TIP CLEARANCE FLOW INTERACTION WITH CIRCUMFERENTIAL GROOVE CASING TREATMENT IN A TRANSONIC AXIAL COMPRESSOR

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Abstract

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Experimental and computational studies were conducted to study the role of the tip leakage flow in axial compressor stall and the relationship between the tip clearance flow flow field and surge margin extension from circumferential groove casing treatment.

The CFD results were used to identify the existence of an interface between the approach flow and the tip-leakage flow. The experiments used a surface streaking visualization method to identify the time-averaged location of this interface as a line of zero axial shear stress at the casing. The axial position of this line, denoted $x_{zs}$, moved upstream with decreasing flow coefficient in both the experiments and computations. The line was consistently located at the rotor leading edge plane at the stalling flow coefficient, regardless of inflow boundary condition. These results were successfully modeled using a control volume approach that balanced the reverse axial momentum flux of the tip-leakage flow with the momentum flux of the approach fluid. Non-uniform tip clearance measurements demonstrated that movement of the interface upstream of the rotor leading edge plane leads to the generation of short length scale rotating disturbances. Therefore, stall was interpreted as a critical point in the momentum flux balance of the approach flow and the reverse axial momentum flux of the tip-leakage flow.
Experimental measurements of surge margin extension from seven CGCT configurations with a fixed groove geometry demonstrated that the contribution of individual grooves in a multi-groove casing to surge margin extension is an \( (a) \) additive and \( (b) \) linear function of the smooth wall tip clearance axial momentum flux at the location of each groove. Extending the axial momentum model to include the influence of a CGCT showed that circumferential grooves reduce the tip leakage flow axial momentum through radial transport. The equivalent force due to a circumferential groove was demonstrated to be related to the smooth wall tip clearance axial momentum flux through a coefficient of drag that had a log-linear dependence on groove aspect ratio.
“As the ancient myth makers knew, we are children equally of the earth and the sky. In our tenure on this planet we’ve accumulated dangerous evolutionary baggage – propensities for aggression and ritual, submission to leaders, hostility to outsiders – all of which puts our survival in some doubt. But we’ve also acquired compassion for others, love for our children and desire to learn from history and experience, and a great soaring passionate intelligence – the clear tools for our continued survival and prosperity.” – Sagan
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SYMBOLS

\( A_g \)  Axial Momentum Flux of the Tip Leakage Jet at the Location of a Groove
\( C \)  Rotor Blade Chord
\( C_{ax} \)  Rotor Blade Axial Chord
\( C_{d_g} \)  Coefficient of Drag for a Groove
\( C_{d_{cv}} \)  Coefficient of Drag for 1-D Analysis Control Volume
\( c_p \)  Specific Heat at Constant Pressure
\( D \)  Compressor Annulus Outer Diameter
\( K_{A_c} \)  Actual-to-Approximated Tip Leakage Jet Axial Momentum Ratio
\( \dot{m}_c \)  Corrected Mass Flow
\( \dot{m} \)  Physical Mass Flow
\( N \)  Physical Shaft Speed
\( N_c \)  Corrected Shaft Speed
\( n_i \)  Blade Count, Inlet Guide Vane
\( n_r \)  Blade Count, Rotor
\( n_s \)  Blade Count, Stator
\( P \)  Rotor Power
\( P_{t_1} \)  Inlet Total Pressure
\( P_{std} \)  Standard Day Pressure, 101.3 kPa
\( Q_o \)  Approach Flow Momentum Flux Per Unit Area
\( \tilde{Q} \)  Approximated Tip Leakage Jet Axial Momentum Flux Per Unit Area
\( T_{std} \)  Standard Day Temperature, 15 °C
\( U_m \)  Rotor Blade Speed at Blade Meanline
\( U_{\text{tip}} \)  Rotor Blade Speed at Blade Tip

\( x_o \)  Virtual Origin of the Tip Leakage Jet

\( x_{zs} \)  Axial Location of the Line of Zero Axial Shear

\( \delta \)  Ratio of Inlet Total Pressure to Reference Pressure, \( P_{\text{t1}}/P_{\text{std}} \)

\( \gamma \)  Ratio of Specific Heats

\( \theta \)  Ratio of Inlet Total Temperature to Reference Temperature, \( T_{\text{t1}}/T_{\text{std}} \)

\( \sigma_i \)  Solidity, Inlet Guide Vane

\( \sigma_r \)  Solidity, Rotor

\( \sigma_s \)  Solidity, Stator

\( \tau \)  Tip Clearance Height

Total Temperature Ratio
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CHAPTER 1

INTRODUCTION

1.1 Motivation

The stable operation of an axial compressor at a given shaft speed is limited at low mass flows by a phenomenon known as compressor stall. Compressor stall is characterized by a decrease in overall pressure rise and non-axisymmetric through-flow. At the stalling operating point, regions of spoiled flow develop in one or more of the rotor passages and rotate around the annulus. These regions, often called stall cells, negatively impact the pressure rise and efficiency of the machine. Compressor stall can lead to the flow phenomenon known as compressor surge. Compressor surge is characterized by axisymmetric fluctuations in mass flow. In some cases, these fluctuations are severe and lead to flow reversal. The transition to compressor surge exposes the machine to large fluctuations in material stress and sustained surge can damage or destroy the compressor. Both compressor stall and surge are detrimental to the performance and the health of the compressor and should be avoided.

The figure of merit for a compressor is the total pressure ratio across the machine. The total pressure ratio is typically plotted as a function of mass flow rate at a fixed shaft speed to form a speed line or compressor characteristic. Several compressor characteristics for different shaft speeds plotted together form a compressor performance map. Figure 1.1 is provided as an illustration of the map of a typical high pressure ratio axial compressor. As shown, the locus of lowest mass flow stable operating points are joined and often called the stall or surge line of the compressor.
Further, the locus of steady state operating points to which the compressor is confined when operating as part of a given gas turbine engine is often included on the compressor’s map and referred to as the compressors working line.

![Figure 1.1. Illustration of a High Pressure Ratio Compressor Map](image)

In most modern axial compressors, the working line associated with maximum efficiency at the design shaft speed are close to the compressor’s stall line. Mass flow transients due to spool acceleration/deceleration or inlet flow distortion due to gusts or maneuvering lead to deviations from the working line. Thus, for a compressor operating on the maximum efficiency working line, an unanticipated decrease in mass flow may force the compressor to an operating point beyond the stall line.

Compressor designers utilize a factor of safety known as \textit{surge margin} or \textit{stall margin} to quantify the “distance” between a given operating point and the stall limit operating point. Often, designers must exchange peak efficiency and performance
at the compressor’s design point in order to provide adequate surge margin between the compressor’s working line and its stall line. Clearly, rather than making this exchange, it would be advantageous to increase surge margin on the peak efficiency and performance working line.

Casing treatments have been known to increase the surge margin of axial compressors since the late 1960s. Casing treatments are geometric modifications to the casing over the rotor which can be in the form of grooves, holes, or other configurations. These treatments modify the tip gap flow field such that a compressor can operate at lower mass flows before reaching their stall limit. Early studies showed that, in general, while casing treatments increase stall margin there is a roughly linear drop in efficiency of the compressor stage. Circumferential groove casing treatments (CGCT) are of particular interest since CGCT configurations have been shown to be equally successful in extending surge margin as other casing treatment designs with less efficiency penalty.

The following sections summarize the current understanding of compressor stall inception mechanisms and how casing treatments modify these mechanisms to extend stall margin.

1.2 Stall Inception

The compressor stall is characterized by a decrease in the pressure ratio of a compressor stage. Stall initiates with the formation of one to several regions in which the axial flow of the fluid is lower than that of adjacent passages. In most modern compressors, short length scale disturbances in pressure evolve, grow, and collect to form these “cells” of spoiled flow.

McDougall et al. [22] investigated the transient stalling process of an axial compressor and suggested that stall inception occurred at the blade tip. This conclusion was supported by later work by Day [12] which investigated the performance of com-
pressors with varied tip clearances and mass injection. The study found that a change in tip clearance height altered the pressure rise and stall point of an axial compressor. Further, the study showed that mass injection at the tip affected mass flow of the stalling operating point. Later, Camp and Day [6] introduced the concept of critical incidence. The work asserted that when a critical incidence angle was exceeded at the blade tip, small scale rotating disturbances or “spikes” were created in the blade row. If the critical incidence angle is reached prior to the peak pressure the rotor blades can develop a three-dimensional separation in the tip region.

These works established that the blade tip is the source for spike stall inception, however the actual stall inception flow physics were not well known. Some initial computational work studying the the blade-to-blade variations were completed by Hoying et al. [19]. In an eight-blade computational investigation, Hoying observed spike type stall initiation and attributed this to the spillage of the tip clearance vortex ahead of the leading edge of the rotor blades.

Hoying’s work was later revisited and improved by Vo et al. [34]. Again, the study implemented a multi-blade computation of the rotor passage flow field to describe the onset of stall. Vo advanced two criteria for the formation of short length scale disturbances that initiate stall. First, the interface between the approach fluid and the reverse flow in the tip gap moved forward of the leading edge of the following blade. Second, flow which originated from the trailing edge of a given blade moved upstream and stagnated on the pressure surface of the next blade. Figure 1.2 is a schematic that illustrates this conceptual framework. Two operating conditions are shown. The first depicts the general topology of the flow pattern at a nominal design flow coefficient. The interface between the approach fluid and the tip gap reverse flow is highlighted as a darkened streamline. In the second sketch the flow coefficient is reduced and the interface is ahead of the leading edge of the trailing blade. The hypothesis that spike-type stall is initiated by leading edge spillage of the interface
was supported by other contemporary numerical results (e.g. [9, 11]).

Recently, Chen et al. [8] computed the time-resolved flow through NASA Rotor 35 during stall inception. The reverse tip-leakage flow, marked by entropy, was shown to move upstream of the rotor leading edge plane in the tip region as the flow coefficient was reduced. The reverse flow occurred in packets that were 2-4 passages in pitchwise extent and was limited to the outer spanwise locations of the blade. The region of backflow initially rotated at 85% of rotor speed, and then slowed as the backflow region grew into fully developed stall cells.

Figure 1.2. Schematic of Velocity Vectors at Rotor Tip Plane as Proposed by Vo et al. [34]
Lin et al. [21] presented a full-annulus unsteady simulation of a low-speed compressor rotor with 94 blades. The study focused on the details of the flow structure that accompanies a short length-scale disturbance. During a transient disturbance, the flow through the blade passages was found to reverse direction in the tip region and move ahead of the rotor leading edge plane. Analysis of the pressure field and streamline patterns showed that the rotor passages circumferentially ahead of the disturbance exhibited a positive pressure fluctuation. The passages where backflow was present displayed a vortex pattern that appeared to result from leading edge flow separation. Due to the concentrated vorticity, a negative pressure fluctuation was present in these passages. This circumferential pattern of high pressure followed by low pressure is a typical attribute that has been observed in previous experimental measurements of short length-scale disturbances (e.g. [13, 20]).

Deppe et al. [14] utilized unsteady pressure measurements to experimentally investigate the relationship between tip clearance flow and spike-type stall inception. Most notably, the study made simultaneous pressure measurements upstream and downstream of the compressor rotor. The results suggested the presence of both leading edge spillage and trailing edge backflow at stall, as Vo hypothesized.

Saathoff and Stark [28] experimentally observed time-averaged endwall shear patterns in a low-speed axial compressor via a oil flow technique applied to the rotor casing. Three distinct regions (viewed from the stationary reference frame) were noted: a band near the rotor leading edge plane characterized by a large void in the marking material, a reverse flow region which extended over the majority of the blade axial chord, and a “reattachment” line located at the rotor trailing edge plane. The band near the leading edge plane was hypothesized to be a result of the interaction between the approach fluid and the reverse flow present in the tip clearance region. The axial location of the band near the rotor leading edge moved upstream as the flow coefficient was reduced. In their conclusion, Saathoff and Stark connect
the upstream movement of the separation line with a decrease in flow coefficient but no measurements or conclusions were made regarding stall. However, these findings were later cited as experimental evidence of the interface observed in Vo’s study [34].

