NONLINEAR BLUFF-BODY AERODYNAMICS

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Teng Wu

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Ahsan Kareem, Director

Graduate Program in Civil Engineering and Geological Sciences
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Abstract

by

Teng Wu

Wind induced effects on structures governed by the Navier-Stokes equations are not adequately represented by the conventional linear analysis framework. This shortcoming is becoming important for contemporary structures, as their increasing span-lengths and heights make them more sensitive to nonlinear and unsteady aerodynamic/aeroelastic load effects. To address this challenge this study focuses on responding to following key questions:

(1) What are the typical nonlinear behaviors observed from wind tunnel studies and full-scale observations and their underlying physics?

(2) What are the effects of nonlinearity and unsteadiness on bluff-body aerodynamics?

(3) What is the ability of existing nonlinear models to capture nonlinear and unsteady effects?

(4) Is it possible to go beyond the current nonlinear models, and establish more effective nonlinear unsteady low-dimensional modeling techniques?
In this context, the higher-order spectral approach is utilized to identify nonlinearity in bluff-body aerodynamics observed in wind-tunnel experiments. Physical sources of aerodynamic nonlinearities are investigated by analyzing the aerodynamic forces on a suite of cross-sections obtained through computational fluid dynamics (CFD). The effects of nonlinearity and unsteadiness on bluff-body aerodynamics are evaluated by comparing aerodynamic responses derived from different analytical models. Current models set in the conventional analysis framework are reviewed to understand their ability in simulating nonlinear unsteady aerodynamics; also, an improved model within the same framework is proposed. Several advanced low-dimensional modeling techniques, characterized by different levels of analysis of nonlinearity and unsteadiness, are then proposed. These include an approach based on artificial neural networks, a nonlinear moving average model (within the framework of Volterra theory), and a Volterra series-based model in which the kernels are identified using impulse functions. The fidelity with which the proposed approaches are able to simulate nonlinear bluff-body aerodynamics and aeroelasticity is verified through wind-tunnel or CFD-based data.

This study allowed to better understand the nonlinear and unsteady features of bluff-body aerodynamics and aeroelasticity and to establish an effective analysis framework which, although mainly developed in the context of cable-supported bridges, has also immediate applications to stay cables, super-tall buildings, airfoils in the transonic region or with high angle of attack, and wind turbines near dynamic stall conditions.
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Structures in the atmospheric boundary layer experience wind-related dynamic effects as a consequence of wind-structure interactions. While, according to common sense, both the structure and the wind are in motion, Dajian Huineng (638-713), the Sixth and Last Patriarch of Chán Buddhism, ignored the perception of outside world and attributed it to the mind set. Apparently, Werner Heisenberg (1901-1976) must have totally overlooked Huineng's theory when he established modern quantum mechanics, in which only relations between observable quantities are considered. While the writer sides with the common sense that the only appreciable observations in the area of fluid-structure interaction are the motions involving fluids and structures. Only the relative motion between the fluid and structure contributes to their interaction, as reflected by the principle of relativity [Sir Isaac Newton (1642-1727)].

If the philosophical point of view is abandoned for the more practical task of investigating and quantifying the interaction between the (turbulent) fluid and the structure, it is even difficult to believe that such a task could ever be accomplished. Usually, things have to be simplified. The first thing which is needed is to set up a form for the functional relationship between inputs and outputs in the dynamic system representing the fluid-structure interaction. In classical mechanics, the motion of a body is directly related to the forces exerted on it, hence it is logical to select the forces as the interim outputs in the dynamic system. Ironically, the motion of the fluid and structure
have to be selected as inputs, but they are also supposed to be the final outputs. The complexity of the solution for this homogenous system results from its chaotic property. It could be possible to select a proper status at which this dynamic system would converge, however its initial conditions are unknown. This actually indicates that it is necessary to understand the dynamic nature of fluids and of structures separately before any interaction takes place. Accordingly, it would be possible to select the individual dynamic status of fluids and structures as the original input to the interaction system and solve this self-evolving dynamic system until convergence is reached. 

In many aerodynamics studies, the detailed investigation of the structural properties is usually ignored. Instead, the structure immersed in the flow is typically assumed to be a rigid body. The dynamic properties of the second-order structural system are relatively easy to understand (though chaotic behavior is occasionally observed due to nonlinearities of structural origin). Two independent metrics for the energy in the structure, which are related to the body acceleration and position, exchange energy in the structure. Besides, the structural dynamic system may contain additional dissipative elements, which are related to the body velocity. On the other hand, a complete understanding of the nature of fluid motion is almost impossible. This is especially true for the turbulent wind in the atmospheric boundary layer. Traditionally, to overcome this difficulty and make some progress in wind-related dynamic issues concerning structures, the fluctuations in the flow are either ignored or replaced with some random numbers where the power distribution among the frequency components is simulated. Recent developments in computational fluid dynamics have opened the door to a new possibility of modeling these fluctuations. Supposing to have a sufficient understanding of fluid
dynamics and structural dynamics and hence that the initial inputs to the fluid-structure interaction dynamic system are known, any combination of (relative) motions in the initial inputs and during the system self-evolution can be decomposed into a relative impulse motion. It will be, therefore, a fundamental advance to develop a functional relationships between the elementary relative impulse motion and the forces on the fluid and structure. At this juncture, however, the progress in this direction is rather marginal.

Another path which is being explored in an attempt to understand the wind-related dynamic issues on structures consists in breaking up the overall dynamic system into several individual sub-dynamic systems. This strategy is primarily based on the assumption of linearity. As a result, the effects of wind-structure interaction are divided into aerodynamic effects (induced by the wind fluctuations), aeroelastic effects (induced by the structural motions in the uniform flow) or their combinations. In bridge aerodynamics, Robert H. Scanlan (1914-2001) and Alan G. Davenport (1932-2009) established a well-known linear analysis framework for the aerodynamic and aeroelastic effects on a bridge deck. This simplified approach has been very successful due to its simplicity and applicability to most of engineering problems. On the other hand, the shortcomings of this linear analysis framework have begun to surface as bridges with longer spans are being considered.

In this dissertation, an attempt has been made to better understand and model nonlinear and unsteady effects in aerodynamics. Several nonlinear approaches are presented, as it is apparent that a single approach to model a typical engineering problem does not exist. These approaches result from the trade-off between the effort to achieve accurate estimates of load effects and the attendant benefits.
Chapter 1 is basically a literature review, in which the existing linear and nonlinear frameworks for the analysis of bridge aerodynamics are systematically reviewed with a focus on the relationships among them and on their shortcomings.

Chapter 1 is based on the following papers:


Chapter 2 is focused on the existence of nonlinearity in bridge aerodynamics. Wind tunnel experiments showing nonlinear behavior are discussed and the use of higher-order spectra for the identification of nonlinearities is illustrated. Chapter 2 is based on the following paper:


Chapter 3 assesses the ability of currently existing nonlinear models and of a proposed nonlinear model, which are basically within the analysis framework established by R.H. Scanlan and A.G. Davenport, to capture fluid memory and nonlinear effects in bridge aerodynamics. Generally, these nonlinear models cannot fully characterize nonlinear bridge aerodynamics. Chapter 3 is based on the following paper:


Chapter 4 introduces application of an artificial neural network framework with embedded cellular automata scheme to model aerodynamic nonlinearities. This model
could be treated as a "black box" for aerodynamic simulation as the configuration of artificial neural network is established without integrating the underlying physics of aerodynamics. Chapter 4 is based on the following paper:


Chapter 5 presents a finite-dimensional nonlinear moving average model, which is within the framework of Volterra theory, to simulate nonlinear bridge aerodynamics and attendant response under turbulent fluctuations. This sparse third-order Volterra model could be treated as a "gray box" for aerodynamic simulation since the fundamental aerodynamic mechanisms are utilized in the model development. Chapter 5 is based on the following paper:


Chapter 6 investigates the theoretical basis of the Volterra series-based simulation of nonlinear bridge aerodynamics, in which the linear convolution scheme is extended to the nonlinear convolution scheme involving higher-order (nonlinear) kernels for the treatment of nonlinear bridge aerodynamics using a "peeling-an-onion" type approach. Since the Volterra kernels are identified based on the impulse function concept, hence retaining the complete underlying physics of the aerodynamic system, the developed nonlinear analysis framework could be treated as a "white box" for the aerodynamic simulation. Besides the efficacy of the Volterra theory-based nonlinear analysis framework in simulating nonlinear aerodynamic and aeroelastic effects, it has been
demonstrated how the kernels are identified with a phenomenological model governing the bridge aerodynamics. Chapter 6 is based on the following paper:


Chapter 7 demonstrates the efficacy of the Volterra theory-based nonlinear analysis framework in simulating vortex-induced vibration (a combination of nonlinear aerodynamics and aeroelasticity). In particular, a Van der Pol type phenomenological model governing the fluid-structure interaction system is utilized to identify the Volterra kernels based on the impulse function concept. Chapter 7 is based on the following paper:


Chapter 8 systematically examines the efficacy of the Volterra theory based nonlinear analysis framework in simulating nonlinear bluff-body aerodynamics based on computational fluid dynamics approach, in which the Navier-Stokes equations are utilized to identify the Volterra kernels with the impulse function concept. Chapter 8 is based on the following paper:


Concluding remarks and future directions of this work are finally presented in Chapter 9.

These chapters are followed by several related papers recently published or in preparation by the candidate, which are given in the Appendices. These papers provide
supplementary information that not only serves as a building block for this research but also helps to mesh up various components of the research to reach the desired objective.

If the time-domain analysis is utilized in the linear or nonlinear modeling of bridge aerodynamics, the knowledge of the indicial or impulse response is indispensable as they represent the fundamental building blocks for the convolution integral. Hence, Appendix A systematically reviews two elementary response features of bridge aerodynamics, namely the effective unit-step (indicial) and unit-impulse response functions from theoretical, experimental and numerical perspectives. Appendix A is based on the following paper:


The discussion of the influence of fluid memory and nonlinear effects on bluff-body aerodynamics presented in Chapter 3 mainly concerned a typical bridge deck cross-section with a long afterbody. Another typical bluff body is the circular cylinder with a circular cross-section. Hence, Appendix B investigates the fluid memory and nonlinear effects on the bluff-body aerodynamics of a circular cross-section with a small surface protrusion in the context of rain-wind induced vibration study. Appendix B is based on the following paper:


When investigating the aerodynamic properties of vortex-induced vibration, most studies have focused on the cross-section of the circular cylinder. Another case study of fundamental importance is the typical bridge deck cross-section characterized by a long
afterbody. Hence, Appendix C presents a critical literature review of vortex-induced vibration of bridge decks from wind-tunnel experiments, full-scale observations, semi-empirical models and computational fluid dynamics perspectives, in which highlights of physical mechanisms central to vortex-induced vibration (a combination of nonlinear aerodynamics and aeroelasticity) from a renewed perspective are provided. Appendix C is based on the following paper:


In parallel with the development of the Volterra theory-based nonlinear analysis framework in time domain, Appendix D introduces a frequency domain approach for nonlinear bridge aerodynamics, based on the Volterra series expansion. The Volterra frequency-response functions are formulated utilizing a topological assemblage scheme and allow to obtain a qualitative insight to the nonlinear bridge aerodynamics. Appendix D is based on the following paper:


In an engineering application, analysis based on two-dimensional (2-D) idealization needs to be extrapolated to three-dimensional (3-D) full-scale situation, wherein an accurate understanding of the spanwise correlation (or coherence) of the wind-induced pressures or forces is critical. Such an understanding also aids to reveal the underlying physics of 3-D aeroelastic model results and helps the selection of an optimal spanwise size of the computational domain in 3-D computational fluid dynamics simulation. In this context, Appendix E presents the correlation structure of the pressure
field and integrated forces on a stationary/oscillating prism with various incident wind characteristics (e.g., different mean wind velocities, turbulence intensities and turbulence integral scales). Appendix E is based on the following paper:

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CHAPTER 1:
INTRODUCTION AND OVERVIEW

Developments made over the past several decades in bluff body aerodynamics and aeroelasticity have enhanced our abilities to better understand and capture the effects of turbulent wind on structures. These developments had implicit assumptions of stationarity, Gaussianity and linear features while observations in storms and the attendant aerodynamic effects clearly show a departure from these tacit assumptions. In view of this, there is a need to revisit the current paradigms and to look for improved understanding concerning the nature of wind fields, the associated aerodynamics and the resulting load effects. This work focuses on the departure from conventional linearized analysis frameworks in bluff-body aerodynamics.

Specifically, the primary goal of this study is to develop effective analysis tools to better understand and capture nonlinear and unsteady features concerning bluff-body aerodynamics (gust-induced effects) and aeroelasticity (motion-induced effects) mainly in the context of cable-supported bridges, with immediate applications to wind-induced effects on stay cables, super tall buildings, airfoils in the transonic region or with high angle of attack, and wind turbines near dynamic stall conditions. These features are not fully captured in the current state-of-the-art analysis procedures. To accomplish this goal, a systematic approach is proposed that focuses on identification and characterization of nonlinearity and unsteadiness in bluff-body aerodynamics and aeroelasticity, assessment
of their impact on the performance, and development of advanced analysis frameworks for modeling and analysis. In view of the complexity and intractability of nonlinear and unsteady fluid-structure interactions using governing equations of fluid and structural motions, low-dimensional models will be utilized and their efficacy will be assessed.

1.1 Conventional Linear Approaches: A Synopsis

There are generally three main wind-induced considerations for the long-span bridges, namely buffeting (aerodynamic), flutter (aeroelastic) and vortex-induced vibration (aerodynamic/aeroelastic). Conventional schemes consider the analyses of these effects in isolation, involving different mechanisms for illustration and models for simulation. To date, all of the models used to simulate the wind-induced bridge behaviors are empirical/semi-empirical models, e.g., Davenport's analysis framework for buffeting (Davenport 1962), Scanlan's analysis framework for flutter (Scanlan and Tomko 1971), and Van der Pol oscillator for vortex-induced vibration (Scanlan 1981), where model parameters involved are empirically derived from experimental data. It should be noted that the use of flutter and buffeting analyses in isolation is valid only in the linear analysis framework. For this reason, the current linear aerodynamic and aeroelastic analysis schemes cannot take into account the interactions between flutter and buffeting beyond linear superposition. In reality, buffeting will change the amplitude of bridge response, which will affect the flutter behavior described as an amplitude dependent self-excited phenomenon. Also, the flutter will change the effective dynamic properties of the wind-bridge system, e.g., the effective frequencies and damping ratios of the bridge deck, which will in turn affect the buffeting behavior described as forced vibration.
1.1.1 Aerodynamic Analysis

Sears function (Sears 1941) is the most elementary building block in the analysis of gust-induced effects on bridges. Liepmann (1952) applied Sears function in the statistical buffeting analysis of thin airfoils, which was later extended to buffeting analysis of bridges (Davenport 1962). In this study, a linear random vibration based scheme, in which the aerodynamic transfer function was measured in a wind-tunnel test (or based on the quasi-steady theory), was utilized (Davenport 1962).

It has been generally recognized that there is a close similarity between the gust-induced (aerodynamic) and motion-induced (aeroelastic) effects on the flow-structure interaction system (Scanlan 2000). For example, the contribution of vertical gust-induced effects to the aerodynamic forces could be accounted for by using the contribution of vertical structural motion-induced effects including chordwise correlation of vertical gusts. Actually, Küssner (1940) used the terminology "partial motion" to describe the vertical gust entry effect. Hence, for the sake of brevity, this study will focus on the aeroelastic analysis while the aerodynamic analysis is touched upon occasionally (e.g., in terms of the turbulence effects on aeroelastic instability). It should be noted that, once the nonlinear consideration is involved, the aerodynamic effects are naturally incorporated into the aeroelastic issues.

1.1.2 Aeroelastic Analysis

Since the pivotal failure of Tacoma Narrows Bridge in 1940, the aerodynamic/aeroelastic behavior of long-span bridges, and especially the aeroelastic instability (flutter), has drawn remarkable attention in the fields of structural engineering and physics, launching the modern era of bridge aeroelasticity (e.g., Ammann et al. 1941; Dunn 1941;
Farquharson 1949-1954). Bleich (1949) for the first time analyzed the bridge flutter behavior based on Theodorsen theory of thin airfoils with the potential flow assumption. This direct application of a theory developed for thin airfoils fails in the cases of bridge decks that are more bluff in their profile than a typical airfoil. During the same period, empirical formulas based only on the bridge structural properties emerged to directly obtain the critical flutter wind velocity without involving any consideration of the flow field (e.g., Selberg 1963; Klöppel and Thiele 1967; Van der Put 1976). Among these, Selberg’s formula (Selberg 1963) is most widely applied. Although these empirical schemes are actually primarily applicable for streamlined decks, they explicitly delineate the effects of structural properties per se on the flutter condition, which has guided the bridge design practice in the preliminary stage.

During that period, Pugsley (1949) made the prophetic suggestion that, unlike the case of thin airfoils for which there exists a linear theoretical solution, the investigation of wind-bridge interaction should rely on the experimental approach. Following this, Scanlan and his colleagues building upon Theodorsen theory (Theodorsen 1935) introduced a semi-empirical linearized analysis framework based on the so-called aeroelastic coefficients (flutter derivatives) (e.g., Scanlan and Tomko 1971). The current bridge aeroelastic analysis schemes are primarily based on this framework. This model based on data-driven properties requires wind-tunnel experiments to extract section specific flutter derivatives. These flutter derivatives can be extracted from the section model testing involving forced vibration tests (Halfman 1952; Loiseau and Széchenyi 1975) or free vibration tests (Scanlan and Sabzevari 1969). Since their introduction significant advances have been made in the experimental arrangements and system
identification techniques to extract accurately flutter derivatives (e.g., Partha P. Sarkar et al. 1994; Haan et al. 1998). In addition to their simplicity and expediency, section models, due to the relatively large scale, dampen the concerns regarding Reynolds number effect to some extent and permit taking into account the geometric details of the bridge deck, to which the wind-induced effects are very sensitive (Scanlan 1981).

1.2 Nonlinear Analysis Schemes: A Overview

Although the linearized analysis methodologies cannot reveal some wind-induced behaviors clearly, they have been proven to be valid with reasonable accuracy in the wind velocity range of interest when utilized in bridge aerodynamic/aeroelasticity studies with the data-driven aerodynamic properties. In these cases, many factors which are not explicitly involved in the model are actually automatically incorporated due to the “let it happen” characteristics of the system identification technique (Scanlan 1993). However, there remain several unnegligible issues that the linearized schemes are not able to address due to inherent limitations. For example, though a linearized scheme could account for linear memory effects, it cannot simulate the hysteresis phenomenon, which corresponds to higher-order memory effects in the wind-bridge interaction. Besides, the linearized scheme cannot predict the potential existence of the limit cycle oscillation (LCO), which is a unique feature of nonlinear systems (Dowell and Tang 2002). Furthermore, the situation can become more complicated by the interaction of aerodynamic/aeroelastic nonlinearities with the potential nonlinearities of structural origin due to large deformations, material nonlinearities or complex damping properties (e.g., Xiang and Ge 2000; Salvatori and Borri 2007; Arena and Lacarbonara 2012).
1.2.1 Conventional Approaches

The nonlinearity of wind-induced effects on the bridge deck essentially results from flow separation, reattachment around the deck and three-dimensional (3-D) wake dynamics. Some aeroelastic (and aerodynamic) nonlinear phenomena have recently been studied in literature from both experimental and analytical points of view (e.g., Diana et al. 2005; Kareem and Wu 2012). To take the nonlinear effects into account, the quasi-steady (QS) theory has been applied in the aerodynamic/aeroelastic analysis of bridges (e.g., Diana et al. 1999a; 1999b; 2005; Denoël 2009). The QS theory was first introduced to model galloping phenomenon (e.g., Parkinson and Brooks 1961). Within this approach, the wind-induced forces are nonlinear functions of the dynamic angle of attack (a combination of the bridge deck motions and approaching turbulence). Generally, the QS theory-based model perform well when the reduced wind velocity \( U_r = U/fB \) (where \( U \) is the horizontal mean wind velocity; \( f \) is the oscillating frequency of the bridge deck; \( B \) is the width of the bridge deck) is greater than 10, where the wind-induced forces are assumed frequency independent. Usually, the error introduced by the QS assumption is insignificant since the interested wind velocity accompanied by the aeroelastic instability is typically very high. However, careful investigation should be carried out since the high-frequency vibration, which are unsuccessfully predicted based on the QS theory, can be particularly important, due to the fact that the damping ratios of the higher-order modes are usually smaller than those associated with the lower-order modes. As for example in the case of the 1:250 Messina Bridge model, the solutions obtained using QS approach have large discrepancies with the observed results (Diana et al. 1999a).
The frequency-independence assumption of the QS theory actually indicates that the wind-induced forces in this scheme are “static” nonlinear functions with respect to the dynamic angle of attack. Therefore, the QS approach cannot consider the fluid memory effects. In fact, the additional vorticity created by the motions of the bridge sheds into the wake downstream and then impact the bridge deck which, in turn, induces a corresponding force. The time to convect downstream introduces fluid memory, represented through the complex time lag between the wind-induced forces and the structural responses. In order to partially involve the fluid memory effects, the corrected quasi-steady (CQS) theory has been introduced to account for the dependence on the reduced wind velocity in the low reduced wind velocity range (Diana et al. 1993). A further detailed scheme named “band superposition” has also been developed to improve the simulation accuracy of the CQS theory-based model under the turbulent flow (Diana et al. 1995).

The QS theory-based models could take into account nonlinearity, but not the fluid memory, whereas conventional models based on Scanlan’s flutter derivatives could account for the fluid memory effects, but not the nonlinearity. In light of these features, a hybrid scheme has been proposed to capture both the unsteady and nonlinear effects of wind-bridge interactions (Chen and Kareem 2003). The basic premise of this semi-empirical, nonlinear approach is that the low-frequency aerodynamic/aeroelastic effects are simulated by the QS theory while the contributions at high frequencies are represented by conventional semi-empirical linear model. Though the hybrid model reasonably accounts for the nonlinear effects, it sacrifices the inclusion of unsteady effects at the low frequencies. Besides, the exact demarcation between the low- and high-
frequency fluctuations currently lacks a rigorous treatment. The semi-empirical, linear, unsteady model is actually a time-linearized method, where a steady-flow field (statically nonlinear) is first determined, and then a small perturbation (linear) is added on this base flow (Dowell and Hall 2001); while the hybrid model extends the steady-flow field to a slowly fluctuating base flow. Although this nonlinear analysis approach assumes that all the nonlinearities in the wind-induced effects could be considered based on the dependence on the slowly-changing (low-frequency part) amplitudes, unfortunately, this methodology does not always have a satisfactory representation of the full nonlinear equations which govern the fluid-structure interactions (Kareem and Wu 2012).

The conventional nonlinear models discussed in the preceding content actually focus on the consideration of nonlinearity as the amplitude dependence. For the QS theory based model the amplitude dependence is introduced through the nonlinear relationship between the steady-state coefficients and the dynamic angle of attack that results from a combination of the structural motions and approaching turbulence (no fluid memory consideration). Whereas, for the hybrid model the amplitude dependence is introduced through the dependence of flutter derivatives on the low-frequency dynamic angle of attack. In order to capture the nonlinear hysteretic effects of amplitude on the instability, a rheological element is utilized to introduce an equivalent model (Diana et al. 2010). Other models have emerged for the simulation of dynamic stall of wind turbines, such as the Beddoes-Leishman, ONERA, Øye, Risø and Boing-Vertol models, which can also capture nonlinear hysteretic effects. It was shown that the fidelity of most models was insufficient in terms of faithful prediction due to their semi-empirical nature (Leishman 2002). Besides there is a need for a comprehensive validation of these models,
as they heavily rely on the unique independent variable, i.e., the dynamic angle of attack, which assumes that the contribution of each input-parameter to be equal in the entire range of reduced wind velocity. This poses an unwarranted constraint in describing wind-structure interaction. In the case of bridges, conventional nonlinear models together with the rheological model have been proven to be unable to fully analyze bridges under wind loads characterized by aerodynamic and aeroelastic nonlinearities (Wu and Kareem 2012a), thus limiting the utility of the state-of-the-art conventional analysis procedures and requiring a comprehensive nonlinear analysis framework.

1.2.2 Advanced Nonlinear Analysis Frameworks

Thus far, as seen in the previous section, few conventional nonlinear models are available with very limited ability to take nonlinear effects into account, partly due to the complexity involved and the associated modeling intractability. In the case of wind turbines, if the nonlinearity and unsteadiness are significant, conventional blade element momentum (BEM) theory for aerodynamic analysis is not suitable. Instead, the unsteady wake modeling, such as vortex wake model, need to be utilized. The current state of the art is to combine the vortex wake model (which is based on the incompressible potential flow theory) and the dynamic stall modeling (which indicates massive separation and intensive viscosity effects). Although this scheme shows a great promise of simulating aerodynamics of wind turbines, it may predict power outputs that are considerably lower than those actually measured (Leishman 2002). An alternate way is to utilize computational fluids dynamics. However, the computational effort is too high considering the three-dimensional nature of wind turbine aerodynamics. In light of this, low-dimensional modeling of the aerodynamics may offer a promise approach.
In light of the high computational efficiency and ability to retain essential physics, the low-dimensional models have been rapidly developed in this context over the last several decades (e.g., Dowell and Hall 2001; Silva et al. 2001; Raveh 2001; Lucia et al. 2004). There are a number of low-dimensional models which have been successfully applied in engineering, such as the describing function, trajectory piece-wise linearization, artificial neural network, autoregressive moving average, Volterra series and proper orthogonal decomposition. Improvement in the efficiency and robustness of these low-dimensional models is a topic of cutting-edge research in aerodynamics community (e.g., Silva 2005; Amsallem and Farhat 2008; Amsallem et al. 2012; Balajewicz et al. 2013). Besides, selection of a proper scheme among a wide range of low-dimensional models to simulate nonlinear unsteady bluff-body aerodynamics and aeroelasticity is quite challenging.

An interesting methodology proposed in this study in response of the said issues is to simulate the wind-bridge interactions utilizing an artificial neural network (ANN), which is a computational (mathematical) model that mimics the structure and/or functionality of a biological neural network (Wu and Kareem 2010). This model, unlike the models that utilize the concept of dynamic angle of attack, has the ability to incorporate the relative contributions of different inputs individually. Thus it offers more versatility in capturing the observed nonlinear behavior of bluff-body aerodynamics. However, the ANN is usually labeled as a nonparametric model because it is difficult to elucidate the physical meaning of the various weighting functions employed in the network. The data-driven characteristics of the ANN may classify it as a “black-box” model.
The time-delay neural network has been shown to be equivalent to a Volterra series (Wray and Green 1994). The Volterra series, consisting of linear and higher-order convolutions, can represent the complex mapping rules (static linear/nonlinear relationships) and time lags (fluid memory effects) between the aerodynamic/aeroelastic inputs and outputs, the hallmark of bluff-body aerodynamics and aeroelasticity. It is noted that the discrete-time Volterra model with a finite nonlinear degree and dynamic order actually, as it unfolds, belongs to a larger class of finite-dimensional nonlinear moving average (NMA) models (Pearson 1995; Pearson et al. 1996) with rich literature on system identification techniques. The most challenging issue with the Volterra model is the high computational demand as the higher-order Volterra kernels are involved, hence, the Volterra model may be pruned from aerodynamic point of view, which significantly reduces computational effort (Wu and Kareem 2013a). As a result, a sparse or pruned Volterra model may be classified as a "gray-box" model since the fundamental aerodynamic mechanisms are utilized in the model development.

Basically, various aerodynamic and aeroelastic sources which contribute to the wind-induced effects on structures (bridges, buildings, aircrafts, or wind turbines) could be decomposed into perturbations of wind velocities, and translational and torsional motions of the structures. In order to improve the quality of nonlinear and unsteady modeling with emphasis on capturing physical significance, Volterra theory-based model is proposed to model the bluff-body aerodynamics and aeroelasticity, where the Volterra kernels are identified based on the impulse inputs of gusts or motions (e.g., Schetzen 1965; Silva 1997; Raveh 2001; Marques and Azevedo 2007; Balajewicz et al. 2010; Wu and Kareem 2011; 2012b; 2012c; 2013b). In this case, Volterra theory-based scheme
could be treated as a “white-box” model (parametric model) after the identified kernels are parameterized based on a selected function. It is noted that for linear cases the Volterra-theory-based model is reduced to conventional linear analysis framework in time domain.

The aforementioned linear and nonlinear analysis approaches for nonlinear bluff-body aerodynamics are systematically presented in Figure 1.1, where dash lines indicate the relative connections among various approaches.

![Figure 1.1. An overview of linear and nonlinear approaches](image-url)
CHAPTER 2:
CHARACTERIZATION AND IDENTIFICATION OF NONLINEAR FEATURES

Modern bridge decks exhibit nonlinear behaviour in wind tunnel experiments which has placed increasing importance on the nonlinear bridge aerodynamics/aeroelasticity considerations for long-span bridges. In this context, various observed nonlinear phenomena in wind tunnels are discussed and their identification through high-order spectrum is illustrated.

2.1 Introduction

The worldwide need to bridge wide river basins, straits and even parts of oceans has led to spans that are progressively increasing with aerodynamic deck configurations that exhibit aerodynamic/aeroelastic nonlinearities (Chen and Kareem 2003; Diana et al. 2005). The nonlinear fluid-structure interactions involving flow separation, reattachment around the deck, 3D wake dynamics and influence of turbulence are critical features that shape aerodynamic behavior of bridges. Traditionally, it is difficult to delineate the contributions of linear and nonlinear effects, therefore, the unexplainable observations in bridge aerodynamics have been often attributed to nonlinear effects (e.g., Dowell and Ilgamov 1988). Recent observations in wind tunnels have highlighted various nonlinear phenomena in bridge aerodynamics/aeroelasticity, which can be categorized as: (i). non-proportional relationship between input and output; (ii). multiple frequencies excited by a
single-frequency; (iii). amplitude dependence of aerodynamic and aeroelastic forces and (iv). hysteretic feature of aerodynamic/aeroelastic behavior versus the angle of attack. These features are summarized in the following section.

2.2 Nonlinear Features

2.2.1 Nonproportional Relationship between Input and Output

The non-proportional relationship between amplitudes of input and output is routinely observed in bridge aerodynamics/aeroelasticity in wind tunnel. In a recent study, related to the influence of turbulence on the bridge response, it was noted that doubling the amplitude of a harmonic fluctuation in the inflow resulted in ten times amplification of the bridge deck response (Diana et al. 2010). This nonlinear feature will significantly increase the aerodynamic responses as compared to the prediction by the linear theory. Furthermore, due to this nonlinear feature, a bridge deck may experience aeroelastic instability which the linear theory based scheme may fail to predict (Diana et al. 2010).

2.2.2 Multiple Frequencies Excited by a Single-Frequency

The nonlinear phenomenon related to the excitation of multiple frequencies by a single-frequency is known to introduce not only superharmonics but also causes harmonic distortion. Figure 2.1 presents experimental data of a typical modern bridge deck. The harmonic distortion in bridge aerodynamics is shown in Figure 2.1(a), i.e., the input turbulent wind velocity (idealized in terms of harmonic gusts) has only two frequency components at 0.37 Hz and 1.50 Hz. However, the torsional response contains not only response corresponding to 0.37 Hz and 1.50 Hz, resulting from the linear transformation,
but also the superharmonic components at 0.74 Hz and 1.13 Hz as a consequence of harmonic distortion. The emergence of superharmonics in bridge aeroelasticity is shown in Figure 2.1(b). The input torsional response has only one frequency component at 1.48 Hz. However, the motion-induced drag force contains not only the corresponding component at 1.48 Hz, from the linear transformation, but also the superharmonic component at 2.97 Hz.

This nonlinear feature may result in serious issues concerning bridge aerodynamics and aeroelasticity, especially if these frequency components generated by superharmonic (or subharmonic) or harmonic distortion fall at or near one of the modal frequencies of the bridge deck resulting in response that may not be predicted by conventional linear theory.

2.2.3 Amplitude Dependence of Aerodynamic and Aeroelastic Forces

The amplitude dependence of aerodynamic and aeroelastic forces is well known in bridge aerodynamics/aeroelasticity, where the steady-state force coefficients and the aerodynamic/aeroelastic transfer functions are dependent on the angle of attack (the angle between the chord line of bridge deck and the oncoming uniform wind). These features of dependence on the angle of attack can be explained by simple aerodynamic considerations. For example, dependence of flutter derivatives on the angle of attack is shown in Figure 2.2. The sensitivity of flutter derivatives at different angles of attack has a direct bearing on the critical flutter wind velocity. At 0° angle of attack the experimental critical flutter wind velocity of this bridge deck is estimated to be 89 m/s, while it decreases to 76 m/s for -3° angle of attack. Therefore, estimates based on the zero degree angle of attack may over predict flutter speed.
Figure 2.1. Nonlinear transformation of bridge aerodynamics and aeroelasticity: (a) turbulent wind input (data from the Politecnico di Milano); (b) torsional displacement input (data from the Hunan University)
Figure 2.2. A bridge deck aeroelastic coefficients under various angles of attack (data from TJ-1 Wind Tunnel in Tongji University): (a) flutter derivative $A_2^f$; (b) flutter derivative $H_2^f$.

2.2.4 Hysteretic Feature of Aerodynamic/Aeroelastic Behavior versus the Angle of Attack

The hysteretic feature is well known in vortex induced vibration of bluff bodies since it was initially observed experimentally by Feng (1968). Recently, the hysteretic phenomenon in the aerodynamic/aeroelastic forces on both the bridge decks and the cable of cable-stayed bridges has been a cause for concern (Diana et al. 2010; Wu et al. 2012).
Typically, the steady-state aerodynamic coefficients are modeled by a nonlinear polynomial in terms of the angle of attack. However, the hysteretic behavior can be best described by a higher-order (nonlinear) polynomial involving the dynamic angle of attack (Resulting from turbulence components and deck motions) and its derivative, e.g.,

\[ c_{l}^{\text{hyst}}(\bar{\theta}, \dot{\theta}) = \sum_{j,k} \eta_{j,k} \bar{\theta}^j \dot{\theta}^k \]  

(2.1)

where subscript "l" represents "L", "D" or "M", which denotes coefficients related to lift, drag and pitch, respectively; \( \eta_{j,k} \) is the coefficient corresponding to \((j+k)\)th-order term; \( 2n \) is the highest order of the polynomial possible to parsimoniously model hysteretic behavior. The actual order of the model depends on the data used for fitting and the identification scheme used. Besides, the contributions of some of the terms in the polynomial may be negligible which leads to simplified expression. For example, for the lift coefficient of a stay cable of a cable-stayed bridge, the following model can represent the hysteretic behavior:

\[ c_{L}^{\text{hyst}}(\bar{\theta}, \dot{\theta}) = \left[ c_{L}(\theta_0) \right] + \eta_{0,0} \bar{\theta} + \eta_{1,0} \dot{\theta} + \eta_{2,0} \ddot{\theta} + \eta_{1,1} \bar{\theta} \dot{\theta} + \eta_{3,0} \dddot{\theta} + \eta_{4,0} \ddddot{\theta} \]  

(2.2)

where the steady-state coefficient \( c_{L}(\theta_0) \) corresponding to the equilibrium position is extracted from the constant term in order to emphasize the difference between the steady-state and the hysteretic cases. A similar model with a third-order polynomial, in the context of a rheological model, has been proposed by Diana et al. (2010) to represent lift coefficient of a bridge deck. Figure 2.3 presents the hysteretic phenomena observed in the lift forces acting on a bridge deck and on a typical stay cable based on wind-tunnel
studies together with the numerically fitted model utilizing Moore-Penrose pseudoinverse identification scheme, which show a good agreement.

Figure 2.3. Modeling and simulation of the hysteretic behavior related to the lift coefficient: (a) hysteretic effect on the bridge deck (data from the Politecnico di Milano); (b) hysteretic effect on the stayed cable (data from the Hunan University)

The hysteretic behavior that appears in bridge aerodynamics/aeroelasticity suggests a higher-order memory in the system. Details of the hysteresis phenomenon, common to many fields, may be found in the literature (e.g., Brilliant 1958). The
numerical examples carried out by the researchers indicate that the wind-induced response of a typical modern bridge deck and a stayed cable is significantly increased by introducing the hysteretic behavior in the respective response prediction schemes (Wu and Kareem 2012a; Wu et al. 2012).

2.3 Tools for Detection of Nonlinear Features

In light of the uncertainties outlined, this section focuses on the higher-order spectrum as a most effective tool to identify and quantify nonlinearities and its application to bridge aerodynamics/aeroelasticity.

2.3.1 Higher-Order Spectrum

The higher-order spectrum is the Fourier transform of the higher-order correlation. It identifies nonlinear interactions among frequency components and preserves the phase information, which is generally a more sensitive marker to identify the departure from linearity as compared to information related to amplitude (Ueda and Dowell 1984). For a linear system, the second-order information embedded in a power spectrum represents distribution of energy at different frequencies, which fully characterizes a linear system in the frequency domain. The normalized value of the cross-power spectrum captures the phase relation at the same frequency between two different signals. However, for nonlinear system higher-order spectrum is needed because the power spectrum cannot portray the energy transformation between different frequency components which is a typical feature of nonlinear systems. Among these higher-order spectra, the bispectrum, the Fourier transform of triple correlation can capture quadratic nonlinearities. Further higher-order interactions can be tracked by trispectra and beyond, but each additional
order adds significantly to the computational demand and need for added length of data
sets (e.g., Kim and Powers 1979; Gurley et al. 1996; Tognarelli 1999; Nayfeh et al. 2003;
Hajj and Silva 2004). A bispectrum is easy to visualize since it can be displayed as a
three-dimensional (3-D) plot, whereas, further higher-order spectra can be only viewed in
3-D by slicing the hyper-dimension at different frequencies.

The auto-bispectrum for a stationary, real-valued, zero-mean signal \( a(t) \) can be
estimated by (e.g., Kim and Powers 1979)

\[
\tilde{B}_{aa}^{(k)}[f_1, f_2] = \frac{1}{N} \sum_{k=1}^{N} \left| a_r^{(k)}[f_1 + f_2]a_r^{*(k)}[f_1]a_r^{*(k)}[f_2] \right| \tag{2.3a}
\]

where \( a_r^{(k)}[f] \) is the Fourier transform of the \( k \)th ensemble of the time series \( a(t) \) taken
over a time \( T \) and \( N \) is the number of these ensembles. The nontrivial value of the
bispectrum could be treated as an indication of quadratic nonlinearity in the system. The
normalized auto-bispectrum is given by Equation 2.3b and can be viewed as auto-
bicoherence

\[
\tilde{b}_{aa}^{(k)}[f_1, f_2] = \frac{1}{N^2} \sum_{k=1}^{N} \left| a_r^{(k)}[f_1 + f_2]a_r^{*(k)}[f_1]a_r^{*(k)}[f_2] \right|^2 \left[ \frac{1}{N} \sum_{k=1}^{N} \left| a_r^{(k)}[f_1 + f_2] \right|^2 \right]^{-1} \left[ \frac{1}{N} \sum_{k=1}^{N} \left| a_r^{(k)}[f_1] \right|^2 \right]^{-1} \left[ \frac{1}{N} \sum_{k=1}^{N} \left| a_r^{(k)}[f_2] \right|^2 \right]^{-1} \tag{2.3b}
\]

The higher-order spectrum is generally complex valued and a multi-dimensional
function of frequency, e.g., the bispectrum is a two-dimensional function of frequency.
The magnitude of the auto-bispectrum \( \tilde{B}_{aa}[f_1, f_2] \) is determined by the phase relationship
between the frequency component \( (f_1 \pm f_2) \) and the frequencies component \( f_1 \) and \( f_2 \).
Usually, the nontrivial values of higher-order spectrum correspond to the harmonic
distortion which represents a typical nonlinear feature. For example, if the frequency component \((f_1 \pm f_2)\) is induced by the two frequency components \(f_1\) and \(f_2\) through harmonic distortion, there must be a certain phase relationship between them. As a result, the value of the corresponding bispectrum at that frequency combination will be nontrivial. On the contrary, if the frequency component \((f_1 \pm f_2)\) is not from harmonic distortion but original input (linear transformation), the phase relationship between frequency component \((f_1 \pm f_2)\) and the frequency components \(f_1\) and \(f_2\) are random. As a result, the corresponding bispectrum at that frequency combination will be trivial as it represents an average value of random quantities.

Analogous to the auto-bispectrum, the cross-bispectrum detects the quadratic nonlinearity that may exist between two different signals. The cross-bispectrum and the corresponding cross-bicoherence is given in Equations 2.4a and 2.4b, respectively, for two stationary, real-valued, zero-mean signals \(a(t)\) and \(b(t)\).

\[
\tilde{B}_{baa}[f_1, f_2] = \frac{1}{N} \sum_{k=1}^{N} \left| p_{T}^{(k)}[f_1 + f_2] a_{T}^{(k)}[f_1] a_{T}^{(k)}[f_2] \right| \tag{2.4b}
\]

\[
\tilde{b}_{baa}^2[f_1, f_2] = \frac{1}{N^2} \sum_{k=1}^{N} \left| p_{T}^{(k)}[f_1 + f_2] a_{T}^{(k)}[f_1] a_{T}^{(k)}[f_2] \right|^2 \frac{1}{N} \sum_{k=1}^{N} \left| a_{T}^{(k)}[f_1] a_{T}^{(k)}[f_2] \right|^2 \tag{2.4b}
\]

2.3.2 Illustrative Example

A simple numerical example is presented to illustrate the contribution of the higher-order spectrum to the identification of nonlinearity. Considering a harmonic nonlinear signal
\[ a(t) = \sin(\omega_1 t + \theta_1) + \sin(\omega_2 t + \theta_2) + \frac{1}{2} \sin(\omega_3 t + \theta_3) + w(t) \]

\[ + \sin(\omega_1 t + \theta_1) \sin(\omega_2 t + \theta_2) + \left[ \sin(\omega_3 t + \theta_3) \right]^3 \]

where \( \omega_1=0.3 \times 2\pi \), \( \omega_2=1.0 \times 2\pi \) and \( \omega_3=\omega_1+\omega_2 \), and \( w(t) \) is a small amplitude, zero mean white noise process. The three phase angles \( \theta_1-\theta_3 \) for different frequency components are independent and uniformly distributed random variables. Figures 2.4(a) and (b) show the time history and the corresponding frequency components of the signal via a Fourier spectrum. As depicted in Figure 2.4(b), there are five frequency components in this nonlinear time signal at 0.3 Hz, 0.7 Hz, 1.0 Hz, 1.3 Hz, and 3.9 Hz. In order to test the nonlinear detection capability of the second-order spectrum, Figure 2.5(a) presents the auto-bicoherence of the linear part of the harmonic nonlinear signal and Figures 2.5(b) and (c) show the auto-bicoherence of the linear and quadratic parts of the signal together with their contour plot. Figures 2.5(d) and (e) present the auto-bicoherence of the linear, quadratic and cubic nonlinear parts of the harmonic nonlinear signal together with their contour plot. As expected, the bicoherence value of a linear system is trivial as noted in Figure 2.5(a). The noise floor results from the white noise in this system and from the numerical errors. On the other hand, the product of harmonic components in the nonlinear signal generates harmonic distortion, i.e., the sum and difference of different frequency components. The bicoherence at frequencies \( (\omega_1, \omega_2) \) and \( (-\omega_1, \omega_2) \) should be nontrivial due to the nonlinear harmonic distortion as indicated in Figures 2.5(b) and (c). There are certain phase relationships between frequency components \( \omega_1, \omega_2 \) and \( \omega_1+\omega_2 \), and between \( \omega_1, \omega_2 \) and \( \omega_2-\omega_1 \). However, since there is a harmonic signal component with frequency component \( \omega_3 = \omega_1+\omega_2 \) that exists in the original signal, for which the
bicoherence value at the frequency combination \((\omega_1, \omega_2)\) is trivial, the value of bicoherence at frequencies \((\omega_1, \omega_2)\) should be less than 1 (around 0.5 for this set of parameters). As shown in Figures 2.5(d) and (e) the value around frequency component 3.9 Hz is trivial, which reaffirms that the second-order spectrum cannot capture cubic nonlinearity. Since the cubic nonlinear part contains linear transformation of frequency component 1.3 Hz, the value of bicoherence at \((\omega_1, \omega_2)\) should be less than 0.5. It should also be noted that the frequency components of interest should be smaller than the Nyquist frequency. Besides, due to the symmetric property of the second-order spectrum it suffices to only present the result in the first quadrant (e.g., Kim and Powers 1979; Nikias and Petropulu 1993).

Figure 2.4. A harmonic nonlinear signal and its spectral density function: (a) time history of the investigated signal; (b) frequency components of the investigated signal
Figure 2.5. Auto-bicoherence of the harmonic linear/nonlinear signal: (a) the auto-bicoherence of the linear part of the nonlinear signal; (b) the auto-bicoherence of the linear and quadratic parts of the nonlinear signal; (c) the contour plot of Fig. 5 (b) with contour levels set at 0.15, 0.3, 0.4, 0.6, and 0.9; (d) the auto-bicoherence of the linear, quadratic and cubic parts of the nonlinear signal; (e) the contour plot of Fig. 5 (d) with contour levels set at 0.15, 0.3, 0.4, 0.6, and 0.9 (p25-27)
2.3.3 Application to Bridge Aerodynamics/Aeroelasticity

Conventionally, the second-order spectrum technique is utilized to analyze aerodynamic/aeroelastic data obtained in wind tunnels. The auto-/cross-bispectrum and auto-/cross-bicoherence could be applied to both the pressure signal or the force signal obtained in wind-tunnel experiments.

Figure 6(a) presents the Fourier components of pressure fluctuations on a cable surface of a cable-stayed bridge being displaced harmonically at 1 Hz. As indicated in this figure, apart from the 1 Hz (0.9869 Hz) frequency component, there are several other
significant frequency components both in the low frequency and high frequency parts. Figure 6(b) presents the corresponding auto-bispectrum. The large spectrum of nontrivial values suggests nonlinear coupling or interaction that exists in this pressure fluctuation signal. For example, the point at coordinates (0.9869, 0.9869, 22.48) in this figure indicates that at least part of the value corresponding to the frequency component 1.974 in the Figure 6(a) results from the quadratic self interaction at the frequency component 0.9869. As a result, it is reasonable to point out that the frequency components lower or higher than the frequency component at 1 Hz partly results from nonlinear interactions among frequencies. Instead of presenting the contour figure typically used in the literature, Figure 6(c) presents the auto-bicoherence of two frequency components with all the frequencies within the range of interest, which clearly presents the nonlinear coupling phenomenon among different frequency components. It is noted in this figure, there is a high degree of nonlinear coupling between 1 Hz or 1.5 Hz components and the frequency region between 0-4 Hz. After this range, the nonlinear coupling among these frequency components decreases significantly. There is almost no nonlinear coupling between 1 Hz or 1.5 Hz and the frequency region higher than 8 Hz. Overall, the level of nonlinear coupling of 1.5 Hz with other frequency components is lower than that of 1 Hz. Figure 7 presents the second-order spectrum of an aerodynamic force acting on a typical bridge deck obtain in a wind tunnel. As mentioned in the preceding, the nontrivial values in the figure indicate nonlinear interactions between the frequency components, suggesting the presence of such interactions in the aerodynamics of bridge decks.

The higher-order spectrum technique is a promising scheme to detect potential existence of nonlinear behavior and has been extensively used in the areas of turbulence
Figure 2.6. Linear and higher-order spectral analysis of pressure fluctuations: (a) magnitude of frequency response; (b) auto-bispectrum; (c) auto-bicoherence
and offshore mechanics. Furthermore, it is also possible that by combining the information theory with higher-order spectral analysis, one can further uncover the directionality of nonlinear energy transfer (e.g., Jamšek et al. 2010). In the case of nonstationarity, the wavelet-based bicoherence could be utilized to detect the intermittent nonlinearities (e.g., Gurley et al. 2003; Chabalko et al. 2006).

2.4 Concluding Remarks
This study presents several nonlinear phenomena observed in wind tunnels concerning bridge aerodynamics/aeroelasticity. Analysis techniques have been shown to efficiently detect these nonlinearities, e.g., the higher-order spectrum scheme for the detection of quadratic coupling and the Moore-Penrose pseudoinverse scheme, though not detailed here, for modeling nonlinearities that result from hysteretic aerodynamic/aeroelastic behavior. It is highlighted that the higher-order spectrum technique affords delineation of the linear and nonlinear effects with high fidelity. Accurate modeling of the
aerodynamic/aeroelastic force on bridge decks is always a critical issue in ensuring the safety of long-span bridges. The demonstration of nonlinearities in aerodynamics and aeroelasticity of cable-supported bridges here calls for improved nonlinear modeling capabilities in this area. The application of quasi-steady theory is a well-known approach applied to model nonlinearity in bridge aerodynamics that accounts for the "static" nonlinearities. Accordingly, the amplitude dependent features of bridge aerodynamics/aeroelasticity could be simulated with a band-superposition or a hybrid formulation scheme (Diana et al. 1995; Chen and Kareem 2003). In order to take into account the "memory" effects, i.e., the hysteretic behavior, a rheologic scheme was proposed by Diana et al. (2010), whereas Wu and Kareem (2010) developed a higher-order artificial neural network to improve the hysteretic modeling capability. Another promising scheme for modeling nonlinear bridge aerodynamics/aeroelasticity is based on Volterra series. As an extension of Taylor series with memory, it provides an accurate description of the nonlinearity while preserving memory effects absent in the static transformation based on the Taylor series approach (Wu and Kareem 2011).
CHAPTER 3:
EFFECTS OF NONLINEARITY AND UNSTEADINESS

Accurate modeling of wind-induced loads on bluff bodies is critical to ensure the functionality and survivability of structures. It is often believed that nonlinearity usually has favorable effects on the aeroelastic systems due to limit-cycle oscillations. For wind turbines, nonlinearity and unsteadiness may alleviate the flapwise instability as the mean slope of the lift force becomes less negative under the dynamic stall condition (Hansen et al. 2006). On the other hand, nonlinearity also could result in unfavorable effects on the aeroelastic systems (Dowell and Tang 2002). For a specific aerodynamic or aeroelastic system, it is important to first make a preliminary investigation to better understand general effects of nonlinearity and unsteadiness (fluid memory) on the gust-, and motion-induced responses with recently developed analytical tools.

Over the last few decades, several schemes have emerged to model bridge behavior from an aerodynamic/aeroelastic perspective. These models include: quasi-steady (QS) theory-based model, corrected QS theory-based model, linearized QS theory-based model, semi-empirical linear model, and hybrid model. In order to account for the nonlinear fluid memory effects, a modified hybrid model is developed in the study, where the steady-state coefficients are modified based on their hysteretic features observed in the wind tunnel. The ability of these models to capture fluid memory and nonlinear effects either individually or collectively is examined, and the simulation bridge response

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based on various models demonstrates the effects of nonlinearity and unsteadiness on bridge aerodynamics and aeroelasticity.

3.1 Introduction

The quasi-steady (QS) theory is central to a large class of aerodynamic and aeroelastic analysis frameworks. It was first introduced to model galloping phenomenon, where the unsteady aerodynamic forces exerted on an oscillating structure were modeled by utilizing the steady-state coefficients (Parkinson and Brooks 1961; Gjelstrup and Georgakis 2011). The high amplitude oscillations of the original Tacoma Narrows Bridge deck were similar to galloping in a torsional degree-of-freedom (usually referred to as torsional flutter). Accordingly, it is convenient to illustrate this phenomenon based on the QS theory. However, an obvious shortcoming of the QS theory is that it cannot take into consideration the unsteady features inherent in fluid-structure interaction with attendant fluid memory effects. In a first attempt to take into account the fluid memory effects based on the static wind-tunnel tests, Steinman (1950) introduced phase correction factors resulting from pressure distribution graphs of stationary tilted straight models (for the effects of vertical velocity and wind angle of attack) and horizontal curved models (for the effects of angular velocity). A more appropriate approach to consider the fluid memory effects is however to modify the steady-state coefficients with unsteady effects measured in the wind tunnel (Diana et al. 1993). This, referred to as the corrected QS theory, could partially involve the consideration of fluid memory effects. The fluid memory indicates that the flow around the structure not only depends on the current relative states (structural motions and flow fluctuations) but also on their time histories.
On the other hand, to circumvent dealing with nonlinear differential equations, the QS theory-based formula is usually linearized to qualitatively evaluate the wind-induced aerodynamic/aeroelastic forces. The parameters (consisting of steady-state coefficients and their first-order derivatives) of the linearized QS theory-based formula for bridge decks have their corresponding equivalent analytical expressions for airfoils, i.e., the asymptotic form of the Sears/Theodorsen function (Sears 1941; Theodorsen 1935). The Sears/Theodorsen function, as a linear analytical function of wind velocity, could account for the unsteady effects. However, it is mathematically intractable to obtain corresponding analytical expressions for bridge deck cross-sections, that are usually bluff, due to flow separation and other aerodynamic considerations. This led to the development of unsteady parameters, derived from dynamic wind-tunnel tests, commonly referred to as aerodynamic admittances and flutter derivatives (Davenport 1962; Scanlan and Tomko 1971). Since these early studies, flutter analysis based on the linear analysis framework has been carried out through numerical and closed-form solutions including a so-called step-by-step method (e.g., N.P. Jones et al. 1998; Chen and Kareem 2006; Matsumoto et al. 2010; Wu 2010). This semi-empirical, linear, unsteady scheme has been applied extensively both in bridge research and design to investigate aerodynamic and aeroelastic behavior of bridges under winds.

It is important to note that the QS theory-based model could take into account the nonlinearity but not the fluid memory effects whereas the conventional semi-empirical model based on Scanlan's flutter derivatives could account for the fluid memory effects but not the nonlinearity. In light of these features, a hybrid scheme has been proposed to capture both the unsteady and nonlinear effects of wind-bridge interactions (Chen and
Kareem 2003). The basic premise of this approach is that the low-frequency aerodynamic/aeroelastic effects are simulated by the QS theory while the contributions at high-frequencies are represented by the conventional semi-empirical model. Though the hybrid model reasonably accounts for the nonlinear effect, it sacrifices the inclusion of unsteady effects at the low frequencies. Besides, the exact demarcation between the low- and high-frequency fluctuations currently lacks a rigorous treatment. In this study, a modified hybrid model is presented, in which the steady-state coefficients are modified based on their hysteretic features observed in the wind tunnel. As a result, this recently proposed model includes the higher-order memory effects through the hysteretic characteristics and the nonlinear feature by the QS theory.

Apart from the capability of considering both linear and nonlinear aerodynamic damping and stiffness, another remarkable advantage of the QS theory-based model in bridge aerodynamic/aeroelastic simulation is its ability to straightforwardly include the influence of turbulence on the aeroelastic instability. This is accomplished through generalizing the angle of attack as the dynamic angle of attack, which results from a combination of the bridge deck motions and approaching turbulence (incidence angle is also included). The inclusion of turbulence is very important since turbulence effects on bridge deck may become increasingly significant due to extreme sensitivity of the wind-induced response to the dynamic angle of attack for some bridge decks currently being considered for longspan bridges (Chen and Kareem 2003). Under turbulent wind conditions, some observations point out that the aeroelastic instability does not happen suddenly with a distinct flutter boundary as the semi-empirical linear analysis predicts (Irwin 1977; Chen and Kareem 2003). One plausible explanation is that turbulence leads
to exchange of energy between different structural modes, which result in a gradual built up of response towards flutter (Scanlan and N.P. Jones 1990; Chen and Kareem 2003). Under the semi-empirical linear analysis framework, generally there are two possible mechanisms of turbulence effect on bridge aerodynamics/aeroelasticity, namely the loss of coherence of the self-excited force and possible changes in flutter derivatives. Recent investigation of turbulence effects on the aerodynamics of an oscillating prism through the measurement of pressure on a bridge deck model indicates that the turbulence intensity and turbulence scale will simultaneously affect the flutter derivatives with different mechanisms (Haan and Kareem 2009). The former mainly changes the flutter derivatives which are sensitive to the pressure phase such as $A_2^*$ (weighting the contribution to the moment from torsional velocity) while the latter mainly changes the flutter derivatives which are sensitive to the pressure amplitude such as $A_3^*$ (weighting the contribution to the moment from torsional displacement) (Haan and Kareem 2009).

3.2 Implementation of Various Models

The coordinate system of the wind-bridge interaction study is shown in Figure 3.1. A unit-length section of a bridge deck subjected to the turbulent wind flow is considered here and the aerodynamic/aeroelastic loads are established based on the “strip theory”. The incident turbulent wind consists of mean wind velocity $U$ and $W$ and associated fluctuations $u$ and $w$ in the horizontal and vertical direction, respectively. The horizontal, vertical and torsional motions of the bridge section under turbulent wind are expressed as $p$, $h$ and $a$, respectively. The wind-induced effects considered are lift force (positive downward), drag force (positive downstream) and moment (positive clockwise). Only the
loads on the bridge deck due to turbulent winds are here included, whereas secondary loads on the cables and their effects are neglected without loss of generality.

In general, the bridge deck motion is modeled as a two-degree-of-freedom system with vertical and rotational motions. Accordingly, the governing equations of motion are given by

\[ m \left( \ddot{h} + 2 \zeta \omega \dot{h} + \omega^2 h \right) = F_y \]  \hspace{1cm} (3.1a)

\[ I \left( \ddot{\alpha} + 2 \zeta \omega \dot{\alpha} + \omega^2 \alpha \right) = M_z \]  \hspace{1cm} (3.1b)

where the over dot indicates derivative with respect to time, \( \zeta \) and \( \omega \) represent damping ratio and circular frequency of bridge deck, respectively; \( m \) is the effective mass and \( I \) the effective moment of inertia; \( F_y \) and \( M_z \) are wind-induced effects (aerodynamic/aeroelastic forces or moments), whose simulation is addressed in this study.
3.2.1 Quasi-Steady (QS) Theory-based Model

A QS theory-based model describes the aerodynamic load through a static nonlinear relationship between the incident flow and the flow-induced forces on the structure. The instantaneous status of the system at current time is mapped to the corresponding status at infinite time, such that the wake has convected far from the body, for evaluating the flow-induced forces. In other words, no fluid memory effects are considered in the QS theory. As a result, the QS theory is only applicable if the time necessary to the incident flow to pass around the structure and be convected far enough downstream is much less than the time it takes to the structure to respond to the changes in the surrounding flow. Hence, the QS theory is usually utilized for the low-frequency oscillation of the structure (or equivalently for the high convected wind velocity). Actually, in order to utilize static wind-tunnel test, to obtain the aerodynamic force, one has to define a steady state status of the structure aerodynamically equivalent to the unsteady status (van Oudheusden 1995). The aerodynamically equivalent steady state status is straightforward to define for the translational motions but not for the rotational motions since the angular velocity corresponding to the angular rotation will results in different values for various points on the deck (Wu et al. 2012).

Based on the QS theory, the lift force and torsional moment per unit span in the global bridge coordinates are expressed as (Kovacs et al. 1992; Miyata et al. 1995)

\[
F_y = F_L \cos(\phi) - F_D \sin(\phi) \\
M_z = M
\]

(3.2a)

where
\[ F_L = -\frac{1}{2} \rho V_r^2 B C_L(\alpha_e) \] (3.3a)

\[ F_D = \frac{1}{2} \rho V_r^2 B C_D(\alpha_e) \] (3.3b)

\[ M = \frac{1}{2} \rho V_r^2 B^2 C_M(\alpha_e) \] (3.3c)

where \( \rho \) is the air density; \( B \) is the bridge deck width; \( C_L, C_D \) and \( C_M \) are non-dimensional steady-state coefficients which are conveniently obtained with static wind-tunnel tests, where \( C_M \) is defined with respect to the center of the cross-section; \( V_r \) is the relative wind velocity given by

\[ V_r = \sqrt{(U + u)^2 + (W + w + \dot{h} + m \dot{\alpha} B)^2} \] (3.4)

and \( \alpha_e \) is the dynamic angle of attack expressed as

\[ \alpha_e = \alpha_s + \alpha + \phi \] (3.5)

where \( \alpha_s \) is the wind angle of attack when the bridge deck is at the equilibrium position (also involving possible incident wind angle); \( \alpha \) is the torsional displacement of the bridge deck under turbulent wind; \( \phi \) is the "dynamic" angle of attack induced by the bridge deck motions and wind fluctuations (here the terminology "effective angle of attack" refers to the situation that only the deck motions contribute the angle of attack) and could be calculated as

\[ \phi = \arctan \left( \frac{W + w + \dot{h} + m \dot{\alpha} B}{(U + u)} \right) \] (3.6)
The role of the parameter $m_1$, which takes into account averaged angular velocity-induced effects, will be discussed in detail in Section 3.5.

3.2.2 Corrected QS Theory-based Model

An obvious shortcoming of the QS theory-based model is that it cannot take into account the unsteady effects, which is only valid in the high wind velocity range, e.g., $(U/fB) > 10$, where $f$ is the oscillating frequency of the bridge deck. In order to improve this model, a corrected QS theory-based model has been advanced where a modified coefficient is introduced to account for unsteady effects (Diana et al., 1993). Based on the corrected QS theory, the lift force and torsional moment per unit span in the global bridge coordinates should be calculated based on Equation 3.2, where

\[
F_L = -\frac{1}{2} \rho V_r^2 B \left\{ C_L (\alpha_s) + \int_{\alpha_s}^{\alpha_s} k_{1i} \frac{d[C_L(\tilde{\alpha})]}{d[\tilde{\alpha}]} d\tilde{\alpha} \right\} \quad \text{(3.7a)}
\]

\[
F_D = \frac{1}{2} \rho V_r^2 B \left\{ C_D (\alpha_s) + \int_{\alpha_s}^{\alpha_s} k_{1D} \frac{d[C_D(\tilde{\alpha})]}{d[\tilde{\alpha}]} d\tilde{\alpha} \right\} \quad \text{(3.7b)}
\]

\[
M = \frac{1}{2} \rho V_r^2 B^2 \left\{ C_M (\alpha_s) + \int_{\alpha_s}^{\alpha_s} k_{1M} \frac{d[C_M(\tilde{\alpha})]}{d[\tilde{\alpha}]} d\tilde{\alpha} \right\} \quad \text{(3.7c)}
\]

The modified coefficients $k_{1i}$ ($i=L, D$ or $M$) are introduced to take into account part of the fluid memory effects which are usually functions of the reduced wind velocity and the angle of attack, and could be evaluated based on the unsteady information available from the dynamic wind-tunnel experiment.
3.2.3 Linearized QS Theory-based Model

Calculations involving bridge aerodynamic response using either the QS or corrected QS theory-based model involve nonlinear differential equations. In order to eliminate the need to solve nonlinear differential equations, the QS theory-based formula is usually linearized at the static equilibrium position to evaluate the wind-induced aerodynamic/aeroelastic forces.

Based on the linearized QS theory, the lift, drag and torsional coefficients are approximated as

\[ C_L(\alpha) \approx C_L(\alpha_s) + (\alpha + \phi)C'_L \mid_{\alpha_s} \quad (3.8a) \]

\[ C_D(\alpha) \approx C_D(\alpha_s) + (\alpha + \phi)C'_D \mid_{\alpha_s} \quad (3.8b) \]

\[ C_M(\alpha) \approx C_M(\alpha_s) + (\alpha + \phi)C'_M \mid_{\alpha_s} \quad (3.8c) \]

where the prime indicates derivative with respect to the angle of attack. After some manipulation, the lift force and torsional moment per unit span based on the linearized QS theory in the global bridge coordinates could be expressed as

\[ F_y = -\frac{1}{2} \rho U^2 B \left[ C_L \left[ \left( C_L + C_D \right) \frac{w}{U} + 2C_L \frac{u}{U} \right] + \left( C_L + C_D \right) \frac{h + m_B \alpha}{U} + C_L \alpha \right] \mid_{\alpha_s} \quad (3.9a) \]

\[ M_z = \frac{1}{2} \rho U^2 B \left[ C_M \left[ \left( C_M \right) \frac{w}{U} + 2C_M \frac{u}{U} \right] + \left( C_M \right) \frac{h + m_B \alpha}{U} + C_M \alpha \right] \mid_{\alpha_s} \quad (3.9b) \]

where the terms in square brackets represent the static, aerodynamic and aeroelastic effects, respectively. It should be noted that the only information used by the linearized
QS theory-based model is given by the steady-state coefficients at the static equilibrium position $\alpha_s$. Hence, even if this model is approximate, it offers a very good illustration of bridge aerodynamics and aeroelasticity.

3.2.4 Semi-Empirical Linear Model

A linearized model typically contains a first-order term added to the static or mean term. For a streamlined cross-section, like in the case of an airfoil, theoretical expressions for the linearized term are available, e.g., the Wagner function (response to a unit step variation in angle of attack) (Wagner 1925), the Theodorsen function (frequency response to sinusoidal motion) (Theodorsen 1935), the Küssner function (response to a sharp-edged gust in incompressible flow) (Küssner 1936), and the Sears function (frequency response to a sinusoidal gust) (Sears 1941). As indicated in Equations 3.9a and 3.9b, the linearized force is usually separated into the buffeting and flutter components, which arise from the buffeting action of the incident turbulence and the bridge deck motions, respectively. No interaction between the aerodynamic and aeroelastic effects is included in this representation.

However, in the case of bridge aerodynamics/aeroelasticity, the bluff nature of the deck precludes the formulation of an analytical expression for the linearized terms. In order to overcome this difficulty, the so-called semi-empirical data-driven scheme was introduced by Scanlan (e.g., Scanlan and Tomko 1971) by utilizing sectional model test data to account for the aeroelastic effects. Similarly, for the buffeting part Davenport (1962) introduced an expression with coefficients derived from wind-tunnel tests (or the linearized QS theory may be invoked).
Accordingly, the lift force and torsional moment in the frequency domain per unit span based on the semi-empirical linear model in the global bridge coordinates can be expressed as

\[
F_y = -\frac{1}{2} \rho U^2 B \left( [C_L] + [F_{u_b}] + [-F_{u_s}] \right) \alpha_x \tag{3.10a}
\]

\[
M_z = \frac{1}{2} \rho U^2 B^2 \left( [C_M] + [M_{u_b}] + [M_{u_s}] \right) \omega_x \tag{3.10b}
\]

where the subscript \( b \) indicates the buffeting forces (or moment) while \( s \) indicates the self-excited forces (or moment). The buffeting forces (or moment) induced by wind turbulence are expressed as (e.g., Davenport 1962)

\[
F_{u_b} = (C'_L + C_D) \chi_{L_w} \frac{w(t)}{U} + 2C_L \chi_{L_u} \frac{\mu(t)}{U} \tag{3.11a}
\]

\[
M_{u_b} = C'_M \chi_{M_w} \frac{w}{U} + 2C_M \chi_{M_u} \frac{\mu}{U} \tag{3.11b}
\]

while the self-excited forces (or moment) induced by the bridge deck motions are expressed as

\[
F_{u_s} = K H_1'(K) \frac{\dot{h}}{U} + K H_2'(K) \frac{B \dot{\alpha}}{U} + K^2 H_3'(K) \dot{\alpha} + K^2 H_4'(K) \frac{h}{B} \tag{3.12a}
\]

\[
M_{u_s} = K A_1'(K) \frac{\dot{h}}{U} + K A_2'(K) \frac{B \dot{\alpha}}{U} + K^2 A_3'(K) \dot{\alpha} + K^2 A_4'(K) \frac{h}{B} \tag{3.12b}
\]

where \( \chi_{L_w}, \chi_{L_u}, \chi_{M_w} \) and \( \chi_{M_u} \) are aerodynamic transfer functions (whose moduli are referred to as aerodynamic admittance functions) between wind fluctuations and buffeting forces (or moment), which are functions of \( K \), wind angle of attack and deck
shape; $K = \frac{B \omega}{U}$ is the dimensionless reduced frequency; the coefficients $H_i'(K)$ and $A_i'(K)$ ($i=1\ldots4$) are aeroelastic transfer functions (flutter derivatives), which are also functions of $K$, wind angle of attack and deck shape (e.g., Scanlan and Tomko 1971). As noted earlier, aerodynamic and aeroelastic transfer functions are measured using section model tests in a wind tunnel (e.g., Scanlan and Tomko 1971; Kareem and Cermak 1979). Actually, the linearized QS theory-based model is a simplified version of the semi-empirical linear model, where the flutter derivatives and aerodynamic admittance functions are replaced by steady-state coefficients evaluated at the static equilibrium position.

