UNSTEADY MEASUREMENT TECHNIQUES FOR TURBOMACHINERY FLOWS

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Abstract

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Accurate unsteady measurements are required for studying the flows in high speed turbomachines, which rely on the interaction between rotating and stationary components. Using statistics of phase locked ensembles simplifies the problem, but accurate frequency response in the 10-100 kHz range significantly limits the applicable techniques. This research advances the state of the art for phase resolved measurement techniques using for high speed turbomachinery flows focusing on the following areas: development, validation, and uncertainty quantification. Four methods were developed and implemented: an unsteady total pressure probe, the multiple overheat hot-wire method, the slanted hot-wire method, and the phase peak yaw hot-wire method. These methods allow for the entire phase locked average flow field to be measured (temperature, pressure, and velocity components, swirl angle, etc.). No trusted reference measurement or representative canonical flow exists for comparison of the phase resolved quantities, making validation challenging. Five different validation exercises were performed to increase the confidence and explore the range of applicability. These exercises relied on checking for consistency with expected flow features, comparing independent measurements, and cross validation with CFD. The combined uncertainties for the measurements were quantified using uncertainty estimates from investigations into the elemental error sources. The frequency response
uncertainty of constant temperature hot-wire system was investigated using a novel method of illuminating the wire with a laser pulse. The uncertainty analysis provided estimates for the uncertainty in the measurements as well as showing the sensitivity to various sources of error.
This dissertation is dedicated to my family and friends. My parents have provided nothing but support, enthusiasm, and love. My brother Peter never fails to remind me to stop worrying, enjoy life, and laugh like an idiot. My closest friends Danny and Katy have always been there for me when I needed them, even when it means answering the phone at 4 am or joining me on a 13 mile run. This work is as much their accomplishment as it is mine.
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CHAPTER 1

INTRODUCTION AND RESEARCH OBJECTIVES

1.1 Motivation

The study of high-speed turbomachinery, including compressors and turbines, is important for continued advancement of many technological applications. Examples include gas-turbine engines for power production or propulsion, turbochargers, etc. The high flow speeds, high mechanical speeds, interactions between rotating and non-rotating components, and mixing of multiple fluid streams result in an extremely complicated environment for either experiments or numerical simulations. This dissertation describes the advancement of several experimental methods for measuring the velocity components, total temperature, and total pressure in high-speed turbomachinery flow fields.

Accurate measurements are useful for studying flow phenomena through experiments and validated simulations. New measurement capabilities enable improved investigation into various fluid mechanics problems. Examples of turbomachinery-related fluid mechanics problems where improved measurement capabilities are significantly impacted include: rotor-stator interactions, stator-rotor interactions, rotor-duct boundary layer interactions, tip clearance flows, rotor secondary flows, wake tracking, cooling flow-rotor interactions, rotor tip injection effects, rotor casing treatment effects, and many others. New measurement techniques should be developed, the uncertainty quantified, and the accuracy validated.

Turbomachinery flows are particularly challenging to study without accurate experiments. Advancements in turbomachinery and advancements in measurement
Numerical techniques are limited by the computational resources needed for CFD simulations of sufficient fidelity. The flow field complexity, necessary grid resolution, flow unsteadiness, and boundary condition sensitivity make accurate simulations of turbomachinery flows challenging and resource intensive. The rapid improvements in computational and modeling ability in the past two decades have supported, rather than displaced, experiments as a tool for analysis and design. Experiments and simulations complement each other as tools for studying turbomachinery flows.

1.2 Flow Field Description

Turbomachinery flow fields are unsteady, compressible, three dimensional, non-isothermal, and contain regions with large gradients. In addition, the lack of physical access, lack of visual access, long thermal transients, high temperatures, high rotational speeds, and high levels of vibration in the testing environment greatly add to the difficulty in making accurate and precise measurements. Many turbomachinery phenomena are also very sensitive to small changes in boundary conditions making them difficult to study.

The high rotational speeds lead to high frequency response requirements for measurements made in the stationary laboratory reference frame. A location in the stationary reference frame observes a fixed location relative to the rotor passing by at the rotor rotational speed with frequency \( f_{\text{rotor}} \). For a rotor with \( N \) blades, a rotor blade passes a fixed point in the stationary reference frame at the blade passing frequency \( (f_{\text{blade}}) \)

\[
f_{\text{blade}} = N f_{\text{rotor}}.
\]  

(1.1)

High speed compressors and turbines often rotate at speeds between 10,000 rpm and 20,000 rpm, which, typically corresponds to a blade passing frequency \( (f_{\text{blade}}) \) be-
tween 5 kHz and 10 kHz. In order to resolve the details of the blade passage flow, a sensor must have a time response that is many times greater than the blade passing frequency. A resolution of 10 points per blade passage corresponds to a sensor cutoff frequency that is typically between 50 kHz and 100 kHz. The frequency response below the cutoff frequency should be either constant or the transfer function accurately quantified in order to accurately resolve the blade passage flow features. Very few transducers are capable of this high frequency response. Constant temperature hot-wires and silicon diaphragm pressure transducers are the two most commonly used transducers.

Modern high speed compressors and turbines typically operate in the transonic range where the compressibility of the fluid cannot be neglected. Local relative Mach numbers are often greater than one. Typical values for the relative blade tip Mach numbers for modern compressors are around 1.3 to 1.4 \cite{14}. Shock waves are present in some regions of the flow. The subsonic regions of flow are typically at Mach numbers where compressibility effects create significant differences between static and total quantities. Typical values of the axial Mach number for modern compressors are between 0.3 and 0.6 \cite{14}. From a measurement perspective, the local fluid properties near a sensor are often different from the free-stream conditions (i.e. evaluated at the film temperature). This becomes an issue even for some steady sensors requiring some form of Mach number correction independent of any Reynolds number effects (i.e. a recovery factor).

The difficulty in gaining sufficient optical or physical access to the flow remains a practical challenge for measurement techniques in turbomachinery flows. Turbo-machines utilize closely spaced rows of rotating and stationary blades to transfer energy between the shaft and the fluid. Typical values of the axial gap spacing for modern axial compressors are approximately 25 percent of the axial chord \cite{14}. This axial gap spacing is roughly $\frac{1}{2}$ inch or less for real engines. With so little room be-
tween bladerows, the ability to make accurate and precise measurements is a practical concern.

1.3 Phase Locked Statistics

Ensembles of data can be generated by phase locking to a time series or a periodic or intermittent signal. In turbomachinery flows, the phase locking is based on a trigger from a signal occurring once per rotor revolution or once per blade passing. By generating phase locked ensembles, sensors can essentially take measurements of an axisymmetric flow as if they were rotating with the rotor and sampling at a frequency of once per revolution.

Understanding phase locked statistics is important for the research objectives and reviewing previous phase resolved measurement work for high speed turbomachinery flows. The unsteadiness of turbomachinery flows resulting from the interactions between rotating and stationary components leads to the concept of generating phase locked ensembles for statistical analysis. Figure 1.1 shows an example of phase locked ensemble statistics. A tachometer trigger signal was generated as a fixed feature on the rotating shaft passes a stationary sensor. The phase location $\tau$ relative to the trigger was used to conditionally sample the signal voltage to generate the ensemble for phase locked statistics.

For a quantity $E$ measured discretely over a length of time, the time series was divided into $N$ rotor revolutions from a once-per-revolution trigger signal. Rotor revolution number was denoted by index $j$. Rotor phase location for discrete data was divided into $M$ phase locations based on the time lag between once-per-revolution trigger events. The rotor phase location was denoted by index $i$. The instantaneous quantity $E$ was decomposed into a Phase Locked Average (PLA) quantity

$$\hat{E}_i = \frac{\sum_{j=1}^{N} E_{i,j}}{N},$$

(1.2)
with a fixed location relative to the rotating reference frame, and a Phase Locked Unsteady quantity (PLU)

\[ E'_{i,j} = E_{i,j} - \bar{E_i}. \]  \hfill (1.3)

The PLA quantity can be decomposed into a Time Average (TA) quantity

\[ \bar{E} = \frac{\sum_{j=1}^{M} \sum_{i=1}^{N} E_{i,j}}{NM}, \]  \hfill (1.4)

with a fixed location relative to the non-rotating reference frame and a Phase Locked Steady quantity (PLS)

\[ \widetilde{E}_i = \bar{E}_i - \bar{E}. \]  \hfill (1.5)

For this decomposition

\[ E = (\bar{E} + \bar{E}') + E' = \bar{E} + E', \]  \hfill (1.6)

and shown in Figure 1.1. By definition, the PLS and PLU quantities have a zero mean.

1.4 Research Objectives

This research had three objectives:

1. Develop hot-wire and silicon diaphragm pressure transducer based techniques for measuring the phase locked statistics of total temperature, swirl angle, total pressure, and velocity components behind rotors of high speed turbomachines.

2. Validate the measured flow quantities from the different techniques in high speed turbomachinery flows.

3. Quantify the uncertainty of the flow quantities from each measurement technique.

These objectives were selected to advance the state of the art for unsteady measurements in high speed turbomachinery flows and make the implementation practical on a increased scale. The development of hot-wire and silicon diaphragm pressure
Figure 1.1. Example of instantaneous and phase-locked statistics in a compressor rotor wake flow.
transducer based techniques was intended to give people studying high speed turbomachinery unsteady flows additional methods with increased utility and capability. The validation was important for advancing the state of the art and assessing the credibility of the measurements within a typical implementation of the methods. The uncertainty quantification estimated the limits of the accuracy of the methods and investigated the sources of uncertainty in the measurements.

1.4.1 Development

The multiple overheat hot-wire method, slanted hot-wire method, phase peak yaw method, and an unsteady $P_t$ probe were developed. Chapter 4 presents the concepts, methodology, and implementation of these methods. The underlying physics behind each method are shown, calibrations are described, and a practical implementation of the method is presented.

Constant temperature hot-wires are used as the transducer in the first three methods primarily because they can have frequency responses in the 100-300 kHz range, which is above the range on interest for high speed turbomachinery flows. The primary disadvantage is that they are sensitive to multiple flow variables, making it challenging to separate the impact of each flow quantity on the sensor output. The multiple overheat method, slanted hot-wire method, and phase peak yaw method all take advantage of the sensitivity to different variables. These methods are most effective at measuring average flow quantities. Historically, for flows in which the slanted hot-wire and multiple overheat methods were used, the average quantities of interest were time averages. The frequency response of a sensor is not significant for time average measurements. In high speed turbomachinery flows, the phase locked average quantites can be used, which take advantage of the high frequency response of constant temperature hot-wires. These methods are novel because they expand existing measurement concepts to take advantage of this principle.
1.4.2 Validation

Chapter 5 was intended to provide confidence that the measured quantities from the measurement techniques accurately measure the physical flow quantities. Validation of a measurement technique is the process of assessing the credibility of the measurement within its domain of applicability by estimating the degree to which the measurement accurately represents reality from the perspective of its intended use [45]. Formally, validation is an ongoing process for testing the accuracy and limits of applicability for a measurement technique. The validation process for a new measurement typically involves comparing the measured value with a reference value. This reference is either from theory for a canonical problem, from a validated numerical simulation, or from another validated measurement.

When no reference is available, validation is challenging. In this case, comparing two different un-validated independent measurements with each other is also an effective validation exercise. The different independent measurements would be unlikely to agree if one or both were incorrect. Qualitative visualization comparison of measurements with expected flow features is an additional validation exercise that has found widespread use in the development of unsteady measurement techniques for high speed turbomachinery flows.

Validation typically requires a known representative flow field or a reference method for quantitative comparison. In general, no known flow field accurately represents the conditions in a high speed compressor or turbine, due to the high blade passing frequencies and high subsonic Mach numbers. Shock tubes have been used since the sharp gradients across the moving shock contain high frequency components as seen by stationary sensors. However, they are not representative of high speed turbomachinery flows. In many ways, the midspan location behind a high speed axial turbomachinery rotor operating at near design incidence condition (when the rotor relative velocity is aligned with the rotor blade camber line at leading edge) is a rep-
resentative and relatively well understood flow. The flow phenomena are relatively well characterized away from the endwall and tip clearance regions near the design incidence operating point.

The unsteady $P_t$ probe involved integrating a miniature silicon diaphragm pressure transducer in the head of a Kiel probe. The transducer response and concept of building unsteady pressure probes by integrating the transducers in the heads of conventional style pressure probes has been characterized by previous work in literature. The dynamic response of the unsteady $P_t$ probe was checked using a shock tube. No additional validation was performed for the total pressure measurements from the unsteady $P_t$ probe.

Comparison with a reference method is a common method of validating a new measurement technique. No validated measurement technique can measure $T_t$ with precision of order 1 degree Kelvin across the 1-100 kHz range for use as a reference for comparison with the multiple overheat method. Methods exist for measuring the remaining phase locked average velocity components (i.e. Laser Doppler Velocimetry, Particle Image Velocimetry, etc.), however the challenges of implementing these methods with limited space and optical access made them impractical. The swirl angle derived from the PLA velocity components measured by the slanted hot-wire method was compared with the phase peak yaw method used as a reference.

Five different validation exercises were performed to assess how accurately the different phase resolved measurement techniques measure the rotor relative flow field behind high speed axial turbines and compressors.

1. The measured phase locked average quantities from various turbomachinery flows were investigated for consistency with the expected flow structures. This consistency was an important starting point for trusting that the measured quantities from the new techniques are accurate.

2. Averages of the PLA quantities across phase location were compared to the time average measurements behind a high speed turbomachine ($\bar{E} = \frac{\sum_{i=1}^{N} E_i}{N}$). Time average total temperature, total pressure, swirl angle, and velocity components
were compared to averages of the phase locked average quantities in a high speed turbomachinery flow. The phase locked averages were measured using the different phase resolved techniques developed. The time averages were measured using thermocouples and pressure probes at the same axial and radial positions.

3. The phase locked average quantities were compared with the rotor relative average flow using a CFD simulation which was validated using time average measurements. A high fidelity phase resolved CFD simulation of a low pressure turbine rotor (LPT) was used for validating the phase lock average measurements. The time average measurements were used to validate the CFD simulation. The CFD simulation was used to validate the phase lock average measurements.

4. The phase locked average swirl angle measurements were compared between the slanted hot-wire method and the phase peak yaw method at the same operating conditions. The measurements from the slanted hot-wire method are completely independent. The measurements from the phase peak yaw method are useful as a reference for comparison because it is inherently insensitive to any amplitude errors (calibration drift, compressibility effects, temperature non-uniformity, frequency attenuation, etc.).

5. The phase locked average total temperature and swirl velocity were compared to each other using the relationship between total temperature change and flow turning in an axial turbomachine shown in the Euler Turbomachinery Equation.

1.4.3 Uncertainty Quantification

The combined standard uncertainty was estimated for the quantity of interest from the measurements in Chapter 6. A detailed uncertainty analysis for new measurement techniques involves the following steps [12].

1. Determining the functional relationship between the measured quantities (such as hot-wire voltage) and the quantities of interest (such as total temperature).

2. Determine the sensitivity coefficients from the functional relationships evaluated at the nominal values.

3. Identify, classify, and estimate the elemental uncertainties from each error source.

4. Propagate each of the elemental uncertainties into the combined standard uncertainty.

The first two steps were completed by calibrating the sensors. Step four was completed by following the detailed uncertainty analysis procedure outlined in [12] using
the elemental uncertainties and sensitivity coefficients in the equations for combining the elemental errors.

The third step was the most involved, since it requires an analysis of the various measurement techniques in order to identify and accurately estimate the elemental uncertainties associated with each error source. The errors from the hot-wire and silicon diaphragm pressure transducer-based techniques must be analyzed separately since each transducer relies on different operating principles.

The overall accuracy of the hot-wire based techniques have been found to be primarily the result of calibration errors, with the frequency response uncertainty becoming significant at low overheats. The calibration errors are quantified and controlled during the calibration procedure which is described and analyzed in the development and methodology chapter. Studying and quantifying the hot-wire frequency response uncertainty lead to development of a novel experimental method, the laser pulse test. The results from this study are shown in the investigation of constant temperature hot-wire frequency response chapter.
CHAPTER 2

LITERATURE REVIEW

2.1 Hot-wire Measurement Techniques

2.1.1 Constant Temperature Hot-wire Theory

A hot-wire probe consists of a thin wire between two metal prongs. Various hot-wire designs are shown in Figure [2.1]. Typical wire materials are tungsten or platinum. Wires are usually between 1 and 5 mm long and approximately 5 microns in diameter.

![Various Hot-wire Probe Designs](image)

Figure 2.1. Various Hot-wire Probe Designs (from [32])
Hot-wires can be operated in constant temperature or constant current modes. Typical hot-wire anemometers are operated in constant temperature mode because of the higher frequency response. Constant temperature hot-wires operate by balancing the thermal convection and conduction from the wire at an elevated temperature through Joule heating to maintain a constant average wire temperature. This heat balance is shown in Figure 2.2.

Figure 2.2. Differential Hot-wire Heat Balance (from [6])
Integrating along the wire, assuming the conduction term
\[ B = -\frac{k_w \pi d_w^2}{2} \frac{\partial T_w}{\partial x} x=0.5L \]
from the wire to the prongs is nearly constant for a constant overheating, using Ohms law to relate the wire voltage and current, and assuming steady state yields the hot-wire governing equation from [6]

\[ \frac{E_w^2}{R_w} = l\pi d h(T_w - T_r) + B. \]  (2.1)

The convection coefficient \((h)\) and recovery temperature \((T_r)\) are a function of the flow conditions. The convection coefficient and recovery temperature relations must be studied experimentally. Hot-wires are effectively only sensitive to velocity components perpendicular to the wire axis.

Standard hot-wire techniques such as x-wire probes and slanted wires are commonly used to decompose the relative velocity components. In high speed turbo-machinery flows, one of the major challenges is accurately measuring the absolute velocity magnitude and total temperature. The remainder of the hot-wire literature review is focused on the potential error sources with these measurements, particularly with respect to uncertainties associated with mapping hot-wire voltages to flow variables in compressible-to-transonic flow regimes as well as uncertainties associated with the frequency response at high frequencies.

### 2.1.2 Hot-wire Dimensional Analysis

Dimensional analysis indicates that the convection coefficient \((h)\) from an infinitely long constant temperature hot-wire in a uniform flow normal to the wire is a function of 10 independent dimensional variables \((h = f(\mu, k, c_p, c_v, d_w, T_w, g, U, \rho, T_t))\).

There are four dimensions relevant to the heat transfer problem: mass, length, time and temperature. The Buckingham Pi Theorem shows that six independent non-dimensional variables are required.
The dependent variable is non-dimensionalized as Nusselt number $\textit{Nu}$. The independent non-dimensional variables are Reynolds number ($\textit{Re}$), Mach number ($\textit{M}$), Grashof number ($\textit{Gr}$), Prandtl number ($\textit{Pr}$), specific heat ratio ($\gamma$), and overheat ratio $\tau_w$. Knudsen number ($\textit{Kn}$) or Recovery factor ($\eta$) are often substituted for Mach number. Wire temperature loading is sometimes substituted for overheat ratio. For most aerodynamic hot-wire measurements in air, several of these variables are constant to good approximation. Grashof number can be neglected for all flows except when Reynolds number is extremely low. Specific heat ratio and Prandtl number are both constants for air except at extremely high temperatures. As a result, only three non-dimensional independent variables remain. The dimension of the independent variable space is three.

There is no published consensus about which non-dimensional independent variables are most suited for the correlations [48] [52] [37] [3]. Reynolds number, Knudsen number, and overheat ratio are primarily used for studying the hot-wire functional relationship in this presented work. Dimensionally, these correspond to $\rho_t U$, $T_t$, and $P_t$ for a constant wire temperature. There is also no published consensus about the conditions at which fluid properties should be evaluated. Fluid properties are evaluated at stagnation conditions for our implementation of hot-wire based techniques.

The hot-wire heat balance shows that the energy into the wire from joule heating and measured by the hot-wire voltage is balanced by energy leaving the wire through forced convection and conduction to the prongs. For a given wire aspect ratio and nominal overheat (constant average wire temperature), the conduction is approximately constant. The dependent variable, Nusselt number, was approximated by

$$\textit{Nu} = \frac{hd_w}{k} \approx \frac{E^2}{\pi L k R_w (T_w - T_t)}.$$

Dimensionally, the dependent variable is just $E^2$.

