimuthal range of crossflow-induced transition expanded to include lower azimuths as angle of attack increased. Figures 5.3c and 5.3d show angle-of-attack variation at \( Re = 11.1 \times 10^6 \) /m. Transitional vortices, characterized by rapid increases in heat flux, were observed for both angles of attack. At a 4° angle of attack, rapid heat-flux increases were observed between \( \phi = 120° \) and the leeward ray. These streaks reached higher heat-flux values compared to the 6° angle-of-attack run. On the windward side of a cone, heat flux increases for increasing angle of attack, but the opposite trend is true on the leeward side. The fluid from the boundary layer edge pushed closer to the wall by the vortices at higher azimuth was processed by a stronger shock at lower angle of attack, hence the higher heat flux.

The data from Figure 5.3 were analyzed using the wavelet analysis technique with spanwise heat-flux profiles (Figure 5.4). The same amplitude threshold of 50 was applied. For both freestream unit Reynolds numbers, similar wavenumbers were generally observed at each angle of attack between \( \phi = 100° \) and 145°. The wavenum-
bers were more uniform at 4° angle of attack compared to the 6° cases.

To further investigate the hot streak behavior with angle-of-attack variation, the relative angles between the streaks and the local line of constant azimuth, $\zeta_{\text{exp}}$, were computed. Figure 5.5 gives the resulting absolute values of the hot-streak angles for each run. At each unit Reynolds number, the angle of attack had a large effect on the hot-streak angles. Figures 5.5a and 5.5b show hot-streak angles at 4° and 6° angle of attack at $Re = 7.5 \times 10^6$ /m. Both runs show the highest hot-streak angles occurred at the furthest upstream locations. The hot-streak angles decreased downstream and near the leeward ray; the angles reached low values as the hot streaks approached the separation line. Near the separation line, the angle of attack did not affect the hot-streak angles. The largest effect of angle of attack on the hot-streak angles occurred near the upstream portion of the images towards the windward ray. All these observations are explained by the effect of the angle of attack on the circumferential pressure gradient: a stronger pressure gradient causes greater inclination of the vortex paths. Substantial spatial variation is visible when surveying the global heat-flux contour — simply reporting a mean or representative streak angle would not characterize the flow field well.
Figure 5.3. Effect of $\alpha$ on surface heat flux.
Figure 5.4. Effect of $\alpha$ on wavenumber.
Figure 5.5. Effect of $\alpha$ on relative streak angle.

(a) $\alpha = 4^\circ$, $Re = 7.5 \times 10^6 /m$

(b) $\alpha = 6^\circ$, $Re = 7.5 \times 10^6 /m$

(c) $\alpha = 4^\circ$, $Re = 10.5 \times 10^6 /m$

(d) $\alpha = 6^\circ$, $Re = 10.5 \times 10^6 /m$
5.3 Effect of Tunnel Noise on Transition

It has been documented that tunnel noise effects can have a large impact on the transition location observed on test models. Under noisy tunnel conditions, transition occurs earlier than in quiet-flow conditions akin to the flight environment [82, 89]. Low speed experiments conducted by Wlezian indicated that both freestream disturbances and roughness play important roles in the transition process [98]. Pate correlated boundary layer transition behavior in conventional noisy tunnels on sharp cones using tunnel wall boundary layer parameters [67–69]. This related a noise parameter to the mean turbulent skin friction coefficient in the test section, independent of Mach number. King took measurements on a yawed circular cone in the NASA Langley Mach-3.5 Quiet Tunnel [48]. He observed that transition occurred further downstream for quiet-flow runs. This tunnel noise effect was reduced as angle of attack was increased. Juliano et al. showed that tunnel noise affected not only the transition location, but also the structure of the heating distribution within the region of crossflow-induced transition on the HIFiRE-5 geometry [41]. Borg collected surface pressure data on the HIFiRE-5 geometry in the BAM6QT under noisy and quiet conditions using Kulite XCQ-062-15A and XCE-062-15A pressure transducers [6]. The transition Reynolds number was taken to be the lowest value for which the pressure spectra did not relax back to the electronic noise levels at high frequencies. He found that reducing the freestream noise levels (from noisy to quiet) increased the transition Reynolds number in the instrumented region by a factor of 2.4.

The transition process of stationary crossflow vortices is influenced by receptivity to surface roughness. There has been speculation in the community that receptivity to surface roughness could dominate any tunnel noise effects. Therefore, the current work investigated whether tunnel noise affected crossflow-induced boundary-layer transition on the model. Tests were conducted at Mach 6 at a 6° angle of attack at different Reynolds numbers. Because noisy runs were expected to possibly move the
transition locations upstream relative to the quiet runs, IR thermography data were collected on the model over a larger axial range by moving the model relative to the viewing area in the IR window.

Figure 5.6 shows surface heat flux under quiet flow for two freestream unit Reynolds numbers. Figures 5.6a and 5.6b show runs at $Re = 9.5 \times 10^6$ /m. Both images show evidence of stationary crossflow vortices based on clear, distinct hot streaks in the data. Tests were conducted with the model in two different axial positions relative to the IR window to obtain surface heat-flux data over a larger axial range than was achievable in one run; this gave an up- or downstream field of view. The IR camera collected data between $x = 160$ mm and 300 mm for Configuration 1 and between $x = 295$ mm and 405 mm for Configuration 2. For Configuration 1, transition was not observed, as the vortices maintained their structure and spacing and did not exhibit increases in surface heat flux increase. Configuration 2 shows evidence of transition for vortices between 120° and 160°. These vortices had transition locations between $x = 330$ and 380 mm, translating to a transitional Reynolds number of between 3.1 and 3.7 $\times 10^6$. At a higher freestream unit Reynolds number (Figure 5.6c and 5.6d), similar behavior was observed. Configuration 1 shows mostly laminar vortices, with some hot streaks appearing transitional near 140°. Configuration 2 shows many transitional vortices, with large increases in surface heat flux and spreading of the hot streaks observed. These runs resulted in transitional Reynolds numbers between 3.0 and 3.8 $\times 10^6$.

Runs were repeated for the same model axial positions and freestream unit Reynolds numbers, but under noisy rather than quiet conditions (Figure 5.7). The surface heating rates differed greatly quantitatively and qualitatively compared to the quiet runs. At the lower freestream unit Reynolds number, transition occurred at axial stations between $x = 245$ and 300 mm, depending on the azimuth (Figure 5.7a). This resulted in a transition Reynolds number between 2.0 and 2.9 $\times 10^6$. For Configuration 2,
most of the flow field was already transitional. For the higher unit Reynolds number test case, transition again occurred in Configuration 1. Transition Reynolds number between 1.8 and $2.9 \times 10^6$ were observed. Configuration 2 was already transitional over the entire image. Hot streaks, presumably the footprint of stationary crossflow vortices, are visible in both the laminar and transitional regions. Indeed, they persist beyond peak heating into the turbulent region beyond the end of transition. These features closely resemble those observed at Mach 7 in a conventional noise wind tunnel with infrared thermography by Willems et al. [97]. As a point of comparison to the data in Figure 5.7, Figure 5.8 shows results from Willems et al. on the HIFiRE-1
7° half-angle cone geometry. Similar hot streaks were not observed on HIFiRE-5, but that may be due to the comparatively limited sensitivity of temperature-sensitive paint [44].

Figure 5.9 shows the crossflow transition fronts under noisy and quiet flow in the BAM6QT. Tunnel noise had a large effect on transition — $Re_x$ at transition was about 1.75 times larger under quiet flow than noisy from $\phi = 110$–$150^\circ$. At lower azimuths, transition was not observed under quiet flow before the end of the model, so the transition delay cannot be ascertained. Transition Reynolds numbers grew from between 1.8 and $3.0 \times 10^6$ for the noisy runs to between 3.1 and $4.0 \times 10^6$ for
Figure 5.8. Figure 8 from reference [97]. $Re = 9.24 \times 10^6 /m$, $\alpha = 6^\circ$, noisy flow.
the quiet runs. Additionally, the hot streak behavior was altered between noisy and quiet runs. Quiet runs resulted in clear, distinct vortices. Here, transition occurred along each individual hot streak, with the streaks independently undergoing increases in surface heat flux, resulting in a jagged transition front. As the streaks progressed further downstream, they would eventually spread and form a turbulent wedge. In contrast, the noisy runs resulted in large, indistinct regions of heating. The vortices were not as clear or distinct in these IR data. The transition process for these runs was also characterized by an increase in heat flux along large spatial regions rather than along an individual hot streak. This resulted in smoother transition fronts. These effects of tunnel noise on the crossflow transition front are very similar to those observed on the HIFiRE-5 model; see Figure 7 in ref. [47].

Figure 5.9. Effect of tunnel noise on transition front, $\alpha = 6^\circ$. 
5.4 Experimental Relative N-Factors

Kroonenberg et al. developed a data reduction technique to experimentally derive relative N-factors of the stationary crossflow waves \cite{12}. They calculated global heat-flux values from IR thermography temperature data and applied a high-pass filter to remove low-frequency spatial periods in the circumferential direction of the data. They computed an amplitude by fitting a sine function to the filtered heat-flux data. Equation (5.1) gives the definition of the relative N-factor:

\[
\Delta N_i = \ln \left( \frac{A_i}{A_{\text{ref}}} \right),
\]

where $\Delta N_i$ is the relative N-factor, $A_i$ is the amplitude at position $i$, and $A_{\text{ref}}$ is chosen as the most upstream value on the infrared image of the cone at the azimuth of interest. In this work, a similar technique was implemented to compute relative N-factors from the experimental data. Instead of fitting a sine function to the data, à la Kroonenberg et al., the peak amplitude from the wavelet analysis at a particular $(x, \phi)$ location was used as $A_i$. The wavelet amplitude at a fixed axial station and varying azimuth, $(x = 295 \text{ mm}, \phi)$, was used as $A_{\text{ref}}$.

Figure 5.10 gives the relative N-factors for the data from Figure 5.1. The data exhibit small regions of high $\Delta N$ between $\phi = 150^\circ$ and $170^\circ$. These are artifacts arising from heat conducted into the PEEK shell by eight Kulite pressure transducers. For $Re = 7.9 \text{ /m}$ and $8.8 \times 10^6 \text{ /m}$, the relative amplification rates were low, with values less than $\Delta N$ of 0.5. As Reynolds number was increased, relative N-factor values increased. For $Re = 9.8 \times 10^6 \text{ /m}$, the highest amplification rates ($\Delta N = 0.8$) were seen near the back edge of the model and between $\phi = 100^\circ$ and $140^\circ$. This corresponds to the region of the model where the crossflow velocity component reaches its maximum \cite{2}. As Reynolds number was further increased, higher amplification rates ($\Delta N = 1.4$) were observed. For $Re = 11.1 \times 10^6 \text{ /m}$, the regions with the
highest amplification rate corresponded to regions where high heating rates were also observed. This is a promising technique to extract amplitude ratios from global experimental data.

5.5 Pressure Measurements

To corroborate the boundary-layer state identified from the IR thermography, eight Kulite XCQ-062-15A pressure transducers were installed in the model’s PEEK shell, and simultaneous measurements of surface temperature and pressure were made. The PEEK shell could be reoriented relative to the windward ray between runs,
enabling measurement of surface pressure at varying azimuth. For this experiment, the placement of the Kulites was motivated by the need to maximize the amount of the PEEK shell that could be observed through the IR window. This meant the sensors were positioned in two rows at $\phi = 145^\circ$ and $160^\circ$, respectively. Because the mechanism of transition on this configuration is different near the leeward ray of the model than in the crossflow region [55], the data from the four sensors at $\phi = 160^\circ$ were not used in the analysis. The Kulites were sampled at 1 MHz. Due to electronic noise associated with the plasma actuators, plasma-on Kulite data are unavailable; non-physical spectra were observed, with no peaks and power levels several orders of magnitude higher than plasma-off runs.

Figure 5.11 shows power spectral densities (PSD) for the four Kulites at $\phi = 145^\circ$ with the steel nosetip. Each plot corresponds to a freestream condition, and each curve corresponds to an axial station. For the lowest Reynolds number ($5.7 \times 10^6 /m$), the spectra exhibit low power (Fig. 5.11a). For the two farther upstream locations no peaks are visible. In the heat-flux data, the hot streaks have low amplitude. Farther downstream, the streaks become more distinct, and the Kulites located at $x = 385$ and 410 mm detected a pressure disturbance with a peak frequency between 20 and 40 kHz. This peak is attributed to traveling crossflow waves because the frequencies match those computed by Balakumar and Owens in their DNS study of a cone at very similar conditions [3]. They computed travelling crossflow growth rates at discrete axial stations and azimuths on a circular cone. At $x = 300$ mm and $\phi = 150^\circ$ (the closest computed location in relation to the Kulites), they computed that the maximum growth rates occurred at 35 kHz. This showed excellent agreement with the experimentally measured spectra. At higher freestream unit Reynolds numbers, the waves between 20 and 40 kHz grew in amplitude, and broadened to cover a wider range of frequencies, suggesting that the boundary layer was transitional (Figs. 5.11b–5.11e). Finally, for $Re = 11.1 \times 10^6 /m$, broadband spectra with higher power
were observed for each sensor (Fig. 5.1f), indicating the boundary layer was fully turbulent.

Figure 5.1 shows the surface heat-flux distributions corresponding to the Kulite PSDs in Figure 5.11. The hot streaks in the surface heat-flux data are associated with stationary crossflow vortices. At \( Re = 5.7 \times 10^6 \) /m, the IR data indicated fully-laminar flow (Fig. 5.1a). The Kulites appear to confirm this, as low power was seen in the spectra, with some evidence of instability amplification at the two sensors farther downstream. Increasing \( Re \) to 7.0 \( \times 10^6 \) /m and again to 7.9 \( \times 10^6 \) /m, laminar flow was still observed in the IR data (Figs. 5.1b and 5.1c). The Kulites began to measure peaks associated with the travelling vortices. This indicated the instabilities associated with the travelling modes were growing, but the amplitude of both the stationary and travelling modes was not high enough to cause transition. At \( Re = 8.9 \times 10^6 \) /m, hot streaks in the crossflow region remained laminar, and the Kulites showed continued disturbance amplification (Fig. 5.1d). At \( Re = 9.8 \times 10^6 \) /m, increased surface heating suggests the vortices were becoming transitional (Fig. 5.1e). The peaks in the PSD data no longer are growing in amplitude, but there is more broadband content. Finally, at \( Re = 11.1 \times 10^6 \) /m, stationary hot streaks with high heating rates passed over the sensors (Fig. 5.1f). The IR data indicated that transition occurred upstream of the sensors. The Kulites exhibit large pressure fluctuation amplitude at all frequencies, indicating that the boundary layer was fully turbulent at this condition. Only the sensor farthest upstream shows a residual peak at the instability frequency.

Previous experiments have also shown corresponding agreement between pressure spectra and surface heating rates. Casper made similar measurements on a 7° half-angle cone at 0° angle of attack under quiet-flow conditions in the BAM6QT using PCB piezoelectric pressure sensors [9]. PSDs were computed for sensors at a constant location for varying freestream conditions. The spectra increased in amplitude as
Increased. Juliano et al. conducted experiments on a 7° half-angle cone at 6° and 9° angle of attack in the H2K hypersonic wind tunnel, a conventional-noise facility, to support the HIFiRE-1 flight test [40]. PCBs were installed at azimuths from the windward to leeward rays in 45° increments and pressure-fluctuation power spectra were computed for a constant freestream condition and varying axial positions along lines of constant azimuth. Amplitude increased as $x$ (and subsequently $Re_x$) increased. These new results exhibit qualitatively similar behavior.
Figure 5.11. Kulite power spectral density for the steel nosetip at $\phi = 145^\circ$, $\alpha = 6^\circ$. 

(a) $Re = 5.7 \times 10^6 /m$.  
(b) $Re = 7.0 \times 10^6 /m$.  
(c) $Re = 7.9 \times 10^6 /m$.  
(d) $Re = 8.9 \times 10^6 /m$.  
(e) $Re = 9.8 \times 10^6 /m$.  
(f) $Re = 11.1 \times 10^6 /m$.  

$PSD = \frac{(p')^2}{(P_{mean})^2} \text{Hz}$
CHAPTER 6

STABILITY ANALYSIS WITH EXPERIMENTALLY OBSERVED WAVENUMBERS

Dr. Matthew Tufts, AFRL/RQHF, was a key contributor to this portion of the project. The results of the wavelet analysis and previous studies show that over the surface of a yawed cone, the stationary crossflow vortices are not uniformly spaced azimuthally around the cone’s surface [64, 70, 85, 99, 100]. Vortices tend to emanate near the attachment line on the windward ray of the cone and become unequally distributed moving towards the leeward ray. The unequal distribution of the vortices results in variation of the local wavenumber over the surface of the cone. Because the local amplification rate is a function of wavenumber, properly modeling the spatial wavenumber variation of the vortices is a critical step in integrating those amplification rates to accurate N-factors. In this work, three different models for the spatial wavenumber variation were employed and compared: the computed locally most amplified wavenumber of the stationary waves, a hybrid technique using computed vortex spreading [64] anchored by experimentally observed wavenumbers, or the experimentally observed largest amplitude wavenumbers.

6.1 Computational Methods

The LASTRAC3d [12-13] stability suite was used to solve the LST/PSE stability equations. An in-house Fortran script converted the Overflow2.2n formatted solution to a LASTRAC3d formatted mean flow file without interpolation. The solution was projected to the wall-normal direction while calculating the wall-normal distance in...
accordance with the LASTRAC3d manual [13]. LASTRAC3d was used to calculate the spatial growth rates of the stationary crossflow instabilities for a range of wavenumbers. The wavenumber was defined as the number of waves that would be present on the surface of the cone if the local wavelength was constant about the cone’s circumference. This spacing was measured orthogonal to the cone’s axis, not the flow’s.

The LASTRAC3d study also output the group velocity pathline of the stationary waves with an optimal spatial growth rate. The group velocity pathline is defined as the trajectory defined by the local group velocity at each point. This is the actual propagation direction of a wave packet. It is defined mathematically as $\frac{d\Omega}{dk}$, where $\Omega$ is the complex frequency and $k$ is the complex wavenumber. For stationary crossflow waves, this is the direction such that the crossflow velocity profile has its inflection point at zero velocity. The group velocity pathline marching direction was predicted to correspond with the stationary crossflow vortex paths. Additionally, inviscid streamline paths were computed for each test case. Dr. Matthew Tufts provided comprehensive information about the governing LST and LPSE equations used in LASTRAC3d (see Appendix B).