As all these models will be compared in the time domain, it is necessary to recast the lift force and torsional moment (buffeting and self-excited loading in the frequency domain) for the time domain analysis. The buffeting and self-excited force (or moment) coefficients in the time domain are usually approximated using the corresponding coefficients in the frequency domain (e.g., aerodynamic admittances and flutter derivatives) measured in a wind tunnel. There are several data fitting schemes available to approximate these coefficients in the time domain, e.g., a rational function (Roger 1977; Chen and Kareem 2003) or an indicial function approximation (Scanlan et al. 1974). In this study, the indicial function approximation is utilized ("semi-inverse" approach) as this approximation offers better physical interpretation of the wind-induced effects (Wu and Kareem 2012d). Usually, the approximated indicial function (here referred to as the effective unit-step response) is normalized by the corresponding steady-state values.
The buffeting forces (or moment) induced by turbulent wind in the time domain are expressed as

\[
F_L(s) = 2C_L \left\{ \psi_u (s) \frac{u(0) + \int_0^s \psi_u (s - \sigma) \frac{u'(\sigma)}{U} d\sigma}{U} \right\} \\
+ C_L \left\{ \psi_w (s) \frac{w(0) + \int_0^s \psi_w (s - \sigma) \frac{w'(\sigma)}{U} d\sigma}{U} \right\} \tag{3.13a}
\]

\[
M_L(s) = 2C_M \left\{ \psi_M^u (s) \frac{u(0) + \int_0^s \psi_M^u (s - \sigma) \frac{u'(\sigma)}{U} d\sigma}{U} \right\} \\
+ C_M \left\{ \psi_M^w (s) \frac{w(0) + \int_0^s \psi_M^w (s - \sigma) \frac{w'(\sigma)}{U} d\sigma}{U} \right\} \tag{3.13b}
\]

and the self-excited forces (or moment) induced by the bridge deck motions in time domain are expressed as

\[
F_L(s) = C_L \left\{ \phi_L (0) \frac{\alpha(s) + \int_0^s \phi_L (s - \sigma) \alpha(\sigma) d\sigma}{B} \right\} \\
+ C_L \left\{ \phi_L (0) \frac{\dot{h}(s) + \int_0^s \phi_L (s - \sigma) \frac{\dot{h}(\sigma)}{B} d\sigma}{B} \right\} \tag{3.14a}
\]

\[
M_L(s) = C_M \left\{ \phi_M (0) \frac{\alpha(s) + \int_0^s \phi_M (s - \sigma) \alpha(\sigma) d\sigma}{B} \right\} \\
+ C_M \left\{ \phi_M (0) \frac{\dot{h}(s) + \int_0^s \phi_M (s - \sigma) \frac{\dot{h}(\sigma)}{B} d\sigma}{B} \right\} \tag{3.14b}
\]

where \( s = Ut/B \) is the non-dimensional time; here the prime indicates derivative with respect to non-dimensional time. \( \psi(s) \) represents the effective unit-step response to buffeting forces (or moment), which is determined indirectly through the respective aerodynamic transfer functions. The asymptotic values of these four non-dimensional effective unit-step response functions \( \psi_{L(u/U)} \), \( \psi_{L(w/U)} \), \( \psi_{M(u/U)} \) and \( \psi_{M(w/U)} \), which correspond
to unit-step gust input, are all equal to one since these variables are normalized with respect to the steady-state values. These functions have an analytical equivalent version for airfoil sections described as the Küssner function. $\phi(s)$ represents the effective unit-step response to self-excited forces (or moment), which is determined indirectly through the respective flutter derivatives. The asymptotic values of these four non-dimensional effective unit-step response functions $\phi_{L\alpha}$, $\phi_{Lh/B_1}$, $\phi_{M\alpha}$ and $\phi_{Mh/B_1}$, which correspond to unit-step bridge deck motion input, are all equal to one since these variables are also normalized with respect to the steady-state values. These functions have an analytical equivalent version for airfoil sections known as the Wagner function. The input for the semi-empirical linear model in both frequency and time domains will be discussed in detail in Section 3.5.

3.2.5 Hybrid Model

It is obvious that the semi-empirical model could take into account unsteady effects without any nonlinearity while the QS theory-based model could model nonlinearity without any unsteady effects, though corrected QS theory-based model attempts to take into consideration unsteady phenomena by introducing a parameter $k_1$, which is identified based on wind-tunnel tests. It should be noted that the determination of the proper reduced wind velocity in the corrected QS theory is not possible. Therefore, only a unique value of the reduced velocity is utilized. In order to overcome this shortcoming of the corrected QS theory-based model, a band superposition model has been advanced (Diana et al. 1995). Within this approach, the incident turbulent wind velocity (and the corresponding response) is separated into low-frequency and high-frequency parts. For
the low-frequency part, the QS theory-based model is utilized. The high-frequency part, where the parameters are dependent on the instantaneous angle of attack obtained from the low frequency part, is further divided into different sub-ranges. A different unique value of the reduced velocity could be defined by the frequency component of the sub-range turbulence for the linear analysis. However, this type of band superposition scheme is not easy to implement for the time-frequency mixed equations of motion and the measurement of flutter derivatives under the incident wind condition characterized by narrow-band turbulent fluctuations in a band superposition scheme is not feasible. Besides, inclusion of possible interaction between each sub-range is not possible.

In order to further improve the band superposition scheme, a framework, which utilizes a rational function approximation scheme to obtain an efficient computational procedure, was proposed by Chen and Kareem (2003). This scheme has a clear connection with conventional linear analysis framework (here referred to as semi-empirical linear model). Indeed, it simply combines the QS theory-based model and the semi-empirical linear model. The QS theory-based model is utilized when the reduced wind velocity is relatively high (the low-frequency part) while the semi-empirical linear model is utilized when the reduced wind velocity is relative low (the high-frequency part). Basically, the hybrid model linearizes the wind-induced force around a dynamic equilibrium angle of attack (or instantaneous angle of attack with low-frequency components) as opposed to the static angle of attack implied in conventional analysis. In the current study, instead of a rational function approximation, an indical function approximation is utilized to implement the high-frequency part of the wind-bridge interaction system.
Based on the hybrid model, the lift force and torsional moment per unit span in the global bridge coordinates are calculated by combining the forms of Equations 3.2a and 3.2b and Equations 3.10a and 3.10b and may be expressed as

\[
F_y = -\frac{1}{2} \rho B \left\{ V_{i}^{1/2} \left\{ C_{L} (\alpha_{i}') \cos(\phi') + C_{D} (\alpha_{i}') \sin(\phi') \right\} \right. \\
\left. + U^2 \left\{ [C_L] + [F_{L}] \right\} \right\}_{l} (3.15a)
\]

\[
M_z = \frac{1}{2} \rho B^2 \left\{ V_{i}^{1/2} \left\{ C_{m} (\alpha_{i}') \right\} \right. \\
\left. + U^2 \left\{ [C_m] + [M] \right\} \right\}_{l} (3.15b)
\]

where superscript \( l \) indicates the low-frequency part based on a presumed range of low frequency and

\[
V_{i}^{l} = \sqrt{(U + u')^2 + (W + w' + \hat{h}' + m_B \hat{\alpha}')^2} (3.16)
\]

\[
\alpha_{i}' = \alpha, + \alpha' + \phi' (3.17)
\]

\[
\phi' = \arctan \left( \frac{W + w' + \hat{h}' + m_B \hat{\alpha}'}{U + u'} \right) (3.18)
\]

In essence, parameters in the high-frequency part should be measured in a simulated incident turbulent flow in a wind tunnel. Since they are dependent on the instantaneous angle of attack resulting from the low-frequency part, it is necessary to calculate the low-frequency part first (also the static equilibrium position). The role of the cut-off frequency dividing the turbulent wind into low-frequency and high-frequency parts will be discussed in detail in Section 3.5.
3.2.6 The Proposed Modified Hybrid Model

The so-called "static" wind-tunnel tests are adopted to determine steady-state coefficients while the "dynamic" wind-tunnel tests are used to assess the aerodynamic or aeroelastic transfer functions. An alternative way to define these static and dynamic features is to describe the results of both "static" and "dynamic" wind-tunnel tests using the same parameters, namely the steady-state and dynamic-state coefficients, respectively. Both steady- and dynamic-state coefficients are nonlinear with respect to the angle of attack. The difference is that the steady-state coefficients vary monotonically with the angle whereas the dynamic-state coefficients exhibit a hysteretic loop with respect to the angle. Like flutter derivatives (or aerodynamic admittance functions), the dynamic-state coefficients with hysteretic features are not only dependent on the equilibrium position of the bridge deck, but also on the reduced wind velocity (e.g., Alonso et al. 2012). In the wind-bridge interaction, if it is assumed that each contributing factor, like deck motion or wind fluctuation component, has equal weighting as implied in the QS theory-based model, then the bridge aerodynamic/aeroelastic behavior could be described utilizing the dynamic-state coefficients with hysteresis (Diana et al. 2010). This assumption may need careful validation because the error introduced may not always be acceptable. This apparent shortcoming has been minimized recently by introducing higher-order artificial neural network (ANN) that accounts for different weights for each contributing factor to the dynamic angle of attack (Wu and Kareem 2010). However, this nonparametric model lacks in its ability to elucidate the physical significance of various weighting factors that result from the training of the ANN. Another critical issue when utilizing the dynamic-state coefficients in bridge aerodynamics/aeroelasticity is that it is very difficult to find a
proper frequency to define the reduced wind velocity, especially when wind fluctuations are involved. One possible way of resolving this issue is to determine an averaged frequency for defining the reduced wind velocity at each wind velocity. The concept of band superposition offers an alternative approach, where different reduced wind velocities could be defined at each sub-range to apply appropriate dynamic-state coefficients measured in the wind tunnel. Besides, this approach could improve the hybrid model which compromises the consideration of unsteady effects in the low-frequency part to account for the nonlinear turbulence effect on the aeroelastic instability.

Based on the modified hybrid model, the lift force and torsional moment per unit span in the global bridge coordinates should be modified as

\[
F_y = -\frac{1}{2} \rho B \left\{ U^2 \left[ C_L \right] + V_r^{\text{hy}} \left\{ C_L^{\text{hy}}(\alpha_e) \cos(\phi^') + C_D^{\text{hy}}(\alpha_e) \sin(\phi^') \right\} + V_r^{h_1} \left\{ C_L^{\text{hy}}(\alpha_e^{h_1}) \cos(\phi^{h_1}) + C_D^{\text{hy}}(\alpha_e^{h_1}) \sin(\phi^{h_1}) \right\} \bigg|_{\alpha_e^{h_1}} + \ldots \right. \\
\left. + V_r^{h_n} \left\{ C_L^{\text{hy}}(\alpha_e^{h_n}) \cos(\phi^{h_n}) + C_D^{\text{hy}}(\alpha_e^{h_n}) \sin(\phi^{h_n}) \right\} \bigg|_{\alpha_e^{h_n}} \right\}
\]

(3.19a)

\[
M_z = \frac{1}{2} \rho B^2 \left\{ U^2 \left[ C_M \right] + V_r^{\text{hy}} \left\{ C_M^{\text{hy}}(\alpha_e) \right\} + V_r^{h_1} \left\{ C_M^{\text{hy}}(\alpha_e^{h_1}) \right\} \bigg|_{\alpha_e^{h_1}} + \ldots \right. \\
\left. + V_r^{h_n} \left\{ C_M^{\text{hy}}(\alpha_e^{h_n}) \right\} \bigg|_{\alpha_e^{h_n}} \right\}
\]

(3.19b)

where superscript \( hys \) indicates the dynamic-state coefficients with hysteresis (that will be discussed in Section 3.3); \( h_n \) denotes that the wind fluctuations in the high-frequency part are divided into \( n \) sub-ranges, and
Due to a lack of experimental data, only the dynamic-state coefficients with hysteresis in the sub-range with low-frequency fluctuation components are utilized in this study.

3.3 Identification of Model Parameters in Time Domain

In this section, identification of various model parameters is carried out for utilizing in the time domain simulation.

3.3.1 Low- and High-Frequency Parts of Wind Fluctuations

The bridge aerodynamic/aeroelastic analysis in time domain requires fluctuating wind loads as input. The time histories of wind fluctuations at the center of the bridge deck are simulated utilizing the spectral representation scheme with prescribed power spectral density (PSD) function and turbulence integral scales and intensities. The mean wind velocity adopted for the following analyses is 60 m/s, the turbulence integral scales $L_u, L_w$ are 120 m and 60 m, the turbulence intensities $I_u, I_w$ are 15% and 8%, respectively and the von Kármán PSD is used.

In order to implement the hybrid and modified hybrid models, wind fluctuations should be separated into the low-frequency and the high-frequency parts. A low-pass and a high-pass elliptic filter were utilized and the cut-off frequency pinpointing the low- and high-frequency parts is fixed at $f_p = 0.104$ Hz which corresponds to the first natural
frequency of the deck. Usually, the contribution of deck motions to the dynamic angle of attack is negligible as compared to the fluctuations in winds. As a result, the fluctuation contributions to the dynamic angle of attack $\phi$ (Equation 3.18) could be approximated as

$$\phi' \approx \arctan \left( \frac{w'}{(U + u')} \right)$$

(3.23)

3.3.2 Steady-State and Dynamic-State Coefficients

Steady-state coefficients of an example bridge deck used in this study are obtained based on static wind-tunnel tests with angles of incidence ranging from $-10^\circ$ to $10^\circ$ (Figure 3.2). The experimental data for $C_L$ (normalized to the bridge width), $C_D$ (normalized to the bridge height), and $C_M$ (normalized to the bridge width) is fitted by polynomials, as shown in the figure. Based on the steady-state coefficients, the nonlinear aerostatic response $\alpha_s$ is calculated and shown in Figure 3.3. No torsional divergence is observed for the mean wind velocity as high as 200 m/s.

Dynamic-state coefficients with varying angle of incidence must be obtained utilizing dynamic wind-tunnel tests. Currently, limited data is available for the dynamic-state coefficients exhibiting hysteretic loops. For the sake of illustration, in this study the dynamic-state coefficients are adopted from Diana et al. (2010). Just like the hysteretic characteristics of materials, the dynamic-steady coefficients with hysteresis could be represented by various models established for simulating hysteretic materials properties.
Figure 3.2. Steady-state coefficients

Figure 3.3. Aerostatic response
Diana et al. (2010) utilized a rheological model to simulate the hysteretic effects, whereas Wu and Kareem (2010) used a third-order ANN to simulate the hysteretic loops observed involving bridge decks. Based on earlier work by Diana et al. (2010), a new higher-order numerical model is applied to simulate the dynamic-state coefficients as (Wu et al. 2012)

\[
C^j_{sys}(a_e, \dot{a}_e) = \left[ C_j(\alpha_j) \right] \\
+ \eta_1 + \eta_2 a_e + \eta_3 \dot{a}_e \\
+ \eta_4 a_e^2 + \eta_5 a_e \dot{a}_e + \eta_6 \dot{a}_e^2 + \eta_7 a_e^3 + \eta_8 a_e^2 \dot{a}_e + \eta_9 a_e \dot{a}_e^2 + \eta_{10} \dot{a}_e^3 + \eta_{11} a_e^4 + \eta_{12} a_e^5
\] (3.24)

where subscript \( j \) represents \( M, L \) or \( D \), respectively. This higher-order numerical model is the optimized one resulting from a much higher-order numerical model based on a data-driven technique. The Moore-Penrose pseudoinverse scheme (e.g., Wu et al. 2012) is applied to identify the coefficients of the numerical model for the dynamic-state coefficients with hysteresis. The experimentally derived dynamic-state coefficients and the numerically fitted results are shown in Figure 3.4.

3.3.3 Effective Unit-Step Response of Bridge

For the bluff bridge deck, a theoretical solution of the unit-step response to the deck motion and gust inputs is not tractable. In order to obtain the effective unit-step response \( \phi_{la}, \phi_{l(h/B)}, \phi_{Ma} \) or \( \phi_{M(h/B)} \) through discrete values of flutter derivatives, it is assumed to have the form similar to R.T. Jones’ approximation (1940) for the Wagner function, and expressed as (Scanlan et al. 1974)

\[
\phi_{ys}(s) = 1 - \sum_{i=1}^{n} a_{ys} e^{-b_{ys} s}
\] (3.25)
Figure 3.4. Dynamic-state coefficients with hysteresis: (a) lift force coefficient; (b) torsional moment coefficient
where subscript $x$ denotes the input variables $\alpha$ or $h/B$ and subscript $y$ denotes the response $L$ or $M$; here prime indicates a derivative with respect to non-dimensional time; the parameters $a$ and $b$ are constants to be identified; the value of $n$ is dependent on the data available and the aerodynamic properties of the bridge deck. The relationship between the Fourier transforms of the effective unit-step response and the flutter derivatives is presented as (Scanlan et al. 1974; Wu and Kareem 2012d)

\[
\begin{align*}
\{ \tilde{\phi}_{Lx}(0) + \tilde{\phi}_{Lx}(K) \} &= \frac{K^2 \left( iH_x^*(K) + H_x^*(K) \right)}{dC_L \, d\alpha} \\
\{ \tilde{\phi}_{L^h_B}(0) + \tilde{\phi}_{L^h_B}(K) \} &= \frac{K \left( H_x^*(K) - iH_x^*(K) \right)}{dC_L \, d\alpha} \\
\{ \tilde{\phi}_{Mx}(0) + \tilde{\phi}_{Mx}(K) \} &= \frac{K^2 \left( iA_x^*(K) + A_x^*(K) \right)}{dC_M \, d\alpha} \\
\{ \tilde{\phi}_{M_B^h}(0) + \tilde{\phi}_{M_B^h}(K) \} &= \frac{K \left( A_x^*(K) - iA_x^*(K) \right)}{dC_M \, d\alpha}
\end{align*}
\] (3.26a)

where an over bar denotes the Fourier transform operator and $i = \sqrt{-1}$.

Figure 3.5 presents flutter derivatives and several effective unit-step responses of the example deck at various wind angles of attack. The theoretical lift force on a thin airfoil induced by a sudden change in the angle of attack is also included in the figure for comparison [Figure 3.5(d)].
Figure 3.5. Comparison of the identified effective unit-step response with various angles of incidence: (a) flutter derivatives at various angles of incidence; (b) fitted flutter derivatives at 0° angle of incidence; (c) fitted flutter derivatives at -3° angle of incidence; (d) comparison of the effective unit-step response

The effective unit-step response due to a gust input is also assumed to have the form similar to the R.T. Jones’ approximation for Küssner function

$$\psi_{yz}(s) = 1 - \sum_{i=1}^{n} c_{0z} e^{-d_{0z}s}$$  \hspace{1cm} (3.27)

where subscript $z$ denotes input variables $u/U$ or $w/U$ and subscript $y$ denotes the responses $L$ or $M$; the parameters $c$ and $d$ are constants to be identified; again, the value of $n$ is dependent on the data available and the aerodynamic properties of the bridge deck.
Once the aerodynamic transfer function in the frequency domain is identified through wind tunnel tests akin to what is done for the aeroelastic transfer functions $H'_e(K)$ or $A'_e(K)$, it would be easy to identify the effective unit-step response due to a gust input using various data fitting techniques (e.g., "semi-inverse" approach). However, the measurement of aerodynamic transfer functions is difficult in wind tunnel as compared to that of the flutter derivatives. As a result, due to a lack of experimental data for aerodynamic transfer functions, the Küssner function (the Sears function in frequency domain) is utilized in this study to calculate turbulence induced lift force and torsional moment. The approximate explicit analytical expression of the Küssner function is (R.T. Jones 1940)

$$\psi(s) = 1.00 - 0.236e^{-0.058s} - 0.513e^{-0.364s} - 0.171e^{-2.420s}$$

(3.28a)

and it is usually approximated by a simpler form (Bisplinghoff and Ashley 1962)

$$\psi(s) = 1 - 0.5e^{-0.130s} - 0.5e^{-s}$$

(3.28b)

It should be noted that in this study the aerodynamic transfer functions are assumed to be insensitive to the angle of incidence.

3.4 Bridge Response Analysis Using Different Models

In this section, the aerodynamic and aeroelastic responses of a long-span cable supported bridge have been calculated with the six models discussed in the preceding sections. The geometric shape of the example bridge deck is presented in Figure 3.1, and typical aerodynamic properties have been given in the previous section. The structural parameters used in the calculation are as follows: the effective mass and polar moment of
inertia per unit span are 32,000 kg/m and 8,770,000 kg·m, respectively; the width of the bridge deck is 41 m; the natural frequencies of fundamental vertical and torsional modes are 0.195 Hz and 0.531 Hz, respectively; the damping ratio for both modes is 0.005; the density of air is 1.225 kg/m³.

3.4.1 Aerodynamic Analysis

3.4.1.1 Comparison of Aerodynamic Response

The aerodynamic response of a bridge deck under turbulent wind conditions is investigated using the various models previously discussed. Figure 3.6 presents the root-mean-square (rms) vertical and torsional displacements at the mean wind velocity of 60 m/s with horizontal turbulence intensity $I_u$ equal to 15% and vertical turbulence intensity $I_w$ equal to 8%. As some of these models under investigation are based on the QS theory, the selection of a relatively high mean wind velocity is necessary to conduct a reasonable comparison. Since the semi-empirical linear model is widely used for bridge aerodynamics/aeroelasticity studies, it is treated here as a reference model for comparison. As discussed in the preceding sections, the QS theory-based model takes into account nonlinear effects but fails to include fluid memory effects; the corrected QS theory-based model considers nonlinear effects with linear fluid memory effects at a fixed reduced frequency; the linearized QS theory-based model accounts for the linear effects without the fluid memory effects; the semi-empirical linear model considers linear effects with linear fluid memory effects; the hybrid model includes nonlinear effects with linear fluid memory effects and the modified hybrid model accounts for nonlinear effects with nonlinear memory (or higher-order memory effects featured by hysteresis).
Based on the aerodynamic response calculated using each model, in general it may be concluded that: (i) the aerodynamic response based on the hybrid or modified hybrid models is close to the semi-empirical linear model, which indicates that the nonlinearity or higher-order fluid memory effects captured by these two models in this case are small for the example bridge; (ii) the aerodynamic response based on the corrected QS model is closer to the semi-empirical linear model (larger for the vertical response and smaller for the torsional response in this case) compared to that based on the QS model, which indicates the corrected QS model could not necessarily improve the response by taking into account the fluid memory at a fixed reduced frequency; (iii) the inclusion of fluid memory effects is more significant for the torsional degree-of-freedom rather than the vertical degree-of-freedom as the relative difference between the torsional response calculated by the linearized QS model and the semi-empirical linear model is large compared to the relative difference between the vertical response; (iv) the static nonlinearity is more significant for the vertical degree-of-freedom as compared to the
torsional degree-of-freedom as the relative difference between the vertical response calculated by the linearized QS model and QS theory-based model is large compared to the relative difference between the torsional response. In essence, based on these results, the contribution to the aerodynamic response from the fluid memory effects is higher than that resulting from nonlinear effects. It should be noticed that the relatively large torsional response based on the modified hybrid model is due in part to the contribution from the limit cycle oscillation (LCO) response resulting from subcritical behavior of the bridge. This suggests that the aeroelastic instability results from the low-frequency part in the hybrid and modified hybrid models.

Since each model has a unique way of defining aerodynamic response, therefore, for a detailed comparison time histories of responses in turbulent flow needs to be examined. Figure 3.7 presents a comparison of time histories of the calculated aerodynamic response based on each model at the mean wind velocity of 60 m/s with $I_u=15\%$ and $I_w=8\%$. This comparison of aerodynamic responses further indicates that the inclusion of fluid memory effects has larger impact on the torsional response since there are significant differences both for the amplitudes and phases in the torsional response based on the QS theory, corrected QS and linearized QS models compared with that based on the reference model. On the other hand, the vertical response based on the hybrid or modified hybrid model involves more frequency components compared to the torsional response due in part to the interaction between low-frequency and high-frequency parts.
Figure 3.7. Time histories of the calculated response based on each model: (a) comparison of results obtained from various models; (b) comparison of results obtained from QS theory-based and semi-empirical linear models; (c) comparison of results obtained from corrected QS theory-based and semi-empirical linear models; (d) comparison of results obtained from linearized QS theory-based and semi-empirical linear models; (e) comparison of results obtained from hybrid and semi-empirical linear models; (f) comparison of results obtained from modified hybrid and semi-empirical linear models
3.4.1.2 Comparison of Aerodynamic Response in Turbulent Wind Conditions

Figure 3.8 presents the rms of the aerodynamic response based on various models under different mean wind velocities with turbulence intensities of $I_u=15\%$ and $I_v=8\%$. Based on this figure, it is obvious that these models could be divided into two groups where one group cannot take into account the fluid memory effects (QS theory-based and linearized QS models) and the other group accounts for the fluid memory effects (semi-empirical linear, hybrid and modified hybrid models). The ability of the corrected QS model to reasonably account for the fluid memory effects is not assured. The results obtained from the methods that do not include fluid memory are obviously larger than those of the group of methods with fluid memory, which indicates that the fluid memory effects reduce the aerodynamic response at different wind velocities. On the other hand, the hybrid model-based results are close to those calculated from the semi-empirical linear model, which further demonstrates that the hybrid model may not fully account for the nonlinear effects. Besides, the comparison of response based on the QS theory and linearized QS theory at various wind velocities further demonstrates that the static nonlinearity (without coupling fluid memory effects) has significant effects on the vertical aerodynamic response at various wind velocity while its influence on the torsional responses is lower. However, the same cannot be said for the dynamic nonlinear effects (with coupling fluid memory effects). Furthermore, it should be noted that these observations are based on comparison with the semi-empirical model, which actually may not provide the "true" baseline for the bridge aerodynamic/aeroelastic issues. The differences noted for various models are more significant at higher wind velocities. The jaggedness in the corrected QS theory-based results at some wind velocities is due to
relatively abrupt changes in the modified coefficient \( k_1 \) while the large oscillations of the modified hybrid-based results at wind velocity 60 m/s and 65 m/s are due to the subcritical behavior.

Figure 3.8. RMS response of various models under different wind velocity \( (I_w=8\%) \): (a) RMS of vertical displacements; (b) RMS of torsional displacements
Figure 3.9 presents the rms of the aerodynamic response calculated through various models under different turbulence intensities at the mean wind velocity of 40 m/s. As indicated in this figure, in general the increase in the aerodynamic response based on the models which can consider the fluid memory effects is smaller than that based on the models which cannot account for the fluid memory effects. Besides, when the turbulence intensity $I_u$ is higher than 15%, there is a disproportionate increase in the aerodynamic response for most models.

Figure 3.9. RMS response of various models under different turbulent conditions, $U=40$ m/s (QS=QS theory-based model; CQS=corrected QS theory-based model; LQS=linearized QS theory-based model; SEL=semi-empirical linear model; H=hybrid model; MH=modified hybrid model): (a) RMS of vertical displacements; (b) RMS of torsional displacements
Figure 3.10 presents the rms of the aerodynamic response calculated through various models at different turbulence intensities for the mean wind velocity 60 m/s. Observations similar to those in Figure 3.9 are noted. However, there are significant differences for the response calculated through the modified hybrid model. A possible explanation for the fact that the torsional rms values obtained from the modified hybrid model are smaller at high turbulence intensity (15% and 25%) is the disappearance of the subcritical behavior which is sustained at lower levels of turbulence intensity (5% and 10%).

Figure 3.10. RMS response based on various models under different turbulent conditions, $U=60$ m/s: (a) RMS of vertical displacements; (b) RMS of torsional displacements
3.4.1.3 Motion-Induced Effects on the Aerodynamic Response

It is well established that the motion-induced effects change the effective structural parameters (mass, stiffness and damping ratio) which in turn impact the aerodynamic response as noted in the preceding section. In order to investigate the motion-induced effects, the aerodynamic response coupled with the aeroelastic response is compared to the aerodynamic response where the motion-induced effects are neglected, as shown in Figure 3.11. It is obvious that the motion-induced effects are more significant at higher wind velocities and more pronounced for the vertical degree-of-freedom for the example bridge. In general, the motion-induced effects will reduce the aerodynamic response. However, for the torsional response calculated through the semi-empirical linear and hybrid models the motion-induced effects increase the aerodynamic response. The errors introduced by neglecting the motion-induced effects for the aerodynamic response based on the models that do not consider the fluid memory are higher than those for the aerodynamic response based on the models considering fluid memory.

3.4.2 Aeroelastic Analysis

3.4.2.1 Critical Flutter Wind Velocity

The stability issue is critical in bridge aeroelasticity, as seen in the case of the Tacoma Narrows Bridge failure, attributed by most researchers to flutter. It is well known that the aeroelastic instability results from negative effective damping (the difference between positive structural damping and aerodynamic damping). At relatively low wind velocities, the effective damping is typically positive due to a contribution from aerodynamic damping that is either positive or insignificant. As the wind velocity increases typically
the aerodynamic damping becomes negative and accordingly the effective damping for the wind-bridge interaction becomes smaller. The critical flutter wind speed in the time domain corresponds to the lowest wind velocity at which the effective damping of the wind-bridge interaction system reduces to zero, which represents onset of flutter. At this stage, one observes typical LCO. Figure 3.12 presents the critical flutter wind velocity $U_{cr}$ calculated utilizing these models under uniform wind conditions. From these results it
can be concluded that: (i) the consideration of the linear memory effects could increase the aeroelastic response (reduce the critical wind velocity) since $U_{cr}$ calculated through the linearized QS theory-based model (without memory) is higher than $U_{cr}$ calculated through the semi-empirical linear model (with linear memory); (ii) the consideration of nonlinear memory effects may reduce the aeroelastic response (increase the critical wind velocity) since $U_{cr}$ calculated through the modified hybrid model (nonlinear memory) is higher than $U_{cr}$ calculated through the hybrid model (linear memory); (iii) the consideration of nonlinear effects involved in the hybrid scheme could reduce the aeroelastic response (increase the critical wind velocity) since $U_{cr}$ calculated through the hybrid model (nonlinear) is higher than $U_{cr}$ calculated through the semi-empirical linear model (linear). In essence, the contribution to the aeroelastic response from the fluid memory effects is higher than that resulting from the consideration of nonlinear effects based on the comparison of the results of these models. However, it should be noted that while the fluid memory effects (especially the linear memory effects) are accounted for by means of the unsteady parameters (e.g., flutter derivatives) identified in wind tunnel, the nonlinear effects are not fully encompassed in such an approach.

![Figure 3.12. Critical flutter wind velocity in uniform flow](image)

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Figure 3.13 shows the damped, critical and undamped (divergent) cases in the simulation for each model. There are several interesting features observed in these response signatures. Since the wind-bridge interaction in reality may experience one or a combination of the aeroelastic response, it is necessary to detect pre-flutter, flutter and post-flutter regimes. While the detailed analysis for the dynamic behaviors of the aeroelastic response is not given here for the sake of brevity, it is worthy to mention two main features. One of these is the post-flutter behavior calculated by the QS theory-based and corrected QS theory-based models. Instead of a typical divergent response, the LCO appears due to the consideration of nonlinear effects as indicated in Figure 3.13(a) and Figure 3.13(b). The other is the subcritical aeroelastic response in the modified hybrid model. As previously mentioned, the hybrid model basically combines the QS theory and the semi-empirical linear-based models. The aeroelastic response calculated by the hybrid model shows that the critical flutter wind velocity is determined by the semi-empirical linear part. Whereas, the results based on the modified hybrid model show that the $U_{cr}$ is determined by the QS theory-based part. The $U_{cr}$ resulted from QS theory-based part is 53 m/s as indicated in Figure 3.13(g) while $U_{cr}$ resulted from the semi-empirical part is 79 m/s as indicated in Figure 3.13(f). However, due to the LCO behavior, the torsional oscillation amplitude (double) of the critical response (here this smaller critical response is referred to as the subcritical response) at a wind velocity of 53 m/s is 1.13° and vertical oscillation amplitude (double) is 0.21 m, which are quite small. Furthermore, the LCO behavior in this case indicates that the amplitude of LCO will not increase rapidly as the wind speed increases. The torsional LCO amplitude (double) of the critical response at a wind velocity of 79 m/s is 5.02° and the vertical LCO amplitude (double) is 0.40 m. As a
result, the critical response resulting from the semi-empirical part is defined as the "ture" critical wind velocity $U_{cr}$. 
Figure 3.13. Characteristics of aeroelastic response for each model:
(a) aeroelastic response of QS theory-based model, (i) damped case ($U=82$ m/s), (ii) critical case ($U=87$ m/s), (iii) undamped case ($U=92$ m/s); (b) aeroelastic response of corrected QS theory-based model, (i) damped case ($U=102$ m/s), (ii) critical case ($U=107$ m/s), (iii) undamped case ($U=112$ m/s); (c) aeroelastic response of linearized QS theory-based model, (i) damped case ($U=83$ m/s), (ii) critical case ($U=88$ m/s), (iii) divergent case ($U=93$ m/s); (d) aeroelastic response of semi-empirical linear model, (i) damped case ($U=68$ m/s), (ii) critical case ($U=73.7$ m/s), (iii) divergent case ($U=78$ m/s); (e) aeroelastic response of hybrid model, (i) damped case ($U=72$ m/s), (ii) critical case ($U=77.5$ m/s), (iii) divergent case ($U=82$ m/s); (f) aeroelastic response of modified hybrid model for the semi-empirical linear part, (i) damped case ($U=74$ m/s), (ii) critical case ($U=79$ m/s), (iii) divergent case ($U=84$ m/s), (iv) LCO of QS theory-based response ($U=79$ m/s); (g) aeroelastic response of modified hybrid model for the QS theory-based part, (i) sub-damped case ($U=48$ m/s), (ii) sub-critical case ($U=53$ m/s), (iii) sub-undamped case ($U=58$ m/s) (p73-75)
3.4.2.2 Effect of Turbulence on Flutter

Despite several studies on the effects of turbulence on the wind-bridge interactions, its influence and the underlying basic mechanisms are still not fully understood. In this study, the turbulence effects on the instability aspect are viewed from two perspectives as pointed out earlier, i.e., its effects on the nonlinearity and on the fluid memory. As the linearized QS model can neither take into account the nonlinearity nor the fluid memory, turbulence will obviously have no effect on the instability within this model. Though the semi-empirical linear model takes into account the fluid memory effects, turbulence will
not significantly alter the instability because the aerodynamic and aeroelastic responses are simply additive. Generally, turbulence will increase the critical flutter wind velocity as its influence on the nonlinearity plays a major role in impacting the aeroelastic response. This is demonstrated in the results obtained from the QS and corrected QS models as shown in Figure 3.14. This observation indicates that turbulence reduces the aeroelastic response due to static nonlinearity. On the other hand, if turbulence effects on the fluid memory give more significant contribution to the aeroelastic response, the situation becomes more complicated. Typically, turbulent fluctuations reduce the critical flutter wind velocity, however, as turbulence intensity increases, the corresponding critical flutter wind velocity slightly increases, as shown in the results for the hybrid and modified hybrid models. This observation indicates that turbulence effects increase the aeroelastic response due to fluid memory, which is stronger for the smaller turbulence intensity cases. Another possible mechanism for this complex situation in the hybrid and modified hybrid models is that the turbulence effect on the nonlinearity involved in these models increases significantly as the turbulence intensity increases, which results in a slight increase in the critical flutter speed as compared to the speed with smaller turbulence intensity. Besides, the higher-order fluid memory will significantly reduce the instability based on the QS theory (low-frequency part) as the subcritical wind velocity in the modified hybrid model is 53 m/s under the uniform inflow condition while that in the hybrid model is 87 m/s. On the other hand, the low-frequency components of the wind fluctuations with small turbulence intensity (e.g., $I_u=5\%$) will slightly change (either reduce or increase) the subcritical wind velocity (from 53 m/s to 52 m/s in the modified hybrid model and from 87 m/s to 88 m/s in the hybrid model) while the low frequency
components of the wind fluctuations with larger turbulence intensity (e.g., $I_u=15\%$) will increase the subcritical wind velocity (from 53 m/s to 59 m/s in the modified hybrid model and from 87 m/s to 91 m/s in the hybrid model).

![Critical wind velocity under various inflow conditions](image)

Figure 3.14. Critical wind velocity under various inflow conditions

Turbulence modifies dynamic response around the critical flutter wind velocity as noted in Figure 3.15 in which the aerodynamic response around the critical flutter wind velocity in uniform and turbulent flows calculated through the various models is presented. As indicated in this figure, the amplitudes of the LCO based on the QS theory and corrected QS models increase significantly under turbulent flow condition as compared to those under uniform flow. The amplitudes of LCO increase even further for the vertical response. This may contribute to explain the reason behind the observation that in the wind tunnel the aeroelastic instability does not occur suddenly with a distinct flutter boundary under turbulent flow condition (Irwin 1977). It should also be noted that under turbulent flow conditions the critical flutter wind velocity is more difficult to
determine than in uniform flow conditions as the turbulence appears to slow down the divergence rate of the responses.

Figure 3.15. Aerodynamic response around the critical flutter wind velocity under various inflow conditions: (a) RMS of vertical displacements around $U_{cr}$; (b) RMS of torsional displacements around $U_{cr}$
3.5 Discussion on the Critical Model Parameters

3.5.1 Parameter \( m_1 \) in the QS Theory-based Model

As mentioned in the preceding section, apart from the fundamental requirement of high wind velocity, it is necessary to determine an equivalent steady state for applying the QS theory. This becomes essential as the QS theory utilizes the steady state of one point in the fluid-structure interaction space to approximate the aerodynamic states of the entire fluid-structure interaction system. In a way, the high wind velocity satisfies the accuracy with respect to the time scale of the fluid-structure interaction system when using the QS theory assumption while the equivalent steady state satisfies the accuracy in a geometric sense. For the vertical degree-of-freedom, it is straightforward to define an equivalent steady state since the relative velocity under this translation motion is the same at each point on the deck cross-section. However, for the torsional degree-of-freedom the relative velocity is different at each point of the cross-section due to the angular motion. It is necessary to determine a reasonable value of the parameter \( m_1 \) in the QS theory for defining the effective steady state.

For streamlined cross-sections, such as thin airfoils, the aeroelastic forces are only dependent on the downwash (the vertical component of the flow) at the \( \frac{3}{4} \) chord point of the airfoil (rear neutral point). As a result, the value of \( m_1 \) is set as 0.25 (corresponding to B in this study). Another situation to circumvent the need for the selection of \( m_1 \) in the case of an airfoil is that the axis of rotation, around which the structure oscillates, is far from the geometric center of the cross-section. In this situation, the rotational motion tends to be a pure translation motion. However, for the bluff cross-section of typical bridge decks, the aeroelastic forces are not only dependent on the downwash at one point
of the bride deck. Besides, the axis of rotation for the bridge deck oscillation is at its geometric center or very close to it. As a result, the selection of $m_1$ in the QS theory becomes critically challenging.

It is reasonable to assume that the effective range of $m_1$ value may fall between -0.5 and 0.5. A straightforward approach to determine $m_1$ for the bridge deck is to simply assume the same value as for the airfoil. However, as indicated in the linearized QS theory-based model, the large positive value (0.25) induces large negative self-excited damping in the torsional degree-of-freedom (Equation 3.9b), which results in extremely low critical wind velocity. This situation is relaxed for the thin airfoil since one of the components of the apparent mass lift forces, which is actually a circulatory force resulting from the centrifugal nature of the force (Fung 1993), is usually not neglected in the description of aeroelastic forces. The moment $(-l/2) \pi \rho Ub' \dot{\alpha}$ corresponding to this apparent mass lift force component results in a positive effective damping. Since the apparent mass effects are usually neglected in bridge aerodynamics/aeroelasticity it is more reasonable to set $m_1$ at a negative value. For example, Farquharson (1949-1954) investigated the bluff body aerodynamics based on the QS assumption by utilizing the local angle of attack at the leading edge (corresponding to $m_1$=-0.5). If the unsteady aeroelastic parameters (e.g., flutter derivatives) are available, it is straightforward to determine the value of $m_1$ by comparing the terms corresponding to the vertical motion and angular velocity in Equation 3.9 with those in Equation 3.12 as indicated by Diana et al. (1993). As a result, $m_1$ changes as the wind velocity varies. Usually, the value of $m_1$ in the torsional degree-of-freedom is different than that of the vertical degree-of-freedom since, unlike the case for the thin airfoil, the resultant lift force of the bridge deck does
not always act at the ¼ chord point of the body (forward neutral point). The value of $m_1$ in the torsional degree-of-freedom in this study is taken around -0.2. It should be noted that $m_1$ calculated from the available flutter derivatives is based on the linearized scheme which is not necessarily accurate for the nonlinear analysis.

3.5.2 Aeroelastic Parameters in the Semi-Empirical Linear Model

In the case of aerodynamic force modeling, it is obvious that the input terms should be the fluctuating components of the incoming flow considered. For example, for 2D cases, the aerodynamic input terms will be the horizontal and vertical fluctuation components. However, it is not straightforward to determine the input terms when modeling the aeroelastic forces. It is well known that the fluid-structure interaction is dependent on the relative velocities, which includes the translation velocity resulting from the translation motion $\dot{h}$ and orientation of the cross-section $\alpha$ and angular velocity induced by the rotational motion $\dot{\alpha}$.

For the streamlined cross-section, the contributions to the aeroelastic forces from the translation motion $\dot{h}$ and orientation of the cross-section $\alpha$ are identical since the flow passing by the structure is always attached on the solid surface. However, for the bluff cross-section, the contributions from the orientation of the cross-section $\alpha$ become complicated due to the flow separation. There is no clear explanation to demonstrate the assumption of an equivalent physical origin for these two contributions to the aeroelastic forces. Besides, the identified contribution of the orientation of the cross-section $\alpha$ in wind tunnel automatically involves the apparent mass effects (also named apparent moment of inertia effects) of the rotational motion based on harmonic oscillations. Different values of the identified flutter derivatives in the wind tunnel corresponding to
the translation motion \( \dot{h} \) and orientation of the cross-section \( \alpha \) also indicate that it is necessary to separate their contributions in bridge aerodynamics/aeroelasticity and to retain both of them as input terms (Scanlan et al. 1997). In order to take into account the apparent mass effects of the vertical motion, the vertical displacement is retained as an input term under the assumption of harmonic oscillation in the semi-empirical linear model.

3.5.3 Cut-Off Frequency Separating the Low and High Frequency Parts

The concept that the structural response is given by the superposition of the low-frequency and high-frequency parts like in the hybrid model is well developed in the buffeting theory, where the background and resonant components are added together. However, the cut-off frequency dividing the background and resonant parts is not clearly defined. Originally, Davenport (1967) calculated the background part up to the reduced frequency equal to 3/8, where the joint acceptance becomes significantly small. In order to avoid the overlap between the background and resonant parts, a cut-off frequency slightly lower than the fundamental natural frequency of the structure is usually chosen. Actually, in the case of the equivalent load simulation the importance of the selection of the cut-off frequency is not significant as there is little contribution to the background part from the high-frequency components.

The cut-off frequency in the band-superposition scheme is set equal to the value which guarantees that the reduced wind velocity is high enough for the low part response calculation based on the QS theory (Diana et al. 1995), while the cut-off frequency in the hybrid model is set to the first natural frequency of the structure based on the premise that the selection of the cut-off frequency needs an in-depth examination (Chen and Kareem...
However, it should be noted that the roles that the cut-off frequency has in the equivalent load simulation and in the hybrid model are different since in the hybrid model there is a direct interaction between the low-frequency and high-frequency components. In other words, the cut-off frequency in the hybrid model should not only guarantee that the reduced wind velocity is high enough for the QS theory to hold but also it should accurately reflect the effective interaction between the low-frequency and high-frequency parts. It is convenient to treat the hybrid model as a Taylor expansion (only linear terms are retained) of the fluid-structure interaction system governed by a nonlinear equation at each time instant. The difference between the hybrid and semi-empirical linear models is that the expansion point of the Taylor expansion in the hybrid model is dynamic instead of the static expansion point in the semi-empirical linear model (Wu and Kareem 2012a). As a result, the interaction between the low-frequency and high-frequency parts is simulated by the dependence of flutter derivatives (strictly, this should be determined under turbulent incident wind condition with high-frequency components) on the instantaneous angle of attack resulting from the low-frequency part. However, since the Taylor expansion in the hybrid model is dynamic, there is no reason for the flutter derivatives to be only dependent on the magnitude of this instantaneous angle of attack while neglecting the frequency information on the dynamic expansion point. On the other hand, the dependence of the flutter derivatives on this instantaneous angle of attack is obtained from the static wind tunnel. Hence, there is no appropriate approach to take into account the dependence of the flutter derivatives on the frequency of this instantaneous angle of attack. As a result, a compromise is reached between fully characterizing the
nonlinearity and the accuracy in utilizing the dependence of flutter derivatives on the instantaneous angle of attack resulting from the low-frequency part.

In this study a parametric investigation is carried out utilizing different values of the cut-off frequency. Specifically, the response of the hybrid model based on the cut-off frequency set as the fundamental natural frequency $f_1$, $0.1 f_1$, $0.25 f_1$, $0.5 f_1$, $0.75 f_1$, $1.25 f_1$, $1.5 f_1$, $1.75 f_1$, $2 f_1$ and $5 f_1$ are presented in Figure 3.16, where the fundamental natural frequency is considered to be the first natural frequency of the lateral mode (around 0.1 Hz); $2 f_1$ is around the first natural frequency of the vertical mode and $5 f_1$ is around the first natural frequency of the torsional mode. The solid symbols indicate the cut-off frequency utilized in this study while the red horizontal dash line indicates the relevant response using the semi-empirical linear model. The QS assumption is valid up to around $2 f_1$ in this case. As indicated in the figure, for a cut-off frequency smaller than the fundamental natural frequency the variance of the response is small for both vertical and torsional degrees-of-freedom. However, for the cut-off frequency larger than $1.5 f_1$ the vertical response significantly increases while the torsional response just slightly increases. Since the QS assumption is not valid, the error at a cut-off frequency of $5 f_1$ is large. It should be noted that the large magnitude of the instantaneous angle of attack due to the high cut-off frequency is not fully characterized as the available data of flutter derivatives under various angle of attack is very limited. As a result, the increase in the response may mainly result from the frequency change in the instantaneous angle of attack which the hybrid model may not accurately encompass at this time.
3.6 Concluding Remarks

Six models for predicting the aerodynamic/aeroelastic behavior of a bridge deck, five existing and one new, are compared. These models include: the QS theory-based model (nonlinear effects without fluid memory), the corrected QS theory-based model (nonlinear effects with linear fluid memory at a fixed reduced wind velocity), the linearized QS theory-based model (linear effects without fluid memory), the semi-empirical linear model (linear effects with linear fluid memory), the hybrid model (nonlinear effects with linear fluid memory), and the modified hybrid model (nonlinear

Figure 3.16. Comparison of the response based on various cut-off frequencies ($f_{co}$-cut-off frequency): (a) RMS of vertical displacements; (b) RMS of torsional displacements
effects with nonlinear fluid memory). Their predictive features are examined considering linear and nonlinear unsteadiness (fluid memory) and nonlinearity in wind-bridge interactions. It is noted that for the example studied, the contribution to the aerodynamic/aeroelastic response resulting from the fluid memory effects (typically reduces the bridge deck response) is higher than the contribution resulting from nonlinear effects. Specifically, for the aerodynamic response, static nonlinearity effects are more significant for the vertical degree-of-freedom than for the torsional degree-of-freedom while fluid memory effects are more significant for the torsional degree-of-freedom than for the vertical degree-of-freedom. For the aeroelastic response, consideration of nonlinear effects involved in the hybrid scheme results in increasing the critical flutter wind velocity. Opposing trends emerge regarding the influence of memory effects. On the one hand, the linear memory effects will reduce the critical flutter wind velocity, while on the other hand, the nonlinear memory effects will increase the critical velocity.

The coupling between aerodynamic (buffeting) and aeroelastic (motion-induced) responses generally leads to an overall reduction in the aerodynamic response as a consequence of motion-induced effects. The introduction of turbulence, however, influences aerodynamic response, which, due to coupling, leads to complicated effects on aeroelastic response. In a particular model, if the effects of turbulence on nonlinearity significantly influence the aeroelastic response, then the resulting prediction of the critical flutter velocity is increased. On the other hand, if in a model turbulence manifests itself through fluid memory effects, then the resulting critical flutter velocity is reduced at low turbulence intensities (e.g., $I_u=5\%$), but slightly increased at higher turbulence intensities.
A new approach to model aerodynamic nonlinearities in the time domain utilizing an artificial neural network (ANN) framework with embedded cellular automata (CA) scheme has been developed. This nonparametric modeling approach has shown good promise in capturing the hysteretic nonlinear behavior of aerodynamic systems in terms of hidden neurons involving higher-order terms. Concurrent training of a set of higher-order neural networks facilitates a unified approach for modeling the combined analysis of flutter and buffeting of cable-supported bridges. Accordingly, the influence of buffeting response on the self-excited forces can be captured, including the contribution of damping and coupling effects on the buffeting response. White noise is intentionally introduced to the input data to enhance the robustness of the trained neural network embedded with optimal typology of CA. The effectiveness of this approach and its applications are discussed by way of modeling the aerodynamic behavior of a single-box girder cross-section bridge deck (2-D) under turbulent wind conditions. This approach can be extended to a full-bridge (3-D) model that also takes into account the correlation of aerodynamic forces along the bridge axis. This novel application of data-driven modeling has shown a remarkable potential for applications to bridge aerodynamics and other related areas. The data-driven characteristics of the ANN may classify it as a “black-box” model.
4.1 Introduction

Traditionally, wind induced flutter and buffeting responses have been predominantly evaluated in the frequency domain in which the aerodynamic forces are linearized at the mean displaced position of the bridge deck. This linear aeroelastic analysis approach is not suited for capturing the emerging concerns in bridge aerodynamics introduced by aerodynamic nonlinearities and the effects of turbulence (Chen and Kareem 2003). These issues may become critical for bridges with increasing spans and/or with aerodynamic characteristics sensitive to the dynamic angle of incidence, which depends on the structural motions and the incoming fluctuations in the wind. While the quasi-steady (QS) theory can take into account the nonlinearities in the aerodynamic forces, it fails to include the unsteady fluid memory effect inherent in hysteretic behavior.

The nonlinearity of bridge aerodynamics occurs primarily from the flow separation around the bridge cross-section, which is usually accompanied by the higher-order fluid memory effect. The additional vorticity created by the deck motion sheds into the wake downstream, which induces a corresponding force with hysteresis. This convected flow feature downstream induces fluid memory, represented through the complex and uncertain time lag between the aerodynamic forces and bridge response. Furthermore, the aerodynamic nonlinearities could interact with the potential nonlinearities of structural origin due to large deformations, material nonlinearities, or complex damping properties, which complicates the situation (e.g., Xiang and Ge 2000; Salvatori and Borri 2007; Arena and Lacarbonara 2012).

In order to address the issue of nonlinearity in unsteady aerodynamic forces, attempts have been made recently to express these aerodynamic forces as a function of
the dynamic angle of incidence, which can be further separated into its low-frequency (large scale) and high-frequency (small scale) components (Diana et al. 1999b; Chen and Kareem 2001; 2003). The low-frequency dynamic angle of attack is based on the QS theory while conventional flutter and buffeting schemes are retained to model high-frequency components; however, one of the unresolved issues with this approach concerns the identification of the demarcation between the low and high frequencies (Chen and Kareem 2003). Alternatively, a numerical approach has been proposed that takes into account the aerodynamic nonlinear effects by means of a rheological model in the time domain (Diana et al. 2006a; 2006b; 2007; 2008a; 2008b; 2010). The modeling of the aerodynamic forces measured on the deck sectional models by a mechanical hysteretic lag system is central to this approach. This involves a rheological model that represents the hysteretic behavior characterizing aerodynamic forces versus dynamic angle of attack and its derivatives. The definition of dynamic angle of attack itself is questionable, especially within high-frequency range. Since the dynamic angle of attack is the unique, independent variable in this model, it is assumed that all other interacting components (bridge deck motions and turbulences) that are represented by this independent variable have equal weighting on the bridge aerodynamic behavior.

The use of artificial neural networks (ANNs) to determine the behavior of nonlinear dynamical systems is a reliable alternative which has been widely explored over the last few decades in many fields of engineering. ANNs have also been applied to simulate the wind field and related processes (Gurley et al. 1996; Martínez-Vázquez and Rodríguez-Cuevas 2007). In this study, the nonlinear behavior describing the bridge aerodynamics is modeled through ANNs. This data-driven approach relies on the data
derived from a bridge deck section model test in a wind tunnel or computational fluid dynamics (CFD) simulation. This information is used to train the ANN model. Once trained, it can be used to predict the response of this bridge for a range of similar type of excitations. The proposed model, which is a canonical feed-forward ANN, is shown to reproduce the hysteretic nonlinear behavior very well since the hidden neurons in this ANN are composed of first- and higher-order terms, where the latter are derived from the first-order neurons. Moreover, the intrinsic attributes of the ANN approach make it particularly suitable for modeling dependencies on other parameters, e.g., the influence of amplitude of motions, the mean angles of attack, etc. The standard back propagation of error (BPE) training algorithm is introduced to update the network weights using experimental data.

The proposed ANN’s configuration is critical. The input and output information could be obtained through the bridge aerodynamics analysis while the number of neurons in the hidden layer is not easy to determine. In most cases, the hidden layer and its connections with the input and output layers are constructed through a trial and error process. The configuration obtained is usually not optimal and is very time consuming. In order to improve this scheme, the cellular automata (CA) scheme, which was originally proposed by von Neumann (1966) to model self-reproducing organisms and later developed by Wolfram (1984) and others in the fields of physics and mathematics (Palash Sarkar 2000), is applied to construct the proposed ANN.

Conventional approaches conduct flutter and buffeting studies in isolation without accounting for all the necessary feedbacks and interactions. Concurrently training a set of higher-order ANNs automatically facilitates integration of the flutter and buffeting
analysis in a unified manner. The effects that buffeting response has on the self-excited forces can be included along with the damping and coupling that the self-excited forces introduce to the buffeting response since the input information to the ANN involves the bridge deck motions and turbulences at each time step. A first step in this direction would be to apply this scheme to a 2-D bridge deck section based on the strip theory formulation. It could subsequently be extended to 3-D full bridge simulation that accounts for the aerodynamic force correlation along the bridge axis.

4.2 Conventional Scheme

4.2.1 Sectional Bridge Aerodynamics

A unit section of a bridge deck subjected to a turbulent wind flow ($U$ is the mean wind, $u$ and $w$ are horizontal and vertical fluctuations, respectively) is considered here and the aerodynamic loads are established using “strip theory” (Figure 4.1). The coordinates X, Y, and Z represent the horizontal, vertical, and spanwise directions, respectively. The bridge cross-section has horizontal motion $s$, vertical motion $h$, and torsional motion $\alpha$ under the turbulent wind. Here only loads on the bridge deck due to turbulent wind are included, whereas secondary loads on the cables and their effects are neglected without loss of generality.

4.2.2 Conventional Wind Load Model

Typically, wind loads are approximated using the linear part of Taylor expansion with respect to a static expansion point. This conventional wind load model is actually a time-linearized method with respect to the bridge motions and turbulence. The basic approach
is to linearize the nonlinear equations under small-perturbation theory. A steady-flow field (nonlinear) is first determined, and then a small perturbation (linear) is added on this base flow (Dowell and Hall 2001). For simplicity, only the linear term in the dynamic perturbation is retained with respect to the time-dependent unknowns (motions or turbulences) with coefficients nonlinearly dependent on the base flow.

In the aerospace field, streamlined cross-section of airfoils permits flow simulation based on the potential flow theory, e.g., Wagner function (response to a unit step variation in angle of attack), and its extension forms, such as Theodorson theory (frequency response to sinusoidal motion), Küssner theory (response to a sharp-edged gust in incompressible flow), and Sear’s function (frequency response to a sinusoidal gust) for which the retaining linear terms are described by analytical expressions. However, in the bridge aerodynamics field, the aerodynamics of such bluff bodies does not permit utilization of the “linearized” potential flow theory. Accordingly, there is no analytical expression for bridge applications. To overcome this shortcoming, Sabzevari and Scanlan (1968) introduced the so-called semi-empirical scheme. In this formalism,
the linearized force is usually separated into flutter, which arises from the motion of the bridge per se, and buffeting, which results from the impinging turbulences. The aerodynamic coefficients in Scanlan’s formulation are determined through sectional model experiments in a wind-tunnel, and system identification techniques are used to extract these derivatives. These data-driven estimates of coefficients assure that the linearized scheme is valid with reasonable accuracy in the wind velocity range of interest when used in most bridge aerodynamic studies; however, there remain several issues that the linearized scheme has not been able to address due to its inherent limitations. For example, the linearized scheme cannot predict the potential existence of a limit oscillation cycle (LOC) phenomenon, which is a unique feature of a nonlinear system (Dowell and Tang 2002). In frequency domain, it is well known that for a linear system one harmonic component only excites the corresponding harmonic response of the system; however, for a nonlinear system, this simple situation is not valid. The nonlinear transformation results in some new frequency components, which cannot be modeled by the linearized scheme. This type of aerodynamic nonlinearity has been recently shown both analytically and in the wind tunnel (e.g., Diana et al. 2010; Kareem and Wu 2012). Examples of this type of nonlinear features are also observed in hydrodynamics (Kareem et al. 1998).

In order to account for nonlinearity in the bridge aerodynamic system, an advanced modeling scheme has been presented (Chen and Kareem 2001; 2003). This model takes into account the aerodynamics manifested by the dynamic angle of attack, which results from a combination of the bridge deck motions and turbulences in the incoming wind (incidence angle is also included). The dependence on the dynamic angle
of attack is in part modeled by measuring the aerodynamic coefficients (flutter
derivatives or aerodynamic coefficients) at respective angles of attack. This approach
results in a need to separate turbulence into low frequency and high frequency parts.
However, it is not easy to pinpoint demarcation between the low and high-frequency
fluctuations. Actually, this scheme uses expansion of the aerodynamic force term about a
dynamic position of the deck as opposed to a static position, which represents the mean
displaced position of the deck as employed in a conventional scheme. In other words, this
framework represents piecewise linearization in a dynamic scheme as opposed to the
static approach. The conventional scheme can be referred to as a parametric model
because each coefficient has a particular physical meaning, which has an advantage when
it is utilized in a modern control theory framework. Figure 4.2 illustrates a flow chart of
conventional scheme.

![Figure 4.2. A flow chart of conventional linear and nonlinear flutter analysis](image-url)
In this figure,

\[ X = X_0 + X_l + X_h \]  \hspace{1cm} (4.1)

is the displacement vector of the bridge deck, where \( X_0 \) is the static displacement of the equilibrium position, \( X_l \) is the motion of the low frequency part and \( X_h \) is the motion of the high frequency part. Similarly, the turbulent wind speed is expressed as follows:

\[ \tilde{U} = U + \tilde{u}_l + \tilde{u}_h \]  \hspace{1cm} (4.2)

where \( U \) is the mean wind velocity, \( \tilde{u}_l \) is the low frequency turbulence part and \( \tilde{u}_h \) is the high frequency turbulence part. The subscript \( e \) represents relative angle of attack that includes contribution of the low frequency fluctuations in wind.

It is well known that the linear conventional scheme cannot take into account the nonlinearity and accompanying higher-order memory in aerodynamic loads. The piecewise linearization scheme proposed by Chen and Kareem (2001; 2003) could in part consider the nonlinearity, since the aerodynamic loads induced by the low frequency part of bridge deck motions and turbulence are obtained through the QS theory. However, it is difficult to capture hysteretic bridge aerodynamic behavior with a QS theory based on the Taylor series expansion.

4.3 ANN based Nonlinear Aerodynamic Model

4.3.1 ANN based Hysteresis Simulation

A new approach to modeling hysteresis behavior is through the implementation of an ANN. There are several successful applications of an ANN to simulate the hysteretic
behavior in engineering (Serpico and Visone 1998; Veeramani et al. 2009). If the input of an ANN is the dynamic angle of attack and the output is the force or moment coefficient of the bridge under study, the hysteretic behavior of bridge aerodynamics is observed. The plots in Figure 4.3 represent simulated hysteretic behavior between the lift force and moment coefficients and the dynamic angle of attack by an ANN.

![Figure 4.3. An example description of force and moment coefficients vs. dynamic angle of attack](image)

An obvious advantage of the ANN compared to the rheological model is that it can be easily expanded for a single term input parameter to several input parameters through introduction of added weights and neurons. This enhanced capability makes the ANN a promising model for simulating hysteretic nonlinear behavior in bridge aerodynamics based on different input parameters as illustrated schematically in Figure 4.4. Besides, this data-driven model is determined by input-output information, which
automatically captures intrinsic aerodynamic/aeroelastic nonlinearity and structural nonlinearity of the wind-bridge interaction system, as shown in Figure 4.5.

Figure 4.4. Schematic of the neural network for modeling hysteretic nonlinearity

Figure 4.5. Integration of structural and aerodynamic nonlinearities via neural network model
4.3.2 ANN Background

An ANN is a computational model that mimics the structure and/or functionality of a biological neural network. The mathematical foundation for applying ANNs for establishing functional approximation can be found in literature (e.g., Cybenko 1989; Hornik et al. 1989). Recent developments permit simulation of any nonlinear system using a multilayer ANN even when employing one hidden layer if enough neurons are introduced. A finite summation of continuous sigmoidal functions in the hidden layer serves as an approximation of a continuous scalar function. It is comprised of a combination of linked groups of artificial neurons and functions that facilitate simulation of a desired system.

In most cases an ANN is an adaptive system that changes its structure based on the external or internal information that flows through the network during the training phase. The data-driven characteristics of the ANN highlight the importance of the quality of the data used in the training phase and the configuration of the ANN. An ANN is usually composed of three basic layers: input, hidden and output, as shown in Figure 4.4.

Usually, the input and output layers are determined for each particular problem to be solved. The number of neurons in the hidden layer and their connections to the input and output layers are critical in constructing a proper network configuration. As a numerical model, ANN also has a significant computational advantage over say a CFD based scheme. In an ANN, the computational work is mainly focused on identifying the proper weights in the network. Once the training phase is completed, the output of the simulated system could be obtained through a simple arithmetic operation with any desired input information. Whereas, in the case of a CFD scheme each new input scenario
requires complete reevaluation of the fluid-structure interaction over the discretized domain. The data-driven characteristics of an ANN may classify it to be a semi-empirical model. ANN is also usually labeled as a nonparametric model because it is difficult to elucidate the physical meaning of the various weighting functions employed in the network.

4.3.3 Algorithm of ANN

The training of an ANN by the back propagation algorithm involves three stages: the feedforward input training pattern, the back propagation of the associated error, and the adjustment or tuning of the weights. In this study, the activation function for each hidden neuron is described using a bipolar sigmoid, within the range of (-1, 1):

\[
s_n(z) = \frac{2}{1 + \exp(-z)} - 1
\]  

(4.3)

In the output layer, neurons are activated with the linear function \( s_o(y) = y \), with a constant derivative, \( s'_o(y) = 1 \). The derivative of Equation 4.3 is shown in Equation 4.4:

\[
s'_n(z) = \frac{1}{2} [1 + s_n(z)][1 - s_n(z)]
\]  

(4.4)

After defining the input unit as \( X_i \), the hidden unit as \( Z_j \) and the output unit as \( Y_k \), the weighting function values, which serve as a connection among neurons, are constantly updated during the training phase as \( w_{new} = w_{old} + \Delta w \). The updating of the weighting function \( \Delta w \) is given in Equations 4.5 and 4.6 (Fausett 1994):

\[
\Delta w_{jk} = \alpha \hat{e}_k z_j
\]  

(4.5)
\[
\Delta w_{ij} = \alpha_r \left( \sum_{k=1}^{m} \delta_k w_{jk} \right) s'_i (z_{inj}) x_i
\]  \hspace{1cm} (4.6)

where \( \alpha_r \) is the learning rate and \( \delta_k \) is the error information term, which is determined using Equation 4.7:

\[
\delta_k = (t_k - y_k) s'_n (y_{in_k})
\]  \hspace{1cm} (4.7)

In this equation, \( t_k \) is output target, \( z_{inj} \) is the net input to hidden neurons and \( y_{in_k} \) is the net input to output unit. The biases in this ANN have a similar updating scheme as well.

The training process is stopped when the mean square error, which is defined by Equation 4.8, falls within a pre-established interval:

\[
E = \frac{1}{2} \sum_{k=1}^{K} (t_k - y_k)^2
\]  \hspace{1cm} (4.8)

Generally, the data is divided into three sets of data; they are used in the training, validating and testing processes. Both the training data and validation data sets may be used to modify the weights in the ANN.

4.4 Simulation of Bridge Aerodynamics based on ANN

4.4.1 Formulation of Bridge Aerodynamics

One of the main goals of this study is to be able to predict the nonlinear response of a bridge under turbulent wind conditions. Such a predictive feature is also an essential attribute for machine learning applications based on inductive learning hypotheses. Using the limited input-output measurements of the bridge system, identification is carried out to model the bridge aerodynamics. Based on this model, the future outputs of this system
can then be predicted for any given future input which is within the regime of the employed inputs in the original training stage. The fundamental input-output pairs to be considered in this study are incoming wind velocity (mean and fluctuation components in longitudinal and vertical directions) and the bridge deck acceleration response.

In general, the two dimensional equation of motion of a bridge system can be written in the following form (Scanlan 1978):

\[ \ddot{h} + 2\zeta_\phi \omega_\phi \dot{h} + \omega_\phi^2 h = F_L / M \]  \hspace{1cm} (4.9)

\[ \ddot{\alpha} + 2\zeta_\alpha \omega_\alpha \dot{\alpha} + \omega_\alpha^2 \alpha = M_\alpha / I \]  \hspace{1cm} (4.10)

where \( h \) represents the vertical displacement of the observed bridge deck section and \( \alpha \) the torsional displacement. \( \zeta \) and \( \omega \) represent the damping ratio and natural frequency, respectively. \( M \) is the mass and \( I \) the mass moment of inertia. The aeroelastic force \( F_L \) and moment \( M_\alpha \) are functions of motions of the system:

\[ F_L = \frac{1}{2} \rho U^2 BC_L (h, \dot{h}, \ddot{h}, \alpha, \dot{\alpha}, \ddot{\alpha}, U) \]  \hspace{1cm} (4.11)

\[ M_\alpha = \frac{1}{2} \rho U^2 B^2 C_M (h, \dot{h}, \ddot{h}, \alpha, \dot{\alpha}, \ddot{\alpha}, U) \]  \hspace{1cm} (4.12)

Also, it is well known that the aerodynamic forces are functions of turbulence \( u \) and \( w \). If the available response measurements are accelerations, we can rewrite the above equation of motion at the time instance \( t_n \) in the following form:

\[ \ddot{h}^n = \mathcal{R}_L^n + f_L^n (h, \dot{h}, \ddot{h}, \alpha, \dot{\alpha}, \ddot{\alpha}, U, U, w) \]  \hspace{1cm} (4.13)

\[ \ddot{\alpha}^n = \mathcal{R}_M^n + f_M^n (h, \dot{h}, \ddot{h}, \alpha, \dot{\alpha}, \ddot{\alpha}, U, U, w) \]  \hspace{1cm} (4.14)
where \( f_L^n \) and \( f_M^n \) represent unknown nonlinear excitation force functions involving fluctuations in wind components, which manifest modifications in the instantaneous angles of attack at time instance \( t_n \). \( R_L^n \) and \( R_M^n \) denote the restoring force functions that are considered to be a function of several unknown quantities.

Prior work on predicting restoring forces for dynamic systems has noted that the idea of mapping restoring force in terms of displacement and velocity alone is often found to be insufficient to model hysteretic behavior (Benedettini et al. 1995; Smyth et al. 2002). A better representation of the restoring force in a hysteretic system may be expressed in a nonlinear differential equation form (Smyth et al. 2002)

\[
\dot{R}_L^n = R_L^n(\delta R_L^n, h, \dot{h}, \alpha, \dot{\alpha})
\]

\[
\dot{R}_M^n = R_M^n(\delta R_M^n, h, \dot{h}, \alpha, \dot{\alpha})
\]

where \( R_L^n \) and \( R_M^n \) are unknown nonlinear functions that are not exhaustive; more terms could be added into the list of variables, such as the amplitude of motion and the mean angle of attack, etc.