Dewey in 1964 [22] plotted and correlated many different sets of convective heat
transfer and recovery temperature data from large aspect ratio circular cylinders in compressible flows. Nusselt number is plotted against Reynolds number for different Mach numbers in Figure 2.3. Nusselt number is primarily a function of Reynolds number, although Mach number effects are increasingly significant at low Reynolds and Mach numbers. At higher Reynolds numbers, the Mach number effects are smaller and appear to change only the magnitude, not the shape of the Reynolds-Nusselt relationship.

![Figure 2.3. Nusselt-Reynolds Correlation (from [22])](image)

Normalized temperature recovery factor \( \eta_s = \frac{\eta - \eta_c}{\eta_f - \eta_c} \) and is plotted versus Knudsen number in Figure 2.4. The free-molecular flow limit is denoted \( \eta_f \eta_f = \eta_c + \)
The high Reynolds number continuum limit is denoted \( \eta_c = 0.2167\left(\frac{M^{2.8}}{0.8521 + M^{2.8}}\right) \). Behrens in 1970 [4] plotted and correlated additional sets of convective heat transfer and recovery temperature data for circular cylinders, concentrating on higher Mach numbers. Mach number sensitivity becomes negligible for supersonic Mach numbers. In supersonic flows, a bow shock is present ahead of the wire. The locally subsonic flow conditions around the wire but downstream of the shock is not sensitive to changes in free-stream conditions. This simplification allows hot-wires to be easily used in supersonic flows over a wide range of flow conditions without requir-
ing complicated calibration maps of Reynolds and Mach numbers. Simplifications to the hot-wire relations can also be made for low Mach number flows. Incompressible aerodynamic flows of interest are also quite often isothermal, where fluid temperature and density are assumed to be constant, allowing further reduction of the number of independent variables.

Theoretically, the Nusselt number correlations are universal and could be applied to any hot-wire to determine the calibration a-priori. In practice, this is not effective. Deposition of particles and dust from the air, deformation due to thermal and mechanical stresses, and oxidation of impurities on the heated wire are all potential causes of calibration drift. It is important to realize that hot-wires are non-linear sensors and do not typically have stable calibrations. For accurate measurements, hot-wires must be calibrated frequently, approximately every few hours of testing. The correlations provided by Dewey and Behrens only contain a subset of the available data. Some data sets appear to be outliers and were omitted by the authors.

2.1.3 Hot-wire Heat Transfer Relations

King developed the functional form of the hot-wire relations in 1914 25

\[(T_w - T) \pi \frac{d}{L} = A + B \sqrt{U}.\]  

(2.3)

Since that time, many different functional forms have been developed for specific ranges of applicability (e.g. [44 6 13 50]).

In 1959, Collis and Williams studied heat transfer relations for hot-wires across Reynolds numbers and overheat. They found that large uncertainties in absolute flow quantities were due in part to the use of inaccurate heat transfer relations. They calibrated and correlated sets of hot-wire data and discovered a non-linear temperature loading (overheat) effect that caused much of the data to collapse. Figure
2.5 shows the temperature loading effect and how it was corrected. They attributed this temperature loading correction to changes in the temperature boundary condition at the surface of the wire. They also found a small discontinuity in the slope of the hot-wire relation which was attributed to the onset of vortex shedding.

Figure 2.5. Nusselt-Reynolds Correlation with Temperature Loading Correction (from [8])

The functional form

\[
Nu \left( \frac{T_f}{T_l} \right)^m = A + B \times Re^n,
\]

is one of the most commonly used hot-wire relations in literature. The exponent \(m\) on the temperature loading term was found to be equal to approximately -0.17 for wires with very large aspect ratios. The film temperature is an average of the wire and fluid temperatures \((T_f = \frac{T_w + T_{fl}}{2})\).
Morkovin in 1956 [37] extensively studied the use of hot-wires in compressible flows. The heat transfer relations used were

\[ Nu = Nu(M, Re, \tau), \]
\[ \eta = g(M, Re) = \frac{T_r}{T_t}, \]
\[ \tau = \frac{T_w}{T_t}. \]

(2.5) \hspace{1cm} (2.6) \hspace{1cm} (2.7)

The non-dimensional steady state heat balance for a hot-wire in a compressible flow is \( Q_w = lk(T_w - \eta T_t) Nu(M, Re, \tau) \). Morkovin used this functional form to study the sensitivities of a hot-wire.

Constant temperature hot-wires of finite length have different wire temperature distributions and therefore different amounts of conduction into the prongs. When comparing finite length hot-wires to infinite wire correlations, it is necessary to correct the Nusselt number \( Nu_c \) to the apparent mean Nusselt number \( Nu_m \) by accounting for the conduction into the prongs. Equation 2.8 is an approximation for wires with temperature distributions that are not flat.

\[ Nu_c = Nu_m - 1.14\sqrt{Nu_m} \]

(2.8)

Morkovin found that the uncertainties of mean and fluctuating quantities tended to be quite different. As a result, he found that the absolute mean flow measurements and the relative flow measurements should be treated very differently. Morkovin also suggested measuring the cold resistance of the wire frequently to analyze the stability of the hot-wire properties. An independent measurement of time average total temperature located near the wire in the flow was found to be useful in studying the stability of the hot-wire calibration.

Kovasznay in 1950 [28] and 1953 [27] used hot-wires to study supersonic turbulent flows. The steady and fluctuating components of each hot-wire variable were
decomposed into different modes and then compared using the sensitivities and the correlations between them.

Koch and Gartshore in 1972 [26] studied the effects of fluid temperature on the heat transfer relations. They found a similar Reynolds-Nusselt collapse with the one shown by Collis and Williams [8] using a temperature loading parameter \( \left( \frac{T_f}{T_t} \right) \) based on film temperature \( T_f = \frac{T_t + T_{\infty}}{2} \) with the same form but raised to a different power (a value of 0.67 as opposed to the value of -0.17 found by Collis and Williams). The different value was attributed to the increased end conduction effects of the hot-wire probe with a smaller aspect ratio.

Horstman and Rose [16] studied the use of hot-wires in transonic flows. Hot-wires are not easily used in transonic flows because none of the simplifications used for supersonic flows or incompressible flows can be used to reduce the number of variables to which a hot-wire is sensitive. Fundamentally, the problem that is the derivatives of Nusselt number with recovery factor and Mach number are not zero over the transonic range. No simplifications can be made and modal analysis techniques developed by Morkovin [37] and Kovasznay [27] cannot be applied to measure the fluctuating flow quantities and their correlations.

Morrison in 1974 [39] studied how a hot-wire response was effected by variations in fluid properties. The sensitivities to fluid property changes more closely matched the heat transfer relation from Collis and Williams [8] than the relation from Koch and Gartshore [26]. This difference was likely due to the differing hot-wire probe geometry.

2.1.4 Hot-wire Frequency Response

Smits and Perry in 1980 [47] studied the effect of overheat on the dynamic response of a constant temperature hot-wire anemometer. The hot-wire probe and constant temperature circuit were analyzed analytically and tested experimentally.
by perturbing the anemometer electronics and measuring the constant-temperature anemometer output. The dynamic response of the anemometer is governed by the poles and zeros of the linear system. At high overheats, the frequency response is dominated by the properties of the electronic control circuit. At low overheats, the frequency response is dominated by the properties of the local hot-wire heat balance. The roll-off frequency drops non-linearly with resistance ratio \( \frac{R_s}{R_0} \). Additionally, the ratio of the sensitivity to temperature fluctuations to velocity fluctuations increases non-linearly with decreasing resistance ratio.

In 1999, Khoo et al. [23] studied the dynamic response of constant temperature hot-wires and hot-films using two different methods. The first method was the square wave voltage perturbation test which is the current standard for verifying the frequency response of a constant temperature hot-wire. The estimated cut-off frequency from the square wave test \( f_s \) predicts the frequency at which the amplitude has been attenuated by -3 dB. The overshoot time is related to the cut-off frequency by

\[
f_s = \frac{1}{1.3 \tau_s}.
\]

The second method compared the hot-wire response in a nearly Couette flow between two rotating disks in order to compare the results with a velocity field of known amplitude and frequency. The disks were closely spaced and rotated at various speeds to study the dynamic response of the hot-wire/film across the frequency range. Figure 2.6 shows the hot-wire output signal relative to the disks.

Khoo et al. found that the dynamic frequency response \( f_D \) and square wave frequency response \( f_s \) were not equivalent. Both \( f_D \) and \( f_s \) showed the same trends, however \( f_D \) was consistently low by a full order of magnitude. Analysis of the assumptions in the hot-wire model equations points to differences between the mechanisms for heating and cooling the wire. This work raises uncertainty about the level of
Figure 2.6. Hot-wire Frequency Response for Periodic Couette Flow (from [23])
absolute accuracy of hot-wire measurements under high frequency fluctuations.

Moen and Schnieder in 1993 [36] found similar frequency response values from shock tube tests to those that they had predicted using the square wave test. Clearly errors from the frequency response of a hot-wire/film system need to be carefully accounted for in a detailed uncertainty analysis of a hot-wire based measurement technique.

Morris and Foss in 2003 [38] performed a series of numerical simulations of a constant temperature hot-wire anemometer by modeling the anemometer electronic circuit as well as the one dimensional time dependent heat transfer problem. The anemometer circuit is able to maintain a constant average temperature along the wire. However, the distribution of temperature along the wire is determined by the amount of convective heat transfer and the conduction into the prongs. Analytically, this phenomena was studied by Freymuth [15].

Abrupt changes in the amount of convection or conduction into the prongs causes the wire temperature distribution to change, even though the average wire temperature is held constant by the hot-wire circuit. For the temperature distribution to change, differential elements of the wire must heat up or cool down to reach the local steady state temperature. This heating and cooling has a time lag associated with it that will cause the instantaneous wire temperature distribution to differ from the steady state distribution. As a result the amount of conduction to the prongs will not match the calibration condition and the instantaneous hot-wire response will be biased. This effect is a function of the frequency of the fluctuation and is referred to as the thermal frequency response of the hot-wire.

The thermal frequency response is shown in Figure 2.7 for a wide range of frequencies. The response ratio ($\phi$) is the ratio of the RMS of the output signal over the RMS of the input signal. The thermal frequency response of the hot-wire decreases with increasing frequency but asymptotes at a value of approximately ($\phi(f = \infty) = 0.93$).
There is no phase lag associated with the thermal frequency response of the hot-wire. The dimensionless frequency \( f^* = f \tau_{\text{thermal}} \) is related to the time constant of the first eigenmode of the wire [42]

\[
\tau_{\text{thermal}} = \frac{4L^2}{\pi^2 \alpha},
\]  

Equation 2.10 shows this non-dimensional frequency related to the wire properties.

Figure 2.7. Thermal Frequency Response (from [38])

A typical 5 \( \mu \text{m} \) diameter tungsten wire with length 1 mm has a time constant \( \tau_{\text{thermal}} = 11.8 \text{ ms} \). For this wire, the thermal frequency response began to deviate from unity above about 100 Hz. Any fluctuations at these higher frequencies would be attenuated. The thermal frequency response was not dependent on the hot-wire
overheat ratio.

Li in 2004 [33] derived analytical solutions modeling the hot-wire system response to turbulent velocity fluctuations for a range of frequencies and wave numbers. The end conduction losses play a dominant role in the hot-wire frequency response at high frequencies. The cut-off frequency of the combined hot-wire and anemometer circuit was found to be about an order of magnitude less than that of the anemometer circuit alone, which is in agreement with the experimental findings of Khoo et al. [23]. This cut-off frequency is dependent on the hot-wire overheat, wire material, wire aspect ratio, and the wire Reynolds number.

Weiss, Knauss, and Wagner in 2001 [54] presented a method for estimating the transfer function of a hot-wire system. The transfer function was used to correct the dynamic response. The electronic square wave frequency response test is performed on the hot-wire and the time series of the system response is recorded. Figure 2.8 shows the output of the hot-wire system to the square wave. The frequency content of the square wave is known and can also be measured.

A transfer function can be determined by comparing the input to output voltages of the hot-wire system in the frequency domain. The transfer function can then be used to correct the response of a hot-wire output to match the actual input for fluctuations across the frequency spectrum. For fluctuations at frequencies where the signal to noise ratio of the hot-wire output becomes too low, the corrected hot-wire signal is no longer meaningful.

2.1.5 Constant Temperature Hot-wire Temperature-Velocity Decoupling

Constant temperature hot-wires in compressible-to-transonic flows are most sensitive to Reynolds number and overheat ratio. In dimensional variables, the constant temperature hot-wire voltage \( E \) is a function of fluid total temperature and mass flux perpendicular to the wire \( \rho U_w \). Corrsin in 1947 [10] was the first to outline the
Figure 2.8. Frequency Response of Hot-wire to Square Wave Test and Corresponding Transfer Function (from [54])
The basic methodology behind decoupling the temperature and velocity using the output of constant temperature hot-wires. The functional relationship is set by the overheat, shown as

$$E_i = f_i(T_t, \rho U_w).$$

(2.11)

The multiple overheat hot-wire method involves operating a hot-wire at N different over-heats under the same average flow conditions. This yields N independent measurements of hot-wire voltage for 2 independent variables. The dual parallel hot-wire method involves placing two constant temperature hot-wires close enough that they see the same instantaneous flow conditions. The two hot-wires are operated at different over-heats in order to provide 2 independent measurements of hot-wire voltage for the 2 independent flow variables. The resulting system of coupled nonlinear equations can be solved for the two flow variables. For the multiple overheat hot-wire method with more than 2 different over-heats, the over constrained system can be solved in the least squares sense. Figure 2.9 shows how the mean flow quantities are found using the mean hot-wire voltages for different over-heats (denoted $a_w$). Each hot-wire voltage has a locus of possible temperature and velocity values that could have generated it. The point where all these curves intersect must be the mean temperature and velocity. The search of the solution to this system of coupled non-linear equations can be performed numerically using an iterative method. An illustration of the search procedure that minimizes the error ($\epsilon_i$) in the solution of the system of equations is shown in Figure 2.10.

Typically, however, the multiple overheat method was used to solve for higher order statistics of the fluctuating quantities about the mean. The fluctuating hot-wire voltage ($e$) is a function of the fluctuating flow variables ($t$ and $u$) and the mean hot-wire sensitivities ($S_t$ and $S_u$). The rms of the fluctuating voltage can then be
Figure 2.9. Multiple Overheat Hot-wire Total Temperature-Velocity Decoupling (from [40])

Figure 2.10. Multiple Overheat Hot-wire Total Temperature-Velocity Decoupling Solution Search (from [40])
written as a function of the second order statistics of the fluctuating variables

\[ e = S_u u + S_t t = u \frac{\partial E}{\partial U} + t \frac{\partial E}{\partial T}, \quad (2.12) \]

\[ \frac{\tilde{e}^2}{S_t^2} = \left( \frac{S_u}{S_t} \right)^2 \tilde{u}^2 + 2 \frac{S_u}{S_t} \tilde{u} \tilde{t} + \tilde{t}^2. \quad (2.13) \]

Hot-wires at different overheats allow for the second order statistics of the fluctuating variables to be solved in the same manner. When solving for higher order statistics of the fluctuating quantities, the correlation between the fluctuating quantities is an additional unknown.

These techniques have been used almost exclusively to measure the second order statistics of the fluctuating quantities in turbulent unsteady flows with varying degrees of success. Liendard and Helland in 1989 [34] used the dual parallel hot-wire method to measure statistics of fluctuating temperature with a cold wire for comparison. They found that the measurements were only accurate at low turbulence intensities. At high turbulence intensities, the errors were unrealistically high. Walker and Ng in 1988 [53] used both the multiple overheat hot-wire method and dual parallel hot-wire methods to measure the mean and fluctuating quantities in a supersonic shear layer. Steady sensors were used to measure the mean values for comparison. The mean mass flux measured by the hot-wire techniques matched those taken by the steady sensors. The mean total temperature measured by the hot-wire techniques was significantly less accurate. This inaccuracy was attributed to the errors in repeatability of the experiment. Ndoye et al. in 2010 [40] developed a circuit for performing the multiple overheat hot-wire method efficiently by periodically stepping the hot-wire through a set of many different overheats.
2.2 Current Unsteady Measurement Techniques for Turbomachinery

2.2.1 Overview

The current state of the art for unsteady measurement techniques for turbomachinery flows is summarized by Sieverding et al. [10]. The largest group of phase resolved measurement techniques for turbomachinery flows are silicon diaphragm pressure transducer-based pressure probes often referred to as fast response aerodynamic probes (FRAP). Cold wire resistance thermometers have been used to measure total temperature fluctuations in turbomachines, but sufficient frequency response poses a major impediment. Cold wires are frequency limited to approximately 2-6 kHz [11]. The dual wire aspirating probe was designed by Ng and Epstein in 1983 [11] to measure time resolved total temperature and total pressure using two hot-wires inside a probe. The aspirating probe has a contraction after the hot-wires and is connected to a vacuum pump to choke the flow at the throat. Thin film temperature probes have been used to measure unsteady temperature in turbomachinery flows using the same two constant temperature sensors at different overheat method initially proposed by Corrsin [10] for hot-wires. These thin film temperature sensors have since been integrated with silicon diaphragm pressure transducers to simultaneously measure total pressure and total temperature (referred to as a fast response entropy probe) [35]. A fibre optic probe was used to measure total temperature by Kidd et al. [24] behind a compressor. Sieverding concludes that silicon diaphragm pressure transducer-based probes are becoming increasingly miniaturized to the point of being useful for resolving features of interest, but the unsteady aerodynamic errors experienced have not been adequately studied or quantified. Unsteady temperature measurement techniques have improved vastly in recent years, but the uncertainty of these measurements are not well characterized or quantified. Sieverding attributes frequency response errors to the hot-film based sensors as the primary impediment.
In general, current state of the art unsteady measurement techniques for turbomachinery flows are considered qualitative rather than quantitative due to a lack of understanding about the different error sources as well as the difficulty validating the new measurements.

An overview of experimental work on transonic flow in turbomachinery is presented in [1]. Work from engine company test programs, research labs, and universities is included. An overview of unsteady measurement techniques used to study turbomachinery flows are presented in [50]. Advancing the state of the art for unsteady measurement techniques in turbomachinery flows is also hindered by the lack of reporting in open literature [1]. Low speed experimental rigs at universities and research labs are responsible for much of published unsteady measurement technique development work. The measurement science research using low speed turbomachinery rigs does not reflect conditions in high speed turbomachinery flows due to the added compressibility effects, large temperature gradients, and higher necessary frequency response. Much of the published work presents turbomachinery flow data without effectively quantifying the uncertainty of the unsteady measurements. Surprisingly little work has been published relating to the science behind the unsteady measurement techniques for studying turbomachinery flows.

2.2.2 Silicon Diaphragm Pressure Transducers

The development of the silicon diaphragm pressure transducer, beginning in the late 1960’s, enabled much of the development of unsteady pressure measurement techniques for turbomachinery flows [16]. Piezo-resistors are formed on the silicon diaphragm using photolithographic methods. These piezo-resistors are assembled in a Wheatstone bridge. The voltage across the bridge is proportional to the change in resistance which is proportional to the pressure across the diaphragm. Silicon diaphragm pressure transducers have high stiffness, small size, and low mass which
gives them unusually high frequency response and sensitivities [29]. For comparison, most standard resistance strain gauges have gage factors between 2 and 4. Silicon diaphragm pressure transducers have gauge factors between 50 and 200. The frequency response of the silicon diaphragm pressure transducer is flat up to approximately $\frac{1}{5}$ of the natural frequency. Most silicon diaphragm pressure transducers have natural frequencies between 150 kHz and 400 kHz. Silicon diaphragm pressure transducers are sensitive to high temperatures. Above about 150 degrees C the n-p junction between the piezoresistors and the silicon diaphragm break down and the voltage is no longer proportional to pressure [29].