6.2 Comparison of Computational Marching Directions with Experimental Data

The group velocity pathline marching direction was predicted to correspond with the stationary crossflow vortex paths. Additionally, inviscid streamline paths were computed for each test case. Figure 6.1 examines which computationally generated trajectory — inviscid streamlines or group velocity pathlines — more accurately models the trajectories of the hot streaks observed in the experiments. The experimentally observed hot streaks are presumed to arise from the stationary crossflow vortices. The comparison was done by first measuring the angle of the inviscid streamlines, group velocity pathlines, and experimentally observed hot-streak pathlines relative to the
Figure 6.1. Difference between computationally derived marching directions with observed hot streak trajectories. $\alpha = 6^\circ$, $Re = 10.5 \times 10^6$/m.

cone axis. The relative angle between the experimental and computational data ($\zeta_{\text{exp}} - \zeta_{\text{comp}}$) was computed by finding the difference between these two angles.

The computed relative angles between the computationally derived inviscid streamlines (figure 6.1a) and experimentally observed hot streaks are relatively large, $\approx 30^\circ$ at $x = 300$ mm, decreasing to $\approx 10^\circ$ by the end of the model. This shows that the inviscid streamlines do not correspond well with the trajectories of the hot streaks observed in the experiment. In contrast, the computed relative angles between the group velocity pathlines and experimentally observed hot streaks are small ($\approx 5^\circ$ or less) at nearly every observed location on the model (figure 6.1b). That is, the computed group velocity pathlines agree well with the trajectories of the experimentally observed hot streaks. For this reason, stability analyses of stationary crossflow vortices should use group velocity pathlines as the marching direction. N-Factors for this study were integrated along the group velocity paths in all cases.

Figure 6.2 overlays the group velocity pathlines for two cases with the experimental heat-flux contours. Case 1, given in Figure 6.2a, shows experimental data from two different wind-tunnel runs at the same freestream unit Reynolds number and an-
Figure 6.2. Comparison of the computed group velocity pathlines and experimental heat-flux data.

(a) Case 1: $\alpha = 6^\circ$, $Re = 10.5 \times 10^6 /m$

(b) Case 2: $\alpha = 4^\circ$, $Re = 10.5 \times 10^6 /m$

gle of attack. Conducting two separate runs was desirable to image further upstream on the model, as optical access was limited by the IR window. The computed group velocity pathlines for this case showed very good agreement with the hot streaks observed in the experiment. The group velocity pathlines and hot streaks emanated near the windward ray and curved towards the leeward ray moving downstream. Case 2 is given in Figure 6.2b. Again, very good agreement was demonstrated between the computed group velocity pathlines and hot streaks. Figure 6.2 also shows labels for the group velocity pathlines. Wavenumber variation, and subsequently N-factors, were computed along these pathlines.

For each integration path, the spatial growth rate output from LASTRAC3d was used to create a piecewise cubic-spline interpolating function across the surface of the cone. The function returned the spatial growth rate as a function of axial distance $x$, the azimuthal angle $\phi$, and the local wavenumber $k$. This made it possible to integrate an N-Factor using an arbitrarily varying wavenumber along a given pathline.
6.3 N-Factor Results

The N-factor is the spatial amplification rate of a disturbance integrated along its path. The amplification rate is a function of the local wavenumber of the stationary crossflow waves and many other variables. The local wavenumber was determined using three techniques and the resulting N-factors are compared for each. For the first technique, the locally most unstable (highest amplification rate) wavenumbers were used at every step along the integration path. This method yields the maximum possible stationary crossflow vortex amplitude for a given pathline. Because of its conservative nature, this is the conventional method used to calculate N-factors in three-dimensional flow fields.

The second technique determined the local spreading of neighboring vortices based on the computed vortex pathlines \[64]\). In this approach, the basic state is adjusted to model the evolution of the wavenumber in the direction perpendicular to the marching path. In the current work, the marching path is along group velocity pathlines. The technique assumes a physical number of spanwise waves remain constant between neighboring vortex paths. As the paths progress downstream, the azimuthal spacing between them may increase or decrease. This change, along with the local orientation of the disturbance path, is used to measure the normal distance between paths. This gives values for the local effective wavenumber.

Using this technique, a non-dimensional ratio of the local wavenumber along each group velocity pathline was computed. The wavenumber ratio was defined as the local wavenumber divided by the wavenumber defined at the first available data point along the integration paths. Because the wavenumber ratios defined a relative rather than absolute wavenumber, this technique required dimensionalizing the computed values, which was done with experimental data. Along a given integration path, a single reference wavenumber at one axial position was used to normalize the wavenumber ratio. This axial station was selected separately for each path in order
to yield the best fit between the experimental wavenumbers and the dimensionalized computational wavenumbers in the least-squares sense. The wavenumber used to compute the amplification rate at each location is thus a hybrid of the computed and experimental data.

The best-fit wavenumber distribution is plotted with the experimentally measured values for three representative pathlines in Figure 6.3. Figure 6.3a shows wavenumber data along three paths: 16, 19, and 21. The labels of these paths correspond to labelled pathlines in Figure 6.2a. At \( x = 0.4 \) m, these paths are all located between \( \phi = 100^\circ \) and \( 140^\circ \). Path 16 is located closest to the leeward ray; higher path numbers correspond to pathlines located closer to the windward ray. Likewise, the paths in Figure 6.3b correspond to the labelled pathlines in Figure 6.2b. They are located in a similar azimuthal range at \( x = 0.4 \) m and follow the same naming convention.

The best-fit computed wavenumber profiles fit the experimental results well for both cases — the trends are accurately captured and the computed profiles exhibit much less scatter than the experimental data. Most importantly, they permit extrapolation upstream, to where the vortex amplitudes are too low to affect surface heating sufficiently to be discerned by IR thermography. In contrast, the locally most unstable wavenumbers do not correspond well with the experimentally observed wavenumbers of largest amplitude (Fig. 6.4). The LST computations from this technique yield the most unstable wavenumbers, i.e., the ones with the largest amplification rate. This method relies only on computed amplification rates. The experimental measurements, on the other hand, are of the wavenumbers with the largest local amplitudes. These two properties are closely related, but they are not identical, so the lack of correspondence between the experimental data and most unstable wavenumbers was expected. Figure 6.4 shows that the most unstable wavenumbers were much higher than the largest amplitude wavenumbers for each case.

The third technique to compute N-factors uses the amplification rate calculated
for crossflow waves with wavenumbers equal to the experimentally observed largest amplitude hot streaks along each integration path. Computed vortex path spreading is not used. Because this method is agnostic of the wavenumber upstream of the IR camera’s field of view, the N-factor at that station \((x = 295 \text{ mm})\) was assumed to equal the N-factor for the hybrid computational/experimental method.

Figure 6.5 shows N-Factors over the area with a defined experimental wavenumber. The first technique — selecting the most unstable wavenumbers — resulted in the N-factor at the end of the cone about 1 larger than the other two methods. The hybrid computational/experimental (‘computationally varying’) wavenumbers and the experimentally observed wavenumbers results in N-factors that matched closely. These behaviors were observed for each of the three test cases.

The similarity of N-factors for the hybrid computational/experimental and experimental-only wavenumber distributions suggests that either can be successfully employed in the stability analysis of a three-dimensional flow field. The hybrid method is a good choice if one has limited experimental data — for example, an FFT calculated for a spanwise heating distribution, or a set of discrete sensors at one axial
(a) Case 1: $\alpha = 6^\circ$, $Re = 10.5 \times 10^6 /m$

(b) Case 2: $\alpha = 4^\circ$, $Re = 10.5 \times 10^6 /m$

Figure 6.4. Comparison of computationally most unstable wavenumber with experimental largest amplitude wavenumber.

station. The experimental-only method is good if one has a complete map of stationary crossflow wavenumber evolution, but less sophisticated computational analysis of the vortex spreading. In contrast, using the computed most unstable wavenumbers introduces error into the calculated N-factor. The error is systematic and will always result in a too-large N-factor.
(a) Case 1: $\alpha = 6^\circ$, $\text{Re} = 10.5 \times 10^6 / \text{m}$
(b) Case 2: $\alpha = 4^\circ$, $\text{Re} = 10.5 \times 10^6 / \text{m}$
(c) Case 3: $\alpha = 4^\circ$, $\text{Re} = 8.52 \times 10^6 / \text{m}$

Figure 6.5. N-Factors calculated with different wavenumber variations.
CHAPTER 7

DIELECTRIC-BARRIER-DISCHARGE PLASMA-ACTUATED FLOW CONTROLLERS

7.1 Overview

A dielectric barrier discharge (DBD) plasma actuator was used in this research as the means of controlling crossflow-induced boundary layer transition. A DBD plasma actuator consists of a dielectric barrier and two electrodes, one exposed to the fluid medium and one buried within the dielectric [18]. The electrodes are arranged asymmetrically. To generate plasma, the buried electrode is supplied with an AC voltage source while the exposed electrode is connected to ground. During the first half of the AC cycle, electrons leave the exposed electrode and move towards the dielectric barrier [17]. The electrons are attracted to the buried electrode, but cannot pass through the dielectric barrier due to its high dielectric strength. They are trapped at the dielectric barrier during this part of the AC cycle. In the second half of the cycle, electrons are supplied by surface discharges at the dielectric barrier and move back towards the exposed electrode [17]. At a high enough AC voltage input, air over the buried electrode (adjacent to the exposed electrode) will weakly ionize. The simplest design for such a given DBD plasma actuator is given in Figure 7.1, shown from the work of Corke et al. [18].

DBD plasma actuators offer several advantages as an active flow controller. They have no moving parts, have a low mass, and have a fast response time [17]. They also eliminate the need to add cavities or permanent roughness features on the surface of an aerodynamic vehicle while potentially achieving the same effects as these...
features. Previous projects seeking to manipulate hypersonic boundary layer transition have utilized passive roughness elements, which were a permanent feature on the model. These designs could introduce problems on a real flight vehicle since the roughness features would only be most effective at a few specific flight conditions. A plasma actuator based system could be actively manipulated during flight to adjust for freestream conditions, taking advantage of different electrode geometries to adapt to the situation.

![Diagram of DBD plasma actuator](image)

Figure 7.1. Basic design for a flat DBD plasma actuator [18]

The basic design of the DBD plasma actuator seen in Figure 7.1 was fabricated on a 7° half-angle cone. A similar design was fabricated by Schuele et al. [84] on the same geometry for a crossflow study at Mach 3.5 in quiet flow. This design was inadequately documented, and substantial time was spent to redevelop a similar system for application in this project. The DBD plasma actuators seek to delay crossflow-induced boundary layer transition by introducing a forcing into the flow near the neutral point of the most amplified stationary waves [84]. To introduce this forcing, disturbance elements must be appropriately spaced azimuthally around
the model. In the case of roughness elements, these can be adhered at the desired spacing. However, for a surface electrode in a DBD plasma actuator system to yield a similar forcing effect, plasma must be generated in the desired azimuthal spacing.

To achieve the proper spacing, Schuele et al. introduced a comb surface electrode configuration shown in Figure 7.2. Plasma is generated normal to the surface electrodes for any actuator, and a normal force is generated at the electrode edges [95]. For the given configuration, the normal force generated at the comb fingers will result in fluid collisions between the fingers. The collisions yield pairs of co- and counter-rotating vortices [84]. These vortices will introduce the forcing into the flow. Therefore, the forcing can be controlled by selecting the spacing of the electrodes.

![Figure 7.2. Comb pattern of the surface electrode.](image)
It is known that the crossflow instability is sensitive to both patterned and random roughness on a conical model [95]. In the current work, the surface electrode of the DBD plasma actuators must be thin enough to not excite the crossflow modes in the absence of plasma generation as a roughness element. As a result, the surface electrode thickness should be less than maximum random surface roughness of the Macor tip.

To generate plasma on the model, a high-voltage wire was run through a hollow sting into the back of the model, through the channel in the steel frustum, into the nosetip. A connector in the steel frustum allowed for nosetips to be swapped. The plasma actuators required a dielectric barrier placed between the two electrodes, one connected to a high-voltage source and the other to an electrical path to ground. The nosetip served as the dielectric barrier and was fabricated from Macor, a machinable ceramic developed and sold by Corning, Inc. Macor was chosen primarily for its high dielectric strength (45.0 kV/mm [20]) and machinability. The surface electrode consisted of a copper coating applied to the outside of the test model in a finger pattern of the desired azimuthal spacing. The azimuthal spacing determined which stationary crossflow mode was forced [84].

In this work, the surface electrode was fabricated on the Macor nosetips using a liftoff photolithography process in conjunction with copper vapor deposition in the Notre Dame Nanofabrication Lab (NDNF). The technique was chosen due to the accuracy and precision required to fabricate this electrode geometry. The neutral point of the most-amplified crossflow waves was located $\approx 50$ mm from the model tip. In order to fabricate an electrode with the geometry seen in Figure 7.2 this close to the tip, high resolution and repeatability of the technique are required. Also, to ensure the surface electrode does not act as a roughness element on the model, a thickness of 60 nm was chosen. Schuele et al. demonstrated a 60 nm thickness is thin enough to prevent forcing, but sufficient to generate plasma [84]. These fabrication
techniques ensure the proper surface geometry and thickness will be achieved on the nozetips.

7.2 Axial Placement of the Surface Electrodes

The axial location of the actuator design was chosen based on a stability analysis. Discrete roughness, whether fixed roughness or plasma actuators, work by seeding a disturbance of a given wavenumber. The controller will be most effective if placed near the first neutral point of the disturbances it is intended to affect. The neutral point is the location where disturbances first begin to amplify. If the actuators are placed too far downstream of the first neutral points, instabilities will already have grown and the controllers will be less effective. If the controllers are placed too far upstream, the instabilities seeded by the plasma actuators may decay before they are able to modify the flow sufficiently to control the larger wavelength disturbances. In order to accelerate transition, a controller with critically spaced plasma actuators equal to the wavenumber of the most amplified stationary crossflow waves was designed. To delay transition, a controller with sub-critically spaced actuators $1.5 \times$ the wavenumber of the most amplified stationary crossflow waves was designed [84].

7.2.1 Simulations from Linear Stability Theory

Dr. Matthew Tufts of the Air Force Research Laboratory provided computational support for this portion of the project. The LASTRAC3d [12, 14] stability suite was used to solve the Linear Stability Theory (LST) stability equations for a $7^\circ$ half-angle circular cone at $6^\circ$ angle of attack at freestream unit Reynolds numbers of 8.0 and $10.5 \times 10^6$ /m (see Appendix B for a detailed discussion of the LST governing equations). The nozetip diameter of the cone was modelled as 1.0 mm, the smallest that could be reliably machined in Macor. The Overflow2.2n flow solver, using 3rd order accurate HLLE++ fluxes, was used to solve unperturbed, steady Navier-Stokes
solutions. For both the laminar heat-flux calculations and mean-flow solutions to be used in later stability calculations, no turbulence models were needed. For the turbulent heat-flux calculations, the Spallart-Allmaras model (SA-noft2) with default coefficients was used over the entire geometry. For all cases, the flow was solved as a steady-state flow. Boundary conditions for the model surfaces were modeled as no-slip, isothermal, uniform-temperature walls.

An in-house Fortran script converted the Overflow2.2n formatted solution to a LASTRAC3d formatted mean flow file without interpolation. The solution was projected to the wall-normal direction while calculating the wall-normal distance in accordance with the LASTRAC3d manual [13]. For the stationary crossflow instabilities, LASTRAC3d was used to calculate the spatial growth rates for wavenumbers between \( m = 12 \) and 60. The wavenumber was defined as axisymmetric and perpendicular to the axis of the cone.

### 7.2.1.1 Inviscid Streamlines

The results of these simulations are presented in a \( x-\phi \) coordinate system, showing spatial N-factor evolution along the marching paths. Figure 7.3 shows the maximum N-factor along inviscid streamlines for wavenumbers between 12 and 60. The instabilities emanate near the windward ray of the model (shown in the figure near \( x \approx 20 \text{ mm} \) between \( \phi = 15^\circ \) and \( 60^\circ \)), and curve towards the leeward ray as they progress downstream. The highest N-factors were predicted to evolve along streamlines to azimuths between \( \phi = 130^\circ \) and \( 160^\circ \) between \( x = 300 \text{ mm} \) and \( 400 \text{ mm} \). These were of particular interest in designing the plasma flow controllers; the stability calculations show that these disturbances first begin to amplify at approximately 20 mm from the nosetip. Placing the plasma actuators at this axial location would be most effective in controlling these instabilities. This location is also desirable from a fabrication standpoint, as the required dielectric barrier thickness can be properly
machined at this axial station. Therefore, each plasma-actuated nosetip tested in this work featured actuators 20 mm from the tip.

Figure 7.3. Streamline integration, $Re = 8.03 \times 10^6 /m$, $\alpha = 6^\circ$, LST.

Updated computations were conducted changing the nosetip diameter to 1 mm to match what would be used in the experiments. This was important because the plasma actuators needed to be placed as close as possible to the neutral points of the instabilities. The first neutral points of the original simulations ($d = 0.4$ mm) were computed and compared with the updated ones ($d = 1$ mm) and those from Li et al. [55]. This comparison was done by plotting the predicted first neutral point location as a function of azimuthal angle, where zero is taken to be the tip position for each different nosetip diameter, not the tip position for a nominally sharp nosetip. An azimuth’s neutral point was only plotted if that streamline subsequently reached
N-factors of at least 6. This result, given in Figure 7.4, shows the neutral point can vary significantly with azimuth. However, there is a strong concentration of disturbances originating near \( x = 20 \) mm. No streamlines originating at \( \phi < 15^\circ \) or \( \phi > 70^\circ \) experienced disturbance amplification.