More recently, Bennington et al. [2] used a real-time streaking technique to visualize the interface location in a low-speed fan and a high-speed compressor operating at part speed. A circumferential line marking the axial location where the axial component of wall shear stress was zero was observed in the casing visualization. Bennington determined that the line was an accurate representation of the time-averaged location of the interface described by Vo. Furthermore, the axial location of the line was shown to be a function of flow coefficient and was consistently found to be very near the rotor leading edge plane at near-stall conditions.

1.3 Casing Treatments

Casing treatments were first found to favorably alter the stall point of axial compressors in the late 1960s. This discovery led to further investigation by NASA which established many of the casing treatment configurations that are studied currently. Early NASA experiments were completed in a transonic facility (e.g., [1, 24, 25]). However, later experiments showed that casing treatments positively impacted stall margin in low speed compressors as well as transonic (e.g., [26]). Therefore, low speed experiments account for a majority of early works since low speed testing eliminated both the complications associated with shocks and the limitations of facility requirements to operate a transonic compressor.

1.3.1 Early Work

Since the discovery of casing treatment as passive method for controlling stall inception, several novel designs have been produced and investigated. Two of the more successful designs examined are the axial skewed slot casing treatment and
circumferential groove casing treatment. Schematics of these two casing treatments are shown in Figure 1.3.

Osborn et al. [25] sized the open area of slot type casing treatments so that they were acoustically tuned to the blade passing frequency at the desired flow conditions. The open area ratio of the circumferential groove was then matched to that of the slot cases. While axial skewed slots were seen to be the most effective in increasing stall margin, circumferential grooves matched or improved upon smooth wall efficiency in a transonic rig [25, 26]. However, the mechanisms which allowed each of these methods to be successful were not clear.

Takata and Tsukuda [32] and later Fujita and Takata [15] completed several experiments which examined the effectiveness of various casing treatment geometries. These investigations mostly focused on the pressure rise characteristic associated
with each casing treatment. The studies investigated five different casing treatments types with various geometries. Among axial slots, blade angle slots, circumferential grooves, reverse axial skewed slots and axial skewed slots, axial skewed slots were appeared to have greatest effect on surge margin and resulting characteristics exhibited higher peak pressure rises. This is consistent with the earlier NASA studies. It is of interest to note that while axial skewed slots increased surge margin, reverse axial skewed slots caused the compressor to stall at higher flow rates.

Takata and Tsukuda [32] utilized hot wires to investigate the flow field within an axial skewed slot. They observed a jet like flow originating out of the slots at leading edge and middle of the slot and inflow at the slot trailing edge. This process was understood to be driven by the pressure difference across the slot created by the rotor. The study concluded that the increase in surge margin due to axial skewed slot was attributable to the jet located at the leading edge of the slot. In fact, the study suggests that the jet facilitated improvement of the surge margin by means of the momentum interchange at the rotor tip.

Fujita and Takata [15] investigated the effectiveness of three different casing treatment types (axial skewed slot, circumferential groove and axial slot) with a total of 30 different changes in individual geometry. Efficiency and stall margin extension associated with each casing treatment were measured. The study showed a roughly linear trend in efficiency penalty with surge margin extension. This result is reproduced in Figure 1.4. While this is a significant conclusion, the study did not provide physical insight into this trend.

Greitzer et al. [17] examined the effectiveness of casing treatments for rotors which stalled at the hub and the tip by varying the blade solidity (and, subsequently, the D-factor) of a single compressor rotor and observing the effect of a casing treatment. Pressure rise characteristics were measured for both a smooth wall case and a axial skewed slot casing treatment for both solidities. The investigation found that the
Figure 1.4. Maximum Efficiency as a Function of Stall Margin Extension Due to Casing Treatment[15]

high solidity case had a greater response to the casing treatment compared to the low solidity case. Greitzer concluded that casing treatments would only positively impact the stall margin for compressors where stalled initiated at the blade tip.

Smith and Cumpsty [30] examined the effects of a axial skewed slot casing treatment on a low-speed compressor via measurements behind the rotor, in the rotor passages, as well as in the treatment slots. They observed blockage at the tip and hub in the smooth wall case with probes downstream of the rotor. There was no evidence of this tip blockage for the treated case at the same flow coefficient. The study suggests that the alleviation of this blockage is responsible for the increase in stall margin. Relative dynamic head contours inside the rotor passages show contour lines near parallel with the leading edge of the compressor blade close to stall in the untreated case. In the treated case, at the same flow coefficient, these contour
lines are within the blade passage. Additionally, Smith and Cumpsty observed flow entering the treatment slot at the slot trailing edge and ejected near the slot’s leading edge, which is consistent with Takata and Tsukuda’s 1977 findings. The fluid injection from the casing slots influenced the rotor passage flow up to 30% radially inward from the blade tip. However, the impact of this radial flow disruption was not examined in relation to efficiency. Finally, this study also investigated the effect of casing treatments with varied tip gap. The axial skewed slot they investigated extended the stall margin at least 20% for tip gaps $1\% < (\tau/C) < 6\%$. No appreciable Reynolds number dependence for stall margin increase was observed.

1.3.2 Circumferential Grooves

Recently, several investigations have utilized computational fluid dynamics to study the complex flow physics associated with the tip clearance flow and the effect of circumferential grooves on this region.

Wilke and Kau [35] utilized steady RANS to study the effect of casing treatments on the tip clearance vortex. They contend that the tip leakage flow is driven by the pressure differences over the blade and impinges on the surface of the next blade which in turn causes blockage and stall. They investigated both axial slots and circumferential grooves. Wilke and Kau observed that both types of casing treatment affect the tip clearance flow and the ability of the tip clearance vortex to form. Circumferential grooves cause recirculation, sending tip clearance flow downstream. According to Wilke and Kau, this recirculation eliminates a driving force for the vortex roll up by removing any perpendicular (to the blade) velocity component, reducing rotational intensity. Wilke and Kau investigated two circumferential groove configurations, one with a “small” number of grooves (4) and a “large” number of grooves (11). In the 11 groove case, the tip leakage fluid that entered the grooves “jumped” back upstream instead of recirculating downstream. The study showed that
the grooves at the trailing edge for both groove counts did not affect the tip vortex generation and did nothing to aid in stall margin extension. This will be shown to be in agreement with further investigations which suggest that circumferential grooves should only be added to specific portions of the rotor casing in order to extend the stall margin without unnecessarily decreasing the rotor efficiency.

Rabe and Hall [27] numerically investigated the effects of circumferential grooves on a transonic rotor. This work is unique as this is one of the few computational efforts utilizing circumferential grooves in a high-speed compressor. The computational work was compared to experimental data to ensure the accuracy of the computational model. In all cases, the experimental stall margin extension was smaller than observed in simulation. Three different groove configurations were tested in this study. The first consisted of five grooves which were placed over 15 to 55% of blade chord with a depth of 20.25 mm (22.75τ). The same number and placement of grooves was tested with a shallow depth of 1.25 mm (1.4τ). Finally, the number of shallow grooves was reduced to the two most upstream grooves of the five groove configuration. All three were tested at 0.83N due to the large changes in stall margin seen at that shaft speed in previous experimental efforts. In both experimental and numerical cases the two shallow grooves produced nearly the same stall margin extension as both five groove configurations with a smaller efficiency penalty. Furthermore, Rabe observed additional radial and tangential flow in the casing treatment studies. Rabe concludes that these flows explained the efficiency penalty for all three cases. This study also showed tip clearance velocity fields consistent with Vo’s hypothesis.

Shabbir and Adamczyk [29] investigated the flow mechanisms which cause casing treatments to extend stall margin by combining steady RANS calculations for a low speed axial compressor with a simplified control volume analysis. Shabbir and Adamczyk primarily studied a five groove treatment with the first groove extending from 10% of axial chord to slightly over 20% and the five grooves extending over
90% of the blade axial chord. The groove depths were altered in order to vary the aspect ratio of the groove cross section. As in Rabe’s 2002 study, varying the groove aspect ratio did not appear to alter the stall margin but did slightly decrease the pressure rise of the rotor close to stall. Through the control volume momentum budget approach, Shabbir concludes that it is the radial transport of axial momentum due to the presence of the groove that leads to stall margin extension.

Shabbir observed that the net increase of axial force via radial transport of axial momentum decreased with groove distance from the rotor leading edge plane. Again, the it is clear from this study that the placement of a circumferential groove is important to its effect on the stall margin. Shabbir attributes this to groove location relative to the rotor blade’s center of pressure but does not outline a relationship between the two.

Recent work by Houghton and Day [13] combined experiments and computations in an investigation on the effect of a single circumferential groove on the performance and stall margin of a low-speed axial compressor. In this study, a single circumferential groove was installed at a number of axial conditions and the resulting measurements were compared to smooth wall results. Stall margin improvement was found to be greatest for groove locations of 8% and 50% of axial chord. However, the groove at 50% axial chord resulted in little to no efficiency loss unlike the 8% placement. The effects of the groove placement on the flow field was examined computationally and corroborated with experimental results. A groove placement at 8% of axial chord interacted with the tip clearance flow, decreasing the reverse flow magnitude at the tip, while also affecting the blade loading near the groove. While this was beneficial for stall margin improvement, the “strong interaction” between the near-casing flow field and casing groove resulted in larger quantities of entropy than compared to the smooth wall case. The groove placement at 50% chord was able improved stall margin with minimal loss and impact on the blade loading. This
study offers insight into the loss mechanisms associated with implementing a circumferential groove casing treatment, but offers no reasoning on why the maximum stall margin improvement is located at the aforementioned axial locations.

1.4 Summary and Research Objectives

These studies and others clearly demonstrate that the rotor tip clearance region fluid mechanics are fundamental to the understanding of spike-type stall inception. Significant efforts have been made over the last several decades in order to provide a more complete understanding of this complex, three-dimensional, unsteady phenomenon. Several studies have demonstrated that casing treatments alter the tip clearance flow such that stall margin is increased. Early stall margin extension research established that casing treatments are effective means of stall margin increase in both subsonic and transonic compressors. Since, several casing treatment geometries have been investigated experimentally and numerically. Circumferential groove casing treatments have become of particular interest due to their comparatively simple geometry and often equal or better stall margin increase. It has been shown that circumferential groove casing treatments’ effectiveness and efficiency impact depends largely on the geometry of the treatment as well as its placement with respect to the compressor rotor.

This dissertation has the following objectives:

1. To identify and describe the fluid dynamic mechanism that governs the generation of short length scale disturbances that lead to spike-type stall inception,

2. To further explore the relationship between surge margin extension due to a circumferential groove and the tip clearance fluid mechanics and establish a functional relationship between the two, and

3. To determine the relationship between circumferential groove casing treatment geometry and surge margin extension.

Chapter 2 will describe the experimental and numerical methods utilized in these
investigations. The first objective will be addressed in Chapter 3. With Vo’s findings and Bennington’s methods as a starting point, the location of the approach flow/tip leakage flow interface is observed, modeled, and compared to experimental compressor performance and stall inception measurements. The second and third objectives are considered in Chapter 4. Through a combination of experimental and numerical efforts, this chapter describes the individual contributions of grooves in a multi-groove casing to overall surge margin extension. A relationship between a compressor’s smooth wall tip clearance fluid mechanics and the surge margin extension due to a groove is also established and addresses the final objective through a parametric experimental study of several circumferential groove casing treatment geometries. The dissertation concludes in Chapter 5 with an overall summary and recommendations for future work.
CHAPTER 2

EXPERIMENTAL AND NUMERICAL METHODS

2.1 Facility and ND Stage 03/04

Experimental measurements were acquired in Transonic Axial Compressor Facility at the University of Notre Dame (ND-TAC). The facility was designed to be a modular experimental facility for single-stage, high-speed compressors [4]. A one-dimensional schematic and scale drawing of the facility is shown in Figure 2.1. The components of interest are identified with a capital letter for reference.