2.2.3 Fast Response Pressure Probes

The development of silicon diaphragm pressure transducer-based probes has been driven by the turbomachinery labs at the Von Karman Institute (VKI) in Brussels, the Eidgenossische Technische Hochschule (ETH) in Zurich, and Oxford University in the UK. Single sensor probes have been developed to measure static or total pressure. Multiple sensor probes have been developed to measure not only total pressure, but also flow angles. FRAP probes are typically custom built and therefore quite expensive, although commercial unsteady pressure probes have recently begun to be produced commercially for the aerospace industry [9]. These probes are relatively robust and stable compared to hot-wire based probes. Spatial resolution and unsteady aerodynamic effects remain significant sources of errors with these measurement techniques. Remote mounted silicon diaphragm pressure transducer measurement techniques are much more robust and have better spatial resolution, however acoustic transmission line errors must be corrected for in the transfer function. High frequency fluctuations are more heavily attenuated with remote mounted transducer methods.

Kupferschmied et al. [30] and Ainsworth et al. [2] provide an overview of the de-
velopment of the fast-response aerodynamic probe as a means of measuring unsteady flow variables in turbomachines. A variety of probe designs have been developed and used to measure these unsteady flow variables based primarily on existing steady measurement designs. The trade offs in designs are due to the desire to increase the frequency bandwidth, spatial resolution, and the number of variables measured. The three-hole straight cylindrical probe is used by Kupferschmied et al. to measure unsteady phenomena and compare to other measurement systems.

Pressure, temperature, flow velocity magnitude and direction all contain both steady and unsteady components. For most turbomachinery applications, the blade passing frequencies are on the order of 5 kHz. In order to resolve most secondary flow structures, it is necessary to resolve up to ten times the blade passing frequency. Miniature piezoresistive pressure transducers are the sensors used in nearly all fast-response aerodynamic probes because of their ability to accurately measure the steady and unsteady pressure. They are also used because of their small size (less than 1 mm). As a result, most probes are measuring some combination of total and static pressure. The pressure transducers are also slightly temperature sensitive.

The design of fast-response aerodynamic probes have taken the form of wedge, cylinder, and Pitot probe designs. They often contain multiple pressure transducers in order to measure multiple flow variables at the same time. Figures 2.11 and 2.12 show two of these designs. In order to maintain the frequency bandwidth needed, the pressure transducers are placed as close to the pressure tap as possible, often mounting the sensor on or just below the probe surface. Any cavity between the measurement location and sensor itself can decrease frequency bandwidth by the signal lag and cavity resonance. The size and placement of the pressure transducers in the probe head places severe restrictions on the spatial resolution and probe design.

Most fast-response aerodynamic probes measure some component of the flow velocity vector by comparing multiple instantaneous pressures to a calibration map.
Figure 2.11. Wedge Design Fast Response Aerodynamic Probe (from [7])

Figure 2.12. Cylindrical Design Fast Response Aerodynamic Probe (from [5])
Three hole probe designs are used to measure two velocity components. To measure the third velocity component, at least one additional measurement is needed. These calibrations are difficult and time consuming since they require realistic steady and unsteady variation of individual independent flow variables.

Finally, Kupferschmied et al. compares the data measured with a three-hole straight cylindrical fast-response aerodynamic probe to other measurement techniques. The time averaged measurements for total pressure and flow angle from the fast-response probe was found to be in good agreement with the pneumatic probe measurements, implying that the fast-response probe design was effective in quantitatively measuring the steady component of the flow variable. The unsteady flow velocity measured by the fast-response probe was compared to LDV measurements of flow velocity. Both methods were able to resolve the blade wake structure implying that the fast-response probe was effective in qualitatively measuring the unsteady component of the flow variable.

2.2.4 Fast Response Entropy Probe

Mansour at el. describe the development of a measurement technique to collect time resolved entropy measurements in a high speed turbomachinery flow. The guiding motivation was to study loss mechanisms in turbomachines. Entropy generation is considered a natural measurement for aerodynamic loss and is a function of total temperature ($T_t$) and total pressure ($P_t$) shown as

$$\Delta s = c_p \ln \frac{T_t}{T_{ref}} - R \ln \frac{P_t}{P_{ref}}.$$  \hspace{1cm} (2.14)

The problem of measuring unsteady entropy can be broken down into measuring total temperature and total pressure with high temporal resolution, on the order of tens of kHz for a turbomachine with a blade passing frequency of a few kHz.
The unsteady total temperature sensor contains two thin film resistance thermometers operating at different constant currents. The heat balance for both thin film resistance thermometers is

\[ \dot{q}_{\text{conv}1} = h(T_t - T_{f1}), \]  \hspace{1cm} (2.15)  
\[ \dot{q}_{\text{conv}2} = h(T_t - T_{f2}). \]  \hspace{1cm} (2.16)  

By assuming the same total temperature \((T_t)\) and convection coefficient \((h)\) at both sensors, the total temperature can be solved explicitly using

\[ T_t = \frac{\dot{q}_{\text{conv}1}(T_{f2} - T_{f1})}{\dot{q}_{\text{conv}1} - \dot{q}_{\text{conv}2}}. \]  \hspace{1cm} (2.17)  

The heat flux due to Joule heating was balanced out by heat flux due to convection and conduction

\[ \dot{q}_{\text{conv}} = V I - \dot{q}_{\text{cond}}. \]  \hspace{1cm} (2.18)  

Joule heating was calculated based on the measured voltage \((V)\) and current \((I)\). The conduction heat flux was modeled by an unsteady heat conduction model which was the limiting factor on the frequency response of the unsteady total temperature sensor.

The unsteady total pressure sensor contains a silicon diaphragm pressure transducer directly beneath a static pressure tap. The pressure transducer had a frequency response of up to 48 kHz. Mounting the transducer inside the probe head effectively removed any transmission line errors. Figure 2.13 shows a picture of the fast response entropy probe head.

The fast response entropy probe was used to measure flow fields behind a centrifugal compressor, in a film cooling facility, and behind an axial turbine. The flow features shown by the fast response entropy probe are qualitatively consistent with
theory and other measurement techniques. The probe was capable of collecting phase averaged entropy generation plots for compressors and turbines with blade passing frequencies between 2.5 and 3 kHz.

The uncertainty in the measurements was dependent on the probes’ operating conditions. The calculated uncertainty for the fast response entropy probe was around ±2.5 percent relative uncertainty in the time average total temperature measurement for operation in several different facilities. Although the fast response entropy probe was compared to thin film gauge temperature measurements, there was little comparison of results between the probe and other measurement techniques. The overall uncertainty of the phase resolved measurements was never effectively quantified.

2.2.5 Aspirating Probe

Ng and Epstein [41] developed an aspirating probe to measure unsteady total temperature and total pressure for turbomachinery flows. Figure 2.14 shows a diagram of the aspirating probe design. The probe contains two closely spaced parallel hot-wires inside the probe tube. There is a contraction behind the wires that chokes when the probe is connected to vacuum.

The hot-wires are operated in constant temperature mode at different overheats,
taking advantage of the hot-wire Reynolds and temperature sensitivities. The hot-wire functional relationship proposed by Collis and Williams [8] is used to relate the hot-wire voltage to the Reynolds number and fluid total temperature. The choked orifice sets the Mach number at the throat and reduces the number of one dimensional flow variables down to two for a given area ratio between the throat and the hot-wire measurement plane. The two hot-wire voltages \( E_i \) can be used to solve for the total temperature and total pressure

\[
E_i^2 = \left( C_i \left( \frac{P_i}{\sqrt{T_i}} \right)^{n_i} + D_i \right) (T_{w_i} - rT_i).
\]  

(2.19)

The aspirating probe relies on spatially uniform flow at the hot-wires as well as different wire operating temperatures in order to decouple the total temperature and total pressure from the hot-wire voltages. The aspirating probe was calibrated using a heated jet with Mach numbers between 0.05 and 0.9. The various steady state error sources were studied as well. Day-to-day hot-wire calibration drift was approximately 0.2 percent. The two hot-wire calibrations collapsed to within 4 percent average error using the Collis and Williams functional form. Misalignment between the flow and the
probe was negligible within a 15 degree acceptance angle. The steady error sources typically produced a 1.6 percent uncertainty in total temperature and a 2.6 percent uncertainty in total pressure. In order to help validate the unsteady measurements, the probe was placed behind a cylinder with vortex shedding and the power spectra were compared. There was no noticeable attenuation up to 8 kHz, which was the highest shedding frequency generated. The frequency response was measured to be 20 kHz, based on the highest visible harmonics of the blade passing frequency.

2.2.6 Hot-wire/Hot-Film Measurements in Turbomachinery

Hot-wire based measurement techniques have been used in turbomachinery flows since the 1970’s. However, their usefulness has been largely restricted to qualitative measurements in high speed rigs and quantitative measurements in low speed experiments where the hot-wire relationships simplify. For example, some of the most extensive use of hot-wire techniques to study turbomachinery flows in open literature has been performed at Penn State by Lakshminarayana [31], however it is important to note that these were conducted in low speed turbomachinery rigs [1]. In low speed turbomachinery rigs, the frequency response demands on the sensor are greatly reduced. Additionally, the flow can be treated as incompressible and isothermal. The hot-wire voltage can be directly related to velocity without having to consider the effects of fluid temperature or density in either the time average or instantaneous voltage. Wire breakage is also not a significant concern for low speed rig testing due to the reduced force on the wire.

Flows in high speed turbomachines are fundamentally different from a hot-wire measurement standpoint. Reynolds number, Mach number, overheat, and flow direction all impact a hot-wire response in transonic flows. As an order of magnitude estimate, the total temperature variation across a typical high speed compressor rotor is approximately 15 degrees Kelvin (based on [41] and [21]). Even for hot-wires
operating at very high overheats, the hot-wire response is not invariant to temperature variations of that amplitude. Japikse [20] cites an absolute accuracy of velocity measurements of ±10% for hot-wires in transonic turbomachinery flows.

Methods do exist for using hot-wires in highly unsteady compressible non-isothermal flows to measure temperature and velocity components, most notably the multiple overheat hot-wire method and the yawed slanted hot-wire method. These methods are useful for decoupling the different flow variables from the hot-wire voltage. These methods are not commonly used because they measure mean flow quantities. Hot-wires are used because of their high frequency response, which is not necessary for measuring time average temperature and velocities. If frequency response was not an issue, there exist many reliable and robust methods for measuring time average temperature and velocity components. The high blade passing frequencies demand sensors with high frequency responses. By using phase locked ensembles, the temperature and velocity components can be decoupled using these methods on the phase locked mean values. Hot-wires, hot-films, and fiber-film probes can all be used with these methods.

Johnston and Fleeter [21] used a hot-film anemometer to measure the total temperature field behind a high speed compressor rotor using the multiple overheat method. A hot-film was used in order to increase the robustness of the probe. The heat conduction effect of the film substrate was known to reduce the frequency response of the sensor, but this effect was not quantified. The hot-film was calibrated using a heated uniform jet in which flow velocity, fluid temperature, fluid density, hot-film overheat, and hot-film orientation could all be changed. The hot-wire sensitivities were calibrated over a range of values. The hot-film was then traversed behind a high speed compressor rotor at three different overheats. Different data reduction procedures were used to decouple the phase locked average total temperature and mass flux from the hot-film voltages shown in Figures 2.15 and 2.16.
Figure 2.15. Phase Locked Average Total Temperature (from [21])

Figure 2.16. Phase Locked Average $\rho U$ (from [21])
Hsu and Wo \[17\] measured all three components of phase locked average velocity behind an axial compressor using the slanted hot-wire method. A single slanted hot-wire was operated at different yaw angles to decompose the phase locked average velocity components behind a low speed compressor.

2.2.7 Cold Wires

Cold wires use a hot-wire probe operated without any overheat as a resistance thermometer. The cold wire is used as a resistor in a Wheatstone bridge circuit. Voltage across the Wheatstone bridge changes proportionally with the change in average wire temperature. The cold wire is not sensitive to velocity or flow direction. The frequency response of the cold wire is limited by the thermal inertial of the wire. Frequency roll off begins at frequencies of only a few kHz for even very small platinum wires \[46\]. Cold wires do not have the frequency response necessary to resolve the blade to blade variations in the phase locked average temperature field.

2.2.8 Yawed Phase Peak Method

Japikse \[20\] mentions one specific use of a hot-wire anemometer in turbomachinery flows which has been effectively carried out to measure phase locked average swirl angle in high speed turbomachinery flows. A hot-wire was placed axially inside a 120 degree arc cylindrical shield in order to increase the yaw sensitivity of the hot-wire. When the incoming flow is aligned with the edges of the shield, the flow can pass over the hot-wire and the hot-wire response is at a maximum. When flow is not aligned with the edges of the cylindrical shield, the flow must deflect around the shield and the hot-wire is effectively seeing stagnant air. The hot-wire response with low velocities is much lower. Figure 2.17 shows a diagram of this process.

The mean flow direction can be determined by yawing the probe and comparing the relative hot-wire response. The yaw angle at which the hot-wire response is a
maximum will correspond to the direction of the mean flow. By using this technique in a turbomachinery flow, the phase locked mean hot-wire responses can be compared across different yaw angles. Figure 2.18 shows phase locked traces of hot-wire voltage for a constant yaw angle in a turbomachinery test. The phase location of the peaks correspond to phase location of the current probe yaw angle.
The methodology is actually far more universal and applicable than presented by Japikse. In order for this yawed phase peak method to work, the sensor only needs two things are needed: sufficient frequency response and sufficient yaw sensitivity. Only relative transducer outputs are compared, meaning it is never actually necessary to calibrate against velocity magnitude. Sensor non-linearity, high frequency attenuation, calibration drift, and temperature drift are largely irrelevant as potential error sources. The phase peak method has far fewer potential uncertainties allowing it to be used to help validate other phase resolved measurements in turbomachinery flows. No error analysis nor additional references were presented by Japikse.
CHAPTER 3

FACILITIES

3.1 ND-TAC Compressor Test Facility

The Notre Dame Transonic Axial Compressor (ND-TAC) facility is used for high-speed single stage compressor experiments. It is an open loop tunnel shown in Figure 3.1. The drive system includes a 300kW electric motor, an interchangeable speed increasing gear set, and an actively controlled magnetic bearing system. The maximum rotor speed is approximately 17000 rpm. Valves at the inlet and exit of the compressor flow path allow for throttling and inlet suppression to precisely control the operating conditions. The facility features advanced measurement capabilities including a torque-meter, tachometer, Venturi flow meter, telemetry system, tip clearance sensors, and an EDAS real-time high speed data acquisition and monitoring system. The high hub-to-tip ratio compressor stage is representative of an engine relevant rear stage high pressure compressor. Probes were traversed radially in the axial gap between the rotor and stator bladerows. Rotor blade passing frequencies at the design operating condition were approximately 8 kHz.

3.2 ND-FSCC Compressor Test Facility

The Notre Dame Front Stage Core Compressor (ND-FSCC) facility is used for high-speed, single-stage compressor experiments. It is a semi-closed loop tunnel. The drive system includes a 520kW electric motor, a speed increasing gearbox, and an actively controlled magnetic bearing system. The facility can be used for aerody-
dynamic performance and aeromechanics measurements of modern, low hub-to-tip ratio compressors and fan hub frame ducts. The drive system has a maximum rotational speed over 27000 rpm. Compressor stages can include individually controllable active variable stagger inlet guide vanes and stators. The compressor stage features a low hub-to-tip ratio compressor representative of an engine relevant front stage core compressor. Probes were traversed between the rotor and stator bladerows. The compressor was operated at part speed in various off-design conditions, including stable rotating stall.

3.3 ND-TRT Turbine Test Facility

3.3.1 ND-TRT Overview

The Notre Dame Transonic Research Turbine (ND-TRT) facility is a continuous operation research turbine designed for aerodynamic and heat-transfer experiments. It is a semi-closed loop tunnel shown in Figure 3.3 with the turbine shaft coupled with
the compressor motor to re-use the energy extracted by the test turbine. The TRT can operate single stage turbines with pressure ratio up to 2.4 and rotor rotational speed up to 15000 rpm. The turbine rotor is levitated on an active magnetic bearing system which allows for precise clearance control and nearly tare-free torque measurements. Previous experiments have demonstrated continuous running with tip clearances as small as 0.15 mm. Repeatability in the measurement of stage efficiency including a full hardware re-build has been demonstrated to be better than ± 0.15 percent.
3.3.2 High Pressure Turbine Stage Description

The high pressure turbine stage (HPT) features an un-shrouded rotor of medium blade loading representative of an engine relevant high pressure turbine stage. The approximate blade passing frequency at the test operating point was around 12 kHz. The stage test design operating point information is shown in Table 3.1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet $T_i$</td>
<td>210</td>
<td>degree F</td>
</tr>
<tr>
<td>Rotor Speed</td>
<td>10050</td>
<td>RPM</td>
</tr>
<tr>
<td>$P_i$ Ratio</td>
<td>2.200</td>
<td>–</td>
</tr>
</tbody>
</table>

3.3.3 Low Pressure Turbine Stage Description

The low pressure turbine (LPT) stage features a shrouded rotor of very high blade loading representative of an engine relevant low pressure turbine stage. A cross section of the LPT stage is shown in Figure 3.4. One unique feature of the LPT rotor is that one of the blade trailing edges was damaged and removed during manufacturing. This damaged blade is shown in Figure 3.5.
Figure 3.4. Low Pressure Turbine Turbine Stage (From [43])

Figure 3.5. Low Pressure Turbine Damaged Blade
3.4 Calibration Jet Facility

3.4.1 Calibration Jet Overview

The schematic of the calibration jet facility is shown in Figure 3.6. The calibration jet facility was a novel design enabling for rapid, accurate, and precise calibrations while easily transitioning to performing traverses and measurements in the various turbomachinery rigs. The calibration jet is comprised of several components: the calibration jet assembly and the controls. The flow was driven by the pressure difference between the supply pressure and exhaust pressure. Depending on the facility and desired flow conditions, this pressure difference can be driven by bleeding air into or out of the turbomachinery rig during operation or supplied by a compressed air system. The LPT and HPT rigs were designed to operate at sub atmospheric pressure, therefore the supply pressure line was open to atmosphere and the exhaust pressure was connected to the turbine facility flow path just upstream of the compressor.

![Figure 3.6. Calibration Jet Facility Schematic](image)

A cross section of the calibration jet assembly is shown in Figure 3.7. The calibration jet assembly mounts directly to the various turbomachinery rig measurement locations
enabling the hot-wires to be traversed behind the rig for acquiring data and retracted into the jet assembly for calibration without changing any part of the measurement system or rig operating point. A linear and rotary traverse mounts to the top of the assembly and moves the hot-wire probe in the radial and yaw directions. All connections are sealed with o-rings to prevent air leakage. A actuator mounted in the base of the Calibration Jet assembly rotates to seal off the calibration jet from the rig flow during calibration.

The calibration jet inlet contains a honeycomb and a fine wire mesh screen for flow conditioning. The inlet has a thermocouple, Kiel probe, and static pressure tap to measure $T_r$, $P_r$, and $P$ respectively. The nozzle has a smooth radius transition geometry to accelerate the flow with a jet diameter of 0.375 inches. Static pressure taps on the face of the nozzle plate are used to measure the pressure at the jet exit. The hot-wire probe is located less than half the jet diameter downstream of the exit. Due to space limitations, the assembly exit flow comes out of the back of the plenum radially using 3-4 smaller diameter hoses with a combined exit flow area nearly equal to plenum cross section. A screen near the back of the plenum further reduces the upstream impact of the non-uniform exit flow on the calibration jet.

The calibration jet controls enable compressible hot-wire calibrations over a wide range of flow conditions. Precision valves upstream and downstream of the calibration jet assembly are controlled individually allowing for the velocity and pressure of the calibration jet flow to be controlled independently. Inline electric air heaters upstream of the calibration jet assembly are controlled through a temperature controller using solid state relay. The heaters and temperature controller allow for precise and stable control of the calibration jet flow temperature.
Figure 3.7. Calibration Jet Assembly Cross Section
3.5 Shock Tube Facility

3.5.1 Shock Tube Overview

The shock tube facility is shown in Figure 3.8 and was designed for dynamic calibration of pressure sensor systems by generating nearly instantaneous step change in pressure across the initial shock wave. A 0.001” mylar membrane was scored in a cross pattern to help ensure a clean diaphragm burst. The membrane covers one end while the pressure inside the shock tube is evacuated down to approximately 0.1 to 0.3 psia at the other end using a vacuum pump. Pressure transducers measure the pressure inside the shock tube as well as outside. A solenoid actuator was used to hit the membrane and initiate the diaphragm burst in a repeatable manner.