The actuators used in this work all have constant axial station. This choice was made for several reasons. It simplifies the actuator’s design because the axial station and spacing of the electrodes are the only two parameters needed. Actuators at constant axial station can be designed more easily and fabricated more repeatably. Finally, the stability analysis indicates that restricting the design to a single axial station does not substantially handicap control authority. For the 1 mm diameter nosetip, the neutral point is predicted to be at \( 20 \pm 2 \) mm for three-quarters of the relevant azimuthal range. Several nosetips were designed with actuators at \( x = 20 \) mm based on this LST result.

The original simulations were conducted with a nosetip diameter of 0.4 mm, as this was the planned geometry. Due to manufacturing challenges, the actual nosetip diameters were 1.0 mm. The first neutral points of the original simulations \( (d = 0.4 \) mm) were computed and compared with the updated ones \( (d = 1 \) mm) and those from Li et al.\[55\] (Figure 7.4). This comparison was done by plotting the first neutral point location as a function of azimuthal angle, where zero is taken to be the tip position for each different nosetip diameter, not the tip position for a nominally sharp nosetip. Disturbances with wavenumbers of 25 to 40 were checked to find the location of earliest amplification. An azimuth’s neutral point was only plotted if that streamline subsequently reached N-factors of at least 6. While the neutral point can vary significantly with azimuth, for a large range of azimuth the neutral point is located near \( x = 20 \) mm. Each plasma-actuated nosetip tested in this work featured actuators 20 mm from the tip, near most of the predicted neutral points in Figure 7.4.
7.2.2 Group Velocity Pathlines

The LASTRAC3d study output the group velocity pathlines of the stationary waves with an optimal spatial growth rate. This marching direction corresponded with the stationary crossflow vortex paths. The results of these simulations are presented in a $x$–$\phi$ coordinate system, showing spatial N-factor evolution along the marching paths.

Figure 7.5 shows the N-factor along group velocity pathlines obtained by integrating the maximum amplification rate for wavenumbers between 12 and 60, inclusive. As shown in Ref. [102] (specifically subsection 6.1), group velocity pathlines align more closely than inviscid streamlines with surface hot streaks. The instabilities emanate near the windward ray of the model (shown in the figure between approximately $x = 25$ and $110$ mm and between $\phi = 20^\circ$ and $60^\circ$), and curve towards the leeward ray as they progress downstream. Placing the plasma actuators along the neutral points of these pathlines would be most effective in controlling the instabilities. The
Macor nosetip design also limited the axial location of the actuators because a certain dielectric thickness needed to be achieved.

![Figure 7.5. Group velocity pathline integration, $Re = 10.5 \times 10^6$ /m, $\alpha = 6^\circ$.](image)

Ideally, the plasma flow controllers would be placed in a swept configuration along the neutral points of the group velocity pathlines shown in Figure 7.5. This would ensure that actuation was achieved for each stationary wave. The plasma actuators used in this work were adhered to the cone at a fixed axial station ($x = 20$ mm) due to fabrication challenges. This meant that the actuators were located at the neutral point of some, but not all of the instabilities. Figure 7.5 shows that group velocity pathlines that emanate near $x = 20$ mm reach the separation line at 160° well upstream of the field of view of the IR camera. The group velocity pathlines
that reach highest N-factors on the model emanate between axial stations of 35 and 95 mm. In general, the closer the group velocity pathline trajectories are to the windward ray, the further downstream their neutral points are located. Therefore, one would expect to see a decreased effectiveness of the plasma actuators for hot streaks closer to the windward ray. Schuele et al. quantified this behavior in terms of a “dividing streamline” [85]. At azimuths below this streamline (towards the windward ray), flow controllers cannot affect crossflow-induced transition.

7.3 Plasma-Actuated Nosetip Designs

7.3.1 $m = 68, 45, 30$ Nosetip Design

The first plasma-actuated nosetip was designed to place the plasma actuators at an axial station near the neutral points of the instabilities based on the inviscid streamline LST results with a predicted neutral point of 20 mm. In this design, the Macor nosetip served as the dielectric barrier and a high-voltage wire epoxied into the nosetip served as the buried electrode. A schematic is given in Figure 7.6 and a photograph in Figure 7.7.

The author previously tested the cone model in the BAM6QT with a steel nosetip and computed the local wavenumbers of the largest amplitude hot streaks (Chapter 5) [102]. Wavenumbers between $m = 35$ and 55 were observed over the relevant range of freestream conditions. Therefore, actuators were designed to provide super-critical ($m = 30$), critical ($m = 45$), and sub-critical ($m = 68$) forcing. The $m = 45$ controller was designed to accelerate transition, the $m = 68$ controller to delay transition, and the behavior induced by the $m = 30$ controller was unknown.

7.3.2 $m = 140, 210$ Nosetip Design

Figure 7.5 indicates that the most amplified stationary waves, as observed experimentally through the IR window, all originate in a small area between $x = 20$ and
110 mm and $\phi = 20^\circ$ and $60^\circ$. This brings up an open question in the research community regarding actuator design: should the spacing of the actuators correspond to the wavenumber of the crossflow vortices at their neutral point or the wavenumber observed further downstream where data is collected? Based on a measured wavenumber of $m = 45$ at $x = 350$ mm from the nosetip, if 45 actuators were places circumferentially around the nosetip, only one or two of them would be located in the area where the most amplified stationary waves originate. This is further illustrated in Figure 7.9 which gives the wavenumber ratio as a function of azimuth. This non-
dimensional parameter gives the ratio of the local wavenumber along a vortex path at a specified axial distance relative to the local wavenumber at the neutral point. Given the computed global wavenumber ratio and an experimental observation of the wavenumber of the vortices at a particular axial station, one can infer the wavenumber at the neutral point. This also makes it clear that the wavenumber of the vortices changes as it evolves downstream.

Corke et al. [18] had success manipulating transition location using a traditional approach, where the number of actuators corresponded to the wavenumber of the crossflow vortices as observed downstream near their transition locations. A new design proposes having a number of actuators corresponding to the critical wavenumber at the neutral point. This accounts for the spreading of the vortices as they progress downstream. This means that for a critical wavenumber $m = 45$, 45 actuators should be placed within the relevant azimuthal range. Consequentially, several hundred actuators will need to be placed around the circumference using this technique. Due to experimental evidence that the first design technique worked, but also a compelling argument that the second technique may be more effective, both design techniques will be implemented.

The second nosetip design was pursued to move the location of the actuators downstream while maintaining the same dielectric barrier thickness (Fig. 7.8). This was done for two reasons. First, the updated stability analysis using group velocity pathlines (corresponding to vortex paths) indicated that the neutral point of the instabilities was further downstream than originally predicted. This meant controllers needed to be placed near $x = 55$ mm based on this analysis. Second, accounting for the spreading of the vortices as they progressed downstream meant that several hundred electrodes needed to be placed around the circumference of the cone. Due to limitations with the photolithography technique, it was not possible to fabricate this many electrodes about the cone’s circumference at $x = 20$ mm. Moving the
controllers downstream increased the overall size of the electrodes, aiding in the fabrication process.

In this second design, the buried electrode consisted of a stainless steel nosetip with a high-voltage wire running into it. A Macor insert served as the dielectric barrier. Another potential future benefit of this new design was that sharper nosetips could potentially be tested in the future. Because of its brittleness, it is very difficult to machine Macor tips with diameters much less than 1 mm. However, all data presented herein were collected with 1.0 mm diameter nosetips. These nosetips were fabricated with $m = 140$ and 210 actuators, respectively. These spacings corresponded to the $m = 45$ and 68 designs when accounting for spreading of the vortices.

Figure 7.8. $m = 140, 210$ nosetip design.
7.4 Lift-off Photolithography Techniques

7.4.1 General Photolithography and Vapor Deposition

Photolithography is a laboratory technique used to accurately fabricate patterns with geometrical features with scales on the order of nanometers. The electrode patterns for this research will see length scales as small as 20 µm, so the technique is desirable for this application. The general technique, also shown in Figure 7.10, is outlined as follows. First, the substrate is cleaned to remove dust or other imperfections on the substrate surface that could affect smooth coating. Next, photoresist, a light sensitive material is applied to the substrate. Depending on whether the positive or negative image of the pattern is desired to be kept, either positive or negative photoresist can be used. The substrate is spun in a centrifuge in order to evenly coat the photoresist. Next, the photoresist is baked at a high temperature in order to cure
The desired pattern is created on a photomask, typically made of film or quartz. The photomask is laid on top of the substrate and then the piece is exposed to ultraviolet light. The sample is next placed in a developing solution which is designed to remove photoresist from either the negative or positive side of the pattern. After this step, the sample is ready for vapor deposition.

The vapor deposition, or sputtering process, is a momentum transfer process where atoms of a desired material (for example copper) from a target are driven off via bombarding ions. The free atoms then travel until they strike a substrate. The bombarding ions in the process are an inert gas. An inert gas must be used because the free atoms are chemically active and could react with the bombarding agent. The process is carried out for a given amount of time to achieve the desired thickness. In this research, the desired sputter thickness was 100 nm. This thickness was demonstrated to be sufficiently thick for an electrical connection between the Macor and steel tips shown in Figure 3.1. A minimum thickness was also desirable to prevent unwanted surface roughness on the nose tips.

Once this step is complete, the entire substrate is coated with the material from vapor deposition. The substrate is washed in acetone, a solvent that will dissolve photoresist. Through agitation, the photoresist under the deposited material will break free, thus removing the material in locations in accordance with the desired pattern. This will leave behind the deposited material forming the desired pattern.

7.4.2 Photomask Design

The first step in developing a photolithography process to adhere the surface electrodes to the Macor nosetips was to design and fabricate a series of photomasks. The role of a photomask in the photolithography process is to create a pattern on the substrate that will expose the photoresist to ultraviolet (UV) light in the corresponding pattern. The masks for this work each featured a unique pattern according to
the desired actuator design for a given nosetip. The first iteration of photomasks were printed on 7 mil silver-halide emulsion film by PhotomaskPORTAL \[8\]. This resulted in a durable mask with a minimum resolution of 0.6 $\mu$m (an example image of the photomask design is given in Figure 7.11). These masks were high quality but presented some challenges in the photolithography process due to their thickness.

Initially, the masks were adhered to the Macor nosetips using Kapton tape. This was ineffective because the masks tended to bend up off the nosetips rather than fitting tightly to the surface. The mask needed to be adhered as tightly as possible to the substrate since UV light can refract in the gap between the pattern and substrate, thus degrading the resolution of the pattern. To alleviate this problem, the first solution was to switch from holding the masks on with Kapton tape to a series of varying diameter rubber o-rings. This change improved the issue, but the masks were still not fully adhering to the cone.

It was determined that the first generation photomask was too thick to properly wrap around the nosetips at the desired location. A new photomask was fabricated on # 770 harddot camera film from Advance Reproductions \[73\]. This film material’s thickness is 4 mil, compared to 7 mil for the silver-halide film. This change resulted in masks that could fully adhere to the Macor nosetips. As such, they were used to manufacture all of the Macor nosetips that were tested in the BAM6QT. Additionally, the new masks were less expensive than the original design. The first design cost $150
Figure 7.11. Example detailed drawing of the $m = 45$ photomask.

per mask. The new design cost approximately $100 for a 8.5 \times 11$ inch sheet that fit 12 masks.

7.4.3 Curved DBD Plasma Actuator Fabrication

A curved, 3-D geometry presented several challenges in the actuator design and fabrication process. First, photolithography is designed to work on flat surfaces or rough 2D surfaces with height variation on the order of microns. Both the 3-D nature and high curvature of the Macor tips make steps in the photolithography process challenging. First, the spin coater used to adhere photoresist to the flat plasma actuators was not designed to accommodate a tall piece such as a cone. The
spin coater maximum clearance was measured and found to be 1.1 inches. However, the Macor tips were 1.86 inches in length. To avoid this problem, the spin coater software was bypassed to allow the machine to operate with the lid up. An adaptor was designed and built to hold the Macor tips vertically in a base bolted to the centrifuge. Initially, the Macor tips were spun using AZ4620 because this yielded the best result with the flat Macor pieces. Using a spin setting of 2500 rpm for 90 seconds, a uniform coating of AZ4620 could be applied to the Macor tips. However, the thickness of the photoresist was found to be too thick and would not properly expose or develop using the techniques of photolithography.

Once it was determined that AZ4620 was poorly coating the 3-D pieces, the author switched to an aerosol positive photoresist. The new photoresist is called Microspray and is manufactured by Microchem \[21\]. It comes in an aerosol can and is sprayed onto a substrate in a manner similar to spraypaint. Microspray is designed to adhere to substrates with severe topography and curvature. It is also designed to react with UV light at 350–450 nm. There are many advantages of the Microspray. The Microspray adheres evenly between and in cavities due to the spray-on application technique. This is advantageous because Macor has high porosity compared with silicon. This means traditional photoresist will have a nonuniform thickness when spun onto the material, since many cavities will be skipped over, even on a flat sample. Second, spin coating results in thick and nonuniform coatings on 3-D surfaces. The Microspray application also works around this problem and avoids having to use spin coating. Finally, the Microspray is designed to react with UV light at a shorter wavelength and over a wider range than AZ4620 photoresist. This is ideal for the light source being used, because the exposure times of the Macor tips are shorter, resulting in better resolution of the electrode pattern. The Microspray photoresist coated the Macor tips uniformly and thinly, as desired. Figure \[7.12\] is a photograph of the tips immediately after applying the spray.
Once the Macor tips were covered with Microspray, they were baked in an oven at 110°C for 4 minutes and allowed to cool. The bake process removed solvents from the photoresist, and subsequently hardened it onto the surface of the cones. This bake time followed the recommended bake time of the Miocrospray on Silicon. However, the time was proven insufficient because the pieces were baked in an oven rather than on a hot plate or in a convection oven. This could be seen when the photoresist did not properly develop after being exposed to a UV light source. The bake time was increased to 1 hour, which fixed the problem. Next, the photomask design was wrapped around the Macor tips at the desired axial location with a series of rubber o-rings of varying diameter. Caution needed to be taken to avoid breaking the Macor tips. The Macor test article was exposed to UV light with the desired photomask placed on the wafer using a light source manufactured by OAI. It is a collimated illumination system source used to expose samples to UV light. The OAI light emits in the deep ultraviolet range of the spectrum (200–220 nm). Once the sample was exposed to UV light, it was washed in a developing agent, leaving behind the desired pattern to be deposited. Once this step was complete, the samples were developed in
Tetramethylammonium hydroxide (TMAH) solution for 3 minutes. The intermediate result yields the desired pattern with photoresist on the Macor tip.

Next, vapor deposition was used to adhere copper to the test article using the Perkin-Elmer 2400 Sputtering System, shown in Figure 7.13. This particular system offers many advantages for the scope of this project. First, the PE 2400 had a deposition chamber large enough to accommodate future cone models, which take up much more room than typical flat wafers. Second, the system allowed for the deposition of both copper and titanium on the same sample. Copper adheres excellently to titanium and this may be useful to adhere the electrodes to future cone models. Finally, this machine was designed to work with various substrates including metals and ceramics such as Macor.

To operate the PE 2400, the chamber was first vented to atmospheric pressure. During this time, the samples were placed in the fabrication chamber under the desired source, in this case copper. The Macor tip was placed into an angle bracket adaptor to hold it horizontally in the PE 2400. Next, the chamber was closed and pumped down to near vacuum ($10^{-6}$ torr). Next, the sputtering process began. Sputtering is a momentum transfer process where atoms on a target are driven off by bombarding ions of the desired material. The copper thickness depends on the sputtering process run time and the substrate’s distance from the source. Argon was used to carry out this bombarding process because it is inert and therefore did not react with the copper ions before they hit the substrate [32]. When the sputtering process was complete, the system was returned to atmospheric pressure and the sample was removed. The sample was then cleaned with acetone to remove any remaining photoresist and unwanted pieces of copper.
7.5 DBD Plasma Actuator Vacuum Chamber Testing

Static pressure tests were conducted to characterize the activation voltage of the plasma actuators at static pressures that would be encountered in the BAM6QT. The experimental setup consisted of a function generator, two audio amplifiers, a power transformer, a high-voltage probe, and an oscilloscope. To generate plasma on the test pieces, first a low AC voltage was selected on the function generator. Next, the gain on the amplifiers was selected. The amplifiers were used to drive the transformer, which increased the low-voltage input signal to the desired high-voltage output to be sent to the plasma actuator. A high-voltage probe was connected between the transformer and test piece to measure the voltage being sent to the actuator on the oscilloscope. The high-voltage input was connected to the buried electrode of the test actuators. Finally, the high-voltage probe and surface electrode were connected to ground.

The experiment sought to test the actuator at static pressures exhibited in the
BAM6QT behind a seven-degree half-angle conical model bow shock for a Mach 6 freestream. Using the isentropic and conic shock relations, the BAM6QT static pressure behind the bow shock will be between 11 and 15.25 torr based on driver tube stagnation pressures of 130 psia and 180 psia. The vacuum chamber was configured to hold the test actuators at these pressures.

In Figure 7.14 breakdown voltage is plotted as a function of pressure. This occurs when the electric field exceeds the electric field strength of the air [91]. Operation voltage is the voltage for which plasma is formed across the entire row of teeth, which is desired for the model. The data reflects the fact that activation voltage decreases non-linearly with decreasing pressure. This agrees with the work of Valerioti and Corke [91]. The frequencies tested represent the operating range of the power transformer to be used in the experiments.

The static pressure conditions that were expected in the operational environment of the plasma actuators were computed by solving the conic shock relations on a 7° half-angle cone at 0° angle of attack. Taking into account the expected freestream unit Reynolds number in the BAM6QT (10.5 × 10^6 /m), the conic shock relations gave a static pressure of \( p = 9.23 \text{ torr} \) behind the bow shock on the surface of the cone. The relevant static pressures in Figure 7.14 are shown near the far left side of the plot. The results indicate that at the static pressures relevant in the experiment, an AC frequency of 3 kHz resulted in the lowest breakdown voltage. Using an input of 3 kHz, plasma will be generated at a lower peak-to-peak voltage as compared with other frequencies.

It was important to quantify the performance of the plasma actuators in order to predict the input settings for frequency and voltage needed during operation of the BAM6QT. This was because the nosetip was not visible during a run due to limited optical access and that the CaF\(_2\) IR window has low transmissivity of the visible light from a plasma discharge. However, plasma generation was confirmed during tunnel
runs by measuring the current of the buried electrode of the DBD plasma actuator. Plasma generation was observed as spikes in the current signal.