Figure 2.1. Schematic and Layout of ND-TAC
Air is drawn into the facility through a large outdoor inlet box (Figure 2.1, location A). Flow passes through a venturi flow meter. At location B the air is throttled to sub-atmospheric total pressure by an electromechanically actuated butterfly valve. After the upstream throttle, the flow is decelerated to low velocity in the upstream plenum (location C). The flow in the plenum is conditioned by adjustable screens and accelerates through the compressor inlet duct. The default setup includes a single perforated plate for turbulence generation. The plate is 16 gauge steel with a porosity of 63% placed 20.32 cm (8 in) upstream of the rotor leading edge. The perforated plate has a pressure drop equal to 1.5 times the dynamic pressure in the inlet annulus and generates a turbulence intensity of 3% at the rotor inlet. After the turbulence screen, the flow enters the test article (location D) where the total pressure is raised to slightly above atmospheric pressure. The compressor exhausts into a small receiving plenum which directs the flow through a second throttle (location E) and then to the outdoor outlet of the facility (location F).

The test article rotor shaft of the ND-TAC is levitated by a five-axis magnetic bearing system and controller. The radial bearings each have a static load capacity of 2.2 kN (500 lbf) and each thrust bearing can produce a static force of 5.8 kN (1300 lbf). The nominal air gap of the magnetic bearings is 508 \( \mu \text{m} \) (0.020 in) radially and 889 \( \mu \text{m} \) (0.035 in) axially. The design gap between the auxiliary bearings and the landing sleeves on the rotor shaft are 254 \( \mu \text{m} \) (0.010 in) radially and 508 \( \mu \text{m} \) (0.020 in) axially. Thus, static rotor movement of approximately 178 \( \mu \text{m} \) (\( \sim 0.007 \) in) radially and 381 \( \mu \text{m} \) (\( \sim 0.015 \) in) axially are possible during operation. The system controller permits robust control of rotor whirl and rotor shaft centerline location during compressor operation. These capabilities were utilized to maintain rotor whirl to less than 7 \( \mu \text{m} \) in all experiments and can be used to statically offset the rotor shaft to produce an asymmetric tip clearance.

The focus test article for this dissertation is ND Stage 03/04, a transonic axial
compressor stage which can be operated with a counter-swirling IGV (Stage 03) and without (Stage 04). The same rotor design is used for both stage configurations. The rotor has variable forward sweep, which has been shown to improve stall margin, efficiency, and clearance sensitivity in conventional compressor rotors [23]. Relevant geometry and flow parameters are summarized in Table 2.1 and a meridional cross section of the Stage 04 configuration is shown as Figure 2.2.

TABLE 2.1

ND STAGE 03/04 DESIGN PARAMETERS

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value</th>
<th>Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tip Dia. [cm]</td>
<td>45.72</td>
<td>Hub Dia. Inlet [cm]</td>
<td>34.29</td>
</tr>
<tr>
<td>$N_c$ [RPM]</td>
<td>14684</td>
<td>Hub Dia. Exit [cm]</td>
<td>36.83</td>
</tr>
<tr>
<td>$\sigma_i$</td>
<td>0.31</td>
<td>$n_i$</td>
<td>17</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>1.21</td>
<td>$n_r$</td>
<td>20</td>
</tr>
<tr>
<td>$\sigma_s$</td>
<td>1.49</td>
<td>$n_s$</td>
<td>43</td>
</tr>
<tr>
<td>$C_{ax}$ [cm]</td>
<td>3.556</td>
<td>$\tau$ [mm]</td>
<td>0.762</td>
</tr>
<tr>
<td>$U_{tip}$ [m/s]</td>
<td>352</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Rotor only total pressure ratio characteristics for Stage 03 and Stage 04 with a smooth casing and uniform inlet at 70%, 90%, and 100% corrected shaft speed are shown in Figure 2.3. The design point for each stage is the 100% $N_c$ operating point that yields approximately 13% surge margin when the smooth wall casing is installed.
Stage 03 has a relatively flat characteristic at 70% corrected shaft speed and displays choked behavior at higher mass flows on the 100% characteristic. At 90% and 100% corrected shaft speeds, the slope of the characteristic is close to zero at mass flows near the stalling mass flow. The design point for Stage 03 with a uniform inlet condition is a corrected mass flow of 22.60 lbm/sec for a rotor only total pressure ratio of 1.47 and a defined stall margin of 13.9%.

At 70% corrected shaft speed, Stage 04 shows a typical characteristic curve for a subsonic compressor with a stalling mass flow of approximately 12 lbm/sec, and a relatively “flat” performance curve up to the higher mass flow values. At 100% corrected shaft speed, the compressor produces a peak pressure ratio of approximately 1.5 at a mass flow very near to the stalling mass flow. Furthermore, the 100% characteristic also exhibits choked behavior at higher mass flow as indicated by the steep slope at a mass flow of approximately 22 lbm/sec. The design point for Stage 04 with a uniform inlet condition is a corrected mass flow of 21.31 lbm/sec for a rotor
Figure 2.3. ND-Stage 03/04 Rotor Only Performance Curves

only total pressure ratio of 1.46.

The stall inception behavior of ND Stage 03/04 was studied by Cameron et al. [3, 5] using casing-mounted Kulite pressure transducers located 0.1\(C_{ax}\) upstream of the rotor leading edge plane. These data demonstrated that stall in this stage is initiated by disturbances of small circumferential extent which rotate at approximately 0.7\(N\). Examination of these data with spatial Fourier mode amplitude and Traveling Wave Energy methods [33] did not indicate any measurable large length scale pre-stall activity. Furthermore, the slope of the total-to-static pressure characteristic for both configurations is negative at the onset of stall. These observations suggest that ND Stage 03/04 stalls via spike-type inception as described by Day [12].

2.2 Optical Measurements and Flow Visualization

A smooth wall casing with a polished Plexiglas insert serving as a segment of the casing wall over the rotor was available for experiments requiring optical access. The frame for the optical window is integrated into the rotor casing and the window was
toleranced to ensure that the concave window surface is concentric with the rotor casing. This casing has a viewing area that is $25^\circ$ of the circumferential extent. A schematic of the optical setup is shown in Figure 2.4.

![Figure 2.4. Schematic of Flow Visualization Setup with Optical Window in Rotor Casing](image)

Small droplets of streaking fluid were injected through a 2.54 mm (0.100 in.) diameter hole in the rotor casing at an axial position $0.1C_{ax}$ downstream of the rotor leading edge plane. The surface streaking pattern and rotor blade tip were illuminated by diffuse laser light. A 2048 by 2048 pixel CCD camera captured real-time images of the endwall surface streaking pattern. The uncertainty in the location of the region of zero axial shear (discussed in Chapter 3) was estimated to be 100 $\mu$m. The reported axial location of the region of zero axial shear is the average location of 50 realizations at each operating point.
2.3 High Frequency Pressure Measurements

Fluctuations in static pressure at the rotor casing were acquired for a number of test cases to examine the stall inception behavior of ND Stage 04. In order to investigate blade-to-blade pressure fluctuations, multiple data points per blade passage must be acquired. Thus, for these studies, 16 high frequency response Kulite pressure transducers were installed flush mounted 0.10\(C\) upstream of the rotor leading edge. The sensors were equally-spaced around the circumference of the annulus. All signals were acquired simultaneously at a sampling frequency of 50kHz in order to accurately trace fluctuations in static pressure at each location around the annulus. These data were analyzed to examine the stall inception behavior of all stall inception events for each test configuration using the same methods outlined by Cameron et al. [3].

2.4 Casing Treatments

Fifteen circumferential groove casing treatment geometries were tested in the experiments described in the subsequent chapters. All tested casings were centered on the axial length of the rotor blades, \(C_{ax}\), and the distance from the leading edge of the most upstream groove to the trailing edge of the most downstream groove was 80\% \(C_{ax}\) in axial extent.

The 3-groove casing, VCT3, is modular and permits the construction of seven casing configurations with at least one “open” groove. For example, the VCT3 casing can be constructed with all three grooves open, the two most upstream grooves open (and the most downstream closed), or only the most upstream groove open (and the two downstream grooves closed). Geometry details for the VCT are summarized in Figure 2.5. Nomenclature for the different constructions are discussed in detail in Chapter 4.

Additionally, four 7-groove casings and four 4-groove casings were tested. The
designations for these casings and the aspect ratios of their respective grooves are summarized in Table 2.2. In both Figure 2.5 and Table 2.2, aspect ratio for a particular casing is defined as the ratio of a single groove’s depth to single groove’s axial length.

![Diagram](image_url)
2.5 Operating Point Data Acquisition and Uncertainty

Compressor operating point was determined from measurements of mass flow, rotor torque, total temperature, and static and total pressure. Temperature and pressure instrumentation were placed at several axial, radial, and circumferential locations. There are four primary axial instrumentation locations: Station 1 is upstream of the stage, Station 2 is just upstream of the rotor leading edge plane, Station 3 downstream of the compressor rotor and upstream of the stator and Station 4 downstream of the entire stage. A scale schematic of these locations is shown as Figure 2.6. The outlet box of the facility, downstream of the test article and upstream of the outlet throttle shown as E in Figure 2.1 is considered Station 5.

The compressor mass flow is measured by the facility flow meter at the location shown in Figure 2.1. Radial sweeps of total pressure at Station 1 were used to
calibrate the flow meter’s discharge coefficient which was found to be 0.9450 at the maximum facility shaft speed. Reported values of mass flow are corrected to Standard Day conditions. Corrected mass flow is defined as

\[ \dot{m}_c = \dot{m} \sqrt{\frac{\theta}{\delta}}, \]  

(2.1)

where

\[ \delta = \frac{P_{t_1}}{P_{std}} \]  

(2.2)

\[ \theta = \frac{T_{t_1}}{T_{std}} \]  

(2.3)

and where overbars indicate mass-averaged quantities.

Rotor torque is measured in some experiments via a torquemeter affixed to the rotor shaft. The torquemeter operates using shaft strain measurements powered through a RF telemetry system. The torquemeter range is +/- 169 N-m (125 ft-lbf) scaled to +/- 10 volt DC output. The RMS error from the calibration was 0.095 (0.07 ft-lbf). Windage losses were estimated by spinning the shaft without the rotor and were found to be up to 2.24 kW (3 Hp) at maximum facility shaft speed.

Total temperature profiles were measured with type K thermocouples installed on the multi-element Kiel rakes at Stations 1 and 4 for all test cases. Most test cases include total temperature measurements at Station 5.

Static pressure is measured at Stations 1 and 4 at six equally-spaced circumferential locations at both axial stations.

Stage inlet total pressure profiles were measured with multi-element radial Kiel-probe rakes at two circumferential locations spaced 60° apart. Each radial probe location on these rakes were at the center of a ring of equal area. The rake total
pressure measurements were supplemented by single immersion Kiel probes at four equally-spaced circumferential locations at the mean line of the passage in order to measure any circumferential variations of total pressure. Stage exit (Station 4) total pressure information is obtained from a similar instrumentation layout.

The instrumentation at Station 1 does not completely capture the incoming boundary layers. Measurements from the facility flow meter are used find the true mass-averaged stage inlet total pressure. Mass-averaged total pressure is the sum of the static pressure and mass averaged dynamic pressure,

\[ P_t = P_s + P_{dyn}. \] (2.4)

The mass-averaged dynamic pressure was calculated as:

\[ P_{dyn} = \frac{1}{2} \rho \left( \frac{\dot{m}}{\rho A} \right)^2. \] (2.5)

The static pressure and temperature measurement at Station 1 was used with the ideal gas law to obtain the density and the physical mass flow rate is given by the facility flow meter. The resulting repeatability for the corrected mass flow rate and the total pressure ratio was approximately 0.5% .

Rotor exit total pressure profiles were measured via two multi-element stator leading edge rakes located on opposite sides of the test article. These rakes’ elements were also Kiel type probes installed at radii at the center of a rings of equal area. The Kiel probes at all axial measurement stations had similar insensitivity to inlet flow angle, +/-30 degrees, as the Kiel probes on multi-element rakes in wind tunnel tests.