![Shock Tube Instrumentation Section](image)

Figure 3.8. Shock Tube Facility

The shock tube was composed of three tubes that can be assembled in any order depending on the distance required for the shock wave to coalesce. The time between
the initial shock wave and the reflected shock was the testing time. One of the tubes was designed for measurements. The measurement tube has threaded holes around the circumference at various equally spaced axial locations for flush mounted silicon diaphragm pressure transducers. It also has larger threaded holes spaced axially for mounting pressure probes in the shock tube. The EDAS realtime high speed data acquisition system was used to monitor and record the measurements.

3.6 High Speed Instrumentation

An AA Labs AN-1003 constant temperature hot-wire anemometer system with high frequency tuning option was used to power and control the hot-wire probes. Hot-wire probes used for testing were Dantec 55P11 miniature straight normal hot-wire probes, 55P12 miniature slanted hot-wire probes, 55P13 miniature parallel hot-wire probes, and 55R02 slanted fiber-film probes. A National Instruments PXI system with NI-PXI-6133 S-Series data acquisition cards with 8 channels and 2.5 Ms/s capability was used to acquire high speed simultaneous data. Rig testing data was acquired at 250 kHz for between 10-15 seconds per data point. This corresponded to more than 1000 rotor revolutions for statistical convergence of the phase locked ensemble averages based on the ambient noise levels. Custom Labview software controlled traverse motions and data acquisition.
CHAPTER 4
DEVELOPMENT AND METHODOLOGY

4.1 Multiple Overheat Hot-wire Method

Hot-wires operating at a constant temperature are most sensitive to wire normal effective velocity magnitude and fluid total temperature. At high overheats, hot-wires are more sensitive to velocity. At low overheats, hot-wires are more sensitive to temperature. The multiple overheat method takes advantage of the different relative sensitivities at different overheats to decouple temperature from velocity. Hot-wires were operated in constant temperature mode to maximize the sensor bandwidth.

The multiple overheat method uses a single wire hot-wire probe operating in a statistically stationary flow. The hot-wire is operated at a range of nominal overheat ratios, typically between 1.1 and 2.0 with the same orientation. The phase locked ensemble average (PLA) voltages from the wire at each nominal overheat (\(\hat{E}_i\)) were used to decouple the PLA total temperature \(\hat{T}_t\) from the PLA effective wire Reynolds number (dimensionally \(\hat{\rho}_t\hat{U}_w\)).

4.1.1 Methodology and Implementation

A simple flowchart for processing the multiple overheat data is shown in Figure 4.1. The multiple overheat method function takes in the PLA total pressure processed from the unsteady total pressure probe and the PLA hot-wire voltage from the hot-wire operating at different overheats with the same orientation in the same flow. The system of non-linear calibration equations are solved for the corresponding PLA \(T_t\) and \(\rho_t\hat{U}_w\). The function outputs the PLA total temperature field.
The generalized solution procedure for the multiple overheat hot-wire method requires solving a system of $N$ non-linear equations corresponding to the different hot-wire overheat settings for each phase location shown as

$$ F(x) = y. \quad (4.1) $$

The flow variables

$$ x = (\rho_t U_w, T_t, P_t). \quad (4.2) $$

The calibration function

$$ F = \begin{pmatrix} f_1() \\ \vdots \\ f_N() \end{pmatrix} \quad (4.3) $$

maps the flow variables into the corresponding hot-wire response $y$ for each wire operating condition (wire resistance setting and flow orientation) where

$$ y = \begin{pmatrix} E_1^2 \\ \vdots \\ E_N^2 \end{pmatrix}. \quad (4.4) $$

The solution procedure numerically solves the system of non-linear equations by
minimizing the magnitude of the residual error

\[ e = y_m - y_p, \]  

(4.5)

between the measured phase locked ensemble averaged hot-wire responses \( y_m \) and the predicted hot-wire responses from the calibration function using the guess for the phase locked ensemble average flow variables \( y_p \). An initial guess for the flow variables is supplied and the numerical method iterates until the error is within the specified convergence tolerance. Many numerical methods (Newton’s method, Levenberg-Marquardt algorithm, Trust-Region-Reflective algorithm, Trust-Region-Dogleg algorithm, etc...) can be used to solve this system of equations. Unlike many previous explicit solution methods presented in [41] and [21], no restrictions were needed for the calibration functional form in order to make the explicit solution algebraically tractable. This methodology was initially presented in [19].

The number of hot-wire overheat settings \( N \) must be greater than or equal to the number of flow variables in order for the system of equations to be solvable. If they are equal, the system of equations has a unique solution corresponding to a single point of intersection between the locus of possible flow variables. If \( N \) is greater than the number of flow variables, the system is overdetermined and there will be a local minimum in the error, corresponding to the flow variables. For an overdetermined system, the residual error is effectively a result of the elemental and implementation errors present.

The multiple overheat solution methodology is shown graphically in Figure 4.2. For constant pressure, a hot-wire voltage is a function of two flow variables (\( T_i \) and effective wire Reynolds number \( Re_w \) are shown). There are a locus of possible flow variable combinations that can produce a given hot-wire voltage. These combinations are independent for different wire temperatures (nominal overheats) due to
the different sensitivities. If the hot-wire is operated in a statistically stationary
flow the intersection of the average voltage curves yields the average flow conditions.
For an overdetermined system (more overheats than flow variables), the solution is
effectively an averaged intersection point among all the curves. For phase locked
average voltages, the multiple overheat method yields the phase locked average flow
quantities.

Figure 4.2. Multiple Overheat Methodology: Locus of Possible $Re_w$ and $T_t$
for Constant Voltage

For the best overall precision, the curves for different overheats should be closer
to orthogonal. The idealized case with one overheat being only sensitive to $T_t$ and
one overheat only sensitive to $\rho_t U_w$ would appear as a horizontal and vertical locus
of points. The sensitivity of the multiple overheat method to the number and values
of overheat selected is shown in Table 4.1 for data in a high speed low pressure axial
turbine flow with blade passing frequency of approximately 10 kHz and total pressure and temperature ratios of approximately 1.75 and 1.155 respectively. The true PLA $T_t$ and $\rho_t U_w$ field is not known, so the result from the multiple overheat method using all the overheat (1.1, 1.3, 1.5, 1.7, and 1.9) was used as the reference for comparison. The differences between the output $T_t$ and $\rho_t U_w$ of the multiple overheat method and the reference valued with different combinations of overheat data as inputs are shown in Table 4.1. These differences are used as estimates of the various uncertainties. The time average and phase locked steady uncertainties in both $T_t$ and $\rho_t U_w$ are shown in units of Kelvin and $\frac{kg}{m^2 s}$ respectively. The time average uncertainties are divided into absolute and relative quantities, effectively the average offset and average difference in the time average profile. The phase locked steady differences are difference between the variance of the PLS quantity and the reference ($\bar{T}_t = 1.4$ and $\bar{\rho_t U_w} = 49.7$).

One significant finding is how effectively the highest and lowest overheat limit the estimated uncertainty. As long as the data from overheats of 1.1 and 1.9 are used, the number of additional overheat data included has relatively little impact on the estimated uncertainty. The absolute time average uncertainty estimate is less than 1 degree Kelvin and 2.5 $\frac{kg}{m^2 s}$. The relative time average uncertainty estimate is less than 0.2 degree Kelvin and 1 $\frac{kg}{m^2 s}$. The PLS uncertainty estimates are an order of magnitude lower as well. The lowest overheat used (1.1) greatly improved the time average accuracy, while not significantly reduce the frequency response and impacting the PLS accuracy.

4.1.2 Compressible Hot-Wire Calibration

The compressible hot-wire calibration effectively relates one dependent non-dimensional variable with three independent non-dimensional variables. The compressible hot-wire calibration should span the range of the 3 independent flow variables for the
TABLE 4.1

DIMENSIONAL DIFFERENCE BETWEEN MEASURED QUANTITIES
FROM THE MULTIPLE OVERHEAT HOT-WIRE METHOD IN LOW
PRESSURE TURBINE USING VARIOUS OVERHEATS.

<table>
<thead>
<tr>
<th>Overheats Used</th>
<th>$\mathcal{T}_{abs}$</th>
<th>$\rho_t U_{wabs}$</th>
<th>$\mathcal{T}_{rel}$</th>
<th>$\rho_t U_{wrel}$</th>
<th>$\tilde{T}_t$</th>
<th>$\rho_t U_w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1, 1.3, 1.5, 1.7, 1.9</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1.1, 1.3, 1.7, 1.9</td>
<td>0.0747</td>
<td>-0.7739</td>
<td>0.0189</td>
<td>0.1701</td>
<td>0.0031</td>
<td>-0.1895</td>
</tr>
<tr>
<td>1.1, 1.5, 1.9</td>
<td>0.6026</td>
<td>2.2626</td>
<td>0.1518</td>
<td>0.7826</td>
<td>-0.0498</td>
<td>0.1651</td>
</tr>
<tr>
<td>1.3, 1.5, 1.7, 1.9</td>
<td>-2.3086</td>
<td>-10.5310</td>
<td>0.4311</td>
<td>2.0931</td>
<td>0.2207</td>
<td>-1.0538</td>
</tr>
<tr>
<td>1.1, 1.9</td>
<td>0.8548</td>
<td>0.4701</td>
<td>0.1572</td>
<td>0.9583</td>
<td>-0.0437</td>
<td>-0.2451</td>
</tr>
<tr>
<td>1.3, 1.9</td>
<td>-2.0581</td>
<td>-10.2092</td>
<td>0.5874</td>
<td>2.8579</td>
<td>0.2917</td>
<td>-1.0889</td>
</tr>
<tr>
<td>1.5, 1.9</td>
<td>-3.9449</td>
<td>-16.8144</td>
<td>0.7493</td>
<td>3.5267</td>
<td>0.2102</td>
<td>-2.8023</td>
</tr>
<tr>
<td>1.7, 1.9</td>
<td>-2.8254</td>
<td>-12.1069</td>
<td>2.929</td>
<td>10.9195</td>
<td>2.1657</td>
<td>2.2062</td>
</tr>
</tbody>
</table>

The calibration jet facility is designed to calibrate across these 3 degrees of freedom by varying flow velocity, total temperature, and total pressure independently using the upstream and downstream valves as well as the inlet air heater.

The calibration function of 3 variables is fit to the calibration data. No consensus exists for the general compressible hot-wire calibration functional form. As long as the calibration function accurately fits the calibration data over the operating range of interest, the calibration function will accurately map the flow conditions with the hot-wire voltage. For simplicity, a separable form of the calibration function was selected and is consistent with many of the commonly used calibration functional forms used in literature. In non-dimensional form (using $Nu$, $Re$, $\tau_w$, and $Kn$), the...
calibration function is separable form is

\[ Nu = f_1(Re) f_2(\tau_w) f_3(Kn). \]  \hspace{1cm} (4.6)

Dimensionally the separable calibration function is

\[ E^2 = f_1(\rho U_w) f_2(T_t) f_3(P_t) \]  \hspace{1cm} (4.7)

The effect of each independent flow variable is investigated by performing a calibration across one variable while holding the other two constant, shown in Figure 4.3. The reference condition shown is useful for normalizing the hot-wire voltage to compare the relative sensitivities to each independent variable.

![Figure 4.3. Non-Dimensional Calibration Space](image)

Figure 4.3. Non-Dimensional Calibration Space
A typical calibration procedure involves estimating the approximate flow conditions in order to calibrate over the correct range for a constant wire temperature. Calibration data is acquired and the flow conditions are varied in all three degrees of freedom around the expected approximate flow conditions and over the expected range of flow conditions. The selected calibration function for a constant wire temperature is

\[ E^2 = f_1(\rho_t U_w) f_2(T_t) f_3(P_t) = (A + B \cdot (\rho_t U)^C) (T_w - T_t) (a + b \cdot P_t^c). \quad (4.8) \]

The calibration function Equation 4.8 is fit to the data using the parameters \( A, B, C, T_w, a, b, \) and \( c. \)

4.1.2.1 Dimensional Calibration Effects

It is useful to use hot-wire calibrations in dimensional quantities for several reasons: the dependent variable becomes only a function of the hot-wire output signal \( (E^2 \) as opposed to \( Nu), \) one independent flow variable becomes \( P_t \) which is a direct output of the unsteady total pressure probe, and another independent flow variable becomes \( T_t \) which is the desired output of the multiple overheat method.

In dimensional form, the Reynolds number calibration data is shown in Figure 4.4. The hot-wire voltage has been normalized by the hot-wire voltage at a representative flow condition for each wire temperature. This allows for easy comparison of the relative sensitivities to flow conditions. The effects of this normalization for comparison is shown for the Reynolds number calibration in dimensional form by

\[ \frac{E^2}{E_{ref}^2} = \frac{f_1(\rho_t U_w) f_2(T_w)_{ref} f_3(P_t)_{ref}}{f_1(\rho_t U_w)_{ref} f_2(T_w)_{ref} f_3(P_t)_{ref}} = \frac{f_1(\rho_t U_w)}{f_1(\rho_t U_w)_{ref}}. \quad (4.9) \]
The functional form used is

\[ f_1(\rho_i U) = A + B \ast (\rho_i U)^C. \]  

(4.10)

Figure 4.4. \( \rho_i U \) Calibration

In dimensional form, the overheat ratio calibration data is shown in Figure 4.5. The hot-wire voltage has been normalized by the hot-wire voltage at a representative flow condition for each wire temperature. The relative sensitivities to \( \rho_i U \) and \( T_i \) are shown to be similar over the expected ranges. Lower nominal overheats \((OH)\) are more sensitive to temperature changes than higher nominal overheats.

Figure 4.6 shows the relationship between the hot-wire temperature \((T_w)\) and the wire resistance \((R_w)\) that sets the nominal overheat. The resistance setting of the anemometer is the total resistance \((R_{total})\), which includes the wire resistance and the
lead resistance \((R_L)\) of the contacts at the welds between the wire and the prongs of the hot-wire probe. The resistances are combined in series \(R_{total} = R_w + R_L\). The wire temperature calculated as a hot-wire calibration parameter using the calibration data was compared with the estimated wire temperature using Equation 4.11 in Figure 4.6. The difference between the estimated wire temperature from the theory and the calibration best fit can be tens of degrees Kelvin. The wire temperature cannot be accurately estimated with enough precision from the wire temperature-resistance relation

\[
R_w = R_{ref}(1 + \alpha_{ref}(T_w - T_{ref})). \tag{4.11}
\]

The wire resistance setting \((R_w)\) corresponding to the wire temperature setting \((T_w)\) is related to the wire resistance and temperature at a reference condition \((R_{ref} \text{ and } T_{ref})\), usually 293 degrees Kelvin. The linear approximation of the temperature coefficient of resistance evaluated at reference conditions \((\alpha_{ref})\) is a property of the material and the cross sectional area of the wire or film. The nominal overheat set-
Figure 4.6. Hot-wire Probe Wire Temperature-Resistance Relation

ting is therefore \( \frac{T_w}{T_{ref}} \approx \frac{R_w}{R_{ref}} \). Uncertainties in the contact resistances of the welds between the wire and prongs as well as the cross sectional area of the wire increase the uncertainty in a-priori estimating the wire temperature. As a result, the wire temperature is treated as an unknown parameter that is evaluated from this dimensional calibration data. The functional form

\[
    f_2(T_t) = T_w - T_t
\]

was used. The wire temperature \( T_w \) is the calibration parameter using this functional form. \( T_w \) was evaluated by fitting the overheat calibration data in the dimensional form \( \frac{E^2}{E_{ref}^2} = \frac{T_w - T_t}{T_w - T_{ref}T_t} \).

In dimensional form, the Knudsen number calibration data is shown in Figure 4.7. The relative sensitivity to \( P_t \) is clearly lower than either \( \rho_tU \) or \( T_t \), but not insignificant over the expected range. Higher nominal overheats (i.e. wire temperature) are more
sensitive to pressure changes. The power law functional form was used

\[ f_3(P_t) = a + b \cdot P_t^c. \]  \hspace{1cm} (4.13)

![Figure 4.7. $P_t$ Calibration](image)

4.1.2.2 Non-Dimensional Calibration Effects

It can be useful to look at the non-dimensional hot-wire calibration relationships to highlight the different effects. Reynolds number, overheat ratio, and Knudsen number were the three non-dimensional quantities selected which spanned the non-dimensional calibration space. The Reynolds number effects were investigated by calibrating across a range of $\rho_t U$ while holding $T_t$ and $P_t$ constant. The power law functional form \( f(Re) = a + bRe^c \) was fit to the calibration data as shown in Figure
This is a commonly used functional form in literature and is based on King’s Law.

The overheat ratio effects were investigated by calibrating across a range of $T_i$ while holding $\rho_i U$ and $P_i$ constant. The linear functional form $(f(\tau_w) = a + b\tau_w)$ was fit to the calibration data as shown in Figure 4.8. The Overheat ratio vs Nusselt number relationship shows only the temperature effects of the fluid properties in the flow (i.e. the effect of the change in fluid properties evaluated near the wire). The wire and flow temperature difference $T_w - T_i$ is in the Nusselt number approximation shown in Equation 2.2. As the air temperature near the wire increases, the fluid thermal conductivity increases. Over a small temperature range, this relationship is approximately linear. The fluid thermal conductivity is in the denominator in the Nusselt number definition, therefore Nusselt number should decrease linearly with
increasing overheat ratio as shown in Figure 4.9.

![Figure 4.9. Overheat Ratio Calibration (Nominal Overheat Ratio = 1.1)](image)

The Knudsen effects were studied by calibrating across a range of $P_t$ while holding $\rho_tU$ and $T_t$ constant. The linear functional form $(f(Kn) = a + bKn)$ was fit to the calibration data is shown in Figure 4.10.

4.2 Slanted Hot-wire Method

Hot-wires are most sensitive to the velocity component normal to the wire axis. Wires with different orientations in the same flow can be used to decompose the various average components of velocity. The slanted hot-wire method rotates a slanted hot-wire probe, shown in Figure 4.11 to different orientations. The averaged voltages are then used to calculate the average velocity components.
Figure 4.10. Knudsen Number Calibration (Nominal Overheat Ratio = 1.9)

(b) Straight prongs, sensor at angle of 45° to probe axis.

Figure 4.11. Slanted Hot-wire Probe (From [13])
A single slanted hot-wire was operated sequentially at several different wire orientations with the same overheat. In compressible non-isothermal flows, the PLA $T_t$ from the multiple overheat method and the PLA $P_t$ from the unsteady total pressure probe were used to compensate the PLA voltage of each phase location at each wire orientation for the fluid properties. The PLA voltages were transformed into PLA effective wire velocities at each phase location. The slanted hot-wire equations were used to decouple the PLA velocity components from the PLA effective wire velocity with different orientations.

4.2.1 Methodology and Implementation

A flowchart for processing the slanted hot-wire data is shown in Figure 4.12. The PLA $P_t$, $T_t$, and hot-wire voltage from different wire orientations are inputs into the Fluid Thermodynamics Property Corrector function. The PLA $P_t$ comes from the unsteady total pressure probe data. The PLA $T_t$ and hot-wire voltage come from the hot-wire data at different overheats and probe yaw angles respectively. The PLA wire normal effective velocity $U_w$ for each hot-wire probe yaw angle is the output of the function. The Fluid Thermodynamics Property Corrector function solves the system of non-linear hot-wire calibration equations using the same methodology as the Multiple Overheat Method function, with two differences: the PLA $T_t$ is a known quantity and the overheat is held constant so that only one hot-wire calibration function is used. The Fluid Thermodynamics Property Corrector function corrects the hot-wire voltage for the fluid properties and applies the hot-wire calibration to transform the PLA voltages into effective velocities for each probe yaw angle.