![Figure 7.14. Plasma actuator breakdown voltage at low static pressures.](image)

7.6 Mechanisms of AC Dielectric-Barrier-Discharge Plasma Actuators

AC-driven DBD plasma actuators introduce a velocity perturbation into the flow. The induced velocity is small, on the order of 1 m/s [84, 88], but if properly selected it will grow according to linear stability theory. Balakumar performed DNS on a 7° half-angle cone at 4.2° angle of attack at a Mach number of 3.5 and showed that the perturbation amplitude at the location of the neutral points is sufficient to achieve the desired control effect if it is just 0.5% of the freestream velocity [2].

It is not clear from the literature whether the momentum or thermal perturbation by a plasma actuator is responsible for its influence on boundary-layer transition.
The question is important, because if thermal energy input is primarily influential, actuators may be ineffective at flight enthalpy levels. Regardless, they are useful as a laboratory tool for investigating the physics of boundary-layer stability.

Several experimental studies have characterized the momentum transfer processes associated with AC-driven DBD plasma actuators. Corke et al. report that for sinusoidally driven DBDs, the primary mechanism of control is generated through a body-force vector field that couples with the momentum in the external flow [18]. They derived an expression that gives the time-dependent body force produced by the plasma as a function of the electric potential and net charge density (see equation 5 in Ref. [18]). The predicted body force from this model scales as $V^{3.5}$, which agreed with AC experiments performed in a static environment [65]. Valerioti and Corke tested AC-driven DBD actuators in a static pressure chamber at pressures between 0.17 and 9.0 bar [91]. Experiments were conducted to determine the effect of static pressure on the properties of an SDBD plasma actuator. The average body force was related to the thrust generated by the plasma actuator in experiments. The sub-atmospheric pressure where the peak body force occurred depended on the actuator voltage, with higher voltages causing the peak to occur at lower pressures.

Schuele et al. conducted experiments with passive roughness elements and an AC-driven DBD actuator on a cone at angle of attack in Mach-3.5 flow [85]. The body force vector field was designed to cause the formation of counter-rotating vortices that were similar to that produced by a span wise array of discrete roughness elements. For AC frequencies between 1 and 3 kHz, the plasma roughness scaled linearly with the AC voltage frequency, which was a predicted trend based on the modeling of the body force. The stationary crossflow modes were found to be as receptive to the plasma array as they were to passive roughness. This result was important, because if the plasma actuators primarily added temperature to the flow, then passive roughness wouldn’t produce the same effect.
Matlis collected Schlieren images of a plasma sensor at Mach 3.5 [60]. Despite the fact that this sensor was much hotter than a DBD due to its lack of a dielectric layer, no evidence of a thermal contribution was observed. Matlis noted that a DBD generates a “cold plasma,” where the only source of heating is the dissipation in the system, which produces ohmic heating at the dielectric layer. For the current work, Matlis also attributed the small thermal effects to the low power and low current of the system in comparison to the high mass flow rates at Mach 6.

Although experiments have demonstrated the presence of a body force in association with DBD actuators, most of these studies do not comment on the thermal effects; the presence of a body force does not negate the possibility of thermal effects. The actuators could also release energy to translational degrees of freedom of the molecules, resulting in thermal energy addition to the flow. Starikovskiy and Alexandrov analyzed the results of two observations of non-equilibrium plasma produced by high-voltage nanosecond discharges [88]. These results involved the measurement of the velocity of a shock wave that propagates through air heated by an impulse discharge at 20 torr and the experimental study of a SDBD in atmospheric-pressure air. The electron power transferred into heat in air plasmas was estimated in high (103 Td) electric fields. It was shown that around 50% of the discharge power can be transferred into heat for a short period of time (≈1 µs) at atmospheric pressure and 30% at 20 torr. Correale et al. developed a method to quantify the efficiency of the first two operational stages of a nanosecond dielectric barrier discharge (ns-DBD) plasma actuator [22]. They calculated the fraction of discharged energy that actively contributed to the thermal effect induced by a ns-DBD plasma actuator for varying dielectric thickness. Between 50% and 60% of energy was lost due to thermal effects. It should be noted that the current work implements an AC-DBD actuator, where thermal effects are less important than in a ns-DBD. Overall, the majority of research pertaining to AC-driven SDBD plasma discharges supports the hypothesis
that momentum transfer effects are the dominant mechanism of flow control. More work is needed to fully understand the role that thermal effects play in this process.

7.7 Nosetip Roughness Measurements

The crossflow instability is known to be extremely sensitive to roughness effects [26, 86, 95]. Consequentially, it was desirable to minimize and characterize the roughness of the nosetips used in the experiments. Prior to running wind tunnel tests, each nosetip was polished to a smooth finish using a multi-step process consisting of 1000, 2000, and 3000 grit dry sandpaper and a diamond polishing agent. Figure 7.16 shows images taken before and after this polishing process. The goal of this process was to reduce the roughness of the Macor nosetips to a level similar to that of the steel nosetip. The root-mean-square (rms) surface roughness of each nosetip was measured using an Olympus LEXT OLS4100 Confocal Microscope with an uncertainty of ±0.01 µm. The measurements were conducted 20 mm from the nosetip. Figure 7.15 shows an example image from the microscope. This image shows one of the actuators on the $m = 45$ nosetip deposited onto the Macor. The viewing area in this image is 640×640 µm$^2$. Table 7.1 shows rms surface roughness characteristics of each nosetip. These values were computed from a 150×150 µm$^2$ area on each surface at a location 20 mm downstream of the nosetip. This location was selected because it coincided with the axial position of the plasma actuators. This measurement area was registered to remove surface curvature effects. The Macor nosetips had more roughness than the steep nosetips ($\approx 1.0$ µm compared to 0.7 µm). The polishing process was very effective, as the unpolished Macor nosetip had rms roughness of 3.0 µm. The $m = 45$ and 30 nosetips featured similar roughness characteristics, while the $m = 140$ nosetip had a slightly higher rms roughness ($\approx 1.3$ µm compared to 0.9 µm).

It was desirable to compare the roughness characteristics of the nosetips with a flow scaling parameter. This was accomplished by computing the local boundary-
layer thickness (see Section A) as a function of azimuthal angle at the same axial station (20 mm) for different angles of attack and different Reynolds numbers (figure 7.17). The thinnest boundary layer at the $x = 20$ mm station is 0.15 mm ($\phi = 0^\circ$, $\alpha = 6^\circ$, $Re = 10 \times 10^6$ /m). The ratio of the measured rms surface roughness to the computed boundary-layer thickness is $6.1 \times 10^{-3}$.

![Plasma actuator at 20× optical zoom (640 × 640 µm² viewing area).](image)

**Figure 7.15.** Plasma actuator at 20× optical zoom (640 × 640 µm² viewing area).

### TABLE 7.1

**NOSETIP ROUGHNESS CHARACTERISTICS.**

<table>
<thead>
<tr>
<th>RMS Roughness (µm)</th>
<th>unpolished Macor</th>
<th>m = 45</th>
<th>m = 30</th>
<th>m = 140</th>
<th>steel</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3.040</td>
<td>0.918</td>
<td>0.944</td>
<td>1.282</td>
<td>0.698</td>
</tr>
</tbody>
</table>
Figure 7.16. Nosetip roughness characterization before and after polishing.

Figure 7.17. Boundary-layer thickness for different flow conditions at $x = 20$ mm.
CHAPTER 8

RESULTS WITH PLASMA ACTUATORS

The experimental results discussed in this work were all taken on a 7° circular cone with 1.0 mm diameter nosetips and a 101.6 mm (4 in.) base diameter. All wind-tunnel runs were conducted at a freestream Mach number of 6. Freestream unit Reynolds number, the model nosetip, and the applied voltage were varied to observe their effect on the wavenumber and transition location of the stationary waves. Interchangeable nosetips permitted testing of various actuator spacings and designs. To better quantify change in transition location, the data for all runs were mapped from a Cartesian pseudo x–y plane to a x–ϕ plane using an in-house image processing code [73, 77]. This mapping allowed for vortex paths and lines of constant ϕ and x to be analyzed, thus better quantifying the effect of the plasma actuators and input voltage on transition. In the mapping, ϕ = 0° corresponds with the windward ray and ϕ = 180° corresponds with the leeward ray.

It has been well documented that tunnel noise effects can have a large impact on the crossflow-induced transition location observed on cones at angle of attack in supersonic and hypersonic flows [41, 47, 48, 82, 89]. The results with plasma actuators presented in this work were all collected under quiet-flow conditions. These runs resulted in distinct hot streaks. Transition occurred along each individual hot streak, with the streaks independently undergoing increases in surface heat flux. This resulted in jagged transition fronts, which made defining the exact transition location difficult. In contrast, noisy runs resulted in wide regions of heating without discrete streaks (see Fig. 5.9). This difference is thought to be due to the relatively large amplitude
of unsteady disturbances in conventional-noise wind tunnels.

8.1 Plasma Effect on Surface Heat Flux and Transition Location

8.1.1 \( m = 68 \) Noisetip

The \( m = 68 \) nosetip was designed to space the plasma actuators at 1.5\( \times \) the naturally most amplified wavenumbers of the stationary crossflow vortices near their transition locations \[^{[101]}\]. This design was expected to delay crossflow-induced transition. Figure 8.1 shows three surface heat-flux maps taken on the model: a plasma-off case with the Macor nosetip, a plasma-on case with the Macor nosetip, and a test case with the steel nosetip to serve as a baseline. Each test case resulted in several transitional hot streaks, based on large increases in surface heat flux along each path. Each test case shows several transitional hot streaks between 100° and 160°.

To quantify a transition location for the experimentally observed hot streaks, heat flux along them was extracted. A transition location was found by computing linear fits for two portions of the data: one where heat-flux values were decreasing moving downstream and one where heat-flux values were increasing downstream, if observed. The intersection point of these fits was concluded to be an estimate of the transition location \[^{[44]}\]. Uncertainty estimates were computed by varying the domains of these two regions. Figure 8.2 shows the results from Figure 8.1 with a particular vortex path indicated by solid black lines. This streak was present for both the plasma-off and -on cases at the position on the model. It is of interest because the transition location was affected by the plasma actuators. Figure 8.3 shows the heat flux along this path, with a transition location denoted by the green marker. The error bar shows the uncertainty in the transition location. The surface heat flux initially decreased downstream for both the plasma-off and -on cases. Near \( x = 338 \) mm, the hot streak from the plasma-off run saw a sharp rise in heating. This was concluded to correspond to a transition point. The plasma-on case appeared fully
Figure 8.1. Surface heat flux. $m = 68$, $\alpha = 6^\circ$, $Re = 10.5 \times 10^6 /m$.

laminar moving further downstream. A transition location for this case was computed
at $x = 365$ mm. The plasma-off vortex saw constant heating rates between $x = 385$
and 400 mm, suggesting the flow had become turbulent. Meanwhile, the plasma-on
streak did not plateau, suggesting the end of transition was not reached. Plasma
actuation delayed transition by $\approx 30$ mm for this case.

A map of the transition front was constructed based on the surface heat flux (Fig-
ure 8.6). Markers with corresponding uncertainty bars are shown for each hot streak
that appears to be transitional. An estimated transition front was constructed whenever adjacent hot streaks were deemed transitional. This is given by the solid black
Figure 8.2. Vortex paths. $m = 68, \alpha = 6^\circ, Re = 10.5 \times 10^6 /m$.

Figure 8.3. Heat flux along vortex. $m = 68, \alpha = 6^\circ, Re = 10.5 \times 10^6 /m$. 

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line. Five transition locations were computed for the plasma-off case, and six each for the plasma-on and steel test cases. When the plasma flow controller was activated, transitional vortices were still observed, but their respective transition points moved downstream. This was a promising result, because it provided experimental evidence that the plasma actuators can successfully delay crossflow-induced transition.

(a) $V_a = 0 \text{ kV}$

(b) $V_a = 4 \text{ kV}$

(c) Steel nosetip

Figure 8.4. Heat flux and transition fronts. $m = 68$, $\alpha = 6^\circ$, $Re = 10.5 \times 10^6 /m.$
Results were also collected at a lower freestream unit Reynolds number, $9.5 \times 10^6 / m$. These data showed less transitional vortices, making it difficult to quantify changes in transition location at most azimuths. Transition appeared slightly delayed for a hot streak near $\phi = 140^\circ$. These results were less conclusive than the higher $Re$ test cases because transition couldn’t be quantified in most cases.

![Graphs showing heat flux and transition fronts.](image)

(a) $V_a = 0$ kV  
(b) $V_a = 4$ kV  
(c) Steel nosetip

Figure 8.5. Heat flux and transition fronts. $m = 68$, $\alpha = 6^\circ$, $Re = 9.5 \times 10^6 / m$. 

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8.1.2 $m = 45$ Nosetip

The $m = 45$ Macor nosetip was designed to provide critical forcing and expected to advance crossflow-induced boundary-layer transition on the model. Based on experimental results with the steel nosetip, this spacing corresponded to the wavenumber of the largest amplitude hot streaks. Figure 8.6 shows the results of three runs conducted at the same freestream conditions with transition fronts superimposed with the data ($Re = 10.5 \times 10^6 /m$). Figures 8.6a and 8.6b show the experimental results for the $m = 45$ Macor nosetip without and with the effects of the plasma flow controller, respectively. Figure 8.6c shows the results with the steel nosetip. Hot streaks are clearly visible in each IR image. Some vortices appear transitional based on the steep increase in heat-flux values observed along their paths. With the actuators on, hot streaks developed between $\phi = 100^\circ$ and $140^\circ$ near the downstream portion of the image (Figure 8.6b). These hot streaks appear to increase in heat flux and widen as they develop. In contrast, with the actuators off, there was less evidence of transitional vortices in this region; streaks of high heating did not increase as sharply and did not widen near the back edge of the model (Figure 8.6a). The transition locations of the hot streaks between $\phi = 80^\circ$ and $110^\circ$ were most affected. In this region, transition moved upstream when the flow controller was activated. The corresponding steel nosetip case also showed signs of transition, but only between $\phi = 100^\circ$ and $160^\circ$ (Figure 8.6c).

For all three nosetips, steep increases in heating were observed for hot streaks between $\phi = 160^\circ$ and $\phi = 180^\circ$. The development of these vortices was negligibly affected by the plasma actuators. For an axisymmetric cone in hypersonic flow at angle of attack, multiple transition mechanisms exist and second mode, rather than crossflow, is expected to dominate the transition process near the leeward ray [55]. Therefore, the actuators weren’t expected to affect transition location in this region.

Data were also collected with the $m = 45$ nosetip at $6^\circ$ angle of attack at a
lower freestream unit Reynolds number, $9.5 \times 10^6$/m. Hot streaks were observed (Figure 8.7) and two transition locations were computed for the Macor nosetip runs. Transition was advanced for the plasma-on run, although this was within the uncertainty of the analysis.
8.1.3 $m=30$ Nosetip

Prior testing has found that the wavenumber of the largest amplitude hot streaks on a 7° half-angle cone at 6° angle of attack at $Re = 11.1 \times 10^6$ /m without actuation was between 30 and 60 per circumference. Therefore, nosetips with actuator spacings corresponding to these wavenumbers would be expected to accelerate transition. In these experiments, a nosetip featuring 30 actuators spaced evenly around the azimuth was tested in addition to the nosetip with 45 actuators. Figure 8.8 shows the surface heat flux observed at these conditions for the Macor $m=30$ and steel nosetips.
Transition locations are superimposed on the heat-flux contours. Between $\phi = 90^\circ$ and $140^\circ$, streaks for each run exhibited an increase in heat flux as they moved downstream, even for the steel nosetip (Fig. 8.8c) and the Macor nosetip without the flow controller activated (Fig. 8.8a). When the plasma was turned on for this configuration and test condition (Fig. 8.8b), a large increase in heat flux was observed for the hot streaks between $\phi = 90^\circ$ and $110^\circ$, and they began to spread laterally. The plasma-based flow controller was seen to advance transition compared to the plasma-off case, and transition was observed over a larger azimuthal range when the controller was activated.

The $m = 30$ nosetip was also tested at a lower freestream unit Reynolds number, $9.8 \times 10^6 /m$ (Figure 8.9). In general, heat-flux values were lower across the entire spatial range compared to the higher Reynolds number test case, as expected. Distinct hot streaks were observed for these runs, although for some azimuths, they were only faintly visible near the upstream portion of the images. Figure 8.9a shows the results with the $m = 30$ nosetip and no flow controller. Two hot streaks, between $\phi = 100^\circ$ and $130^\circ$, saw sharp increases in heating along their paths. These two streaks also appear to merge together as they progress downstream. This suggests that the streaks are transitional. Compared to the Macor nosetip data, the steel nosetip data (Figure 8.9c) did not feature hot streaks that appear transitional. Figure 8.9b shows the results with the $m = 30$ nosetip when the plasma actuators were turned on. The hot streaks that saw a steep increase in heat flux in Figure 8.9a do not appear to transition in this data. For the plasma-on test case, these two streaks are visible, but appear fully laminar, as evidenced by their lower heating rates. Thus, for these conditions, actuation at $m = 30$ delayed transition.

Interestingly, this configuration had a different effect depending on the freestream conditions. For $Re = 11.1 \times 10^6 /m$, the controller accelerated the onset of transition, while for $Re = 9.8 \times 10^6 /m$, the controller delayed transition. Ward saw a similar
Figure 8.8. Heat flux and transition fronts. $m = 30, \alpha = 6^\circ$, $Re = 11.1 \times 10^6 \text{ /m}$.

effect using passive roughness elements on a similar configuration in the BAM6QT [95]. A 50-dimpled roughness insert was used in his experiments to manipulate transition location of the crossflow vortices on a $7^\circ$ half-angle cone at $6^\circ$ angle of attack under quiet-flow conditions in the BAM6QT. Global heat-flux measurements were made using TSP. In Ward’s experiments, the 50-dimple passive roughness insert produced larger amplitude stationary waves compared to the smooth insert at $Re = 8.1 \times 10^6 \text{ /m}$. However, at a higher Reynolds number ($10.5 \times 10^6 \text{ /m}$), the 50-dimple insert had the opposite effect.
8.1.4 $m = 140$ Nosetip

A nosetip implementing the new flow-controller design methodology was implemented in these experiments, based on the computational wavenumber ratio (Fig. 7.9). Runs were conducted at similar freestream conditions as the $m = 68$, 45, and 30 nosetip tests. Figure 8.10 shows surface heat flux for three cases: the steel nosetip and the $m = 140$ Macor nosetip with plasma on and off. The hot streaks develop similarly for the plasma-off (Fig. 8.10a) and no-actuator (Fig. 8.10c) cases. Hot streaks that exhibit the most spreading and steep increases are observed between $\phi = 100^\circ$...
and 130°. Plasma actuation increased the amplitude of the hot streaks (Fig. 8.10b).