Adiabatic efficiency was calculated from rotor only total pressure ratio and total temperature ratio,

\[ \eta = \frac{\pi_{31}^{\gamma-1}/\gamma - 1}{\tau_{51} - 1}, \] (2.6)
or, when available, from rotor only total pressure ratio and compressor shaft power (derived from the rotor torque measurement),

\[
\eta = \frac{\tau_{31}^{\gamma - 1} / \tau - 1}{\dot{m}_c \dot{p}_{1}} \tag{2.7}
\]

The stall margin values reported in the document are as given by Cumpsty [10, chap. 9]:

\[
SM = 1 - \left( \frac{\dot{m}_{c,\text{surge}} \Pi_{\text{design}}}{\dot{m}_{c,\text{design}} \Pi_{\text{surge}}} \right). \tag{2.8}
\]

The uncertainty in total pressure ratio, total temperature ratio, adiabatic efficiency, and mass flow are listed in Table 2.3.

<table>
<thead>
<tr>
<th>TABLE 2.3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>UNCERTAINTY OF OPERATING POINT DATA</strong></td>
</tr>
<tr>
<td>Quantity</td>
</tr>
<tr>
<td>( \pi )</td>
</tr>
<tr>
<td>( \tau )</td>
</tr>
<tr>
<td>( \eta )</td>
</tr>
<tr>
<td>( \dot{m}_c )</td>
</tr>
<tr>
<td>( SM )</td>
</tr>
</tbody>
</table>
2.6 Tip Clearance Calibration and Measurement

Capacitance tip clearance sensors were utilized to dynamically measure the tip gap for all cases studied. These measurements were used in conjunction with shaft whirl readouts to accurately measure the location of the rotor and quantify the tip clearance. The values from this sensor were referenced to static (non-rotating) tip gaps manually measured with shims of specific width at various circumferential locations to ensure both the accuracy of the single point dynamic measurements and concentricity of the casing.

2.7 Simulation and Validation

Simulations of a single rotor pitch of ND Stage 04 at 100% corrected shaft speed were obtained via collaboration with Dr. Haixin Chen of Tsinghua University and Dr. Juan Du of The Chinese Academy of Sciences. This section briefly reviews both simulation techniques and demonstrates validation of the two numerical methods with experimental data.

2.7.1 Tsinghua University Simulation

The following summarizes the details of a CFD simulation of a single rotor passage and tip clearance gap of ND Stage 04 conducted with the NSAWET (Navier-Stokes Analysis based on Window-Embedment Technology) code developed at Tsinghua University [7].

The inlet of the computational domain is the Station 1 axial location depicted in Figure 2.6. The measured inlet total pressure profile is applied as the inlet boundary condition. Similarly, the exit of the computational domain is at the axial location of Station 4 in Figure 2.6. The radial momentum balance equation is used to obtain the exit static pressure distribution there.
Rotor only and stage performance is then computed by interpolating the simulation data to the Station 3 and 4 axial location. The interpolated values are circumferentially averaged and then interpolated to the spanwise location of the experimental probe measurements. Pressure ratios derived from the simulation are calculated from these computational “probe” values using a processing method identical to those used to process the experimental data.

Simulated performance curves (solid lines) for ND Stage 04 with a smooth wall casing and the CT7B circumferential grooves are compared to the experimentally measured performance curves (solid circles) in Figure 2.7. There is adequate agreement between the two on operation range of mass flows and stalling mass flow and pressure rise.

Figure 2.7. Tsinghua Performance Validation
2.7.2 Chinese Academy of Sciences Simulation

The following details the simulation of a single rotor passage and tip clearance gap of ND Stage 04 by solving the steady and unsteady three-dimensional, Reynolds-averaged Navier-Stokes equations with FLUENT, a commercial solver package. The solver is a three-dimensional, time-accurate code with an implicit second-order scheme. Turbulence was simulated by using a standard $k-\epsilon$ turbulence model and standard wall function.

The mesh for the rotor only simulation was generated by ANSYS TURBOGRID. The computational domain and grid resolution of one blade passage, including enlarged views of the grid near the blade tip leading edge (LE) and trailing edge (TE), are shown in Figure 2.8.

Figure 2.8. Chinese Academy of Sciences Simulation Grid
Mesh studies were conducted for the steady simulations by increasing and decreasing the mesh size in order to ensure the computational results were independent of grid distribution. The grid used in this work was a compromise between accuracy and computational demands and had approximately 0.92 million elements. The grid was a J-mesh with 91 nodes streamwise along the blade chord and 79 nodes in the spanwise direction including 20 nodes in the tip clearance region. The static pressure was specified with radial equilibrium at the outlet. No-slip and adiabatic conditions were imposed on all solid walls. For the time-accurate simulations, 40 physical time steps per blade passage and 20 inner iterations for each time step were calculated.

The simulated and measured rotor only performance curves are plotted in Figure 2.9 as a function of flow coefficient. Operating points A and B indicated in the figure were selected for detailed analysis in Chapter 3.

Figure 2.9. Chinese Academy of Sciences Validation
The simulation results were obtained by averaging the solutions at the same spatial locations as the experimental sensors. The mass flow rate at both choke and stall conditions agreed reasonably well between the experiments and computations. The predicted pressure ratio was also close to the experimentally observed values at both low and high mass flow but were under-predicted through the intermediate mass flow rate values. It was determined that this was likely related to an overly rapid mixing of the wake in the numerical simulations. Numerical values obtained closer to the trailing edge of the blade did show improved agreement with the measurements. No “tuning” was attempted since these details did not affect the momentum distribution in the tip region.
CHAPTER 3

COMPRESSOR STALL AND MODELING THE TIP CLEARANCE
MOMENTUM BALANCE

3.1 Overview

The present work employed experimental and numerical methods to examine tip-leakage flow and its relation to stall in a transonic axial compressor. The high-speed compressor experiments included performance, unsteady casing pressure measurements, and real-time casing streaking visualization. The independent variables considered included the mass flow rate, tip-gap size, as well as varied inlet velocity profiles. In combination, these data will show that there is a deterministic link between the interface location, spike generation, and stall inception. Analysis of the measurements and numerically generated data will further demonstrate that the axial location of the approach-flow/tip-leakage interface is governed by a momentum balance in the tip clearance region. It will be shown that the spillage of the interface and, therefore, stall initiated by spike-type disturbances is a critical point in this balance. The section closes with a discussion of the results and conclusions.

3.2 Tip Clearance Flow

Three-dimensional views of the tip clearance flow at operating points A and B are shown in Figure 3.1. Entropy contours are shown at multiple slices through the computational domain. The slices are parallel to the radial and axial directions and
the radial extent of the contours represents the casing radius. Thin black lines on several of these slices show the location of the blade tip.

Figure 3.1. Axial-radial slices of entropy

The contours for the higher mass flow (operating point A) show entropy that is consistent with the typical understanding of a tip clearance flow at low loading conditions. Very little leakage fluid is observed over the first 50% of rotor chord. The majority of the fluid that forms the leakage vortex originates from the aft part of the blade. The interface between the leakage fluid and the approach fluid is nearly parallel with the circumferential direction over most of the pitch. The leakage flow intersects the pressure side of the adjacent blade at approximately 50% of the blade chord.

The contours show a significant increase in both the pitchwise and the spanwise extent of the tip-leakage fluid at the lower mass flow rate (operating point B).
interface between the approach flow and the leakage flow is oriented nearly parallel to the circumferential direction. The axial location of the interface is observed to be close to the leading edge plane. The intersection of the interface with the pressure side of the following blade for operating point B was found to be at about 10% of the chord from the leading edge. Note that the leakage fluid at this point originated from the lower blade at about 60% of blade chord. This is approximately what would be found if it were assumed that the leakage flow direction was normal to the blade camber-line in the relative frame of reference.

Figure 3.2 shows contours of entropy and axial shear stress at 99% of span for operating points A and B.

The entropy contours for operating point A show the leading edge shock upstream of the blade row and evidence of the passage shock interacting with the leakage flow where the maximum entropy was observed. The entropy contours for operating point B show similar features, but the intersection of the leakage fluid with the passage shock occurs closer to the leading edge plane as shown in Figure 3.1. Contours of axial shear stress at the casing are also shown in Figure 3.2. These results show that negative axial shear stress is present over a significant portion of the passage, consistent with the regions of elevated entropy. That is, the tip-leakage flow has both increased entropy and reverse axial velocity. More importantly, the interface between the tip-leakage fluid and the approach fluid, as marked by entropy, is similarly marked by a line of zero axial shear stress at the casing. This observation allows for a simple experimental evaluation of the time-averaged interface location based on shear-stress visualization.
Figure 3.2. Entropy and axial shear contours for operating points A and B
3.3 Time-Averaged Location of Approach Flow/Leakage Flow Interface

Visualization of surface streaking on the casing over the compressor rotor was obtained. An example realization is shown in Figure 3.3(a). The abscissa represents the axial direction with the origin placed at the rotor leading edge plane. The streaking pattern shows three distinctive regions as in Saathoff and Stark [28]. The flow upstream of the rotor was observed to have a positive axial wall shear stress consistent with the direction of the approach flow. The region over the rotor was characterized by a circumferential component of wall shear that was in the direction of rotor rotation and an axial component that was in the negative axial direction. A third region downstream of the blade row displayed positive axial wall shear with a swirl component consistent with the design stator incidence angle.

![Figure 3.3](image)

(a) Experiment  
(b) Simulation

Figure 3.3. Comparison of a surface streaking realization with numerically constructed casing surface streamlines. Inflow is left to right. The rotor leading edge plane is at $x/C_{ax} = 0$ and the rotor trailing edge plane is at $x/C_{ax} = 1$. 

The axial location of the approach fluid/reverse tip-leakage flow interface was taken to be the where marking fluid accumulated on the casing due to zero axial shear stress. This location is denoted as \( x_{zs} \). The image shown in Figure 3.3(a) was obtained at a flow coefficient of \( \Phi = 0.291 \) which was the lowest flow coefficient obtained without stalling the compressor. The value of \( x_{zs} \) obtained from the Figure 3.3(a) was 0.36mm, or \( x_{zs}/C_{ax} = 0.01 \). The location of the interface was consistently located a fraction of a millimeter downstream of the rotor leading edge plane when the compressor operating point was set very close to (but greater than) the stalling mass flow. When flow coefficient was reduced from this point, the compressor immediately entered rotating stall. Simultaneously, the \( x_{zs} \) line rapidly moved upstream of the rotor leading edge plane and out of the camera’s field-of-view. This movement of the streaking fluid cannot be easily studied in a quantitative way since this involved unsteady, two-phase, surface-flow phenomena. However, it was qualitatively clear that stall inception was accompanied by a large reverse flow region at the blade tip that extended far upstream of the rotor leading edge plane. This large reverse flow region has also been observed numerically (e.g. Figure 6 of Chen et al. [8]).

A circumferential average of the wall shear stress was computed from the single-passage simulation data. This information was used to simulate the casing streakline patterns. The numerically constructed shear stress lines shown in Figure 3.3(b) are very similar to the time-averaged pattern observed in the laboratory. The three regions are observed, and the interfaces between them are found at nearly the same axial location as in the experiment. Furthermore, the computed angles of the shear stress vectors over the rotor agree very well with the streaking pattern. These results are evidence that the magnitude and direction of the tip-leakage flow was computed accurately.

Figure 3.4 shows the computed pitch-averaged axial shear stress as a function of the streamwise coordinate for a selection of flow coefficients.
Figure 3.4. Pitch-averaged axial shear stress as a function of axial location from CFD simulation. Positive axial shear stress is in the through-flow direction. The rotor leading edge plane is located at $x/C_{ax} = 0$.