The PLA effective wire velocities for the hot-wire probe with different orientations in the same flow are inputs for the Slanted Hot-wire Method function. The system of non-linear equations are solved for the corresponding PLA velocity components. The PLA velocity component field is the output from the Slanted Hot-wire Method
The general solution procedure for the slanted hot-wire method requires solving a system of $N$ non-linear equations corresponding to the different slanted hot-wire orientations for each phase location shown as

$$
\mathbf{F}(\mathbf{x}) = \mathbf{y}. \quad (4.14)
$$

The velocity components

$$
\mathbf{x} = (U_r, U_\theta, U_z). \quad (4.15)
$$

The calibration function

$$
\mathbf{F} = \begin{pmatrix}
    f_1() \\
    \vdots \\
    f_N()
\end{pmatrix} \quad (4.16)
$$
maps the velocity components onto the corresponding effective velocity $y$ for each slanted hot-wire orientation where

$$y = \begin{pmatrix}
U_{w_1} \\
\vdots \\
U_{w_N}
\end{pmatrix}.$$  \hspace{1cm} (4.17)

The solution procedure numerically solves the system of non-linear equations by minimizing the magnitude of the error

$$e = y_m - y_p,$$  \hspace{1cm} (4.18)

between the measured phase locked ensemble averaged slanted hot-wire effective velocity $y_m$ and the predicted slanted hot-wire effective velocity from the calibration function and wire orientation using the guess for the phase locked ensemble average flow variables $y_p$. An initial guess for the flow variables is supplied and the numerical method iterates until the error is within the convergence tolerance (selected to be a value of $1^{-16}$ which is below the voltage resolution of the data acquisition system).

It is possible for there to be additional erroneous solutions to the system of non-linear equations. Since the slanted hot-wire equations are derived from trigonometric relations which are not monotonic, the slanted hot-wire method can have multiple solutions (local error minima). An example of this is shown in Table 4.2. Time average velocity components (in m/s) were calculated from the slanted hot-wire method with different initial guesses in the iterative numerical solver. Both solutions used the same data from the high pressure turbine and are converged solutions to the system of non-linear equations based on the convergence criteria. Only one of these solutions is physical. Due to this phenomena, it is very useful to have some other time average velocity component measurements or trusted computations to compare against and
use for supplying an initial guess. It can be seen that non-physical solutions tend to be very different from the physical solution, and can usually be identified and excluded fairly easily.


**TABLE 4.2**

**SLANTED HOT-WIRE METHOD: SOLUTION COMPARISON**

*(SENSITIVITY TO INITIAL GUESS)*

<table>
<thead>
<tr>
<th>Solution</th>
<th>$U_z$ (init)</th>
<th>$U_\theta$ (init)</th>
<th>$U_r$ (init)</th>
<th>$U_z$ (sol)</th>
<th>$U_\theta$ (sol)</th>
<th>$U_r$ (sol)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Physical</td>
<td>110</td>
<td>30</td>
<td>-10</td>
<td>107.97</td>
<td>24.88</td>
<td>-5.71</td>
</tr>
<tr>
<td>Non-physical</td>
<td>-30</td>
<td>50</td>
<td>100</td>
<td>-67.85</td>
<td>-67.64</td>
<td>44.77</td>
</tr>
</tbody>
</table>

An initial guess based on time average measurements reduces the chances that the numerical method will converge to a non-physical solution. The Slanted Hot-wire Method can also be made more robust and less susceptible to converging to non-physical solutions by aligning the hot-wire with the mean flow swirl angle and removing any slanted hot-wire data from orientations outside the acceptance angle of the hot-wire (typically a range of approximately $\pm 45$ degrees).

4.2.2 Calibration

The geometry and coordinate system for a slanted hot-wire probe aligned in the radial direction is shown in Figure 4.13. The slanted hot-wire probe has one prong longer than the other, causing the hot-wire to have a slant angle $\alpha$ relative to the probe axis. The hot-wire probe can be rotated about its axis over a yaw (or roll)
angle $\gamma$. If the probe is not inserted radially into the turbomachinery flow, it will require additional trigonometry to transform the probe relative coordinate system presented into the turbomachinery flow coordinate system.

The slanted hot-wire calibration equations are:

\begin{align*}
U_n &= U_z \cos \alpha + (U_r \cos \gamma - U_\theta \sin \gamma) \sin \alpha, \quad (4.19) \\
U_t &= -U_z \sin \alpha + (U_r \cos \gamma - U_\theta \sin \gamma) \cos \alpha, \quad (4.20) \\
U_b &= U_r \sin \gamma + U_\theta \cos \gamma, \quad (4.21) \\
U_w^2 &= U_n^2 + h^2 U_b^2 + k^2 U_t^2. \quad (4.22)
\end{align*}

They are a combination of the coordinate transformation between the probe and wire using trigonometry (Equations 4.19 through 4.21) and the commonly used equation for effective wire velocity 4.22. Equation 4.22 has two empirical parameters ($h$ and $k$) with established values of approximately 1 and 0.1 respectively.
A slanted hot-wire yaw calibration is shown in Figure 4.14. For this calibration, the hot-wire was left at a constant nominal overheat and the calibration jet operating conditions were held constant. The hot-wire probe was yawed relative to the jet. The normalized effective velocity ($\frac{U_{\text{wire}}}{U_{\text{jet}}}$) is shown over the range of probe yaw angles. The measured wire effective velocity from the hot-wire voltage is labeled Yaw Cal Data. The curves $\alpha = 20$ through $\alpha = 50$ degrees represent the wire effective velocity based on Equations 4.19 through 4.22 using the measured jet velocity and the probe yaw angle. The deviation of the data from the slanted hot-wire calibration equations in the range $\gamma = 0 - 45$ degrees is due to prong interference when the longer prong of the hot-wire probe is directly upstream and the wire is in its wake.

![Slanted Hot-wire Yaw Calibration](image)

**Figure 4.14. Slanted Hot-wire Yaw Calibration**

The two parameters varied and used to fit the calibration equations with the yaw calibration data are the slant angle $\alpha$ and the yaw offset angle $\gamma_{\text{offset}}$. The
slant angle takes into account the individual hot-wire probe geometry as well as the effects of any slack in the wire between the prongs. The yaw offset angle (measured to be 12.6 degrees) is a misalignment between the hot-wire probe holder and the traverse during installation. It shows up as the maximum when the entire velocity is perpendicular to the wire (ideally at $\gamma = 90$ degrees). From the curve fit, this maximum is offset at $\gamma = 102.6$ degrees. It is simply an offset that can be removed by resetting the coordinates in the traverse software. Outside the prong interference region, the slanted hot-wire equations fit the calibration data with an average error of less than 1.7 percent in effective wire velocity.

4.3 Unsteady $P_t$ Probe

A miniature silicon diaphragm pressure transducer was integrated into the head of a Kiel style total pressure probe in order to measure unsteady total pressure without sensitivity to local instantaneous velocity misalignment with the probe. The probe head is shown in Figure 4.15. For scale, the diameter of the probe head Kiel shroud is 3.175 mm. The miniature silicon diaphragm pressure transducer in the center of the probe head is 1.65 mm in diameter. The entire probe head is less than 5.1 mm long and the probe fits into a 6.35 mm diameter radial hole in the compressor or turbine casing.

4.3.1 Methodology and Implementation

Figure 4.16 shows a diagram of conceptually how flow misalignment can bias the measured pressure from the unsteady $P_t$ probe in an unsteady flow. The unsteady total pressure probe was designed to measure total pressure even when misaligned with the flow over a significant range (acceptance angle limits are approximately $\pm 25$ degrees). Outside these acceptance angle limits, the total pressure measurement is biased low. The probability density of the flow angle is shown centered around the
time average angle of 20 degrees. The distribution of the probability density function corresponds to the relative percentage of phase locations with a given PLA swirl angle. Case 1 corresponds to the unsteady $P_t$ probe being aligned with the time average flow. All the phase locations lie within the probe acceptance angle limits so there is no bias error. Case 2 corresponds to a small but significant misalignment (relative to the probe acceptance angle limits) between the probe and time average flow. The time average experiences a small bias error, but a small percentage of the phase locations experience a significant bias error. Case 3 corresponds to a significant misalignment where the alignment with the time average flow is outside the acceptance angle limits of the probe. The time average is biased significantly low. A large percentage of the phase locations experience a significant bias error.

To highlight the impact of the flow misalignment with the probe, the unsteady total pressure probe was traversed radially at a constant yaw angle of 0 degrees. The probe was traversed downstream of a high pressure turbine. The phase locked average pressure measurements from this traverse were compared with the reference phase locked average total pressure. The reference phase locked average total pressure field was calculated from the composite of five radial traverses with the yaw angle spanning ±60 degrees (probe aligned to -60, -30, 0, 30, and 60 degrees). The difference was
normalized by the local phase locked average total pressure and the error is shown in Figure 4.17. Over the lower 60 percent span, the PLA flow angle lies within the probe acceptance angle limits for all phase locations. From approximately 60 to 80 percent span, the time average flow angle lies within the probe acceptance angle, but the phase locked average flow angle lies outside the acceptance angle some of the phase locations. For the top 20 percent span, the time average flow angle lies outside the probe acceptance angle.

![Figure 4.16. Local Flow Probe Misalignment Error Diagram](image)

In order to correct for this potential bias error, the unsteady total pressure probe is traversed at several yaw angles. A simple flowchart for processing this data is shown in Figure 4.18. The $P_t$ Probe Yaw Corrector function uses the measured phase locked average data from the unsteady total pressure probe from multiple probe yaw angles as inputs and returns the correct phase locked average total pressure field as an output. For each radial and phase location, the function compares the measured...
probe pressures across the different yaw angles and selects the maximum, which is equal to the total pressure as long as the phase locked average flow angle lies within the acceptance angle limits of the probe for one of the probe yaw angle data sets.

\[ P_t^{\text{Probe Yaw Corrector}} = \max(P_t^{\text{Multiple Yaw Angles}}) \]

\( \tilde{P}_t(\tau, r) = \text{PLA Unsteady } P_t \text{ Probe Pressure (Multiple Yaw Angles)} \)

Figure 4.17. PLA \( P_t \) Bias Error Due to Probe Misalignment with PLA Flow (Percent)

Figure 4.18. Unsteady \( P_t \) Probe PLA Processing Flowchart
4.4 Phase Peak Yaw Method

A sensor that responds at high frequency and is sensitive to the yaw angle between the sensor and flow can be used to resolve the phase locked average swirl angle using the phase peak yaw method. Hot-wires are most sensitive to velocity perpendicular to the wire. By inserting a normal straight hot-wire probe, shown in Figure 4.19 radially into the flow and rotating about the probe axis, the component of velocity relative to the wire changes. The yaw angle at which the average signal is at a local maximum corresponds to the average swirl angle of the flow being perpendicular to the wire.

![Figure 4.19. Straight Hot-wire Probe (From [13])](image)

The advantage of the phase peak yaw method is that this method is not sensitive to many of the major sources of error, since only the relative signal magnitude is used. Calibration errors and high frequency attenuation only act to scale the signal magnitude with respect to yaw angle, not to change the phase position of the local maximum. Fluctuation errors can impact the result from the phase peak yaw method. However, these errors are shown to be negligible in the uncertainty analysis chapter. The probe yaw angle corresponding to the phase location of the local maximum is
the only important information from the data, not the absolute magnitude of the hot-wire signal. Knowing only the geometry and orientation of the hot-wire probe, it is possible to effectively use the phase peak yaw method without having to do any calibration or high frequency tuning.

4.4.1 Methodology and Implementation

A normal straight hot-wire probe was rotated over a range of yaw angles and traversed radially. The maximum hot-wire voltage occurs when the flow is primarily perpendicular to the wire. The phase peak yaw method compares the PLA voltage from the wire across a range of yaw angles. For each phase location, the yaw angle with the highest voltage yields the PLA swirl angle. Fitting a curve based on the known functional form of the hot-wire yaw sensitivity shown in Equation 4.23 allows for the peak location corresponding to the swirl angle to be estimated between the discrete probe yaw angle measurements. Figure 4.20 shows the phase peak yaw method solution for a single radial position and phase location from data in a high pressure turbine.

4.4.2 Calibration

The relationship between the hot-wire measured effective velocity has been shown to be effectively approximated by neglecting the tangential component of velocity. The trigonometric relations shown simplify to

\[ U_w \approx \|U\| \cos(\gamma - \alpha_2). \]  \hspace{1cm} (4.23)

For a normal straight hot-wire probe inserted radially downstream of a turbomachinery rotor, the highest effective velocity measured by the hot-wire occurs when the wire axis is aligned perpendicular to the flow swirl angle ($\alpha_2$). The only unknown to
Figure 4.20. Phase Peak Yaw Method Solution Example

be calibrated is the potential yaw offset during the hot-wire probe holder installation. The yaw calibration from the slanted hot-wire probe, was used to measure the offset (12.56 degrees).
CHAPTER 5
RESULTS AND VALIDATION

Five validation techniques were performed. First, the time average results from the multiple overheat method, slanted hot-wire method, and unsteady $P_t$ probe were compared to independent time average measurements. The independent time average absolute and relative measurements are compared separately in order to highlight the sources and types of uncertainty. Second, the PLS swirl angle measurements from the slanted hot-wire method were compared with the phase peak yaw method used as the reference. Third, the multiple overheat and slanted hot-wire measurements were compared with each other using the theoretical relationship from the Euler Turbomachinery equation. Fourth, the PLA measurements are shown to be qualitatively consistent with the expected compressor and turbine rotor flow structures. Lastly, CFD numerical simulations were validated using independent time average measurements and then the validated simulations were compared with the PLA measurements.

5.1 Results

5.1.1 Turbine Results

5.1.1.1 High Pressure Turbine Results

The multiple overheat method, slanted hot-wire method, unsteady $P_t$ probe, and phase peak yaw method were all used to acquire phase resolved measurements behind a high pressure turbine (HPT) rotor in the ND-TRT facility. The high pressure turbine rotor was un-shrouded. Probe traverse measurements were taken at two
different circumferential locations ($\theta = 0$, and $\theta = 180$ degrees). The hot-wire and fiber-film probes were all traversed at the $\theta = 0$ location. The unsteady $P_t$ probe, 5-hole pressure probe, and thermocouple probe were all traversed at the $\theta = 180$ degree location. Due to the odd number of nozzle vanes, the clocking of these traverse locations relative to the nozzles did not match, although circumferential traverse data indicate relatively small nozzle blade wake effects at the rotor exit plane.

The PLA and PLS total pressure from the unsteady $P_t$ probe are shown for the HPT rotor in Figures 5.1 and 5.2 respectively. The high total pressure at the tip is consistent with the tip leakage flow over the open rotor tips. The flow structure underneath the tip leakage flow is consistent with the strong secondary flow structures found in turbine rotor flows. The non-uniformity in the total pressure field from the blade passages is clearly visible in the PLS data.

![Graph showing total pressure data](image)

Figure 5.1. High Pressure Turbine PLA $P_t$ (Pascals)
The PLA and PLS total temperature from the multiple overheat method are shown in Figures 5.3 and 5.4 respectively. The region of high total temperature at the tip is consistent with the tip leakage flow. The higher enthalpy fluid is able to flow over blade tips without having energy extracted by the rotor in the form of work. There is very little non-uniformity in the total temperature field from the blade passages visible in the PLS data. For a well designed turbine of moderate blade loading operating at near design incidence (such as the HPT), one would not expect large non-uniformities in the total temperature, since total temperature is related to flow deviation from the Euler Turbomachinery equation. There is an unusual feature showing up in the PLS data in the near hub region. The PLS $T_t$ seems to vary with every other blade passage.

The PLA and PLS axial velocity from the slanted hot-wire method (using the fiber-film probe) are shown in Figures 5.5 and 5.6 respectively. The low velocity region near the hub is expected. The PLS axial velocity measurements show similar flow structures as the PLS total pressure shown in Figure 5.2. The same structures appearing in two independent sets of measurements ($U_z$ from the slanted hot-wire
Figure 5.3. High Pressure Turbine PLA $T_t$ (Kelvin)

Figure 5.4. High Pressure Turbine PLS $T_t$ (Kelvin)
method and $P_t$ from the unsteady total pressure probe) indicates that the measurements are resolving actual physics of the flow.

![Figure 5.5. High Pressure Turbine PLA $U_z$ (m/s)](image)

The PLA and PLS swirl velocity from the slanted hot-wire method are shown in Figures 5.7 and 5.8 respectively. The swirl velocity measurements resemble the total temperature data for both the PLA and PLS quantities, which is consistent with the physics of the Euler Turbomachinery equation relating changes in fluid enthalpy with changes in swirl velocity of the flow (for axial turbomachines). The same structures appearing in two independent sets of measurements ($T_i$ from the multiple overheat method and $U_\theta$ from the slanted hot-wire method) indicates that the measurements are physical. The high swirl velocity near the tip is consistent with the tip leakage flow and the un-extracted potential work left over from the highly swirled flow exiting the nozzle.
Figure 5.6. High Pressure Turbine PLS $U_z$ (m/s)

Figure 5.7. High Pressure Turbine PLA $U_{\theta}$ (m/s)
The PLA and PLS radial velocity from the slanted hot-wire method are shown in Figures 5.9 and 5.10, respectively. The probe was inserted radially. The hub and shroud at the rotor exit were at constant radii. There was no flow through any of the purge seals at this operating condition. The low magnitude of the radial velocity matches the continuity equation, since there should be no bulk mass flux in the radial direction. The PLS radial velocity near the hub shows the same ever other blade passage feature that appeared in the total temperature. Once again, flow structures are visible in independent measurements ($U_r$ from the slanted hot-wire and $T_t$ from the multiple overheat method).

5.1.1.2 Low Pressure Turbine Results

The multiple overheat method and unsteady $P_t$ probe were used to acquire phase resolved measurements behind a low pressure turbine (LPT) rotor in the ND-TRT facility. The low pressure turbine rotor was shrouded and had a very high blade loading design. The LPT rotor also had one damaged blade from manufacturing with the aft section of the blade removed. A 5-hole pressure probe, a steady Kiel
Figure 5.9. High Pressure Turbine PLA $U_r$ (m/s)

Figure 5.10. High Pressure Turbine PLS $U_r$ (m/s)
$P_t$ probe, and a thermocouple probe were also traversed behind the LPT rotor. All traverses were at the same measurement location. High quality CFD simulations of the LPT rotor were available for comparison.

The PLA total pressure field measured by the unsteady $P_t$ probe is shown in Figure 5.11 and the PLA total temperature field measured by the multiple overheat method is shown in Figure 5.12. The effects of the damaged rotor blade is clearly visible. The non-uniformity from the damaged blade impacts the flow around the surrounding blades and impacts a significant sector of the entire rotor flow field. This unusual, but expected, feature in the flow is present in both independent measurements.

![Figure 5.11. Low Pressure Turbine PLA $P_t$ (Pascals)](image.png)
5.1.2 Compressor Results

5.1.2.1 High Hub-to-Tip Ratio Compressor Results

The multiple overheat method was performed using a straight normal hot-wire probe yawed at 55 degrees behind a high hub-to-tip ratio (high HTTR) compressor rotor in the ND-TAC facility. No additional probes were traversed due to testing schedule constraints.

The PLA total temperature measured by the multiple overheat method is shown in Figure 5.13. The blade wakes and tip leakage flow are clearly visible. The sharp change in total temperature across the blade wake region corresponds to resolving approximately a 7 degree Kelvin change in $T_i$ at 80 kHz.

The PLA $\rho_tU_w$ measured by the multiple overheat method is shown in Figure 5.14. The blade wakes and tip clearance flow are clearly visible. Unlike the turbine data, the rotor blade wakes are significant and clearly defined. This is consistent with the global adverse pressure gradient present in compressors. No unsteady total pressure probe was available for testing, so it was not possible to remove the density from the
measurement without assuming zero PLS $P_t$. Likewise, only the one hot-wire probe orientation was used, therefore the wire effective velocity magnitude $U_w$ could not be decomposed into its components except in the midspan region assuming zero radial velocity.

5.2 Time Average Validation

Time average measurements of $P_t$, $T_t$, swirl angle, $U_z$, $U_\theta$, and $U_r$ were compared with the averages of the phase locked average quantities for validation from the multiple overheat method, slanted hot-wire method, phase peak yaw method, and unsteady $P_t$ probe. The measurements were taken approximately one chord downstream of the high pressure turbine rotor.