Figure 8.10. Heat flux and transition fronts. $m = 140$, $\alpha = 6^\circ$, $Re = 11.1 \times 10^6$ /m.

Figure 8.11 shows the same nosetips tested at a lower freestream unit Reynolds number, $9.8 \times 10^6$ /m. Transitional vortices were observed between $\phi = 100^\circ$ and $120^\circ$ for the runs with the Macor nosetip. The transition location was slightly affected by the plasma actuators.
8.1.5 $m = 210$ Nosetip

A nosetip with 210 actuators was predicted to delay crossflow-induced transition based on the new design methodology. By placing 210 actuators around the azimuth of the cone at an axial station near the neutral points of the stationary crossflow waves, 68 actuators would be in the azimuthal range where the vortices emanated from. Therefore, this actuator was predicted to force a wavenumber of 68 at the transition location. Thus, the actuator was designed to delay transition. Figure 8.12 shows surface heat flux contours for the $m = 210$ nosetip for plasma-off and -on test.
cases. Transitional vortices were observed for both test cases. The plasma-off test case featured several transitional vortices between 100° and 140°. These vortices resulted in a jagged transition front. When the plasma was turned on, more hot streaks were observed (see subsection 8.2), suggesting that the actuator was successful in forcing a higher wavenumber. Transition was delayed along vortices near 110°. The changes in heat flux and the spacing of the vortices are evidence that the controller influenced the behavior of the hot streaks. Data with the $m = 210$ nosetip were unavailable at a lower freestream unit Reynolds number due to an issue with the dielectric barrier of this actuator, which developed a series of cracks after multiple tunnel runs. This reduced the insulating properties of the dielectric, resulting in arcing between the exposed and buried electrodes at the tip.
8.1.6 Transition-Front Summary

In Figure 8.13, the transition fronts for the plasma-off, plasma-on, and steel nosetip test cases have been superimposed, to facilitate direct evaluation of the flow controllers. Figure 8.13a shows the plasma-off, plasma-on, and steel nosetip results of the $m = 30$ nosetip at a freestream unit Reynolds number of $11.1 \times 10^6$ /m. For these tests, many transitional vortices were observed. For the plasma-off run, a transition front was between $\phi = 90^\circ$ and $120^\circ$. The steel nosetip run resulted in a similar transition front. For the plasma-on run, a transition front was between $\phi = 80^\circ$ and $118^\circ$. 

Figure 8.12. Heat flux and transition fronts. $m = 210$, $\alpha = 6^\circ$, $Re = 10.5 \times 10^6$ /m.
140°. The plasma widened the transition front and also accelerated transition. Thus, the $m = 30$ nosetip had the opposite effect at two different stagnation conditions: at $Re = 9.8 \times 10^6 /m$ transition was delayed, while at $Re = 11.1 \times 10^6 /m$, transition was accelerated. This was an unexpected, yet promising result as it demonstrated the controllers can successfully accelerate and delay transition.

Figure 8.13b shows the transition fronts for the $m = 45$ nosetip at the same freestream unit Reynolds number. Many transitional vortices were observed for both the plasma-off and -on tests. Vortices between $\phi = 130^\circ$ and $150^\circ$ were unaffected by the plasma actuators. The largest effect on transition was between $\phi = 100^\circ$ and $130^\circ$, where it was advanced by $\approx 30$ mm.

Figure 8.13c shows data from the $m = 68$ nosetip and corresponding steel nosetip data for a nominally identical freestream unit Reynolds number. Several transitional hot streaks were observed for each run between $100^\circ$ and $150^\circ$. Comparing the transition locations between the plasma-off and steel nosetip cases, the results showed good agreement with each other; most of the transition locations for each case fell within the uncertainty of the analysis. When the plasma was turned on, the transition locations between $120^\circ$ and $130^\circ$ were most affected by the actuation. Transition was moved downstream from 335 mm to 370 mm. This translated to a 10% delay in the transition location. The controller appeared less effective at very low azimuths. Hot streaks at lower azimuths had neutral points well downstream of the axial location of the plasma actuators, hence a decrease in transition control authority.

Figure 8.13d shows results for the $m = 140$ nosetip at $Re = 11.1 \times 10^6 /m$. In this plot, transition fronts were identified between $\phi = 85^\circ$ and $\phi = 120^\circ$. The transition fronts are nearly identical, with a slight difference between $\phi = 90^\circ$ and $100^\circ$. Although the actuator did not significantly affect transition, it had a large effect on the heat flux due to the transitional vortices. With plasma actuation, the hot streaks near $\phi = 100^\circ$ at the back edge of the model exhibit higher heating rates.
and lateral spreading (Fig. 8.10b) than they do without actuation (Fig. 8.10a). So while transition location was only mildly affected, it appears the hot streaks break down much more quickly in the plasma-on case. More work is needed to explain and predict this phenomenon.

Figure 8.13e shows the results using the \( m = 210 \) nosetip with the corresponding steel nosetip data. Many transition locations were identified between 100° and 140°. The transition locations of hot streaks between 100° and 125° moved downstream when the plasma was turned on; a delay of 20 mm was observed at 125°. These results were more subtle compared to the \( m = 68 \) nosetip runs, but a delay in transition was still observed. Compared to the plasma-off case, the plasma-on case also resulted in a larger quantity of transitional hot streaks, suggesting that the wavenumber of the streaks was affected by the plasma actuators. Interestingly, the controller also had an improved effect at lower azimuths (≈ 80°) compared to the \( m = 68 \) nosetip design. This may be due to the axial position of the plasma actuators for each respective nosetip. The \( m = 68 \) nosetip featured actuators at \( x = 20 \) mm, well upstream of the neutral points of these instabilities. The \( m = 210 \) nosetip featured actuators at \( x = 55 \) mm, which was closer to the neutral points of these instabilities (see Figure 7.5). Moving the axial location of the actuators further downstream moves the dividing streamline to lower azimuths, hence an improvement in the actuator effectiveness at lower azimuths.

Figure 8.14 shows the transition-front results for each nosetip at a lower freestream unit Reynolds number. Again, data from the plasma-off, plasma-on, and steel nosetip runs are superimposed to evaluate the flow controllers’ effectiveness. Figure 8.14a shows the measured transition locations and corresponding uncertainty for the \( m = 30 \) nosetip case at \( Re = 9.5 \times 10^6 /m. \) For this case, some transitional vortices were observed, while the majority of vortices remained laminar. For the plasma-off test, three transitional vortices were observed. Two of these vortices were adjacent,
(a) $m = 30$, $Re = 11.1 \times 10^6$ /m.

(b) $m = 45$, $Re = 10.5 \times 10^6$ /m.

(c) $m = 68$, $Re = 10.5 \times 10^6$ /m.

(d) $m = 140$, $Re = 11.1 \times 10^6$ /m.

(e) $m = 210$, $Re = 10.5 \times 10^6$ /m.

Figure 8.13. Effect of plasma actuation on transition front, high $Re$. 
and a transition front was estimated between $\phi = 100^\circ$ and $120^\circ$. When the plasma was activated, two transitional vortices were identified. The vortex at $\phi = 140^\circ$ was slightly affected, although the effect was within the range of uncertainty. Transition between $\phi = 100^\circ$ and $120^\circ$ was delayed by approximately 25 mm, which exceeds the experimental uncertainty.

Figure 8.14b shows the plasma-off and -on transition locations for the $m = 45$ nosetip $Re = 9.5 \times 10^6 /m$. Two adjacent transitional vortices were observed for each run. The plasma slightly accelerated transition, but this was within the experimental uncertainty. For the $m = 45$ nosetip, tests at $Re = 9.5 \times 10^6 /m$ did not show transitional vortices for either the plasma-off or -on cases, although the actuators did affect the heating rates along a laminar vortex.

Figure 8.14c shows the results for the $m = 68$ nosetip at $Re = 9.5 \times 10^6 /m$. Two transition points were identified for each run, with most vortices remaining fully laminar. The hot streak near $\phi = 140^\circ$ saw a delay of approximately 15 mm. This demonstrated the controller still had an effect at delaying transition for this lower Reynolds number. Figure 8.14d shows the transition fronts for the $m = 140$ nosetip.

The transition fronts were similar for each test and the actuator had no influence on transition location. Data for the $m = 210$ nosetip were unavailable at this Reynolds number.

Figure 8.15 shows transition fronts for three nominally identical Macor nosetips at similar freestream conditions. Figure 8.15a shows results at $Re = 9.5$ and $9.8 \times 10^6 /m$. The transitional hot streaks were concentrated between $\phi = 90^\circ$ and $120^\circ$. Similar transition fronts were identified for each nosetip. Figure 8.15b plots transition fronts at higher $Re$. The transition fronts were spread over a larger azimuthal range compared to the lower $Re$ case. These results show that different nosetips influence the location of the hot streaks on the model. However, in comparing the transition fronts, good agreement was seen. Dinzl and Candler showed computationally that
Figure 8.14. Effect of plasma actuation on transition front, low $Re$.

The stationary crossflow instability is highly sensitive to roughness near the nose of the HIFiRE-5 model under quiet flow [26]. This experimental result provides additional evidence of that sensitivity. Although nominally identical nosetips are seeding stationary crossflow waves at slightly different azimuths, the axial position and shape of the transition front is not strongly affected.

Table 8.1 summarizes the transition-front results from the plasma-actuated nosetip test cases. The “natural wavenumber” is the wavenumber of the largest amplitude stationary crossflow waves as indicated by the wavelet analysis of surface heat flux. It was calculated for each individual nosetip (with plasma off), which is why the
observed wavenumber varies slightly even for consistent test conditions. The range stated is the range of wavenumbers detected from $\phi = 90$ to $160^\circ$ and $x = 300$ to $400$ mm. The $m = 30$ actuators were designed based on older, ambiguous surface heat-flux data; although intended to provide critical forcing, it was in actuality supercritical. Recall that critical forcing is expected to accelerate transition and subcritical is expected to delay it; the literature is unclear on the effect of supercritical forcing. The $m = 45$ and 68 actuators were designed assuming the critical wavenumber does not evolve along the length; the $m = 140$ and 210 actuators assume vortex spreading. The maximum change in transition location corresponds to the particular hot streak that exhibited the largest change between plasma-off and -on transition. It is reported as a percentage of the plasma-off vortex transition location.
TABLE 8.1

ACTUATOR EFFECTS ON TRANSITION

<table>
<thead>
<tr>
<th>Subfigure</th>
<th>$Re$ (× $10^6$ /m)</th>
<th>$\alpha$ (deg.)</th>
<th>Natural Wavenumber</th>
<th>Actuator Wavenumber</th>
<th>Expected Effect</th>
<th>Maximum % Change in Transition Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.14a</td>
<td>9.5</td>
<td>6</td>
<td>40–46</td>
<td>30</td>
<td>Uncertain</td>
<td>Delay 11%</td>
</tr>
<tr>
<td>8.13a</td>
<td>11.1</td>
<td>6</td>
<td>40–48</td>
<td>30</td>
<td>Uncertain</td>
<td>Accelerate 8%</td>
</tr>
<tr>
<td>8.14b</td>
<td>9.5</td>
<td>6</td>
<td>44–49</td>
<td>45</td>
<td>Accelerate</td>
<td>Accelerate 4%</td>
</tr>
<tr>
<td>8.13b</td>
<td>10.4</td>
<td>6</td>
<td>39–49</td>
<td>45</td>
<td>Accelerate</td>
<td>Accelerate 16%</td>
</tr>
<tr>
<td>8.14c</td>
<td>9.5</td>
<td>6</td>
<td>42–49</td>
<td>45</td>
<td>Delay</td>
<td>Delay 16%</td>
</tr>
<tr>
<td>8.13c</td>
<td>10.5</td>
<td>6</td>
<td>41–50</td>
<td>68</td>
<td>Delay</td>
<td>Delay 16%</td>
</tr>
<tr>
<td>8.14d</td>
<td>9.8</td>
<td>6</td>
<td>44–52</td>
<td>140</td>
<td>Accelerate</td>
<td>Negligible</td>
</tr>
<tr>
<td>8.13d</td>
<td>11.1</td>
<td>6</td>
<td>45–54</td>
<td>140</td>
<td>Accelerate</td>
<td>Negligible</td>
</tr>
<tr>
<td>8.13c</td>
<td>10.5</td>
<td>6</td>
<td>40–56</td>
<td>210</td>
<td>Delay</td>
<td>Delay 8%</td>
</tr>
</tbody>
</table>

8.2 Plasma Effect on Wavenumber

In addition to their effect on transition location, the effectiveness of the plasma actuators were assessed by their impact on the wavenumbers of the largest amplitude stationary crossflow vortices. Data from the steel nosetip runs provided a basic understanding of the naturally largest amplitude waves at 6° angle of attack (Figure 5.2). To quantify the impact of the actuators on wavenumbers of the hot streaks, runs with the plasma-actuated nosetips were analyzed with the wavelet analysis. The first goal of this analysis was to calculate the wavenumbers of the largest amplitude hot streaks on the plasma-actuated nosetips with the plasma turned off and compare these results with the steel nosetip data. It was important to do this comparison to ensure that the changes in wavenumber observed in the plasma-on runs were due to the plasma and not an undesired effect. For example, roughness associated with the height of the actuators or different surface roughness characteristics of the Macor vs.
steel nosetips could introduce undesirable disturbances into the boundary layer. At a freestream unit Reynolds number of $10.5 \times 10^6 / m$, wavenumbers between 40 and 50 were observed for the steel nosetip and for each of the Macor nosetips (Figs. 8.17a, 8.17c, and 8.17e). This consistency provided good evidence that the inactive plasma actuators were not affecting receptivity.

The next goal of the wavelet analysis was to study the effect of the plasma flow controllers on the wavenumbers of the largest amplitude hot streaks. Figures 8.17a and 8.17b show results from the $m = 45$ nosetip for plasma-off and -on runs. This actuator was designed to accelerate transition by exciting the same wavenumber at the naturally most-amplified frequency. The data show that between the plasma-off and -on runs, no significant change in the wavenumber was observed. Both runs showed wavenumbers concentrated near 45 and the corresponding transition fronts showed an advance in transition location with the plasma turned on. The plasma-on run saw a slight azimuthal shift in the locations of the largest amplitude wavenumbers. This could be influenced by the change in surface heat-flux between the plasma-off and -on runs. These combined results demonstrate the the $m = 45$ actuator introduced the proper forcing and had the desired effect on the hot streaks.

The anticipated effect of the $m = 30$ nosetip was unknown because it introduced a supercritical forcing, which is not well documented in the literature. The wavelet analysis revealed that this actuator design did effectively alter the largest amplitude wavenumber, from 45 with plasma off (Fig. 8.17c) to 30 with plasma on (Figure 8.17d). Furthermore, the amplitude of hot streaks with plasma on was increased compared to the plasma-off case.

The $m = 68$ nosetip was designed to delay transition with an actuator spacing 1.5 times the critical wavenumber of 45. This actuator did substantially affect the wavenumber distribution (Figure 8.17f). Not only was the amplitude of waves at $m = 68 \pm 10$ significantly increased, the $m = 45$ waves were substantially suppressed.
It seems likely that the transition delay created by this actuator (Fig. 8.13c) results from this redirection of energy to less amplified wavenumbers, as desired.

The $m = 140$ nosetip was designed to accelerate transition based on the new actuator design methodology — matching the wavenumber at the neutral point to account for the spreading of the vortices. The surface heat-flux data revealed this actuator affected the amplitude of the heating along certain hot streaks, but their transition locations remained largely unaffected. Figures 8.17g and 8.17h examine the effect of this nosetip on wavenumber. These data show that the effect on wavenumber was small. Since this actuator was designed to introduce critical forcing, it was promising that the wavenumber was similar between runs. The $m = 210$ nosetip was designed to delay transition based on this same methodology. Figures 8.17i and 8.17j show results for this nosetip. When the plasma was turned on, energy at subcritical modes was increased.

Figure 8.17 shows results from the same nosetips (with the exception of $m = 210$) at lower freestream unit Reynolds numbers. The disturbance amplitudes resolved in the wavelet analysis were lower for each configuration tested at these lower values of $Re$. This was an expected result, as most of the vortices exhibited lower, fully laminar heating rates. At these lower Reynolds number, there was again good evidence that the controllers affected the wavenumbers of the hot streaks. The $m = 45$ actuator did not affect the largest amplitude wavenumbers, the $m = 30$ actuator input lower wavenumbers near 30, and the $m = 68$ actuator input higher wavenumbers near 68. The $m = 140$ nosetip also had no substantial effect on wavenumber, as expected.
Figure 8.16. Effect of plasma actuation on wavenumber, $x = 340$ mm, High $Re$ (pages 129-130).
(a) $m = 45, V_a = 0$ kV, $Re = 10.5 \times 10^6$/m.

(b) $m = 45, V_a = 4$ kV, $Re = 10.5 \times 10^6$/m.

(c) $m = 30, V_a = 0$ kV, $Re = 11.1 \times 10^6$/m.

(d) $m = 30, V_a = 4$ kV, $Re = 11.1 \times 10^6$/m.

(e) $m = 68, V_a = 0$ kV, $Re = 10.5 \times 10^6$/m.