At the highest mass flow rate, the pitch-averaged shear stress increased to a local maximum at approximately 15% of axial chord and falls to zero at 42% of axial chord. The axial location where the curve crosses through zero represents the computed value of $x_{zs}$. After this point, the pitch-averaged shear stress reached a local minimum and increased over the aft portion of the rotor. The region where the shear stress was negative represents the time-averaged region where the tip leakage fluid was moving in the negative axial direction. As the mass flow rate was reduced, the $x_{zs}$ value decreased towards zero (the rotor leading edge plane) and the streamwise extent of the reverse flow region grew. The $x_{zs}$ location was nearly aligned with the rotor leading edge plane at a flow coefficient of $\Phi = 0.291$. When the pressure boundary conditions were set to lower the mass flow further, the numerical solution quickly diverged. This divergence is often used as a method of predicting compressor stall and was used originally by Vo et al. [34] in their analysis of the entropy interface location and stall.
The $x_{zs}$ values compiled from both the experiments (i.e. Figure 3.3(a)) and the CFD solutions (i.e. Figure 3.4) are shown in Figure 3.5 as a function of flow coefficient. Both the CFD and experimental measurements of $x_{zs}$ show very good agreement overall, with a monotonic movement of the interface location towards the leading edge plane as the flow coefficient is reduced. Both the CFD and the experiment show a value of $x_{zs}$ very close to zero at the stalling flow coefficient.

![Figure 3.5. Axial location of the region of zero axial shear ($x_{zs}$) as a function of $\Phi$.](image)

During the interrogation of the visualization images it was noted that observations near $\Phi \approx 0.315$ were found to have a small region of finite axial extent where the axial shear stress appeared close to zero. This axial extent is shown as “error bars” on the experimental values in Figure 3.5. Interestingly, the CFD results shown in Figure 3.4
also indicate an extended region where the axial shear stress was very close to zero similar flow coefficients. The qualitative and quantitative agreement shown in Figure 3.5 suggests that the fluid mechanics of the tip leakage flow were accurately modeled with the CFD, and thus the results presented in Figures 3.1 and 3.2 are reasonable representations of the actual tip leakage flow.

3.4 Axial Momentum Based Model for the Location of Zero Axial Shear

The numerical and experimental results suggest that the location of the \(x_{zs}\) line at a given flow coefficient depends on a momentum flux balance between the approach fluid at the casing and the axial reverse tip-leakage flow. As the flow coefficient is decreased, the momentum of the approach fluid at the tip decreases. Flow coefficient reduction also increases incidence to the rotor, resulting in increased loading at the blade tip and therefore increased axial reverse tip-leakage flow momentum. The resulting balance moves the line of zero axial shear closer to the rotor leading edge plane. A model for the \(x_{zs}\) location can be derived based on this concept.

A wall-bounded free-stream flow interacting with a counter-flow wall jet is shown in Figure 3.6. We will consider this model a simplified analog of the compressor tip-gap flow. The free-stream has velocity \(U_o\), and the jet has a height \(h\) and velocity \(U_j\). The counter-flow wall jet represents the tip-leakage flow and the free-stream represents the approach flow. It is clear that a wall separation will occur in this model flow at the interface between the free-stream flow and the wall jet flow. The separation streamline is represented by a darkened line in Figure 3.6. The distance between the wall jet exit and the separation point is \(l_o\). The distance \(l_o\) is taken to be the characteristic length scale of the control volume drawn in Figure 3.6.
The drag force per unit depth that is exerted on this control volume due to the approach flow will be proportional to $\rho U_o^2 l_o$. This drag is balanced by the reverse flow momentum flux from the wall jet, $\rho U_j^2 h$. We assume $h << l_o$, and thus the momentum flux leaving the control volume is small. Thus,

$$F'_x \propto \rho U_o^2 l_o \propto \rho U_j^2 h,$$

which simplifies to:

$$\frac{l_o}{h} \propto \left( \frac{U_j}{U_o} \right)^2.$$

The variables in Equation 3.2 are now related to the flow in the tip region of the axial compressor. The free-stream velocity, $U_o$, is taken to be the approach flow velocity near the casing. The jet velocity, $U_j$, can be given in terms of the pressure

Figure 3.6. Schematic of free-stream and counter-flow wall jet interaction.
difference across the blade which drives the gap flow. Assuming that the acceleration of the fluid into the tip-leakage jet is isentropic, an average of the axial component of the tip-leakage velocity can be approximated by:

\[
U_j^2 = \frac{2}{\gamma - 1} \left[ \frac{P_{t3}}{P_1} (\Phi, \tau)^{\gamma-1/\gamma} - 1 \right] \gamma RT \sin^2(\lambda),
\]

where \( P_{t3} \) is the downstream total pressure, \( P_1 \) is upstream static pressure, \( R \) is the gas constant for air, \( T \) is the upstream static temperature and \( \lambda \) is the blade stagger angle.

The final step of the analysis is to relate the origin of the wall jet in the model flow to an axial location that represents an effective centroid of the compressor tip leakage jet. With the origin at the rotor tip leading edge plane, the centroid of the tip leakage jet is downstream at a location \( x_o \), thus

\[
x_{zs} = x_o - l_o.
\]

The unknown value of \( x_o \) will be found using the experimentally measured \( x_{zs} \) values. Substituting Equations 3.2 and 3.3 into Equation 3.4 gives:

\[
x_{zs} = x_o - K \tau \left( \frac{2}{\gamma - 1} \left[ \frac{P_{t3}}{P_1} (\Phi, \tau)^{\gamma-1/\gamma} - 1 \right] \gamma RT \sin^2(\lambda) \right) U_o^2,
\]

where \( \tau \), the compressor tip gap height, takes the place of \( h \) and \( K \) is a constant of proportionality. Equation 3.5 has been included as a solid curve in Figure 3.5. An \( x_o \) value of 0.642\( C_{ax} \) and a \( K \) value of 3.6 was used to best fit the experimental data. The theory shows an increase in \( x_{zs} \) with increased flow coefficient that is within the uncertainty of the experimental data. The shape of the theoretical curve, including the increase in slope at higher flow coefficients, is very similar to the curves given by experiment and simulation. The fitted values of \( x_o \) and \( K \) prevents the theory

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from predicting the \( x_{zs} \) location or the stalling mass flow for an arbitrary compressor. However, the close agreement of the model’s slope with both the experimental and computational results supports the hypothesis that the location of the zero axial shear line is a result of the momentum flux balance between the approach flow and the reverse-flow tip leakage flow.

### 3.5 Effect of Radial Inlet Distortion

The experiments, computations, and theory presented in the previous sections have demonstrated that the interface between the approach fluid and the reverse tip-leakage flow is

1. primarily influenced by the momentum flux of the two flows and
2. that the interface was observed to be at the leading edge of the compressor rotor just prior to stall. Experimental measurements with two additional inflow boundary conditions are described in this section. Specifically, radial distortion screens were used to create a momentum deficit at the inner radius (‘hub-weak’) and at the outer radius (‘tip-weak’). The intent was to alter the momentum balance between the approach and leakage flows at a given flow coefficient in order to further validate the proposed hypothesis.

The normalized inlet velocity profiles for the radial distortion cases are compared with that of the uniform inlet case in Figure 3.7(a). The hub-weak inlet distortion produced an inlet velocity profile that increased in the radial direction with a maximum velocity near the tip that was approximately 30% larger than the mean. The tip-weak inlet distortion produced an inlet velocity profile that decreased in the radial direction with a minimum velocity near the tip that was approximately 20% lower than the mean. The spanwise distributions of the rotor exit total pressure ratio are presented in Figure 3.7(b). These data exhibit similar radial variation in that a lower local inlet velocity produces a higher local pressure ratio.

The pressure rise coefficient \((\Psi)\) is plotted as a function of the flow coefficient in...
Figure 3.7. Inlet velocity profile and rotor total pressure ratio for radial inlet distortion cases

Figure 3.8(a) for the three test conditions. The pressure rise coefficient was higher for both distortion cases when compared to that of the uniform inflow case. Tip-weak inlet distortion resulted in a stalling flow coefficient higher than that observed with an uniform inlet boundary condition. Conversely, hub-weak inlet distortion resulted in a stalling flow coefficient lower than that of the uniform inflow case. The slope of the total-to-static characteristic (not shown) was negative at the last stable operating point of all test cases.

The value of $x_{zs}$ is plotted as a function of flow coefficient in Figure 3.8(b). At a given flow coefficient, when compared to the location observed with uniform inflow, the line of zero axial shear was generally observed farther downstream with hub-weak inlet distortion and farther upstream with tip-weak inlet distortion. These results can be explained in light of the analysis of the previous section. Under the tip-weak inlet
condition, the approach flow near the casing has a reduced momentum flux. Further, the increased incidence increases blade loading and thereby increases the tip-leakage momentum flux. The resultant momentum flux balance places the interface location farther upstream at a given flow coefficient. Similarly, for a given flow coefficient, the hub-weak inlet condition yields a higher approach flow momentum flux at the casing and a lower tip-leakage momentum flux, and the resultant momentum flux balance places the interface downstream of the location observed with a uniform inlet.

Figure 3.8. (a) Pressure rise and (b) $x_{zs}$ as a function of flow coefficient for uniform inlet, tip-weak, and hub-weak distortion
Figure 3.8(b) shows that the line of zero axial shear was found to be adjacent to the rotor leading edge plane when the compressor was set near the stalling flow coefficient under all three inflow conditions. Any further reduction of flow coefficient from this operating point resulted in the movement of the line upstream of the rotor leading edge plane and simultaneous compressor stall. This result provides additional support to the concept that stall is initiated by the movement of this interface upstream of the rotor leading edge plane, regardless of the blade loading distribution or the slope of the characteristic.

The solid lines in Figure 3.8(b) represent Equation 3.5, where the numerical values for $U_0$ and the total pressure ratio were found from the measurements obtained near the tip radius as shown in Figure 3.7. The value of $x_o$ was $0.642C_{ax}$ for the uniform inlet condition, $0.609C_{ax}$ for the tip-weak inlet condition, and $0.686C_{ax}$ for the hub-weak inlet condition. The value of $K$ was 3.6 for all three inlet conditions. Again, the theory captures the overall trends (slope and changes in slope) that are present in the data. This is additional support for the hypothesis that the location of the zero axial shear line and, consequently, stall inception are dependent on the balance of the approach fluid momentum flux at the tip and the reverse axial momentum flux of the tip-leakage fluid.

The observed changes in stalling flow coefficient with inlet distortion are qualitatively consistent with the critical-incidence model proposed by Camp and Day [6]. However, the rotor tip incidence angles at the stalling flow coefficient are not. The incidence angle variations at the stall point were calculated from inlet velocity triangles using the profiles shown in Figure 3.7. The stall point incidence angle at the rotor tip was 0 degrees for the tip-weak case, 2 degrees for the uniform case, and 4 degrees for the hub-weak case. Although these differences are small, they are beyond the variations found by Camp and Day. This is evidence that stall inception is more accurately described by the tip momentum flux balance rather than a
fixed incidence angle. Hence, it may be more appropriate to consider stall inception to occur at a critical momentum balance rather than a critical incidence angle. In cases where variations in boundary conditions result in changes in the tip loading, the incidence at which the leakage back-flow occurs may vary significantly. However, for a given compressor with limited variations in boundary conditions, these will not be distinguishable criteria for stall, since the tip incidence will in fact determine the momentum balance.

3.6 Spike Generation and Interface Location

Observations of the time-averaged axial position of the interface \((x_{zs})\) have established the link between stall and the movement the interface beyond rotor leading edge plane. This section will demonstrate a direct link between the movement of the interface upstream of the leading edge and the generation of rotating, spike-type disturbances in the compressor. An experiment was conducted in which a non-uniform tip clearance was created by offsetting the rotor shaft within the circular casing. It was hypothesized that, in this configuration, there would be a stable operating point near the stalling flow coefficient where the interface would be upstream of the rotor leading edge plane at circumferential locations where the tip gap was large. Similarly, the interface would be downstream of the rotor leading edge plane where the tip gap was locally smaller.