Time averages from the multiple overheat method (from a hot-wire and fiber-film probe) is compared to thermocouples in rakes shown in Figure 5.15. The thermocouple rake measurements were selected to most closely match the nozzle clocking of the multiple overheat method probe measurement location ($\theta = 0$ degrees). There
were two sets of rakes at different axial positions. Rakes were located at the traverse plane (PL42) and at a plane approximately one axial chord downstream of the traverse plane (PL43). The multiple overheat method accurately captures the radial $T_i$ profile shape to better than one degree Kelvin in relative total temperature for both the hot-wire and fiber-film probes. The hot-wire probe experiences an absolute error of about 3 degrees Kelvin, which appears as an offset in $T_i$. This is consistent with hot-wire calibration drift issues. It is reasonable that for a constant wire resistance setting, the wire temperature may drift by about 3-6 degrees Kelvin over the course of a day. The fiber-film probe, however, appears to be much more stable and less sensitive to calibration drift. There is no significant offset in the $T_i$ measured by the multiple overheat method with the fiber-film probe.

Time averages from the unsteady $P_t$ probe is compared to $P_t$ measured by a 5-hole pressure probe and steady $P_t$ Kiel rakes shown in Figure 5.16. Both the unsteady $P_t$ probe and 5-hole pressure probe were traversed at the $\theta = 180$ degree measurement location. The unsteady $P_t$ probe shows the same shape as the 5-hole pressure probe.
data, however it is offset in the absolute time average. This offset is indicative of the calibration and implementation errors associated with the miniature silicon diaphragm pressure transducer integrated into the unsteady $P_t$ probe. The 5-hole probe data also shows an offset in the absolute magnitude of the time average total pressure compared to the rakes.

Time averages of swirl angle from the slanted hot-wire method (using the fiber-film probe), the phase peak yaw method, and the 5-hole pressure probe are shown in Figure 5.17. The absolute and relative error in swirl angle is approximately 1-3 degrees between the measurements. The differences between the slanted hot-wire method data and phase peak yaw data are likely the result of implementation errors.

Time averages of axial velocity from the slanted hot-wire method (using the fiber-film probe), and the 5-hole pressure probe are shown in Figure 5.18. The slanted hot-wire method time average data agrees with the 5-hole probe data in absolute
Figure 5.16. HPT Time Average $P_t$ Comparison (Rake Data from [49])

Figure 5.17. HPT Time Average Swirl Angle Comparison
magnitude and relative shape over the first 70 percent of span. At the top 30 percent span, the results from the methods differ. It is unclear if this is the result of the different nozzle clocking relative at the different measurement locations. This region corresponds to the tip leakage flow, which has significantly higher gradients and levels of unsteadiness. Steady 5-hole pressure probes are known to be significantly less accurate in these types of flows.

Figure 5.18. HPT Time Average Axial Velocity Comparison

Time averages of swirl velocity from the slanted hot-wire method (using the fiber-film probe), and the 5-hole pressure probe are shown in Figure 5.19. The slanted hot-wire method time average results show similar shape and trends, but with about a 5-10 m/s offset in absolute swirl velocity. It is suspected that implementation errors are the source of this absolute uncertainty.
Time averages of radial velocity from the slanted hot-wire method (using the fiber-film probe), and the 5-hole pressure probe are shown in Figure 5.20. The absolute magnitude of the measured radial velocity agrees quite well and the relative differences are approximately 1-5 m/s. Five-hole pressure probes are known to become significantly inaccurate in flows with large gradients relative to the probe head size. The tip leakage flow and the hub boundary layer are both regions where this is of particular concern, consequently, it is not known if the time average velocity components measured by the slanted hot-wire method or the 5-hole pressure probe represent the true time average velocity.

5.3 CFD Validation

High quality CFD simulations of the low pressure turbine were performed by Perez [43] and used to help validate the phase resolved measurement techniques. The CFD was validated from the time average measurements shown in Figures 5.21 and 5.22.
Figure 5.20. HPT Time Average Radial Velocity

for the total pressure and total temperature profiles respectively. All the probes were traversed at the same measurement location. The time average CFD validation was used to gain confidence that the CFD simulations were correctly representing the real physics. The CFD simulations were then used to help validate the PLA measurements through comparison. The confidence that the CFD simulations represented the physical flow is then used to gain confidence that the PLA measurements also are representing the correct physical flow quantities.

The time average total temperature ratio profile displayed in Figure 5.21 show good agreement in shape over the majority of the span. The CFD does not show the drop off in $T_t$ ratio very near the tip because the simulation did not incorporate the leakage flow over the rotor shroud through the labyrinth seal. The local minimum near the bottom 30 percent span appears more pronounced in the CFD compared to the experimental measurements. The multiple overheat method was performed with a hot-wire probe which appeared to experience approximately 7 degree Kelvin drift in absolute temperature. This amount of drift is reasonable, considering the short
hot-wire burn-in time and fact that the hot-wire broke after acquiring this data set so no post calibration correction was possible. This apparent offset was removed for the purposes of comparison.

![Figure 5.21. Low Pressure Turbine Time Average CFD $T_t$ Ratio Comparison](image)

The time average total temperature ratio profile displayed in Figure 5.21 shows very good agreement in shape and magnitude over the span. The steady $P_t$ Kiel probe and the 5-hole pressure probe show excellent agreement with each other in both absolute magnitude and shape. The unsteady $P_t$ probe data appeared to have a 0.45 psia offset in absolute magnitude (corresponding to only 10 mV), but the relative $P_t$ matches to within 0.01 psia. The offset is reasonable considering the signal from the miniature silicon diaphragm pressure transducer was not amplified, exposed to large amount of EMF electrical noise from the magnetic bearings, and was operating
well outside its specified range.

![Graph showing comparison of CFD and measured total pressure ratio]

**Figure 5.22. Low Pressure Turbine Time Average CFD $P_t$ Ratio Comparison**

The comparison between the rotor total temperature ratio field calculated from the CFD simulations and measured from the multiple overheat method are shown in Figures 5.23 and 5.24 respectively. The structures corresponding to regions of lower total temperature ratio near the hub and tip as well as the blade passages are clearly visible in both the CFD and the PLA measurements. In the PLA measurements, the variation in total temperature ratio across a blade passage is approximately 0.005 across the lower 75 percent span, corresponding to approximately 2 degrees Kelvin. In the top 25 percent span the variation in total temperature ratio increases by approximately a factor of three. The CFD shows the same features and variations
in total temperature, but with greater amplitude corresponding to approximately 1 degree Kelvin. This is consistent with the differences between the time average comparison between the CFD and experimental measurements.

![Diagram](image)

**Figure 5.23. Low Pressure Turbine CFD $T_t$ Ratio (CFD Data from [43])**

The comparison between the rotor total pressure ratio field calculated from the CFD simulations and measured from the multiple overheat method are shown in Figures 5.25 and 5.26 respectively. The structure corresponding to regions of low total pressure ratio near the tip is clearly visible in both the CFD and PLA $P_t$ measurements. Similarly, the structures corresponding the regions of high total pressure ratio near the hub and across the blade passage is distinct in the CFD and PLA $P_t$ measurements from the unsteady $P_t$ probe. The CFD data and the PLA total pressure ratio measurements show the same features and magnitude variations across a blade passage. In the top 10 percent of the span, there is no significant variation in
$P_t$ across the blade passage. There is also no significant variation in $P_t$ in the near hub region (the unsteady $P_t$ probe was not traversed within the bottom 10 percent of span). The variation in total pressure ratio across the blade passage is between 0.1 and 0.15 (corresponding to 7-11 kPa) between 20 to 90 percent span. There is no significant difference in the amplitude of these variations. This is consistent with the time average comparison between the CFD and experimental measurements.

5.4 Phase Resolved Swirl Angle Measurement Comparison

In the HPT, both the slanted hot-wire method and the phase peak yaw method were performed and can be used to measure the phase resolved swirl angle. Both methods were performed at the same measurement location and the same operating condition. Due to the insensitivity of the phase peak yaw method to absolute magnitude and frequency response uncertainties, there is far more confidence in its measurements and can be effectively used as a reference for the purpose of validation. Furthermore, the data from the phase peak yaw method was completely independent.
Figure 5.25. Low Pressure Turbine CFD $P_t$ Ratio (CFD Data from [43])

Figure 5.26. Low Pressure Turbine PLA $P_t$ Ratio
of the slanted hot-wire method data, since it was acquired on a different day with a different probe.

The PLA and PLS swirl angle measured by the phase peak yaw method are shown in Figures 5.27 and 5.28. The PLA and PLS swirl angle measured by the slanted hot-wire method is shown in Figures 5.29 and 5.30. The PLA swirl angle data agree quite well between the two methods. The time average comparison shown in the earlier validation section (Figure 5.17) shows a difference in absolute and relative magnitude of about 2 degrees. The PLS swirl angle data between the two methods are similar, although the small variations in swirl angle present are nearly in the noise associated with the implementation of the methods. A more in depth validation would ideally use a flow with a much higher amplitude and distinct variations across a blade passage (such as the high hub-to-tip ratio compressor).

Figure 5.27. HPT PLA Swirl Angle from Phase Peak Yaw Method
Figure 5.28. HPT PLS Swirl Angle from Phase Peak Yaw Method

Figure 5.29. HPT PLA Swirl Angle from Slanted Hot-wire Method
The PLS swirl angle measured by the slanted hot-wire method and the phase peak yaw method are shown in Figure 5.3. The amplitude of the swirl angle variation measured by both methods match quite well, even though there is less than 2 degree variation in swirl angle in the HPT flow. The noise present in the PLS signals appears to be the uncertainty due to implementation errors (turbine operating point stability, convergence of phase locked statistics, and the stationarity of the signal).

5.5 Phase Resolved $\Delta T_t$ and $\Delta U_\theta$ Comparison

5.5.1 Euler Turbine Equation for PLA measurements

The Euler Turbomachinery Equation

$$ h_{t2} - h_{t1} = \omega(r_2C_{\theta2} - r_1C_{\theta1}), $$

is derived in the general case by applying conservation of energy and conservation of angular momentum (derivation of general Euler Turbine Equation in differential form is shown in [51]).
Figure 5.31. HPT PLS Swirl Angle Comparison

For the purpose of validation, the special case of an axial turbomachine with no radial transport is considered. The derivation of this special case is based on stagnation enthalpy along a streamtube in the rotating reference frame.

Stagnation enthalphy \( h_t \) is defined as \( h_t \equiv h + \frac{V^2}{2} \). The flow in the rotor relative reference frame is shown in Figure 5.5.1 In the rotor relative reference frame, the blades are stationary, so they do not impart work into the fluid. The flow can be assumed to be adiabatic, so there is no heat transfer into the fluid. As a result, there is no mechanism for increasing the relative total enthalpy of the fluid and it remains constant through the streamtube from the upstream conditions (1) to the downstream conditions (2). This is shown as

\[
h_{t,r} = h_1 + \frac{W_1^2}{2} = h_2 + \frac{W_2^2}{2}. \tag{5.2}
\]

The square of the relative velocity magnitude \( W^2 \) is replaced with the sum of the
squares of the velocity components \((W^2 = W_r^2 + W_\theta^2)\), which yields the relationship

\[
h_1 + \frac{W_{z1}^2}{2} + \frac{W_{\theta1}^2}{2} = h_2 + \frac{W_{z2}^2}{2} + \frac{W_{\theta2}^2}{2}. \quad (5.3)
\]

The velocity components are transformed from the relative reference frame to the absolute reference frame using the relations shown in the velocity triangles \(W_z = C_z\) and \(W_\theta = C_\theta - U_{\text{blade}}\) to get the relation

\[
h_1 + \frac{C_{z1}^2}{2} + \frac{C_{\theta1}^2}{2} - C_{\theta1}U_{\text{blade}} = h_2 + \frac{C_{z2}^2}{2} + \frac{C_{\theta2}^2}{2} - C_{\theta2}U_{\text{blade}}. \quad (5.4)
\]

The velocity components and static enthalpy in the first three terms on each side are combined into the stagnation enthalpy in the absolute reference frame yielding

\[
h_{t1} - U_{\text{blade}}C_{\theta1} = h_{t2} - U_{\text{blade}}C_{\theta2}. \quad (5.5)
\]
Rearranging the terms results in Equation 5.6 across a constant radius

\[ h_{t2} - h_{t1} = U_{blade}(C_{\theta2} - C_{\theta1}). \]  

(5.6)

5.5.2 High Hub-to-Tip Ratio Compressor Comparison

The multiple overheat method produces the phase locked average total temperature and wire effective mass flux \((\rho_t U_w)\). No unsteady \(P_t\) probe was available, therefore the PLA \(P_t\) was assumed to be equal to the time average \(P_t\). This approximation, while necessary, neglects the PLS \(P_t\) variation across a blade passage, which has been measured in similar compressors to be on the order of tens of kPa. In the midspan region of the high hub-to-tip ratio compressor, there was assumed to be no significant radial velocity component. The straight normal hot-wire probe was traversed radially with a known yaw angle (55 degrees). The swirl component of velocity at the rotor exit \(C_{\theta2}\) was calculated from the PLA effective wire velocity \(U_w\). The upstream swirl velocity \(C_{\theta1}\) was estimated from the inlet guide vane exit metal angle and the inlet velocity and was constant across the rotor blade pitch.

The multiple overheat method measured the PLA total temperature at the rotor exit \((T_{t2})\). The inlet total temperature was measured from the inlet rakes \((T_{t1})\) and was constant across the rotor blade pitch. The blade velocity was calculated from the radial position of the probe and the rotational speed of the rotor measured from the tachometer \((U_{blade} = r\omega)\). There was assumed to be no radial transport at the midspan location \((r_1 = r_2)\), which is reasonable for a high hub-to-tip ratio axial compressor. The relationship comparing the measured quantities derived from the Euler Turbomachinery equation is

\[ h_{t2} - h_{t1} = U_{blade}(C_{\theta2} - C_{\theta1}). \]  

(5.7)

\[ c_p(T_{t2} - T_{t1}) = U_{blade}(C_{\theta2} - C_{\theta1}). \]  

(5.8)
The left and right hand sides of Equation 5.8 are shown for the average rotor blade passage at midspan for the high HTTR compressor in Figure 5.32. Even under all the simplifications required, the PLA measurements match quite well in the time average as well as the PLS amplitude. For reference, the offset between the two curves in the first 40 percent of blade pitch corresponds to approximately one degree Kelvin.

Figure 5.32. High HTTR Compressor PLA $\Delta T_t - \Delta U_\theta$ Comparison
CHAPTER 6

UNCERTAINTY QUANTIFICATION

6.1 Hot-wire Uncertainty Analysis

6.1.1 Overall Uncertainty

There are four basic classes of elemental errors for using hot-wires to measure PLA flow variables: fluctuation errors, calibration errors, frequency response uncertainties and implementation errors. Calibration errors are differences between the actual mapping between flow conditions and the hot-wire voltage with the functional representation. Implementation errors are not formal error sources so they are not included in the general uncertainty estimate, but are the result of difficulties implementing the method such as non-stationary boundary conditions.

Frequency response uncertainties are magnitude errors in the PLS quantities (and subsequently the PLA quantities). The hot-wire system response is limited by the electronic system response and the hot-wire heat balance response. The electronic pulse test is the standard method for estimating the frequency response of the electronic system, but does not accurately characterize the frequency response due to the hot-wire heat balance.

Measurements can be either absolute or relative, which effect their associated uncertainty. A change in the offset in a transducer calibration is an example of an error in an absolute measurement. The absolute value of the transducer measurement changes, but the relative sensitivity of the transducer remains the same. A change in the slope of a transducer calibration is an example of an error in a relative measure-
ment. The absolute value of the transducer measurement and the relative sensitivity both change. The uncertainty of absolute and relative measurements are typically not the same. Absolute measurements typically have a much higher uncertainty than relative measurements. This is an important distinction for hot-wire measurements due to the effect of calibration drift on the uncertainty.

6.1.1.1 Multiple Overheat Overall Uncertainty

The uncertainty of a result \( (u_r) \) such as \( T_i \) or \( \rho U \) is approximated by

\[
 u_r^2 \approx \sum_{i=1}^{N} \theta_i^2 S_{x_i}^2
\]  

(6.1)

assuming errors are uncorrelated. The PLA voltage for the hot-wire at each different nominal overheat setting is the measurand \( x_i \). The sensitivities \( \theta_i = \frac{\partial r}{\partial x_i} \). There are \( N \) different overheat settings, therefore there are \( N \) different measurands. There are \( M \) elemental uncertainties of each measurand

\[
 S_{x_i}^2 = \sum_{k=1}^{M} (S_{x_i})_k^2.
\]  

(6.2)

For the case of the multiple overheat hot-wire method with four nominal overheat settings (1.2, 1.4, 1.6, 1.8) corresponding to the average voltages \( E_1 \) through \( E_4 \). The three basic elemental uncertainties (fluctuation errors \( S_{fl} \), calibration errors \( S_{cl} \), and frequency response uncertainties \( S_{fr} \)) are combined as \( S_{x_i}^2 = (S_{x_i})_{fl}^2 + (S_{x_i})_{cl}^2 + (S_{x_i})_{fr}^2 \). The calibration error uncertainty estimate based on the absolute or relative measurement. Conservative estimates for the values of these elemental uncertainties are shown in Table 6.1. Fluctuation errors have a much smaller impact on the overall uncertainty than calibration errors or frequency response uncertainty. Improvements in accuracy should be focused on reducing these two elemental error sources. Calibration errors are often effectively offsets due to calibration drift effecting the uncertainty
of absolute quantities while having a much smaller effect on the uncertainty of relative quantities. Frequency response uncertainty only affects the amplitude of the PLS variation, primarily effecting the uncertainty of relative quantities and having a smaller effect on the uncertainty of absolute quantities.

**TABLE 6.1**

**ESTIMATED VALUES FOR MULTIPLE OVERHEAT UNCERTAINTY ANALYSIS**

<table>
<thead>
<tr>
<th>$x_i$</th>
<th>Overheat</th>
<th>$\frac{\partial T_t}{\partial E^2}$</th>
<th>$\frac{\partial \rho_t U_w}{\partial E^2}$</th>
<th>$S_{fl}$</th>
<th>$S_{fr}$</th>
<th>$S_{cl}$ (absolute)</th>
<th>$S_{cl}$ (relative)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1^2$</td>
<td>1.2</td>
<td>-6.4</td>
<td>24.9</td>
<td>0.0021</td>
<td>0.0258</td>
<td>0.412</td>
<td>0.0515</td>
</tr>
<tr>
<td>$E_2^2$</td>
<td>1.4</td>
<td>-6.8</td>
<td>16.2</td>
<td>0.0031</td>
<td>0.0398</td>
<td>0.477</td>
<td>0.0731</td>
</tr>
<tr>
<td>$E_3^2$</td>
<td>1.6</td>
<td>-7.1</td>
<td>12.7</td>
<td>0.0040</td>
<td>0.0505</td>
<td>0.404</td>
<td>0.0808</td>
</tr>
<tr>
<td>$E_4^2$</td>
<td>1.8</td>
<td>-8.8</td>
<td>10.8</td>
<td>0.0050</td>
<td>0.0595</td>
<td>0.237</td>
<td>0.0790</td>
</tr>
</tbody>
</table>

The estimated combined standard uncertainty is about 5.5 degrees Kelvin absolute $T_t$ measurement and about 1.2 degrees Kelvin for the relative $T_t$ measurement after correcting for . The estimated combined standard uncertainty is 14.0 $\frac{kg}{m^2s}$ for the absolute $\rho_t U_w$ measurement and about 2.5 $\frac{kg}{m^2s}$ for the relative measurement. These are conservative estimates for the combined standard uncertainties evaluated at time average flow conditions ($T_t \approx 305$ Kelvin and $\rho_t U_w \approx 60 \frac{kg}{m^2s}$) representative of the high pressure turbine. The hot-wire calibration functions are non-linear, therefore the sensitivities $\theta_i$ are dependent on the mean flow conditions.