(f) $m = 68, V_a = 4$ kV, $Re = 10.5 \times 10^6$/m.
(g) $m = 140, V_a = 0 \text{ kV, } Re = 11.1 \times 10^6 / m.$

(h) $m = 140, V_a = 4 \text{ kV, } Re = 11.1 \times 10^6 / m.$

(i) $m = 210, V_a = 0 \text{ kV, } Re = 10.5 \times 10^6 / m.$

(j) $m = 210, V_a = 4 \text{ kV, } Re = 10.5 \times 10^6 / m.$
Figure 8.17. Effect of plasma actuation on wavenumber, $x = 340\, \text{mm}$, Low $Re$ (pages 132-133).
(a) $m = 45, V_a = 0 \text{kV}$, 
$Re = 9.5 \times 10^6 \text{/m}$.

(b) $m = 45, V_a = 4 \text{kV}$, 
$Re = 9.5 \times 10^6 \text{/m}$.

(c) $m = 30, V_a = 0 \text{kV}$, 
$Re = 9.5 \times 10^6 \text{/m}$.

(d) $m = 30, V_a = 4 \text{kV}$, 
$Re = 9.5 \times 10^6 \text{/m}$.

(e) $m = 68, V_a = 0 \text{kV}$, 
$Re = 9.5 \times 10^6 \text{/m}$.

(f) $m = 68, V_a = 4 \text{kV}$, 
$Re = 9.5 \times 10^6 \text{/m}$.
(g) $m = 140$, $V_a = 0$ kV, $Re = 9.8 \times 10^6$ /m.

(h) $m = 140$, $V_a = 4$ kV, $Re = 9.8 \times 10^6$ /m.
CHAPTER 9

CONCLUSIONS

9.1 Summary

A 7° half-angle cone model with interchangeable nosetips was designed, fabricated, and tested in the BAM6QT to investigate crossflow-induced transition behavior. Tests with a steel nosetip were conducted under quiet-flow conditions for varying stagnation conditions and angles of attack to inform the design of a plasma-based flow control system and stability analysis. Data were collected using IR thermography to calculate surface heating, while Kulite pressure transducers were used to further characterize the boundary layer. A wavelet analysis was used to compute the local largest amplitude wavenumber of the stationary waves, which manifested themselves as hot streaks in the surface heat flux.

The wavelet analysis provided a quantitative description of the spatial heat flux distribution, presented as wavenumber. This analysis is an improvement to the state of the art, which formerly employed Fourier analysis, a technique offering a more limited description of the spatial variation of the heating. One application of the wavelet analysis was to inform a stability analysis. N-factors were integrated along group velocity pathlines with the local wavenumber obtained three different ways. The traditional method, using the most unstable wavenumbers in the absence of experimental results, led to integrating amplification rates for wavenumbers that were not observed experimentally and relatively large N-factors. Global measurement of the wavenumber distribution is the most challenging because of the experimental and data reduction requirements. A hybrid method for computing crossflow N-factors is
recommended as the best path forward: compute the non-dimensional wavenumber ratios from group velocity pathlines and then dimensionalize them with experimental data. This would achieve very good agreement in N-factor with the experimentally more demanding method, but could better leverage results from literature, obtained with less comprehensive instrumentation, or for slightly different flow conditions.

Measurements of crossflow wavenumbers were also needed to design the nosetips with plasma actuators. Five Macor nosetip designs were fabricated and tested. Three designs were in the first generation of nosetips, all based on the assumption that the disturbance wavenumber input near the neutral point should equal the largest amplitude waves observed near the end of the model. These three designs had supercritical \((m = 30)\), critical \((m = 45)\), and subcritical \((m = 68)\) actuators. The second-generation nosetips accounted for the evolving spatial distribution of crossflow vortices — 140 evenly spaced actuators near the neutral point would seed the \(m = 45\) vortices observed downstream. A complementary subcritical actuator had a wavenumber of 210.

Tests were conducted at two freestream unit Reynolds numbers in the BAM6QT at Purdue University. Results at the higher \(Re \approx 10.5 \times 10^6 /m\) indicated that the \(m = 45\) actuation accelerated transition and the \(m = 68\) nosetip delayed transition. When the plasma was activated, transition was accelerated by up to 16% and delayed by up to 11% relative to the plasma-off transition locations. The results for the \(m = 45\) and 68 actuators are highly encouraging: the actuators proved to have the same effect in hypersonic quiet flow that they have in supersonic quiet flow and hypersonic noisy flow. Results for the lower \(Re \approx 9.5 \times 10^6 /m\) showed less clear results, with many of the vortices remaining fully laminar. This made a change in transition location difficult to quantify. Still, the controllers had the desired effect on individual hot streaks.

Wavenumber alteration proved to be a more sensitive indication of actuator in-
fluence than their impact on transition location because it permitted evaluation of actuators at Reynolds numbers too low for transition onset. Forcing at the critical wavenumber did not change the wavenumber (as expected). The \( m = 30 \) and 68 actuators forced the largest amplitude disturbances to have those wavenumbers; the \( m = 68 \) forcing delayed transition, presumably due to the diversion of energy to less unstable modes. There is evidence both for and against the effectiveness of actuators designed to account for vortex evolution. The \( m = 210 \) nosetip forced wavenumbers of about 68 near the transition location. Moreover, transition was delayed. On the other hand, the \( m = 140 \) second-generation nosetip did not affect transition, and it did not excite the \( m = 45 \) mode.

Experiments were also conducted to investigate the effect of tunnel noise on crossflow-induced boundary-layer transition. Tunnel noise had a large effect on transition — \( Re_x \) at transition was about 1.75 times larger under quiet flow than noisy at azimuths where crossflow was the dominant transition mechanism. Additionally, the hot streak behavior was altered between noisy and quiet runs. Quiet runs resulted in clear, distinct vortices with transition occurring along each individual hot streak. In contrast, the noisy runs resulted in large, indistinct regions of heating. The vortices were not as clear or distinct in the noisy IR data. The transition process for noisy runs was also characterized by an increase in heat flux along large spatial regions rather than along an individual hot streak, resulting in smoother transition fronts.

Kulite pressure transducers were used to corroborate the boundary-layer state as indicated by infrared thermography. Stages of the boundary layer from fully laminar to fully turbulent were observed. Increasing instability amplitudes corresponded with strengthening laminar hot streaks; increasing broadband amplitude corresponded with transition as indicated by surface heating. A peak in the spectra between 20 and 40 kHz was associated with the travelling crossflow modes. These results showed good agreement with predictions from simulations and measurements from other
experiments on conical geometries.

Overall, the results demonstrated that plasma-actuated flow control effective at lower Mach numbers or higher noise levels is also effective in Mach-6 quiet flow. En route, other meaningful results were obtained. Global heat-flux measurement with infrared thermography was developed. These high spatial resolution, highly sensitive data sets were well suited to wavelet analysis. This work represents a novel application for that tool, which could be applied to other flows featuring complex heat-flux distributions.

9.2 Future Work

The progress made during this project provides a starting point for several recommendations of future work.

9.2.1 Plasma Flow Controllers

The actuators used in this work were designed based on LST and wavelet analysis results at one set of conditions. The wavenumber of the naturally most-amplified stationary crossflow waves, which informs the spacing of the critical and subcritical actuators, will vary at different test conditions (i.e. different Reynolds numbers and angles of attack). Future flow controllers could be designed that are tailored to each experimental condition.

Another recommendation is to design, fabricate, and test actuators with more complex geometries (e.g., not at a fixed axial station). The stability analysis results indicated that the neutral points of the instabilities were not at a fixed axial station (Fig. 7.5). The ideal actuator design would place the comb pattern at the specific neutral point of each vortex path in a swept pattern. The design of the nosetips used in this work made it difficult to fabricate actuators in this manner because the dielectric thickness varied with axial station. This meant the plasma would behave
differently at different axial positions. A new design could be pursued to achieve this swept design. This would greatly reduce the azimuthal extent of the dividing streamline, below which the actuators are ineffective in controlling transition location.

Plasma generation was confirmed during wind-tunnel runs by measuring current spikes on an oscilloscope from the AC power supply. Future tests should seek to visually observe the operation of the plasma actuators during a tunnel run. These observations will confirm that plasma is generated uniformly about the azimuth of the nosetip. Such observations could also be used to examine the behavior of the plasma flow controllers in the testing environment.

9.2.2 AFOSR/Notre Dame Large Mach-6 Quiet Tunnel Tests

A future goal of the plasma flow-control experiments is to conduct tests in the AFOSR/Notre Dame Large Mach-6 Quiet Tunnel (ANDLM6QT). The tunnel is operational and is currently undergoing preliminary tests. As of October 2019, a surrogate nozzle fabricated out of aluminum is installed. This nozzle is polished, but not to the same extent as the final stainless steel nozzle. The facility is expected to deliver quiet-flow conditions up to some stagnation pressure with the surrogate nozzle, although the exact performance is still being characterized. The facility will likely run with conventional noise at the stagnation pressures required to resolve hot streaks on the model. The first goal of these tests should be to conduct runs with the original test model with both the steel and Macor nosetips, similar to what was done in the BAM6QT. The goal of these tests will be to further validate the effectiveness of the plasma flow controllers and measurement techniques and to ensure the results are facility independent.

Future experiments should also be conducted with the model extension, currently under fabrication, to take full advantage of the large open-jet test section that the ANDLM6QT provides. Natural transition in the flight environment occurs at $Re_x$. 
= 20 – 40 \times 10^6 \text{[37]. To achieve Reynolds numbers of this order, the freestream unit Reynolds number could be increased, or alternatively larger models could be tested. The ANDLM6QT features a quiet-flow test core of 3.6 m in length \text{[37]. The plasma flow controllers should be tested on the larger model, which will be 1.2 m in total length, translating to a maximum } Re_x \text{ value of } 13 \times 10^6. \text{ With conventional noise, these results will compare with those obtained at Mach 7 in H2K as part of the HIFiRE-1 post-flight research effort \text{[40, 42, 97]. Under quiet flow at low unit Reynolds number, the length Reynolds number will match the BAM6QT experiments, allowing investigation of scale effects. Low unit Reynolds number was collected with the steel nosetip in the BAM6QT, which could offer a point of comparison. Under quiet flow at higher unit Reynolds number, the conditions will match the Reynolds number and noise level of the HIFiRE-1 flight test \text{[87]. In these tests, the stationary waves would be expected to fully break down to turbulence, a behavior that could not be observed in the BAM6QT or with the smaller configuration of the model.}

As shown in Table \text{8.1} the actuators tested \textit{usually} had the anticipated effect, but not always. Discovering the root of the discrepancies in the ANDM6QT may lead to further insight into crossflow-induced transition. In particular, the question of which is the better strategy for inputting control — matching wavenumber or accounting for vortex spreading — has not been resolved conclusively.
APPENDIX A
MEAN FLOW COMPUTATIONAL METHODS AND RESULTS

Dr. Matthew Tufts, AFRL/RQHF, provided the mean flow computational methods and results presented in this section. Computations were conducted to predict the heating rates for fully laminar and fully turbulent boundary-layer state at the test conditions. These results gave confidence in the experimental data-acquisition techniques. The Overflow2.2n flow solver, using 3rd order accurate HLLE++ fluxes, was used to solve unperturbed, steady Navier-Stokes solutions. For both the laminar heat-flux calculations and mean-flow solutions to be used in later stability calculations, no turbulence models were needed. For the turbulent heat-flux calculations, the Spallart-Allmaras model (SA-noft2) with default coefficients was used over the entire geometry. For all cases, the flow was solved as a steady-state flow. Boundary conditions for the model surfaces were modeled as no-slip, isothermal, uniform-temperature walls. Flow conditions were selected to model conditions as run in the BAM6QT. These conditions are summarized in table A.1, where $Re$ is the freestream unit Reynolds number, $L$ is the model length, $\rho_\infty$, $p_\infty$, $T_\infty$, and $M$ are the freestream density, pressure, temperature, and Mach number, $T_w$ is the wall temperature, and $T_0$ is the stagnation temperature. In this work, the freestream unit Reynolds number is defined as $\frac{Re_x}{x}$, where $Re_x$ is the length Reynolds number defined as $\frac{u_\infty x}{\nu_\infty}$. Here, $u_\infty$ and $\nu_\infty$ are the freestream velocity and kinematic viscosity, respectively and $x$ is the distance from the nosetip.

Structured, hexahedral, multi-block overset grids were created using an in-house Fortran program. Connectivity and grid metric data were computed using Pegasus.
TABLE A.1

NOMINAL WIND-TUNNEL FLOW PARAMETERS.

<table>
<thead>
<tr>
<th></th>
<th>Re (1/m)</th>
<th>α</th>
<th>ρ∞ (kg/m³)</th>
<th>p∞ (Pa)</th>
<th>T∞ (K)</th>
<th>M</th>
<th>Tw (K)</th>
<th>Tw/T₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>10.50 × 10⁶</td>
<td>6°</td>
<td>0.0412</td>
<td>624.5</td>
<td>52.80</td>
<td>6.0</td>
<td>304</td>
<td>0.70</td>
</tr>
<tr>
<td>Case 2</td>
<td>8.52 × 10⁶</td>
<td>4°</td>
<td>0.0334</td>
<td>506.69</td>
<td>52.80</td>
<td>6.0</td>
<td>304</td>
<td>0.70</td>
</tr>
<tr>
<td>Case 3</td>
<td>10.50 × 10⁶</td>
<td>4°</td>
<td>0.0412</td>
<td>624.5</td>
<td>52.80</td>
<td>6.0</td>
<td>304</td>
<td>0.70</td>
</tr>
</tbody>
</table>

5.2. The computational model’s surface was completely smooth (no modeled surface roughness) with a spherical tip. The nose was represented by projecting a square mesh onto the sphere and subsequently smoothing the projected shape to blend smoothly with a conical grid placed on the frustum.

The computational domain covers only a half-cone (180° azimuthal extent) to reduce computational expense, while allowing for the addition of angle of attack. From the surface, the domain extends to capture the full shock. Additional overset grids were added to the system to better refine the shock after an initial solution was calculated. For both computational and experimental data the x-axis (axial distance) is defined as positive along the streamwise direction. The y-axis is defined such that the windward ray is negative y, and the z-axis is the cross product. Azimuthal angle is defined as arctan(z, y) + 180° such that the windward ray is located at an azimuthal angle of 0°.

Grid resolution details are summarized in table A.2. In table A.2 i represents axial (±x), j represents azimuthal points, while k represents wall-normal. The surface grid topology at the overlapped nose can be seen in Figure A.1. Wall-normal spacing in terms of y⁺ was a maximum of 0.8 anywhere on the surface of the cone for the worst case (6° angle of attack, turbulent.)
Figure A.1. Surface grid topology at nose.

TABLE A.2

GRID RESOLUTION DETAILS (ONE-HALF CONE)

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$i$</th>
<th>$j$</th>
<th>$k$</th>
<th>Total Pts.</th>
<th>Approx. Pts. in B.L.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nose Grid</td>
<td>122</td>
<td>61</td>
<td>301</td>
<td>2240042</td>
<td>50</td>
</tr>
<tr>
<td>Conical Grid</td>
<td>1541</td>
<td>122</td>
<td>301</td>
<td>56588602</td>
<td>75</td>
</tr>
<tr>
<td>Shock Capture 1</td>
<td>82</td>
<td>41</td>
<td>71</td>
<td>238702</td>
<td>N/A</td>
</tr>
<tr>
<td>Shock Capture 2</td>
<td>587</td>
<td>164</td>
<td>71</td>
<td>6835028</td>
<td>N/A</td>
</tr>
<tr>
<td>Shock Capture 3</td>
<td>220</td>
<td>291</td>
<td>71</td>
<td>4545420</td>
<td>N/A</td>
</tr>
<tr>
<td>Shock Capture 4</td>
<td>412</td>
<td>545</td>
<td>71</td>
<td>15942340</td>
<td>N/A</td>
</tr>
<tr>
<td>Shock Capture 5</td>
<td>693</td>
<td>1053</td>
<td>71</td>
<td>51810759</td>
<td>N/A</td>
</tr>
</tbody>
</table>
Figure A.2. Non-dimensional density contours.

An overview of the computational domain and contours of density for Case 1 can be seen in Figure A.2. The lines present on the symmetry plane show the location of the additional shock-capture grids. One can see the difference in shock angle from windward to leeward that causes the circumferential pressure gradient. Note also the “mushroom” flow feature and associated boundary layer thickening that begins at 160° of azimuth near the leeward ray. The influence of this flow feature is apparent in many later results.