The ND-TAC active magnetic bearing system was utilized to offset the rotor shaft centerline. Since the clear casing insert used for the \(x_{zs}\) observations had a fixed azimuthal location, the magnetic bearing system was used to move, or “clock”, the circumferential position of the tip gap offset relative to the insert. This effectively simulated a circumferential traverse of the clear casing and camera over the rotor. Thus, the interface location was observed over the entire circumference of the rotor with an asymmetric tip clearance.
The spatially non-uniform tip clearance resulted in boundary conditions that were unsteady in the rotor reference frame and steady in the laboratory frame. The centerline offset (or, equivalently, the maximum deviation in clearance gap) was \( \epsilon = 0.15 \tau \) (0.1 mm). The variation in tip gap around the annulus is shown in Figure 3.9(a). Asymmetric variation in tip clearance has been shown previously to cause a circumferential variation in the local flow coefficient \([5, 16]\). The local flow coefficient was measured via upstream total pressure measurements and circumferentially distributed static pressure taps located 4.0 cm upstream of the rotor leading edge plane. These measurements are shown in Figure 3.9(b). These data show a phase shift, as predicted by the theory presented in \([16]\).

The circumferential variation in the \( x_{zs} \) value is shown in Figure 3.9(c). As anticipated, the interface was observed upstream of the rotor leading edge (i.e., negative values) plane where the local flow coefficient was low and the clearance was large. Where flow coefficient was larger and the clearance smaller, the interface was located downstream of the rotor leading edge plane. The compressor remained in stable operation until further reduction in average flow coefficient moved the interface beyond the rotor leading edge plane in the region where tip clearance was a minimum.

Time-resolved casing pressure measurements were acquired during pre-stall and stall inception for both axisymmetric and asymmetric tip clearance configurations. An equally spaced circumferential array of 16 Kulite transducers was placed 0.10\( C_{ax} \) upstream of the rotor leading edge plane. The data were recorded simultaneously at a frequency of 50 kHz \((204N)\). This provided approximately 10 samples per blade passage period. The voltage signals were converted to pressure values using calibration measurements obtained immediately prior to the experiment. The data were high pass filtered to remove low frequency transients (e.g., throttle movement) and normalized to a single standard deviation value.

The methods for processing and analyzing pressure time series obtained from a
Figure 3.9. Circumferential variation in (a) rotor offset, (b) flow coefficient, and (c) $x_{zs}$ due to rotor offset.

circumferential array of transducers varies considerably in the literature. A review of many of these techniques was given by Cameron et al. [3]. The Spatial Correlation Measure (SCM) was found to be a useful method for examining rotating disturbances during both pre-stall and stall inception. The SCM is a scalar metric defined as:

$$\chi(\theta, t_0) = \int_{\tau_{\min}}^{\tau_{\max}} R_{\theta, \theta + \Delta\theta} d\tau;$$  \hspace{1cm} (3.6)

where the windowed, two-point cross correlation of casing surface pressure is defined as
The variable $\chi(\theta, t_0)$ represents the magnitude of pressure disturbances that rotate in the rotor direction at a given range of angular velocity. The values of $\tau_{\text{min}}$ and $\tau_{\text{max}}$ represent the time-delay range that corresponds to the desired rotational speed range. Disturbances that are random, uncorrelated between sensors, or rotating outside of the specified range of angular velocity will not contribute to the net $\chi$ value. The rotating speed range was selected to be between 0.3 and 1.0 times the compressor-rotor rotational speed. The window size was determined by $w$, and determines how long of the time series is included in the correlation measure. Longer values were found to provide additional smoothing of noise and random disturbances. Shorter values provide increased contrast in the magnitude of $\chi$ when a spike-type disturbance was present. A value of $w$ corresponding to 0.25 of a rotor revolution was used.

The $\chi$ time series were computed from the pressure time series data. The results are shown as contour plots for 50 rotor revolution prior stall in Figure 3.10. The abscissa represents time elapsed in rotor revolutions with $t = 0$ defined at the time of stall inception. Stall inception was specified as the time at which the amplitude of the first spatial Fourier mode was 80% of the final value observed in fully developed stall.
The symmetric clearance condition showed few rotating disturbances prior to stall. Some relatively weak disturbances were found throughout the time series, but their circumferential and temporal position appeared to be stochastic. Most of these disturbances rotated at the same angular velocity as the rotor, with some disturbances rotating slightly slower (0.7N to 1.0N). At approximately 7 revolutions prior to stall, a significant disturbance rotating at 0.7N was observed but decayed after about one full revolution. At \( t = -3 \), several similar disturbances appeared and grew quickly.
into fully developed stall. After several revolutions in rotating stall, the compressor system surged.

Contours of $\chi$ for the asymmetric clearance condition (Figure 3.10(b)) show many short length scale rotating disturbances that travel around the annulus at approximately $0.7N$, suggesting that they are spike type disturbances. These spikes generally originated from a circumferential location of $270^\circ$ and propagated around the annulus until diminishing near a circumferential location of $90^\circ$. That is, the generation and propagation of spike type disturbances occurred over the section of the rotor annulus where the tip-leakage interface was found to be upstream of the rotor leading edge plane (see Figure 3.9). Rotating disturbances were found to decrease quickly in magnitude over circumferential locations where the zero axial shear line was found downstream of the rotor leading edge plane.
CHAPTER 4

SURGE MARGIN EXTENSION AND GROOVE INTERACTION WITH THE TIP CLEARANCE FLOW FIELD

4.1 Overview

The previous chapter indicates that spike-generation in an axial compressor is a critical point in the axial momentum balance in the rotor tip clearance. Based on this result, it is reasonable to conjecture that circumferential groove casing treatments (CGCT) increase stall margin by altering the axial momentum balance through their interaction with the tip clearance flow field. This chapter examines this hypothesis through the findings of several experiments conducted on ND Stage 04 with the Variable Casing Treatment (VCT) described in Chapter 2 in combination with information available from simulations of the compressor with a smooth wall (also described in Chapter 2).

The chapter will first describe the performance and stall margin extension results from the VCT geometries. The analysis of the data will provide insight into the contribution of the individual grooves to the overall surge margin extension of the multi-grooved casing treatment.

Second, the one-dimensional momentum balance developed in the previous chapter will be extended to include the impact of a groove on the momentum balance. The chapter concludes with a short examination of the effect of groove geometry on the momentum balance.
4.2 Performance and Efficiency Results and Analysis

Experimental performance and efficiency measurements were acquired for seven configurations of the Variable Casing Treatment. The nomenclature selected for these configurations are illustrated in Figure 4.1. The VCT was assembled into four multi-groove configurations (011, 101, 110, and 111) and three single-groove configurations (001, 010, and 100). The leading edge of the rotor is on the left of each illustration.

![Figure 4.1. VCT3D Configurations and Nomenclature](image)

Figure 4.1. VCT3D Configurations and Nomenclature

Figure 4.2 shows the 98% $N_c$ rotor only characteristics of the smooth wall baseline and the seven VCT constructions. The dashed line on Figure 4.2 represents the performance of a zero deviation compressor with the same geometry and inlet condition as ND Stage 04 and a constant adiabatic efficiency corresponding to the measured value at ND Stage 04’s aerodynamic design point.

Three key observations can be made based on the results shown in Figure 4.2.
First, the characteristics nearly collapse at mass flows greater than \( \dot{m}_c > 20.5 \text{ lbm/sec} \). In this operating range, it appears that the casing treatment has little to no effect on the compressor’s performance. Second, the stall points of all VCT constructions are at lower mass flows and higher pressure ratios. Thus, all VCT constructions extended the stall margin of the compressor. Third, the VCT constructions that provided the greatest stall margin extension also have characteristics that remain near the zero deviation compressor trend for the widest operating mass flow ranges. It is noted that the locus of the stall points forms a line that is nearly parallel to the zero deviation compressor trend.

The surge margin extension (SME) from each casing are summarized in Table 4.1. The table also includes changes in design point and maximum adiabatic efficiency.
TABLE 4.1

VCT3D RESULTS AT 98% $N_C$

<table>
<thead>
<tr>
<th>Casing</th>
<th>SME</th>
<th>$\Delta\eta_{adp}$</th>
<th>$\Delta\eta_{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>001</td>
<td>1.35%</td>
<td>+0.5%</td>
<td>+0.2%</td>
</tr>
<tr>
<td>010</td>
<td>4.37%</td>
<td>-0.7%</td>
<td>-0.9%</td>
</tr>
<tr>
<td>011</td>
<td>5.85%</td>
<td>-3.4%</td>
<td>-3.3%</td>
</tr>
<tr>
<td>100</td>
<td>8.33%</td>
<td>-1.2%</td>
<td>-1.2%</td>
</tr>
<tr>
<td>101</td>
<td>9.16%</td>
<td>-0.5%</td>
<td>-1.1%</td>
</tr>
<tr>
<td>110</td>
<td>11.7%</td>
<td>-0.5%</td>
<td>-1.0%</td>
</tr>
<tr>
<td>111</td>
<td>14.4%</td>
<td>-1.6%</td>
<td>-1.9%</td>
</tr>
</tbody>
</table>

Change in maximum efficiency is plotted against surge margin extension in Figure 4.3. These data are supplemented with similar sets of data acquired at 90% and 70% $N_c$.

The change in maximum efficiency generally follows a negative trend with increasing surge margin extension. The solid line in Figure 4.3 is a least-squares fit of all the data in the figure with a slope of -0.118. The dotted lines bracketing the least-squares fit represents the $\pm 0.5\%$ (in efficiency points) repeatability of the efficiency measurement. Most of the data lies within this band with a few clear outliers. The dashed line on the figure is the fit of Fujita’s 1984 low-speed rig results and has a slope of -0.123. The close agreement between the slopes of the two data sets suggests a general rule for the relationship between surge margin extension and change in maximum adiabatic efficiency in any compressor, especially in the context that the present data set includes subsonic and transonic rotor tip results and that Fujita’s results are from a compressor with a comparatively much slower tip speed.
The surge margin extension data in Table 4.1 has been organized into the bar graph shown in Figure 4.4. Surge margin extension results from 70% and 90% \( N_c \) measurements are also included for comparison.

It is observed that for most VCT constructions there is greater surge margin extension at higher shaft speeds. This may suggest that surge margin extension from a given CGCT geometry has a dependence on blade tip Mach number. With focus on the 98% \( N_c \) data, there are two important observations that can be made from Figure 4.4:
1. **SME due to a multi-groove CGCT appears equivalent to the summation of the SME of the individual grooves which comprise it.** The surge margin extension of the 111 construction is 14.4%. The surge margin extensions of the 100, 010, and 001 constructions are 8.3%, 4.4%, and 1.4% respectively. The sum of these single-groove constructions is 14.1%. A qualitative inspection of Figure 4.4 suggests similar results for other multi-groove/single-groove comparisons.

2. **SME due to a single groove appears sensitive to the groove’s placement.** The magnitude of SME due to a single groove with a fixed geometry depends on its placement over the rotor. Again, a single groove in the 100 configuration yields a surge margin extension nearly double that of a single groove in the 010 configuration. This result echoes Houghton’s 2010 findings.

Based on these observations, it is hypothesized that the surge margin extension due to a single groove of fixed geometry is governed by a function which

\( (i.) \) has the property of superposition and

\( (ii.) \) depends on a physical quantity derived from the rotor tip clearance flow field. The following section explores this hypothesis.
4.3  CGCT Surge Margin Extension Dependence on SW Tip Clearance Flow Field

First, Observation 1 from the previous section is examined via a closer consideration of the 98% \( N_c \) VCT surge margin extension results. The SME data shown in Table 4.1 has been organized into the stacked bar chart shown as Figure 4.5.

![Figure 4.5. VCT Surge Margin Results and Combinations (Boxed Values are the Percent Difference Compared to Respective Baselines)](image)

All seven VCT constructions are represented in Figure 4.5 according to the scheme indicated in the legend at the top of the figure. There are four groupings of information, one for each of the multi-groove configurations of the VCT (111, 110, 101, and 011). The measured SME for each of these four “baseline” configurations is at the left of each grouping. The stacked bars to the right of each
of these baselines represent summations of SME from combinations of VCT constructions that, when superpositioned, have the same geometry as the baseline (e.g. \(\text{SME}_{111} = \text{SME}_{100} + \text{SME}_{010} + \text{SME}_{001}, \text{SME}_{111} = \text{SME}_{100} + \text{SME}_{011}\), etc.). The number at the center of each of the stacked bars is the measured surge margin extension for that VCT construction.