Assuming that during the sampling time \( t_{n+1} - t_n \) can be taken as a constant \( \Delta t \) for a given data set, one has the first-order forward difference equation

\[
\dot{R}_L^n = (R_L^{n+1} - R_L^n) / \Delta t
\]

\[
\dot{R}_M^n = (R_M^{n+1} - R_M^n) / \Delta t
\]

After some mathematical manipulations, the acceleration response can be expressed as

\[
\ddot{h}^{n+1} = Q_L(h^n, \dot{h}^n, \ddot{h}^n, \alpha^n, \dot{\alpha}^n, \dddot{h}^n, u^n, w^n, U^{n+1}, u^{n+1}, w^{n+1})
\]
\[
\ddot{\alpha}^{n+1} = \mathbb{Q}_M(h^n, \dot{h}^n, \ddot{h}^n, \alpha^n, \dot{\alpha}^n, U^n, u^n, w^n, U^{n+1}, u^{n+1}, w^{n+1})
\] (4.20)

where \(\mathbb{Q}_l\) and \(\mathbb{Q}_M\) are unknown nonlinear functions which can best be captured based on an ANN scheme. Actually, as discussed in the preceding content, these two unknown nonlinear functions automatically combine the effects of aerodynamic transfer function and structural response transfer function, which are usually used in conventional scheme, into one black nonlinear filter.

4.4.2 Pre-Processing for ANN

In an ANN, pre-processing is required to estimate displacements and velocities \(h^n, \alpha^n\) and \(\dot{h}^n, \dot{\alpha}^n\) from measured acceleration. This information is used for the identification of the model using a part of the input. In this study, a simple explicit integration scheme is used to estimate displacement and velocity from measured acceleration. It can be expressed in the following form:

\[
\tilde{h}^n = \tilde{h}^{n-1} + \Delta t \cdot \dot{\tilde{h}}^n
\] (4.21a)

\[
\tilde{h}^n = \tilde{h}^{n-1} + \Delta t \cdot \dot{\tilde{h}}^n
\] (4.21b)

\[
\tilde{\alpha}^n = \tilde{\alpha}^{n-1} + \Delta t \cdot \dot{\tilde{\alpha}}^n
\] (4.22a)

\[
\tilde{\alpha}^n = \tilde{\alpha}^{n-1} + \Delta t \cdot \dot{\tilde{\alpha}}^n
\] (4.22b)

where the symbol ‘~’ represents the estimated velocity and displacement. A similar explicit scheme used in the literature (Pei et al. 2004) has shown good stability in the training of ANN process. The initial values of these two quantities are set equal to zero as
the system is at rest prior to the beginning of the excitation. A higher order and/or implicit integration scheme could be conveniently applied for added accuracy.

4.4.3 ANN Information for Two Dimensional Bridge System

From the above problem formulation, the input vector in the input layer of ANN is expressed as

\[
X_{1n} = [h^n, \dot{h}^n, \ddot{h}^n, \alpha^n, \dot{\alpha}^n, \ddot{\alpha}^n, U^n, u^n, U^{n+1}, u^{n+1}, w^n, w^{n+1}]^T
\]

(4.23)

\[
X_{2n} = [h^n, \dot{h}^n, \ddot{h}^n, \alpha^n, \dot{\alpha}^n, \ddot{\alpha}^n, U^n, u^n, U^{n+1}, u^{n+1}, w^n, w^{n+1}]^T
\]

(4.24)

which are obtained from pre-processed signals, where the subscripts 1 and 2 represent the ANNs for the vertical and torsional responses, respectively. As mentioned earlier, white noise is intentionally added in the input data to render the ANN scheme more robust (The MATLAB EXPO 1993). There is only one output neuron in the output layer for each ANN, i.e., $h^{n+1}$ and $\dot{a}^{n+1}$, which is obtained directly from the original measured data.

4.5 CA based ANN for Bridge Aerodynamics

4.5.1 CA Background

The input and output vectors could usually be determined through some operations similar to the above description while the number of neurons in the hidden layer depends highly on a tedious trial-and-error process. In this study, an automatic scheme is introduced to determine the neurons in the hidden layer and the connections between the hidden layer and the input and output layers. This automated method is an indirect constructive encoding scheme which is based on the CA representation (Galván et al.
Originally proposed by von Neumann (1966), CA models are formally self-reproducing organisms. They were later significantly developed by Wolfram (2002) in the fields of physics and mathematics. A classic CA model is a bottom-up approach which dynamically evolves in discrete space and time. This spatio-temporal scheme is especially suitable for modeling aeroelastic behavior though there is no prior evidence of its applications. Cells which constitute the cellular system are usually binary in nature. This dynamical system evolves using a local rule (transition rule) belonging to a class of Boolean function. The new state of a cell is completely determined by its own and its neighbors’ states by means of a simple local rule. The local rule is applied to a CA system homogeneously in space and synchronously in time. The states of all the cells in a cellular system portray the global behavior of the system being simulated. Since the CA system is autonomous, there is no external input for a conventional system. The most important feature of CA is that it could simulate very complicated phenomenon with a simple local rule and some initial states. One major problem of applying the CA in practice is the search for explicit local rules for the system under study, which is time consuming because there are a large number of potential rules controlling the CA system even if the system is simple. For example, for a CA system only containing 3 cells, the potential rule tables will be $2^3$.

### 4.5.2 CA Scheme

The basic idea of a CA scheme concerns the number of neurons in a hidden layer and the connections among the hidden layer and the input and output layers, which are represented by a two-dimensional CA system consisting of a binary matrix $C$ of $C_i \times C_j$.

The size of this matrix depends on the number of input and output nodes of the ANN and
the selected number of hidden neurons to be considered. Typically, \( C_i \) is equal to the number of input and output nodes while \( C_j \) corresponds to the selected number of hidden neurons. To ensure completeness in the search for an optimal configuration, usually the number of hidden neurons is selected as the maximum number, which could be approximated as the square of the row elements of the matrix. For example, if there are 12 elements in the row of the matrix, the maximum number of the hidden neurons is 144. To illustrate this scheme conceptually, the number of the hidden neurons represented by the proposed cellular system is simply selected as twice the number of elements in the row of the matrix in the problem. Since the ANN designed for the bridge aerodynamics in this study has only one output, it is reasonable to assume all hidden neurons are connected to this output, which makes \( C_i \) equal to the number of the input nodes. The construction of two dimensional CA is described in Figure 4.6. The position of each cell is defined with the two coordinates \((i, j)\). Various fill patterns represent different evolving seeds in the initial configuration with a value \( G_m \) (Equation 4.26). White cells represent no connection between corresponding hidden neurons and input nodes; their value is zero.

4.5.3 CA based ANN Configuration

The CA based ANN could be designed mainly through two sequential cellular systems. They are growing cellular and pruning cellular systems, respectively (Galván et al. 2000). The first step is to evolve the two dimensional CA matrix using the growing cellular system with the randomly positioned growing seeds \( G_m \). The second step is to prune the evolved CA matrix using the pruning cellular system with the randomly positioned
pruning seeds $p_x$. The final configuration of the ANN could be obtained by replacing the growing cells with black ones, which represent a connection between the corresponding input and hidden neurons, and replacing the pruning cells with white ones, which represent no connection between them. The number of growing seeds and pruning seeds are parameters of the cellular systems. The configuration of the ANN obtained through the above description will be examined using a “fitness” index $f$, which is a function of learning cycles and connections of the ANN (Gutiérrez et al. 2005). Both the learning cycle and connections contribute to the complexity of the network.

The neighborhood structure of the CA system is chosen as a “nine-neighborhood square,” which is defined by the square region around a central cell (Gutiérrez et al. 2005). In this situation, the state of the cells in this two dimensional CA system could be determined as

$$C_{i,j}^{n+1} = \gamma\left\{C_{i,j}^n, C_{i+1,j}^n, C_{i-1,j}^n, C_{i,j+1}^n, C_{i,j-1}^n, C_{i+1,j+1}^n, C_{i+1,j-1}^n, C_{i-1,j+1}^n, C_{i-1,j-1}^n\right\}$$

(4.25)
where \( \gamma \) represents the local rules of this cellular system; superscript \( n \) represents each time step. In order to launch the expansion procedure of the growing seeds, it is necessary to replicate the growing seeds over their quadratic neighborhood. The local rules for the growing cellular system are given as follows (Gutiérrez et al. 2005):

If \( C_{i,j}^n = 0 \) and \( \left\{ C_{i-1,j-1}^n = C_{i-1,j}^n = C_{i,j-1}^n = G_m \right\} \)

or \( \left\{ C_{i-1,j-1}^n = C_{i,j}^n = C_{i+1,j}^n = G_m \right\} \)

or \( \left\{ C_{i-1,j}^n = C_{i,j}^n = C_{i+1,j-1}^n = G_m \right\} \)

or \( \left\{ C_{i+1,j-1}^n = C_{i,j}^n = C_{i+1,j}^n = G_m \right\} \)

or \( \left\{ C_{i+1,j}^n = C_{i,j}^n = C_{i,j-1}^n = G_m \right\} \)

or \( \left\{ C_{i,j}^n = C_{i,j+1}^n = C_{i,j}^n = G_m \right\} \)

then \( C_{i,j}^{n+1} = G_m \)

otherwise \( C_{i,j}^{n+1} = C_{i,j}^n \)

The final configuration of the growing cellular system is given in Figure 4.7. There are 283 total connections between input and hidden layers, which result from applying the growing cellular system with 5 growing seeds.

The pruning seeds with the value \( P_n \) are then placed randomly on the final configuration of the growing cellular system. The original growing seeds or white cells are replaced by these pruning seeds. The local rules for the pruning cellular system are given as follows (Gutiérrez et al. 2005):
The final configuration of the pruning cellular system is given in Figure 4.8. There are a total of 131 connections between input and hidden layers following the application of the pruning cellular system with 5 pruning seeds.

There exist 288 original connections between the input and hidden layers with 24 hidden neurons if one utilizes a connection characterized as “fully”, where “fully” refers to the condition in which each input node is connected to all the hidden neurons and vice
versa. The CA based ANN configuration only contains 131 connections, and 19 effective neurons are located in the hidden layer. Given a certain level of training error, the CA based ANN significantly reduces the computational work compared to the “fully” connected ANN. In Figure 4.8 it has been shown that the maximum number of connections takes place at positions representing torsional acceleration and turbulence. These are the most important variables influencing the bridge aerodynamic response. This implies that the CA based encoding is able to identify the most significant input information.

4.6 ANN of Bridge Aerodynamics with Higher-Order Terms

For a system with hysteric behavior, the higher-order neurons in the ANN are critical for capturing this feature (Giles and Maxwell 1987; Kosmatopoulos et al. 1995). For this reason, hidden neurons in the proposed ANN are composed of the first- and higher-order terms, where the latter can be considered the power terms of the first-order terms.
themselves as well as their cross terms. Since the second-order and third-order terms are used in this system, the effective input information to output neurons is not only a linear combination of the components $z_j$, but also of their products $z_j z_l$ and $z_j z_l z_n$. It is interesting to note that the higher-order concept has also been applied in the rheological model proposed by Diana et al. (2008b) and Diana et al. (2010) to simulate bridge aerodynamics involving nonlinearity in terms of dynamic angle of attack. In the rheological model, the aerodynamic forces can be expressed as third-order polynomial functions of the dynamic angle of attack and its derivatives. However, since this dynamic angle of attack approach utilizes the combined effects of bridge deck motions and turbulence, motions in different degrees of freedom and turbulence effects have equal weighting on the aerodynamic force on the bridge deck. This poses an unwarranted constraint in the aerodynamic force description. By contrast, the higher-order ANN can account for different weights for various components of the dynamic angle of attack. Thus it offers more versatility in capturing the observed hysteretic behavior of bridge aerodynamics.

The outline concerning the construction of the CA based ANN is described here. Layer 1 spans from the input to the hidden layer in which the hidden neurons include the first and higher-order terms. Here, the hidden neurons are not “fully” connected to all the input nodes. The connections between these two layers are based on the final structure of the CA matrix. The corresponding weights constitute the weighting matrix $W$, whose terms are $w_{ij}$. Layer 2 spans from the hidden to the output layers. The output nodes are fully connected to all the hidden neurons. The corresponding weights result in another weighting matrix $W$, whose terms are $w_{jk}$. 

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Based on the architecture of a universal approximator, all input and hidden nodes can be considered to be linear and nonlinear basis functions, respectively, in which the coefficients represent the linear weighting matrix $W$. The mathematical basis for the back propagation technique is the first-order optimization algorithm known as gradient descent. The gradient of a function (in this case, the function is the error between the simulated results and targets, and the independent variables are the weights of the ANN) provides the direction in which the function increases more rapidly; the negative gradient provides the direction in which the function decreases most rapidly (Fausett 1994).

The choice of proper values for the design constants, e.g., the learning rate, also presents a significant challenge. This could be done through a trial and error approach, which can be made more effective through the use of some rules of thumb that are available in literature (Müller and Reinhardt 1990).

As described earlier in this study, two ANN have been constructed and will be trained concurrently since the input of one ANN relies on the output of the other ANN and vice versa, providing coupling between different degrees of freedom. Training these two higher-order ANNs simultaneously automatically combines the flutter and buffeting analysis in a unified approach since the input information to the ANN involves the bridge deck motions and turbulences at each time step. Accordingly, the influence of buffeting response on the self-excited forces can be included along with the damping and coupling effects that the self-excited forces have on the buffeting response. The overall ANN system constructed in this study is shown in Figure 4.9.
4.7 Example

4.7.1 Example Parameters

An example bridge with single-box section is used to investigate the methodology presented above. The turbulent wind is simulated and measured in the Tongji-1 Wind Tunnel at State Key Lab for Disaster Reduction in Civil Engineering, Tongji University. The recorded responses of the bridge deck section under the simulated turbulent wind include the time histories of vertical and torsional accelerations. The mean wind velocity $U$ is around 9 m/s. The time step $\Delta t$ used in the ANN model is selected to be 0.01 second, which is also the sampling rate of the measurements in the wind tunnel. The learning rate
α is selected based on trial and error. In this example a value of 0.08 is selected. There are 12 nodes in the input layer, 19 effective first-order neurons in the hidden layer, and one node in the output layer for each ANN. The number of the connections between the input and hidden layers is 131. Figure 4.10 gives the time histories of the input turbulence, i.e., u and w.

Figure 4.10. Time histories of input turbulence: (a) u (including the mean wind velocity U) fluctuation component; (b) w fluctuation component
4.7.2 Simulation Results based on First-Order ANN

Only first-order neurons are used in this initial ANN based simulation. The comparison between the target accelerations (vertical and torsional components) and the simulated accelerations for the training process are represented in Figure 4.11(a).

![Figure 4.11(a)](image1)

![Figure 4.11(b)](image2)

Figure 4.11. Time-history of the target (red line) and simulated (blue line) accelerations in the training process: (a) first-order ANN simulation, (i) torsional acceleration component, (ii) vertical acceleration component; (b) higher-order ANN simulation, (i) torsional acceleration component, (ii) vertical acceleration component

In the first-order ANN, the total effective neurons are 19. However, this ANN oversimplifies the simulated system. The simulation results in the training process appear to be lacking in effectively representing the target. Figure 4.12(a) gives the time history of a selection of training weights in the first-order ANN based simulation. The time
histories of the training weights are still unstable even after about 5000 learning cycles (50 seconds), which means this system is not well trained for subsequent use.

4.7.3 Simulation Results based on Higher-Order ANN

In light of the inability of the first-order ANN to faithfully represent the desired system, higher-order neurons are included in the ANN. There are a total of 442 effective new neurons created, which includes both the second- and third-order features. The second-order neurons are given here in which, for the sake brevity, superscript “n” has been omitted:
As a consequence of the symmetry, the number of additional neurons, including the contribution of the second-order terms, is only 78 as opposed to 144. The third-order neurons also exhibit symmetry, which reduces the number of effective third-order neurons to 364. The third-order neurons are not shown here for the sake of brevity.

For this CA based higher-order ANN, the “fitness” used to examine the complexity is defined as following:

$$f_i = \frac{1}{(conn_{ij} + conn_{jk}) \cdot cyc}$$  \hspace{1cm} (4.29)

where $conn_{ij}$ is the number of connections between input and hidden layers, which is represented by the two-dimensional cellular system; $cyc$ represents the number of learning cycles; $conn_{jk}$ represents the number of connections between hidden and output layers, which could be approximated as

$$conn_{jk} = \frac{neu^3}{6} + neu^2 + 2neu$$  \hspace{1cm} (4.30)

where $neu$ is the number of effective hidden neurons. Without the optimized process made possibly by the cellular system, the “fully” connected ANN has $conn_{ij}$ equal to 288
and neu equal to 24 while the corresponding values of conn, and neu for the CA based ANN are 133 and 19, respectively. The learning cycles cyc for both cases are about 310, which are at the same error level in the training process. The “fitness” of the “fully” connected ANN is about $1.003 \times 10^{-6}$ while the “fitness” of the CA based ANN is about $1.928 \times 10^{-6}$, which means the complexity of the CA based ANN, represented by the “fitness” function, is almost half of the “fully” connected one. The computational burden for the activation function of each neuron also needs to be accounted for if considering computational efficiency of the ANN.

The simulation results in the training process are improved significantly after including the second- and third-order neurons in the ANN hidden layers, as shown in the Figure 4.11(b). The time histories of the training weights have shown good stability after approximately 310 learning cycles [Figure 4.12(b)]. This suggests that the system is appropriately trained (primed) for the next stage. Figure 4.13 shows the torsional component response and compares it to the target results. Though the accuracy of this prediction is acceptable, additional refinements are possible.

4.8 Concluding Remarks

The nonlinearity in bridge aerodynamics cannot be captured by the conventional linear aerodynamics approach currently employed. Even with the current nonlinear parametric models, which rely heavily on the concept of dynamic angle of attack, the simulation of bridge aerodynamics has shown some shortcomings. A new nonparametric scheme to model nonlinear interactions between the bridge and the approach turbulent wind field is proposed. Unlike those models that utilize the concept of dynamic angle of attack, the
Figure 4.13. Time-history of a selection of training weights: (a) first-order ANN simulation; (b) higher-order ANN simulation

model has the ability to incorporate the effects of different bridge deck motions, angles of attack of wind and turbulence individually. Other factors, such as the amplitude of responses and/or the mean angle of attack, which affect the bridge aerodynamic response, could be incorporated straightforwardly in this approach. The cellular automata (CA) based system is used to optimize the artificial neural network (ANN) configuration. The potential of combined ANN and CA is underscored here for applications to aerodynamics. An example using data from a wind tunnel study has shown the efficacy of a CA based ANN to simulate nonlinear interactions between the bridge deck and the approach turbulence field, especially when the higher-order neurons in the hidden layer are incorporated. This study presents initial findings and the salient features of this novel application of a data-driven model for capturing nonlinear aerodynamic behavior of bridge decks under turbulent flow field. Immediate applications of this approach are
numerous in wind effects on structures, e.g., vortex induced vibrations and propagation of damage in structures and communities.
CHAPTER 5:

SPARSE THIRD-ORDER VOLTERRA SERIES: A GRAY-BOX MODEL

A sparse third-order Volterra model is utilized to simulate nonlinear bridge aerodynamics and attendant response under turbulent fluctuations. It is noted that the discrete-time Volterra model with a finite nonlinear degree and dynamic order actually, as it unfolds, belongs to a larger class of finite-dimensional nonlinear moving average (NMA) models. The Volterra model is pruned based on aerodynamic considerations, which significantly reduces computational effort needed for nonlinear analysis. The first-, second- and third-order Volterra kernels are identified using least-squares with the input-output pairs obtained from a numerical simulation and a wind-tunnel experiment. Both studies involving a simulation and an experiment show that the proposed sparse third-order Volterra model can adequately simulate nonlinear bridge aerodynamics with high fidelity. Finally, the robustness of the sparse Volterra model is verified by comparing it to an upgraded Volterra scheme. A sparse or pruned Volterra model may be classified as a "gray-box" model since the fundamental aerodynamic mechanisms are utilized in the model development.

5.1 Introduction

Innovative bridge decks being employed for long-span bridges exhibit nonlinear behavior in wind tunnel tests which has led to increased interest in nonlinear bridge aerodynamics
of long-span bridges (Kareem and Wu 2013). Accordingly, a number of studies have been focused on identifying the sources of nonlinearity and its modeling (e.g., Diana et al. 1995; Chen and Kareem 2003; Wu and Kareem 2010). The main physical sources of nonlinearity in bridge aerodynamics may result from the flow separation, reattachment around the deck, and the three-dimensional (3-D) nature of the ensuing wake. The situation can become more complicated by the interaction of aerodynamic nonlinearities with the potential nonlinearities of structural origin due to large deformations, material nonlinearities or complex damping properties (e.g., Xiang and Ge 2000; Salvatori and Borri 2007; Arena and Lacarbonara 2012). However, in this study only the aerodynamic nonlinearities are considered in terms of two-dimensional bridge deck sectional models. These models can be eventually extended to full 3-D bridge structure for overall response.

A conventional approach to take into account nonlinearity is to utilize the quasi-steady (QS) theory, which ignores frequency dependence of aerodynamic loads, hence assumption in the QS theory cannot account for fluid memory effects. In addressing this concern, corrected quasi-steady (CQS) theory has been introduced (Diana et al. 1993) to include fluid memory effects. However, this scheme has very limited provision for accommodating fluid memory considerations since it is intractable to select an appropriate frequency (and hence the reduced wind velocity) to account for the unsteadiness of the aerodynamic forces. Accordingly, a more detailed scheme, “band superposition” was proposed by Diana et al. (1995), in which the aerodynamic forces are linearized about a reference value of the dynamic angle of attack (a combination of the bridge deck motions and approaching turbulence). With a large number of sub-ranges to
accommodate the frequency spectrum in this approach, the "band superposition" scheme may become complex in its implementation. Chen and Kareem (2003) proposed a numerical analysis framework (referred herein as the hybrid scheme) which utilizes a rational function approximation scheme to obtain more efficient computational procedures. The hybrid scheme models bridge aerodynamics utilizing the linearized approach based on dynamic expansion points represented by the dynamic angle of attack. This is opposed to the static expansion point characterized by the mean displaced position of the deck applied in conventional linearized scheme. Accordingly, this scheme can be treated as a piecewise (dynamically) linearized method, which could take into account some of the nonlinearities in bridge aerodynamics featured as amplitude dependent (Kareem and Wu 2012). The hybrid scheme shows promise in simulating nonlinearity in bridge aerodynamics, but it has its own limitations at this juncture stemming from the challenge of pinpointing the separation between the low- and high-frequency parts of turbulent fluctuations. Generally, these nonlinear models mentioned above cannot fully characterize nonlinear bridge aerodynamics (Wu and Kareem 2012a).

In order to capture the nonlinear hysteretic effects in bridge aerodynamics, a rheological model has been utilized with a threshold, where the numerical model is related to a mechanical system (Diana et al. 2010). Since this model heavily relies on a unique independent variable, i.e., the dynamic angle of attack, it assumes the contribution of turbulence components and different degrees of bridge deck motions to the aerodynamic forces to be equally weighted in the entire range of reduced wind velocity. This inherently poses an unwarranted constraint in describing the wind-bridge interactions using this model. Wu and Kareem (2010) proposed a new non-parametric
scheme based on an artificial neural network (ANN) to model nonlinear bridge aerodynamics. This model, unlike the models that utilize the concept of dynamic angle of attack, has the ability to incorporate the contribution of different bridge deck motions, angles of attack of wind and turbulence components individually. Thus it offers more versatility in capturing the observed nonlinear behavior of bridge aerodynamics. Recently, another promising nonlinear model based on Volterra series has been exploited to simulate nonlinear bridge aerodynamics (e.g., Wu and Kareem 2011; 2012b; 2012c). It has also been shown that a neural network with a specific architecture, namely the time-delay neural network is equivalent to a Volterra series (Wray and Green 1994). The Volterra series, consisting of linear and higher-order convolutions, can represent the complex mapping rules (static linear/nonlinear relationships) and time lags (fluid memory effects) between the aerodynamic inputs and outputs, the hallmark of bridge aerodynamics. The most challenging issue with the Volterra model is the high computational cost as the higher-order Volterra kernels are involved. In this study, a sparse third-order Volterra model is employed to simulate nonlinear bridge aerodynamics, wherein the input is turbulent fluctuations and the output is deck response. The Volterra model is pruned from aerodynamic point of view, which significantly reduces computational effort in nonlinear modeling. The first-, second- and third-order Volterra kernels are identified using least-squares techniques with input-output pairs obtained from a hybrid simulation scheme or a wind-tunnel test. Both numerical and experimental examples show that the proposed sparse third-order Volterra model can simulate nonlinear bridge aerodynamics with high fidelity. A comparison with an upgraded Volterra scheme suggests that the simulation might be robust, although the
rigorous proof of this fact is beyond the scope of this study and not presented here. The data-driven characteristics of the ANN may classify it to be a “black-box” model, whereas, the sparse third-order Volterra model here may be classified as to be a "gray-box" model since the fundamental aerodynamic mechanisms are utilized in the model development (Doyle III et al. 2002).

5.2 An Unconstrained Volterra Model

A Volterra model provides an accurate description of nonlinearities in a system while preserving memory effects lost in static transformations. The basic premise of the Volterra theory is that a large class of nonlinear systems can be modeled as a sum of multidimensional convolution integrals of increasing order (Volterra 1959). For nonlinear systems with fading memory, a truncated Volterra series may be utilized (Boyd and Chua 1985).

5.2.1 Basic Volterra Theory

Bridge aerodynamics is typically time-invariant and causal, with fading fluid memory. Therefore, the bridge deck response \( y(t) \) under an arbitrary aerodynamic input \( x(t) \) may be represented as (Rugh 1981)

\[
y(t) = h_0 + \int_0^t h_1(t - \tau)x(\tau)d\tau \\
+ \sum_{p=2}^P \int_0^t \ldots \int_0^t h_p(t - \tau_1, \ldots, t - \tau_p)x(\tau_1) \ldots x(\tau_p)d\tau_1 \ldots d\tau_p
\]

where \( h_0 \) means steady-state term which satisfies system initial condition; \( h_1 \) represents the first-order kernel which describes the linear behavior of system; \( h_p \) the higher-order
terms which represent nonlinear behavior existing in the system; \( P \) denotes the nonlinear degree of the Volterra model.

The identification of Volterra kernels is critical when applying this model to simulate a nonlinear system. The first attempt to obtain the Volterra kernels is to derive them from analytical expressions describing the nonlinear system under investigation (e.g., Carassale and Kareem 2010). For example, the Duhamel integral is one of the simplest analytical expressions for a linear system. For higher-order kernels such expressions or equivalent computational descriptions are available for hydrodynamic loading on offshore structures (Li and Kareem 1990; Tognarelli et al. 1997). Similarly, expressions have been developed for aerofoil aerodynamics and aeroelasticity in the literature (e.g., Marzocca et al. 2000; 2001). For most other engineering problems of interest such analytical expressions are not available. Therefore, an alternate approach is to identify the Volterra kernels based on system identification techniques, where the discrete-time model is utilized rather than the continuous-time model. As a result, the bridge deck response \( y[n] \) under an arbitrary aerodynamic input \( x[n] \) is given by (e.g., Clancy and Rugh 1979)

\[
y[n] = h_0 + \sum_{k=0}^{M} h_k[n][n-k] + \sum_{p=2}^{P} H_{M}^{p}[n]
\]

\[
H_{M}^{p}[n] = \sum_{k_1=0}^{M} \ldots \sum_{k_p=0}^{M} h_p[k_1, \ldots, k_p][n-k_1] \ldots [n-k_p]
\]

where the integer \( n \) indicates the time step; \( M \) denotes the dynamic order of the Volterra model, which describes the duration of the fluid memory effects in the bridge aerodynamic system.

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5.2.2 A General Approach to Identify Volterra Kernels

It is noted that the discrete-time Volterra model with finite nonlinear degree and dynamic order actually, as it unfolds, belongs to a larger class of finite-dimensional nonlinear moving average (NMA) models (Pearson 1995; Pearson et al. 1996) with rich literature on system identification techniques. The kernel identification of a discrete-time Volterra model using the general input-output pairs from computational simulations or experimental tests is actually a linear problem (Rugh 1981)

\[ Y_{N\times 1} = X_{N\times Q} H_{Q\times 1} \]  
(5.3)

where the steady-state term \( h_0 \) is neglected due to the focus on dynamic response. \( N \) indicates the number of input-output pairs; \( Q \) denotes the number of kernel terms in the discrete-time Volterra model. \( Y \) is a vector which consists of the output data

\[ Y_{N\times 1} = [y[0], y[1], ..., y[n]]^T \]  
(5.4)

where \( N=n+1 \); the symbol “T” denotes the matrix transpose. \( X \) is a matrix which consists of the input data combinations

\[ X_{N\times Q} = \left[ X_{N\times Q_1}, ..., X_{N\times Q_p} \right] \]  
(5.5a)

\[ X_{N\times Q_1} = \begin{bmatrix} x[0] & 0 & \cdots & 0 \\ x[1] & x[0] & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ x[n] & x[n-1] & \cdots & x[n-M] \end{bmatrix}_{Q_1=(M+1)} \]  
(5.5b)
where $Q_p$ ($p=1, \ldots, P$) indicates the number of the input data combinations corresponding to the $p^{th}$-order Volterra kernel. It should be noted that, since the bridge aerodynamic system is causal, the term $x[n-k]$ is equal to zero if $[n-k]$ becomes smaller than zero. $\mathbb{H}$ is a vector which consists of the corresponding kernel terms

\[
\mathbb{H}_{Q_1} = [\mathbb{H}_{Q_1}^1, \ldots, \mathbb{H}_{Q_1}^p]^T
\]  

\[
\mathbb{H}_{Q_1}^1 = \left[ h[0], h[1], \ldots, h[M] \right]^T_{Q_1 = (M+1)^i}
\]  

\[
\mathbb{H}_{Q_1}^2 = \left[ h[0,0], h[0,1], \ldots, h[0,M], \ldots, h[M,0], h[M,1], \ldots, h[M,M] \right]^T_{Q_1 = (M+1)^i}
\]  

\[
\mathbb{H}_{Q_1}^p = \left[ h[0,\ldots,0], h[0,0,\ldots,1], \ldots, h[0,0,\ldots,M], \ldots, h[M,0,\ldots,0], h[M,0,\ldots,1], \ldots, h[M,M,\ldots,0], h[M,M,\ldots,1] \right]^T_{Q_1 = (M+1)^i}
\]  

As it can be inferred from Equation 5.3, only if the number of input-output pairs $N$ is equal to the number of kernel terms $Q$, the input matrix $X_{N\times Q}$ is a square matrix and hence is possibly invertible, otherwise, if $N$ is larger or smaller than $Q$, or $X_{N\times Q}$ is not invertible, then one needs to introduce pseudo-inversion (e.g., Moore-Penrose pseudo-
inverse) based on the least-squares techniques for kernel identification (Rugh 1981). Accordingly, the kernels in the discrete-time Volterra model could be identified as

$$H_{Q=1} = \left( X_{N \times Q} \right)^+ Y_{N \times 1}$$

(5.7)

where the symbol “+” represents the Moore-Penrose pseudo-inverse.

There are various other approaches to identify the Volterra kernels, such as harmonic probing method (e.g., Bedrosian and Rice 1971), identification based on Wiener orthogonal kernels (e.g., Lee and Schetzen 1965) and identification based on impulse function concept (e.g., Schetzen 1965). For these techniques, the input sequences are required to be harmonic components, Gaussian white noise and impulse function, respectively. On the other hand, there is no special requirement of input sequence for the aforementioned general identification technique. However, as indicated in Equation 5.6, the kernel terms increase exponentially with the degree of nonlinearity of the discrete-time Volterra model. Hence, there is a serious disadvantage of this general identification scheme in terms of the computational effort, which limits most of the applications based on the Volterra model up to the second order. A straightforward scheme to reduce the kernel results from commutative multiplication of terms, where the nonlinear system could be modeled by using the triangular kernel. The triangular kernel has the property (Rugh 1981)

$$h_{tr_i} (t - \tau_1, t - \tau_2, \cdots, t - \tau_p) = 0 \text{ if } \tau_i < \tau_{i+j}$$

(5.8)

where $i, j \in \text{positive integers};$ the subscript "$tr$" denotes the triangular kernel. The kernel terms need to be identified are reduced substantially, however, it should be noted that they still increase exponentially with the nonlinear degree.
5.3 A Sparse Volterra Model

Computational demand of the discrete-time Volterra model has precluded its application to many problems. Therefore, there has been efforts to reduce this demand. Specifically, computational complexity may be reduced by eliminating the insignificant kernels (e.g., Yao et al. 1992), or by the sparse-interpolation scheme where an ad hoc recreation procedure for the kernel terms is introduced (e.g., Batista et al. 2010). It should be noted that there is no underlying metric for most of pruning schemes utilized for simplifying the Volterra model. In this study, an unconstrained Volterra model is pruned from aerodynamic point of view, where the degree of nonlinearity and the dynamic order are based on physical considerations.

5.3.1 Selection of Degree of Nonlinearity

It is well known that the even-order convolution captures the even-order super-harmonics and asymmetric nonlinearity, whereas the odd-order convolution captures the odd-order super-harmonics and symmetric nonlinearity. For a bridge deck, typically the cross-section is not symmetric with respect to the chord line, hence, the third-order Volterra model (consisting of first-, second- and third-order kernel terms) may offer the lowest order nonlinear model for comprehensively simulating the nonlinearities in bridge aerodynamics. On the other hand, for a symmetric airfoil, the Volterra model retaining only the first- and third-order kernel terms may be sufficient in modeling nonlinear aerodynamics (e.g., Balajewicz 2010). Similar treatment of the Volterra kernels in case of waves excited offshore structures can be found in Tognarelli et al. (1997).

In a rheological model to describe nonlinear aerodynamic forces acting on a bridge deck involved the use of third-order polynomial functions of the dynamic angle of
attack and its derivatives (Diana et al. 2010). A similar higher-order terms concept has also been introduced in an ANN-based bridge aerodynamics nonlinear simulation, where the second- and third-order neurons in the hidden layer are incorporated to simulate the hysteretic nonlinearity in bridge aerodynamics (Wu and Kareem 2010). In spite of the challenges as mentioned in the preceding discussion, both rheological and ANN models, which utilize the nonlinear schemes up to the third order, show promise in simulating nonlinearity in bridge aerodynamics. Hence, in this study, the third-order Volterra model is utilized to simulate nonlinear bridge aerodynamics, where $P=3$ in Equation 5.1.

5.3.2 Selection of Dynamic Order

The wake dynamic features behind a bridge deck continue to contribute to the bridge aerodynamics until these are convected far downstream, which leads to the fluid memory effects. The duration of the fluid memory effects on the aerodynamic forces in a Volterra system can be represented by the dynamic order $M$. For streamlined sections like an airfoil, the linear aerodynamic forces induced by the unit-step gust input were theoretically derived by Küssner (1936). The approximated explicit analytical solution of the Küssner function is usually expressed as (Bisplinghoff and Ashley 1962)

$$
\psi(s) = 1 - 0.5e^{-0.130s} - 0.5e^{-s}
$$

(5.9)

where $s=Uit/b$ is the nondimensional time; $U$ is the mean wind velocity; $b$ is the half of chord width. The Küssner function $\psi(s)$ is plotted in Figure 5.1.

As shown in the figure, the Küssner function rapidly approaches the asymptotic value of unity, which indicates the linear fluid memory effects fade away with time. Actually, as the nondimensional time is at 20, the linear fluid memory effects is less than
0.5%. The Küssner function cannot be directly utilized to calculate aerodynamic forces on bridge decks due to the separation of flow in this case. Whereas, since the wake is convected downstream by the mean wind velocity, it may be reasonable to assume for the bridge decks to have linear fluid memory effects similar to the airfoil in terms of the nondimensional time.

In this study, the dynamic order of the linear convolution is determined by assuming that the duration of the linear fluid memory effects is 20 nondimensional time. Generally, the fading time of the nonlinear fluid memory effects is much shorter compared to that of the linear fluid memory effects (Doyle III et al. 2002), hence, the dynamic order of the $(p+1)^{\text{th}}$-order convolution is selected to be half of that of the $p^{\text{th}}$-order convolution. Besides, to obtain sparse model the third-order Volterra system could be further pruned by removing insignificant off-diagonal kernel terms. The nonlinear contribution of the $p^{\text{th}}$-order convolution, which could be treated as a deviation from the linear superposition, results from the nonlinear coupling effects among the responses due to sequential inputs $x[n-k_i]$ (where $i=1, ..., p$ and $k_i=0, ..., M$). It is assumed that the
nonlinear coupling effects fade away significantly fast with time lags among the inputs. As a consequence, a large number of kernel terms could be ignored according to the following expression:

\[ h_p[k_1, ..., k_p] = 0 \quad \text{for} \quad \sum_{j_1=1}^{j_1=p} \sum_{j_2=1}^{j_2=p} \text{abs}(k_{j_1} - k_{j_2}) > NC \]  

(5.10)

where NC is a threshold determined by the sampling interval and the predefined effective time lags beyond which the nonlinear coupling effects may be ignored. In this study, the exact value of NC is selected by trial and error. A more precise selection of NC needs in-depth investigation from theoretical considerations and additional exercises involving more comprehensive study cases.

5.4 Simulation of Nonlinear Bridge Aerodynamics

Two examples are utilized in this study to show the fidelity of the simulation of nonlinear bridge aerodynamics based on the proposed sparse third-order Volterra model. In the first example, the input-output pairs are obtained from a numerical simulation of a hybrid scheme (Chen and Kareem 2003; Wu and Kareem 2012a), where input is the vertical wind fluctuations and the output is the vertical displacement of the bridge deck. As mentioned earlier, the hybrid scheme is a numerical analysis framework which could take into account some of the nonlinearities in bridge aerodynamics featured as amplitude dependent. In the second example, the input-output pairs are obtained from an experimental test in a wind tunnel, where input is the vertical wind fluctuations and the output is the torsional displacement of the bridge deck. Both these examples involve two-dimensional bridge deck models.
5.4.1 Example 1: Hybrid Scheme-based Simulation

The first example considers a long-span cable-stayed bridge with a single box girder. The nonlinear aerodynamics in this case results from the amplitude dependence of aerodynamic force coefficients on the dynamic angle of attack. Figure 5.2 shows a comparison between the linear and nonlinear simulations, which are based on conventional linearized model (Scanlan and Tomko 1971) and the hybrid scheme (Chen and Kareem 2003), respectively. The comparison indicates that the bridge aerodynamic nonlinearity not only increases the amplitude of bridge deck response significantly, but also alters the phase.

The input wind fluctuations were simulated utilizing spectral representation technique with turbulence intensity of 15%; the aerodynamic coefficients under different incident wind angles were measured in a wind tunnel. The structural parameters used in the numerical calculation were as follows: the width of the bridge deck was 41 m; the effective mass per unit span was 32,000 kg/m; the natural frequency of fundamental
vertical mode was 0.195 Hz; the damping ratio for the fundamental vertical mode was 0.005. These data are based on the designed parameters of the example long-span cable-stayed bridge. The mean wind velocity was set at 60 m/s. The sampling interval was 0.125 s, accordingly, the nondimensional sampling interval was 0.4. Hence, the dynamic order $M_1$ for the linear convolution was set to be 50, while the dynamic orders $M_2$ and $M_3$ for the second-order and third-order convolutions were taken to be 25 and 12, respectively. The threshold NC for further pruning of the sparse third-order Volterra model was set to be 2. As a result, the kernel terms that needed to be identified have been reduced from 135303 of the unconstrained third-order Volterra model to 260 of the sparse third-order Volterra model.

Figure 5.3 presents the identified first-order kernel and Figure 5.4 shows the identified second-order kernel. It is noted that both the first- and second-order kernels converge rather rapidly to zero with time, which indicates the linear and nonlinear fluid memory effects fade away quickly.
The identified first-, second- and third-order kernels are used to simulate the bridge deck response according to Equation 5.2. The input wind fluctuations are given in Figure 5.5(a) and Figure 5.5(b) shows simulation results based on the sparse linear, second-order and third-order Volterra model. The results indicate that the fidelity of simulation is significantly increased by including the higher-order convolutions.

5.4.2 Example 2: Experimental Simulation

The second example considers a long-span suspension bridge with a multi-box girder. For this deck section strong nonlinear aerodynamics has been observed in a wind tunnel (Diana et al. 1995).

The input wind fluctuations were generated by an active turbulence generator made up of 10 NACA 0012 carbon fiber wings equipped with pitching rotation driven by two computer controlled brushless motors (Diana et al. 2004). The wind fluctuations were measured by a hot film anemometer positioned at the half chord upwind the bridge.
Figure 5.5. Input and output of the numerical simulation example: (a) input data; (b) output data

deck model, and the bridge deck response was recorded by laser displacement transducers (Diana et al. 2004). The structural parameters of the bridge deck model were as follows: the width of the bridge deck was 1 m; the polar moment of inertia was 8.3 kg·m²; the natural frequency of fundamental torsional mode was 0.99 Hz; the damping ratio for the fundamental torsional mode was 0.003. These data are based on the designed parameters of the example bridge deck model in the wind tunnel at Politecnico di Milano. The mean wind velocity was around 8.25 m/s. The sampling interval was around 0.06 s, accordingly, the nondimensional sampling interval was 1.0. Hence, the dynamic order $M_1$ for the linear convolution was set to be 20, while the dynamic orders $M_2$ and $M_3$ for the second-order and third-order convolutions were taken to be 10 and 5, respectively. The
threshold NC for further pruning of the sparse third-order Volterra model was set to be 2. As a result, the kernel terms that needed to be identified were reduced from 9723 of the unconstrained third-order Volterra model to 106 of the sparse third-order Volterra model.

Figures 5.6 and 5.7 show the identified first-order and second-order kernels, respectively and Figure 5.8 provides the diagonal terms $h_3[n, n, n]$ of the identified third-order kernel. It is noted that the first-, second- and third-order kernels rapidly converge to zero with time, which suggests that the linear and nonlinear fluid memory effects fade away quickly.

![Figure 5.6. First-order kernel of the experimental test example](image)
Figure 5.7. Second-order kernel of the experimental test example

Figure 5.8. Diagonal term of third-order kernel of the experimental test example
There are totally 240 input-output pairs utilized to identify the proposed sparse third-order Volterra model based on the least-squares technique described in the preceding discussion. A larger number of input-output pairs are employed to identify the Volterra kernels, where poor results are obtained due to over-fitting issues. The input data in the training stage is shown in Figure 5.9(a) and Figure 5.9(b) presents the deck response based on the sparse linear, second-order and third-order Volterra model in the training stage and the exact results.

![Figure 5.9(a)](image1)

![Figure 5.9(b)](image2)

Figure 5.9. Input and output of the training stage of the experimental test example: (a) input data; (b) output data

The identified first-, second- and third-order kernels are used to simulate the bridge deck response according to Equation 5.2. The input data used in the test is given in
Figure 5.10(a) and Figure 5.10(b) presents the simulation results based on the sparse linear, second-order and third-order Volterra model in the testing stage. Generally, the sparse third-order Volterra model shows promise in simulating nonlinear bridge aerodynamics. It should be noted that the first 20 predicted response points are not shown here since the input data in the training stage will be involved due to the fluid memory effects.

![Figure 5.10. Input and output of the prediction of the experimental test example: (a) input data; (b) output data](image)

In order to further improve the simulation capability, the proposed sparse third-order Volterra model is updated at each time step, where the 240 input-output pairs in the training stage are updated according to the prediction time step. Suppose the prediction
response is at time step $n_{pe}$, the input-output pairs utilized to identify the Volterra kernel terms are as follows:

$$X_{240×1} = [x[n_{pe} - 240], x[n_{pe} - 239], ..., x[n_{pe} - 1]]^T \quad (5.11a)$$

$$Y_{240×1} = [y[n_{pe} - 240], y[n_{pe} - 239], ..., y[n_{pe} - 1]]^T \quad (5.11b)$$

where $[n_{pe} - 240]$ is larger than or equal to zero. Figure 5.11 presents the simulation results based on the updated sparse third-order Volterra model in the testing stage. The simulation results do not show any superiority compared to the simulation results in Figure 5.10(b). This observation suggests that the identified sparse third-order Volterra model is reasonably robust, where the kernel terms do not change according to different input-output training data sets.

![Figure 5.11. Simulation results based on the updated Volterra model](image)

Figure 5.11. Simulation results based on the updated Volterra model
5.5 Conclusions

A sparse third-order Volterra model is proposed to simulate nonlinear bridge aerodynamics and associated response. Specifically, the linear kernel terms are pruned based on the Küssner function, while the nonlinear kernel terms are reduced based on the premise that the nonlinear fluid memory effects fade away more rapidly than the linear fluid memory effects. Besides, the off-diagonal terms of the higher-order kernels were dropped based on the observation that the nonlinear coupling effects, which could be treated as the deviation from the superposition theory, decrease significantly as the time lag among the sequential inputs increases. It is shown that the computational complexity of the proposed sparse third-order Volterra model is substantially reduced compared to the unpruned scheme.

Two bridge aerodynamic examples involving a numerical simulation and experimental data from a wind-tunnel test are utilized to verify the fidelity of the proposed sparse third-order Volterra model. The first-, second- and third-order Volterra kernels are identified based on the least-squares technique. The identified results indicate that both the linear and nonlinear fluid memory effects fade away rapidly with time. Both examples suggest that the proposed sparse third-order Volterra model has the promise of capturing and simulating nonlinear bridge aerodynamics. In addition, the results also reflect that the proposed scheme is robust. It should be noted that the single-input single-output (SISO) Volterra model is investigated in this study to basically illustrate the identification technique for the proposed sparse third-order Volterra model. A multi-input multi-output (MIMO) Volterra model is an immediate extension of the SISO case, where the cross kernels contribute to the overall system response and need to be identified.
A linear convolution scheme involving first-order (linear) kernels for linear bridge aerodynamics is first reviewed and the significance of the selection of proper input parameters is emphasized. Following the concept of nonlinear indicial response function, the linear convolution scheme is extended to the nonlinear convolution scheme involving higher-order (nonlinear) kernels for the treatment of nonlinear bridge aerodynamics using a "peeling-an-onion" type procedure. Utilizing an impulse function as input, a comprehensive kernel identification scheme is developed. A numerical example of a long-span suspension bridge, where the gust- and motion-induced response is simulated based on a phenomenological model, is investigated to verify the fidelity of the proposed nonlinear convolution scheme. The nonlinear convolution scheme is actually represented utilizing a Volterra-type formalism. Once the Volterra kernels are identified using impulse function-based scheme, they could be utilized in predicting the response of the investigated nonlinear system under arbitrary inputs (harmonic, random, or other signals). This Volterra theory-based scheme could be treated as a “white-box” model (parametric model) after the identified kernels are parameterized based on a selected function.
6.1 Introduction

A main source of nonlinearity in bridge aerodynamics results from flow separation around the deck. For streamlined sections like an airfoil, flow separation occurs only in the case of large angles of attack (dynamic stall) or the shock motions in the transonic region (the shock motion itself also induces nonlinearity). For bluff sections like a bridge deck, flow separation is prevalent as the fluid motion around the deck cannot negotiate sudden changes in the deck profile. The resulting nonlinearity can be viewed from four viewpoints: (i). non-proportional relationship between amplitudes of input and output; (ii). single-frequency input exciting multiple frequencies; (iii). amplitude dependence of aerodynamic and aeroelastic forces and (iv). hysteretic behavior of aerodynamic forces versus angles of attack (Wu and Kareem 2011). Nonlinear effects are usually exploited to offer a possible explanation for any differences observed between the linear analysis results and experiments (Dowell and Ilgamov 1988) although it is difficult to delineate their relative contributions.

In order to take into account the increasing nonlinear behaviour of bridge aerodynamics observed in wind-tunnel tests, several numerical schemes such as the "band superposition" (Diana et al. 1995), "hybrid" (Chen and Kareem 2003), "rheological" (Diana et al. 2010) and "artificial neural network" (Wu and Kareem 2010) have been proposed over the last decade to advance conventional linear analysis framework (Davenport 1962; Scanlan and Tomko 1971). Generally, these numerical schemes have been unable to represent completely nonlinear bridge aerodynamics (Wu and Kareem 2011; 2012b), which limits their utility and calls for a comprehensive nonlinear analysis framework.
The consideration of nonlinearity is usually carried out in the time domain benefitting from its ability to take into account the nonlinear effects readily. In the time domain, the convolution of a linear kernel, e.g., the unit-step response function, is well known as the Duhamel's integral. In this study, the linear convolution scheme concerning first-order kernels for linear analysis of bridge aerodynamics is reviewed together with a selection of proper input variables. Then, it is extended to the nonlinear convolution scheme involving higher-order kernels for nonlinear analysis of bridges under winds based on the concept of nonlinear indicial response function. A nonlinear convolution scheme is represented utilizing a Volterra-type formalism, which ensures convergence of its truncated form. To facilitate this formalism, a comprehensive kernel identification scheme is developed utilizing the impulse function as input. Finally, a numerical example of a long-span suspension bridge with vertical and torsional degrees of freedom is investigated to verify the fidelity of the simulation based on the proposed nonlinear convolution scheme, where the amplitude dependence of kernels is also discussed.

6.2 Linear Convolution Scheme

This study focuses on the simulation based on a two-dimensional (2-D) representation of the deck and the strip theory.

6.2.1 Input Information of Bridge Aerodynamics

The selection of proper input variables for bridge aerodynamics based on convolution integrals is a critical issue. In the case of gust-induced effects, the input information is straightforward, i.e., the gust fluctuations in each degree of freedom. However, in the
case of motion-induced effects, the input information is often misunderstood in bridge aerodynamics.

It is well known that the motion-induced forces on a bridge deck are dependent on the orientation of local coordinate system and the relative motion between the wind and the deck (Yoshimura and Nakamura 1979). In order to simplify the analysis, the case of a bridge deck moving in the absence of flow is first investigated (Figure 6.1). As shown in Figure 6.1, the orientation of local coordinate system (as denoted by the subscript "b") can be represented by the angle \( \theta \) defined as the pitching angle, which is the angle between the local coordinate abscissa axis \( x_b \) and the global coordinate abscissa axis \( x \); while the relative motion between the wind and the deck can be decomposed into the relative translatory motion represented by \( V \), which is the time derivative of the displacement \( \mathbf{s} \), and the relative rotational motion represented by \( \dot{\theta} \) (velocity of angle of pitch). \( V \) is represented by the velocity magnitude \( V \) and velocity direction \( \gamma \), which is the angle between the velocity direction and the global coordinate abscissa axis \( x \). It is noted that \( \alpha = \gamma + \theta \), where \( \alpha \) defined as the angle of attack, is the angle between the velocity direction and the local coordinate abscissa axis \( x_b \). As a result, the motion-induced forces on the bridge deck are functions of \( V, \theta, \dot{\theta} \) and \( \alpha \).

For a bridge deck harmonically oscillating in vertical and torsional degrees of freedom under a stationary uniform wind flow (constant wind velocity), translation direction \( \gamma \) is a constant. For such a case, \( \alpha \) and \( V \) can be represented by the variable \( \theta \) and \( \dot{h} \), respectively, where \( h \) is the vertical translatory displacement of the deck. Hence, the bridge deck motion can be decomposed into \( \dot{h}, \theta \) and \( \dot{\theta} \), as shown in Figure 6.2,
which contribute to the motion-induced forces on the bridge deck. As indicated in Figure 6.2, the translatory motion coupled with $\theta$ has a negative equivalent that is coupled with $\dot{\theta}$, hence, combination of motions $\theta$ and $\dot{\theta}$ presents a pure harmonic rotary oscillation. It should be noted that, for the streamlined cross-section, the contributions to the motion-induced forces from the translatory motion $\dot{h}$ and the orientation of the cross-section $\theta$ are identical since the flow passing by the structure is always attached on the solid
surface. Whereas, for the bluff cross-section, the contribution from $\theta$ becomes complicated due to the flow separation. There is no clear explanation to demonstrate the assumption of an equivalent physical origin for these two contributions to the aeroelastic forces. Besides, as the harmonic oscillation is applied to the deck, the identified contribution of the orientation of the cross-section $\theta$ in a wind tunnel naturally involves the apparent moment of inertia effects of the rotational motion. Different values of the identified flutter derivatives in the wind tunnel corresponding to the translatory motion $h$ and orientation of the cross-section $\theta$ also indicate that it is necessary to separate their contributions in bridge aerodynamics and to retain both of them as input variables (Scanlan et al. 1997).

6.2.2 Convolution Scheme with Indicial Response Function

With the inclusion of chord-wise correlation, the gust-induced effects (mainly vertical) are directly related to the motion-induced case (von Kármán and Sears 1938) and are not presented here for the sake of brevity. Generally, the motion-induced forces are expressed as

$$F(t) = f\left(\theta, \dot{h}, \dot{\theta}, t\right)$$

(6.1)

where $F$ denotes motion-induced forces, i.e., the motion-induced lift force $L$ or torsional moment $M$; $f$ represents a general nonlinear function. The linear part of the Taylor expansion of the nonlinear motion-induced lift force increment $\Delta L(t)$ and torsional moment increment $\Delta M(t)$, due to an infinitesimal change in the input variables at time $\tau$, could be represented as (Nixon 1989)
\[ \Delta L(t) = - \left[ \left( \frac{\partial L(t, \tau)}{\partial \theta} \right)_{\theta = \theta_0} \left( \frac{d\theta}{d\tau} \Delta \tau \right) \right] \]
\[ + \left[ \left( \frac{\partial L(t, \tau)}{\partial \dot{h}} \right)_{h = h_0} \left( \frac{d\dot{h}}{d\tau} \Delta \tau \right) \right] \]
\[ + \left[ \left( \frac{\partial L(t, \tau)}{\partial \dot{\theta}} \right)_{\dot{\theta} = \dot{\theta}_0} \left( \frac{d\dot{\theta}}{d\tau} \Delta \tau \right) \right] \]  
\[ (6.2a) \]

\[ \Delta M(t) = \left[ \left( \frac{\partial M(t, \tau)}{\partial \theta} \right)_{\theta = \theta_0} \left( \frac{d\theta}{d\tau} \Delta \tau \right) \right] \]
\[ + \left[ \left( \frac{\partial M(t, \tau)}{\partial \dot{h}} \right)_{h = h_0} \left( \frac{d\dot{h}}{d\tau} \Delta \tau \right) \right] \]
\[ + \left[ \left( \frac{\partial M(t, \tau)}{\partial \dot{\theta}} \right)_{\dot{\theta} = \dot{\theta}_0} \left( \frac{d\dot{\theta}}{d\tau} \Delta \tau \right) \right] \]  
\[ (6.2b) \]

where \( \frac{\partial y(t, \tau)}{\partial x} \) (\( y= L, \) or \( M \) and \( x= \theta, \) \( \dot{h}, \) or \( \dot{\theta} \)) denotes the rate of change of \( F(t) \) with input at time \( \tau. \) It is obvious that there are two time scales involved in describing the time dependent characteristics of linear wind-bridge interactions, i.e., the time \( \tau \) at which the boundary conditions (input variables) change and the time \( t \) at which the lift force or torsional moment is measured.

As a time invariant system, the time dependent characteristics of the linear wind-bridge interactions could be described only using one time scale, i.e., the time difference \( (t-\tau) \) representing duration since the change in boundary conditions. Hence, as \( \Delta \theta, \) \( \Delta \dot{h} \) and \( \Delta \dot{\theta} \) converge towards infinitesimal, the accumulated lift force or torsional moment on the bridge deck up to \( t \) is given with the linear convolution integral (Wu and Kareem 2013c)
where \( C_M, C_L, C_D \) are steady-state moment, lift and drag coefficients, respectively; \( b \) is the half width of the bridge deck; \( s \) and \( \sigma \) represent non-dimensional time corresponding to \( t \) and \( \tau \), respectively; \( \Phi_{L\theta}, \Phi_{L\theta'}, \Phi_{M\theta}, \Phi_{M\theta'}, \Phi_{Mh'/b} \) and \( \Phi_{M\theta'} \) denote non-dimensional rate of change of \( F(s) \) with input at non-dimensional time \( \sigma \), which are also known as non-dimensional motion-induced indicial (unit-step) response functions.

As indicated in Equation 6.3, there are totally six unit-step response functions for simulating the motion-induced lift force or torsional moment on the bridge deck. On the other hand, it is noted that the motion-induced lift force on the thin airfoil only depends on the downwash (the vertical component of the flow) at the \( \frac{3}{4} \) chord point of the airfoil (rear neutral point). Furthermore, the resultant lift force on the airfoil always acts at the \( \frac{1}{4} \) chord point of the airfoil (forward neutral point). Hence, in the case of a thin airfoil with small disturbance, only one unit-step response function, namely the lift force induced by
the unit-step function of angle of attack is sufficient to characterize the motion-induced
effects. This unique unit-step response function is usually called Wagner lift-growth
function $\Phi_w(s)$, which was theoretically derived by Wagner (1925) for a thin airfoil. In
the case of a bridge deck, there is no analytical expression for these unit-step response
functions, hence, they need to be directly identified in a wind tunnel or evaluated using
computational fluid dynamics (CFD) (e.g., Yoshimura and Nakamura 1979; Turbelin and
Gibert 2001). Alternatively, these could be identified indirectly using the indicial or
rational function approximations (e.g., Scanlan et al. 1974; Chen and Kareem 2003; Wu
and Kareem 2012d).

6.3 Nonlinear Convolution Scheme

In this study, the discussion on the simulation of nonlinear bridge aerodynamics involves
only one input variable to illustrate the concept clearly without the loss of generality.

6.3.1 Nonlinear Indicial Response Function

In order to capture the nonlinear bridge aerodynamics features, a "hybrid" scheme was
proposed by Chen and Kareem (2003), in which the motion-induced unit-step response
functions were amplitude dependent. Hence, if only $\theta$ is involved as the input variable,
the motion-induced nonlinear lift force and torsional moment are given by

$$L(s) = -\frac{1}{2} \rho U^2 2b \left[ \frac{dC_L}{d\theta} \left\{ \Phi_{L\theta} \{\theta(0); s\} \theta(0) + \int_0^\prime \Phi_{L\theta} \{\theta(s); s - \sigma\} \theta(s)ds \right\} \right] \quad (6.4a)$$

$$M(s) = \frac{1}{2} \rho U^2 2b \left[ \frac{dC_M}{d\theta} \left\{ \Phi_{M\theta} \{\theta(0); s\} \theta(0) + \int_0^\prime \Phi_{M\theta} \{\theta(s); s - \sigma\} \theta(s)ds \right\} \right] \quad (6.4b)$$
where $\Phi_{L\theta}$ and $\Phi_{M\theta}$ are functionals of the input variable $\theta(\sigma)$. In order to improve the simulation accuracy of nonlinear bridge aerodynamics, Wu and Kareem (2012a) developed a "modified hybrid" scheme, where the motion-induced unit-step response functions are dependent on $\theta(\sigma)$ and its derivative with time. Furthermore, Tobak and Pearson (Tobak and Pearson 1964) considered a more generalized situation where the motion-induced unit-step response functions depended not only on the motion $\theta(\sigma)$ at time $\sigma$ at which the unit-step input was made, but also on all the past values of $\theta$. Accordingly, the motion-induced nonlinear lift force and torsional moment should be expressed (Tobak and Pearson 1964)

$$L(s) = -\frac{1}{2} \rho U^2 2b \left[ \frac{dC_L}{d\theta} \{ \Phi_{L\theta} \{ \theta(0); s, 0 \} \theta(0) + \int_0^t \Phi_{L\theta} \{ \theta(\kappa); s, \sigma \} \theta'(\sigma) d\sigma \} \right]$$

$(6.5a)$

$$M(s) = \frac{1}{2} \rho U^2 2b^2 \left[ \frac{dC_M}{d\theta} \{ \Phi_{M\theta} \{ \theta(0); s, 0 \} \theta(0) + \int_0^t \Phi_{M\theta} \{ \theta(\kappa); s, \sigma \} \theta'(\sigma) d\sigma \} \right]$$

$(6.5b)$

where $\Phi_{L\theta} \{ \theta(\kappa); s, \sigma \}$ and $\Phi_{M\theta} \{ \theta(\kappa); s, \sigma \} \ (0 \leq \kappa \leq s)$ denote the nonlinear indicial (unit-step) response functions. The nonlinear unit-step response function is a functional and could be defined by the functional derivative (Fréchet derivative) with respect to the input $\theta$ (Reisenthel 1996)

$$\Phi_{L\theta} \{ \theta(\kappa); s, \sigma \} = \lim_{\Delta \theta \to 0} \frac{\Delta L(s)}{\Delta \theta} = \lim_{\Delta \theta \to 0} \left[ \frac{L \{ \theta(\kappa) + H(\kappa - \sigma) \Delta \theta \} - L \{ \theta(\kappa) \}}{\Delta \theta} \right]$$

$(6.6a)$

$$\Phi_{M\theta} \{ \theta(\kappa); s, \sigma \} = \lim_{\Delta \theta \to 0} \frac{\Delta M(s)}{\Delta \theta} = \lim_{\Delta \theta \to 0} \left[ \frac{M \{ \theta(\kappa) + H(\kappa - \sigma) \Delta \theta \} - M \{ \theta(\kappa) \}}{\Delta \theta} \right]$$

$(6.6b)$
where the input incremental $\Delta \theta$ is applied at time $s=\sigma$ and $H$ denotes the Heaviside step function.

6.3.2 Nonlinear Convolution Scheme Involving Higher-Order Kernels

The concept of nonlinear unit-step response function offers a general framework to simulate nonlinear aerodynamics, however, its translation to applications is intractable. One possible approach to obtain a practical nonlinear analysis framework for practical problems is to utilize a "peeling-an-onion" type approach, in which the nonlinear effects of wind-bridge interactions are extracted from a nonlinear unit-step response function using a "step by step" procedure.

Obviously, if the motion-induced effects are linear and time-invariant, $\Phi_{L\theta}(\theta(\kappa); s, \sigma)$ and $\Phi_{M\theta}(\theta(\kappa); s, \sigma)$ reduce to $\Phi_{L\theta}(s-\sigma)$ and $\Phi_{M\theta}(s-\sigma)$, respectively, which are characterized by one time scale $(s-\sigma)$. Hence, the nonlinear unit-step response functions could be expressed as

$$
\dot{\Phi}_{L\theta}(\theta(\kappa); s, \sigma) = \left[ \Phi_{L\theta}(s-\sigma) + \int_0^s \dot{\Phi}_{L\theta}^{\text{non}}(\theta(\kappa); s, \sigma) \dot{\theta}(\kappa) d\kappa \right] 
$$

(6.7a)

$$
\Phi_{M\theta}(\theta(\kappa); s, \sigma) = \left[ \Phi_{M\theta}(s-\sigma) + \int_0^s \dot{\Phi}_{M\theta}^{\text{non}}(\theta(\kappa); s, \sigma) \dot{\theta}(\kappa) d\kappa \right] 
$$

(6.7b)

where it is assumed that $\theta(0)=0$ to provide a parsimonious model. It should be noted that there are infinite time scales in nonlinear unit-step response functions given in Equation 6.7 since $\kappa$ may attain an arbitrary value in the time interval $[0, s]$. Suppose one extracts a part of the time-invariant nonlinear coupling effects due to two time scales from the remaining nonlinear unit-step response functions $\dot{\Phi}_{L\theta}^{\text{non}}(\theta(\kappa); s, \sigma)$ and $\dot{\Phi}_{M\theta}^{\text{non}}(\theta(\kappa); s, \sigma)$ by
adding a unit-step function input at time $\kappa_1$, a possible form of the nonlinear functional
\[ \tilde{\Phi}_{L\theta}^{\text{non}} \{ \theta(\kappa); s, \sigma \} \] and
\[ \tilde{\Phi}_{M\theta}^{\text{non}} \{ \theta(\kappa); s, \sigma \} \] with respect to the input $\theta$, similar to the expressions of Equation 6.7, could be

\[ \tilde{\Phi}_{L\theta}^{\text{non}} \{ \theta(\kappa); s, \sigma \} = \left[ \Phi_{L\theta_1} (s-\sigma, s-\kappa_1) + \int_0^s \tilde{\Phi}_{L\theta_2}^{\text{non}} \{ \theta(\kappa_{-1}); s, \sigma \} \theta(\kappa_2) d\kappa_2 \right] \quad (6.8a) \]

\[ \tilde{\Phi}_{M\theta}^{\text{non}} \{ \theta(\kappa); s, \sigma \} = \left[ \Phi_{M\theta_1} (s-\sigma, s-\kappa_1) + \int_0^s \tilde{\Phi}_{M\theta_2}^{\text{non}} \{ \theta(\kappa_{-1}); s, \sigma \} \theta(\kappa_2) d\kappa_2 \right] \quad (6.8b) \]

where the subscript "-1" denotes step 1 and indicates that there are some limitations added to the nonlinear unit-step response functions and the variable $\kappa$ due to the extraction of information from the previous nonlinear unit-step response functions. Furthermore, one could extracts a part of the time-invariant nonlinear coupling effects due to three time scales from the remaining nonlinear unit-step response functions
\[ \tilde{\Phi}_{L\theta_1}^{\text{non}} \{ \theta(\kappa_{-1}); s, \sigma \} \] and \[ \tilde{\Phi}_{M\theta_1}^{\text{non}} \{ \theta(\kappa_{-1}); s, \sigma \} \] by adding a unit-step function input at time $\kappa_2$, hence, a possible form of the nonlinear functional
\[ \tilde{\Phi}_{L\theta_1}^{\text{non}} \{ \theta(\kappa_{-1}); s, \sigma \} \] and
\[ \tilde{\Phi}_{M\theta_1}^{\text{non}} \{ \theta(\kappa_{-1}); s, \sigma \} \] with respect to the input $\theta$ could be

\[ \tilde{\Phi}_{L\theta_1}^{\text{non}} \{ \theta(\kappa_{-1}); s, \sigma \} = \left[ \Phi_{L\theta_1} (s-\sigma, s-\kappa_1, s-\kappa_2) + \int_0^s \tilde{\Phi}_{L\theta_2}^{\text{non}} \{ \theta(\kappa_{-2}); s, \sigma \} \theta(\kappa_3) d\kappa_3 \right] \quad (6.9a) \]

\[ \tilde{\Phi}_{M\theta_1}^{\text{non}} \{ \theta(\kappa_{-1}); s, \sigma \} = \left[ \Phi_{M\theta_1} (s-\sigma, s-\kappa_1, s-\kappa_2) + \int_0^s \tilde{\Phi}_{M\theta_2}^{\text{non}} \{ \theta(\kappa_{-2}); s, \sigma \} \theta(\kappa_3) d\kappa_3 \right] \quad (6.9b) \]

By analogy, one can obtain
\[ \Phi_{\text{non}}^{\text{L}} \{ \theta(\kappa_i); \ s, \ \sigma \} = \left[ \Phi_{\text{L}_0} (s - \sigma, \ s - \kappa_i, \ ..., \ s - \kappa_{i+1}) \right. \\
\left. + \int_0^s \Phi_{\text{non}}^{\text{L}_i} \{ \theta(\kappa_{(i-1)}); \ s, \ \sigma \} \theta' (\kappa_{i+2}) d\kappa_{i+2} \right] \] (6.10a)

\[ \Phi_{\text{non}}^{\text{M}} \{ \theta(\kappa_i); \ s, \ \sigma \} = \left[ \Phi_{\text{M}_0} (s - \sigma, \ s - \kappa_i, \ ..., \ s - \kappa_{i+1}) \right. \\
\left. + \int_0^s \Phi_{\text{non}}^{\text{M}_i} \{ \theta(\kappa_{(i-1)}); \ s, \ \sigma \} \theta' (\kappa_{i+2}) d\kappa_{i+2} \right] \] (6.10b)

where \( i = 1, ..., +\infty \). Substituting Equations 6.7, 6.8 and 6.10 into Equation 6.5, one can obtain a possible form of the nonlinear lift force and torsional moment as

\[
L(s) = -\frac{1}{2} \rho U^2 b \left[ \frac{dC_L}{d\theta} \left( \int_0^s \Phi_{\text{L}_0} (s - \sigma, \theta' (\sigma)) d\sigma \right) \\
+ \int_0^s \int_0^s \Phi_{\text{L}_2} (s - \sigma, \ s - \kappa_1) \theta' (\sigma) \theta' (\kappa_1) d\sigma d\kappa_1 \\
+ \int_0^s \int_0^s \int_0^s \Phi_{\text{L}_3} (s - \sigma, \ s - \kappa_1, \ s - \kappa_2) \theta' (\sigma) \theta' (\kappa_1) \theta' (\kappa_2) d\sigma d\kappa_1 d\kappa_2 \\
+ \cdots \right] \] (6.11a)

\[
M(s) = \frac{1}{2} \rho U^2 b^2 \left[ \frac{dC_M}{d\theta} \left( \int_0^s \Phi_{\text{M}_0} (s - \sigma, \theta' (\sigma)) d\sigma \right) \\
+ \int_0^s \int_0^s \Phi_{\text{M}_2} (s - \sigma, \ s - \kappa_1) \theta' (\sigma) \theta' (\kappa_1) d\sigma d\kappa_1 \\
+ \int_0^s \int_0^s \int_0^s \Phi_{\text{M}_3} (s - \sigma, \ s - \kappa_1, \ s - \kappa_2) \theta' (\sigma) \theta' (\kappa_1) \theta' (\kappa_2) d\sigma d\kappa_1 d\kappa_2 \\
+ \cdots \right] \] (6.11b)

A typical "peeling-an-onion" procedure to analyze a time-invariant nonlinear aerodynamic system is summarized in Figure 6.3.