Figure A.3 plots contour lines of heat flux at 6° angle of attack and nominal wind-tunnel conditions calculated with the fine and coarse grids. The largest changes are near the leeward ray, far downstream. The grids used for calculations in the current study are judged to be sufficient based on convergence of the surface heat transfer. Global heat-flux maps were constructed from the mean-flow computational results. Figure A.4 shows the computational laminar and turbulent surface heating rates for
Case 1 (recall table A.1). These conditions are nominally identical to the baseline experimental conditions. As will be done when presenting experimental data, the colorbar scale for this figure has been chosen to more easily see variation in heat-flux data over the primary region of interest (where crossflow is the dominant instability mechanism). The boundary layer grows downstream, and the circumferential pressure gradient causes the boundary layer to grow towards the leeward ray. For these reasons, heat flux decreased axially downstream and azimuthally towards the leeward ray. The boundary layer separates along a line near the leeward ray ($\phi \approx 165^\circ$) \[3\]. This behavior results in heating rates being lowest near the leeward ray for both the laminar and turbulent cases. The turbulent heating rates are up to five times greater than the laminar heating rates. The mean-flow computations do not convey the development of the stationary crossflow vortices because they begin at a very low initial amplitude in the model. The mean-flow computations also assume that there is no surface roughness and that the freestream is quiescent. The simulations are run non-time accurate, therefore there is no instability growth in the model.
Figure A.4. Computational heat flux. Case 1 flow conditions.
APPENDIX B

LST/LPSE GOVERNING EQUATIONS

B.1 Linear Stability Theory

Dr. Matthew Tufts, AFRL/RQHF, provided the following mathematical discussion that was used in the stability analysis. To solve for the stability characteristics, we define a solution vector \( \mathbf{Q} \) containing the primitive variables.

\[
\mathbf{Q} = [u, v, w, \rho, t]^T
\]

(B.1)

One may then write the full nonlinear Navier-Stokes equations as an operator \( \mathcal{N} \) which gives the general form of the equations.

\[
\frac{\partial \mathbf{Q}}{\partial t} = \mathcal{N} \mathbf{Q}
\]

(B.2)

By writing the equations in this form, we can perform derivations without the loss of generality. If we then decompose the solution vector \( \mathbf{Q} \) into a non-time dependent base state \( \overline{\mathbf{Q}} \) and a time-varying perturbation \( \mathbf{q}' \) assumed to be of small magnitude \( O(\varepsilon) \) such that:

\[
\mathbf{Q} = \overline{\mathbf{Q}} + \mathbf{q}'
\]

(B.3)

and separate the linear \( O(\varepsilon) \) terms from the nonlinear \( O(\varepsilon^2) \) terms, we can define
the Linear Navier Stokes Equations (LNSE). These can be written as an initial value problem in terms of operators.

\[ B \frac{\partial q'}{\partial t} = A q' + O(\varepsilon^2) \]  

(B.4)

Depending on the desired application, these equations may also be written separating out spatial derivatives in a nonlinear form.

\[ B_1 \frac{\partial q'}{\partial x} + B_2 \frac{\partial^2 q'}{\partial x^2} = A q' + O(\varepsilon^2) \]  

(B.5)

Starting from equation (B.4), we can define our assumptions about \( q \) and the form of \( q' \). For LST, we define \( q = \bar{q}(y) \), meaning the base flow solution varies only in the wall-normal direction, and is homogeneous in the other two directions. This lends itself to the form of disturbances given by the assumption of normal modes. The disturbances should be discretized in the same dimensions as the underlying basic state, and represented in Fourier space in the homogeneous directions as well as in time. For example, using the form for one inhomogenous direction results in the set of equations typically called the Linear Stability Theory equations:

\[ q' = \tilde{q}(y)e^{i(\alpha x + \beta z - \omega t)} + \text{c.c.} \]  

(B.6)

In general, \( \alpha, \beta, \) and \( \omega \) are all complex and c.c. is the complex conjugate of the first term on the right hand side [50]. Using this mode is equivalent to a Fourier transform in time and a Laplace transform in space. Substitution of this ansatz into equation (B.4) (along with removing the base state solution, which is identically zero,
and discarding the $O(\varepsilon^2)$ or higher terms) allows for the creation of a generalized, linear, temporal eigenvalue problem of the form:

$$(\omega B - A)\hat{q} = 0$$ \hfill (B.7)$$

In this formulation, $\alpha$ and $\beta$ are taken to be real parameters, and $\omega$ is complex and part of the solution. To derive the LST equations, an assumption was made that one or more directions were spatially discretized, and the remaining dimensions were locally homogeneous. However, given the derivation of the LNSE separation of scales, it is possible to consider a “slowly varying” direction such that the spatial gradients are not $O(1)$ but rather $O(\varepsilon)$. This slowly varying direction is typically taken to be the streamwise direction. However, the spatial problem is of more interest than the temporal problem. Examining the LNSE operator as written in equation B.5 reveals that the equations are nonlinear in terms of spatial derivatives, resulting in a quadratic eigenvalue problem of the form:

$$(\alpha B_1 + \alpha^2 B_2 - A)\hat{q} = 0$$ \hfill (B.8)$$

This is then converted to a linear eigenvalue problem using the companion matrix method, wherein $\hat{q}$ is redefined as $\hat{q}^+$ containing the first of the quadratic terms. For example, if $\hat{q} = [u, v, w, p]^T$, then $\hat{q}^+ = [u, v, w, p, \alpha u, \alpha v, \alpha w, \alpha p]^T$. Doing so results in an eigenvalue problem of the form:

$$(\alpha B - A)\hat{q}^+ = 0$$ \hfill (B.9)$$
In this formulation, \( \omega \) and \( \beta \) are taken to be real parameters, and \( \alpha \) is complex and part of the solution. Equation \( \text{[B.9]} \) is the form solved in LASTRAC for the current work’s LST results, where the operators \( A \) and \( B \) are given by the compressible form of the Navier-Stokes equations. For further details on LASTRAC’s specific implementation see \([12]\) and \([13]\).

B.2 Linear Parabolized Stability Equations

To derive the Parabolized Stability Equations, one replaces the normal mode ansatz using constant wavenumber parameters (e.g., Equation \([\text{B.6}]\)) with an ansatz that contains a phase function that will allow for both “slow” changes in the basic state and “slow” changes in the shape of the disturbance in one or more directions. In this formulation, both the wave function \( \alpha(x) \) and the shape function \( \hat{q} \) are considered to vary with the \( x \) direction. These variations are defined as “slow” with \( x \), such that their derivatives are \( O(\varepsilon) \). This distinction is denoted by replacing \( x \) with the dummy variable \( \psi \).

\[
q' = \hat{q}(x, y) e^{i \int_{\psi}^{x} \alpha(\psi) d\psi} e^{i(\beta z - \omega t)} + \text{c.c.} \tag{B.10}
\]

After substituting in (B.10), the equations given by (B.4) then take on a form given by:

\[
L\hat{q} + M \frac{\partial \hat{q}}{\partial x} = F \tag{B.11}
\]

Because the term \( \frac{\partial \hat{q}}{\partial x} \) is now non-zero, this form is no longer separable into an eigenvalue problem (even in the linear form where \( F = 0 \)). In practice, this is solved as an initial value problem using the spatial direction as a pseudo-time step. The
initial condition is typically an unstable eigenmode derived from LST. Further detail on the implementation in LASTRAC is found in [12] and [13].

B.3 Solving for Group Velocity

In general, the group velocity direction is the propagation direction of a wave packet. Mathematically this direction is found by finding the dispersion relation between individual wave components, found by solving for the real part of the quantity:

\[
\frac{d\omega}{d\beta} \frac{d\omega}{d\alpha} = \frac{d\alpha}{d\beta}
\]  

(B.12)

In this case, \( \omega \) represents the temporal wavenumber, while the spatial wavenumber is given by \( \alpha \) and \( \beta \) in the streamwise and spanwise directions, respectively. Within the framework of LASTRAC, this is found by first allowing \( \beta \) to be real (consistent with the assumptions given by LST). Then complex \( \alpha \) is solved for by finding the solution with \( \frac{d\alpha}{d\beta} = 0 \). The corresponding group velocity direction is then given by \( \frac{d\alpha_R}{d\beta} \). Further detail is found in [13]. The LASTRAC3d study output the group velocity pathline of the stationary waves with an optimal spatial growth rate. For stationary crossflow waves, this is the direction such that the crossflow velocity profile has its inflection point at zero velocity.
APPENDIX C

MATLAB CODES

C.1 Image Processing

% Master program for pixel inverse perspective calculation
% Applied to Harrison's cone data, but intended to be generally
% applicable

% Author: Michael Thompson, March 2017
% Advisors: Carson Running, Prof. Juliano

%clear
close all
clc
tic

% general data
% actual coordinates of registration points relative to tip of cone
% done on theoretical cone at edges
\texttt{reg\_pt1 = [98 50];}\% change to 50 for post process
\texttt{reg\_pt2 = [486 62];}\%
\texttt{top\_left = [105 180]; \%[row, col], pixel coordinates}
\texttt{top\_right = [160 566]; \%pixels on top/bottom edges}
\texttt{bot\_left = [477 164];}
\texttt{bot\_right = [426 545];}
\texttt{i\_crop = [1 640];}
\texttt{k\_crop = [30 510];}

\texttt{angle = 77; \%deg, angle between cone and camera axis}
\texttt{cam\_dist = 100; \%mm, distance from cone to camera}

\texttt{images\_folder = '/Users/harrisyates/Desktop/March2019/Run36}
\texttt{ '}; \%name of folder which has the images
\texttt{filename\_fixed = 'Run36_0600.asc'; \% name of fixed frame}
\texttt{picture (reference picture)}
\texttt{data\_file\_name = 'Run31.mat'; \%name of matlab workspace to}
\texttt{save results of program}

%%% load data
\texttt{filename\_fixed = fullfile(images\_folder, filename\_fixed);}

\texttt{load\_fixed\_set\_regpts_har;}

%%% create and map cone
\texttt{create\_cone\_harrison;
pers_trans_initial_har;

invpers_ref_gen_pts_har;

post_process;

analyze wind on frames

data_file_name = fullfile(images_folder, data_file_name);

filename_moving = fullfile(images_folder, '*.asc');

% Uncomment next line when ready to run entire set
% loop_moving_frames; %read asc images, loops through to
% register im_data
%

make plots

make_plots;
%
% toc

load_fixed_set_regpts_har.m
% load fixed reference picture, rotate only if necessary, and
% set registration
% point locations for pixel-mm conversion
% Author: Michael Thompson, March 2017
% Advisors: Carson Running, Prof. Juliano

alldata = importdata(filename_fixed, '\t');
data = alldata.data;

fixed = qdotdata(:, :, 963); % data; % 788 corresponds to 2 sec, 963 to 2.5 sec.
fixed_orig = fixed;

%%%% set registration mark positions
% now done in master program

rotation_angle = 0;

%%%% rotate data (comment out section if not necessary)
% pick two points that should be vertical

rotation_angle = (asind((top_left(1)-top_right(1))/(top_left(2)-top_right(2)))-asind((bot_left(1)-bot_right(1))/(bot_right(2)-bot_left(2))))/2;
% asind((reg_pt2(1)-reg_pt1(1))/(reg_pt2(2)-reg_pt1(2)));

data_rot = imrotate(fixed, rotation_angle, 'nearest', 'crop')
; % rotate image
data_rot(data_rot==0) = NaN;
mat_rot = [\text{cosd}(\text{rotation\_angle}), \text{sind}(\text{rotation\_angle})]
\hspace{1cm} -\text{sind}(\text{rotation\_angle}), \text{cosd}(\text{rotation\_angle})];

\text{reg\_pt1} = \text{round}\left( (\text{reg\_pt1} - \text{size}\left(\text{fixed}\right)/2) \ast \text{mat\_rot} + \text{size}\left(\text{fixed}\right)/2 \right);
\text{reg\_pt2} = \text{round}\left( (\text{reg\_pt2} - \text{size}\left(\text{fixed}\right)/2) \ast \text{mat\_rot} + \text{size}\left(\text{fixed}\right)/2 \right);
\%
\text{reg\_pt3} = \text{round}\left( (\text{reg\_pt3} - \text{size}\left(\text{fixed}\right)/2) \ast \text{mat\_rot} + \text{size}\left(\text{fixed}\right)/2 \right);
\%
\text{reg\_pt4} = \text{round}\left( (\text{reg\_pt4} - \text{size}\left(\text{fixed}\right)/2) \ast \text{mat\_rot} + \text{size}\left(\text{fixed}\right)/2 \right);

\text{fixed} = \text{data\_rot};
\text{on\_shift} = \text{fixed};

%%% create\_cone\_harrison.m

%%% %Author: Michael Thompson, March 2017
%%% %Advisors: Carson Running, Prof. Juliano

%%% Harrison create theoretical cone

\text{l\_cone} = 160; \text{\text{\textit{\textmu}m}}
\text{l\_tot} = \text{l\_cone};
\text{r\_max} = 49.23; \text{\text{\textmu}m}
cam_x = -cam_dist*cosd(angle);
cam_y = cam_dist*sind(-angle);

%resolution settings
res_phi = 400; %number of points on each concentric circle
res_x = 500; %fineness on x-coordinate on sphere part

%define x-vector
x_vect = linspace(0, l_tot, res_x);

%find radius(x)
radius = zeros(1, length(x_vect));

for i = 1:length(radius)
    radius(i) = r_max-(l_tot-x_vect(i))*tand(7);
end

%make r and phi meshgrid
[r, phi] = meshgrid(radius, linspace(0, 2*pi, res_phi));

%make the X meshgrid
x = repmat(x_vect, length(r(:, 1)), 1);

%transform from polar to cartesian
y = r.*cos(phi);
z = r.*sin(phi);
% create and plot cone surface
figure('visible', 'off')
cone = surf(x, y, z, 'linestyle', 'none');

%% pers_trans_initial_har.m

% crop and do 2–d projection of cone

% Author: Michael Thompson, March 2017
% Advisors: Carson Running, Prof. Juliano

% camera position relative to cone tip
cam_x = -cam_dist*cosd(angle);
cam_y = cam_dist*sind(angle);

% create normal vectors
disp('creating vectors/dot products')
normals = cone.VertexNormals;

[m, n] = size(x);
dot_products = zeros(size(x));
view_ang = zeros(size(x));
vis_count = 1;
% create dot products
for i = 1:n
for k = 1:m
    nom = [normals(k, i, 1) normals(k, i, 2) normals(k, i, 3)];
    cam_vect = [x(k, i)-cam_x y(k,i)-cam_y z(k,i)];
    dot_products(k, i) = dot(nom, cam_vect);
    view_ang(k, i) = acosd(dot_products(k, i)/norm(
        cam_vect));
end
end

% find positive dot products
[vis_m, vis_n] = find(dot_products<0);
ind = sub2ind(size(x), vis_m, vis_n);

x_crop = x; x_crop(ind) = NaN;
y_crop = y; y_crop(ind) = NaN;
z_crop = z; z_crop(ind) = NaN;

% create projection matrix
disp('creating initial 2d transform')

% for i = 1:n
%    for k = 1:m
%        th_p = 270-angle; % atan2((cam_y-y_crop(k,i)),(x_crop(k,i)-cam_x))*180/pi;
%        phi_p = 0;%atand(z_crop(k,i)/(cam_y-y_crop(k,i)));
%        %
% mat = viewmtx(th_p, phi_p);
%
% pts = [x_crop(k,i), y_crop(k,i), z_crop(k,i), 1]';
%
% pt2 = mat * pts;
%
% x_2d(k,i) = pt2(1);
% y_2d(k,i) = pt2(2);
% r3_2d(k,i) = pt2(3);
% r4_2d(k,i) = pt2(4);
% end
% end

nom = max(max(x_crop));

mat = viewmtx(270-angle, 0);

pts = [x_crop(:), y_crop(:), z_crop(:), ones(length(x_crop(:)),1)]';

pt2 = mat * pts;

x_2d = pt2(1,:)';
y_2d = pt2(2,:)';
r3_2d = pt2(3,:)';
r4_2d = pt2(4,:)';

isValid = ~isnan(x_2d);
r3_interp = scatteredInterpolant(x_2d(isvalid), y_2d(isvalid), r3_2d(isvalid));
r4_interp = scatteredInterpolant(x_2d(isvalid), y_2d(isvalid), r4_2d(isvalid));

r3_interp.ExtrapolationMethod = 'none';
r4_interp.ExtrapolationMethod = 'none';

% top back corner of cone
[y_max_temp, I_1] = max(y_2d);
[y_max, I_2] = max(y_max_temp);

x_top = x_2d(I_1(I_2), I_2);
y_top = y_max;

% bottom back corner of cone
[y_min_temp, I_1] = min(y_2d);
[y_min, I_2] = min(y_min_temp);

x_bot = x_2d(I_1(I_2), I_2);
y_bot = y_min;

% back of cone
x_back_max = min(x_2d);
%% figure
%% plot(x_2d, y_2d)
%% axis equal

%% figure
%% surf(x_crop, y_crop, z_crop, 'linestyle', 'none')
%% axis equal

%% invpers_ref_gen_pts_har.m
%% Match picture data to 3-D coordinates using the cone interpolant and
%% perspective transformation matrix

%% Author: Michael Thompson, March 2017
%% Advisors: Carson Running, Prof. Juliano

%% start mapping process
[m, n] = size(fixed);

%% reference and registration point distances

pix2pts_y = abs(norm([y_top-y_bot] [x_top-x_bot])/norm([reg_pt1(1)-reg_pt2(1) reg_pt1(2)-reg_pt2(2)]));
pix2pts_x = pix2pts_y;

x_ref_mm = x_top;
y_ref_mm = y_top;

x_ref_pix = reg_pt1(2);
y_ref_pix = reg_pt1(1);

%initialize
x_pixel = NaN(size(fixed));
y_pixel = NaN(size(fixed));
z_pixel = NaN(size(fixed));

x_pos = NaN(size(fixed));
y_pos = NaN(size(fixed));

%iteration limits
i_start = i_crop(1);
i_end = i_crop(2);
k_start = k_crop(1);
k_end = k_crop(2);

t1 = tic;
view_mat = viewmtx(270-angle, 0);
for i = i_start:i_end %back_c:tip_c
    xp = pix2pts_x*(i-x_ref_pix) + x_ref_mm; %linear x-
position on 2-D synthesized cone orthographic projection

for k = k_start:k_end % 1:length(fixed(:,1))
    yp = pix2pts_y*(y_ref_pix−k) + y_ref_mm; %linear y−position

    x_pos(k, i) = xp;
    y_pos(k, i) = yp;

    r3_pos = r3_interp(x_pos(k, i), y_pos(k, i));
    r4_pos = r4_interp(x_pos(k, i), y_pos(k, i));

    xyz_temp = view_mat\[x_pos(k, i), y_pos(k, i), r3_pos
        , r4_pos]\';

    x_pixel(k,i) = xyz_temp(1);
    y_pixel(k,i) = xyz_temp(2);
    z_pixel(k,i) = xyz_temp(3);
end

if mod(i, 10) == 0
    t2 = toc(t1);
    disp([\'column \ num2str(i) \ ', elapsed loop time \ num2str(t2) \ ', estimated remaining time \ ...
        num2str(t2/(i−i_start+1)*(i_end−i)) \ s\']]}
end
end

% disp('Mapping done')
% toc
% disp('')

%%% load_shift_moving_frame.m
% load moving frame and perform linear shift and rotation

% Author: Michael Thompson, March 2017
% Advisors: Carson Running, Prof. Juliano

alldata = importdata(filename, '\t');
data = alldata.data;

moving = data;
off = fixed_orig;
on = moving;

% translate image to align with reference image
precision = 100; % e.g., 1/100 pixel precision
[output, fft_on_shift] = dftregistration(fft2(off),fft2(on),
  precision);
x_shift=output(4);
y_shift=output(3);
on_shift_orig = abs(ifft2(fft_on_shift)); %this is the shifted wind on image

on_shift = imrotate(on_shift_orig, rotation_angle, 'nearest', 'crop'); %rotate image
on_shift(on_shift==0) = NaN;

disp([filename ' : pixel x_shift , num2str(x_shift) , y_shift ' num2str(y_shift)])