The boxed numbers near the top of Figure 4.5 are the percent difference between the baseline measurements and the summation results. The percent difference between various configurations was found to be within \(\pm 9\%\). This range corresponds to a SME of approximately \(\pm 1\%\). This range is considered to be within the range of measurement uncertainty of SME. In the ND-TAC facility, the compressor operating point is set by throttles with discrete settings. Thus, it is possible for the true stalling operation point to lie on a throttle line between two available throttle settings. In these cases, the measured stall margin would be biased low. This bias is estimated to be no more than -1% surge margin for ND Stage 04 in the ND-TAC facility.

The excellent agreement between the “baseline” SME measurement for each of the multi-groove VCT constructions and the corresponding combination summations strongly demonstrates that the hypothesis that the total surge margin extension of a multi-groove casing is the superposition of the extensions due the individual grooves.

Next, Observation 2 is considered in the context of the findings of Chapter 3 and the literature. Chapter 3 demonstrated that compressor stall is a critical point in the axial momentum balance. It is reasonable to conclude that CGCT increase stall margin by reducing the reverse momentum flux from the rotor tip leakage flow. In this light, Shabbir’s 2005 results may be interpreted in the following way: radial transport of axial momentum is the fluid dynamic mechanism through which a circumferential groove reduces the reverse axial momentum flux due to the tip leakage jet. Assuming that this fluid dynamic mechanism is unchanged by a groove’s axial location over a rotor, Observation 2 suggests that the “output” of a fixed-geometry single groove,
surge margin extension, is a *response* to some physical quantity in the smooth-wall rotor tip clearance corresponding to where the groove would be placed.

Considering the conclusion of previous chapter, it is a rational choice to consider axial momentum flux as this physical quantity. The following analysis will use computational and the experimental VCT results to seek a functional relationship between surge margin extension and the smooth wall tip clearance axial momentum flux.

Axial momentum flux can be understood as the area under a distribution of axial momentum flux density. It is convenient to define *normalized tip clearance axial momentum flux density* for a single rotor blade $\alpha(s)$ as:

$$\alpha(s) \equiv \frac{\int_{\text{case}}^{\text{tip}} \rho u (V \cdot n) \, dr}{\rho_1 U_{\text{tip}}^2 \tau} \quad (4.1)$$

where the s-coordinate axis is aligned with the camberline of the rotor blade. Simulation data was used to construct the distribution of normalized axial momentum flux density through the smooth wall tip clearance for ND Stage 04 at an operating point very close to stall shown in Figure 4.6. For these data, the axial component of the surface normal always pointed downstream, and the negative magnitudes indicate that the flux direction is upstream.

Figure 4.6 shows that, at this operating point, the axial momentum flux density is skewed toward the rotor leading edge and has the greatest magnitude near $x/C = .2$. It is interesting to note that in the band $.05 < x/C < .25$ the actual momentum flux density is up to 40% greater than a momentum flux density based on inlet density and rotor tip velocity.
The total smooth wall tip clearance axial momentum flux at the location of a given groove $A_g$ is given by integrating Equation 4.1 from the groove’s leading edge to the groove’s trailing edge and multiplying by the number of rotor blades,

$$A_g \equiv n_r \rho_1 U_{tip}^2 \tau \int_{gLE}^{gTE} \alpha_{sw}(s) \, ds. \quad (4.2)$$

The dashed lines on Figure 4.6 indicate axial locations of the leading and trailing edges of the VCT grooves in the 111 construction. The arrows indicate the integration regions for each groove. For a multi-groove casing,

$$\text{AMF} = \sum_{i=1}^{g} A_{g_i}. \quad (4.3)$$
where the limit $g$ is the total number of grooves. For example, the summation
\[ \text{AMF} = \sum_{i=1}^{g} A_{gi} \]
for the 111 construction is the summation of the integrals of these three integration regions. Similarly, the summation for the 101 construction is the summation of the integrals of the most upstream and the most downstream integration regions only, the summation term for the 100 construction is simply the integral of the most upstream region, etc.

Figure 4.7 shows the 98% $N_c$ VCT surge margin extension results listed in Table 4.1 plotted against the corresponding values of AMF computed according to Equation 4.2 using the distribution shown in Figure 4.6. (The absolute value of AMF is used strictly for presentation.)

![Figure 4.7. VCT Surge Margin Extension v. Total SW Axial Momentum Flux](image-url)
The dashed line included in Figure 4.7 is included for illustrative purposes only. Surge margin extension from a circumferential groove casing treatment is linear function of the axial momentum flux through smooth-wall rotor tip clearance at corresponding location of the grooves. Furthermore, the data in Figure 4.7 appear to have the form $y = mx$, a functional form that has the property of superposition. These results support the hypothesis that surge margin extension from a circumferential groove is a response to the smooth wall tip clearance flow field.

4.4 One-Dimensional Momentum Balance With Groove

The previous section draws a functional relationship between the surge margin extension due to a casing treatment and the smooth wall tip clearance flow field. However, a compressor designer will be interested in surge margin extension to be gained from an arbitrary circumferential groove casing treatment given a smooth wall axial momentum flux density distribution. Therefore, the designer requires a relationship between a groove geometry and its influence on the tip clearance axial momentum flux balance. This section will extend the one-dimensional momentum balance model presented in Chapter 3 to include the influence of a CGCT and use the resulting formulation to explore this relationship.

The model problem geometry is shown in Figure 4.8. The compressor tip gap flow is again modeled as a wall bounded flow (left-to-right) and a counter-flow wall jet. The effect of the casing treatment will be modeled by considering the single groove just outside of wall jet exit.

Control volume boundaries are drawn as in the previous chapter. Linear momentum applied to this control volume reduces to

$$F_d = A_c - F_g,$$  \hspace{1cm} (4.4)
where $F_d$ is the total drag on the control volume, $A_c$ is the total axial momentum flux from the tip leakage jet through the tip clearance, and $F_g$ is the force of the groove on the control volume via to radial transport of axial momentum. The objective of the following is to link the three terms in equation 4.4 to quantities that can be obtained from either experiment or from smooth wall CFD results.

According to the results of the previous chapter, the term $F_d$ can be expressed in terms of an experimentally determined coefficient of drag for the control volume, $C_{dcv}$,

$$C_{dcv} \rho_o U_o^2 \pi D (x_o - xzs) = A_c - F_g$$

$$C_{dcv} Q_o \pi D (x_o - xzs) = A_c - F_g,$$  \hspace{1cm} (4.5)
where $\pi D$ is the compressor’s circumference, and $Q_o \equiv \rho_o U_o^2$ represents the momentum flux of the approach flow per unit area. The values of the constants $C_{dv}$ and $x_o$ were determined in Chapter 3 from the smooth wall conditions.

All terms on left-hand side of Equation 4.5 aside from the dependent variable $x_{zs}$ are known from compressor geometry or from previous work. The terms on the right hand side of Equation 4.5 are now defined in terms of information that is available from smooth wall experiment or simulation results.

As in Chapter 3, the total axial momentum flux of the tip leakage jet, $A_c$, is modeled by the equation

$$A_c \equiv n_r K_{A_c} \bar{\rho}_j \bar{U}_j^2 \tau C_{ax}$$

where the over-tilde values are the experimentally determined surrogates discussed in the previous chapter. The coefficient $K_{A_c}$ relates the true tip clearance axial momentum flux to the surrogate quantities according to the definition

$$K_{A_c} \equiv \left. \int_0^C \int_{\text{tip}}^{\text{case}} \rho u (\mathbf{V} \cdot \mathbf{n}) \ dr \ ds \right|_{sw} \frac{\bar{\rho}_j \bar{U}_j^2 \tau C_{ax}}{} \right.$$

Figure 4.9(a) shows the numerator (computation) and the denominator (experiment) of Equation 4.7 as a function of corrected mass flow. The computational data has been interpolated to the experimental mass flows to produce Figure 4.9, the quotient of the two quantities.

The value of $K_{A_c}$ rises a value of approximately 0.78 as the stalling operation point is approached and stays in the neighborhood of this value in the vicinity of the stall point. As this is the operating range of interest, a value of 0.78 was selected for the following analysis. It is assumed that the value of $K_{A_c}$ is insensitive to the casing being modeled.

After substituting Equation 4.7 Equation 4.5 becomes
Corrected Mass Flow, [lbm/sec]  
Tip Gap Axial Momentum Flux, [lbf]  

Computation  
Exp. Estimate  
Interpolation  

(a) Tip Clearance Axial Momentum Flux  

(b) $K_{A_c}$  

Figure 4.9. Determining the Actual-to-Approximated Tip Clearance Axial Momentum Flux Ratio (Smooth Wall Experiment and Simulation Data)  

\begin{align*}  
C_{d_{c0}}Q_o\pi D (x_o - x_{zs}) &= n_r K_{A_c} \hat{\rho}_j \hat{U}_j^2 \tau C_{ax} - F_g  
\end{align*}

where $\hat{Q}$ represents the momentum flux of the leakage jet per unit area, $\hat{\rho}_j \hat{U}_j^2$.  

The final step in the analysis is to relate the net axial force $F_g$ on the control volume due to the groove to the smooth wall axial momentum flux through the tip clearance. It is proposed that the force $F_g$ is related to the smooth wall axial momentum flux through the tip clearance at the groove location $A_g$, through a groove geometry dependent coefficient of drag $C_{d_g}$,  

\begin{align*}  
F_g &= C_{d_g} A_g  
&= C_{d_g} n_r \rho_1 U_{tip}^2 \tau \int_{s=0}^{9TE} \alpha_{sw} (s) \, ds,  
\end{align*}

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Following the results of Section 4.3, \( F_g \) for a multi-grooved casing can be written as a summation of individual groove forces:

\[
F_g = \sum_{i=1}^{g} f_{gi}
\]

where \( g \) is the number of grooves, \( f_i \) represents the force due to the \( i \)th groove, and \( A_{gi} \) retains the definition shown in Equation 4.2. For a multi-groove casing composed of grooves of all the same aspect ratio, the coefficient of drag is fixed and can be moved outside of the summation:

\[
F_g = C_{dg} \sum_{i=1}^{g} A_{gi}.
\]  

(4.11)

Substituting Equation 4.11 into Equation 4.8 produces

\[
C_{dce} Q_o \pi D (x_o - x_{zs}) = n_r K A_c \tilde{Q} \tau C_{ax} - C_{dg} \sum_{i=1}^{g} A_{gi}.
\]  

(4.12)

4.4.1 Determination of Groove Coefficient of Drag

Equation 4.12 was used in conjunction with ND Stage 04 smooth wall simulation data and the VCT performance measurements to determine the groove drag coefficient \( C_{dg} \) for the VCT groove geometry.

It has been demonstrated that \( x_{zs} \approx 0 \) in ND Stage 04 at the operating point just prior to stall. Thus Equation 4.12 can be solved for \( C_{dg} \) in terms of compressor geometry, coefficients known from the previous smooth wall investigation, the experimentally measured quantities \( Q_o \) and \( \tilde{Q} \) at stall, and the summation \( \sum_{i=1}^{g} A_{gi} \) which comes from the smooth wall simulation data.
Equation 4.12 was applied to the stall point performance measurements of all VCT configurations using the appropriate summation for each casing. The computed groove drag coefficient is plotted against VCT configuration in Figure 4.10.