It is interesting to note that based on the relationship between the unit-step response function \( \Phi(s) \) and unit-impulse response function \( I(s) = \Phi(0)\delta(s) + \Phi(s) \) (where \( \delta(s) \) is a Dirac delta function) the nonlinear convolution integrals of Equation 6.11 could be represented by a Volterra-type formalism.
Figure 6.3. A schematic of the proposed "peeling-an-onion" type approach

\[
L(s) = -\frac{1}{2} \rho U^2 2b \left[ \int_0^s I_{L0}(s-\sigma)\theta(\sigma)d\sigma + \int_0^s \int_0^s I_{L0}(s-\sigma, s-\kappa_1)\theta(\sigma)\theta(\kappa_1)d\sigma d\kappa_1 + \int_0^s \int_0^s \int_0^s I_{M0}(s-\sigma, s-\kappa_1, s-\kappa_2)\theta(\sigma)\theta(\kappa_1)\theta(\kappa_2)d\sigma d\kappa_1 d\kappa_2 + \ldots \right]
\]

\[
M(s) = \frac{1}{2} \rho U^2 2b^2 \left[ \int_0^s \Phi_{M0}(s-\sigma)\theta'(\sigma)d\sigma + \int_0^s \int_0^s \Phi_{M0}(s-\sigma, s-\kappa_1)\theta'(\sigma)\theta'(\kappa_1)d\sigma d\kappa_1 + \int_0^s \int_0^s \int_0^s \Phi_{M0}(s-\sigma, s-\kappa_1, s-\kappa_2)\theta'(\sigma)\theta'(\kappa_1)\theta'(\kappa_2)d\sigma d\kappa_1 d\kappa_2 + \ldots \right]
\]

(6.12a)

(6.12b)
where the steady-state information is included in the unit-impulse response functions. The abovementioned "peeling-an-onion" procedure is enlightened by the "successive substitution" procedure, which is utilized to derive the Volterra equation of the second kind (Tricomi 1957). The mathematical properties of the Volterra series are discussed comprehensively by Volterra (Volterra 1959). The basic premise of the Volterra theory of nonlinear systems is that a large class of nonlinear systems can be approximated as a sum of multidimensional convolution integrals of increasing order (Volterra 1959). On the other hand, it should be noted that the "peeling-an-onion" procedure, utilized for deriving Equation 6.12, suggests that there may be some information missed at each step in this procedure. Actually, it has been demonstrated that the Volterra series is a subset of the nonlinear functional expansion of the nonlinear unit-step response functions (Tobak and Chapman 1985). Besides, in the case of the multiple input variables, the principle of superposition is not valid in the nonlinear situation, hence, cross terms which involve coupling effects of different input variables should be added to accurately simulate nonlinear aerodynamics.

6.4 Kernel Identification Scheme

Wind-bridge interactions have a fading memory, which suggests that the wind-induced forces on a bridge deck are not dependent on the infinite past inputs. Boyd (1985) demonstrated that the finite-order truncated Volterra series will converge in the simulation of a fading memory nonlinear system. Hence, the nonlinear bridge aerodynamics could be simulated using finite terms of the Volterra series, which indicates that finite steps of the "peeling-an-onion" procedure may be carried out. In this
study, the second-order truncated Volterra series is applied to simulate nonlinear bridge aerodynamics as the first step beyond conventional linear simulation. To this end, the motion-induced nonlinear lift force and torsional moment could be expressed as

\[
L(s) = -\frac{1}{2} \rho U^2 2b \left[ \int_0^s I_{Lo}(s-\sigma)\theta(s-\sigma)d\sigma \right. \\
+ \left. \int_0^s \int_0^s I_{Lo}(s-\sigma, s-\kappa)\theta(s-\kappa_1)d\sigma d\kappa_1 \right]
\]

(6.13a)

\[
M(s) = \frac{1}{2} \rho U^2 2b^2 \left[ \int_0^s \Phi_{M\theta}(s-\sigma)\theta'(\sigma)d\sigma \right. \\
+ \left. \int_0^s \int_0^s \Phi_{M\theta}(s-\sigma, s-\kappa)\theta'(s-\kappa_1)d\sigma d\kappa_1 \right]
\]

(6.13b)

In order to simplify the presentation of the proposed kernel identification scheme, a typical single-input single-output (SISO) second-order Volterra system is expressed as (Rugh 1981; Carassale and Kareem 2010)

\[
y(t) = \int_0^t h_1(t-\tau)x(\tau)d\tau + \int_0^t \int_0^t h_2(t-\tau_1, t-\tau_2)x(\tau_1)x(\tau_2)d\tau_1d\tau_2
\]

(6.14)

where \(x(t)\) and \(y(t)\) are input and output of the system, respectively; \(h_1\) represents the first-order (linear) kernel which describes the linear behavior of the system; \(h_2\) the second-order (nonlinear) kernel which represents the nonlinear behavior existing in the system. There are various approaches to identify the Volterra kernels. In this study, the identification of Volterra kernels with impulse function as inputs is introduced, which is based on the earlier work by Rugh (1981).

Suppose two input signals are applied to the second-order terms of the Volterra series, the nonlinear response is represented as below
\[
y_2[x_1(t) + x_2(t)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_2(\tau_1, \tau_2)[x_1(t - \tau_1)
+ x_2(t - \tau_1)]dx_1(t - \tau_1)d\tau_1d\tau_2
\]
\[
= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_2(\tau_1, \tau_2)x_1(t - \tau_1)x_1(t - \tau_2)d\tau_1d\tau_2
+ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_2(\tau_1, \tau_2)x_1(t - \tau_1)x_2(t - \tau_2)d\tau_1d\tau_2
+ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_2(\tau_1, \tau_2)x_2(t - \tau_1)x_1(t - \tau_2)d\tau_1d\tau_2
+ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_2(\tau_1, \tau_2)x_2(t - \tau_1)x_2(t - \tau_2)d\tau_1d\tau_2
\]
\[
= y_2[x_1(t)] + y_2[x_2(t)]
+ 2\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_2(\tau_1, \tau_2)x_1(t - \tau_1)x_2(t - \tau_2)d\tau_1d\tau_2
\]

(6.15)

where the subscript “2” with the output \(y(t)\) indicates the response resulting from the second-order of nonlinear origin. The symmetric property of the kernels is used here to obtain the final result. Suppose the input signals are impulse functions, i.e.,

\[
x_e(t) = \beta_e\delta(t - \tau_e)
\]

(6.16)

where \(e\) equals to 1 or 2; \(\beta_1\) and \(\beta_2\) are selected constants. Utilizing the property of the impulse function

\[
\int_{-\infty}^{\infty} h_1(\tau)\delta(t - \tau)d\tau_1 = h_1(t - \tau_1)
\]

(6.17)

\[
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_2(\tau_1, \tau_2)\delta(t - \tau_1)\delta(t - \tau_2)d\tau_1d\tau_2 = h_2(t - \tau_1, t - \tau_2)
\]

(6.18)

Equation 6.15 is employed to calculate the second-order kernel, which could be expressed as

\[
h_2(t - \tau_1, t - \tau_2) = \frac{1}{2\beta_1\beta_2}\{y_2[\beta_1\delta(t - \tau_1) + \beta_2\delta(t - \tau_2)]
- y_2[\beta_1\delta(t - \tau_1)] - y_2[\beta_2\delta(t - \tau_2)]\}
\]

(6.19)
For an inhomogeneous system, the responses resulting from the first-order and higher-order terms have to be considered simultaneously. There is a typical linear relationship involving the first-order kernel with the impulse function inputs, given by

\[ y_i[x_1(t) + x_2(t)] - y_i[x_1(t)] - y_i[x_2(t)] = 0 \] (6.20)

As a result, Equation 6.19 could be presented as a more practical form

\[
h_2(t - \tau_1, t - \tau_2) = \frac{1}{2\beta_1\beta_2} \left\{ y[\beta_1\delta(t - \tau_1) + \beta_2\delta(t - \tau_2)] \\
- y[\beta_1\delta(t - \tau_1)] - y[\beta_2\delta(t - \tau_2)] \right\} \] (6.21)

Using the property of power series shared by the Volterra series, the first-order kernel could be obtained with an interpolation scheme. Suppose a new impulse function \( \alpha\delta(t-\tau) \), where \( \alpha \) is not equal to unit, is applied to the system, the corresponding response is

\[ y[\alpha\delta(t-\tau)] = \alpha h_1(t-\tau) + \alpha^2 h_2(t-\tau, t-\tau) \] (6.22)

Hence, a system of equations is obtained as

\[
\begin{bmatrix}
  y[\delta(t-\tau)] \\
y[\alpha\delta(t-\tau)]
\end{bmatrix} = \begin{bmatrix} 1 \quad 1 \\ \alpha \quad \alpha^2 \end{bmatrix} \begin{bmatrix} h_1(t-\tau) \\
h_2(t-\tau, t-\tau) \end{bmatrix} \] (6.23)

Solving Equation 6.23 for kernels gives

\[ h_1(t-\tau) = \frac{1}{\alpha^2 - \alpha} \left( \alpha^2 y[\delta(t-\tau)] - y[\alpha\delta(t-\tau)] \right) \] (6.24)

\[ h_2(t-\tau, t-\tau) = \frac{1}{\alpha - \alpha^2} \left( \alpha y[\delta(t-\tau)] - y[\alpha\delta(t-\tau)] \right) \] (6.25)
With the Equations 6.21 and 6.24, for the case of $\beta_1=1$, $\beta_2=1$ and $\alpha=2$, a popular formulation used for the identification of first- and second-order kernels is obtained as following (Rugh 1981)

\[
h_1(t - \tau_1) = 2y[\delta(t - \tau_1)] - \frac{1}{2}y[2\delta(t - \tau_1)] \tag{6.26}
\]

\[
h_2(t - \tau_1, t - \tau_2) = \frac{1}{2}[y[\delta(t - \tau_1) + \delta(t - \tau_2)] - y[\delta(t - \tau_1)] - y[\delta(t - \tau_2)]] \tag{6.27}
\]

It should be noted that a nonlinear system usually features input-amplitude dependence if it is simulated utilizing a truncated nonlinear model. Hence, in the kernel identification of a truncated Volterra series based model, the selection of the parameters $\alpha$, $\beta_1$ and $\beta_2$ is significantly important and is dependent on the modeled nonlinear system. This will be investigated in the following section by a numerical example. Besides, Equation 6.24 indicates that the first-order kernel may be also input-amplitude dependent and could be utilized to measure the degree of discrepancy to the linear superposition principle which is only valid for a linear system. The input-amplitude dependent property of the first-order kernel also suggests that the first-order kernel for a nonlinear system may be different from the linear unit-impulse response function describing purely a linear system, while the nontrivial values of the higher-order kernels directly present the nonlinearity of a system.
6.5 Numerical Example

6.5.1 Mathematical Model

Since the catastrophic failure of the Tacoma Narrows Bridge, the aerodynamic behavior of long-span bridges has been drawing remarkable attention in the engineering and science fields. Pugsley (1949) made a prophetic suggestion that the investigation of wind-bridge interactions might rely on the experimental approach. Following this concept, Scanlan and his coworkers (e.g., Scanlan Tomko 1971) set up a so-called semi-empirical analysis framework based on the aerodynamic coefficients which were measured from wind-tunnel tests. An alternate approach, as considered in several studies, models the wind-induced issues by investigating the interactions between the convected vortex and bridge deck (e.g., Larsen 2000). However, the semi-analytical schemes employed in such analysis fail to take into account nonlinear effects. CFD based approaches show promise to model nonlinear bridge aerodynamics, but they have their own limitations at this juncture stemming from the lack of robust turbulence models for engineering applications and extensive computational demands. On the other hand, some attempts have been made to look for possible mathematical expressions, involving nonlinear considerations, for wind-bridge interactions (e.g., McKenna and Tuama 2001).

In this study, a nonlinear analytical model is introduced to describe the simplified nonlinear wind-bridge interactions, based on which the kernel identification scheme with impulse function inputs, the fidelity of the simulation based on nonlinear convolution integrals and the amplitude dependence of kernels are discussed. The proposed nonlinear model is not very sensitive to the initial conditions as compared to the model developed by McKenna and Tuama (2001), which is given here
\[
I_1 \frac{d^2 y_1}{dt^2} + c_1 \frac{dy_1}{dt} + k(y_1 + \varepsilon y_2 + \varepsilon_1 y_1^2) + dy_1 y_2 + \zeta_1 \cos(y_1) (\gamma/\eta) \left[ \exp(\eta(y_2 - \kappa \sin(y_1))) - \exp(\eta(y_2 + \kappa \sin(y_1))) \right] = x_1(t)
\]

(6.28a)

\[
m_2 \frac{d^2 y_2}{dt^2} + c_2 \frac{dy_2}{dt} + k(\varepsilon y_1 + y_2 + \varepsilon_2 y_2^2) + dy_1 y_2 + \zeta_2 (\gamma/\eta) \left[ \exp(\eta(y_2 - \kappa \sin(y_1))) + \exp(\eta(y_2 + \kappa \sin(y_1))) \right] = \lambda x_1(t)
\]

(6.28b)

where \(y_1\) and \(y_2\) represent the torsional and vertical displacement, respectively; \(I_1\) and \(m_2\) are moment of inertia and mass of the bridge deck, respectively; \(c_1\) and \(c_2\) are the viscosity damping in the torsional and vertical degrees of freedom, respectively; \(x_1(t)\) is motion input (namely torsional motion of the bridge deck) while \(x_2(t)\) is gust input (namely vertical wind fluctuations); \(k, \varepsilon, \varepsilon_1, d, \zeta_1, \zeta_2, \gamma, \eta, \kappa\) and \(\lambda\) are physical constants whose values are based on the bridge structural and aerodynamic properties. Specifically, these values are tuned to mimic amplitudes and frequencies of large torsional and vertical oscillations as noted in the original report of the Tacoma Narrows disaster (Ammann et al. 1941). In order to focus on a single-input multi-output (SIMO) case, \(x_1(t)\) and \(x_2(t)\) are not set equal to nonzero value concurrently. It should be noted that the linear and nonlinear unit-impulse response functions are bridge deck responses (displacements) instead of the motion-induced or gust-induced aerodynamic forces, hence, the identified kernels herein actually integrated the contributions from both the aerodynamics and structural dynamics perspectives.

6.5.2 Simulation Results

Suppose the motion input (unit-impulse function of the angle of attack) is applied to this nonlinear wind-bridge interaction system. The first-order, the diagonal term of second-
order and the second-order motion-induced kernels are shown in Figure 6.4(a) and Figure 6.4(b) for a specific group of parameters ($\Delta t=0.02s$). The superscripts $i_1i_2$ represent the kernel of output $i_1$ under input $i_2$. The identified kernel is called as direct-kernel if $i_1$ is equal to $i_2$ or cross-kernel if $i_1$ is not equal to $i_2$. As shown in the figure, the motion-induced second-order kernel is several orders of magnitude smaller than that of the motion-induced first-order kernel. Besides, both motion-induced first- and second-order impulse response functions decay rapidly with time, which suggests computational efficiency of the Volterra series based reduced-order modeling for the motion-induced bridge aerodynamics.

Figure 6.4. Identified motion-induced kernels of the nonlinear wind-bridge interaction system: (a) motion-induced direct-kernels; (b) motion-induced cross-kernels
The torsional harmonic input signal (simulation of a torsional motion input) is applied to verify the identified motion-induced impulse response functions and to show the necessity of using higher-order motion-induced impulse response functions for the simulation of motion-induced responses in this nonlinear system. Figure 6.5(a) presents the time history of the torsional harmonic input; Figure 6.5(b) and Figure 6.5(c) show the corresponding torsional and vertical responses, respectively. The linear approximation response in the figures indicates that the response is calculated only involving the first-order kernel; the nonlinear approximation denotes the response is obtained by utilizing both the first- and second-order kernels; the exact solution represents the response is calculated using the fourth-order Runge-Kutta scheme.

As presented in Figure 6.5(b), there is notable difference in the torsional output $y_1$ (amplitude) between the linear approximation response and exact solution, while the nonlinear approximated response collapses to the exact solution. This observation suggests that the second-order kernel is necessary and sufficient to simulate the nonlinear torsional-motion-induced torsional response. On the other hand, in the case of vertical output $y_2$, as shown in Figure 6.5(c), the linear response shows notable discrepancy (both amplitude and phase) as compared to the exact solution. Although the inclusion of the second-order kernel significantly enhances the simulation accuracy of the vertical response (both amplitude and phase) for this nonlinear wind-bridge interaction system, there remains some discrepancy between the nonlinear approximation and exact solution. This observation indicates that the second-order kernel is necessary but may not be sufficient to simulate the nonlinear torsional-motion-induced vertical response of a bridge sensitive to flutter conditions.
Figure 6.5. Motion input and corresponding outputs of the nonlinear wind-bridge interaction system: (a) harmonic torsional motion input $x_1$; (b) torsional output $y_1$; (c) vertical output $y_2$

Suppose the gust input (unit-impulse function of vertical fluctuations) is applied to this nonlinear wind-bridge interaction system. The first-order, the diagonal term of second-order and the second-order gust-induced kernels are shown in Figure 6.6(a) and Figure 6.6(b) for the same group of parameters (time interval $\Delta t=0.02s$). As shown in the figure, the gust-induced second-order kernel is several orders of magnitude smaller than that of the gust-induced first-order kernel. Besides, both gust-induced first- and second-
order impulse response functions decay significantly rapidly with time, which suggests computational efficiency of the Volterra series based reduced-order modeling for gust-induced bridge aerodynamics. It should be noted that, although the gust-induced linear cross-kernel $h_{12}^{12}$ is close to the motion-induced linear cross-kernel $h_{21}^{21}$, they are not exactly the same.

Figure 6.6. Identified gust-induced kernels of the nonlinear wind-bridge interaction system: (a) gust-induced cross-kernels; (b) gust-induced direct-kernels

The vertical random input signal (simulation of a vertical gust input) is applied to verify the identified gust-induced impulse response functions and to identify the need for using higher-order gust-induced impulse response functions for the simulation of gust-induced responses in this nonlinear system. Figure 6.7(a) presents the time history of
vertical random input; Figure 6.7(b) and Figure 6.7(c) show the corresponding torsional and vertical responses, respectively.

Figure 6.7. Gust input and corresponding outputs of the nonlinear wind-bridge interaction system: (a) white-noise vertical gust input $x_2$; (b) torsional output $y_1$; (c) vertical output $y_2$

As presented in Figure 6.7(b), there is no significant difference among the linear approximation, nonlinear approximation and the exact solution, which indicates that the
torsional output $y_1$ may be appropriately simulated using a linear model. On the other hand, in the case of vertical output $y_2$, as shown in Figure 6.7(c), the linear response shows notable discrepancy as compared to the exact solution while there is insignificant difference between the nonlinear approximation response and exact solution. This suggests that the second-order kernel is necessary and sufficient to simulate the nonlinear vertical-gust-induced-vertical response.

6.5.3 Discussion on Amplitude Dependence

As mentioned in the preceding discussion, due to possible contributions of the truncated terms of Volterra series to nonlinear bridge aerodynamics, the identified kernels may be amplitude dependent. The amplitude dependence of the first-order kernels is investigated using Equation 6.24, where various values of $\alpha$ are selected. Figures 6.8(a) and 6.8(b) present the motion-induced and gust-induced first-order kernels in torsional and vertical degrees of freedom, respectively. As indicated in the figures, the amplitude dependence of the first-order kernels are negligible in this study.

The amplitude dependence of the second-order kernels is investigated using Equation 6.21, where various values of $\beta_1$ and $\beta_2$ are selected. Figures 6.9(a) and 6.9(b) present the motion-induced and gust-induced second-order kernels (diagonal values) in torsional and vertical degrees of freedom, respectively. As noted from the figures, there are substantial changes among the identified second-order kernels with different values of $\beta_1$ and $\beta_2$. This indicates that the amplitude dependence of the second-order kernels are significant. It should be noted that, even though both the linear and nonlinear approximations present small discrepancy as compared to the exact solution, as in the cases of the motion-induced torsional response and the gust-induced vertical response,
Figure 6.8. Amplitude dependence of first-order kernels: (a) motion-induced first-order kernels; (b) gust-induced first-order kernels.
Figure 6.9. Amplitude dependence of second-order kernels: (a) motion-induced second-order kernels; (b) gust-induced second-order kernels
the amplitude dependence of the corresponding second-order kernels are very notable. The weak nonlinearity in bridge aerodynamics of this specific numerical example, where the second-order kernels are several orders of magnitude smaller than that of the first-order kernels, may have been the reason for this observation.

Figure 6.10 presents the errors of the simulated nonlinear motion-induced responses with the kernels identified using various values of $\beta_1$ and $\beta_2$, where the error is defined as

$$error = \sum \frac{(y_{exact}[n] - y_{appr}[n])^2}{(y_{exact}[n])^2}$$

(6.29)

As indicated in Figure 6.10, the errors of the nonlinear motion-induced torsional and vertical responses are monotonically increasing functions with the values of $\beta_1$ and $\beta_2$. Figure 6.11 presents the errors of the simulated nonlinear gust-induced responses with the kernels identified using various values of $\beta_1$ and $\beta_2$. As indicated in Figure 6.11, the errors of the nonlinear gust-induced torsional and vertical responses are smallest for $\beta_1$ equal to $\beta_2$. It should be noted that the smallest values of $\beta_1$ and $\beta_2$ result in the largest errors in this case, which may indicate that the magnitudes of the impulse function inputs are insufficient to excite the nonlinear bridge aerodynamics. Since the amplitude dependence of the kernels and corresponding simulation results are case sensitive, additional examples are needed for further delineation of these observations.
6.6 Concluding Remarks

The nonlinear simulation of bridge aerodynamics is comprehensively investigated within the framework of a nonlinear convolution scheme. A group of parameters, namely the translatory motion $\dot{h}$, angle of pitch (or angle of attack) $\theta$ and angular velocity $\dot{\theta}$ for the motion-induced effects and the fluctuation component of wind in each degree of freedom for the gust-induced effects are used as input variables for the convolution integrals. The
flow separation around the bridge deck is a main physical basis for treating $h$ and $\theta$ as two independent variables in bridge aerodynamics. The linear convolution scheme concerning the first-order kernels for linear analysis of bridge aerodynamics is reviewed. Based on the concept of nonlinear indicial response function, the nonlinear convolution scheme involving higher-order kernels for nonlinear analysis of bridge aerodynamics is formulated with a "peeling-an-onion" type procedure. The nonlinear convolution scheme is represented utilizing a Volterra-type formalism, hence, the convergence of its truncated expression is guaranteed. A comprehensive kernel identification scheme is developed based on the impulse function input. This also facilitates the evaluation of amplitude dependency of kernels.

A numerical example of a long-span suspension bridge with vertical and torsional degrees of freedom is investigated to verify the fidelity of simulation based on the proposed nonlinear convolution scheme for nonlinear bridge aerodynamics. The mathematical model of this numerical example is designed to mimic amplitudes and frequencies of the large torsional and vertical oscillations of the Tacoma Narrows Bridge near collapse. It is shown that accurate simulations of the motion-induced torsional (direct), motion-induced vertical (cross) and gust-induced vertical (direct) responses need to include the nonlinear convolution scheme, while the gust-induced torsional (cross) response could be simulated adequately using the linear convolution scheme. The amplitude dependency of kernels is investigated based on the proposed comprehensive kernel identification scheme. Linear kernels show negligible amplitude dependence, whereas nonlinear kernels exhibit significant dependence on the amplitude.
CHAPTER 7:
IDENTIFICATION OF VOLterra MODELS: A PHENOMENOLOGICAL APPROACH

To assess the fidelity of Volterra models for simulating nonlinear bluff-body aerodynamics, the model parameters will be derived from an existing nonlinear phenomenological model of a vortex-induced vibration (VIV) system. Once the Volterra system identification procedure has been fine tuned and vetted, it could be conveniently applied to CFD-based and wind-tunnel data.

A brief overview of VIV of bridge decks is presented highlighting special VIV features concerning bridge decks, where a popular VIV model (Van de Pol type model) for bridge decks is examined in detail. Then, a truncated Volterra series based nonlinear oscillator is introduced to model the VIV system. Typical features of VIV such as the "limit cycle oscillation" (LCO), “frequency shift”, "hysteresis" and "beat phenomenon" are parsimoniously and accurately captured in the proposed nonlinear model. As a functional expansion of a nonlinear system, Volterra series is convenient for estimating the linear and nonlinear contributions to VIV. It is demonstrated that the relative contribution of nonlinear effects in VIV is around 50 percent of the total response for a range of bridge cross-sections. The efficacy of Volterra series as a reduced order model (ROM) in capturing aerodynamic nonlinearities eliminates the need for reliance on
conventional phenomenological models as it promises to offer a unified framework for nonlinear wind effects on long span bridges, e.g., VIV, buffeting and flutter.

7.1 Background

For bluff bodies with separated flows vortices are shed with a resulting wake that impacts the body with some periodicity. The fluctuating pressures around the bluff body generated by the periodic vortex shedding result in the cross-wind force with a dominant frequency given by the Strouhal number $S_t = fD/U$, where $f$ is the dominant frequency of vortex shedding; $D$ is the projected area of the structure per unit length; $U$ is the oncoming flow velocity. As the frequency of vortex-induced forces approaches a frequency of the bluff body, “lock-in” sets in with relatively large response. Due to nonlinear fluid-structure interactions, this vortex-induced vibration (VIV) exhibits limit cycle oscillations (LCOs). Though VIV response does not always result in catastrophic events, it seriously impacts the fatigue life and in the case of tall stacks and super tall buildings a loss of desired functionality.

Since the pioneer work of Strouhal (1878) and Rayleigh (1896), a large number of studies have been carried out on the fundamental understanding of the VIV mechanism. Among them, von Kármán and Rubach (1912) identified the famous “vortex streets” phenomenon with the Kármán constant $K = h/a$, where $h$ is the transverse distance between the two rows of vortices and $a$ is the longitudinal space period. Gerrard (1966) on the other hand, revealed the underlying physical mechanisms of vortex formation in the wake of bluff bodies. Central to Gerrard's (1966) study was the formation region in which the irrotational flow from outside the wake crosses the wake axis. This flow is
divided into three parts. One part is entrained by the growing vortex on the opposite side; the second part is entrained by the shear layer upstream of the vortex; the third part reverses to the interior of the formation region. More recent developments on this topic are well documented in several comprehensive reviews (e.g., Sarpkaya 1979; Bearman 1984; Williamson 1996; Sarpkaya 2004; Williamson and Govardhan 2004).

It should be noted that the focus of most studies has been on the VIV of circular cylinders. On the one hand, the findings reveal some fundamental mechanisms of VIV as the circular cylinder is an important and representative bluff body. While on the other hand, VIV of bridge decks, as bluff bodies characterized by a significant afterbody shows substantial departure from features noted for circular cylinders. For example, there are fixed separation points for bridge decks instead of moving separation points for circular cylinders, which spatially shift as Reynolds number changes with attendant modification to shedding frequency and its strength. The lifting surface resulting from a long afterbody also introduces torsional loads on bridge decks due to wake dynamics.

During past several decades, wind-tunnel experiments have underscored the difference in the VIV features between circular cylinders and long afterbody bluff bridge decks. For example, while the emergence of multiple “lock-in” regions is attributed to the “frequency demultiplication” phenomenon (Van der Pol 1927) by most of the studies on circular cylinders, Shiraishi and Matsumoto (1983) suggested a different scenario regarding bluff bodies with large aspect ratio. They postulated that the smaller-peak oscillation at lower wind velocity region resulted from the arrival of separated vortices from the leading edge to the trailing edge after \( n \) cycles of the heaving motion (where \( n \) is a natural number) or \((2n-1)/2\) cycles of the torsional motion. In addition, there are notable
differences in the sensitivity of each section to Reynolds numbers (e.g., Larsen et al. 2004; Sarpkaya 1978; Griffin and Koopmann 1977). It has also been observed that in the case of bridge decks a decrease in the cross-sectional aspect ratio (length to width) enhances vortex-induced effects (Nakamura and Mizota 1975); whereas, no such comparable possibility exists for changes in the cross-sectional aspect ratio for circular cross-sections. Besides, the decrease in aspect ratio between the spanwise length to the size of the cross-section weakens the vortex-induced forces (Fail et al. 1959; Wootton 1969).

7.2 Semiempirical Models for VIV of Bridge Decks

Recent full-scale observations suggest that many bridge decks are prone to VIV in a relatively low wind velocity range for winds approaching almost perpendicular to the bridge axis with low turbulence intensity (e.g., Smith 1980; Kumarasena et al. 1989; 1991; Owen et al. 1996; Ronaldo and Michèle 2000; Larsen et al. 2000; Frandsen 2001; Fujino and Yoshida 2002; Li et al. 2011). This poses an increasing demand on advance simulation capability for VIV analysis of bridge decks.

7.2.1 Nonlinear Semiempirical Models

The concept of wake oscillator has been utilized to model vortex-induced effects since the early work of Bishop and Hassan (1964a; 1964b). In this scheme, the VIV is simulated by a system of two coupled equations, in which the structural motion is modeled by a second-order linear mechanical oscillator excited by the vortex-induced force represented by a second-order nonlinear wake oscillator (Vol del Pol type oscillator) coupled with the structural motion (Hartlen and Currie 1970). This phenomenological model over the years has been further developed by many researchers
(e.g., Skop and Griffin 1973; Landl 1975; Iwan and Blevins 1974; Dowell 1981; Billah 1989). These models rely on several experimentally derived parameters needed to characterize the system and to describe coupling between the linear mechanical and the nonlinear wake oscillators. The experimental procedures involved to identify these parameters need to be sophisticated and usually have considerable element of uncertainty (e.g., Sarpkaya 1979).

To simplify the parameter identification procedure, Scanlan (1981) proposed an explicit function for the vortex-induced force for application to VIV of bridge decks. In other words, the VIV of bridge decks is simulated with a single ordinary differential equation, where the vortex-induced force is explicitly expressed as a sum of the self-excited term (induced by structural motion) and the forced term (induced by vortex shedding). The Scanlan’s proposal is given by

\[
m(\ddot{y} + 2\zeta\omega_0 \dot{y} + \omega_0^2 y) = \frac{1}{2} \rho U^2 (2D) \left[ Y_1(K) \left( 1 - \epsilon \frac{y^2}{D^2} \right) \frac{\dot{y}}{U} \right.
\]
\[
\left. + Y_2(K) \frac{y}{D} + \frac{1}{2} C_L(K) \sin(\omega t + \theta) \right]\tag{7.1}
\]

where \(m\) is the mass per unit span length; \(y\) is the displacement of the cross-wind degree of freedom; \(\zeta\) is the mechanical damping ratio to critical; \(\omega_0\) is the mechanical circular frequency; \(\rho\) is the air density; \(U\) is oncoming mean wind velocity; \(D\) is the cross-wind dimension of the structure; \(K = \omega D/U\) is the reduced frequency of vortex shedding (where \(\omega\) is the circular frequency of vortex shedding determined using the Strouhal relationship outside the “lock-in” or “frequency entrainment” behavior inside the “lock-in”); \(Y_1, \epsilon, Y_2\) and \(C_L\) are parameters in this VIV model, which could be identified based on the
experimental observation. Equation 7.1 in a dimensionless form can be expressed as (Ehsan and Scanlan 1990)

\[ \eta'(s) + 2\zeta K_0 \eta'(s) + K_0^2 \eta(s) = m_r \left[ Y_1 \{1 - \varepsilon \eta^2(s)\} \eta'(s) 
+ Y_2 \eta(s) + \frac{1}{2} C_L(K) \sin(Ks + \theta) \right] \] (7.2)

where \( s = Ut/D \) is dimensionless time; \( \eta = y/D \) is normalized cross-wind displacement; \( K_0 = \omega_0 D/U \) is the reduced frequency; \( m_r = \rho D^2/m \) is the air-structure mass ratio; the prime indicates the differentiation with respect to the dimensionless time \( s \). Experimental observations show that, in the “lock-in” region, the vortex-induced lift force is negligible compared to the motion-induced force (Ehsan and Scanlan 1990). As this model concerns large-amplitude oscillations at "lock-in", the forced term \( C_L \sin(Ks + \theta)/2 \) may be dropped. In addition, experimentally identified results show that there is only insignificant variation between the natural frequency of the structure and the frequency of the system at "lock-in" (Ehsan and Scanlan 1990). Hence, the parameter \( Y_2 \), which represents the "frequency shift", could also be dropped. As a result, a simplified model can be expressed as (Ehsan and Scanlan 1990)

\[ \eta'(s) + 2\zeta K_0 \eta'(s) + K_0^2 \eta(s) = m_r \left[ Y_1 \{1 - \varepsilon \eta^2(s)\} \eta'(s) \right] \] (7.3)

As shown in Equation 7.1, for Scanlan’s VIV model: (1) there is no direct interplay between the vortex-induced and motion-induced effects; (2) inclusion of nonlinear motion-induced effects \( (\varepsilon \neq 0) \) results in a nonlinear VIV system; (3) the self-limiting feature is materialized by the nonlinear damping which depends on the vibration amplitude instead of the phase change in the external vortex-induced force. It is plausible
that Scanlan’s VIV model over emphasized the motion-induced effects, as Sarpkaya (1979) noted that VIV is a forced oscillation with self-excited character. Another feature of Scanlan's VIV model is that the limit cycle oscillation (LCO) amplitude is only dependent on a single parameter, the mass-damping parameter. Although Sarpkaya (1978) has demonstrated that, for structures with small values of the mass-damping parameter, the nondimensional mass and damping affect the structural response independently, for structures with large values of the mass-damping parameter (a normal situation for the bridge deck) the structural response is dependent on a combination parameter comprising of the mass ratio and damping, which are represented effectively in the Scruton number (Zdravkovich 1982). Griffin and Ramberg (1982) showed that, for similar values of the mass-damping parameter, the maximum LCO amplitudes are nearly equal and occur at similar ranges of reduced wind velocity.

In contrast to Skop and Griffin’s model (1973) in which both empirical parameters simply show a logarithm relationship with the mass-damping ratio (represented by the parameter $\zeta/\mu$ in their study, where $\zeta$ means the mechanical damping coefficient and $\mu$ is the mass ratio of the displaced mass of fluid to the mass of the structure), the parameters in Scanlan’s model change significantly and irregularly with the mechanical damping ratio (and therefore the mass-damping ratio) (Ehsan and Scanlan 1990). Hence, Scanlan’s model seems to have rather limited predictive capability for VIV of bridge decks with different mass-damping ratios.

Scanlan (1981) proposed a parameter identification scheme based on the energy conservation in the LCO state. For the simplified model as shown in Equation 7.3, the identification of the two parameters needs a pair of experiments with different amplitudes
of the LCO state, which could be obtained by changing the mechanical damping. Ehsan and Scanlan (1990) pointed out the shortcomings of this identification method and then proposed a new identification approach which can identify all the parameters in the model with a single experiment by utilizing the transient portion of the response time history. This so-called “decay-to-resonance” (or “growth-to-resonance”) technique is based on the assumption that the VIV system is a weakly nonlinear system. Hence, the solution of the nonlinear equation could be approximated by slowly varying parameters (Van der Pol 1920), i.e., the steady-state amplitude and phase angle have negligible change during a single period of $2\pi/K$. In order to reduce the uncertainties in the estimated parameters resulting from the transient portion of the response signal, the linear regression scheme was utilized in the parameter identification procedure. Gupta et al. (1996) presented a general parameter identification approach in the time domain for Scanlan’s model. Marra et al. (2011) investigated Scanlan’s model in detail and pointed out that the identification results for the model parameters significantly depend on the amplitude of the LCO state. In addition, they claimed that the model parameters could be directly identified utilizing the nonlinear differential equations instead of utilizing the slowly varying parameters approximation (Van der Pol 1920).

Goswami et al. (1993a; 1993b) further developed Scanlan's single-degree-of-freedom (SDOF) VIV model based on Billah's (1989) coupled wake oscillator model. Larsen (1995) generalized Scanlan’s VIV model by introducing a power multiplier $\nu$ to modify the curvature of the predicted steady-state response versus the Scruton number. The so-called generalized VIV model presents a downward curvature, which is consistent with the experimental results and is opposite to the predictions of the Scanlan’s model.
Parameter identification schemes based on either the steady-state response or the transient response are given for this generalized VIV model. Recently, Diana et al. (2006b) proposed a new numerical model for the VIV of bridge decks by representing the fluid-structure interaction with a second-order nonlinear mechanical system coupled with the structural motion through the linear and cubic terms.

7.2.2 Linear Semiempirical Model

In order to reconcile the analysis of VIV with the well-known Scanlan's linear analysis framework (Scanlan and Tomko 1971), the vortex-induced force was expressed as (Scanlan 1998)

\[ F_{vi} = \frac{1}{2} \rho U^2 B \left( \frac{KH_i^*}{U} \tilde{y} - \tilde{C}_L \sin \omega t \right) \] (7.4)

where \( H_i^* \), which represents the aerodynamic damping contribution, is one of the flutter derivatives utilized in Scanlan's linear flutter analysis framework; the coefficient \( \tilde{C}_L \) indicates the active forcing term in the model. The parameter \( H_i^* \) in the Equation 7.4 can be determined by the transition portion of the displacement response decaying to the steady state, while \( \tilde{C}_L \) could be obtained from the sustained amplitude of the LCO of VIV. Whereas, it seems to be inappropriate to investigate the VIV based on a linear analysis framework since the vibration presents significant nonlinearity. Actually, in Scanlan's study the LCO behavior of the VIV system based on the "self"-limiting property is replaced by the "forced"-limiting property as a consequence of the observation that VIV is purely driven by vortex-induced force in the "lock-in" region. Besides, identified \( H_i^* \) based on the "decay-to-resonance" scheme in many cases are
negative, which indicates that the decay of the structural response is accelerated by the positive aerodynamic damping, however, if the “growth-to-resonance” scheme is applied, then the identified results of $H_i$ are completely the opposite, which indicates that the decay of the structural response is decelerated.

7.3 Volterra Series-based Model for VIV of Bridge Decks

In the absence of an analytical treatment of the flow around a stationary or an oscillating structure, a closed-from description of the VIV phenomenon will remain mathematically intractable. In lieu of this, semi-empirical models have been advanced to model the VIV of bridge decks, but due to their phenomenological origin, they often have limited predictive capability. Experimental VIV studies using scaled models have been limited to low Reynolds numbers, especially for the decks of super long-span bridges. CFD based approaches show promise (e.g., Fujiwara et al. 1993; Nomura 1993; Lee et al. 1997; Sarwar and Ishihara 2010), but they have their own limitations at this juncture stemming from the lack of robust turbulence models for engineering applications and extensive computational demands. In order to close the gap between a reliable numerical simulation and the demands of practical applications, reduced order models (ROMs) offer high fidelity at much reduced computational demand. As there are much lower-order degrees of freedom involved in an ROM as compared to CFD, it can be tailored to meet the demands placed by the fundamental physics of the application (e.g., Raveh 2001; Lucia et al. 2004). Among various ROMs, Volterra series based ROM, which is a form of Taylor series with memory effects, has the promise of modeling the VIV system. The complex mapping rules (static linear/nonlinear relationships) and time lag (fluid memory effects)
between the aerodynamic/aeroelastic inputs and outputs, the hallmark of VIV, can be represented by the superposition of scaled and time shifted fundamental responses, i.e., convolution. These features are captured elegantly by Volterra series thus it qualifies as an ideal candidate for ROM modeling of a VIV system, hence it is explored here.

7.3.1 Volterra Theory Background

The basic premise of the Volterra theory is that a large class of nonlinear systems can be modeled as a sum of multidimensional convolution integrals of increasing order (Volterra 1959). The Volterra theory was first successfully applied in electrical engineering (e.g., Wiener 1942; Alper 1965; Ewen and Weiner 1980; Boyd 1985), which led to its comprehensive development (Schetzen 1980; Rugh 1981). In the field of offshore engineering, the Volterra series has been utilized to simulate non-Gaussian loading processes (e.g., Næss 1985; Winterstein et al. 1994, Gurley et al. 1997) and to evaluate stationary response of systems with mechanical nonlinearities as well as load-structure feedback interactions (e.g., Donley and Spanos 1990; Li and Kareem 1990; Tognarelli et al. 1997; Carassale and Kareem 2010). Silva (1997) applied the Volterra theory to the aerospace problems, based on which a series of new findings were presented (e.g., Silva et al. 2001; Lucia et al. 2004; Silva 2005). Recently, Wu and Kareem (2011; 2012b; 2012c; 2012e) utilized the Volterra theory to develop models for simulating bridge aerodynamics/aeroelasticity. A VIV system typically is time-invariant and causal. For such a system, the response $y(t)$ under an arbitrary input $x(t)$ may be represented as (Rugh 1981)
\[ y(t) = h_0 + \int_0^t h_1(t-\tau)x(\tau)d\tau + \sum_{n=2}^{\infty} \int_0^t \int_0^t \cdots \int_0^t h_n(t-\tau_1, \ldots, t-\tau_n)x(\tau_1) \cdots x(\tau_n)d\tau_1 \cdots d\tau_n \]

where \( h_0 \) means the steady-state term which satisfies system initial condition; \( h_1 \) represents the first-order kernel which describes the linear behavior of the system; \( h_n \) the higher-order terms which represent the nonlinear behavior existing in the system.

Earlier models by Skop and Griffin (1973) and Ehsan and Scanlan (1990) were based on Van der Pol type equation exploiting the approximation of slowly varying parameters, which suggests that VIV could be classified as a weakly nonlinear system. This observation facilitates the use of truncated Volterra series with finite terms for modeling of VIV. In this study, the second-order Volterra series is used to simulate the VIV system, which is motivated by the following observations: i) the VIV system could be treated as a weakly nonlinear system; ii) the second-order term means a “first step beyond linearity”, which could capture important nonlinear phenomena (if they exist) without involving complexity of identifying the new model; iii) the identification of higher-order kernels is currently difficult to became a part of a practical solution scheme.

For a nonlinear system modeled with the second-order Volterra series, the response \( y(t) \) under an arbitrary input \( x(t) \) could be represented as (Rugh 1981)

\[ y(t) = \int_0^t h_1(t-\tau)x(\tau)d\tau + \int_0^t \int_0^t h_2(t-\tau_1, t-\tau_2)x(\tau_1)x(\tau_2)d\tau_1d\tau_2 \]  

where the steady-state term \( h_0 \) is neglected due to the focus on the dynamic responses of the VIV system. Based on the earlier work by Rugh (1981), a generalized impulse-
function-based kernel identification scheme was developed, where the kernels of the second-order Volterra system could be expressed as (Wu and Kareem 2011)

\[ h_1(t - \tau) = \frac{1}{\alpha^2 - \alpha} \left( \alpha^2 y[\delta(t - \tau)] - y[\alpha \delta(t - \tau)] \right) \]  \hspace{1cm} (7.7)

\[ h_2(t - \tau_1, t - \tau_2) = \frac{1}{2\kappa_1 \kappa_2} \left[ y[\kappa_1 \delta(t - \tau_1) + \kappa_2 \delta(t - \tau_2)] - y[\kappa_1 \delta(t - \tau_1)] - y[\kappa_2 \delta(t - \tau_2)] \right] \]  \hspace{1cm} (7.8)

where \( \delta(t) \) represents the Dirac delta function (unit-impulse function); \( y[\delta(t)] \) indicates the unit-impulse response; \( \alpha, \kappa_1 \) and \( \kappa_2 \) are selected constants.

7.3.2 Simulation of VIV System

As detailed in the kernel identification scheme, the higher-order kernel identification requires concurrent multi-input signals, which makes it experimentally very challenging. Accordingly, no data under multi-input inputs has been obtained experimentally thus far. Alternatively these kernels can be identified by a CFD technique, but it would entail considerable challenge as well. For example, if an impulse displacement is utilized to identify the relevant aeroelastic Volterra kernels, there will be several intractable issues since a displacement is accompanied with velocity and acceleration (Raveh 2001). While a CFD based scheme is under development, in this study for the sake of demonstrating the effectiveness of Volterra based approach the kernels are identified based on the data obtained from experimentally fitted Scanlan's VIV model. As a result, the complexity of impulse motion input could be circumvented by utilizing the external impulse force input. The simulation results indicate that the Volterra series based nonlinear model could capture typical features of the VIV of bridge decks, such as the LCO, "frequency shift",
hysteresis and beat phenomenon. Also examined herein is the contribution of nonlinear terms to the VIV of bridge decks. These features are illustrated in the following.

7.3.2.1 LCO Simulation at Lock-In

A rectangular prism with the width 0.3 m has the following aspect ratios: $B/D = 4:1$ and $L/B = 3.28:1$. The mass per unit length of the model = 6.085 kg/m; the natural frequency in the across-wind direction = 13.48 Hz; the Strouhal number = 0.136; the mass ratio = 0.0011; damping ratio = 0.0021 which results in the Scruton number of 6.0 (Marra et al. 2011). The nondimensional structural response at the wind velocity of 8.5 m/s (“lock-in” region) is shown in Figure 7.1. The parameters of simplified Scanlan’s model are identified based on the “decay-to-resonance” technique, where $Y_1$ is equal to 6.27; $Y_2$ is -5.70; $\varepsilon$ is 1082.2 at the wind velocity of 8.5 m/s (Marra et al. 2011). Accordingly, the “lock-in” behavior of the VIV system could be reasonably described using the following approximate equation:

$$\ddot{y} + 0.3544 \dot{y} + 7120.5y = 0.8047\left(1 - 192391y^2\right)\dot{y} - 82.906y$$  \hspace{1cm} (7.9)$$

Based on the aforementioned impulse-function identification scheme, the kernels of the Volterra series based model at “lock-in” are obtained. Figure 7.2 shows the identified first- and second-order kernels (time interval $\Delta t=0.002s$). In order to demonstrate that the truncated Volterra series based model is able to simulate the “lock-in” behavior, the response at “lock-in” of this VIV system is obtained utilizing the identified Volterra kernels and compared to the reference response obtained using the fourth-order Runge-Kutta scheme, as shown in Figure 7.3. The linear approximation response is based on the first-order kernel only while the nonlinear approximation represents the response
obtained by utilizing both the first- and second-orders kernels. As presented in Figure 7.3(b), the linear approximation response shows notable discrepancy as compared to the reference results while there is indiscernible difference between the nonlinear approximation response and the reference data. This observation indicates that the second-order kernel is necessary and sufficient to simulate the “lock-in” of a VIV system, as used in this example.

Figure 7.1. Nondimensional displacement record at “lock-in” [data from Marra et al. (2011)]

Figure 7.2. The linear and nonlinear kernels of the VIV system
Figure 7.3. Impulse input and corresponding "lock-in" response of the VIV system: (a) input signal; (b) the corresponding response; (c) close-up of the response.
It is well-known that the truncated Volterra series could well represent the nonlinear dynamic system with “fading” memory, where the output of the system considered is not dependent on the infinite past of the input (Boyd 1985). The “fading” memory system indicates that the linear and nonlinear kernels as a function of time lags should converge to zero. Whereas, it is interesting that both the first- and second-order kernels of the "lock-in" response do not diminish with time, as noted in Figure 7.2. Here, the VIV system is referred to as the “lasting” memory system. By ignoring the coupling effects between the vortex shedding and structural motion, which cannot be directly simulated by Scanlan’s model, it is reasonable to assume the “lasting” memory of the VIV system may result from either contributions of the external forcing and the motion-induced effects individually or collectively. On the one hand, it has been shown that the far field flow effects induced by the fading of vortex street are in the order of \((1/r)\) where \(r\) is the distance from the vortex street to the body (Wiehs 1972), which indicates that the external forcing effect possesses a “fading” memory. On the other hand, as a spontaneous behavior, the motion-induced effect of the VIV can be labelled as a “lasting” memory system. In addition, the LCO feature of this motion-induced “lasting” memory is a critical reason that the truncated Volterra series provides an excellent approximation of the VIV system at "lock-in". In order to distinguish the "memory" contributions of the external forcing and motion-induced effects, the first- and second-order kernels of the linear mechanical system in Equation 7.9 are identified and shown in Figure 7.4. As expected, the mechanical system under the external impulse force presents “fading” memory. The "memory" resulting from the motion-induced effects is obtained by the subtracting forcing-effect-induced memory (shown in Figure 7.4) from total memory.
Since this mechanical system is linear, the second-order kernel has trivial value theoretically. Small values of the second-order kernel shown in Figure 7.4 stem from numerical errors. It should be noted that the identification of Volterra kernels corresponding to the motion-induced effects is not based on the impulse displacement, instead, it is triggered by the impulse force. Since a new homogeneous system will be constructed after combining the motion-induced terms with the original mechanical terms (stiffness and damping), the impulse force actually results in the motion-induced kernels (after subtracting the “fading” memory effects). As mentioned in the preceding section, the illustrative identification scheme utilized in this study to demonstrate ability of a Volterra based ROM for modeling VIV could circumvent the issues surrounding CFD based identification approach at this time.

![Figure 7.4. The linear and nonlinear kernels of the linear mechanical system](image)

7.3.2.2 Simulation of Frequency Shift at Lock-In

In order to verify the capability of the Volterra series based model to simulate the “frequency shift” at “lock-in”, the frequency components of the reference, linear and
nonlinear responses via the Fourier spectra are shown in Figures 7.5(a), 7.5(b) and 7.5(c), respectively. It shows that both the simulation results based on the linear and nonlinear approximations can capture the "frequency shift" at "lock-in" [from 13.48 Hz (natural frequency) to 13.50 Hz].

Figure 7.5. The frequency components of each response: (a) reference response; (b) linear response; (c) nonlinear response

7.3.2.3 Simulation Hysteresis at Lock-In

The hysteresis phenomenon in VIV systems has being investigated since Feng’s experimental results (1968) that introduced the presence of hysteretic behavior. The hysteresis phenomenon indicates that there are two amplitudes of the LCO in a certain wind velocity range. Currie et al. (1974) observed that the larger amplitude at “lock-in” region is much higher than that based on the linear restoring force. Hence, Currie et al. (1974) introduced a cubic nonlinear term \(-\varepsilon_k y^3\), where \(\varepsilon_k\) is a constant much smaller than 1 (e.g., 1%), in the restoring force to take into account the hysteretic effects. Wood and Parkinson (1977) investigated the hysteresis of VIV systems by empirically adding a
nonlinear motion-dependent damping term, which can be conveniently merged into Scanlan’s VIV model. Consideration of nonlinear stiffness and damping renders a linear mechanical system to a nonlinear one. Therefore, the VIV system is investigated by calculating a nonlinear wake oscillator coupled with a nonlinear mechanical system. Although it is difficult to reveal the underlying physical mechanism of hysteresis in the VIV based on the consideration of the nonlinear mechanical system, this approach serves as a good representative phenomenological model.

The hysteresis effects indicate the presence of higher-order memory in the dynamic system. As a functional series representation, Volterra series based model inherently involves higher-order memory in terms of higher-order convolution terms. Therefore, Volterra series based model is a promising approach to simulate the hysteresis phenomenon in VIV systems regardless of the underlying physics of its origin. An example involving the Original Tacoma Narrows Bridge, whose VIV model parameters are presented in Table 7.1, is used to illustrate the model. Figure 7.6 presents the identified kernels of the Volterra series for the VIV system involving hysteresis effects (attributing to the nonlinear stiffness). Figure 7.7 shows the simulation results of the VIV system at "lock-in" with hysteresis effects using the identified Volterra kernels. As shown in the figure, the nonlinear approximation converges to the reference results. The first-and second-order Volterra kernels of the nonlinear mechanical system are also calculated as presented in Figure 7.8, where the second-order kernel has non-trivial values. The "memory" resulting from the motion-induced effects is obtained by the subtracting forcing-effect-induced memory (shown in Figure 7.8) from total memory (shown in Figure 7.6).
TABLE 7.1
PARAMETERS FOR THE CALCULATION CASES [DATA FROM SCANLAN (1998)]

<table>
<thead>
<tr>
<th>Model</th>
<th>$\zeta$</th>
<th>$K$</th>
<th>$m_r$</th>
<th>$Y_1$</th>
<th>$Y_2$</th>
<th>$C_{non}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>H-section</td>
<td>0.00497</td>
<td>0.6308</td>
<td>0.001327</td>
<td>10.45</td>
<td>-5.70</td>
<td>50.3%</td>
</tr>
<tr>
<td>Rectangular section</td>
<td>0.00352</td>
<td>0.6720</td>
<td>0.001416</td>
<td>6.49</td>
<td>-5.70</td>
<td>52.5%</td>
</tr>
<tr>
<td>Deer Isle Bridge section</td>
<td>0.00710</td>
<td>0.7984</td>
<td>0.001386</td>
<td>8.91</td>
<td>-5.70</td>
<td>51.7%</td>
</tr>
<tr>
<td>Original Tacoma Narrows</td>
<td>0.00370</td>
<td>0.6418</td>
<td>0.000617</td>
<td>12.07</td>
<td>-5.70</td>
<td>46.6%</td>
</tr>
</tbody>
</table>

Figure 7.6. Identified kernels for hysteresis simulation

Figure 7.7. Response of the VIV system involving hysteresis effects
7.3.2.4 Simulation of Beat Observation at Non-Lock-In

For the approach wind velocity at just before or past the “lock-in” region of a VIV system, two distinct frequencies would surface in the response signature. One of these relates to the mechanical vibration while the other corresponds to vortex shedding. As a result, the beat phenomenon could occur at the “non-lock-in” regions, which has been recorded in several studies (e.g., Goswami et al. 1993a). In order to investigate the capability of a Volterra series based model to simulate the beat phenomenon at the “non-lock-in” range, the vortex-induced force term $C_L \sin(Ks+\theta)/2$ corresponds to the dominant frequency of vortex shedding is set at 15.00 Hz. The identified first- and second-order Volterra kernels are shown in Figure 7.9 while the VIV response is presented in Figure 7.10. The simulated time history demonstrates that the Volterra series based model that involves the second-order convolution could faithfully simulate the beat phenomenon at “non-lock-in” region.
7.3.3 Nonlinear Contribution in VIV System

As highlighted in aforementioned sections, VIV of bridge decks presents noticeable nonlinearity, which can be captured by the functional expansion comprised of linear and nonlinear convolution terms, represented by Volterra series. Figure 7.11 presents the linear and nonlinear approximations together with the reference solution at “lock-in” for the cases adopted from Scanlan (1998), i.e., an H-section with the aspect ratio of 4:1,
solid rectangle section with the aspect ratio 4:1, Deer Isle Bridge deck section with the aspect ratio of 3.5:1 and Original Tacoma Narrows Bridge deck section with the aspect ratio of 5:1. The nondimensional time interval in the numerical calculation is $\Delta s=0.2$. The relevant parameters are presented in Table 7.1, which were originally reported in Scanlan (1998). In order to account for the “frequency shift” effects, the parameter $Y_2$ is assumed reasonably to be -5.70 for all cases. Table 7.1 presents the estimated nonlinear contribution to the VIV response at "lock-in", which is calculated using the difference between the values of the maximum displacement of the linear approximation and the maximum displacement of the reference solution divided by the values of the maximum displacement of the reference solution, i.e.,

$$C_{non} = \frac{\text{max}[\text{linear}_{\text{dis}}] - \text{max[reference}_{\text{dis}}]}{\text{max}[\text{reference}_{\text{dis}}]}$$  \hspace{1cm} (7.10)$$

where $C_{non}$ presents the relative contribution of the nonlinearity to the VIV response at "lock-in"; $\text{max}[\text{linear}_{\text{dis}}]$ = the maximum displacement of the linear approximation and $\text{max}[\text{reference}_{\text{dis}}]$ = maximum displacement of the reference solution.

As shown in the results for four cases (Figure 7.11), the truncated Volterra series can simulate the VIV response with reasonable accuracy. The nonlinearity contributes around 50% of the reference response for all four cases. Due to the limited sampling frequency in the study, the linear approximation presents a very slight frequency shift compared to the reference response or nonlinear approximation. Although the nonlinear contribution to the response is relatively large, the VIV phenomenon could still be treated as a weakly nonlinear system since the second-order Volterra series reproduces the reference response accurately.
Figure 7.11. Linear and nonlinear approximations of the VIV response at "lock-in" for various cases: (a) Case 1: H-section; (b) close-up of Case 1 response; (c) Case 2: rectangular section; (d) close-up of Case 2 response; (e) Case 3: Deer Isle Bridge section; (f) close-up of Case 3 response; (g) Case 4: original Tacoma Narrows Bridge section; (h) close-up of Case 4 response
7.4 Closing Remarks

A brief overview of vortex-induced vibration (VIV) of bridge decks is presented highlighting its departure from conventional observed VIV of circular cylinders. A Volterra series based nonlinear model is introduced to simulate VIV of bridge decks. The kernel identification scheme based on the impulse function input is utilized to identify the first- and second-order kernels of Volterra series. Both the "fading" and "lasting" memory of the VIV system at “lock-in” could be accurately modeled by the truncated Volterra series. The limit cycle oscillation (LCO) of the motion-induced “lasting” memory is a critical reason for the truncated Volterra series to provide an excellent approximation of the “lock-in” behavior of VIV systems. Additionally, it has also been demonstrated that the Volterra series based model is a promising scheme to simulate the "frequency shift" and hysteresis features in the “lock-in” region and to model the observed beat phenomenon in the “non-lock-in” region.

As a functional expansion of a nonlinear system, Volterra series is utilized to estimate the linear and nonlinear contributions to VIV of bridge decks. It is noted that the nonlinearity contributes are around 50 percent of the VIV response of various bridge decks studied. It is also demonstrated that the attempt to set up a unified analysis framework based on Scanlan's linear approach, where VIV is simulated with a linear semi-empirical model, may need additional examination before a viable model emerges. On the other hand, Volterra series based model per se is a nonlinear analysis framework. Success in the application of Volterra series based model to VIV system ensures the development of a unified analysis framework for wind-induced effects on bridge decks (Wu and Kareem 2012c), where the linear convolution term is utilized to portray
conventional flutter and buffeting analysis models while the combination of the linear and nonlinear terms are utilized to model nonlinear flutter (also post-flutter behavior) and buffeting and VIV issues.
Volterra kernels will be identified by the solution of the Navier-Stokes equations governing fluid-structure interactions using computational fluid dynamics (CFD). If the Volterra kernels are identified based on CFD, the input information in this phase will be the smoothed-ramped pulse function (an approximation of the unit-impulse function) or unit-sample function (an equivalent of unit-impulse function in discrete-time domain) of vertical turbulent component for bluff-body aerodynamics, and vertical velocity, torsional displacement, or angular velocity for bluff-body aeroelasticity. The output information could be the forces or pressures on the structures. The experience obtained in the CFD-based estimation of the Volterra model could help in the Volterra kernel identification based on the wind-tunnel tests.

The sources of nonlinear aerodynamics involved in wind-structure interactions represented by the Navier-Stokes equations are first investigated. Computational approaches employed in this study are validated through theoretical consideration, i.e., Blasius solution for the steady-state simulation and Theodorsen solution for the dynamic-state simulation. A nonlinear analysis framework for bluff-body aerodynamics based on Volterra theory is utilized to capture the linear and nonlinear aerodynamic effects. The Volterra kernel identification is based on the impulse function concept which is
illustrated through computational fluid dynamics. The simulation based on this reduced-order modeling scheme is obtained by the convolution of the identified kernels and the external inputs. It is demonstrated that the Volterra theory based nonlinear analysis framework of bluff-body aerodynamics is promising in capturing the essential features of bluff-body aerodynamics and offers an accurate approximation of the Navier-Stokes equations with minimal computational effort.

8.1 Introduction
A physically realizable dynamic system could be generally represented as Figure 8.1(a), where \(x(t)\) denotes the input signal and \(y(t)\) the output signal. The simplest input-output relationship is linear, any mapping operators beyond the linear regime could be treated as nonlinear relationship (Bendat and Piersol 1980). Hence, the dynamic system could be considered as a summation of linear and nonlinear mapping operators, as indicated in Figure 8.1(b). Besides, any outside disturbances could be represented as an exterior noise \(n_e(t)\), which is usually assumed to be uncorrelated with the output of the dynamic system. For many engineering applications, the dynamic system is ordinarily approximated by linear mapping operators based on the premise that the error introduced is acceptable. In the case of bridge aerodynamics, the aerodynamic transfer functions (aerodynamic admittances) and aeroelastic transfer functions (flutter derivatives) are traditionally utilized to characterize the dynamic relationships between gusts (inputs) and forces on the bridge deck (outputs), and between bridge deck motions (inputs) and forces on the bridge deck (outputs), respectively (Davenport 1962; Scanlan and Tomko 1971). Both of the aerodynamic admittances and flutter derivatives are linear mapping operators in the
frequency domain. Since the instantaneous output $y(t)$ depends on the input $x(t)$ and its history, these linear mapping operators are functions of frequency which ensures capturing the system memory effects. The time domain equivalents of these frequency transfer functions are also utilized to obtain simulation of bridge aerodynamics based on the linear convolution scheme (Scanlan et al. 1974).

![Diagram](image-url)

Figure 8.1. Schematic representation of a dynamic system: (a) a general dynamic system; (b) the summation of general linear and nonlinear mapping operators; (c) the summation of time-invariant linear and nonlinear mapping operators
Rapid increase in the bridge spans and the attendant innovative bridge deck cross-sections demand that the bridge aerodynamic system may not be represented by linear mapping operators (Diana et al. 2010; Kareem and Wu 2013). Recent observations in wind tunnels have highlighted various nonlinear phenomena in bridge aerodynamics such as: (i) non-proportional relationship between input and output; (ii) multiple frequencies excited by a single-frequency; (iii) amplitude dependence of aerodynamic and aeroelastic forces and (iv) hysteretic feature of aerodynamic/aeroelastic behavior versus the angle of attack (Wu and Kareem 2011). In order to capture nonlinear bridge aerodynamics, a number of studies have been focused on identifying the sources of nonlinearity and their modeling (e.g., Diana et al. 1995; Chen and Kareem 2003; Wu and Kareem 2011). In general, these nonlinear models mentioned in the above literature cannot fully characterize nonlinear bridge aerodynamics (Wu and Kareem 2012a). Another promising approach recently developed to consider nonlinear bridge aerodynamics is based on the Volterra theory (e.g., Wu and Kareem 2011; 2012b; 2012c; 2012e), where the linear convolution scheme is logically extended to the summation of linear and nonlinear convolution schemes. In such a case, the response $y(t)$ under an arbitrary input $x(t)$ may be represented as (Rugh 1981)

\[
y(t) = h_0 + \int_0^t h_1(t - \tau)x(\tau)d\tau \\
+ \sum_{n=2}^{\infty} \int_0^t \cdots \int_0^t h_n(t - \tau_1, ..., t - \tau_n)x(\tau_1) \cdots x(\tau_n)d\tau_1 \cdots d\tau_n
\]

(8.1)

where $h_0$ represents steady-state term which satisfies system initial condition; $h_1$ represents the first-order kernel which describes the linear behavior of system; and $h_n$ the higher-order terms representing nonlinear features of the system. Due to the fading
memory, a characteristic of the bridge aerodynamics, a truncated Volterra system could be utilized. In this study, the second-order Volterra representation \((n=2)\) is employed as the first attempt beyond linearity in the simulation of wind-structure interactions.

The traditional application of Volterra theory is based on the assumption that the dynamic system involved is time invariant, otherwise several intractable revisions need to be considered (Rugh 1981). However, a typical wind-structure interaction system governed by the Navier-Stokes equations may result in aerodynamic force that is time variant despite time-invariant input (either gusts or deck motions). This is a consequence of wake instability (Hopf bifurcations) (Jackson 1987) and laminar-turbulence transition (period-doubling bifurcations) (Karniadakis and Triantafyllou 1992). These internal disturbances could be treated as an interior noise \(n_e(t)\), whose properties and its correlation with the output are still not well understood. For the case of streamlined-body aerodynamics, the integrated forces on the structure induced by the interior noise are typically negligible, while for bluff-body aerodynamics these are generally significant due to the separation and the ensuing wake dynamics. It should be noted that in the case of bridge aerodynamics the separation occurs not only at the trailing edge but also at the leading edge. Besides, the separation at the leading edge usually leads to unsteady reattachment as bridge decks present a long afterbody. Hence, both the unsteadiness in the wake and around the bridge deck contribute to the interior noise. Figure 8.2(a) and 8.2(b) presents the lift force coefficient time history of a stationary zero-thickness flat plate and a stationary rectangular prism with aspect ratio of 1:5, respectively. Both cases are at Reynolds number of \(10^4\) (chord length based), at which the zero-thickness flat plate aerodynamics is still substantially laminar flow while the rectangular prism aerodynamics
is turbulent. As shown in the figure, the magnitude of the lift force coefficient of the zero-thickness flat plate is several order smaller compared to that of the rectangular prism. In this study the interior noise $n_e(t)$ is either negligible or assumed to be uncorrelated with the motion-induced or gust-induced effects, hence, both the linear and nonlinear mapping operators could be treated as time invariant, as indicated in Figure 8.1(c).

![Figure 8.1](image1.png)  
(a) Stationary zero-thickness flat plate aerodynamics; (b) Stationary rectangular prism aerodynamics.

The identification of Volterra kernels is critical when applying this model to simulate bluff-body aerodynamics. There are various approaches to identify Volterra kernels, such as harmonic probing method (e.g., Bedrosian and Rice 1971), identification based on Wiener orthogonal kernels (e.g., Lee and Schetzen 1965), identification based on impulse function concept (e.g., Schetzen 1965) and identification based on system identification techniques (e.g., Rugh 1981). For the first three techniques, the input
sequences are required to be harmonic components, Gaussian white noise and impulse function, respectively. On the other hand, there is no special requirement of input sequence for a general system identification technique. Among these identification schemes, the one based on the impulse function concept retains the complete underlying physics of the kernels within the original concept of Volterra theory (Volterra 1959). Besides, the kernels identified using impulse function-based scheme could be utilized in predicting the response of the nonlinear system under arbitrary input (Schetzen 1980), and hence is employed in this study. Based on the earlier work by Rugh (1981), Wu and Kareem (2012c) developed a generalized impulse function-based kernel identification scheme, where the first- and second-order kernels of a truncated second-order Volterra system could be expressed as (Wu and Kareem 2012c)

\[
h_1(t - \tau) = \frac{1}{\beta^2 - \beta} \left( \beta^2 y[\delta(t - \tau)] - y[\beta\delta(t - \tau)] \right) \tag{8.2a}
\]

\[
h_2(t - \tau_1, t - \tau_2) = \frac{1}{2\kappa_1\kappa_2} \left[ y[\kappa_1\delta(t - \tau_1) + \kappa_2\delta(t - \tau_2)] - y[\kappa_1\delta(t - \tau_1)] - y[\kappa_2\delta(t - \tau_2)] \right] \tag{8.2b}
\]

where \( \delta(t) \) represents the unit-impulse function (Dirac delta function); \( y[\delta(t)] \) indicates the unit-impulse response; \( \beta, \kappa_1 \) and \( \kappa_2 \) are selected constants. It should be noted that, although the linear response of a linear system and a nonlinear system are conceptually the same, the latter based on the first-order kernel is amplitude dependent. This indicates that the first-order kernel is an averaged representation of the linear mapping operator of a nonlinear system (Rugh 1981). In this study, the response based on the first-order kernel is referred to as the first-order approximation. The first attempt to obtain the
Voltterra kernels is to derive them from analytical expressions describing the nonlinear aerodynamic system (e.g., Wu and Kareem 2012b; 2012e). It should be noted that these analytical expressions are basically phenomenological models in bridge aerodynamics. Hence, while the success of Volterra theory-based models in these applications contributes significantly to the feasibility of simulating nonlinear bridge aerodynamics with the Volterra theory-based nonlinear framework, it cannot depict the bridge aerodynamics beyond the empirical information integrated in these analytical, phenomenological models. In order to reveal the underlying mechanism of gusts- and motion-induced forces in bridge aerodynamics and hence obtain better prediction capability, the kernels need to be identified in wind tunnel or based on computational fluids dynamics (CFD) principles. In this study, the kernel identification based on CFD is systematically investigated with a focus on the two-dimensional (2-D) motion-induced aerodynamics.