Calibration errors, especially in the absolute quantities, are the most significant
source of error. One method for improving the uncertainty is to use a probe with a more stable calibration. Fiber-film probes experience about an order of magnitude less calibration drift than hot-wire probes, meaning the absolute calibration error uncertainty is effectively the relative combined uncertainty shown in Table 6.1.

Several types of hot-wire and film probes are available commercially shown in Figure 6.1 with estimated frequency response and survivability limits shown. Miniature hot-wire probes (typically made with tungsten wires) are known to provide the best spatial and temporal resolution with minimal interference with the local flow, however they are susceptible to breaking, especially in the high dynamic pressure unsteady flows characteristic in turbomachines. It has been found that wires tend to break more frequently when the prongs are aligned with the local flow and the wake from the upstream prong interacts with the downstream prong.

<table>
<thead>
<tr>
<th>SENSOR TYPE</th>
<th>Sensor material</th>
<th>Sensor dimensions</th>
<th>Thickness of quartz coating</th>
<th>Thermal resistance (Ω)</th>
<th>Max. sensor temperature</th>
<th>Max. ambient pressure</th>
<th>Max. velocity</th>
<th>Min. velocity</th>
<th>Frequency limit (kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold-plated wire sensors</td>
<td>Plated tungsten</td>
<td>5 µm dia. 125 mm long</td>
<td>—</td>
<td>3.5 Ω</td>
<td>300°C</td>
<td>150°C</td>
<td>—</td>
<td>0.2 m/s</td>
<td>90 kHz</td>
</tr>
<tr>
<td>Miniature wire sensors</td>
<td>Plated tungsten</td>
<td>5 µm dia. 125 mm long</td>
<td>—</td>
<td>3.5 Ω</td>
<td>300°C</td>
<td>150°C</td>
<td>—</td>
<td>0.2 m/s</td>
<td>150 kHz</td>
</tr>
<tr>
<td>Wire sensors for temperature measurements</td>
<td>Platinum</td>
<td>1 mm dia. 0.4 mm long</td>
<td>—</td>
<td>50 Ω</td>
<td>150°C</td>
<td>—</td>
<td>—</td>
<td>60 m/s</td>
<td>2 kHz</td>
</tr>
<tr>
<td>Fiber-film sensors</td>
<td>Nickel</td>
<td>70 µm dia. 1.25 mm long</td>
<td>—</td>
<td>0.5 µm</td>
<td>300°C</td>
<td>100°C</td>
<td>—</td>
<td>0.2 m/s</td>
<td>90 kHz</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 µm</td>
<td>6 Ω</td>
<td>0.40%/K</td>
<td>50°C</td>
<td>70°C</td>
<td>—</td>
<td>0.01 m/s</td>
<td>10 m/s</td>
</tr>
</tbody>
</table>

Figure 6.1. Probe Comparison Chart (From [13])

Fiber-film probes are structurally similar to hot-wires. They are comprised of a 70 µm diameter quartz cylindrical fiber and are covered with a nickel thin film.
The total wire length is 3 mm, with about 1.5 mm active sensing length. Fiber-film probes are much more robust due to their larger thickness and the quartz rod substrate and effectively act like robust hot-wires. They have also been found to be much more stable sensors with smaller amounts of calibration drift and are less sensitive to Knudsen number variation. There has, however, been some concern about their frequency response due to the presence of the quartz substrate, especially at low overheats. In order to test the impact of this, the multiple overheat method was performed with a miniature hot-wire probe and a fiber-film probe for the same nominal overheats (1.2, 1.4, 1.6, and 1.8) in the same operating point behind the HPT rotor. The average blade passage PLS $T_i$ is shown averaged across the midspan region in Figure 6.2. There appears to be a very small amplitude PLS total temperature variation across the blade passage (less than $\frac{1}{3}$ of a degree Kelvin). There does not appear to be any significant difference between the two probes, implying that both probes have sufficient frequency response even at low overheats.

### 6.1.1.2 Slanted Hot-wire Method Overall Uncertainty

The uncertainty of a result ($u_r$) such as $U_z$, $U_\theta$, or $U_r$ is approximated by

$$u_r^2 \approx \sum_{i=1}^{N} \theta_i^2 S_{x_i}^2$$

(6.3)

assuming errors are uncorrelated. The PLA effective wire velocity ($U_w$) from hot-wire at each different yaw angle overheat setting is the measurand $x_i$. The sensitivities $\theta_i = \frac{\partial r}{\partial x_i}$. There are $N$ different yaw settings, therefore there are $N$ different measurands. For the case of the slanted hot-wire method with five yaw angles (-60, -30, 0, 30, 60) corresponding to the average wire effective velocity $U_{w1}$ through $U_{w5}$.

Estimates for the values of the absolute and relative uncertainties are based on the uncertainties from the hot-wire velocity at the overheat of 1.8 from the multiple over-
heat combined standard uncertainty. Table 6.2 shows sensitivities and uncertainty estimates for the different yaw angles.

The absolute combined standard uncertainties for the velocity components are between approximately 6 and 14 m/s. The relative combined standard uncertainties for the velocity components are approximately 3 m/s.

6.1.2 Calibration Errors

Calibration errors are the result of differences between the actual mapping between flow conditions during testing and the hot-wire voltage with the functional representation fit from calibration. They are a significant source of error, especially in the absolute magnitude of the time average quantities. Hot-wire calibration drift and stability are primary concerns for calibration errors, while the quality of the calibration functional fit is secondary. Calibration errors can be caused by differences
between the calibration conditions and the flow conditions (deposition of particulates/stuff on wire, wire strain, hot-wire prong temperature, non-uniform flow along hot-wire, thermal radiative environment, etc.). Hot-wires operated for a long time in the various turbomachinery facilities have clearly shown deposition of particulates on the probe. Fiber-film probe calibrations have shown to be much more stable, but at the cost of frequency response.

It is well known that hot-wires are not stable sensors and need to be calibrated frequently in order to maintain accuracy in the absolute quantities. This was part of the motivation behind designing and building the Calibration Jet Facility. Hot-wire calibration drift can be seen over time by a change in the cold resistance of the wire. An example of hot-wire calibration drift is shown in Figure 6.3 for the same hot-wire between two days. It is also apparent that the calibration drift effects are different for different nominal overheats, with lower overheats being more sensitive to calibration drift. This is consistent with the wire temperature drifting for a constant resistance setting. Calibration drift can be approximated by scaling the calibration data to account for small variations in the wire temperature. If the wire temperature

### TABLE 6.2

**ESTIMATED VALUES FOR SLANTED HOT-WIRE UNCERTAINTY ANALYSIS**

<table>
<thead>
<tr>
<th>$x_i$</th>
<th>Yaw Angle</th>
<th>$\frac{\partial U_w}{\partial U_z}$</th>
<th>$\frac{\partial U_w}{\partial U_0}$</th>
<th>$\frac{\partial U_w}{\partial U_r}$</th>
<th>$S_u$ (absolute)</th>
<th>$S_u$ (relative)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_{w1}$</td>
<td>-60</td>
<td>0.9502</td>
<td>0.0405</td>
<td>0.331</td>
<td>7.03</td>
<td>2.04</td>
</tr>
<tr>
<td>$U_{w2}$</td>
<td>-30</td>
<td>0.8047</td>
<td>0.0456</td>
<td>0.5997</td>
<td>7.03</td>
<td>2.04</td>
</tr>
<tr>
<td>$U_{w3}$</td>
<td>0</td>
<td>0.6552</td>
<td>0.3637</td>
<td>0.686</td>
<td>7.03</td>
<td>2.04</td>
</tr>
<tr>
<td>$U_{w4}$</td>
<td>30</td>
<td>0.6917</td>
<td>0.5847</td>
<td>0.4756</td>
<td>7.03</td>
<td>2.04</td>
</tr>
<tr>
<td>$U_{w5}$</td>
<td>60</td>
<td>0.8506</td>
<td>0.5164</td>
<td>0.199</td>
<td>7.03</td>
<td>2.04</td>
</tr>
</tbody>
</table>
is assumed constant, the calibration drift appears as a bias error in the temperature sensitivity (approximately 5 Kelvin between days based on Figure 6.3 and the temperature calibration). For the multiple overheat method, calibration drift will show up as a bias error in the time average absolute quantities, most significantly in total temperature. The relative time average quantities and PLS quantities are far less sensitive to calibration drift. As a result, it is recommended that when using the multiple overheat method, a time average sensor with good absolute accuracy should be included for comparison.

![Figure 6.3](image)

Figure 6.3. Calibrations from different days for the same hot-wire probe under the same nominal operating conditions.

A typical hot-wire anemometer has a precision of 0.1 Ohms in the wire resistance setting, corresponding to approximately 8-9 degrees Kelvin. Hot-wire probe cold resistance can drift by approximately 0.1 Ohms over a day of testing. Fiber-film
probe cold resistance can drift by approximately 0.1 Ohms over 1-2 weeks of testing.

The difference between the functional representation of the mapping between the flow conditions and hot-wire response using the calibration data is another example of a calibration error. These differences are much smaller than the calibration drift. They are the result only of how well the functional form of the calibration equations represents the actual mapping, the quality of the calibration data, and how close the actual operating flow conditions are from the calibration conditions.

6.1.3 Fluctuation Errors

Fluctuation errors cause a bias error in average voltage as a result of fluctuations about the mean flow conditions. These errors occur because of the non-linear relationship between voltage and the flow quantities of interest. They are the result of the averaging occurring prior to transforming the voltage into flow quantities. This is necessitated by running the hot-wires at different instants in time, making it impossible to decompose the instantaneous flow quantities from the voltage time-series.

In order to study the impact of these fluctuation errors on the phase resolved measurements from the multiple overheat method, a Monte Carlo numerical simulation was performed. A representative mean and standard deviation in $T_t$ and $\rho U$ was selected. Since the hot-wire calibration function is a two to one mapping between $T_t$ and $\rho U$ to voltage for each nominal overheat, an additional correlation coefficient between the flow quantities was needed to generate the simulated time-series for all quantities ($T_t$, $\rho U$, and voltage for each nominal overheat). With the simulated time-series in both $T_t$ and $\rho U$ the actual bias error resulting from the fluctuations could be calculated.

The effects of the total temperature fluctuation amplitude on the resulting bias error from the Monte Carlo simulation of the multiple overheat hot-wire method is shown in 6.4. There was no $\rho U$ fluctuation and the mean values remained constant.
As expected, the effect of the fluctuation amplitude on the resulting bias error in the mean is clearly non-linear and increasing.

![Graph](image)

Figure 6.4. Fluctuation errors due to uncorrelated $T_t$ fluctuations from Monte Carlo simulation.

The effects of the $\rho U$ fluctuation amplitude on the resulting bias error from the Monte Carlo simulation of the multiple overheat hot-wire method is shown in 6.5. There was no $T_t$ fluctuation and the mean $\rho U$ and $T_t$ were constant. It is clear that $\rho U$ fluctuations appear to have a much smaller impact on the mean quantities than the $T_t$ fluctuations.

The effects of the correlation between $\rho U$ and $T_t$ fluctuations across the range of amplitudes on the resulting bias error from the Monte Carlo simulation of the multiple overheat hot-wire method are shown in 6.6. The mean $\rho U$ and $T_t$ were constant. It is clear that the correlation between the $T_t$ and $\rho U$ fluctuations has no significant impact on the magnitude of the fluctuation bias error in the $\rho U$ mean. For
the mean $T_t$, the fluctuation bias error is higher for a positive correlation and lower for a negative correlation between the $T_t$ and $\rho U$ fluctuations. In turbomachinery flows, changes in total temperature are positively correlated with changes in swirl velocity through the Euler Turbo Equation, which gives a mechanism by which the fluctuations may be positively correlated. A more complete understanding of the bias effects of fluctuation errors may provide future opportunities for extracting additional information about the flow.

6.1.4 Frequency Response Uncertainty

The frequency response uncertainty of constant temperature hot-wires is investigated extensively in Chapter 7 using the electronic pulse test as well as a novel laser pulse test. Based on this investigation, the electronic anemometer circuit response was not shown to roll off in the frequency range 1-100 kHz even at low overheats. The only attenuation expected was related to perturbations in the wire temperature distribution. The frequency response uncertainty was approximated to be equal to
Figure 6.6. Fluctuation errors due to correlated $T_t$ and $\rho U$ fluctuations from Monte Carlo simulation.

95 percent of the PLS voltage amplitude for each overheat.

6.1.5 Implementation Errors

Implementation errors are uncertainties associated with the experiment specific implementation of the methods. Examples of implementation errors include: operating condition drift due to tunnel unsteadiness, uncertainties associated with ambient electronic noise and the data acquisition, and uncertainties due to the setup such as the tolerances of locating and aligning the probe relative to the flow path. These errors are the

The effect of operating condition drift on multiple overheat method measurements were investigated by simulating the effect of drift in velocity and total temperature between the two sets of overheat measurements. The same mean operating conditions and hot-wire calibrations from the Monte Carlo investigation of fluctuation errors were used. The impact of velocity drift is shown in Figures 6.7 and 6.8.
6.8 shows how the velocity drift biases the $T_t$ measured by the multiple overheat method. Figure 6.7 shows how the velocity drift biases the velocity measured by the multiple overheat method. Velocity drift effects the measured $T_t$ with approximately the same sensitivity regardless of which overheat experiences the drift. The measured $U$ is increasingly sensitive to velocity drift with increasing overheat. At a nominal overheat of 1.8 ($OH = 1.8$) the velocity drift effectively biases the measured $U$ directly. At a nominal overheat of 1.2, the velocity drift has a reduced impact on the measured $U$.

![Graph showing effect of velocity drift on measured U from multiple overheat method.](image)

Figure 6.7. Effect of velocity drift on measured $U$ from multiple overheat method.
Figure 6.8. Effect of velocity drift on measured $T_t$ from multiple overheat method.

The impact of total temperature drift is shown in Figures 6.9 and 6.10. Figure 6.10 shows how the total temperature drift biases the $T_t$ measured by the multiple overheat method. Figure 6.9 shows how the total temperature drift biases the velocity measured by the multiple overheat method. The effect of temperature drift is similar to the effect of velocity drift, except that measured $U$ has approximately the same sensitivity regardless of overheat experiencing the drift. The measured $T_t$ is effectively offset by the temperature drift when the lowest overheat experiences the drift. The sensitivity to drift is reduced when the highest overheat experiences the drift. The drift in operating condition velocity or temperature effects the multiple overheat method results by biasing the measurement. The magnitude of the drift is effectively an upper bound on the resulting implementation error.
Figure 6.9. Effect of $T_t$ drift on measured $U$ from multiple overheat method.

An estimate of the magnitude of an implementation error associated with the data acquisition was tested by connecting a precision voltage supply source to several different data acquisition systems. The difference between the precision voltage source and the measured voltage from several data acquisition systems was approximately 5-10 mV offset. For the un-amplified unsteady total pressure probe output signal at full scale, this corresponds to approximately 2 kPa.

6.2 Unsteady $P_t$ Probe Uncertainty Analysis

6.2.1 Overall Uncertainty

There are three basic classes of elemental errors for using an unsteady total pressure probe to measure PLA total pressure: probe interference errors, frequency response uncertainty, and transducer errors.

The combined uncertainty of $P_t$ measured by the unsteady total pressure probe
Figure 6.10. Effect of $T_l$ drift on measured $T_l$ from multiple overheat method.
(\( u_r \)) is approximated by

\[
  u_r^2 \approx \theta^2 S_x^2
\]

assuming errors are uncorrelated. The PLA voltage for the unsteady total pressure
probe is the measureand \( x \). The sensitivity \( \theta = \frac{\partial r}{\partial x} \). There are \( M \) elemental uncer-
tainties of the measureand

\[
  S_x^2 = \sum_{k=1}^{M} (S_x)_k^2.
\]

The three basic elemental uncertainties (probe interference errors \( S_{pi} \), transducer
errors \( S_{tr} \), and frequency response uncertainties \( S_{fr} \)) are combined as

\[
  S_x^2 = (S_x)_{pi}^2 + (S_x)_{tr}^2 + (S_x)_{fr}^2.
\]

Conservative estimates for the values of these elemental uncertainties are shown in
Table 6.3. Probe interference errors have a larger impact on the combined uncertainty
than transducer errors or frequency response uncertainty. One mechanism for this
elevated uncertainty is that the instantaneous flow can be outside the acceptance
angle of the probe, causing the measured total pressure to be biased low. To correct
for this error, the unsteady total pressure probe was traverses at several different yaw
angles and the maximum measured total pressure at each radial and phase location
was selected from across the traverses. Transducer errors are often effectively offsets
due to calibration drift effecting the absolute uncertainty while having a much smaller
effect on the relative uncertainty. Transducer errors will primarily impact the time
average quantities, not the PLS quantities. Frequency response uncertainty only
effects the amplitude of the PLS quantities, not the time average quantities.

The estimated combined standard uncertainty for \( P_t \) is about 0.1 psia, which
 corresponds to about 1 percent in the time average absolute total pressure.
6.2.2 Probe Interference Errors

Probe interference errors are caused by the probe’s presence altering the local flow field. As a result, the instantaneous total pressure measured by the probe is not equal to the total pressure at the measurement location in the undisturbed flow. Yaw sensitivity is an example of a steady probe interference error source. The Kiel probe head is designed to alter the local flow so that the flow is aligned with the inner Pitot tube or pressure transducer, allowing the probe to measure the total pressure of the flow, even when the flow is misaligned outside the probe acceptance angle. Figure 6.11 shows the yaw sensitivity of the probe. There is an acceptance angle of approximately ±25 degrees shown by the flat section of the yaw sensitivity curve. In this region, the measured pressure by the probe is effectively equal to the flow total pressure. Within the acceptance angle of the probe, the error in total pressure is less than 2 percent, which is used as an estimate for the probe interference elemental uncertainty.

6.2.3 Frequency Response Uncertainty

Frequency response uncertainties are caused primarily by the acoustic response of the cavity inside the probe head and the pressure transducer. Pressure fluctuation attenuation and acoustic resonances are the two major causes of phase and amplitude uncertainty in the transfer function. The current design of the unsteady total pressure
probe integrates the miniature silicon diaphragm pressure transducer into the Kiel probe head to minimize these pressure fluctuation attenuation acoustic resonances and phase lag. The response time of the probe was investigated in the shock tube facility.

The shock tube response is shown in Figure 6.11. The shock wave is effectively a step change in total pressure. The damped oscillations in the unsteady total pressure probe response shows that the system is behaving like a second order linear system (as expected from the transducer frequency response). There does not appear to be any significant low frequency acoustic resonances or viscous attenuation, since the transducer is mounted directly in the probe head. The response of the probe when yawed relative to the shock wave does not appear to change. The frequency response of the unsteady total pressure probe is estimated by comparing the measured pressure time-series during the shock with the theoretical pressure change. For an estimated frequency range of interest (around 50 kHz), the corresponding response time (2 microseconds) was used to estimate the amplitude of the measured signal.

Figure 6.11. Unsteady $P_t$ Probe Yaw Test (From [18])
with respect to the theoretical pressure (approximately 80 percent of the pressure fluctuation).

6.2.4 Transducer Errors

Transducer errors can result from calibration drift, temperature biasing the mean response, as well as transducer vibration contaminating the unsteady response. The silicon diaphragm pressure transducer has a built-in temperature compensation circuit to remove the bias due to changing temperature. The transducer vibration contaminating the unsteady pressure signal can be ignored due to the stiffness and high natural frequency of the miniature silicon diaphragm used in the transducer. The small mass and high stiffness of the silicon diaphragm make it insensitive to the small deflections resulting from vibration. The natural frequency of the diaphragm (100's of kHz) is much higher than the frequencies of interest giving a flat frequency response up to around 50 kHz. The manufacturer has designed and tested these
transducers to minimize the impact of these transducer error sources.

The major source of transducer error comes from calibration drift. The particular transducers available for use in the unsteady total pressure probe (Kulite Model XCQ-95-062-5A) were scaled only for 0-5 psia, meaning that the transducer was routinely operating well outside the specified range. While these transducers were intended to remain linear well outside this range, the manufacturer could not specify the extent of the linear range and burst pressures, meaning calibration drift is a potentially significant source of transducer error. Two calibrations of the transducer are shown in Table 6.4. The difference in measured voltage at atmospheric pressure was used as an estimate of the transducer error elemental uncertainty.