![Graph](image)

Figure 4.10. Groove Coefficient of Drag vs. VCT3D Build

The data in Figure 4.10 appear to be scattered about a constant value. The high groove drag coefficient for the 001 construction is likely due to the sensitivity of the model to values of $\sum_{i=1}^{q} A_{g_i}$ near zero. The dashed horizontal line is drawn at the value of the mean of the data excluding the 001 point. This value is $\overline{C_{d_g}} = 0.472$ and is taken to be the groove drag coefficient for the groove geometry of the VCT casing according to the definition of Equation 4.9.
It was noted above that the groove coefficient of drag $C_{dg}$ is likely a function of groove geometry. In order to explore this hypothesis, the analysis described above was repeated for eight additional CGCT geometries, four with a 4-groove construction and four with a 7-groove construction. Although eight additional casings were examined, only 4 additional groove aspect ratios were represented. These geometry details are summarized in Section 2.4. The computed groove drag coefficients for the eight additional casing treatments and the VCT are shown as a function of the logarithm of groove aspect ratio in Figure 4.11.

![Figure 4.11. $C_{dg}$ as a Function of the Logarithm of Groove Aspect Ratio](image)

The data appear to collapse to a positive linear trend with some scatter (most
apparent at AR ≈ 1) in this log-linear space. The trend suggests that grooves which are deeper compared to their axial extent exert a greater drag force. This is intuitive in the light of the model problem illustrated in Figure 4.8: a larger aspect ratio grooves has more “room” for radial transport of axial momentum, the mechanism producing the groove force. Furthermore, it must be noted that these data represent (a) 3 groove geometries with markedly different axial extents, therefore (b) 3 different values for the summation \( \sum_{i=1}^{g} A_{g_i} \), and, though not presented, (c) very different trends in surge margin extension. Under this consideration, the collapse of these results suggests that this is the proper nondimensional space for these data.

4.4.2 Stall Margin Extension Prediction Via 1-D Momentum Balance

With a known \( C_{d_g} \) value for a given groove geometry, an appropriate model for a compressor’s performance at operating points beyond smooth wall stall point, and smooth wall simulation data, Equation A.1 can be used to estimate the surge margin extension from a given circumferential groove casing treatment.

A potential simple model for compressor performance at operating points beyond the smooth wall stall point is to suppose that: (a) the momentum flux of the tip leakage flow per unit area \( \tilde{Q} \) and (b) the total-to-static pressure ratio \( \frac{P_t}{P_1} \) remain constant and equal to their respective magnitudes at the smooth wall stall point. These elements are considered the flat total-to-static pressure ratio model. The step-by-step methodology for using Equation 4.12 with this model of compressor performance to estimate surge margin extension for a casing treatment is included as Appendix A.

The expected surge margin extension from the 4-groove, 7-groove, and VCT constructions as predicted by the flat total-to-static pressure ratio model in ratio with the measured surge margin extensions is plotted as a function of AMF in Figure 4.12.

The predicted surge margin extension ranges from a \( \approx 10\% \) underestimate to a \( \approx 40\% \) overestimate. The flat total-to-static model progressively overpredicts SME
from the VCT casing with increasing AMF. The estimate for the 7-groove casing ranges from a very slight underprediction to a $\approx 35\%$ overprediction and the 4-groove casing SME is overestimated by up to $\approx 35\%$ with most estimates in the neighborhood of $+15\%$.

The range of this result is expected. The flat total-to-static model is an over-simplified model that is unlikely an adequate model for this compressor as surge margin extension increases: the true momentum flux per unit area of the tip leakage jet will increase as rotor loading increases. That is, the flat total-to-static characteristic model necessarily underestimates the magnitude of the tip leakage flow. The resultant estimated momentum balance will tend toward stalling mass flows that are too low. However, though over-simplified, this model provides estimates of SME that are of the same order of the true SME. Under this consideration, it is likely
that Equation 4.12 combined with the groove drag coefficient trend shown in Figure 4.11 and more refined model for compressor performance beyond the smooth wall stall point can be used as a first-order estimate of surge margin extension from an arbitrary circumferential groove casing treatment.
CHAPTER 5

CONCLUSION

The experiments and computations presented in this document were designed to study the role of the tip leakage flow in axial compressor stall and the relationship between the tip clearance flow flow field and surge margin extension from circumferential groove casing treatment.

In Chapter 3, measurements of compressor performance and unsteady casing pressure in conjunction with surface streaking visualization produced a better understanding of how the tip leakage flow influences stall in a transonic axial compressor. The CFD was conducted in order to provide detailed visualization of the tip-leakage flow inside the blade passages.

The first conclusion drawn from this study was that a very distinct interface exists in the blade tip region where the approach gas meets the reverse direction tip leakage flow. This interface was observed in all of the computational and experimental results. In all cases, the time-averaged interface location moved toward the rotor leading edge plane as flow coefficient was reduced. The compressor stalled when this interface reached the rotor leading edge plane, regardless of compressor speed or radial inlet distortion. A simplified model was developed to describe the observed changes in $x_{zs}$ with changes in flow coefficient. The model was based on a momentum flux balance between the approach flow and the tip-leakage flow. The model accurately captured changes in the slope of the $x_{zs}$ vs $\Phi$ curves for cases with and without inlet distortion.

Asymmetric tip clearance measurements provided additional insight into the fluid mechanics of the interface location. Specifically, the near-stall streaking measure-
ments showed that the interface was located upstream of the rotor leading edge plane at circumferential locations where the tip gap was largest. The interface was found downstream of the rotor leading edge plane where the gap was smallest. Unsteady pressure measurements at the compressor casing confirmed that the regions where the interface was ahead of the leading edge was also the region where short length scale disturbances originated. In the region where the interface was located downstream of the leading edge, the disturbances quickly decayed. Furthermore, when average flow coefficient was reduced, the interface was moved upstream of the rotor leading edge plane at all circumferential locations, leading to stall. This finding not only confirmed that spillage ahead of the rotor leading edge plane is a criterion for spike disturbances, but also demonstrated that spillage of the tip-leakage fluid is in fact the disturbance itself. This experimental result confirms what was observed numerically by Chen et al. [8] and Lin et al. [21].

The numerical and experimental work presented in Chapter 3 supports the conclusion that the interface location, and hence the stalling mass flow rate of the compressor, can be considered to be a direct result of the momentum flux balance between the approach fluid and the reverse-direction tip leakage fluid. This conclusion provides insight into previously demonstrated stall control techniques. For example, the efficacy of tip injection (e.g. [31]) can be understood as a result of the momentum flux addition to the approach fluid, altering the momentum flux balance in a beneficial way and moving the interface location further downstream compared to the non-controlled case.

In the context of these findings, circumferential groove casing treatments were considered as a means of altering the tip clearance momentum flux balance by reducing the momentum of the tip leakage jet. Chapter 4 investigated the relationship between groove geometry and placement and surge margin extension through experimental and numerical efforts.
Experimental measurements of surge margin extension from seven CGCT configurations with a fixed groove geometry demonstrated that the contribution of individual grooves in a multi-groove casing to surge margin extension is an \((a)\) additive and \((b)\) linear function of the smooth wall tip clearance axial momentum flux at the location of each groove.

Given this result, the axial momentum flux balance was extended to include the influence of a circumferential groove. Analysis of this new model showed that the force of a circumferential groove was related to the smooth wall tip clearance axial momentum flux through a coefficient of drag that has a log-linear dependence on groove aspect ratio.

These findings and methods described above are progress in answering the research questions listed in Chapter 1. They also invite continued refined work and new questions. Recommendations for future work can be broken into short term and long term categories:

1. **Short Term** The new axial momentum flux model developed in Chapter 4 should be applied to additional compressors to confirm or refine the model. Furthermore, there is a rich database of ND Stage 04 performance measurements at different corrected shaft speeds and inlet boundary conditions that can also be used to confirm or refine the model. Additionally, Chapter 4 closed with a demonstration of how the model could be used in concert with the groove drag coefficient trend with groove aspect ratio to predict the surge margin extension from an arbitrary CGCT geometry. A more refined model for compressor behavior beyond the smooth wall stall point must be developed and applied.

2. **Long Term** Examination of the Tsinghua computational data available for ND Stage 04 reveals a tip clearance flow field topology that suggests that blade tip design may be critical to the efficacy of a circumferential groove casing treatment. Surge margin extension due to a given casing treatment may be dependent on the geometry of the blade tip or the ratio of tip clearance height to groove depth, etc. A closer examination of these details is highly recommended.
APPENDIX A

METHOD FOR SME PREDICTION VIA 1-D MOMENTUM BALANCE AND FLAT TOTAL-TO-STATIC PRESSURE RATIO

The following outlines the procedure for predicting surge margin extension from a given circumferential groove casing geometry if given experimental and numerical smooth wall compressor performance data. The method can be very easily adapted for a case where only numerical data is available.

Equation 4.12 can be rearranged such that \( \frac{x_{zs}}{C_{ax}} \) is the dependent variable:

\[
\frac{x_{zs}}{C_{ax}} = \frac{x_o}{C_{ax}} - \frac{n_x}{\pi} K_{A_e} \tilde{Q} \tau \frac{\tau}{D} + \frac{1}{\pi} C_{d_g} \sum_{i=1}^{g} A_{g_i}. \tag{A.1}
\]

1. Assume that the tip clearance jet momentum flux per unit area as determined from the surrogate values described in Chapter 2, \( \tilde{Q} \), and the compressor’s total-to-static pressure ratio, \( \frac{P_3}{P_1} \), maintain the same value as at the smooth wall stall point for all mass flows lower than the smooth wall stall point. These assumptions define the “flat total-to-static pressure ratio” model.

2. To compute a surge margin extension according to Equation 2.8, we assume that any change to the design point mass flow and total pressure rise, \( \dot{m}_d \) and \( \pi_d \) respectively, due to the casing treatment is negligible. Smooth wall stalling mass flow and total pressure rise, \( \dot{m}_{s1} \) and \( \pi_{s1} \) respectively, are known from either experiment or simulation.

3. Select inlet total pressure and inlet total temperature. It is most convenient to use Standard Day Conditions. Assume that \( \gamma = 1.4 \).

4. Given a CGCT geometry, \( \sum_{i=1}^{g} A_{g_i} \) is known from smooth wall simulation data near stall. The grooves’ drag coefficient \( C_{d_g} \) is dependent on groove geometry and must be known beforehand. For the analysis in Chapter 4 this value was the experimentally determined value \( \bar{C}_{d_g} \).
5. Assume that the constants $x_0$, $K_{Ac}$, and $C_{d_{cv}}$ which were determined from smooth wall measurements remain unchanged.

6. At stall $x_{zs} = 0$, thus Equation [A.1] can be solved for $Q_o$ in terms of a known compressor geometry, constants known from previous work, the modeled value for $\bar{Q}$, and the summation $\sum_{i=1}^{9} A_{g_i}$ known from simulation.

7. The approach flow momentum flux per unit area $Q_o$ can be related to the approach Mach number via manipulation of the Mass Flow Function. This relationship can be numerically solved for Mach number with $Q_o$ known from the previous step, given approach total pressure.

8. The approach static pressure, $P_1$, is determined by the isentropic relationship between the known approach Mach number and approach total pressure.

9. The total pressure downstream of the rotor, $P_{t3}$, is the product of the assumed total-to-static pressure ratio and the now known approach static pressure. The total pressure rise over the rotor at stall $\pi_{s2}$ can now be computed.

10. The approach static temperature, $T_1$, is determined by the isentropic relationship between the known approach Mach number and approach total temperature.

11. With $T_1$ and $P_1$ known, the approach flow density $\rho_1$ follows from the Ideal Gas equation of state.

12. With $\rho_1$ known, the flow velocity $c_x$ can be computed from the approach flow momentum flux per unit area.

13. The inlet cross-sectional area is combined with $\rho_1$ and $c_x$ to determine the stalling mass flow $\dot{m}_{s2}$.

14. Surge margin extension is the difference between the surge margin for the compressor with the casing treatment (subscript 2) and that for the compressor with the smooth wall (subscript 1). Assuming the same design point performance,

$$SME = \frac{\pi_d}{\dot{m}_d} \left( \frac{\dot{m}_{s1}}{\pi_{s1}} - \frac{\dot{m}_{s2}}{\pi_{s2}} \right).$$  \hspace{1cm} (A.2)