8.2 Physical Sources of Nonlinearity
As mentioned in the preceding section, several important nonlinear features observed in wind tunnel have strengthen the demand for nonlinear considerations in bridge aerodynamics. On the other hand, underlying physical sources of nonlinear mapping operators in the aerodynamic system has not been systematically established. von Kármán and Sears (1938) introduced a linear theory of aerodynamics based on four important assumptions, namely a thin flat plate (zero thickness), straight-line wake vortices (undeformed wake), potential flow (without viscosity effects) and infinitesimal disturbance (small amplitude of structural oscillations or gusts). In order to meet these
assumptions in a viscous flow, the case of a zero-thickness flat plate with a relatively high Reynolds number Re could be selected. Specifically, Re=10^4 (chord length based) is employed for two reasons: (1) Re is sufficiently high that the boundary layer is so thin that its viscosity effects are negligible; (2) Re is sufficiently low that generally the transition from laminar to turbulence does not occur. Besides, the disturbance due to the wake instability results in the lift force and torsional moment coefficients with magnitude of O(10^{-4}) and O(10^{-5}), respectively, which are several order smaller compared to those induced by the structural motion with a small amplitude (e.g., 3% of chord length).

8.2.1 Baseline Aerodynamic System

In this section, the case of a zero-thickness flat plate at Re=10^4 is selected as the baseline aerodynamic system. The simulation results shown here are based on the computational approach described in the next section. Figure 8.3 presents the drag force, lift force and torsional moment coefficients resulting from the vertical plate oscillation with an amplitude of 3% of the chord length, where the Reynolds number Re=10^4 and the reduced oscillating frequency \( f_r = 0.5 \) (i.e., reduced wind velocity \( U_r = 2 \)). Since the nonlinear effects generally change the shape of the waveform, the spectra of the input motion and output forces are utilized as a primary tool in the nonlinearity investigation. Figure 8.4 shows the power spectrum density (PSD) of the input motion, drag force, lift force and torsional moment.

One noticeable feature in Figure 8.4 is that the dominant frequency component of the drag force is double of the input frequency, which probably results from the nonlinear self-interaction of the unique input frequency component. In order to verify this conjecture, the bispectrum is utilized to investigate the drag force signal. The bispectrum
Figure 8.3. Output time histories of the zero-thickness flat plate at Re=10^4 and $U_r=2$

Figure 8.4. Input and output spectra of the zero-thickness flat plate at Re=10^4 and $U_r=2$
is the Fourier transform of triple correlation which can capture quadratic nonlinearities in one signal (auto-bispectrum) and between several signals (cross-bispectrum) (Kim and Powers 1979; Wu et al. 2012). Figure 8.5 presents the auto-bispectrum of the drag force signal. The spike in the figure straightforwardly suggests that the dominant frequency component of the drag force results from the quadratic nonlinear self-interaction of the reduced frequency component 0.5, which is the input frequency component. This unique feature of the drag force signal is worth further investigation, however, in this study the lift force and torsional moment are focused as these two outputs are generally more important in bridge aerodynamics. As shown in Figure 8.4, the first main frequency component of both the lift force and torsional moment is the input frequency component. While the second main frequency component in these output signals is triple of the input frequency component, which is interpreted here as the nonlinear coupling between the linear lift force or torsional moment and the drag force.

Figure 8.5. Auto-bispectrum of the drag force of the zero-thickness flat plate
8.2.2 Physical Sources of Nonlinear Effects

In the following, the nonlinear effects of thickness, oscillating frequency, viscosity and oscillating amplitude are investigated through the comparison with the baseline aerodynamic system. Table 8.1 shows the main contributions (relative to the first main frequency component) of the nonlinear effects in the output force signals for four various aerodynamics cases together with the baseline system.

<table>
<thead>
<tr>
<th>Case</th>
<th>$H/B$</th>
<th>$A/B$</th>
<th>$fr_i$</th>
<th>Re</th>
<th>$1^{st}$ Main $fr_o$</th>
<th>$2^{nd}$ Main $fr_o$</th>
<th>$3^{rd}$ Main $fr_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$(1 \times fr_i)$</td>
<td>$(2 \times fr_i)$</td>
<td>$(3 \times fr_i)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Lift Moment</td>
<td>Lift Moment</td>
<td>Lift Moment</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>3%</td>
<td>0.5</td>
<td>$10^4$</td>
<td>100 100</td>
<td>0 0</td>
<td>2.6 4.3</td>
</tr>
<tr>
<td>1</td>
<td>1:200</td>
<td>3%</td>
<td>0.5</td>
<td>$10^4$</td>
<td>100 100</td>
<td>0 0</td>
<td>2.8 2.8</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>3%</td>
<td>2</td>
<td>$10^4$</td>
<td>100 100</td>
<td>2.3 8.5</td>
<td>1.2 10.5</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>3%</td>
<td>0.5</td>
<td>200</td>
<td>100 100</td>
<td>0 0</td>
<td>0 0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>30%</td>
<td>0.5</td>
<td>200</td>
<td>100 100</td>
<td>0 0</td>
<td>6.9 10.9</td>
</tr>
</tbody>
</table>

(Note: $H$=height of the structure; $B$=chord length of the structure; $A$=oscillating amplitude; $fr_i$=reduced oscillating frequency of input signal; $fr_o$=reduced oscillating frequency of output signal)

It is well known that, as the thickness of a flat plate increases, the structure is changed to a bluff body. In such a case, the dynamic system is qualitatively changed from
a streamlined-body to a more complex bluff-body aerodynamics where the mapping rules between aerodynamic inputs and outputs are sensitive to the unsteady separations. In order to confine the aerodynamic system in the same regime of baseline case, a very small aspect ratio (1:200) is designed to account for the effects of thickness (Case 1). The results in Table 8.1 suggests that there is no abrupt change of the aerodynamics due to the added small thickness. The effects of a large thickness in the case of an airfoil, which is still in the regime of streamlined-body aerodynamics, were investigated by Giesing (1968). According to Giesing (1968), the nonlinear effects due to the large thickness generally reduce the force outputs. Case 2 is designed to take into account the effects of oscillating frequency. The high oscillating frequency and hence the relatively low convective flow speed leads to the gathered wake vortices just behind the structure, which could be treated as a strong wake deformation. As indicated in Table 8.1, the high oscillating frequency not only increases the nonlinear coupling between the torsional moment and drag force, but also generates the nonlinear self-interaction for both lift force and torsional moment. It is also well recognized that the unsteady effects are significantly important as the oscillating frequency becomes high (low reduced wind velocity). Both of the nonlinearity and unsteadiness observed at such a low reduced wind velocity may contribute to the underlying mechanism for the well-known vortex-induced vibration of bridge deck, which usually occurs at reduced wind velocity less than unit (e.g., Larsen et al. 2000; Diana et al. 2004; Wu and Kareem 2012e). Case 3 is designed to account for the viscosity effects. As the Reynolds numbers becomes smaller the boundary layer increases to relatively thicker, and thus the viscosity effects are more important in the aerodynamic system. The viscosity is usually utilized to explain the possible nonlinear effects,
however, the results here suggest that the viscosity itself actually reduces the nonlinear effects. Case 4 is designed to account for the effects of oscillating amplitude. As suggested in Table 8.1, the larger oscillating amplitude increases the nonlinear coupling between the drag force and lift force or torsional moment. Nevertheless, the nonlinear self-interaction of lift force and torsional moment does not occur under such a large oscillating amplitude. It should be noted that, although all the nonlinear effects in the abovementioned aerodynamic system are relatively weak as they basically have small deviations from the linear aerodynamics, the physical sources of the nonlinear effects in an aerodynamic system are well illustrated.

8.2.3 Rectangular Case
For the streamlined-body aerodynamics, the nonlinearity mainly results from the wake due to the attached flow condition along the surface. On the other hand, for bluff-body aerodynamics, both the flow patterns in the wake and near the structural surface (separated shear layer) significantly contribute to the nonlinear effects due to unsteady separation. In order to emphasis the motion-induced effects in bluff-body aerodynamics, a rectangular prism with aspect ratio of 1:5 is investigated at Reynolds number of 200. In this case, the wake instability and the laminar-turbulence transition generally do not occur. Figure 8.6 presents the time histories of the lift force and torsional moment coefficients under the vertical oscillation input with $fr=0.5$ and $A/B=3\%$ (labeled as Case 5). Comparing with the results of Case 3, the results in Figure 8.6 suggest that the thickness itself actually cannot generate the aerodynamic nonlinearity even in the situation that the significant increase of thickness results in the steady separation. The simulation results of the same case with a larger oscillation amplitude ($A/B=30\%$)
suggests that the increase of oscillating amplitude also increases the nonlinear coupling between the drag force and lift force or torsional moment.

![Graph](image)

Figure 8.6. Output time histories of the rectangular prism at Re=200 and \( U_r = 2 \)

The last case under investigation is a rectangular prism with aspect ratio of 1:5 at the Reynolds number of \( 10^4 \). In such a case, the wake instability and the laminar-turbulence transition substantially exist, hence, the interior noise significantly contributes to the total force on the structure. Figure 8.7 and Figure 8.8 present the time histories of the lift force and torsional moment coefficients under the vertical oscillating input with \( f_r = 0.5 \) and \( A/B = 3\% \) (labeled as Case 6) before and after subtracting the effects of interior noise, respectively. It is shown that both of these cases present significant nonlinear aerodynamics. Although the magnitudes of the force or moment induced by the interior noise and the motion-induced effects are usually at the same order, the underlying mechanism is quite different. The former falls under the forced effects, which usually
impacts the fatigue life of the structures, while the latter is a self-excited case, which may result in catastrophic events at high wind velocity.

Figure 8.7. Output time histories of the rectangular prism at $\text{Re}=10^4$ and $U_r=2$

Figure 8.8. Output time histories of the rectangular prism at $\text{Re}=10^3$ and $U_r=2$ without interior noise
8.3 Computational Approach

As mentioned in Chapter 8.1, the efficacy of Volterra theory based models in simulating nonlinear bridge aerodynamics has been demonstrated in the literature based on several phenomenological models, which are basically governed by ordinary differential equations in time domain (Wu and Kareem 2012b; 2012e). In this study, efforts are made to apply the Volterra based analysis framework to nonlinear aerodynamics governed by Navier-Stokes equations in time-space domain. In such a case, the computational domain needs to be discretized first, and then the time step of the large system of ordinary differential equations arising therefrom is advanced. In the case of the motion-induced aerodynamic effects, the input is the time-dependent boundary due to the moving rigid body in the computational domain and the primary output is selected to be the integrated forces on the structure.

8.3.1 Computational Domain Description

As the focus of this study is to establish an analysis framework of nonlinear aerodynamics, the simulation of Case 0 with Volterra theory based model, which is utilized to validate the computational procedure of the proposed scheme mentioned in Chapter 8.1, will be elaborated. Besides, in order to investigate the contribution of the higher-order kernels and the corresponding nonlinear convolutions to the response of bluff-body aerodynamic system, the simulations of Case 5 and Case 6 with Volterra theory-based model will also be discussed.

The computational domain size is chosen as $-15.5B \leq Lx \leq 30.5B$ and $-15B \leq Ly \leq 15B$ for all investigated cases, as indicated in Figure 8.9. The far field and structural
surface boundary conditions are also presented in the figure, wherein the Dirichlet pressure condition at outlet is based on the presumption that the computational domain is sufficiently large. Figure 8.10 shows the mesh of central part of the computational domain for Cases 5 and 6. The distance of the first grid layer to the structural surface is 0.0012B, and the total cells of the computational domain are 261,000. Case 0 (with 21600 total cells) has a very similar mesh field with that shown in Figure 8.10.

![Figure 8.9. The computational domain with boundary conditions](image)

![Figure 8.10. The mesh of central part of the computational domain](image)
8.3.2 Governing Navier-Stokes Equations Nested in an Arbitrary Lagrangian-Eulerian Framework

Generally, the Eulerian description is preferred in fluid dynamics while a Lagrangian description is preferred in structural dynamics. As there is a moving interface between the fluid and the rigid body in the simulation, an arbitrary Lagrangian-Eulerian (ALE) description for the fluid domain is employed (Noh 1963; Hirt et al. 1974; Demirdžić and Perić 1990). As a result, fluids near the rigid body are described utilizing Lagrangian framework while the traditional Eulerian description of fluids far from the rigid body remains. In this study, the computational fluid dynamics (CFD) is carried out based on the Open Source Field Operation and Manipulation (OpenFOAM) C++ class library, where the spatial domain is discretized utilizing Finite Volume Method (FVM). Hence, the computational domain consists of a set of discrete, non-overlapped, control volumes (cells). The values of independent variables are stored at the centers of control volumes, and the fluxes at the surface of the control volumes are based on the interpolation of cell-center values.

The formal conservation representation of the governing Navier-Stokes equations for the incompressible, viscous flow in the ALE framework could be expressed as (Noh 1963; Demirdžić and Perić 1988; Ferziger and Peric 2002)

\[
\frac{d}{dt} \int_{\Omega} \rho \Phi d\Omega + \int_{S} \rho \Phi (U - U_s) \cdot n dS = \int_{S} \rho \nabla \cdot \Phi \cdot n dS + \int_{\Omega} Q d\Omega
\]

(8.3a)

\[
\frac{d}{dt} \int_{\Omega} d\Omega = \int_{S} U_s \cdot n dS
\]

(8.3b)
where $\Omega$ and $S$ represent volume and surface for an arbitrary control volume; $\rho$ is the flow density; $\Phi$ means the scalar field; $\mathbf{n}$ is the outward-facing unit normal vector to the surface; $\mathbf{U}$ is the fluid velocity, $\mathbf{U}_S$ is the velocity of the moving boundary; $\chi$ is a constant or variable coefficient of diffusion, e.g., the flow kinematic viscosity $\nu$; $Q$ indicates the intensity of the internal source/sink of $\Phi$. Typically, the first term of Equation 8.3a represents the rate of change of the total amount of $\Phi$ within the control volume; the second term indicates the convective flux through the boundary of the control volume; the third term means the diffusive flux through the boundary; Equation 8.3b represents the relationship between the rate of change of the volume and the velocity of the moving boundary of the control volume.

8.3.3 Dynamic Mesh Strategy

In the Eulerian description the mesh is fixed, while in the Lagrangian description the motions of the mesh and fluid particles are synchronous. On the other hand, the employment of the ALE based CFD actually allows that the motions of the mesh and fluid particles are calculated independently, where the difference between the velocities of the mesh and the fluid particles are reflected by the relative velocity in the convection term of Equation 8.3a. Generally there are three sets of velocities needed to be determined, which are due to the structure, fluid particle and mesh motions. In this study, the trajectory of the structural motion is prescribed, and the motion of fluids is calculated utilizing momentum and mass conservations as described in the preceding section. Besides, it is noted that the advancement in time for the Navier-stokes equations in an ALE framework needs a priori knowledge of the mesh velocity in the entire computational domain at each time step.
Suppose the mesh velocity at the interface coincides with the prescribed moving rigid body velocity (to ensure the high resolution in the boundary layer) and the mesh velocity at far field is zero, the internal mesh velocity needs to be calculated. The deformed mesh may introduce extra discretization errors, hence, the primary concern in the selection of the dynamic mesh strategy is the grid quality at each time step with a reasonable computational cost. Among several popular strategies such as the spring analogy method (Batina 1989), the Laplacian or pseudo-solid smoothing techniques (Johnson and Tezduyar 1994; Löhner and Yang 1996) and the scheme based on radial basis functions (Bos 2009), the Laplacian smoothing technique is utilized here. Hence, the mesh velocity \( V \) in the computational domain could be calculated as (Jasak and Tuković 2007)

\[
\nabla \cdot (\kappa \nabla V) = 0
\]

(8.4)

with the boundary conditions at the moving interface and far field mentioned above. The diffusion coefficient \( \kappa \) can be a constant or a variable. In this study, \( \kappa \) is a function of the distance to the moving interface \( d_r \) and can be calculated as

\[
\kappa = \frac{1}{d_r^2}
\]

(8.5)

The deformed mesh fields in the vertical motion (amplitude of 0.6B) and torsional motion (amplitude of 25°) in the simulation of Case 5 with a coarse grid is presented in Figure 8.11. Re-mesh is not necessary for both motions.
Figure 8.11. The deformed mesh fields of Case 5 with a coarse grid: 
(a) deformed mesh field in vertical motion; (b) deformed mesh field in torsional motion

8.3.4 Numerical Algorithm

OpenFOAM uses cell-center FVM, hence, the calculation of convective flux and diffusive flux through the boundary of the control volume needs interpolations of cell-center values. For the convective flux, the third-order Quadratic Upstream Interpolation of Convective Kinematics (QUICK) scheme is utilized; for the diffusive flux, the second-order central difference scheme is employed. The time-marching utilizes first-order implicit Euler scheme. The pressure-velocity coupling is accomplished based on the
Pressure Implicit with Splitting Operators (PISO) algorithm. Each time step is separated into two time sub-steps: the velocity components are first solved sequentially based on the momentum equations without pressure gradient term involved; then the correct pressure distribution is searched and correction of velocity components are made to satisfy the incompressible condition; the maximum correction number is set to 3. At each time step/sub-step, a set of ordinary differential equations need to be solved. In this study, the conjugate-gradient method, with the diagonal incomplete-LU preconditioning, is employed to solve the velocity; and the generalized method of geometric-algebraic multi-grid (GAMG), with the Gauss-Seidel smoothing, is utilized to solve the pressure distribution. Typically, the computational time is substantially controlled by the iteration of the pressure to obtain convergence. All simulations are carried out until the equation residual falls below the tolerance of $10^{-7}$.

8.4 Computational Scheme Validation

The representation of fluids in the ALE framework can actually recover to the traditional Eulerian description of Navier-Stokes equations for a steady-state simulation. In order to validate the computational approach mentioned in the preceding section, the steady-state simulation of the zero-thickness flat plate at $Re=10^4$ is first carried out. The boundary layer velocity distribution is compared with the analytical solution of Blasius (1950). Then, the lift force and torsional moment of Case 0 are simulated and compared to the analytical solution of Theodorsen (1935) to validate the dynamic-steady simulation in this study. The validation procedure is presented in Figure 8.12. The convergent steady
state is utilized to be the initial condition of the dynamic-state simulation, while the initial condition for the steady-state simulation is a trivial issue for the laminar flow case.

Figure 8.12. The validation procedure of steady-state and dynamic-state simulations

8.4.1 Steady-State Simulation

For the 2-D case that a zero-thickness flat plate places parallel to a uniform flow \((U_0,0)\), Blasius showed the similarity solution of velocity distribution in the laminar boundary layer as (Blasius 1950)

\[
\frac{u}{U_0} = f(\eta) \tag{8.6a}
\]

\[
\eta = \frac{y}{\delta(x)} \tag{8.6b}
\]

where \(u\) is the velocity at point \((x, y)\) in the boundary layer; \(\delta(x)\) is the boundary layer thickness at the location \(x\), which has the unique form \(\delta \sim \sqrt{vx/U_0}\) according to dimensional argument; \(f(\eta)\) is governed by a third-order ordinary differential equation with three straightforward boundary conditions, namely \(f(0) = f'(0) = 0\) and \(f''(\infty) = 1\).
Hence, the similarity solution of velocity distribution could be calculated with the Runge-Kutta numerical technique. Figure 8.13 presents the exact velocity distribution of Blasius and the simulated velocity distribution based on the computation approach described in the preceding section for the case of a zero-thickness flat plate at Re=10^4, where master and slave sides mean the upper and lower surfaces of the zero-thickness flat plate, respectively. There totally 12 cases, which cover a broad range of locations in the upstream and downstream parts of the flat plate, are investigated. As suggested in the results of Figure 8.13, the simulations of all investigated locations in the boundary layer are quite good.

![Figure 8.13. Velocity distribution in the boundary layer of a steady-state, laminar flow](image)

8.4.2 Dynamic-State Simulation

For the 2-D case that a zero-thickness flat plate harmonically oscillates (vertically or torsionally) in a uniform potential flow \((U_0,0)\) with a zero angle of attack, Theodorsen
presented an analytical solution for the lift force and torsional moment as (Theodorsen 1935)

\[ L = \pi \rho b^2 \left[ \dot{h} + U_0 \alpha \right] + 2 \pi \rho U_0 b C(k) \left[ \dot{h} + U_0 \alpha + \frac{b}{2} \dot{\alpha} \right] \]  

(8.7a)

\[ M = \pi \rho b^2 \left[ -U_0 \frac{b}{2} \dot{\alpha} - \frac{b^2}{8} \dot{\alpha} \right] + \pi \rho U_0 b^2 C(k) \left[ \dot{h} + U_0 \alpha + \frac{b}{2} \dot{\alpha} \right] \]  

(8.7b)

where \( b = B/2 \) is the half chord length of the flat plate; \( h \) and \( \alpha \) represent the vertical and torsional displacement, respectively; \( k = b\omega / U_0 \) is the dimensionless, reduced circular frequency with \( \omega \) as the circular frequency of the flat plate vibration; \( C(k) \) is the complex Theodorsen circulation function expressed as \( C(k) = F(k) + iG(k) \). The Theodorsen circulation function is originally expressed in terms of Bessel or Hankel functions, which could be calculated using R.T. Jones' approximation (R.T. Jones 1940). Figure 8.14 presents Theodorsen solutions of lift force coefficient \( [C_L = L / (\rho U_0^2 B^2) ] \) and torsional moment coefficient \( [C_M = M / (\rho U_0^2 B^2) ] \) together with the corresponding simulation results in Case 0. The results shown in Figure 8.14 suggest that the dynamic-state simulation based on the computation approach mentioned in the preceding section is excellent. The frequency transfer functions (flutter derivatives) \( H_1(k) \) and \( A_1(k) \) could be calculated through the Fourier transform of the lift force and torsional moment time histories in Figure 8.14. The results indicate that the accuracy of simulation of the dynamic state in this study is improved compared to that in the literature (e.g., Walther and Larsen 1997; Le Maître et al. 2003; Fransos and Bruno 2006).
8.5 Volterra Kernel Identification based on Impulse Function

One of the most important features of the impulse function is that it is the most elementary component of any signal, hence, the response based on the impulse function input can be utilized, within the same dynamic system, to simulate the response resulting from any other input (Schetzen 1980). The demonstration of this important concept in the application of Volterra theory based models has recently been presented (e.g., Silva 1997; Wu and Kareem 2011).

On the other hand, the Volterra kernel identification based on the impulse function has its own limitations in the applications of bluff-body aerodynamics governed by the Navier-Stokes equations. From the mathematical point of view, the Navier-Stokes equations (a hybrid elliptic/parabolic-hyperbolic type) present a diverse range of properties with various values of their parameters, e.g., Reynolds number. As mentioned
in the preceding content, with various Reynolds numbers three basic regions could be recognized, namely the stationary, wake-instability and chaos regions. Each region should be treated as an individual dynamic system. There is no reason that the Volterra kernels identified in a certain region (dynamic system) can be employed in another region (dynamic system). Even at a fixed Reynolds number, it is well known that the flow patterns surrounding the structure in the vortex-induced vibration are significantly different for various oscillation amplitudes (velocities) (e.g., Williamson and Govardhan 2004; Kareem and Wu 2012). Besides, as suggested in Section 8.2, even in the case of the zero-thickness flat plate nonlinear aerodynamics occurs as the oscillation amplitude (velocity) becomes large, hence, the identified Volterra kernel under a certain oscillation amplitude (velocity) set can only apply to a limited range of oscillation amplitudes (velocities). This may indicate that the time-dependent boundary conditions of the Navier-Stokes equations suggest that it may not be sufficient to characterize mathematical properties by the traditional definition of the Reynolds number. Actually, a typical definition of the Reynolds number in the research of hovering insect flight involves the amplitude and frequency of the moving boundary (Wang 2000). Furthermore, if the Volterra kernels are identified based on CFD principles, from the numerical point of view each computational approach with a given mesh field has limited space and time resolutions, which derogate the simulation results based on the impulse function input.

As this study attempts to establish a nonlinear analysis framework of aerodynamics by employing the Volterra theory based models, the abovementioned mathematical and numerical issues are not investigated in detail. Instead, the physical
parameters (e.g., the Reynolds number and the oscillating amplitude and frequency) are systematically selected so that for each numerical case the aerodynamic systems at the kernel identification and response simulation stages could be treated to belong to the same regime.

8.5.1 Impulse Function Design

A purely mathematical description of the impulse function cannot be implemented in real or computational world due to its infinitely-high amplitude at an infinitely-small value-defined region. Instead, the smoothed-ramped function is usually utilized to approximate the impulse function, where the summation of its values is equal to one. It should be stressed that it is the velocity of the moving boundary \( V_b \) that results in the aerodynamic effects. Hence, the time trace of \( V_b \) needs to be designed as an impulse function rather than the displacement of the moving boundary. This argument is frequently ignored in the literature. On the other hand, the displacement trajectory of the moving boundary \( S_b \) controls the moving mesh field and the acceleration magnitude \( A_b \) results in fluid added mass. As a result, all of these three quantities should be investigated for the designed smoothed-ramped impulse functions. Three smoothed-ramped impulse functions popularly employed in the literature, which are based on the sinusoidal, polynomial and exponential functions (e.g., Bakhle et al. 1991; Lesieutre et al. 1994; Fransos and Bruno 2006; Marques and Azevedo 2007), respectively, are expressed as follows

\[
V_{b-sin} = \begin{cases} 
0 & t < t_0 \\
\frac{\pi a_s}{2 T_s} \sin \left( \frac{\pi t}{T_s} \right) & t_0 \leq t < T_s \\
0 & t \geq T_s 
\end{cases}
\]  

(8.8a)
where $a_s$ and $T_s$ denote the amplitude and duration of the smoothed-ramped impulse function, respectively; $t_0$ is the beginning time; $b$ and $c$ in Equation 8.8c have similar functions of $T_s$ and $t_0$, respectively. However, there is no clear duration and beginning time of the smoothed-ramped impulse function based on the exponential function.

Figure 8.15 shows the displacement, velocity and acceleration of the designed functions based on Equations 8.8a, 8.8b and 8.8c together with the PSD of velocity. The red dash lines in the figures present the PSD of the smoothed-ramped impulse functions with the duration of $2*T_s$. As indicated in Figure 8.15, the duration determines the frequency component range that the smoothed-ramped impulse function can cover, hence, the selection of the duration is case dependent. In the case of wind engineering, Fransos and Bruno (2006) suggested $T_s*U/B$ should be around one. In this study, the maximum value of $T_s*U/B$ is selected to be 0.2 as it is shown that the longer duration significantly affects the phase of the simulation response, which will be discussed in the next section. On the other hand, it should be noted that, in order to sufficiently resolve the smoothed-ramped impulse function in the CFD analysis the selected time step in the simulation usually needs to be proportion to the duration time. There is no specific
requirement for the amplitude $a$, so long as the aerodynamic system remains in the same region, as discussed in the previous section. Hence, a reasonable choice of amplitude of the smoothed-ramped impulse function may be close to that of the input velocity at the response simulation stage.

Figure 8.15. Characteristics of various smoothed-ramped impulse functions: (a) sinusoidal basis; (b) polynomial basis; (c) exponential basis
Generally, there is no distinct advantage among the smoothed-ramped impulse functions described in Figure 8.15. Nevertheless, since the beginning time and duration of the smoothed-ramped impulse function based on the exponential function can not be explicitly assigned, it is not employed here. The properties of displacement and velocity of these cases are basically similar, however, there is a discontinuity in the acceleration at the beginning or end time in the case of the sinusoidal function. Hence, the smoothed-ramped impulse function based on the polynomial function is utilized in the simulations. On the other hand, the unique discontinuous feature of the acceleration in the case based on the sinusoidal function, where the velocity is equal to zero, is employed to identify the added mass effects in the fluid-structure interaction. It is intractable to distinguish the circulatory effects (induced by the velocity) (Bisplinghoff et al. 1957) and added mass effects (induced by the acceleration) in other cases. It is shown that the added-mass effects induced by the structural motion in the limited computational domain has a error less than 0.2% compared to the theoretical solution based on the assumption of an infinite liquid flow domain (Lamb 1932). This also suggests that the selected computational domain in this study is sufficiently large. The added mass effects are relatively large and significantly important in the Volterra kernel identification based on the impulse function concept.

8.5.2 Volterra Kernele Identification and Aerodynamic Response Simulation

The Volterra kernels are obtained in kernel identification stage utilizing Equations 8.2a and 8.2b, and the aerodynamic response (integrated force or moment on the structure) are calculated in the response simulation stage using Equation 8.1. The direct CFD
simulation response is referred to as the exact solution and utilized to examine the fidelity of the convolution response based on the identified Volterra kernels.

8.5.2.1 First-Order Kernels and Response Approximations

8.5.2.1.1 Steamin-body Linear Aerodynamics: Case 0

Figure 8.16 presents the linear impulse response and the first-order kernel of the torsional moment coefficient under a vertical velocity impulse on zero-thickness flat plate at Re=$10^4$. The linear impulse response is obtained based on the single smoothed-ramped impulse function input with an amplitude equal to the velocity amplitude $A_V$ of the harmonic input at the response simulation stage. The first-order kernel is calculated based on Equation 8.2a, where the smaller input amplitude is equal to $A_V$ and the other input amplitude is 1.33 times larger. It should be noted that both linear impulse response and first-order kernel given in Figure 8.16 are only due to circulatory effects. The asymptotic values of both linear impulse response and first-order kernel, after several nondimensional time steps, approach zero. Figure 8.17 shows the linear response and first-order approximation together with the exact solution of the torsional moment coefficient in Case 0. Both the linear response and first-order approximation are calculated based on the convolution scheme as indicated in Equation 8.1, while the former is the convolution response of linear impulse response and the latter is the convolution response of first-order kernel. The results presented in Figure 8.17 suggest that the first-order approximation has better accuracy compared to that of the linear response. The identified Volterra kernels can be utilized to calculate the aerodynamic response for a wide range of oscillating frequencies, hence, the computational workload is significantly reduced.
Figure 8.16. Torsional moment coefficient under impulse vertical velocity of plate at Re=10^4

Figure 8.17. Torsional moment coefficient under harmonic vertical velocity of plate in Case 0

Figure 8.18 shows the first-order kernel of the lift force coefficient under a vertical velocity impulse on zero-thickness flat plate at Re=10^4, where only circulatory effects are presented. The asymptotic value of the first-order kernel, after several nondimensional time steps, approaches zero. Figure 8.19 presents the first-order approximation together with the exact solution of the lift force coefficient in Case 0. The results presented in Figure 8.19 suggest that the first-order approximation has a great
accuracy as compared to the exact solution. It should be noted that both the first-order approximation and exact solution result from the summation of circulatory and added-mass effects. In order to investigate the contribution of the added-mass effects, the first-order approximation involving only circulatory effects are presented in Figure 8.19, which indicates a significant contribution of the added-mass effects in this case.

Figure 8.18. Lift force coefficient under impulse vertical velocity of plate at $\text{Re}=10^4$

Figure 8.19. Lift force coefficient under harmonic vertical velocity of plate in Case 0
In order to investigate the effects of the time duration of the smoothed-ramped impulse function on the convolution response, Figure 8.20 presents the first-order kernels based on nondimensional time duration of 0.2 and 0.4, while Figure 8.21 shows the corresponding convolution responses based on these two first-order kernels. The results in Figure 8.21 suggests that the phase of the convolution response is sensitive to the duration time of the smoothed-ramped impulse function. In addition, it should be noted that the frequency transfer functions (flutter derivatives) are extremely sensitive to the phase of the response signal.

Figure 8.20. First-order kernels based on different duration time

8.5.2.1.2 Bluff-Body Linear Aerodynamics: Case 5 and Case 6

Figure 8.22 presents the first-order kernel of the torsional moment coefficient under a vertical velocity impulse on 1:5 rectangular prism at Re=200. Figure 8.23 shows the first-order approximation together with the exact solution of the torsional moment coefficient in Case 5. The results presented in Figure 8.23 suggest that the first-order approximation accurately compares with the exact solution.
Figure 8.21. First-order approximations based on the first-order kernels with different duration time

Figure 8.22. Torsional moment coefficient under impulse vertical velocity of rectangle at Re=200

Figure 8.24 presents the first-order kernel of the torsional moment coefficient under a vertical velocity impulse on 1:5 rectangular prism at Re=10^4. It should be noted that the effects of interior noise has been subtracted in the Volterra kernels presented in this study. As indicated in the figure, there is an upward secondary bump in the first-order kernel for bluff-body aerodynamic system at high Reynolds number. The asymptotic value of the first-order kernel, after several nondimensional time steps,
Figure 8.23. Torsional moment coefficient under harmonic vertical velocity of rectangle in Case 5 approaches zero. Figure 8.25 shows the first-order approximation together with the exact solution of the torsional moment coefficient in Case 6. The results presented in Figure 8.25 suggest that the first-order approximation may not be sufficient to accurately represent the exact, nonlinear solution of the bluff-body aerodynamics at high Reynolds number.

Figure 8.24. Torsional moment coefficient under impulse vertical velocity of rectangle at Re=10^4
8.5.2.2 Second-Order Kernels and Nonlinear Response

8.5.2.2.1 Steamlined-Body Nonlinear Aerodynamics: Case 0

As indicated in the simulation results given in the preceding section, the first-order approximation appears to be sufficiently accurate for both the lift force and torsional moment in Case 0. Hence, the nonlinear contribution is obviously negligible for the streamlined-body aerodynamics. Figure 8.26 presents the diagonal term of the second-order kernel of the lift force coefficient together with the first-order kernel under impulse vertical velocity of the zero-thickness flat plate at \( \text{Re}=10^4 \) and Figure 8.27 gives the torsional moment coefficient. As shown in the figures, the diagonal terms of the second-order kernels are significantly smaller compared to the first-order kernels. Besides, the fading time of the former is significantly shorter compared to that of the latter. Also, typically the value of diagonal term is larger than that of the off-diagonal term.

It should be noted that the force or moment investigated here is actually a global aerodynamic response, which is obtained by integrating the pressures over the entire
structural surface. Hence, it may not present the nonlinearity of the local aerodynamic response due to the summation operation. Figure 8.28 presents PSD of the relative pressure at a point located at the center line in the wake and just behind the flat plate \((x=0.51B, \ y=0)\) in Case 0. The results suggest that there are significant nonlinear frequency components in the relative pressure signal due to the vertical harmonic oscillation of the flat plate. Figure 8.29 shows the diagonal term of the second-order
kernel of the relative pressure together with the first-order kernel. The figure suggests the diagonal term of the second-order kernel is even larger compared to the first-order kernel. The accurate simulation of the localized aerodynamics is important in the active controls where a flap action is introduced and aerodynamic optimization applications where localized shape changes may be influenced.

Figure 8.28. PSD of the relative pressure at the point located at (0.51B, 0)

Figure 8.29. First-order kernel and diagonal term of the second-order kernel of relative pressure
8.5.2.2.2 Bluff-Body Nonlinear Aerodynamics: Case 5 and Case 6

Figure 8.30 presents the diagonal term of the second-order kernel of the torsional moment coefficient together with the first-order kernel under a vertical velocity impulse on 1:5 rectangular prism at Re=200. As the figure suggests, the diagonal terms of the second-order kernel are significantly smaller compared to the first-order kernel. Figure 8.31 shows the torsional moment coefficient under two sequential vertical velocity impulses (time lag=1.6*T<sub>s</sub>) on 1:5 rectangular prism at Re=200, which are utilized in the calculation of the off-diagonal term of the second-order kernel. The results in the figure suggest that there is no obvious difference between the two impulse responses, which indicates that the off-diagonal term is negligible.

![Figure 8.30. Diagonal term of the second-order kernel of torsional moment coefficient of bluff-body aerodynamics at low Reynolds number](image)

Figure 8.32 presents the diagonal term of the second-order kernel of the torsional moment coefficient together with the first-order kernel under a vertical velocity impulse on 1:5 rectangular prism at Re=10<sup>4</sup>. As the figure suggests, the contribution of the diagonal term of the second-order kernel in the case of bluff-body aerodynamics at high
Figure 8.31. Response of two sequential impulse inputs of bluff-body aerodynamics at low Reynolds number

Reynolds number to the total response is relatively large compared to those in the case of streamlined-body aerodynamics (Case 0) and bluff-body aerodynamics at low Reynolds number (Case 5). Figure 8.33 shows the torsional moment coefficient under two sequential vertical velocity impulses (time lag=1.6*T₀) on 1:5 rectangular prism at Re=10⁴ to evaluate the off-diagonal term of the second-order kernel. The results in the figure suggest that the impulse response of the second impulse input presents a secondary trough, which is opposite to that of the first-order kernel and suggests that the contribution of this off-diagonal term is non-negligible. A nonlinear simulation of the bluff-body aerodynamic system in Case 6 needs more identifications of the off-diagonal terms of the second-order kernel.

8.5.3 Revisit Impulse Function in Discrete Time Domain

As discussed in the preceding section, in order to obtain the linear and nonlinear impulse responses in time domain or linear frequency transfer functions in frequency domain with high accuracy, the time duration of the smoothed-ramped impulse function should be
Figure 8.32. Diagonal term of the second-order kernel of torsional moment coefficient of bluff-body aerodynamics at high Reynolds number

Figure 8.33. Diagonal term of the second-order kernel of torsional moment coefficient of bluff-body aerodynamics at high Reynolds number
sufficiently short. Accordingly, the time step in the computational simulation is usually extremely small to achieve a high resolution for the smoothed-ramped impulse function, which leads to a heavy computational workload. On the other hand, if the duration time is selected as the unit time step $\Delta t$, there is a unique nontrivial version of the smoothed-ramped impulse function, as indicated in Figure 8.34. The PSD suggests that the frequency components are equally distributed at all frequencies, like a white noise, for the smoothed-ramped impulse function with duration of $\Delta t$. Besides, since there is only a unique nontrivial point, the computation workload in the simulation is significantly reduced. In such a case, the impulse function (Dirac delta function) in the continuous time domain is actually replaced by the sample function (Kronecker delta function) in the discrete time domain. The sample function for discrete time signals plays an equivalent role as the impulse function for continuous time signals (Oppenheim et al. 1999).

![Figure 8.34](image_url)

**Figure 8.34.** Sample function of moving boundary velocity and the related displacement, acceleration and PSD

Basic properties of the sample function in the application of Navier-Stokes equations was discussed by Silva (1997). Figure 8.35 presents the first-order kernel and the diagonal term of the second-order kernel of the torsional moment coefficient based on
the sample function of the plate vertical velocity at $\text{Re}=10^4$. The results suggest that both first-order and second-order kernels fade away quickly, which further reduces the computational workload, especially in the identification of higher-order kernels where the simulation needs to be carried out for each time lag. In order to investigate the fidelity of the Volterra kernel identification based on the sample function, Figure 8.36 shows the first-order approximation based on the identified first-order kernel together with exact solution of the torsional moment coefficient for Case 0. The results suggest that the simulation phase of the convolution response is more accurately represented as compared to that based on the smoothed-ramped impulse function. On the other hand, the relatively small discrepancy in amplitude may be improved by tuning the amplitude of the sample function. It should be noted that the Volterra kernel identification based on the sample function is quite sensitive to the numerical environment (Raveh 2001). Besides, the added-mass effects is critical in such applications.

![Figure 8.35. First-order kernel and diagonal term of second-order kernel based on sample function](image)

Figure 8.35. First-order kernel and diagonal term of second-order kernel based on sample function
8.6 Discussion

In light of this presentation, there are several issues worth further investigation. First is the assumption that the disturbance due to the wake instability, turbulence, vortex shedding and unsteady separation reattachment (interior noise) is uncorrelated with the motion-induced or gust-induced effects. The mechanism of the interaction between the interior noise and the motion-induced or gust-induced effects is not well understood (e.g., Hunt et al. 1990). Generally, in the case that the reduced wind velocity is very high (e.g., approaching to the critical flutter wind velocity), this assumption may be acceptable as the frequency components of the interior noise are relatively higher compared to those of the motion-induced or gust-induced effects. On the other hand, in the case that the reduced wind velocity is low (e.g., in the synchronization or "lock-in" region with large oscillation amplitude), this assumption definitely needs a further look.

Second is the absence of the turbulence modeling in this study. There are some studies which claim that, in the direct numerical simulation of fluid-structure interaction,
turbulence modeling may be insignificant even in the case that the grid size is not sufficiently small to resolve the Kolmogorov length scale (e.g., Tamura et al. 1988; Fujiwara et al. 1993; Nobili et al. 2008), however, its role needs to be further clarified. Although there is no turbulence modeling in this study, the numerical errors resulting from the discretization of Navier-Stokes equations (e.g., the third-order scheme for convection flux) may play a role in turbulence modeling (altering the effective viscosity of the flow) (e.g., Kawamura et al. 1986), whereas the connection of the numerical errors and turbulence effects on the solutions is not transparent. Besides, the effects of the absence of turbulence modeling on the aerodynamic nonlinearity is not clear. It should be noted the time-resolved computational simulation in turbulent modeling is extremely time consuming.

Third is the range of application of the Volterra theory-based nonlinear analysis framework in the simulation of nonlinear bluff-body aerodynamics. As mentioned in the preceding content, the mathematical properties of the governing Navier-Stokes equations present a significantly diverse scenarios according to the system parameters, such as Reynolds number, oscillation amplitude and frequency. Figure 8.37 presents the first-order kernel of the torsional moment coefficient under a vertical velocity impulse on 1:5 rectangular prism at Re=200 based on the smoothed-ramped impulse functions with different maximum velocity amplitudes, where the reference case is the one utilized in the preceding section while Case A has four times larger maximum velocity amplitude. It is shown that there is an extra fluctuation in the first-order kernel in the case of larger velocity amplitude, which may suggest that the flow pattern surrounding the rectangular prism is changed (e.g., a new vortical structure in the separated flow). Figure 8.38 shows
the first-order kernel of the torsional moment coefficient under a vertical velocity impulse on 1:5 rectangular prism at \( \text{Re}=10^4 \) based on the smoothed-ramped impulse functions with different maximum velocity amplitudes, where the reference case is the one utilized in the preceding section while Case A and Case B have the ten times larger and double maximum velocity amplitudes, respectively. It is shown that the time trace of the torsional moment coefficient in Case A is significantly different from those in other cases, which may suggest that the flow pattern surrounding the rectangular prism is altered. On the other hand, it is probable that the flow patterns in Case B and the reference case have negligible difference, which indicates that these two cases could be treated as being in the same aerodynamic regime. It is necessary to systematically investigate this issue to determine the range of application of each identified Volterra model. The generalized Volterra kernel identification scheme developed by Wu and Kareem (2011), where arbitrary amplitudes of the impulse functions can be employed, is a good candidate for such a systematic investigation.

Figure 8.37. Torsional moment coefficients at \( \text{Re}=200 \) with difference maximum velocity amplitudes in the identification
Figure 8.38. Torsional moment coefficients at Re=$10^4$ with difference maximum velocity amplitudes in the identification

8.7 Concluding Remarks

The physical sources of nonlinear aerodynamics are first investigated. The results indicate that the thickness of the structure and the viscosity of the fluids may not directly result in aerodynamic nonlinearities. On the other hand, the oscillation frequency and amplitude of a structure in the flow have significant effects on the generation of nonlinear aerodynamics. A nonlinear analysis framework of bluff-body aerodynamics is systematically established based on the Volterra theory. If a linear mapping is sufficient to describe the aerodynamic system, only first-order kernel of Volterra series is utilized as the higher-order kernels' contribution is negligible. The Volterra kernels, which are identified based on computational fluids dynamics schemes with impulse function input concept, can be utilized to calculate the aerodynamic response for a wide range of oscillation frequencies. Hence, the computational workload is significantly reduced compared to the conventional linear analysis in the frequency domain. If a linear mapping
is not sufficient to describe the aerodynamic system, the linear and nonlinear convolutions based on the first-order and second-order Volterra kernels, respectively, are utilized in this study to investigate the nonlinear aerodynamics as a first step beyond the linear analysis. The simulation results indicate that the streamlined-body and bluff-body aerodynamics at low Reynolds number are basically linear, while the nonlinear contribution cannot be ignored in bluff-body aerodynamics at high Reynolds number.

In order to establish the criteria for appropriate selection of the highest order of Volterra series needed in the simulation of bluff-body aerodynamics, more cases need to be simulated and investigated. In the estimation of nonlinear convolutions, an efficient numerical algorithm would help to expedite the simulation. The computational procedure of two-dimensional case study discussed here could be easily expanded into three-dimensional case. Immediate applications of the established nonlinear analysis framework of aerodynamics are abound in the area of flutter and post-flutter analyses, buffeting, vortex-induced limit cycle vibration, active control of bluff-body aerodynamics and insect flight.
CHAPTER 9:
GENERAL CONCLUSIONS AND FUTURE DIRECTIONS

The focus of this study was to address several critical issues related to nonlinear bluff-body aerodynamics by responding to the following questions: (1) What are the typical nonlinear behaviors observed from wind tunnel studies and full-scale observations and their underlying physics? (2) What are the effects of nonlinearity and unsteadiness on bluff-body aerodynamics? (3) What is the ability of existing nonlinear models to capture nonlinear and unsteady effects? (4) Is it possible to go beyond the current nonlinear models, and establish more effective nonlinear unsteady low-dimensional modeling techniques?

General conclusions drawn from this study and possible future extensions of this work are presented in the following sections.

9.1 Concluding Remarks

(1) Nonlinear features in bluff-body aerodynamics can be viewed from four perspectives: (i) non-proportional relationship between amplitudes of input and output; (ii) single-frequency input exciting multiple frequencies; (iii) amplitude dependence of aerodynamic and aeroelastic forces; (iv) hysteretic behavior of aerodynamic forces versus angles of attack. In addition, limit cycle oscillations of bluff bodies resulting from nonlinear aerodynamic effects are observed frequently in the wind tunnel or in full-scale. Sources
of nonlinearity stemming from body thickness, oscillation frequency and amplitude of the cross-section, and viscosity of the surrounding fluid were examined using computational fluid dynamics (CFD) techniques. It was demonstrated that the body thickness by itself cannot be responsible for aerodynamic nonlinearity even in the case when a significant increase in thickness results in steady flow separation. The high frequency of body oscillations not only increases nonlinear coupling between the torsional moment and drag force, but also generates contribution from nonlinear self-interaction for both the lift force and torsional moment. Besides, it was also well recognized that unsteady effects become significantly important as the oscillation frequency increases (low reduced wind velocity). Larger amplitudes of oscillations increase the nonlinear coupling between the drag force and lift force or torsional moment. Nonetheless, nonlinear self-interaction of lift force and torsional moment do not increase with the amplitude of oscillations. Finally, the viscosity by itself tends to inhibit nonlinear effects.

(2) It is noted that for the example bridge deck that has been considered as a case study, the contribution to the aerodynamic/aeroelastic response resulting from the fluid memory (unsteady) effects (typically reducing the bridge deck response) is larger than the contribution given by nonlinear effects. Specifically, for the aerodynamic response, the effects of static nonlinearity are more significant for the vertical degree of freedom than for the torsional degree of freedom while the fluid memory effects are more significant for the torsional degree of freedom than for the vertical degree of freedom. For the aeroelastic response, consideration of nonlinear effects involved in the hybrid scheme results in increasing the critical flutter wind velocity. Opposing trends emerge regarding the influence of fluid memory effects. On the one hand, the linear memory effects reduce
the critical flutter wind velocity, while on the other hand, the nonlinear memory effects increase the critical velocity. In a particular model, if the effects of turbulence on nonlinearity significantly influence the aeroelastic response, the resulting critical flutter velocity is increased. On the other hand, if in a model turbulence manifests itself through fluid memory effects, then the resulting critical flutter velocity is reduced at low turbulence intensities (e.g., $I_u=5\%$), but slightly increased at higher turbulence intensities.

(3) Six models for predicting the aerodynamic/aeroelastic behavior of a bridge deck (five existing and one new), all of which are based on the conventional analysis framework, were systematically reviewed. These models include: the quasi-steady (QS) theory-based model (nonlinear effects without fluid memory); the corrected QS theory-based model (nonlinear effects with linear fluid memory at a fixed reduced wind velocity); the linearized QS theory-based model (linear effects without fluid memory); the semi-empirical linear model (linear effects with linear fluid memory); the hybrid model (nonlinear effects with linear fluid memory); the modified hybrid model (nonlinear effects with nonlinear fluid memory). Predictive features of these models were examined considering linear and nonlinear unsteadiness (fluid memory) and nonlinearity in wind-bridge interactions. In general, these models are not able to represent all the characteristic features of nonlinear fluid-structure interactions.

(4) To accurately model the wind-induced linear/nonlinear effects on bluff bodies, a cache of nonlinear and unsteady modeling tools comprised of a series of low-dimensional models that are characterized by different levels of simulation was developed. These models were grouped under the three categories of "black-box", "gray-box" and "white-box". The fidelity of the proposed artificial neural network (ANN)
model ("black-box"), nonlinear moving average model (within the framework of Volterra theory) ("gray-box"), and Volterra series-based model in which the kernels are identified using impulse-functions ("white-box") to simulate nonlinear bluff-body aerodynamics have been verified with numerical examples involving wind-tunnel or CFD-based data. The Volterra series-based model comprised of single and multi-convolutions in terms of the aerodynamic or aeroelastic inputs to simulate the wind-induced effects on bluff bodies was discussed in detail. The theoretical basis of this model for the simulation of nonlinear aerodynamics was investigated by means of a "peeling-an-onion" type approach. Accordingly, a generalized Volterra kernel identification scheme was developed, and the linear and nonlinear kernels of various cross-sections were identified utilizing existing nonlinear phenomenological models or CFD-based simulations.

9.2 Future Directions

(1) Improvement in the efficiency and robustness of low-dimensional models is a topic of cutting-edge research in the field of aerodynamics. Although the models reviewed and proposed in this work give a significant contribution in this direction, further research effort is needed to enhance their capabilities in simulating the nonlinear aerodynamic behavior of bluff bodies. Since the cache of nonlinear unsteady modeling tools for bluff-body aerodynamics has been developed in this work, a significant step will consist in integrating other low-dimensional models (e.g., describing function, trajectory piece-wise linearization, autoregressive moving average, Uryson model, projection-pursuit model, proper orthogonal decomposition, wavelet-based scheme, Chen-Fliess series) into the library of nonlinear aerodynamics simulation schemes. Besides, the selection of the most
appropriate modeling scheme among a wide range of low-dimensional models to simulate nonlinear unsteady bluff-body aerodynamics and aeroelasticity is quite challenging. To this aim, it would be advantageous to define a nonlinearity index which would guide the selection of the level of nonlinear and unsteady simulation, namely linear schemes in the basic level (conventional linear schemes), nonlinear schemes at the basic level (conventional nonlinear schemes), level 1 ("black-box"), level 2 ("gray-box") or level 3 ("white-box"). To be effective, such a nonlinearity index would have to be based on the knowledge available or easily obtained, to end users.

(2) Part of this work focused on the Volterra kernel identification (using impulse function concept) based on the existing nonlinear phenomenological models or CFD schemes. It is necessary to develop a specially designed forced vibration system utilized for the linear and nonlinear Volterra kernel identification through wind tunnel experiments. A challenging issue to accomplish this task lies in the fact that the higher-order kernel identification requires simultaneously applied multi-input signals.

(3) In this study, the second-order truncated Volterra series has been applied (using impulse function concept) to simulate nonlinear bluff-body aerodynamics as a first step beyond conventional linear simulation. It is well known that the even-order convolution captures the even-order super-harmonics and asymmetric nonlinearity, whereas odd-order convolution would be necessary to capture the odd-order super-harmonics and symmetric nonlinearity. Therefore, the third-order Volterra model (consisting of first-, second- and third-order kernel terms) may offer the lowest order nonlinear model for comprehensively simulating nonlinearities in bridge aerodynamics. Hence, a future development of this work will be to establish a nonlinear analysis
framework based on the third-order truncated Volterra series. Critical to this aim will be the development of an effective third-order Volterra kernel identification scheme.

(4) In discrete time/frequency domain, there are mainly four popular Volterra identification techniques, which are the general identification method, the harmonic probing method, identification based on Wiener orthogonal kernels, and identification based on impulse function concept. While the first identification technique does not require a specific input sequence, the last three techniques require as input sequences harmonic components, Gaussian white noise and impulse function, respectively. It is very important to investigate characteristics of the input sequence used for the Volterra kernels identification, especially when the identified kernels are of higher-order. Besides, it will be interesting to systematically review and compare the efficacy of these kernel identification schemes in the context of nonlinear bluff-body aerodynamics.

(5) There are several essential assumptions made for pruning the Volterra model (using general identification method), which need to be verified using wind-tunnel or CFD-based data. Besides, it is necessary to develop corresponding strategy for pruning the cross kernels of the Volterra model based on the aerodynamic considerations in the case when multi-input multi-output (MIMO) nonlinear bluff-body aerodynamics is considered.

(6) Another interesting subject of future study will be the investigation of the linkage between each model belonging to different classifications ("black-box", "gray-box" and "white-box"). For example, it can be shown that a neural network with a specific architecture, namely the time-delay neural network, is equivalent to a Volterra series. In general, the training process of ANN is computationally more efficient as
compared to the kernel identification of Volterra series, while the Volterra theory-based model capture better physical significance in the simulation of bluff-body aerodynamics. The investigation of the potential linkage between Volterra series and ANN may improve both of these nonlinear models.

(7) Conventional nonlinear models, e.g., the hybrid model, actually focus on the consideration of nonlinearity with amplitude dependence, which is introduced through the dependence of the flutter derivatives on the low-frequency dynamic angle of attack (resulting from a combination of the structural motions and approaching turbulence). The nonstationarity of the approaching wind and its effects (together with wind direction effects) on the dynamic angle of attack in the simulation of nonlinear bluff-body aerodynamics should be considered. Besides, the exact demarcation between the low- and high-frequency fluctuations currently lacks a rigorous treatment. In particular, there is no appropriate approach to take into account the dependence of the flutter derivatives on the frequency of the low-frequency dynamic angle of attack. The empirical mode decomposition or wavelet analysis in nonstationary wind analysis could be applied to investigate this intractable issue. Instead of using the high-pass and low-pass filters, where the fluctuating wind signal is divided into two parts, the empirical mode decomposition or wavelet analysis techniques could decompose the broadband signal into a collection of successively more band limited components.

(8) Due to the complexity of the measurement of aerodynamic transfer functions in wind tunnel, the dependence of aerodynamic admittance functions on the low-frequency dynamic angle of attack is not included in conventional nonlinear aerodynamics/aeroelasticity analysis framework. It should be noted that there is an
underlying connection between the motion-induced and gust-induced effects. For the streamlined-body aerodynamics, there is an analytical relationship between the Wagner function (or Theodorsen function) and Küssner function (or Sears function). Hence, the dependence of flutter derivatives on the angle of attack measured in the wind tunnel may shed light on the amplitude dependence of aerodynamic admittances.

(9) Conventionally, aerodynamic/aeroelastic transfer functions (e.g., flutter derivatives) are assumed to vary monotonically with the angle of attack. Recent observations, however, indicate that they may exhibit a hysteretic loop with respect to the angle of attack. As a function of the angle of attack and the reduced wind velocity, the advanced aerodynamic/aeroelastic transfer functions could be described as a "potato" like variable in the three-dimensional space. Corresponding wind-tunnel experiments for conveniently obtaining hysteretic data and semi-empirical analytical models (e.g., the modified hybrid model proposed in this study) need to be established for developing this advanced analysis scheme.

(10) The study focused on the nonlinearity of aerodynamics without considering nonlinearities of structural origin. In the case of super-tall buildings, super-long bridges or stay cables, the structural nonlinearity (especially the geometric nonlinearity) could be very important. Based on Hamilton's principle, the basic governing equations of structural dynamics under wind loads, where the nonlinearities of both aerodynamic and structural origins are involved, can be easily derived. It will be interesting to investigate the interaction between aerodynamic and structural nonlinearities.

(11) In the study of cable-stayed bridge aerodynamics, the aerodynamic or aeroelastic response of decks is typically examined without considering the dynamic
cable-deck interactions. In light of the observed nonlinear behavior, it is critical to assess the dynamics of combined cable-bridge dynamics under winds.

(12) The approach consisting in introducing perforations in the bridge deck as an effective passive aerodynamic control strategy to enhance the aeroelastic stability of the long-span bridges has been widely investigated. Also, schemes to actively control aerodynamics of airfoils have been systematically studied using the synthetic jet technology. Both of these passive and active aerodynamic control approaches are based on the "dynamic virtual shaping" concept. An examination of the aerodynamics of bridges and buildings using suction or ejection of air would be fundamental for the implementation of such a promising means for shaping the aerodynamics of structures.
APPENDIX A:

STEP AND IMPULSE RESPONSE FUNCTIONS OF BRIDGE AERODYNAMICS

Two elementary response features of bridge aerodynamics, namely unit-step (indicial) and unit-impulse response functions, as the fundamental building blocks for the convolution integral, are systematically reviewed. The comparison of these elementary response functions is carried out from theoretical, experimental and numerical perspectives. Theoretically, their new interpretation is given in the context of bridge aerodynamics. Experimentally, the underlying mechanism concerning these elementary response functions as applied to bridge aerodynamics is investigated, e.g., the "overshooting" feature and the applications to the simulation of gust- and motion-induced forces. Numerically, a comparison of the indicial and rational function approximations, often utilized in the indirect identification of the effective unit-step and unit-impulse response functions, respectively, is highlighted by underscoring the underlying physics in the associated mathematical derivation. A numerical example of a long-span bridge is presented to demonstrate the fidelity of simulation based on these numerical approximations.

A.1 Introduction

There is an increasing demand to advance the time domain analysis of bridge aerodynamics to effectively account for nonlinear effects and for real-time control
applications. Generally, a quasi-steady (QS) theory based analytical model is utilized in the time domain analysis. The basic premise of the QS model is that the reduced fluid speed is sufficiently high for the wake fluctuations to influence aerodynamic forces. Therefore, the aerodynamic forces depend only on the instantaneous status of the surrounding flow while any potential fluid memory effects are neglected. The QS assumption does not only alter the magnitude of the aerodynamic forces but also introduces a phase lag, which may be more critical. For example, in the case of a thin airfoil, even if the reduced frequency is as low as 0.1 the phase lag between the motion-induced lift force and the input motion is large (around 11°), which indicates a significant discrepancy between the QS theory based and exact results. Since the energy transformation between the structure and the surrounding flow is sensitive to the phase angle (Bisplinghoff et al. 1957), the QS theory should be employed for bridge aerodynamics with care.

In order to accurately consider the fluid memory effects in the time domain, the complex mapping rules (static linear/nonlinear relationships) and time lags (fluid memory effects) between the aerodynamic inputs and outputs need to be considered by a superposition of scaled and time shifted fundamental responses, i.e., convolution. While the linear convolution scheme will be the focus of this study for linear bridge aerodynamics, it can be extended to the nonlinear convolution scheme, which is represented utilizing a Volterra-type formalism, to account for nonlinear bridge aerodynamics (Wu and Kareem 2012b). The fundamental (elementary) response is conventionally taken to be the unit-step or unit-impulse response function which makes their identification critical. Unlike the case of an airfoil, a theoretical treatment of these
elementary response components of bridge decks is currently intractable. To address this issue, there have been some attempts to directly (experimentally) measure these elementary response functions of bluff cross-sections in wind tunnels (e.g., Caracoglia and N.P. Jones 2003b) or utilizing computational fluid dynamics (CFD) (e.g., Turbelin and Gibert 2001). A direct experimental identification approach needs additional refinement before it may become a routine identification scheme. On the other hand, the frequency domain identification of flutter derivatives and aerodynamic admittance functions is routinely carried out for conventional linear analysis framework (e.g., Scanlan and Tomko 1971). Hence, one of the promising indirect (numerical) identification approaches is to utilize the inverse Fourier transform of the measured flutter derivatives or aerodynamic transfer functions in the wind tunnel to obtain the equivalent unit-step response (Scanlan et al. 1974) or unit-impulse response function (Bucher and Lin 1988) (here referred to as effective unit-step or unit-impulse response function) by employing the indicial and rational function approximations, respectively. As a result, the multiplication between the input and transfer function (flutter derivative or aerodynamic transfer function) in the frequency domain is transformed into the convolution integral with respect to the input through effective unit-step or unit-impulse response function. One of the earliest performance analysis of bridge aerodynamics in the time domain was carried out by Béliveau et al. (1977) by utilizing the effective unit-step response function. More recently, Chen et al. (2000) utilized an effective unit-impulse response function approach for their analysis in time domain.

In this study, a comparison of the unit-step and unit-impulse response functions is made from theoretical, experimental and numerical perspectives. In Section A.2, the
basic theoretical background of the convolution integral based on the unit-step and unit-impulse response functions is presented. A new interpretation of these elementary response functions is given with specific reference to the bridge aerodynamics. Section A.3 focuses on the underlying mechanism as these elementary response functions are directly (experimentally) identified. Specific characteristics of these response components for bridge deck sections are investigated in light of airfoil sections and their simulation of gust- and motion-induced forces is discussed. In Section A.4, a comparison of the indicial and rational function approximations, which are employed in the indirect (numerical) identification of the effective unit-step and unit-impulse response functions, respectively, is emphasized through presentation of the physical significance of various steps involved in the mathematical derivation. Finally, a numerical example of a long-span bridge is provided to illustrate the fidelity of simulation based on these numerical approximations.

A.2 Theoretical Background

A.2.1 Basic Properties of Step and Impulse Response Functions

There are numerous comprehensive references that discuss theoretical background of the step and impulse response functions (e.g., von Kármán and Biot 1940). In this study, the relevant theoretical basis is presented in a slightly different fashion for convenient application to bridge aerodynamics. These two elementary functions, namely unit-step function $s(t)$ and unit-impulse function $\delta(t)$ are defined as

$$s(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$

(A.1a)
\[
\delta(t) = \begin{cases} 
+\infty, & t = 0 \\
0, & t \neq 0
\end{cases}
\] (A.1b)

The unit-impulse function is also known as Dirac delta function. It is obvious that \(\delta(t)\) is the derivative of \(s(t)\) with respect to time. If the response due to a unit-step function (unit-step response function) \(a(t)\) or the response due to a unit-impulse function (unit-impulse response function) \(h(t)\) is known, then the response \(y(t)\) of a time invariant, causal, linear system under arbitrary input \(x(t)\) can be obtained in the time domain by

\[
y(t) = a(t)x(0) + \int_{0}^{t} a(t-\tau)\dot{x}(\tau)d\tau \\
\text{or} \quad y(t) = x(t)a(0) + \int_{0}^{t} \dot{a}(t-\tau)x(\tau)d\tau
\] (A.2a)

\[
y(t) = \int_{0}^{t} h(t-\tau)x(\tau)d\tau
\] (A.2b)

where the dot denotes a derivative with respect to time. Equation A.2a is well known as Duhamel’s integral. The latter formula of Equation A.2a is obtained through integration by parts. The convolution integrals corresponding to Equations A.2a and A.2b are graphically presented in Figure A.1, where they are discretized into equivalent summations for the sake of illustration. Red parts denote that the increment or the magnitude of the input variable is negative.

As indicated in Figure A.1, an important difference between Equation A.2a and Equation A.2b is that the former is the superposition of response under the incremental input value at each infinitesimal time interval while the latter is the superposition of the response to the input amplitude at each infinitesimal time interval. The convolution integral based on the unit-step response function [Figure A.1(a)] is helpful to illustrate the
Figure A.1. Sketch of the convolution integrals through unit-step and unit-impulse response functions: (a) unit-step response function; (b) unit-impulse response function
gust-induced forces on a bridge deck as will be discussed in the following section, while the convolution integral based on the unit-impulse response function [Figure A.1(b)] straightforwardly presents the duration of the fluid memory effects of the system. In the frequency domain, the input of unit-impulse function is well known to uniformly excite all the frequency components (the inverse Fourier transform of unit-impulse function is a constant) while the input of the unit-step function excites the frequency components inhomogeneously (the inverse Fourier transform of unit-step function is \[1/(i\omega+\pi\delta(\omega))\] where \(\omega\) is the frequency component). It should be noted that one needs to introduce a mathematical manipulation, such as the operational rules (e.g., Sears 1940), to obtain the Fourier integral representation of the unit-step function since it is not a convergent integral.

With the property of Dirac delta function

\[x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(\tau - t)d\tau\]  \hspace{1cm} (A.3)

one could obtain a relationship between the unit-impulse and unit-step response functions as following

\[h(t) = a(0)\delta(t) + \dot{a}(t)\]  \hspace{1cm} (A.4)

Hence, although the abstract function \(\delta(t)\) is the derivative of \(s(t)\) with respect to time, this simple relationship has an exception at \(t=0\) as applied to the case of \(h(t)\) and \(a(t)\). This feature will be emphasized in the discussion of the relationship between the effective unit-step and unit-impulse response functions in bridge aerodynamics.
A.2.2 A New Interpretation of Step and Impulse Response Functions

Suppose a bridge deck is immersed in a uniform or fluctuating flow field as shown in Figure A.2. The wake induced by the motion and/or by the gust will continue to contribute to the aerodynamic force until it is convected far downstream, which results in the fluid memory effect. The wake flow convected velocity is approximately equal to the uniform flow velocity (e.g., von Kármán and Sears 1938). If the time scale of the wake flow convected far downstream is much faster compared to that of the structure oscillation or oncoming flow fluctuation, then the fluid memory effects induced by the wake may be insignificant, in which case the QS theory is valid. Otherwise, the transient unsteady features cannot be neglected since a good portion of the dynamic contribution to the aerodynamic forces, at each time instant, results from the fluid memory effects.

![Diagram](image)

Figure A.2. Two-dimensional (2-D) sketch of the bridge deck immersed in a uniform or fluctuating flow

In a fading memory system, in which the duration of the fluid memory effects is limited, the unsteady forces at each time instant will converge to the QS forces with time.
There are several approaches to describe this asymptotic behavior. For example, as the "little-o notation", which is a typical asymptotic notation utilized to compare the relative values of two functions, applied, one could express the aerodynamic forces with an abrupt input at time instant \( \tau \) as

\[
f_{UN}(t) = f_{QS}(t) + o(f_{QS}(t)) \quad \text{(as} \ t - \tau \to \infty) \quad \text{(A.5a)}
\]

or, suppose the QS response is nonzero, therefore, the asymptotic behavior could be expressed as

\[
f_{UN}(t)/f_{QS}(t) = 1 + o(1) \quad \text{(as} \ t - \tau \to \infty) \quad \text{(A.5b)}
\]

where the subscript "UN" and "QS" denote the unsteady and QS forces, respectively. Conventionally, the nondimensional unit-step response function is expressed in the form of Equation A.5b while Equation A.5a is employed for the nondimensional unit-impulse response function. Hence, identification of the unit-step or unit-impulse response function is reduced to the search for the solution of \( o(f_{QS}(t)) \) or \( o(1) \).

A.3 Step and Impulse Response Functions in Bridge Aerodynamics

A.3.1 Special Characteristics in Bridge Aerodynamics

For a thin airfoil with small disturbances, \( o(1) \) in Equation A.5b, in the case of motion-induced lift force, has the analytical solution given by (R.T. Jones 1940)

\[
o(1) = -0.165e^{-0.045s} - 0.335e^{-0.300s} \quad \text{(A.6)}
\]

where \( s=Ut/b \) is nondimensional time; \( U \) is the uniform wind velocity; \( b \) is the half chord of the airfoil. Equation A.6 actually is the R.T. Jones approximation of the transient part
of the Wagner function (Wagner 1925). On the other hand, for the bridge deck, there is no analytical expression for $o(f_{QS}(t))$ or $o(1)$, hence, direct (experimental) or indirect (numerical) identification schemes need to be introduced. Figure A.3 presents direct identification results of the unit-step response functions adopted from the available literature together with the Wagner function (R.T. Jones approximation) for a thin airfoil. In these cases, the input is the angle of attack whereas the output represents the lift force. As indicated in Figure A.3, a significant difference between the nondimensional unit-step response functions of an airfoil and of a bridge deck is that the latter presents an "overshooting" behavior, which generally compromises the stability of wind-bridge interactions (Yoshimura and Nakamura 1975). Actually, with the increase in the bluff nature of a body, the "overshooting" feature (either positive or negative) in the nondimensional unit-step response functions becomes more significant. For example, the T-section (with a shape of "├", where a vertical plate is attached to the windward edge of the horizontal plate) with aspect ratio 8.33 has more significant "overshooting" feature compared to T-section with aspect ratio 11.11.