<table>
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<th>Calibration Date</th>
<th>Slope (psia/Volt)</th>
<th>Offset (psia)</th>
<th>Linear Fit $R^2$</th>
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<td>1.000</td>
</tr>
<tr>
<td>2014-07-11</td>
<td>54.971</td>
<td>0.143</td>
<td>1.000</td>
</tr>
</tbody>
</table>
CHAPTER 7

INVESTIGATION OF CONSTANT TEMPERATURE HOT-WIRE FREQUENCY RESPONSE

7.1 Experimental Investigation Setup

The frequency response uncertainty is a significant unknown for the uncertainty analysis for hot-wire methods, particularly for the multiple overheat method which requires constant temperature hot-wire operation at low overheats (hot-wire frequency response is known to decrease non-linearly with overheat). Signal attenuation in the 1-100 kHz range can be a significant source of error in phase locked steady measurements in high speed turbomachinery flows due to the high blade passing frequencies, and is difficult to identify because the time averages will not be affected. This source of error is particularly difficult to quantify because of the lack of an effective testing method for studying the real hot-wire system response at these frequencies.

The laser pulse test was developed as a novel method of characterizing the response of the entire constant temperature hot-wire system, including the heat balance of the hot-wire probe as well as the constant temperature anemometer circuit. The laser pulse perturbs the hot-wire system by nearly instantaneously changing the heat balance of the hot-wire through the wire surface heat flux through radiation, just like a change in convection due to a change in velocity. It is important to note that neither the duration of the laser pulse nor the time between laser pulses has any impact on the frequency response at high frequencies. Only the sharpness of the laser pulse matters. The laser pulse duration is approximately 9 nanoseconds, effectively producing a delta function perturbation in the hot-wire system.
To perform the laser pulse hot-wire experiment, a hot-wire probe was mounted at the exit of a calibration jet similar to the Calibration Jet Facility. The nozzle exiting to atmosphere without a plenum in order to provide the best optical access for the laser. The flow was driven by a compressed air system as part of the ND-TRT Facility. The flow was throttled precisely using an upstream butterfly valve, a bleed air valve, and a pressure regulator on the compressed air tank.

An AA-Labs 1003 Constant Temperature Anemometer system was used to operate the hot-wire and fiber-film probes as well as generate the electronic pulse test. A class 4 NdYAG pulsed laser from a Particle Image Velocimetry system was used to generate the laser pulse. The laser controller unit only had high and low intensity settings. Data were acquired using a high speed PXI system sampling at 1MHz. The laser beam was aligned, focused, and masked off to illuminate only the wire of the hot-wire probe. In order to do this, the beam needed to be narrowed and aligned with the hot-wire to fractions of a millimeter. The laser alignment with the wire only illuminated is shown in Figure 7.1. The laser illuminating the wire and prongs are shown in Figure 7.2. For scale, the wire length is approximately 1.25 millimeters.

With the the laser pulse illuminating only the wire, the temperature boundary conditions enforced at the wire ends by the prongs is unaffected. Since the laser pulse duration is nearly instantaneous (multiple orders of magnitude faster than the system response time), the hot-wire is still governed by the typical heat balance equations after the pulse. The perturbation simply sets the initial conditions to something other than the steady-state solution to the equations.

With the the laser pulse illuminating the wire and the prongs, the temperature boundary conditions at the wire ends are affected by the prong temperature. The prongs heat up from the laser pulse and cool down from convection over the prongs back to the equilibrium temperature. Due to the thermal heat capacity of the prongs, increased laser pulse intensity affects the prong temperature to a larger degree. Com-
Figure 7.1. Laser Hot-wire Pulse (Wire only Illuminated)

Laser focused and masked to only illuminate wire

Figure 7.2. Laser Hot-wire Pulse (Wire and Prongs Illuminated)

Laser unfocused and to illuminate wire and prongs
paring the hot-wire system response with and without illuminating the prongs and varying the laser intensity allows for the influence of the wire boundary conditions to be studied.

Figure 7.3. Individual triggered pulses and the trigger locked average pulse from electronic pulse test.

In order to effectively study the pulse response quantitatively, a normalized trigger locked ensemble average voltage \( \overline{E_i} \) was generated from the individual pulse voltages. For voltage \( E \) acquired over a length of time corresponding to \( N \) pulse events with index \( j \) and \( M \) trigger locked time lags with index \( i \), the trigger locked average voltage

\[
\overline{E_i} = \frac{\sum_{j=1}^{N} E_{i,j}}{N}.
\]  

(7.1)

The trigger locked average voltage was normalized by the pulse peak response \( \text{max} \left( \overline{E_i} \right) \)
and the unperturbed steady state hot-wire voltage $\tilde{E}_0$ with

$$\tilde{E}_i^* = \frac{\tilde{E}_i - \tilde{E}_0}{\max(\tilde{E}_i) - \tilde{E}_0}. \tag{7.2}$$

A simple threshold triggering was used to identify each pulse. Each pulse was normalized by the relative peak amplitude, and each normalized pulse was offset in time to match the normalized trigger threshold. The trigger locked individual pulses were ensemble averaged to average out electronic noise and flow unsteadiness. The resulting normalized trigger locked ensemble average along with several individual normalized pulses from the electronic pulse test are shown in 7.3. Electronic noise is visible in the individual pulses, but are averaged out in the trigger locked average.

Figure 7.4. Conceptual pulse response divided into regions with important points identified.
In order to better interpret the hot-wire system response to the laser pulse it is important to identify common features in the response waveform. A diagram of the common pulse response features is shown in Figure 7.4. The response can be divided into a rising region and a falling region. Point 0 represents the time the laser pulse begins and energy is entering the wire through radiation. Point 2 represents the time the laser pulse ends and no more energy is entering the wire through radiation. After point 1, the heat balance of the wire is governed by the typical hot-wire equation with a different initial condition. Point 2 is the maximum change in response to the perturbation. Point 3 is the point at which the response has dropped back to within a specified threshold, typically 80 percent of the difference between the peak and the steady state response.

7.2 Electronic Pulse Test

The electronic pulse test has been used as the standard method for testing the response of the constant temperature anemometer electronics by perturbing the feedback circuit with a nearly instantaneous change in bridge voltage (typically with a delta function or step function waveform). The shape of the circuit response (usually the response time and system natural frequency) is used to estimate the rolloff frequency.

A comparison between the response times from the hot-wire and fiber-film probes is shown in Figures 7.5 and 7.6 respectively. The laser pulse was tested for eight different wire overheats between 1.1 and 1.8 at three different flow Mach numbers between 0.1 and 0.3. The electronic pulse was tested for the same wire overheats with Mach number of 0.3. The response times were measured in both the rising and falling regions. The fiber-film probe was also tested with the wire and prongs illuminated. The electronic pulse test response time is of the same order as the laser pulse test rising response time, indicating that the same physics are relevant in both.
electronic pulse response time is not of the same order as the laser pulse test falling response time, indicating that different physics are relevant in each. The laser pulse falling response time data collapses with hot-wire voltage. Through the hot-wire heat balance, this voltage is a measure of how quickly heat is being convected away by the flow over the wire. This is under the reasonable assumption that the rate of heat leaving the wire through convection is much greater than the rate of heat leaving the wire through conduction to the prongs. The response times of the hot-wire and fiber film probes are similar.

![Figure 7.5. Hot-wire probe response time versus steady state hot-wire voltage for a range of flow velocities and overheats.](image-url)
Figure 7.6. Fiber-film probe response time versus steady state hot-wire voltage for a range of flow velocities and overheats.

7.3 Laser Pulse Test Results

If the simplistic hot-wire system frequency response based on the electronic pulse test accurately described the response of the real hot-wire system, then all the pulse responses would look similar for a constant hot-wire resistance, tuning, and flow condition. This does appear to be approximately true in the rising region, but not in the falling region. The response time associated with the system response in the falling region is an order of magnitude faster according to the electronic pulse test than the laser pulse. This indicates that different system dynamics are dominant in the falling region between the electronic pulse and laser pulse tests. The response time associated with the rising region is very similar between all the pulses, indicating that the dominant system dynamics are the same between the electronic and laser pulse tests. For the purpose of estimating the frequency response uncertainty from the constant temperature hot-wire system at various overheats, it is important to
identify the relevant system dynamics.

The response shape between the laser pulse at different intensities are similar. The response times in the falling region are different but of the same order of magnitude. This indicates that the system dynamics are governed by the same physics, but it is dependent on the amplitude of the perturbation. The amplitudes of the electronic pulse and the low power laser pulse were similar, translating into approximately a 10 percent change in velocity using the hot-wire calibration. The high power laser pulse perturbation amplitude was much larger (outside the range of the hot-wire calibration). The response shape in the falling region of the laser pulse response appears to be a decreasing exponential function.

Figure 7.7. Laser versus electronic pulse test response comparison.
A comparison between the laser pulse response with and without illuminating the prongs at different intensities shows the impact of the conduction at the wire ends in Figure 7.8. For the low intensity laser pulse, the response is nearly identical, which is consistent with the small change in prong temperature resulting from the lower amount of energy released during the low intensity laser pulse. The high intensity laser pulse shows a much longer response time when the prongs are illuminated, which is consistent with the large change in prong temperature. The shape of the response in the falling region is different for the 100 µs before it transitions back to the exponential decay. This indicates that the changing temperature boundary condition at the wire ends has an impact on physics of the system response, but is not responsible for the typical response.

![Figure 7.8. Hot-wire laser pulse response with different prong illumination with different intensities,](image)

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The system response to the laser pulse for a range of flow velocities (non-dimensionalized as Mach number) is shown in Figure 7.9. The time constant for the rising range is nearly constant. The time constant for the falling range varies with flow velocity. The response time decreases with increasing flow velocity (with one outlier). The highest flow velocity (Mach = 0.3) response doesn’t follow the trend, likely due to a change in perturbation amplitude from a laser alignment issue. The high dynamic pressure on the hot-wire probe from the high flow velocity likely caused a slight deflection affecting the alignment of the laser on the wire.

Figure 7.9. Hot-wire laser pulse response for various flow Mach numbers.

The system response to the laser pulse for a range of wire overheats (shown as wire resistance) is shown in Figure 7.10. The time constant for the rising range is effectively
constant. The time constant for the falling range varies with overheat, with increasing overheat decreasing the response time.

![Graph showing hot-wire laser pulse response for various overheat levels.](image)

Figure 7.10. Hot-wire laser pulse response for various overheat levels.

7.4 Proposed Mechanism

A hot-wire temperature distribution from the laser pulse perturbation is shown conceptually in Figure 7.11. The average temperature along the wire is $T_w$. The wire temperature is normalized by $\frac{T_w - T_1}{T_w - T_t}$. For comparison, the laser pulse response regions in Figure 7.4 are referenced. The hot-wire begins at the steady state temperature distribution prior to the laser pulse with the Wheatstone bridge balanced and the average wire temperature held constant by the electronics (Point $T_0$). The laser pulse
instantaneously raises the local wire temperature through radiation (Point #1). The mean wire temperature is elevated from the laser pulse, unbalancing the Wheatstone bridge and causing the electronics to reduce the current in order to bring the mean wire temperature back to equilibrium. The electronic response is shown when the mean temperature has been returned to the steady state, even though the temperature distribution is not at steady state (Point #2). At Point #3, the mean wire temperature remains constant, but the temperature distribution continues to relax back towards the steady state distribution, as shown in the heat transfer response in Figure 7.11. For constant mean wire temperature, the effect of the wire temperature distribution impacts the bridge voltage through the changes in the conduction to the prongs.

Figure 7.11. $T_w(x)$ change to 5% perturbation in $T_w$ from laser pulse shown conceptually.
For the laser pulse, the rising region is characterized by a change in the mean wire temperature with a response time governed primarily by the response time of the electronic constant temperature anemometer circuit. The falling region is characterized by a change in the wire temperature distribution while the mean wire temperature is held constant with the response time governed primarily by the local heat transfer of the wire. It is important to note that the electronic circuit can only input heat into the wire, so it can only raise the local wire temperature. Assuming that the rate of heat flux due to convection is much greater than the rate of heat flux due to conduction, the local wire temperature can only be reduced through convection to the flow. The response time is limited by how quickly convection can locally remove heat from the wire. For the high power laser pulse test data shown, the amplitude of the perturbation was estimated to be about a factor of 10 higher than shown schematically in Figure 7.11. In the high speed turbomachinery flows, the hot-wires were operated in high velocity flow ranges, where the heat flux due to convection was large, even for low overheats.

The motivation for investigating the constant temperature hot-wire system response using the laser pulse test was to find an estimate for the signal attenuation in the 10-100 kHz frequency range (corresponding to about 100-10 microsecond response time). The perturbations expected from the hot-wires in the turbomachinery flow are expected to be more uniform along the wire and be a smaller amplitude than the high power laser pulse. A hot-wire temperature distribution from a uniform perturbation is shown conceptually in Figure 7.12. The perturbation is an instantaneous step change in convection heat flux which brings the wire from an initial distribution steady state distribution 1 to the perturbation distribution. An increase in convection causes a negative perturbation and a decrease in convection causes a positive perturbation in mean wire temperature. Once the system electronics have responded by bringing the mean temperature back to equilibrium, the distribution is
back to the electronic response distribution. From this point, the mean wire temperature remains constant, but the temperature distribution changes towards the new steady state distribution 2 though the local wire heat transfer. This change in wire temperature distribution is significantly smaller and has a much smaller impact on the response times. This is the mechanism described by Freymuth \[15\] as well as Morris and Foss \[38\]. In order to change from one wire temperature distribution to another, the local wire temperature must be raised or lowered by through the local joule heating (controlled by the electronic circuit response) and convection heat transfer respectively.

Figure 7.12. $T_w(x)$ change to 5 % uniform perturbation in $\bar{T}_w$ shown conceptually.
7.5 Conclusions

Several conclusions can be drawn from the investigation into the constant temperature hot-wire system frequency response using the laser pulse test. First, the laser pulse test is a novel method for studying the overall hot-wire system response. The response in the rising region is consistent with the response of the anemometer circuit. The response in the falling region is consistent with the local heat balance as the wire temperature distribution is perturbed. The response times in the falling region from the laser pulse test are not representative of the expected system response from the hot-wire in a turbomachinery flow because the large amplitude and non-uniformity of the perturbation. Second, in the frequency range of 10-100 kHz, the circuit response does not appear to be attenuating the system response significantly. For the flow conditions expected in high speed turbomachinery flows, the system response appears to see less than 10 percent attenuation in the range of 10-100 kHz, even at the low overheats.
CHAPTER 8

CONCLUSIONS

8.1 Conclusions

The motivation guiding this research was to provide better experimental phase resolved measurement techniques suitable for studying high speed turbomachinery flows. This research had three specific objectives:

1. Develop hot-wire and silicon diaphragm pressure transducer based techniques for measuring the phase locked statistics of total temperature, swirl angle, total pressure, and velocity components behind rotors of high speed turbomachines.

2. Validate the measured flow quantities from the different techniques in high speed turbomachinery flows.

3. Quantify the uncertainty from each measurement technique.

This work has helped to advance the state of the art from the industry standard time average measurements, shown in Figure 5.21 to the current phase locked average measurements, shown in Figure 5.24. The impact of this advancement in measurement capabilities is shown from the types of high speed turbomachinery flow phenomena that can now be effectively studied experimentally, such as the effect of a damaged rotor blade shown in Figure 5.12.

8.1.1 Development

Four methods were developed:

1. The multiple overheat method was expanded for phase locked average quantities, adapted for high speed turbomachinery flows, and the fundamental principles were
investigated. A straightforward but general implementation was described. The multiple overheat method was shown to be effective at measuring phase resolved $T_t$. The lowest and highest overheat set the accuracy, with nominal overheat of 1.1-1.2 and 1.8-1.9 being effective. Fiber-film probes are significantly more stable and are less sensitive to time average offsets resulting from calibration drift. The relative error in time average total temperature is approximately 1 degree Kelvin. Phase locked steady $T_t$ resolution is less than one degree Kelvin.

2. The slanted hot-wire method was expanded for phase locked average quantities, adapted for high speed turbomachinery flows, and a straightforward but general implementation was described. The method appears to resolve approximate shapes and magnitudes correct, but needs additional development on the implementation in order to improve the accuracy. The current implementation is accurate within roughly 5-10 m/s for the time averages.

3. The phase peak yaw method was significantly expanded and the fundamental principles were described. An implementation using a straight normal hot-wire probe was presented. The phase peak yaw method is shown to be a new and effective way to measure phase resolved swirl angle within a few degrees of the time average and capable of resolving fluctuations of approximately 1 degree.

4. An unsteady total pressure probe was created and a straightforward implementation for high speed turbomachinery flows was presented. The unsteady $P_t$ probe is shown to be effective at measuring phase resolved $P_t$ even in unsteady three dimensional flows.

The underlying principles and methodology were investigated to clarify how each method works and a practical implementation was outlined. It is now straightforward for these methods to be applied and accurate phase resolved measurements can be made. Investigating the underlying principles helps guide future improvements.

8.1.2 Validation

This research significantly improved the confidence that the phase resolved measurements represent the real physical quantities. Very little validation of phase resolved measurements had been attempted in high speed turbomachinery flows in previous work beyond arguing that the results appear reasonable. Five validation exercises were performed:
1. The phase resolved measurements from various high speed turbomachinery flows were compared for consistency with the expected flow features. One key improvement is the ability to compare across independent measurements, techniques, and related flow quantities.

2. The time averages from the various phase resolved measurement techniques were compared to various steady measurements. The time average total pressure matched within approximately 1 kPa. The time average total temperature matched within approximately 1 degree Kelvin after correcting for calibration drift of the hot-wire probe. The time average velocity components matched within about 10 m/s.

3. The phase locked average quantities were compared to results from time average validated CFD simulations. The measured PLA total pressure matched the CFD within approximately 10 kPa. The measured relative PLA total temperature matched the CFD within approximately 1 degree Kelvin.

4. The PLA and PLS swirl angle was compared between two independent phase resolved measurement techniques; the phase peak yaw and slanted hot-wire methods. They were shown to agree to within about 2 degrees in the time average and 1 degree in the PLS swirl angle.

5. The phase locked average total temperature and swirl velocity components from the multiple overheat method were validated against each other using the relationship between total temperature change and flow turning in an axial turbomachine. The change in PLA swirl velocity and change in PLA total temperature matched to within approximately 1 degree Kelvin across a blade pitch.

These validation exercises significantly increase the confidence that the phase resolved flow quantities from these methods are the real physical quantities of interest.

8.1.3 Uncertainty Quantification

This research went significantly more in depth in investigating the underlying physics of the various elemental uncertainties, compared with previous work in literature. A novel method for investigating the frequency response of constant temperature hot-wire system using a pulsed laser was tested. Reasonable estimates for the elemental uncertainties were made and incorporated into the combined standard uncertainty from each method:

1. The accuracy of the hot-wire based methods (multiple overheat, slanted hot-wire, and phase peak yaw) is primarily the result of calibration errors. For the multiple
overheat method, the absolute magnitude in the time average can experience offsets of approximately 3-8 degrees Kelvin likely due to hot-wire calibration drift. The relative accuracy of the PLA total temperature from the multiple overheat method is approximately 1 degree Kelvin. The absolute accuracy of the phase peak yaw method is approximately 2 degrees in swirl angle. The relative accuracy of the slanted hot-wire method is approximately 2-10 m/s. Fiber-film probes are much more stable and do not see a significant absolute temperature offset over the course of testing. Implementation errors are not a formal error source for the method, but present a limit on the accuracy of the measurements.

2. The accuracy of the unsteady total pressure probe is primarily a result of transducer errors, with an estimated combined standard uncertainty of approximately 0.1 psia. Implementation errors present a limit on the accuracy of the measurements.

This uncertainty quantification investigation gave estimates for the uncertainties in the measurements, identified the dominant sources of error, showed the impact of errors in the way the methods were implemented, and explained the underlying physics of the elemental error sources which helps guide future improvements.
BIBLIOGRAPHY