%%% loop_moving_frames.m
% load and register wind on frames

% Author: Michael Thompson, March 2017
% Advisors: Carson Running, Prof. Juliano

list = dir(filename_moving);
%filename = 'run5_160psia_0001_468.asc';

sz_list = length(list);
[m, n] = size(fixed);

im_data = NaN(m, n, sz_list);
im_names = cell(sz_list, 1);

for count = 1:sz_list
    filename = fullfile(images_folder, list(count).name);
    % code continues...
load_shift_moving_frame;

im_data(:, :, count) = on_shift;
im_names(count) = cellstr(filename);

end

% disp('saving data')
% save(data_file_name, 'im_data', 'im_names', 'pixel_phi', 'pixel_emis', 'pixel_x', 'X', 'x_crop', 'y', 'z', 'phi_crop', 'row_pts', 'col_pts')
% disp('data saved')

%%% post_process.m

% Find viewing angle for each pixel (plus other parameters as needed)

% Author: Michael Thompson, March 2017
% Advisors: Carson Running, Prof. Juliano

disp('post process')

% phi calculation
phi_pixel = atan2(z_pixel, y_pixel);
% viewing angle calculation
cone_pixel = surf(x_pixel, y_pixel, z_pixel, 'linestyle', 'none', 'visible', 'off');
axis equal

normals_pixel = cone_pixel.VertexNormals;

[m, n] = size(x_pixel);
dot_products_pixel = zeros(size(x_pixel));
view_ang_pixel = zeros(size(x_pixel));

% create dot products
for i = 1:n
    for k = 1:m
        nom = [normals_pixel(k, i, 1) normals_pixel(k, i, 2) normals_pixel(k, i, 3)];
        cam_vect = [x_pixel(k, i)−cam_x y_pixel(k, i)−cam_y z_pixel(k, i)];
        dot_products_pixel(k, i) = dot(nom, cam_vect);
        view_ang_pixel(k, i) = acosd(dot_products_pixel(k, i)/norm(cam_vect));
    end
end
C.2 Heat-Flux Calculations

```matlab
%% heatflux.m
% Code to calculate global heat flux on model
% 3/3/17 (updated 3/20/19)
% Harrison Yates

clc
close all

cd '/Users/harrisonyates/Desktop/March2019/Run32'

%%
% sampling inputs
frame_rate = 355; % (Hz) camera’s frame rate
total_frames = 750; % total number of frames
t = linspace(0,total_frames/frame_rate,total_frames); % (sec) time
num_rows = 512; % row camera resolution
num_columns = 640; % column camera resolution

row_min = 1;
row_max = num_rows;
row = row_max - row_min;
column_min = 1;
column_max = num_columns;
column = column_max - column_min;
```

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% material properties (from materials.m)
materialID = 'PEEK'; % material ID number. See materials.m
frame_end = total_frames;

%% zero all matrices
frame_end=total_frames;
thick = zeros(row_max,column_max);
radius = zeros(row_max,column_max);
coord = zeros(row_max,column_max);
dx = zeros(row_max,column_max);
TF=zeros(row_max,column_max,frame_end);
TB=zeros(row_max,column_max,frame_end);
qdotdata=zeros(row_max,column_max,frame_end);
TBisothermal=zeros(row_max,column_max);
TBadiabatic=zeros(row_max,column_max,frame_end);
TFmov=zeros(row_max,column_max,frame_end);
TBmov=zeros(row_max,column_max,frame_end);
qdotdatamov=zeros(row_max,column_max,frame_end);

%% Compute thickness at some axial distance
start = 0;
d=linspace(16.29,12.09,column_max-start); % distances from tip
to edges of visible image, with 529 pixels
thick1=(tand(7).*d-0.9).*0.0254;
d2=linspace(12.09,11.76,39);

thick2=tand(7).*d2.*0.0254;
for g=1:row_max
    for h=1:column_max
        %if h <= 640 && h > start
            thick(:,h) = (thick1(h-start));
        %elseif h > 581 && h <= 640
            thick(:,h) = thick2(h-581);
        %else
            thick(:,h) = (thick1(1));
    end
end

%%%
tic
for g=1:512
    for h=1:640
        for x=1:750
            TF(g,h,x) = im_data(g,h,x);
        end
    end
    disp(g)
end
toc

tic
for g = 418
for h = 1:610 %:column_max
    %TF(g,h,:);
    %TB(g,h)=TF fra(g,h,1);
    radius(g,h) = thick(g,h);
    qcalc;
    qdotdata(g,h,:) = qdot(:);
end
disp(g)
end
toc

%%% qcalc.m
% Thomas Juliano, edited by Harrison Yates
% 10–11–12, edited 5/7/17
% qcalc.m translated and enhanced from qcalc96.for
% calculates heat transfer from wall and backface
  temperatures

%%% set up
% tic
% clear all
% close all

%%% set material propeties
% materialID = 4; % material ID number. See materials.m
% cd Run3/Run3
[cp, k, rho] = materials(materialID);
thermdiff = k / rho / cp; % (m^2/s) thermal diffusivity (alpha)

%% define geometry
% thick = 0.02; % (m) material thickness
% radius = 2; % (m) radius of curvature for cylindrical or spherical case
nodes_user_max = 500; % highest number of nodes to use

%% set coordinate system
% coord = 0; % flat plate (Cartesian)
coord = 1; % cylindrical
% coord = 2; % spherical

%% choose back face boundary condition
% backface = [1 0 0]; % back face thermocouple
backface = [0 1 0]; % adiabatic
% backface = [0 0 1]; % isothermal
% backface = [0.5 0 0.5]; % mutant hybrid

% n.b.: TBabiabatic is only calculated correctly when
backface = [0 1 0]

%% compute number of nodes based upon time step and stability criteria
N = length(t); % number of data points
dt = [0; diff(t)']; % (s) time step
sampling = (N-1) / (t(end) - t(1)); % (Hz) mean sampling rate
dx_min = sqrt(thermdiff*max(dt)*2); % (m) minimum thickness step
nodes_stability_max = floor(0.9*thick(g,h)/dx_min); % maximum nodes in wall
I = 500;%min([nodes_user_max, nodes_stability_max]); % nodes
%1000
dx(g,h) = thick(g,h) / (I-1); % (m) thickness step size
%depth = linspace(0, thick, I); % (m) node depth from surface

%% define radius for cylindrical or spherical coordinate calculations
if exist ('radius_inside', 'var') == 1
    r = linspace(radius(g,h), thick(g,h), I); % (m) local radius of curvature
elseif isinf(radius(g,h))
    r = inf*ones(1, I); % (m) local radius of curvature
else
    r = linspace(radius(g,h), radius(g,h) - thick(g,h), I); % (m) local radius of curvature
end
if radius(g,h) < 0
    r = -r; % (m) radius
end

%% initialize variables
T = zeros(N, I); % (K) temperature; each row is one time,
each column, one location

\[
T(1, :) = \text{linspace}(TF(g,h,1), TB(g,h,1), I); \text{ \% (K) initial temperature}
\]

\[
T(:, 1) = TF(g,h,:); \text{ \% (K) boundary condition from TC data (camera data)}
\]

\[
T(:, I) = TB(g,h,:); \text{ \% (K) boundary condition from TC data (backface)}
\]

TBadiabatic(g,h,:) = TB(g,h)*ones(N, 1);  
TBisothermal(g,h) = TB(g,h); \text{ \% changed}

if backface(3)==1
    TB(g,h,:) = TBisothermal(g,h)*ones(N, 1);
end

qdot = zeros(N, 1); \text{ \% (W/m^2) heat flux into front face}
qdotB = zeros(N, 1); \text{ \% (W/m^2) heat flux into back face}

%% loop through time and location
% tic

for n = 2:1:N \% time index
    for i = 2:1:I-1 \% location index
        \[T(n,i) = T(n-1,i)\]
        \[+ \ \text{thermdiff*dt(n)/dx(g,h)}^2 * (T(n-1,i+1) - 2*T(n-1,i) + T(n-1,i-1))\]
        \[- \ \text{coord} * \ \text{thermdiff*dt(n)/dx(g,h)}/r(i)/2 * (T(n-1,i+1) - T(n-1,i-1));\]
    end

TBadiabatic(g,h,n) = (4*T(n-1, I-1) - T(n-1, I-2)) / 3; \text{ \% adiabatic definition of back face T}
\[ T(n, 1) = \text{sum}(\text{backface} \times [T_{\text{B}}(g, h, n) T_{\text{B}}(g, h, n) T_{\text{B}}(g, h, n)]) ; \]
\[ qdot(n) = -0.5 \frac{k}{dx(g, h)} (-3T(n, 1) + 4T(n, 2) - T(n, 3)) ; \% \text{(W/m}^2\text{) heat flux} \]
\[ qdotB(n) = -0.5 \frac{k}{dx(g, h)} (-3T(n, I) + 4T(n, I - 1) - T(n, I - 2)) ; \% \text{(W/m}^2\text{) heat flux at back face} \]
end
\%
% toc;

if not(backface(2) == 1) \% if adiabatic BC not applied,
    T_{\text{B}}(g, h) = \text{nan} ; \% wipe out false adiabatic TB
end

%% heafluxalongvortex.m
close all
bound1 = 620 ; \% these are the column bounds of an image in
terms of pixels
bound2 = 60 ;

% Determine points along vortex path from scatter data
x = [62.5 100 138 157.3] ;
y = [.29 .16 0 -.06] ;
xb = linspace(165,30,640) ;
p = polyfit(x,y,2) ;
yb = polyval(p,xb) ;
% THIS IS THE CORRECT CODE FOR HEAT-FLUX ALONG A VORTEX PATH

for i = 1:512
    for j = 1:bound1-bound2
        if phi_pixel2(i,j+bound2)-(yb(round(length(yb)*j/(bound1-bound2)))) >= -0.005
            && phi_pixel2(i,j+bound2)-(yb(round(length(yb)*j/(bound1-bound2)))) <= 0.005

        path1_x7(i,j) = x_pixel2(i,j+bound2);
        path1_phi7(i,j) = phi_pixel2(i,j+bound2);
        path1_heatflux7(i,j) = on_shift2(i,j+bound2);

    end
end

span_path_x7 = path1_x7(:);
phi_path_x7 = path1_phi7(:);
span_pathheatflux7 = path1_heatflux7(:);
span_path_x7 = nonzeros(span_path_x7(:));
phi_path_x7 = nonzeros(phi_path_x7(:));
span_pathheatflux7 = nonzeros(span_pathheatflux7(:));
span7 = [span_path_x7 span_pathheatflux7];
span7 = sortrows(span7);
heatfluxsmooth7 = smooth(span_path_x7,span_pathheatflux7
 ,0.1,'rloess');

figure
plot(span_path_x7(1:end),span_pathheatflux7(1:end),'b')
xlabel('Distance from Nosetip (mm)', 'FontSize',16)
ylabel('Heat Flux (W/m^2)', 'FontSize',16)
grid on
set(gca, 'FontSize',16)
C.3 Wavelet Analysis

%%% Wavelet Analysis
% author: Harrison Yates
% written: 5/4/18

%clear
clc
close all
%

%%% Loop to determine span at each station
clear span_path_heatflux span_path_phi span_path_x f_max

start = 1; % choose start/stop columns of mapped data
stop = 150;
cutoff = 50; % set cutoff of magnitude

span_path_phi = zeros(512,640,stop-start);
span_path_heatflux = zeros(512,640,stop-start);
span_path_x = zeros(512,640,stop-start);

for k = start:stop-1
    for i = 1:512
        for j = 1:640
if x_pixel(i,j) >= k && x_pixel(i,j) <= k+1

span_path_phi(i,j,k-(start-1)) = phi_pixel(i,j);
span_pathheatflux(i,j,k-(start-1)) = on_shift(i,
     j);

end
end
end

span_path_phi_new = span_path_phi(:,:,k-(start-1));
span_path_phi_2(:,k-(start-1)) = span_path_phi_new(:,);
span_pathheatflux_new = span_pathheatflux(:,:,k-(start-1));
span_pathheatflux_2(:,k-(start-1)) = span_pathheatflux_new(:,)
     ;

span(:,:,k-(start-1)) = [span_path_phi_2(:,k-(start-1))
     span_pathheatflux_2(:,k-(start-1))];
span(:,:,k-(start-1)) = sortrows(span(:,:,k-(start-1)));

phi_final(1:length(nonzeros(span_path_phi_new(:,))),k-(start
     -1)) = nonzeros(span(:,1,k-(start-1)));

[wt,f,coi] = cwt(nonzeros(span(:,2,k-(start-1))));

%Find max wavelength at each azimuth
clear w_max f_max

f_max = zeros(length(nonzeros(span(:,2,k-(start-1)))),length(f));

% Delete data if magnitude is below cutoff

for i = 1:length(nonzeros(span(:,2,k-(start-1))))
    if abs(wt(j,i)) == max(abs(wt(:,i))) && abs(wt(j,i)) > cutoff;
        f_max(j,i) = f(1)-j.*(f(1)-f(end))./length(f);
    elseif abs(wt(j,i)) == max(abs(wt(:,i))) && abs(wt(j,i)) < cutoff;
        f_max(j,i) = NaN;
    end
end
end

f_max = nonzeros(f_max(:,));

% for i = 1:length(f_max)
    if f_max(i) >= 0.13
\[ f_{\text{max}}(i) = \text{NaN}; \]
\[ \text{end} \]
\[ \text{end} \]

% delete data if it's under the COI

\[
\text{for } i = 1: \text{length}(f_{\text{max}}) \\
\quad \text{if } f_{\text{max}}(i) \leq \text{coi}(i) \\
\quad \quad f_{\text{max}}(i) = \text{NaN}; \\
\quad \text{end} \\
\text{end} \\
\]

\[ f_{\text{max}_\text{final}}(1: \text{length}(f_{\text{max}}), k-(\text{start} - 1)) = f_{\text{max}}; \]
\[ \text{end} \]

\[
[m \ n] = \text{size}(f_{\text{max}_\text{final}}); \\
\]

%%% See fmax on cwt plot

\%
\[
\text{xbound} = \text{linspace}(0, \text{length}(\text{nonzeros(span(:,2,k-(start-1))))), \text{length}(\text{nonzeros(span(:,2,k-(start-1)))))); \\
\text{figure} \\
\text{imagesc((1:}\text{numel(\text{nonzeros(span(:,1,k-(start-1))))}),f,\text{abs(wt)})
\]

\[ 181 \]
hold on
plot(1:numel(nonzeros(span(:,1,k-(start-1)))),(coi),'k-','LineWidth',3)

a = area(xbound,(coi),'FaceColor','k')

%plot(f_max,'r','LineWidth',2) % this is the max amplitude line

%%% Continous Contour Plot

f_contour2 = f_max_final;
x_contour2 = zeros(m,stop-start);
phi_contour2 = zeros(m,stop-start);

for i = 1 : m
    for j = 1 : stop-start
        x_contour2(:,j) = (300+j).*ones(m,1);
        phi_contour2(i,:) = i;
    end
end

%%% Shift positions of elements in f_contour vector

f_contour_cal = zeros(m,stop-start);
f_contour_cal_final = zeros(m,stop-start);
wave_length = zeros(m,stop-start);
wave_no = zeros(m,start-stop);
phi_final_cal = zeros(m,stop-start);
um_zeros = zeros(1,stop-start);

for i = 1:(stop-start)
    num_zeros(i) = length(f_contour2(:,i)) - length(nonzeros(f_contour2(:,i)));
    if mod(num_zeros(i),2) == 0
        f_contour_cal(:,i) = circshift(f_contour2(:,i),
                                   num_zeros(i)/2,1);
        phi_final_cal(:,i) = circshift(phi_final(:,i),
                                        num_zeros(i)/2,1);
    else
        f_contour_cal(:,i) = circshift(f_contour2(:,i),
                                        num_zeros(i)+1)/2,1);
        phi_final_cal(:,i) = circshift(phi_final(:,i),
                                        num_zeros(i)+1)/2,1);
    end
end

%Wavelength
for i = 1:m
    for j = 1:stop-start
if f_contour_cal(i,j) == 0
    f_contour_cal(i,j) = NaN;
    wave_length(i,j) = NaN;
else
    wave_length(i,j) = 1/(f_contour_cal(i,j));
end
end
end

for i = 1:m
    for j = 1:stop-start
        if f_contour_cal(i,j) == 0
            f_contour_cal(i,j) = NaN;
            wave_no(i,j) = NaN;
        else
            wave_no(i,j) = 360/(wave_length(i,j));
        end
    end
end
end

x_final_cal = phi_contour2;

%% Optional section to find an edge
% clear x_line phi_line x_line2 phi_line2
%
184
% bottom = 900;
% top = 1200;
% left = 1;
% right = 83;
%
% x_line = zeros(top-bottom,right-left);
% phi_line = zeros(top-bottom,right-left);
%
% for i = bottom:top
% for j = left:right
% if phi_final_cal(i,j) - phi_final_cal(i,j) <= .01
% &&...
% abs(wave_no(i,j) - wave_no(i,j+1)) >= 18
%
% x_line(i,j) = x_contour2(i,j);
% phi_line(i,j) = phi_final_cal(i,j) -.15;
% end
%
% end
%
% x_line = nonzeros(x_line(:));
% phi_line = nonzeros(phi_line(:));
%
% bottom = 1;
% top = 900;
% left = 1;
% right = 90;
%
% x_line2 = zeros(top-bottom,right-left);
% phi_line2 = zeros(top-bottom,right-left);
%

%%% Contour and imagesc plots

close all
figure
scatter(x_contour2(:,), phi_final_cal(:,), 10, wave_no(:,,'*'))
xlabel('Axial Position (mm)', 'FontSize', 14)
ylabel('Azimuthal Angle (Degrees)', 'FontSize', 14)
title('', 'FontSize', 14)
a=colorbar;
ylabel(a, 'Wavelength (Degrees)', 'FontSize', 14)
L = get(gca, 'YLim');
set(gca, 'CLim', [30,80])
map = jet(16);
colormap(map)
Figure D.2: Steel Frustum Detailed Drawing