It is generally believed that the "overshooting" phenomenon is due to the flow separation from the structure. This inference could be well illustrated by utilizing the experimental data of a stalled wing obtained by Francis and Cohen (1933). Figure A.4 shows the nondimensional results of the circulation around a wing which starts impulsively form rest at a high incidence angle of 0.48 (This is equivalent to a sudden change of angle of attack). The angle of attack is so large that the stall (which indicates the flow separation from the wing) occurs before the steady state is obtained. The red point in Figure A.4 indicates the first sign of stall of the wing. Instead of the original
representation of the experimental results, the nondimensional value in Figure A.4 is obtained by normalizing with respect to the measured steady-state circulation to explicitly present the "overshooting" feature of the circulation of the stalled (flow separation) system. Since the circulation is directly relative to the lift force of the wing, the results shown in Figure A.4 also indicate the "overshooting" of the lift force of the wing. The experimental data is available up to the nondimensional time around 10 (as indicated by the red dash line), where the circulation value begins to drop (Francis and Cohen 1933). In order to present the concept that, if the measured data is sufficient, the result will converge to the steady state, the data after nondimensional time 10 is made up (as indicated by the black dash line). The theoretical lift force on the thin airfoil induced
by a sudden change of angle (no flow separation) is also presented in Figure A.4 for comparison. These two curves explicitly show that the "overshooting" feature is accompanied by the flow separation (stall).

Figure A.4. Nondimensional circulation of a stalled wing induced by the sudden change of angle

A.3.2 Applications to the Gust- and Motion-Induced Forces

In bridge aerodynamics, in the case of gust input, the unit-step or unit-impulse response function is referred to as the aerodynamic unit-step or unit-impulse response function, whereas it is referred to as the aeroelastic uni-step or unit-impulse response function when the motion is the input. If a direct identification scheme is employed, the aeroelastic unit-impulse response function may be preferred over the aeroelastic unit-step response function. First, the time history of the motion input may impose some restrictions on the differentiability, as indicated in Equation A.2a, if the superposition of the response is based on the unit-step response function $a(t)$. Hence, the inevitable noise of the input signal in the wind tunnel results in amplified noise in the output (Silva et al. 2005).
Besides, for a bridge deck immersed in the flow, the sudden nose-up unit-step torsional angle of attack induces surrounding flow pattern that differs from the sudden nose-down unit-step torsional angle of attack due to the asymmetry of the bridge deck. On the contrary, the application of unit-impulse function actually can capture the effects of this asymmetry where the bridge deck first experiences nose up and then nose down to make it back to the original position.

On the other hand, the aerodynamic unit-step response feature may be superior to illustrate the mechanism of gust-induced forces. The connection between the gust-induced and motion-induced aerodynamic forces could be illustrated in frequency domain or in time domain where the unit-step aerodynamic response function is employed, as shown in Figure A.5. As indicated in Figure A.5(a), in the frequency domain the fluctuating vertical velocity could be decomposed into components with various frequencies. For the very low frequency component (large eddy scale), namely the component (I) has the same effect with that induced by the structural motion. Whereas, for the high frequency components (small eddy scales), namely the components (II), (III) and (IV) may not impact the bridge deck concurrently. In other words, there is a coherence reduction for the high frequency components. However, it should be noted that the coherence reduction cannot involve the transient effect since this consideration is in frequency domain. For example, the lift force induced by the vertical fluctuation and the drag force induced by the horizontal fluctuations have similar analysis scheme in frequency domain. On the other hand, as indicated in Figure A.5(b), in time domain the fluctuation vertical velocity could be divided into components with different time instants. Each component could be treated as a sharp-edged gust, whose transient effect

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could be represented by the effective angle of attack represented by the "broken-line bridge deck". von Kármán and Sears (1938) used this "broken-line airfoil" to illustrate the connection between the motion-induced and sharp-edged gust-induced effects. As a result, Duhamel’s integral scheme can be utilized to obtain the total response induced by the vertical fluctuations, as illustrated in Figure A.1(a). The application of unit-impulse response function may not be able to give this physical insight of the connection between the gust-induced and motion-induced forces in time domain.

Figure A.5. Illustration of gust-induced force on the bridge deck in both frequency and time domain
A.4 A Comparison of Indicial and Rational Function Approximations

There have been some attempts to directly measure these elementary response functions of bluff cross-sections in wind tunnels (e.g., Yoshimura and Nakamura 1975; Caracoglia and N.P. Jones 2003b) or based on CFD (e.g., Brar et al. 1996; Turbelin and Gibert 2001). However, the direct identification approach needs additional refinement before it may become a routine identification scheme. On the other hand, it is prevalent to measure the parameters in frequency domain, i.e., flutter derivatives and aerodynamic admittances, which are utilized in the conventional linear analysis framework (e.g., Scanlan and Tomko 1971). Hence, one of the promising indirect identification approaches is to utilize the inverse Fourier transform of the measured flutter derivatives or aerodynamic transfer functions in the wind tunnel to obtain the effective unit-step response function (Scanlan et al. 1974), where the indicial function approximation is exploited, or effective unit-impulse response function (Bucher and Lin 1988) where the rational function approximation is employed. The measurement of the aerodynamic transfer functions in a wind tunnel is challenging compared to that of the flutter derivatives. Hence, in this study, the comparison of the effective aeroelastic unit-step and unit-impulse response functions are investigated without the loss of generality.

A.4.1 Connection between the Effective Aeroelastic Unit-Step Response Functions and Flutter Derivatives

Suppose the independent variables of motion-induced forces acting on a bridge deck are \( h \), \( \theta \) and \( \dot{\theta} \) where \( h \) is vertical displacement and \( \theta \) torsional displacement. According to Equation A.2a, the lift force \( L \) and torsional moment \( M \) in terms of unit-step response functions may be expressed as
\[ L(t) = - \left\{ \Phi_{dl\theta}(t)\theta(0) + \int_0^t \Phi_{dl\theta}(t-\tau)\dot{\theta}(\tau)d\tau \right\} \\
+ \left\{ \Phi_{dlh}(t)\dot{h}(0) + \int_0^t \Phi_{dlh}(t-\tau)\ddot{h}(\tau)d\tau \right\} \\
+ \left\{ \Phi_{dl\theta}(t)\dot{\theta}(0) + \int_0^t \Phi_{dl\theta}(t-\tau)\ddot{\theta}(\tau)d\tau \right\} \]  
(A.7a)

\[ M(t) = \left\{ \Phi_{dM\theta}(t)\theta(0) + \int_0^t \Phi_{dM\theta}(t-\tau)\dot{\theta}(\tau)d\tau \right\} \\
+ \left\{ \Phi_{dMh}(t)\dot{h}(0) + \int_0^t \Phi_{dMh}(t-\tau)\ddot{h}(\tau)d\tau \right\} \\
+ \left\{ \Phi_{dM\theta}(t)\dot{\theta}(0) + \int_0^t \Phi_{dM\theta}(t-\tau)\ddot{\theta}(\tau)d\tau \right\} \]  
(A.7b)

where \( \Phi(t) \) denotes the aeroelastic unit-step response function; the subscript \( d \) indicates that the variables are dimensional. The equivalent nondimensional form may be expressed as

\[ L(s) = -\frac{1}{2} \rho U^2 2b^2 \left[ \frac{dC_L}{d\theta} \left\{ \Phi_{L\theta}(s)\theta(0) + \int_0^s \Phi_{L\theta}(s-\sigma)\dot{\theta}(\sigma)d\sigma \right\} \\
+ \left\{ \frac{dC_L}{d\theta} + C_D \right\} \left\{ \Phi_{L}\dot{h}(0) + \int_0^s \Phi_{L}(s-\sigma)\ddot{h}(\sigma)d\sigma \right\} \\
+ \left\{ \frac{dC_L}{d\theta} + C_D \right\} \left\{ \Phi_{L\theta}(0)\dot{\theta}(0) + \int_0^s \Phi_{L\theta}(s-\sigma)\dot{\theta}(\sigma)d\sigma \right\} \right] \]  
(A.8a)

\[ M(s) = \frac{1}{2} \rho U^2 2b^2 \left[ \frac{dC_M}{d\theta} \left\{ \Phi_{M\theta}(s)\theta(0) + \int_0^s \Phi_{M\theta}(s-\sigma)\dot{\theta}(\sigma)d\sigma \right\} \\
+ \left\{ \frac{dC_M}{d\theta} \right\} \left\{ \Phi_{M}\dot{h}(0) + \int_0^s \Phi_{M}(s-\sigma)\ddot{h}(\sigma)d\sigma \right\} \\
+ \left\{ \frac{dC_M}{d\theta} \right\} \left\{ \Phi_{M\theta}(0)\dot{\theta}(0) + \int_0^s \Phi_{M\theta}(s-\sigma)\dot{\theta}(\sigma)d\sigma \right\} \right] \]  
(A.8b)
As indicated in Equation A.5b, the asymptotic values of these six nondimensional unit-step response functions $\Phi_{L_0}$, $\Phi_{L_0'/B}$, $\Phi_{L_0'}$, $\Phi_{M_0}$, $\Phi_{M_0'/B}$ and $\Phi_{M_0'}$ are all equal to one. On the other hand, the motion-induced force and moment exerted on the bridge deck section in a mixed frequency and time domain could be expressed as (Scanlan and Tomko 1971)

\[
L_f = -(1/2) \rho U^2 B \left[ KH_1^*(K) \left( \hat{h}/U \right) + KH_2^*(K) \left( B \hat{\theta}/U \right) + K^2 H_3^*(K) \theta + K^2 H_4^*(K) \left( h/B \right) \right]
\]

(A.9a)

\[
M_f = (1/2) \rho U^2 B^2 \left[ KA_1^*(K) \left( \hat{h}/U \right) + KA_2^*(K) \left( B \hat{\theta}/U \right) + K^2 A_3^*(K) \theta + K^2 A_4^*(K) \left( h/B \right) \right]
\]

(A.9b)

where the subscript $f$ indicates the expressions are based on the flutter derivatives; $K= Bo \omega / U$ is nondimensional reduced frequency with $\omega$ as circular frequency of bridge deck vibration; $B=2b$ is the width of bridge deck; the coefficients $H_i^*(K)$ and $A_i^*(K)$ ($i=1…4$) are flutter derivatives, which are functions of $K$, wind angle of attack and deck shape. Generally, the flutter derivatives under a certain wind incident angle are extracted from the section-model tests in a wind tunnel. The equivalent expressions in frequency domain are given as

\[
\bar{L}_f (K) = -(1/2) \rho U^2 B K^2 \left[ (i H_2^*(K) + H_3^*(K)) \bar{\theta} + (i H_1^*(K) + H_4^*(K)) \left( \bar{h}/B \right) \right]
\]

(A.10a)

\[
\bar{M}_f (K) = (1/2) \rho U^2 B^2 K^2 \left[ (i A_2^*(K) + A_3^*(K)) \bar{\theta} + (i A_1^*(K) + A_4^*(K)) \left( \bar{h}/B \right) \right]
\]

(A.10b)

where the overbar denotes the Fourier transform operator.
In order to conveniently establish the connection between the effective aeroelastic unit-step response functions and flutter derivatives, the lift force and torsional moment of bridge deck, Equations A.8a and A.8b are simplified to

\[
L(s) = -\frac{1}{2} \rho U^2 B \frac{dC_L}{d\theta} \left[ \phi_{L0}(0)\theta(s) + \int_0^s \phi_{L0}^\prime(s-\sigma)\theta(\sigma)d\sigma \right] \\
+ \left\{ \phi_{L0}^\prime_B(0) \frac{h^\prime(s)}{B} + \int_0^s \phi_{L0}^\prime_B(s-\sigma) \frac{h^\prime(\sigma)}{B} d\sigma \right\}
\]

(A.11a)

\[
M(s) = \frac{1}{2} \rho U^2 B^2 \frac{dC_M}{d\theta} \left[ \phi_{M0}(0)\theta(s) + \int_0^s \phi_{M0}^\prime(s-\sigma)\theta(\sigma)d\sigma \right] \\
+ \left\{ \phi_{M0}^\prime_B(0) \frac{h^\prime(s)}{B} + \int_0^s \phi_{M0}^\prime_B(s-\sigma) \frac{h^\prime(\sigma)}{B} d\sigma \right\}
\]

(A.11b)

where the function \( \phi(s) \), which possesses the asymptotic form of Equation A.5b, represents effective nondimensional aeroelastic unit-step response function and are identified through an indirect approach by utilizing the known flutter derivatives. Here, the nondimensional time \( s \) is defined as \( Ut/B \) to maintain the conventional expressions in bridge aerodynamics. As indicated in the above equations, the input variable \( \theta' \) is deleted, which means the contribution of the angular velocity to the force or moment is integrated into the contribution of the angle of attack. However, it should be noted that the mechanism of \( \theta' \)-induced force (or moment) is different from \( \theta \)-induced force (or moment). For example, \( \theta' \) will induce nonuniform vertical velocity while \( \theta \) will induce uniform vertical velocity over the bridge deck section. Besides, the asymptotic lift force induced by the angular velocity (\( [dC_l/d\theta' + CD] \) as indicated in Equation A.8a) is different from that induced by the angle of attack (\( [dC_l/d\theta] \) as indicated in Equation A.8a), which cannot be taken into account using this simplified expression. Although it is attempted to
separate the contribution of the input \( h'/B \) to the unsteady force (or moment) from that of the input \( \theta \) in the simplified expression, their steady-state contributions are treated as the same since the slopes of the steady-state coefficients \( C_L \) and \( C_D \) with respect to \( h'/B \) are replaced with those with respect to \( \theta \). In the aerofoil theory, the contribution of \( h'/B \) are treated the same with that of \( \theta \) since both of them produce the uniform vertical velocity over the thin airfoil. However, for bridge deck, due to the complex surrounding flow the contributions of these two input variables are conventionally treated different, which can also be concluded from the different values of the corresponding flutter derivatives (Scanlan et al. 1997). It should be noted that once relevant information is available one can utilize three input variables instead of two to obtain more reasonable expressions of effective aeroelastic unit-step response functions, and hence more accurate results. The equivalent expressions of Equations A.11a and A.11b in the frequency domain are given as

\[
\bar{L}(K) = -\left(1/2\right) \rho U^2 B \left( dC_L / d\theta \right) \left[ \left\{ \bar{\phi}_{L\theta}(0) + \bar{\phi}_{L\theta}(K) \right\} \bar{\theta} + \left\{ \bar{\phi}_{Lh/B}(0) + \bar{\phi}_{Lh/B}(K) \right\} (iK\bar{h}/B) \right] \quad \text{(A.12a)}
\]

\[
\bar{M}(K) = \left(1/2\right) \rho U^2 B^2 \left( dC_M / d\theta \right) \left[ \left\{ \bar{\phi}_{M\theta}(0) + \bar{\phi}_{M\theta}(K) \right\} \bar{\theta} + \left\{ \bar{\phi}_{Mh/B}(0) + \bar{\phi}_{Mh/B}(K) \right\} (iK\bar{h}/B) \right] \quad \text{(A.12b)}
\]

Comparing Equations A.12a and A.12b with Equations A.10a and A.10b, the relationships between the Fourier transform of the effective aeroelastic unit-step responses and the flutter derivatives are presented as (e.g., Caracoglia and N.P. Jones 2003a; Zhang et al. 2011)
\[
\left\{ \tilde{\phi}_{L\theta}(0) + \tilde{\phi}_{L\theta}^{*}(K) \right\} = K^2 \left( iH_2^*(K) + H_3^*(K) \right) / \left( dC_L / d\theta \right) \tag{A.13a} \\
\left\{ \tilde{\phi}_{Lh'/B}(0) + \tilde{\phi}_{Lh'/B}^{*}(K) \right\} = K \left( H_1^*(K) - iH_4^*(K) \right) / \left( dC_L / d\theta \right) \tag{A.13b} \\
\left\{ \tilde{\phi}_{M\theta}(0) + \tilde{\phi}_{M\theta}^{*}(K) \right\} = K^2 \left( iA_2^*(K) + A_3^*(K) \right) / \left( dC_M / d\theta \right) \tag{A.13c} \\
\left\{ \tilde{\phi}_{Mh'/B}(0) + \tilde{\phi}_{Mh'/B}^{*}(K) \right\} = K \left( A_1^*(K) - iA_4^*(K) \right) / \left( dC_M / d\theta \right) \tag{A.13d} 
\]

In order to efficiently obtain the effective nondimensional aeroelastic unit-step response functions \( \phi_{L\theta}, \phi_{Lh'/B}, \phi_{M\theta}, \) and \( \phi_{Mh'/B} \) through the discrete values of the flutter derivatives, the indicial function approximation is employed in the indirect identification procedure. Hence, \( o(1) \) in Equation A.5b are assumed to have form similar to the transient part of the Wagner function, wherein the effective nondimensional aeroelastic unit-step response functions can be expressed as (Scanlan et al. 1974)

\[
\phi_{yx}(s) = 1 - \sum_{i=1}^{n} a_{yx} e^{-b_{yx}s} \tag{A.14} 
\]

where \( x \) denotes input variables \( \theta \) or \( h'/B \) and \( y \) the responses \( L \) or \( M \); the parameters \( a \) and \( b \) are constants to be identified; the value of \( n \) is dependent on the available data and the aerodynamic properties of the bridge deck. Typically, for a streamlined bridge deck, \( n=2 \) is sufficient while for a bluff bridge deck \( n \) is usually larger than 3 (e.g., Caracoglia and N.P. Jones 2003a). The nonlinear least-square technique is conventionally utilized to minimize the identification error.
A.4.2 Connection between the Effective Aeroelastic Unit-Impulse Response Functions and Flutter Derivatives

It is well known that there is a direct relationship between the unit-step and unit-impulse response functions as discussed in Section A.2. On the other hand, the lift force and torsional moment in terms of an effective aeroelastic unit-impulse response function can also be obtained by utilizing the inverse Fourier transform of Equations A.10a and A.10b. According to Equation A.2b, the lift force and torsional moment in terms of the unit-impulse response function can be expressed as

\[
L(t) = - \left[ \int_0^t \Psi_{dL\theta}(t-\tau)\theta(\tau)d\tau \right] + \left[ \int_0^t \Psi_{dLh}(t-\tau)\dot{h}(\tau)d\tau \right] + \left[ \int_0^t \Psi_{dL\theta}(t-\tau)\dot{\theta}(\tau)d\tau \right]
\]

(A.15a)

\[
M(t) = \left[ \int_0^t \Psi_{dM\theta}(t-\tau)\theta(\tau)d\tau \right] + \left[ \int_0^t \Psi_{dMh}(t-\tau)\dot{h}(\tau)d\tau \right] + \left[ \int_0^t \Psi_{dM\theta}(t-\tau)\dot{\theta}(\tau)d\tau \right]
\]

(A.15b)

where \(\Psi(t)\) denotes the aeroelastic unit-impulse response function. In order to conveniently establish a connection between the effective aeroelastic unit-impulse response functions and flutter derivatives, the lift force and torsional moment of the bridge deck, Equations A.15a and A.15b are simplified to

\[
L(s) = -\frac{1}{2} \rho U^2 B \left[ \int_0^s \tilde{I}_{L\theta}(s-\sigma)\theta(\sigma)d\sigma \right] + \left[ \int_0^s \tilde{I}_{Lh/B}(s-\sigma)\dot{h}(\sigma)/B d\sigma \right]
\]

(A.16a)

\[
M(s) = \frac{1}{2} \rho U^2 B^2 \left[ \int_0^s \tilde{I}_{M\theta}(s-\sigma)\theta(\sigma)d\sigma \right] + \left[ \int_0^s \tilde{I}_{Mh/B}(s-\sigma)\dot{h}(\sigma)/B d\sigma \right]
\]

(A.16b)
where the dynamic pressure is extracted from the effective aeroelastic unit-impulse response function $I(s)$. As indicated in the above equations, the input variable $\theta'$ is also deleted, which means the contribution of the angular velocity to the force or moment is incorporated into the contribution of the angle of attack. The equivalent expressions of Equations A.16a and A.16b in frequency domain are expressed as

$$
\tilde{L}(K) = -(1/2) \rho U^2 B \left[ \tilde{I}_{L\theta}(K) \tilde{\theta} + \left\{ \tilde{I}_{Lh/B}(K) \left( iK\tilde{h}/B \right) \right\} \right] 
$$

(A.17a)

$$
\tilde{M}(K) = (1/2) \rho U^2 B \left[ \tilde{I}_{M\theta}(K) \tilde{\theta} + \left\{ \tilde{I}_{Mh/B}(K) \left( iK\tilde{h}/B \right) \right\} \right] 
$$

(A.17b)

The available flutter derivatives are also employed in the indirect identification of the effective aeroelastic unit-impulse response function.

Comparing Equations A.17a and A.17b with Equations A.10a and A.10b, the relationships between the Fourier transform of the effective aeroelastic unit-impulse response functions and the flutter derivatives are presented as

$$
\tilde{I}_{L\theta}(K) = K^2 \left( iH^*_2(K) + H^*_3(K) \right)
$$

(A.18a)

$$
\tilde{I}_{Lh/B}(K) = K \left( H^*_1(K) - iH^*_4(K) \right)
$$

(A.18b)

$$
\tilde{I}_{M\theta}(K) = K^2 \left( iA^*_2(K) + A^*_3(K) \right)
$$

(A.18c)

$$
\tilde{I}_{Mh/B}(K) = K \left( A^*_1(K) - iA^*_4(K) \right)
$$

(A.18d)

Comparing Equations A.18a, A.18b, A.18c and A.18d with Equations A.13a, A.13b, A.13c and A.13d, it shows that the effective unit-step response functions and the
corresponding effective unit-impulse response functions possess the relationship indicated in Equation A.4 in frequency domain.

Since the effective unit-impulse response function fades out with respect to time (i.e., the asymptotic value is equal to zero), the asymptotic form of Equation A.5a is utilized. Accordingly, the unit constant in the effective unit-step response function as indicated in Equation A.14 should be replaced with the QS force (or moment) in the effective unit-impulse response function. As is well known, the QS force (or moment) only depends on the instantaneous input variables, which indicates the QS part of the lift force (or torsional moment) could be presented as

\[
[y_{x}(s)]_{QS} = (1/2) \rho U^2 B \left[ g_{yx} x(s) \right]
\]  

(A.19)

where \( g \) denotes the coefficient corresponding to the QS part. Since the torsional angle-induced force (or moment) actually involve both contributions from the angle of attack \( \theta \) and angular velocity \( \dot{\theta} \), once the force (or moment) relates to the torsional degree of freedom, it is more reasonable to rewrite Equation A.19 as

\[
[y_{\theta}(s)]_{QS} = (1/2) \rho U^2 B \left[ g_{1y\theta} \theta(s) + g_{2y\dot{\theta}} \dot{\theta}(s) \right]
\]  

(A.20)

It should be noted that it is inappropriate to interpret the second term in Equation A.20 as the unsteady effect of the angle of attack, e.g., in Bucher and Lin (1988), as it is the QS effect of the angular velocity. Hence, the QS part of the effective aeroelastic unit-impulse response in time domain is expressed as

\[
[I_{xy/h_B}(s)]_{QS} = \left[ g_{1y/h_B} \delta(s) \right]
\]  

(A.21a)
The corresponding expressions in frequency domain are

\[
[I_{\theta}(s)]_{QS} = \left[ g_{1\theta\phi}\delta(s) + g_{2\theta\phi}\delta'(s) \right] \quad (A.21b)
\]

\[
[I_{\theta|d,k}(K)]_{QS} = g_{1\theta/k}\delta(s) \quad (A.22a)
\]

\[
[I_{\theta}(K)]_{QS} = \left[ g_{1\theta\phi} + g_{2\theta\phi}iK \right] \quad (A.22b)
\]

It is stated that the unsteady effects could be adequately approximated by several transfer functions of linear filters, which are well known as rational functions (Roger 1977). In order to efficiently obtain the effective aeroelastic unit-impulse response functions through the discrete values of the flutter derivatives, the rational function approximation is employed in the indirect identification procedure. Hence, the unsteady part of the effective aeroelastic unit-impulse response function, namely \( o(f_{QS}(t)) \) in Equation A.5a in frequency domain could be expressed as

\[
[I_{\theta}(s)]_{QS} = \left[ g_{1\theta\phi}\delta(s) + g_{2\theta\phi}\delta'(s) \right] \quad (A.23)
\]

where \( g \) denotes the coefficient corresponding to the unsteady part. These linear filters are usually referred to lag terms as each rational function represents the unsteady component that lags behind the corresponding input term and permits an approximation of the time delay resulting from the fluid memory effects. The parameter \( d_{lyx} \) is usually selected to be positive in order to satisfy the stability requirement of the filter (e.g., Chen and Kareem 2003). Combining Equations A.22a and A.22b and Equation A.23, the effective aeroelastic unit-impulse response functions in frequency domain are obtained as
The parameters of the effective aeroelastic unit-impulse response functions based on the rational function approximation could be identified straightforwardly given the measured flutter derivatives using the Equations A.18a, A.18b, A.18c and A.18d and Equations A.24a and A.24b. The value of $n$ is dependent on the available data and the aerodynamic properties of the bridge deck. Typically, for the streamlined bridge deck $n=1$ or 2 is sufficient while for the bluff bridge deck $n$ is larger than 3 (e.g., Fujino et al. 1995). Suppose the parameters $d_{iy\alpha}$ is fixed, linear least-square technique could be applied in the identification procedure as usually implemented in bridge aerodynamics (Boonyapinyo et al. 1999). The corresponding time domain equivalent of Equations A.24a and A.24b is expressed as

$$I_{y'h/B}(s) = g_{1y'h/B} \delta(s) + \sum_{i=3}^{n} g_{iy'h/B} \left( \delta(s) - \left( d_{iy'h/B} e^{-\left( d_{iy'h/B}\right)s} \right) \right)$$ (A.25a)

$$I_{\gamma\theta}(s) = \left[ g_{1y\theta} \delta(s) + g_{2y\theta} \delta'(s) \right] + \sum_{i=3}^{n} g_{iy\theta} \left( \delta(s) - \left( d_{iy\theta} e^{-\left( d_{iy\theta}\right)s} \right) \right)$$ (A.25b)

The second term in Equations A.25a and A.25b actually presents the presupposed form of $o(f_{QS}(t))$ in Equation A.5a. Although the parameters $g_{1y'h/B}$, $g_{1y\theta}$ or $g_{2y\theta}$ is introduced to take into account the QS part in the effective aeroelastic unit-impulse response functions, it should be noted that the conventional identification scheme is only based on the flutter
derivatives which presents the unsteady effects. In order to identify the effective aeroelastic unit-impulse responses with both QS and unsteady effects, in this study the parameters $g_{1yB}$, $g_{1y\theta}$ and $g_{2y\theta'}$ in Equations A.25a and A.25b are first determined from the slope of steady-state coefficients $dC_{ij}/d\theta$. Since the information on the slopes of the steady-state coefficients with respect to the variable $h'/B$ and $\theta'$ is not available, they are assumed to be the same as $dC_{ij}/d\theta$. In order to verify the reliability of the identified effective aeroelastic unit-impulse response function and the corresponding effective aeroelastic unit-step response function, one may expect that the summation of $g_{ijy}$ should be equal to the value of $\phi_{ij}(0)$ by recalling Equation A.4. Besides, it should be noted that the non-circulation apparent mass effects may be involved in the identified effective unit-step or unit-impulse response functions as the flutter derivative $H_{ij}(K)$ and $A_{ij}(K)$ are considered in the identification process.

A.4.3 Numerical Example

A numerical example of a long-span bridge, which employs a single-box girder with aspect ratio $(B/D)$ 11, is presented to investigate indirectly identified effective aeroelastic unit-step and unit-impulse response functions. The steady-state coefficients of this long-span bridge are $C_{L}(0^o)=-0.1066$ and $C_{M}(0^o)=0.0282$ and the corresponding slopes of the steady-state coefficients are $dC_{L}(0^o)/d\theta=5.0019$ and $dC_{M}(0^o)/d\theta=0.9540$. The experimentally derived flutter derivatives are given in Figure A.8. The effective aeroelastic unit-step response functions can be calculated utilizing Equations A.13a, A.13b, A.13c and A.13d and Equations A.14a and A.14b, while the effective aeroelastic unit-impulse response functions can be obtained using Equations A.18a, A.18b, A.18c and A.18d and Equations A.24a and A.24b. A nonlinear least-square technique was
utilized to minimize the identification error. Figure A.6 presents the identified effective aeroelastic unit-step response functions based on the indicial function approximation while Figure A.7 shows the identified effective aeroelastic unit-impulse response functions based on the rational function approximation where the singularity values are not presented. The unit-step or unit-impulse response functions of the airfoil are also presented for comparison. Obviously, both of these effective aeroelastic unit-step and unit-impulse response functions present the "overshooting" feature. As indicated in the figures, the vertical motion has more significant unsteady contribution to both the lift force and torsional moment compared to the torsional motion. Besides, the duration of the fluid memory effects on the torsional moment due to the vertical motion is significantly higher compared to that due to the torsional motion. Comparing with the effective aeroelastic unit-step response functions, the effective aeroelastic unit-impulse response functions increase the complexity of manipulation and interpretation due to the Dirac delta functions in the expressions, whereas, there is no essential superiority between these two effective elementary responses as applied in bridge aerodynamics.

Once the effective aeroelastic unit-step or unit-impulse response functions are identified, the flutter derivatives can be calculated based on Equations A.13a, A.13b, A.13c and A.13d or Equations A.18a, A.18b, A.18c and A.18d to verify the fidelity of the indirect identification scheme discussed in this study. The fitted flutter derivatives based on the identified effective aeroelastic unit-step response function with indicial function approximation and on the identified effective aeroelastic unit-impulse response function with rational function approximation are given in Figure A.8 together with the original
Figure A.6. Nondimensional effective aeroelastic unit-step response functions of a long-span bridge: (a) unit-step response function of lift force; (b) unit-step response function of torsional moment
Figure A.7. Nondimensional effective aeroelastic unit-impulse response functions of a long-span bridge: (a) unit-impulse response function of lift force; (b) unit-impulse response function of torsional moment
Figure A.8. Comparison of indicial function and rational function approximations

tested values. The results demonstrate that numerical approximations provide satisfactory results. It should be noted that the effective unit-step or unit-impulse response function carry no additional information than contained in the flutter derivatives or aerodynamic admittance functions. Actually, the assumptions and shortcomings in the linear frequency domain analysis, such as limited applicable frequency range, are adopted in this time domain analysis based on the indirect identification schemes. Besides, extra uncertainties result from the identification procedure based on the nonlinear (linear) least-square technique. Hence, the effective aeroelastic unit-step or unit-impulse response functions based on the indirect identification scheme may not be able to delineate the "true" distribution (both magnitude and duration) of the fluid memory effects in bridge aerodynamics. Obviously, the relationship between unit-step and unit-impulse responses represented in Equation A.4 is not valid for the case of the identified effective unit-step
and unit-impulse response functions as indicated in Figures A.6 and A.7. On the other hand, it is emphasized that indirectly identified effective aeroelastic unit-step or unit-impulse response functions may preserve majority of useful aerodynamic information included in the directly identified aeroelastic unit-step or unit-impulse response function due to the presupposed forms of unsteady effects, namely the indicial or rational function approximation. Hence, it is expected that the effective aeroelastic unit-step or unit-impulse response function should represent the "averaged" fluid memory effect so that the bridge deck responses obtained through the convolution integrals in terms of the effective unit-step or unit-impulse response function are reliable.

Figure A.9 presents calculated motion-induced damped, critical and divergent responses based on the identified effective aeroelastic unit-step response functions as shown in Figure A.6 to present the computational fidelity of the convolution scheme. The calculated motion-induced responses based on the identified effective aeroelastic unit-impulse response functions are similar. Figure A.10 shows time histories of wind-induced vertical and torsional responses of the bridge deck based on the identified effective aeroelastic unit-step response functions and the analytical aerodynamic unit-step response function $K(s)=1.0-0.236e^{-0.058s}-0.513e^{-0.364s}-0.171e^{-2.420s}$ (Küssner 1936). (With the inclusion of chord-wise correlation, the identification of the effective aerodynamic unit-step or unit-impulse response functions for the gust-induced unsteady effects could be developed parallel to the motion-induced case and are not presented for the sake of brevity.) The mean wind velocity $U$ is 40 m/s with turbulence intensity $I_u=0.15$ and $I_w=0.08$. It should be noted that nonlinear considerations from the geometric, material and aerodynamic aspects could be conveniently implemented in this time domain analysis.
Figure A.9. Motion-induced responses using convolution integral: (a) damped case \((U=68 \text{ m/s})\); (b) critical case \((U=73.7 \text{ m/s})\); (c) divergent case \((U=78 \text{ m/s})\)
A.5 Concluding Remarks

Two elementary response features of bridge aerodynamics, namely unit-step (indicial) and unit-impulse response functions couched in a convolution integral, are systematically reviewed in this study. Theoretically, there is a close relationship between the unit-step and unit-impulse response functions while the latter shows marginal advantage as the impulse function uniformly excites all the frequency components. It is noted that the convolution integral based on the unit-step response function is the superposition of the response under the incremental input value at each infinitesimal time interval. Whereas the approach based on the unit-impulse response function is the superposition of the response to the input amplitude at each infinitesimal time interval. This observation indicates that the unit-step response function is helpful in illustrating the physical origin of the connection between the motion-induced and gust-induced aerodynamic forces. On the other hand, the unit-impulse response function is believed to show superiority in the application of motion-induced aerodynamic forces due to its property of "round-trip". The "overshooting" feature of these elementary response functions is investigated and
attributed to flow separation as being main physical source and the degree of "overshooting" is related to the bluff nature of the cross-section.

The indirect identification of effective unit-step and unit-impulse response functions is emphasized in this study, where the physical significance is presented with clarification of numerous issues surrounding the effective unit-step or unit-impulse response function in their mathematical derivations. Generally, there is no superiority between the two as the indirect identification is employed based on the indicial or rational function approximations, respectively. It should be noted that, although the identified effective unit-step and unit-impulse response functions preserve little physical origin and cannot delineate the "true" distribution of the fluid memory effects in bridge aerodynamics, both could be efficiently utilized in the time domain analysis. A numerical example of a long-span bridge is utilized, where the simulation results show the computational fidelity of the convolution scheme based on the indirectly identified effective elementary response functions.
APPENDIX B: UNSTEADY AND HYSTERETIC NONLINEAR FEATURES OF RAIN-WIND INDUCED VIBRATION OF CABLES

Models for simulating rain-wind induced vibrations (RWIVs) are proposed that capture often ignored basic excitation mechanisms involving unsteady aerodynamics and hysteretic nonlinearity. The need for these features becomes important since conventional quasi-steady (QS) theory based model cannot account for these aerodynamic features adequately due to the absence of any consideration of the fluid memory effects. In light of this shortcoming, a semi-empirical model parallel to Scanlan's analysis framework for simulating self-excited forces on bridge decks is developed to take into account the unsteadiness. Whereas an optimally-chosen higher-order (nonlinear) polynomial with the Moore-Penrose pseudoinverse identification scheme is exploited for capturing the hysteretic nonlinearity. Parameters related to unsteadiness and nonlinearity in these models are derived from the mapping of pressure field and attendant measured integrated loads on an oscillating cable-rivulet section model. Numerical examples with unsteady and hysteretic nonlinear features are illustrated based on models derived from wind-tunnel data. In addition, the higher-order spectrum is utilized to analyze nonlinear coupling features in RWIV. These models and observations of RWIV promise to serve as building blocks for a holistic model for RWIV as additional research is carried out to further refine the role of each feature and how these can be combined.
B.1 Introduction

Modern cable-stayed bridges, due to their economic and aesthetic advantages, have become a form of choice in the past few decades for bridges in the medium-to-long-span range. Cables of this type of bridges have low inherent structural damping and a wide range of natural frequencies (because of different lengths of cables), hence, they are susceptible to excitation under natural wind. Among various types of wind-induced vibrations of cables of cable-stayed bridges, such as ice galloping, Kármán vortex shedding induced vibration, wake induced vibration, vortex-induced oscillation at high wind velocity, rain-wind induced vibration (RWIV), RWIV has a dominant role due to its large amplitude and frequent occurrence. Since the first observation of RWIV during the construction of a cable-stayed bridge (Hikami and Shiraishi 1988), focus has been on this topic to better understand and model the RWIV.

The physical significance of RWIV has been investigated typically through field observation, wind-tunnel experiment, theoretical modeling, and recently the computational fluid dynamics (CFD). On the one hand, consensus does exist among researchers regarding the formation of water rivulet on the cable surface due to moderate rain (Hikami and Shiraishi 1988) and the attendant axial flow formation (Matsumoto et al. 2005) as two main factors responsible for RWIV. However, despite a large volume of research effort, no consensus exists regarding the underlying mechanism responsible for RWIV due to the complex nature of the air-water-solid interaction. Actually, because the observed behavior of RWIV is extremely sensitive to the system parameters and to the environment, the conclusions drawn from publications based on various field observations and wind-tunnel experiments are contradictory, e.g., the effects of
turbulence (Matsumoto 1998; Peil and Dreyer 2007), the intensity of precipitation (Cosentino et al. 2003a; Zuo and N.P. Jones 2010), and the shape and size of the rivulet (Bosdogianni and Olivari 1996; Gu et al. 2002) on RWIV.

One typical feature of RWIV is that the vibration exists in a limited range of wind velocities and amplitudes. As the variation of lift force coefficient of the cable with rivulet versus angle of attack presents significant negative slope, the mechanism of classical galloping is a plausible explanation of RWIV. Consequently, the circumferential motion of the rivulet on the cable, which changes the effective cross-section of the cable, is critical to properly model RWIV. There are various assumptions about the motion of the rivulet, for example, simple sinusoidal (Xu and Wang 2003) and stochastic (Cao et al. 2003) models have been developed recently. A more representative model may simulate various forces on the rivulet, namely the weight of water, the inertial forces due to rivulet itself and the cable vibration, the friction force, and the aerodynamic force, for establishing the equation of motion of the rivulet (e.g., Gu et al. 2009). A dynamically coupled cable-rivulet model, where the aerodynamic force on the rivulet was based on the QS assumption, successfully reproduced the wind velocity and amplitude limited characteristics of RWIV (e.g., Gu et al. 2009). However, it is interesting to note that the “amplitude limited” property of RWIV could actually be easily reproduced regardless of the motion of rivulet. For example, with the assumption of rivulet oscillating harmonically or as a narrow banded stochastic process, the simulated cable vibrations still present the “amplitude limited” feature with the same order of the magnitude as compared to the results based on the QS theory in a similar range of wind velocities. Furthermore, even if the rivulet rests on the cable at the equilibrium position, which
belongs to the classical galloping instability, the results of the example in this study indicate that at the same range of wind velocities, the magnitude of the limited amplitude is of the same order as that obtained from the harmonic/stochastic/QS models.

The QS assumption obviously leaves room for improvement due to extremely complicated flow field around the rivulet. In this study, the unsteady effect is considered with a scheme parallel to Scanlan’s analysis framework for simulating self-excited forces on bridge decks (Scanlan and Tomko 1971). Actually, observation in the wind tunnel has pointed out that RWIV bears self-excited characteristics (Verwiebe and Ruscheweyh 1998). The aerodynamic coefficients (flutter derivatives) of the rivulet in the proposed unsteady model are identified in the wind tunnel based on the measurement of pressure field on an oscillatory cable-rivulet section model. Although RWIV is recognized to have dynamic nonlinear interaction among the solid, water, and air, neither the QS model, which is able to simulate static nonlinearity, nor the proposed unsteady model, which is able to simulate the linear fluid memory effect, can take into account the dynamic (hysteretic) nonlinearity feature, i.e., higher-order fluid memory. Hence, an optimally-chosen higher-order (nonlinear) polynomial, whose parameters are identified based on the Moore-Penrose pseudoinverse scheme, is exploited to account for the hysteretic nonlinearity of RWIV in this study. A detailed comparison among the results based on the QS, unsteady, and hysteretic nonlinear models is given to investigate the excitation mechanisms of RWIV. Besides, the higher-order spectrum is utilized to analyze the nonlinear coupling features in RWIV. Specifically, the wind-induced pressure on the rivulet and on the cable under forced oscillations is investigated.
B.2 Analytical Models

This study focuses on a two-dimensional (2-D) analysis of RWIV in a plane, while the consideration of the out-of-plane coupling effect is adopted from the earlier research by Li et al. (2013). A convenient scheme recently developed treats the cable and rivulet as a coupled dynamic system, hence, a coupled system of equations is utilized to simulate the 2-D motions of the cable and rivulet (Gu et al. 2009)

\[
\ddot{y} + 2\xi_y \omega_y \dot{y} + \omega_y^2 y = -F_y / M \\
m \left( \frac{D}{2} \right) \ddot{\theta} + F_0 \text{sign}(\dot{\theta}) + c_r \left( \frac{D}{2} \right) \dot{\theta} = F_r + m\ddot{y} \cos(\theta) - mg \cos(\phi) \cos(\theta)
\]

where \( M \) and \( m \) are the masses per unit length of cable and rivulet, respectively; \( y \) is the vertical displacement of cable; \( \omega_y \) and \( \xi_y \) are circular frequency and damping ratio of the cable, respectively; \( D \) is the diameter of the cable, \( \theta \) is the position of the rivulet (Figure B.2); \( c_r \) and \( F_0 \) are the linear damping coefficient and the Coulomb damping force between the rivulet and cable surface, respectively; \( \phi \) is the cable inclined angle; \( F_y \) and \( F_r \) are the aerodynamic forces on the cable and rivulet, respectively (Figure B.2). The surface tension on the rivulet-cable interface is neglected (Cosentino et al. 2003b). RWIV is sensitive to any small change in the rivulet location, hence, it is critical to accurately simulate various forces on the rivulet. While other forces on the rivulet especially the Coulomb damping force is important and their impact needs to be investigated in a future research. In this study, the focus is primarily on the simulation of aerodynamic force.
B.2.1 Conventional Quasi-Steady Model

Both aerodynamic and kinematic perspectives need to be investigated for the proper application of the QS theory. Based on the aerodynamic point of view, the aerodynamic forces acting on an oscillating structure could be modeled by utilizing the steady-state situation without the fluid memory (unsteadiness) consideration. Based on the kinematic point of view, it should be convenient to define an equivalent steady-state situation for the aerodynamic system (van Oudheusden 1995). Since it is intractable to define an equivalent steady-state situation for the torsional oscillations, as the relative velocity (local relative angle of attack) is different at each point of the cross-section due to the angular motion, the fidelity of the QS theory based simulation is usually degraded for the torsional motion case. Although it is never explicitly stated in the literature, the premise for the applicability of the QS assumption in RWIV, where the torsional motion is critical, includes: (1) for the cable only one local relative angle of attack is sufficient to define the equivalent steady-state situation of the cable since the rotational motion is uniquely determined by the rivulet position and (2) for the rivulet the rotational center is far from the geometric center where the situation approaches the translational motion case.

Based on the QS assumption, the aerodynamic forces on the cable and rivulet could be represented as (Gu et al. 2009)

\[
F_y = \frac{1}{2} \rho DU_{rel}^2 \left[ C_L(\phi')\cos(\phi) + C_D(\phi')\sin(\phi) \right] \tag{B.2a}
\]

\[
F_r = \frac{1}{2} \rho RU_{rel}^2 \left[ c_i(\phi')\cos(\phi') + c_d(\phi')\sin(\phi') \right] \tag{B.2b}
\]
where \( U_{rel} \) is the relative wind velocity; \( R \) is the characteristic size of the rivulet; \( \rho \) is the air density; \( C_L \) and \( C_D \) are lift and drag force coefficients of the cable, respectively; \( c_l \) and \( c_d \) are lift and drag force coefficients of the upper rivulet, respectively; \( \phi \) represents the angle between relative wind velocity and x axis; and \( \phi' \) the angle between relative wind velocity and position of upper rivulet.

B.2.2 Unsteady Model of RWIV

Suppose the unsteadiness generated by the local separation due to the rivulet is weak for the cable, the aerodynamic force on the cable may be adequately modeled by the QS assumption since RWIV usually happens at a relatively high reduced wind velocity, where the aerodynamics of the circular cross-section with/without the small protuberance is primarily controlled by the far wake. However, the QS assumption may not be appropriate for the upper rivulet since the flow around it usually becomes very unsteady due to the local separation and the interference of the cable. The strong unsteadiness could be observed form the significant changes in the wind pressure coefficients around the rivulet with various locations, as shown in Figure B.1, where the data is obtained from the wind-tunnel tests to be discussed in the following section. As shown in Figure B.1(a) of wind pressure coefficients on the cable, for the rivulet positions (also angles of attack as indicated in Figure B.2) in the range of 25° to 58° there is a sudden change between the pressures at points before and after the rivulet, which may indicate the local flow separates around the rivulet. Beyond this range, the difference gradually decreases with respect to the increasing angle of attack, which indicates flow separation from the cable ahead of the rivulet position. A similar observation is also reported in a recent study (Gu et al. 2009). The distribution of wind pressure coefficients on the rivulet further
reaffirms this observation. As shown in Figure B.1(b) of wind pressure coefficients on
the rivulet, for the rivulet positions (also angles of attack as indicated in Figure B.2) in
the range of 25° to 58° there is a sudden phase change between the pressure at two points,
one of which is on the windward side and the other of which is on the leeward side, of the
rivulet. A CFD based study utilizing the direct numerical simulation (DNS) also shows a
similar trend in the flow around the rivulet (Li and Gu 2006). Specifically, the flow
separates at the location of the rivulet for both large and small angles of attack, however,
as the angle of attack is small, the separated flow reattaches soon (Li and Gu 2006). In
addition, the rivulet shape, which is not explicitly described to date in the literature, may
feature a long afterbody, which may intensify the unsteady effects.

In order to simulate the unsteady effects, a semi-empirical model parallel to
Scanlan's analysis framework for simulating self-excited forces on bridge decks is
developed, where the unsteady aerodynamic force on the rivulet is represented as

\[
F'_r = \frac{1}{2} \rho RU^2 \left[ c_d(\theta_0) \sin(\theta_0) + c_i(\theta_0) \cos(\theta_0) \right] + \left[ -L \cos(\theta_0) + D \sin(\theta_0) \right] \quad \text{(B.3)}
\]

\[
L = \frac{1}{2} \rho U^2 (2R) \left[ K_y H_{R1}(K_y) \frac{\dot{y}}{U} + K_\theta H_{R2}(K_\theta) \frac{\dot{\theta} R}{U} + K_\theta^2 H_{R3}(K_\theta) \dot{\theta} + K_y^2 H_{R4}(K_y) \frac{y}{R} \right] \quad \text{(B.4a)}
\]

\[
D = \frac{1}{2} \rho U^2 (2R) \left[ K_\theta P_{R2}(K_\theta) \frac{\dot{\theta} R}{U} + K_\theta^2 P_{R3}(K_\theta) \dot{\theta} + K_y P_{R5}(K_y) \frac{\dot{y}}{U} + K_y^2 P_{R6}(K_y) \frac{y}{R} \right] \quad \text{(B.4b)}
\]
Figure B.1. Wind pressure coefficients on the cable and rivulet with different angles of attack: (a) wind pressure coefficients on the cable; (b) wind pressure coefficients on the rivulet.
where $F_r^w$ represents the wind-induced force on the rivulet using unsteady theory; $\theta_0$ means the equilibrium position of rivulet and $\dot{\theta} = \theta - \theta_0$ is the dynamic circumferential displacement of rivulet on the cable; $H_{R1}^*, H_{R2}^*, H_{R3}^*, H_{R4}^*, P_{R2}^*, P_{R3}^*, P_{R5}^*$ and $P_{R6}^*$ are aerodynamic coefficients (flutter derivatives) with respect to the reduced frequency $K_\omega = \omega R/U$ or $K_\theta = \omega_\theta R/U$. These aerodynamic coefficients play the role of a transfer-function in this unsteady model, for example, $H_{R1}^*$ takes into account the lift force exerted on the rivulet induced by the vertical motion velocity. In the case of a bridge deck, due to the extremely small aerodynamic stiffness compared to the bridge deck stiffness, typically the aerodynamic coefficient $H_i^*$ of the vertical displacement is insignificant compared to other aerodynamic coefficients. However, this observation cannot apply to the RWIV system since the stiffness of the rivulet is negligible as mentioned in the preceding content.

B.2.3 Hysteretic Nonlinear Model

The hysteretic type nonlinearity observed in bridge decks has been investigated recently (e.g. Diana et al. 2010; Wu and Kareem 2010). Similar to the bridge deck, strong hysteretic nonlinearity is observed in the wind-rivulet-cable interaction, which indicates that the higher-order fluid memory in the RWIV system is another important feature that cannot be ignored. The aforementioned unsteady model actually can only take the linear fluid memory effect into account. In order to account for the nonlinear unsteadiness, the hysteretic loops need to be simulated. Typically, the steady-state aerodynamic coefficients are modeled by a nonlinear polynomial in terms of the angle of attack, however, the hysteretic behavior can be best described by higher-order (nonlinear)
polynomial involving the dynamic angle of attack (resulting from turbulence components and structure motions) and its derivative (Wu et al. 2012)

\[ c_{i}^{\text{hyst}}(\dot{\theta}, \ddot{\theta}) = \sum_{j,k}^{n} \eta_{j,k} \dot{\theta}^{j} \dot{\theta}^{k} \]  

(B.5)

where subscript "L" represents "L", "D" or "M", which denotes coefficients related to lift force, drag force and pitching moment, respectively; \( \eta_{j,k} \) is the coefficient corresponding to \((j+k)^{th}\)-order term; 2n is the highest order of the polynomial possible to parsimoniously model hysteretic behavior. The actual order of the model is optimally chosen based on the data used for fitting. Besides, the contributions of some of the terms in the polynomial may be negligible which leads to a simplified expression. For example, for the lift force coefficient of the rivulet in this research, the hysteretic nonlinearity can be well represented by the following model

\[ c_{L}^{\text{hyst}}(\dot{\theta}, \ddot{\theta}) = [c_{L}(\theta_{0})] + \eta_{0,0} + \eta_{1,0} \dot{\theta} + \eta_{1,1} \dot{\theta} \ddot{\theta} + \eta_{2,2} \ddot{\theta}^{2} + \eta_{2,3} \dot{\theta}^{2} \ddot{\theta} + \eta_{3,3} \ddot{\theta}^{3} + \eta_{3,4} \dot{\theta}^{3} \ddot{\theta} + \eta_{4,4} \ddot{\theta}^{4} + \eta_{4,5} \dot{\theta}^{4} \ddot{\theta} + \eta_{5,5} \ddot{\theta}^{5} + \eta_{6,6} \dot{\theta}^{5} \ddot{\theta} + \eta_{5,6} \ddot{\theta}^{6} + \eta_{6,7} \dot{\theta}^{6} \ddot{\theta} + \eta_{7,7} \ddot{\theta}^{7} \]  

(B.6)

where the steady-state coefficient \( c_{L}(\theta_{0}) \) corresponding to the equilibrium position is extracted from the constant term in order to emphasize the difference between the steady-state and the hysteretic cases. A similar model with a third-order polynomial, in the context of a rheological model, has been proposed by Diana et al. (2010) to represent lift coefficients on a bridge deck.
B.3 Parameter Identification

A series of wind-tunnel tests have been carried out in HD-2 wind tunnel, Hunan University, to extract the parameters utilized in conventional QS, unsteady, and hysteretic nonlinear models.

B.3.1 Wind-Tunnel Test Set-Up

There are two schemes popularly utilized to simulate the rivulet on the cable. One is to generate ‘natural’ rivulet using the water-spray system; and the other is to use ‘artificial’ rivulet by sticking a solid rod on the cable. The ‘artificial’ rivulet is applied in this study since the pressure on the rivulet needs to be measured. The lower rivulet, typically located inside the wake formed behind the cable, has negligible effect on the cable aerodynamics, hence, only the upper rivulet is simulated. End plates are installed at each end of the rigid cable to control the end flow conditions. The cable-rivulet section model is mounted on a forced vibration system. Inflow conditions have minimal level of turbulence in this study.

The overall aerosteady forces on the cable-rivulet system were measured at various angles of attack, while the pressures on the cable and upper rivulet surfaces were obtained under the forced pitching and plunging motions. The frequency of both pitching and plunging motions was 1 Hz. The peak-to-peak amplitudes were 4 mm for plunging and 10 degrees for pitching. There were a total of four sections of the rigid cable in which the pressure taps were installed, and on each section 63 pressure taps were installed. The position of these pressure taps is defined by the angle \( \psi \), which is determined from the stagnation point to the center of taps (Figure B.2). Since the pressure on the rivulet surface was being monitored, the size of the rivulet and the corresponding cable was...
amplified proportionately for experimental convenience. The rigid cable was made of acryl glass with the diameter of 350 mm and the length of 1540 mm while the rigid rivulet was made of polyethylene. The shape of the rivulet is one section of an arc of 42 mm which belongs to a circle with a 25 mm radius (Figure B.2). The schematic illustrations of the test section coordinates, the positions of rivulet and pressure holes, and the rivulet and cable sizes are presented in Figure B.2. The angle $\theta$ represents the position of upper rivulet and $\alpha$ the relative angle of attack induced by vertical motion of cable.

Figure B.2. The coordinate system and arrangement of pressure taps on cable and upper rivulet
B.3.2 Steady-State Coefficients Utilized in Quasi-Steady Model

The steady-state coefficients for the cable and rivulet were identified through the experimentally obtained aerostatic forces and are shown in Figure B.3. In order to conveniently obtain their derivatives in terms of the angle of attack, which will be used in the calculation of QS theory based RWIV, these steady-state coefficients were fitted using a nonlinear least square scheme.

As indicated in Figure B.3, the negative slope of the lift force coefficient with respect to the angle of attack in the certain ranges is extremely large. Since the equilibrium position of the rivulet (angle of attack) is changed as the wind velocity increases, the position of the rivulet is in the negative slope range of the lift force coefficient in a small range of wind velocities where the instability occurs based on the classical galloping theory. It should be noted that the “amplitude limited” property of RWIV may result in part from the extremely narrow range of the negative slope of the lift force coefficient. Assuming that the cable vibrates harmonically with an amplitude $a_c$ and frequency $b_c$, the velocity magnitude of the cable, which contributes to the “relative” angle of attack, is proportional to $a_c \cdot b_c$. Hence, as the amplitude of the vibration increases, the “total” angle of attack is very easy to escape out of the narrow range of negative slope, especially for the high frequencies of vibration or in higher modes of the cable.

B.3.3 Aerodynamic Coefficients Utilized in Unsteady Model

There are generally two different approaches for the identification of aerodynamic coefficients, namely the free and forced oscillation methods. In each scheme the forces can be either directly measured or through integration of surface pressures. For the
Figure B.3. Experimental and numerical steady-state coefficients: (a) steady-state coefficients of cable; (b) steady-state coefficients of rivulet
aerodynamic coefficients of the rivulet, the identification scheme is based on measured pressures, which may be the only practical approach. It is postulated that the contribution of the rotational motion of the cable without artificial rivulet to the aerodynamic forces is negligible, hence, the circumferential oscillation of the rivulet on the cable may be adequately simulated by the rotational motion of the cable-rivulet system. A special triggering system was designed to synchronize the structural motion and wind-induced pressure signals. In order to validate the identified aerodynamic coefficients based on measured pressures, a comparison of the force time histories obtained from the original data and from the identified aerodynamic coefficients is presented in Figure B.4. No significant differences between these time histories of forces on the rivulet are noted. The small difference at the peaks may be due to the higher-order frequency components resulting from nonlinear effect (Wu and Kareem 2011). Besides, there is indiscernible phase shift between these time histories, which indicates the trigger successfully synchronized the motion and pressure data.

The selected identified aerodynamic coefficients of the upper rivulet are presented in Figure B.5. As shown in the figures, the aerodynamic coefficients vary significantly with respect to the wind angle of attack (e.g. the position of rivulet). As the angle of attack is in the range of 58° - 62°, the values of these aerodynamic coefficients become extraordinarily large with significant changes with respect to the reduced wind velocity, which indicates presence of strong unsteadiness in this range. This range is critical for RWIV since it is the region of significant negative slope of the lift force coefficient of the cable within this range that leads to large amplitude oscillations. It should be noted that
Figure B.4. Comparison of force time histories on the rivulet: (a) time history of lift force $C_y$ on the rivulet; (b) time history of drag force $C_x$ on the rivulet
the aerodynamic coefficient $H_a$ of the vertical displacement is not negligible compared to other aerodynamic coefficients.

![Graphs showing aerodynamic coefficients](image)

Figure B.5. Selected identified aerodynamic coefficients for upper rivulet: (a) aerodynamic coefficients of vertical motion; (b) aerodynamic coefficients of torsional motion

B.3.4 Hysteretic Loops Utilized in Nonlinear Hysteretic Model

The Moore-Penrose pseudoinverse scheme is utilized to identify the coefficients of the hysteretic nonlinear model. The Equation B.6 could be rewritten in the matrix form as

$$ C_{L,N \times 1} = \Theta_{N \times Q} \eta_{Q \times 1} $$

(B.7)
where $\mathbf{C}_L$ is a vector which consists of the output data and $\mathbf{\Theta}$ is a matrix which consists of the input data; $N$ indicates the number of input-output pairs; $\eta$ is a vector which consists of the coefficients of the hysteretic nonlinear model with the number of $Q$. As it can be inferred from Equation B.7, only if the number of input-output pairs $N$ is equal to the number of the coefficients of the hysteretic nonlinear model $Q$, the input matrix $\mathbf{\Theta}_{N \times Q}$ is a square matrix and hence is possibly invertible, otherwise, if $N$ is larger or smaller than $Q$, or $\mathbf{\Theta}_{N \times Q}$ is not invertible, then one needs to introduce pseudoinversion based on the least-squares techniques for the coefficient identification. Accordingly, the coefficients of the hysteretic nonlinear model could be identified as

$$\eta_{Q \times 1} = (\mathbf{\Theta}_{N \times Q})^+ \mathbf{C}_{L_{N \times 1}}$$

(B.8)

where the symbol “$^+$” represents the Moore-Penrose pseudoinverse. At the reduced wind velocity of 9.5 m/s, the measured and numerically simulated hysteretic loops for the aerodynamic lift and drag forces of the rivulet are presented in Figure B.6 (no turbulence contribution is considered in this study). Due to the limitation of the testing platform in the wind tunnel, where the maximum peak-to-peak amplitude is limited to $10^\circ$ for pitching, the results shown here therefore, portray a limited description of hysteretic nonlinearity of RWIV.

B.4 Numerical Example

A numerical example is utilized to demonstrate the effects of unsteadiness and hysteretic nonlinearity on RWIV. The cable-rivulet models used in the numerical example are described in the preceding sections. The natural frequency of the cable is set equal to the
Figure B.6. Hysteretic simulation of rivulet ($\theta_0=66^\circ$): (a) hysteretic nonlinearity in lift force; (b) hysteretic nonlinearity in drag force
frequency of forced vibration for the identification of aerodynamic coefficients. The mass and damping ratio of the cable are 6 kg per unit length and 0.001, respectively. Based on an earlier study, the linear damping coefficient and the Coulomb damping force between rivulet and cable are set as 0.0008 N·s/m² and 20% of the weight of the rivulet, respectively (Li et al. 2013). The mass of the rivulet is set 0.0098 kg per unit length according to the density of water and the assumed area of the upper rivulet on the cable. Besides, a realistic range of the circumferential motion of the rivulet is assumed to fall between 0 and π/2. For the unsteady model, the equilibrium position of the rivulet on the cable needs to be established first. It is assumed that the results obtained from the QS model would serve as a good estimate of the equilibrium condition. Although the equilibrium position based on the QS assumption shows slight difference with the observation in the wind tunnel, fortunately the simulation of the unsteady model proposed in this research are not sensitive to the equilibrium position of rivulet. As long as this equilibrium position is in a reasonable range, the calculation of this unsteady model converges to the same result. The equilibrium position of the rivulet on the cable based on the QS assumption used in this example could be easily replaced in the proposed unsteady model if additional reliable data based on the wind-tunnel tests becomes available. Some high-quality records using an ultrasonic transmission thickness measurement system (UTTMS) have been presented in a recent study (e.g., Li et al. 2010).

RWIV is simulated over a range of wind velocities utilizing the aforementioned models, namely the QS, unsteady, and hysteretic nonlinear models. The calculated response of the cable based on the QS, unsteady and hysteretic nonlinear models at the
wind velocity of $Uf/D=9.5$ m/s are shown in Figure B.7 while the responses of the rivulet are presented in Figure B.8. As shown in the figures, the overall behavior of the cable and rivulet obtained from the QS, unsteady and hysteretic nonlinear models are similar while the vibration amplitude of the unsteady model is smaller and the vibration amplitude of the hysteretic nonlinear model is larger compared to the calculations based on the QS model. This comparison indicates that the linear unsteadiness tends to ameliorate RWIV while the hysteretic nonlinearity intensifies RWIV. A close examination of the time histories of the vibration of the cable and rivulet reveals that the motion of the cable is a purely harmonic with a unique significant frequency component, while the rivulet signature presents relaxation type oscillations, which indicates the motion of the rivulet undergoes a nonlinear effect.

Figure B.9 shows the time histories of the forces acting on the rivulet in the tangential direction of the cross-section of the cable based on the QS, unsteady, and hysteretic nonlinear models. A comparison of results with the QS model reveals no significant qualitative differences in the overall behavior of the forces based on the hysteretic nonlinear model with the exception of some quantitative discrepancies. Whereas there are three significantly different features in the forces based on the unsteady model. First, since the proposed unsteady model is based on a dynamically linear analysis framework, the attendant aerodynamic forces are described by a pure harmonic oscillation with a unique significant frequency component. Second, the aerodynamic force based on the unsteady model has a negative minimum value, which is reasonable considering the vibration behavior of the cable and rivulet. Third, the Coulomb damping force, whose role in the cable-rivulet system is not fully understood to
Figure B.7. Comparison of calculation results of the cable based on various models: (a) QS model; (b) unsteady model; (c) hysteretic nonlinear model
Figure B.8. Comparison of calculation results of the rivulet based on various models: (a) QS model; (b) unsteady model; (c) hysteretic nonlinear model
date, has a smaller relative contribution. In order to further shed light on the effect of forces exerted on the rivulet, Figure B.10 presents the force-induced energy per unit time based on the QS, unsteady, and hysteretic nonlinear models. Compared to the QS results, there is no significant difference in the overall behavior of the force-induced energy per unit time based on the hysteretic nonlinear model with the exception of the quantitative variations, whereas there are significantly different features noted for the unsteady results, i.e., the aerodynamic force essentially does positive work.

Figure B.11 presents a comparison of the amplitudes of the cable and of the wind velocity range in which the large limited amplitude oscillation appears between the numerical results based on the QS and unsteady models (here the comparison with the nonlinear hysteretic results is not presented due to limited data available). As indicated in the figure, both of these models can obtain wind velocity limited and amplitude limited results. The limited wind velocity is almost the same for these two models while the limiting amplitude obtained from the unsteady model is around half of the one obtained based on the QS model. It should be noted that the oscillation frequency of the rivulet is set to be 1 Hz during the calculation process with the unsteady model. In other words, the reduced frequency $\kappa_\gamma$ and $\kappa_\phi$ are not changed under a certain wind velocity. A more sophisticated procedure involving iterative calculation may give the unsteady results with higher accuracy, in which the oscillation frequency of the rivulet is treated as an unknown. However, based on the experience of the flutter analysis on bridge decks, such a simplification only changes the results negligibly.
Figure B.9. Comparison of various forces exerted on the rivulet based on the QS, unsteady, and hysteretic nonlinear models: (a) QS model; (b) unsteady model; (c) hysteretic nonlinear model (p322-324)
Figure B.10. Comparison of various force-induced energy per unit time based on the QS, unsteady and hysteretic nonlinear models: (a) QS model; (b) unsteady model; (c) hysteretic nonlinear model (p325-328)
(a)
B.5 Detection of Nonlinear Features

The fluid-structure interaction behavior varies significantly in their complexity depending on the rivulet position. As mentioned in the preceding sections, nonlinearity is portrayed in the pressure/force signal represented by the local peaks. It is interesting to investigate the nonlinear pressure/force signal based on the higher-order spectrum technique, where the underlying mechanism of the nonlinear interaction in RWIV could be explained. For a linear system, the second-order information embedded in a power spectrum represents distribution of energy at different frequencies which fully characterizes a linear system in the frequency domain. The normalized value of the cross-power spectrum captures the phase relation at the same frequency between two different signals. However, for a nonlinear system higher-order spectrum is needed because the power spectrum cannot
portray the energy transfer between different frequency components which is a typical feature of nonlinear systems.

Among these higher-order spectra, the bispectrum, the Fourier transform of the triple correlation can capture quadratic nonlinearities (e.g., Kim and Powers 1979). Usually, the normalized bispectrum, the bicoherence is actually utilized to analyze the nonlinear signal, where the unit value could be treated as an indication of fully quadratic nonlinearity in the signal. In this study, the nonlinear coupling phenomenon among different frequency components for the pressure data on the cable and rivulet (where the oscillation frequency is 1 Hz) is analyzed. Instead of presenting the contour format typically used in the literature, Figure B.12 presents the bicoherence of two frequency components with all the frequencies within the range of interest (a) for a point located on the rivulet, (b) for a point located on the cable, and (c) between the above-mentioned two points. The peaks in the figures are due to the background noise in the wind tunnel. It is noted in Figure B.12(a) that there is a high degree of nonlinear coupling between 1 Hz or 1.5 Hz component and the frequency region between 0-4 Hz. Beyond this range, the nonlinear coupling among these frequency components decreases significantly. There is almost no nonlinear coupling between 1 Hz or 1.5 Hz component and the frequency region higher than 8 Hz. Overall, the level of nonlinear coupling at 1.5 Hz with other frequency components is lower than that of 1 Hz. The level of nonlinear coupling behavior for the point on the cable and between the points on the rivulet and the cable, as respectively indicated in the Figure B.12(b) and Figure B.12(c), is not as high as that for the point on the rivulet. The identified nonlinear coupling features among different frequency components of aerodynamic pressures (forces) based on the auto- and cross-
bispectrum analysis gives new insight into origin of some higher-order harmonics of forces in the RWIV system. This reinforces the need to include nonlinearity and it offers future avenues to further investigate the fundamentals concerning the excitation mechanism of RWIV.

B.6 Concluding Remarks

The complexity of flow around a rivulet in the rain-wind induced vibrations (RWIVs) of a cable that exhibits unsteady and nonlinear features is highlighted. The results based on the higher-order spectrum analysis indicate that the nonlinear coupling behavior in the aerodynamics of rivulet is more significant than that in the aerodynamics of cable. Current models for calculating the RWIV are customarily based on the quasi-steady (QS) assumption, which fails to capture unsteadiness and hysteretic nonlinearity. In order to capture unsteadiness and nonlinearity, an unsteady model, which is parallel to Scanlan’s analysis framework for the bridge aerodynamics, and a hysteretic nonlinear model, which is optimally chosen based on the available data, respectively, are presented. The aerodynamic coefficients needed for the unsteady model are identified based on the mapping of pressure field on an oscillating cable-rivulet section model, while the parameters for the hysteretic nonlinear model are identified using measured integrated loads based on the Moore-Penrose pseudoinverse scheme. Both models take into account the fluid memory in the aerodynamic forces exerted on the rivulet. Specifically, the unsteady model considers the linear fluid memory effects while the hysteretic nonlinear model accounts for the higher-order (nonlinear) fluid memory effects. The simulated results of RWIV based on the QS, unsteady and nonlinear hysteretic models exhibit
Figure B.12. Higher-order spectrum analysis of the pressure fluctuations: (a) auto-bicoherence of a point on the rivulet (point 58 in Figure B.2); (b) auto-bicoherence of a point on the cable (point 7 in Figure B.2); (c) cross-bicoherence of two points (points 58 and 7 in Figure B.2)
(a) 

(b) 

(c)
different values. A comparison indicates that the linear unsteadiness tends to ameliorate RWIV while conversely the hysteretic nonlinearity intensifies RWIV.
APPENDIX C:

VORTEX-INDUCED VIBRATION OF BRIDGE DECKS

A brief overview of vortex-induced vibration (VIV) of circular cylinders is first given as most of VIV studies have been focused on this particular bluff cross-section. A critical literature review of VIV of bridge decks that highlights physical mechanisms central to VIV from a renewed perspective is provided. The discussion focuses on VIV of bridge decks from wind tunnel experiments, full-scale observations, semi-empirical models and computational fluids dynamics (CFD) perspectives. Finally, a recently introduced reduced order model (ROM) based on truncated Volterra series is introduced to model VIV of long-span bridges. This model captures successfully salient features of VIV at "lock-in" and unlike most phenomenological models offers physical significance of the model kernels.

C.1 Introduction

Flow around bluff bodies typically separates resulting in shedding of vortices that impact the body with some periodicity. The fluctuating pressures around the bluff body generated by the periodic vortex shedding result in the fluctuations in the cross-wind force characterized by a dominant frequency described by the Strouhal number $S_t = fD/U$, where $f$ is the dominate frequency component of the vortex-induced force; $D$ is the front projection area of the structure cross-section per unit length; $U$ is the oncoming flow
velocity. The Strouhal number is a function of structural geometry and Reynolds numbers \( R_e = U/B/\nu \), where \( B \) is the characteristic size of the structure and \( \nu \) is the kinematic viscosity of the media around the structure. As the frequency of vortex-induced forces approaches the modal frequency of the bluff body, “lock-in” a range of wind velocity in which the appearance and disappearance of complex wave-forms is observed (Bishop and Hassan 1964a), emerges with relatively large associated response. Due to nonlinear fluid-structure interactions, the vortex-induced vibration (VIV) exhibits limit cycle oscillations (LCOs). Although VIV response does not always result in catastrophic failures, it seriously impacts the fatigue life and a loss of desired functionality in the case of buildings.

It should be noted that the focus of most studies in the literature has been on the VIV of circular cylinders. On the one hand, the findings reveal some fundamental mechanisms of VIV as the circular cylinder is an important and representative bluff body. While on the other hand, VIV of bridge decks, as a typical bluff body of structural engineering significance characterized by a significantly long afterbody, exhibits substantially different flow features from circular cylinders. For example, there are often fixed separation points for bridge decks instead of moving separation points for circular cylinders, which vary with Reynolds numbers and body oscillation frequency and amplitude. Besides, the torsional force induced by the wake vortices may be important for bridge-deck-like bluff bodies as the structural geometry features a long afterbody. The angle of wind incidence also becomes an important parameter of VIV of bridge decks due to asymmetric bridge deck cross-section. It is well known that the Kármán "vortex street" resulting from the flow separation is the main physical origin of VIV for circular
cylinders. The underlying VIV mechanisms of bridge-deck-like bluff bodies may belong to one of the following four types, i.e., the leading-edge vortex shedding, the impinging leading-edge vortices, the trailing-edge vortex shedding and the vortex shedding alternating between edges (Naudascher and Wang 1993).

C.2 VIV of Circular Cylinders

Since the pioneering work of Strouhal (1878) and Rayleigh (1896), a large number of studies have been carried out on the fundamental mechanisms around VIV with a focus on circular sections. A chronological development of the subject is well documented in several reviews (e.g., Sarpkaya 1979; Bearman 1984; Williamson 1996; Billah 1989; Sarpkaya 2004; Williamson and Govardhan 2004). A brief overview of VIV is provided here as a primer for VIV of bridge decks.

C.2.1 Fundamental Studies of VIV

von Kármán and Rubach (1912) characterized the famous “vortex streets” phenomenon with the Kármán constant $K=h/a$, where $h$ is the transverse distance between the two rows of vortices and $a$ is the longitudinal space period, as indicated in Figure C.1. There are two vortex sheets at each side of the body stemming from the boundary layer separation, which roll up into the “vortex streets” in the wake. The Kármán “vortex streets” were deduced in ideal situations, e.g., the nonviscous fluid and the concentration of vorticity. Actually, in a viscous flow, in order to keep the moment of vorticity, which is equal to the mean vorticity per unit length multiplied by $h$, as a constant, the transverse distance $h$ has to increase with time since the mean vorticity decreases with time due to
the interaction of the two vortex rows (two separating shear layers) with vorticities of opposite sign (Birkhoff 1953; Abernathy and Kronauer 1962).

![Figure C.1. Kármán "vortex street" behind the circular cylinder](image)

Gerrard (1966) on the other hand, revealed the underlying physical mechanism of vortex formation in the wake of bluff bodies. Central to Gerrard's (1966) study is the formation region in which the irrotational flow from outside the wake crosses the wake axis. This flow is divided into three parts, as presented in Figure C.2. One part is entrained by the growing vortex on the opposite side; the second part is entrained by the shear layer upstream of the vortex; the third part reverses to the interior of the formation region.

![Figure C.2. Filament-line sketch of the formation region [after Gerrard (1966)]: arrows showing reverse flow c and entrainment a and b](image)
Recently, the VIV area has benefitted from development in innovative experimental procedures and increasing computational capability, such as Particle image velocimetry (PIV) and the direct numerical simulation (DNS) techniques. A more detailed behavior of the wake vortices, e.g. the different wake patterns such as 2S (two single vortices per cycle of motion), 2P (two vortex pairs per cycle of motion) and P+S modes, have been observed, as presented in Figure C.3. The relevant underlying physical mechanisms have been deduced, which have been presented in Williamson and Govardhan (2004).

Figure C.3. Various vortex formation modes [after Williamson and Govardhan (2004)]: (a) 2S mode; (b) 2P mode; (c) P + S mode
C.2.2 Modeling VIV of Circular Cylinders

Increasing availability of experimental data has confirmed some general features of VIV, such as the self-excited and self-limiting vibration at "lock-in", the frequency of wake vortices entrained by the structural frequency at “lock-in”, the nonlinear hysteresis behavior and the frequency demultiplication. The vortex-induced forces and the attendant structural response (especially in the "lock-in" region in the cross-wind direction) can be estimated utilizing empirical models, developed over the years based on experimental observations. It should be noted that, apart from the fluctuating transverse force, the fluctuating in-line force is also induced by the wake vortices with roughly double the dominant frequency-component and with a lower intensity. Most of the models reviewed in this study are based on the assumption of full correlation of the forces spaced along the spanwise direction of the structure.

Bishop and Hassan (1964a; 1964b) systematically carried out a series of experiments to establish a solid foundation for the development of VIV mathematical models based on the experimental observations. The experimental results of the stationary circular cylinder with typical features of the mean and fluctuating drag and lift coefficients were presented. Similar to the \((1/S_t)-\Re_c\) curve, these aerodynamic coefficients are also characterized by critical positions in terms of Reynolds numbers as these plots exhibit major changes in trends. Besides, the experimental observations suggested that it may be appropriate to utilize a conventional mechanical oscillator for simulating the circular cylinder wake behavior. Whereas in the case of oscillating circular cylinders experimental results show “hysteresis loop” and frequency demultiplication phenomena. Besides, the experimental results showed that there was a sudden change in the force
amplitude and the phase difference between the force and the structural motion at a certain “critical” frequency. Furthermore, these experimental observations suggested that the wake behavior behind the circular cylinder could be simulated using a nonlinear self-excited oscillator (wake oscillator).

Stimulated by the suggestion in Bishop and Hassan's (1964a; 1964b) study, Hartlen and Currie (1970) simulated the VIV with a second-order linear mechanical oscillator and a second-order nonlinear wake oscillator which was coupled with the structural motion. The governing equations of their model were substantially derived based on the experimental data. It was shown that the proposed wake oscillator was a Von der Pol type oscillator which captured typical features of VIV at "lock-in", such as the self-excited and self-limiting properties, the Strouhal relationship and the interaction between the wake and the structural motion. This phenomenological approach based wake oscillator, however, failed to offer a clear and relevant physical significance of its empirical parameters. Skop and Griffin (1973) improved the model by assigning physical meaning to each empirical parameter, which showed added advantages as it demonstrated good predictive capability of VIV. Typically, both empirical parameters $h$ and $G$ in the improved wake oscillator simply showed a logarithmic relationship with the mass-damping ratio (represented by the parameter $\zeta/\mu$ in their study, where $\zeta$ means the mechanical damping coefficient and $\mu$ is the mass ratio of the displaced mass of fluid to the mass of the structure), as shown in Figure C.4. Based on the similar idea, Landl (1975) also improved Hartlen and Currie's model by introducing a fifth-order aerodynamic damping term.
Iwan and Blevins (1974) presented a model based on basic fluid-mechanics principles and the momentum conservation theory which directly related parameters in their model to physical constants of a VIV system. The identification of the model parameters was basically based on stationary and forced-vibration, which suggested that the underlying assumption of fluid memory effects on VIV system was significantly weak. Employing a quite different approach compared to Hartlen and Currie’s scheme, this model, however, also showed a Vol de Pol type nonlinear behavior. Similarly, Dowell (1981) offered another self-consistent nonlinear VIV model based on basic fluid mechanics considerations, which was verified using G.W. Jones et al.’ (1969) experimental results at high Reynolds numbers.
The models based on the wake-oscillator concept obtained significant success as applied to practical engineering problems. Birkhoff (1953), on the other hand, presented an attractive observation that the behavior of wake swinging from side to side was similar to the tail of a swimming fish. The impetus of Birkhoff's concept resulted in several interesting nonlinear models (Nakamura 1969; Tamura and Matsui 1979). Additionally, Basu and Vickery (1983) proposed another type of VIV model under the framework of linear random-vibration theory. In their study, the vortex-induced effects were simulated with a narrow-band random force while the motion-induced effects were considered using a nonlinear damping term which could capture the amplitude-dependent and Reynolds number effects. The concept of vortex-induced stochastic aerodynamic force and of motion-induced deterministic aeroelastic force was further developed by d’Asdia et al. (1993) through correlating the along-wind and cross-wind aerodynamic forces.

C.3 VIV of Bridge Decks

Basic understanding of VIV of bridge decks has been improving during the past several decades as it has been a subject of focused wind-tunnel experiments, full-scale observations, and development of semi-empirical models and computational fluid dynamics (CFD) schemes. A review focusing on VIV of bridge decks highlighting physical mechanisms with a renewed perspective is provided. Only a selected few studies highlighting the progress in the VIV of bridge decks are included for the sake of brevity.

C.3.1 Wind-Tunnel Experimental Results

Nakamura and Mizota (1975) investigated spring-mounted rectangular prisms with various aspect ratios under smooth wind conditions. They noted that, at zero incidence of
wind flow, the boundary layers separated at two front edges with no downstream reattachment for the models with the aspect ratios of 1:1 and 1:2 while the separated shear layers reattach somewhere on the side faces for the model with an aspect ratio of 1:4. Their experimental results in terms of base pressures and lift force coefficients suggested an enhancement in vortex-induced effects with decreasing aspect ratio of length (along-wind direction) to width (cross-wind direction) of rectangular prisms. Whereas the decreasing aspect ratio of the spanwise length to the size of the cross-section weakened vortex-induced forces (Fail et al. 1959; Wootton 1969). All models with different aspect ratios presented VIV with a notable “lock-in” region. In addition, Nakamura and Mizota (1975) explained the physical mechanisms of “lock-in” phenomenon. The abrupt phase change of the unsteady lift force, which closely correlated with the phase change of the near-wake velocity field, resulted in negative aerodynamic damping near the “lock-in” range and led to large vortex-induced motion amplitudes. Similar observations have been reported for circular cylinders as discussed in the preceding section (Bishop and Hassan 1964a; 1964b).

Komatsu and Kobayashi (1980) carried out a series of experiments of various cross-sections, such as the L-shaped, T-shaped, H-shaped and rectangular cylinders with various aspect ratios. Both free and forced harmonic oscillations were used together with smoke-wire flow visualization technique. They noted that Kármán vortex street (generated from the trailing edge) and motion-induced vortices (generated from the leading edge) were the two main sources of VIV of bluff prismatic cylinders. They also reported an empirical formula for the critical wind velocity corresponding to the maximum VIV response, which is a linear function of the aspect ratio of bluff cylinders.
Shiraishi and Matsumoto (1983) investigated a series of cross-sections such as rectangular, H-shaped, trapezoidal and hexagonal sections with various aspect ratios in both heaving and torsional degrees of freedom, including the consideration of various flow angles of attack. Three generation mechanisms of VIV were classified, i.e., separated vortices from leading edge due to structural motion, secondary vortices at trailing edge due to structural motion and separated vortices from trailing edge due to Kármán vortex shedding. Five simplified bridge deck cross-sections with various aspect ratios were investigated to guide the deck optimization for reducing VIV response. The results indicated that bridge decks with long afterbody are more effective in mitigating VIV. While the emergence of multi-“lock-in” regions was attributed to the “frequency demultiplication” phenomenon (Van der Pol 1927) by most of the researchers, Shiraishi and Matsumoto (1983) claimed that the smaller-peak oscillation in the lower wind velocity region resulted from the arrival of the separated vortices from the leading edge to the trailing edge after $n$ cycles of heaving motion (where $n$ is a natural number) or after $(2n-1)/2$ cycles of torsional motion. Accordingly, the onset critical reduced wind velocity for the smaller-peak oscillation of the torsional mode was $[2V_{cr}/(2n-1)]$ where $V_{cr}$ was the onset critical reduced wind velocity for the larger-peak oscillation. Zhang et al.’s (2008) experimental data, as shown in Figure C.5, sheds a different view point, where the lower critical reduced wind velocity for the smaller-peak oscillation of the torsional mode was noted to be around $V_{cr}/2$.

The secondary vortex was also observed by Nakamura and Nakashima (1986) and it was attributed to being a characteristic of the elongated sharp-edged bluff bodies (e.g. typical bridge decks) since no similar phenomenon was observed for oscillating circular
Figure C.5. VIV response versus reduced wind velocity for a typical bridge deck [after Zhang et al. (2008)]

cylinders (Koopman 1967). In the study by Nakamura and Nakashima (1986), both stationary and oscillating experiments of bluff prisms with elongated H-shaped, -shaped and rectangular cross-sections were carried out in a wind tunnel and in a water tank with a hydrogen-bubble visualization technique. They noted that VIV occurred even if a splitter plate, which prevented the interaction between shear layers, was inserted in the wake. Hence, both the Kármán vortices (double-layer flow instability) and the impinging-shear-layer instability (single-layer flow instability) were believed to be the contributing mechanism for VIV response.

Matsumoto et al. (1993) investigated the interaction between Kármán and motion-induced vortices and the effects of turbulence on VIV for a rectangular cylinder with an aspect ratio of 4:1 and a hexagonal box girder with/without handrails. On the one hand, the experimental results indicated that VIV of both models was essentially excited by the
motion-induced vortices which were mitigated by the presence of Kármán vortices. While on the other hand, in order to explain the physical mechanism of "frequency entrainment" in the "lock-in" region, Sarpkaya and Shoaff (1979b) emphasized the role of structural motion in enhancing vortex strength.

According to the results in Matsumoto et al. (1993), the effects of turbulence on VIV is rather complicated. While the turbulence stabilizes the VIV of rectangular cylinders, its effects on the hexagonal box girder depended on several other features such as the solidity ratio of the handrail and the Scruton number. The effects of turbulence on the motion-induced force is uncertain due to the limited understanding of this issue (e.g., Scanlan 1997; Haan and Kareem 2009; Kareem and Wu 2012). Whereas, one deterministic effect of turbulence is that it increases vorticity diffusion and thus reduces vortex strength. Similar observation has been advanced by Sarpkaya (1979a; 1979b). Besides, experimental and field observations indicate that turbulence decreases the spanwise correlation, which concomitantly decreases VIV response of the structure.

Suppose the motion-induced vortices are not affected by turbulence, one possible approach to examine the final effects of turbulence on VIV (stabilization or destabilization), which is illustrated in Figure C.6, may depend on the relative intensity of Kármán vortices compared to the motion-induced vortices. If Kármán vortices dominate VIV, then the turbulence decreases structural response (Sarpkaya 1979a; 1979b); on the other hand, if the motion-induced vortices dominated VIV, then the turbulence may increase the structural response as the enhancement of structural response due to weakening mitigative effects of Kármán vortices on the motion-induced vortices is significant. More experimental data sets are necessary to validate this conjecture.
Larose et al. (2003) investigated VIV of Stonecutters bridge deck (twin-box) based on a 1:20 scaled sectional model. Two distinct "lock-in" regions were observed for the torsional mode while there was only one well-defined "lock-in" region for the heaving mode. The Reynolds number effects on VIV of the bridge deck was examined in their study. It appeared that the bridge deck appendages (e.g., longitudinal guide vanes and maintenance gantry rails) were less efficient in mitigating VIV response at low Reynolds numbers as compared to at high Reynolds numbers.

Larsen et al. (2004) studied VIV of Storebælt (Great Belt East) bridge deck (single-box) using a large scaled model (1:30) with a high Reynolds number (around 500,000, bridge deck width based). On the one hand, the experimental results indicated that, in the high Reynolds numbers range, the Strouhal number was slightly higher at Reynolds number around 500,000 compared to that at the Reynolds number around 100,000. While on the other hand, in the low Reynolds numbers range, Sarpkaya (1978)
successfully predicted the maximum amplitudes of vortex-induced forces at Reynolds numbers between 300 and 1000, measured by Griffin and Koopmann (1977), utilizing the data measured over the Reynolds number range from 5000 to 25,000. This successful prediction suggested that the Reynolds number may not be an important parameter for oscillating circular cylinders. Larsen et al. (2004) also investigated in detail the mitigating effects of guide vanes on VIV utilizing this large-scale bridge sectional model.

Diana et al. (2006b) observed VIV of a multi-box bridge deck in detail in the wind tunnel based on Messina Strait suspension bridge. The experimental results presented a strong dependence on the VIV amplitude, which indicated significant nonlinear behavior. Two “lock-in” ranges were identified for the heaving mode with the bridge deck model in their study. Besides, the bridge deck shape was optimized to mitigate VIV response based on experimental observations.

Recently, Zhang et al. (2008) investigated VIV of Xihoumen bridge deck (twin-box) with various scaled models. The experimental results indicated that the "lock-in" emerges at higher and wider-range of damping ratio with larger amplitude and wider wind velocity range for low Reynolds number cases compared to for high Reynolds numbers cases. This suggested that the predicted response based on experimental results of scaled model at low Reynolds numbers was over-conservative. Similar conclusion was obtained by Schewe and Larsen (1998). Besides, the effects of guide vanes on VIV was studied at various wind angles of attack. It was shown that the guide vanes mitigate VIV response for the zero angle of attack case effectively. If the attack angle changes (±3°), the guide vanes on the one hand mitigate VIV response at high Reynolds numbers cases while it enhances VIV response for low Reynolds number cases. Two distinct "lock-in"
regions were observed for the torsional mode in their study while one well-defined "lock-in" region was identified for the heaving mode.

C.3.2 Full-Scale Observations

Smith (1980) presented field measurements of Wye Bridge, a cable-stayed box girder bridge with a 235 m span. The records indicated that the large amplitude response occurred when the wind velocity (1-minute mean value) was in the range of 7-8 m/s, the wind direction was approximately perpendicular to the bridge axis, the wind inclination was in the range of -5°-0°, and the turbulence intensity was low (around 5.0%). This large amplitude vibration was considered as VIV since the vibration frequency was close to the natural frequency of the bridge first bending mode. The critical wind speed for VIV was close to the prediction based on the wind-tunnel sectional model while the amplitude of the response was significantly lower than that derived from the wind-tunnel results. The full-scale observations supported the wind-tunnel based argument that the predicted response based on the experimental results of scaled model at low Reynolds numbers was over-conservative.

Kumarasena et al. (1989; 1991) investigated the wind-induced motion of Deer Isle Bridge with field observations, wind-tunnel experiments and finite element analyses. On-site data showed that VIV occurred at a wind velocity of 9.11 m/s, with a high response level. Kumarasena et al. (1991) pointed out that the ordinary vortex shedding of the stationary deck was affected significantly by the turbulence even with a low intensity level as it changed from being the narrow-banded (periodic) to the broad-banded.

Owen et al. (1996) monitored Kessorck Bridge with an "open cross-section" and with a 240 main span. The observation indicated that VIV occurred when the wind
velocity was in the range of 22-25 m/s. Specifically, a 3-minute duration of the large amplitude motion of the bridge deck in first mode, which was excited by the vortex shedding, was observed based on the one-hour records. The observations indicated that the emergence of VIV was very sensitive to many factors such as the wind velocity, wind direction and turbulence effects. “Lock-in” phenomenon was identified using the field measurements, where the influence of turbulence was diminished. In addition, Owen et al. (1996) pointed out that it needed a certain time to build up the large amplitude response of VIV after the corresponding vortex shedding occurred. The torsional VIV was identified utilizing the theory developed by Shiraishi and Matsumoto (1983).

To decrease the intense VIV response observed on Rio-Niterói Bridge with a steel twin-box bridge deck, which had been shut down at occasions when the wind velocity approached 14 m/s, Ronaldo and Michèle (2000) designed passive and active control devices which significantly decreased (the level of peak to peak amplitude is around 4.2 cm) or ceased VIV from a much larger intensity observed on site (the level of peak to peak amplitude is around 50 cm).

Larsen et al. (2000) monitored wind-induced behavior of the full-scale Storebælt suspension bridge. The observations indicated that the unacceptable wind-induced oscillations occurred as the wind velocity reached the range of 4-12 m/s with a direction perpendicular to the bridge axis and with a rather low turbulence intensity. The field measurements showed that the large amplitude oscillations were almost sinusoidal motions with a primary frequency component, which confirmed the phenomenon as VIV. The observed critical wind velocity of VIV on site had a good agreement with the result based on the scaled section model and scaled taut strip model, but slightly higher
compared to that derived from the full-scale aeroelastic model test. The guide vanes were designed and installed to mitigate VIV of Storebælt suspension bridge based on the field measurements. Excellent effectiveness was achieved so that VIV ceased and the flutter stability was also improved. Frandsen (2001) also investigated the Storebælt suspension bridge on site. VIV was found at a low wind velocity (around 8 m/s) with the wind direction almost normal to the bridge axis and with the low turbulence intensity. The “lock-in” phenomenon lasted about two hours based on a 4.5-hour record with the peak-to-peak amplitude around 0.7 m. The time histories of pressure on the deck surface and the accelerations of the bridge deck were simultaneously recorded and indicated high correlation in the “lock-in” range.

Fujino and Yoshida (2002) carried out full-scale measurements on a ten-span continuous steel box-girder bridge of the Trans-Tokyo Bay Highway Crossing (the measuring tool was installed on the center with the longest span of 240 m). VIV was observed under the prevailing winds (13-18 m/s for the first mode VIV and around 23 m/s for the second mode VIV) with the wind direction almost normal to the bridge axis (within ±20°) and with low turbulence intensity. VIV response of the scaled model in wind tunnel and that of field observation were in good agreement with respect to the amplitude and wind velocity range of the “lock-in”. Whereas, it should be noted that the “equivalent” full-scale amplitudes based on wind-tunnel results were simply obtained utilizing the nondimensional amplitude of scaled model multiplied by the actual bridge deck size, which may be questionable in light of nonlinearity in VIV.

Recently, Li et al. (2011) studied VIV of a suspension bridge with a twin-box girder and with a 1650 m center span. VIV was observed in the wind velocity range of 6-
10 m/s with the direction almost perpendicular to the bridge axis and with low turbulence intensity. Time-frequency S transform technique was utilized to identify the vortex-shedding pattern around the bridge deck. It was identified that the structural motion increased the vortex-shedding intensity. Similar observations have been reported for circular cylinders as discussed in the preceding section (Sarpkaya and Shoaff 1979b). Hence, the vortex shedding in the "lock-in" stage not only occurred in the gap between the two bridge decks and at the tail end of the downstream deck section as in the early stage of VIV but also occurred at the tail end of the upstream deck section and around the entire lower surface of the downstream deck. Based on the spanwise installed pressure sensors, it was observed that the spanwise correlation remained constant, with relatively small value as compared to Wilkinson's data (1981). Besides, the estimated Strouhal number based on the field measurements remained constant within the Reynolds number range of $1.74 \times 10^7 \sim 2.40 \times 10^7$ (bridge deck width based).

It should be noted that the in-line VIV is not reported during all the VIV investigations (experiments and full-scale observations) discussed above.

C.3.3 Semi-Empirical Models

The concept of wake oscillator has been utilized to model vortex-induced effects since the early work of Bishop and Hassan (1964a; 1964b). In this scheme, VIV is simulated by a system of two coupled equations, in which the structural motion is modeled by a second-order linear mechanical oscillator excited by the vortex-induced force modeled by a second-order nonlinear wake oscillator (Vol del Pol type oscillator) coupled with the structural motion (Hartlen and Currie 1970). The models rely on several experimentally derived parameters needed to characterized the system and to describe coupling between
the linear mechanical and the nonlinear wake oscillators. The experimental procedures involved to identify these parameters need to be sophisticated and care should be exercised as considerable element of uncertainty exists in such experiments (e.g., Sarpkaya 1979).

To simplify the parameter identification procedure, Scanlan (1981) proposed an explicit function for the vortex-induced force for application to VIV of bridge decks. In other words, the VIV of bridge decks was simulated with a single ordinary differential equation, where the vortex-induced force was explicitly expressed as the sum of the self-excited terms (induced by structural motion) and the forced terms (induced by vortex shedding). The Scanlan’s proposal is given by

\[
m(\ddot{y} + 2\zeta\omega_0\dot{y} + \omega_0^2 y) = \frac{1}{2}\rho U^2 (2D) \left[ Y_1(K) \left( 1 - \varepsilon \frac{y^2}{D^2} \right) \frac{\dot{y}}{U} + Y_2(K) \frac{y}{D} + \frac{1}{2} C_L(K) \sin(\omega t) \right]
\]

where \( m \) is the mass per unit span length; \( y \) is the displacement of the cross-wind degree of freedom; \( \zeta \) is the mechanical damping ratio to critical; \( \omega_0 \) is the mechanical circular frequency; \( \rho \) is the air density; \( U \) is oncoming mean wind velocity; \( D \) is the cross-wind dimension of the structure; \( K = \omega D/U \) is the reduced frequency of vortex shedding (where \( \omega \) is the circular frequency of vortex shedding determined using the Strouhal relationship outside the “lock-in” or “frequency entrainment” behavior inside the “lock-in”); \( Y_1, \varepsilon, Y_2 \) and \( C_L \) are parameters in this VIV model, which could be identified based on the experimental observation.
As shown in Equation C.1, for Scanlan’s VIV model: (1) there is no direct interplay between the vortex-induced and motion-induced effects; (2) consideration of the nonlinear motion-induced effects with a nonzero value of $\varepsilon$ results in the nonlinear VIV system; (3) the self-limiting feature was obtained with the nonlinear damping depended on the vibration amplitude instead of the phase change in the external vortex-induced force. It is possible that the Scanlan’s VIV model overweighed the motion-induced effects, in light of Sarpkaya's (1979) observation that VIV was a forced oscillation with self-excited character. Another feature of Scanlan's VIV model was that the LCO amplitude was only dependent on a single parameter, the mass-damping parameter. Although Sarpkaya (1978) has demonstrated that, for structures with small values of the mass-damping parameter, the nondimensional mass and damping affect the structural response independently. For structures with large values of the mass-damping parameter (a normal situation for the bridge deck) the structural response depends on the combination parameter of the mass ratio and damping, which is represented effectively by the Scruton number (Zdravkovich 1982). Griffin and Ramberg (1982) showed that, for similar values of the mass-damping parameter, the maximum LCO amplitudes are nearly equal and occur at similar ranges of the reduced wind velocity, as indicated in Figure C.7, where $Y$ is the cross-flow amplitude and $n$ is the natural frequency of the structure.

In contrast to Skop and Griffin’s model (1973) in which both empirical parameters simply show a logarithm relationship with the mass-damping ratio as shown in Figure C.4, the empirical parameters $Y_1$ and $\varepsilon$ in Scanlan’s model change significantly and irregularly with the mechanical damping ratio (and therefore the mass-damping ratio) (Ehsan and Scanlan 1990) as shown in Figure C.8, where each symbol indicates a
difference bridge deck. Hence, Scanlan’s model seems to have rather limited predictive capability for VIV of bridge decks with various mass-damping ratios. It should be noted that, since Scanlan’s model is a nonlinear one, the prototype results are not simply obtained by scaling model results. Often to such scaling issues from wind-tunnel to full-scale predictions are not reported.

Goswami et al. (1993a) further developed Scanlan’s single-degree-of-freedom (SDOF) VIV model based on Billah’s (1989) coupled wake oscillator model. Larsen (1995) generalized Scanlan’s VIV model by introducing a power multiplier \( \nu \) to modify the curvature of the predicted steady-state response versus the Scruton number. The so-called generalized VIV model presents a downward curvature, which is consistent with the experimental results and is opposite to the predictions of Scanlan’s model. Parameter identification schemes based on either the steady-state response or the transient response are given for this generalized VIV model. Recently, Diana et al. (2006b) proposed a new
numerical model for VIV of bridge decks by representing the fluid-structure interaction with a second-order nonlinear mechanical system coupled with the structural motion through linear and cubic relationships.

C.3.4 Computational Fluid Dynamics Simulations

With the burgeoning growth in computational capability, CFD is becoming a powerful tool for the analysis of wind-induced effects on the structures. Hence, it is a promising approach to simulate VIV of bridge decks. Most of the literature on the numerical simulation of VIV of bridge decks, with relatively large Reynolds number, is limited to 2D domain.

Fujiwara et al. (1993) investigated VIV of bridge decks with three types of elastically-mounted edge-beam cross-sections in 2D domain using direct integration of
Navier-Stokes equations (height based Reynolds number range was 2100-4000). The incident wind had a 5° angle of attack. The numerical displacement results were compared to that of scaled model in the wind tunnel. The agreement between numerical and experimental results varied with the shape of the deck cross-section.

Nomura (1993) investigated VIV of a Tacoma-like bridge deck (thin-H section of aspect ratio 5) in 2D domain based on the finite element method using the arbitrary Lagrangian-Eulerian formulation. Two sudden increases in the computed oscillation amplitudes and lift coefficients were identified around Reynolds number of 1000 and 2400. Typical flow pattern around the bridge deck were reported to be similar to the experimental results by Nakamura and Nakashima (1986).

Lee et al. (1997) studied VIV of two bridge decks, namely the Namehae bridge deck which represents a streamlined cross-section and the Seohae bridge deck which represents a bluff cross-section, using Reynolds-averaged Navier-Stokes (RANS) equations. The bridge deck remained stationary during the CFD simulation under the assumption that the structural motion would not significantly affect the flow domain if the cross-wind vibration amplitude was within 10% of the structural size. Good agreement was achieved between the drag and lift coefficients of CFD simulations and those of wind-tunnel tests for the Namehae bridge. For the Seohae bridge, both the computational and experimental VIV amplitudes showed dual displacement peaks.

Sarwar and Ishihara (2010) investigated VIV of a rectangular and a box girder cross-section using three-dimensional (3D) large-eddy simulation (LES) approach. Both forced and free oscillations of the structure in the flow were simulated with the sliding mesh technique. The computational simulation of forced oscillations for the rectangular
section obtained the “lock-in” region in which an abrupt change of the phase angle of the unsteady lift force, as an important physical mechanism of VIV, was detected. The computational response of the free oscillations for the rectangular section presented two “lock-in” regions, showing good agreement with the experimental results. Besides, aerodynamic mechanisms of countermeasuring VIV of bridge decks were studied using pressure distributions. Typically, they found the fairing had negative while the double flaps had positive effects on VIV.

Recently, the use of discrete-vortex method (DVM) to study VIV has gained attention. The development of DVM to calculate the vortex shedding of bluff bodies is based on Abernathy and Kronauer’s pioneering work (1962) in 2D domain. Sarpkaya and Shoaff (1979a; 1979b) developed the DVM comprehensively based on the interactions between the potential flow and the boundary layer around the stationary and oscillating circular cylinder. Larsen and Walther (Larsen and Walther 1997; Walther and Larsen 1997; Larsen and Walther 1998) applied DVM to the computational bridge engineering for the aeroelastic analysis with 2D viscous incompressible flow while VIV of bridge decks using DVM has also been studied by Frandsen (1999) and Morgenthal (2000).

C.4 Volterra Series based Model for VIV of Bridge Decks

In the absence of an analytical treatment of the flow around a stationary or an oscillating structure, a closed-form representation of the VIV phenomenon remains mathematical intractable. In lieu of this, semi-empirical models have been advanced to model VIV of bridge decks, but due to their phenomenological origin, their accuracy of their predictions is not ensured. In the experimental area, VIV studies, with scaled models, is limited to
low Reynolds numbers, especially for the decks of super long-span bridges. A promising approach involves CFD, however, this has its own limitation at this juncture stemming from lack of robust turbulence models for engineering applications and high demand on computational resources. In order to close the gap between a reliable numerical simulation and the need to have predictive model for practical applications, reduced order models (ROMs) traditionally offer predictions with high fidelity with reduced computational effort. As there are much lower-order degrees of freedom involved in an ROM compared to CFD, it can be tailored to meet the demands placed by the fundamental physics of the application (e.g., Raveh 2001; Lucia et al. 2004). Among various ROMs, Volterra series based model, which is a form of Taylor series with memory effects, has the promise of effectively modeling the VIV system. The complex mapping rules (static linear/nonlinear relationships) and time lag (fluid memory effects) between the aerodynamic/aeroelastic inputs and outputs, the hallmark of VIV, can be represented by the superposition of scaled and time shifted fundamental responses, i.e., convolution. Since each feature is captured elegantly by the Volterra Series that makes it an ideal candidate for ROM modeling of a VIV system.

Earlier models by Skop and Griffin (1973) and Ehsan and Scanlan (1990) were based on Van der Pol type equation with the approximation of slowly varying parameters that suggested classification of VIV as a weakly nonlinear system. This feature suggests the use of truncated Volterra series with finite terms for VIV modeling. The second-order Volterra series has been selected to simulate VIV of bridge decks by Wu and Kareem (2012e). For a nonlinear system modeled with the second-order Volterra series, the response $y(t)$ under an arbitrary input $x(t)$ could be represented as follows (Rugh 1981):
\[ y(t) = \int_0^t h_1(t-\tau)x(\tau)d\tau + \int_0^t \int_0^t h_2(t-\tau_1, t-\tau_2)x(\tau_1)x(\tau_2)d\tau_1d\tau_2 \]  
(C.2)

where the steady-state term \( h_0 \) is neglected due to the focus on the dynamic response of the VIV system; \( h_1 \) represents the first-order kernel which describes the linear behavior of the system; \( h_n \) the higher-order terms which indicates the nonlinear behavior existing in the system. Based on earlier work by Rugh (1981), a generalized impulse-function-based kernel identification scheme is developed, where the kernels of the second-order Volterra system could be expressed as (Wu and Kareem 2011)

\[
h_1(t-\tau) = \frac{1}{\alpha^2 - \alpha} \left( \alpha^2 y[\delta(t-\tau)] - y[\alpha \delta(t-\tau)] \right) \]  
(C.3)

\[
h_2(t-\tau_1, t-\tau_2) = \frac{1}{2\kappa_1 \kappa_2} \left[ y[\kappa_1 \delta(t-\tau_1) + \kappa_2 \delta(t-\tau_2)] - y[\kappa_1 \delta(t-\tau_1) - y[\kappa_2 \delta(t-\tau_2)] \right] \]  
(C.4)

where \( \delta(t) \) represents the Dirac delta function (unit-impulse function); \( y[\delta(t)] \) indicates the unit-impulse response; \( \alpha, \kappa_1 \) and \( \kappa_2 \) are selected constants.

Figure C.9 shows the identified first- and second-order kernels (time interval \( \Delta t=0.002s \)) of VIV of a rectangular prism. As shown in the figure, the magnitude of the second-order kernel is several orders smaller than that of the first-order kernel. In order to demonstrate that the truncated Volterra series based model is able to simulate the “lock-in” behavior, the response at “lock-in” of this VIV system is obtained utilizing the identified Volterra kernels and compared to the reference response obtained using the fourth-order Runge-Kutta scheme, as shown in Figure C.10. The linear approximation response is based on the first-order kernel only while the nonlinear approximation response is...
represents the response obtained by utilizing up to the second-order Volterra series. As presented in Figure C.10, the linear approximation response shows notable discrepancy as compared to the reference results while there is indiscernible difference between the nonlinear approximation response and the reference data. This observation indicates that the second-order kernel is necessary and sufficient to simulate the “lock-in” phenomenon of a VIV system.

Figure C.9. The linear and nonlinear kernels of the VIV system

Figure C.10. "Lock-in" response of the VIV system
C.5 Closing Remarks

A critical overview of the literature on vortex-induced vibration (VIV) of bridge decks is presented based on wind-tunnel experiments, full-scale observations, semi-empirical models and computational fluid dynamics (CFD). Several physical mechanisms surrounding VIV of bridge decks are elucidated in new light, such as the complicated role of turbulence and Reynolds number effects on VIV. Conventional Scanlan’s model (Van de Pol type) for the VIV of bridge decks is examined in detail. In addition, the Volterra series based nonlinear model is introduced to simulate VIV of bridge decks. The kernel identification scheme based on the impulse function input is utilized to identify the first- and second-order kernels of Volterra series. It is demonstrated that the VIV system at “lock-in” could be accurately modeled by truncated Volterra series.

A selected summary of VIV of bridge decks based on the review presented in this study is given here.

1) The decreasing aspect ratio of length (along-wind direction) to width (cross-wind direction) of a prismatic cross-section enhanced the vortex-induced effects; whereas a decreasing aspect ratio of the spanwise length to the size of the cross-section weakens the vortex-induced forces.

2) For the bluff body with a large aspect ratio, the smaller-peak oscillation in the lower wind velocity region resulted from the arrival of the separated vortices from the leading edge and advancing to the trailing edge.

3) The final effects of turbulence on VIV (stabilization or destabilization) may depend on the relative intensity of the Kármán vortices as compared to the motion-induced vortices.
4) Reynolds numbers effects on VIV may be more important in the high Reynolds numbers range as compared to the low Reynolds numbers range.

5) The predicted VIV response based on scaled experiments at low Reynolds numbers was over-conservative when compared to the full-scale observations.

6) Many bridge decks are prone to VIV in a relatively low wind velocity range for wind direction almost perpendicular to the bridge axis and low turbulence intensity.

7) Scanlan’s VIV model seems to have rather limited predictive capability for VIV of bridge decks with various mass-damping ratios.

8) The second-order kernel of the Volterra series based model is necessary and sufficient to simulate the “lock-in” phenomenon of a VIV system.
APPENDIX D:

SIMULATION OF NONLINEAR AERODYNAMICS BASED ON VOLTERRA

FREQUENCY RESPONSE FUNCTIONS

A frequency domain approach for nonlinear bridge aerodynamics and aeroelasticity, based on the Volterra series expansion, is introduced in this study. The Volterra frequency-response functions (VFRFs) and the associated linear equations are formulated utilizing a topological assemblage scheme and are identified utilizing an existing full-time-domain nonlinear bridge aerodynamics and aeroelasticity analysis framework. A two-dimensional sectional model of a long-span bridge is used to illustrate this approach. The results show a good comparison between the time-domain simulation and the proposed frequency-domain model. The availability of VFRFs enables to gain a qualitative insight to the nonlinear bridge aerodynamics and aeroelasticity.

D.1 Introduction

Wind-induced forces on bridge decks are traditionally represented as the sum of buffeting (aerodynamic) force, related to the approach wind velocity fluctuations, and the self-excited (aeroelastic) force, generated by the bridge deck motions. Conventional schemes consider the analyses of these forces in isolation, where both of them are modelled by linear operators (with memory) whose Frequency Response Functions (FRF) are estimated through specific wind-tunnel tests (e.g., Davenport 1962; Scanlan and Tomko...
It should be noted that the use of aerodynamic and aeroelastic analyses in isolation is valid only in the linear analysis framework. For this reason, the traditional linear schemes cannot take into account the interactions between buffeting and self-excited forces beyond linear superposition. In order to take the nonlinear effects into account, the quasi-steady (QS) theory has been applied in the aeroelastic analysis of bridges. Within this approach, the aeroelastic forces are nonlinear functions of the dynamic angle of attack (a combination of the bridge deck motions and approaching wind velocity fluctuations). Generally, the QS theory-based models perform well when the reduced wind velocity $U_r = U/nb$ (where $U$ is the horizontal mean wind velocity; $n$ is the oscillating frequency of the bridge deck; $b$ is the width of the bridge deck) is greater than 10, where the aeroelastic forces are assumed frequency independent. In such a case, the time scale of the structural motions is much lower than the time scale related to the approaching wind. The frequency-independence assumption of the QS theory actually indicates that the wind-induced forces in this scheme are “static” nonlinear functions (memory-less) with respect to the dynamic angle of attack. Therefore, the QS approach cannot consider the fluid memory effects.

To partially involve the fluid memory effects in the nonlinear consideration, the corrected quasi-steady (CQS) theory has been introduced to account for the dependence on the reduced wind velocity in the low reduced wind velocity range (Diana et al. 1993). A further detailed scheme named “band superposition” has also been developed to improve the simulation accuracy of the CQS theory-based model in turbulent flows (Diana et al. 1995). In this approach, the QS model is used to represent the low-frequency part of the forces, while the high-frequency part (both buffeting and self-excited) is
modelled through the traditional linear models with memory, whose parameters are updated according to the instantaneous low-frequency dynamic angle of attack (Diana et al. 1995). Following this concept, the high-frequency response is provided by a differential equation whose parameters depend on both time and frequency. To solve this problem with more efficient computational procedures, Chen and Kareem (2003) transformed the mixed frequency- and time-domain "band superposition" scheme into a full-time-domain formulation by means of a rational-function approximation and solved the equations of motion in time domain through an integrated approach. This solution, though mathematically rigorous, consists essentially of a calculation procedure and as such cannot serve as a model, thus does not assist in offering any qualitative assessment of the nonlinear bridge aerodynamics and aeroelasticity.

To investigate the underlying mechanisms of bridge aerodynamics, governed by the abovementioned nonlinear input-output relationships in the time domain, the concept of harmonic-band superposition is introduced in a frequency-domain based on the Volterra series expansion of both buffeting and self-excited forces. To this end, the governing equations (e.g., Chen and Kareem 2003) are re-casted into a block-diagram format invoking, whenever necessary, polynomial approximations. The synthesis of the resulting Volterra system (up to the 3rd order) is carried out adopting the topological assemblage scheme proposed in Carassale and Kareem (2010).

This study is organized as follows: Section D.2 provides some background on Volterra series, with particular reference to its multi-variate form (Carassale and Kareem 2012); Section D.3 briefly recalls the nonlinear approach proposed in Chen and Kareem (2003); Section D.4 shows the synthesis of the Volterra series approximation of the
governing equations described in Section D.3; Section D.5 presents a numerical application for the aerodynamic analysis of a long-span bridge based on the resulting Volterra system.

D.2 Volterra Series: Theoretical Background

The basic premise of the Volterra theory is that a large class of nonlinear systems can be modeled as a sum of multidimensional convolution integrals of increasing order (Volterra 1959). The Volterra theory-based model could provide an accurate description of the nonlinearities while preserving fluid memory effects missed in static transformations, e.g., in the QS theory-based models. The Volterra theory was first successfully applied in electrical engineering (e.g., Wiener 1942; Alper 1965; Ewen and Wiener 1980; Boyd and Chua 1985), which led to its comprehensive development (Schetzen 1980; Rugh 1981). In the field of offshore engineering, the Volterra series has been utilized to model non-Gaussian loading processes (e.g., Næss 1985; Li and Kareem 1990; Winterstein et al. 1994; Gurley et al. 1997) and to evaluate stationary response of systems with mechanical nonlinearities as well as load-structure feedback interactions (e.g., Donley and Spanos 1990; Li and Kareem 1990; Tognarelli et al. 1997; Carassale and Kareem 2010). Silva (1997) applied the Volterra theory to aerospace problems, based on which a series of new findings were presented (e.g., Silva et al. 2001; Lucia et al. 2004; Silva 2005). Recently, Wu and Kareem (2011; 2012b; 2012c; 2012e) utilized the Volterra theory to develop models for simulating bridge aerodynamics and aeroelasticity.

Let us consider a nonlinear system represented by the following equation:

\[
x(t) = \mathcal{H}[u(t)]
\]  

(D.1)
where \( \mathbf{u}(t) = [u^{(1)}(t) \ldots u^{(n)}(t)]^T \) and \( \mathbf{x}(t) = [x^{(1)}(t) \ldots x^{(m)}(t)]^T \) are vectors in \( \mathbb{R}^n \) and \( \mathbb{R}^m \), respectively, representing the input and output; \( t \) is the time. If the operator \( \mathcal{H}[\bullet] \) is time-invariant and causal, its output \( \mathbf{x}(t) \) can be expressed, far enough from the initial conditions, through the Volterra series expansion (e.g., Rugh 1981; Carassale and Kareem 2010):

\[
\mathbf{x}(t) = \sum_{j=0}^{\infty} \mathbf{x}_j(t) = \sum_{j=0}^{\infty} \mathcal{H}_j[\mathbf{u}(t)]
\]  

(D.2)

where \( \mathbf{x}_j = [x_j^{(1)}(t) \ldots x_j^{(m)}(t)]^T \), \( j = 0, 1, \ldots \) is the output of the time-invariant operator \( \mathcal{H}_j \), referred to as the \( j \)th-order Volterra Operator (VO) of \( \mathcal{H} \). The 0th-order term \( \mathbf{x}_0 = \mathcal{H}_0 \) is a constant output independent of the input; the \( j \)th-order terms with \( j \geq 1 \) are defined through the expression (Carassale and Kareem 2012):

\[
\mathbf{x}_j(t) = \mathcal{H}_j[\mathbf{u}(t)] = \int_{\tau_j \in \mathbb{R}^j} h_j(\tau_j) \otimes \mathbf{u}(t-\tau_j) \, d\tau_j
\]  

(D.3)

where \( \tau_j = [\tau_1 \ldots \tau_j]^T \) is a vector containing \( j \) integration variables; the functions \( h_j \) are called Volterra kernels and have values in \( \mathbb{R}^{m \times n^j} \); the product of the vectors \( \mathbf{u}(t-\tau_r) \), represented by the symbol \( \otimes \), must be interpreted as a sequence of Kronecker products, i.e.,:

\[
\bigotimes_{r=1}^{j} \mathbf{u}(t-\tau_r) = \mathbf{u}(t-\tau_1) \otimes \cdots \otimes \mathbf{u}(t-\tau_j)
\]  

(D.4)

The 1st-order term of the Volterra series is the convolution integral typical of linear dynamical systems, with \( h_1 \) being the impulse response function. The higher-order terms are multiple convolutions involving products of the input values for different time
delays. For nonlinear systems with fading memory, which is the hallmark of wind-bridge interaction, a truncated Volterra series may be utilized (Boyd and Chua 1985). It is well known that the even-order convolution captures the even-order super-harmonics and asymmetric nonlinearities, whereas the odd-order convolution captures the odd-order super-harmonics and symmetric nonlinearities. For a bridge deck, typically the cross-section is not symmetric with respect to the chord line, hence, the second-order Volterra model is technically the simplest possible nonlinear model, however, also basing on the experience of wing aerodynamics a third-order model is believed more comprehensive for the case of bridge decks (Wu and Kareem 2013a). A similar higher-order terms concept has been introduced in an Artificial Neural Network (ANN)-based nonlinear simulation of bridge aerodynamics, where the second- and third-order neurons in the hidden layer have been incorporated to simulate the hysteretic nonlinearity in bridge aerodynamics (Wu and Kareem 2010). These approaches that utilize the nonlinear schemes up to the third-order, show promise in simulating nonlinearity in bridge aerodynamics.

A Volterra system is entirely determined by its Volterra kernels. Hence, the identification of the kernels of the Volterra series is critical when applying this approach to a nonlinear system. Typically, the Volterra kernels are identified in the time domain based on the impulse input functions (e.g., Rugh 1981). An alternative representation in the frequency domain involves the Volterra frequency-response functions (VFRFs) representing the multi-dimensional Fourier transforms of the Volterra kernels (e.g., Schetzen 1980; Carassale and Kareem 2010):
\[ H_j(\Omega_j) = \int_{\tau_j \in \mathbb{R}^j} e^{-i\Omega_j \tau_j} h_j(\tau_j) d\tau_j \tag{D.5} \]

where \( \Omega_j = [\omega_1 \ldots \omega_j]^T \) is a vector containing the \( j \) circular frequency values corresponding to \( \tau_1, \ldots, \tau_j \) in the Fourier transform pair. The VFRFs are functions with values in \( \mathbb{C}^{m \times n_j} \).

The expression of the VOs provided by Equation D.3 can be slightly generalized by defining the multi-linear operators:

\[ \mathcal{H}_j(u_1, \ldots, u_j) = \int_{\tau_j \in \mathbb{R}^j} h_j(\tau_j) \sum_{r=1}^{j} u_r(t-\tau_r) d\tau_j \tag{D.6} \]

where \( u_r \in \mathbb{R}^n \) \((r = 1, \ldots, j)\) are, in general different, inputs. The output of these multi-linear operators is invariant with respect to a permutation of their inputs \( u_r \). This implies some symmetry conditions to the Volterra kernels and to the VFRFs. For the case of Single-Input-Single-Output (SISO) systems this symmetry simply reflects into the invariance of the kernels with respect to the order of the time delay variables \( \tau_r \) and the invariance of the VFRFs with respect to the order of the frequency variables \( \omega_r \) (e.g., Schetzen 1980). In the case of Multi-Input-Single-Output (MISO) or Multi-Input-Multi-Output (MIMO) systems the effects of the VO symmetry are more complicated and have been discussed in detail in Carassale and Kareem (2012).

D.3 Nonlinear Approach of Bridge Aerodynamics in Time Domain

In this section the nonlinear approach for the calculation of the aeroelastic response of bridges described in Chen and Kareem (2003) is briefly revisited. The coordinate system of the wind-bridge interaction study is shown in Figure D.1 with reference to the deck.
cross-section. A unit-length section of a bridge deck subjected to turbulent wind is considered here and the wind-induced loads are established based on the “strip theory”. The wind-induced effects considered are the lift force $L$ (positive upward), the drag force $D$ (positive downstream) and the torsional moment $M$ (positive nose-up). Only the loads on the bridge deck due to turbulent winds are included here, whereas secondary loads on the cables and their effects are neglected without loss of generality. Generally, the incident turbulent wind consists of the mean wind velocities $U$ and $W$ and associated fluctuations $u$ and $w$ in the horizontal and vertical direction, respectively. The horizontal, vertical and torsional motions of the bridge section under turbulent wind are expressed as $p$, $h$ and $\alpha$, respectively. For simplicity, the original model is reduced disregarding the effects of the vertical mean wind velocity, the longitudinal turbulence and the sway degree of freedom (Figure D.1) in this study. These features are not included here because the added formal complexity introduced by a more sophisticated bridge model may hide the conceptual structure of the proposed technique. As a result, the dynamic response of the bridge deck is provided by the differential equation:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f}_L + \mathbf{f}_H + \mathbf{f}_{se}$$  \hspace{1cm} (D.7)$$

where the over dot indicates derivative with respect to time; $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass, viscous damping and stiffness matrixes, respectively; $\mathbf{x} = [h, \alpha]^T$ is the displacement vector containing the vertical displacement $h$ (positive downwards) and the torsional rotation $\alpha$ (positive nose up), as shown in Figure D.1. The response $\mathbf{x}$ is divided into a low-frequency component $\mathbf{x}^{(L)} = [h_L, \alpha_L]^T$ and a high-frequency component $\mathbf{x}^{(H)} = [h_H, \alpha_H]^T$, separated by the frequency $n_c$. Accordingly, $\mathbf{f}_L$, $\mathbf{f}_H$ and $\mathbf{f}_{se}$ are, respectively, the low-
frequency force modeled based on a QS nonlinear approach, the high-frequency buffeting force and the self-excited force, respectively. It should be noted that it is a challenging issue to pinpoint demarcation between the low- and high-frequency parts. The fundamental natural frequency of the structure was selected in Chen and Kareem's (2003) work based on the premise that it offers a good starting point and more refined selection of this does not influence the development of this framework. However, a more in-depth analysis of this demarcation has been addressed in Wu and Kareem (2012a).

The low-frequency force is defined as:

$$f_L = \begin{bmatrix} -L \\ M \end{bmatrix} = \frac{1}{2}\rho b l V_r^2 R(\phi) C(\alpha_s)$$

(D.8)

where $\rho$ is the air density, $b$ the deck width, $l$ the length of the strip on which the load is applied, $V_r$ the wind-structure relative velocity expressed as:

$$V_r^2 = U^2 + \left( w_L + \dot{h}_L + m_1 b \dot{\alpha}_L \right)^2$$

(D.9)

where $U$ is the mean wind velocity, $w_L$ is the low-frequency vertical turbulence (i.e., the vertical turbulence, $w$, low-pass filtered at the frequency $n = n_c$); $m_1 b$ is the leg for the...
reduction of the apparent velocity field generated by the torsional velocity; \( \mathbf{R} \) is a rotation matrix defined as:

\[
\mathbf{R}(\phi) = \begin{bmatrix}
-\sin \phi & \cos \phi & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\]  

(D.10)

where \( \phi \) is the apparent (low-frequency) wind angle:

\[
\phi = \arctan \left( \frac{w_L + \dot{h}_L + m_L \dot{\alpha}_L}{U} \right)
\]  

(D.11)

The vector \( \mathbf{C} = [C_D, -C_L, bC_M]^T \) contains the steady-state aerodynamic coefficients, estimated for the dynamic angle of attack \( \alpha_e = \phi + \alpha_L \) through static wind tunnel tests.

The high-frequency buffeting force is modeled as:

\[
f_H = \frac{1}{2} \rho U b l \mathcal{B} [w_H; \alpha_e]
\]  

(D.12)

where \( w_H \) is the high-frequency vertical turbulence (i.e., \( w \) high-pass filtered at \( n = n_c \)) and \( \mathcal{B} \) is a linear operator whose Frequency-Response Function (FRF) depends on the low-frequency response through the dynamic angle of attack \( \alpha_e \) and is expressed as:

\[
\mathbf{B}(\omega, \alpha_e) = \begin{bmatrix}
-C_D(\alpha_e) - C_L'(\alpha_e) & 0 \\
0 & bC_M'(\alpha_e) \\
\end{bmatrix}
\begin{bmatrix}
\chi_{wL}(\omega, \alpha_e) \\
\chi_{wM}(\omega, \alpha_e) \\
\end{bmatrix}
\]

(D.13)

where \( C_L' \) and \( C_M' \) are the prime derivatives of \( C_L \) and \( C_M \), respectively; \( \chi_{wL} \) and \( \chi_{wM} \) are the admittance functions weighting the effects of the vertical turbulence on the lift force and torsional moment, respectively; \( \omega = 2\pi n \) is the circular frequency. Since \( \alpha_e \) is
variable in time due to $w_L$ and $x^{(L)}$, the operator $\mathcal{B}$ may be interpreted as a time-variant linear operator.

The self-excited forces are defined through the model:

$$f_{se} = \frac{1}{2} \rho b^2 L \mathcal{A}
\begin{bmatrix} x^{(H)}; \alpha_e \end{bmatrix}$$  \hfill (D.14)

where $\mathcal{A}$ is a linear operator whose FRF depends on $\alpha_e$ and is defined as:

$$A(\omega; \alpha_e) = \omega^2 \left[ \begin{array}{c} H_j^* (k; \alpha_e) + i H_i^* (k; \alpha_e) \\ b \left( A_j^* (k; \alpha_e) + i A_i^* (k; \alpha_e) \right) \\ b^2 \left( A_j^* (k; \alpha_e) + i A_i^* (k; \alpha_e) \right) \end{array} \right]$$  \hfill (D.15)

where $H_j^* (k; \alpha_e)$ and $A_j^* (k; \alpha_e)$ are the aerodynamic derivatives estimated at the reduced frequency $k = \omega b / U$ and for a mean angle of attack corresponding to $\alpha_e$. Likewise the operator $\mathcal{B}$, also $\mathcal{A}$ is linear, and is time-variant because of the dependency on $\alpha_e$.

D.4 Synthesis of a Third-Order Volterra System

In this section, the model described above is synthesized into a 3rd-order Volterra system. Its VFRFs and Associated Linear Equations (ALEs) are obtained by adopting the topological assemblage approach described in Carassale and Kareem (2010; 2012).

Figure D.2 shows a block diagram of the whole system. The input $w$ is separated into $w_L$ and $w_H$ through the low-pass filter $\mathcal{P}_L$ and the high-pass filter $\mathcal{P}_H$, whose FRFs are $P_L(\omega)$ and $P_H(\omega)$, respectively. The output $x$ is generated by the sum $x^{(L)}$ and $x^{(H)}$ obtained, respectively, as the outputs of the low-frequency and high-frequency stages of the system. A channel delivers the dynamic angle of attack $\alpha_e$ (actually its fluctuation around the static angle $\alpha_0$) from the low-frequency stage of the system to the high-
frequency stage. The rectangular boxes represent operators defined through a constitutive equation; the boxes with rounded corners are operators defined through experimental data; the triangles are gain blocks in which the vectors $b_1 = [0 \ 1]$ and $b_2 = [1 \ m_1 \ b]$ are used to extract or combine the components $h$ and $\alpha$ of the response. The operator $D$ represents the left-hand side of Equation D.7 and its FRF is:

$$D(\omega) = -\omega^2 M + i\omega C + K$$  \hspace{1cm} (D.16)

![Figure D.2. Block diagram of the nonlinear aeroelastic bridge model](image)

The operators $A_j$ and $B_j \ (j= 0, 1, 2)$ are linear time-invariant operators whose FRFs, $A_j(\omega)$ and $B_j(\omega)$, are obtained by a 2\textsuperscript{nd}-order polynomial approximation [see...
Carassale and Kareem (2010)] of \(A(\omega; \alpha_e)\) and \(B(\omega; \alpha_e)\) defined by Equation D.13 and D.15:

\[
A(\omega; \alpha_e) = \sum_{j=0}^{2} A_j(\omega)(\alpha_e - \alpha_0)^j \\
B(\omega; \alpha_e) = \sum_{j=0}^{2} B_j(\omega)(\alpha_e - \alpha_0)^j
\]  

(D.17)

The low-frequency and the high-frequency stages of the system are represented, respectively, by the operators \(L\) and \(H\) providing the low-frequency and high-frequency response \(x^{(L)}\) and \(x^{(H)}\), given the wind velocity fluctuation \(w\). It is assumed that such operators can be approximated by the 3rd-order Volterra series:

\[
x^{(L)} = L[w] = \sum_{j=0}^{3} x_j^{(L)} = \sum_{j=0}^{3} L_j[w] \\
x^{(H)} = H[w] = \sum_{j=0}^{3} x_j^{(H)} = \sum_{j=0}^{3} H_j[w]
\]  

(D.18)

where \(L_j\) and \(H_j\) are the VOs of \(L\) and \(H\) and are defined through the VFRFs \(L_j\) and \(H_j\).

D.4.1 Low-Frequency Stage

To simplify the synthesis of the low-frequency stage \(L\), it is convenient to proceed with the preliminary identification of the five subsystems represented in Figure D.3.

The operator \(g\) provides the apparent wind angle \(\phi\), given the wind velocity fluctuation \(w\). It contains the unknown operator \(L\) and the non-polynomial function \(\arctan(\bullet)\). To proceed with the system synthesis this latter function must be approximated by a polynomial. In this case a polynomialization based on the Taylor series expansion can be conveniently adopted, i.e.,:

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After this approximation, all the building-blocks contained in $G$ are Volterra systems, thus VOs $G_j$ of $G$ can be obtained through the assemblage rules defined in Carassale and Kareem (2010). In particular, it is easy to verify that $G_0 = 0$ since the effect of a possible term $L_0 \neq 0$ is canceled by the differentiation. Besides, the output of $G_j$ (for $j \geq 1$) can be expressed as:

$$
\phi_j = G_j[w] = G_j^{(d)}[w] + G_j^{(f)}[D_j[w]]
$$

(D.20)

where, according to the procedure defined in Carassale and Kareem (2012), the output of the VO $G_j$ has been divided as the sum of the outputs of the operator $G_j^{(d)}$ and of the composite operator $G_j^{(f)}[D_j[\bullet]]$. The operator $G_j^{(d)}$ is referred to as direct term of $G_j$ and is a $j^{th}$-order VO; the operator $G_j^{(f)}$ is called feedback term of $G_j$ and is a linear operator,
though its series combination with \( \mathcal{J} \) gives rise to a \( j \)-th-order VO. They are provided (up to the 3\(^{\text{rd}}\) order) by the expressions:

\[
\begin{align*}
\mathcal{G}_1^{(d)}[w] &= \frac{1}{U} \mathcal{R}_1[w] \\
\mathcal{G}_2^{(d)}[w] &= 0 \\
\mathcal{G}_3^{(d)}[w] &= -\frac{1}{3U^3} \left( \mathcal{R}_1[w] + \mathbf{b}_2 \frac{d}{dt} \mathcal{R}_1[w] \right)^3 \\
\mathcal{G}_j^{(j)}[\mathbf{x}] &= \frac{1}{U} \mathbf{b}_2 \mathbf{x} \quad (j=1, 2, 3)
\end{align*}
\]  

(D.21)

and the corresponding VFRFs are given as:

\[
\begin{align*}
G_0 &= G_2^{(d)} = 0 \\
G_1^{(d)}(\omega) &= \frac{1}{U} P_L(\omega) \\
G_3^{(d)}(\Omega_3) &= -\frac{1}{3U^3} \left[ \prod_{r=1}^{3} P_L(\omega_r) + \prod_{r=1}^{3} i \omega_r \mathbf{b}_2 L_4(\omega_r) \right] \\
G_j^{(j)}(\omega) &= \frac{1}{U} i \omega \mathbf{b}_2 \quad (j = 1, 2, 3)
\end{align*}
\]  

(D.22)

The VFRFs \( G_j^{(j)} \) are functions in \( \mathbb{C}^{1 \times 2} \), since they represent linear 2-IN/1-OUT systems. In this case the operators \( \mathcal{G}_j^{(j)} \) (and their FRF \( G_j^{(j)} \)) are the same for any order \( j \).

The VFRFs of the whole operator \( \mathcal{F} \) can be coupled as:

\[
G_j(\Omega_j) = G_j^{(d)}(\Omega_j) + G_j^{(j)}(\Sigma \Omega_j) L_j(\Omega_j) \quad (j = 1, 2, 3)
\]  

(D.23)

Figure D.3(b) shows the block diagram of the operator \( \mathcal{F} \) providing the dynamic angle of attack \( \alpha_e \), given the wind velocity fluctuation \( w \). It contains the operator \( \mathcal{F} \) defined above and the unknown operator \( \mathcal{L} \). Applying the pertinent assemblage rules, its \( j \)-th-order VO \( \mathcal{F}_j \) is obtained as:
\[ \mathcal{E}_j[w] = \mathcal{G}_j[w] + b_1 \mathcal{J}_j[w] \]  
(D.24)

The 0th-order output \( \mathcal{E}_0 = b_1 \mathcal{J}_0 \) is equal to the static torsional response \( \alpha_0 = b_1 x_0^{(L)} \); for \( j \geq 1 \), the operators can be decomposed as \( \mathcal{E}_j = \mathcal{E}_j^{(d)}[\bullet] + \mathcal{E}_j^{(f)}[\mathcal{J}[\bullet]] \), where:

\[ \mathcal{E}_j^{(d)}[w] = \mathcal{G}_j^{(d)}[w] \]
\[ \mathcal{E}_j^{(f)}[\mathbf{x}] = \mathcal{G}_j^{(f)}[\mathbf{x}] + b_1 \mathbf{x} = \frac{1}{U} b_2 \mathbf{x} + b_1 \mathbf{x} \]  
(D.25)

and the corresponding VFRFs are:

\[ E_j^{(d)}(\Omega_j) = G_j^{(d)}(\Omega_j) \]
\[ E_j^{(f)}(\omega) = G_j^{(f)}(\omega) + b_1 = \frac{1}{U} i \omega b_2 + b_1 \quad (j = 1, 2, 3) \]  
(D.26)

Figure D.3(c) shows the block diagram of the operator \( \mathcal{Y} \) providing the vector \( \mathbf{y} \in \mathbb{R}^2 \) (whose meaning is explained in Figure D.2), given the wind velocity fluctuation \( w \). The output of the block \( \mathbf{R} \) is the rotation matrix \( \mathbf{R}(\phi) \in \mathbb{R}^{2 \times 3} \) (Equation D.10), which is a function of the apparent wind angle \( \phi \) provided by the operator \( \mathcal{G} \). The output of the block \( \mathbf{C} \) is the vector of the aerodynamic coefficients \( \mathbf{C}(\alpha_e) \in \mathbb{R}^3 \) that depends on the dynamic angle of attack \( \alpha_e \) given by the operator \( \mathcal{E} \). The synthesis of \( \mathcal{Y} \) requires that the blocks \( \mathbf{R} \) and \( \mathbf{C} \) are approximated by polynomials; in this case 3rd-order polynomials are adopted:

\[ \mathbf{R}(\phi) = \sum_{j=0}^{3} \mathbf{R}_j \phi^j \]  
(D.27a)
\[
C(\alpha_e) = \sum_{j=0}^{3} C_j (\alpha_e - \alpha_0)^j
\]  
(D.27b)

The matrices \( R_j \in \mathbb{R}^{2 \times 3} \) may be calculated adopting a Taylor series approximation, yielding:

\[
R_0 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]  
(D.28a)

\[
R_1 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\]  
(D.28b)

\[
R_2 = \frac{1}{2} \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\]  
(D.28c)

\[
R_3 = \frac{1}{6} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\]  
(D.28d)

while the vectors \( C_j \in \mathbb{R}^{3} \) must be obtained on the basis of the results of the static aerodynamic tests through some error minimization procedure. The expansion point for the approximation of \( C(\alpha_e) \) is set equal to the static torsional angle \( \alpha_0 \). This is a convenient choice since in this way the 0\(^{th}\)-order output \( \alpha_0 \) of \( \varepsilon \) is cancelled and the assemblage rules remain simplified.

The product between \( R(\phi) \) and \( C(\alpha_e) \) is a standard matrix product, which may be interpreted as the sum of three Kronecker products (in this case the Kronecker product is degenerate since one of the members is a scalar), thus the pertinent assemblage rules can be adopted to obtain the VOs of \( \mathcal{Y} \). These are divided into direct and feedback terms \( \mathcal{Y}_{j}[\bullet] = \mathcal{Y}_{j}^{(d)}[\bullet] + \mathcal{Y}_{j}^{(f)}[\mathcal{Y}_{j}[\bullet]] \), which are given as:
$y_j = R_0 C_0$

$y_1^{(d)}[w] = R_0 \varepsilon_1^{(d)}[w] + \mathcal{R}_1^{(d)}[w] C_0$

$y_2^{(d)}[w] = R_0 \varepsilon_2^{(d)}[w] + \mathcal{R}_1^{(d)}[w] \varepsilon_1[w] + \mathcal{R}_2^{(d)}[w] C_0$

$y_3^{(d)}[w] = R_0 \varepsilon_3^{(d)}[w] + \mathcal{R}_1^{(d)}[w] \varepsilon_2[w] + \mathcal{R}_2^{(d)}[w] \varepsilon_1[w] + \mathcal{R}_3^{(d)}[w] C_0$

$y_j^{(f)}[x] = R_0 \varepsilon_1^{(f)}[x] + \mathcal{R}_j^{(f)}[x] C_0 \quad (j = 1, 2, 3)$

$y_j^{(d)}[w] = R_j \varepsilon_j^{(d)}[w] + \mathcal{R}_j^{(d)}[w] C_0$

$\mathcal{R}_1^{(d)}[w] = R_1 \varepsilon_1^{(d)}[w]$

$\mathcal{R}_2^{(d)}[w] = R_2 \varepsilon_2^{(d)}[w] + R_2 (\varepsilon_1[w])^2$

$\mathcal{R}_3^{(d)}[w] = R_3 \varepsilon_3^{(d)}[w] + 2 R_2 \varepsilon_2[w] \varepsilon_1[w] + R_3 (\varepsilon_1[w])^3$

$\mathcal{R}_j^{(f)}[x] = R_j \varepsilon_j^{(f)}[x] = \frac{1}{U} b_2 x R_1 \quad (j = 1, 2, 3)$

$\varepsilon_1^{(d)}[w] = C_1 \varepsilon_1^{(d)}[w]$

$\varepsilon_2^{(d)}[w] = C_1 \varepsilon_2^{(d)}[w] + C_2 (\varepsilon_1[w])^2$

$\varepsilon_3^{(d)}[w] = C_1 \varepsilon_3^{(d)}[w] + 2 C_2 \varepsilon_1[w] \varepsilon_2[w] + C_3 (\varepsilon_1[w])^3$

$\varepsilon_j^{(f)}[x] = C_1 \varepsilon_j^{(f)}[x] = \left(\frac{1}{U} b_2 x + b_1 x\right) C_1 \quad (j = 1, 2, 3)$

Substituting the last one of Equations D.30 and D.31 into the last one of Equations D.29, the feedback term of the VO $y_j$ is given as:

$y_j^{(f)}[x] = \frac{1}{U} b_2 x (R_0 C_1 + R_1 C_0) + b_1 x R_0 C_1 \quad (j = 1, 2, 3)$
Operating in terms of VFRFs, Equations D.29 to D.31 can be translated into the relationships:

\[
Y_0 = R_0 C_0 \\
Y_1^{(d)}(\omega) = R_0 C_1 E_1^{(d)}(\omega) + R_1(\omega) C_0 G_1^{(d)}(\omega) \\
Y_2^{(d)}(\Omega_2) = R_0 C_2 E_1(\omega_1) E_1(\omega_2) + R_1 C_1 G_1(\omega_1) E_1(\omega_2) + R_2 C_0 G_1(\omega_1) G_1(\omega_2) \\
Y_3^{(d)}(\Omega_3) = R_0 C_3 E_1^{(d)}(\Omega_3) + R_1 C_0 G_1^{(d)}(\Omega_3) + \\
\quad + R_0 C_3 E_1(\omega_1) E_1(\omega_2) E_1(\omega_3) + R_1 C_2 G_1(\omega_1) E_1(\omega_2) E_1(\omega_3) + \\
\quad + R_2 C_1 G_1(\omega_1) G_1(\omega_2) E_1(\omega_3) + R_3 C_0 G_1(\omega_1) G_1(\omega_2) G_1(\omega_3) + \\
\quad + 2R_0 C_2 E_1(\omega_1, \omega_2) E_2(\omega_2, \omega_3) + 2R_2 C_0 G_1(\omega_1, \omega_2) G_2(\omega_2, \omega_3) \\
Y_j^{(f)}(\omega) = R_0 C_1 E_j^{(f)}(\omega) + R_1 C_0 G_j^{(f)}(\omega) \\
Y_j(\Omega_j) = Y_j^{(d)}(\Omega_j) + Y_j^{(f)}(\Omega_j) L_j(\Omega_j) \\
\quad (j = 1, 2, 3)
\]

Figure D.3(d) shows the block diagram defining the operator $\mathcal{V}$ that provides the square of the wind-structure relative velocity $V_r^2$, given the wind velocity fluctuation $w$. Its VOs up to the 3rd order are:

\[
\gamma_0 = U^2 \\
\gamma_1[w] = 0 \\
\gamma_2[w] = \left( b_2 \frac{d}{dt} \mathcal{G}_1[w] + \mathcal{G}_2[w] \right)^2 \\
\gamma_3[w] = 2 \left( b_2 \frac{d}{dt} \mathcal{G}_1[w] + \mathcal{G}_2[w] \right) \left( b_2 \frac{d}{dt} \mathcal{G}_2[w] \right)
\]

where there is no need of distinction between direct and feedback terms since none of the operators $\gamma_j$ depends on the unknown operator $\mathcal{G}$ of the same order (i.e., $\gamma_j$ has only direct term). The corresponding VFRFs are:
\( V_0 = U^2 \)
\( V_1 = 0 \)
\[
V_2(\Omega_2) = P_1(\omega_1)P_1(\omega_2) - \omega_1\omega_2b_2L_1(\omega_1)b_2L_1(\omega_2) \\
+ 2i\omega_2P_1(\omega_1)b_2L_1(\omega_2) \\
V_3(\Omega_3) = 2iP_1(\omega_1)(\omega_2 + \omega_3)b_2L_2(\omega_2,\omega_3) \\
- 2\omega_1(\omega_2 + \omega_3)b_2L_4(\omega_1)b_2L_2(\omega_2,\omega_3)
\]

(D.35)

Figure D.3(e) shows the operator \( F \) providing the low-frequency force \( f_L \), given the wind velocity fluctuation \( w \). Making the usual distinction between direct and feedback terms, its VOs \( F[\bullet] = F^{(d)}[\bullet] + F^{(f)}[\bullet] \) are expressed by the relationships:

\[
F_0 = \frac{1}{2}\rho b l \gamma_0 x_0 = \frac{1}{2}\rho b l U^2 R_0 C_0
\]
\[
F^{(d)}[w] = \frac{1}{2}\rho b l \gamma_0 x_0^{(d)}[w]
\]
\[
F^{(d)}[w] = \frac{1}{2}\rho b l \left( \gamma_0 x_2^{(d)}[w] + \gamma_2[w] x_0 \right)
\]
\[
F^{(d)}[w] = \frac{1}{2}\rho b l \left( \gamma_0 x_3^{(d)}[w] + \gamma_2[w] x_2^{(d)}[w] + \gamma_3[w] x_0 \right)
\]
\[
F^{(f)}[x] = \frac{1}{2}\rho b l \gamma_0 x_{1;j}^{(f)}[x]
\]
\[
= \frac{1}{2}\rho b l U \left[ b_2 x(R_0 C_1 + R_1 C_0) + U b_1 x R_0 C_1 \right] \quad (j=1, 2, 3)
\]

and the corresponding VFRFs are:
The force \( f_L \) can be obtained through the operator \( \mathcal{A} \) defined by Equations D.36, i.e., \( f_L = \mathcal{A}[w] \), as well as through the mechanical system \( \mathcal{D} \), i.e., \( f_L = \mathcal{D}[x^{(L)}] \) (Figure D.2). Equating at each order the VOs defining these two dynamical systems the following equations are obtained:

\[
 x_0^{(L)} = \mathcal{K}^{-1} \mathcal{A}[w] = \frac{1}{2} \rho b l U^2 \mathcal{K}^{-1} R_0 C_0 \tag{D.38}
\]

\[
 \mathcal{D}[x_j^{(t)}] - \mathcal{A}^{(f)}[x_j^{(t)}] = \mathcal{F}^{(d)}[w] \quad (j = 1, 2, 3) \tag{D.39}
\]

Equation D.38 that provides the 0\(^{th}\)-order (steady state) response is nonlinear since the vector \( C_0 \) depends on the static torsional response \( \alpha_0 \) employed as expansion point in Equations D.27. On the other hand, Equations D.39 are linear since, \( \mathcal{D} \) and \( \mathcal{A}^{(f)} \) are linear operators and the VO \( \mathcal{A}^{(d)} \) does not depend on \( x_k^{(L)} \) with \( k \geq j \). For this reason Equations D.39 are called ALEs. The left-hand-side operator of the ALEs is common to any order, has a differential structure and can be easily obtained by combining Equation D.7 and the last one of Equations D.36 as:

\[
 \mathcal{D}[x] - \mathcal{A}^{(f)}[x] = Mx + (C + C_a)x + (K + K_a)x \tag{D.40}
\]
where $C_a$ and $K_a$ are aerodynamic damping and stiffness matrices defined as:

\[
C_a = -\frac{1}{2} \rho b I U (R_0 C_i + R_1 C_0) b_2 \\
K_a = -\frac{1}{2} \rho b I U^2 R_0 C_i b_i
\]  

(D.41)

The VFRFs of the operator $\mathcal{D}$ providing the low-frequency response can be obtained by equating the VFRFs of the operators $\mathcal{D}[\mathcal{D}[\star]]$ and $\mathcal{K}$, leading to the equations:

\[
L_j(\Omega_j) = \left[ D(\sum \Omega_j) - F^{(f)}(\sum \Omega_j) \right]^{-1} F^{(d)}(\Omega_j) \quad (j = 1, 2, 3)
\]  

(D.42)

The equations providing the VFRFs $L_j$ of $\mathcal{D}$ are linear since both $F^{(f)}_j$ and $F^{(d)}$ do not depend on $L_k$ with $k \geq j$ and can be solved one by one starting from $j = 1$.

D.4.2 High-Frequency Stage

The synthesis of the high-frequency stage $\mathcal{H}$ of the system requires the identification of the operators $\mathcal{B}$ and $\mathcal{A}$ providing, respectively the high-frequency buffeting force $f_H$ and the self-excited force $f_{se}$. As far as $\mathcal{B}$ is concerned, its VOs $\mathcal{B}_j$ and VFRFs $\mathcal{B}_j$ can be obtained considering the schema reported in Figure D.4(a), applying the pertinent assemblage rules and taking into account that $\mathcal{B}_k (k = 0, 1, 2)$ are linear operators and that the $0^{th}$-order term $e_0$ of $e$ is canceled by $a_0$. It yields:
Figure D.4. Block diagram of the sub-systems composing the high-frequency stage

$$\mathcal{R}_h = 0$$

$$\mathcal{R}_j[w] = \frac{1}{2} \rho \text{Ubl} \mathcal{R}_h \left[ \mathcal{P}_h [w] \right]$$

$$\mathcal{R}_2[w] = \frac{1}{2} \rho \text{Ubl} \mathcal{R}_h \left[ \mathcal{P}_h [w] \xi_1[w] \right]$$

$$\mathcal{R}_1[w] = \frac{1}{2} \rho \text{Ubl} \left( \mathcal{R}_h \left[ \mathcal{P}_h [w] \xi_2[w] \right] + \mathcal{R}_2 \left[ \mathcal{P}_h [w] \xi_1[w]^2 \right] \right) \quad (D.43)$$

$$\mathcal{B}_0 = 0$$

$$\mathcal{B}_1(\omega) = \frac{1}{2} \rho \text{Ubl} \mathcal{B}_1(\omega) P_h(\omega)$$

$$\mathcal{B}_2(\Omega_2) = \frac{1}{2} \rho \text{Ubl} \left( \mathcal{B}_1(\Sigma \Omega_2) P_h(\omega_1) E_i(\omega_2) \right) \quad (D.44)$$

$$\mathcal{B}_3(\Omega_3) = \frac{1}{2} \rho \text{Ubl} \left( \mathcal{B}_1(\Sigma \Omega_3) P_h(\omega_1) E_2(\omega_2, \omega_3) + \mathcal{B}_2(\Sigma \Omega_3) P_h(\omega_1) E_i(\omega_2) E_i(\omega_3) \right)$$

To synthesize the operator $\mathcal{J}$ defined according to the block diagram reported in Figure D.4(b), its VO $\mathcal{J}_j$ must be decomposed into direct and feedback terms, i.e.,

$$\mathcal{J}_j[\bullet] = \mathcal{J}_j^{(d)}[\bullet] + \mathcal{J}_j^{(f)}[\mathcal{P}[\bullet]]$$

which can be obtained as:
Also in this case the feedback term is the same for any order \( j \). Operating in terms of VFRFs it leads to:

\[
\bar{A}_0 = \bar{A}_1^{(d)}(\omega) = 0
\]

\[
\bar{A}_2^{(d)}(\Omega_2) = \frac{1}{2} \rho b^2 l A_1(\Sigma \Omega_2) H_i(\omega_1) E_i(\omega_2)
\]

\[
\bar{A}_3^{(d)}(\Omega_3) = \frac{1}{2} \rho b^2 l \left( A_1(\Sigma \Omega_3) H_i(\omega_1) E_i(\omega_2, \omega_3) + A_i(\Sigma \Omega_3) H_i(\omega_1) E_i(\omega_2) E_i(\omega_3) \right)
\]

\[
\bar{A}_j^{(f)}(\Omega_j) = \frac{1}{2} \rho b^2 l A_0(\Sigma \Omega_j)
\]

\[
\bar{A}_j(\Omega_j) = \bar{A}_j^{(d)}(\Omega_j) + \bar{A}_j^{(f)}(\Sigma \Omega_j) H_i(\Omega_j)
\]

It can be observed that the \( j^{th} \)-order high-frequency response is influenced by the low-frequency response up to the order \( j-1 \) through the VFRFs \( E_k \) \((k = 1, ..., j-1)\). The dynamic equilibrium of the high-frequency stage is imposed through the equation:

\[
D [x^{(H)}] = \bar{A} [w] + \bar{B} [w]
\]

which can be fulfilled by equating, order by order, the VOs representing the two terms of the equation. This leads to the set of ALEs

\[
D [x_j^{(H)}] - \bar{A}_j^{(f)} [x_j^{(H)}] = \bar{A}_j^{(d)} [w] + \bar{B}_j [w]
\]

\((j = 1, 2, 3)\)
that, in general do not have a differential structure, but since they are linear, can be solved in the frequency domain, proceeding one by one starting from $j = 1$.

The VFRFs of $\mathcal{H}$ can be obtained in a similar way by equating the VFRFs of the two sides of Equation D.47, leading to the equations:

$$
\mathbf{H}_j(\Omega_j) = \hat{\mathbf{D}}(\Sigma\Omega_j)^{-1}\left[\bar{\mathbf{A}}_j^{(d)}(\Omega_j) + \bar{\mathbf{B}}_j(\Omega_j)\right] \quad (j = 1, 2, 3)
$$

(D.49)

where $\hat{\mathbf{D}} = \mathbf{D} - \bar{\mathbf{A}}_j^{(d)}$ is the FRF of the mechanical system modified by the linear term of the system providing the self-excited forces.

D.5 Numerical Application

The accuracy of the Volterra series representation of the nonlinear aeroelastic bridge model described in Section D.3 is demonstrated through the calculation of the response of a long-span bridge. Numerical analyses are carried out adopting the following parameters: $\rho = 1.225$ kg/m$^3$; $U = 40$ m/s, $b = 41$ m; $m_1 = 0.5$; $\mathbf{M} = \text{diag}(m, I)$ with $m = 3.21 \cdot 10^4$ kg and $I = 8.76 \cdot 10^6$ kgm$^2$; $\mathbf{K} = \mathbf{M} \text{diag}(\omega_h^2, \omega_\alpha^2)$ with $\omega_h = 1.22$ rad/s ($n_h=0.19$ Hz) and $\omega_\alpha = 3.33$ rad/s ($n_\alpha=0.53$ Hz); $\mathbf{C} = 2\xi\mathbf{M} \text{diag}(\omega_h, \omega_\alpha)$ with $\xi = 0.005$. The frequency value dividing the low and high-frequency components is set at $\omega_c = 1.0$ rad/s ($n_c = 0.16$ Hz).

Figure D.5 presents the steady-state aerodynamic coefficients $C_D$, $C_L$ and $C_M$ obtained from static wind tunnel tests for angle of attack between -10° and 10° with step 1° (solid lines) and their 3rd-order polynomial approximation (dashed lines) obtained through the procedure reported in Carassale and Kareem (2010). The expansion point is set to zero since the static torsional rotation is very small ($\alpha_0 = 0.02^\circ$); the standard
deviation of the angle of attack is set $\sigma_\alpha = I_w = 0.06$. It can be observed that the polynomial approximations fit very well the experimental data for small angle of incidence, but have some discrepancy when $C_L$ and $C_M$ change slope sharply. This issue could be solved by increasing the order of the approximating polynomials, but it would imply the necessity of extending, accordingly, the Volterra series representing the system.

![Figure D.5. Steady-state aerodynamic coefficients (solid lines) and polynomial approximation (dashed lines)](image)

To define the high-frequency stage of the system, it is necessary to approximate the FRF $B(\omega, \alpha_e)$ of $B$ given by Equation D.13 as a polynomial function of $\alpha_e$. According to Equation D.17 a 2$^{nd}$-order approximation is adopted. Due to lack of experimental data, the admittance functions are modeled through the Sears function and are considered as independent of the angle of attack:
\[ \chi_{Lw} \left( \omega, \alpha_e \right) = \chi_{Mw} \left( \omega, \alpha_e \right) = 1 - \frac{0.236i k}{0.058 + i k} - \frac{0.513i k}{0.364 + i k} - \frac{0.171i k}{2.420 + i k} \quad \left( k = \frac{\omega b}{U} \right) \]  

(D.50)

The polynomial approximation for the derivatives of the steady-state aerodynamic coefficients, \( C_L' \) and \( C_M' \), are obtained by differentiating the polynomial approximations obtained for \( C_L \) and \( C_M \).

The definition of the subsystem \( \mathcal{A} \) needs the polynomial approximation of the dependency with respect to \( \alpha \) of the FRF \( A(\omega, \alpha_e) \) defined in Equation D.15. According to Equation D.17 a 2nd-order approximation is adopted and the polynomial coefficients are calculated applying the procedure described in Carassale and Kareem (2010) to any one of the flutter derivatives contained in \( A(\omega, \alpha_e) \) and for any frequency \( \omega \). Figure D.6 shows the polynomial approximation of the eight flutter derivatives used in the definition of \( A \). The black lines correspond to the values estimated from experiments. The modification of the wind-bridge interaction system produces the variation of the aerodynamic damping and stiffness with respect to the dynamic angle of attack, actually accounting for the amplitude-dependent nonlinearity in the system. For example, the figure related to \( A'_2 \) shows that, comparing with the case of \( \alpha = 0^\circ \), the aerodynamic torsional response may be smaller in the case of \( \alpha = +3^\circ \) due to the relatively large positive aerodynamic damping, while the aerodynamic vertical response may be larger due to the relatively larger negative aerodynamic damping.

Figure D.7 shows the absolute value of the 1st-order VFRF of the whole system (low-frequency and high-frequency stages together). It can be observed that some coupling exists between the two degrees of freedom due to the linear aeroelastic terms and
Figure D.6. Flutter derivatives function of the frequency and of the angle on attack, experimental values (thick solid lines) and polynomial approximation (surfaces)
a discontinuity for $\omega = \omega_c$ corresponding with the junction point between the low-frequency and the high-frequency stages. This discontinuity is almost entirely generated by the admittance function, which has the effect of filtering the high-frequency turbulence component reducing the high-frequency buffeting force.

![Figure D.7. Absolute value of the 1st-order VFRF of the whole system $|L_1(\omega) + H_1(\omega)|$](image)

Figure D.8 shows the absolute value of the 2nd-order VFRF for the vertical (a) and torsional (b) response. The VFRFs vanish when $|\omega_1| > \omega_c$ and $|\omega_2| > \omega_c$ resulting in the cross-shaped diagrams shown in the figure. The internal square (for $|\omega_1| < \omega_c$ and $|\omega_2| < \omega_c$) is related to the low-frequency stage, while the external parts are produced by the high-frequency stage. The zero regions of the VFRF reflect the fact that the 2nd-order high-frequency forces are the result of a bilinear combination between $x_1^{(h)}$ (for the aeroelastic subsystem) or $w_h$ (for the buffeting subsystem), with the first order Volterra
series term of $\alpha_c$. While the former terms ($x_1^{(h)}$ and $w_h$) have non-zero harmonic content only for $\omega > \omega_c$, the latter term has non-zero harmonic content only for $\omega < \omega_c$.

Figure D.8. Absolute value of the $2^{nd}$-order VFRF of the whole system $|L_2(\omega) + H_2(\omega)|$: (a) vertical response; (b) torsional response
The diagonal ridges for \( \omega_1 + \omega_2 = \pm \omega_h \) or \( \pm \omega_\alpha \) correspond to the resonant amplification generated by the mechanical part of the system modified by the linear aeroelastic terms. It can be noted that the amplification involves also the low-frequency stage, even if the frequency value \( \omega_k \) dividing the low and high-frequency ranges has been set below the first natural frequency \( \omega_h \). The ridges for \( \omega_1 \) or \( \omega_2 = \pm \omega_h \) or \( \pm \omega_\alpha \) are produced by the nonlinear feedback related to the 2\(^{\text{nd}}\)-order aeroelastic terms and has the effect of spreading out the spectral energy from the resonance peaks.

Likewise for the 1\(^{\text{st}}\)-order VFRFs a discontinuity at the junction between the low-frequency and the high-frequency stages is obvious. Such discontinuity involves only the amplitude of the VFRFs (again due to the admittance function), while their qualitative shape is substantially continuous. It should be noted that the relatively small value of the selected frequency to pinpoint \( n_c \) significantly reduces the complexity of the higher-order VFRFs as compared to the typical systems with polynomial nonlinearities like the one studied in Carassale and Kareem (2012).

Figure D.9 shows the absolute value of the 3\(^{\text{rd}}\)-order VFRFs for the vertical (a) and torsional (b) response. The figures show 4 slices of the VFRFs for \( \omega_3 = 0, 0.5, 1.0 \) and 1.5 rad/s. The non-zero parts of the VFRFs have the shape of a three-dimensional cross since the 3\(^{\text{rd}}\)-order response is generated by the bilinear combination of couples of terms in which one comes from the low-frequency stage and the other comes from the high-frequency stage. The high-amplitude planes for \( \omega_1 + \omega_2 + \omega_3 = \pm \omega_h \) or \( \pm \omega_\alpha \) are produced by the resonant amplification of the mechanical system modified by the linear aeroelastic terms. The high-amplitude planes for \( \omega_1 \) or \( \omega_2 \) or \( \omega_3 = \pm \omega_h \) or \( \pm \omega_\alpha \) are, likewise for the 2\(^{\text{nd}}\)-order VFRF, are generated by the aeroelastic feedback of the linear response. The
further high-amplitude planes for $\omega_1 + \omega_2$ or $\omega_2 + \omega_3$ or $\omega_3 + \omega_1 = \pm \omega_h$ or $\pm \omega_\alpha$ are generated by the aeroelastic feedback of the 2$^{\text{nd}}$-order response.

Figure D.9. Absolute value of the 3$^{\text{rd}}$-order VFRF of the whole system $|L_3(\omega) + H_3(\omega)|$: (a) vertical response; (b) torsional response
Figure D.10 shows the low-frequency response \( \mathbf{x}^{(L)} = [\eta^{(L)} \quad \alpha^{(L)}]^T \) calculated through the integration of the ALEs of order 1 to 3 (Equations D.39-D.41, solid lines), compared with the exact nonlinear response calculated by a standard ODE solver (dots). The result of the 3\(^{rd}\)-order Volterra series appears quite accurate, in particular if compared to the linear model (1\(^{st}\)-order Volterra series) which fails in reproducing the torsional response mainly due to its implicit symmetry.

![Figure D.10. Low-frequency response calculated by standard ODE solver (dots) and by ALEs of orders 1 to 3 (solid lines)](image)

Figure D.11 shows the high-frequency response calculated through the Volterra model with order from 1 to 3, compared with the solution obtained by the full-time-domain procedure proposed in Chen and Kareem (2003). Also for this response
component the 3rd-order Volterra series approximation provides a response very close to the time-domain solution.

![Graph](image)

Figure D.11. High-frequency response calculated by a time-domain integrated space-state model [Chen and Kareem (2003), dots] and by ALEs of orders 1 to 3 (solid lines)

D.6 Concluding Remarks

The model proposed in this study essentially translates an existing full-time-domain nonlinear analysis procedure to the frequency domain, and therefore it inherits all its conceptual advantages and limitations (in particular, it is based on the hypothesis of small high-frequency oscillation of the bridge around a slow, possibly large, variation of the dynamic angle of attack). The added value of this new formulation is mostly related to its
potential for deriving qualitative interpretation of nonlinear bridge aerodynamics through this model. The identified linear and nonlinear Volterra Frequency Response Functions (VFRFs) straightforwardly present the nonlinear contribution of the aerodynamic system to various frequency components in the response. Besides, their structure is rather simple so that they may be parameterized to serve as a scaffolding in constructing a fully-nonlinear model based on the Volterra series representation, to be identified from ad-hoc experimental procedures.

From a computational point of view, the implementation of the model in the framework of the Volterra series enables the formulation of a very efficient frequency-domain solution based on the concept of Associated Linear Equations (ALEs). Accordingly, the model defined herein can be implemented through only six linear frequency-domain equations that can be conveniently solved in a cascade manner.
This study sought to illuminate the turbulence-induced changes in the aerodynamic correlation structure on the stationary/oscillating rectangular prism. A forced-vibration system was employed with a model instrumented with 66 pressure transducers. The study included the effects of the turbulence intensity, turbulence integral scale and structural motion on the spanwise and streamwise correlation (or coherence) of integrated aerodynamic quantities (lift force or torsional moment) and pressures. These effects were discussed based on the stationary model tests, the oscillating model tests in the low reduced wind velocity range and the oscillating model tests in the high reduced wind velocity range, which were characterized as three different aerodynamic regimes. In general, the turbulence scale effects were more significant than those of the turbulence intensity. Larger turbulence scale usually led to larger spanwise correlation of aerodynamic quantities on the stationary model, while the results of the oscillating model tests in the low reduced wind velocity range indicated a reversal of this trend. The results in the high reduced wind velocity range suggested self-excited force exhibited correlation coefficients greater than 0.95 for every incident flow in the entire spanwise separation range considered (2.4 times the deck width), which supported the assumption common in analytical estimates of fully correlated self-excited forces. Typically, the lift force
correlation was larger than that of the torsional moment. The self-excited force
correlation values were far larger than those of any velocity components and significantly
larger than those of buffeting force. Spanwise correlation of the buffeting force showed
exceptional similarity between stationary and oscillating model tests. The spanwise
correlation of the total wind-induced force (a combination of the self-excited and
buffeting forces) had a value between those of purely self-excited and buffeting cases.

E.1 Introduction
The current investigation of the wind-induced effects on bluff bodies is primarily based
on the semi-empirical analysis frameworks, where wind-tunnel testing is indispensable
due to the complexity of flow-structure interactions. The aerodynamic properties, which
are utilized in the semi-empirical analyses, are typically extracted from the section model
testing involving forced-vibration tests (Halfman 1952; Loiseau and Széchenyi 1975) or
free-vibration tests (Scanlan and Sabzevari 1969). In addition to their simplicity and
expediency, section models, due to the relatively large scale, dampen the concerns
regarding Reynolds number effects to some extent and permit taking into account the
geometric details of the bridge deck, to which the wind-induced effects are very sensitive
(Scanlan 1981). Since section model results must be extrapolated to full-scale estimates,
an accurate understanding of the spanwise correlation (or coherence) of the wind-induced
forces is critical. Such an understanding also aids to reveal the underlying physics of
three-dimensional (3-D) aeroelastic model results and helps the selection of an optimal
spanwise size of the computational domain in 3-D computational fluid dynamics (CFD)
simulation.
The study on spanwise correlation in wind engineering field started from understanding the nature of the wind (e.g., Davenport 1961; Harris 1970). For engineering applications, Davenport (1962) assumed that the spanwise correlation of the aerodynamic (buffeting) force is the same with that of the incident wind velocity, while it is typical to use perfect spanwise correlation of the aeroelastic (self-excited) force (Scanlan 1978). Past investigations on this issue indicated that the latter assumption for self-excited force may be acceptable, however, it is a well-established concept that the spanwise correlation of buffeting force is generally larger than that of the incident wind velocity (e.g., Ektin 1972; Melbourne 1982). Actually, apart from the "pillow effects" proposed by Davenport [in Larose (2003)], which lead to the relatively larger spanwise correlation of aerodynamic force compared to that of incident wind velocity, there are many other factors affecting the spanwise correlation of aerodynamic quantities on the bluff bodies, such as the characteristics of the incident wind and the body-induced flow.

In this study, the wind-tunnel experiments are carried out with a stationary/oscillating rectangular prism to investigate the spanwise correlation of wind-induced forces and pressures. Various turbulence intensities and turbulence integral scales are generated in the wind tunnel to examine the effects of the incident wind characteristics on the spanwise correlation of wind-induced forces. The spanwise correlation of pressures at various streamwise locations is inspected to investigate the effects of the flow structures around the body on the spanwise correlation while the streamwise correlation of pressures is inspected to examine the body-induced flow structures. Besides, it should be noted that, in reality, the buffeting and self-excited forces are usually concurrently exerted on the bridge deck, hence, the motion-induced
(aeroelastic) effects on the spanwise correlation of buffeting force and the gust-induced (aerodynamic) effects on the self-excited force need to be understood.

To investigate the relationship between spanwise correlations of wind velocities and of aerodynamic forces, and the effects of incident wind characteristics on the spanwise correlation of aerodynamic forces, most of studies focused on the stationary model tests. In this study, the spanwise correlation on the stationary model is firstly revisited. The effects of the turbulence intensity and turbulence scale on the spanwise correlation of aerodynamic forces and pressures are delineated independently.

The interplay between the gust-induced and motion-induced effects depends on the aerodynamic features of the wind-bridge interaction system, which can be identified with respect to the wind velocity region. There are generally three main wind-induced considerations for the long-span bridges, namely vortex-induced vibration (VIV), buffeting and flutter. The VIV behavior usually occurs at relatively lower reduced wind velocity range than that of flutter. Current analysis framework of bridge aerodynamics deals with the combination of the aerodynamic and aeroelastic forces in the study of VIV since the VIV is a typical nonlinear aerodynamic/aeroelastic interaction system. While the buffeting and flutter phenomena are investigated separately which indicates an assumption of the linear interaction between aerodynamics and aeroelasticity. Hence, in this study the spanwise correlation of oscillating model tests is investigated based on two different reduced wind velocity regions. In the relatively lower reduced wind velocity range, spanwise correlation of total wind-induced forces (pressures) is studied, where the effects of turbulence intensity and turbulence scale on the spanwise correlation on the oscillating model are investigated. While in the relatively higher reduced wind velocity
range, spanwise correlations of the buffeting and self-excited forces (pressures) are investigated separately, where the turbulence effects on the spanwise correlation of self-excited force and the motion-induced effects on the spanwise correlation of buffeting force can be examined explicitly.

There are various indexes to characterize the aerodynamic correlation, for example, the (normalized) correlation function in time domain, which is used to assess the overall correlation of aerodynamic forces or pressures in time domain, and the (normalized) coherence function in frequency domain, which can also present the correlation information at specific frequency components. Besides, after fitting the correlation function based on the exponential basis, one can obtain the correlation length scale by integration, which can be utilized to estimate a global correlation of aerodynamic forces or pressures. In this study, all these three indexes are utilized. While the correlation of integrated forces is very important for engineering applications, the correlation of pressures at different locations (both in streamwise and spanwise directions) contributes significantly in revealing the underlying mechanism of the aerodynamic correlation structure. Hence, both of these two are used in this study.

E.2 Experimental Setup

This section describes the equipment used for the present study including the wind tunnel facility, the model and its motion-driving system, the computer data acquisition system, and all relevant instrumentation.
E.2.1 Atmospheric Wind Tunnel

The atmospheric wind tunnel at the University of Notre Dame's Hessert Center for Aerospace Research was employed for the present study. Figure E.1 shows a top view of the facility. A 1.52m by 1.52m by 14.9m open-return test section is connected to a centrifugal fan powered by a 30 hp motor. Air entering the inlet passes through two anti-turbulence screens and a set of flow straighteners, through a 3.48:1 contraction section and, finally, through an additional set of flow straighteners. The turbulence intensity of the resulting flow in the test section is less than 0.5%.

![Top view schematic of the atmospheric wind tunnel](image)

Figure E.1. Top view schematic of the atmospheric wind tunnel (dimensions are to scale)

Turbulent flows were generated using two conventional biplane, square-mesh grids, placed at various distances upstream of the model as indicated in Figure E.1. Constructed of wood, these grids each had a solidity ratio of 0.32 where solidity ratio is the projected solid area of the grid per unit total area. The two grids had mesh sizes of 0.19m and 0.54m where mesh size, $M$, is the center to center distance of the grid bars.
The smaller-mesh grid generated $I_u = 6\%$ and $I_u = 12\%$ cases using the two grid positions closest to the model (distances from the model of $x = 6.67M$ and $x = 14.1M$, respectively). The larger-mesh grid generated $I_u = 6\%$ and $I_u = 12\%$ cases from the two further grid positions (distances from the model of $x = 13.5M$ and $x = 21.9M$, respectively). Turbulence quantities for each flow case generated for the present study are shown in Table E.1. Different cases are named with a number and a letter. The number denotes the turbulence intensity and small and large integral scales are indicated by the letters "a" and "b", respectively.

<table>
<thead>
<tr>
<th>Case</th>
<th>$I_u$</th>
<th>$I_w$</th>
<th>$L_{ux}$</th>
<th>$L_{uy}$</th>
<th>$L_{ux}$</th>
<th>$L_{wy}$</th>
<th>$S_u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth</td>
<td>0.4%</td>
<td>0.3%</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>$I_u = 6%$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 6a</td>
<td>6.1%</td>
<td>5.7%</td>
<td>1.81D</td>
<td>1.69D</td>
<td>0.86D</td>
<td>1.34D</td>
<td>2.3</td>
</tr>
<tr>
<td>Case 6b</td>
<td>5.8%</td>
<td>5.1%</td>
<td>4.9D</td>
<td>2.68D</td>
<td>2.56D</td>
<td>2.21D</td>
<td>0.3</td>
</tr>
<tr>
<td>$I_u = 12%$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 12a</td>
<td>11.7%</td>
<td>11.1%</td>
<td>1.34D</td>
<td>1.14D</td>
<td>0.61D</td>
<td>0.83D</td>
<td>53</td>
</tr>
<tr>
<td>Case 12b</td>
<td>11.6%</td>
<td>10.2%</td>
<td>4.89D</td>
<td>3.33D</td>
<td>2.66D</td>
<td>2.52D</td>
<td>6.0</td>
</tr>
</tbody>
</table>

(Note: $I_i$=turbulence intensity for the $i$th component of velocity; $L_{ij}$=integral length scale for the $i$th velocity component in the $j$th direction; $S_u$=small-scale spectral density parameter defined as $S_u= fG_{uu}(f)/U^2(10^6)$ where $G_{uu}$=power spectral density function of the $u$ component of velocity; $f$=frequency associated with scale of size $D/10$ and $U$=mean freestream velocity; N/A=not applicable)
E.2.2 Model

All pressure measurements were made on a model having a rectangular cross-section. Constructed of Plexiglas, the model had a frontal height, $D$, of 38mm, a streamwise width, $B$, of 254mm, and a span, $L$, of 1.07m. End plates of height $7D$ extended $2.5D$ and $5D$ from the model’s leading and trailing edges, respectively. The top face of the model was fixed in place with screws and was removable to allow for placement and connection of pressure sensors and tubing. Linkages driven by electric servo motors drove the motion. The model was mounted on bearings between two guide rails to allow both pitching and vertical motions along guide rails. For the present study, only the pitching motion was used, which was driven by a motor mounted to the outside wall of the wind tunnel. Additional details are available in Haan (2000).

Twenty-one streamwise sets of taps were distributed across the span—both top and bottom. The middle five rows were separated by 1 inch (25.4mm) while the outer rows had a 2 inch (50.8mm) separation. Each row consisted of 32 taps—16 on the top and 16 on the bottom. Six taps were distributed in the first $1D$ of the streamwise width, $B$, while the remaining 10 taps were equally spaced across the rest of the width bottom as shown in Figure E.2. Sixty-four pressure sensors were fixed inside the model connected to the pressure taps with 355mm lengths of plastic tubing (inner diameter 1.3mm), which were synchronously monitored. Because the number of sensors accommodates the taps of two different spanwise positions at once, the taps were constructed with stainless steel tubes to allow for convenient rearranging of the tubing to vary the spanwise separation.
E.2.3 Data Acquisition System

A computer data acquisition system was used for all the experiments. To accommodate the high channel count required for this work, the data acquisition hardware consisted of both a 16-bit data acquisition board (National Instruments model PCI-MIO-16XE-10) residing in a PC and nine boards (NI model SCXI-1140) that simultaneously sample and hold (SSH) multiple channels of voltages in a stand-alone chassis. This system “holds” 72 channels of voltages simultaneously while the computer board digitizes each one sequentially. This holding circuitry allowed each channel to be sampled with minimal time lag between the first and last channels. Because the maximum sampling rate of the analog-to-digital converter (ADC) was 100 kHz, the maximum sampling rate for any one of the 72 channels was 1388 Hz (the sampling rate used for this study was 1000 Hz).
E.2.4 Pressure and Velocity Measurements

Sixty-six pressure transducers were utilized to measure the dynamic and static pressures in the wind tunnel and the pressure distributions about the model. Two types of pressure transducers were used: Honeywell Microswitch sensors (Model 163PC) with a range of 620 Pa and SenSym sensors (Model ASCX01DN) with a range of 6900 Pa. Both sensors measured differential pressures, were temperature compensated, and had onboard voltage amplification. The SenSym transducers were mounted within the model to measure pressure distributions while the Honeywell sensors were used to measure the static and dynamic pressures of the flows with a Pitot-static probe.

The dynamic pressure—used to normalize all the model pressure measurements and to calculate mean flow velocity—was measured using a Pitot probe mounted $8D$ upstream of the model, $6.67D$ down vertically from the center of the tunnel, and $3.33D$ laterally from the center of the tunnel. The static pressure line of this same Pitot probe was connected via long plastic tubing to the static pressure ports of the pressure transducers inside the model. The pressure measurement system was dynamically calibrated and the system transfer function was employed to correct each channel of data before further processing. All pressure data were acquired in ensembles of 4096 points sampled at 1000 Hz. Stationary model tests consisted of 60 ensembles (245.8 seconds) while oscillating model tests consisted of 12 ensembles (49.2 seconds).

Hot film probes with X configurations were used to measure two components of turbulent velocity fluctuations—the streamwise component and the vertical component. A TSI IFA 100 anemometer was used along with TSI Model 1241 X probes. These probes were calibrated using a small, open jet wind tunnel and a look-up table approach.
employing a bilinear interpolation scheme. Profiles of the test section properties were acquired using a horizontal traverse. Velocity coherence measurements—requiring two probes—were made by mounting one probe on a fixed support and the other on the traverse.

E.3 Stationary Model Test

In this section, the spanwise correlation of the integrated aerodynamic quantities (lift force and torsional moment) was first investigated, as shown schematically in Figure E.3(a). Besides, in order to obtain a global understanding of the correlation of the aerodynamic quantities, exponential functions were fit to the spanwise correlation data and then integrated to obtain the spanwise correlation length scales. Then, the spanwise correlation between individual pressure signals was examined to understand the streamwise position dependence of the spanwise correlation, as shown schematically in Figure E.3(b). Finally, the streamwise correlation was discussed as an important parameter in calculating the integrated lift force or torsional moment.

E.3.1 Spanwise Correlation and Coherence of Forces

In order to investigate the correlation and coherence of the lift force and torsional moment on the stationary model, pressure signals were integrated at two different spanwise locations to produce time-varying lift force and torsional moment functions. Figure E.4(a) shows the cross correlation coefficient values for lift force (at zero time lag) in each of the flow conditions. The most notable trend is that the correlation curves show more of a dependence on turbulence scale than on turbulence intensity. Both the "a" pair and the "b" pair of flow cases have similar values throughout the range of spanwise
Figure E.3. Spanwise correlation of aerodynamic quantities: (a) between lift force functions at two different spanwise locations; (b) between individual pressure signals at each streamwise location for various spanwise separations separation considered despite the fact that one of each pair corresponds to a flow having double the other’s turbulence intensity. The increase in spanwise correlation values for greater incident turbulence scale is a trend reported by researchers investigating both general bluff bodies and bridge deck sections (e.g., Larose et al. 1993; Sankaran and Jancauskas 1993; Kimura et al. 1997; Saathoff and Melbourne 1997; Larose and Mann 1998). Figure E.4(b) shows the corresponding spanwise correlation coefficient values for torsional moment in each of the flow conditions. Each of these curves, while following
the same trends as those for lift force, has slightly lower values than their complementary lift force correlation curves.

![Cross correlation coefficient values on the stationary model: (a) lift force case; (b) torsional moment case](image)

Figure E.4. Cross correlation coefficient values on the stationary model: (a) lift force case; (b) torsional moment case

Table E.2 lists the spanwise correlation length scales for both the lift force \( L_{L_{\gamma}} \) and torsional moment \( L_{M_{\gamma}} \). It is noted that the spanwise correlation length scales of the lift force are greater than the integral length scales of the incident turbulent flows (as
presented in Table E.1) by a factor 3 to 4. This ratio drops slightly, to 2 or 3, when considering the spanwise correlation length scales of the torsional moment in the various incident flows.

<table>
<thead>
<tr>
<th>TABLE E.2</th>
<th>SPANWISE CORRELATION LENGTH SCALES FOR LIFT FORCE AND TORSIONAL MOMENT ON THE SECONDARY MODEL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Smooth</td>
</tr>
<tr>
<td>$L_{s\nu}$</td>
<td>$3.59D$</td>
</tr>
<tr>
<td>$L_{s\theta}$</td>
<td>$1.62D$</td>
</tr>
</tbody>
</table>

Correlation functions can illustrate the overall relationships of aerodynamic quantities to each other, however, the prediction of wind-induced dynamic response of the structure with the linear random vibration based spectral scheme (Davenport 1962) requires frequency information. Coherence functions provide such frequency information. Figure E.5(a) shows the coherence functions of the lift forces for the Case 6 flows plotted versus a reduced frequency defined as: $f dy/U$ (where $f$ is frequency in Hz, $dy=\Delta y$ is the spanwise separation, and $U$ is the mean freestream velocity). Each plot corresponds to a different separation distance between pressure measurement locations. As the separation distance increases, the coherence values fall for all reduced frequencies. For a given separation, the coherence values decrease with reduced frequency, however,
it seems that not all the cases decrease exponentially. The larger integral scale of the Case 6b flow results in coherence values larger than those of Case 6a over all reduced frequencies with a more gradual reduction with increasing spanwise separation. On the other hand, the coherence values with larger integral scale seem to reduce more intensely with increasing reduced frequency. Coherence functions of torsional moments on the stationary model in the Case 6 flows are shown in Figure E.5(b). These plots show very similar trends to those observed for lift forces, but as with the correlation values discussed previously, the coherence of torsional moment is slightly lower than that of lift force.

Figures E.6(a) and E.6(b) show coherence functions for lift forces and torsional moments on the stationary model, respectively, in the Case 12 flows. In general, increasing the turbulence intensity to 12% did not seem to affect the coherence values significantly. The coherence values are very similar with those of the Case 6 flows over the entire reduced velocity range suggesting that turbulence scale is a more important parameter for coherence than turbulence intensity. On the other hand, one notable difference does exist between Case 6b and Case 12b values. Case 12b values maintained a higher coherence for greater spanwise separation despite the fact that both incident flows of Case 6b and Case 12b had very similar integral scales. Hence, turbulence intensity effects should not be assumed insignificant in this case.

It should be noted that, even though the coherence values near zero frequency are quite high, they are less than unity as indicated in Figures E.5 and E.6. Similar behavior of the coherence function was also reported in the literature (e.g., Larose and Mann 1998). A discussion of such behavior can be found in Kareem (1987). Besides, the
Figure E.5. Coherence functions on the stationary model in the Case 6 turbulent flows (‘+’ Case 6a; ‘o’ Case 6b): (a) lift force cases at several different spanwise separations; (b) torsional moment cases at several different spanwise separations
Figure E.6. Coherence functions on the stationary model in the Case 12 turbulent flows ('+' Case 12a; 'o' Case 12b): (a) lift force cases at several different spanwise separations; (b) torsional moment cases at several different spanwise separations
coherence functions of the same reduced frequency may present scattered values with different spanwise separations. In order to better present the coherence function, the von Karman parameter, which can take into account this correction, was proposed to replace the reduced frequency parameter (e.g., Roberts and Surry 1973; Larose et al. 1993).

E.3.2 Streamwise Position Dependence of Spanwise Pressure Correlation

To investigate the structure of the spanwise correlation of aerodynamic quantities, spanwise correlation functions were computed between pressure signals at discrete streamwise locations. Exponential curves were fit to each of the pressure correlation curves. From these fits, spanwise pressure correlation length scales were computed and are shown in Figure E.7, which summarizes the results of the spanwise pressure correlation with a concise form. As indicated in Figure E.7, positions from the leading edge to $x/B \approx 0.6$ show the cases with larger turbulence scale having larger spanwise correlation. Beyond this point, however, this trend reverses, and the flows with larger integral scales have somewhat lower correlation values. Not all the pressure correlations, therefore, follow the same scale dependence as the lift force and torsional moment. The spanwise correlations of lift force and torsional moment behave as they do because the windward half of the model, where $x/B \leq 0.6$, experiences larger pressure magnitudes and is thus the greatest contributor to the integrated aerodynamic quantities. It should be noted that the observation based on the spanwise correlation length scales need further investigation since they are only rough estimates due to the small number of spanwise measurement locations.

Besides, two trends in Figure E.7 are worthy of note. On the upstream half of the surface, spanwise pressure correlation length scale behavior of the "b" cases roughly
Figure E.7. Spanwise pressure correlation length scales as a function of streamwise position for the stationary model

consisted of a streamwise increase to a maximum and then a decrease. This is similar to the velocity correlation observations of Kiya and Sasaki (1983) on a blunt, flat plate, where the spanwise correlation of the structures of the separated shear layer increased up to a maximum at reattachment. Kiya and Sasaki (1983) did not measure too far downstream of reattachment, however, the overall behavior of the spanwise correlation length scales shown in Figure E.7 is similar with that in their study. In addition to this behavior on the upstream half of the model, the spanwise correlation length scales increased near the trailing edge. This phenomenon is similar with the observations of Matsumoto et al. (2003) with a 1:5 rectangular prism. On the other hand, this does not occur for blunt, flat plates where the body extends "infinitely" downstream (Kiya and Sasaki 1983), which indicates that the shedding of vortices from the trailing edge may be exerting an organizing influence on the flow near the trailing edge thus increasing the spanwise correlation in that region.
E.3.3 Streamwise Correlation Structure

The lift force or torsional moment coefficients at each spanwise location are obtained by integrating the pressure signals along the streamwise, hence, any reduction in streamwise correlation may lead to a reduction in the lift force and torsional moment coefficients. It is usually to assume a perfect streamwise correlation for the square prism case since almost all the sideface flow separates from the body. On the other hand, for the bluff body with a long afterbody, the streamwise correlation cannot be assumed to be perfect due to the flow reattachment. Actually, Chaplin (1970) proposed a basic shape of the streamwise correlation curve which should be a cosine function with respect to the streamwise separation. Figure E.8 presents the streamwise pressure correlation coefficients with respect to the pressure signal near the leading edge (tap 1) on the stationary model. The cosine function shape of the streamwise correlation curve with respect to the streamwise separation is presented in the figure. In general, the incident turbulence effects lower the streamwise correlation values of the smooth flow condition except near the leading or trailing edges. Similar observation was also concluded by Wilkinson (1981). Besides, it seems that the effects of the incident turbulence on the streamwise correlation are grouped according to the turbulence scale. On the upstream half of the surface, the streamwise pressure correlation values of the "a" cases are smaller than those of the "b" cases.

E.4 Oscillating Model Test at Low Reduced Wind Velocity

A number of researchers investigating vortex-induced vibration (VIV) of bluff bodies with a short afterbody have found the spanwise correlations of both surface pressures and
Figure E.8. Streamwise pressure correlation coefficients with respect to tap 1 on the stationary model

wake velocities depend on the oscillation amplitude and the reduced wind velocity $U_r$ (e.g., Toebes 1969; Wilkinson 1981; Bearman and Obasaju 1982). Spanwise correlations of both pressure and wake velocity fluctuations reach maximum values for $U_r$ within the "lock-in" region. "Lock-in" refers to the phenomenon where the vortex shedding frequency is "captured" by a mode of the structure which has a frequency close to the structure's natural vortex shedding frequency. Due to nonlinear fluid-structure interactions, the VIV exhibits limit cycle oscillations. The magnitudes of these maximum values increase with greater oscillation amplitudes. Based on the spanwise correlation at a range of reduced wind velocities for a 5:1 rectangular cylinder, Ricciardelli (2010) also pointed out that, apart from the amplitude of oscillation, both the degree of freedom in which the body oscillates and the excitation mechanism which is different at various wind velocity ranges, affect the spanwise correlation. The increase in spanwise
correlation at "lock-in" can be attributed to the nonlinear interactions between the vortex shedding and the structural motions (Wilkinson 1981).

For the bluff body with a long afterbody, it should be noted that both the vortex shedding from the trailing edge and the shedding of large-scale vortices from the separation bubble (Kiya and Sasaki 1983) contribute the increase of the spanwise correlation. The cross correlation coefficient values for lift force (at zero time lag) at a spanwise separation of 1.2B with various reduced wind velocities in the present study are shown in Figure E.9(a) while the values for the torsional moment at the same separation with various reduced wind velocities are shown in Figure E.9(b). In order to investigate a more general trend of the spanwise correlation with respect to the reduced wind velocity, the corresponding correlation length scales of lift force and torsional moment as functions of the reduced velocity for each flow case are presented in Figure E.10(a) and E.10(b), respectively.

Due to the design problems with the dynamics of the model's flexible couplings, data of the reduced wind velocities in the range of 3.1 < U_r < 8 are not presented here. As indicated in these figures, however, maximum spanwise correlation occurs between U_r = 3.1 and U_r = 8 followed by a gradual reduction as reduced wind velocity increases. It is noted that the reduced wind velocity U_r is around 6 for which the body experiences "lock-in" in this study (Haan 2000). Besides, observations of the power spectral density plots of the pressure signals along the streamwise direction for both the stationary and oscillating model tests reveal a low-frequency peak near 3.2-3.4 Hz (Haan 2000). When scaled as F_{shed}D/U≈0.02, this frequency is comparable to that of the shedding of large-scale vortices from the separation bubble observed by Kiya and Sasaki (1983) and Cherry
Figure E.9. Cross correlation coefficient values as a function of reduced wind velocity (at a spanwise separation of \( \Delta y = 1.2B \)): (a) lift force case; (b) torsional moment case
et al. (1984). The reduced wind velocity of this large-scale unsteadiness is $U_r = U/F_{shed}B \approx 6.7$ which is also in the range for which the maximum spanwise correlation values occur for the current tests. Exciting the shear layer with forced oscillations near the reduced wind velocity of this large-scale shedding may cause behavior similar to that observed in vortex-induced vibration problems, that is, natural shedding mode of the structure is enhanced by the oscillation increasing the organization of the shedding and increasing the spanwise correlation. Besides, it is believed that both the vortex shedding from the
trailing edge and the shedding of large-scale vortices from the separation bubble enhance the spanwise correlation with a nonlinear, unsteady aerodynamic mechanism, where the linear superposition philosophy may not be applicable. Hence, in this study the spanwise correlation at relatively low reduced wind velocity range is investigated with the total aerodynamic quantities (forces or pressures) which are comprised of both aerodynamic and aeroelastic effects and their interactions.

E.4.1 Spanwise Correlation and Coherence of Forces

Reduced wind velocity $U_r=8$ is selected as a representative case to investigate the spanwise correlation and coherence of the lift force and torsional moment on the oscillating model in the relatively low reduced wind velocity range. Figure E.11(a) presents the cross correlation coefficient values (at zero time lag) of the lift force for $U_r=8$. Two major differences between these values and those for the stationary model (as shown in Figure E.4) present themselves. The first is the much higher values of the spanwise correlation for the oscillating model. Actually, stationary model spanwise correlation length scales were between $0.3B$ and $1.9B$ depending on the turbulence properties while oscillating model correlation length scales were between $5B$ and $20B$ for turbulent flow and far greater for smooth flow. The second difference is the reversal of the trend with incident turbulence scale. While larger turbulence scales increase the spanwise correlation for the stationary cylinder, Figure E.11(a) shows that the opposite is true for the oscillating cylinder at this reduced wind velocity. Both larger-scale "b" flow cases produced significantly lower spanwise lift force correlation than the corresponding smaller-scale "a" cases. The effects of turbulence intensity were much more subtle in the smaller-scale turbulent flow conditions—when comparing the correlations between flows
both having smaller turbulence length scales, the flow with greater turbulence intensity had a slightly lower correlation value. However, the effects of turbulence intensity in the larger-scale turbulent flow conditions on the spanwise correlation are enhanced. The similar behavior is observed for the coherence functions on the stationary model (as shown in Figures E.5 and E.6). The smooth flow correlation values also warrant mention due to the fact that they are higher than those of the turbulent flow cases—a behavior that is opposite to the stationary model behavior. For the stationary model, the smooth flow correlation values were consistently lower than the turbulent flow values. Figure E.11(b) presents the cross correlation values of the torsional moment for $U_r=8$. Similar trends with respect to turbulence are evident comparing these curves to their lift force correlation counterparts. However, lift force correlation values are slightly higher than torsional moment values for this reduced wind velocity.

Coherence functions were also calculated to examine these spanwise correlation issues in terms of frequency dependence as well. Figure E.12(a) shows the coherence functions of the lift forces for the Case 6 turbulent flows at $U_r=8$ for both the stationary and oscillating tests. Except for the coherence at the frequency of the model motion, $f_m$, the results from tests with and without motion are remarkably similar, which indicate the stationary coherence values utilized in the conventional buffeting theory is acceptable. This will be discussed in detail in the following section. At the model oscillation frequency, however, the oscillating-model coherence values are very high, near unity. While the broadband coherence decreases rapidly with spanwise separation, the coherence at $f_m$ decreases only slightly and sustains extremely high values throughout the separation range considered. Coherence functions of the torsional moments for the Case 6
Figure E.11. Cross correlation coefficient values for $U_r=8$ in smooth and turbulent flows: (a) lift force case; (b) torsional moment case

turbulent flows at $U_r=8$ in Case 6 flows is presented in Figure E.12(b). These values behave quite similarly to their lift force coherence counterparts but with slightly lower magnitudes.

Lift force coherence functions for the Case 12 turbulent flows at $U_r=8$ is shown in Figure E.13(a). Like the Case 6 plots, the broadband coherence of the lift forces on the oscillating model were very similar to those on the stationary model—except for the peak at the model oscillation frequency. Torsional moment coherence functions for the Case
Figure E.12. Coherence functions at $U_r=8$ in the Case 6 flows ('+' Case 6a stationary; ' - ' Case 6a oscillating; 'o' Case 6b stationary; '—' Case 6b oscillating): (a) lift force cases at several different spanwise separations; (b) torsional moment cases at several different spanwise separations
12 turbulent flows at $U_r=8$ is presented in Figure E.13(b). These values behave also quite similarly to their lift force coherence counterparts but with slightly lower magnitudes.

As indicated in the abovementioned discussion, it seems that the higher spanwise correlation of the oscillating cases compared to that of the stationary cases mainly results from the extremely high coherence values (near unity) at the oscillation frequency. Hence, it will be interesting to investigate the coherence values at the model oscillation frequency versus the spanwise separation in various flow conditions, which are presented in Figure E.14(a) for the lift force case. With respect to turbulence properties, these coherence values show the behavior similar to that of the correlation values of the total aerodynamic lift force. Smooth incident flow resulted in the highest lift force coherence values, and incident turbulent flow led to lower coherence values. Larger-scale turbulence caused larger decreases in the coherence values. These general statements are true of the spanwise correlation values discussed previously (as shown in Figure E.11).

The main difference for the coherence values is how much smaller the relative changes are in different flow conditions, which indicates that the turbulence effects may alter the broadband coherence much more significantly compared to the change of the oscillation frequency coherence. Because the precision uncertainty in this range of coherence is about 3%, conclusive statements cannot be made concerning turbulence effects on these peak values. However, it should be noted that any turbulence effects on these peak values will significantly contribute the overall spanwise correlation of the total lift forces and torsional moments. Besides, it would take a significantly longer model span to identify the separation lengths for which the coherence values of the oscillation frequency would drop to insignificant levels. Figure E.14(b) shows the torsional moment coherence values...
Figure E.13. Coherence functions at $U_r=8$ in the Case 12 flows (‘+’ Case 12a stationary; ‘- -’ Case 12a oscillating; ‘o’ Case 12b stationary; ‘—’ Case 12b oscillating): (a) lift force cases at several different spanwise separations; (b) torsional moment cases at several different spanwise separations
at the oscillation frequency in various flow conditions. Comparing the torsional moment results with their lift force coherence counterparts reveals very similar behaviors.

![Coherence values at the model oscillation frequency, \( f_m \), for \( U_r = 8 \) in smooth and turbulent flows: (a) lift force case; (b) torsional moment case](image)

**Figure E.14.** Coherence values at the model oscillation frequency, \( f_m \), for \( U_r = 8 \) in smooth and turbulent flows: (a) lift force case; (b) torsional moment case

**E.4.2 Streamwise Position Dependence of Spanwise Pressure Correlation**

As in Section E.3.2 for stationary model tests, this section presents spanwise correlation functions between pressure signals at discrete streamwise locations for the oscillating
model. The structure of the spanwise pressure correlation was thus examined rather than the correlation only of the integrated aerodynamic quantities. An overall picture of the aerodynamic correlation structure can be seen in Figure E.15 where the spanwise pressure correlation length scales are plotted versus streamwise position. As indicated in this figure, these correlation values show a grouping according to turbulence intensity, which is different with the behavior of the stationary model tests with respect to the incident turbulence properties (as shown in Figure E.7). The correlation values in the Case 12 turbulent flows show lower values than those in the Case 6 turbulent flows. Integral scale effects seemed to be greatest between $0.1375B < x < 0.2775B$ (between tap 6 and tap 8) for Case 6 turbulent flows and $0.0875B < x < 0.2775B$ (between tap 4 and tap 8) for Case 12 turbulent flows. An overall observation can be made for the spanwise pressure correlation length scales, which presents an increase to a maximum when proceeding downstream from the leading edge. As discussed for the stationary model spanwise pressure correlation length scales, this behavior may be due to a coalescence or amalgamation of vortices in the shear layer that no longer occurs after reattachment on the body surface. Downstream of this maximum the spanwise scales drop significantly. However, this plot for $U_r=8$ does not show an increase in correlation on the downstream half of the body as the behavior of the stationary case.

E.4.3 Streamwise Correlation Structure

The streamwise pressure correlation coefficients with respect to the pressure signal near the leading edge (tap 1) on the oscillating model ($U_r=8$) is presented in Figure E.16. Compared to the stationary model tests, the streamwise pressure correlation of the oscillating model is generally higher. The incident turbulence effects also lower the
streamwise pressure correlation values of the smooth flow condition except near trailing edges. Besides, it seems that the effects of the incident turbulence on the streamwise pressure correlation of the oscillating model are grouped according to the turbulence intensity, which is different with the behavior of the stationary model with respect to the turbulence properties (as shown in Figure E.8). On the upstream half of the surface, the streamwise pressure correlation values of the Case 12 turbulent flows are smaller than those of the Case 6.

E.5 Oscillating Model Test at High Reduced Wind Velocity

At high reduced wind velocity, aeroelastic instability (flutter) is a critical issue for bridge aerodynamics. As mentioned above, conventional schemes consider the analyses of the buffeting and flutter phenomena in isolation which indicates a linear interaction between aerodynamics and aeroelasticity. Hence, the spanwise correlations of the buffeting and
self-excited forces are investigated separately, which can be utilized to extend the two-dimensional (2-D) conventional analysis to the three-dimensional (3-D) case. Besides, the coherence results in Section E.4 indicate that the increase in spanwise correlation going from the stationary model to the oscillating model may be due almost entirely to the contribution from a single frequency (model oscillation frequency), while broadband coherence levels of the aerodynamic quantities on the stationary and oscillating models were very similar. These observations need to be further investigated by examining the correlation and coherence functions for distinct components (buffeting and self-excited) of the signals measured on the oscillating model in the relatively high reduced wind velocity range.

Reduced wind velocity $U_r=20$ is selected as a representative case to investigate the spanwise correlation and coherence of the lift force and torsional moment on the
stationary and oscillating models in the relatively high wind velocity range. Lift force and torsional moment calculated at a given spanwise position are separated as follows:

\[
L_i(t) = L_{se_i}(t) + L_{b_i}(t) \tag{E.1a}
\]

\[
M_i(t) = M_{se_i}(t) + M_{b_i}(t) \tag{E.1b}
\]

where \(L_i, M_i\) is the total lift force and torsional moment at spanwise position \(i\) (\(i=1, 2\)), respectively; the subscript "se" refers to self-excited and "b" refers to buffeting. The self-excited components, which are accounted for by the flutter derivatives, are those forces that result from the motion of the body itself and require no other outside forcing. The buffeting components are the unsteady forces resulting from the natural unsteadiness of the incident turbulence and are independent of the body motion. It should be noted that the buffeting forces calculated by Equations E.1a and E.1b actually involve the body-induced forces (signature turbulence).

E.5.1 Spanwise Correlation of Buffeting Component

Figure E.17 shows each component of a lift force signal on the model oscillating at \(U_r=20\) in Case 6a flow. The signal processing techniques used to split the signal into self-excited and buffeting components are referred to Haan (2000). Correlation and coherence calculations were performed on each of these components separately. As an example, Figure E.18 presents the cross correlation functions for the lift forces separated by \(\Delta y =1.2B\) on the model oscillating at \(U_r=20\) in Case 12b flow. The cross correlation function of the total lift force is shown on the left, and those of each component, \(L_{se}(t)\) and \(L_{b}(t)\) are shown on the right. The self-excited component is far more highly correlated than the
buffeting component. The resulting correlation of the total lift force then has a value between that of the self-excited force and that of the buffeting force.

Figure E.17. Plots of the total, self-excited, and buffeting components of lift for $U_r=20$ in Case 6a flow

Figure E.18. Cross correlation functions for the total, self-excited, and buffeting components of lift force for $U_r=20$ in Case 12b flow at a separation of $\Delta y = 1.2B$
Figures E.19 shows the spanwise correlation coefficients (at zero time lag) of the buffeting components of the lift force on both the stationary model and the oscillating model \((U_r=20)\). In general, the correlation values for \(L_b(t)\) on the oscillating model are close to those found for the stationary model. In the cases of large spanwise separation, the correlation values of the oscillating cases are slightly larger than those of the stationary cases, while in the cases of very small spanwise separation with larger turbulence scale, the correlation values of the oscillating cases are slightly smaller than those of the stationary cases. The sources of such mismatches may include the effects of harmonics of the model oscillation frequency that were not removed in the splitting analysis, effects of signature turbulence, or spurious correlation induced by non-aerodynamic effects (Haan 2000). However, the overall matches between the stationary and oscillating model tests are quite good suggesting that the correlation structure of the buffeting component on the oscillating model is essentially the same as that on the stationary model. The quantitative analysis of the spanwise coherence on the stationary and oscillating models, which was not presented here, shows that the difference in the coherence values were within the random error of the calculations. It should be noted that the buffeting force magnitudes and spanwise coherences are usually measured on the stationary model and then employed in the overall dynamic analyses of the bridge deck which is, actually, in oscillation. Results of the spanwise correlation/coherence on the stationary and oscillating models observed in this study seem to support this practice at least in terms of the buffeting force calculation.

The results of this splitting analysis have shown that, just like the behavior of the buffeting component on the stationary model, the spanwise correlation values of
buffeting component acting on the oscillating model increase with increasing turbulence scale. Similar observation can be found in the literature (e.g., Larose et al. 2003; Matsumoto et al. 2003). On the other hand, trends of the spanwise correlation values of the total aerodynamic quantities on the oscillating model show that increasing turbulence scale decreases the spanwise correlation (as discussed in Section E.4), which may be attributed to the characteristics of the spanwise correlation of motion-induced (self-excited) aerodynamic quantities and the contribution from nonlinear aerodynamics.

E.5.2 Spanwise Correlation of Self-Excited Component

Calculating spanwise correlation coefficients of the self-excited component of the lift force and torsional moment resulted in values above 0.95 for every incident flow and every separation. Figure E.20 shows an example of the self-excited correlation values for \( U_r = 20 \). The cases with larger turbulence scale showed a slightly lower correlation than
those of smaller scale, however, it should be noted that the estimated 95% confidence intervals of ±0.03 put all the values within the statistical spread of the others.

Figure E.20. Spanwise correlation coefficients of the self-excited components (se) of lift force with the model oscillating at $U_r=20$

The difference between smooth and turbulent flow flutter results for full, aeroelastic models of bridges has quite often been illustrated using the observations from the study of the Lions’ Gate Bridge (Irwin and Schuyler 1977). When examining the root mean square (rms) response of a single location on the deck of a full, aeroelastic bridge model in both smooth and turbulent flows, a distinct flutter boundary was evident for smooth flow while a gradual rise in response amplitudes with no such distinct flutter boundary was observed for turbulent flow. Data such as these have led many researchers to conclude that the flutter boundaries found in smooth flow are conservative. The proposed physical explanation for this has been that turbulent flow reduces the spanwise correlation of the motion-induced forces from their smooth-flow values thus reducing the
possibility of unstable motion, where an exponential reduction in the coherence of the flutter derivatives are typically utilized (e.g., Scanlan 1997). Although this hypothesis has been mentioned by a number of researchers, no direct experimental work has addressed this issue. The experimental results at relatively high reduced wind velocity shown in this study, however, indicate that the self-excited forces are highly correlated in the spanwise direction even at turbulent flow conditions. Hence, the observation from this particular example may not support the hypothesis which is used to illustrate the difference between the flutter boundaries of a full, aeroelastic bridge model in the smooth and turbulent flow conditions. Besides, this hypothesis is based on the single-model analysis. Actually, the effects of the reduction in the spanwise coherence are not so obvious for multi-mode, coupled flutter analyses. While decreases in spanwise coherence may stabilize a deck by reducing negative aerodynamic damping effects, it may also destabilize a deck by reducing favorable aerodynamic damping (Chen and Kareem 2003). The impact of such complex effects will become more important as bridges with longer spans are designed and multi-mode flutter becomes more probable. Experimental models with far greater aspect ratio (defined as span length, $L$, to deck width, $B$) will be needed to identify how far such a high correlation can be sustained. The model studied here had an aspect ratio of only $L/B = 4.2$. Long span bridges, obviously, have much greater aspect ratios—for example, the ratio of the center span to the deck width of the Akashi-Kaikyo Bridge is 56. The spanwise correlation under such cases deserves further attention.

If one examines the spanwise correlation of the total lift force rather than the correlation of the decomposed components, the results, which are presented in Figure E.21 (the spanwise correlation of the total lift force for $U_r=20$), indicate that the
correlation trends for total lift force follow the trends of the self-excited lift force (as shown in Figure E.20) rather than those of the buffeting lift force [as shown in Figure E.19(a)], i.e., the spanwise correlation decreases for increasing turbulent integral scale. These same results were found for total torsional moment as well and for each reduced wind velocity (e.g., see the results shown in Section E.4). Spanwise correlation measurements of the total lift force and torsional moment on the oscillating model were also made by Cheung and Melbourne (2005) based on a full-bridge aeroelastic model in the boundary layer wind tunnel. Their results also indicated a decrease in spanwise correlation with increasing turbulence scale. However, since in their cases the increase in turbulence scale was coupled with a decrease in turbulence intensity, no conclusive statements can be made concerning the effects of turbulence scale or turbulence intensity. Besides, it should be noted that the oscillation amplitude is another important issue affecting the spanwise correlation. The effects of the motion-induced vortices, vortex-shedding from the trailing edge, the shedding of large-scale vortices from the separation bubble, and their interactions each other complicate the aerodynamic behavior of the bluff body in oscillation (Kareem and Wu 2012).
E.6 Conclusions

An experimental study was conducted to examine the aerodynamic correlation structures on the stationary/oscillating rectangular prism. Spanwise correlation was quantified for both total aerodynamic/aeroelastic lift force (or torsional moment) and for self-excited and buffeting components separately. Typically, lift force correlation is larger than that of the torsional moment.

Based on the stationary model tests, it is concluded that the spanwise correlation of the aerodynamic quantities shows more of a dependence on turbulence scale than on turbulence intensity. Both the "a" pair (indicating a smaller turbulence scale) and the "b" pair (indicating a larger turbulence scale) of flow cases have similar values throughout the range of spanwise separation considered despite the fact that one of each pair corresponds to a flow having double the other’s turbulence intensity. However, not all the
pressure correlations follow the same turbulence-scale dependence as the integrated lift force and torsional moment. It is shown that positions from the leading edge to $x/B \leq 0.6$ show the cases with larger turbulence scale having larger spanwise correlation, while beyond this point this trend reverses and the larger incident turbulence scale decreases the spanwise correlation values.

The spanwise correlation in relatively low reduced wind velocity range is investigated with the total aerodynamic quantities. The spanwise correlation values of the oscillating model are much higher than those of the stationary model, which is attributed to the extremely high value (near unity) of the spanwise coherence at the oscillation frequency. While larger turbulence scale increases the spanwise correlation for the stationary cylinder, it is shown that the opposite is true for the oscillating model tests. Both larger-scale "b" flow cases produced significantly lower spanwise lift force correlation than the corresponding smaller-scale "a" cases. This may be attributed to the characteristics of the spanwise correlation of the motion-induced (self-excited) aerodynamic quantities and the contribution from the nonlinear aerodynamics. The effects of turbulence intensity on the spanwise correlation were much more subtle while these effects are enhanced in the larger-scale turbulence flows. Besides, compared to the stationary model tests, the values of spanwise correlation for the oscillating model is much higher. Besides, the turbulence effects may alter the broadband coherence much more significantly compared to the change of the oscillation frequency coherence.

The spanwise correlation in relatively high reduced wind velocity range is investigated with the self-excited and buffeting components separately. Spanwise correlation of the buffeting component showed exceptional similarity between stationary
and oscillating model tests. These results suggest that the conventional buffeting analysis done by so many past researchers on stationary models can be used to predict buffeting behavior on oscillating bodies. Self-excited component showed correlation values greater than 0.95 for every incident flow in the entire spanwise separation range studied (2.4B). These values were far larger than any velocity correlation scale and significantly larger than buffeting force correlation. Only the body motion itself was correlated on the same order as the self-excited force. This supports the assumption common in analytical estimates of fully correlated self-excited forces. However, it does not support the hypothesis that the stabilizing effect of turbulence observed in full aeroelastic tests is due to a turbulence-induced decrease in the spanwise coherence of the self-excited force. In the future, greater spanwise separations must be tested for full understanding of this behavior.


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